

Multi-reference description of open-shell nuclei

SSNET 2024, Thursday 7th 2024

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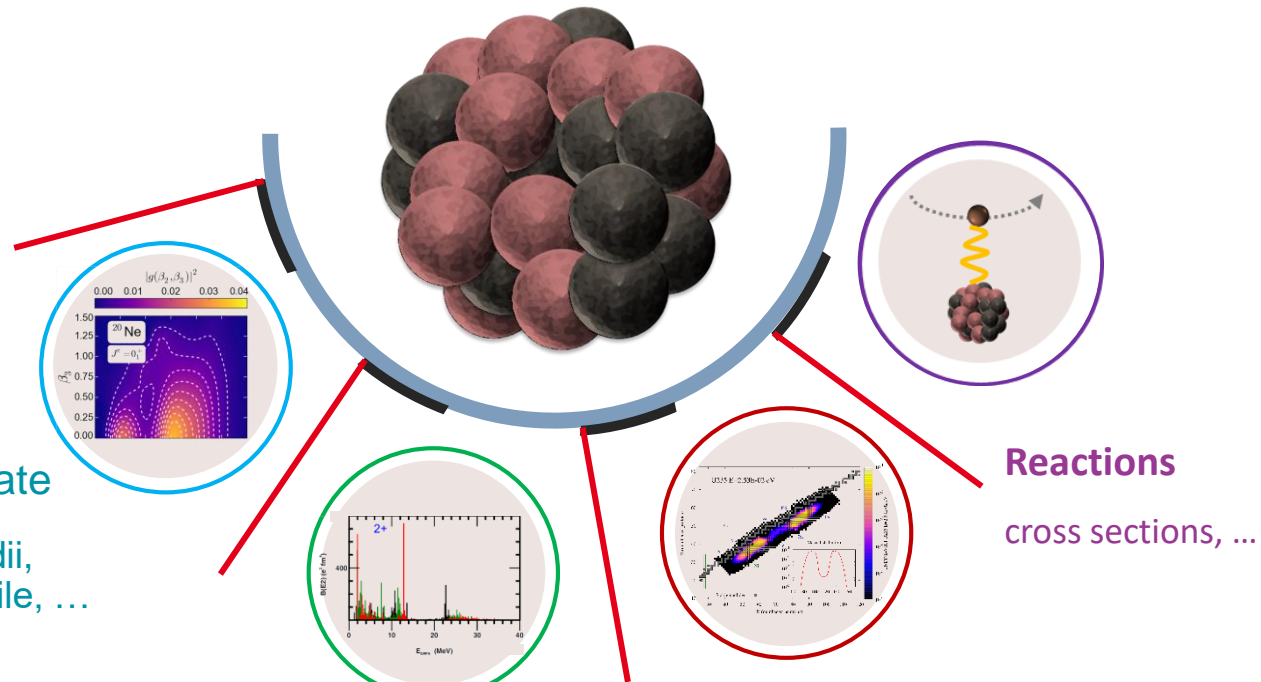


Microscopic models of nuclei

From nucleons to nuclei

Prediction of macroscopic properties from interacting nucleons

Towards a **complete**, **coherent** and **systematically improvable** description of nuclei for nuclear data



Ground-state

masses, radii,
density profile, ...

Excitation spectra

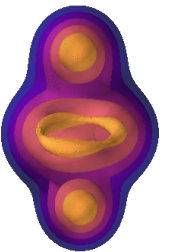
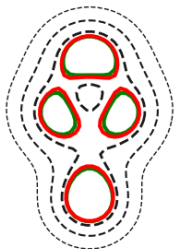
energies, transition probabilities,
response function to electroweak
probes, ...

Decay modes

lifetime, yields, ...

Reactions

cross sections, ...



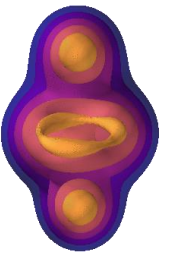
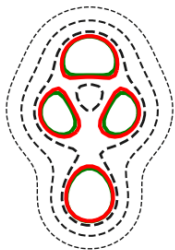
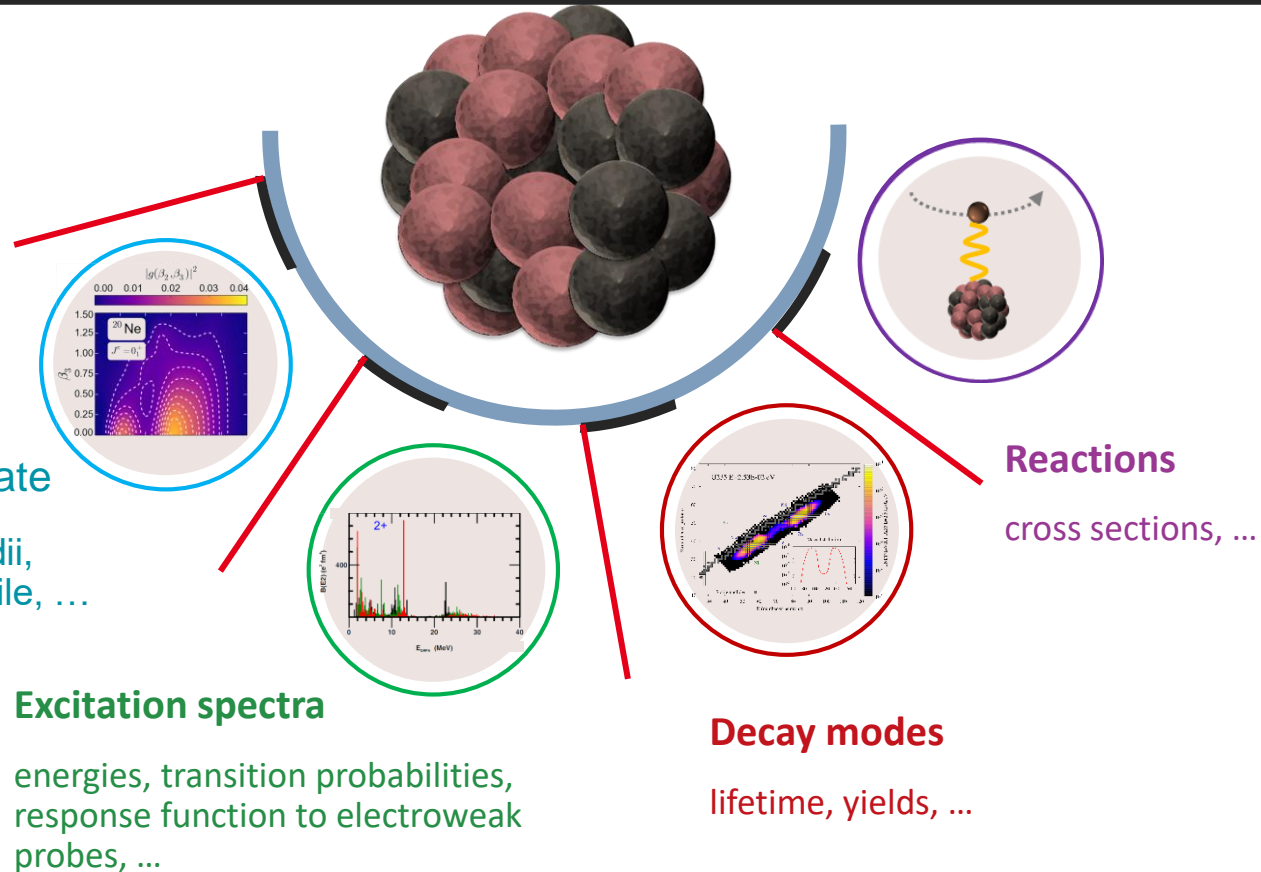
Microscopic models of nuclei

Presentation focused on particular *ab initio* method

Objective : **solve A-body Schrödinger equation** to given accuracy

Multi-reference Perturbation Theory (PGCM-PT)

How can **multi-reference expansion methods** help towards a complete picture?



Outline

1. Expansion methods for open-shell nuclei

Progress of ab initio / in medias res methods

Single and multi-reference expansion methods

Circumventing the complexity of three body interactions

2. Application: from low-lying spectroscopy to giant resonances

PGCM-PT(2) and scale decoupling

Application of PGCM in realistic model space

3. Conclusion



1 ■ Formalism

- Progress of ab initio / in medias res methods
- Single and multi-reference expansion methods
- Circumventing the complexity of three body interactions

Progress of *ab initio* / *in medias res* methods



***Ab initio* methods**

- 1) A structure-less nucleons as degrees of freedom
- 2) Interaction mediated by pions and contact terms (e.g. Weinberg PC)
- 3) **Solve A-body Schrödinger equation to relevant accuracy***
** controlled and improvable way*

Steady progress in the last decades

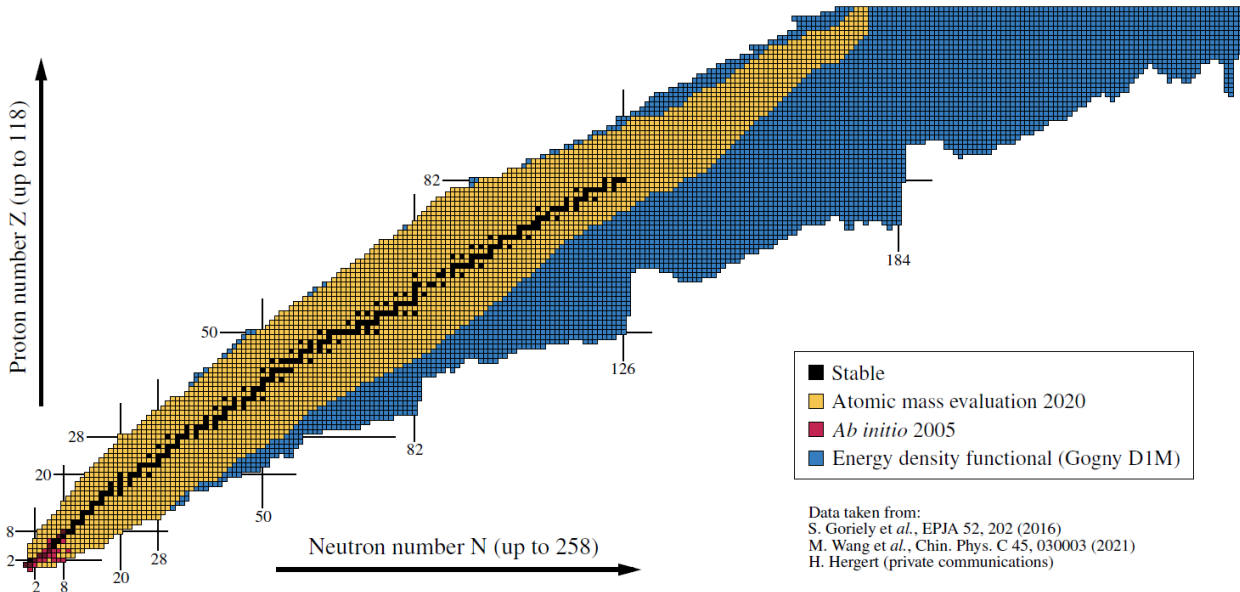
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Light nuclei

Quasi-exact methods

1990's

Exponential scaling

Courtesy of B. Bally

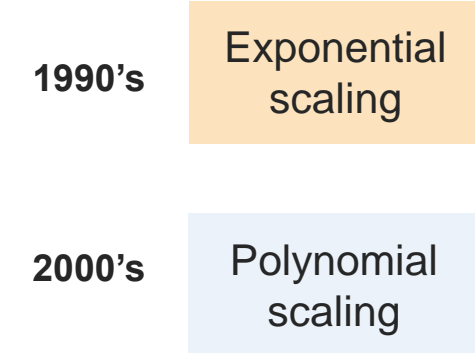
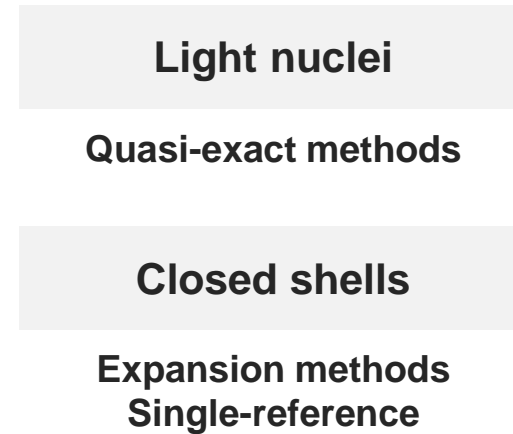
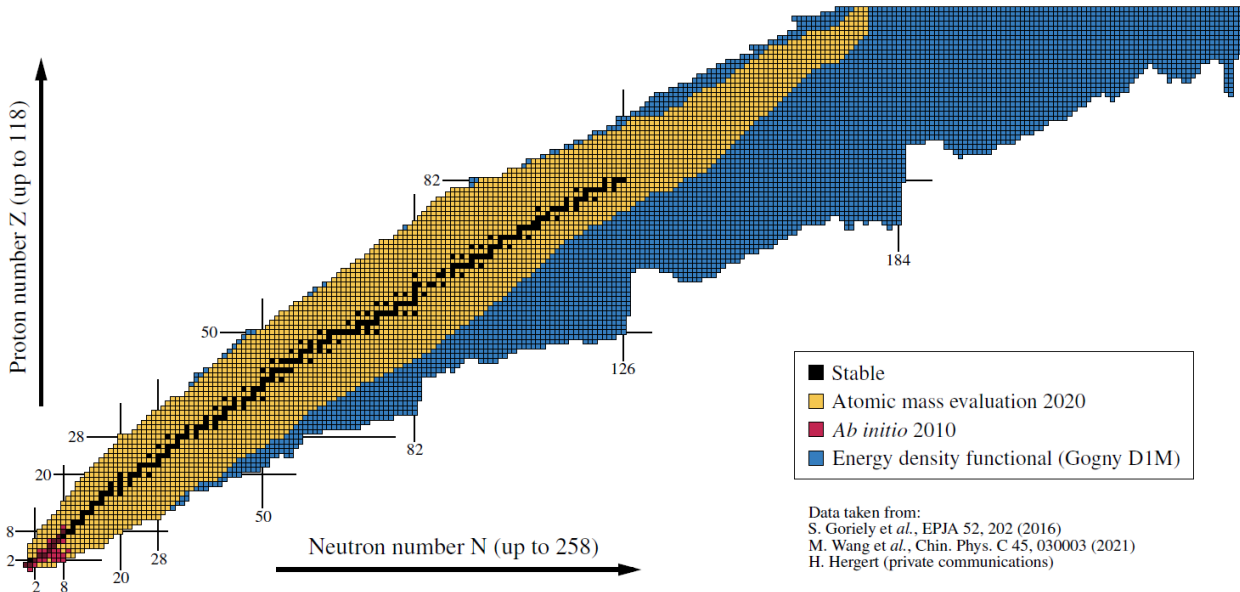
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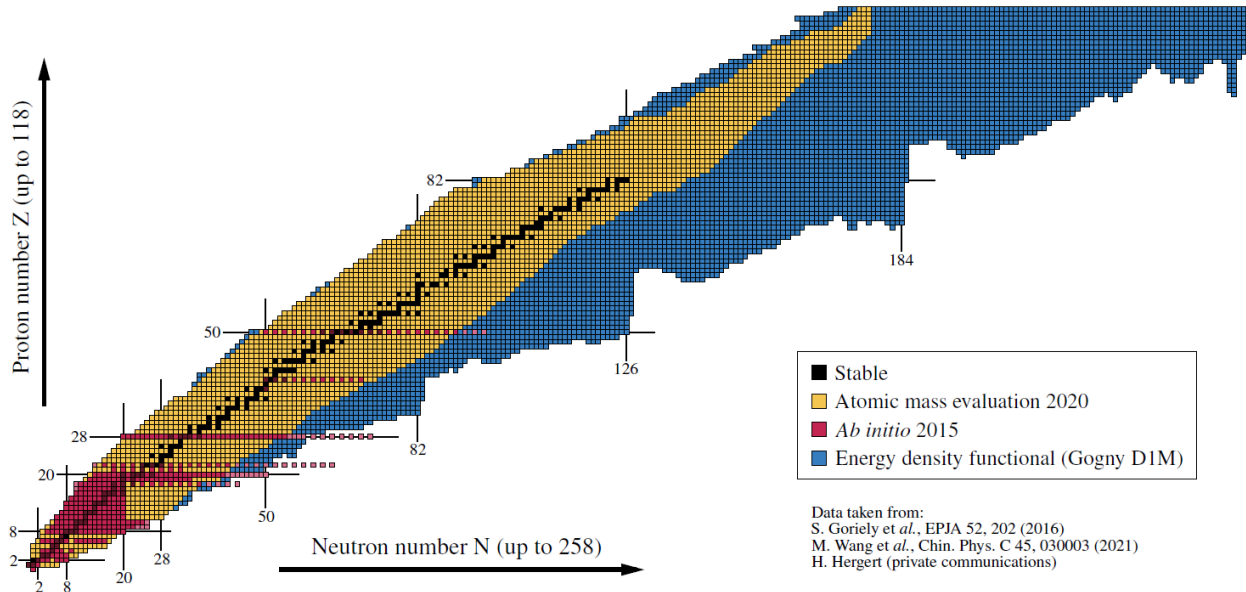
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Steady progress in the last decades

Light nuclei	1990's	Exponential scaling
Quasi-exact methods		
Closed shells	2000's	Polynomial scaling
Expansion methods Single-reference		
Singly open-shells	2010's	Polynomial scaling
Symmetry-breaking Multi-reference		

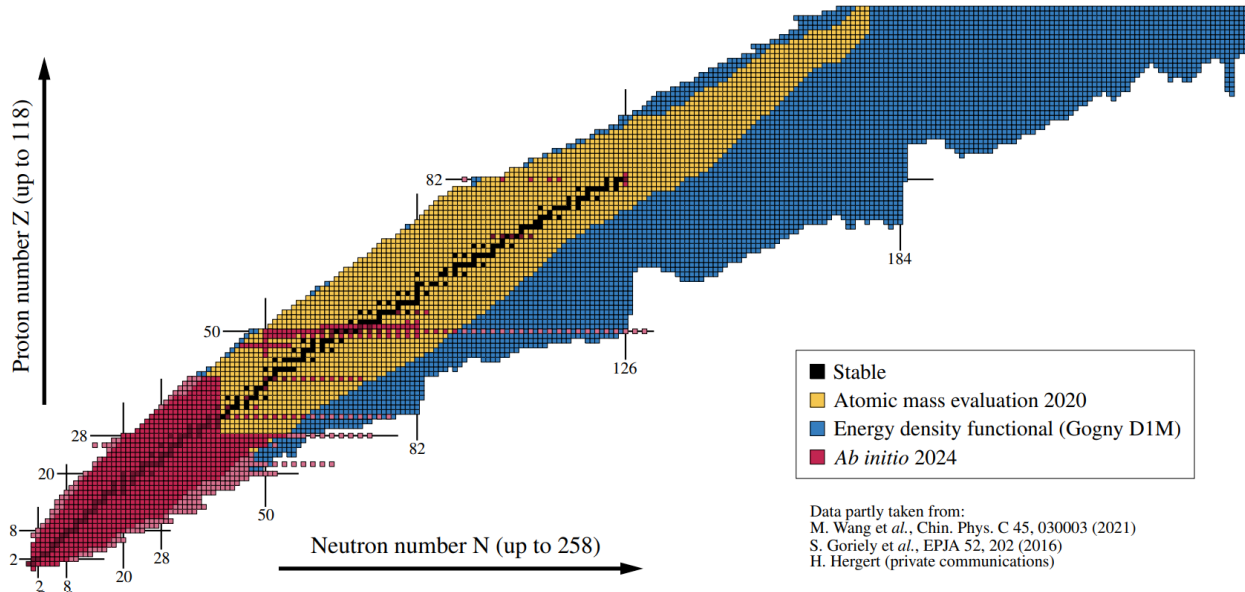
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Progress of *ab initio* / *in medias res* methods



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Singly open-shells		
Symmetry-breaking Multi-reference	2010's	Polynomial scaling
Doubly open-shells		
Valence space Symmetry-breaking Multi-reference	2020-?	Mixed / Polynomial Scaling

Single and multi-reference expansion



$[H, R] = 0$ Expansion methods

Nucleon interaction
 1-,2-,3-... body $H|\Psi^\sigma\rangle = E^\sigma|\Psi^\sigma\rangle$ *Correlated wave-function*
A! parameters



Partitioning

Unperturbed problem
 « Easy » $H \equiv H_0 + H_1$ *Residual interaction*
Treated approximatively

Perturbation Theory
 Coupled Cluster
 IMSRG
 Green's functions



$$\begin{aligned}
 \mathcal{W} &\sim \sum_k H_1^k \\
 &\sim e^T = e^{\mathcal{T}_1 + \mathcal{T}_2 + \dots} \\
 &\dots
 \end{aligned}$$

$|\Psi\rangle \equiv \mathcal{W}|\Theta^{(0)}\rangle$ *Wave operator*
Expressed as tensor network

Expansion method
 → Efficient for **short range**
 → 2p2h, 3p3h, ...

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How to grasp long range
at polynomial cost ?

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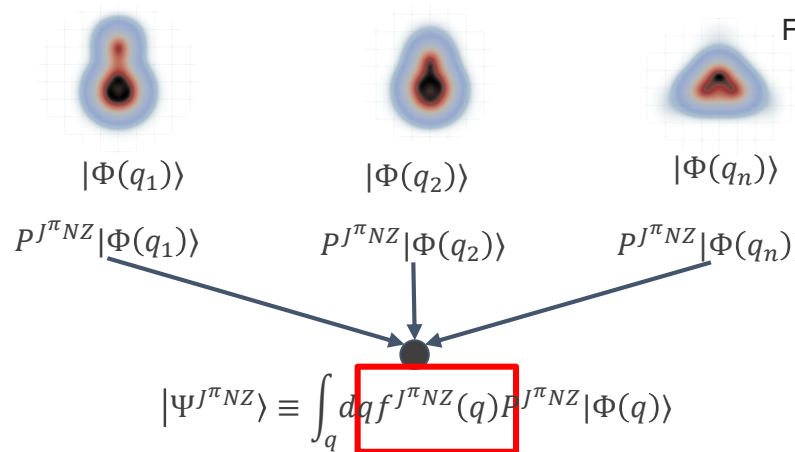
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Projected Generator Coordinate Method

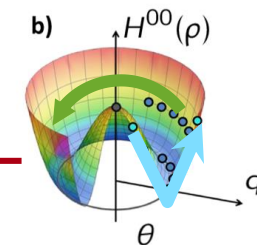
Frosini et al; Eur.Phys.J.A 58 (2022) 4, 62



Projection
Rotation

Shape mixing
Vibration

Variational HWG $\sum_q H_{pq}^\sigma f_\mu^\sigma = \epsilon_\mu^\sigma(q) \sum_q N_{pq}^\sigma f_\mu^\sigma(q)$
Low dimensional Hamiltonian kernel Norm kernel



Symmetry breaking minimum

Shape oscillations / rotations accounting for mpmh effects

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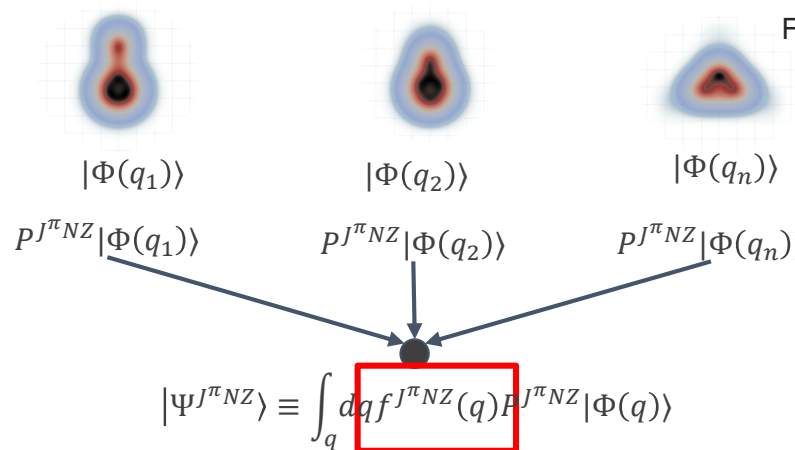
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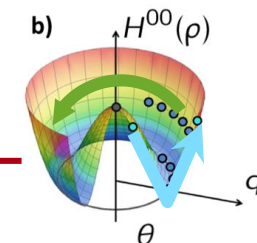
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Multi-reference (non orthogonal) perturbation theory

PGCM + perturbative 2p2h

Strong static / collective correlations captured by PGCM reference
Weak / dynamical correlations captured in perturbation
Versatile but **expansive symmetry conserving** expansion method
Different from PCC
- project + partition + expand / partition + expand & project

Circumventing the problem of three-body interaction

Ab initio Hamiltonian

$$H = T + V^{NN} + V^{NNN} + \dots$$

Similar to other interactions

Neglected for now

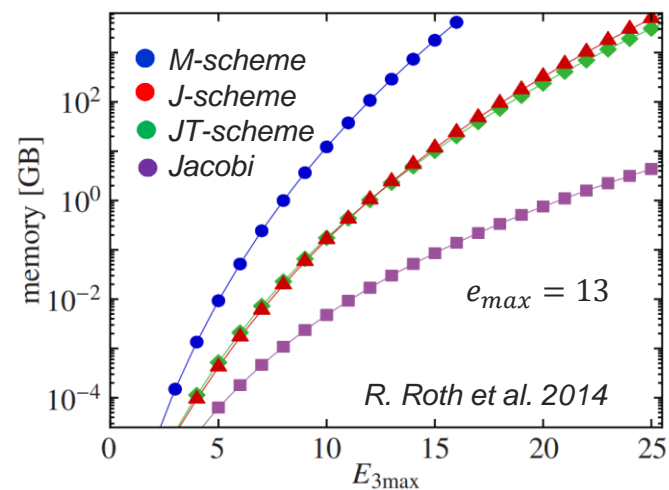
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Essential for predictivity of the theory
True Hamiltonian (no spurisities BMF)



Memory bottleneck ($O(N^6)$ vs $O(N^4)$)
Runtime bottleneck

Cost increase in deformed calculation
too large to be handled

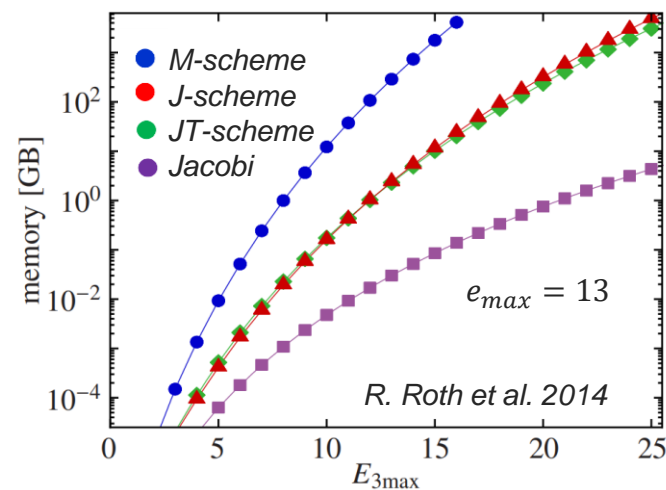
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In medium interactions

Frosini et al, Eur.Phys.J.A 57 (2021) 4, 151

1. Apply same contractions with arbitrary « **well chosen** » ρ
2. **Discard pure three-body terms**
3. Convert **back to single particle basis**

$$\bar{E}_0 \equiv \frac{1}{3!} V^{NNN} \cdot \rho \cdot \rho \cdot \rho$$

ρ chosen to be symmetry conserving

Applications:

$$\bar{T} \equiv T - \frac{1}{2!} V^{NNN} \cdot \rho \cdot \rho$$

- Small error with reasonable ρ

$$\bar{V} \equiv V + V^{NNN} \cdot \rho$$

- Very close to standard NO2B

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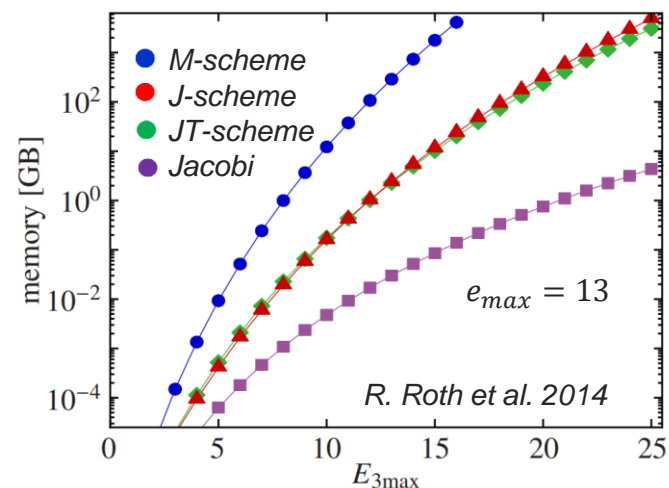
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Limitations of the method

Total Energy Curve in axial calculations at different basis size
hw0 = 10, **E3max=28** (single precision, generated with Nuhamil)

T. Miyagi Eur. Phys. J. A 59, 150 (2023)

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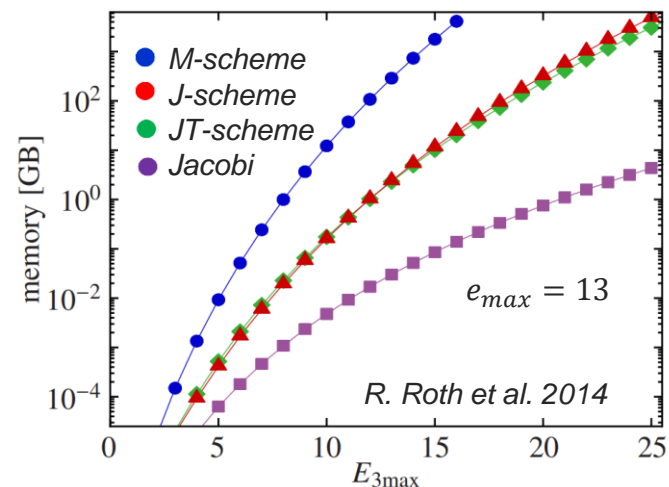
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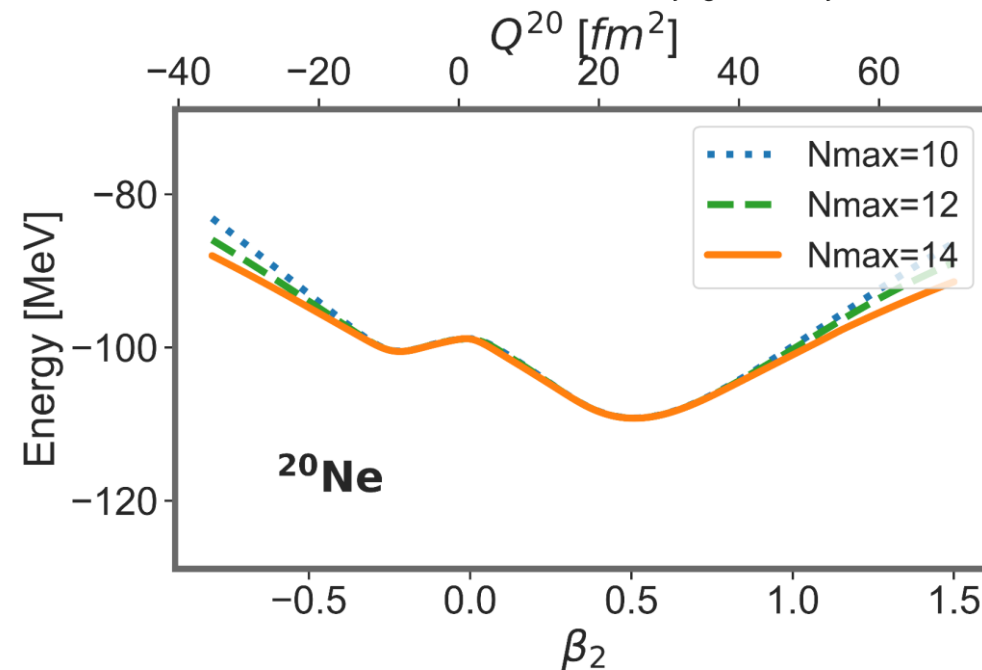
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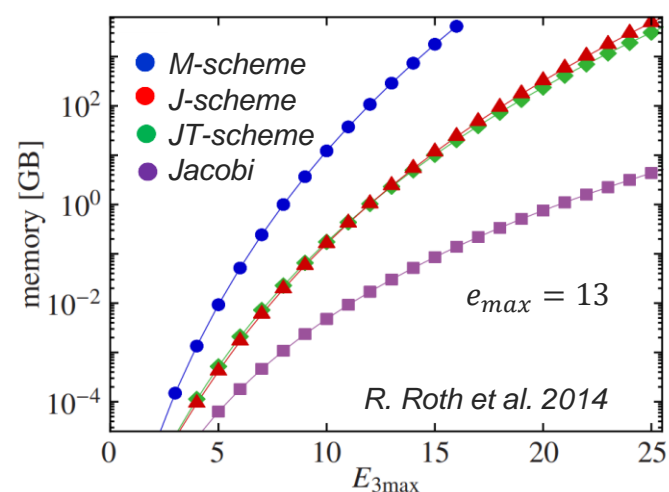
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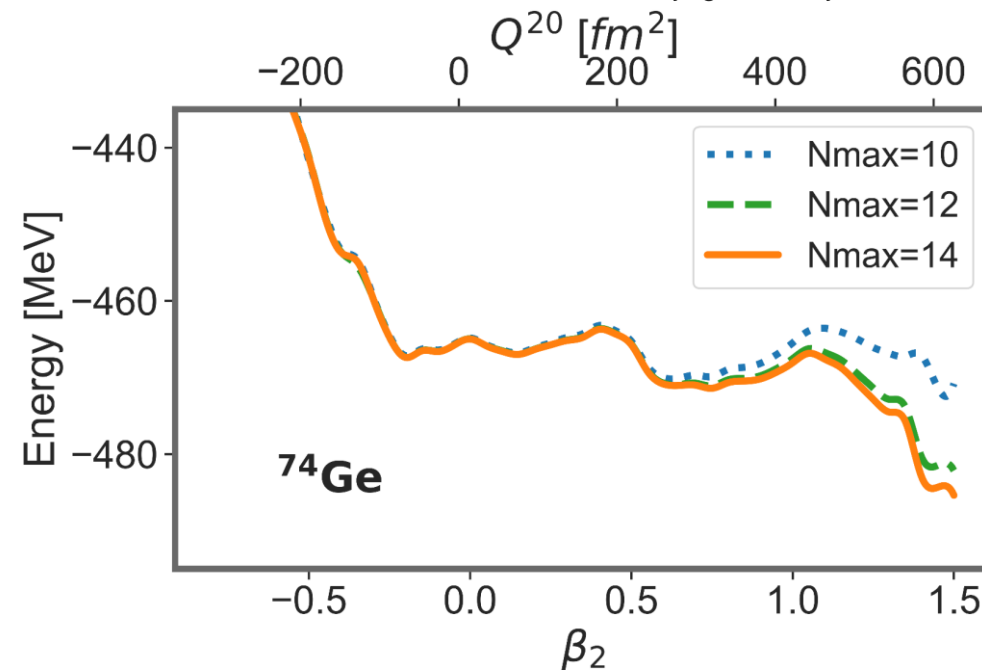
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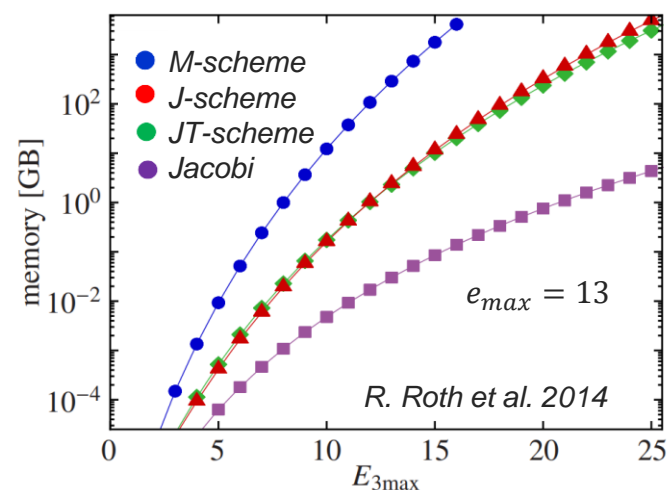
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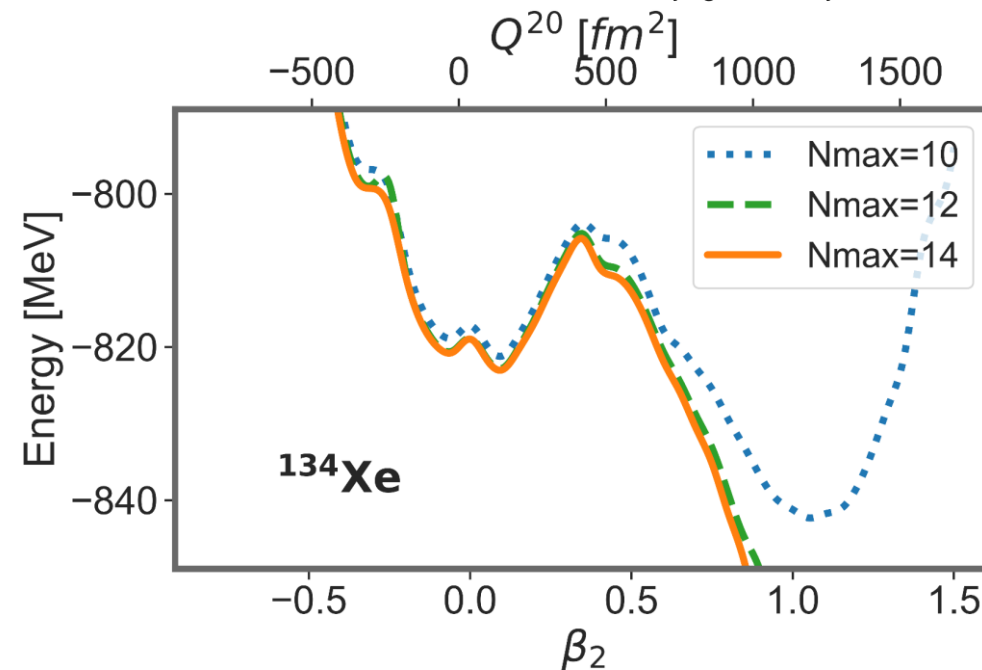
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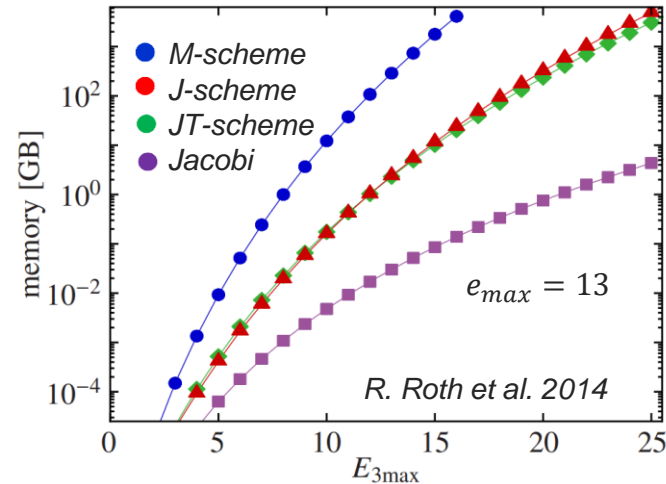
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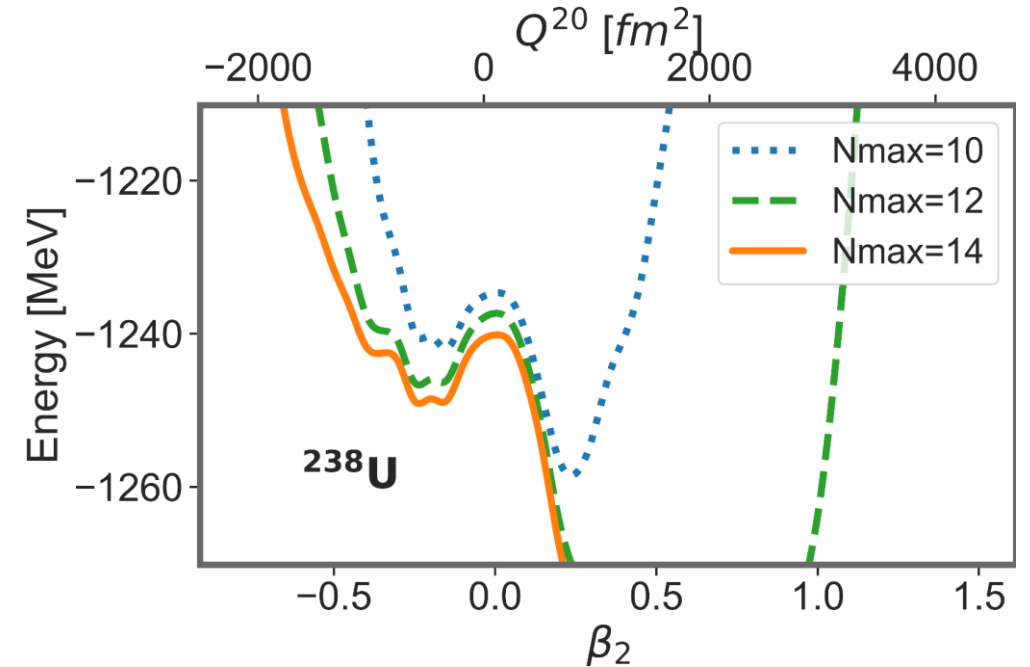
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The exact origin is still under investigation...

- But no doubt approximation we perform on 3-body is responsible
- Already observed by Hergert, Bally, Yao... five years ago!
- Exact 3-body (very expensive) / MR-IMSRG...?

For now, from different tests (deformed), three mass ranges identified

3 – 80 (safe) / 80 – 150 (danger zone) / 150 + not under control



3 ■ Application: from low-lying spectroscopy to giant resonances

- PGCM-PT(2) and scale decoupling
- Application of PGCM in realistic model space

PGCM-PT(2) and effective decoupling of scales

How well long vs. short range physics decouple?

Best way to understand it is to change the scale !

Applying MR-IMSRG evolution to shuffle correlations

Comparing bare PGCM and PGCM-PT(2)

Long (static) \leftrightarrow Short (dynamical)



PGCM-PT(2) and effective decoupling of scales

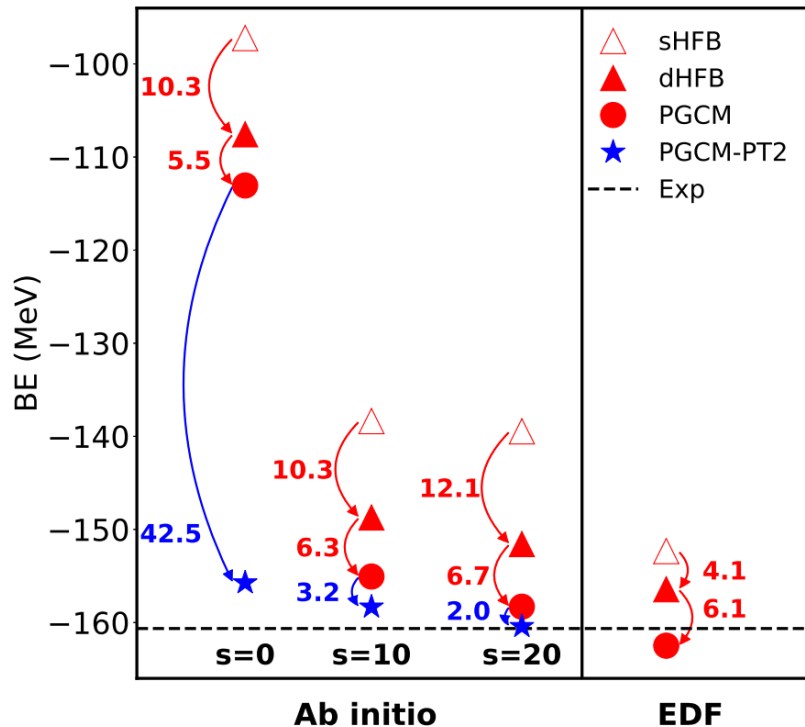
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Reshuffling of correlations

- Much lower mean-field
- **Increase of static** correlations
- Link with EDF?

PGCM-PT(2) dynamical correlations

- Strong decrease due to reshuffling
- Not vanishing (approximate decoupling)
- Higher order effects (PGCM-PT(3))?

PGCM-PT(2) and effective decoupling of scales

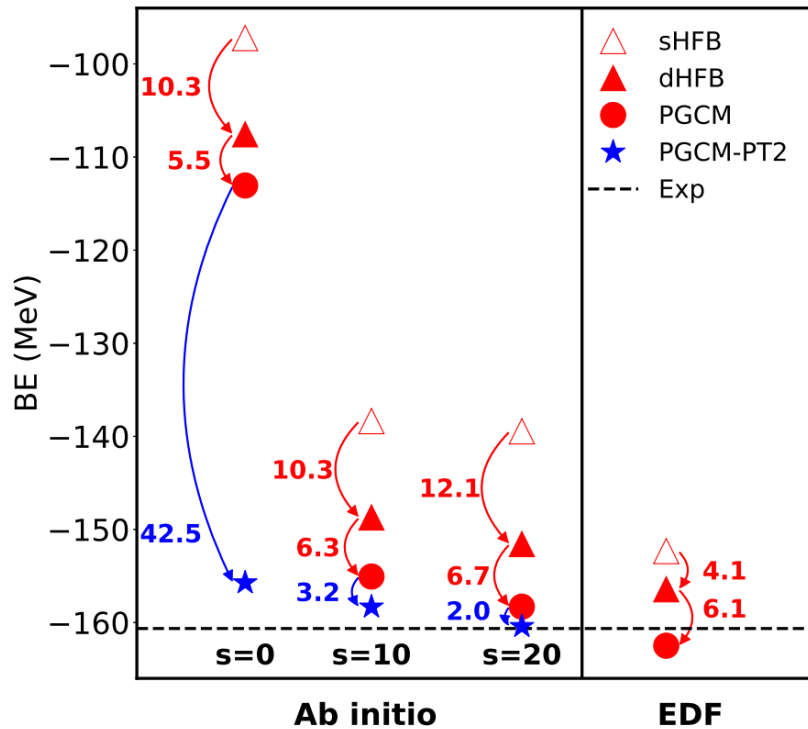
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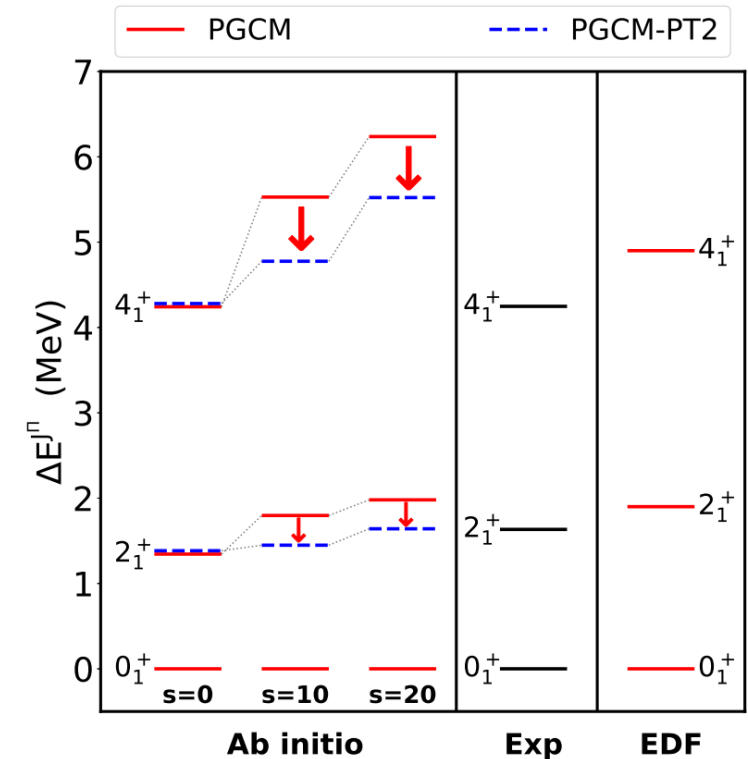
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Effect on excited states

- Dilatation of rotational spectrum
- PGCM-PT(2) contracts back spectra



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Conclusion

Good scale decoupling with vacuum interaction

PGCM reliable for low-lying rotational spectroscopy

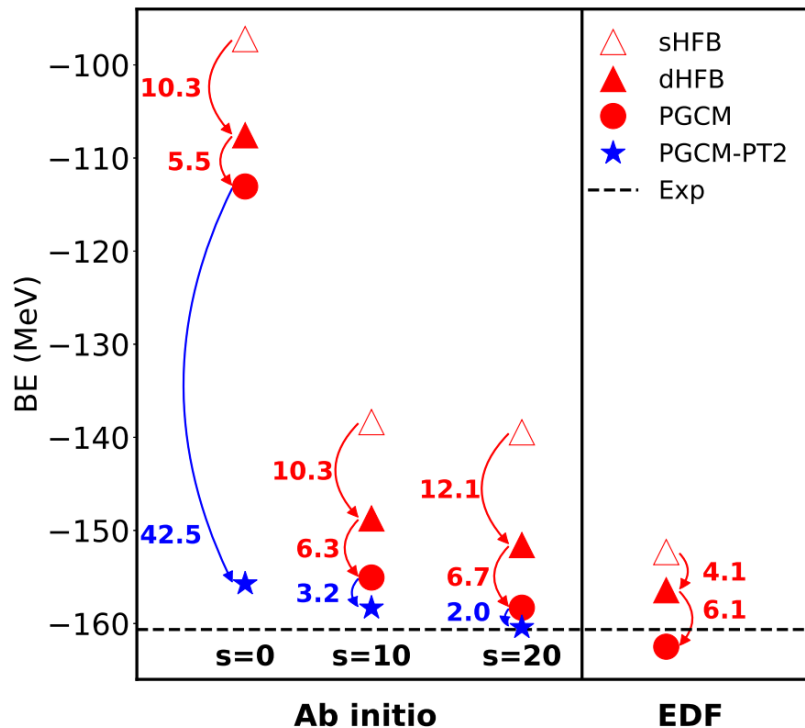
independently confirmed by *Sun et al.*

effective confirmation with QRPA *Porro et al EPJA 60 (2024)*

Gonzalez-Miret et al. (2025)

What about vibrations? Until when can we go?

In the following, calculations at PGCM level...



Reshuffling of correlations

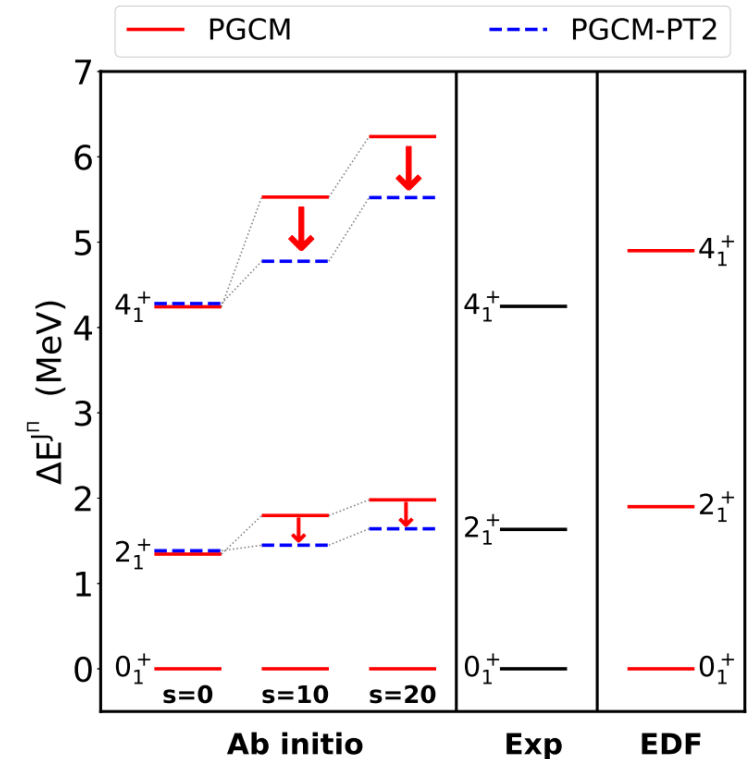
- Much lower mean-field
- **Increase of static** correlations
- Link with EDF?

PGCM-PT(2) dynamical correlations

- Strong decrease due to reshuffling
- Not vanishing (approximate decoupling)
- Higher order effects (PGCM-PT(3))?

Effect on excited states

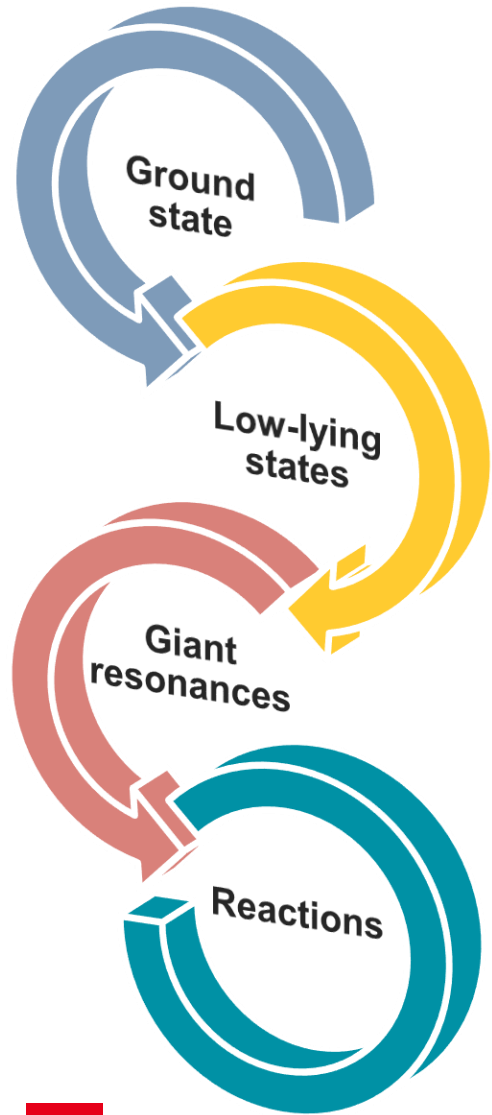
- Dilatation of rotational spectrum
- PGCM-PT(2) contracts back spectra



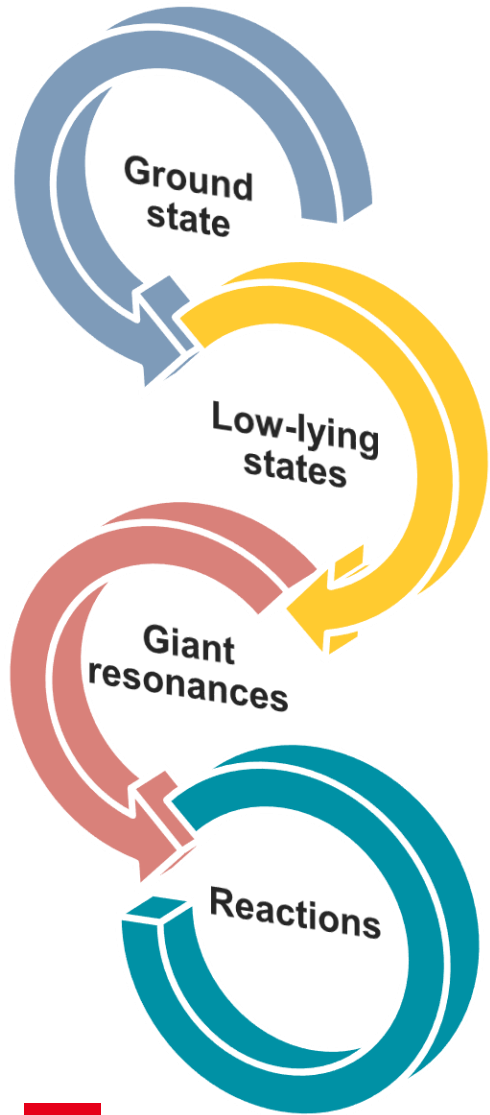
Duguet et al Eur.Phys.J.A 59 (2023)

Frosini et al Eur.Phys.J.A 58 (2022)

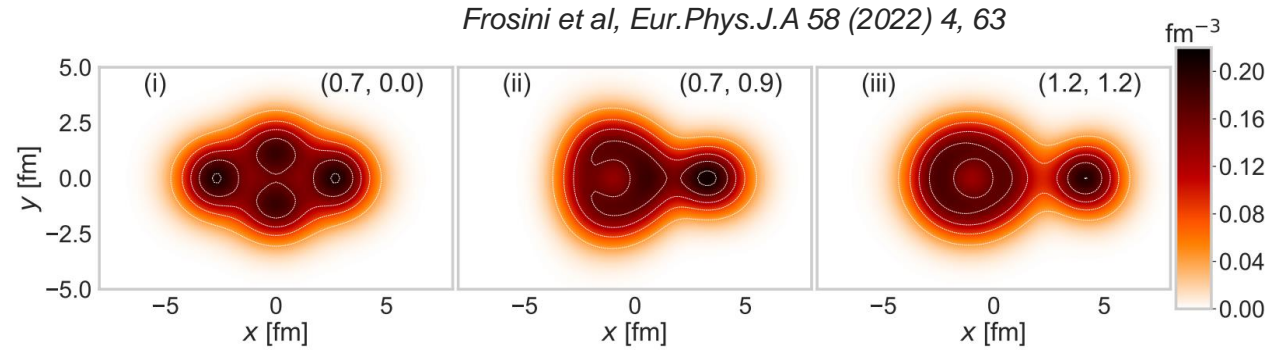
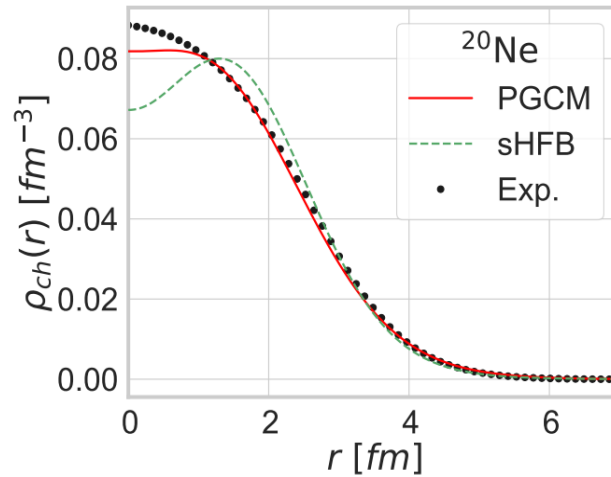
Applications of PGCM in realistic model spaces



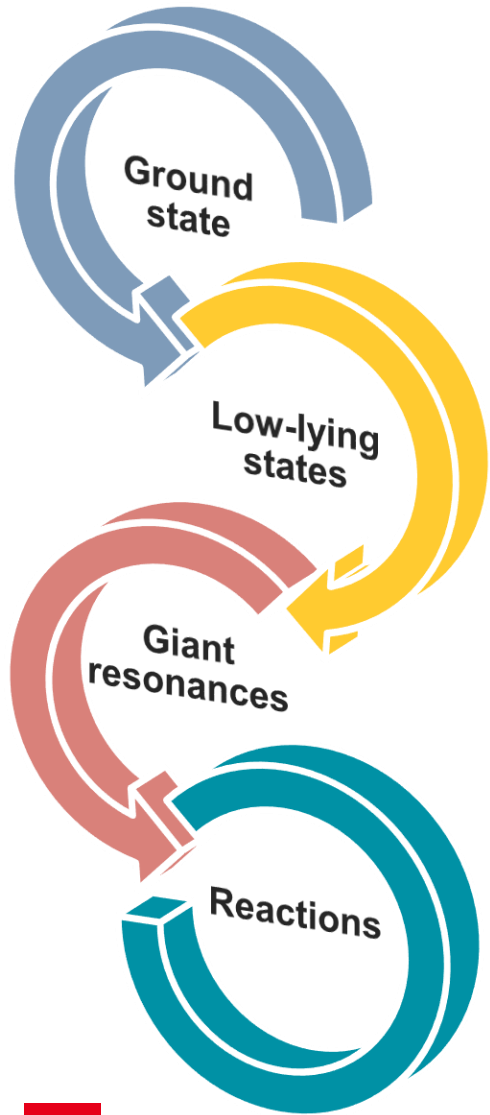
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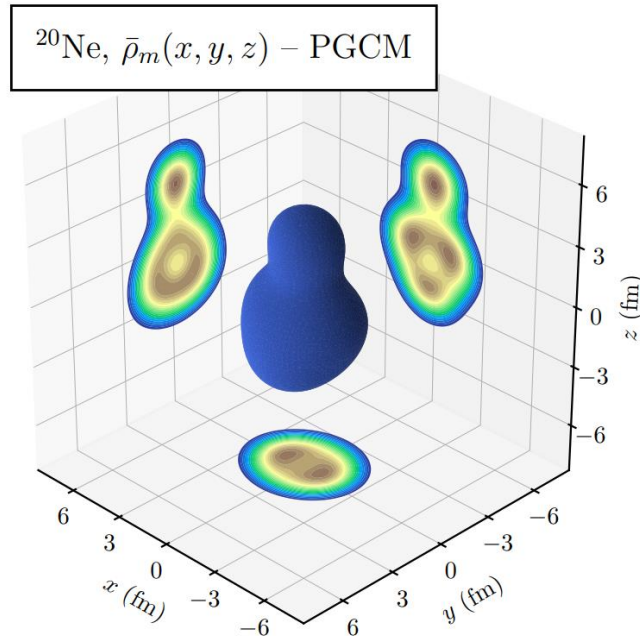
Shape mixing : deep insight on structure
Coherent superposition of clustered states
Improvement wrt spherical mean-field
Applications to heavy-ion collision



Applications of PGCM in realistic model spaces

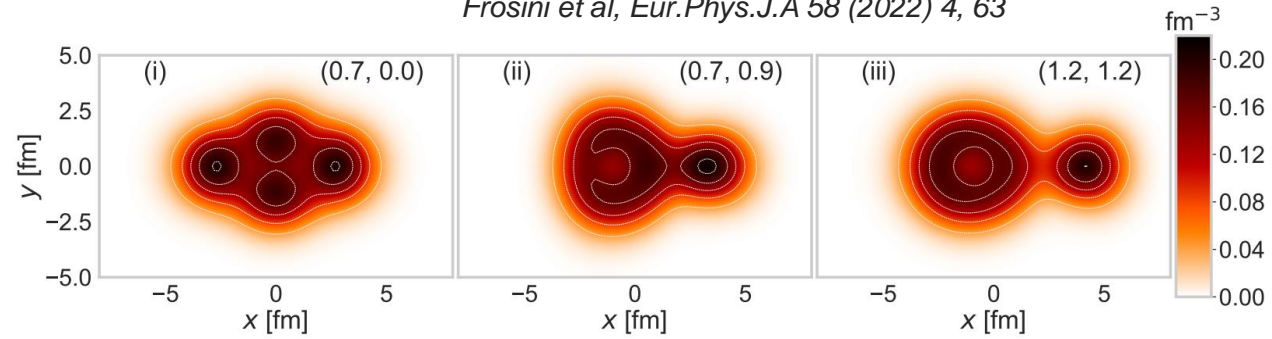


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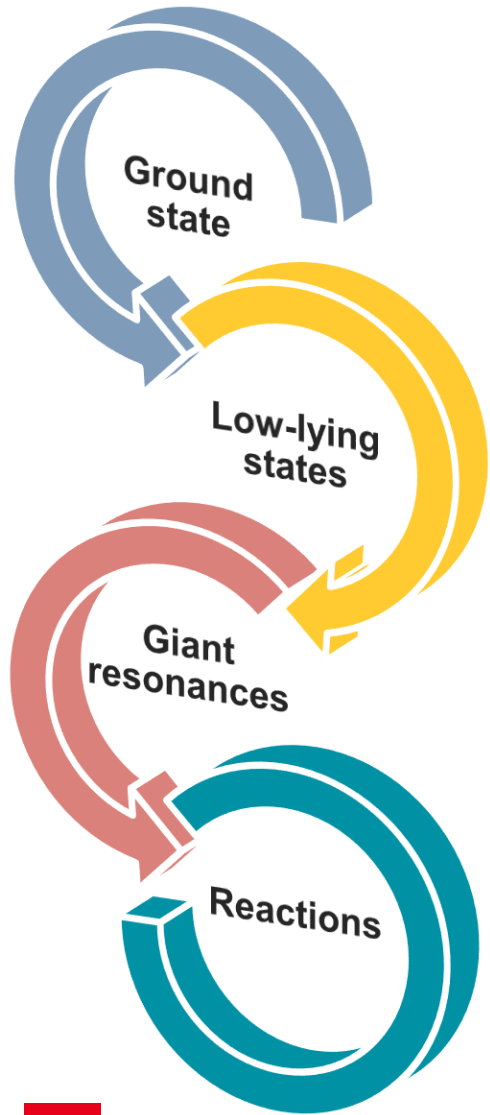


Giacolone, Bally et al. (2024) arxiv 2402.05995

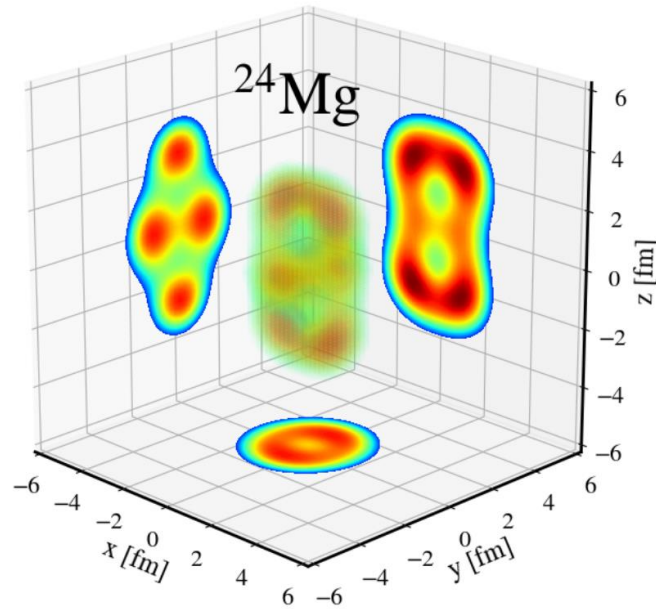
Frosini et al, Eur.Phys.J.A 58 (2022) 4, 63



Applications of PGCM in realistic model spaces



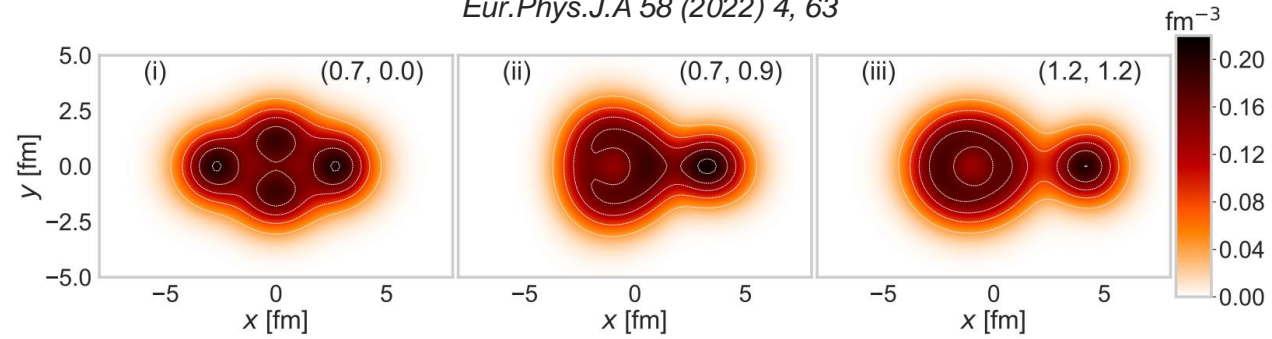
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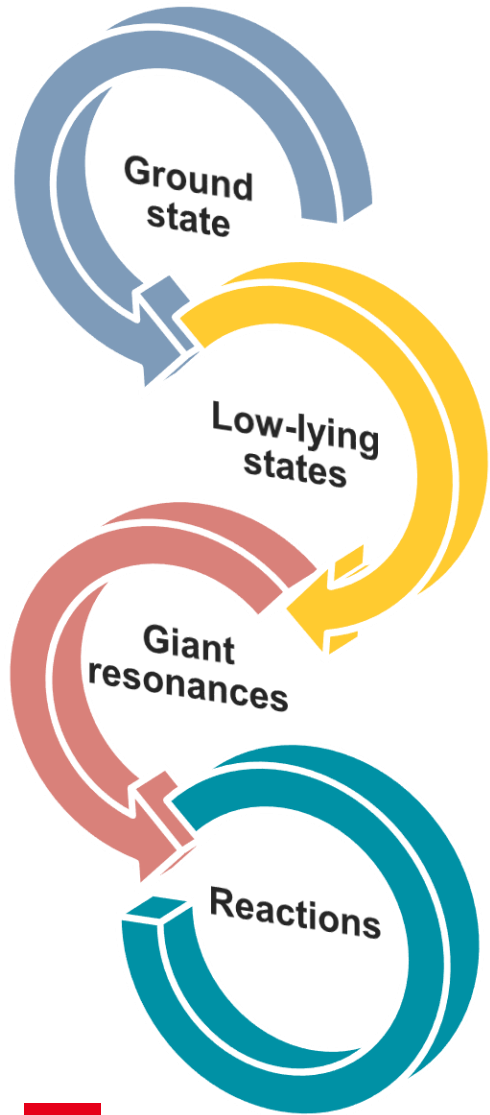
Giacolone, Bally et al. (2024) arxiv 2402.05995

Gonzalez-Miret (in preparation)

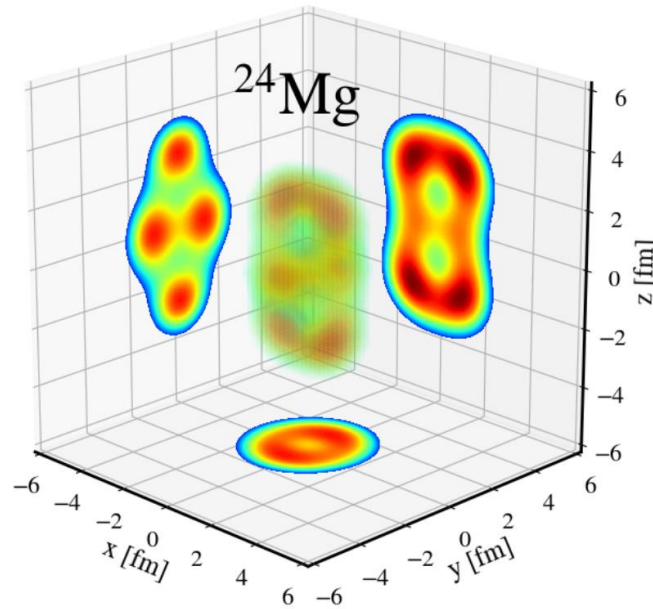
Eur.Phys.J.A 58 (2022) 4, 63



Applications of PGCM in realistic model spaces



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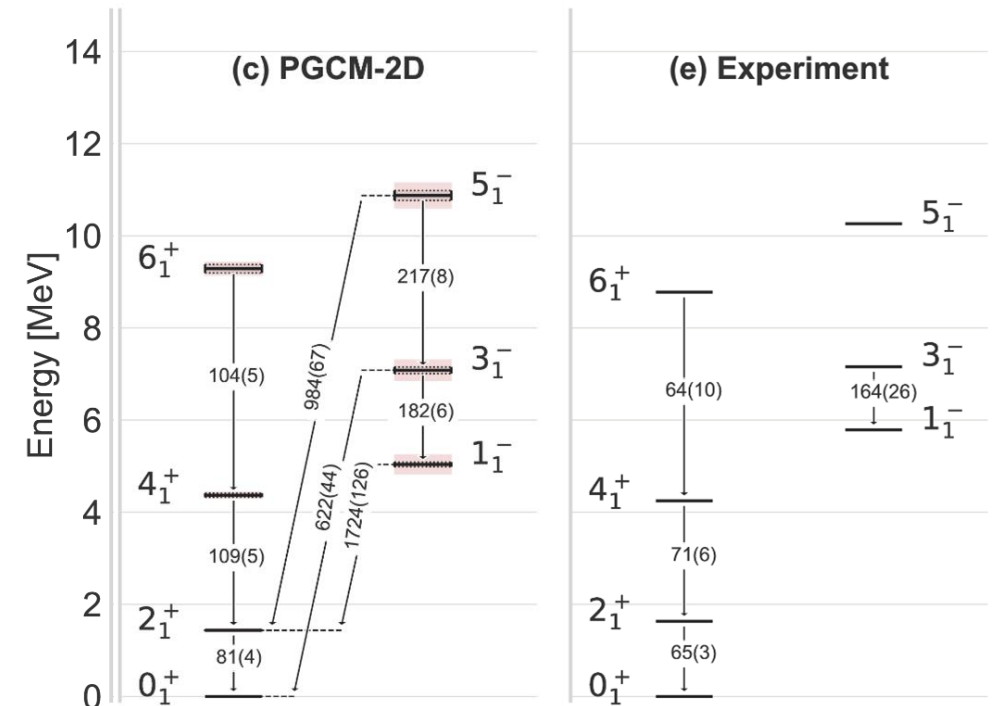
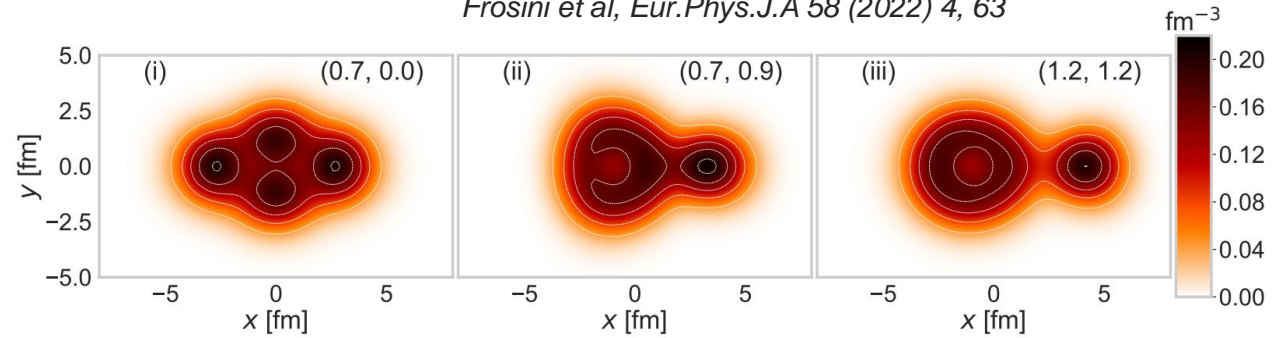


Giacolone, Bally et al. (2024) arxiv 2402.05995

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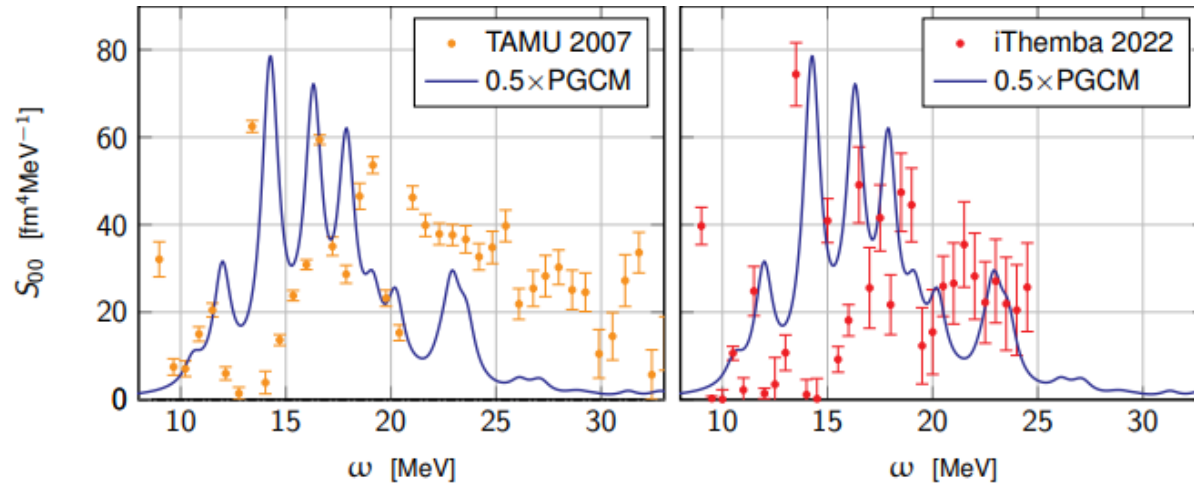
Rotational & vibrational states
 Good agreement with experiment
 Reproduction of B(E2), Q2, etc.

Frosini et al, Eur.Phys.J.A 58 (2022) 4, 63



Applications of PGCM in realistic model spaces

A. Porro et al. *Eur.Phys.J.A* 60 (2024)



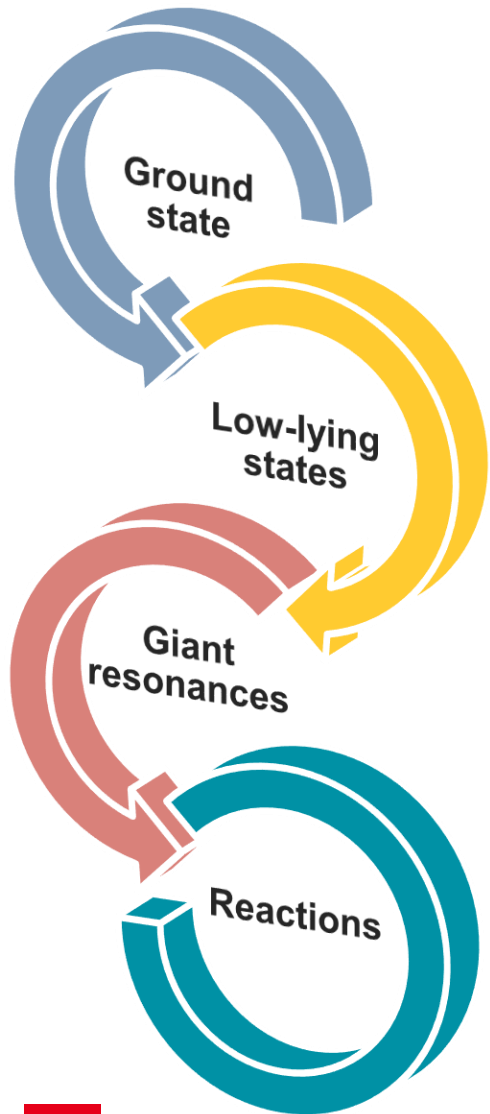
Giant resonance description

Monopole study in $N=Z$ nuclei

PGCM : Good fractioning (wrt QRPA)

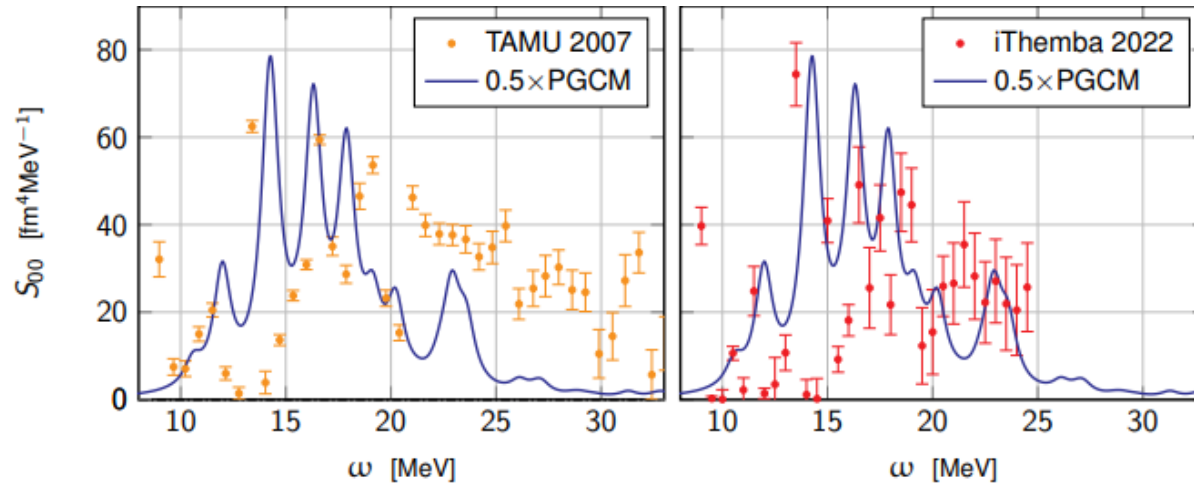
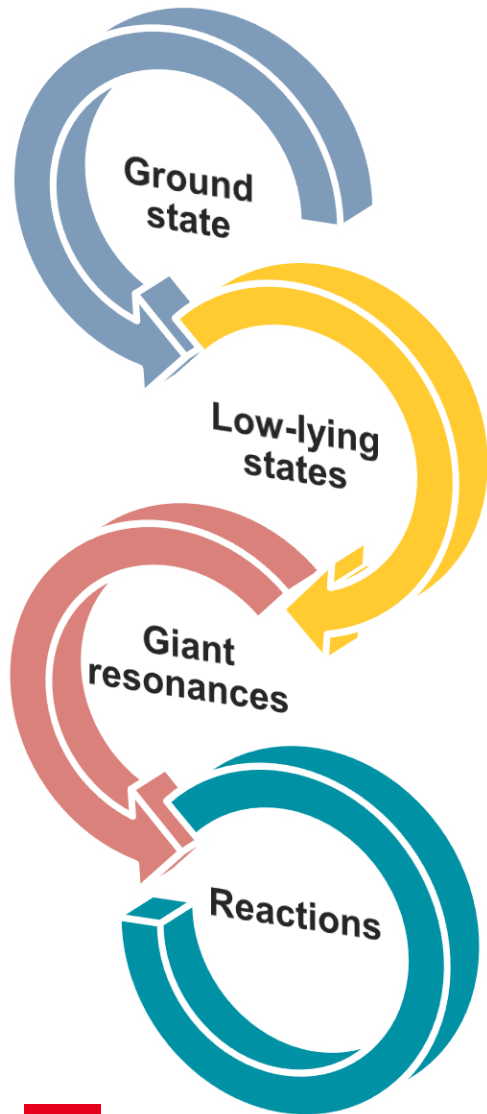
No *ad hoc* shift

When dynamical become important?



Applications of PGCM in realistic model spaces

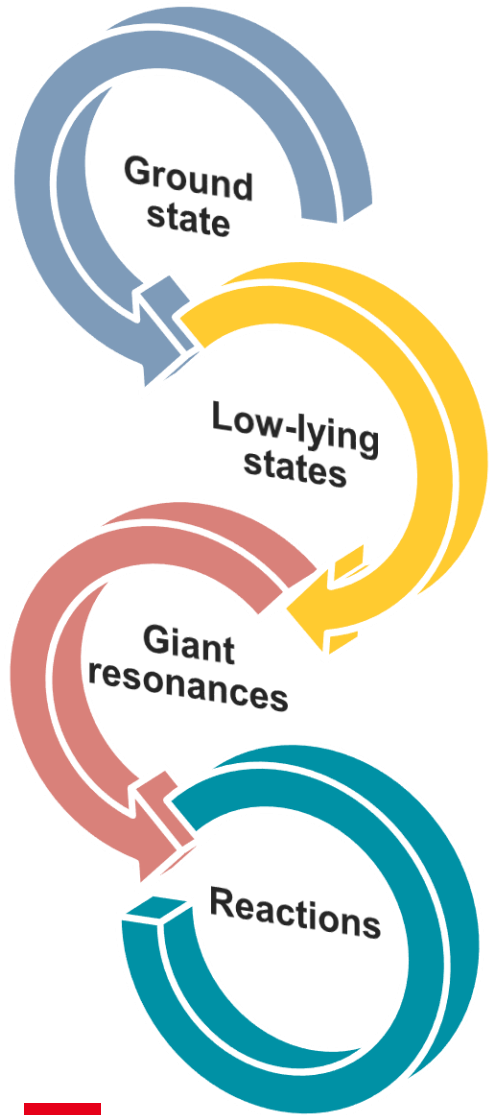
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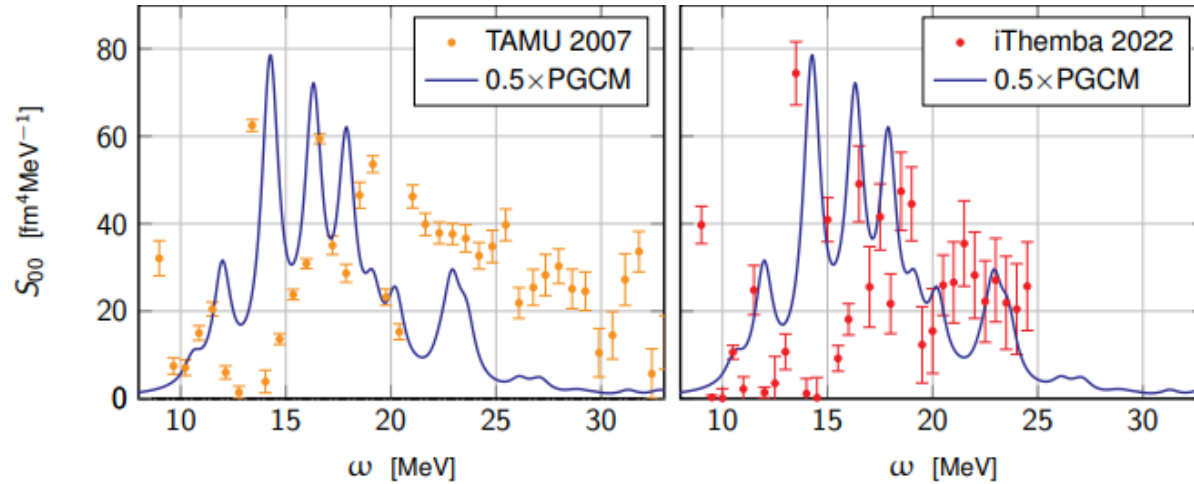
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Generalization to $J \neq 0$
Additional complexity
Need for triaxiality ($K \neq 0$)
Choice of GC? VS benchmarks

Applications of PGCM in realistic model spaces

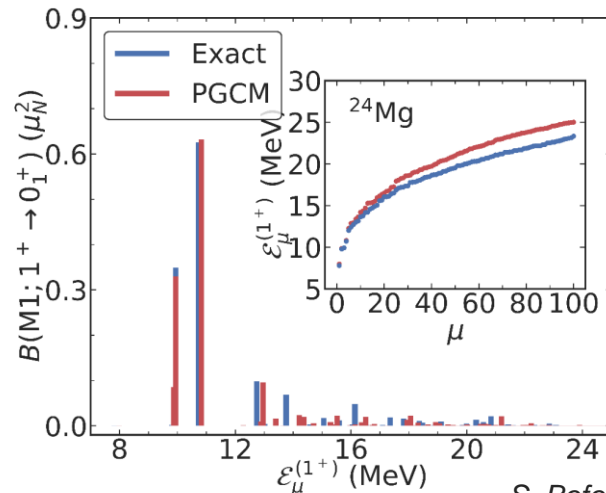


A. Porro et al. *Eur.Phys.J.A* 60 (2024)



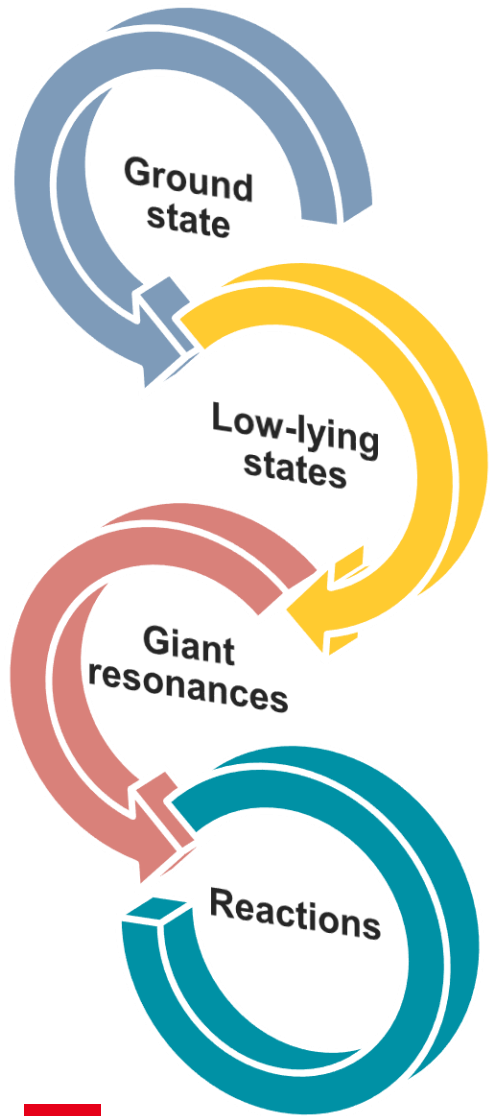
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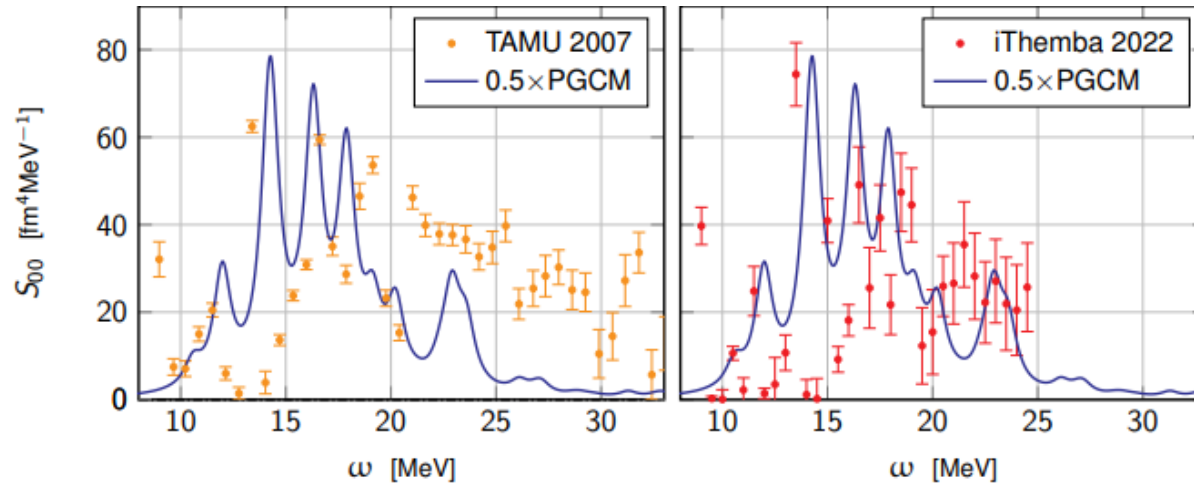


S. Bofos, J. Martinez et al. in preparation

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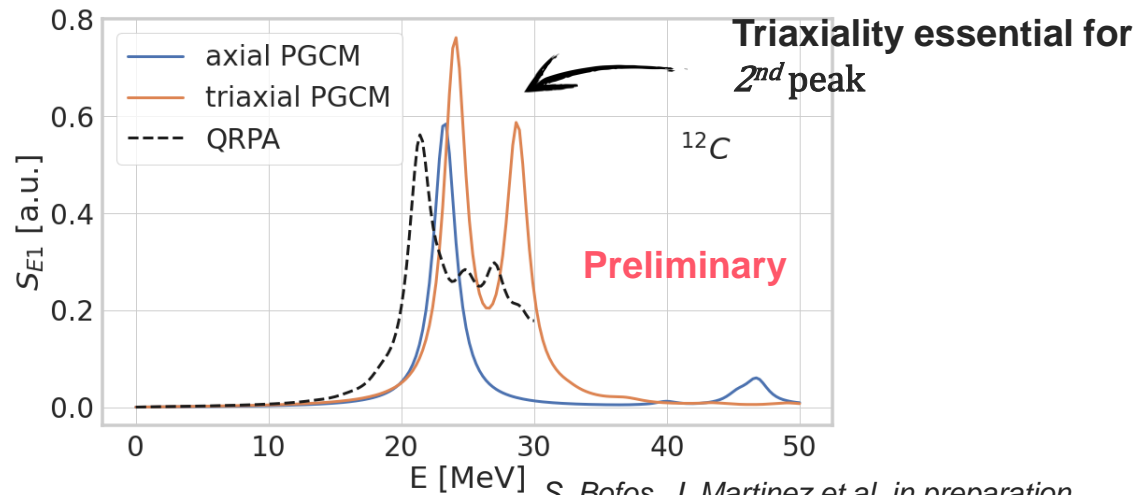


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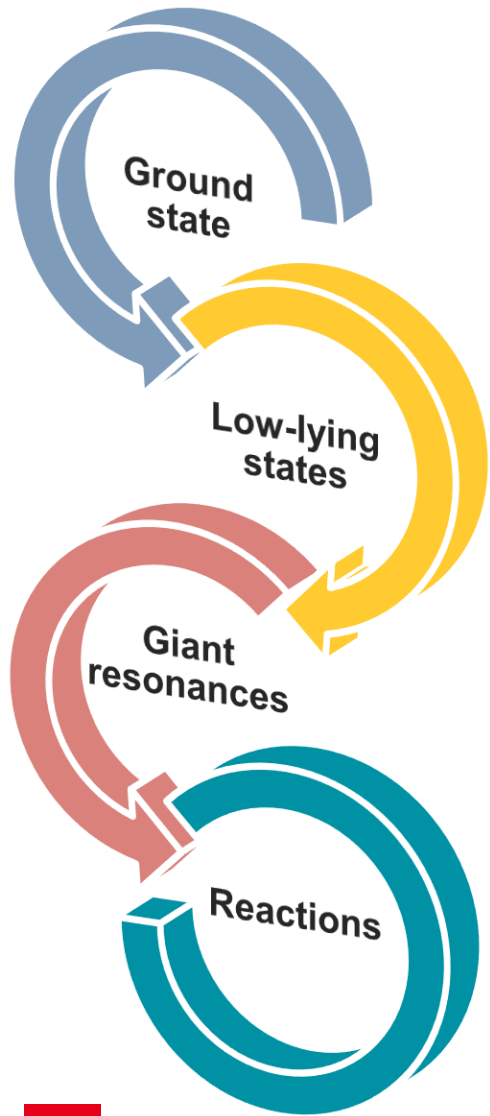
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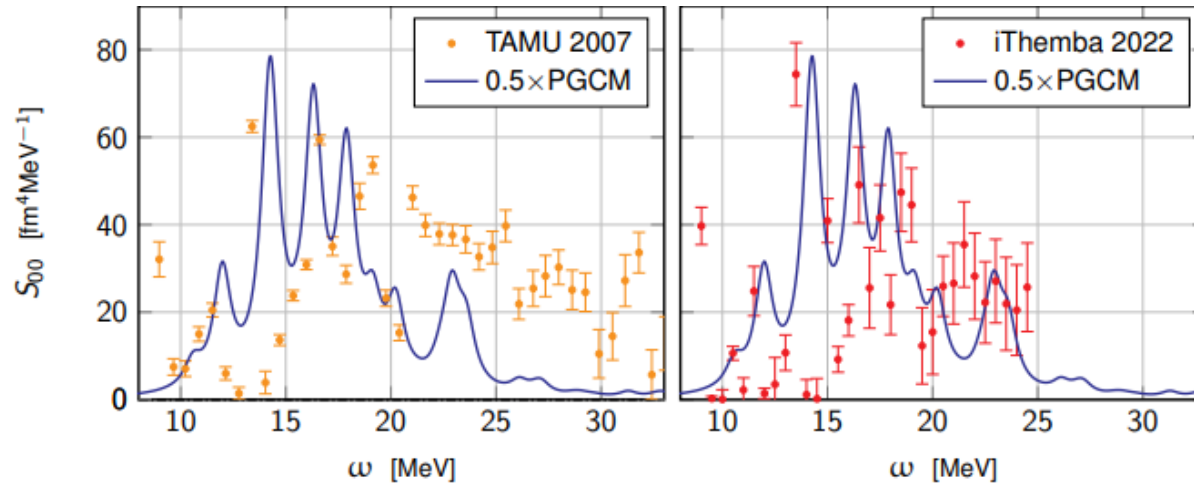


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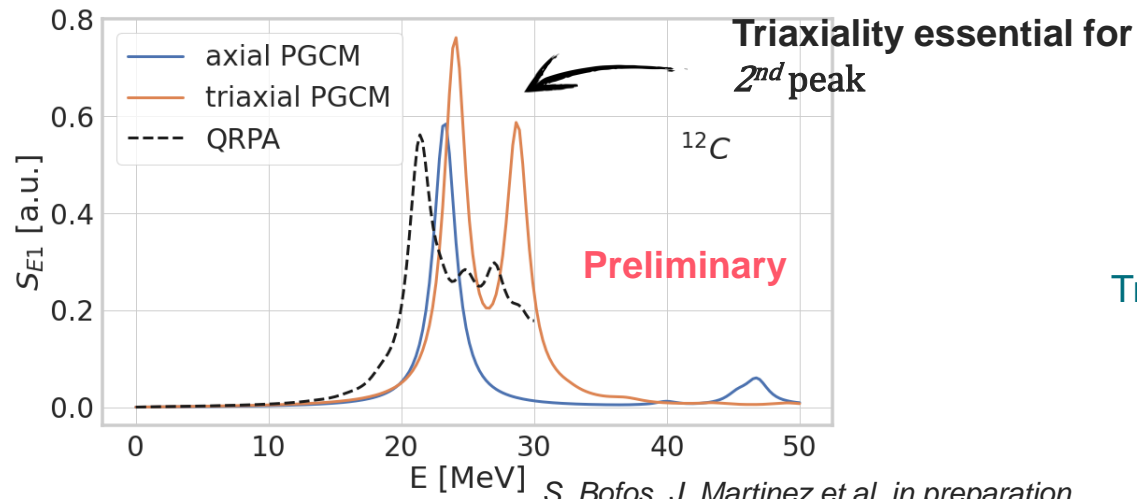


A. Porro et al. Eur.Phys.J.A 60 (2024)

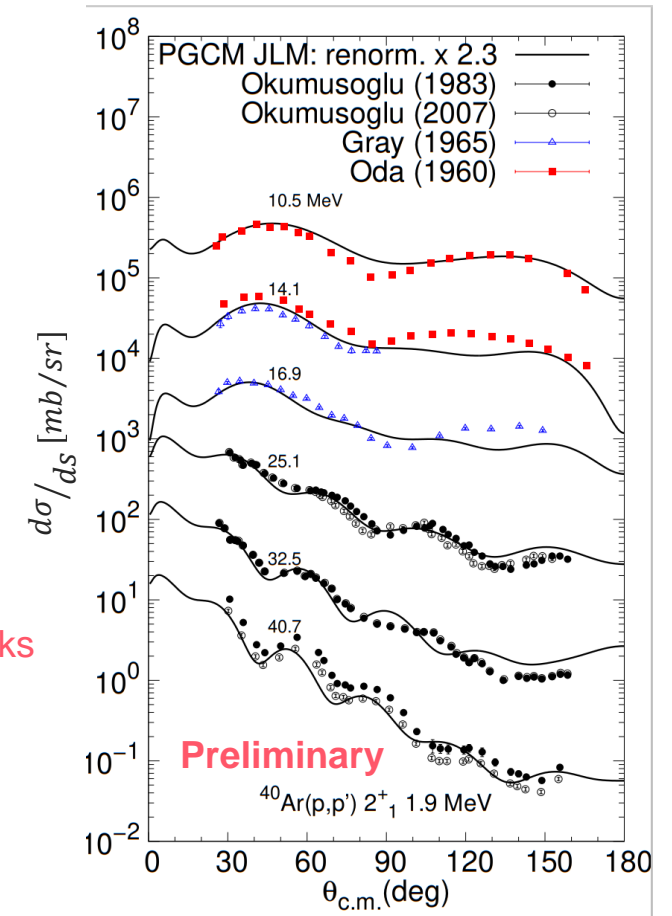


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S. Bofos, J. Martinez et al. in preparation



Application to reactions
 Transitions densities between excited states
 Feeding reaction model with excited states
 Very preliminary results

P. Da Costa, M. Dupuis et al. in preparation



4. Conclusion

Conclusion



Multi-reference methods are a powerful tool towards universal modelling of nuclei

Coherent account of vibrational & rotational physics (PGCM)

Short range dynamical correlations captured perturbatively (PGCM-PT)

Observation of *accidental* scale decoupling between static & dynamical

Many developments needed and ongoing to scale up



Better account of 3-body interaction

Limitations of the current method

Deformed heavy nuclei not reliable yet (**$A > 150$**)

Large E_{max}^3 did not solve the issue...



Optimizing MR-Perturbation Theory

Current implementation still very heavy

Many avenues exist to leverage the cost

Ongoing project being revived

Lowering the cost of PGCM

Restoring symmetries comes at a significant cost

Many nuclei / applications require triaxiality

Applying dimensionality reduction techniques?

S. Bofos (2025)

Thanks for your attention



Rémi Bernard
Olivier Litaize
Gille Noguère
Alessandro Pastore
Pierre Tamagno
Stavros Bofos
Clémentine Azam
Steve Sainato

Thomas Duguet
Vittorio Somà
Benjamin Bally
Gianluca Stelini

Jean-Paul Ebran
Sophie Péru
Lars Zurek
Philippe Da Costa
David Durel
Luis Gonzalez-Miret



Robert Roth
Andrea Porro



Heiko Hergert



Alberto Scalesi



T. R. Rodríguez



Kamila Sieja