International Conference on Shapes and Symmetries in Nuclei: from Experiment to Theory
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Entanglement and coherence of wobbling motion

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Ref: Q. B. Chen and S. Frauendorf, arXiv:2410.08749 (2024)

Outline

- Wobbling motion
- SCS/Azimuthal and SSS plots
- Entanglement entropy
- Quantum coherence
- Summary

Prediction of wobbling motion

- **Rotation of a triaxial nucleus** *Bohr & Mottelson1975, Vol. II, page 190*

$$\hat{H}_{\rm rot} = \frac{\hat{I}_m^2}{2\mathcal{J}_m} + \frac{\hat{I}_s^2}{2\mathcal{J}_s} + \frac{\hat{I}_l^2}{2\mathcal{J}_l}$$

$$\mathcal{J}_m > \mathcal{J}_s, \mathcal{J}_l$$

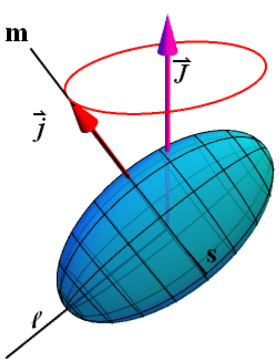
for given I (>>1), the state of lowest energy (yrast) has $|I_m| \sim I$

$$\hat{H}_{\rm rot} = \frac{I(I+1)}{2\mathcal{J}_m} + \boxed{(\hat{n} + \frac{1}{2})\hbar\Omega}$$

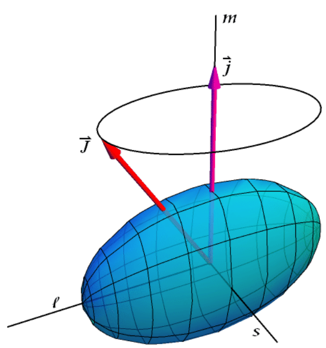
rotation **precession**

The quantum number $\hat{n}$ describes the precessional motion of the axes with respect to the direction of I; for small amplitudes, this motion has the character of a harmonic vibration with frequency ω. If the intrinsic state

$$\hbar\Omega = 2I\sqrt{\left(\frac{\hbar}{2\mathcal{J}_s} - \frac{\hbar}{2\mathcal{J}_m}\right)\left(\frac{\hbar}{2\mathcal{J}_l} - \frac{\hbar}{2\mathcal{J}_m}\right)}$$



LAB frame



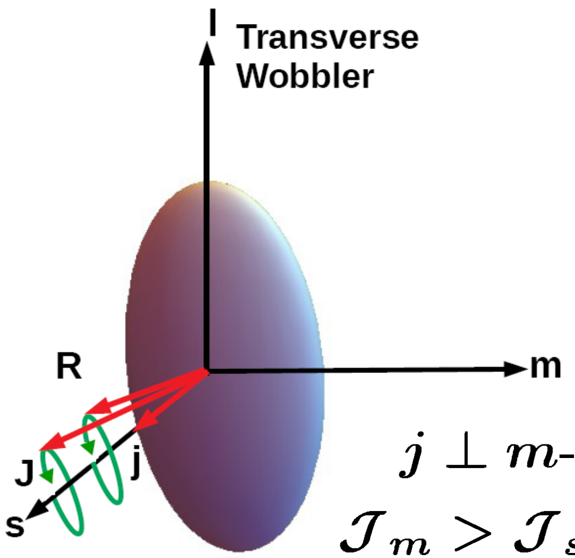
Intrinsic frame

frequency: wobbling frequency/energy
quanta: wobbling phonon quanta

Valence particle influences the wobbling

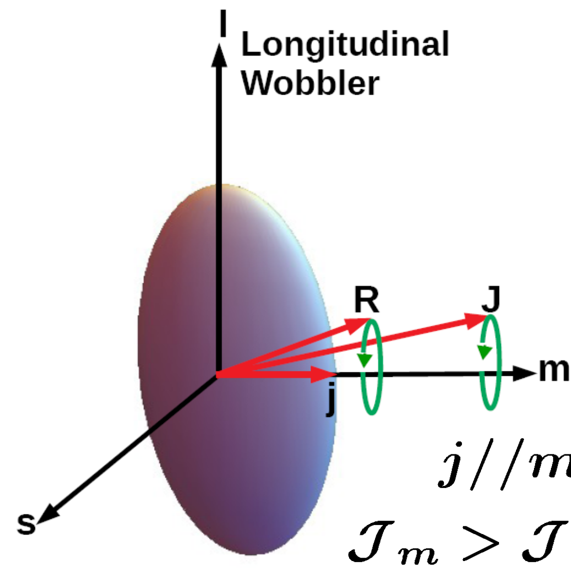
- **Odd-mass nuclei: transverse and longitudinal wobbling**

Frauendorf &Dönau, PRC 89, 014322 (2014)



$j \perp m$-axis,

$\mathcal{J}_m > \mathcal{J}_s, \mathcal{J}_l$



$j // m$-axis

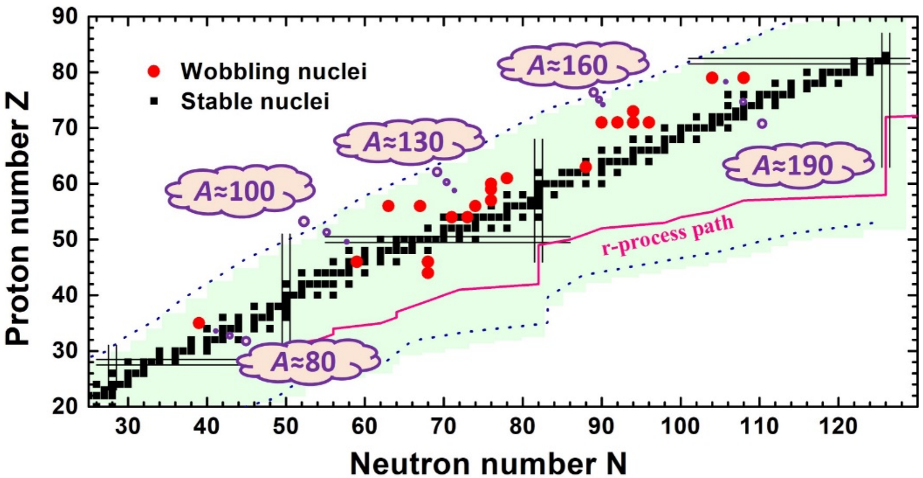
$\mathcal{J}_m > \mathcal{J}_s, \mathcal{J}_l$

With increasing spin *I*, wobbling frequency E_{wob} in

- **TW: decrease**
- **LW: increase**

Experimental observations

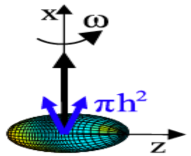
- **Experimental status: 20+ wobbling nuclei were reported in A~80, 100, 130, 160, and 190 mass regions; configurations based on one or two quasiparticles.**

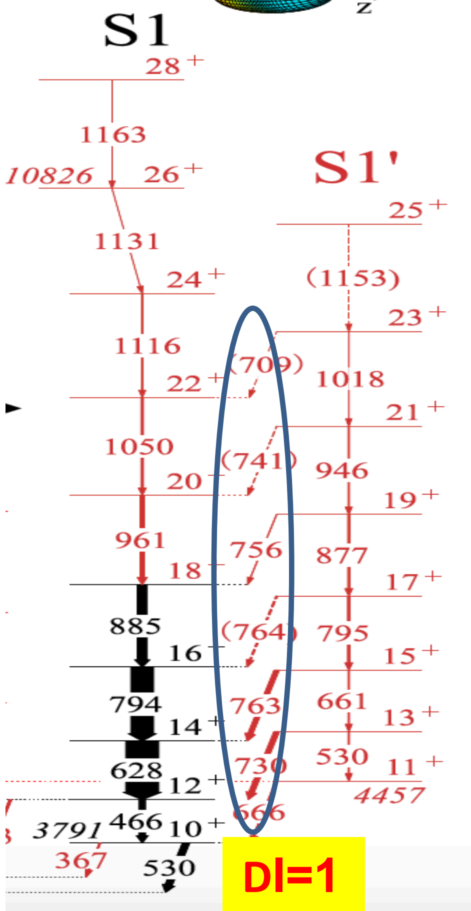


原子核	质子数	中子数	组态	β	γ	核区	参考文献
^{74}Br	35	39	$\pi g_{9/2} \otimes \nu g_{9/2}$	0.45	27.5°	80	[22]
^{105}Pd	46	59	$\nu h_{11/2}$	0.27	25°	100	[23]
^{125}Xe	54	71	$\nu h_{11/2}$	0.16	27°	130	[38]
^{127}Xe	54	73	$\nu h_{11/2}$	-	-	130	[28]
^{130}Ba	56	74	$\pi (h_{11/2})^2$	0.24	21.5°	130	[26, 27]
^{133}Ba	56	77	$\nu h_{11/2}$	-	-	130	[29]
^{133}La	57	76	$\pi h_{11/2}$	0.17	26°	130	[25]
^{135}Pr	59	76	$\pi h_{11/2}$	0.17	26°	130	[20, 24]
^{136}Nd	60	76	$\pi h^2_{11/2}$	0.17	28°	130	[30, 39]
^{151}Eu	63	88	$\pi i_{13/2}$	0.19	20°	150	[36]
^{161}Lu	71	90	$\pi i_{13/2}$	0.40	20°	160	[32]
^{163}Lu	71	92	$\pi i_{13/2}$	0.40	20°	160	[19, 31]
^{165}Lu	71	94	$\pi i_{13/2}$	0.40	20°	160	[33]
^{167}Lu	71	96	$\pi i_{13/2}$	0.43	19°	160	[34]
^{167}Ta	73	94	$\pi i_{13/2}$	0.41	20°	160	[35]
^{183}Au	79	104	$\pi i_{13/2}$	0.29	21.4°	190	[37]
^{183}Au	79	104	$\pi h_{9/2}$	0.3	20°	190	[37]
^{187}Au	79	108	$\pi h_{9/2}$	0.23	23°	190	[21]

Wobbling example: ^{130}Ba

^{130}Ba: $h_{11/2}{}^2$





Petrache et al., PLB 795, 241 (2019)

collective enhancement of the interband B(E2)

TABLE I. Experimental and theoretical mixing ratios δ as well as the transition probability ratios $B(M1)_{\rm out}/B(E2)_{\rm in}$ and $B(E2)_{\rm out}/B(E2)_{\rm in}$ for the transitions from band S1′ to band S1 of ^{130}Ba.

	δ		$\frac{B(M1)_{\rm out}}{B(E2)_{\rm in}}\left(\frac{\mu_N^2}{e^2b^2}\right)$		$\frac{B(E2)_{\rm out}}{B(E2)_{\rm in}}$	
I ($\hbar$)	Expt	PRM	Expt	PRM	Expt	PRM
13	-0.58^{+13}_{-13}	−0.67	0.36^{+19}_{-13}	1.11	0.32^{+18}_{-15}	0.51
15	-0.62^{+10}_{-10}	−0.68	0.38^{+61}_{-16}	0.90	0.36^{+70}_{-19}	0.42
17	-0.62^{+10}_{-10}	−0.68	0.23^{+22}_{-09}	0.76	0.22^{+27}_{-10}	0.35
19	−0.60	−0.66	0.25^{+23}_{-08}	0.67	0.22^{+21}_{-07}	0.29
21	−0.60	−0.63	0.43^{+35}_{-13}	0.63	0.41^{+34}_{-13}	0.25

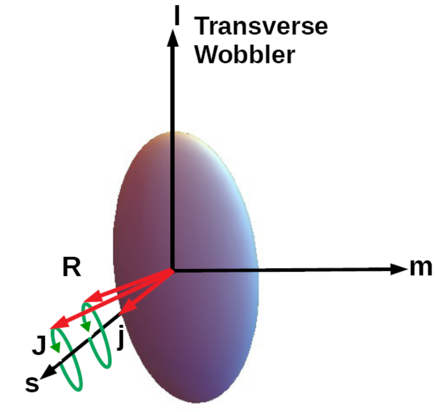
QB, Frauendorf, Petrache, PRC 100, 061301(R) (2019)

Elucidate the physics behind!
---> SCS/Azimuthal and SSS plots

Calculate SCS and SSS plots in particle-rotor model

- **Hamiltonian:**

$$\hat{H}_{\mathrm{PRM}} = \hat{H}_{\mathrm{coll}} + \hat{H}_{\mathrm{intr}}$$



$$\hat{H}_{\mathrm{coll}} = \sum_{i=1}^{3} \frac{\hat{R}_i^2}{2\mathcal{J}_i} = \sum_{i=1}^{3} \frac{(\hat{I}_i - \hat{j}_i)^2}{2\mathcal{J}_i}$$

$$\hat{H}_{\mathrm{intr}} = \sum_{i=1} \sum_{\nu} \varepsilon_{i,\nu} a_{i,\nu}^{\dagger} a_{i,\nu}$$

$$\boldsymbol{I} = \boldsymbol{R} + \boldsymbol{j}$$

d.o.f: *K (**I**) and k (**j**), bipartition*

- **Coupling basis**

$$|IMK\varphi\rangle = \frac{1}{\sqrt{2}}\Big[|IMK\rangle|\varphi\rangle + (-1)^{I-K}|IM-K\rangle|\bar{\varphi}\rangle\Big], \ |\varphi\rangle = |jk\rangle$$

$$|IM\rangle_{\nu} = \sum_{K\varphi} C_{IK\varphi}^{(\nu)} |IMK\varphi\rangle$$

- **Reduced density matrix for total or particle AM**

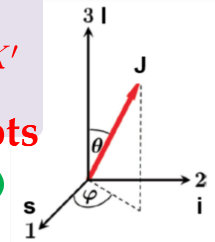
$$\boldsymbol{\rho}_{KK'}^{(\nu)} = \sum_{\varphi} C_{IK\varphi}^{(\nu)} C_{IK'\varphi}^{(\nu)*} \qquad \boldsymbol{\rho}_{\varphi\varphi'}^{(\nu)} = \sum_{K} C_{IK\varphi}^{(\nu)} C_{IK\varphi'}^{(\nu)*}$$

$$\mathcal{P}^{(\nu)}(\theta, \phi) = \sum_{KK'} D_{KI}^{I*}(\theta, \phi, 0) \boldsymbol{\rho}_{KK'}^{(\nu)} D_{K'I}^{I}(\theta, \phi, 0)$$

Azimuthal/Spin coherent state plots

$$\mathcal{P}^{(\nu)}(\phi) = \frac{1}{2\pi} \sum_{KK'} e^{-i(K-K')\phi} \boldsymbol{\rho}_{KK'}^{(\nu)}$$

Spin squeezed state plots



Chen, QB, Luo, Meng, Zhang, PRC 96, 051303(R) (2017); QB, Frauendorf, EPJA 58, 75 (2022)

QB, Frauendorf, PRC 109, 044304 (2024)

Semiclassical visualization and classification

Eur. Phys. J. A (2022) 58:75
https://doi.org/10.1140/epja/s10050-022-00727-5

THE EUROPEAN PHYSICAL JOURNAL A

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Regular Article - Theoretical Physics

SCS/Azimuthal plots

Study of wobbling modes by means of spin coherent state maps

Q. B. Chen[1,2], S. Frauendorf[3,a]

PHYSICAL REVIEW C **109**, 044304 (2024)

SSS plots

Spin squeezed states and wobbling motion in a collective Hamiltonian

Q. B. Chen[1,*] and S. Frauendorf[2,†]

More general classification of TW and LW were introduced:

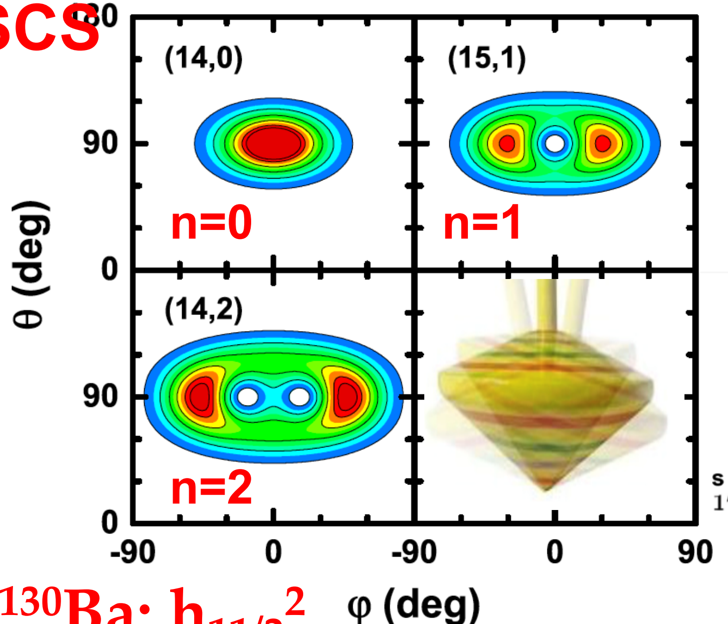
- LW as precession around the mediate axis (with the largest MoI);
- TW as precession about an axis transverse to the m-axis.

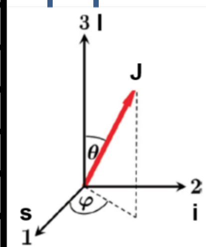
with the key signatures:

- collective enhancement of the interband B(E2);
- TW decreasing wobbling energy and LW increasing wobbling energy.

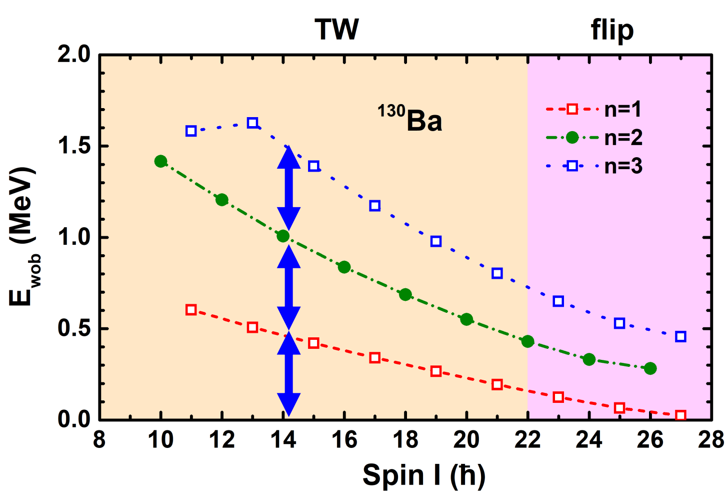
Visualization examples

SCS



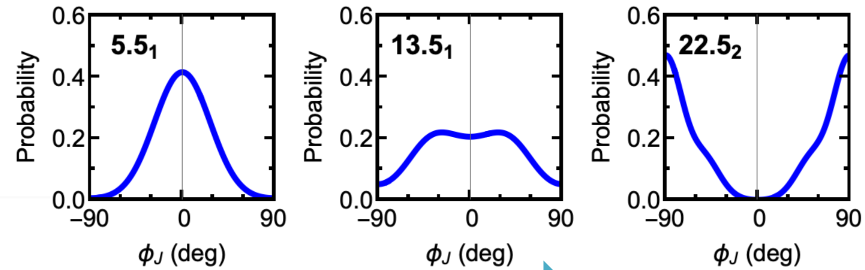


^{130}Ba: $h_{11/2}^2$

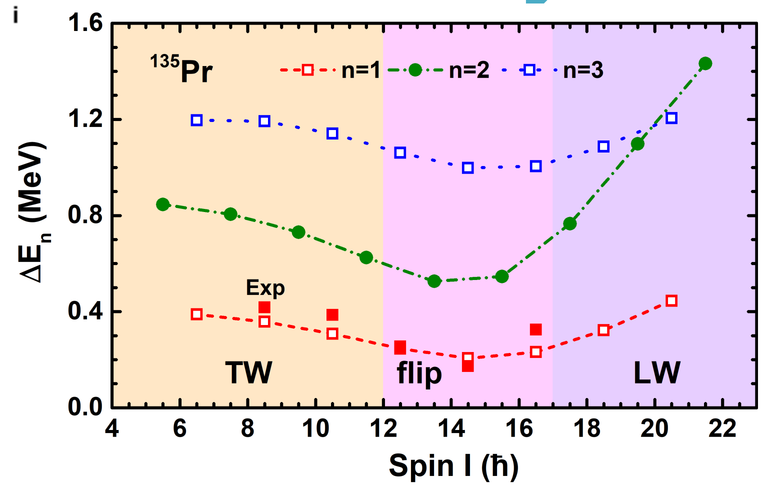


SSS

^{135}Pr: $h_{11/2}^1$



TW: s **flip** **LW: m**



Determined by the coupling/Coriolis interaction between total and particle angular momenta.

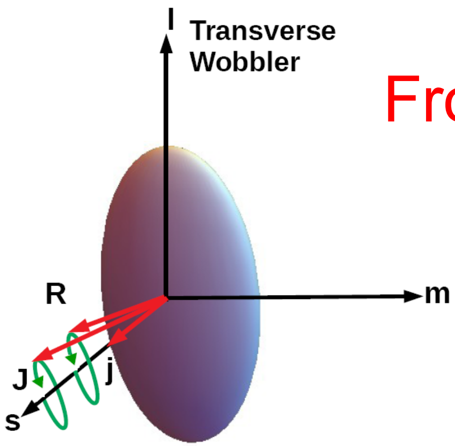
How do we describe this?

Another perspective – back to the origin

PHYSICAL REVIEW C **89**, 014322 (2014)

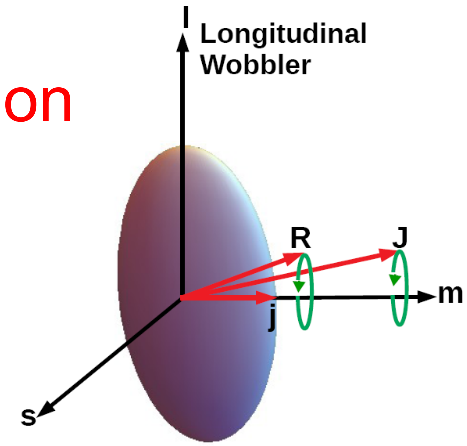
Transverse wobbling: A collective mode in odd-*A* triaxial nuclei

S. Frauendorf[1,*] and F. Dönau[2,†]



Frozen alignment approximation

$$\Psi(\boldsymbol{J},\boldsymbol{j}) = \psi(\boldsymbol{J})\varphi(\boldsymbol{j})$$



To what extend can one interpret the coupled system in terms of a particle and a rotor state?

Quantify their entanglement!

How to quantify the entanglement?

Go back to reduced density matrix and calculate its eigenvalues

$$\rho^{(\nu)}_{KK'} = \sum_{\varphi} C^{(\nu)}_{IK\varphi} C^{(\nu)*}_{IK'\varphi}$$

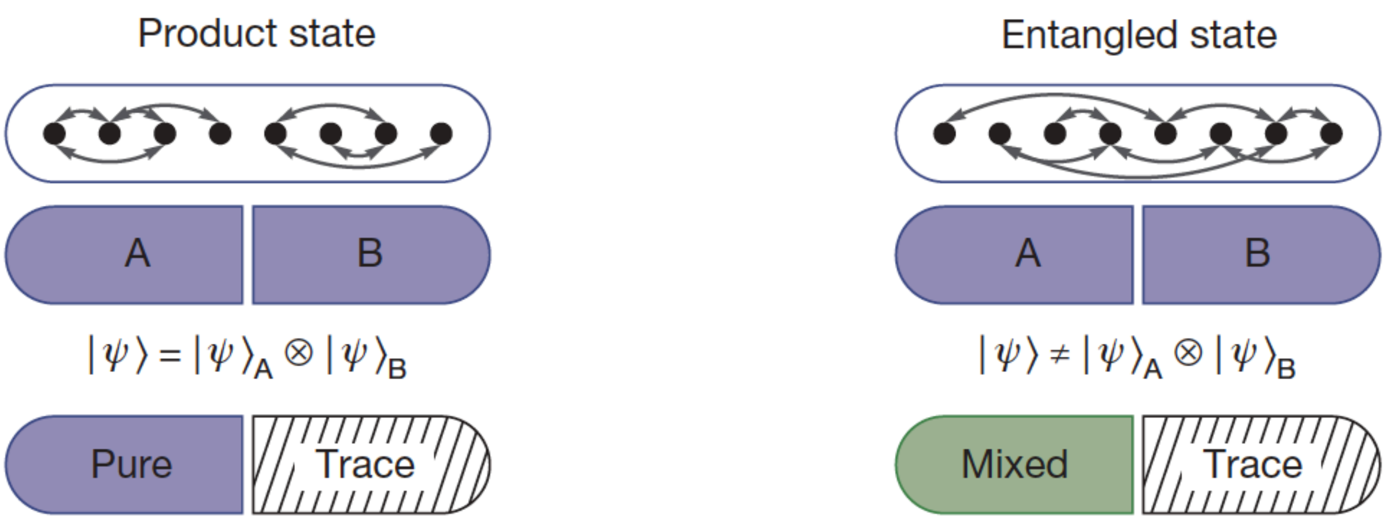
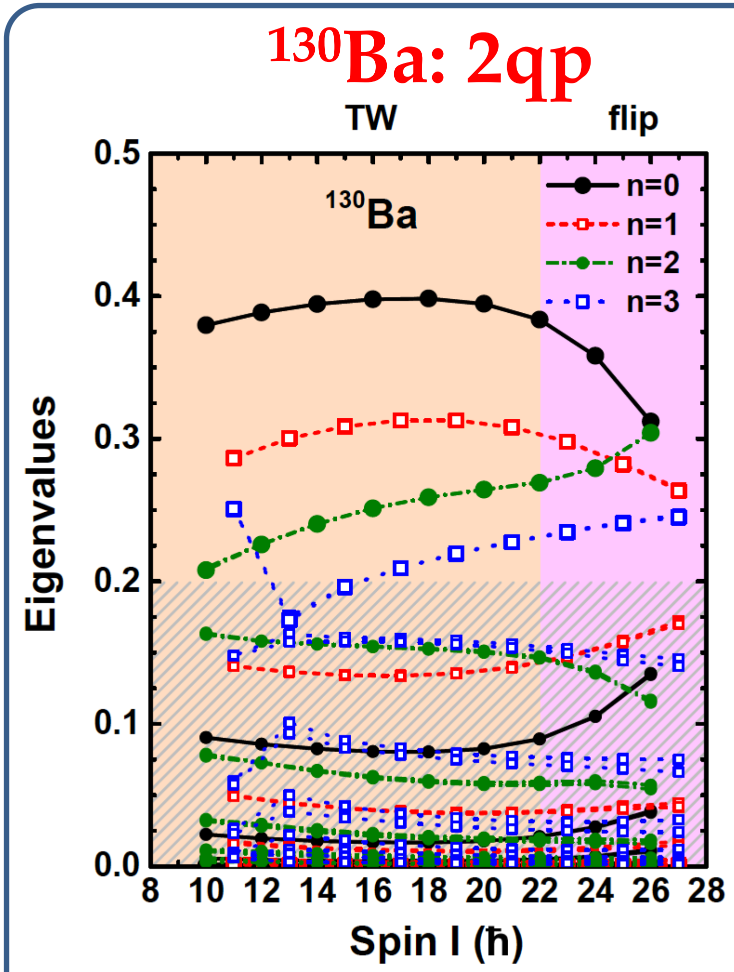


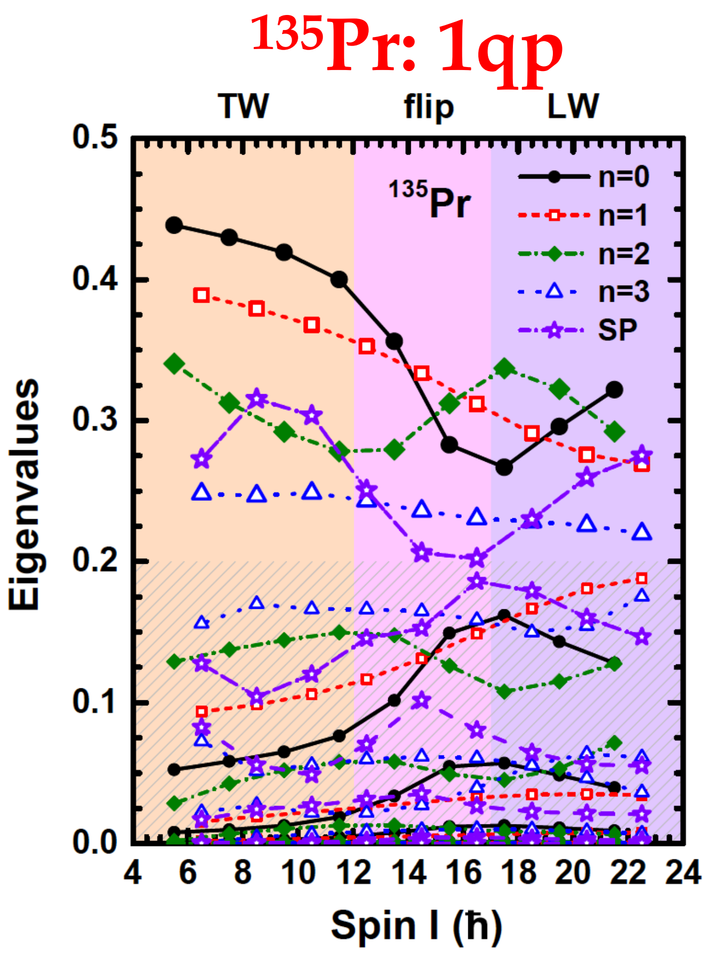
Figure from: Nature 528, 77 (2015)

$\rho^2=\rho$, 1 eigenvalue $e_v=1$ and the rest $e_v=0$: system is pure.

$\rho^2\neq\rho$, more non-zero e_v: two subsystems are entangled.

Eigenvalue spectra





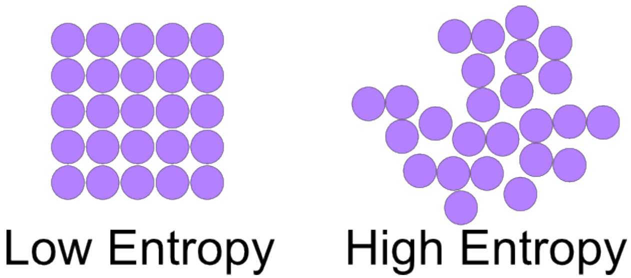
one main state with large eigenvalue

QB, Frauendorf, arXiv: 2410.08749

Particle and Rotor states are entangled! How to describe it quantitatively?

Using entropy and purity to describe entanglement

Entropy: quantitative measure of the degree of quantum entanglement (disorder) between two subsystems **basis independent**



- **Entropy:**

$$S_A = -\mathrm{Tr}\ \rho_A \ln \rho = -\sum_m p_m \ln p_m$$

p_m: eigenvalues

von Neumann entropy

S=0, pure state; S>0, entangled; S_{max}=ln d (d: dimension)

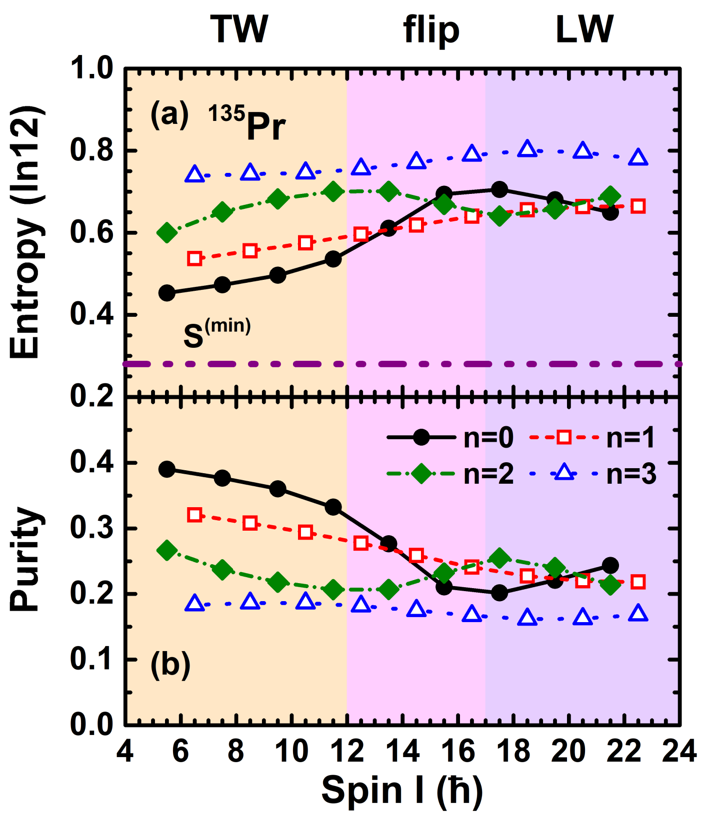
Purity: measure of the degree of mixing

- **Purity:**

$$P = \mathrm{Tr}\ \rho^2 = \sum_m p_m^2$$

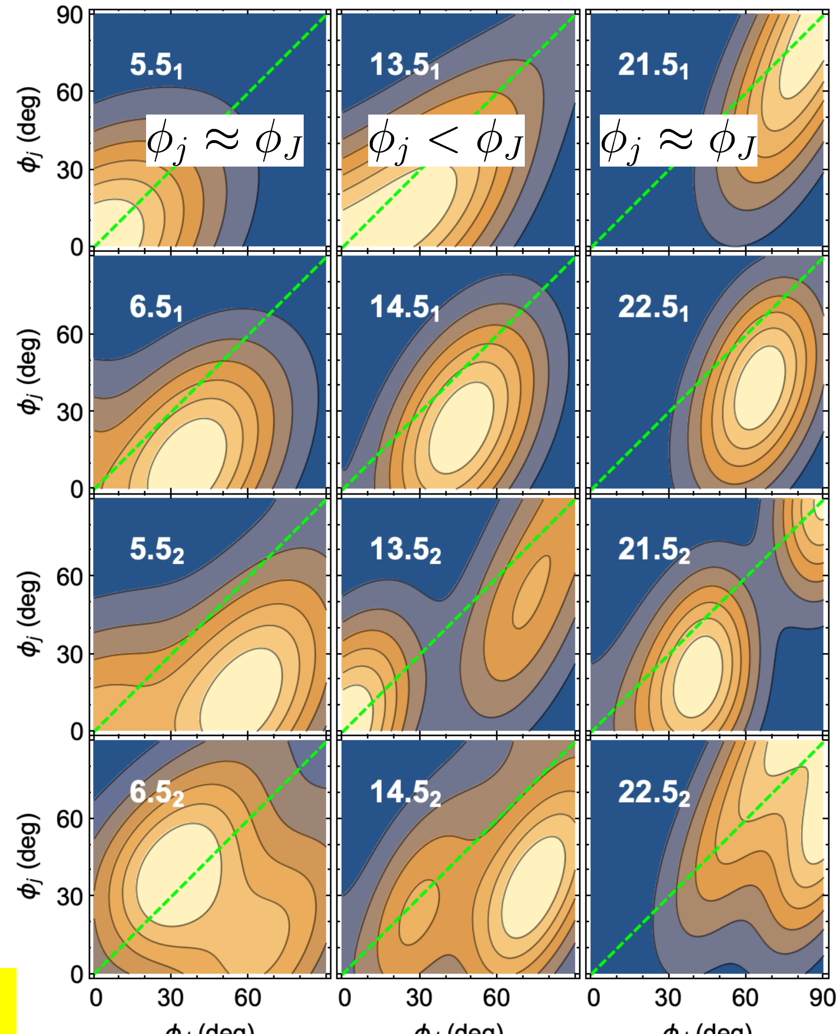
P=1, pure state; P<1, mixed state

Entropy and purity: ^{135}Pr



TW collapses: entropy increases

LW established: entropy decreases

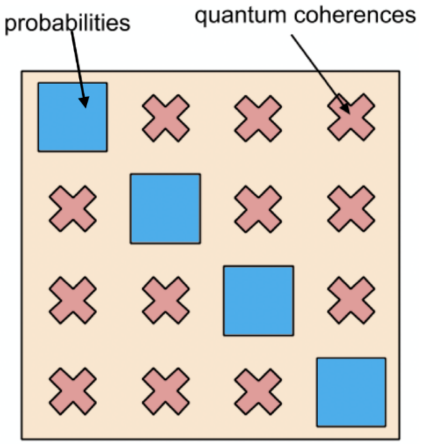


Orientation angle of J and j

The impurity washes out the interference pattern of pure wave function, i.e., causes decoherence.
How to measure the decoherence?

Coherence

Coherence: the correlation and interference between different quantum states in a system; represented by the non-zero off-diagonal elements of the density matrix



Density matrix

basis dependent

- **Intuitive l_1 norm:** $$C_{l_1}(\rho) = \sum_{i \neq j} |\rho_{i,j}|$$

PRL 113, 140401 (2014)

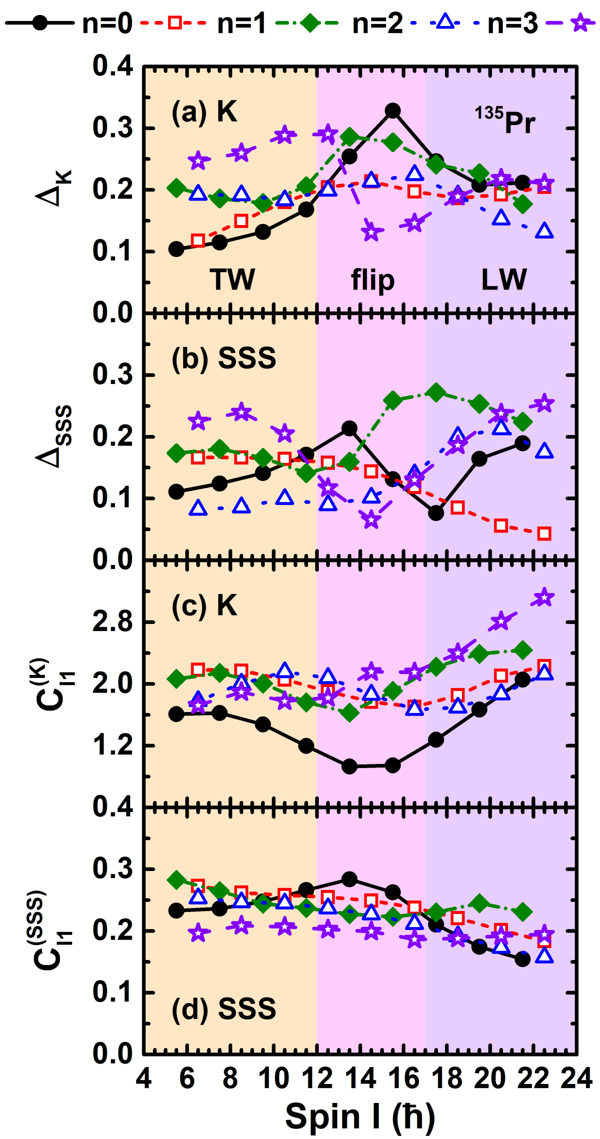
- **K/SSS-plots coherence:**

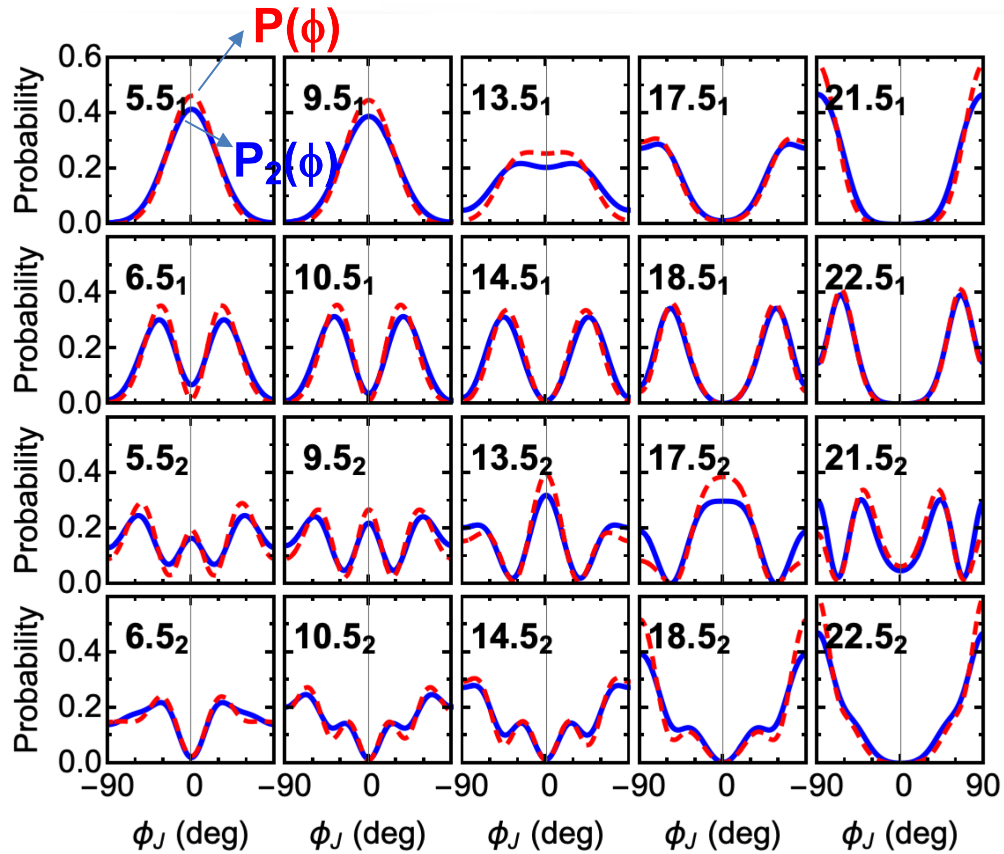
$$P(\phi) = \rho^{(I\nu)}_{\phi,\phi}, \quad P_2(\phi) = \left[\rho^{(I\nu)}\right]^2_{\phi,\phi} / \mathrm{Tr}\left\{\left[\rho^{(I\nu)}\right]^2\right\},$$

$$\Delta_{\mathrm{SSS}} = \int_{-\pi}^{\pi} d\phi \left| P_2(\phi) - P(\phi) \right|.$$

$\rho^2 = \rho$, complete coherence, $\Delta = 0$;

De-coherence in SSS plot





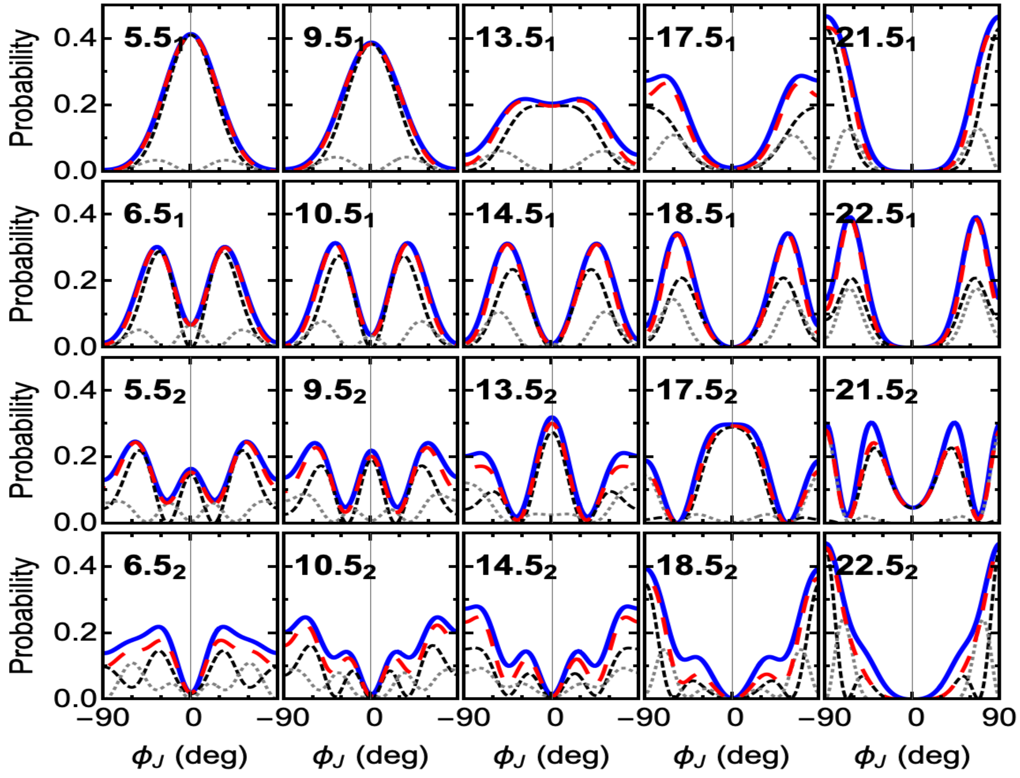
- **P (blue) and P_2 (red) deviations, missing coherence.**
- **Δ_{SSS} are small, scatter between 0.1 to 0.3.**

To understand the nature of de-coherence

- **Schmidt decomposition of reduced density matrix:**

$$\rho = \sum_m \rho^{(m)} \text{ (blue)}, \quad \rho^{(m)}_{KK'} = p_m C^{(m)}_{IK} C^{(m)}_{IK'},$$

$$\rho^{(12)} = \rho^{(1)} + \rho^{(2)} \text{ (black)}, \quad \rho^{(34)} = \rho^{(3)} + \rho^{(4)} \text{ (gray)}, \quad \rho^{(1234)} = \rho^{(12)} + \rho^{(34)} \text{ (red)}$$



- ➢ **The appearance of higher order terms causes the de-coherence.**
- ➢ **The higher order terms are small, so the Δ_{SSS} is small.**

Summary

Entropy and coherence are studied for the wobbling motion in ^{135}Pr and ^{130}Ba in the framework of particle rotor model.

When transverse wobbling collapses: entropy increases; when longitudinal wobbling is established, entropy decreases. Although the entropy changes with number of wobbling quanta n, de-coherence stays about the same.

The physics of entropy and coherence should be further explored. They will be extended to study the chiral modes.

Thank you!