

IJClab, Nov. 06, 2024



Effect of np pairing on proton and alpha decays in N~Z nuclei

Chong Qi Dept. Physics, KTH Royal Institute of Technology, **Stockholm**

In collaboration with Z.C. Gao, Z.J. Lian, Ramon Wyss, Roberto Liotta





Motivation microscopic large-scale shell-model description of intermediate-mass nuclei and np pairing correlation

- → Overwhelming evidence for likeparticle pairing. It manifests itself through:
- Strongly attractive J=0 TBMEs
- Dominance of J=0 pairs in the w.f. of semi-magic nuclei

→No consensus on the role of the np correlation. The interaction itself is also complex:

- Strong monople interaction->drift in single particle energies
- QQ correlation->deformation
- Non-collective residual interaction in odd-odd nuclei
- Highly-mixed wave functions in most cases.



Deformation minima in even-even nuclei by using the *deformed Woods-Saxon potential (not LSSM)*. Z.X. Xu and C. Qi, Phys. Lett. B 724, 247 (2013). Z. Wu et al., Phys. Rev. C 92, 024306 (2015).



- There have been tremendous studies on neutron-proton pairing correlations with various approaches (MF+BCS, SM, Quartet etc) on observables like binding energy, spectrum, np transfer, beta decay and alpha decay
- Like-pairing pairing only spans the seniority-zero states, which we can • comfortably handle with neural guantum states. I. Bonnier, Master thesis, KTH, 2024
- The inclusion of T=0 and 1 np pairing spans however the full SM space but could be essential for N~Z nuclei

Two-body matrix elements in the effective interaction



FIG. 3. Comparison of data from various multiplets with j_1 $=j_2$ and T=1. The values of the matrix elements are divided by $\overline{E} = \sum_{J} [J] E_{J} / \sum_{J} [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

90° θ_{12} FIG. 2. Comparison of data from various multiplets with $j_1 = j_2$ and T=0. The values of the matrix elements are divided by $\overline{E} = \sum_{J} [J] E_{J} / \sum_{J} [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets. $\cos q_{12} = \frac{J(J+1)}{2j(j+1)}$

∆ |p_{3/2}

□ |d_{3/2} 0 ld 5/2

+ | f_{7/2}

× |g_{9/2}

J.P. Schiffer and W.W. True, Rev.Mod.Phys. 48,191 (1976)

T=0

Seniority coupling as a result of strong J=0 pairing

$$|g.s.\rangle = |\nu = 0; J = 0\rangle = (P_j^+)^{n/2} |\Phi_0\rangle |\nu = 2; JM\rangle = (P_j^+)^{(n-2)/2} A^+ (j^2 JM) |\Phi_0\rangle$$



10² in seniority space Easier to include many shells

One can readily solve a half-filled system with upto 36-38 doubly- degenerate orbitals and 18-19 pairs (Dim: 9*109-3.5*10¹⁰, shell-model dimension: 4*10²⁰-7*10²¹).



Computer Physics Communications Volume 259, February 2021, 107349



PairDiag: An exact diagonalization program for solving general pairing Hamiltonians \ddagger ,

☆☆

Xiao-Yu Liu ^{a b c}, Chong Qi ^a 🙎 🖂



- There have been tremendous studies on neutron-proton pairing correlations with various approaches (MF+BCS, SM, Quartet etc) on observables like binding energy, spectrum, np transfer, beta decay and alpha decay
- Like-pairing pairing only spans the seniority-zero states, which we can comfortably handle with neural quantum states. I. Bonnier, Master thesis, KTH, 2024
- The inclusion of T=0 and 1 np pairing spans however the full SM space but could be essential for N~Z nuclei

These nuclides represent the first opportunity to study alpha decay from nuclei where the "valence" neutrons and protons are in the same single-particle level, in this case, the $1g_{7/2}$ level. This may give rise to a kind of "superallowed" alpha decay resulting in large reduced alpha widths. At present, we cannot give any estimates of the alpha reduced widths for Te¹⁰⁷ and Te¹⁰⁸ because the alpha branch-



R. D. Macfarlane and A. Siivola, Phys. Rev. Lett. 14, 114 (1965).

Superallowed α Decay to Doubly Magic ¹⁰⁰Sn

K. Auranen,^{1,*} D. Seweryniak,¹ M. Albers,¹ A. D. Ayangeakaa,^{1,†} S. Bottoni,^{1,±} M. P. Carpenter,¹ C. J. Chiara,^{1,2,§} P. Copp,^{1,3}
 H. M. David,^{1,1} D. T. Doherty,^{4,§} J. Harker,^{1,2} C. R. Hoffman,¹ R. V. F. Janssens,^{5,6} T. L. Khoo,¹ S. A. Kuvin,^{1,7} T. Lauritsen,¹ G. Lotay,[§] A. M. Rogers,^{1,*} J. Sethi,^{1,2} C. Scholey,⁹ R. Talwar,¹ W. B. Walters,² P. J. Woods,⁴ and S. Zhu¹
 ¹ Physics Division, Argonne National Laboratory, 9700 South Cass Avenue, Lemont, Illinois 60439, USA
 ² Department of Chemistry and Biochemistry, University of Maryland, College Park, Maryland 20742, USA
 ³ Department of Physics and Applied Physics, University of Maryland, College Park, Maryland 20742, USA
 ⁴ University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
 ⁵ Department of Physics, Nuiversity of North Carolina at Chapel Hill, North Carolina 27599, USA
 ⁶ Triangle Universities Nuclear Laboratory, Duke University, Durham, North Carolina 27708, USA
 ⁷ Department of Physics, University of Surrey, Guildford GUZ 27KH, United Kingdom
 ⁹ Department of Physics, University of Jyvaskyla, P.O. Box 35, FI-40014 University of Jyvaskyla, Finland

(Received 31 July 2018; revised manuscript received 7 September 2018; published 30 October 2018)

We report the first observation of the 108 Xe \rightarrow 104 Te \rightarrow 100 Sn α -decay chain. The α emitters, 108 Xe $[E_{\alpha} = 4.4(2) \text{ MeV}, T_{1/2} = 58^{+106}_{-216} \mu_8]$ and 109 Te $[E_{\alpha} = 4.9(2) \text{ MeV}, T_{1/2} < 18 \text{ ns}]$ decaying into doubly magic 100 Sn were produced using a fusion-evaporation reaction 54 Fe(58 Ni, 4n) 108 Xe, and identified with a recoil mass separator and an implantation-decay correlation technique. This is the first time α radioactivity has been observed to a heavy self-conjugate nucleus. A previous benchmark for study of this fundamental decay mode has been the decay of 212 Po into doubly magic 208 Pb. Enhanced proton-neutron interactions in the N = Z parent nuclei may result in superallowed α decays with reduced α -decay widths significantly greater than that for 212 Po. From the decay chain, we deduce that the α -reduced width for 108 Xe or 104 Te is more than a factor of 5 larger than that for 212 Po.



Common picture of the alpha decay as a tunneling process

 Alpha decay provides direct evidence for the existence of alpha clustering in g.s. of heavy nuclei- (~ 2500 data in total in Nudat3 database, ~400 gs to gs decays, one expect "indirect" data from (p,pα) reaction)



 $\lambda = \ln 2/T = \nu SP_s$

The spectroscopic factor for alpha particle is not an observable.

The alpha formation amplitude

$$m \rightarrow d + \alpha$$
$$\mathcal{F}_{l}(R) = \int d\mathbf{R} d\xi_{d} d\xi_{\alpha} [\Psi(\xi_{d})\phi(\xi_{\alpha})Y_{l}(\mathbf{R})]^{*}_{J_{m}M_{m}} \Psi_{m}(\xi_{d},\xi_{\alpha},\mathbf{R}),$$

Nuclear structure indicator for particle decay

R is the distance between the center of mass of the cluster and daughter nucleus which divides the decay process into an internal region and complementary external region.

F the formation amplitude;

H the Coulomb Hankel function (for the tail of the alpha wave function)

C. Qi, R. Liotta and R. Wyss / Progress in Particle and Nuclear Physics 105 (2019) 214–251



Simple alpha-emitter examples: ²¹²Po vs ²¹⁰Po

$$|^{212} \operatorname{Po}(\alpha_4)\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2; \alpha_4) |^{210} \operatorname{Pb}(\alpha_2) \otimes^{210} \operatorname{Po}(\beta_2)\rangle$$

If we neglect the proton-neutron interaction between the four valence nucleons (Or pn interaction only being considered at the mean field level)

$$|^{212}$$
Po $(\alpha, g.s.)\rangle = |^{210}$ Pb $(2\nu, g.s.) \otimes |^{210}$ Po $(2\pi, g.s.)\rangle,$

 $|^{210}$ Po $(\alpha, \text{g.s.})\rangle = |^{206}$ Pb $(2\nu^{-1}, \text{g.s.}) \otimes |^{210}$ Po $(2\pi, \text{g.s.})\rangle.$

$$\mathscr{F}_{\alpha}(R;^{212}\operatorname{Po}(\mathrm{gs})) = \int d\mathbf{R}d\xi_{\alpha}\phi_{\alpha}(\xi_{\alpha}) \Psi(\mathbf{r_{1}r_{2}};^{210}\operatorname{Pb}(\mathrm{gs}))\Psi(\mathbf{r_{3}r_{4}};^{210}\operatorname{Po}(\mathrm{gs})), \quad \text{Two-body}$$

$$\mathscr{F}_{\alpha}(R;^{210}\operatorname{Po}(\mathrm{gs})) = \int d\mathbf{R}d\xi_{\alpha}\phi_{\alpha}(\xi_{\alpha})\Psi(\mathbf{r_{1}r_{2}};^{206}\operatorname{Pb}(\mathrm{gs}))\Psi(\mathbf{r_{3}r_{4}};^{210}\operatorname{Po}(\mathrm{gs})). \quad \text{clustering}$$

$$F_{\alpha}(R) = \sqrt{\frac{1}{4\pi}} \int dr_{\alpha} \phi_{\alpha}(r_{\alpha}) \Psi_{2\pi}(\mathbf{r_1}, \mathbf{r_2}) \Psi_{2\nu}(\mathbf{r_3}, \mathbf{r_4}),$$

Alpha particle is formed on the nuclear surface;
Clustering inside the nucleus is suppressed due to Pauli blocking.





- Pair correlations result in a constructive interference of formation amplitudes
- That configuration mixing from higher lying orbits is important for clustering at the surface

$$\Psi_2(\mathbf{r_1}, \mathbf{r_2}) = (\chi_1 \chi_2)_0 \Phi_2(r_1, r_2, \theta_{12}) = (\chi_1 \chi_2)_0 \frac{1}{4\pi} \sum_p \sqrt{\frac{2j_p + 1}{2}} X_p \phi_p(r_1) \phi_p(r_2) P_{l_p}(\cos \theta_{12}),$$

$$G\sum_{i}\frac{2j_i+1}{2\varepsilon_i-E_2}=2.$$

The corresponding wave function amplitudes are





r₁=9fm

A.N. Andreyev et al., PhysRevLett.110.242502 (2013).



Single-particle estimation



Or how do we quantify the formation amplitude in an understandable way

$$\mathcal{F}_c(R) = \int d\hat{R} d\xi_d d\xi_\alpha [\Psi_d(\xi_d)\phi(\xi_\alpha)Y_l(\hat{R})]^*_{J_mM_m}\Psi_m(\xi_d,\xi_\alpha,\hat{R}),$$

The intrinsic α -particle wave function has the form of a n = l = 0 (0s)⁴ harmonic oscillator eigenstate in the neutron-neutron relative distances

$$\phi(\xi_{\alpha}) = \sqrt{\frac{1}{8}} (\frac{\nu_{\alpha}}{\pi})^{9/4} exp[-\nu_{\alpha}(r_{nn}^2 + r_{pp}^2 + 2r_{pn}^2)/4]\xi_{\alpha}$$

The alpha decay of four uncorrelated/independent particles means that the mother nucleus consists of the daughter nucleus times a pure configuration of a neutron pair coupled to zero angular momentum times a similar proton pair

$$\Psi_m(\xi_d, \xi_\alpha, \hat{R}) = (\varphi_\nu(\mathbf{r_1})\varphi_\nu(\mathbf{r_2}))_{00}(\varphi_\pi(\mathbf{r_3})\varphi_\pi(\mathbf{r_4}))_{00}\Psi_d(\xi_d)$$



Single-particle decay unit

We assume that the radial single-particle wave function u(r) is constant inside the mother nucleus, with radius R, which leads to

$$\int_{0}^{R} (u(r)/r)^{2} r^{2} dr = RC^{2} = 1$$

The corresponding formation amplitude acquires the form,

$$F_{\alpha}(R) = \int d\hat{R} \int r_{nn}^{2} dr_{nn} r_{pp}^{2} dr_{pp} r_{pn}^{2} dr_{pn} \sqrt{\frac{1}{8}} (\frac{\nu_{\alpha}}{\pi})^{9/4}$$
$$\times e^{-\nu_{\alpha} (r_{nn}^{2} + r_{pp}^{2} + 2r_{pn}^{2})/4} \frac{1}{\sqrt{4\pi}} \frac{C^{4}}{r_{nn} r_{pp} r_{pn} R}$$
$$= \int r_{nn} dr_{nn} r_{pp} dr_{pp} r_{pn} dr_{pn} \sqrt{\frac{1}{8}} (\frac{\nu_{\alpha}}{\pi})^{9/4}$$
$$\times e^{-\nu_{\alpha} (r_{nn}^{2} + r_{pp}^{2} + 2r_{pn}^{2})/4} \frac{\sqrt{4\pi}}{R^{3}}$$

After the integration, one can get

$$F_{\alpha;\text{pdu}}(R) = \sqrt{\frac{1}{8}} (\frac{\nu_{\alpha}}{\pi})^{9/4} \sqrt{4\pi} \frac{C^4}{R} \frac{4}{\nu_{\alpha}^3} = \frac{\sqrt{8}\nu_{\alpha}^{-3/4} \pi^{-7/4}}{R^3}$$





Fig. 1. α -particle formation probabilities in p.d.u. for the decays of the even-even isotopes as a function of the neutron numbers *N* of the mother nuclei.

That defines the possibility for four "independent" particles to overlap and form an alpha particle at the surface



Proton decay from heavy nuclei

- Nearly 50 events observed for decays from g.s. and isomeric states of odd-Z nuclei with Z>50.
- Sensitive to nuclear deformation and pairing
- The decay formation amplitude has the form:



 $F_{p;\text{pdu}}(R) = \frac{1}{R^{3/2}}.$

np pair transfer reaction cross section



Fig. 2. Left axis: Unit cross-sections for the 0^+ (red) and 1^+ (black) states and Right axis: *np* Weisskopf Units (blue) as a function of the target mass. The results from the approximation of Eq. (8) is shown in a dashed (blue) line. The corresponding orbits being filled are indicated at the top. Experimental data points from Refs. [16–18] are also included.

J.A. Lay, Y. Ayyad and A.O. Macchiavelli Physics Letters B 824 (2022) 136789

KTH The neutron-proton pairing effect?

- If we don't consider np pairing, the alpha clustering is reflected as coupling of 2n and 2p clusters as in pdu
- We have managed to calculate ¹⁰⁴Te in six major shells and all Te isotopes in one major shell w. and w/o the np interaction.
- Naively one can expect an enhancement upto a factor of around 3 for alpha decays when approaching N=Z in our calculations including np interaction
- But one has to be careful which parts of the np interaction is contributing and how they evolve as more n/p added.











data from Nudat3/Nubase2020 and K. Auranen, et al., Phys. Rev. Lett. 121 (2018) 182501.

R. M. Clark et al, Phys. Rev. C 101, 034313 (2020)

M. Patial, R. J. Liotta, and R. Wyss Phys. Rev. C 93, 054326 (2016)



The average proton-neutron interaction

$$V_{pn}(Z, N) = \frac{1}{4} \Big[B(Z, N) + B(Z - 2, N - 2) \\ - B(Z - 2, N) - B(Z, N - 2) \Big]$$

J.-Y. Zhang, R.F. Casten, D.S. Brenner, Phys. Lett. B 227 (1989) 1.

Pairing rotation

$$E_{\rm rot}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{nn}} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{np}} + \frac{(\Delta Z)^2}{2\mathcal{J}_{pp}},$$
$$\mathcal{J}_{np}^{-1}(N,Z) = \frac{1}{4} \left[S_{2n}(N+2,Z) - S_{2n}(N+2,Z+2) \right].$$

N. Hinohara and W. Nazarewicz Phys. Rev. Lett. 116, 152502 (2016)







Z.Y. Zhang et al., Phys. Rev. Lett. 126, 152502 (2021)

KTH Alpha correlation from the binding energy

The pairing correlation is well reflected in the odd-even staggering of bind energy and can be extracted with simple mass filters; Here we extract the alpha correlation/separation energy (the energy one gains by adding one alpha particle or α -like quartet), similar to what was done in G. Dussel, E. Caurier, A.P. Zuker, Atomic Data and Nuclear Data Tables 39, 205 (1988).

which is just opposite to the Q value in $d Q_{\alpha}(d N \in Z) = B(N,Z) - B(N-2,Z-2) - B_{^4He}$

70

60

80

90

110

120



C. Qi, R. Wyss, to be submitted



S.A. Changizi, C. Qi, R. Wyss, Nucl. Phys. A (2015)

N. Hinohara and W. Nazarewicz Phys. Rev. Lett. 116, 152502 (2016) T. Papenbrock, Phys. Rev. C **105**, 044322 (2022)



Quartet-like coupling

Table I. Configurations with the largest probabilities for the state ${}^{92}Pd(0_1^+)$ corresponding to the tensorial products of different two-particle states (upper) and four-particle states (lower).

The wave function of a system with 4 np pairs (4 alphas/quartets) can be nearly identical to the simple coupling of two



C.Qi and R.Wyss, Physica Scripta, 91, 013009 (2016). C. Qi, overview to be published in Int. J. Mod. Phys. E More in N. Sandulescu's talk

KTH VETENSKAP VCH KONST

Proton decay formation amplitude

Does the quantity make sense since the emitting proton already "exists"?

Proton decay F(R) can be expressed as the overlap between the mother, emitting proton and daughter wave functions; It reflects the probability to find the proton in the particular emiting state at R.

$$\mathcal{F}_p(R) = \int d\mathbf{R} d\xi_d [\Psi_d(\xi_d) \chi(\xi_p) Y_l(\mathbf{R})]^*_{J_m M_m} \Psi_m(\xi_d, \xi_p, \mathbf{R})$$

- It is equivalent to the spectroscopic factor if one assume that the mother, daughter and the emitted particle share the same single-particle wave function $S = \frac{1}{2J_i + 1} \sum_{M_i, M_f, m} \left| < \Psi_f^{A-1} J_f M_f \right| \tilde{a}_{k,m} \left| \Psi_i^A J_i M_i > \right|^2$ $= \frac{\left| < \Psi_f^{A-1} J_f \left\| \tilde{a}_k \right\| \left\| \Psi_i^A J_i > \right|^2}{(2J_i + 1)}$ or neutron). $S = u^2 \text{ for the decay of a quasiproton, the formation amplitude is } u \varphi(r)$
- The two quantities are different if the single-particle wave functions are different, say with different deformations for initial and final states.



A shell model description of the proton emission

- Many emitters are deformed but the symmetry needs to be restored. Both SM and MF+projection could be good tools.
- ➤ The largest systems we can handle in gdsh model space are ¹¹²Ba and ¹¹⁸Te (2*10¹⁰).
- Recently we have been studying the proton emitters within the truncated SM (monopole + seniority truncations) and the variation after projection approach (Z.C. Gao, Phys. Lett. B 824 (2022) 136795).



➢ We have managed to go upto emitters ^{112,113}Cs so far.

$$S = \frac{1}{2J_i + 1} \sum_{M_i, M_f, m} \left| < \Psi_f^{A-1} J_f M_f \right| \tilde{a}_{k, m} \left| \Psi_i^A J_i M_i > \right|^2$$
$$= \frac{\left| < \Psi_f^{A-1} J_f \left\| \tilde{a}_k \right\| \Psi_i^A J_i > \right|^2}{(2J_i + 1)}.$$

C. Qi, Z.C. Gao, to be submitted.



1270 nodes with two AMD EPYC[™] Zen2 2.25 GHz 64-core processors 56 GPU nodes with four AMD Instinct[™] MI250X GPUs





Enhanced proton emission from odd-odd nuclei

There is no difference between the spectroscopic factor for emission of a single proton outside the core with and without the presence of an extra neutron. $(j_p j_n)^p vs j_p$



Decay	Spec. Factor
$ \begin{array}{c} (j_p j_n)^J \rightarrow (j_n) \\ (j_p j_n^3)^{J=0} \rightarrow (j_n^3)^{J=7/2} \end{array} $	1 1
$(j_p^3)^{J=7/2} \to (j_p^2)^{J=0}$	0.75
$ \begin{array}{c} (j_p^3 j_n)^{J=0} \to (j_p^2 j_n)^{J=7/2} \\ (j_p^3 j_n)^{J=7} \to (j_p^2 j_n)_i^{J=7/2} \end{array} $	$0 \\ 0.3731, 0.3196$
$(j_p^3 j_n^2)^{J=7/2} \to (j_p^2 j_n^2)^{J=0}$	0.7381
$\begin{array}{c} (j_p^3 j_n^3)^{J=0} \to (j_p^2 j_n^3)^{J=7/2} \\ (j_p^3 j_n^3)^{J=7} \to (j_p^2 j_n^3)^{J=7/2} \end{array}$	$1.6499 \\ 0.7544$



Fig. 3 Proton formation probabilities. These formation probabilities, $|RF_I(R)|^2$, were deduced for the ground-state proton decays in the odd-*Z* elements between Z = 53 and 83 as a function of the proton number *Z*.

W. Zhang et al., Nature Communications Physics 5, 285 (2022).

KTH Particle decay and np pairing correlation

- A simple particle decay unit (pdu) can be introduced to quantify alpha clustering in heavy nuclei
- Observed alpha decays show substantial enhancement in comparison to pdu, especially at midshell, which is also reflected in systematics of like-particle pairing correlation and *alpha correlation*
- The inclusion of np pairing can substantially increase the alpha formation amplitude in theoretical calcualtions but the effect may be hard to separate.
- The np effect can be much more clear in the simplier proton emission.

