From T_d to C_{2v}: Water Molecule Symmetry Identified in an Actinide Nucleus – ^{236}U

Irene Dedes

The Henryk Niewodniczański
Institute of Nuclear Physics
Polish Academy of Sciences

Shape and Symmetries in Nuclei: from Experiment to Theory
4 – 8 November 2024
IJCLab, Orsay, France

In Collaboration with:

Jerzy Dudek

IPHC and University of Strasbourg, France

Andrzej Baran and Andrzej Góźdź

UMCS, Lublin, Poland

Dominique Curien and David Rouvel

IPHC and University of Strasbourg, France

Abdelghafar Gaamouci
IFJ Polish Academy of Sciences, Kraków, Poland

Aleksandra Pędrak

National Centre for Nuclear Research, Warsaw, Poland

Jie Yang

Liaoning Normal University, Dalian, China

Introductory Remarks and Employed Terminology

Our Definition of the Term: Exotic *(Molecular)* Nuclear Symmetries

• Symmetries which do **not** correspond to prolate, oblate or triaxial quadrupole shapes, neither pear-shape octupole deformations

Why Are We Interested in Molecular Symmetries in Subatomic Physics ?

• *Similarities* in the observed spectra in totally different objects: ***Molecules*** composed of relatively distant point particles (atoms) and ***Nuclei*** composed of the tightly packed nucleons interacting with the forces among most complex in the universe

• Exotic symmetries generate unprecedented *degeneracies* in both *individual-nucleonic* and *collective-rotation excitations*, new forms of behaviour and unprecedented hindrance factors

Further Consequences for Future Research in This Domain

• New highway towards exotic nuclei: **Nuclear Isomers** living longer than the ground-states
• Possible exploration directions in astrophysics: **New magic numbers** for nucleosynthesis

Introductory Remarks and Employed Terminology

Our Definition of the Term: Exotic *(Molecular)* Nuclear Symmetries

• Symmetries which do **not** correspond to prolate, oblate or triaxial quadrupole shapes, neither pear-shape octupole deformations

Why Are We Interested in Molecular Symmetries in Subatomic Physics ?

• *Similarities* in the observed spectra in totally different objects: ***Molecules*** composed of relatively distant point particles (atoms) and ***Nuclei*** composed of the tightly packed nucleons interacting with the forces among most complex in the universe

• Exotic symmetries generate unprecedented *degeneracies* in both *individual-nucleonic* and *collective-rotation excitations*, new forms of behaviour and unprecedented hindrance factors

Further Consequences for Future Research in This Domain

• New highway towards exotic nuclei: **Nuclear Isomers** living longer than the ground-states
• Possible exploration directions in astrophysics: **New magic numbers** for nucleosynthesis

Introductory Remarks and Employed Terminology

Our Definition of the Term: Exotic *(Molecular)* Nuclear Symmetries

• Symmetries which do **not** correspond to prolate, oblate or triaxial quadrupole shapes, neither pear-shape octupole deformations

Why Are We Interested in Molecular Symmetries in Subatomic Physics ?

• *Similarities* in the observed spectra in totally different objects: ***Molecules*** composed of relatively distant point particles (atoms) and ***Nuclei*** composed of the tightly packed nucleons interacting with the forces among most complex in the universe

• Exotic symmetries generate unprecedented *degeneracies* in both *individual-nucleonic* and *collective-rotation excitations*, new forms of behaviour and unprecedented hindrance factors

Further Consequences for Future Research in This Domain

• New highway towards exotic nuclei: **Nuclear Isomers** living longer than the ground-states
• Possible exploration directions in astrophysics: **New magic numbers** for nucleosynthesis

Introductory Remarks and Employed Terminology

Our Definition of the Term: Exotic *(Molecular)* Nuclear Symmetries

• Symmetries which do **not** correspond to prolate, oblate or triaxial quadrupole shapes, neither pear-shape octupole deformations

Why Are We Interested in Molecular Symmetries in Subatomic Physics ?

• *Similarities* in the observed spectra in totally different objects: ***Molecules*** composed of relatively distant point particles (atoms) and ***Nuclei*** composed of the tightly packed nucleons interacting with the forces among most complex in the universe

• Exotic symmetries generate unprecedented *degeneracies* in both *individual-nucleonic* and *collective-rotation excitations*, new forms of behaviour and unprecedented hindrance factors

Further Consequences for Future Research in This Domain

• New highway towards exotic nuclei: **Nuclear Isomers** living longer than the ground-states
• Possible exploration directions in astrophysics: **New magic numbers** for nucleosynthesis

Introductory Remarks and Employed Terminology

Our Definition of the Term: Exotic *(Molecular)* Nuclear Symmetries

- Symmetries which do **not** correspond to prolate, oblate or triaxial quadrupole shapes, neither pear-shape octupole deformations

Why Are We Interested in Molecular Symmetries in Subatomic Physics ?

- *Similarities* in the observed spectra in totally different objects: ***Molecules*** composed of relatively distant point particles (atoms) and ***Nuclei*** composed of the tightly packed nucleons interacting with the forces among most complex in the universe

- Exotic symmetries generate unprecedented *degeneracies* in both *individual-nucleonic* and *collective-rotation excitations*, new forms of behaviour and unprecedented hindrance factors

Further Consequences for Future Research in This Domain

- New highway towards exotic nuclei: **Nuclear Isomers** living longer than the ground-states
- Possible exploration directions in astrophysics: **New magic numbers** for nucleosynthesis

Introductory Remarks and Employed Terminology

Our Definition of the Term: Exotic *(Molecular)* Nuclear Symmetries

• Symmetries which do **not** correspond to prolate, oblate or triaxial quadrupole shapes, neither pear-shape octupole deformations

Why Are We Interested in Molecular Symmetries in Subatomic Physics ?

• *Similarities* in the observed spectra in totally different objects: ***Molecules*** composed of relatively distant point particles (atoms) and ***Nuclei*** composed of the tightly packed nucleons interacting with the forces among most complex in the universe

• Exotic symmetries generate unprecedented *degeneracies* in both *individual-nucleonic* and *collective-rotation excitations*, new forms of behaviour and unprecedented hindrance factors

Further Consequences for Future Research in This Domain

• New highway towards exotic nuclei: **Nuclear Isomers** living longer than the ground-states
• Possible exploration directions in astrophysics: **New magic numbers** for nucleosynthesis

Special Case: High-Rank Symmetries – T_d and O_h

- **High-rank Symmetry Groups** (Tetrahedral-T_d and Octahedral-O_h) have 4D irreducible representations, meaning that some levels are 4-fold degenerate, in contrast to the usual 2-fold (Kramers) degeneracy
- It follows: Existence of exotic (16-fold) degeneracies of 2p-2h excitations built out of 4-fold degenerate levels – similarly 32-fold degeneracies in the more complex 4p-4h excitations
- For instance: Exotic degeneracies in rotational bands with positive and negative parities
- For instance: unprecedented forms of the nuclear rotational behaviour - rotational bands without ‘rotational E2-transitions’
- Theory predicts numerous competing configurations in many regions of the Periodic Table manifesting exotic symmetries
- These symmetries may lead to well pronounced potential energy minima generating unprecedented, new nuclear quantum mechanisms in terms of spectroscopy and isomerism
- ... and many more

Special Case: High-Rank Symmetries – T_d and O_h

- **High-rank Symmetry Groups** (Tetrahedral-T_d and Octahedral-O_h) have 4D irreducible representations, meaning that some levels are 4-fold degenerate, in contrast to the usual 2-fold (Kramers) degeneracy

- It follows: Existence of exotic (16-fold) degeneracies of 2p-2h excitations built out of 4-fold degenerate levels – similarly 32-fold degeneracies in the more complex 4p-4h excitations

- For instance: Exotic degeneracies in rotational bands with positive and negative parities

- For instance: unprecedented forms of the nuclear rotational behaviour - rotational bands without 'rotational E2-transitions'

- Theory predicts numerous competing configurations in many regions of the Periodic Table manifesting exotic symmetries

- These symmetries may lead to well pronounced potential energy minima generating unprecedented, new nuclear quantum mechanisms in terms of spectroscopy and isomerism

- ... and many more

Special Case: High-Rank Symmetries – T_d and O_h

- **High-rank Symmetry Groups** (Tetrahedral-T_d and Octahedral-O_h) have 4D irreducible representations, meaning that some levels are 4-fold degenerate, in contrast to the usual 2-fold (Kramers) degeneracy

- It follows: Existence of exotic (16-fold) degeneracies of 2p-2h excitations built out of 4-fold degenerate levels – similarly 32-fold degeneracies in the more complex 4p-4h excitations

- For instance: Exotic degeneracies in rotational bands with positive and negative parities

- For instance: unprecedented forms of the nuclear rotational behaviour - rotational bands without 'rotational E2-transitions'

- Theory predicts numerous competing configurations in many regions of the Periodic Table manifesting exotic symmetries

- These symmetries may lead to well pronounced potential energy minima generating unprecedented, new nuclear quantum mechanisms in terms of spectroscopy and isomerism

- ... and many more

Special Case: High-Rank Symmetries – T_d and O_h

- **High-rank Symmetry Groups** (Tetrahedral-T_d and Octahedral-O_h) have 4D irreducible representations, meaning that some levels are 4-fold degenerate, in contrast to the usual 2-fold (Kramers) degeneracy

- It follows: Existence of exotic (16-fold) degeneracies of 2p-2h excitations built out of 4-fold degenerate levels – similarly 32-fold degeneracies in the more complex 4p-4h excitations

- For instance: Exotic degeneracies in rotational bands with positive and negative parities

- For instance: unprecedented forms of the nuclear rotational behaviour - rotational bands without 'rotational E2-transitions'

- Theory predicts numerous competing configurations in many regions of the Periodic Table manifesting exotic symmetries

- These symmetries may lead to well pronounced potential energy minima generating unprecedented, new nuclear quantum mechanisms in terms of spectroscopy and isomerism

- ... and many more

Special Case: High-Rank Symmetries – T_d and O_h

- **High-rank Symmetry Groups** (Tetrahedral-T_d and Octahedral-O_h) have 4D irreducible representations, meaning that some levels are 4-fold degenerate, in contrast to the usual 2-fold (Kramers) degeneracy
- It follows: Existence of exotic (16-fold) degeneracies of 2p-2h excitations built out of 4-fold degenerate levels – similarly 32-fold degeneracies in the more complex 4p-4h excitations
- For instance: Exotic degeneracies in rotational bands with positive and negative parities
- For instance: unprecedented forms of the nuclear rotational behaviour - rotational bands without 'rotational E2-transitions'
- Theory predicts numerous competing configurations in many regions of the Periodic Table manifesting exotic symmetries
- These symmetries may lead to well pronounced potential energy minima generating unprecedented, new nuclear quantum mechanisms in terms of spectroscopy and isomerism
- ... and many more

Special Case: High-Rank Symmetries – T_d and O_h

- **High-rank Symmetry Groups** (Tetrahedral-T_d and Octahedral-O_h) have 4D irreducible representations, meaning that some levels are 4-fold degenerate, in contrast to the usual 2-fold (Kramers) degeneracy

- It follows: Existence of exotic (16-fold) degeneracies of 2p-2h excitations built out of 4-fold degenerate levels – similarly 32-fold degeneracies in the more complex 4p-4h excitations

- For instance: Exotic degeneracies in rotational bands with positive and negative parities

- For instance: unprecedented forms of the nuclear rotational behaviour - rotational bands without 'rotational E2-transitions'

- Theory predicts numerous competing configurations in many regions of the Periodic Table manifesting exotic symmetries

- These symmetries may lead to well pronounced potential energy minima generating unprecedented, new nuclear quantum mechanisms in terms of spectroscopy and isomerism

- ... and many more

Principal Goals and Strategy of Presented Research

- Large scale mean-field theory calculations addressing the presence of various exotic shape symmetries, their competition and evolution throughout the Mass Table

Principal Methods Used

- We calculate and analyse nuclear energies using one of the most powerful nuclear structure technique: **Realistic Phenomenological Nuclear Mean-Field Theory**

- We combine contemporary powerful **mathematical tools** of **group theory, inverse problem theory and graph-theory** & phenomenological nuclear mean-field theory

- Our modelling employs parameter optimisation based on recent experimental data – using Inverse Problem & Monte Carlo methods to remove parametric correlations

About Physics and Mathematics Tools Used

Principal Goals and Strategy of Presented Research

• Large scale mean-field theory calculations addressing the presence of various exotic shape symmetries, their competition and evolution throughout the Mass Table

Principal Methods Used

• We calculate and analyse nuclear energies using one of the most powerful nuclear structure technique: **Realistic Phenomenological Nuclear Mean-Field Theory**

• We combine contemporary powerful **mathematical tools** of **group theory, inverse problem theory and graph-theory** & phenomenological nuclear mean-field theory

• Our modelling employs parameter optimisation based on recent experimental data – using Inverse Problem & Monte Carlo methods to remove parametric correlations

Principal Goals and Strategy of Presented Research

• Large scale mean-field theory calculations addressing the presence of various exotic shape symmetries, their competition and evolution throughout the Mass Table

Principal Methods Used

• We calculate and analyse nuclear energies using one of the most powerful nuclear structure technique: **Realistic Phenomenological Nuclear Mean-Field Theory**

• We combine contemporary powerful **mathematical tools** of **group theory, inverse problem theory and graph-theory** & phenomenological nuclear mean-field theory

• Our modelling employs parameter optimisation based on recent experimental data – using Inverse Problem & Monte Carlo methods to remove parametric correlations

About Physics and Mathematics Tools Used

Principal Goals and Strategy of Presented Research

• Large scale mean-field theory calculations addressing the presence of various exotic shape symmetries, their competition and evolution throughout the Mass Table

Principal Methods Used

• We calculate and analyse nuclear energies using one of the most powerful nuclear structure technique: **Realistic Phenomenological Nuclear Mean-Field Theory**

• We combine contemporary powerful **mathematical tools** of **group theory, inverse problem theory and graph-theory** & phenomenological nuclear mean-field theory

• Our modelling employs parameter optimisation based on recent experimental data – using Inverse Problem & Monte Carlo methods to remove parametric correlations

Our Choice of Theory Approach:

Phenomenological Mean Field

Deformed Woods-Saxon Hamiltonian

Introducing Deformed Universal Woods-Saxon Hamiltonian

- Phenomenological **Woods-Saxon Hamiltonian** with the so-called **'universal'** parameterisation
  ⇒ **fixed set of parameters for thousands of nuclei!**

- **Central Potential**

$$\mathcal{V}^{\rm WS}_{\rm cent} = \frac{V_c}{1 + \exp\left[\mathrm{dist}_\Sigma(\vec{r}; r_c)/a_c\right]}$$

- **Spin-Orbit Potential**

$$\mathcal{V}^{\rm WS}_{\rm SO} = \frac{2\hbar\lambda_{so}}{(2mc)^2}\left[(\vec{\nabla}V^{\rm WS}_{\rm SO}) \wedge \hat{p}\right] \cdot \hat{s}, \;\text{ with } V^{\rm WS}_{\rm SO} = \frac{V_o}{1 + \exp[\mathrm{dist}_\Sigma(\vec{r}, r_{so})/a_{so}]}$$

- **Isospin distinction** ($+ \leftrightarrow$ protons) and ($- \leftrightarrow$ neutrons)

$$V_c = V_o\left[1 \pm \kappa_c \frac{N-Z}{N+Z}\right]; \quad \lambda_{so} = \lambda_o\left[1 \pm \kappa_{so}\frac{N-Z}{N+Z}\right]$$

- **This potential depends *only* on two sets of 6 parameters ↔ Mass Table ∼ 3 000 nuclei**

$$\boxed{\{V_c, r_c, a_c; \lambda_{so}, r_{so}, a_{so}\}_{\pi,\nu}} \Leftrightarrow \boxed{\{V_o, \kappa_c, r_c^{\pi,\nu}, a_c^{\pi,\nu}; \lambda_o, \kappa_{so}, r_{so}^{\pi,\nu}, a_{so}^{\pi,\nu}\}}$$

Introducing Deformed Universal Woods-Saxon Hamiltonian

- Phenomenological **Woods-Saxon Hamiltonian** with the so-called **'universal'** parameterisation
  ⇒ fixed set of parameters for thousands of nuclei!

- **Central Potential**

$$\mathcal{V}^{\mathrm{WS}}_{\mathrm{cent}} = \frac{V_c}{1 + \exp\left[\mathrm{dist}_\Sigma(\vec{r}; r_c)/a_c\right]}$$

- **Spin-Orbit Potential**

$$\mathcal{V}^{\mathrm{WS}}_{\mathrm{SO}} = \frac{2\hbar\lambda_{so}}{(2mc)^2}\left[(\vec{\nabla}V^{\mathrm{WS}}_{\mathrm{SO}}) \wedge \hat{p}\right] \cdot \hat{s}, \;\text{ with } V^{\mathrm{WS}}_{\mathrm{SO}} = \frac{V_o}{1 + \exp[\mathrm{dist}_\Sigma(\vec{r}, r_{so})/a_{so}]}$$

- **Isospin distinction** (+ ↔ protons) and (− ↔ neutrons)

$$V_c = V_o\left[1 \pm \kappa_c \frac{N-Z}{N+Z}\right]; \quad \lambda_{so} = \lambda_o\left[1 \pm \kappa_{so}\frac{N-Z}{N+Z}\right]$$

- **This potential depends *only* on two sets of 6 parameters ↔ Mass Table ∼ 3 000 nuclei**

$$\boxed{\{V_c, r_c, a_c; \lambda_{so}, r_{so}, a_{so}\}_{\pi,\nu}} \Leftrightarrow \boxed{\{V_o, \kappa_c, r_c^{\pi,\nu}, a_c^{\pi,\nu}; \lambda_o, \kappa_{so}, r_{so}^{\pi,\nu}, a_{so}^{\pi,\nu}\}}$$

Introducing Deformed Universal Woods-Saxon Hamiltonian

- Phenomenological **Woods-Saxon Hamiltonian** with the so-called **'universal'** parameterisation
  ⇒ fixed set of parameters for thousands of nuclei!

- **Central Potential**

$$\mathcal{V}^{\rm WS}_{\rm cent} = \frac{V_c}{1 + \exp\left[\mathrm{dist}_\Sigma(\vec{r}; r_c)/a_c\right]}$$

- **Spin-Orbit Potential**

$$\mathcal{V}^{\rm WS}_{\rm SO} = \frac{2\hbar\lambda_{so}}{(2mc)^2}\left[(\vec{\nabla}V^{\rm WS}_{\rm SO}) \wedge \hat{p}\,\right]\cdot\hat{s}, \;\; \text{with } V^{\rm WS}_{\rm SO} = \frac{V_o}{1 + \exp[\mathrm{dist}_\Sigma(\vec{r}, r_{so})/a_{so}]}$$

- **Isospin distinction** ($+ \leftrightarrow$ protons) and ($- \leftrightarrow$ neutrons)

$$V_c = V_o\left[1 \pm \kappa_c\frac{N-Z}{N+Z}\right]; \quad \lambda_{so} = \lambda_o\left[1 \pm \kappa_{so}\frac{N-Z}{N+Z}\right]$$

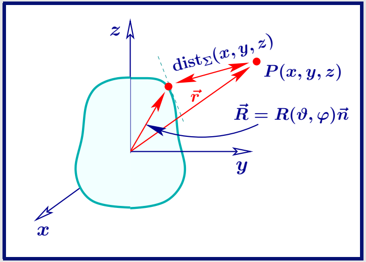
- **This potential depends *only* on two sets of 6 parameters ↔ Mass Table ~ 3 000 nuclei**

$$\boxed{\{V_c, r_c, a_c; \lambda_{so}, r_{so}, a_{so}\}_{\pi,\nu}} \Leftrightarrow \boxed{\{V_o, \kappa_c, r_c^{\pi,\nu}, a_c^{\pi,\nu}; \lambda_o, \kappa_{so}, r_{so}^{\pi,\nu}, a_{so}^{\pi,\nu}\}}$$

Woods-Saxon Mean-Field is a Functional of $\text{dist}_\Sigma(\vec{r})$

Surface Σ : $\quad R(\vartheta,\varphi) = R_o\, c(\{\alpha\})\left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta,\varphi)\right]$

Given surface $\Sigma \Leftrightarrow \text{dist}_\Sigma(\vec{r})$



$\vec{n} = \{\cos\varphi \sin\vartheta, \sin\varphi\sin\vartheta, \cos\vartheta\}$

- WS potential respects the surface-Σ symmetries:

$$V(\vec{r}; V_o, R, a) = \frac{V_o}{1 + \exp[\text{dist}_\Sigma(\vec{r})/a]}$$

- Auxiliary function

$$f(\vartheta,\varphi) \equiv \left[\vec{r} - R(\vartheta,\varphi)\,\vec{n}(\vartheta,\varphi)\right]^2$$

- Distance function

$$\text{dist}_\Sigma(\vec{r}) \equiv \min_{\{\vartheta,\varphi\}} f(\vartheta,\varphi)$$

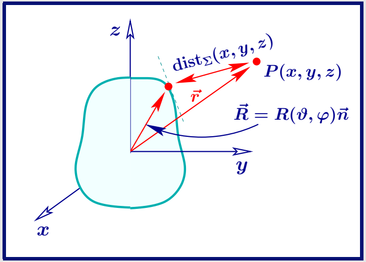
Mean-Field Potential: $\hat{\mathcal{V}}_{\text{m-f}} = \hat{\mathcal{V}}^{\text{WS}}_{\text{cent}} + \hat{\mathcal{V}}^{\text{WS}}_{\text{SO}} + \hat{\mathcal{V}}_{\text{C}}$

Hamiltonian: $\hat{\mathcal{H}}_{\text{m-f}} = \hat{\mathcal{T}} + \hat{\mathcal{V}}_{\text{m-f}}$

Woods-Saxon Mean-Field is a Functional of $\mathrm{dist}_\Sigma(\vec{r})$

Surface Σ : $R(\vartheta,\varphi) = R_o\, c(\{\alpha\})\,[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta,\varphi)]$

Given surface $\Sigma \Leftrightarrow \mathrm{dist}_\Sigma(\vec{r})$



$\vec{n} = \{\cos\varphi \sin\vartheta, \sin\varphi \sin\vartheta, \cos\vartheta\}$

- WS potential respects the surface-Σ symmetries:

$$V(\vec{r}; V_o, R, a) = \frac{V_o}{1 + \exp[\mathrm{dist}_\Sigma(\vec{r})/a]}$$

- Auxiliary function

$$f(\vartheta,\varphi) \equiv [\vec{r} - R(\vartheta,\varphi)\,\vec{n}(\vartheta,\varphi)]^2$$

- Distance function

$$\mathrm{dist}_\Sigma(\vec{r}) \equiv \min_{\{\vartheta,\varphi\}} f(\vartheta,\varphi)$$

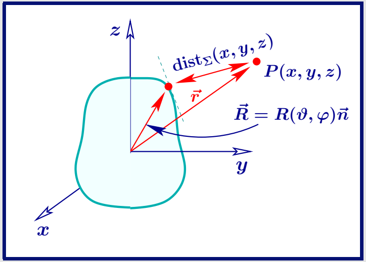
Mean-Field Potential: $\hat{\mathcal{V}}_{\mathrm{m-f}} = \hat{\mathcal{V}}^{\mathrm{WS}}_{\mathrm{cent}} + \hat{\mathcal{V}}^{\mathrm{WS}}_{\mathrm{SO}} + \hat{\mathcal{V}}_{\mathrm{C}}$

Hamiltonian: $\hat{\mathcal{H}}_{\mathrm{m-f}} = \hat{\mathcal{T}} + \hat{\mathcal{V}}_{\mathrm{m-f}}$

Woods-Saxon Mean-Field is a Functional of $\mathrm{dist}_\Sigma(\vec{r})$

Surface Σ : $R(\vartheta,\varphi) = R_o\, c(\{\alpha\}) \left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta,\varphi)\right]$

Given surface $\Sigma \Leftrightarrow \mathrm{dist}_\Sigma(\vec{r})$



$\vec{n} = \{\cos\varphi \sin\vartheta, \sin\varphi \sin\vartheta, \cos\vartheta\}$

- WS potential respects the surface-Σ symmetries:

$$V(\vec{r}; V_o, R, a) = \frac{V_o}{1 + \exp[\mathrm{dist}_\Sigma(\vec{r})/a]}$$

- Auxiliary function

$$f(\vartheta,\varphi) \equiv \left[\vec{r} - R(\vartheta,\varphi)\,\vec{n}(\vartheta,\varphi)\right]^2$$

- Distance function

$$\mathrm{dist}_\Sigma(\vec{r}) \equiv \min_{\{\vartheta,\varphi\}} f(\vartheta,\varphi)$$

Mean-Field Potential: $\hat{\mathcal{V}}_{\mathrm{m-f}} = \hat{\mathcal{V}}^{\mathrm{WS}}_{\mathrm{cent}} + \hat{\mathcal{V}}^{\mathrm{WS}}_{\mathrm{SO}} + \hat{\mathcal{V}}_{\mathrm{C}}$

Hamiltonian: $\hat{\mathcal{H}}_{\mathrm{m-f}} = \hat{\mathcal{T}} + \hat{\mathcal{V}}_{\mathrm{m-f}}$

A Short Historical Perspective

The hunt for exotic high-rank symmetries started several years ago ...

Non-Trivial Exotic Nuclear Shapes: Historical Perspective

- Already in Ancient Greece, they were interested in the *beauty in symmetry*
- **Platonic Solids** (Plato, 428-347 bC): Polyhedra whose faces are identical regular convex polygons $\Longrightarrow$ triangles, squares and regular pentagons
- Thus, there are only five Platonic Solids:

Non-Trivial Exotic Nuclear Shapes: Historical Perspective

- Already in Ancient Greece, they were interested in the *beauty in symmetry*
- **Platonic Solids** (Plato, 428-347 bC): Polyhedra whose faces are identical regular convex polygons $\Longrightarrow$ triangles, squares and regular pentagons
- Thus, there are only five Platonic Solids:

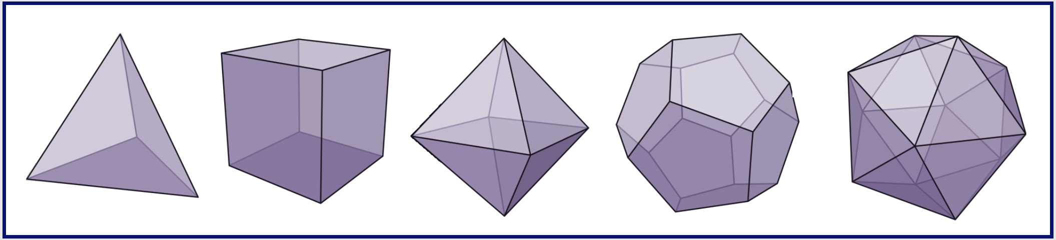
Non-Trivial Exotic Nuclear Shapes: Historical Perspective

- Already in Ancient Greece, they were interested in the *beauty in symmetry*
- **Platonic Solids** (Plato, 428-347 bC): Polyhedra whose faces are identical regular convex polygons $\Longrightarrow$ triangles, squares and regular pentagons
- Thus, there are only five Platonic Solids:

Non-Trivial Exotic Nuclear Shapes: Historical Perspective

- Already in Ancient Greece, they were interested in the *beauty in symmetry*
- **Platonic Solids** (Plato, 428-347 bC): Polyhedra whose faces are identical regular convex polygons $\Longrightarrow$ triangles, squares and regular pentagons
- Thus, there are only five Platonic Solids:

Tetrahedron **Cube** **Octahedron** **Dodecahedron** **Icosahedron**



VOLUME 88, No. 25 PHYSICAL REVIEW LETTERS 22 June 2002

Nuclear Tetrahedral Symmetry: Possibly Present throughout the Periodic Table

J. Dudek, A. Góźdź, N. Schunck, and M. Miśkiewicz

More than half a century after the fundamental, spherical shell structure in nuclei had been established, theoretical predictions indicate that the **shell gaps comparable or even stronger than those at spherical shapes may exist.** *Group-theoretical analysis supported by realistic mean-field calculations indicate that the corresponding nuclei are characterized by the T_d^D ("double-tetrahedral") symmetry group.*

Strong shell-gap structure is enhanced by the existence of the four-fold degenerate levels; *it can be seen as a geometrical effect that does not depend on a particular realization of the mean field. Possibilities of discovering the T_d^D symmetry in experiment are discussed.*

VOLUME 88, No. 25 PHYSICAL REVIEW LETTERS 22 June 2002

Nuclear Tetrahedral Symmetry: Possibly Present throughout the Periodic Table

J. Dudek, A. Góźdź, N. Schunck, and M. Miśkiewicz

More than half a century after the fundamental, spherical shell structure in nuclei had been established, theoretical predictions indicate that the **shell gaps comparable or even stronger than those at spherical shapes may exist.** *Group-theoretical analysis supported by realistic mean-field calculations indicate that the corresponding nuclei are characterized by the T_d^D ("double-tetrahedral") symmetry group.*

Strong shell-gap structure is enhanced by the existence of the four-fold degenerate levels; *it can be seen as a geometrical effect that does not depend on a particular realization of the mean field. Possibilities of discovering the T_d^D symmetry in experiment are discussed.*

[This was a follow-up of an earlier pilot project:
X. Li and J. Dudek, Phys. Rev. C 94, R1250 (1994)]

PHYSICAL REVIEW C VOLUME 97, 021302(R) FEBRUARY 2018

Spectroscopic criteria for identification of nuclear tetrahedral and octahedral symmetries: Illustration on a rare earth nucleus

J. Dudek, D. Curien, I. Dedes, K. Mazurek, S. Tagami, Y. R. Shimizu and T. Bhattacharjee

We formulate criteria for identification of the nuclear tetrahedral and octahedral symmetries and illustrate for the first time their possible realization in a rare earth nucleus ^{152}Sm. We use realistic nuclear mean-field theory calculations with the phenomenological macroscopic-microscopic method, the Gogny-Hartree-Fock-Bogoliubov approach, and general point-group theory considerations to guide the experimental identification method as illustrated on published experimental data. Following group theory the examined symmetries imply the existence of exotic rotational bands on whose properties the spectroscopic identification criteria are based. These bands may contain simultaneously states of even and odd spins, of both parities and parity doublets at well-defined spins. In the exact-symmetry limit those bands involve no E2 transitions. We show that coexistence of tetrahedral and octahedral deformations is essential when calculating the corresponding energy minima and surrounding barriers, and that it has a characteristic impact on the rotational bands. The symmetries in question imply the existence of long-lived shape isomers and, possibly, new waiting point nuclei-impacting the nucleosynthesis processes in astrophysics – and an existence of 16-fold degenerate particle-hole excitations.

This article announces the discovery – Culmination

Selected Molecular Symmetries in Atomic Nuclei

Example: So-called High-Rank*) Symmetries

Tetrahedral T_d and Octahedral O_h

*) The ones with 4D irreducible spinor representations – 4-fold nucleonic degeneracies

Tetrahedral Symmetry: Spherical-Harmonic Basis

- **Reminder:** nuclear surface, Σ: $R(\vartheta,\varphi) = R_o\, c(\{\alpha\})\left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta,\varphi)\right]$

- Only *special combinations* of only odd-order spherical harmonics may form a basis for surfaces with tetrahedral symmetry:

Three Lowest Order Solutions: **Rank ↔ Multipolarity λ**

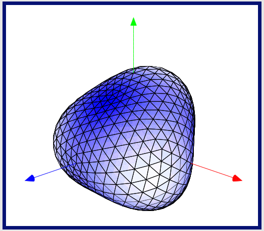
$\lambda = 3:\quad t_1 \equiv \alpha_{3,\pm 2}$

$\lambda = 5:\quad$ **no solution possible**

$\lambda = 7:\quad t_2 \equiv \alpha_{7,\pm 2}$ and $\alpha_{7,\pm 6} = -\sqrt{\tfrac{11}{13}} \cdot \alpha_{7,\pm 2}$

$\lambda = 9:\quad t_3 \equiv \alpha_{9,\pm 2}$ and $\alpha_{9,\pm 6} = +\sqrt{\tfrac{28}{198}} \cdot \alpha_{9,\pm 2}$

$\alpha_{32} \equiv t_1 = 0.2$



- **Problem presented in detail in:**

J. Dudek, J. Dobaczewski, N. Dubray, A. Góźdź, V. Pangon and N. Schunck, Int. J. Mod. Phys. E16, 516 (2007) [516-532].

OBSERVATION:

Tetrahedral symmetry group, T_d, is a sub-group of the octahedral one, O_h

Octahedral Symmetry: Spherical-Harmonic Basis

- **Reminder:** nuclear surface, Σ: $R(\vartheta,\varphi) = R_o\, c(\{\alpha\})\, \big[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta,\varphi)\big]$

- Only *special combinations* of only even-order $\lambda \geq 4$ spherical harmonics may form a basis for surfaces with octahedral symmetry

Three Lowest Order Solutions: **Rank ↔ Multipolarity λ**

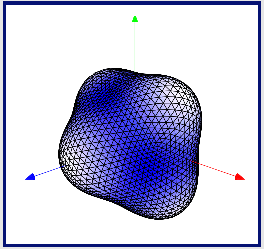
$\lambda = 4$: $o_1 \equiv \alpha_{40}$ and $\alpha_{4,\pm4} = -\sqrt{\frac{5}{14}} \cdot \alpha_{40}$

$\lambda = 6$: $o_2 \equiv \alpha_{60}$ and $\alpha_{6,\pm4} = -\sqrt{\frac{7}{2}} \cdot \alpha_{60}$

$\lambda = 8$: $o_3 \equiv \alpha_{80}$ and $\alpha_{8,\pm4} = \sqrt{\frac{28}{198}} \cdot \alpha_{80}$

and $\alpha_{8,\pm8} = \sqrt{\frac{65}{198}} \cdot \alpha_{80}$

$o_1 = 0.2$



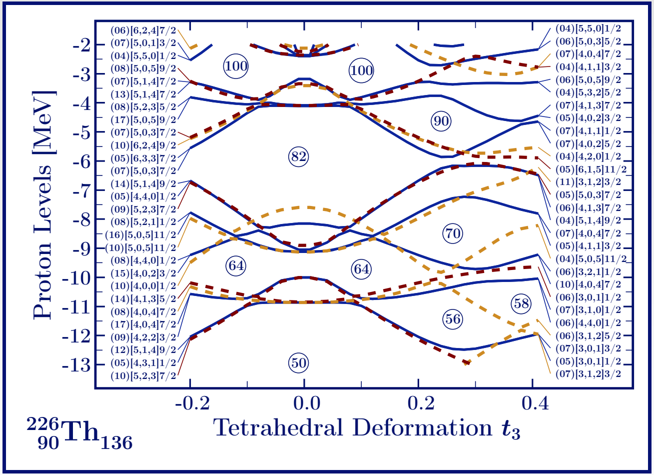
- **Problem presented in detail in:**

J. Dudek, J. Dobaczewski, N. Dubray, A. Góźdź, V. Pangon and N. Schunck, Int. J. Mod. Phys. E16, 516 (2007) [516-532].

Mean Field Theory: Tetrahedral Gaps – Shape Stabilisation I

Double group T_d^D has two 2-dimensional and one 4-dimensional irreducible representations (irreps.)

→ Three distinct families of nucleon levels ←



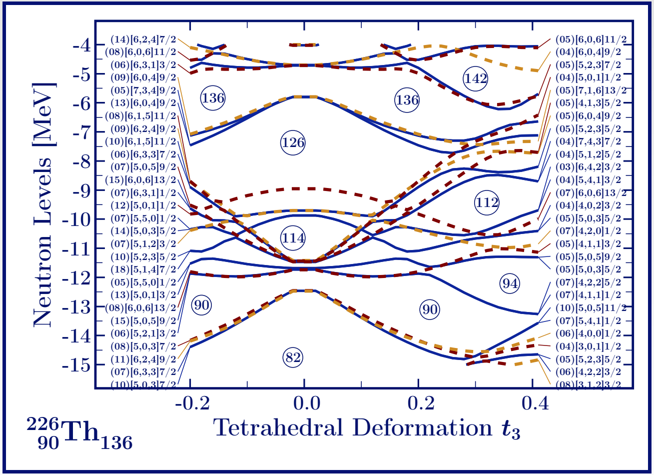
Full lines ↔ one 4D-irreps
Dashed lines ↔ two 2D-irreps

Notice gaps at $Z = 56, 64, 70, 90, 94, 100$.

Mean Field Theory: Tetrahedral Gaps – Shape Stabilisation II

Double group T_d^D has two 2-dimensional and one 4-dimensional irreducible representations (irreps.)

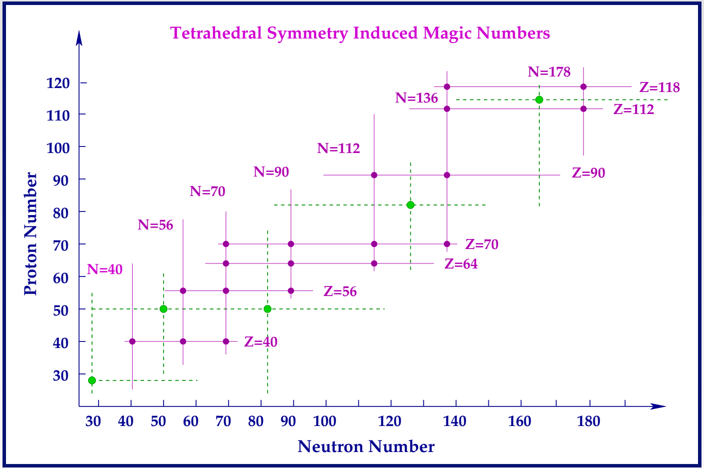
→ Three distinct families of nucleon levels ←



Full lines ↔ one 4D-irreps
Dashed lines ↔ two 2D-irreps

Notice gaps at $N = 90, 94, 112, 136, 142$.

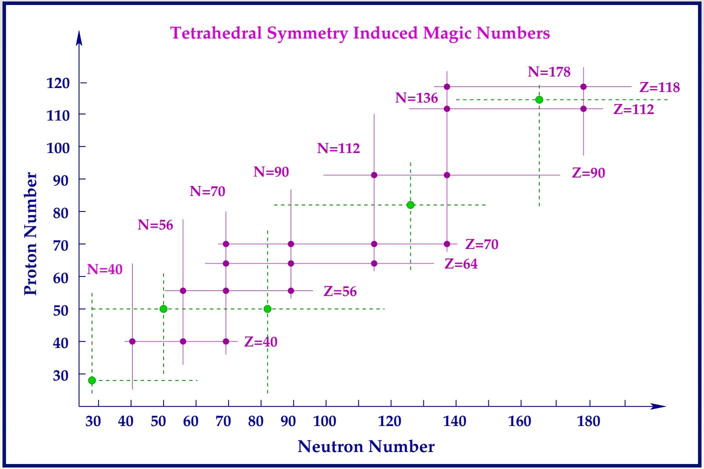
Numerous Tetrahedral Doubly-Magic Nuclei



- Doubly-Magic Tetrahedral Nuclei are more numerous than Doubly-Magic Spherical Nuclei
- Recall: at the exact symmetry limit tetrahedral nuclei emit neither E2 nor E1 transitions

→ ISOMERS

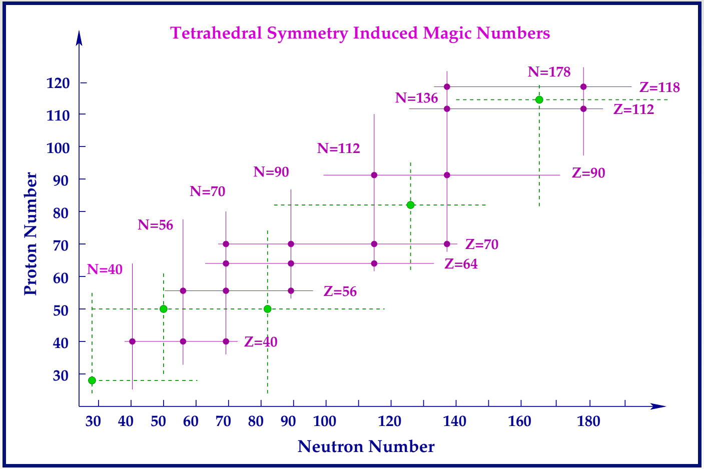
Numerous Tetrahedral Doubly-Magic Nuclei



- Doubly-Magic Tetrahedral Nuclei are more numerous than Doubly-Magic Spherical Nuclei
- Recall: at the exact symmetry limit tetrahedral nuclei emit neither E2 nor E1 transitions

→ ISOMERS

Numerous Tetrahedral Doubly-Magic Nuclei



- **Doubly-Magic Tetrahedral Nuclei** are more numerous than **Doubly-Magic Spherical Nuclei**
- **Recall: at the exact symmetry limit tetrahedral nuclei emit neither E2 nor E1 transitions**

→ ISOMERS

Tetrahedral (octahedral) symmetries generate bands without rotational electromagnetic transitions

“bands of isomers”

⇓

Rotating High-Rank Symmetric Nuclei

Seen Through Group-Representation Theory

[Symmetry Properties of Quantum Rotors]

Tetrahedral (octahedral) symmetries generate bands without rotational electromagnetic transitions

“bands of isomers”

⇓

Rotating High-Rank Symmetric Nuclei

Seen Through Group-Representation Theory

[Symmetry Properties of Quantum Rotors]

Reminders for Theorists: Group and Point Group Theories

• Consider a point-group symmetry characterised by group G. The SO(3)-group representation which mathematically describes rotor wave functions, $D^{(I\pi)}$, with given I^π, can be decomposed in terms of irreducible representations D_i (*irreps.*) of the concerned point-group G:

$$D^{(I\pi)} = \sum_{i=1}^{M} a_i^{(I\pi)} D_i,$$

where the so-called multiplicity coefficients, $a_i^{(I\pi)}$, satisfy *)

$$a_i^{(I\pi)} = \frac{1}{N_G} \sum_{R\in G} \chi_{(I\pi)}(R)\chi_i(R) = \frac{1}{N_G} \sum_{\alpha=1}^{M} n_\alpha \chi_{(I\pi)}(g_\alpha)\chi_i(g_\alpha)$$

→ $\chi_{(I\pi)}$ - characters of the reducible representation $D^{(I\pi)}$ of the SO(3)-group;
→ χ_i - characters of the irreducible representation D_i of a point group;
→ N_G - order of the group G;
→ g - group element;
→ n_α - the number of elements in the class α, whose representative element is g_α.

---

*) M. Hamermesh, *Group Theory and Its Application to Physical Problems*, Addison-Wesley Publishing Company, Inc., 1962

*) Tagami, Shimizu, Dudek, Phys. Rev. C**87**, 054306 (2013)

Reminders for Theorists – Example: Tetrahedral T_d-Group

- T_d-group has **5 irreps.: A_1, A_2, E, F_1, F_2,** and **5 classes: $E, C_3, C_2\ (= S_4^2), \sigma_d, S_4$**
- The characters of irreducible representations χ_i of T_d are listed below *)

T_d	E	$C_3(8)$	$C_2(3)$	$\sigma_d(6)$	$S_4(6)$
A_1	1	1	1	1	1
A_2	1	1	1	−1	−1
E	2	−1	2	0	0
F_1	3	0	−1	−1	1
F_2	3	0	−1	1	−1

- The characters $\chi_{(I\pi)}(g_\alpha)$ for the SO(3) reducible representations are as follows *):

$$\chi_{(I\pi)}(E) = 2I+1, \quad \chi_{(I\pi)}(C_n) = \sum_{K=-I}^{I} e^{\frac{2\pi K}{n} i}, \quad \chi_{(I\pi)}(\sigma_d) = \pi \times \chi_{(I\pi)}(C_2), \quad \chi_{(I\pi)}(S_4) = \pi \times \chi_{(I\pi)}(C_4)$$

- Multiplicity coefficients can be calculated in an elementary fashion

$$a_i^{(I\pi)} = \frac{1}{N_G} \sum_{g\in G} \chi_{(I\pi)}(g)\chi_i(g) = \frac{1}{N_G} \sum_{\alpha=1}^{M} n_\alpha \chi_{(I\pi)}(g_\alpha)\chi_i(g_\alpha);$$

---

*) M. Hamermesh, *Group Theory and Its Application to Physical Problems*, Addison-Wesley Publishing Company, Inc., 1962

*) Tagami, Shimizu, Dudek, Phys. Rev. C**87**, 054306 (2013)

Reminders: Resulting Prediction of the Structure of T_d-Bands

- The number of states $a_i^{(I\pi)}$ within five irreps
- If $a_i^{(I\pi)} = 0 \;\rightarrow$ states not allowed; $a_i^{(I\pi)} = 2 \;\rightarrow$ doubly degenerate, etc.

I^+	0^+	1^+	2^+	3^+	4^+	5^+	6^+	7^+	8^+	9^+	10^+
A_1	**1**	0	0	0	**1**	0	**1**	0	**1**	**1**	**1**
A_2	0	0	0	1	0	0	1	1	0	1	1
E	0	0	1	0	1	1	1	1	2	1	2
F_1	0	1	0	1	1	2	1	2	2	3	2
F_2	0	0	1	1	1	1	2	2	2	2	3
I^-	0^-	1^-	2^-	3^-	4^-	5^-	6^-	7^-	8^-	9^-	10^-
A_1	0	0	0	**1**	0	0	**1**	**1**	0	**1**	**1**
A_2	1	0	0	0	1	0	1	0	1	1	1
E	0	0	1	0	1	1	1	1	2	1	2
F_1	0	0	1	1	1	1	2	2	2	2	3
F_2	0	1	0	1	1	2	1	2	2	3	2

- In this way we find the I^π-sequence for A_1-representation

A_1 : $0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, 11^-, 2\times 12^+, 12^-, \cdots$

- **This is the group-theory prediction of the spin-parity structure of the tetrahedral g.s.b.**

Tetrahedral Bands Are Not Like the Others!

As we have shown using the methods of the point-group representation theory that, for instance, rotational bands based on 0^+ "T_d ground-state" have the structure:

$$A_1: \quad 0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, 11^-, 2\times 12^+, 12^-, \cdots$$

and NOT "ellipsoidal like"

$$\boldsymbol{I^\pi: \quad 0^+, 2^+, 4^+, 6^+, 8^+, 10^+, 12^+, \cdots}$$

Tetrahedral Bands Are Not Like the Others!

As we have shown using the methods of the point-group representation theory that, for instance, rotational bands based on 0^+ "T_d ground-state" have the structure:

$$A_1 : \quad 0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, 11^-, 2\times 12^+, 12^-, \cdots$$

and NOT "ellipsoidal like"

$$\boldsymbol{I^\pi : \quad 0^+, 2^+, 4^+, 6^+, 8^+, 10^+, 12^+, \cdots}$$

Similarly there are **no analogies** of the "octupole bands"

$$\boldsymbol{I^\pi : \quad 3^-, 5^-, 7^-, 9^-, 11^-, 13^-, 15^-, \cdots}$$

Quantum Rotors: Tetrahedral vs. Octahedral

- The **tetrahedral T_d** symmetry group has 5 irreps
- The ground-state $I^\pi = 0^+$ belongs to A_1-representation given by:

$$\underbrace{A_1: \quad 0^+, 3^-, 4^+, \underbrace{(6^+, 6^-)}_{\text{doublet}}, 7^-, 8^+, \underbrace{(9^+, 9^-)}_{\text{doublet}}, \underbrace{(10^+, 10^-)}_{\text{doublet}}, 11^-, \underbrace{2\times 12^+, 12^-}_{\text{triplet}}, \cdots}_{\text{Forming a common parabola}}$$

- Observations: $\rightarrow$ spins $I = 1, 2$ and 5 **missing**
  $\rightarrow$ **parity doublets** at $I = 6, 9, 10\ldots$ with: $E_{6^-} \approx E_{6^+}, E_{9^-} \approx E_{9^+}, \ldots$
  $\rightarrow$ **parity triplet** at $I = 12$

Quantum Rotors: Tetrahedral vs. Octahedral

- The **tetrahedral $\mathbf{T_d}$** symmetry group has 5 irreps
- The ground-state $I^\pi = 0^+$ belongs to A_1-representation given by:

$$A_1: \quad \underbrace{0^+, 3^-, 4^+, \underbrace{(6^+, 6^-)}_{\text{doublet}}, 7^-, 8^+, \underbrace{(9^+, 9^-)}_{\text{doublet}}, \underbrace{(10^+, 10^-)}_{\text{doublet}}, 11^-, \underbrace{2\times 12^+, 12^-}_{\text{triplet}}, \cdots}_{\text{Forming a common parabola}}$$

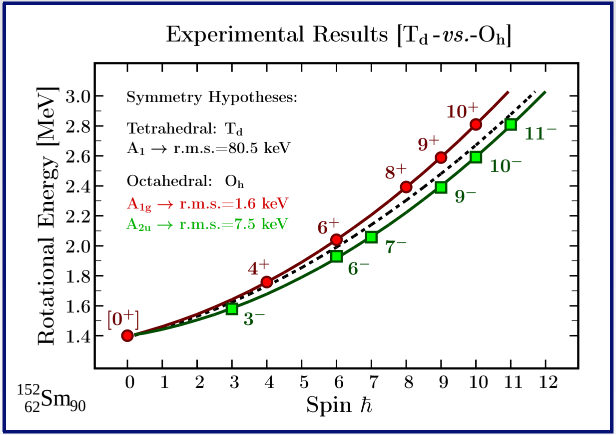
- Observations: $\rightarrow$ spins $I = 1, 2$ and 5 **missing**
  $\rightarrow$ **parity doublets** at $I = 6, 9, 10 \ldots$ with: $E_{6^-} \approx E_{6^+}, E_{9^-} \approx E_{9^+}, \ldots$
  $\rightarrow$ **parity triplet** at $I = 12$
- One shows the analogue structures for the **octahedral $\mathbf{O_h}$** symmetry

$$\underbrace{A_{1g}: \; 0^+, 4^+, 6^+, 8^+, 9^+, 10^+, \; \ldots, \; I^\pi = I^+}_{\text{Forming a common parabola}}$$

$$\underbrace{A_{2u}: \; 3^-, 6^-, 7^-, 9^-, 10^-, 11^-, \ldots, \; I^\pi = I^-}_{\text{Forming another (common) parabola}}$$

Perfect Parabolas Represent Experimental Results

Physical Review C 97, 021302(R) (2018)

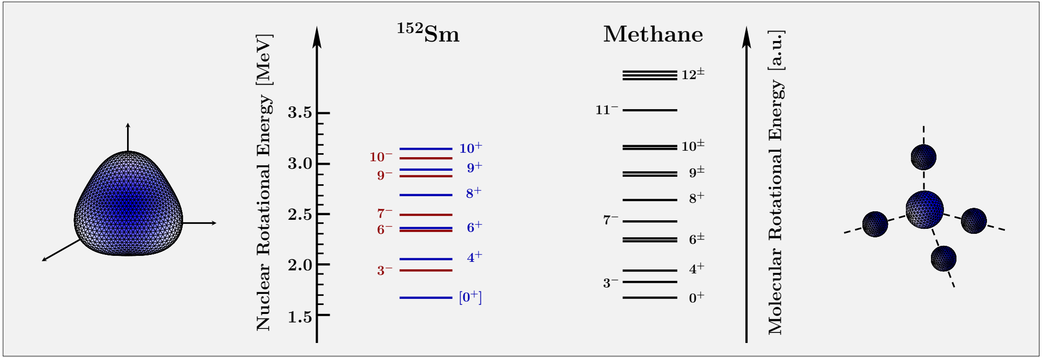


- Parabolic looking sequences are interpreted as **coexistence of tetrahedral and octahedral symmetries.**

Curves represent the parabolic fit and are *not* meant to guide the eye.

This is the first evidence of T_d (dashed) and O_h (two branches) based on the experimental data

Molecular Symmetries in Subatomic Physics: Methane vs. ^{152}Sm



- **Left**: The world first experimental identification of the tetrahedral-rotor band in ^{152}**Sm**
- **Right**: tetrahedral rotor band associated with the **methane CH_4 molecule** *)

---

*) G. Herzberg, *Molecular Spectra And Molecular Structure*, Vol II, (D.van Nostrand Company Inc.,1945)

Continuing the Hunt for Molecular Symmetries Throughout the Nuclear Chart

Possible World First Experimental Evidence of C_{2v} in ^{236}U

After a series of publications

- J. Yang, J .Dudek *et al.*, Phys. Rev. C **105**, 034348 (2022)
- J. Yang, J .Dudek *et al.*, Phys. Rev. C **106**, 054314 (2022)
- J. Yang, J .Dudek *et al.*, Phys. Rev. C **107**, 054304 (2023)

→ See next talk by **J. Yang**,
Exotic Nuclear Symmetries and New Concepts of Magic Numbers: Focus on Heavy Nuclei

Continuing the Hunt for Molecular Symmetries Throughout the Nuclear Chart

Possible World First Experimental Evidence of C_{2v} in ^{236}U

After a series of publications

- J. Yang, J .Dudek *et al.*, Phys. Rev. C **105**, 034348 (2022)
- J. Yang, J .Dudek *et al.*, Phys. Rev. C **106**, 054314 (2022)
- J. Yang, J .Dudek *et al.*, Phys. Rev. C **107**, 054304 (2023)

→ See next talk by **J. Yang**,
Exotic Nuclear Symmetries and New Concepts of Magic Numbers: Focus on Heavy Nuclei

Synthetic View of Octupole Instabilities

- Our calculations show **octupole shape instabilities** for $N = \mathbf{136}$, with big shell gaps and pronounced energy surface minima

- The octupole-shape deformations include $\boxed{\alpha_{\lambda=3,\mu=0,1,2,3}}$ thus leading to 4 independent shape degrees of freedom

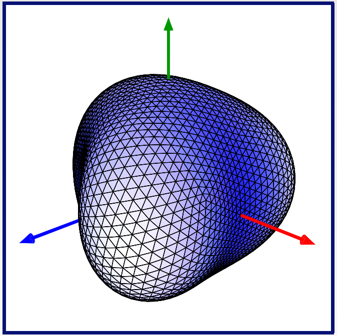
- They generate **Point-Group Symmetries:**

$$\boxed{\alpha_{30} \Rightarrow \mathrm{C}_{\infty \mathrm{v}}, \quad \alpha_{31} \Rightarrow \mathrm{C}_{2\mathrm{v}}, \quad \alpha_{32} \Rightarrow \mathrm{T}_{\mathrm{d}}, \quad \alpha_{33} \Rightarrow \mathrm{D}_{3\mathrm{h}}}$$

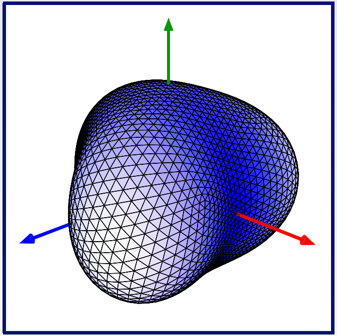
- It turns out that octupole static or dynamic equilibrium configurations are expected to generate specific rotational band structures mixing both parities $\Rightarrow$ What are these structures?

Molecular (Point-Group) Symmetry - $C_{2v} \Leftrightarrow \alpha_{31}$

- Symmetry induced by either ($\alpha_{31} \neq 0$) or ($\alpha_{20} \neq 0, \alpha_{31} \neq 0$)



$\alpha_{31} = 0.25$

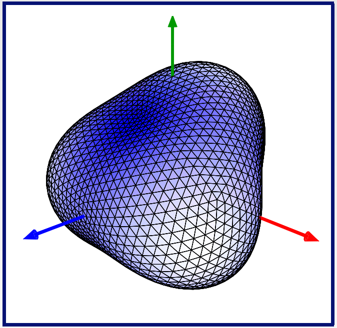


$\alpha_{20} = 0.15, \alpha_{31} = 0.25$

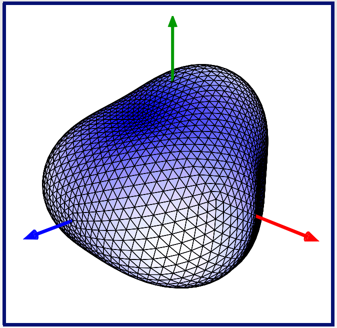
Nuclear C_{2v} Point Group Symmetry Shapes

Molecular (Point-Group) Symmetries - T_d & D_{2d} ⇔ α_{32}

- Symmetries induced by $(\alpha_{32} \neq 0 \rightarrow T_d)$ or $(\alpha_{20} \neq 0, \alpha_{32} \neq 0 \rightarrow D_{2d})$



T_d: $\alpha_{32} = 0.25$

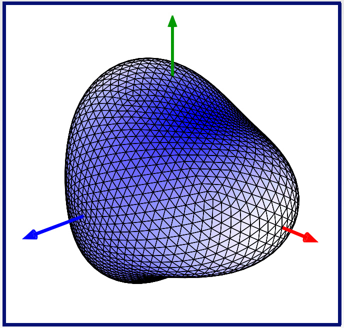


D_{2d}: $\alpha_{20} = 0.15, \alpha_{32} = 0.25$

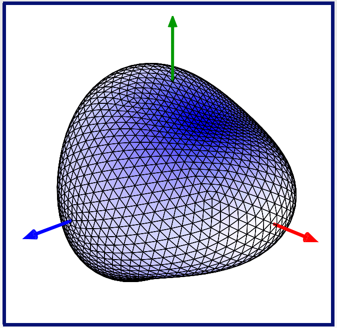
Nuclear T_d and D_{2d} Point Group Symmetry Shapes

Molecular (Point-Group) Symmetries - $D_{3h} \Leftrightarrow \alpha_{33}$

- Symmetry induced by both ($\alpha_{33} \neq 0$) and ($\alpha_{20} \neq 0, \alpha_{33} \neq 0$)



$\alpha_{33} = 0.25$



$\alpha_{20} = 0.15, \alpha_{33} = 0.25$

Nuclear D_{3h} Point Group Symmetry Shapes

How to proceed to experimental identification once we know the point group symmetry of interest?

How to proceed to experimental identification once we know the point group symmetry of interest?

Suggestion:

Formulate the identification criteria with the help of the group representation theory

Resulting Prediction of the Structure of C_{2v}-Bands

- Multiplicity factors for the 4 irreducible representations of C_{2v}-group

I^+	0^+	1^+	2^+	3^+	4^+	5^+	6^+	7^+	8^+	9^+	10^+
A_1	**1**	0	**2**	**1**	**3**	**2**	**4**	**3**	**5**	**4**	**6**
A_2	0	1	1	2	2	3	3	4	4	5	5
B_1	0	1	1	2	2	3	3	4	4	5	5
B_2	0	1	1	2	2	3	3	4	4	5	5
I^-	0^-	1^-	2^-	3^-	4^-	5^-	6^-	7^-	8^-	9^-	10^-
A_1	0	**1**	**1**	**2**	**2**	**3**	**3**	**4**	**4**	**5**	**5**
A_2	1	0	2	1	3	2	4	3	5	4	6
B_1	0	1	1	2	2	3	3	4	4	5	5
B_2	0	1	1	2	2	3	3	4	4	5	5

- In this way we find the spin-parity sequence for A_1-representation

$$A_1: \; 0^+,\, 1^-, 2\times 2^+, 2^-, 3^+, 2\times 3^-, 3\times 4^+, 2\times 4^-, 2\times 5^+, 3\times 5^-, 4\times 6^+, 4\times 6^-, \cdots$$

- Group-theory prediction of the spin-parity structure of the C_{2v} g.s.b.

. . . similarly can be done for D_{2d} and D_{3h}

Experimental Data Selection for C_{2v}

About criteria for the experimental data search

$$C_{2v} \to A_1 : 0^+, 1^-, 2\times 2^+, 2^-, 3^+, 2\times 3^-, 3\times 4^+, 2\times 4^-, 2\times 5^+, 3\times 5^-, 4\times 6^+, 4\times 6^-, \cdots$$

- Avoid rotational bands generated by leading ellipsoidal geometry and characterised by strong $\Delta I = 2$ quadrupole transitions
- Identified yrast-trap or K-isomers and related axial symmetry non-collective particle-hole excitations should be eliminated
- Energy-wise – C_{2v} bands form regular sequences

$$E_I \propto AI^2 + BI + C$$

Experimental Data Selection for C_{2v}

About criteria for the experimental data search

$$C_{2v} \to A_1 : 0^+, 1^-, 2\times 2^+, 2^-, 3^+, 2\times 3^-, 3\times 4^+, 2\times 4^-, 2\times 5^+, 3\times 5^-, 4\times 6^+, 4\times 6^-, \cdots$$

- Avoid rotational bands generated by leading ellipsoidal geometry and characterised by strong $\Delta I = 2$ quadrupole transitions
- Identified yrast-trap or K-isomers and related axial symmetry non-collective particle-hole excitations should be eliminated
- Energy-wise – C_{2v} bands form regular sequences

$$E_I \propto AI^2 + BI + C$$

Experimental Data Selection for C_{2v}

About criteria for the experimental data search

$$C_{2v} \to A_1 : 0^+, 1^-, 2\times 2^+, 2^-, 3^+, 2\times 3^-, 3\times 4^+, 2\times 4^-, 2\times 5^+, 3\times 5^-, 4\times 6^+, 4\times 6^-, \cdots$$

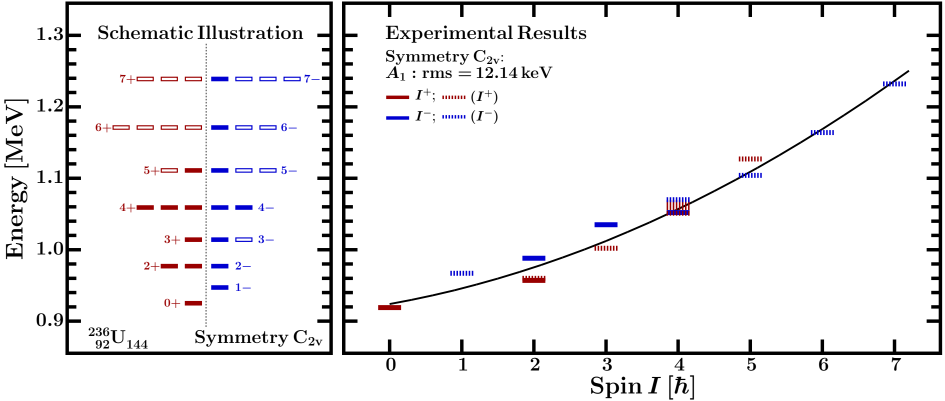
- Avoid rotational bands generated by leading ellipsoidal geometry and characterised by strong $\Delta I = 2$ quadrupole transitions
- Identified yrast-trap or K-isomers and related axial symmetry non-collective particle-hole excitations should be eliminated
- Energy-wise – $\mathbf{C_{2v}}$ bands form regular sequences

$$E_I \propto AI^2 + BI + C$$

Experimental Identification - Recent Results : ^{236}U

- **Rotational band structure of a nucleus according to a C_{2v}-symmetric configuration**

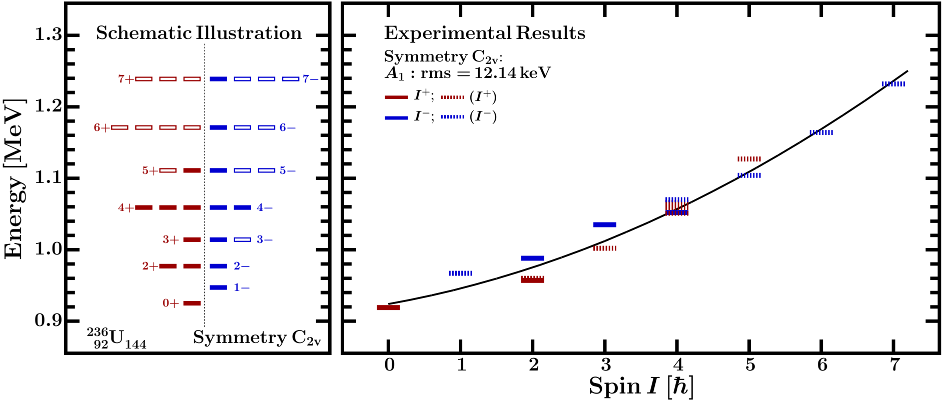
Attention: Experimental degeneracies for ^{236}U according to NNDC



Experimental Identification - Recent Results : ^{236}U

- **Rotational band structure of a nucleus according to a C_{2v}-symmetric configuration**

Attention: Experimental degeneracies for ^{236}U according to NNDC



- **Rotational band constructed employing 16 states; we find r.m.s. deviation 12.14 keV**

[rms(gsb)=3.79 keV]

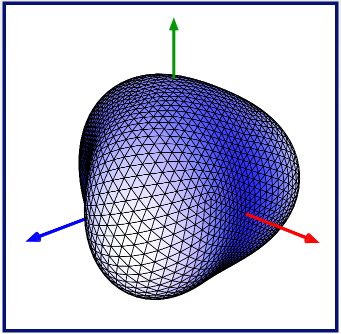
Exotic Symmetries: Nuclei vs. Molecules

- **Rotational band structure of a nucleus according to a C_{2v}-symmetric configuration**

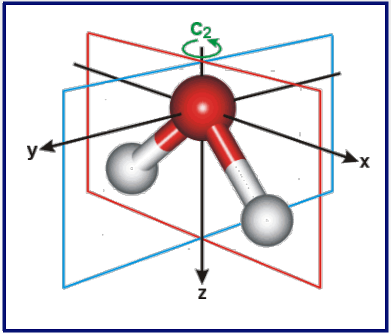
Attention: Experimental degeneracies for ^{236}U according to NNDC

- Notice similarities in the observed spectra (see below) in totally different objects: Molecules are composed of relatively distant point particles (atoms) and nuclei composed of the tightly packed nucleons interacting with the forces among most complex in the universe

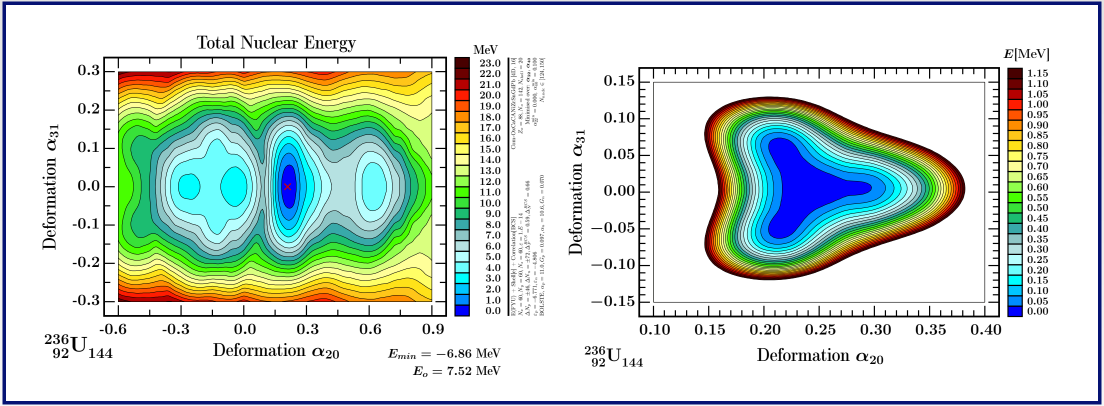
- **$\alpha_{31} = 0.25$**



- **H_2O has C_{2v}-symmetry:**



Exotic Symmetries for ^{236}U – Suspects for C_{2v}



- We associate the prolate minimum at $\alpha_{20}^{th} \sim 0.25$ [r.m.s.$(\alpha_{20}^{exp}) = 0.2821(18)$]$^{*)}$ with the ground-state, ...
- ... and enlarging the scales around the prolate minimum, $\alpha_{31} \neq 0$ is worth noticing

$\Longrightarrow$ associated with the C_{2v} symmetry

$^{*)}$ S. Raman, C. W. Nestor, JR., and P. Tikkanen
Atomic Data and Nuclear Data Tables, Vol. 78, No. 1, May 2001

We have argued that the potential energy landscapes may only give qualitative suggestions about equilibrium deformations → shapes & symmetries

We have argued that the potential energy landscapes may only give qualitative suggestions about equilibrium deformations → shapes & symmetries

We will turn to the s o l u t i o n s of the collective Schrödinger equation!!

Collective Schrödinger Equation

A New Approach to Adiabaticity Concepts in Collective Nuclear Motion: Impact for the Collective-Inertia Tensor and Comparisons with Experiment

PHYSICAL REVIEW C **99**, 041303(R) (2019)

D. Rouvel and J. Dudek

- New concepts of adiabaticity within collective model of Bohr, employing new microscopic method of calculations of collective inertia tensor
- It follows that collective Hamiltonian takes the form:

$$\hat{H}_{\rm coll} = -\frac{\hbar^2}{2}\Delta + V(\alpha_{\lambda\mu}) \;\leftrightarrow\; \Delta \stackrel{df.}{=} \sum_{m,n=1}^{d} \frac{1}{\sqrt{|B|}} \frac{\partial}{\partial q^n}\left(\sqrt{|B|} B^{nm} \frac{\partial}{\partial q^m}\right); \quad (q^m \leftrightarrow \alpha_{\lambda\mu})$$

where $|B|$-determinant of the mass tensor, with the resulting collective Schrödinger equation

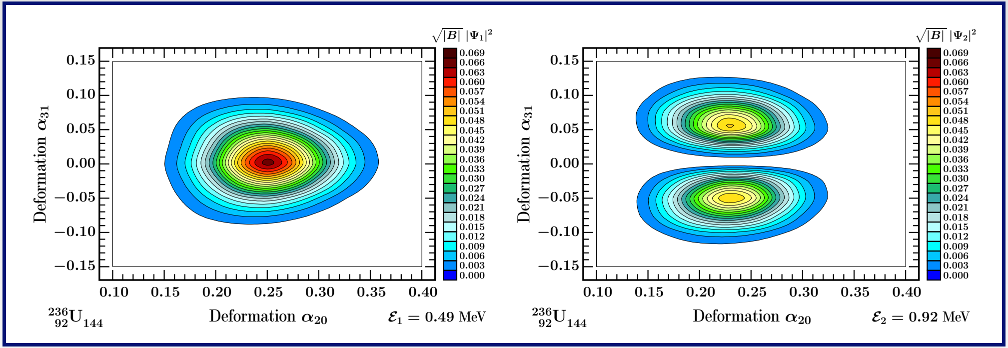
$$\boxed{\hat{H}_{\rm coll}\Psi_{\rm coll} = E_{\rm coll}\Psi_{\rm coll}}$$

- The most probable $\alpha_{\lambda\mu}$ deformation $\leftrightarrow$ the so-called "dynamic equilibrium"

$$\alpha_{\lambda\mu}^{\rm dyn} \;\leftrightarrow\; \langle\alpha_{\lambda\mu}^2\rangle = \int \Psi^*(\alpha_{\lambda\mu})\alpha_{\lambda\mu}^2\Psi(\alpha_{\lambda\mu})d\alpha_{\lambda\mu}$$

2D Collective Schrödinger Equation for C_{2v}

- The most probable deformation ↔ ("dynamic equilibrium") deduced from the following solutions



- Left: Probability density distribution corresponding to the ground-state
- Right: Similar, corresponding to the first excited state with two C_{2v}-symmetry maxima
- Resulting dynamical equilibrium values are close to typical values of the secondary deformations such as the hexadecapole one reported in many nuclei ↔ a typical numerical estimate $\alpha_{\lambda>2,\mu} \approx 0.10$

EURO-Labs Theo4Exp – MeanField4Exp

Collaboration between **IFJ PAN** and **IPHC Strasbourg**

`https://meanfield4exp.ifj.edu.pl`

Woods-Saxon phenomenological mean-field calculations,
developed by **J. Dudek** *et al.*

Currently four (almost six) services:

- Single Particle Energies
- Macroscopic-Microscopic Energy
- (3D Cranking, preliminary stage)
- Nuclear Energy Diagrams
- Shape Evolution with Spin
- (3D Nuclear Surface, preliminary stage)

- **Registration** required through → `https://iam-eurolabs.ijclab.in2p3.fr/login`
- **Contact us:** `meanfield4exp@ifj.edu.pl`
- **Web developer:** Abdelghafar Gaamouci, from IFJ PAN

EURO-Labs Theo4Exp – MeanField4Exp

Collaboration between **IFJ PAN** and **IPHC Strasbourg**

https://meanfield4exp.ifj.edu.pl

Woods-Saxon phenomenological mean-field calculations,
developed by **J. Dudek** *et al.*

Currently four (almost six) services:

- Single Particle Energies
- Macroscopic-Microscopic Energy
- (3D Cranking, preliminary stage)
- Nuclear Energy Diagrams
- Shape Evolution with Spin
- (3D Nuclear Surface, preliminary stage)

- **Registration** required through → https://iam-eurolabs.ijclab.in2p3.fr/login
- **Contact us:** meanfield4exp@ifj.edu.pl
- **Web developer:** Abdelghafar Gaamouci, from IFJ PAN

EURO-Labs Theo4Exp – MeanField4Exp

Collaboration between **IFJ PAN** and **IPHC Strasbourg**

https://meanfield4exp.ifj.edu.pl

Woods-Saxon phenomenological mean-field calculations,
developed by **J. Dudek** *et al.*

Currently four (almost six) services:

- Single Particle Energies
- Macroscopic-Microscopic Energy
- (3D Cranking, preliminary stage)
- Nuclear Energy Diagrams
- Shape Evolution with Spin
- (3D Nuclear Surface, preliminary stage)

- **Registration** required through → https://iam-eurolabs.ijclab.in2p3.fr/login
- **Contact us:** meanfield4exp@ifj.edu.pl
- **Web developer:** Abdelghafar Gaamouci, from IFJ PAN

EURO-Labs Theo4Exp – MeanField4Exp

Collaboration between **IFJ PAN** and **IPHC Strasbourg**

`https://meanfield4exp.ifj.edu.pl`

Woods-Saxon phenomenological mean-field calculations,
developed by **J. Dudek** *et al.*

Currently four (almost six) services:

- Single Particle Energies
- Macroscopic-Microscopic Energy
- (3D Cranking, preliminary stage)
- Nuclear Energy Diagrams
- Shape Evolution with Spin
- (3D Nuclear Surface, preliminary stage)

- **Registration** required through → `https://iam-eurolabs.ijclab.in2p3.fr/login`
- **Contact us:** `meanfield4exp@ifj.edu.pl`
- **Web developer:** Abdelghafar Gaamouci, from IFJ PAN

EURO-LABS Project – MeanField4Exp

Home Contact Us Mean Field Theory

MeanField4Exp

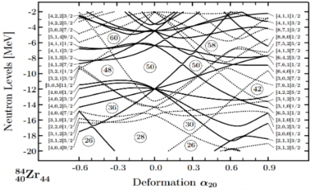
IFJ PAN, KRAKOW, POLAND and IPHC and UNIVERSITY OF STRASBOURG, FRANCE

Logged as Services About Theo4Exp

This website is based on an extensive research program
developed over many years by J. Dudek (IPHC and University of Strasbourg) and his collaborators;
it allows to explore elementary structure properties of atomic nuclei, including shapes, symmetries and excitations,
using **Realistic Phenomenological Nuclear Mean Field Theory Calculations.**

Single Particle Energies

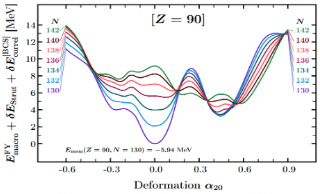
Generating diagrams of single nucleon energies.



Enter

Nuclear Energy Diagrams

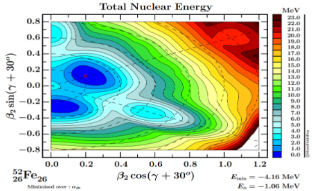
Generate Macroscopic-Microscopic Method nuclear energy diagrams.



Enter

Macroscopic-Microscopic Energy

Generating total energy diagrams according to the Macroscopic-Microscopic approximation.



Enter

EURO-LABS Project – MeanField4Exp

Home Contact Us Mean Field Theory

MeanField4Exp

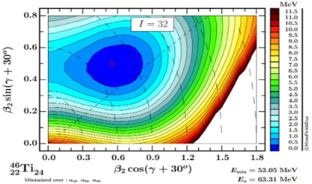
IFJ PAN, KRAKOW, POLAND and IPHC and UNIVERSITY OF STRASBOURG, FRANCE

Logged as Services About Theo4Exp

This website is based on an extensive research program
developed over many years by J. Dudek (IPHC and University of Strasbourg) and his collaborators;
it allows to explore elementary structure properties of atomic nuclei, including shapes, symmetries and excitations,
using **Realistic Phenomenological Nuclear Mean Field Theory Calculations.**

Shape Evolution with Spin

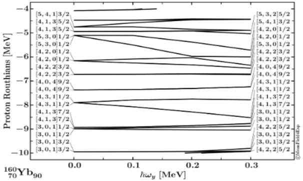
Generating diagrams of shape evolution with spin according to macroscopic energy models.



Enter

3D Cranking

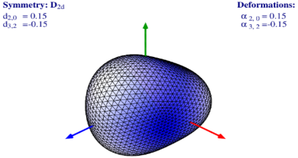
Create diagrams for single-particle Routhians through the Cranking method.



Enter

3D Nuclear Surface

Generate 3d diagrams of the nuclear shapes for customized deformations.



Enter

Summary and Conclusions

• 'Spectroscopy rules' related to point group nuclear symmetries have been presented, based on group representation theory ↔ they address isomerism and new rotational band properties

• Analysis used for the first Tetrahedral Rotational Band found in ^{152}Sm allowed for addressing spontaneous symmetry breaking and a new interpretations related to

Tetrahedral Symmetry Spontaneously Broken by Octahedral one

• Presence of universal magic gaps at $N = 136$ and another one at $N = 198$ generating strong shell effects/minima in heavy nuclei for α_{30}, α_{31}, α_{32} and α_{33} deformations – simultaneously

• We constructed the experimental identification criteria of exotic point-group symmetries in nuclei employing group-, and group representation theories – which lead to the bands with degenerate states

• Exotic symmetries associated with α_{30}, α_{31}, α_{32} or α_{33} are given by groups $C_{\infty v}$, C_{2v}, T_d and D_{3h}

• We have presented the world first identification of the exotic C_{2v} point group symmetry in ^{236}U

• By solving the collective Schrödinger equation in the framework of the theory of Bohr we have been able to establish the most probable dynamical deformations for exotic symmetry configurations