Proton-neutron pairing and α- like quartet condensation in N=Z nuclei

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<u>Outline</u>

- Proton-neutron pairing in the quartet condensation model (QCM)

  relation between QCM and HFB

- Quartet condensation for general two-body forces of shell-model type

  band-like structures in N=Z nuclei built on quartets

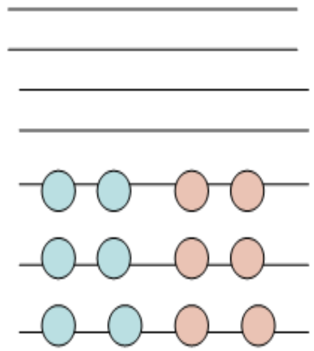
- Proton-neutron pairing in mean-field+QCM calculations

  binding energies in quark-meson-coupling+QCM

Proton-neutron pairing in N=Z nuclei: main issues

6 types of spin-isospin pairs

S=0, T=1			**S=1, T=0**		
$\nu^+_\uparrow \nu^+_\downarrow$	$\pi^+_\uparrow \pi^+_\downarrow$	$\nu^+_\uparrow \pi^+_\downarrow + \pi^+_\uparrow \nu^+_\downarrow$	$\nu^+_\uparrow \pi^+_\downarrow - \pi^+_\uparrow \nu^+_\downarrow$	$\nu^+_\downarrow \pi^+_\downarrow$	$\nu^+_\uparrow \pi^+_\uparrow$



collective pair $\quad \Gamma^+_{\pi\nu} = \sum_i x_i(\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}}) \qquad \Delta^+_0 = \sum_i x_i(\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})$

pair condensate $\quad (\Gamma^+_{\nu\pi})^{N_{\pi\nu}/2} \qquad (\Delta^+_0)^{N_{\pi\nu}/2}$

Long standing questions

there is a “condensate” of pn pairs in nuclei ?

the fingerprints of a pn condensate ?

Theoretical approach

BCS/HFB-type models : - unified descriptions of all types of pairing

- drawback : particle number, spin and isospin are not conserved

restoring the symmetries generate α-like quartet correlations !

Isospin projection

$$\mathcal{P}_{T;T_z=0} = \int_{S^2} \mathrm{d}\hat{n}\, D^{T*}_{00}(\hat{n})\, R(\hat{n})$$

$$\mathcal{P}_{T;T_z=0} \; (\Gamma^+_{\nu\pi})^{\frac{N+Z}{2}} \, |-> \;\; \Rightarrow \;\; (2\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi} - \Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi})^{n_q} \, |->$$

α-like quartet $\quad Q^+ = 2\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi} - \Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi}$

$$\mathcal{P}^{\mathcal{N}=4n_q}_{T=0} |BCS\rangle = (Q^\dagger)^{n_q} |0\rangle$$

a particular case of the Quartet Condensation Model (QCM)

V. V. Baran, D. Nichita, D. Negrea, D. Delion, N. Sandulescu, P. Schuck, PRC 102, 061301®(2020)

Isovector pairing in the QCM approach

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P^+_{i,\tau} P_{j,\tau}$$

$$P^+_{i1} \propto \nu_i^+ \nu_{\bar{i}}^+ \qquad P^+_{i-1} \propto \pi_i^+ \pi_{\bar{i}}^+ \qquad P^+_{i0} \propto \nu_i^+ \pi_{\bar{i}}^+ + \pi_i^+ \nu_{\bar{i}}^+$$

non-collective quartets

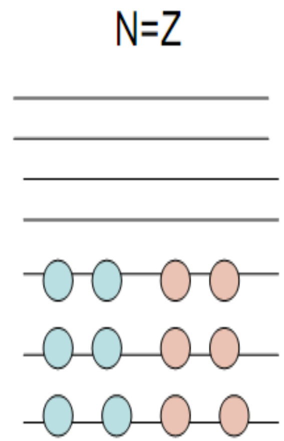
$$Q^+_{ij} = [P^+_{i\tau} P^+_{j\tau'}]^{T=0} \propto P^+_{\nu\nu,i} P^+_{\pi\pi,j} + P^+_{\pi\pi,i} P^+_{\nu\nu,j} - P^+_{\nu\pi,i} P^+_{\nu\pi,j}$$

collective quartet

$$Q^+ = \sum_{ij} x_{ij} [P^+_{i\tau} P^+_{j\tau'}]^{T=0}$$

quartet condensate

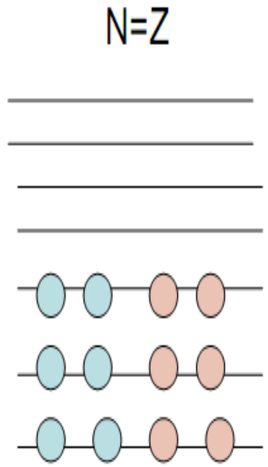
$$|QCM> = Q^{+n_q} |->$$ (has T=0, J=0)



N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

QCM for isovector (J=0) and isoscalar (J=1) pairing

$$H=\sum \varepsilon_i N_i+\sum_{ij} V_{J=0}^{T=1}(i,j)\sum_{\tau} P_{i\tau}^{+}P_{j\tau}+\sum_{ij} V_{J=1}^{T=0}(i,j)\sum_{\sigma} D_{i\sigma}^{+}D_{j\sigma}$$



isovector

$$P_{i,T_z}^{+}=[a_i^{+}a_i^{+}]_{T_z}^{T=1,J=0}$$

isoscalar

$$D_{ij,J_z}^{+}=[a_i^{+}a_j^{+}]_{J_z}^{J=1,T=0}$$

collective quartets

$$Q_\nu^{+(iv)}=\sum_{i,j} x_{ij}^{(\nu)}[P_i^{+}P_j^{+}]^{T=0}$$

$$Q_\nu^{+(is)}=\sum_{ij,kl} y_{ij,kl}^{(\nu)}[D_{ij}^{+}D_{kl}^{+}]^{J=0}$$

generalised quartet

$$Q_\nu^{+}=Q_\nu^{+(iv)}+Q_\nu^{+(is)}$$

ground state

$$|QCM>=Q^{+n_q}|->$$

superposition of T=0 and T=1 quartets

M. Sambataro,, N. S, C. W. Johnson, Phys. Lett. B740 (2015)137

Quartet condensation versus pair condensation

$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

	$(Q^+)^{n_q} \vert - >$	$(\Gamma_{\nu\nu}^{+}\Gamma_{\pi\pi}^{+})^{n_q} \vert - >$	$(\Gamma_{\nu\pi}^{+})^{2n_q} \vert - >$	$(\Delta_0^{+})^{2n_q} \vert 0\rangle$
^{20}Ne	15.985 (-)	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
^{24}Mg	28.595 (0.24%)	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
^{28}Si	35.288 (0.57%)	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
^{44}Ti	7.019 (-)	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
^{48}Cr	11.614 (0.21%)	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
^{52}Fe	13.799 (0.42%)	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
^{104}Te	3.147 (-)	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
^{108}Xe	5.489 (0.20%)	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
^{112}Ba	7.017 (0.34%)	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

- a pure isoscalar pairing condensation is not a good approximation !
- T=1 and T=0 pairing correlations always coexist in quartets

M. Sambataro and N. S , Phys. Rev C93, 054320 (2016)

Relation between QCM and projected - HFB

$$|\Psi_{\text{g.s.}}\rangle = (Q^+)^{n_q}|0\rangle \qquad Q^+ = Q_1^+ + Q_0^+,$$

$$Q_1^+ = \sum_{j_1 j_2} x_{j_1 j_2}[P_{j_1}^+ P_{j_2}^+]^{T=0}, \qquad Q_0^+ = \sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4}[D_{j_1 j_2}^+ D_{j_3 j_4}^+]^{J=0}.$$

$$x_{j_1 j_2} = \bar{x}_{j_1}\bar{x}_{j_2} \qquad y_{j_1 j_2 j_3 j_4} = \bar{y}_{j_1 j_2}\bar{y}_{j_3 j_4}$$

$$\Gamma_{T_z}^+ = \sum_j \bar{x}_j P_{j,T_z}^+, \qquad \Delta_{J_z}^+ = \sum_{j_1 j_2} \bar{y}_{j_1 j_2} D_{j_1 j_2 J_z}^+.$$

$$\bar{Q}_1^+ = 2\Gamma_1^+\Gamma_{-1}^+ - (\Gamma_0^+)^2, \qquad \bar{Q}_0^+ = 2\Delta_1^+\Delta_{-1}^+ - \Delta_0^{+2}.$$

$$|\overline{\text{QCM}}_{T=1}\rangle = (\bar{Q}_1^+)^{n_q}|0\rangle \qquad |\overline{\text{QCM}}_{T=0}\rangle = (\bar{Q}_0^+)^{n_q}|0\rangle.$$

equivalent to isospin projection

equivalent to spin projection

$$|\overline{\text{QCM}}\rangle = (\bar{Q}_1^+ + \bar{Q}_0^+)^{n_q}|0\rangle$$

isospin- spin projection from a mixed HFB state

TABLE II. Correlation energies (19) relative to various calculations for $N = Z$ nuclei described by the Hamiltonian (17). We show the results for the QCM the state (4) as well as for the QCM approximations relative to the quartets (10) and (11), i.e., $|\overline{\mathrm{QCM}}\rangle = (\bar{Q}_1^+ + \bar{Q}_0^+)^{n_q}|0\rangle$, $|\overline{\mathrm{QCM}}_{T=1}\rangle = (\bar{Q}_1^+)^{n_q}|0\rangle$, and $|\overline{\mathrm{QCM}}_{T=0}\rangle = (\bar{Q}_0^+)^{n_q}|0\rangle$. The QM results refer to the state (18) and are taken from Ref. [17]. In brackets we show the relative errors with respect to the exact results obtained by diagonalization. All energies are in MeV.

	Exact	QM	QCM	$\overline{\mathrm{QCM}}$	$\overline{\mathrm{QCM}}_{T=1}$	$\overline{\mathrm{QCM}}_{T=0}$
^{20}Ne	15.985	15.985 (-)	15.985 (-)	15.510 (2.97%)	14.373 (10.08%)	14.930 (6.60%)
^{24}Mg	28.694	28.626 (0.24%)	28.595 (0.34%)	27.764 (3.24%	23.229 (19.04%)	26.299 (8.35%)
^{28}Si	35.600	35.396 (0.57%)	35.288 (0.88%)	33.913 (4.74%)	28.830 (19.02%)	32.067 (9.92%)
^{44}Ti	7.019	7.019 (-)	7.019 (-)	6.302 (10.21%)	6.273 (10.63%)	4.825 (31.26%)
^{48}Cr	11.649	11.624 (0.21%)	11.614 (0.30%)	10.674 (8.37%)	10.582 (10.67%)	7.075 (39.26%)
^{52}Fe	13.887	13.828 (0.42%)	13.799 (0.63%)	12.971 (6.60%)	12.795 (7.92%)	9.589 (30.95%)
^{104}Te	3.147	3.147 (-)	3.147 (-)	3.052 (3.02%)	3.041 (3.37%)	1.512 (51.95%)
^{108}Xe	5.505	5.495 (0.20%)	5.489 (0.29%)	5.279 (4.10%)	5.239 (4.83%)	2.530 (54.04%)
^{112}Ba	7.059	7.035 (0.34%)	7.017 (0.59%)	6.691 (5.21%)	6.609 (6.37%)	4.391 (37.79%)

the isospin-spin projected state $|\overline{\mathrm{QCM}}\rangle$ is less accurate than full QCM

proj-HFB underestimates the pn correlations

M. Sambataro and N. S , Phys. Rev C93, 054320 (2016)

Quartet condensation for general two-body interactions

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A^{+}_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$

sd-shell: USDB

pf-shell: KB3G

Band-like structures in N=Z nuclei based on quartets

- “Intrinsic" ground state: *quartet condensate*

$$|\Theta_g\rangle = (Q_g^+)^n|0\rangle, \quad Q_g^+ = \sum_J \alpha_{g,J}(q_g^+)_{J0}$$ J=0,2,4

ground state and ground state band

$$\hat{P}_J|\Theta_g\rangle$$ J=0,2,4,6 ….

- Intrinsic excited states: *one - broken - quartet states*

$$|\Theta_k\rangle = Q_k^\dagger (Q_g^\dagger)^{(n-1)}|0\rangle, \quad Q_k^\dagger = \sum_J \alpha_{k,J}(q_k^\dagger)_{Jk}$$

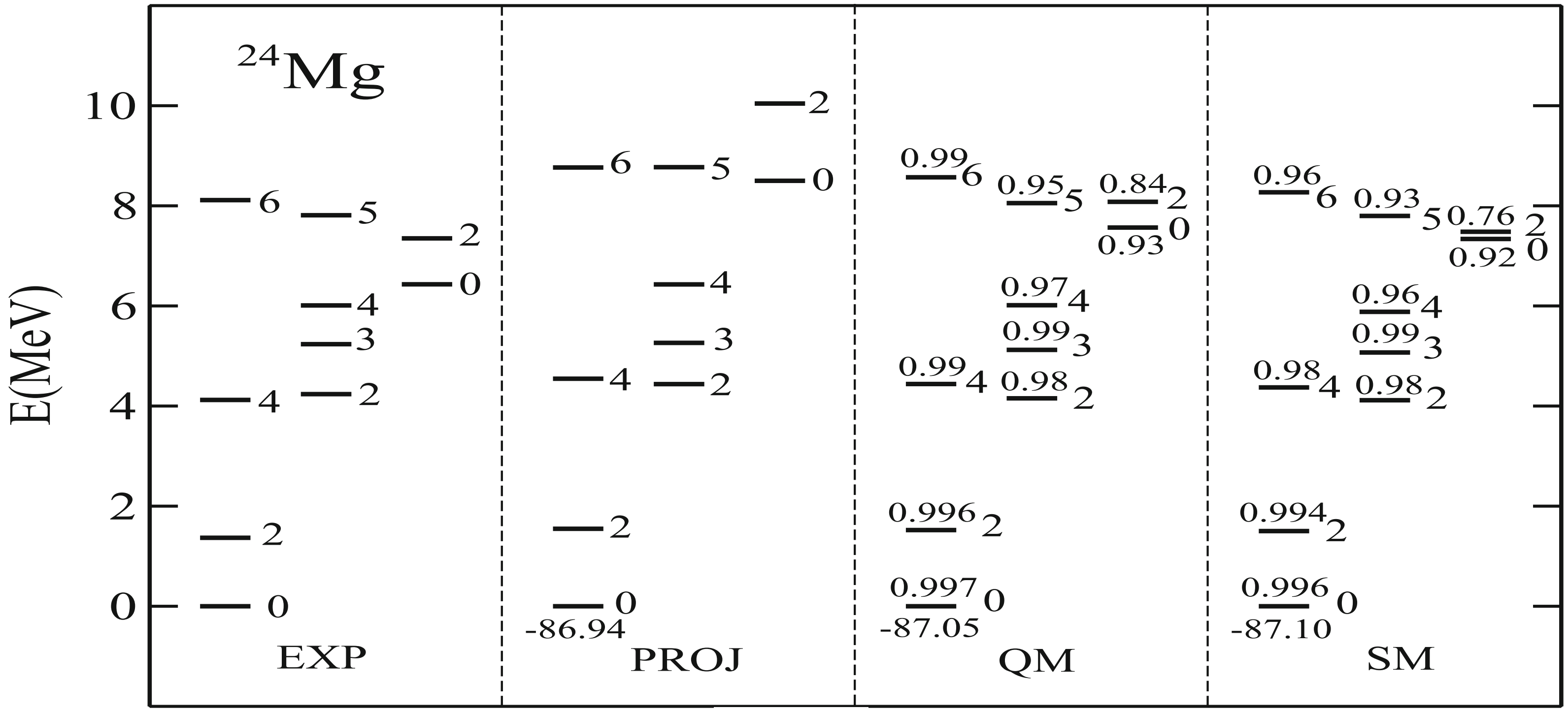
$$k = 0 \rightarrow \text{“}\beta\text{''}, \; k = 2 \rightarrow \text{“}\gamma\text{''}, \cdots$$

band structures

$$\hat{P}_J|\Theta_k\rangle$$

M. Sambataro and N.S, Phys. Lett. B 827 (2022) , EPJA 59 (2023)

Band structures generated from quartet states: ^{24}Mg



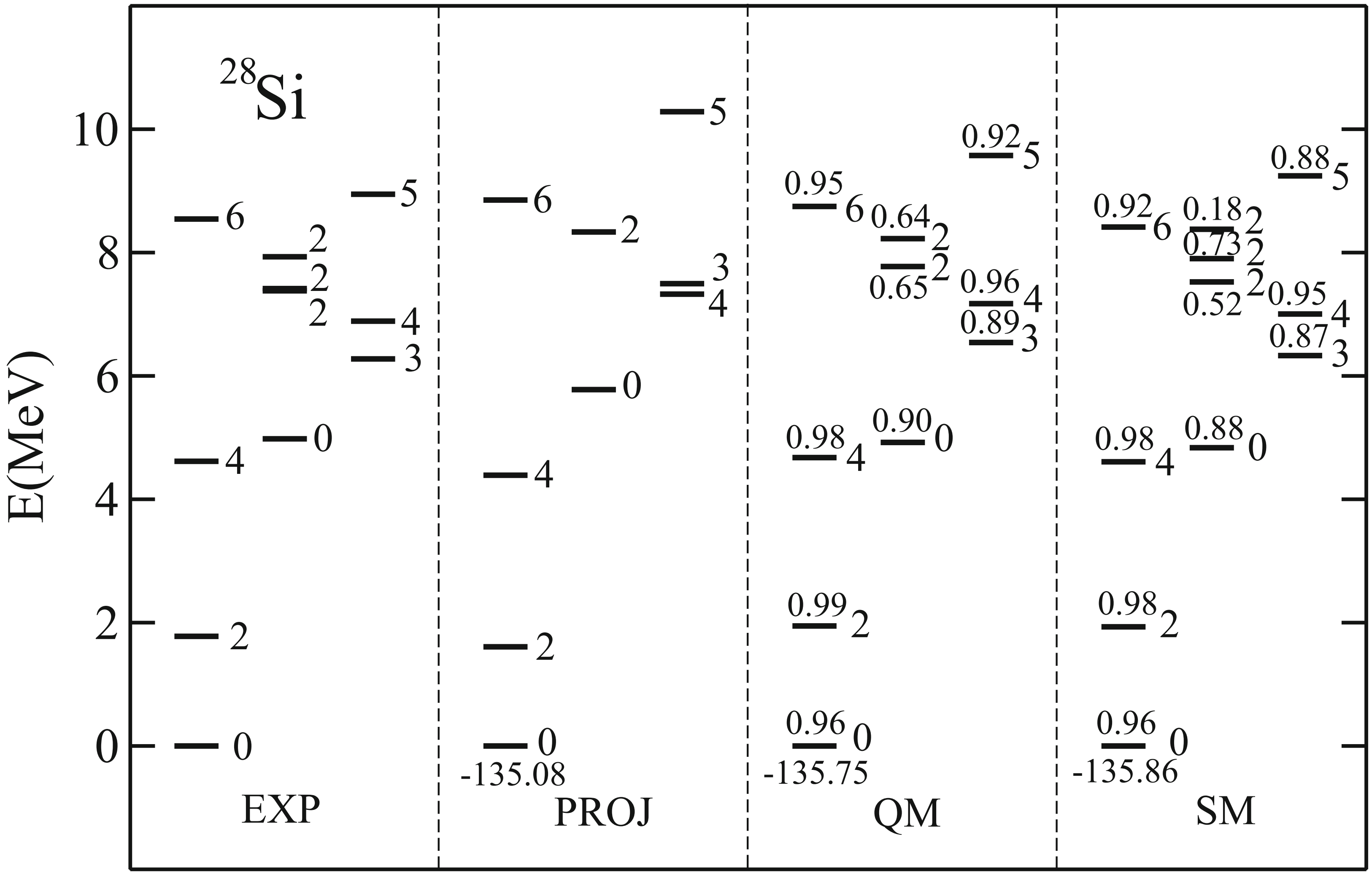
$\hat{P}_J|\Theta_g\rangle$ $\hat{P}_J|\Theta_k\rangle$

diagonalisation in the quartet basis

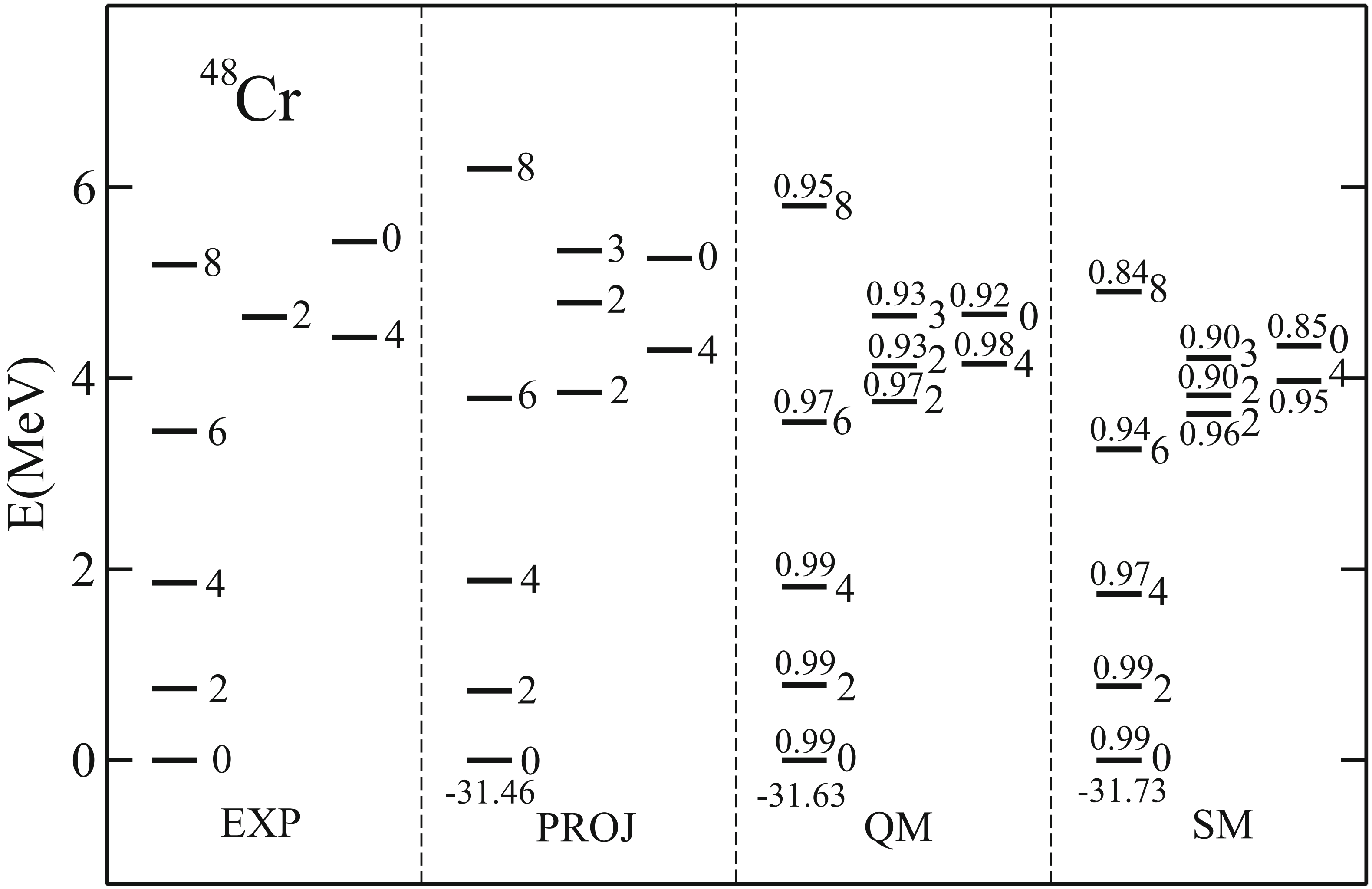
shell model

- band-like structures similar to experimental bands
- large overlaps with the shell-model exact states
- ground state energy very close to exact SM

M. Sambataro and N.S, Phys. Lett. B 827 (2022) , EPJA 59 (2023)



^{28}Si
E(MeV)
0
2
4
6
8
10
EXP
PROJ
QM
SM
-135.08
-135.75
-135.86
0.96
0.99
0.98
0.90
0.95
0.64
0.65
0.92
0.96
0.89
0.96
0.98
0.88
0.92
0.18
0.73
0.52
0.88
0.95
0.87



48Cr
E(MeV)
0
2
4
6
EXP
PROJ
QM
SM
-31.46
-31.63
-31.73
0.99
0.97
0.95
0.93
0.92
0.98
0.84
0.90
0.85
0.96
0.94

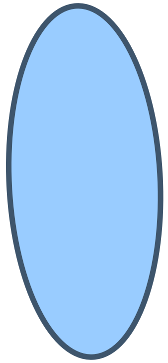
Proton-neutron pairing in self-consistent

mean-field +QCM calculations

Isoscalar – isovector pairing in axially deformed mean-fields

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$

mean-field   pairing in time-reversed deformed states

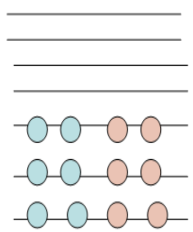


$$Q^{+} = \sum_{ij} x_i x_j [P^{+}_{i\tau} P^{+}_{j\tau'}]^{T=0} \qquad \Delta_0^{+} = \sum y_i D^{+}_{i,0}$$

- Ground state $$|\Psi> = (Q^{+} + \Delta_0^{+2})^{n_q} |->$$

exact solution for degenerate states

errors for correlation energies < 1%



- **Applications :** Skyrme-HF+ QCM calculations (D. Negrea, N.S , D. Gambacurta, PRC105 (2022))

  Quark-meson-coupling+ QCM: work in progress

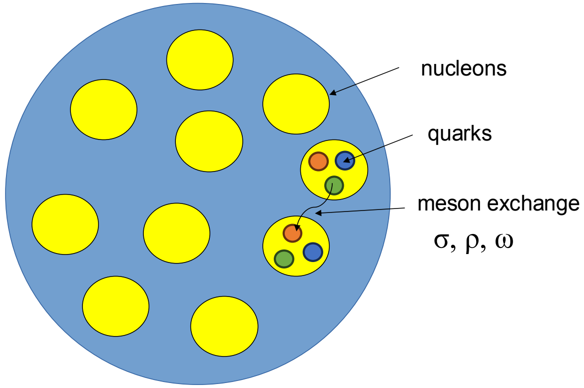
N.S, D.Negrea, D. Gambacurta, Phys. Lett. B (2015)

Quark Meson Coupling model

- Quark-Meson Coupling (QCM) model : basic assumptions

  P. A. M. Guichon, Phys. Lett. B (1988)

  nucleons described as clusters of 3 quarks (MIT bag model)
  the quarks interact by the exchange of σ, ρ and ω mesons



- QMC energy density functional (EDF): Skyrme like structure

  P. Guichon, H. Metosovian, N. Sandulescu, A. W. Thomas,, NPA (2006)

  QMC-EDF depends only of 4 parameters **(m_σ , $g^{\sigma,\omega,\rho}{}_q$)**

  tensor and pairing forces derived within QMC

- Systematic QMC calculations of nuclei :

  J. R. Stone, P. A. M. Guichon, P. G. Reinhard, A. W. Thomas, PRL 116 (2016)
  K. L. Martinez, A. W. Thomas, J. R. Stone, P. Guichon PRC102 (2020)

  accuracy comparable with the best Skyrme calculations !

no proton-neutron pairing !

Proton-neutron pairing interaction in QMC

- Isovector pairing force: pairs with (T=1, Jz=0)

derived from QMC+ a scaling factor (s)

$$V^{T=1} = -s\left(\frac{G_\sigma}{1 + d' G_\sigma \rho(\vec{r})} - G_\omega - \frac{G_\rho}{4}\right)\delta(\vec{r} - \vec{r'})$$

$$G_i = \frac{g_i^2}{m_i^2} \qquad d' = d + \frac{1}{3} g_\sigma \lambda_3$$

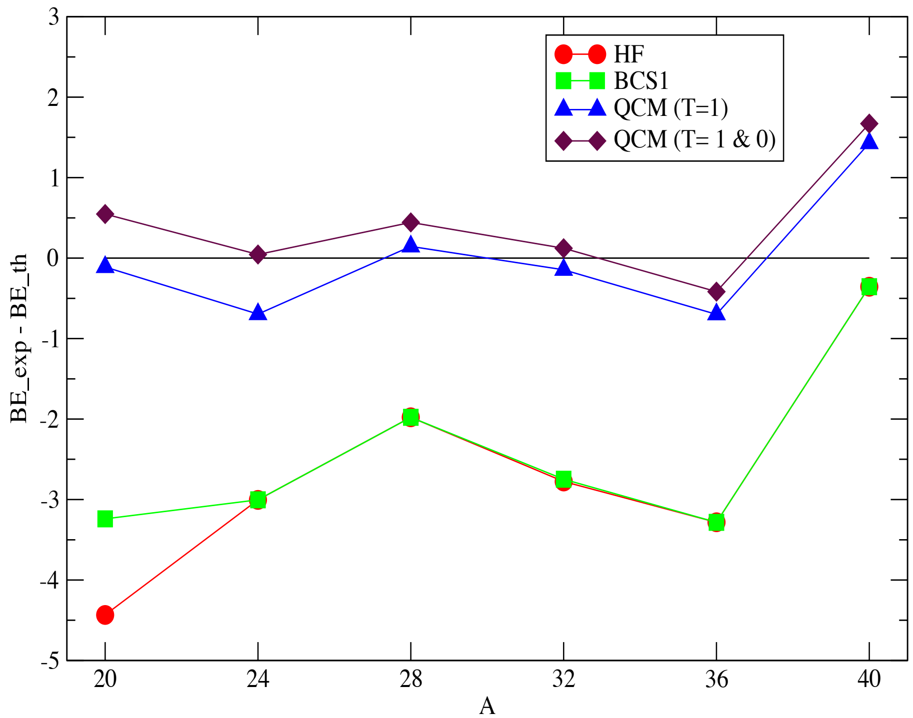
s=1.5 (fixed from the gaps in Sn isotopes)

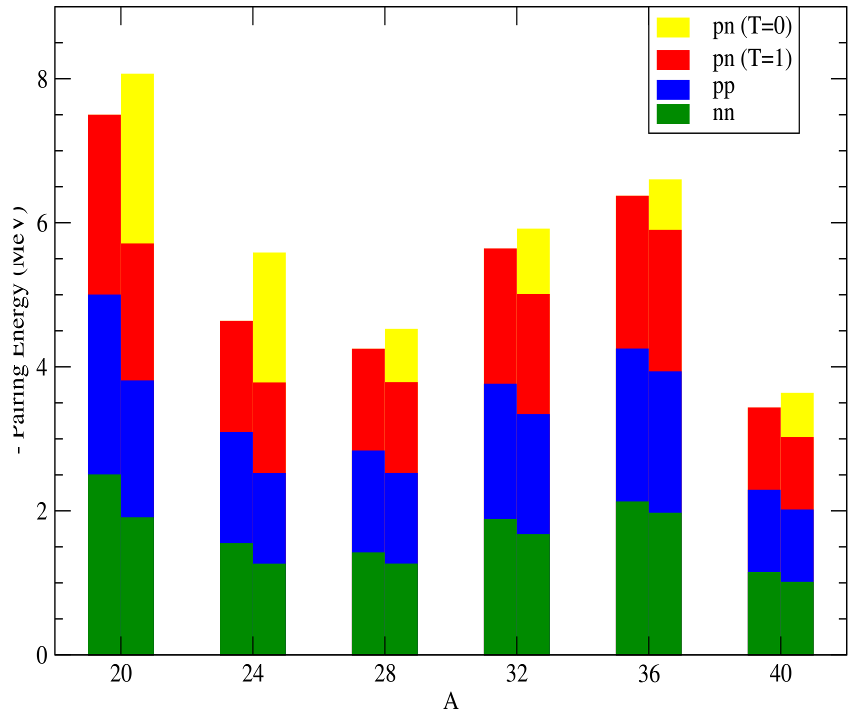
- Isoscalar pairing force: pairs with (T=0,Jz=0), Sz=0,1,-1

$$V^{T=0} = w\ V^{T=1} \qquad w=1.6$$

Binding and pairing energies : sd-shell nuclei

Preliminary results





T=1 & T=0 pairing account for the missing binding energies

dominant contribution from T=1

T=1 & T=0 proton-neutron pairing coexist in all the nuclei

A. Popa, N.S et al, in preparation

Summary and Conclusions

- α-like quartets are the appropriate degrees of freedom for describing pn pairing

  quartets appears naturally by imposing ispin-isospin conservation

- Quartet condensation model (QCM) describes accurately the pn pairing (errors < 1%)

  proj-HFB is a particular approximation of QCM

- Proton-neutron pairing has an important contribution to binding energies of N=Z nuclei

  results of Skyrme+QCM and Quark-meson-coupling +QCM calculations

- The band-structures in N=Z nuclei can be generated from intrinsic quartet condensates

Perspectives

- Probing α-like condensation: α-transfer in N=Z nuclei ?
- Josphson-like effect related to α condensation ?

VOLUME 34, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JANUARY 1975

Systematics of the (d, ^{6}Li) Reaction and α Clustering in Heavy Nuclei*

F. D. Becchetti, L. T. Chua, J. Jänecke, and A. M. VanderMolen
Cyclotron Laboratory, Physics Department, The University of Michigan, Ann Arbor, Michigan 48105
(Received 5 August 1974)

Data for the α-particle pickup reaction (d, ^{6}Li) have been obtained at 35-MeV bombarding energy for even-even nuclei from ^{12}C to ^{238}U. The cross sections for the transitions to the ground states decrease approximately as $1/A_t^3$ where A_t is the target mass. α-particle transfer probabilities have been extracted from the data and are found to be substantially enhanced in heavy nuclei away from shell closures, particularly for deformed nuclei near $A \approx 150$. α-particle correlations appear to be related to two-nucleon pairing effects.

" It has been suggested that heavy-ion reactions involving transfer of two nucleons between superconducting nuclei [...] should exhibit enhancement phenomena similar to those observed in the Josephson effect in ordinary superconductors. Such an effect might also be observed in **the alpha-transfer between alpha-superconducting nuclei.**"

Thank you for your attention !