Topology change and non-geometry at infinite distances

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Based on upcoming work
in collaboration with Thomas Raml and Dieter Lüst

November 30, 2023
LaPTh, Annecy

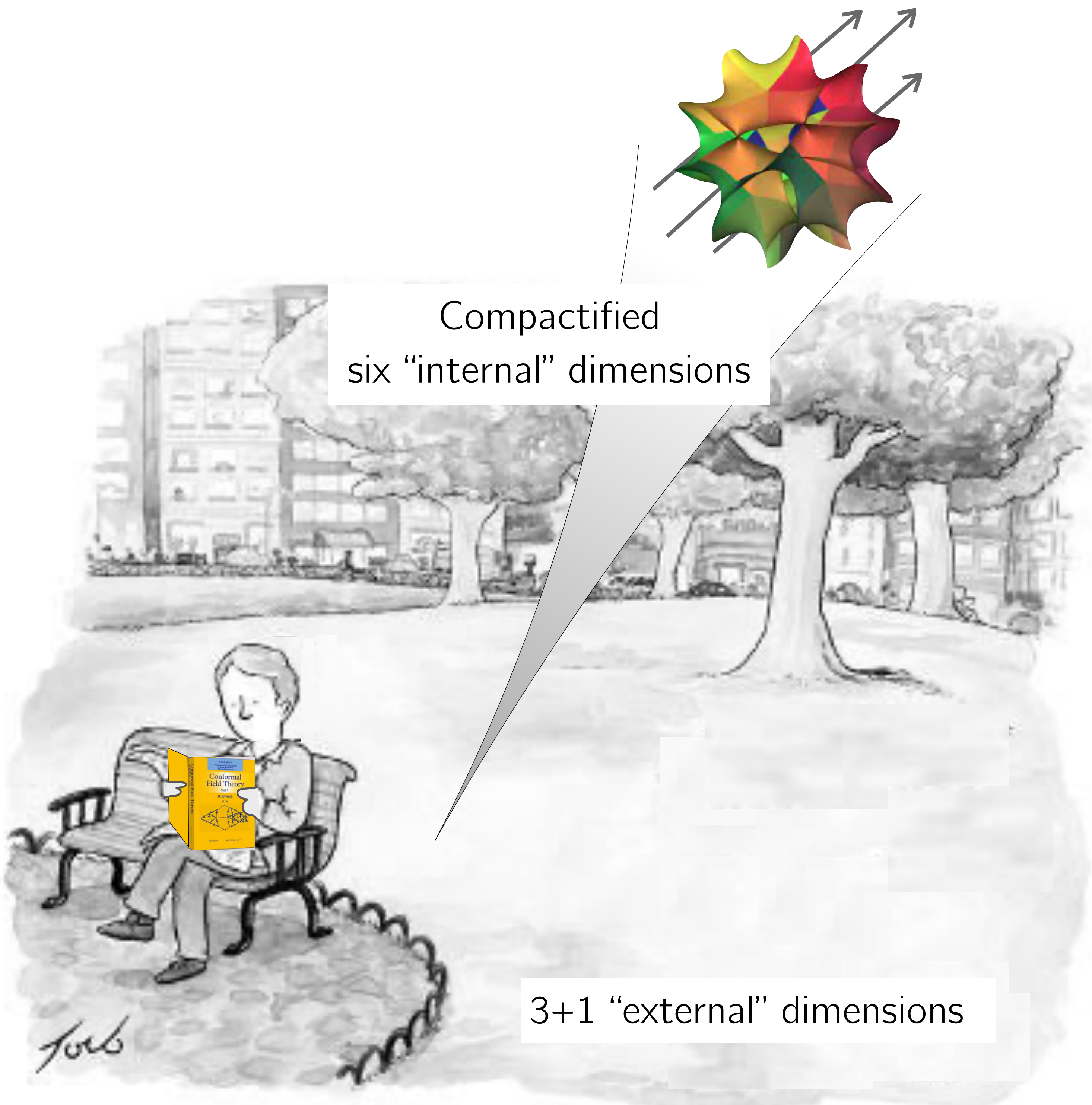
String theory and quantum gravity

Quantum mechanics and gravity
can be unified within string theory

A consistent theory of strings
requires to live in 10 dimensions

↓

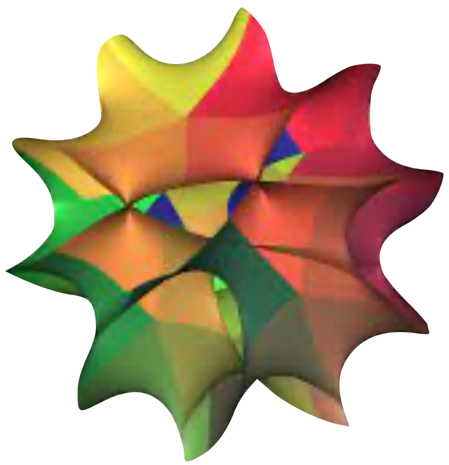
Requires studing compactifications



String compactifications

What kind of string compactification
can arise from a theory of quantum gravity ?

- ▷ What is the **shape** of this internal manifold ?
- ▷ What are their **properties** ?
- ▷ What **physics** do they contain?



Calabi-Yau manifolds are however **very hard to study**....

$\longrightarrow$ instead **study simpler manifolds that display critical properties**

Guiding principles and tools: **T-duality and integrability**

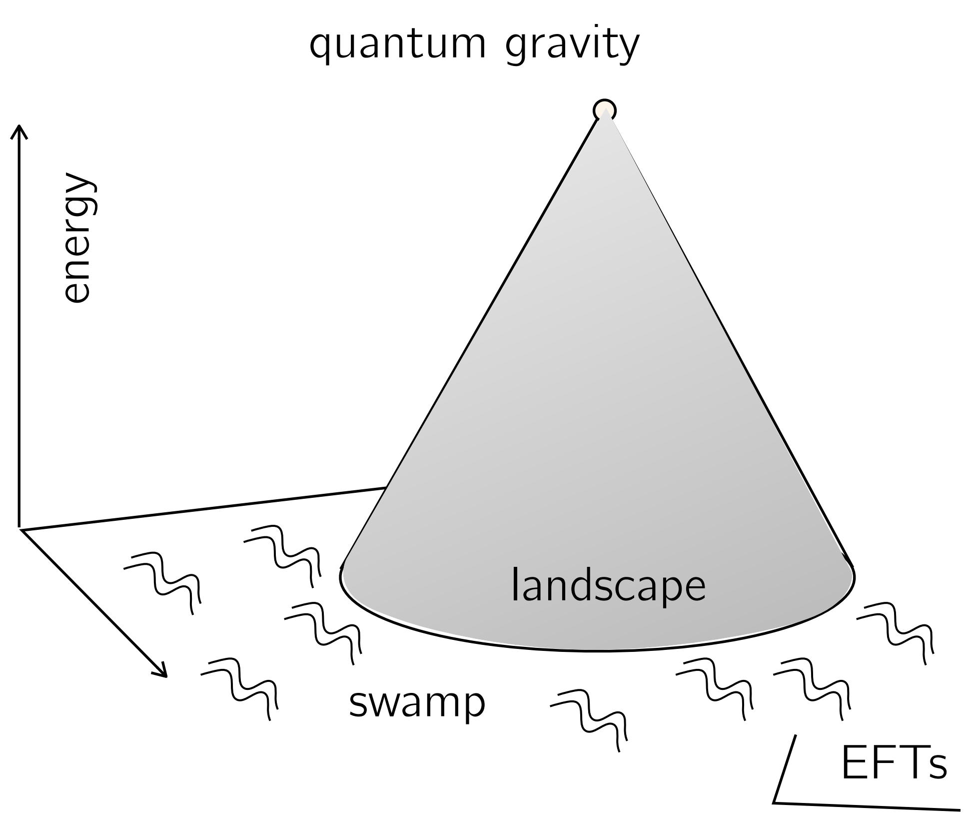
Quantum gravity and the swampland

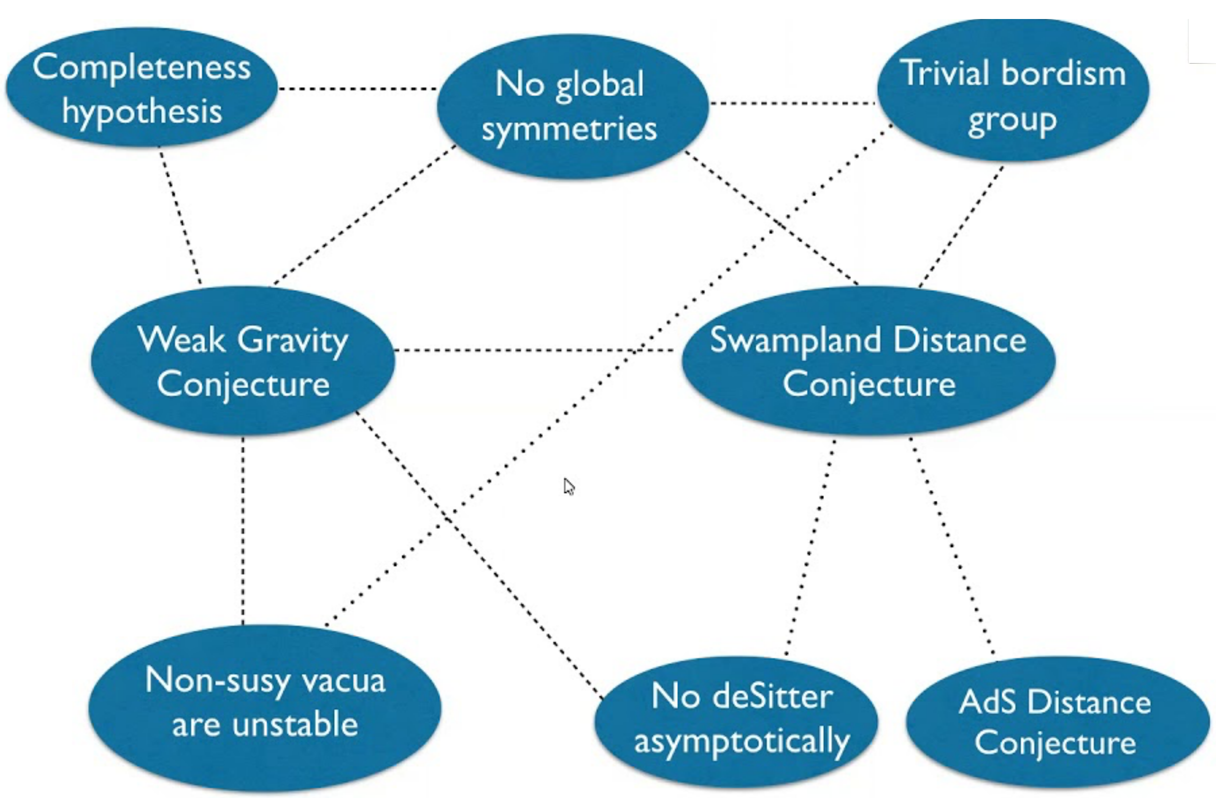
Quantum field theories that emerge in the low energy limit of a quantum gravity theory are **very special**

Swampland program

identify criteria for what **"very special"** concretely means

By identifying a set of defining conjectures





From I.Valenzuela's lecture "Navigating the Web of Swampland Conjectures"

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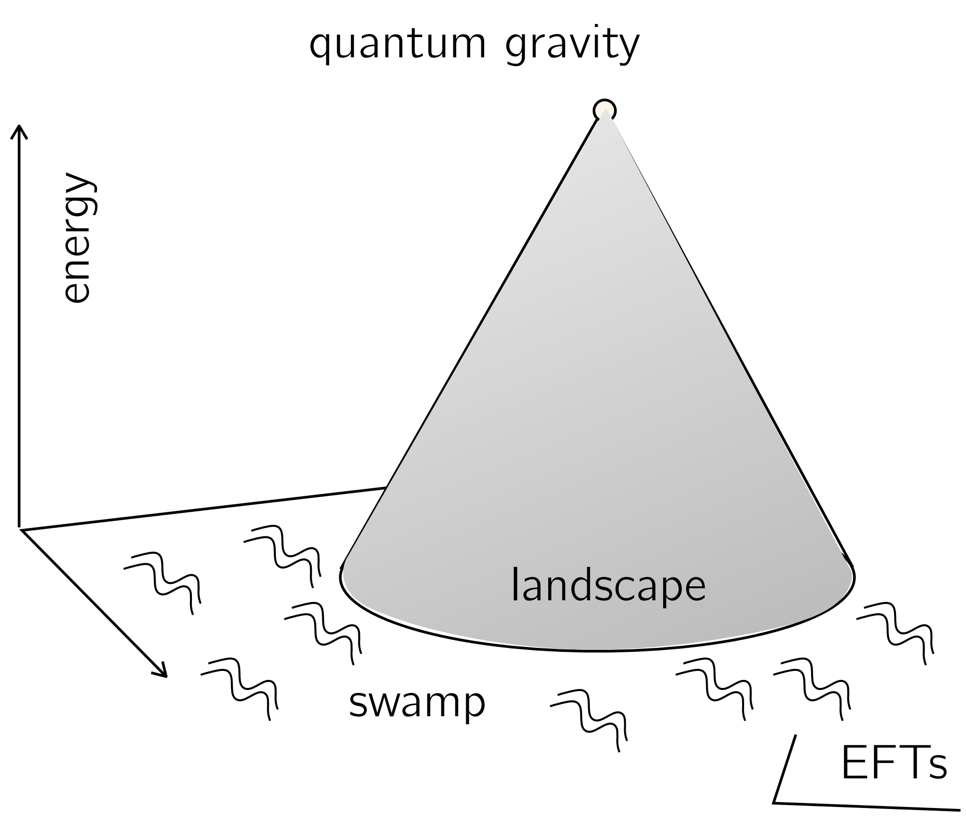

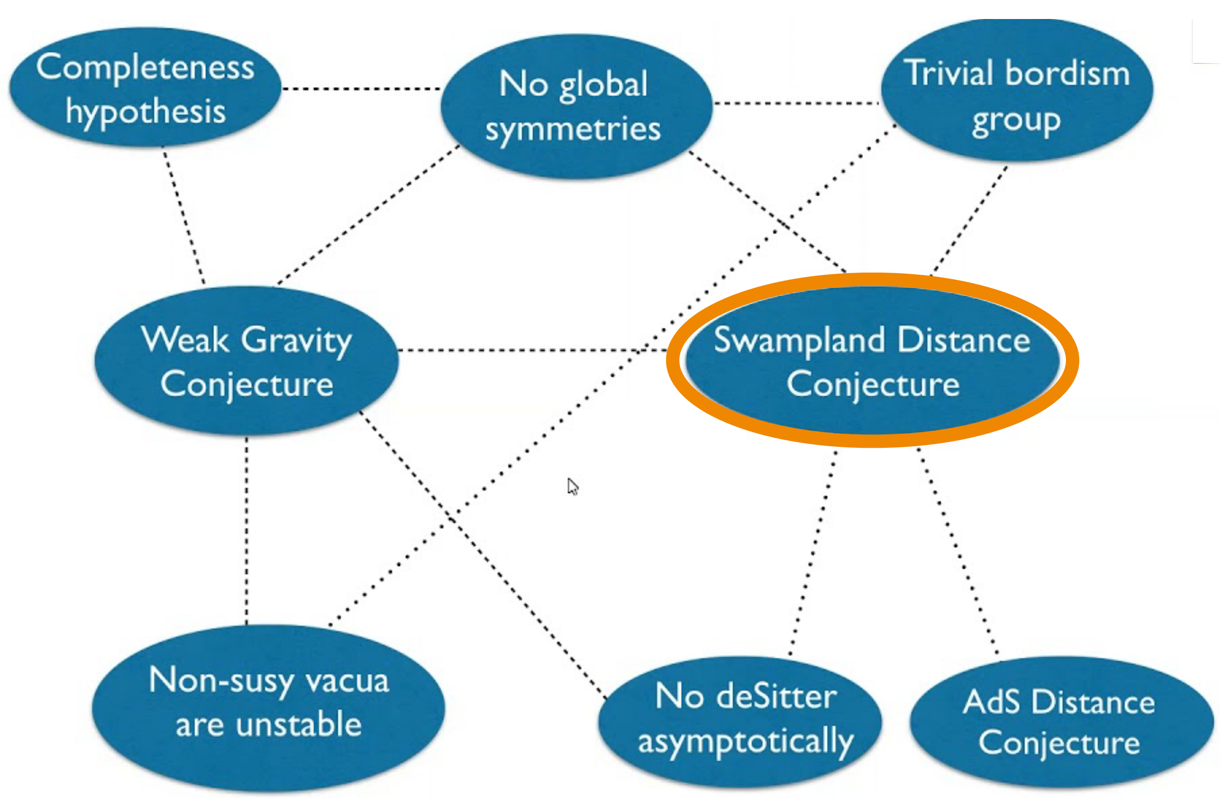

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Outline

$\longrightarrow$ Quick intro to the Swampland distance conjecture

$\longrightarrow$ Relation to T-duality: a simple example

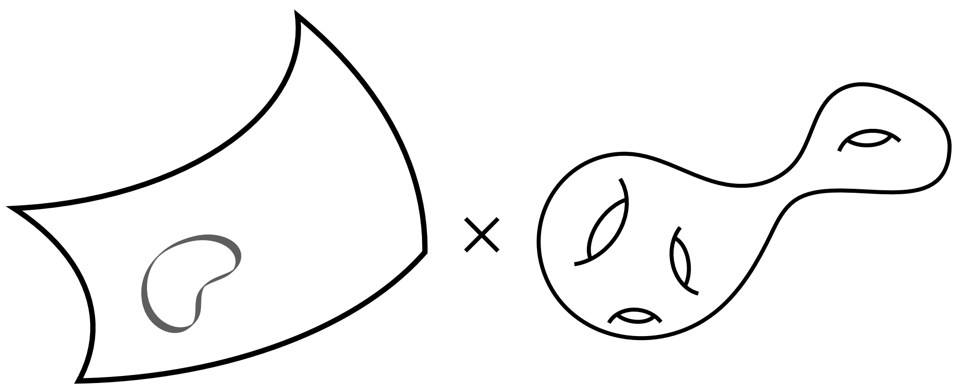
$\longrightarrow$ Challenging the conjecture: change of topology

$\longrightarrow$ Extending the Swampland distance conjecture

[Ooguri, Vafa] ....

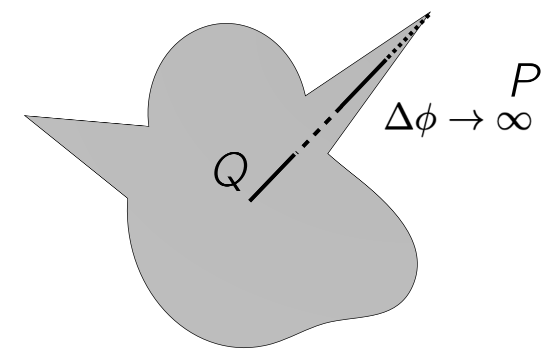
The Swampland distance conjecture

Target space manifold



External space   internal space

Moduli space



describes the parameters
of the internal space

In any consistent theory of quantum gravity:

When going to large distances in its moduli space,
encounter an infinite tower of particles which become light exponentially

$$M(Q) \sim M(P) e^{-\lambda \Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty \quad \text{and} \quad \Delta\phi \equiv d(P,Q)$$

[Ooguri, Vafa] ....

The Swampland distance conjecture

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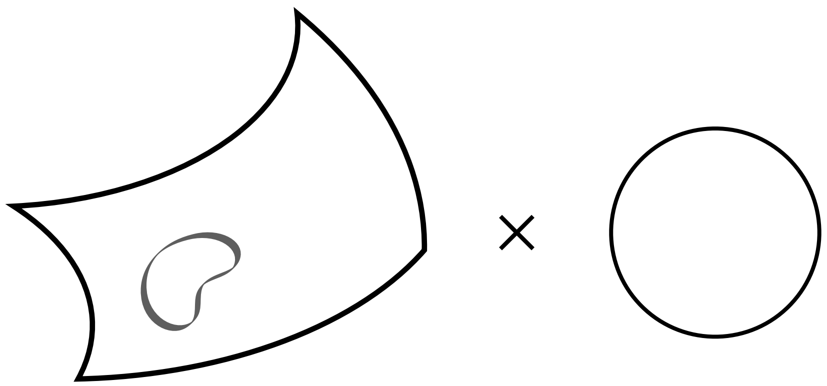

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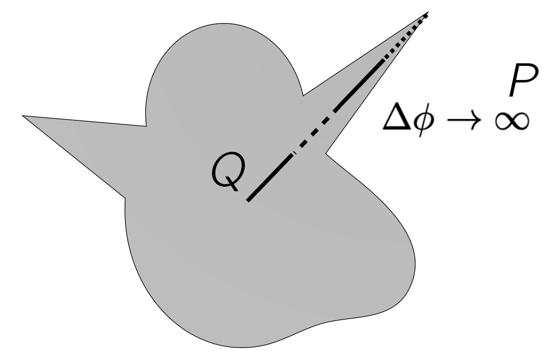

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A very simple example: the free boson on a circle

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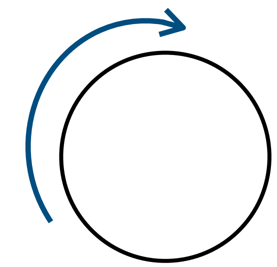
$R_{\mathrm{d}} = 1/R$



Moduli space:

$R = 0$ — infinite distance point

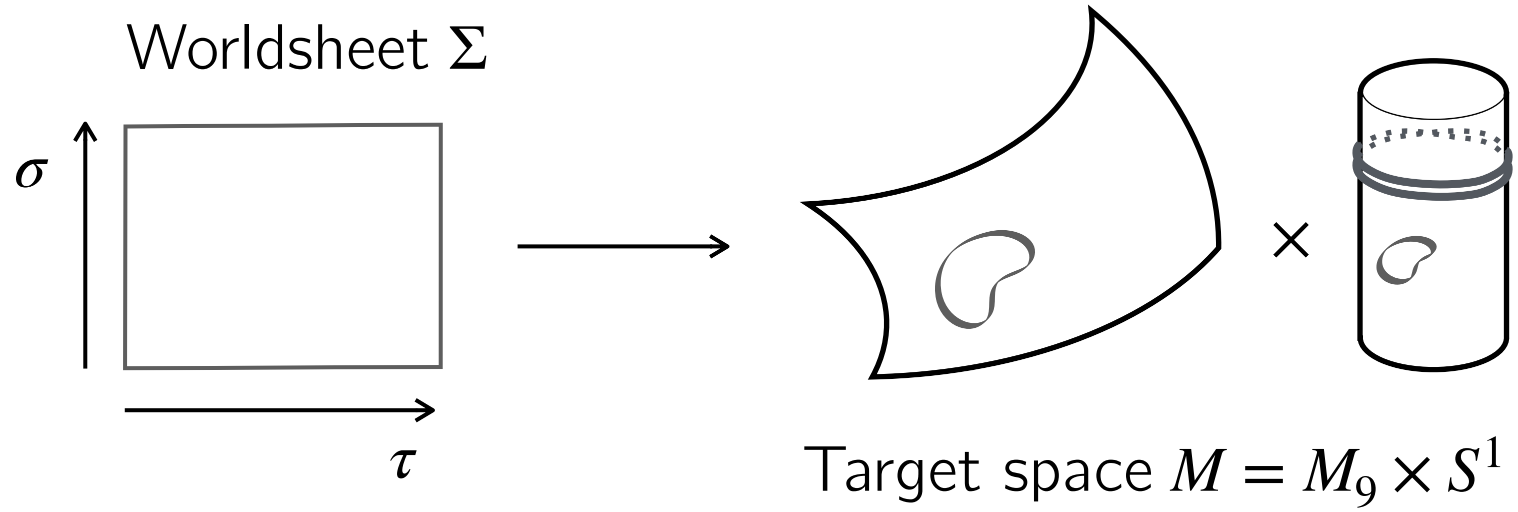
$R \to \infty$ — infinite distance point

At $R \to \infty$
Infinite tower of
massless KK-modes
&
At $R_{\mathrm{d}} = 0$
Infinite tower of
massless winding-modes

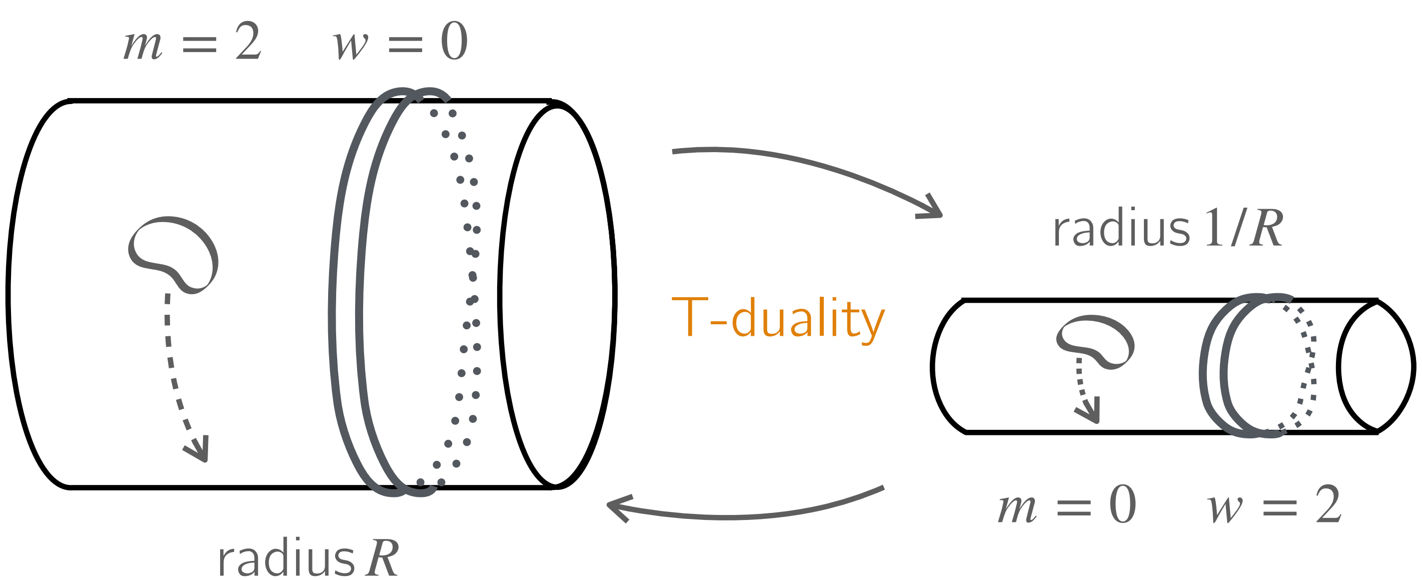
$$(M_{n,w})^2 = \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} \left(\frac{n}{R}\right)^2 + (2\pi R)^{\frac{2}{d-2}} \left(\frac{wR}{\alpha_0'}\right)^2$$

Infinite towers of massless states ✓

Compact dimensions and T-duality

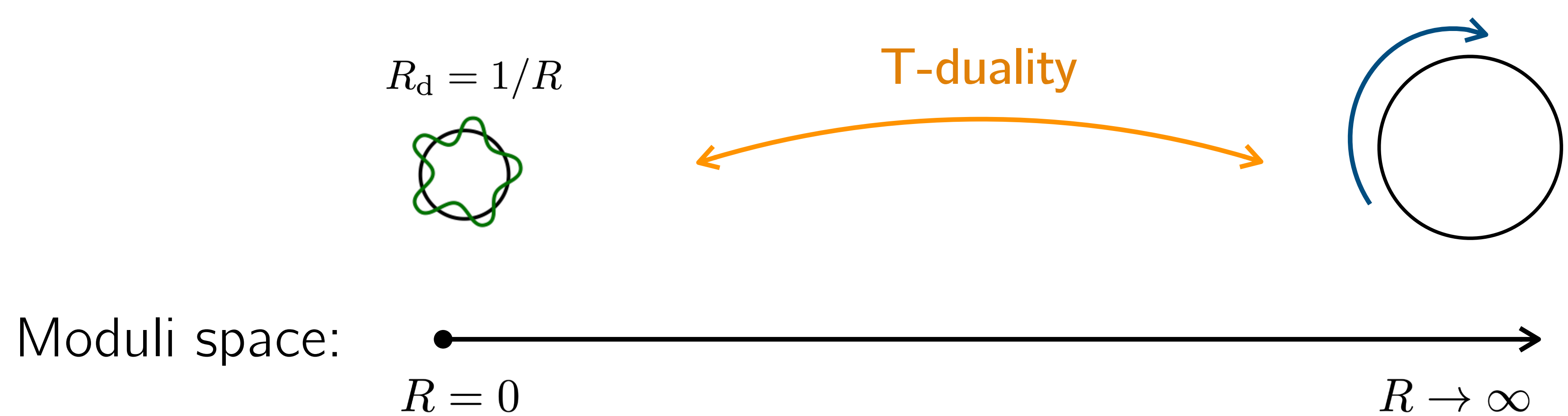


$\rightarrow$ For a string: "large circle" $\equiv$ "small circle" $\Rightarrow$ T(oroidal)-duality



exchange of winding and momentum modes

Swampland distance conjecture and T-duality



infinite distance point

infinite distance point

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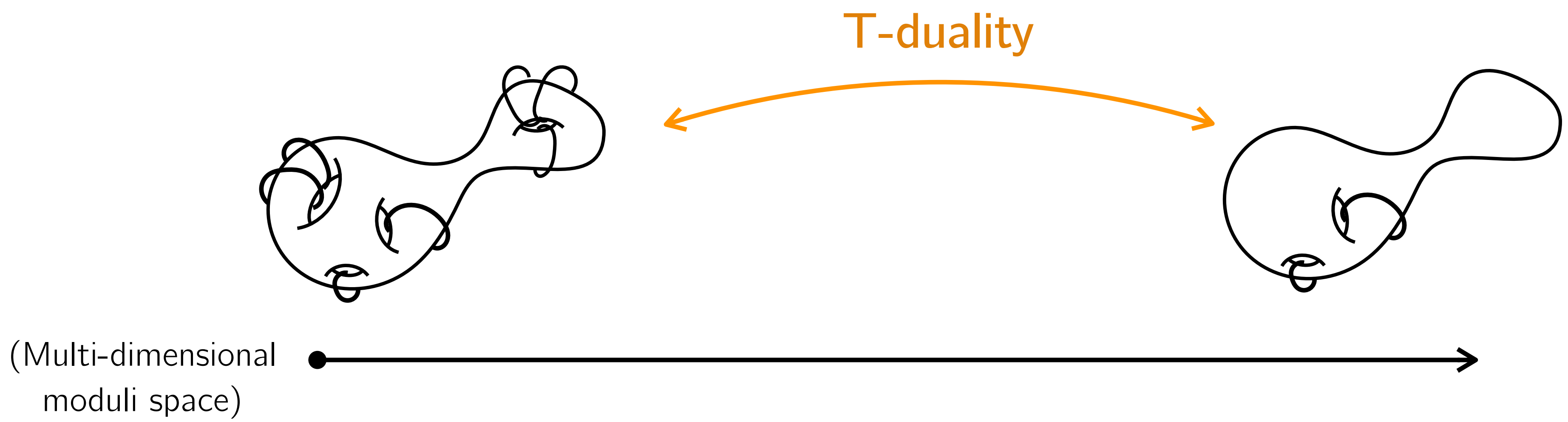
What about more complicated geometries ?

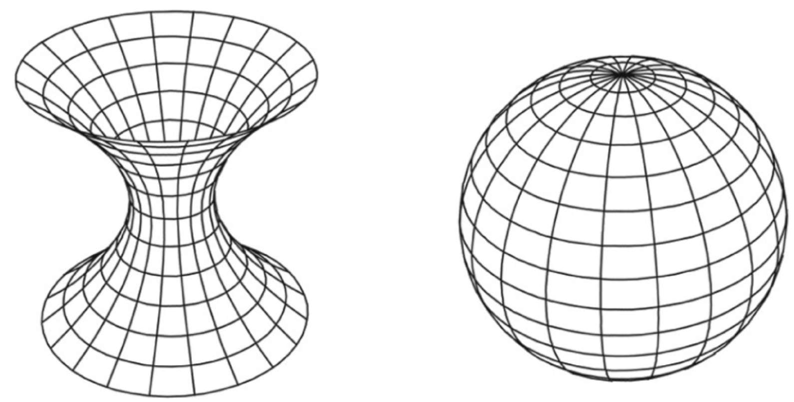


A much more challenging question...

- Backgrounds have now curvature, fluxes: sources a scalar potential
- Under T-duality may display changes in topology
- Non-geometric backgrounds

Presents a challenge to the Swampland Distance Conjecture

Some details on string compactifications



Described by

$G_{ij}\,,\ B_{ij}\,,\ \phi\,,\ F$

Starting form a ten dimensional (supergravity) theory

$$S=\frac{1}{2\kappa_0^2}\int \mathrm{d}^D X\sqrt{-G}e^{-2\Phi}\left(\mathcal{R}(G)-\frac{1}{12}\mathcal{H}_{IJK}\mathcal{H}^{IJK}+4\partial_I\Phi\partial^I\Phi\right)$$

Lower dimensional effectivel field theory

$$\begin{aligned}S=\frac{1}{2\kappa^2}\int \mathrm{d}^d x\mathrm{d}^n y\sqrt{-g}\Omega\Bigg(&\mathcal{R}(g)-\frac{1}{12}\mathcal{H}_{\mu\nu\lambda}\mathcal{H}^{\mu\nu\lambda}e^{\frac{-8\phi}{d-2}}R^{\frac{4\alpha}{d-2}}-\frac{4}{d-2}\partial_\mu\phi\partial^\mu\phi\\&+\left(\mathcal{R}(h)-\frac{1}{12}\mathcal{H}_{ijk}\mathcal{H}^{ijk}+4\partial_i\Phi_y\partial^i\Phi_y\right)e^{\frac{4\phi}{d-2}}R^{\frac{-2\alpha}{d-2}}-\frac{3}{12}\mathcal{H}_{\mu jk}\mathcal{H}^{\mu jk}\\&-2\alpha\frac{d-3}{d-2}R^{-1}(\partial\phi\partial R)-\left(\frac{3}{12}(\partial_R B)^2+\frac{\alpha^2}{d-2}R^{-2}+\frac{1}{4}\mathrm{tr}(k^2)\right)(\partial R)^2\Bigg)\,.\end{aligned}$$

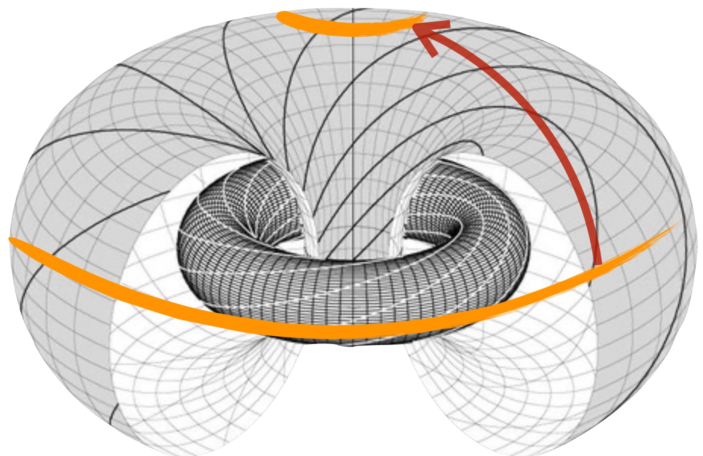
Leading to a non-vanishing potential $V\supset \mathcal{R}\,,\ \mathcal{H}^2\,,\ (\partial\Phi)^2\,,\ \ldots\,.$

curvature, fluxes, dilaton of the compact space

A simple example of change in topology

The three-sphere and its T-dual

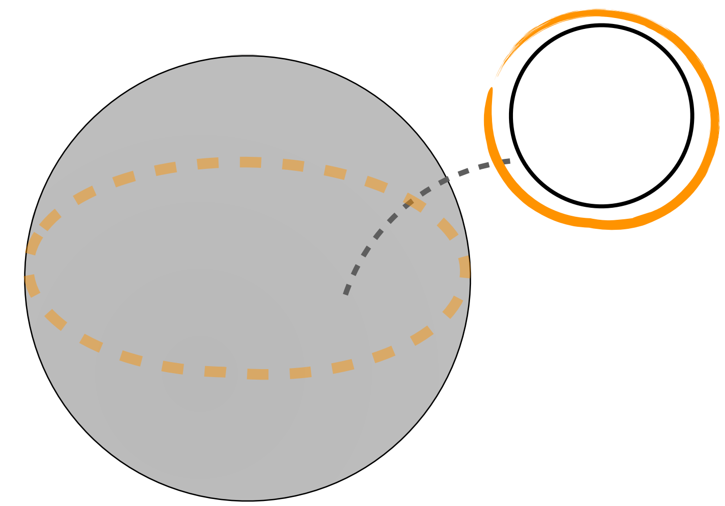
S^3 in the Hopf fibration



No winding
(all closed loops are contractible)

Abelian
T-duality
$\longleftrightarrow$

trivial fibration $S^2 \times S^1$



has winding modes

New phenomenon: winding/(certain) momentum modes are forbidden *!*

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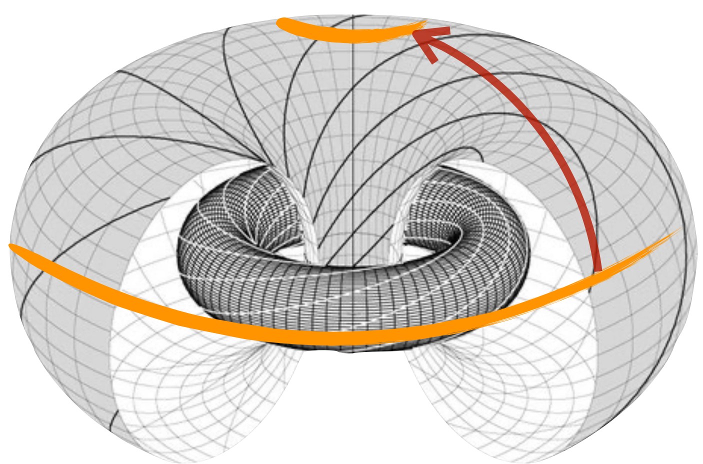

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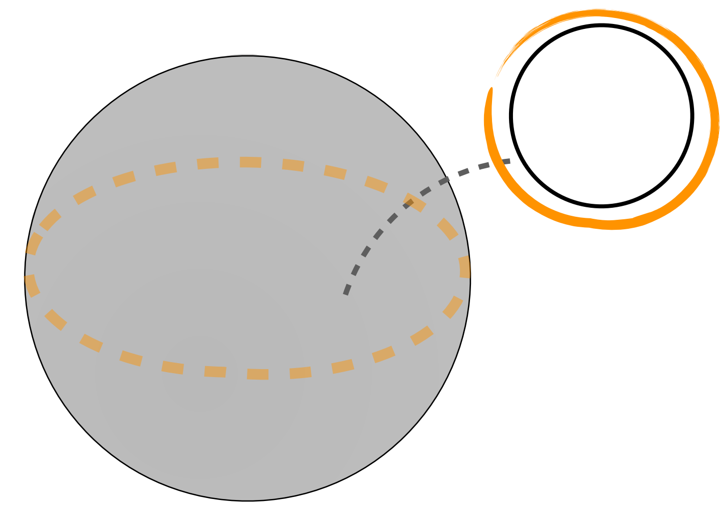

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T-duality can change topology: $\quad \text{fluxes} : \imath_k \mathcal{H} \quad \longleftrightarrow \quad \text{topology} : c_1(E_k)$

Calabi-Yaus also display changes in topology under mirror symmetry [cite]

$\rightarrow$ This poses a puzzle for the Swampland distance conjecture

A simple example of change in topology

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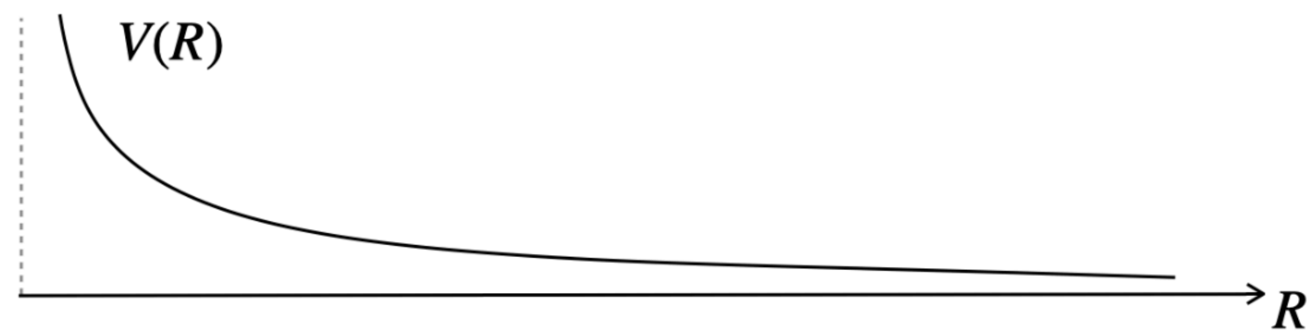

	$R \to 0$	$R \to \infty$	Potential V
$S^3_{\sqrt{k}}, \quad [\mathcal{H}] = k$	w : none p : heavy	w : none p : light	$3/2R^2$
$S^3/\mathbb{Z}_k, \quad [\mathcal{H}] = 1$	w : none* p : heavy	w : $\mathbb{Z}_{k\to\infty}$, heavy p : light	

A simple example of change in topology

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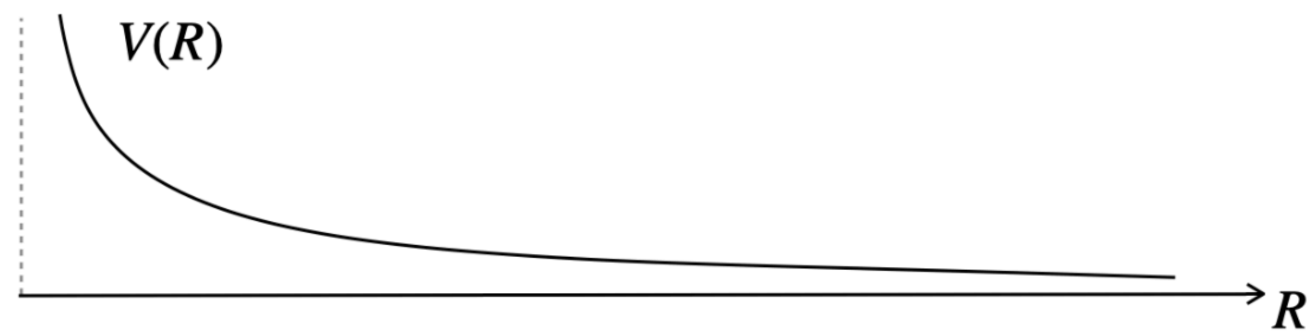

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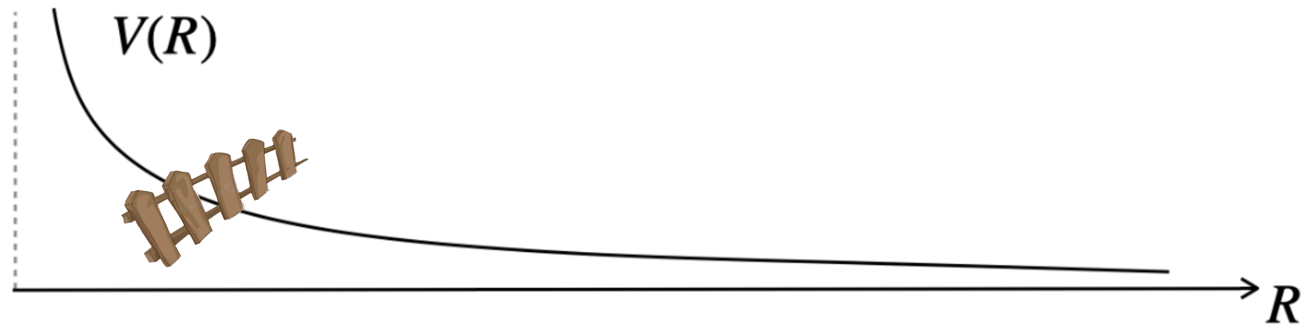
$$m^2 = \frac{l(l+n-1)}{R^2}, \quad l = 0, 1, 2 \dots$$

Extension to the Swampland distance conjecture



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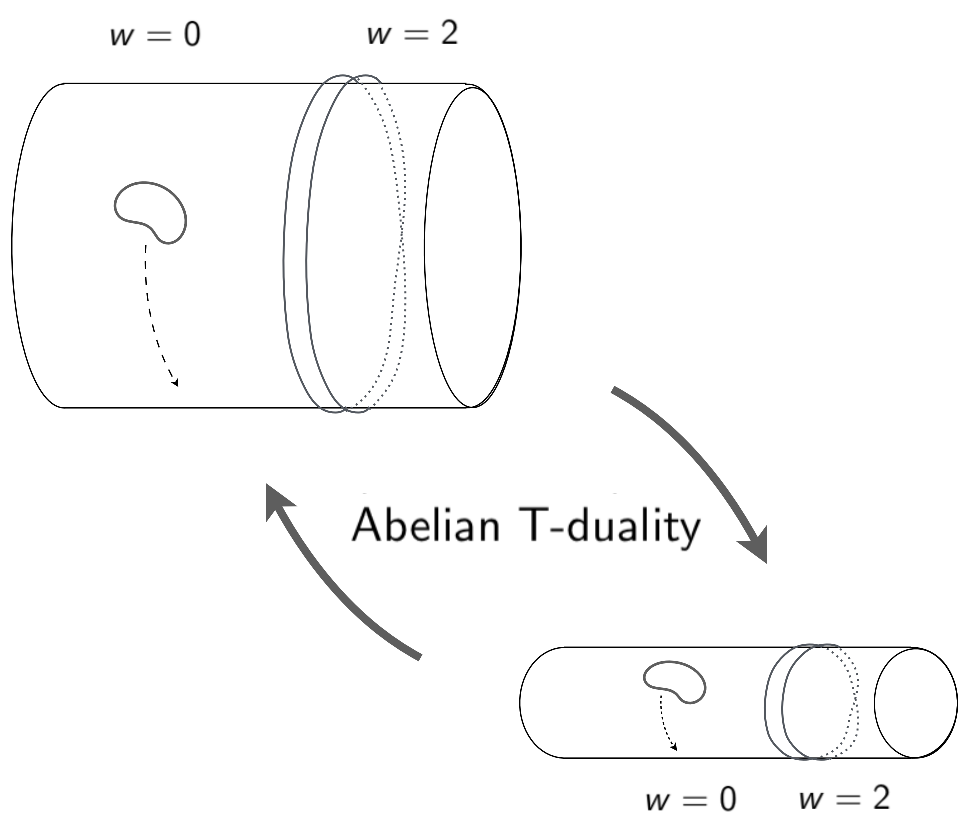
Suggests a generalisation to the Distance Conjecture: [SD, Lüst, Raml]

At infinite distance points of the moduli space
where the geometry cannot carry a light tower of states
the scalar potential will be divergent

That is, the potential effectively screens pathological infinite points.

Generalised T-duality in under 2 mins

Abelian T-duality



- Compactification on a circle
- Apply T-duality

$$1/R \leftrightarrow R$$

- momentum modes $\leftrightarrow$ winding modes

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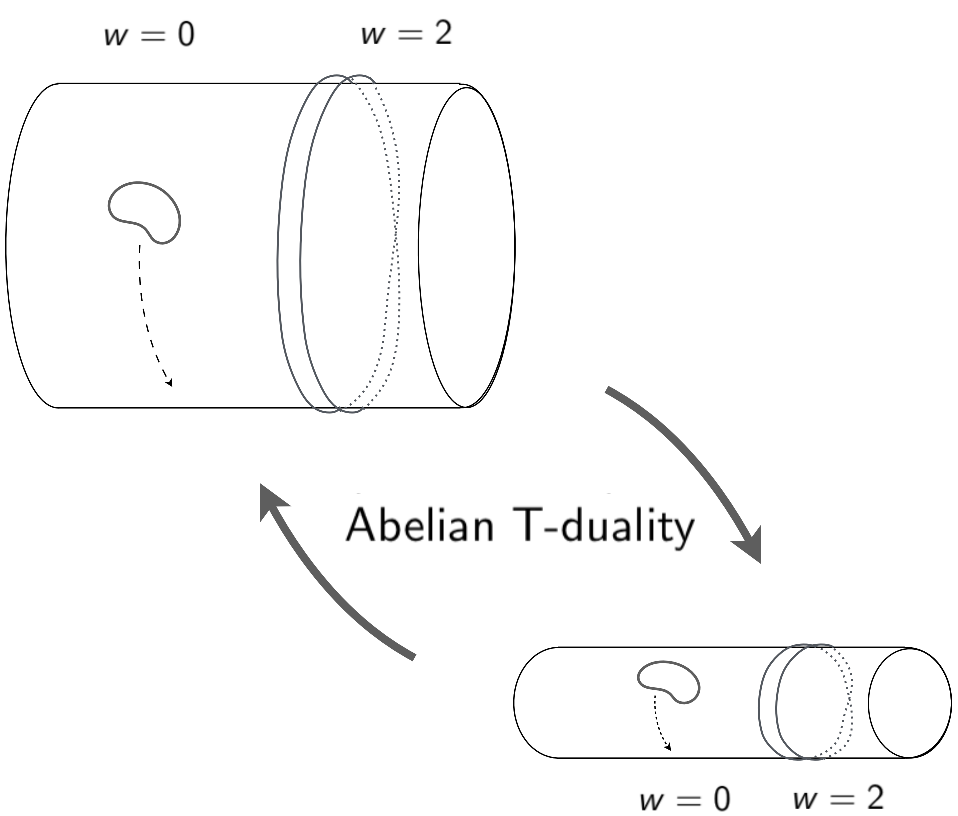

- ▷ Compactification on a circle
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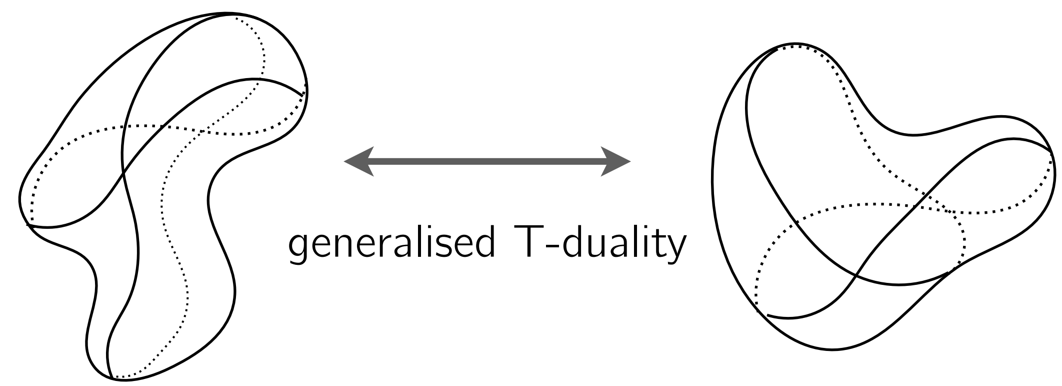
non-Abelian T-duality

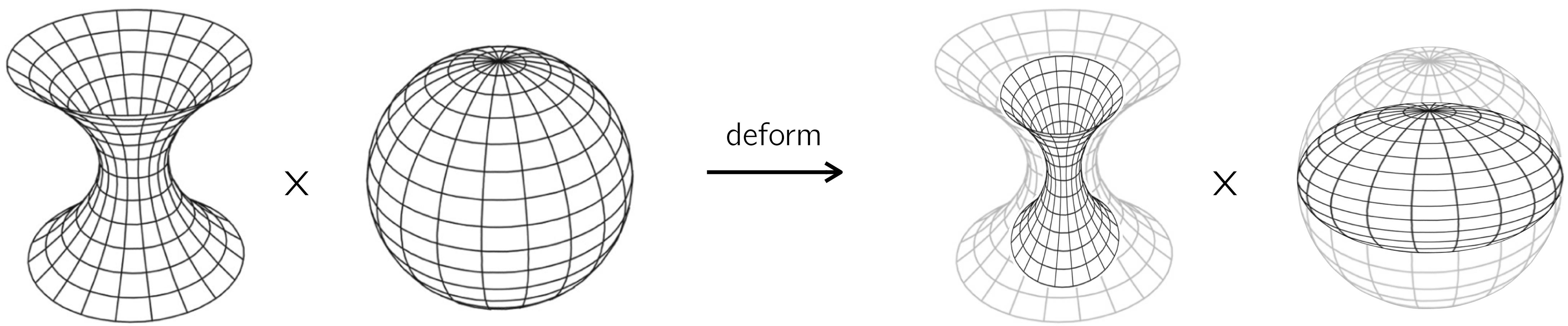


- ▷ Canonical transformation
- ▷ Solution generating technique
- ▷ Useful to generate parametric families of supergravity solutions which can be integrable

Deforming the string theory side:
integrable deformations of string backgrounds

They deform the background (and thus the symmetries), whilst remaining integrable and are closed related to generalised T-duality

For example: $AdS_5 \times S^5 \quad \longrightarrow \quad (AdS_5 \times S^5)_\eta$



En corresponding deformed (g)SUGRA solutions:

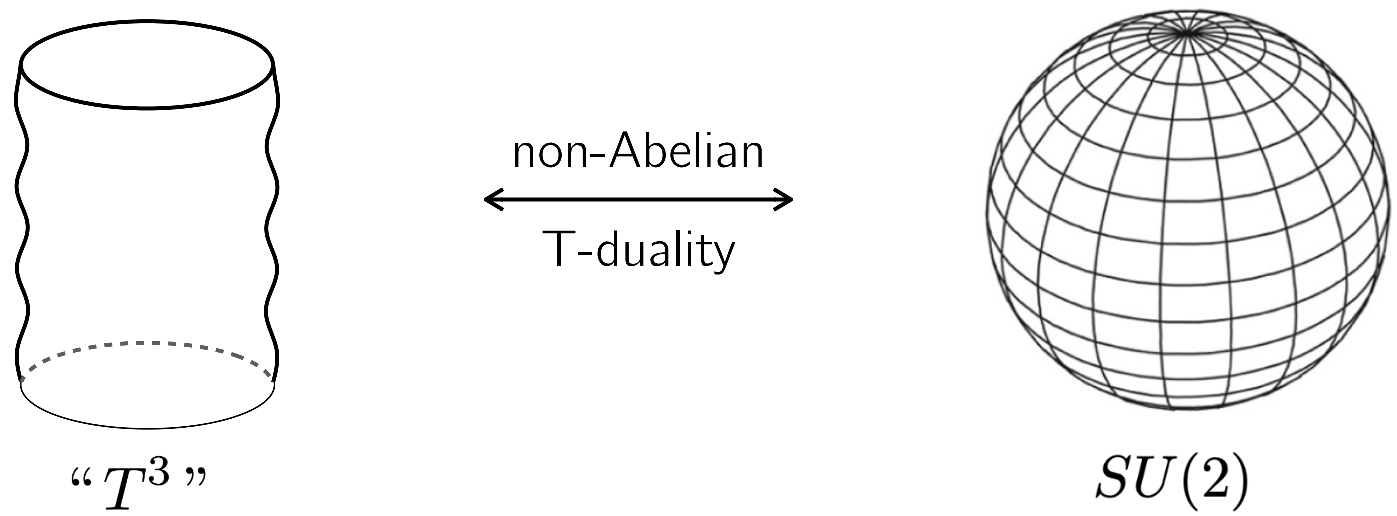
$$G_{ij}\,,\; B_{ij}\,,\; \phi\,,\; F \quad \longrightarrow \quad G^{\eta}_{ij}\,,\; B^{\eta}_{ij}\,,\; \phi^{\eta}\,,\; F^{\eta}$$

Offers the possibility to explore the SDC in less symmetric parametric families of solutions

[SD, Raml; wip]

S³ and non-Abelian T-duality

We take to be a three-sphere $SU(2) \cong S^3$ and its non-abelian T-dual



$$U(1)^3 : w = \mathbb{Z}^{\oplus 3} \text{ and } m = 0 \quad \longleftrightarrow \quad SU(2) : w = 0 \text{ and } m = \mathbb{Z}^{\oplus 3}$$

[Klimčík, Ševera]

$\rightarrow$ encounter a very similar problem: missing zero modes to provide towers

But does non-trivially meets expectations of the SDC !

$$V_{S^3} = \sigma^{-2}\rho^{-1}V_f^0 \sim R^{-2}$$
$$V_{\text{NATD}} = \tilde{\sigma}^{-2}\tilde{\rho}V_Q^0 \sim R^{-2}\,,$$

Summary

→ What compactification spaces lead to consistent Quantum Gravities ?

→ Curvature and fluxes challenge the current form of the Distance Conjecture

→ The moduli space is a feature-full manifold:

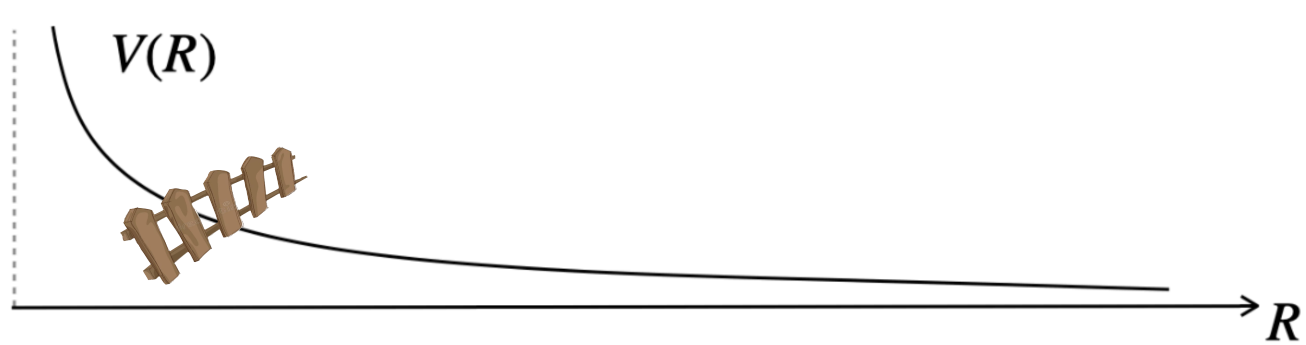
→ The potential cannot be discarded

→ How does it transform under T-duality ?

→ What is global and local structure ?

→ We propose to an extension to the SDC

There is protecting mechanism triggered by the potential
whenever the (global) geometry does not warrant for a tower of states



Outlook

Investigate richer examples with non-geometry and fluxes

Study the geodesic flow on the moduli space

Are two T-dual moduli space always equivalent?

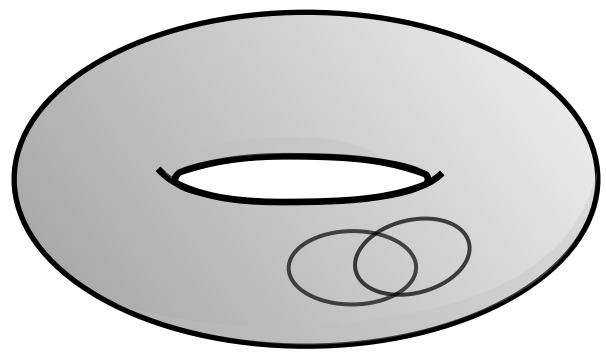
Exploit T-duality and integrability to test (supersymmetric) reductions?

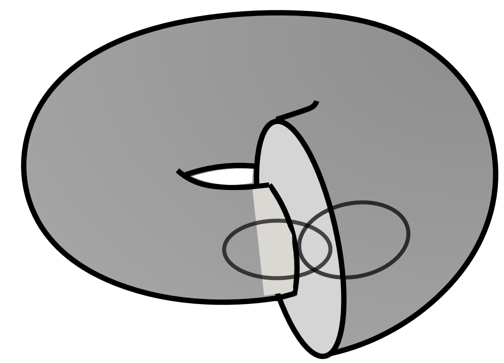
Thank you for your attention !

Backup slides

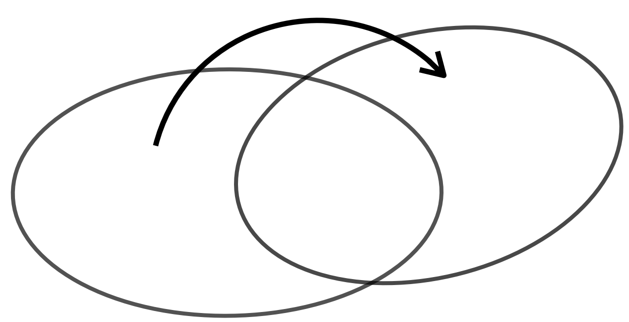
Non-geometric backgrounds

Manifold, e.g. a torus vs. non-geometric background

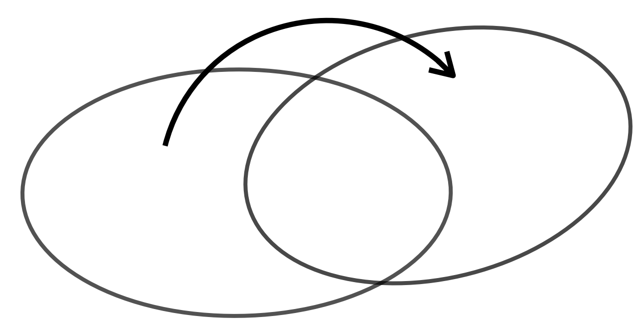




diffeomorphism

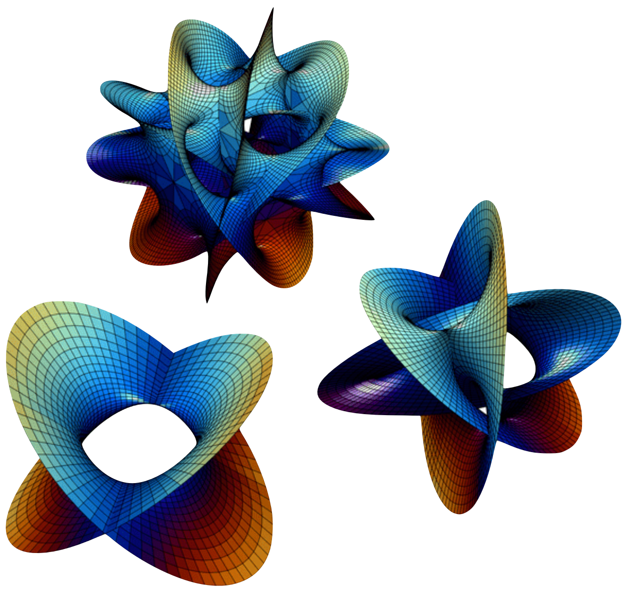


... and T-duality



Are these **valid backgrounds** for **quantum gravity?**

Geometries and moduli



Moduli?

Free parameters, specifying the size and shape of the internal manifold

Why fix moduli?

They are unwanted because they lead to a fifth force,
Might destroy successful prediction of Big Bag nucleosynthesis
Change the gauge and Yukawa couplings
....

How to fix them?

Create potential sourced by curvarture and fluxes

Geometries and moduli

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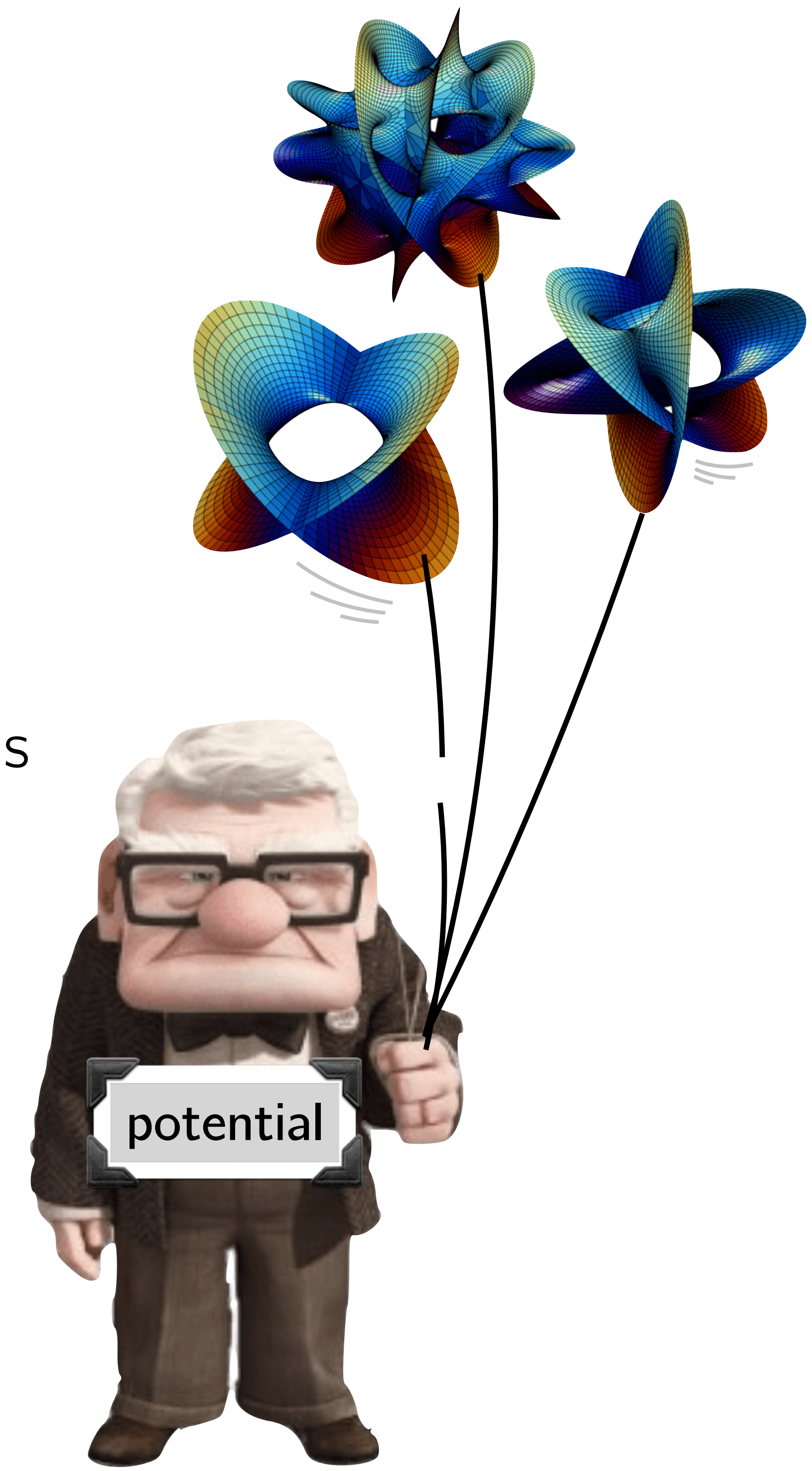

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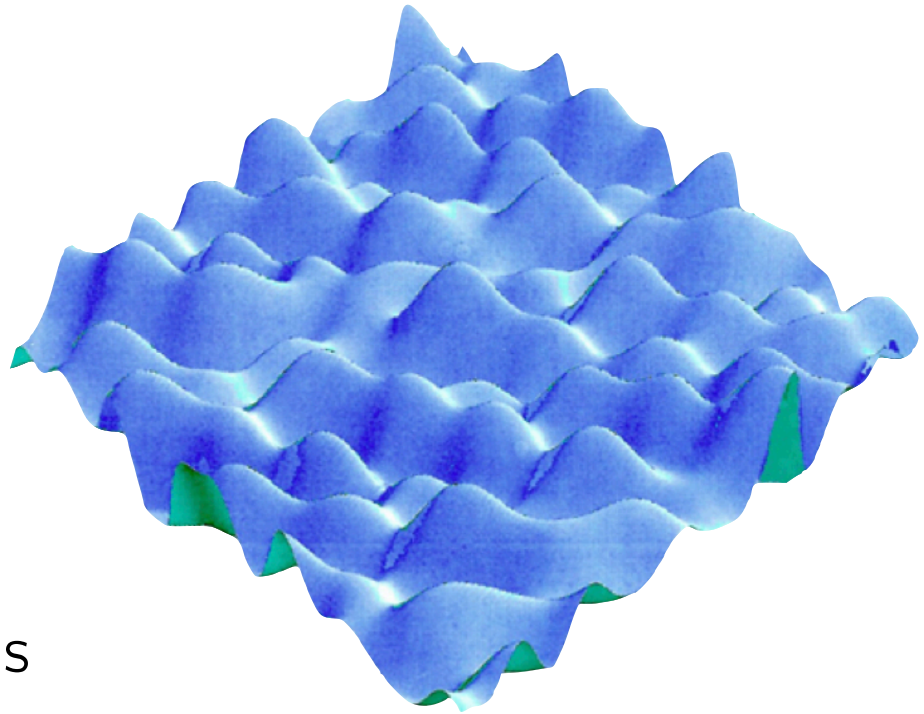

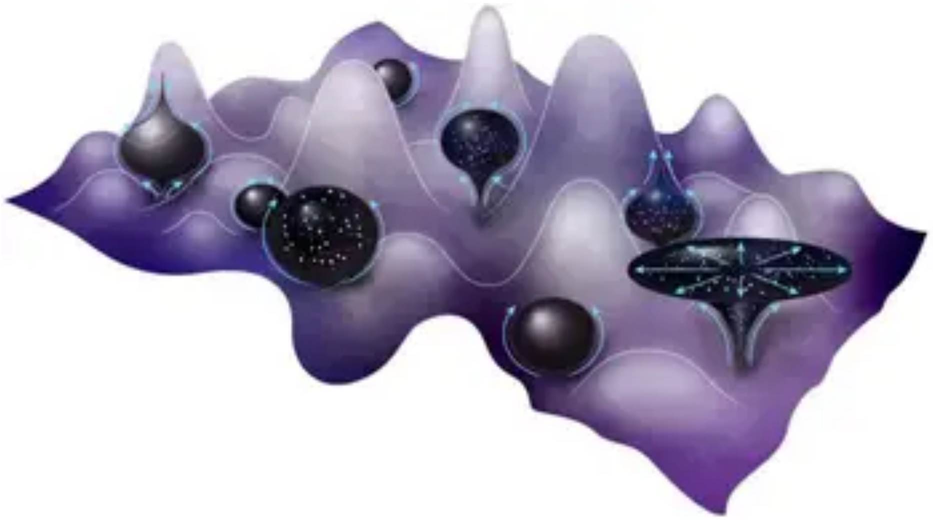

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T-duality and its generalisations

The sigma-model characterisation of T-duality

$$S = \int \mathrm{d}^2\sigma (G_{ij} + B_{ij}) \partial_\mu X^i \partial^\mu X^j = \int \mathrm{d}^2\sigma E_{ij} \partial_\mu X^i \partial^\mu X^j \,, \qquad J_{a,\pm} = k_a{}^i E_{ij} \partial_\pm X^j$$

Abelian T-duality	Abelian isometry	*exact symmetry of string theory*
$[k_a, k_b] = 0$	$L_{k_a} E_{ij} = 0$	$\mathrm{d} \star J_a = 0$
non-Abelian T-duality	non-Abelian isometry	*solution generating technique*
$[k_a, k_b] = f_{ab}{}^c k_c$	$L_{k_a} E_{ij} = 0$	$\mathrm{d} \star J_a = 0$
Poisson-Lie T-duality	non-Abelian ~~isometry~~	*? (and rest of the talk)*
$[k_a, k_b] = f_{ab}{}^c k_c$	$L_{k_a} E_{ij} = \tilde{f}^{bc}{}_a k_b{}^m E_{mi} E_{jn} k_c{}^n$	$\mathrm{d} \star J_a = \tilde{f}^{bc}{}_a J_b \wedge J_c$

Has a natural algebraic interpretation $\rightarrow$ G fits into a **Drinfel'd double** $D = G \bowtie \widetilde{G}$

determines the **dual background**

T-duality and its generalisations

The sigma-model characterisation of T-duality

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Poisson-Lie T-duality	non-Abelian ~~isometry~~	*? (and rest of the talk)*
$[k_a, k_b] = f_{ab}{}^c k_c$	$L_{k_a} E_{ij} = \tilde{f}^{bc}{}_a k_b{}^m E_{mi} E_{jn} k_c{}^n$	$\mathrm{d} \star J_a = \tilde{f}^{bc}{}_a J_b \wedge J_c$

Has a natural algebraic interpretation $\rightarrow$ G fits into a Drinfel'd double $D = G \cdot \widetilde{G}$

Jargon: G and $\widetilde{G}$ are called Poisson-Lie groups

The doubled structure of the η-deformation

The η-deformation/Yang-Baxter model were first constructed

as example of Poisson-Lie-dualisable models [Klimčik,Ševera '95]

.... and "accidentally" turned out to be integrable

$$S_{\eta\mathrm{PCM}} = \frac{1}{2\pi t}\int \mathrm{d}^2\sigma\,\mathrm{tr}\left(\partial_+ g g^{-1}\frac{1}{1+\eta\mathcal{R}}\partial_- g g^{-1}\right) \qquad g:\Sigma\to G$$

The R-matrix defines a second Lie algebra

$$G \equiv \mathrm{Lie}(\mathfrak{g},[-,-]) \qquad G_{\mathcal{R}} \equiv \mathrm{Lie}(\mathfrak{g},[-,-]_{\mathcal{R}}) \equiv \mathrm{Lie}\,\tilde{\mathfrak{g}} = \widetilde{G}$$

Form together the Drinfel'd double $D = G\cdot\widetilde{G}$

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$$S_{\eta\mathrm{PCM}} = \frac{1}{2\pi t}\int \mathrm{d}^2\sigma\,\mathrm{tr}\left(\partial_+ g g^{-1}\frac{1}{1+\eta\mathcal{R}}\partial_- g g^{-1}\right) \qquad g:\Sigma\to G$$

$$S_{\widetilde{\eta\mathrm{PCM}}} = \frac{1}{2\pi t}\int \mathrm{d}^2\sigma\,\mathrm{tr}\left(\partial_+ \tilde{g}\tilde{g}^{-1}\frac{1}{1+\eta\widetilde{\mathcal{R}}}\partial_- \tilde{g}\tilde{g}^{-1}\right) \qquad \tilde{g}:\Sigma\to \widetilde{G}$$

$D = \mathrm{Lie}(\mathfrak{g}\oplus\mathfrak{g}_{\mathcal{R}})$ "doubled space"

"reduction"

YB model on		YB model on
$G \equiv \mathrm{Lie}(\mathfrak{g},[-,-])$	$\longleftrightarrow$	$G_{\mathcal{R}} \equiv \mathrm{Lie}(\mathfrak{g},[-,-]_{\mathcal{R}})$

generalised T-duals

Poisson-Lie T-duality [Klimčik,Ševera '95]

$D = \mathrm{Lie}(\mathfrak{g} \oplus \mathfrak{g}_{\mathcal{R}})$ "doubled space"

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YB model on $G \equiv \mathrm{Lie}(\mathfrak{g}, [-,-])$ $\longleftrightarrow$ YB model on $G_{\mathcal{R}} \equiv \mathrm{Lie}(\mathfrak{g}, [-,-]_{\mathcal{R}})$

generalised T-duals

- Examples

Abelian T-duality	non-Abelian T-duality	Poisson-Lie T-duality
$D = U(1)^N \times U(1)^N$	$D = G \times U(1)^N$	$D = G^{\mathbb{C}} = G \cdot AN$
$G_0 \longleftrightarrow G_0^{-1}$	$E_0 \longleftrightarrow \tilde{E}_0 = [(E_0)_{ab} - f_{ab}{}^c \tilde{x}_c]^{-1}$	$S_\eta \longleftrightarrow S_{\lambda^\star}$

where $E_0 = G_0 + B_0$