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Monte Carlo tools

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https://klaus.pages.in2p3.fr/epos4/physics/lectures -> Monte Carlo tools

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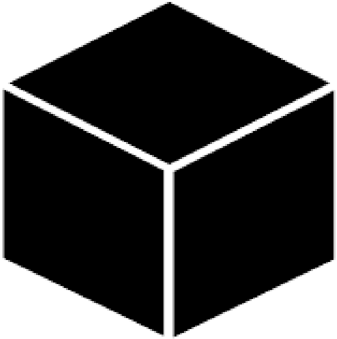
1 Introduction

1.1 What means "Monte Carlo Method"



Not simply a black box producing "events" of particles

based on some complex computer code with many if statements ...

Monte Carlo Method means

- ☐ **a tool to solve well defined mathematical problems**
- ☐ **based on probability theory**
  (random variables and random numbers)
- ☐ **based on equations**

Example: Compute $I = \int_0^1 f(x)dx$, which may be written as

$$I = \int_{-\infty}^{\infty} w(x)\, f(x)\, dx, \text{ with } w(x) = \begin{cases} 1 & \text{for } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

We may interprete w as probability distribution and I as expectation value (or mean value), so

$$I = \langle f \rangle = \underbrace{\frac{1}{N}\sum_{i=1}^{N} f(x_i)}_{MC\,estimate} + O\left(\frac{1}{\sqrt{N}}\right)$$

with uniform (in [0,1]) random numbers x_i

An error of order $1/\sqrt{N}$ is huge, nobody computes an 1D-integral like that, BUT for computing high-dimensional integrals, the formula

$$I = \int w(x_1, ..., x_n)\, f(x_1, ..., x_n)\, dx_1 ... dx_n$$

$$= \underbrace{\frac{1}{N} \sum_{i=1}^{N} f(x_1^{(i)}, ..., x_n^{(i)})}_{MC\ estimate} + O\left(\frac{1}{\sqrt{N}}\right)$$

is very useful.

Attention: generating $x_1^{(i)}, ..., x_n^{(i)}$ randomly arcording to some law w$(x_1, ..., x_n)$ is not trivial.

Monte Carlo Method (as discussed in this talk) means

- ☐ **a tool to compute integrals $\int w(X)\, f(X) dX$ of a multi-dimensional variable X**
- ☐ **as mean value $\langle f(X) \rangle$ with X distributed according to w (with w being a multi-dimensional distribution)**

Generating random numbers:
https://klaus.pages.in2p3.fr/epos4/physics/lectures -> Monte Carlo Simulation

1.2 Monte Carlo and Factorization

The most popular approach to treat HE pp, is based on "factorization", where the di-jet cross section is given as

$$\sigma_{\mathrm{dijet}} = \int \frac{d^3p_3 d^3p_4}{E_3E_4} \sum_{klmn} \int dx_1 dx_2\, f^k_{\mathrm{PDF}}(x_1,\mu_F^2)\, f^l_{\mathrm{PDF}}(x_2,\mu_F^2) \times \frac{1}{32s\pi^2} \bar{\sum} |\mathcal{M}^{kl\to mn}|^2 \delta^4(p_1+p_2-p_3-p_4) \times 1/(1+\delta_{mn}),$$

Integrals can be done[(1)] (4 δ functions).
But usually we need more details.

(1) arXiv:2306.02396, PRC 108, 034904 (also on https://klaus.pages.in2p3.fr/epos4/physics/papers)

Changing variables, integrating out five, dividing by σ_{dijet}:

$$1 = \int dx_1 dx_2 dt \, \frac{1}{\sigma_{\mathrm{dijet}}} \sum_{klmn} \int dx_1 dx_2 \, \underbrace{f^k_{\mathrm{PDF}}(x_1, \mu_F^2) f^l_{\mathrm{PDF}}(x_2, \mu_F^2) \frac{\pi \alpha_s^2}{s^2} \left\{ \frac{1}{g^4} \bar{\sum} |\mathcal{M}^{kl \to mn}|^2 \right\} \frac{1}{1 + \delta_{mn}}}_{w(x_1, x_2, t)},$$

$w(x_1, x_2, t)$ (*probability*)

How to get the cross section σ_y for particle production in the rapidity interval $I_y = [y - \Delta y/2, y + \Delta y/2]$?

First: $\sigma_y = \sigma_{\mathrm{dijet}} \times \bar{f}_y$; $\bar{f}_y$ = fraction of jets with $y_{\mathrm{jet}} \in I_y$

$$\bar{f}_y = \int dx_1 dx_2 dt \;\; w(x_1, x_2, t) \underbrace{\int_{I_y} dy' \, \delta(y_{\text{jet}} - y')}_{f_y(x_1,x_2,t)}$$

Two ways to handle that:

- □ work out $y_{\text{jet}}(x_1, x_2, t)$ and do the integral$^{(1)}$
- □ Monte Carlo method, based on

$$\bar{f}_y = \langle f_y \rangle_{\text{law}\, w}$$

(1) arXiv:2306.02396, PRC 108, 034904 (also on https://klaus.pages.in2p3.fr/epos4/physics/papers)

Monte Carlo method

(A) **Generate N triplets $x_1^{(i)}, x_2^{(i)}, t^{(i)}$ (events) according to the law $w(x_1, x_2, t)$**

not trivial, but doable – done by the MC authors

(B) **Compute the average**

$$\langle f_y \rangle_{\mathrm{law}\,w} = \frac{1}{N} \sum_{i=1}^{n} f_y(x_1^{(i)}, x_2^{(i)}, t^{(i)})$$

with $$f_y(x_1, x_2, t) = \left\{ \begin{array}{ll} 1 & \text{if } y_{jet} \in I_y \\ 0 & \text{otherwise} \end{array} \right\}$$

trivial – done by the MC users

We see again: Monte Carlo Method means

- ☐ **a tool to solve well defined mathematical problems (compute integrals)**
- ☐ **in the same way as classical numerical methods (Gaussian quadrature)**

One uses even very similar techniques in both cases, the same variable changes to get a "well-behaved" function w, to apply

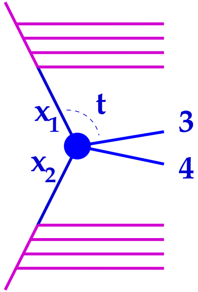
- ☐ the rejection method in case of MC
- ☐ Gaussian quadrature (with $n < 20$)
  in the case of numerical integration

Events

An event in the MC procedure is the set of generated random numbers

like $x_1^{(i)}, x_2^{(i)}, t^{(i)}$ in our example

On may associate a picture



But here the MC event (and the picture) do not correpond to a real physics event

The known "QCD event generators" (Pythia, Herwig,...) generate the "hard processes" in this way — not EPOS

1.3 Monte Carlo Methods and the Ising Model

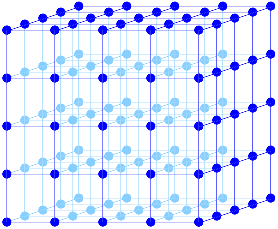
In general, we do not have to deal with triplets, but with n-tuples for large n.

Generating n-tuples distributed according to some given law $w(x_1, ..., x_n)$ is usually very complicated for large n

- **a problem well known in statistical physics since a long time**

- **with intelligent solutions**

Extremely useful: The Ising model of ferromagnetism



Box of $N \times N \times N$ atoms each one carrying a spin with possible values +1 and -1 (spin up, spin down)

- **Anyhow useful to know, one deals with phase trasitions very similar to the QGP phase transition**
- **The MC methods used there are precisely what we need for heavy ion simulations**
- **Good example of a multi-dimensional variable X, being here the N^3 spin values, let us call it a "state"**

The interesting quantity here is the average magnetization $\langle M \rangle$:

$$\langle M \rangle = \sum w(X)\, M(X)$$

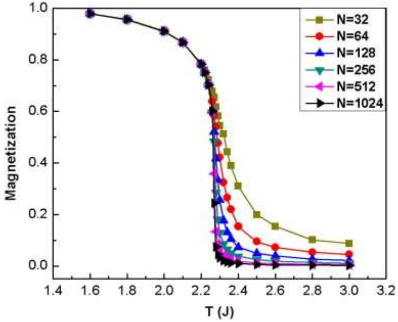
with

$$w(X) = \frac{1}{Z} e^{-\beta E(X)}$$

with

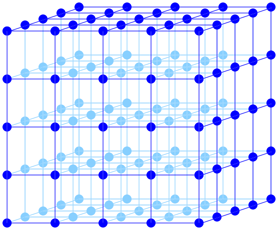
$$E = -\alpha \sum_{\text{neighbors } k,k'} s_k s_{k'}$$

Phase transition
2D Ising model



Journal of Computational Physics, 237, (2013) 224

Why difficult?



For N^3 atoms, the number K of possible states is $2^{\left(N^3\right)}$

$N = 100 : K \approx 10^{300000}$

Solution: Monte-Carlo method :

$$\langle M \rangle = \sum_{i=1}^{K} w(X_i)\, M(X_i) \quad \rightarrow \quad \frac{1}{J} \sum_{j=1}^{J} M(X_j)$$

with "reasonable" J, and X_j distributed according to $w(X)$

... provided we know how to generate X according to w(X)

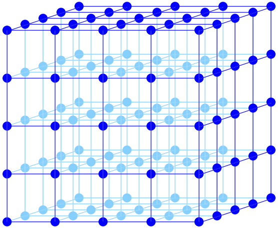
1.4 Ising Model and Markov chains

The problem is:

generate a "state" X according to

$$w(X) = \frac{1}{Z} e^{-\beta E(X)}$$

corresponding to "themal equilibrium"



X is one of the $2^{(N^3)}$ possible states of the lattice

Simple "direct methods" (rejection sampling) do not work.

Idea: Let's copy nature which always finds eventually the "equilibrium distribution"

One considers a stochastic iterative process (Markov chain)

$$w_1 \rightarrow w_2 \rightarrow \ldots$$

A. Markov

with appropriate transitions $w_t \rightarrow w_{t+1}$ (Metropolis) such that w_t converges to $w_\infty = \frac{1}{Z}\, e^{-\beta E(X)}$

(it works, thanks to "fixed point theorems")

Why useful for us ?

- **Markov chain + Metropolis is extremely powerful,**
  it works for ANY distribution and not just Boltzmann distributions

- **It allows to treat "parallel interactions" in high energy scattering**

- **We use it also for microcanonical QGP decay (needed for small systems)**

Crucial for the whole dicussion in this lecture: parallel scattering

1.5 Parallel and sequential scattering in AA

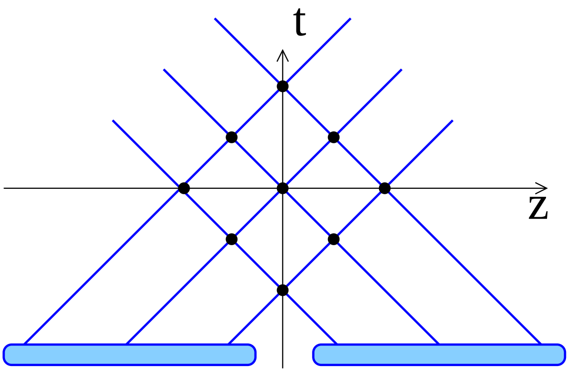
What kind of model do we need depending on the collision energy

Crucial time scales

$\tau_{\text{collision}}$ **is the duration of the AA collision**

$\tau_{\text{interaction}}$ **is the time between two NN interactions**

τ_{form} **is the hadron formation time after the interaction of two nucleons**



A+A collision in space-time

Blue lines: nucleons

Points: possible interactions (assuming that the trajectories are close in transverse direction)

At "low" energy

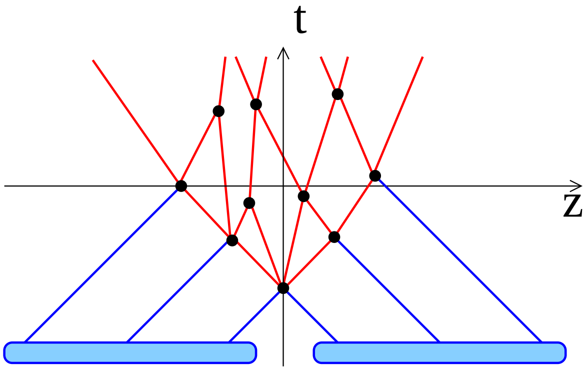
Sequential collisions (cascade)

Condition:

$$\tau_{\mathrm{form}} < \tau_{\mathrm{interaction}}$$

τ_{form} is the particle formation time

$\tau_{\mathrm{interaction}}$ is the time between two NN interactions



A+A collision at low energy
red lines: produced hadrons

At "high" energy

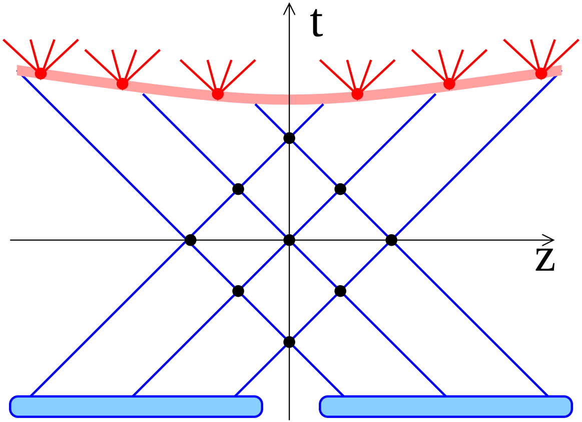
First all *NN* interactions occur, instantaneously, in parallel

Hadron production comes later

Condition:

$$\tau_{\mathrm{form}} > \tau_{\mathrm{collision}}$$

$\tau_{\mathrm{collision}}$ is the duration of the AA collision



A+A collision at very high energy

Collision zone pointlike

Low energy and high energy nuclear scattering are very different, and different theoretical methods are needed

- At high energies, one can completely separate
  - primary interactions (at $t \approx 0$)
  - and secondary interactions (hydro evolution etc)
- High energy approach = parallel primary interactions

What means "high/low energy" ?

Define (E in the sense of $\sqrt{s_{NN}}$):

- High energy thresholds E_{HE} by $\tau_{\mathrm{form}} = \tau_{\mathrm{collision}}$
- Low energy thresholds E_{LE} by $\tau_{\mathrm{form}} = \tau_{\mathrm{interaction}}$

Numerical estimates of thresholds

$\tau_{\mathrm{form}} = \tau^0_{\mathrm{form}}\gamma_{\mathrm{hadr}}$, with $\tau^0_{\mathrm{form}} = 1\,\mathrm{fm/c}$,
and $\gamma_{\mathrm{hadr}} = 1$ ($\rightarrow$ upper limits for energy thresholds)

High energy threshold ($\tau_{\mathrm{form}} = \tau_{\mathrm{collision}} = \frac{2R}{\gamma v}$)
Using $R = 6.5\,\mathrm{fm}$:

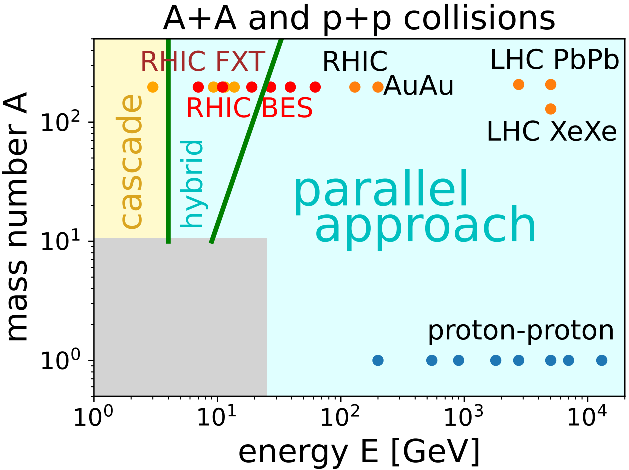
$$E_{\mathrm{HE}} = 24\,\mathrm{GeV} \qquad (\sqrt{s_{NN}})$$

Low energy threshold ($\tau_{\mathrm{form}} = \tau_{\mathrm{interaction}} = \frac{2R/n}{\gamma v}$)
Using $R = 6.5\,\mathrm{fm}$, $n = 7$ (n = nr of nucleons in a row) **:**

$$E_{\mathrm{LE}} = 4\,\mathrm{GeV}$$

The intermediate range $4 < \sqrt{s_{NN}} < 24$ GeV: hybrid

Which approach at what energy?



green lines:

Threshholds $E_{\rm LE}$ and $E_{\rm HE}$

for $A = 200$:

$$E_{\rm LE} = 4\,\text{GeV}$$

$$E_{\rm HE} = 24\,\text{GeV}$$

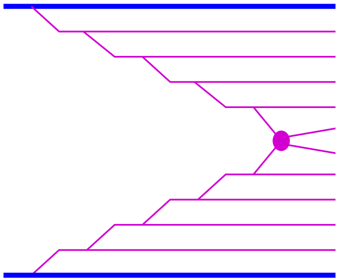
Parallel: EPOS4 = GR$^+$ & pQCD & saturation

1.6 Parallel scattering and factorization in pp

At LHC energy: Interaction: successive parton emissions

Large gamma factors, very long lived ptls

The complete process takes a very long time



pp collision = parton emissions before hard process

If a single collision takes several hundreds of fm/c:

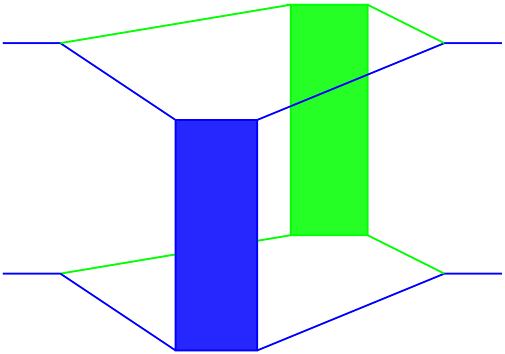
- **Impossible to have several of these interactions in a row**

So also in pp:

- **High energy approach = parallel interactions (as done in EPOS)**

And we know that multiple scattering is important!

So double scattering in pp should look like this:



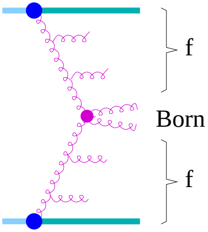
Here two parallel scatterings

No contradictions with respect to timescales

So it seems mandatory to use a parallel scattering scheme, for pp and AA, known since a long time ... but somewhat forgotten nowadays – why ?

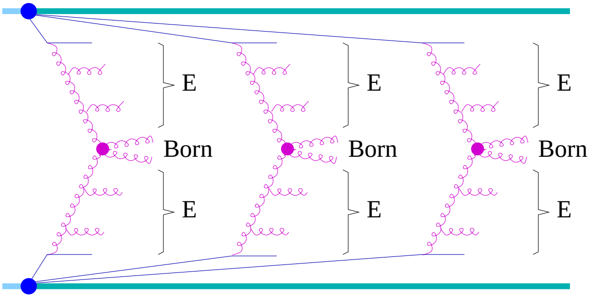
Factorization (MC or not)

First:



(reconnections comes later)

Real event structure



Left: PDFs (f), makes only sense for a single diagram

Both pictures are correct! But depends on the purpose!

- ☐ **Left: MC event, to be used for inclusive cross sections**
- ☐ **Right: Real event, needed for "beyond inclusive"**

We need to understand the relation between

- ☐ **the factorization picture (single diagram, PDFs)**
- ☐ **the parallel scattering picture**

1.7 Some history (parallel scattering, factorization)

Before QCD

- **Gribov-Regge (GR) approach, for pp, pA, AA**
  V. A. Abramovsky, V. N. Gribov, O. V. Kancheli, L. N. Lipatov **(1967-1973)**
- **S-matrix theory, parallel scattering scheme**
- **Exchanged "objects" are called Pomerons**
- **AGK theorem ($\sigma_{\mathrm{incl}}^{AB} = AB \times \sigma_{\mathrm{incl}}^{\mathrm{single\ Pom}}$)**
- **Infinite energy limit**
  (problematic...)

Perturbative QCD for pp

- **Asymptotic freedom**
  D. Gross, F. Wilczek,H. Politzer **(1973)**
- **DGLAP (linear) evolution**
  V. N. Gribov, L. N. Lipatov **(1973)**
  G. Altarelli, G. Parisi **(1977)**, Y. L. Dokshitzer **(1977)**
- **Factorization** J. Collins, D. Soper, G. Sterman **(1989)**
- **Covers only a small fraction of observables** (inclusive, hard)
  NOT covered: Triggering on high multiplicity or on centrality classes
  (in connection with soft or hard probes)

Saturation (CGC, small-x physics,...)

- **Nonlinear evolution**
  L. V. Gribov, E. M. Levin, and M. G. Ryskin **(1984)**
  L. D. McLerran and R. Venugopalan **(1994)**, Y. V. Kovchegov **(1996)**, ...

An attempt to couple GR and pQCD

- ☐ **NEXUS model, earlier EPOS versions**
  H.J. Drescher, M. Hladik, Sergey Ostapchenko, Tanguy Pierog, K. Werner (2001)

- ☐ **Using: Pomeron = pQCD parton ladder**

- ☐ **With energy sharing! (GR^+) ... crucial for MC applications**
  Keeping parallel scattering scheme!!!

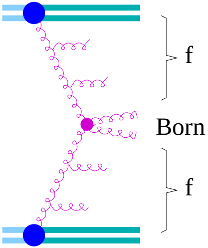
- ☐ **Problem: violates AGK (and binary scaling and factorization)**

Solution: EPOS4 = GR^+ & pQCD & saturation

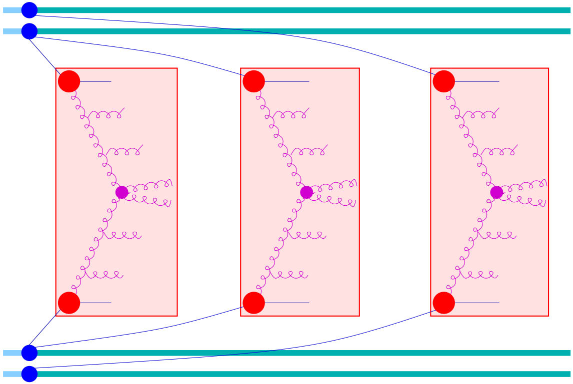
- ☐ **Redefine connection Pomeron <-> pQCD parton ladder**
  by taking into account saturation in a very particular way

- ☐ **Fully recovers AGK (and geometric properties which follow)**
  Parallel scattering scheme, going beyond factorization,
  perfectly covering "observables per event class", soft physics
  but at the same time
  factorization works for inclusive xsections for hard processes

(A) Factorization picture
(inclusive cross sections, ond ONLY for these)



(B) Parallel scattering (considering real events)



EPOS4: fully selfconsistent picture (B) to be used for “event class issues”, which breaks down to (A) for inclusive hard particle production, due to lots of cancellations

Problematic to get from (A) to (B), the multiple scattering information is lost!
(A) to (B) usually based on the eikonal model (from 1958)

Crucial to distinguish between “inclusive” and “beyond inclusive”

The di-jet cross section is an inclusive cross section, i.e. one counts di-jets, not di-jet events, so a 2-di-jet event counts twice

Summing N-di-jet events, we have

$$\sigma_{\mathbf{dijet}} = \sum_N N\,\sigma_{\mathbf{dijet}}^{(N)}$$

whereas the total cross section (forgetting soft for the moment)

$$\sigma_{\mathbf{tot}} = \sum_N \sigma_{\mathbf{dijet}}^{(N)}$$

For inclusive cross sections, enormous simplifications apply, but to understand this we have to first understand "parallel scattering" => Gribov-Regge (GR) approach

Gribov-Regge approach for *pp* scattering, based on

- ☐ **S-matrix theory**
- ☐ **cut diagrams, cutting rules**
- ☐ **Regge poles (in the complex *s*-plane)**

s = Mandelstam variable

In the following:
some very elementary facts about S-matrices, poles, and cuts

1.8 Poles and branch cuts

Even functions $f(x)$ of a **real variable** x
may need to be **continued into the complex plane,**
to understand their properties.

Example
$$f(x) = \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} \left(\frac{x}{2i}\right)^n .$$

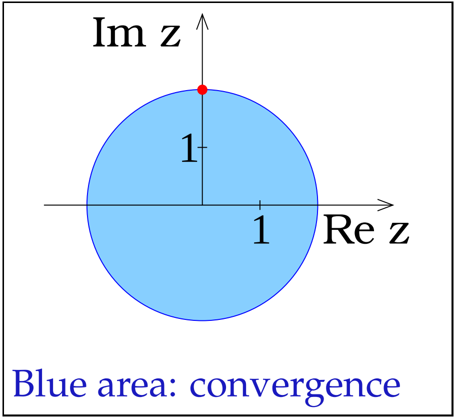
The radius of convergence is

$$\rho = \lim_{n\to\infty} |a_n|^{-1/n} = 2$$

Which is obvious, since f considered as function of a complex variable z, writes

$$f(z) = \frac{1}{1 - z/(2i)}$$

having a **pole** at $z = 2i$,



whereas $f(x)$ has no singularity (for $x \in \mathbb{R}$)

We will see later: the asymptotic behavior of the T-matrix T(s,t) is affected by poles in the complex *s*-plane.

Branch cuts

An example: The logarithm.

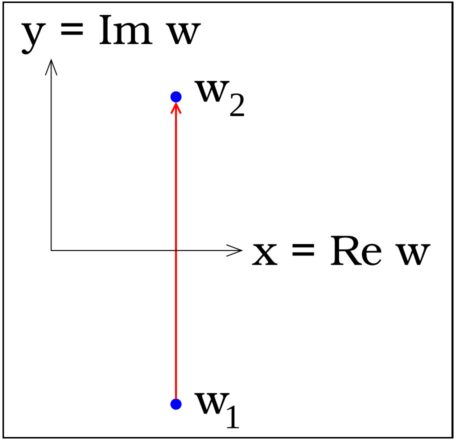
The exponential function defines a mapping M

$$M: \begin{array}{l} \mathbb{C} \rightarrow \mathbb{C} \\ w \rightarrow z \quad = \exp(w) \end{array}$$

which is well defined in the whole complex plane.

Consider $w = x + iy$, with x fixed and y going from $-\pi$ to π.

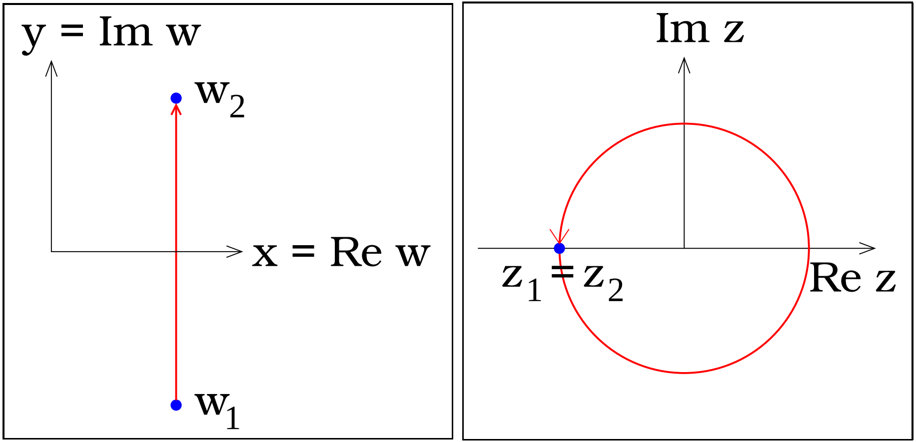
(Trajectory γ going from $w_1 = x - i\pi$ to $w_2 = x + i\pi$)



The mapped trajectory $\gamma' = M(\gamma)$ is given as

$$z = \exp(w) = \exp(x)\exp(iy)$$

=> A circle with start and end point $z_1 = z_2 = -e^x$

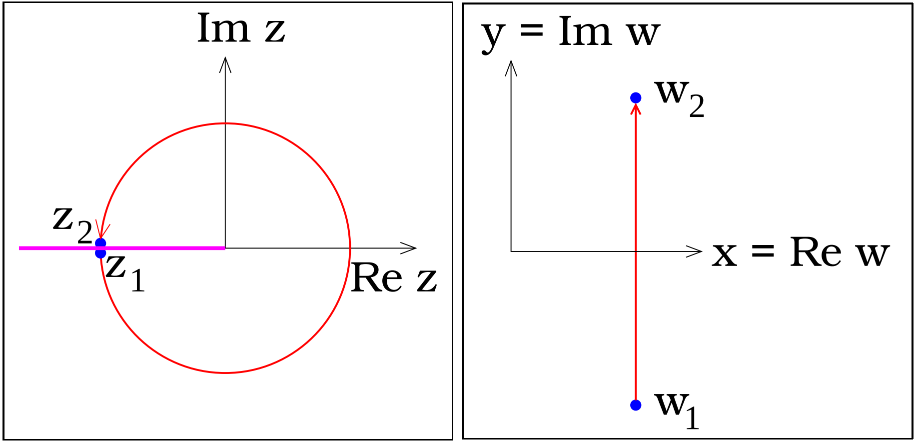


Doing the inverse mapping

$$M^{-1}:\ z \to w = \log(z),$$

we get for $z_1 = z_2$ two different values w_1 and w_2 !!

One has to define **log** in $\mathbb{C} - \mathbb{R}_{\leq \mathbf{0}}$.
The negative real axis is called branch cut.



The discontinuity at $z = -e^x$:

$$\log(z + i\epsilon) - \log(z - i\epsilon) = 2\pi i$$

1.9 Cut diagrams

The scattering operator $\hat{S}$ is defined via

$$|\psi(t=+\infty)\rangle = \hat{S}\,|\psi(t=-\infty)\rangle$$

Unitarity relation $\hat{S}^{\dagger}\hat{S} = 1$ gives (considering a discrete Hilbert space)

$$\begin{aligned} 1 &= \langle i|\,\hat{S}^{\dagger}\hat{S}\,|i\rangle \\ &= \sum_f \langle i|\,\hat{S}^{\dagger}\,|f\rangle\,\langle f|\,\hat{S}\,|i\rangle \\ &= \sum_f \langle f|\,\hat{S}\,|i\rangle^{*}\,\langle f|\,\hat{S}\,|i\rangle \end{aligned}$$

Expressed in terms of the S-matrix:

$$1 = \sum_f S^*_{fi} S_{fi} \qquad (A)$$

Using

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta(p_f - p_i) T_{fi} \qquad (B)$$

one gets from (A):

$$\frac{1}{i}\left(T_{ii} - T^*_{ii}\right) = \sum_f (2\pi)^4 \delta(p_f - p_i) \left|T_{fi}\right|^2 \qquad (C)$$

$$= 2s\,\sigma_{\mathrm{tot}} \quad \text{for } s \to \infty \qquad (D)$$

(see next page)

Be ϕ le current of incoming particles hitting a target of area A containing N particles. The transsition rate τ is

$$\tau = \phi A \, \frac{\sigma N}{A} = \phi \, \sigma \, N,$$

The cross section is
$$\sigma = \frac{\tau}{N\phi} = \frac{\tau}{V\,\phi\,\rho} = \frac{W}{TV\,\phi\,\rho} \equiv \frac{W}{TV\,w}.$$

The transition probability $W = |S_{fi}|^2$ is

$$\left((2\pi)^4\delta^4(p_f - p_i)\right)^2 |T_{fi}|^2 = TV\,(2\pi)^4\delta^4(p_f - p_i)|T_{fi}|^2.$$

The cross section is then
$$\sigma = \frac{1}{w}|T_{fi}|^2\,(2\pi)^4\delta^4(p_f - p_i).$$

with $w = 2E_1 v_1 2E_2$. We need a covariant form of $f = E_1 v_1 E_2$. In the lab frame, we have $f^2 = |\vec{p}_1|^2 m_2^2 = (E_1^2 - m_1^2)m_2^2$, which gives the invariant form $f = \sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}$. With $2p_1 p_2 = s - m_1^2 - m_2^2$, we get $2f = \sqrt{(s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2}$, and thus

$$w = 4f = 2\sqrt{\left(s - (m_1 + m_2)^2\right)\left(s - (m_1 - m_2)^2\right)} \to 2s \text{ for } s \to \infty$$

Using

$$\frac{1}{i}\left(T_{ii}-T_{ii}^{*}\right)=2\mathrm{Im}T_{ii}\,,$$

we get the optical theorem

$$2\mathrm{Im}T_{ii} \;=\; \sum_{f}(2\pi)^{4}\delta(p_{f}-p_{i})\left|T_{fi}\right|^{2}=2s\,\sigma_{\mathrm{tot}}$$

Assume:

- □ T_{ii} is Lorentz invariant $\rightarrow$ use s, t
- □ $T_{ii}(s,t)$ is an analytic function$^{(1)}$ of s, with s considered as a complex variable
(Hermitean analyticity)
- □ $T_{ii}(s,t)$ is real on some part of the real axis

Using the Schwarz reflection principle, $T_{ii}(s,t)$ first defined for $\mathrm{Im}s \geq 0$ can be continued in a unique fashion via $T_{ii}(s^*,t) = T_{ii}(s,t)^*$.

---

(1) locally given by a convergent power series

Defining disc $f(s) = f(s+i\epsilon) - f(s-i\epsilon)$:

$$\frac{1}{i}\left(T_{ii}(s,t) - T_{ii}(s,t)^*\right) = \frac{1}{i}\left(T_{ii}(s,t) - T_{ii}(s^*,t)\right)$$

i.e.:

$$2\mathrm{Im}T_{ii} = \frac{1}{\mathrm{i}}\mathrm{disc}\,T_{ii}$$

In the following $T = T_{ii}$.

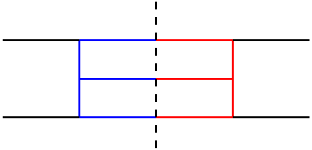
We have finally the following relation between elastic (T) and inelastic processes (T_{fi}) and σ_{tot}:

$$\frac{1}{\mathrm{i}}\mathrm{disc}\,T \;=\; (2\pi)^4\delta(p_f - p_i)\sum_f \left|T_{fi}\right|^2 = 2s\,\sigma_{\mathrm{tot}}$$

Interpretation: $\frac{1}{\mathrm{i}}\mathrm{disc}\,T$ can be seen as a so-called "cut diagram", with modified Feynman rules, the "intermediate particles" are on mass shell.

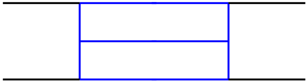
Modified Feynman rules :

- ☐ Draw a dashed line from top to bottom

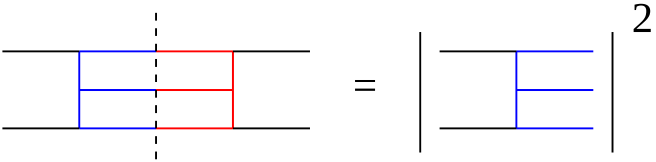


- ☐ Use “normal” Feynman rules to the left
- ☐ Use the complex conjugate expressions to the right
- ☐ For lines crossing the cut: Replace propagators by mass shell conditions $2\pi\theta(p^0)\delta(p^2 - m^2)$

Take a diagram representing **elastic** scattering,

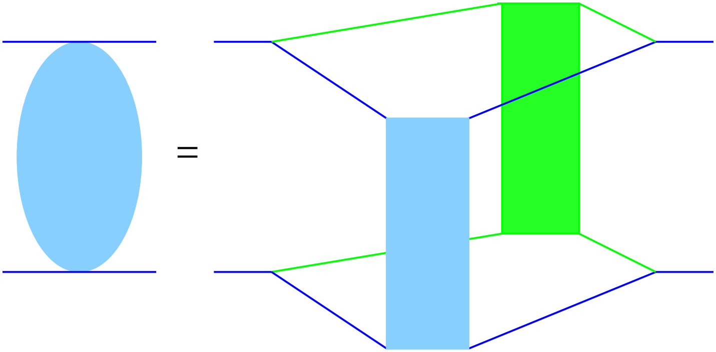
uncut diagram: 

Cutting it corresponds to **inelastic** scattering

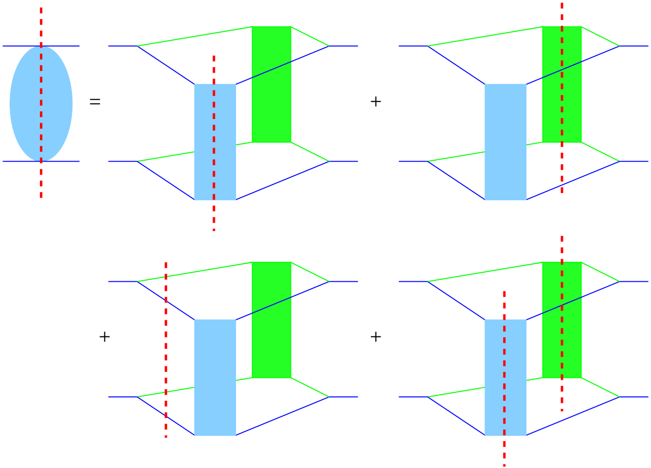
cut diagram: 

To treat inelastic scattering:
simply take the elastic diagram and cut it

Cutting diagrams is extremely useful
in case of substructures:



Precisely the multiple scattering structure
in the GR approach and in EPOS4



Cut diagram
= sum of products of cut/uncut subdiagrams
=> Gribov-Regge approach of multiple scattering

2 Parallel scattering in the GR approach

Parallel multiple scattering in pp, pA, AA scattering

Without energy sharing ($s \to \infty$ limit)

Chapter 5, arXiv:2310.09380, PRC 109, 034918 (also on https://klaus.pages.in2p3.fr/epos4/physics/papers)

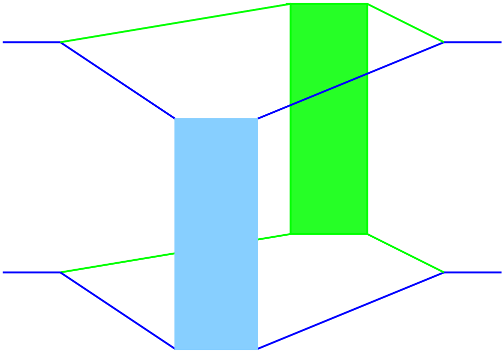
2.1 GR approach

Based on the observation of multiple scattering at HE, and the necessity of parallel scatterings, one postulates a particular form of the (elastic scattering) **T-matrix**

being composed of

multiple "Pomerons" as

$$-i\,\{iT_{\mathrm{Pom}}\times ... \times iT_{\mathrm{Pom}}\}$$



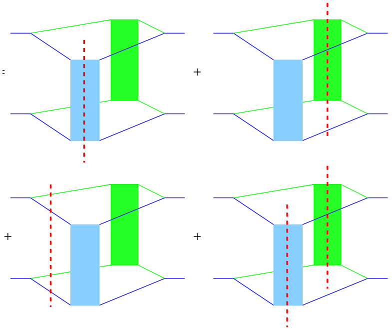
Microscopic structures are hidden in the "boxes" (Pomerons)

2.2 Shadowing

Let us consider GR picture (no energy sharing), using simple assumptions (in impact parameter representation[1])

Consider multiple scattering amplitude $iT = \prod iT_{\mathrm{P}}$

cross section: sum over all cuts



[1] $T = T(s,b)$ is the Fourier transformation of $\mathrm{T}(s,t)$ with respect to the momentum transfer, divided by $2s$.

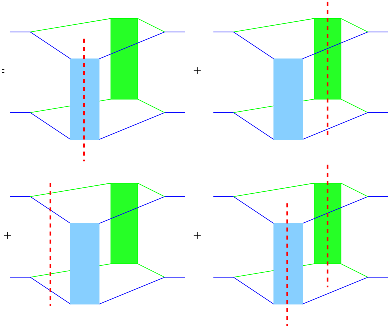
Optical theorem: $\sigma_{\mathrm{tot}} = \int d^2b \, 2\mathrm{Im}T = \int d^2b \, \tilde{\sigma}_{\mathrm{tot}}$

For each cut Pom, assuming imaginary $T_{\mathrm{P}} = \frac{i}{2}G \quad (G > 0)$

$$\frac{1}{i}\mathrm{disc}T_{\mathrm{P}} = 2\mathrm{Im}T_{\mathrm{P}} = G$$

For each uncut one

$$iT_{\mathrm{P}} + \{iT_{\mathrm{P}}\}^* = i\frac{i}{2}G + \left\{i\frac{i}{2}G\right\}^* = -G$$

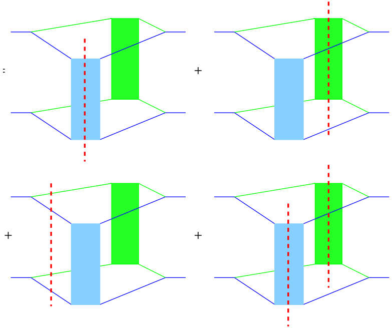


Fundamental relation: **cut Pom. = G => uncut Pom. = -G**

Single cut Pomeron contributions

(upper two graphs)

$$G \times (-G) + (-G) \times G$$
$$= -2G^2 < 0$$



=> absortive contribution / shadowing / screening

2.3 AGK cancellations (crucial!)

Let us assume that each “box” represents di-jet production. Each cut Pomeron produces 1 di-jet.

Be n the number of Poms and k the number of cut Poms.

Inclusive cross section:

$$\tilde{\sigma}_{\mathrm{incl}} = \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{k=0}^{n} k \times \tilde{\sigma}_{nk}$$

Important: the factor k since we consider an inclusive cross section

Consider the contribution for $n = 2$ (without 1/n!):

$$0 \times \tilde{\sigma}_{20} \quad + \quad 1 \times \tilde{\sigma}_{21} \quad + \quad 2 \times \tilde{\sigma}_{22}$$

$$\propto 0 \times G^2 \quad + 1 \times (-2G^2) + \quad 2 \times G^2 \;= 0$$

The absorbtive contribution $\tilde{\sigma}_{21}$ cancels exactly $\tilde{\sigma}_{22}$.

The double-Pomeron contribution

to the inclusive cross section is zero.

Consider the contribution for any $n > 1$:

$$\sum_{k=0}^{n} k \times \sigma_{nk} = \sum_{k=0}^{n} k \times G^k \,(-G)^{n-k} \begin{pmatrix} n \\ k \end{pmatrix} = 0$$

The n-Pomeron contribution

to the inclusive cross section is zero for n>1.

Huge amount of cancellations!

Inelastic cross section: $\sum_{k=1}^{n} G^k \,(-G)^{n-k} \begin{pmatrix} n \\ k \end{pmatrix} \neq 0$

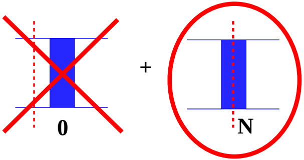
- **Almost all of the diagrams (i.e. n=2, n=3, ...) do not contribute at all to the inclusive cross section**

- **Enormous amount of cancellations (interference)**

- **AGK cancellations**
  Abramovskii, Gribov and Kancheli (1973)

- **Only single-Pomron contribution (n=1)**

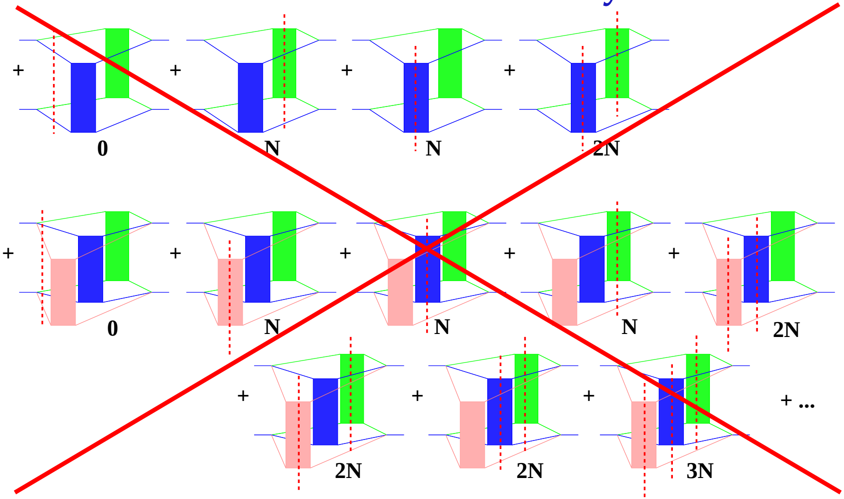
- **Generalization to pA and AA**



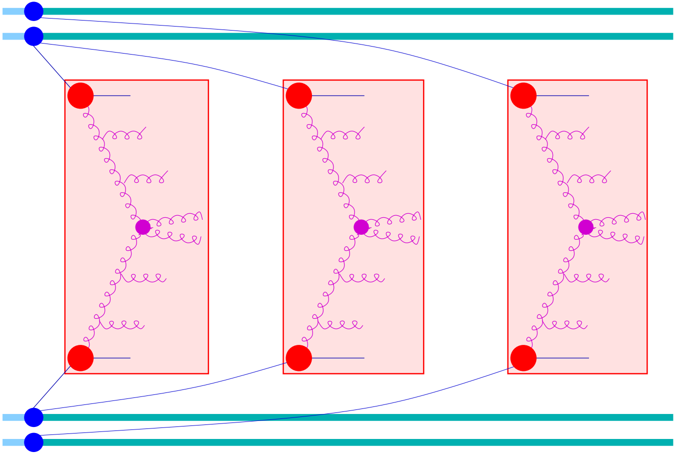
for **inclusive cross sections**

only one diagram contributes

necessary for factorization

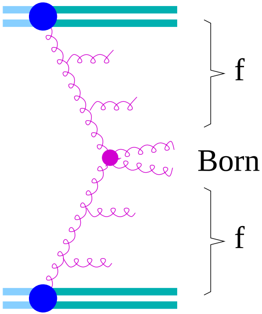


Although reality looks like this



AA scattering withere several Pomeron exchanges
(meaning of red dots ... later)

inclusive cross section calculation are based on a single Pomeron



which allows defing PDFs (and making life easier!)

Going beyond inclusive
=> FULL multiple Pomeron diagram

Models	**Start with**	**then**	**comment**
EPOS4	**Multiple parallel scatterings**	**derive factorization**	**difficult**
Pythia8[(1)]	**Factorization**	**add multiple scattering**	**problematic**

(1) essentially all “QCD Monte Carlo generators”

2.4 Consistency checks

The formalism accomodates elastic and inelastic scattering

$$\tilde{\sigma}_{\mathrm{tot}} = 2\mathrm{Im}T,$$

$$\tilde{\sigma}_{\mathrm{el}} = |T|^2.$$

Using $T_{\mathrm{P}} = iG/2$ with real G:

$$iT = \sum_{n=1}^{\infty} \frac{1}{n!} \prod iT_{\mathrm{P}} = \sum_{n=1}^{\infty} \frac{1}{n!} \left(-\frac{G}{2}\right)^n = \exp\left(-\frac{G}{2}\right) - 1,$$

and so

$$\tilde{\sigma}_{\mathrm{tot}} = 2\left\{1 - \exp\left(-\frac{G}{2}\right)\right\},$$

$$\tilde{\sigma}_{\mathrm{el}} = \left\{1 - \exp\left(-\frac{G}{2}\right)\right\}^2, \quad \tilde{\sigma}_{\mathrm{in}} = \left\{1 - \exp\left(-G\right)\right\}.$$

Using cutting rules

$$\tilde{\sigma}_{\rm in} = \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{m=1}^{n} \left(\begin{array}{c} n \\ m \end{array} \right) G^m (-G)^{n-m} = \sum_{n=1}^{\infty} \frac{1}{n!} \{0 - (-G)^n\} = 1 - \exp{(-G)} \,.$$

For the total cross section, we have to subtract the case where all Pomerons are all to the left or right of the cut

$$\tilde{\sigma}_{\rm tot} = \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{m=0}^{n} \left(\begin{array}{c} n \\ m \end{array} \right) G^m (-G)^{n-m} - 2 \left\{ \exp\left(\frac{G}{2}\right) - 1 \right\} = 2 \left\{ 1 - \exp\left(\frac{G}{2}\right) \right\} .$$

All this "cutting"

- ☐ **not really needed to compute total, elastic, and inelastic cross sections**
- ☐ **but it becomes crucial when we focus on "event classes" (multiplicity triggers)**

2.5 Nucleus-nucleus (A+B) scattering

Define integration over b and transv. nucleon coordiates b_i^A and b_j^B

$$\int db_{AB} = \int d^2b \int \prod_{i=1}^{A} d^2b_i^A \, T_A(b_i^A) \int \prod_{j=1}^{B} d^2b_j^B \, T_B(b_j^B),$$

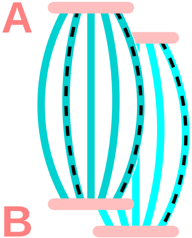
with the nuclear thickness function

$$T_A(b) = \int dz \, \rho_A\left(\sqrt{b^2 + z^2}\right)$$

where ρ_A is the (normalized) nuclear density for nucleus A. Then

$$\sigma_{\mathrm{in}}^{AB} = \int db_{AB} \, \tilde{\sigma}_{\mathrm{in}}^{AB}$$

$\tilde{\sigma}_{\mathrm{in}}^{AB}$ is a sum over all cut (dashed) and uncut (solid) Pomerons between all possible pairs of nucleons of nuclei A and B:



$$\tilde{\sigma}_{\mathrm{in}}^{AB} = \sum_{m_1 l_1} \cdots \sum_{m_{AB} l_{AB}}{}' \prod_{k=1}^{AB} \frac{1}{\mathrm{m_k!l_k!}} (G_k)^{m_k} (-G_k)^{l_k}$$

$\sum ... \sum'$ means at least one m_k being nonzero. We use $G_k = G(b_k)$ with $b_k = |b + b^A_{\pi(k)} - b^B_{\tau(k)}|$ referring to the impact par. of the NN pair k

Summing the uncut Pomerons

$$\tilde{\sigma}_{\mathrm{in}}^{AB} = \sum_{m_1 l_1} \cdots \sum_{m_{AB} l_{AB}}{}' \prod_{k=1}^{AB} \frac{1}{\mathrm{m_k!l_k!}} (G_k)^{m_k} (-G_k)^{l_k}$$

$$= \sum_{m_1} \cdots \sum_{m_{AB}}{}' \underbrace{\prod_{k=1}^{AB} \frac{1}{\mathrm{m_k!}} (G_k)^{m_k} \; \exp(-G_k)}_{P(m_1,\ldots,m_{AB})}$$

Crucial: $\sum P(m_1, ..., m_{AB}) = 1$ => probability interpretation

$P(m_1, ..., m_{AB}) =$ probab. of configuration $\{m_1, ..., m_{AB}\}$

Basis of Monte Carlo treatment

Perfectly parallel scattering scenario! No sequence of collisons.

One considers all possible NN collisions instantaneously.

3 EPOS4 primary interactions

An attempt to do (from the beginning) full parallel scattering, but be compatible with factorization for inclusive cross sections

by taking into account saturation

EPOS4 documentation

- **Oct. 2022 EPOS4.0.0 release** (no “official” EPOS3 release)
  https://klaus.pages.in2p3.fr/epos4/
  thanks Laurent Aphecetche for explaining gitlab pages, nextjs etc
  thanks Damien Vintache for managing installation/technical issues

- **Papers** (https://klaus.pages.in2p3.fr/epos4/physics/papers)
  – **arxiv:2301.12517** **PRC 108, 064903** **EPOS4 Overview**
  – **arxiv:2306.02396** **PRC 108, 034904** **pQCD in EPOS4** with B. Guiot
  – **arxiv:2310.09380** **PRC 109, 034918**
  Parallel scattering formalism, S-matrix theory & pQCD & saturation
  46 pages, systematic and complete presentation of the theoretical basis,
  – **arxiv:2306.10277** **PRC 109, 014910**
  Microcanonical hadronization, core-corona in EPOS4
  – **arxiv:2401.11275** **EPOS4 results on RHIC**
  with J. Jahan, I. Karpenko, T. Pierog, M. Stefaniak, D. Vintache
  – **arxiv:2310.08684** **EPOS4HQ**: Heavy flavor collectivity in pp
  – **arxiv:2401.17096** **EPOS4HQ**: Heavy flavour in HI at RHIC and LHC
  EPOS4HQ: Jiaxing Zhao, Joerg Aichelin, Pol-Bernard Gossiaux, KW

3.1 EPOS4 general structure

(Possible at "high energies")

- ☐ **Primary scatterings (at t = 0)**
  parallel scattering approach based on S-matrix theory
  (Major changes)

- ☐ **Secondary scatterings (at t > 0)**
  - core-corona procedure (New methods)
  - hydro evolution [1]
  - microcanonical decay (New)
  - hadronic rescattering [2]

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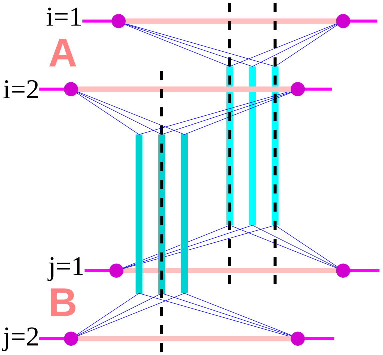
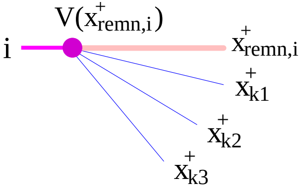
[1]) I. Karpenko et al, Computer Physics Communications 185, 3016 (2014), K. Werner, B. Guiot, I. Karpenko, and T. Pierog, Phys. Rev. C 89, 064903 (2014), 1312.1233,

[2]) S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 225 (1998), M. Bleicher et al., J. Phys. G25, 1859 (1999)

3.2 EPOS4 S-matrix approach

Realistic scenario, (details: arXiv:2301.12517)
parallel scattering, with energy sharing (GR$^+$)





$$x^+_{\mathrm{remn},i} = 1 - \sum_{\substack{k=1 \\ \pi(k)=i}}^{AB} \sum_{\mu=1}^{n_k} x^+_{k\mu}\,, \quad x^-_{\mathrm{remn},j} = 1 - \sum_{\substack{k=1 \\ \tau(k)=j}}^{AB} \sum_{\mu=1}^{n_k} x^-_{k\mu},$$

We generalize the GR formula ($\to$ GR$^+$)

$$\tilde{\sigma}_{\mathrm{in}}^{AB} = \sum_{n_1=0}^{\infty} \cdots \sum_{n_{AB}=0}^{\infty} \quad \sum_{m_1 \le n_1} \cdots \sum_{m_{AB} \le n_{AB}}{}' \int dX_{AB}$$

$$\prod_{k=1}^{AB} \frac{1}{n_k!} \binom{n_k}{m_k} \prod_{\mu=1}^{m_k} G_{k\mu} \prod_{\mu=m_k+1}^{n_k} -G_{k\mu}$$

$$\prod_{i=1}^{A} V(x^+_{\mathrm{remn},i}) \prod_{j=1}^{B} V(x^-_{\mathrm{remn},j})$$

$$G_{k\mu} = G\left(x^+_{k\mu}, x^-_{k\mu}, s, b_k\right)$$

$$\int dX_{AB} = \int \prod_{k=1}^{AB} \prod_{\nu=1}^{n_k} dx^+_{k\nu} dx^-_{k\nu}$$

All possible NN collisions considered instantaneously.

Again, uncut Pomerons are "summed over",
and $\tilde{\sigma}_{\rm in}^{AB}$ can be expressed in terms of

$$P = \prod_{k=1}^{AB} \left[\frac{1}{m_k!} \prod_{\mu=1}^{m_k} G_{k\mu} \right] \times W_{AB}$$

with some known W_{AB} and

$$\sum_{\{m_k\}} \int dX_{AB} P = 1$$

Interpretation:
$P = P(K)$ = probability of configuration $K = \{\{m_k\}, \{x^{\pm}_{k\mu}\}\}$, representing m_k cut Pomerons per pair k, with light-cone momentum fractions $x^{\pm}_{k\mu}$.

Basis of Monte Carlo, one determines K according to $P(K)$, instantaneously

For completeness: W_{AB}

(which amounts to summing over and integrating out all uncut Pomerons)

$$W_{AB} = W_{AB}(\{x_i^+\},\{x_j^-\}) = \sum_{\{l_k\}} \int \prod_{k=1}^{AB} \left(\prod_{\lambda=1}^{l_k} d\tilde{x}_{k\lambda}^+ d\tilde{x}_{k\lambda}^- \right)$$
$$\left\{ \prod_{k=1}^{AB} \left[\frac{1}{l_k!} \prod_{\lambda=1}^{l_k} -G_{\mathrm{QCDpar}}(\tilde{x}_{k\lambda}^+, \tilde{x}_{k\lambda}^-, s, b_k) \right] \right.$$
$$\left. \prod_{i=1}^{A} \left(x_i^+ - \sum_{\substack{k=1 \\ \pi(k)=i}}^{AB} \sum_{\lambda=1}^{l_k} \tilde{x}_{k\lambda}^+ \right)^{\alpha_{\mathrm{remn}}} \prod_{j=1}^{B} \left(x_j^- - \sum_{\substack{k=1 \\ \tau(k)=j}}^{AB} \sum_{\lambda=1}^{l_k} \tilde{x}_{k\lambda}^- \right)^{\alpha_{\mathrm{remn}}} \right\},$$

$\sum_{\{l_k\}}$ means summing all the indices l_k, with $1 \leq k \leq AB$, from zero to infinity.

l_k refers to the number of uncut Pomerons of nucleon-nucleon pair k.

W_{AB} is a function of the remnant LC momentum fractions

$$x_i^+ = 1 - \sum_{\substack{k=1 \\ \pi(k)=i}}^{AB} \sum_{\mu=1}^{m_k} x_{k\mu}^+, \quad x_j^- = 1 - \sum_{\substack{k=1 \\ \tau(k)=j}}^{AB} \sum_{\mu=1}^{m_k} x_{k\mu}^-.$$

Challenging. High-dimensional non-separable integrals. Major issue!

So far

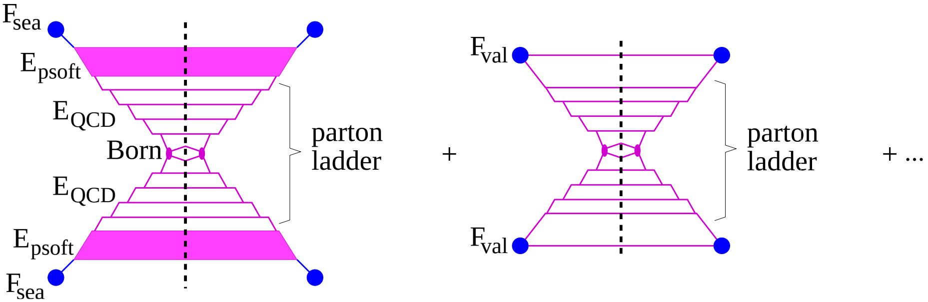
- ☐ **general framework allowing to treat rigorously parallel scattering + energy sharing (the only one...)**
- ☐ **formulas expressed in terms of an elementary "cut diagram" $G = \frac{1}{i}\mathrm{disc}T$**
- ☐ **representing an elementary parton-parton scattering**

What is the connection with QCD?

T = Fourier transform w.r.t. to transv. momentum of the T-matrix element $\mathbf{T}_{ii}$ divided by $2s$; relation S-matrix - T-matrix: $\mathbf{S}_{fi} = \delta_{fi} + i(2\pi)^4\delta(p_f - p_i)\mathbf{T}_{fi})$

First try:

$$G = G_{QCD} = G_{QCD}^{\mathbf{sea-sea}} + G_{QCD}^{\mathbf{val-val}} + G_{QCD}^{\mathbf{sea-val}} + G_{QCD}^{\mathbf{val-sea}}$$



Composed of modules (formulas see arXiv:2306.02396, PRC 108, 034904)

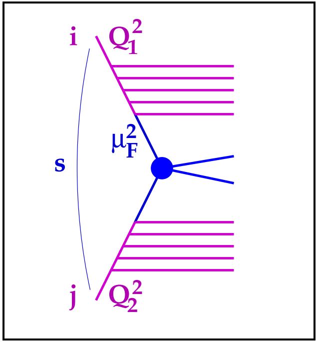
Technical issues: From diagrams to formulas

details: arXiv:2306.02396, PRC 108, 034904

Symbols used in the EPOS4 framework

$\mathbf{T}$	Diagonal element of the elastic scattering T-matrix as defined in standard quantum mechanics textbooks, where the asymptotic state is a system of two protons or two nuclei
T	Fourier transform with respect to the transverse momentum exchange of the elastic scattering T-matrix $\mathbf{T}$, divided by $2s$ (formulas are simpler using this representation)
G	Defined as $G = \mathrm{cut}\,T = 2\mathrm{Im}T = \frac{1}{i}\mathrm{disc}T$ (where “disc” refers to the variable s), referring to the inelastic process associated with the cut of the elastic diagram corresponding to T
σ	Integrated inclusive parton-parton scattering cross section, which is useful because $\mathbf{T}$, T, and G may be expressed in terms of σ

Example: Parton-parton scattering



$$\sigma^{ij}_{\mathrm{hard}}(Q_1^2,Q_2^2,s) = \sum_{klmn}\int dx_1 dx_2 \int \frac{d^3p_3 d^3p_4}{E_3E_4}$$

$$E^{ik}_{\mathrm{QCD}}(x_1,Q_1^2,\mu_{\mathrm{F}}^2)\,E^{jl}_{\mathrm{QCD}}(x_2,Q_2^2,\mu_{\mathrm{F}}^2)$$

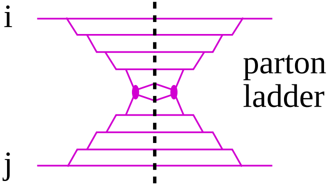
$$\frac{1}{2s}\,\frac{1}{16\pi^2}\bar{\sum}|\mathcal{M}^{kl\to mn}|^2\delta^4(p_1+p_2-p_3-p_4)\frac{1}{1+\delta_{mn}},$$

where the momenta of the outgoing partons (jets) are integrated out.

Very similar to the usual factorization formula

- ☐ but with the PDFs replaced by $E^{ik}_{\mathrm{QCD}}(x_K,Q_K^2,\mu_{\mathrm{F}}^2)$,
- ☐ for parton evolution starting at virtuality Q_K^2 with a distribution $\delta(x-1)\delta_{ki}$,
- ☐ but using the same DGLAP evolution

Corresponding cut diagram, referred to as cut parton ladder



Considering first the corresponding elastic scattering T-matrix, we assume

$$\mathbf{T}^{ij}_{\mathrm{hard}}(Q_1^2,Q_2^2,s,t)=i\,s\,\sigma^{ij}_{\mathrm{hard}}(Q_1^2,Q_2^2,s)\times\exp(R^2_{\mathrm{hard}}t)$$

compatible with the usual relation
$\sigma^{ij}_{\mathrm{hard}}=2\mathrm{Im}\,\mathbf{T}^{ij}_{\mathrm{hard}}(t=0)\,/\,(2s)$.

Assuming a purely transverse momentum exchange $t=-q_\perp^2$ the Fourier transform and division by $2s$ gives

$$T^{ij}_{\mathrm{hard}}(Q_1^2,Q_2^2,s,b)=\frac{1}{8\pi^2 s}\int d^2q_\perp e^{-iq_\perp b}\mathbf{T}^{ij}_{\mathrm{hard}}(s,t)$$

$$=\frac{i}{2}\sigma^{ij}_{\mathrm{hard}}(Q_1^2,Q_2^2,s)\,\frac{1}{4\pi R^2_{\mathrm{hard}}}\exp\left(-\frac{b^2}{4R^2_{\mathrm{hard}}}\right).$$

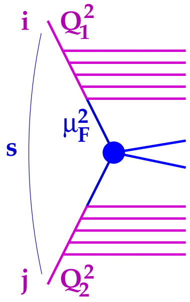
For the corresponding $G = \text{cut}\, T_{\text{hard}} = 2\text{Im}\, T_{\text{hard}}$, we get

$$G_{\text{QCD}}^{\text{hard,ij}}(Q_1^2, Q_2^2, s, b)$$
$$= \sigma_{\text{hard}}^{ij}(Q_1^2, Q_2^2, s)\, \frac{1}{4\pi R_{\text{hard}}^2} \exp\left(-\frac{b^2}{4R_{\text{hard}}^2}\right).$$

So the cut parton ladder expression G is simply

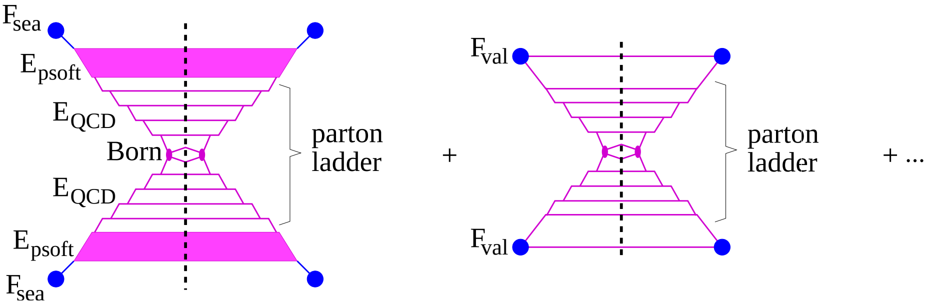
- ☐ the product of a Gaussian impact parameter dependence and
- ☐ the dijet production cross section $\sigma_{\text{hard}}^{ij}$ being of the form

$$\int E_{\text{QCD}} \times E_{\text{QCD}} \times \underbrace{|\mathcal{M}|^2}_{\text{Born}}$$



In a similar way, all contributions in

$$G_{QCD} = G_{QCD}^{\mathrm{sea-sea}} + G_{QCD}^{\mathrm{val-val}} + G_{QCD}^{\mathrm{sea-val}} + G_{QCD}^{\mathrm{val-sea}}$$

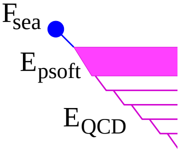


can be expressed in terms of modules like

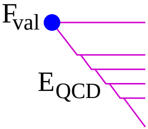
E_{QCD}, **Born**, F_{sea}, **etc** (formulas see arXiv:2306.02396, PRC 108, 034904)

Different ways to rearrange the modules. One may define (and tabulate) a PDF

$f_{\mathrm{PDF}} =$



$+$



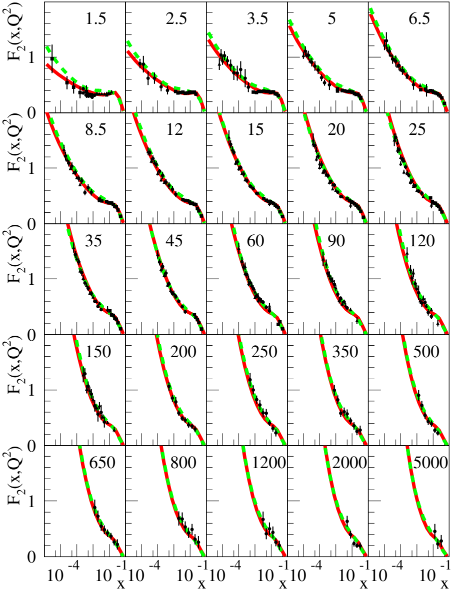
allowing to compute the single Pomeron dijet cross section

$$E_3 E_4 \frac{d^6\sigma}{d^3p_3 d^3p_4}$$

$$= \sum_{klmn} \int\!\!\int d\xi_1 d\xi_2 \, f^k_{\mathrm{PDF}}(\xi_1, \mu_\mathrm{F}^2) f^l_{\mathrm{PDF}}(\xi_2, \mu_\mathrm{F}^2)$$

$$\frac{1}{32 s\pi^2} \bar{\sum} |\mathcal{M}^{kl\to mn}|^2 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{1}{1+\delta_{mn}},$$

Electron-proton scattering F_2 vs x



To check our f_{PDF}, we can compute

$$F_2 = \sum_k e_k^2\, x\, f_{\mathrm{PDF}}^k(x, Q^2)$$

with

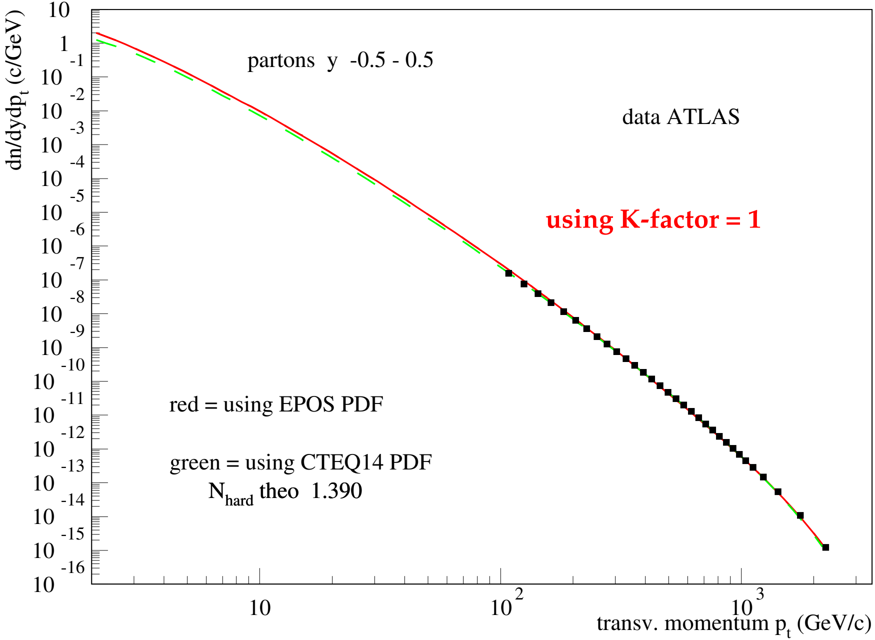
$$x = x_B = \frac{Q^2}{2pq}$$

in the EPOS framework,

and compare with data from ZEUS, H1

and with calculations based on CTEQ14(5f)

Jet cross section vs pt for pp at 13 TeV



Looks good, but

- **Here we showed a 1-Pomeron result**

- **In GR, the full multiple scattering scenario is equal to the 1-Pomeron result for inclusive cross sections (AGK theorem)**

- **Does AGK hold in our case (GR^{+}) ?**

- **And does AGK hold for nuclear scattering (which would amount to binary scaling)?**

Technical issues, for full multiple scattering scenario

details: arXiv:2310.09380, PRC 109, 034918

- Crucial: G_{QCD} **can be parametrized** as

$$G_{\mathrm{QCDpar}}(x^+, x^-, s, b) = \sum_{N=1}^{N_{\mathrm{par}}} \alpha_N \times (x^+ x^-)^{\beta_N},$$

  where α_N and β_N depend on s and b given in terms of a few parameters

- Parametric form inspired by the asymptotic expressions for T-matrices $T(s_{\mathrm{Pom}}, t_{\mathrm{Pom}}) \propto s_{\mathrm{Pom}}^{A(t_{\mathrm{Pom}})}, \quad s_{\mathrm{Pom}} = x^+ x^- s$

- **Integrals in W_{AB} can be done!!!**

- □ Configurations $\boldsymbol{K = \{\{m_k\},\{x^{\pm}_{k\mu}\}\}}$

  representing m_k cut Pomerons per pair k,

  with LC momentum fractions $x^{\pm}_{k\mu}$

  are generated randomly according to the law

$$P(K) = \prod_{k=1}^{AB}\left[\frac{1}{m_k!}\prod_{\mu=1}^{m_k} G_{k\mu}\right] \times W_{AB}$$

- □ **Can be done using Markov chain techniques!!!**

  essentially mapping configurations to a 2D Ising model spin-flip corresponds to adding/removing a Pomeron

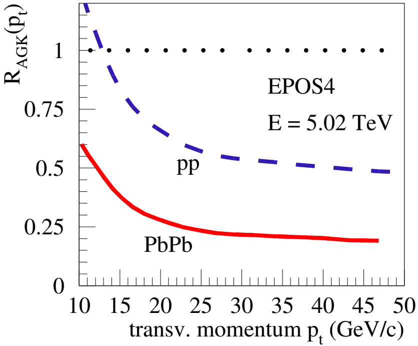
- □ **So the full MC is doable, one can start making tests...**

Validity of AGK Check p_t of partons for minimum bias PbPb and pp scatterings at 5.02 TeV.

Ratio

$$R_{\mathrm{AGK}}(p_t) = \frac{d\sigma_{\mathrm{incl}}^{AB}}{dp_t} \Big/ \left\{ AB \times \frac{d\sigma_{\mathrm{incl}}^{\mathrm{single\,Pom}}}{dp_t} \right\}$$

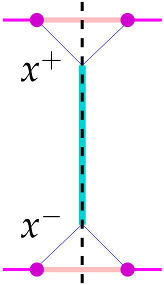
should be unity



AGK badly violated!!!

The problem is the energy sharing among Pomerons

3.3 The effect of energy sharing in EPOS4



Inclusive particle spectra (like p_t) are determined by the distribution of the LC momenta x^+ and x^-.

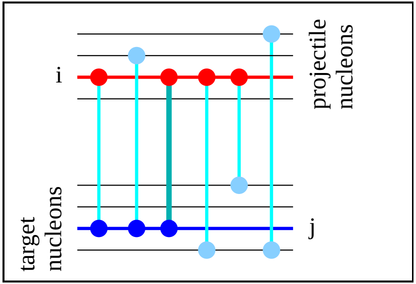
Crucial variable: the Pomeron's squared CMS energy fraction

$$x_{\mathrm{PE}} = x^+ x^- \approx s_{\mathrm{Pom}} / s_{\mathrm{tot}}$$

For a given Pomeron, connecting
projectile nucleon i and
target nucleon j
define:

$$N_{conn} = \frac{N_P + N_T}{2}$$

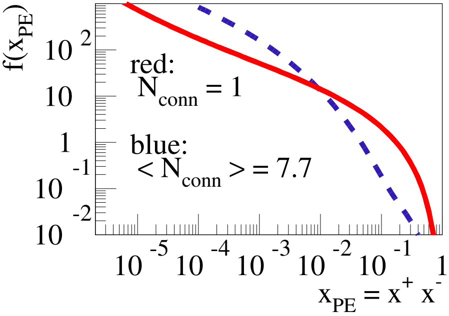
N_P = **number of Pomerons connected to** i
N_T = **number of Pomerons connected to** j



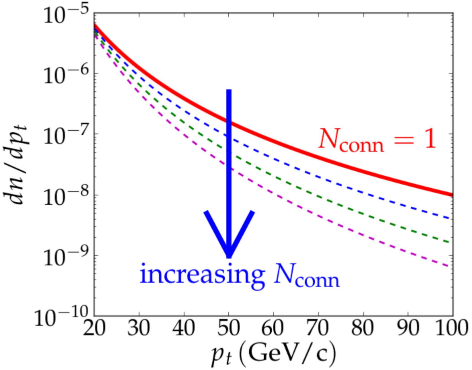
The x_{PE} distributions $f(x_{PE})$ depend on N_{conn}

Large N_{conn} => large x_{PE} suppressed
small x_{PE} enhanced

We will use the notation $f^{(N_{conn})}(x_{PE})$



Large N_{conn} => large x_{PE} suppressed => large p_t suppressed



Min, bias pp or AA = superposition of different N_{conn} contributions

Cannot be equal to the 1-Pomeron case ($N_{\mathrm{conn}} = 1$)
=> violation of AGK

We define the "deformation" of $f^{(N_{\text{conn}})}(x_{\text{PE}})$ relative to the reference $f^{(1)}(x_{\text{PE}})$

$$R_{\text{deform}} = \frac{f^{(N_{\text{conn}})}(x_{\text{PE}})}{f^{(1)}(x_{\text{PE}})}$$

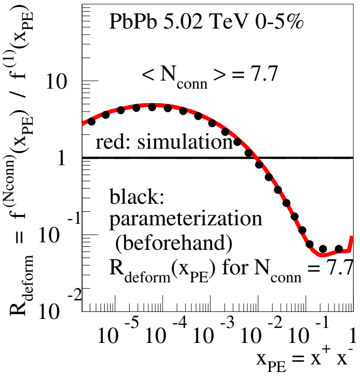
$R_{\text{deform}} \neq 1$ creates the problem

But we are able to parameterize R_{deform} and tabulate it, for all systems, all centrality classes

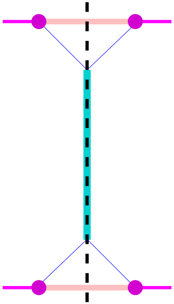
So

$$R_{\text{deform}} = R_{\text{deform}}(N_{\text{conn}}, x_{\text{PE}})$$

can be considered to be known, it is tabulated and available via interpolation (to be used later).



Two problems



single cut Pomeron G is fundamental building block of the multiple scattering formalism

So far one assumes

$$G = G_{\mathrm{QCD}}.$$

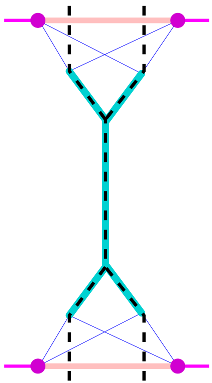
Obviously wrong, it leads to a strong violation of binary scaling at large p_t

Another serious problem

G_{QCD} based on DGLAP, but it is known that saturation phenomena (nonlinear effects) become important.

In the Pomeron language:
diagrams with triple (and more) Pomeron vertices.

Such nonlinear effects are completely missing.



3.4 The solution: Dynamical saturation scales in EPOS4

The two problems:

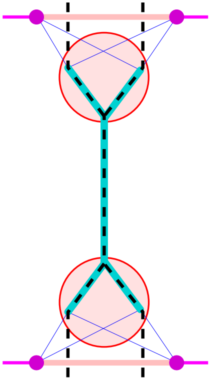
- □ wrong relation $G = G_{\mathrm{QCD}}$
- □ missing saturation

are related,
and can be solved simultaneously.

Saturation phenomena may be characterized by saturation scales

suggesting to treat nonlinear effects
by introducing saturation scales
as the lower limits Q_0^2 of the virtualities
for DGLAP evolutions

(i.e., nonlinear effects (inside the red circles) are "summarized" in the form of saturation scales)



We compute and tabulate $G_{\mathrm{QCD}}(Q_0^2, x^+, x^-, s, b)$
for a large range of Q_0^2 values **(see arXiv:2306.02396)**

For the connection between the basic multiple scattering building block G and the QCD expression G_{QCD} one postulates:

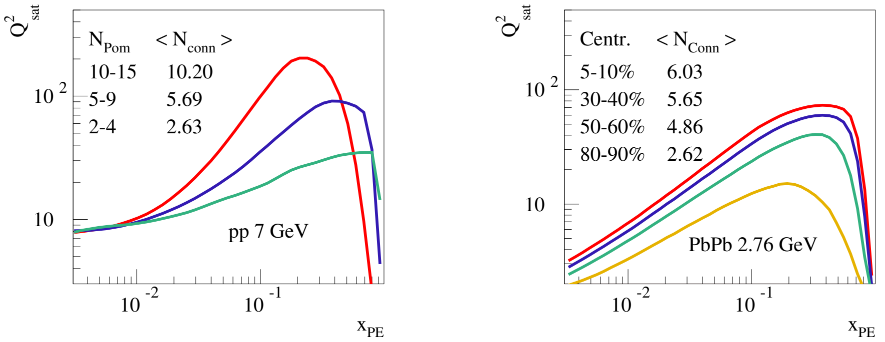
For each cut Pomeron, for given $x^{\pm}$, s, b, and N_{conn}:

$$G(x^+, x^-, s, b) = \frac{n}{R_{\mathrm{deform}}(N_{\mathrm{conn}}, x_{\mathrm{PE}})} \times G_{\mathrm{QCD}}(Q_{\mathrm{sat}}^2, x^+, x^-, s, b)$$

such that G does not depend on N_{conn},
whereas Q_{sat}^2 does depend on $x^+, x^-, N_{\mathrm{conn}}$

n is a normalization constant

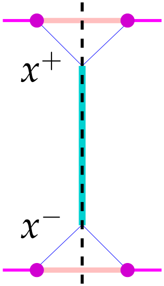
This equation defines Q_{sat}^2



N_{conn} dependence

- ☐ **very strong in *pp***
- ☐ **litte change form semi-peripheral to central *AA***

Recovering AGK



Considering the distribution of the LC momenta x^+ and x^-, one tries to relate the $A+B$ result (for given N_{conn})

$$\frac{d^2\sigma_{\mathrm{incl}}^{AB\,(N_{\mathrm{conn}})}}{dx^+dx^-}$$

to the 1-Pomeron case

Explicitely:

$$\frac{d^2\sigma_{\mathrm{incl}}^{AB\,(N_{\mathrm{conn}})}}{dx^+dx^-} = \sum_{\{m_k\}\neq 0} \int db_{AB} \int dX_{AB}\; P(K)$$
$$\times \Big\{ \sum_{k'=1}^{AB} \sum_{\mu'=1}^{m_{k'}} \delta_{N_{\mathrm{conn}}(k',\mu')}^{N_{\mathrm{conn}}} \delta(x^+ - x^+_{k'\mu'})\delta(x^- - x^-_{k'\mu'}) \Big\}$$

with

$$P(K) = \prod_{k=1}^{AB} \left[\frac{1}{m_k!} \prod_{\mu=1}^{m_k} G_{k\mu} \right] \times W_{AB}(\{x_i^+\},\{x_j^-\})$$

and with

$$W_{AB} = \prod_{i=1}^{A} c_1(x_i^+)^{\alpha_{\mathrm{remn}}} \prod_{j=1}^{B} c_1(x_j^-)^{\alpha_{\mathrm{remn}}} \prod_{k=1}^{AB} \exp\Big(-\tilde{G}(x^+_{\pi(k)} x^-_{\tau(k)}) \Big)$$

On can show (arXiv:2310.09380, PRC 109, 034918)

$$\frac{d^2\sigma_{\mathrm{incl}}^{AB\,(N_{\mathrm{conn}})}}{dx^+dx^-} \propto \frac{d\sigma_{\mathrm{incl}}^{\mathrm{single\,Pom}}}{dx^+dx^-}\left[Q_{\mathrm{sat}}^2(N_{\mathrm{conn}},x^+,x^-)\right]$$

i.e., the A+B cross section (given N_{conn})

- **is equal to the 1-Pomeron case,**
- **but with Q_{sat}^2 corresponding to N_{conn}**

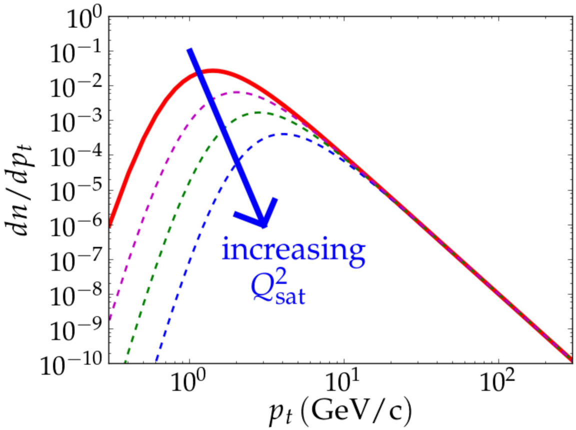
Same relation for p_t distributions (deduced from x^+x^-)

one gets for the min. bias cross section

$$\frac{d\sigma_{\text{incl}}^{AB\,(mb)}}{dp_t} = \sum_{N_{\text{conn}}} w(N_{\text{conn}}) \frac{d\sigma_{\text{incl}}^{\text{single Pom}}}{dp_t} \left[Q_{\text{sat}}^2(N_{\text{conn}}, x^+, x^-)\right]$$

i.e., the A+B cross section is a weighted sum

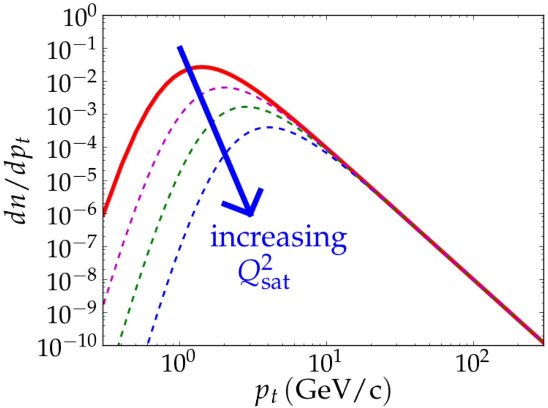
- ☐ **of 1-Pomeron contributions,**
- ☐ **but with Q_{sat}^2 corresponding to N_{conn}**



one expects with increasing N_{conn}

- ☐ **an increasing Q^2_{sat}**
- ☐ **and a reduction at $p_t^2 < Q^2_{sat}$ compared to $N_{conn} = 1$ (red)**

But no change for large p_t



so for p_t squared larger than the relevant Q^2_{sat} values , one replaces

$$\frac{d\sigma^{\text{single Pom}}_{\text{incl}}}{dp_t}\left[Q^2_{\text{sat}}(N_{\text{conn}},x^+,x^-)\right]$$

by

$$\frac{d\sigma^{\text{single Pom}}_{\text{incl}}}{dp_t}\left[Q^2_0\right]$$

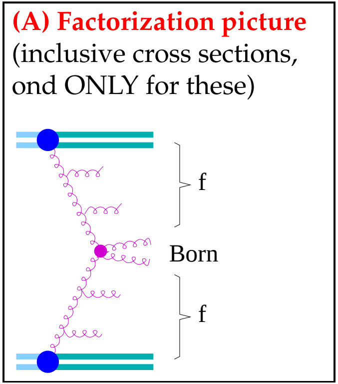
with constant Q^2_0

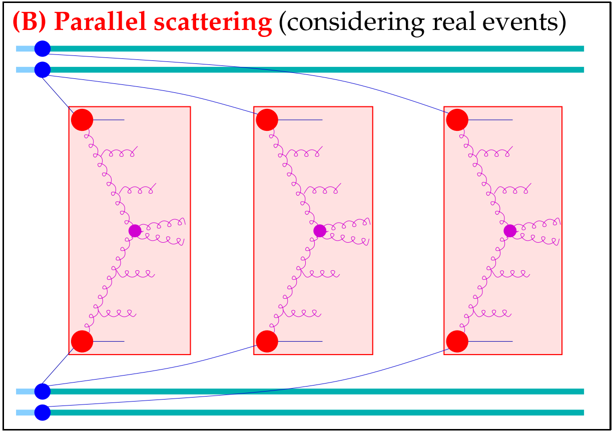
One gets finally

$$\frac{d\sigma_{\mathrm{incl}}^{AB\,(mb)}}{dp_t} = AB\,\frac{d\sigma_{\mathrm{incl}}^{\mathrm{single\,Pom}}}{dp_t}\,\left[Q_0^2\right]$$

but only for p_t^2 bigger than the relevant Q_{sat}^2 values (gAGK theorem)

Corollary: factorization (pp) and binary scaling ($A+B$)

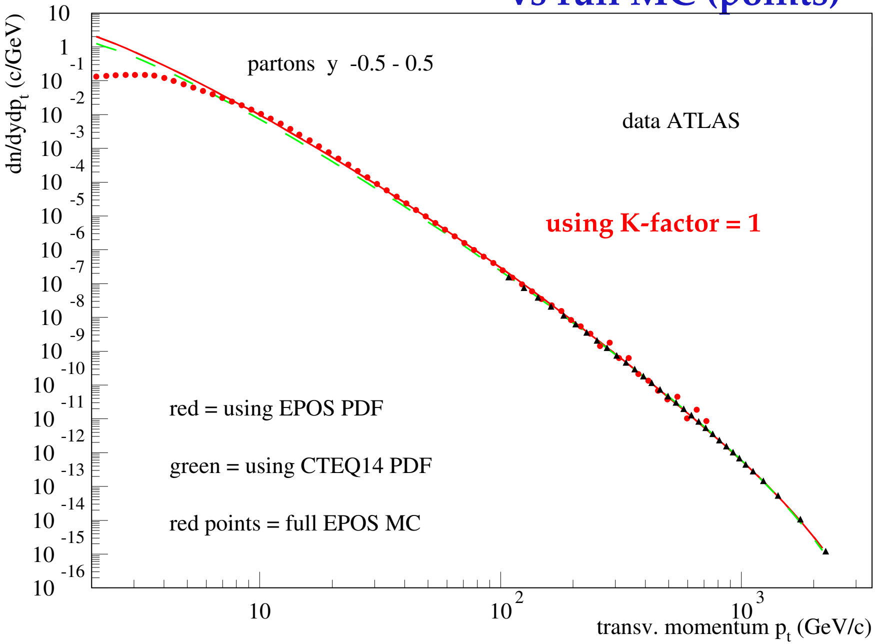




EPOS4:
fully selfconsistent picture (B) to be used for “event class issues”, which breaks down to (A) for inclusive hard particle production.

Crucial: saturation, fixes a problem caused by energy sharing

Jet cross section vs pt for pp at 13 TeV, factorization result vs full MC (points)



3.5 Angantyr (basic)

☐ **Papers** (proposed by Christian Bierlich)

- **arXiv:1806.10820 JHEP 10 (2018) 134**
- **arXiv:2205.11170 Phys.Lett.B 835 (2022) 137571**

☐ **Authors**

- **Christian Bierlich, Smita Chakraborty, Gösta Gustafson, Leif Lönnblad, Harsh Shah**

☐ **General structure**

- **Basic AA model**
  S-matrix theory (Glauber formalism)
  => independent sub-processes
- **String interactions / rescattering**
  - rope model
  - to come: combining ropes with shoving & hadronic cascade

Basic AA model (Glauber model)

- **Elastic scattering S-matrix**

$$S^{AB}(b) = \prod_{i=1}^{A} \prod_{j=1}^{B} S^{ij}(b_j + b - b_i), \quad T^{ij} = 1 - S^{ij}$$

$$\frac{d\sigma^{ij}_{\text{tot}}(b)}{d^2b} = 2T^{ij}, \quad \frac{d\sigma^{ij}_{\text{abs}}(b)}{d^2b} = 2T^{ij} - \left(T^{ij}\right)^2$$

- **Sub-T-matrix** - including nucleon size fluctuations $P(r) \propto r^{k-1} \exp(-r/r_0)$

$$T^{ij} = T(b^{ij}, r_p, r_t) \propto \Theta\left(\sqrt{\frac{(r_p + r_t)^2}{2T_0}} - b^{ij}\right)$$

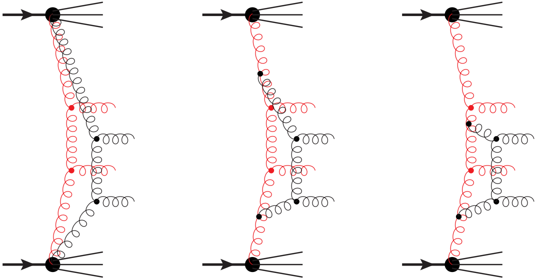
opacity : $T_0 = (1 - e^{-\pi(r_p+r_t)^2/\sigma_t})^{\alpha}$

No energy-momentum arguments! Independent sub-processes

- **For each pair, probability for absorptive interaction $2T^{ij} - (T^{ij})^2$**
  Two states to distinguish between absorptively and diffractively wounded wounded nucleons.

- **MPI: sub-collisions treated as separate QCD $2 \rightarrow 2$ scatterings**
  Parton densities rescaled according to an overlap function assuming some matter distribution in the colliding protons

- **Event with two sub-scatterings of type $gg \rightarrow gg$**
  Choice (a) gives wrong multiplicity dependence of mean pt

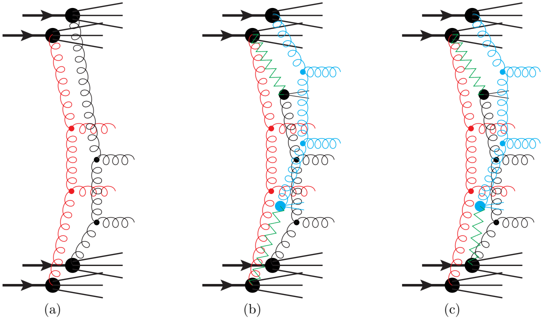


(a) (b) (c)

Better: additional sub-scatterings colour connected to partons in previous sub-scatterings (b) and (c)

EPOS4: choice (a)
+ saturation scale

- **AA scattering: two types of NN scatterings**
  - primary in case of not-yet-wounded nucleons (a)
  - secondary in case of already-wounded nucleons (b,c)



Primary: normal PYTHIA MPI scattering

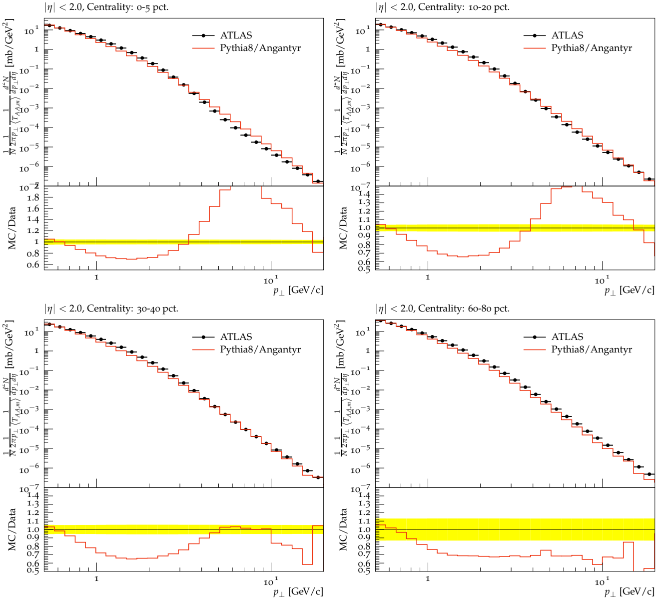
Secondary: diffractive PYTHIA scattering

= IP-N scattering
Pomeron IP = zigzag

EPOS4: same amplitude for wounded / unwounded compensated by "dynamical saturation scale"

- **Interactions ordered wrt increasing NN impact parameter**
- **then treat NN scattering one after the other**
  Several iterations: first absorptive scatterings, primary; second iteration to treat secondary scatterings. If not enough energy, redo / skip

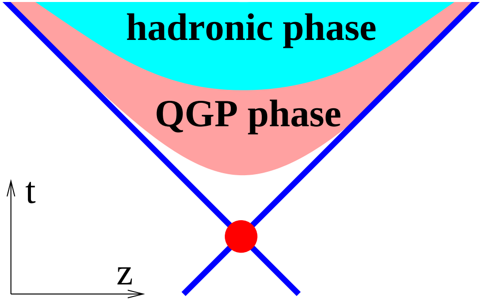
pt distributions charged ptls PbPb 2.76 TeV, different centralities



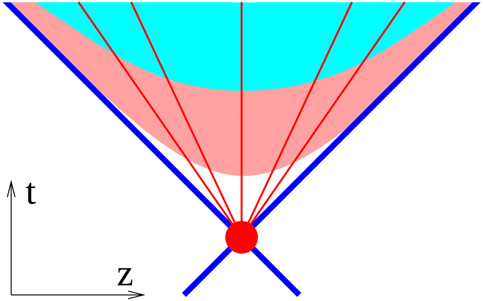
Very small v2 (no string interactions)

4 EPOS4 secondary interactions

So far we discussed primary interactions (the red point)



Milne coordiantes are used to describe evolution



Proper time (hyperbolas)

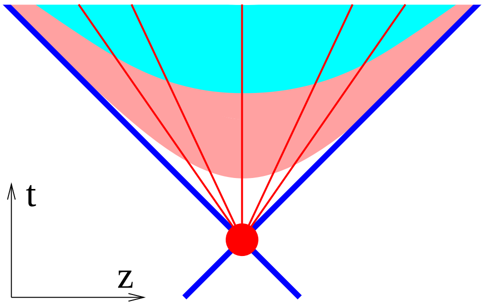
$$\tau = \sqrt{t^2 - z^2}$$

Space-time rapidity (red lines)

$$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$$

(not pseudorapidity)

Primary interactions determine matter distribution in η_s



η_s corresponds (roughly) to the average rapidity (of volume cells): $< y > \approx \eta_s$

so primary interactions determine "essentially" the rapidity distrbution

$$\text{with}\; y = \frac{1}{2} \ln \frac{E + P_z}{E - P_z}$$

Connecting primary and secondary interactions:
the core-corona procedure

4.1 Role of core, corona, remnants

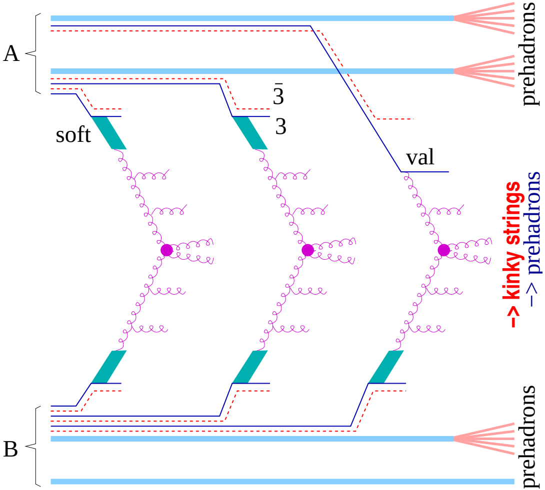
From diagrams to "prehadrons"

arXiv:2306.02396, PRC 108, 034904

A+B scattering (three cut Pomerons)

Remnants always "white" but excited (large masses) -> prehadrons

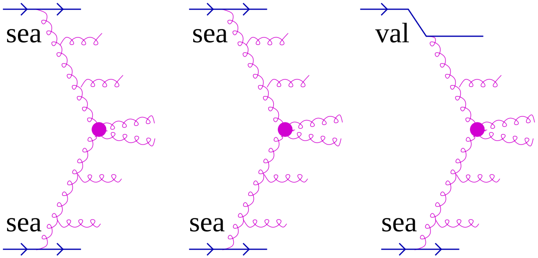
"Pomeron ends" (3 — $\bar{3}$) + parton ladders -> kinky strings constructed following color flow -> prehadrons



light blue: remnants

Colorwise equivalent:

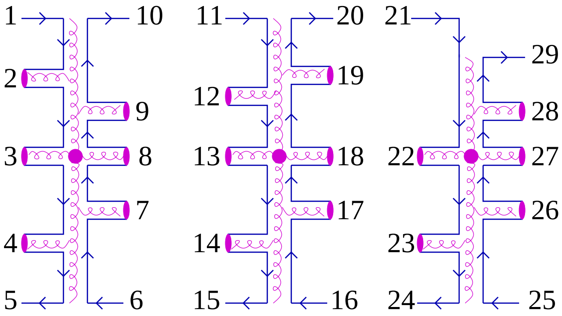
Outgoing antiquarks drawn as incoming quarks (arrows towards vertices)



Color flow

Chains of partons from antiquark to quark 1-2-3-4-5, 6-7-8-9-10 etc

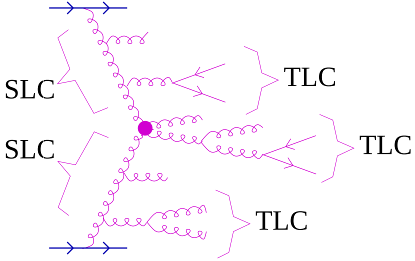
No color reconnections needed !!



Spacelike cascade (SLC)

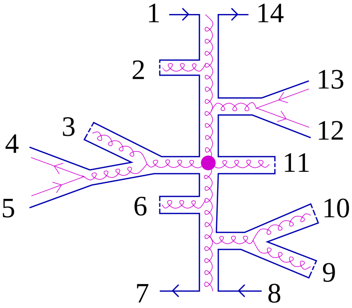
and

timelike cascade (TLC)

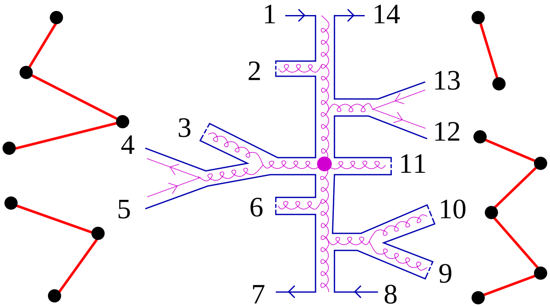


Color flow

Chains of partons from antiquark to quark 1-2-3-4, 5-6-7, 8-9-10-11-12, 13-14



From chains of partons to strings



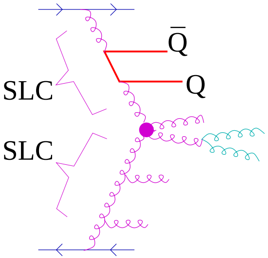
The four color flow chains 1-2-3-4, 5-6-7, 8-9-10-11-12, and 13-14 mapped to kinky strings (red lines), black points indicate the kinks

String breaks into "prehadrons" via area law

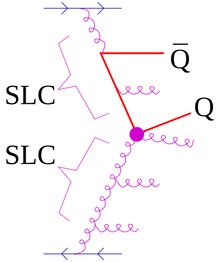
hep-ph/0007198, Phys.Rept. 350 (2001) 93-289

Heavy flavor production

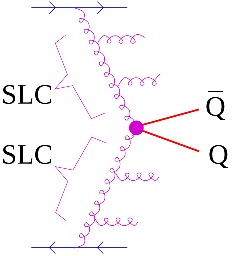
(1)



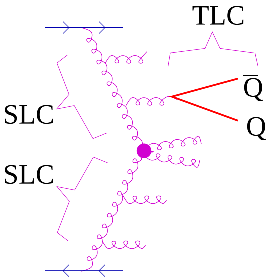
(2)



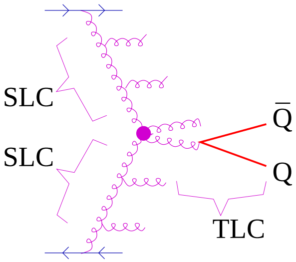
(3)



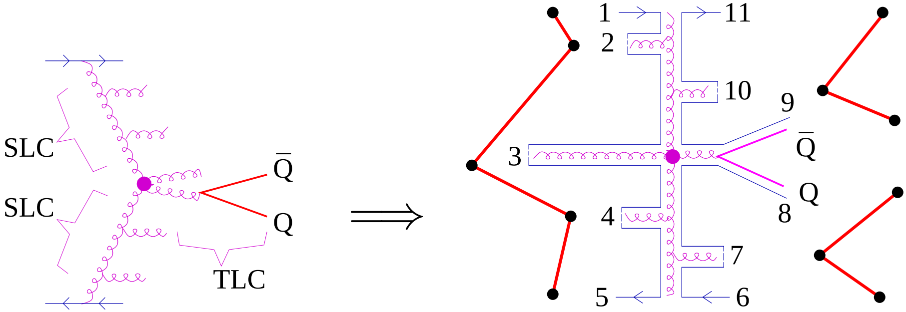
(4)



(5)



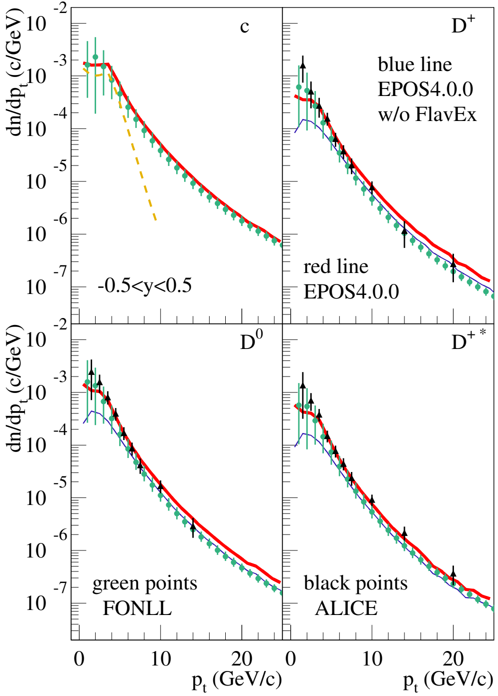
Heavy flavor strings



chains 1-2-3-4-5, 6-7-8, 9-10-11

Again string breaking via area law

hep-ph/0007198, Phys.Rept. 350 (2001) 93-289



close to FONLL

4.0.0 not yet optimized

more recent work, see https://klaus.pages.in2p3.fr /epos4/physics/papers -> The EPOS4HQ project

HQ QGP interaction important even in pp

Core-corona procedure
(Big and small systems)

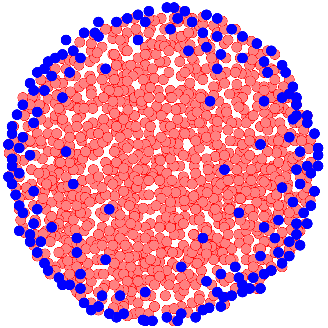
Consider all prehadrons (no charm)

Each prehadron: estimate energy loss ΔE on its way out of this system
(keeping the positons of the others)

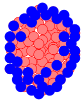
If $\Delta E > E$ -> core prehadron
If $\Delta E < E$ -> corona prehadron

Corona hadrons -> hadrons
Core hadrons -> "the core" (matter)

Big system



Small system



corona = blue **core = red**

Core-corona procedure
for minimum bias proton-proton

Prehadron yield vs space-time rapidity $\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$

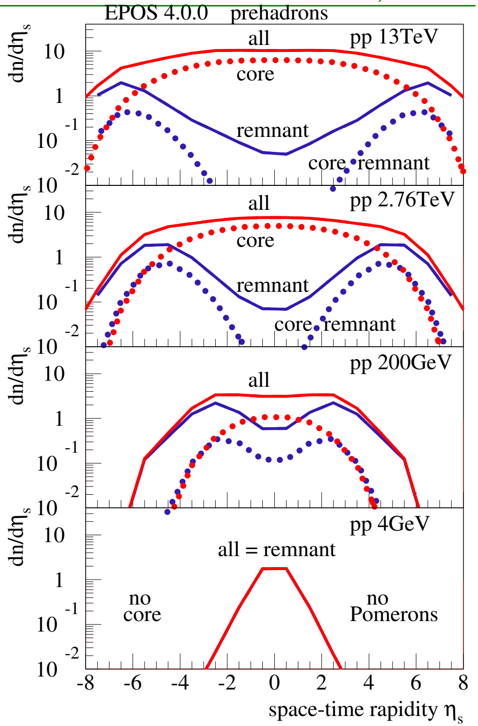
Note: $\eta_s \approx y$ (rapidity)

High energy: big core contribution even for min bias!

From high to low energy:

- **core contribution smaller**
- **remnants more important**
- **contribute at mid-rapidity**

4 GeV : No Pomerons, no core



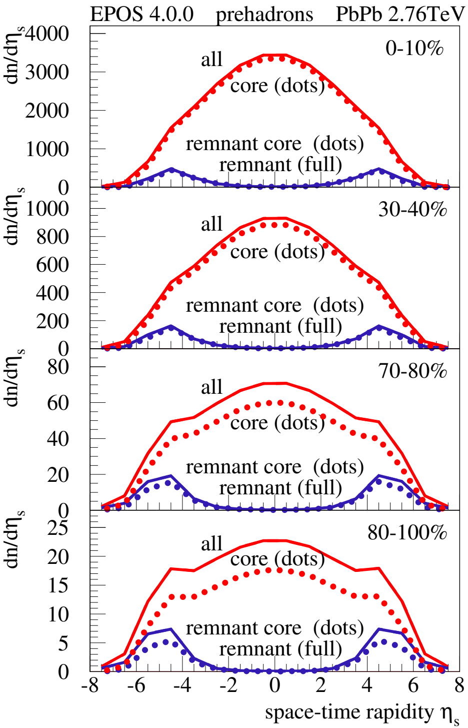
Core-corona procedure
for nucleus-nucleus

Prehadron yield vs η_s

central PbPb:
core almost 100%
even for remnants

From central to peripheral:

- ☐ **core drops at large η_s**
- ☐ **remnants get more important**



Energy density ε
Calculation at hydro start time τ_0

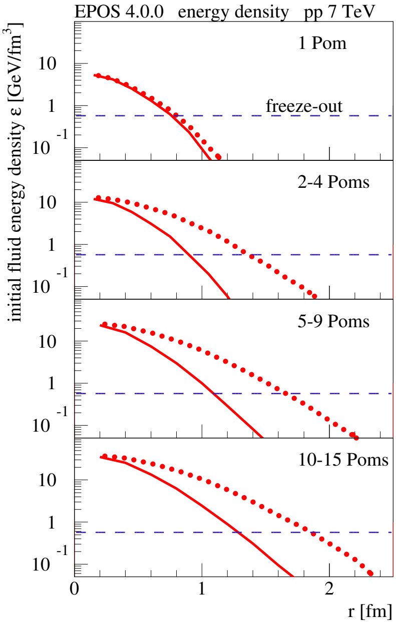
***pp* 7 TeV**

Compute $T^{\mu\nu}$ from prehadrons, boost to comoving frame, extract ε and flow vector

Dashed line:
FO en. density

Different Pomeron numbers

For each event, one determines the event plane angle ψ and rotate the system accordingly (to have after rotation event plane angles zero). The solid lines correspond to azimuthal angles $\phi = 0$, the dotted lines to $\phi = \pi/2$.



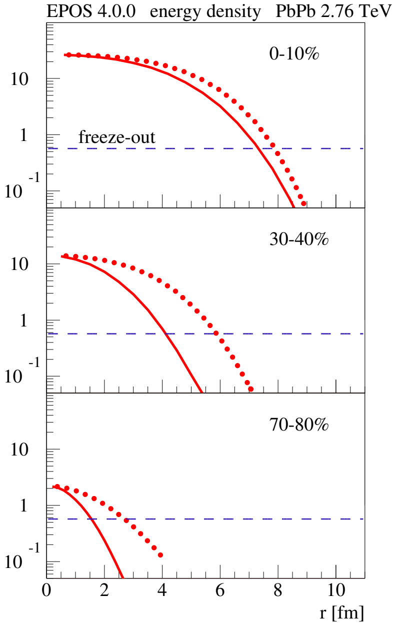
Energy density ε
Calculation at hydro start time τ_0

PbPb 2.76 TeV

Compute $T^{\mu\nu}$ from prehadrons, boost to comoving frame, extract ε and flow vector

**Dashed line:
FO en. density**

Different centralities



Next steps

- □ **Hydrodynamic evolution**
  (code from Iu. Karpenko[1,2])
- □ **Sudden freeze-out (microcanonical) at** $\varepsilon_{\mathrm{FO}} = 0.57\,\frac{\mathrm{GeV}}{\mathrm{fm}^3}$
  (many new features, important for small fluids, in pp and AA)
- □ **Hadronic cascade** (UrQMD[3,4])

---

(1) Werner, K. and Guiot, B. and Karpenko, Iu. and Pierog, T., Phys. Rev. C 89, 6 (2014), pp. 064903

(2) Iu. Karpenko and P. Huovinen and M. Bleicher, Computer Physics Communications 185, 11 (2014), pp. 3016--3027

(3) S. A. Bass and others, Prog. Part. Nucl. Phys. 41 (1998), pp. 225-370.

(4) M. Bleicher and others, J. Phys. G25 (1999), pp. 1859-1896.

4.2 Microcanonical hadronization of plasma droplets

(see arXiv:2306.10277)

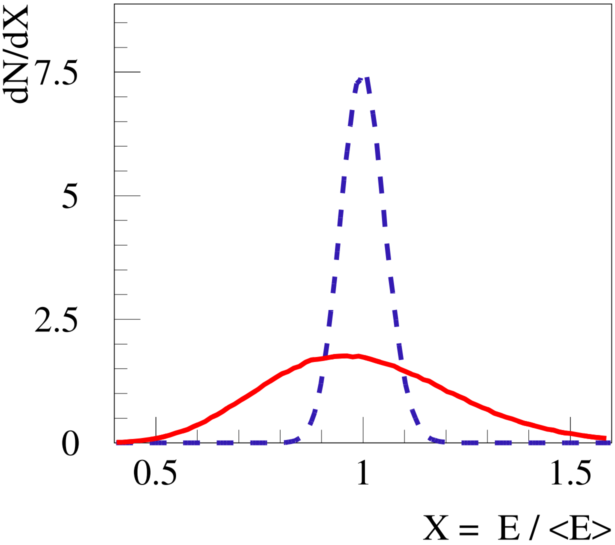
- **Real hadronization (not transition fluid-particles)**
  (sudden statistical decay)

- **Energy and flavor conservation**
  (important for small systems)

- **Extremely fast**
  (major technical impovements in EPOS4)

Grand canonical decay, T = 130 MeV, $f_k = \frac{g_k V}{(2\pi\hbar)^3} \exp\left(-\frac{E_k}{T}\right)$

V=50 fm^3; V=1000 fm^3



Microcanonic decay
of given volume in its CMS into n hadrons

$$dP = C_{\mathrm{vol}}\, C_{\mathrm{deg}}\, C_{\mathrm{ident}} \times \delta(E - \Sigma E_i)\, \delta(\Sigma \vec{p}_i) \prod_A \delta_{Q_A, \Sigma q_{A\,i}} \prod_{i=1}^{n} d^3 p_i$$

$$C_{\mathrm{vol}} = \frac{V^n}{(2\pi\hbar)^{3n}}\,, \quad C_{\mathrm{deg}} = \prod_{i=1}^{n} g_i\,, \quad C_{\mathrm{ident}} = \prod_{\alpha\in\mathcal{S}} \frac{1}{n_\alpha!}$$

(n_α is the number of particles of species α, $\mathcal{S}$ is the set of particle species)

Different from decay rate of a massive particle (using LIPS), where asymptotic states are defined over an infinitely large volume

(see Becattini et al, EPJC35:243-258,2004). **But** $E_i = \sqrt{p_i^2 + m_i^2}$

Microcanonical decay

$$dP \propto d\Phi_{\mathrm{NRPS}} = \delta(M - \Sigma E_i)\, \delta(\Sigma \vec{p}_i) \prod_{i=1}^{n} d^3 p_i$$

- ☐ Hagedorn 1958 methods to compute Φ_{NRPS}
- ☐ Lorentz invariant phase space (LIPS) (James 1968)
- ☐ Hagedorn methods used for decaying QGP droplets (Werner, Aichelin, 1994, Becattini 2003)
- ☐ 2012 (Bignamini,Becattini,Piccinini) compute Φ_{NRPS} via the Lorentz invariant phase space (LIPS)
- ☐ **NEW (EPOS4) 2022:**
  - **Much improved Hagedorn integral method, made very efficient at large n**
  - **use LIPS method only for small n,** (gets time consuming at large n)

Grand canonical limit

For very large M we should recover the “grand canonical limit” for single particle spectra:

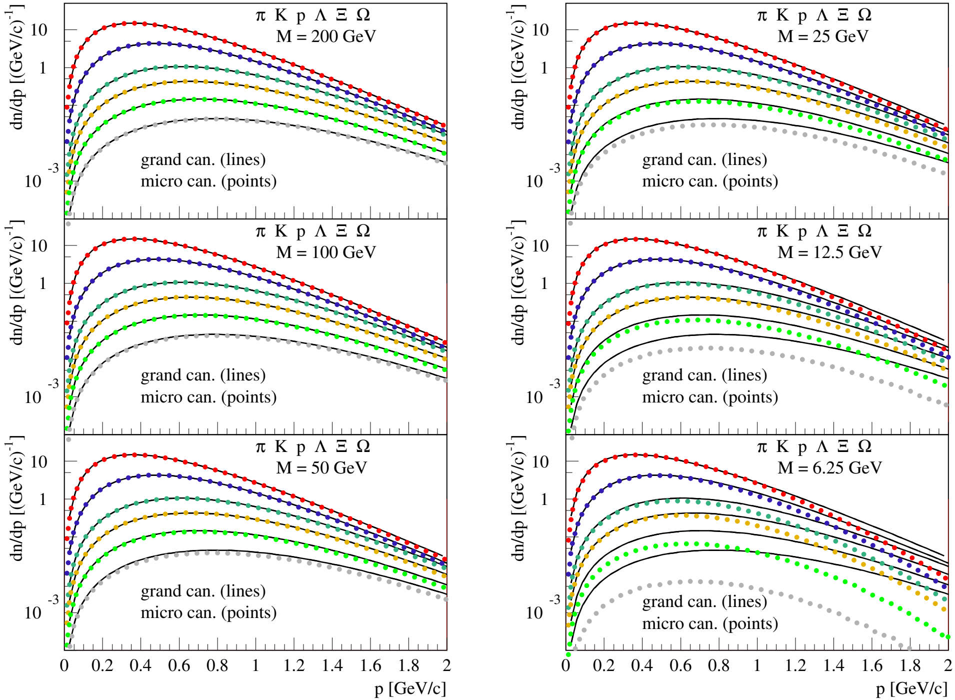
$$f_k = \frac{g_k V}{(2\pi\hbar)^3} \exp\left(-\frac{E_k}{T}\right),$$

The average energy is

$$\bar{E} = \sum_k \frac{4\pi g_k V}{(2\pi\hbar)^3} m_k^2 T \left(3TK_2(\frac{m_k}{T}) + m_k K_1(\frac{m_k}{T})\right).$$

The microcanonical decay of an object of mass M and volume V should converge (for $M \to \infty$) to the GC single particle spectra

with T obtained from $M = \bar{E}$. $T = 167$ **MeV in the following**


π K p Λ Ξ Ω
M = 200 GeV
grand can. (lines)
micro can. (points)
π K p Λ Ξ Ω
M = 25 GeV
grand can. (lines)
micro can. (points)
π K p Λ Ξ Ω
M = 100 GeV
grand can. (lines)
micro can. (points)
π K p Λ Ξ Ω
M = 12.5 GeV
grand can. (lines)
micro can. (points)
π K p Λ Ξ Ω
M = 50 GeV
grand can. (lines)
micro can. (points)
π K p Λ Ξ Ω
M = 6.25 GeV
grand can. (lines)
micro can. (points)
dn/dp [(GeV/c)$^{-1}$]
p [GeV/c]

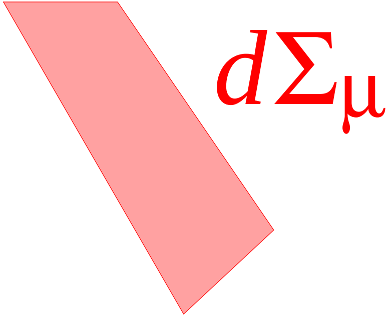
Hadronization on hyper-surface

Hypersurface element:

$$d\Sigma_\mu = \varepsilon_{\mu\nu\kappa\lambda} \frac{\partial x^\nu}{\partial \tau} \frac{\partial x^\kappa}{\partial \varphi} \frac{\partial x^\lambda}{\partial \eta} \, d\tau \, d\varphi \, d\eta$$

Surface:
$x^0 = \tau \cosh \eta$, $x^1 = r \cos \varphi$,
$x^2 = r \sin \varphi$, $x^3 = \tau \sinh \eta$

with $r = r(\tau, \varphi, \eta)$,
representing the
FO condition $\varepsilon = \varepsilon_{\mathrm{FO}}$

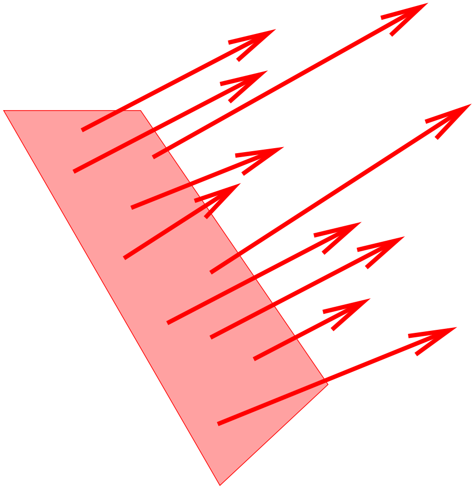


GC particle production via Cooper-Frye

$$E\frac{dn}{d^3p} = \int d\Sigma_\mu p^\mu f(up),$$

assuming a thermalized resonance gas

(adding δf for viscous hydro)



Our approach:

Flow of momentum vector dP^{μ} and conserved charges dQ_A through the surface element:

$$
\begin{aligned}
dP^{\mu} &= T^{\mu\nu} d\Sigma_{\nu}, \\
dQ_A &= J_A^{\nu} d\Sigma_{\nu}.
\end{aligned}
$$

(with $A \in \{C, B, S\}$, corresponding electric charge, baryon number and strangeness)

Construct an **effective mass** by summing surface elements:

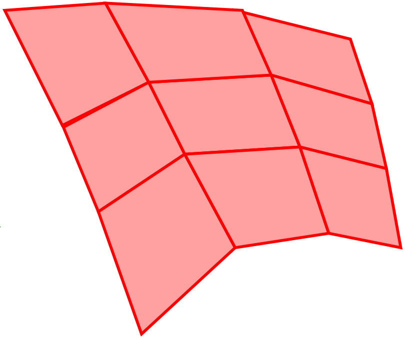
$$M = \int_{\text{surface area}} dM,$$

with

$$dM = \sqrt{dP^\mu dP_\mu},$$

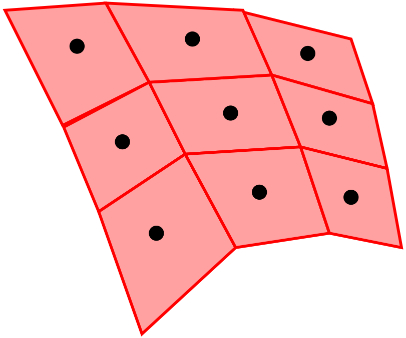
knowing for each element four-velocity

$$U^\mu = dP^\mu / dM,$$



The four-velocity U^μ is NOT equal to the fluid velocity u^μ!

The effective mass decays microcanonically



Particles are distributed on the hyper-surface

$$x^{\mu}(\tau, \varphi, \eta)$$

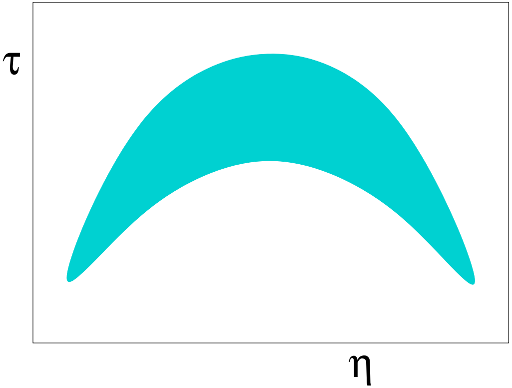
according to the distribution

$$dM(\tau, \varphi, \eta)$$

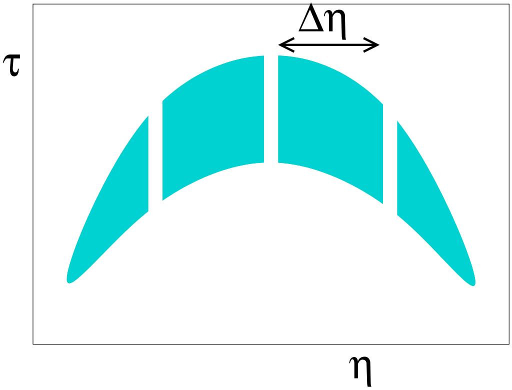
and they are boosted according to the four-velocity

$$\boldsymbol{U^{\mu}(\tau, \varphi, \eta)}$$

Decaying object extended in space-time



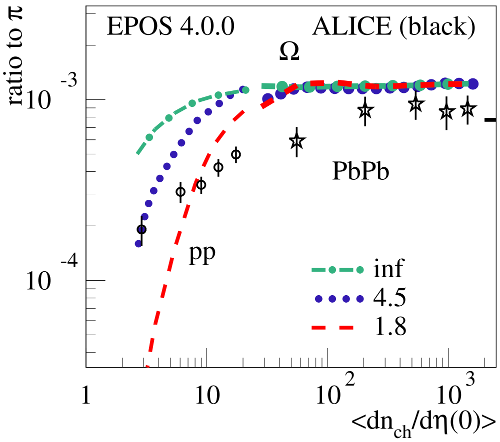
Does is decay as single effective mass M?



... or as several independent objects of width $\Delta\eta$

We will try several choices of $\Delta\eta$

Omega to pion ratio (pure core)



different choices of $\Delta\eta$

$\Delta\eta = \infty$: drops slightly

$\Delta\eta = 1.8$: drops quickly around $dn/d\eta = 10$

4.3 Core/corona contributions to hadrons production

Distinguish:

(A) The “**core+corona**” contribution: primary + core-corona separation + hydrodynamic evolution + microcanonical hadronization, but **without hadronic rescattering**.

(B) The “**core**” contribution: as (A), but considering only core particles.

(C) The “**corona**” contribution: as (A), but considering only corona particles.

(D) The “**full**” EPOS4 scheme: as (A), but in addition hadronic rescattering.

Note: Rescattering concerns core and corona particles

Core, corona, full
pp at 7 TeV

pions, kaons, protons, lambdas (top to bottom)

Green: $\frac{\text{core}}{\text{core+corona}}$

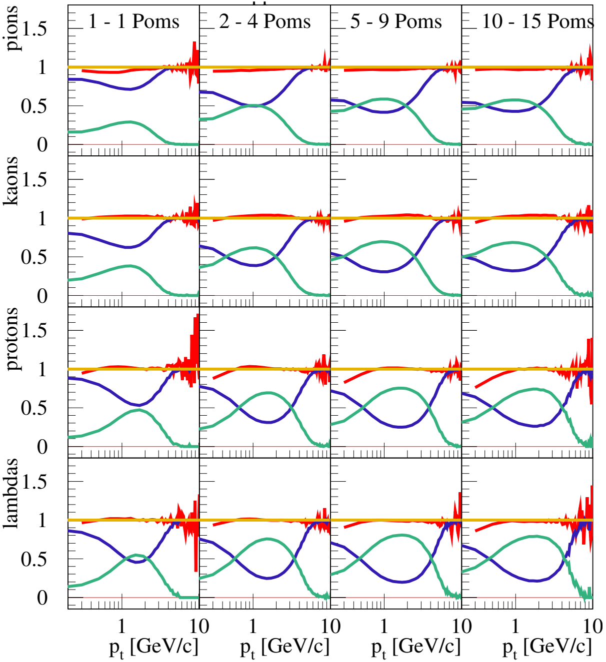
Blue: $\frac{\text{corona}}{\text{core+corona}}$

Red: $\frac{\text{full}}{\text{core+corona}}$

Core reaches to higher pt for baryons

Core has maximum at intermediate pt (flow)

Rescattering not very important



Core, corona, full
PbPb at 5.02 TeV

pions, kaons, protons, lambdas
(top to bottom)

Green: $\frac{\text{core}}{\text{core+corona}}$

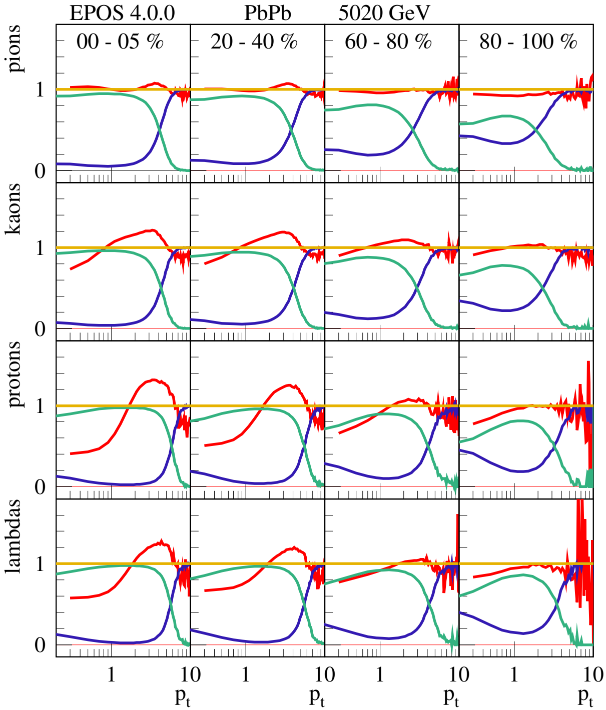
Blue: $\frac{\text{corona}}{\text{core+corona}}$

Red: $\frac{\text{full}}{\text{core+corona}}$

Core reaches to higher pt for baryons

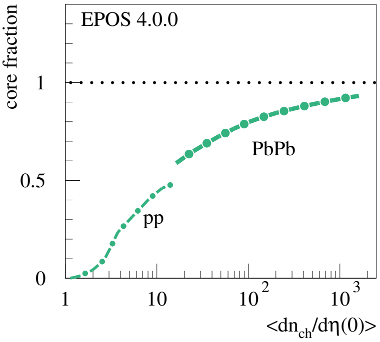
Core has maximum at intermediate pt (flow)

Rescattering important



4.4 Core + corona results - multiplicity dependencies

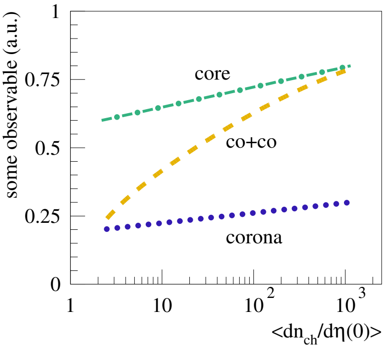
Core fraction



Almost continuous!

see DCCI2, Y. Kanakubo et al
Phys. Rev. C 105 (2022) 2, 024905

Core + corona (co+co) results (sketch)

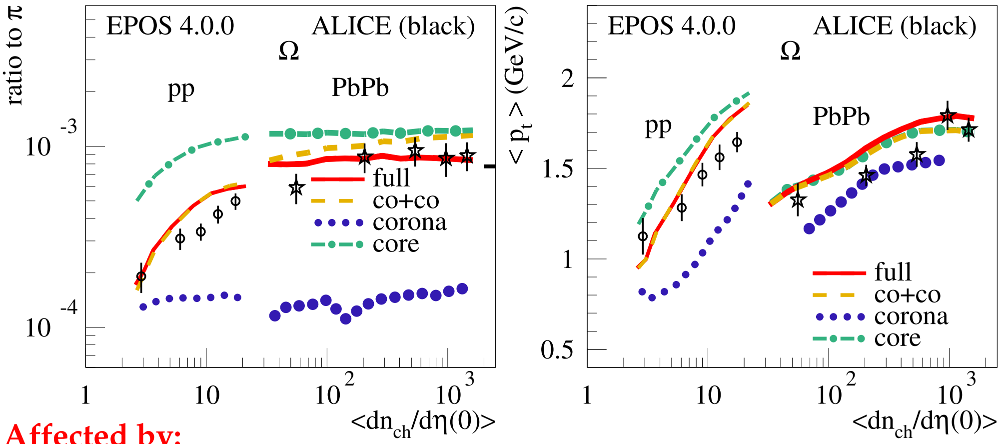


Transition from corona core

Attention ! Core and corona curve continuous ... or not (depends on variable)

On top: effects from hadronic cascade (UrQMD, S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 225 (1998), M. Bleicher et al., J. Phys. G25, 1859 (1999))

continuous curve **jump**



Affected by:

core-corona

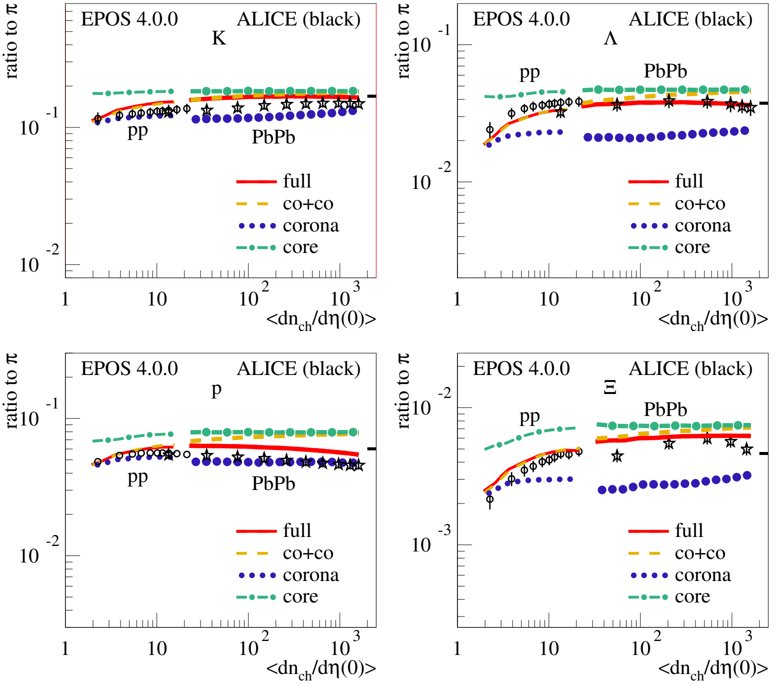
microcanonical

hadronic cascade (UrQMD)

saturation

flow

core-corona



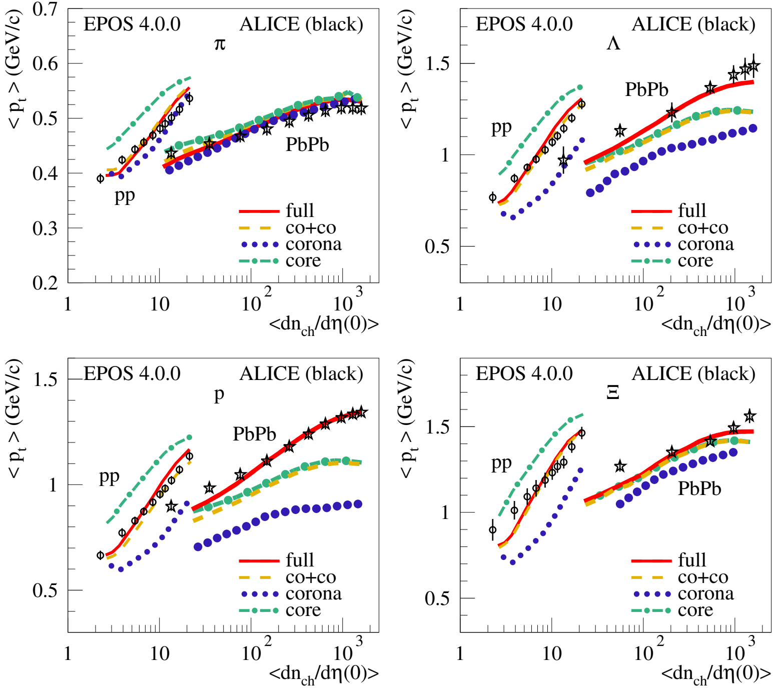
continuous curves

Affected by:

core-corona

microcanonical

hadronic cascade



discontinuities

curves affected by:

saturation

flow

core-corona

hadronic cascade

Multiplicity dependence of charm production

saturation and "hydro" effect

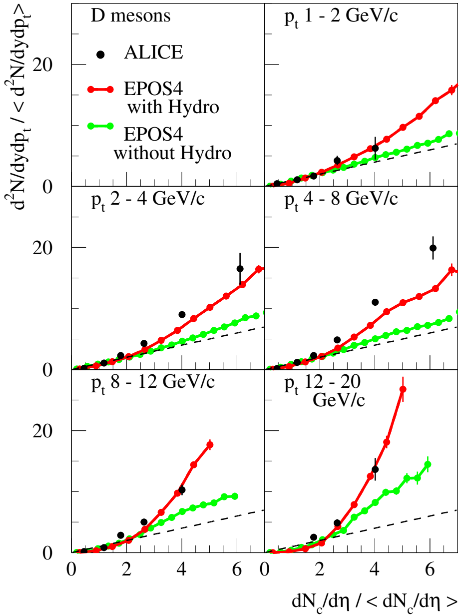
pp 7TeV

Self-normalized
***D* meson multiplicity**

for different transverse momentum ranges

versus self-normalized charged particle multiplity,

compared to ALICE data



More details and many hundreds of plots data-simulation

(https://klaus.pages.in2p3.fr/epos4/physics/papers)

- ☐ **arxiv:2301.12517 PRC 108, 064903** **EPOS4 Overview**
- ☐ **arxiv:2306.02396 PRC 108, 034904** **pQCD in EPOS4** with B. Guiot
- ☐ **arxiv:2310.09380 PRC 109, 034918**
  Parallel scattering formalism, S-matrix theory & pQCD & saturation
  46 pages, systematic and complete presentation of the theoretical basis,
- ☐ **arxiv:2306.10277 PRC 109, 014910**
  Microcanonical hadronization, core-corona in EPOS4
- ☐ **arxiv:2401.11275** **EPOS4 results on RHIC**
  with J. Jahan, I. Karpenko, T. Pierog, M. Stefaniak, D. Vintache
- ☐ **arxiv:2310.08684** **EPOS4HQ**: Heavy flavor collectivity in pp
- ☐ **arxiv:2401.17096** **EPOS4HQ**: Heavy flavour in HI at RHIC and LHC
  EPOS4HQ: Jiaxing Zhao, Joerg Aichelin, Pol-Bernard Gossiaux, KW

Key point: The presented formalism allows,
for $A + B$ collisions (including pp)

- ☐ **to treat in parallel, instantaneously (mandatory at high energy)**
  - **ALL $A \times B$ possible NN interactions**
  - **each one composed of many parton-parton scatterings**

- ☐ **by perfectly conserving energy-momentum (also mandatory)**

- ☐ **being compatible with factorization for inclusive xsections after implementing dynamical saturation scales (forced to do so, for consistent picture, recover AGK!)**

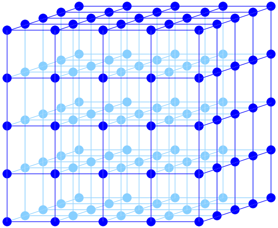
- ☐ **Monte Carlo 100% compatible with theoretical framework**

5 Complements

5.1 Configurations via Markov chains

Let x be a multidimensional random number (better random configuration) distributed according to some law $f(x)$.

Example 1: Ising model



x is one of the $2^{(N^3)}$ possible states of the lattice

The law is

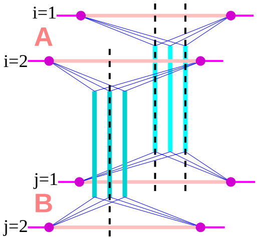
$$f(x) = \frac{1}{Z} e^{-\beta E(x)}$$

with

$$E = -\alpha \sum_{\text{neighbors } k,k'} s_k s_{k'}$$

(thermal equilibrium)

Example 2: Pomeron configuration



x is a multiple Pomeron configuration

$$x = \{\{m_k\}, \{x^{\pm}_{k\mu}\}\}$$

k=1,...,AB, $\mu = 1, ..., m_k$

The law is

$$f(x) = \prod_{k=1}^{AB} \left[\frac{1}{m_k!} \prod_{\mu=1}^{m_k} G_{k\mu} \right] \times W_{AB}$$

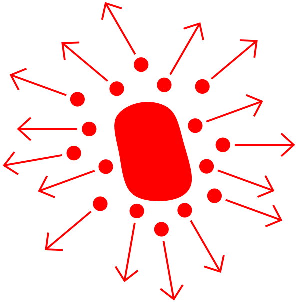
(GR$^+$)

$$G_{k\mu} = G\left(x^{+}_{k\mu}, x^{-}_{k\mu}, s, b_k\right)$$

$$W_{AB} = W_{AB}(x)$$

details:
arXiv:2310.09380, PRC 109, 034918

Example 3: Decaying plasma



x is a set of n particles (up to several hundreds) with given ids and 4-momenta

The law (microcanonical) is

$$f(x) = N \frac{V^n}{(2\pi\hbar)^{3n}} \prod_{i=1}^{n} g_i \prod_{\alpha \in \mathcal{S}} \frac{1}{n_\alpha !}$$

$$\times \delta(E - \Sigma_{i=1}^{n} E_i)\, \delta(\Sigma_{i=1}^{n} \vec{p}_i)$$

$$\times \prod_{A} \delta_{Q_A, \Sigma q_{A\,i}}$$

N overall normalization, g_i degeneracy, $\mathcal{S}$ set of particle species, n_α number of ptls of species α, Q_A conserved quantities (u,d,s)

details:
arXiv:2306.10277, PRC 109, 014910

How to generate x according to some law $f(x)$?

Idea: Let's copy nature which always finds eventually the "equilibrium distribution"

A. Markov

Finally the method works not only for "thermal" distributions but for any law $f(x)$.

For each generation of a "random configuration" x:

One considers a stochastic iterative process (Markov chain)

$$f_1 \to f_2 \to ...$$

with appropriate transitions $f_t \to f_{t+1}$ (Metropolis) such that f_t converges to $f_\infty = f$

One generates corresponding random configurations x_i

$$x_1 \to x_2 \to ...$$

and takes finally $x = x_\infty$

Consider a sequence of such multidimensional random configurations

$$x_1\,,\ x_2\,,\ x_3, ...$$

with f_t being the law for x_t.

A homogeneous Markov chain is defined as

$$f_t(x) = \sum_{x'} f_{t-1}(x')p(x' \to x).$$

with $p(x' \to x)$ being the transition probability (or matrix). Normalization : $\sum_x p(x' \to x) = 1$.

Consider the law f_t for x_t. The law for x_{t+1} is

$$\sum_a f_t(a)\, p(a \to b)\,.$$

One defines an operator T (like <u>T</u>ranslation)

$$Tf_t(b) = \sum_a f_t(a)\, p(a \to b)\,.$$

So $Tf_t = f_{t+1}$ is the law for x_{t+1} when f_t is the law for x_t.

A law is called stationary if $Tf = f$.

Theorem: If a stationary law $Tf = f$ exists, then $T^k f_1$ converges towards f (which is unique) for any f_1.

So to generate random configurations according to some (given) law f,

- **one constructs a T such that $Tf = f$**
- **and then considers $f_1 \rightarrow Tf_1 \rightarrow T^2 f_1$...**
- **and constructs the corresponding random configurations**

One needs, for a given law f,
to **find a transition matrix p such that $Tf = f$**

Sufficient condition (detailed balance):

$$f(a)\,p(a \to b) = f(b)\,p(b \to a)\,,$$

$$\text{Proof :} \qquad Tf(b) = \sum_a f(a)\,p(a \to b) = \sum_a f(b)\,p(b \to a) = f(b) \sum_a p(b \to a) = f(b)\,.$$

Metropolis algorithm

Definitions:

$$\begin{aligned} p_{ab} &= p(a \rightarrow b)\,, \\ f_a &= f(a)\,. \end{aligned}$$

Take

$$p_{ab} = w_{ab}\, u_{ab}\,.$$

with

$$w_{ab} : \text{proposal matrix } (\textstyle\sum_b w_{ab} = 1)$$

$$u_{ab} : \text{acceptance matrix } (u_{ab} \leq 1)$$

This is NOT the simple acceptance-rejection method!!

Detailed balance:

$$f_a\, p_{ab} = f_b\, p_{ba}$$

amounts to

$$f_a\, w_{ab}\, u_{ab} = f_b\, w_{ba}\, u_{ba}\,,$$

which may be written as

$$\frac{u_{ab}}{u_{ba}} = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\,.$$

The equation (detailed balance)

$$\frac{u_{ab}}{u_{ba}} = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}$$

is solved by

$$u_{ab} = F\left(\frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\right),$$

with a function F with

$$\frac{F(z)}{F(\frac{1}{z})} = z\,.$$

$$\text{Proof : With } z \equiv \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}} \text{ one finds : } \frac{u_{ab}}{u_{ba}} = \frac{F(z)}{F(\frac{1}{z})} = z = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}.$$

The F according to Metropolis is

$$F(z) = \min(z, 1)\,.$$

One finds indeed

$$\frac{F(z)}{F(\frac{1}{z})} = \frac{\min(z,1)}{\min(\frac{1}{z},1)} = \left\{ \begin{array}{lll} z/1 & \text{pour} & z \leq 1 \\ 1/\frac{1}{z} & \text{pour} & z > 1 \end{array} \right\} = z\,.$$

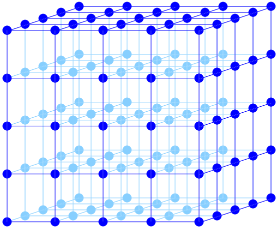
So one proposes for each iteration a new configuation b according to some w_{ab}, and accepts it with probability

$$u_{ab} = \min\left(\frac{f_b}{f_a}\frac{w_{ba}}{w_{ab}}, 1\right).$$

- **Convergence is guaranteed for whatever choice of w!!**

- **Provided we choose w such that starting from a any goal b can be reached with a nonzero probability for a finite (reasonable) number of iterations.**

Example 1: Ising model



a **is one of the** $2^{(N^3)}$ **possible states of the lattice**

The law is

$$f_a = \frac{1}{Z} e^{-\beta E_a}$$

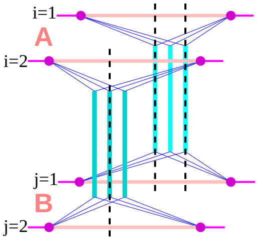
Proposal w_{ab}**:**

Choose randomly an atom of configuration a **and make a spin flip**

$$u_{ab} = \min\left(\frac{f_b}{f_a}, 1\right).$$

if $f_b \geq f_a$: accepted
if $f_b < f_a$: accepted with weight f_b/f_a

Example 2:
Pomeron configuration



x is a multiple Pomeron configuration

$$x = \{\{m_k\}, \{x^{\pm}_{k\mu}\}\}$$

k=1,...,AB, $\mu = 1, ..., m_k$

The law is

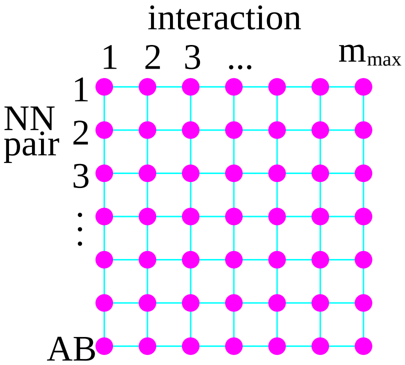
$$f(x) = \prod_{k=1}^{AB} \left[\frac{1}{m_k!} \prod_{\mu=1}^{m_k} G_{k\mu}\right] \times W_{AB}$$

(GR$^+$)

$$G_{k\mu} = G\left(x^+_{k\mu}, x^-_{k\mu}, s, b_k\right)$$

$$W_{AB} = W_{AB}(x)$$

Define interaction matrix



with (for row k)
m_k sites hosting a Pomeron
and $m_{\max} - m_k$ "holes"
considering ordering of Pomerons, purely technical

The law is then

$$f(x) = \prod_{k=1}^{AB}\left[\frac{1}{m_k!}\prod_{\mu=1}^{m_k} G_{k\mu}\right] \times W_{AB}$$

$$/ \prod_{k=1}^{AB} \frac{m_{\max}!}{(m_{\max}-m_k)!\,m_k!}$$

$$/ \prod_{k=1}^{AB} m_k!$$

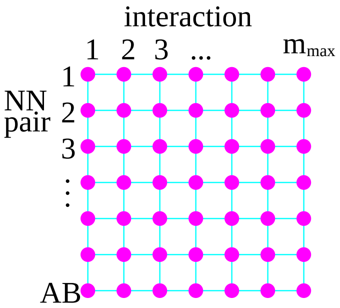
$$G_{k\mu} = G\left(x^+_{k\mu}, x^-_{k\mu}, s, b_k\right)$$

$$W_{AB} = W_{AB}(x)$$

Define w_{ab} such that b changes w.r.t. a only on one lattice site (like Ising model Metropolis),

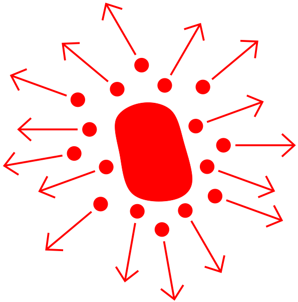
Propose

- ☐ hole with probalility p_0
- ☐ Pomeron with $(1-p_0)G/\int G dx^{=} dx^{-}$



Long iterations, but allows to generate very complex configurations according to a very complex law.

Example 3: Decaying plasma



a is a set of n particles (up to several hundreds) with given ids and 4-momenta

n is not fixed

The microcanonical law is

$$f(a) = N \frac{V^n}{(2\pi\hbar)^{3n}} \prod_{i=1}^{n} g_i \prod_{\alpha \in \mathcal{S}} \frac{1}{n_\alpha !}$$
$$\times \delta(E - \Sigma_{i=1}^{n} E_i)\, \delta(\Sigma_{i=1}^{n} \vec{p}_i)$$
$$\times \prod_A \delta_{Q_A, \Sigma q_{A\,i}}$$

$$df = f \times \prod_{i=1}^{n} d^3 p_i$$

N overall normalization, g_i degeneracy, $\mathcal{S}$ set of particle species, n_α number of ptls of species α, Q_A conserved quantities (u,d,s)

Instead of variable n:
Fixed L (large enough),
but allow holes,
and do ordering

$$\{2p, 1K^+\}$$
$$\rightarrow \{p, \emptyset, K^+, p, \emptyset\}$$

$\rightarrow$ add combinatorial factor
$C=\frac{1}{n!}\ \{\prod_{\alpha\in\mathcal{S}} n_\alpha!\}\ \frac{n!\,(L-n)!}{L!}$

Coordinate trafo

$$\{\vec{p}_i\} \rightarrow \{q_j\},\ q_j \in [0,1]$$

to get rid of δ functions

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Transformed law [known $\tilde{f}$]:

$$d\tilde{f} = \tilde{f}(\{q_i\}) \prod_{j=1}^{3n-4} dq_j$$

Proposal w_{ab}?

First define hadron weights

$$e(h) = \begin{cases} f_h/\{2\sum f_h\} & \text{hadron} \\ 1/2 & \text{hole} \end{cases}$$

f_h being the grand canonical yields

Proposal w_{ab} (arXiv:2306.10277, PRC 109, 014910)

1) to obtain b, chose randomly four hadrons h_i in a, replace them by four hadrons h'_i generated with weights $e(h'_i)$, by conserving flavor

2) in case of change 'hadron to hole' or vice versa, replace one of the q_j by a $[0,1]$ random number

3) compute

$$\frac{w_{ab}}{w_{ba}} = \frac{e(h'_1)e(h'_2)e(h'_3)e(h'_4)}{e(h_1)e(h_2)e(h_3)e(h_4)}$$

4) compute f_b , and (with f_a already known from the step before)

$$u_{ab} = F\left(\frac{f_b}{f_a}\frac{w_{ba}}{w_{ab}}\right)$$

5) accept b with this probability

Summary Markov chains

- Markov chain methods allow to generate very complex random configurations x according to laws $f(x)$, for very different physics problems.

- In case of GR$^+$ for $A+B$ collisions, it allows to treat ALL $A \times B$ possible interactions in parallel, instantaneously,

  each one may amount to up to $n_{\max}$ cut Pomerons, each one characterized by 2 kinematic variables,

  which gives $A \times B \times n_{max} \times 2$ independent variables, highly connected, i.e., for big nuclei: $200 \times 200 \times 20 \times 2$