

# Parton interactions in the medium: theory

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LIP, Lisbon

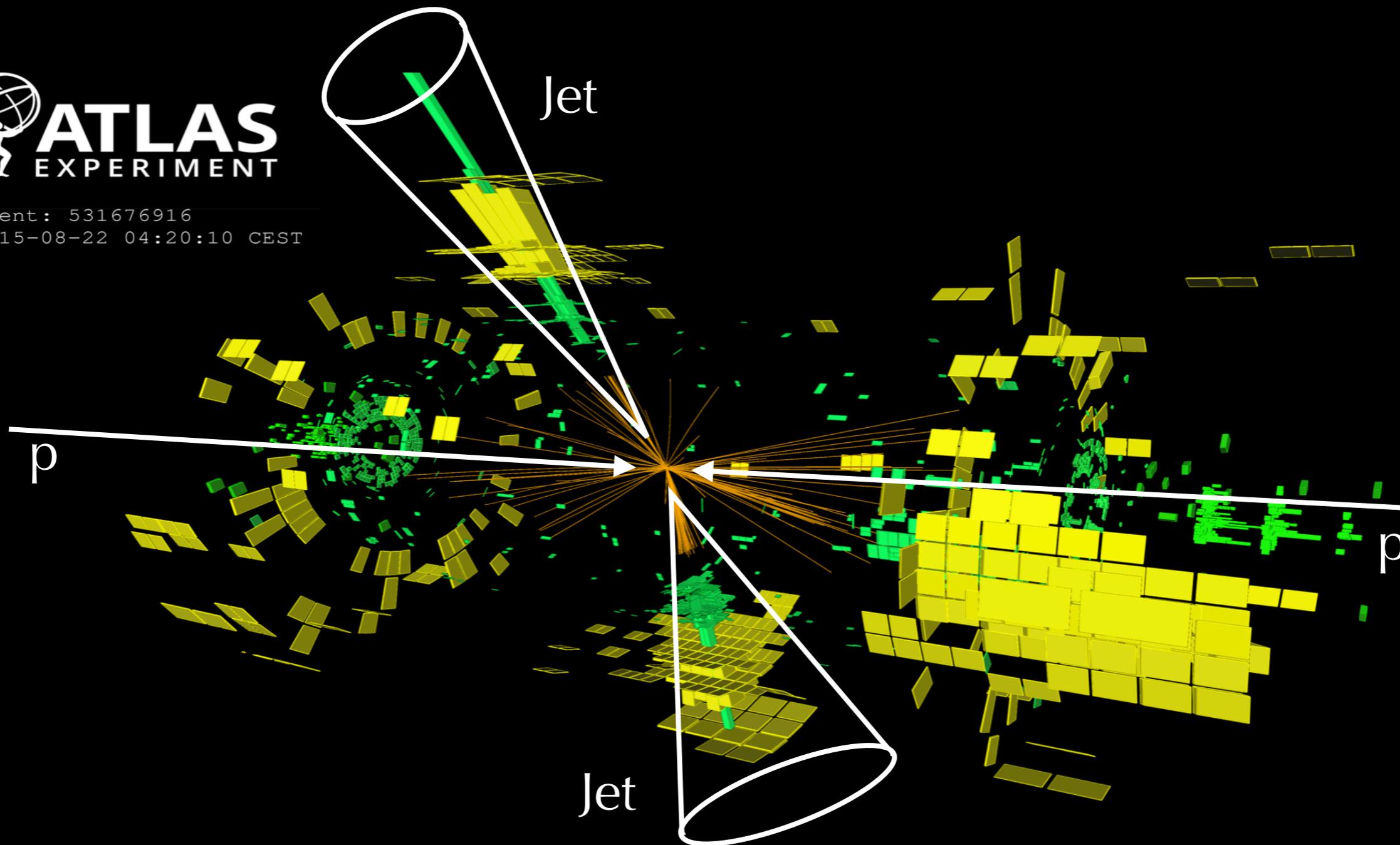
[carlota@lip.pt](mailto:carlota@lip.pt)

GDR QCD Summer School 2024

IJCLab, Orsay, June 9-15, 2024

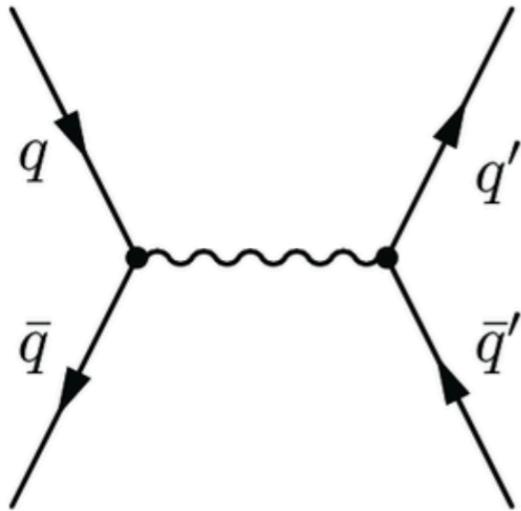


# Some QCD in the vacuum



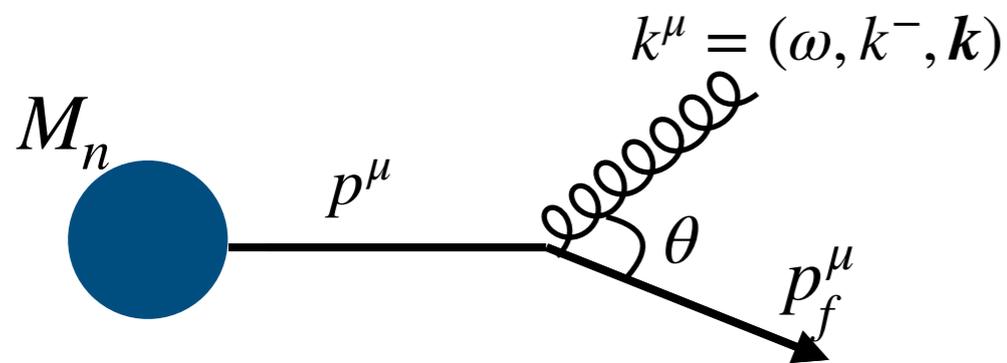
Why do we have jets?

# Gluon emission off a quark



Take an emission off a massless quark:

- Soft:  $\omega \ll \sqrt{s}$
- Collinear w.r.t. emitter:  $\theta \ll 1$



$$M_{n+1} = \bar{u}(p_f) (-ig_s t^a \gamma^\mu) \varepsilon_\mu(k) \frac{i(p_f^\nu + k^\nu) \gamma_\nu}{(p_f + k)^2} M_n$$

$$\downarrow k \ll p$$

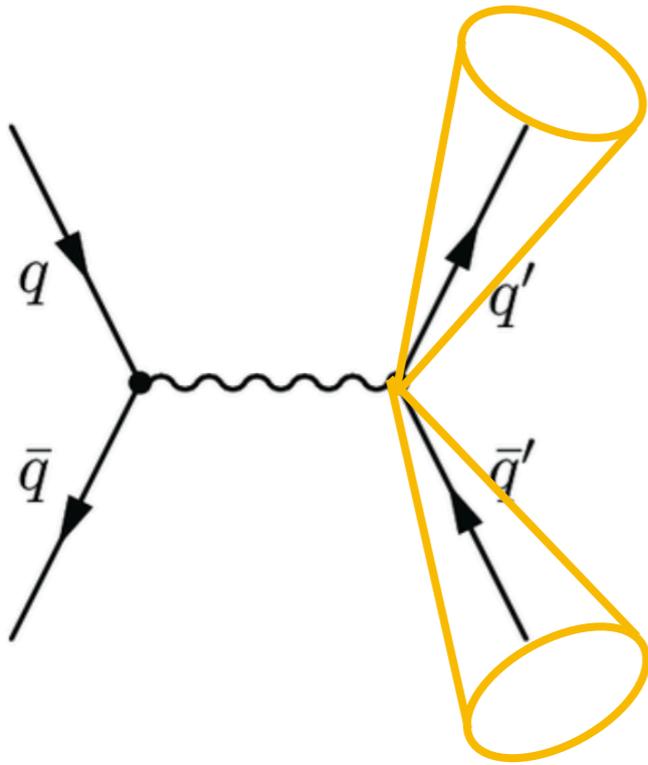
$$M_{n+1} \approx g_s t_a \frac{p_f \cdot \varepsilon}{p_f \cdot k} \bar{u}(p_f) M_n$$

$$\frac{d\omega dk^2}{2\omega(2\pi)^3}$$

$$d\sigma_{n+1} = d\sigma \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$

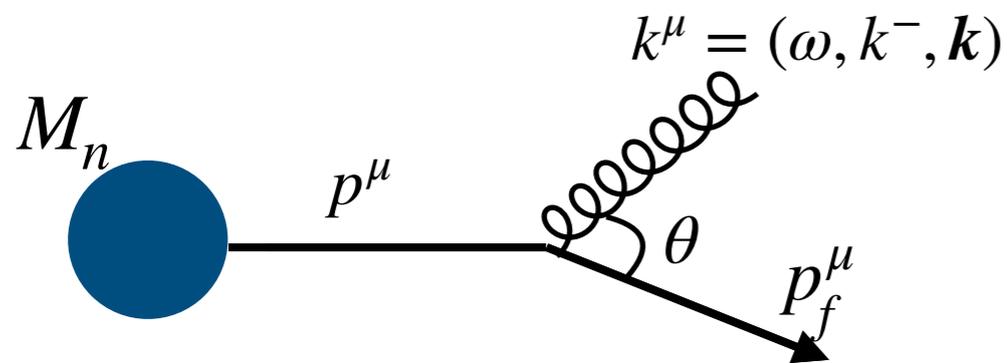
**Soft and collinear divergences!**

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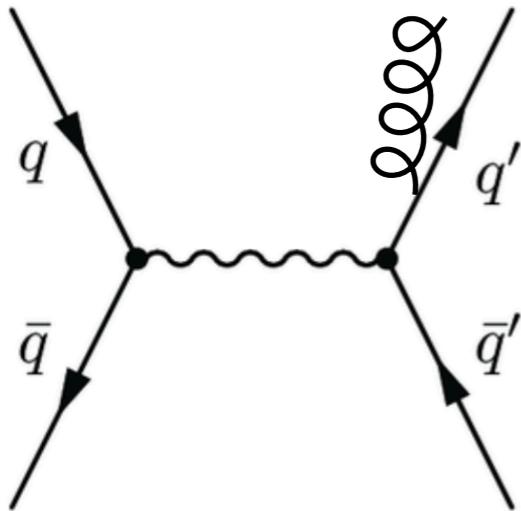
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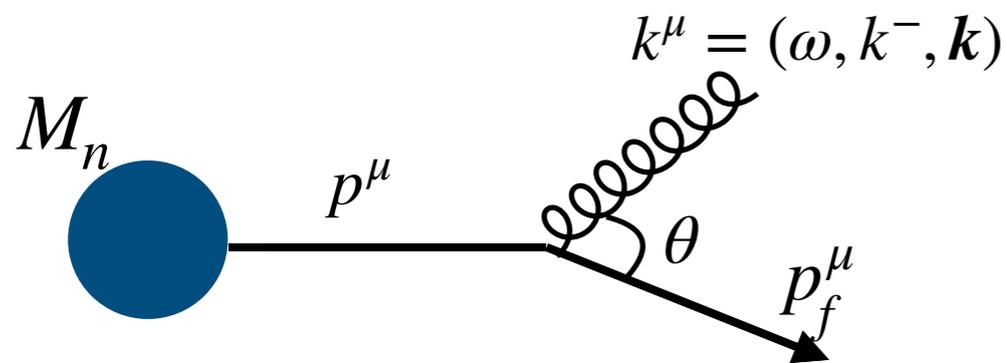
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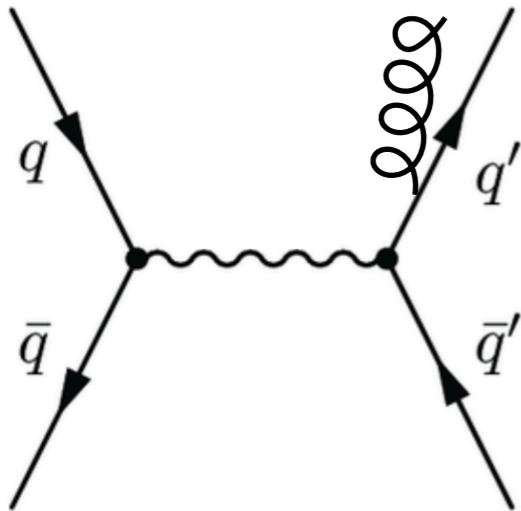
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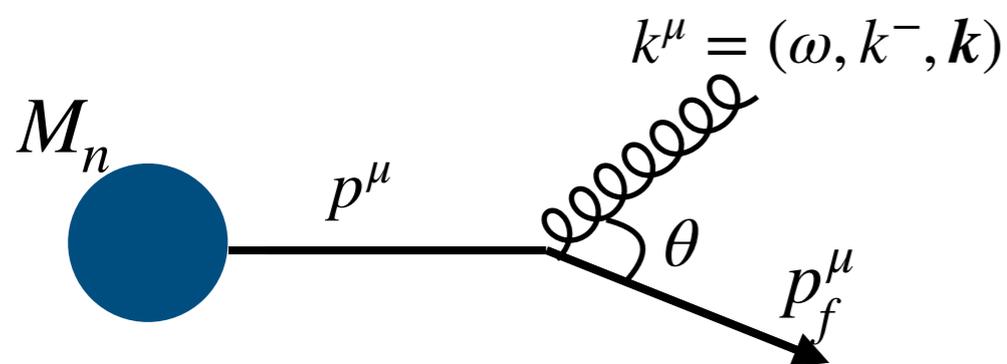
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$$\omega = zE$$

$$d\sigma_{n+1} = d\sigma \frac{2\alpha_s C_F}{\pi} \frac{dz}{z} \frac{d\theta}{\theta}$$

**Soft and collinear divergences!**

# Probability of the emission

Probability of emitting a gluon with energy  $\omega_1$  off a massless quark:

$$P_g \approx \frac{2\alpha_s C_F}{\pi} \int^{\omega_1} \frac{d\omega}{\omega} \int^1 \frac{d\theta}{\theta} \Theta(\omega\theta > Q_0)$$

Transverse momentum of the gluon must be larger than some non-perturbative threshold:  $Q_0$

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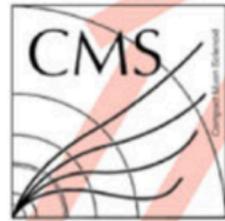
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The result is:

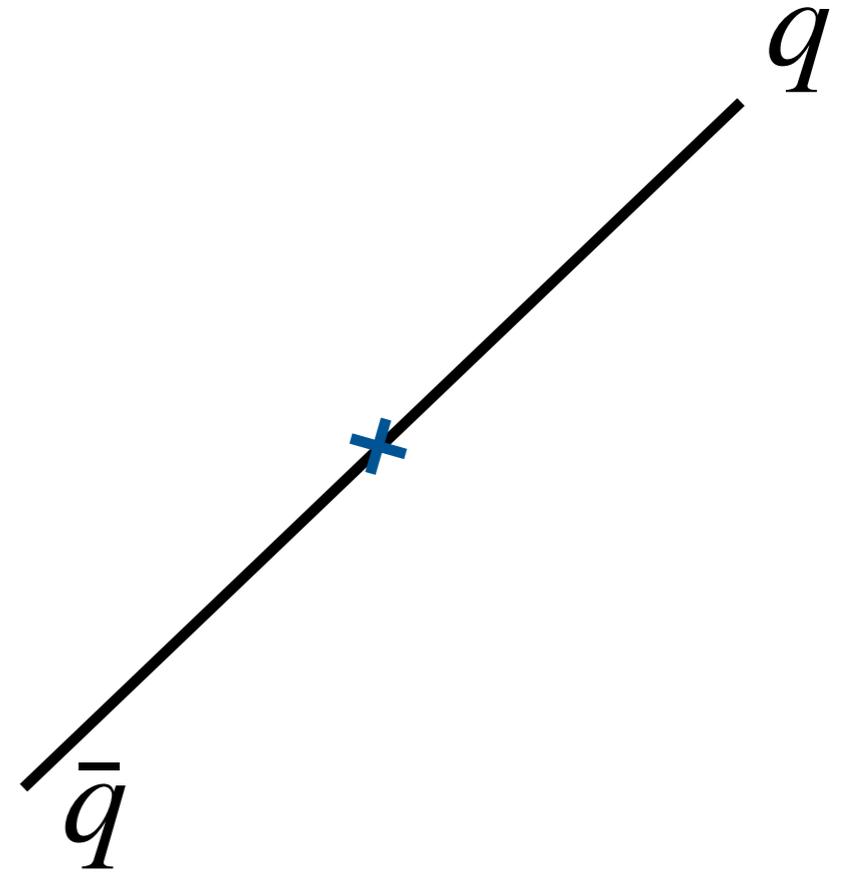
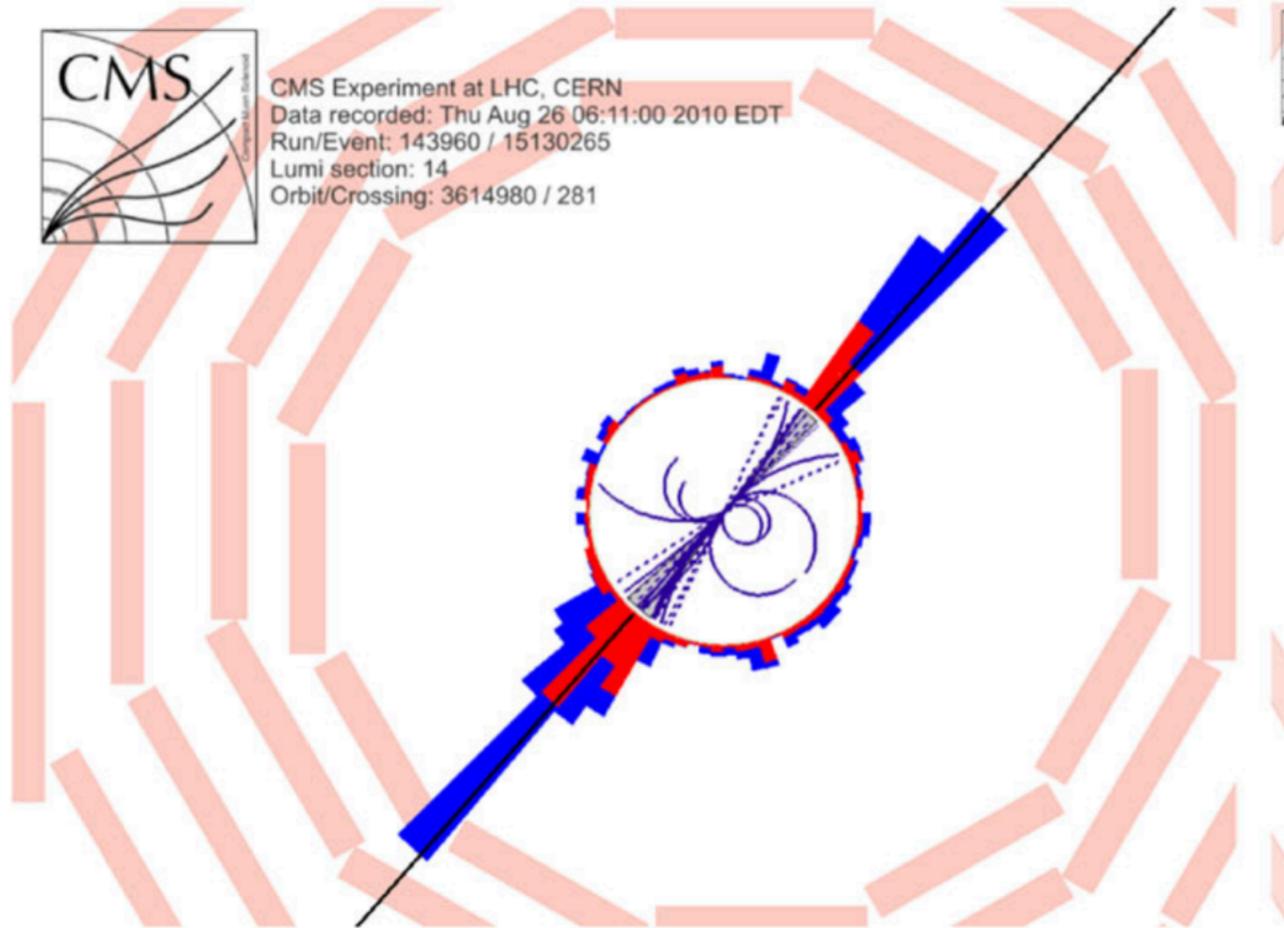
$$P_g \approx \frac{\alpha_s C_F}{\pi} \ln^2 \frac{\omega_1}{Q_0} + \mathcal{O} \left( \alpha_s \ln \frac{\omega_1}{Q_0} \right) \quad \text{This is a double-logarithm!}$$

$$\alpha_s \log^2 \frac{\omega_1}{Q_0} \sim 1 \quad \text{It can be large: resummation}$$

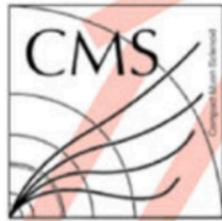
# Dijet event



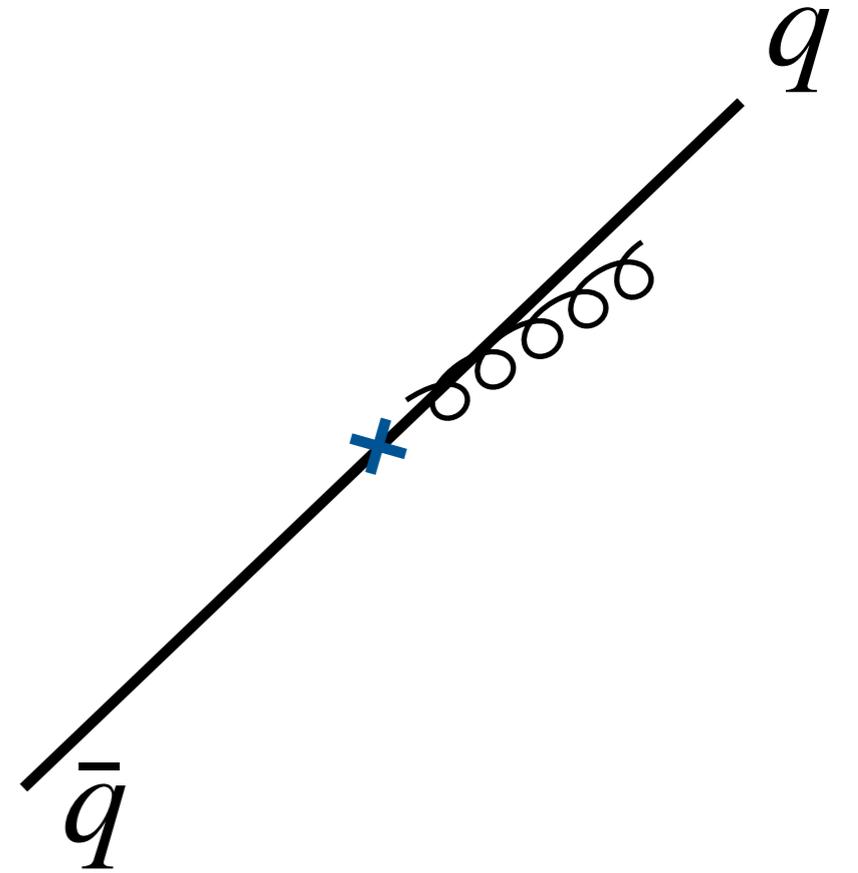
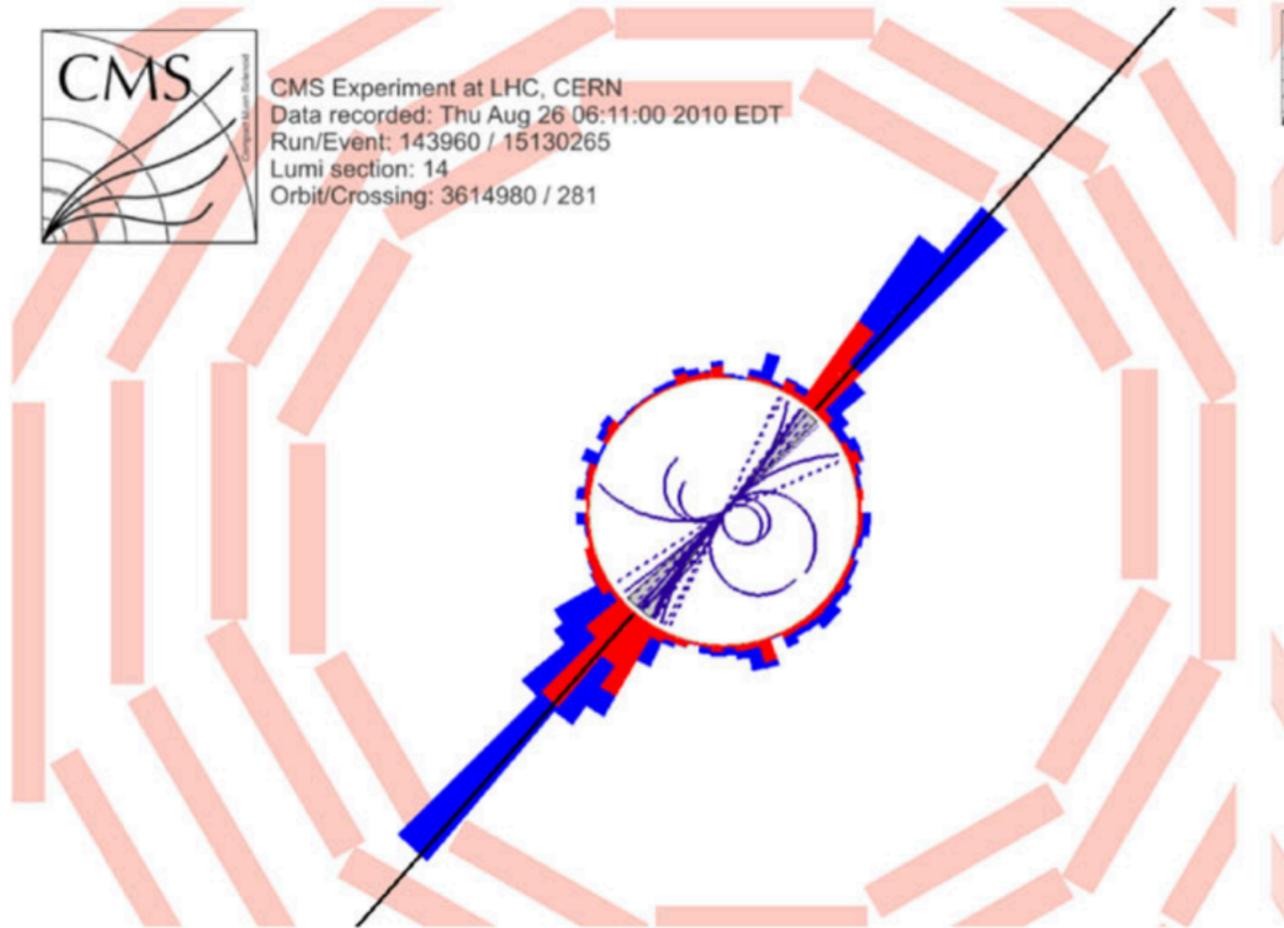
CMS Experiment at LHC, CERN  
Data recorded: Thu Aug 26 06:11:00 2010 EDT  
Run/Event: 143960 / 15130265  
Lumi section: 14  
Orbit/Crossing: 3614980 / 281



# Dijet event



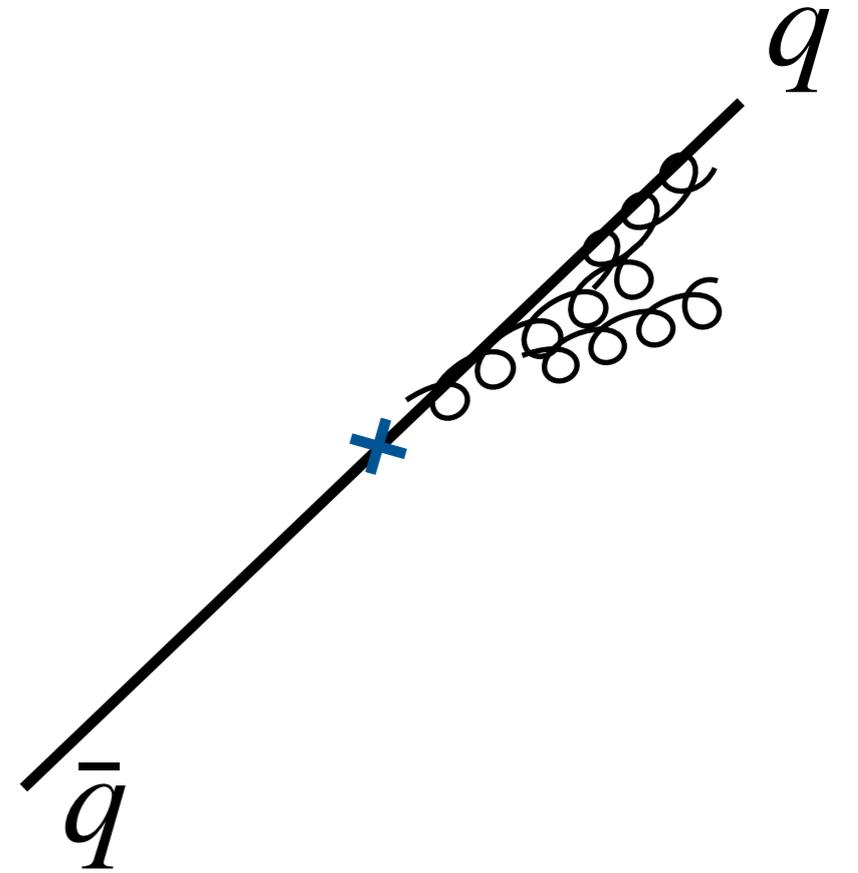
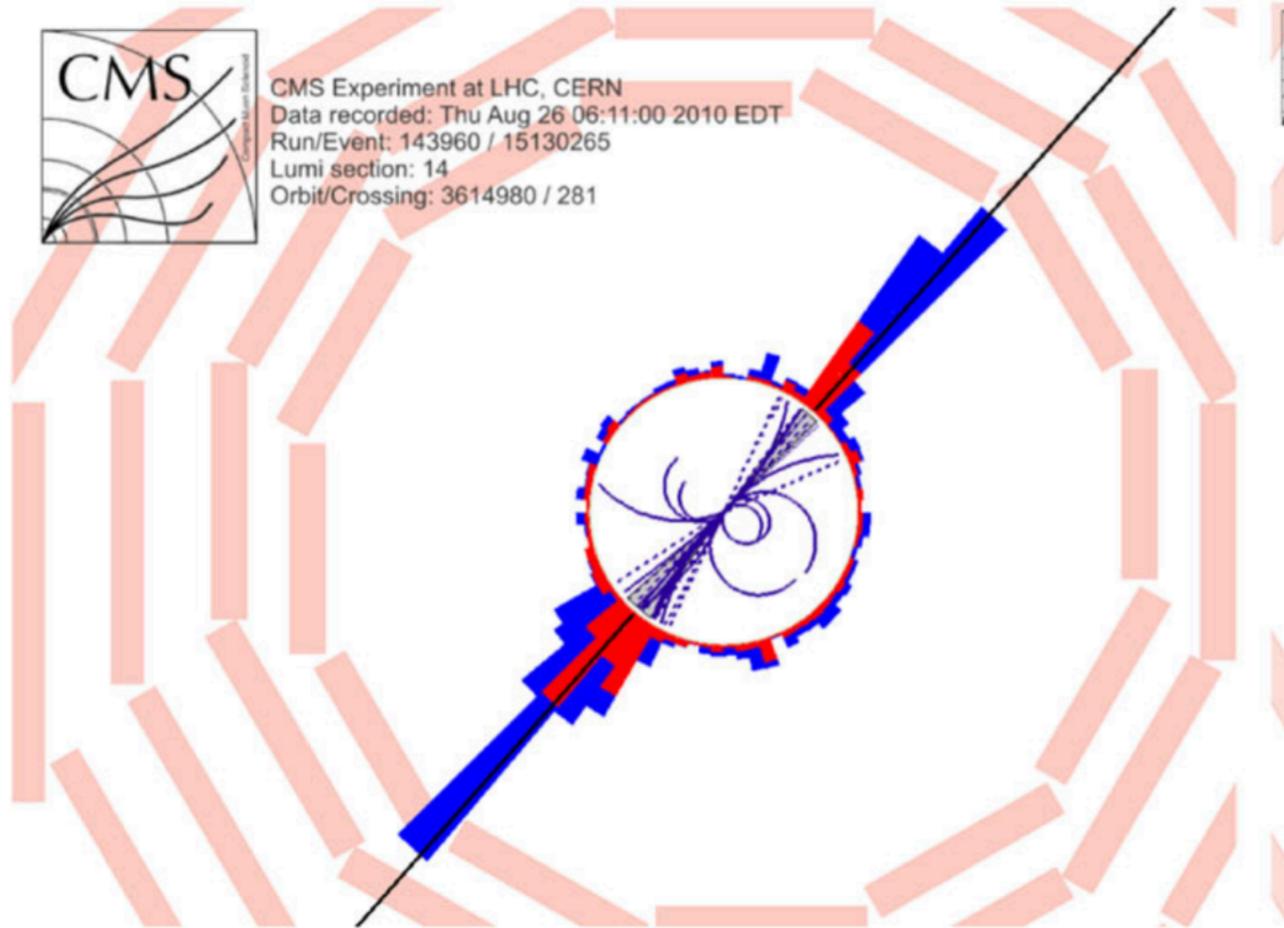
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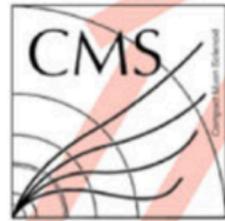
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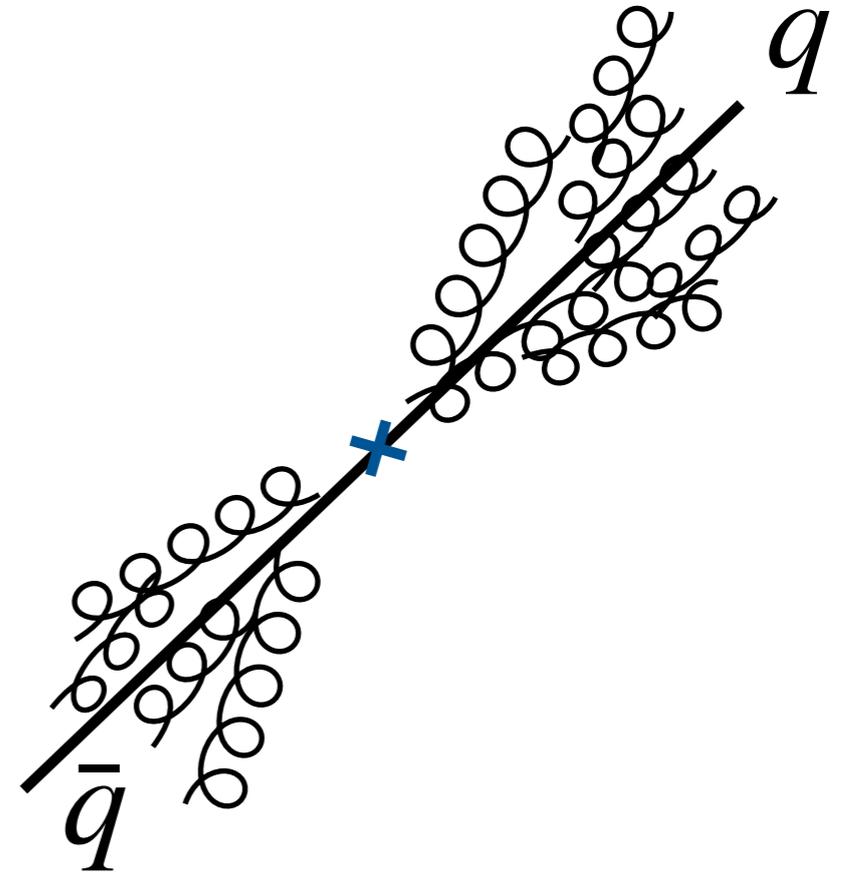
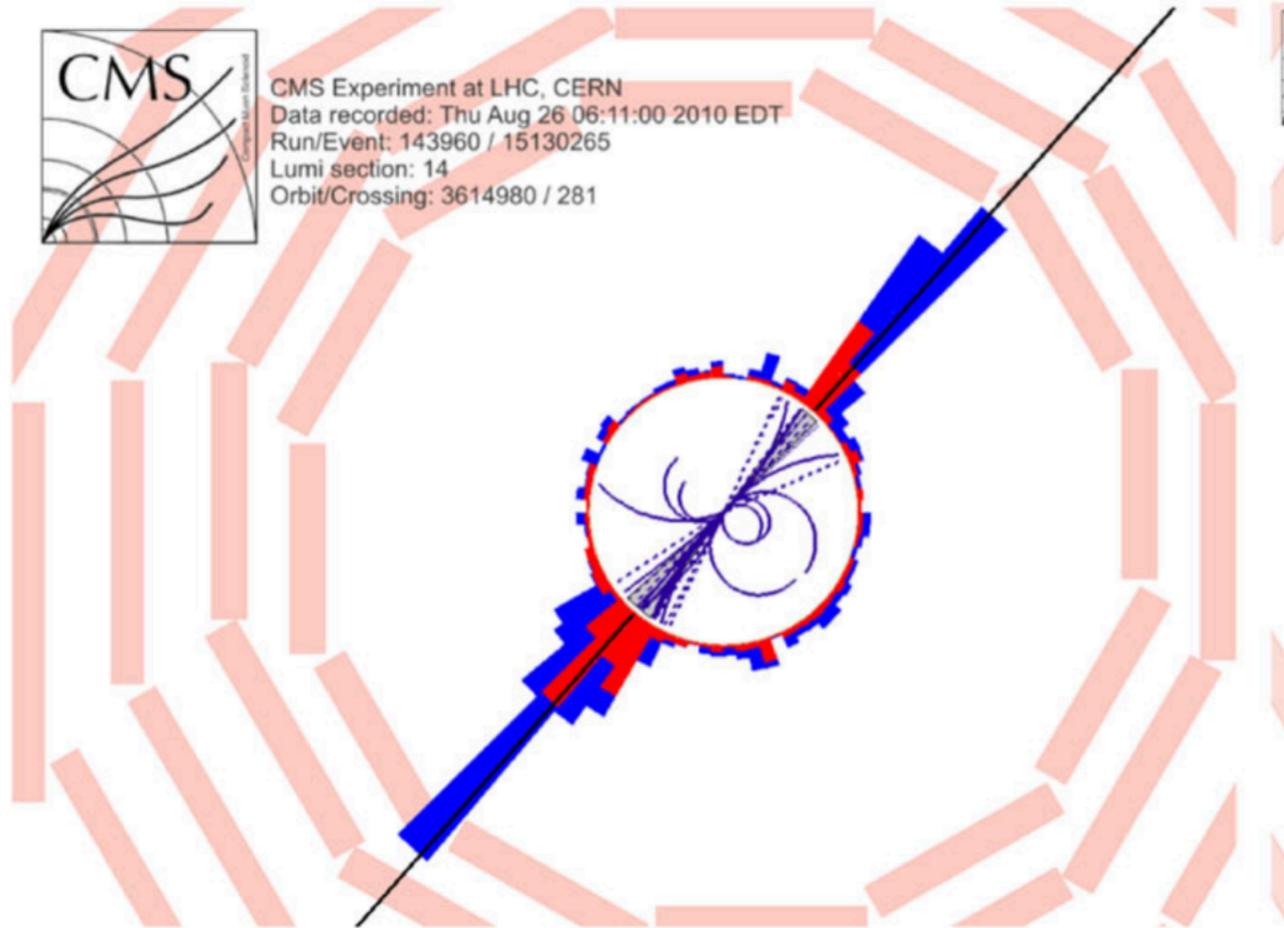
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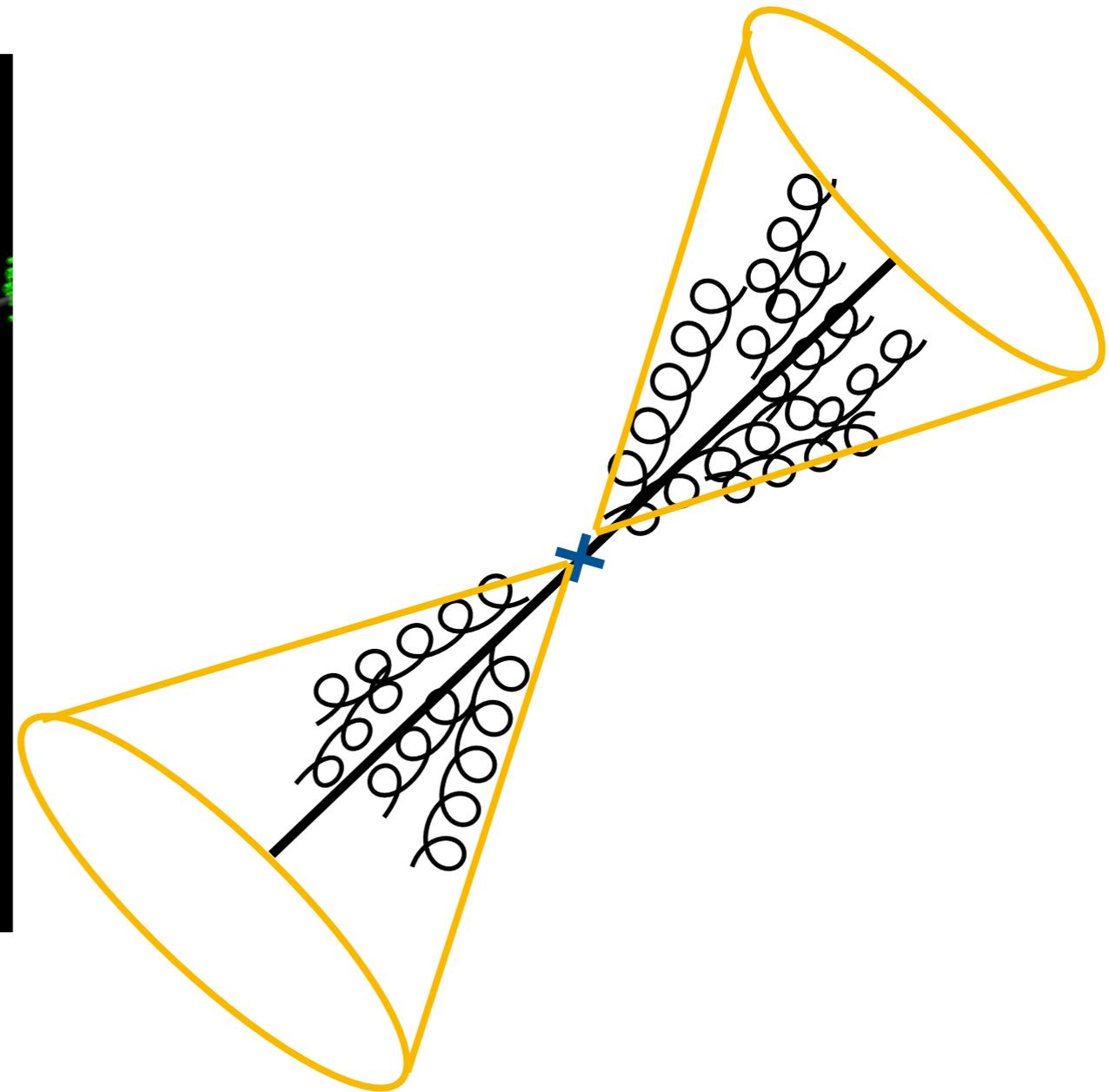
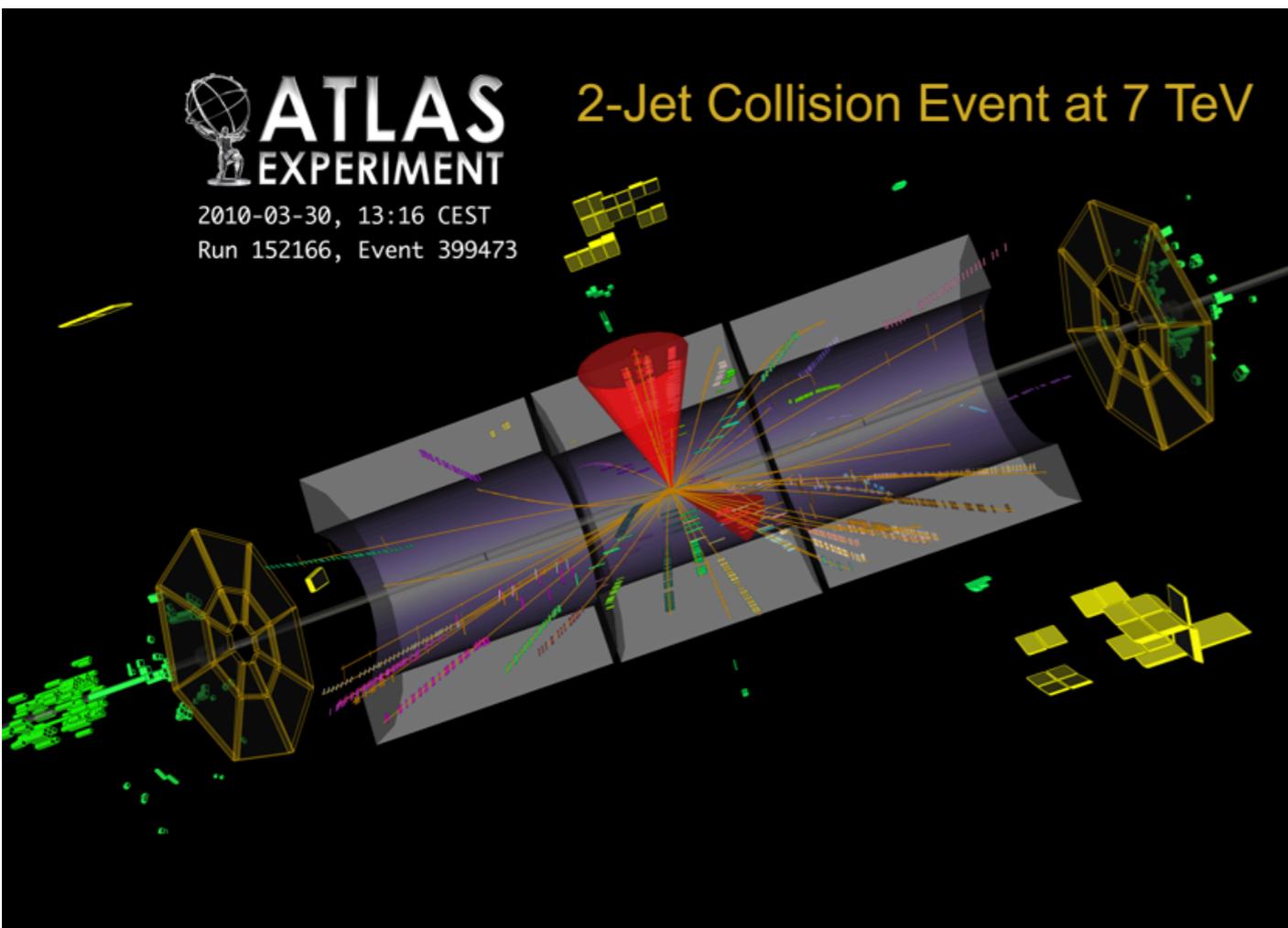
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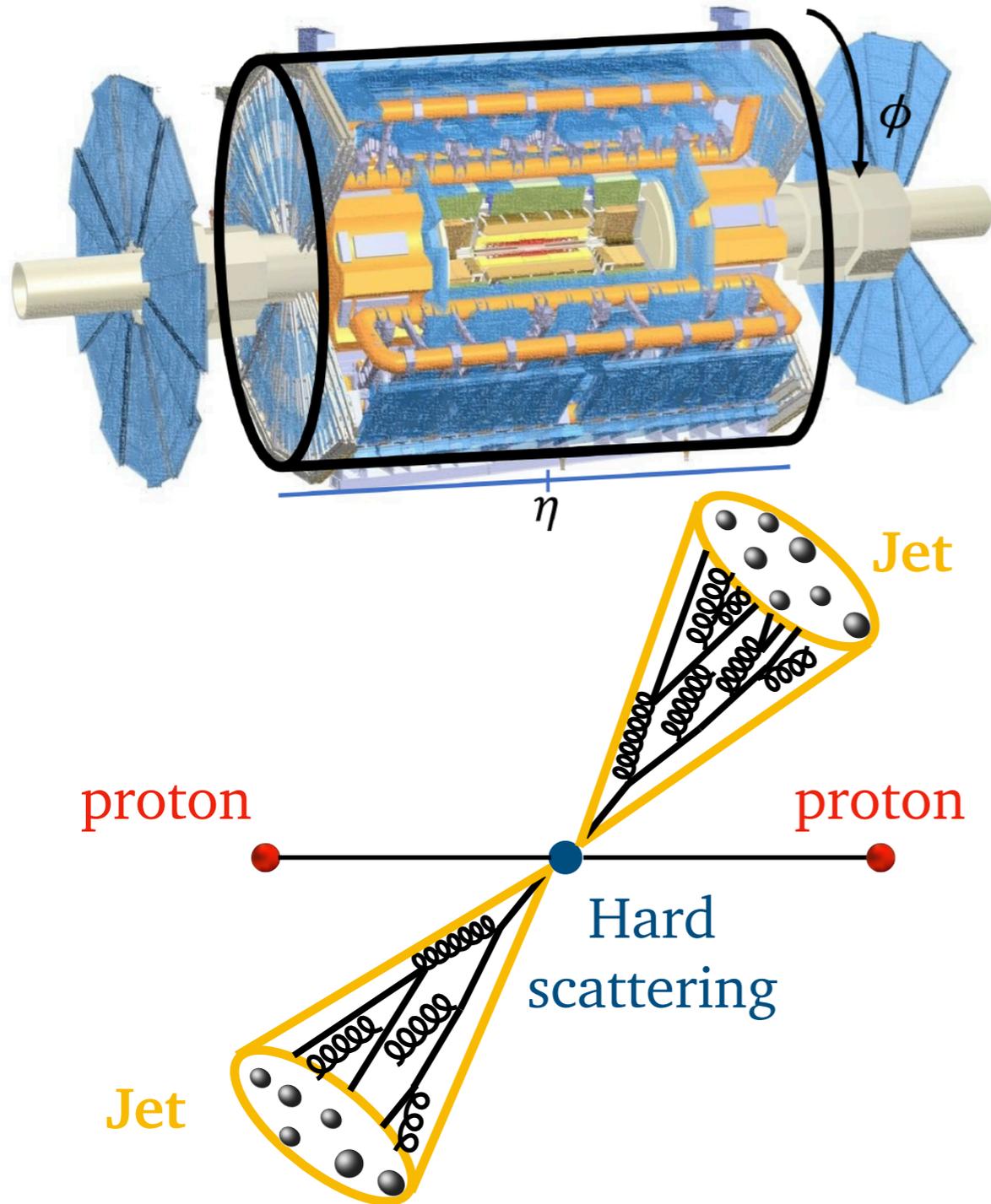
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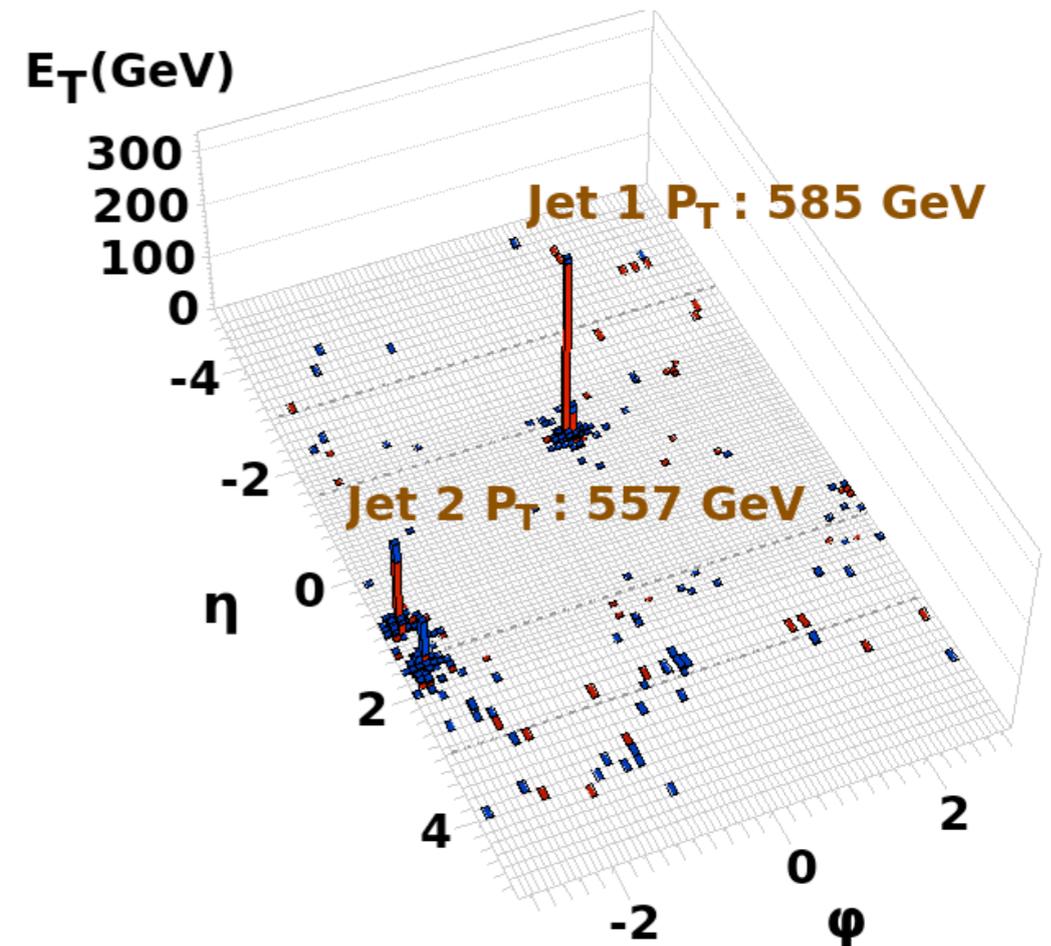
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# What are jets?

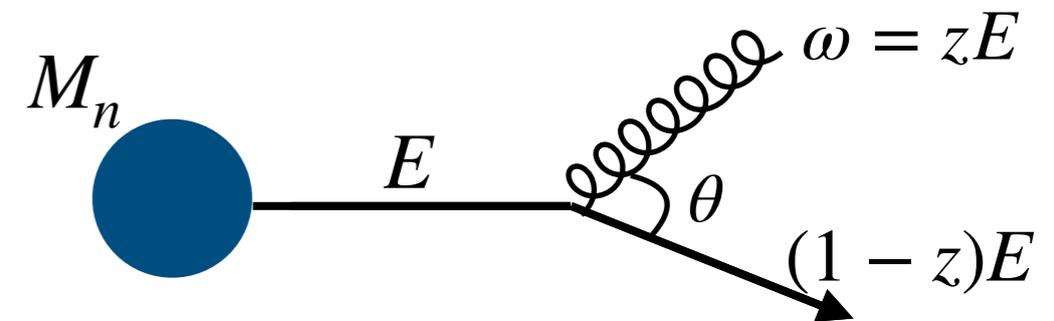
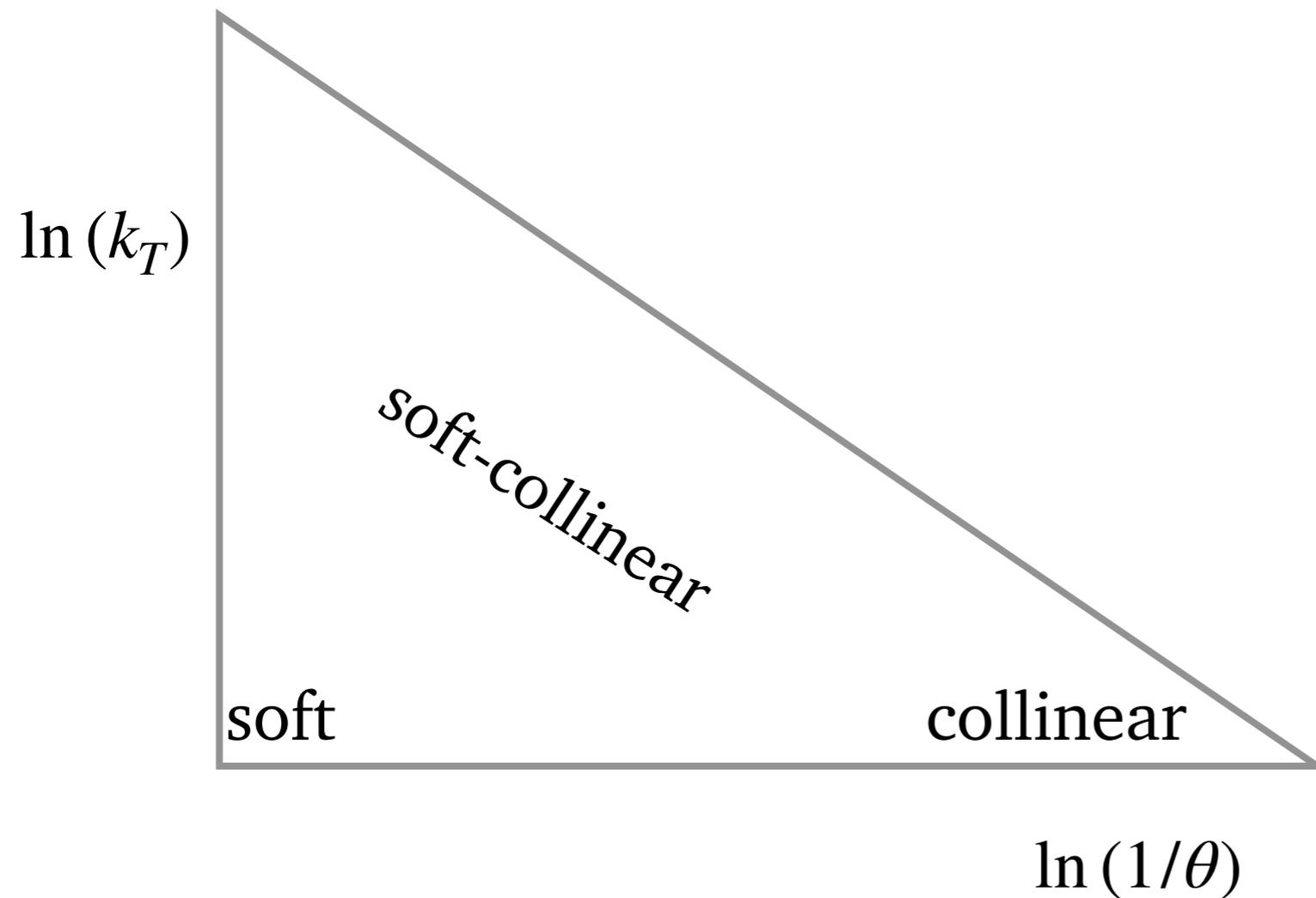


Defined through a clustering algorithm



Jets are proxies of the quarks and gluons produced in the hard scattering

# Lund diagram



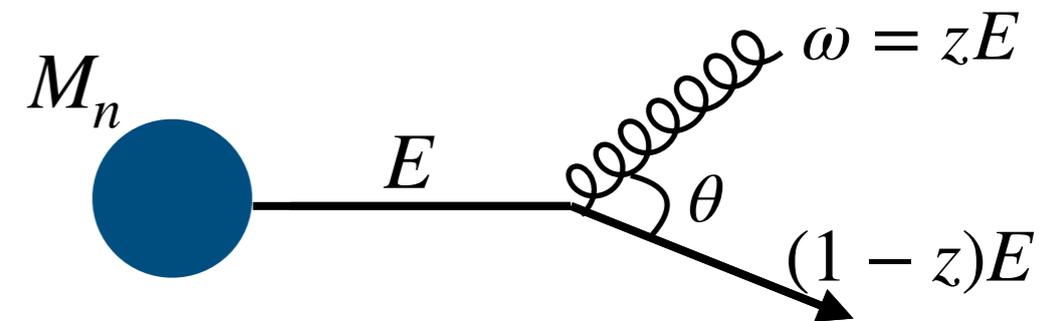
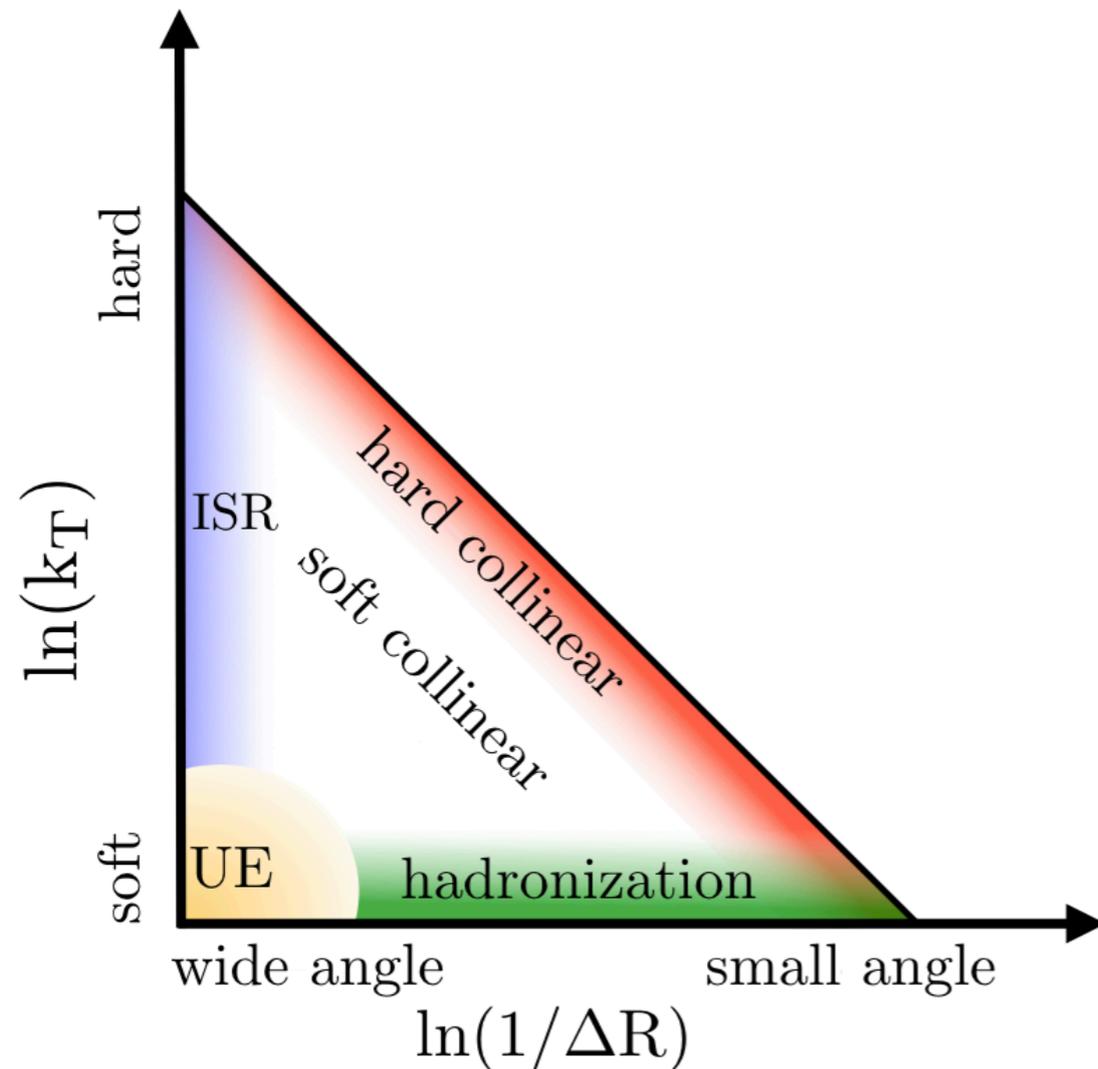
$$k_T = \omega\theta$$

$$d\sigma_{n+1} = d\sigma \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$

Soft and/or collinear emissions are **uniformly** distributed!

Corrections due to higher orders and running coupling apply

# Lund diagram



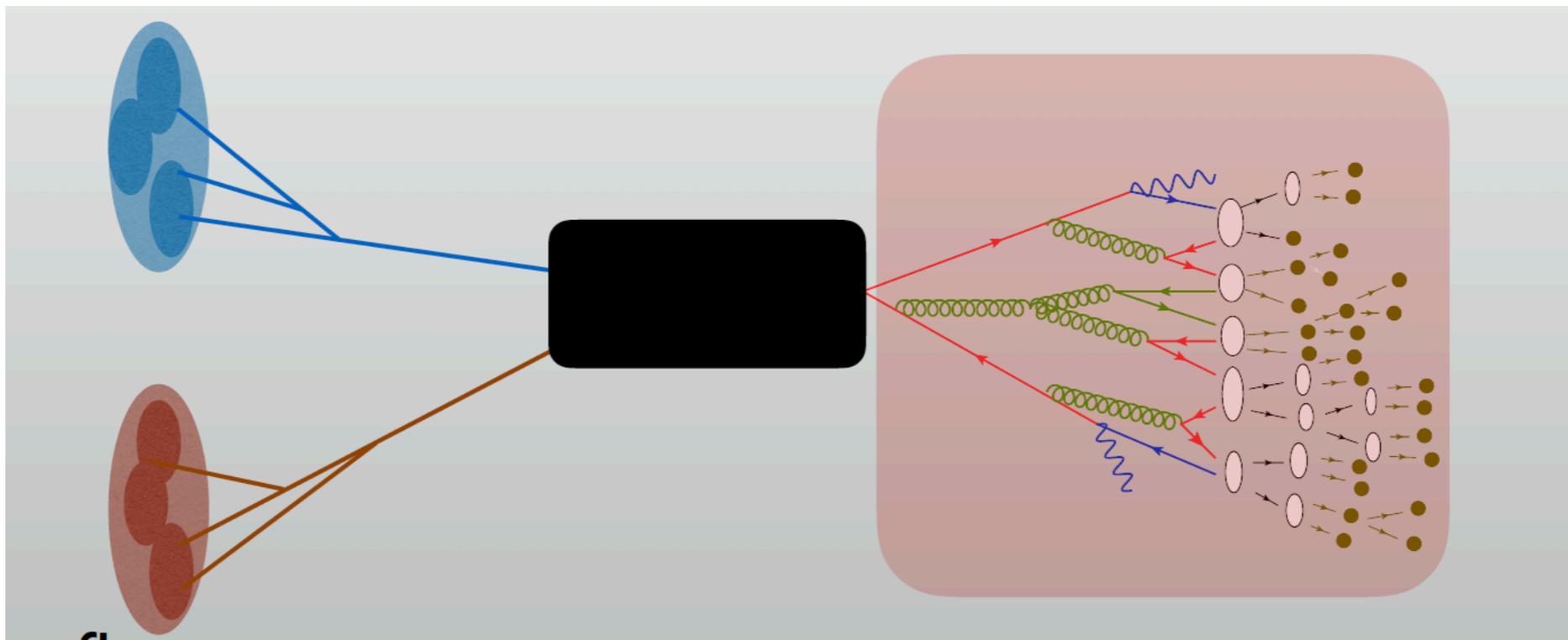
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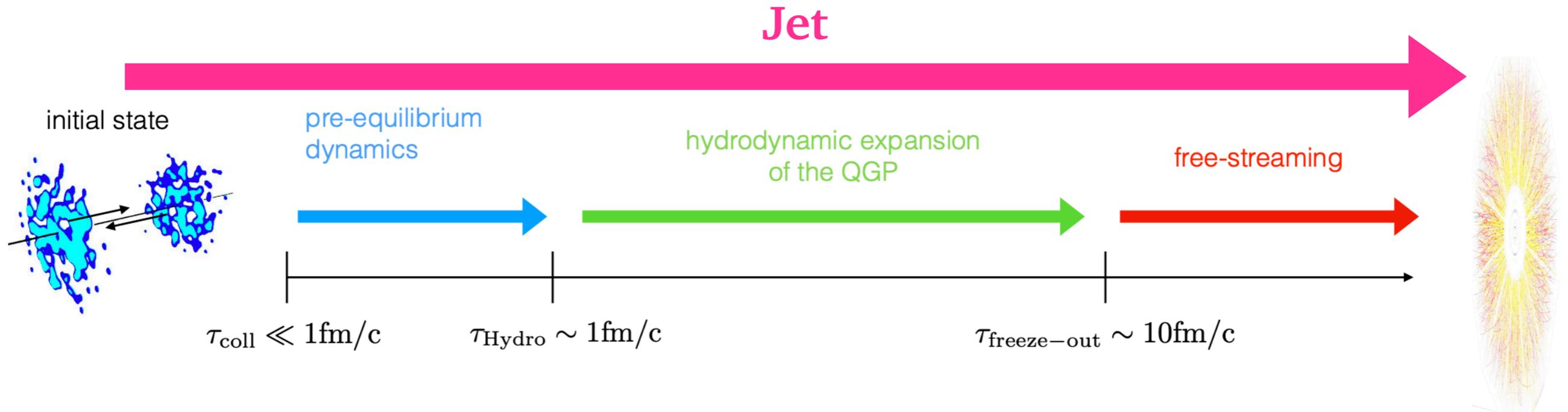
# Some QCD in the medium



# Jets in heavy-ion collisions

- Hard probes/jets ( $Q \sim p_T$ ) are **produced** in the **initial hard scattering**

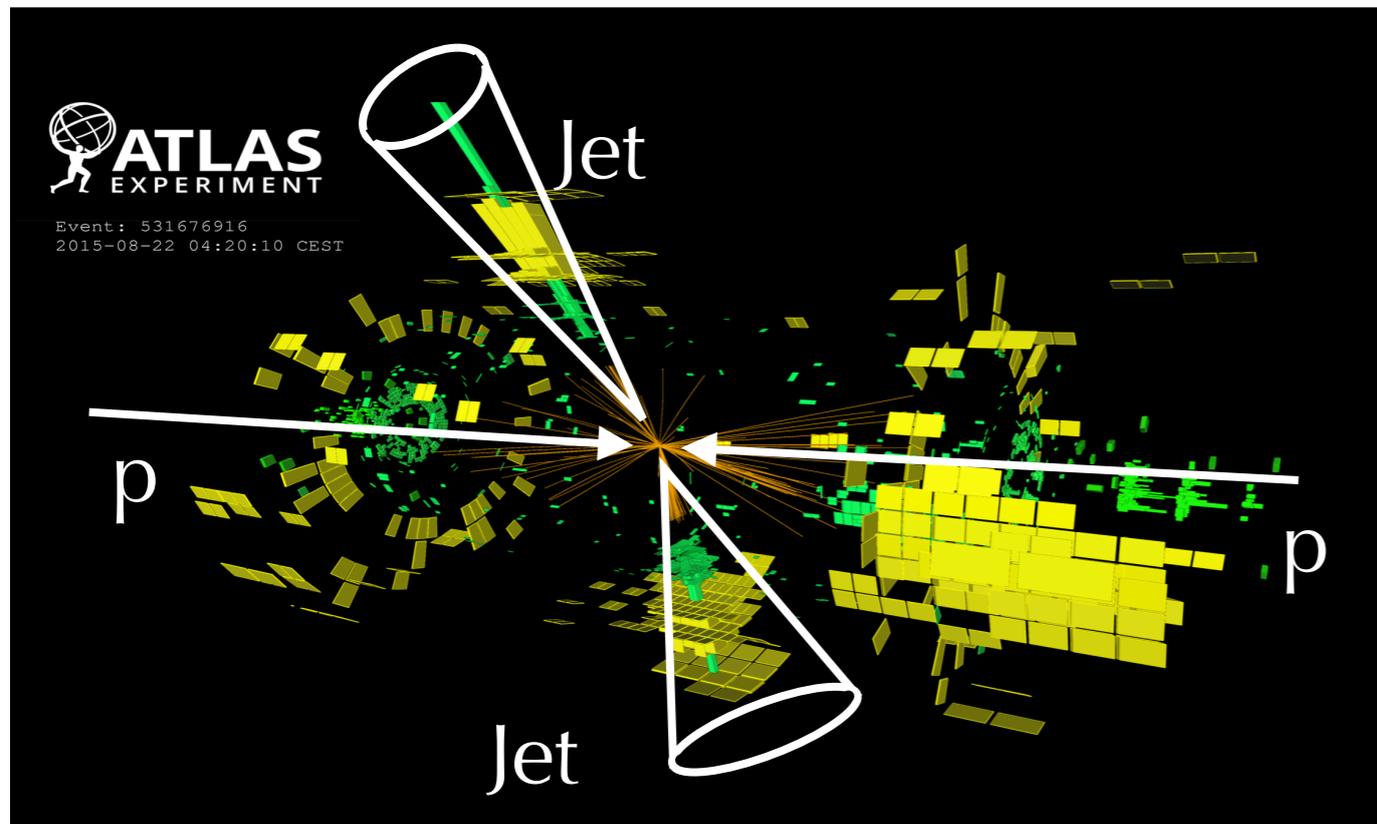
$$\tau_p \sim \frac{1}{Q} \ll \tau_{\text{hydro}} \sim 1 \text{ fm}/c$$



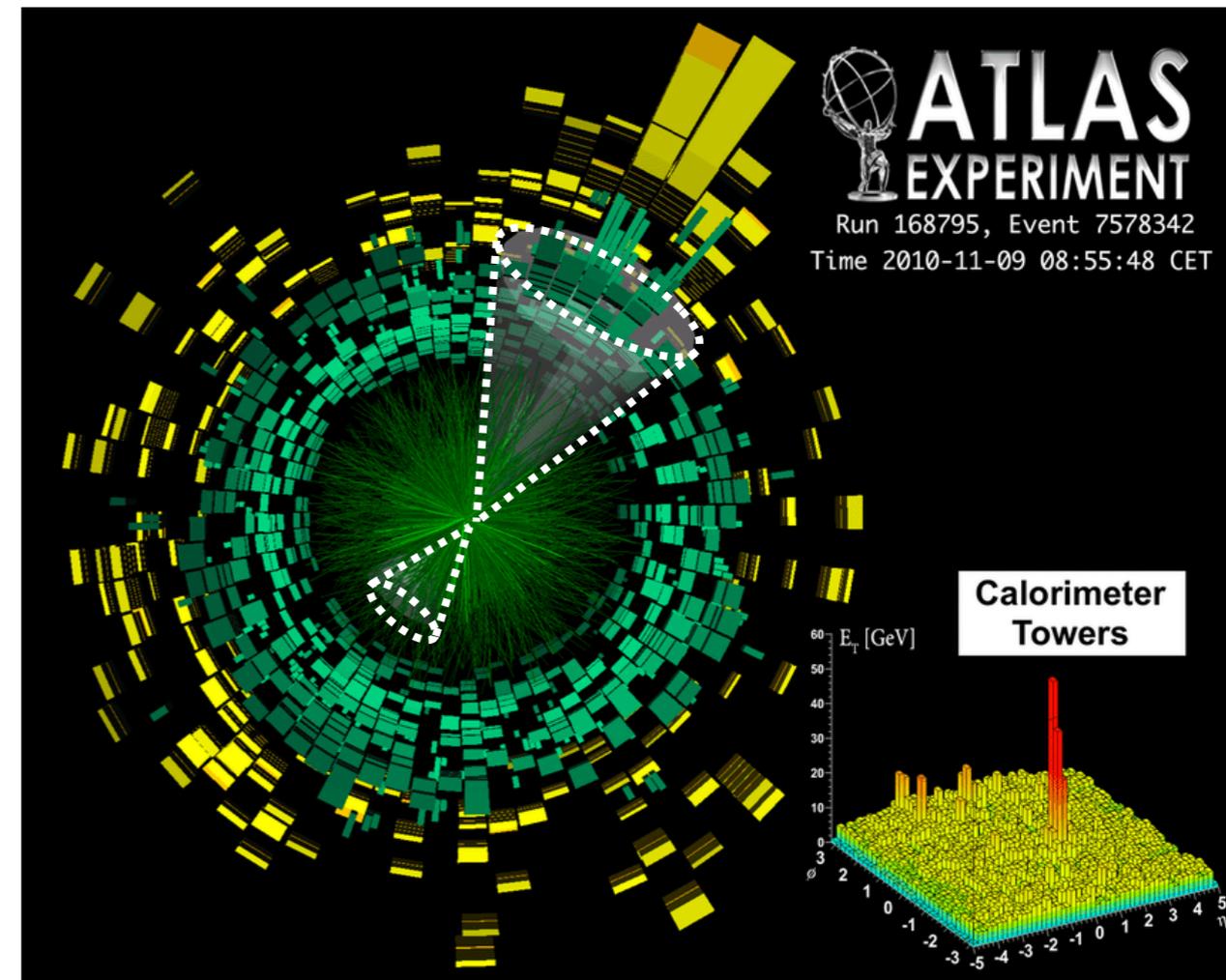
- They **interact with the medium** through the strong interaction

Jets are extended objects: ideal to **probe the medium at different times and resolution scales**

# Jets in p-p vs. A-A

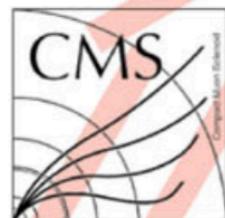


## Jet quenching in A-A

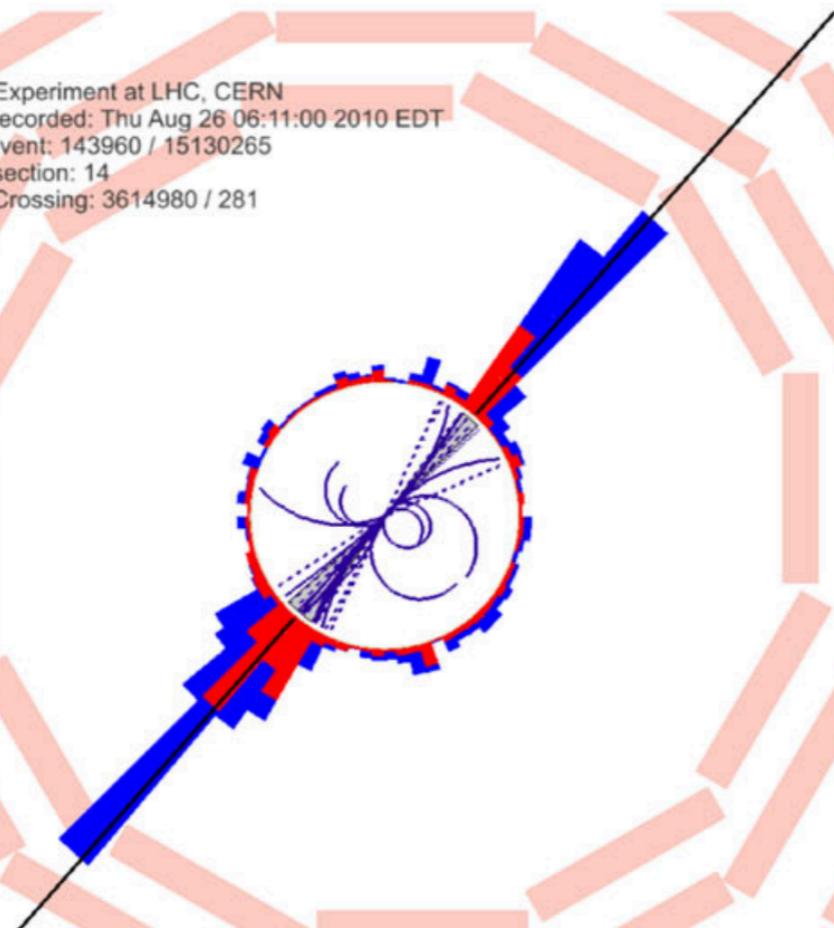


# Energy loss

## Dijet in p-p



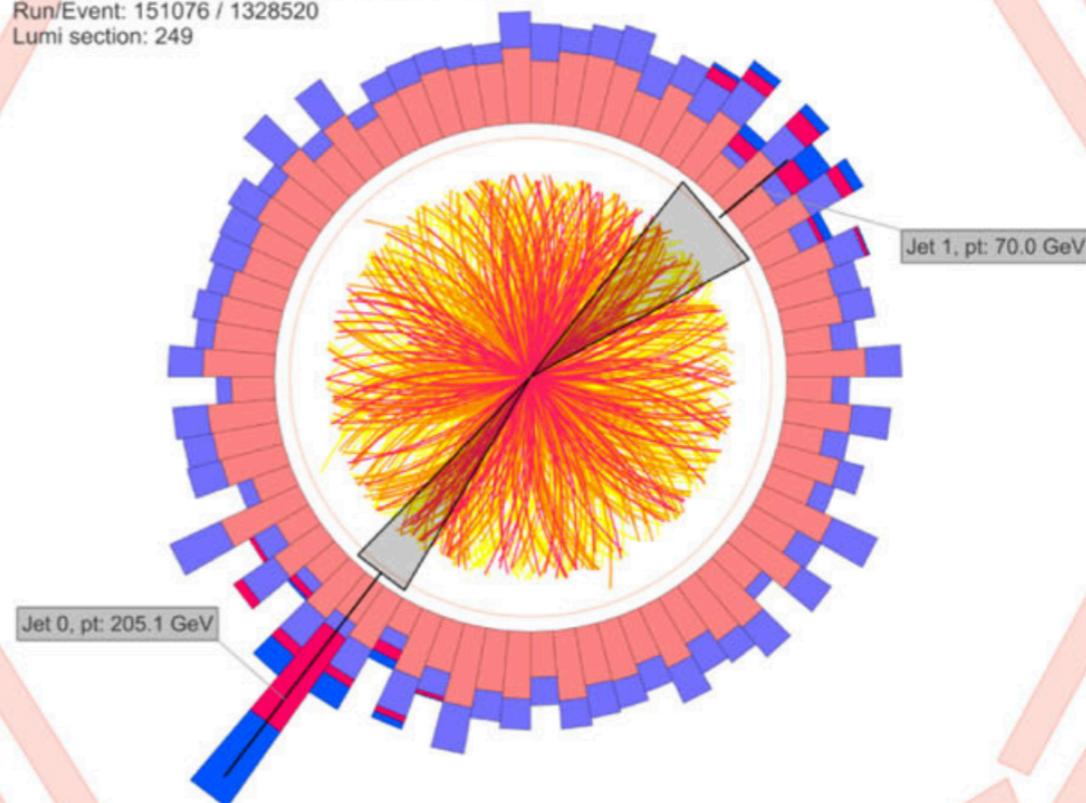
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## Dijet in Pb-Pb

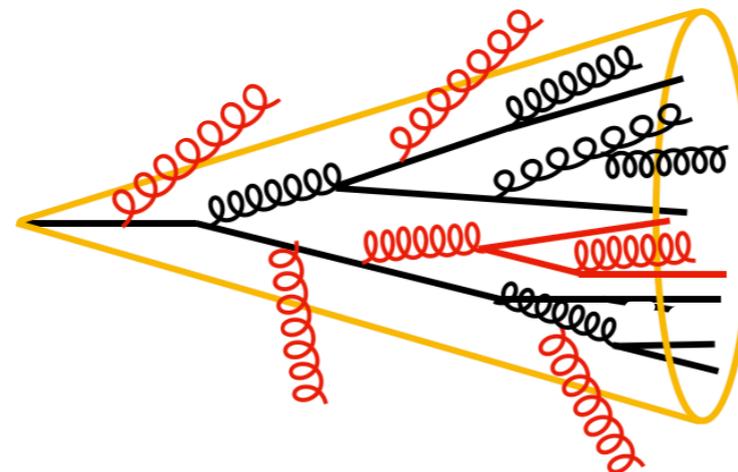
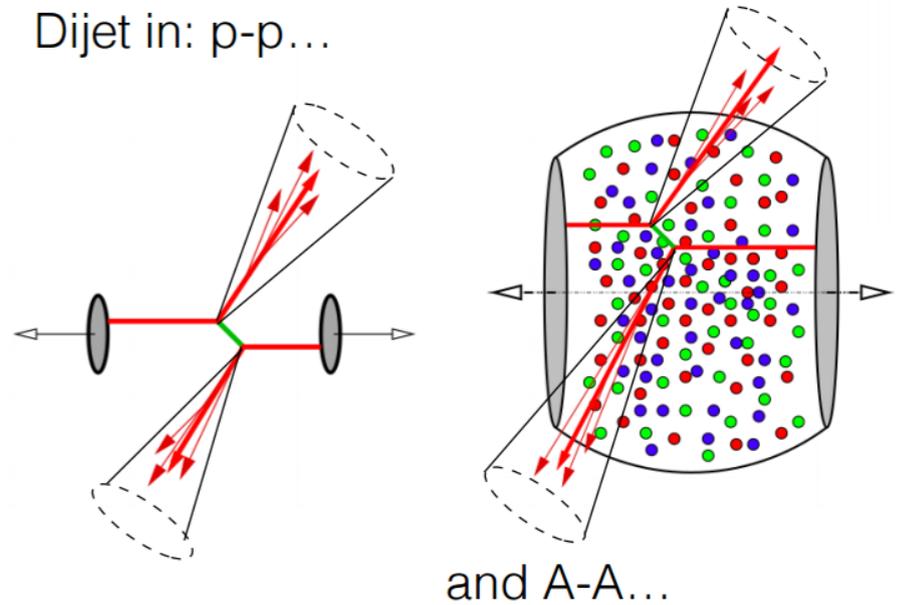


CMS Experiment at LHC, CERN  
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Run/Event: 151076 / 1328520  
Lumi section: 249



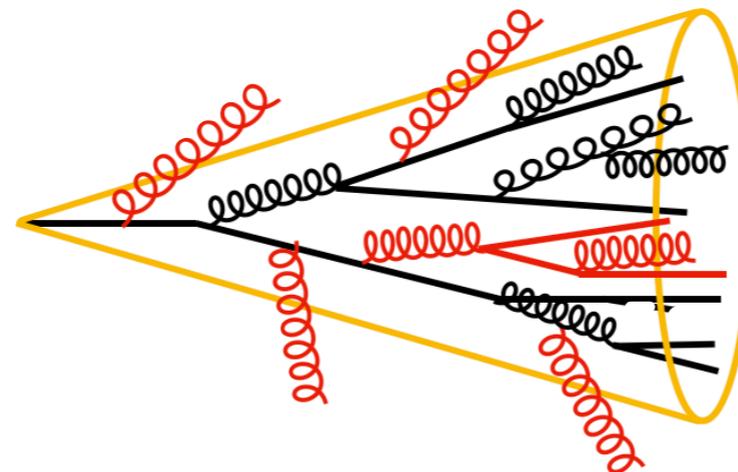
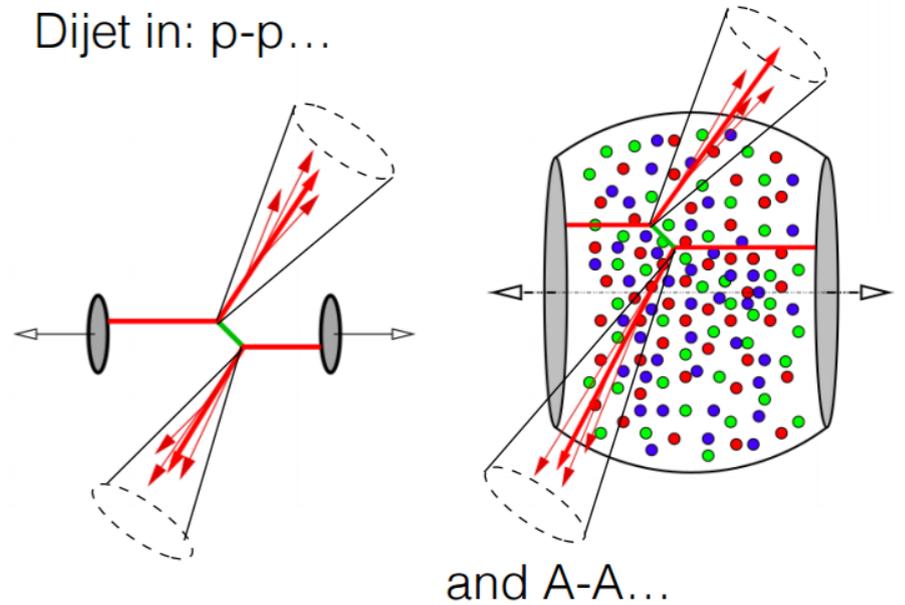
# Jet quenching

- **Jet quenching: partons** interact with the medium losing energy
- How does a parton lose energy in a QCD medium?
  - Collisions - Important for heavy particles
  - **Radiation** - Dominant for light quarks and gluons



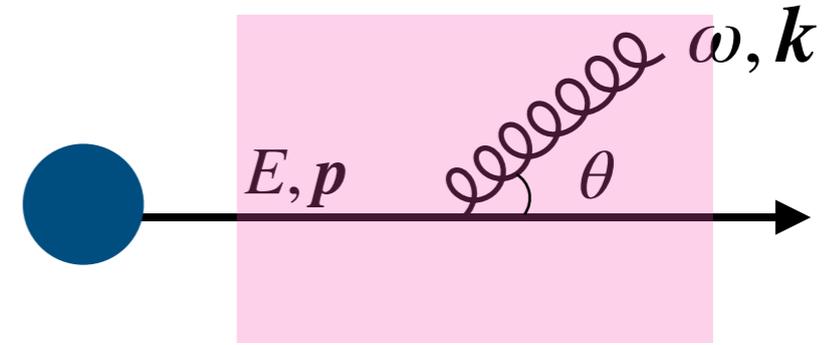
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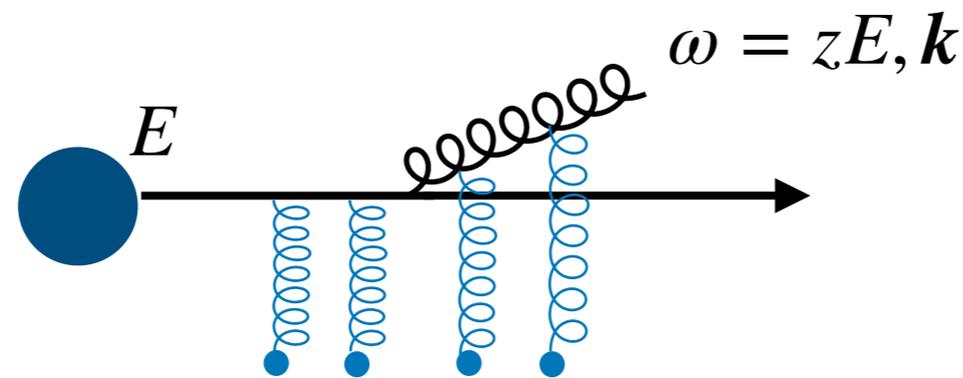


# Medium-induced radiation

# Assumptions



- The gluon is emitted at small angles and  $\mu \ll k, p \ll \omega, E$
- The radiation is due to elastic scatterings mediated by gluons

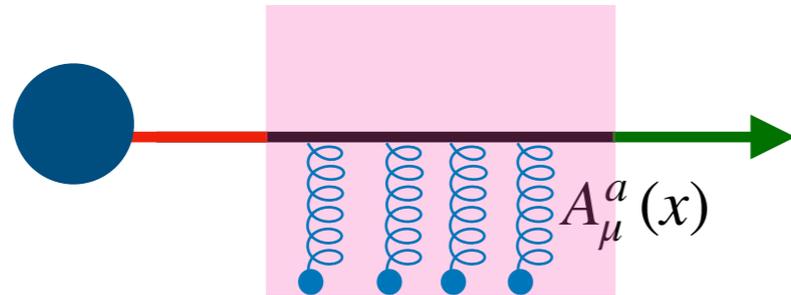


Medium as a  
recoilless  
background field  
 $A_-^a(t, \mathbf{x})$

- The interactions are instantaneous. The medium is seen as recoilless background field

# In-medium parton propagation

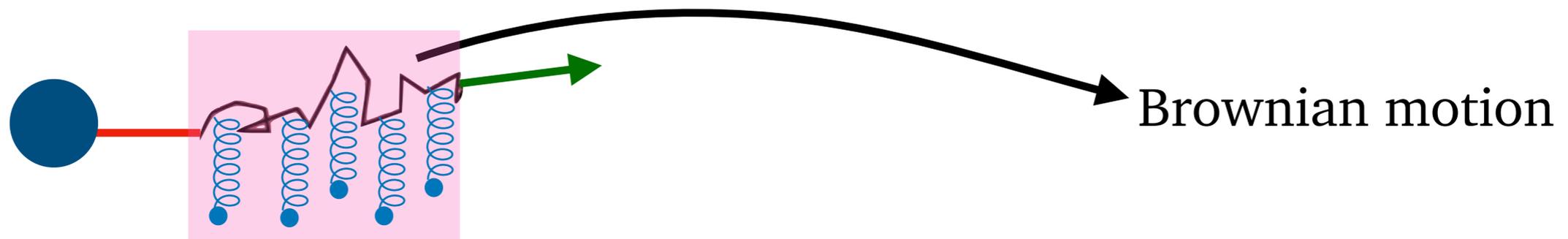
Color rotation



$$W(\mathbf{x}) = \mathcal{P} \exp \left\{ ig_s \int dt A^-(t, \mathbf{x}) \right\}$$

$$p^+ \equiv E \gg \mathbf{p} \gg \mu$$

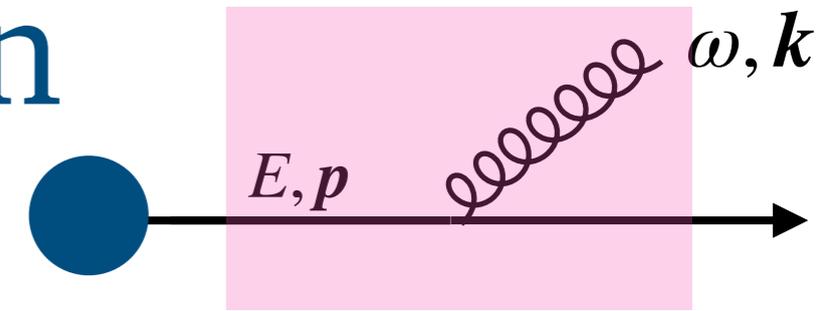
Color rotation +  $\mathbf{p}$ -broadening



$$p^+ \equiv E > \mathbf{p} \gg \mu$$

$$G(t_2, \mathbf{x}_2; t_1, \mathbf{x}_1 | E) = \mathcal{P} \int \mathcal{D}\mathbf{r}(t) \exp \left\{ i \frac{E}{2} \int dt \left[ \frac{d\mathbf{r}}{dt} \right]^2 + ig_s \int dt A^-(t, \mathbf{x}) \right\}$$

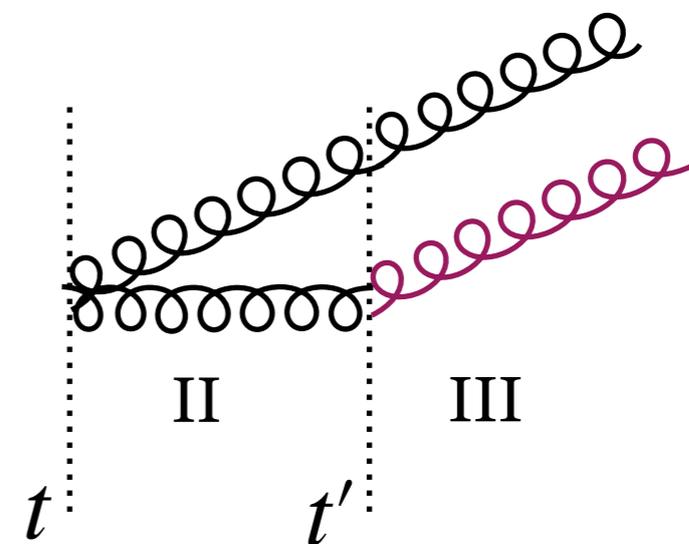
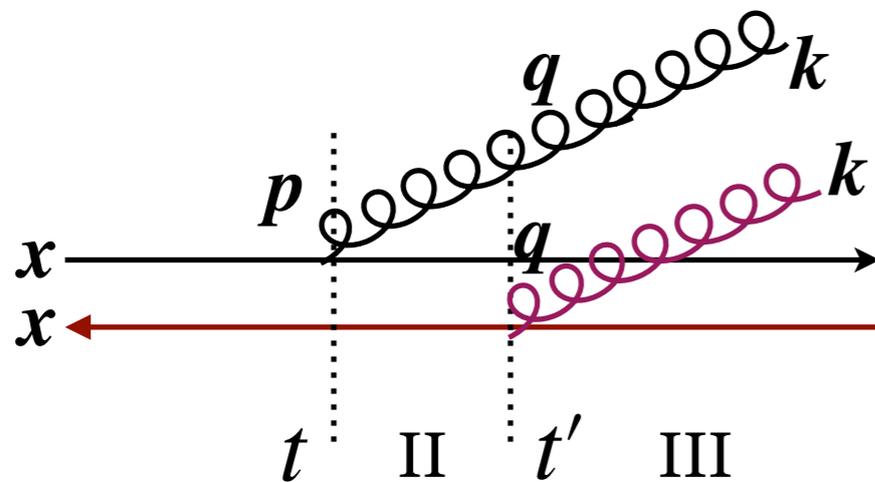
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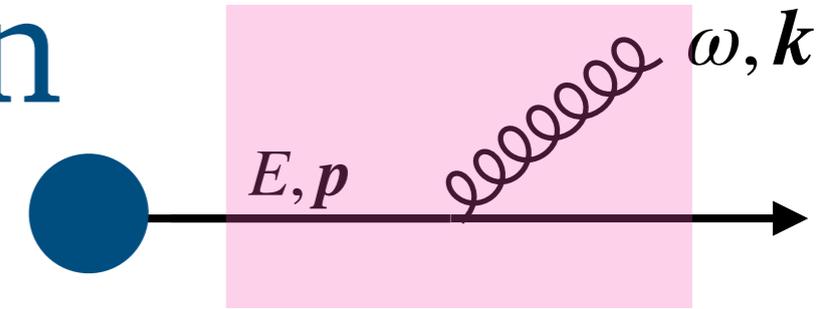
- Emission of a soft gluon off a high energy quark:  $\omega \ll E$

BDMPS-Z formalism (1990's)

$$\omega \frac{dI^{\text{med}}}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega^2} \text{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{pq} \mathbf{p} \cdot \mathbf{q} \tilde{\mathcal{K}}(t', \mathbf{q}; t, \mathbf{p}) \mathcal{P}(\infty, \mathbf{k}; t', \mathbf{q})$$



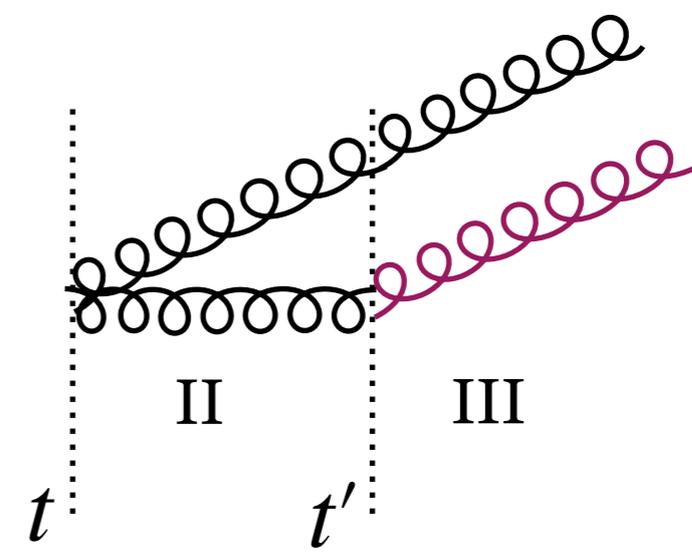
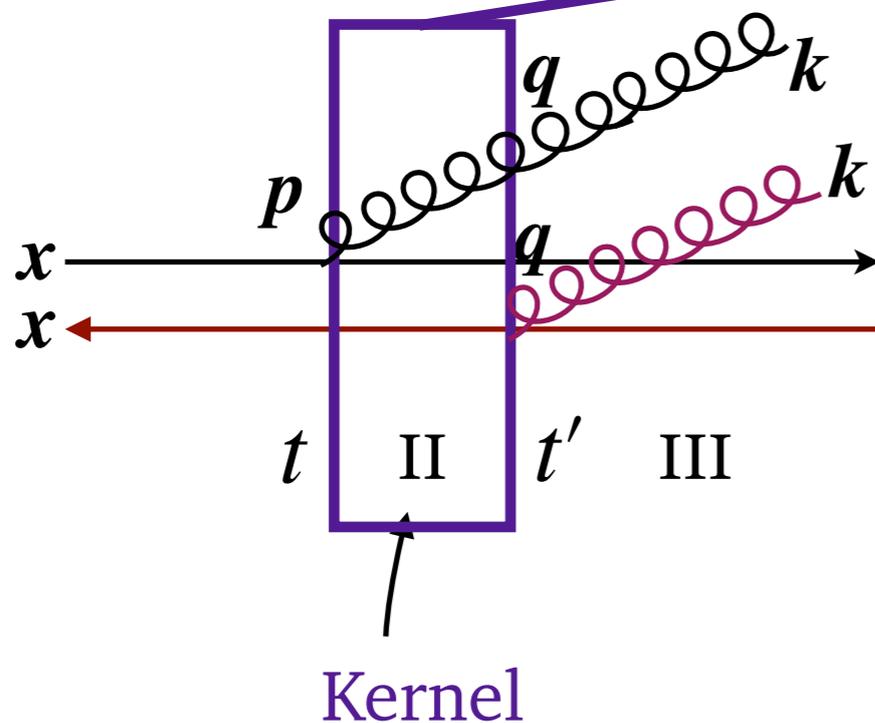
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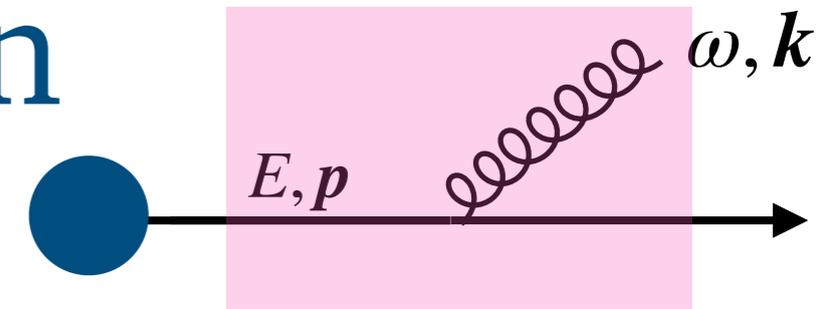
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2-point function  
 $\mathcal{K} \sim \text{Tr} \langle W_A \mathcal{G} \rangle$

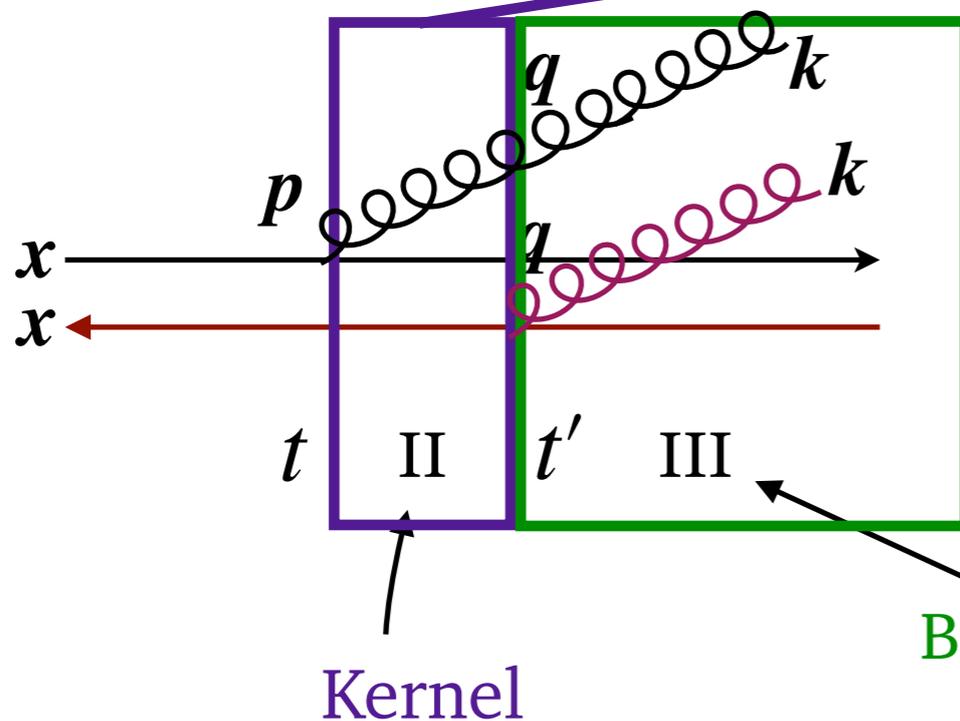
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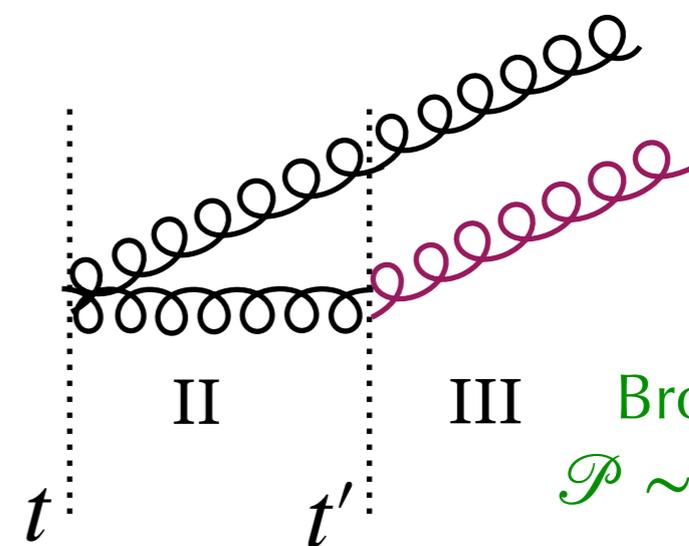
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Broadening

Kernel



2-point function

$$\mathcal{K} \sim \text{Tr} \langle W_A \mathcal{G} \rangle$$

Broadening  
 $\mathcal{P} \sim \text{Tr} \langle \mathcal{G} \mathcal{G}^\dagger \rangle$

# Medium averages

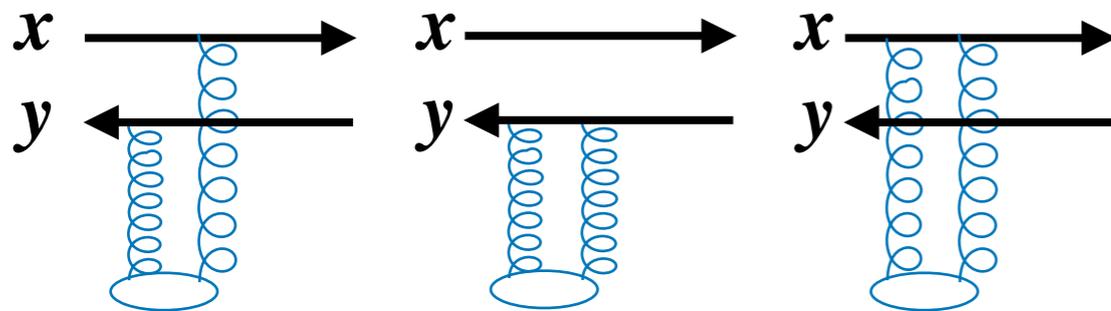
- Medium average:

$$\langle A^{a,-}(t, \mathbf{x}) A^{b,-\dagger}(t', \mathbf{y}) \rangle = \delta^{ab} \delta(t - t') \gamma(\mathbf{x} - \mathbf{y})$$

- Medium average:

$$\frac{1}{N_c} \text{Tr} \langle W(\mathbf{x}) W^\dagger(\mathbf{y}) \rangle = \exp \left\{ -\frac{1}{2} \int ds n(s) \sigma(\mathbf{x} - \mathbf{y}) \right\}$$

medium information



$$\sigma(\mathbf{x} - \mathbf{y}) = 2g_s^2 (\gamma(\mathbf{x} - \mathbf{y}) - \gamma(\mathbf{0}))$$

- Broadening and Kernel

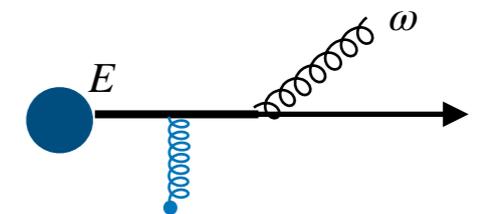
$$\mathcal{P}(\mathbf{x}, t; \mathbf{y}, t') = \exp \left\{ -\frac{1}{2} \int_t^{t'} ds n(s) \sigma(\mathbf{x} - \mathbf{y}) \right\} \quad \mathcal{K}(t', \mathbf{z}; t, \mathbf{y}) = \int_{\mathbf{r}(t)=\mathbf{y}}^{\mathbf{r}(t')=\mathbf{z}} \mathcal{D}\mathbf{r} \exp \left[ \int_t^{t'} ds \left( \frac{i\omega}{2} \dot{\mathbf{r}}^2 - \frac{1}{2} n(s) \sigma(\mathbf{r}) \right) \right]$$

# Medium-induced radiation

- In practice, solved for some approximations

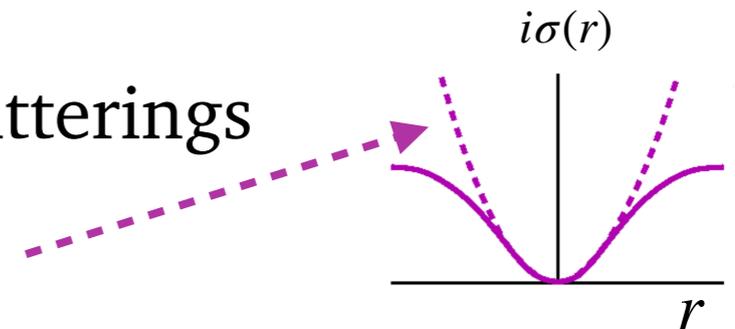
- **Opacity expansion** in the number of scatterings

$N = 1$  : GLV Gyulassy, Levai, Vitev (2000)



- **Harmonic oscillator (HO)**: multiple soft scatterings (Gaussian approximation)

$$n\sigma(r) \approx \frac{1}{2}\hat{q}r^2$$



- **AMY**: infinite length medium Arnold, Moore, Yaffe (2002)

- Recent approaches going beyond these approximations

$$\gamma(q) \propto \frac{1}{q^4}$$

- **Improved opacity expansion**

Semi-analytical expansion around the HO

Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk,  
[1903.00506](#), [2004.02323](#), [2106.07402](#)

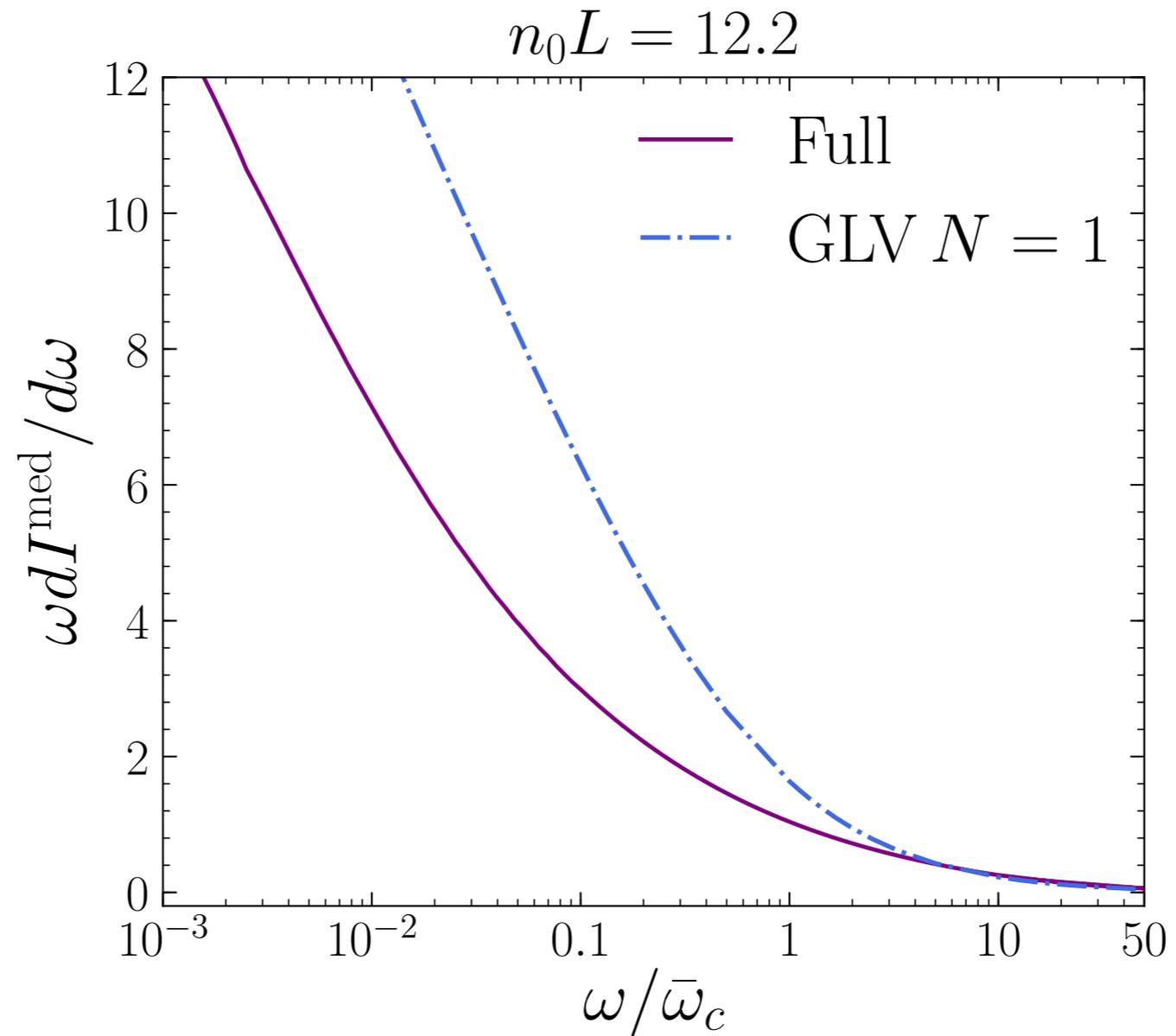
- **Fully resummed spectrum**

Kernel as a time dependent Schrödinger equation

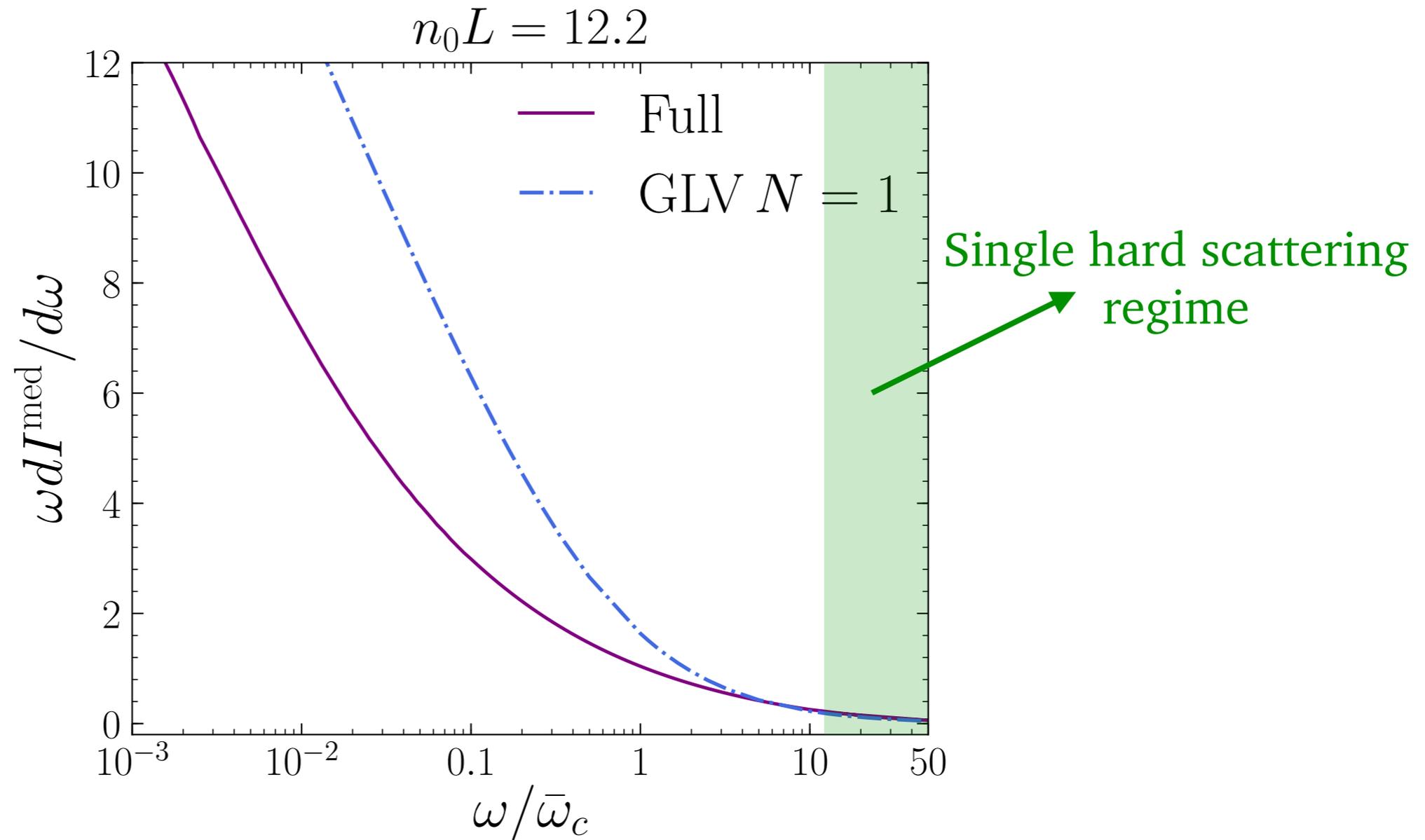
CA, Apolinario, Martinez, Dominguez,  
[2002.01517](#), [2011.06522](#)

Beyond the soft limit but integrated in  $k_T$ :  
Caron-Huot and Gale, [1006.2379](#)

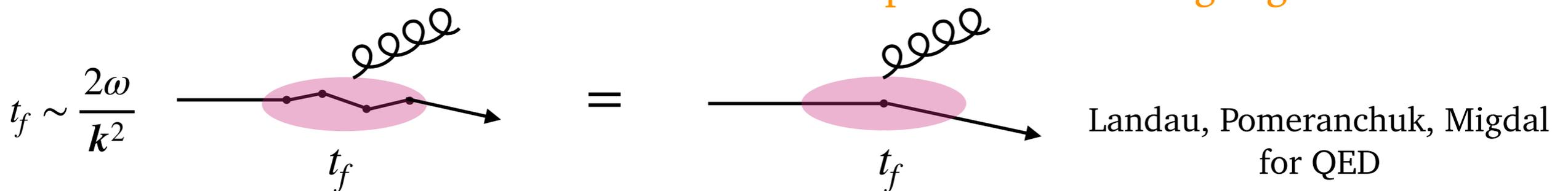
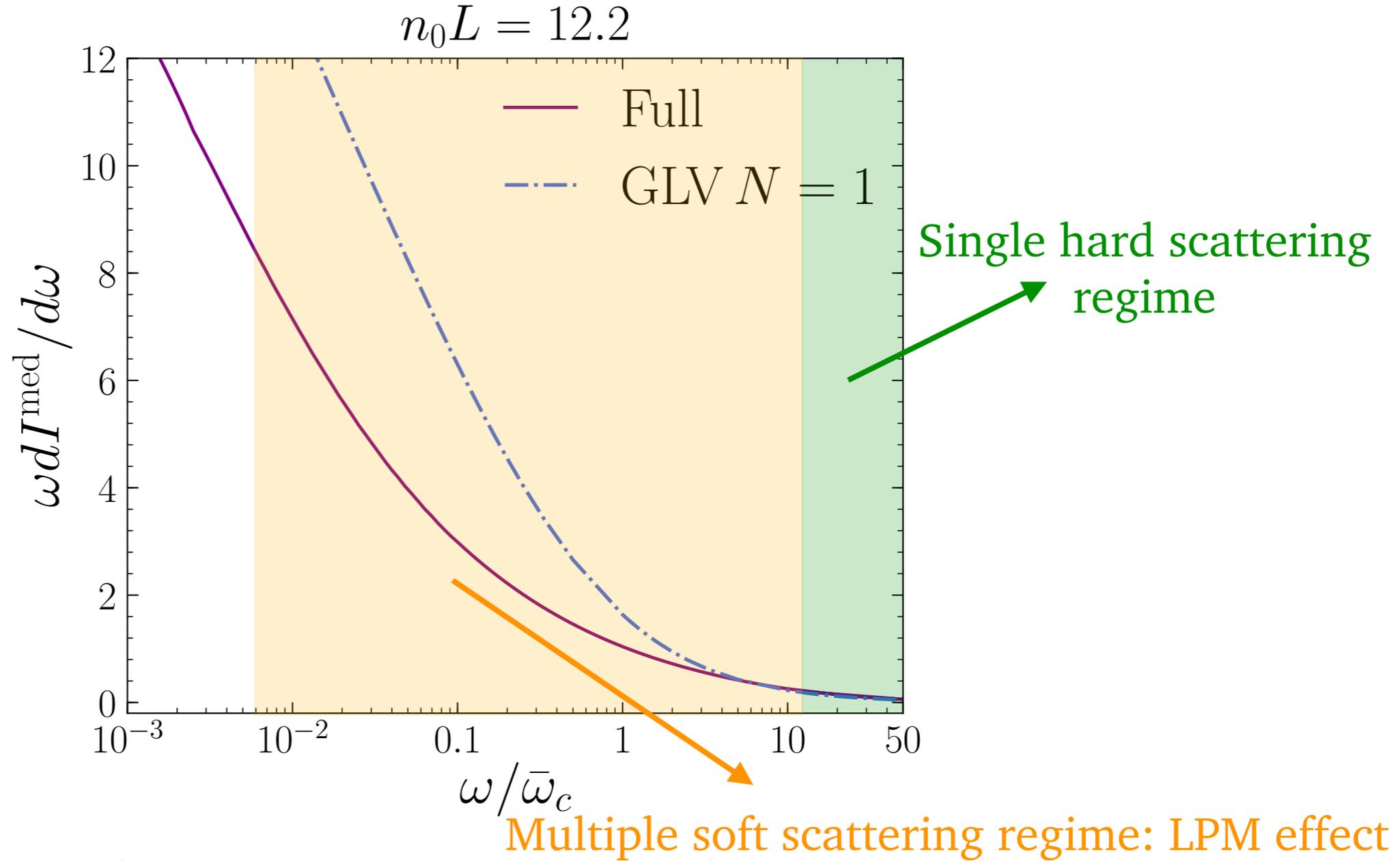
# BDMPS-Z spectrum



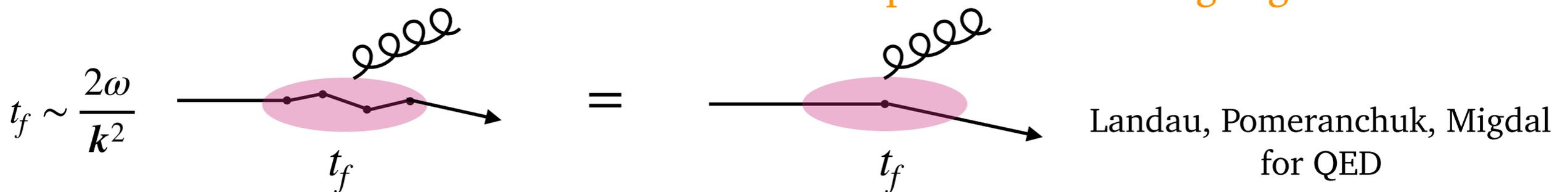
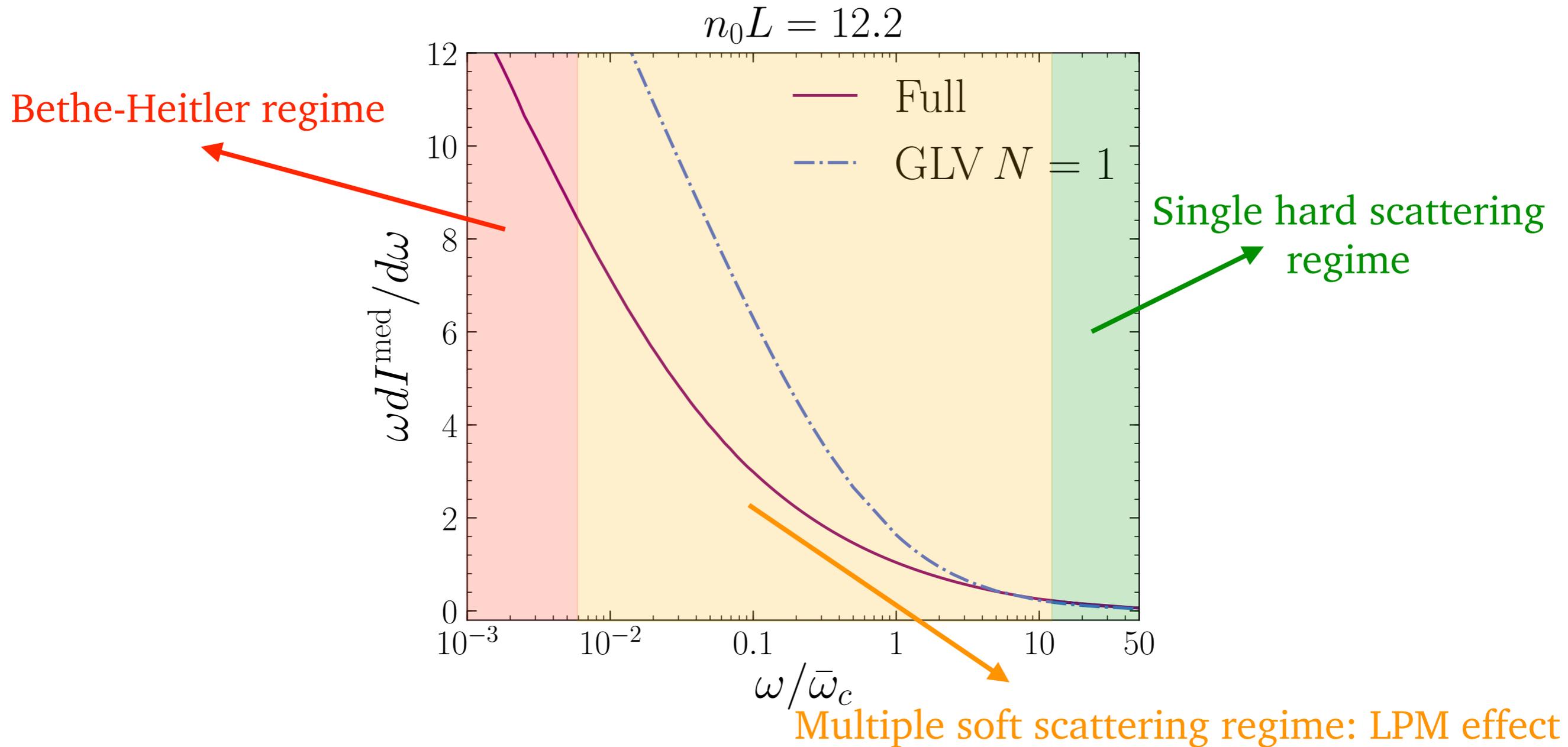
# BDMPS-Z spectrum



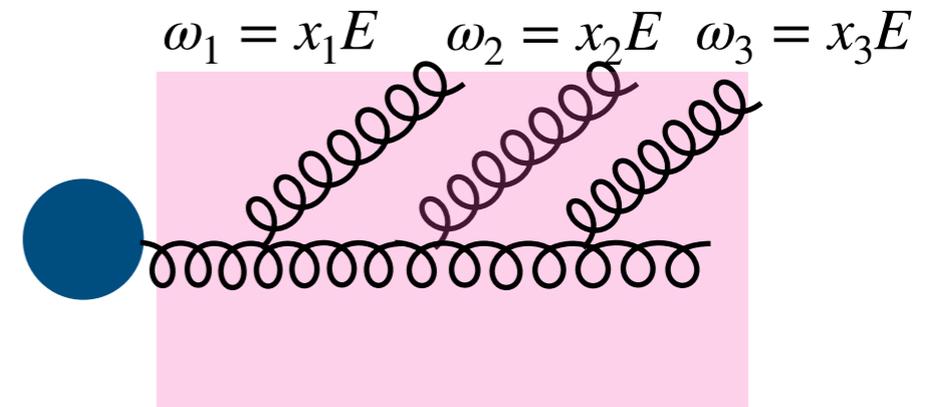
# BDMPS-Z spectrum



# BDMPS-Z spectrum



# Multiple emissions



- Independent emissions:

Jeon, Moore (2005), Blaizot, Dominguez, Iancu, Mehtar-Tani (2014)

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t_*} \int dz \mathcal{K}(z) \left[ \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

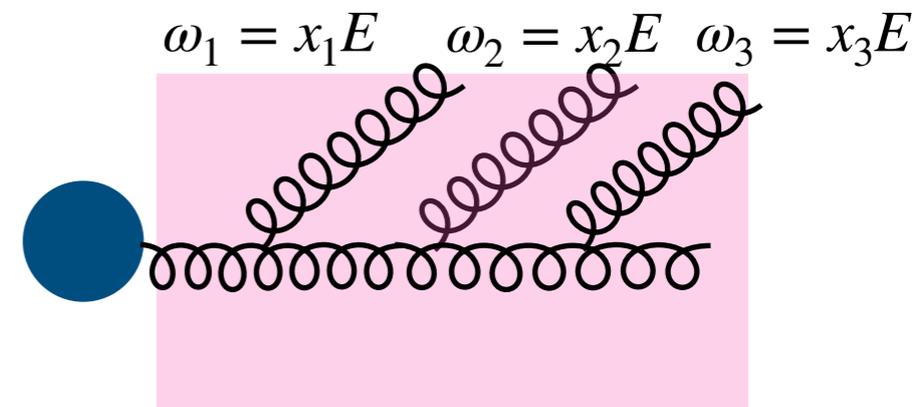
- In the soft limit ( $x \rightarrow 0, E \rightarrow \infty$ )

Baier, Dokshitzer, Mueller, Schiff (2001), Wiedemann, Salgado (2003)

$$P(\epsilon) = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{i=1}^N \left[ \int d\omega_i \frac{dI}{d\omega} \right] \delta \left( \epsilon - \sum_{i=1}^N \omega_i \right) \exp \left[ - \int_0^{\infty} d\omega \frac{dI}{d\omega} \right] \quad \omega_i = x_i E$$

**Probability of energy loss**

# Multiple emissions



- Independent emissions:

Jeon, Moore (2005), Blaizot, Dominguez, Iancu, Mehtar-Tani (2014)

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t_*} \int dz \mathcal{K}(z) \left[ \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

Inclusive gluon distribution
Gain term
Loss term

- In the soft limit ( $x \rightarrow 0, E \rightarrow \infty$ )

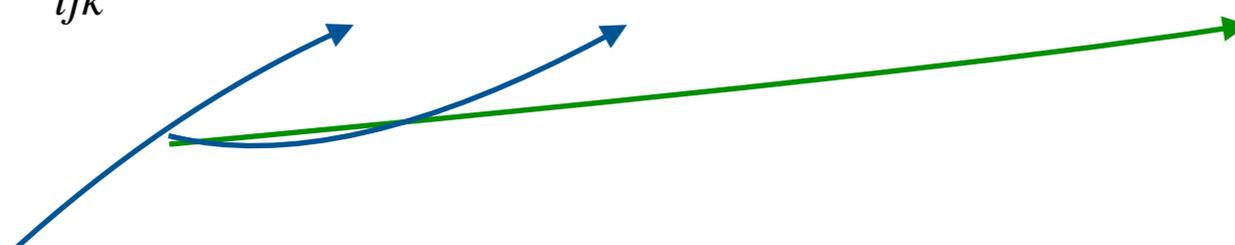
Baier, Dokshitzer, Mueller, Schiff (2001), Wiedemann, Salgado (2003)

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**Probability of energy loss**

# Collinear factorization

$$d\sigma^{pp \rightarrow hX} = \sum_{ijk} f_i(x_1, \mu_f^2) \otimes f_j(x_2, \mu_f^2) \otimes d\sigma_{ij \rightarrow k} \otimes D_{k \rightarrow h}(z, \mu_f^2) + \mathcal{O}(\mu_f^{-2n})$$

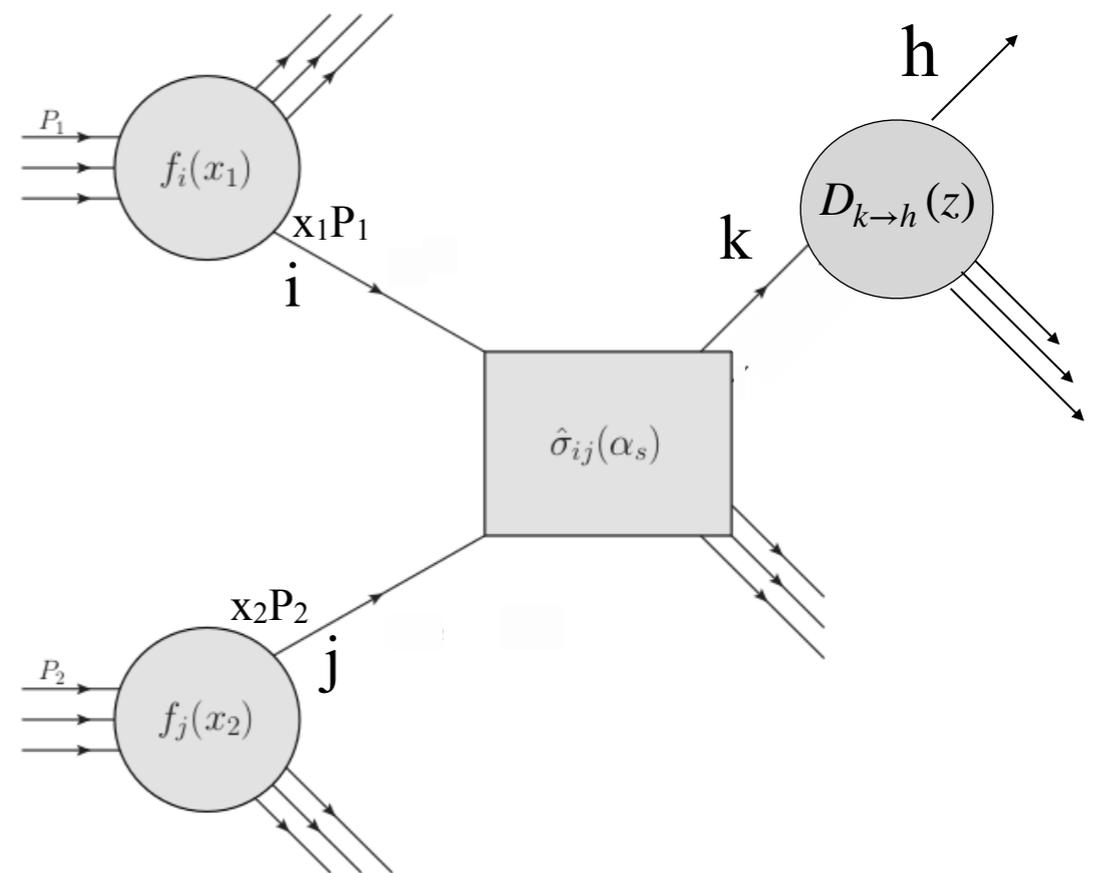


- Non perturbative  
But universal

- Their evolution is perturbative  $\longrightarrow$  DGLAP

$$Q^2 \frac{\partial f_i(x, Q^2)}{\partial Q^2} = \sum_j P_{ij} \otimes f_j(x, Q^2)$$

Splitting functions (pQCD)



# Collinear factorization

$$d\sigma^{pp \rightarrow hX} = \sum_{ijk} f_i(x_1, \mu_f^2) \otimes f_j(x_2, \mu_f^2) \otimes \boxed{d\sigma_{ij \rightarrow k}} \otimes D_{k \rightarrow h}(z, \mu_f^2) + \mathcal{O}(\mu_f^{-2n})$$

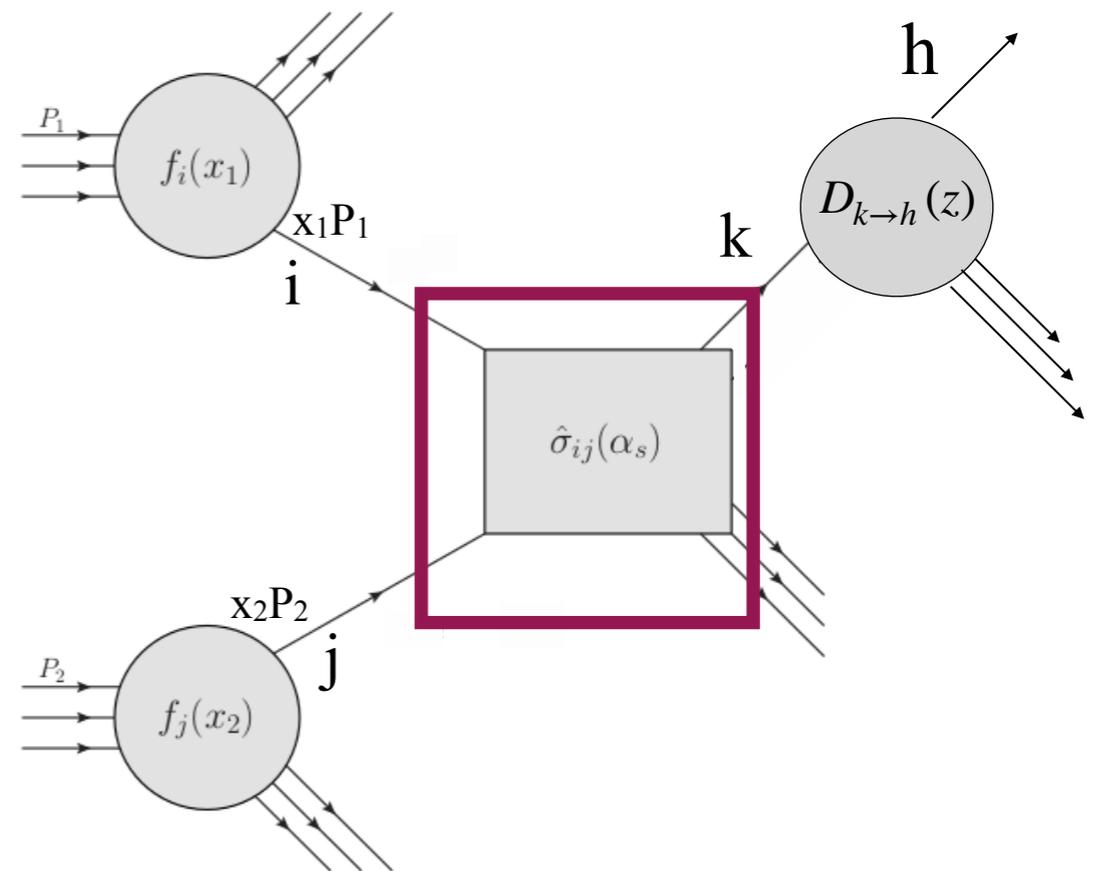
pQCD

- Non perturbative  
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$$Q^2 \frac{\partial f_i(x, Q^2)}{\partial Q^2} = \sum_j \boxed{P_{ij}} \otimes f_j(x, Q^2)$$

Splitting functions (pQCD)



# Collinear factorization

$$d\sigma^{pp \rightarrow hX} = \sum_{ijk} \boxed{f_i(x_1, \mu_f^2)} \otimes \boxed{f_j(x_2, \mu_f^2)} \otimes \boxed{d\sigma_{ij \rightarrow k}} \otimes \boxed{D_{k \rightarrow h}(z, \mu_f^2)} + \mathcal{O}(\mu_f^{-2n})$$

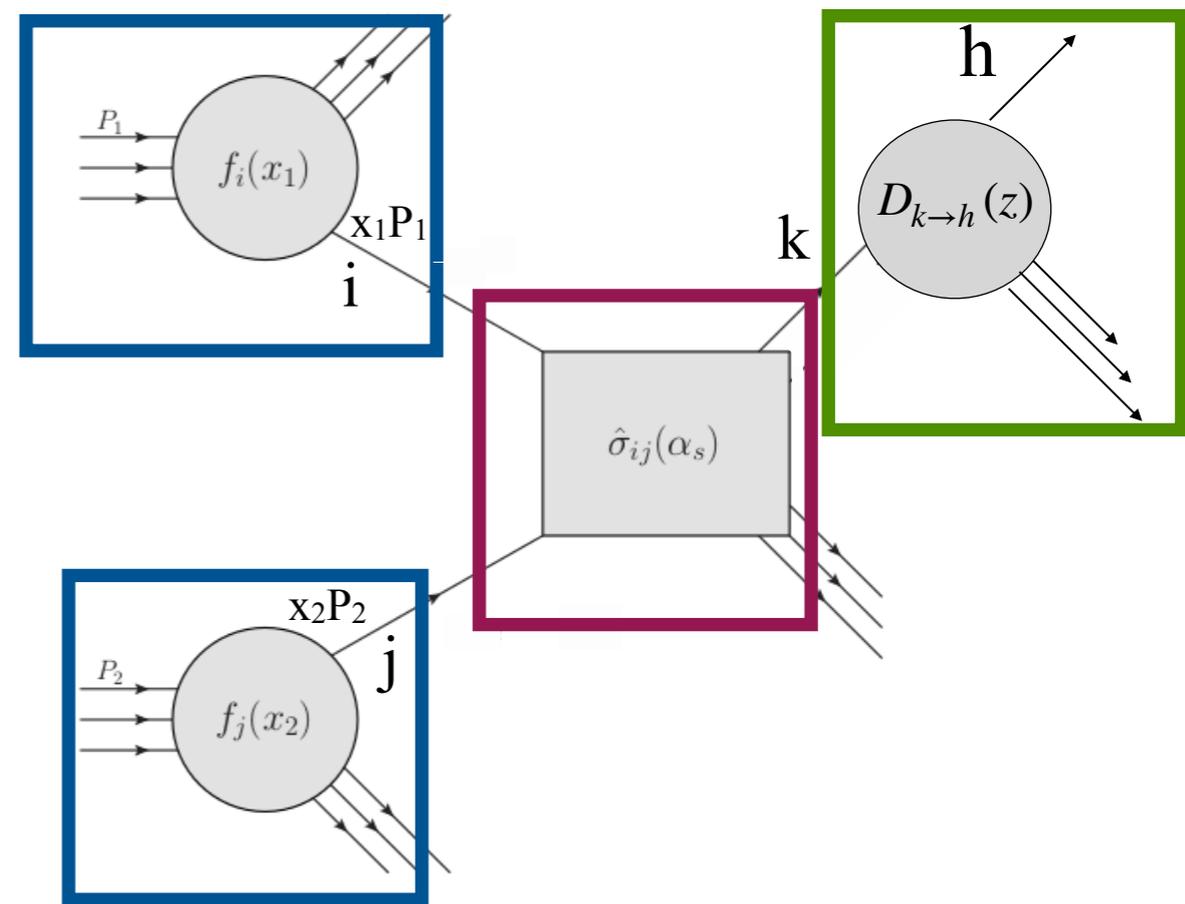
PDFs
pQCD
FFs

- Non perturbative  
But universal

- Their evolution is perturbative  $\longrightarrow$  DGLAP

$$Q^2 \frac{\partial f_i(x, Q^2)}{\partial Q^2} = \sum_j \boxed{P_{ij}} \otimes f_j(x, Q^2)$$

Splitting functions (pQCD)



# Collinear factorization

$$d\sigma^{AA \rightarrow hX} = \sum_{ijk} \boxed{f_i(x_1, \mu_f^2)} \otimes \boxed{f_j(x_2, \mu_f^2)} \otimes \boxed{d\sigma_{ij \rightarrow k}} \otimes \boxed{P(\epsilon)} \otimes \boxed{D_{k \rightarrow h}(z, \mu_f^2)} + \mathcal{O}(\mu_f^{-2n})$$

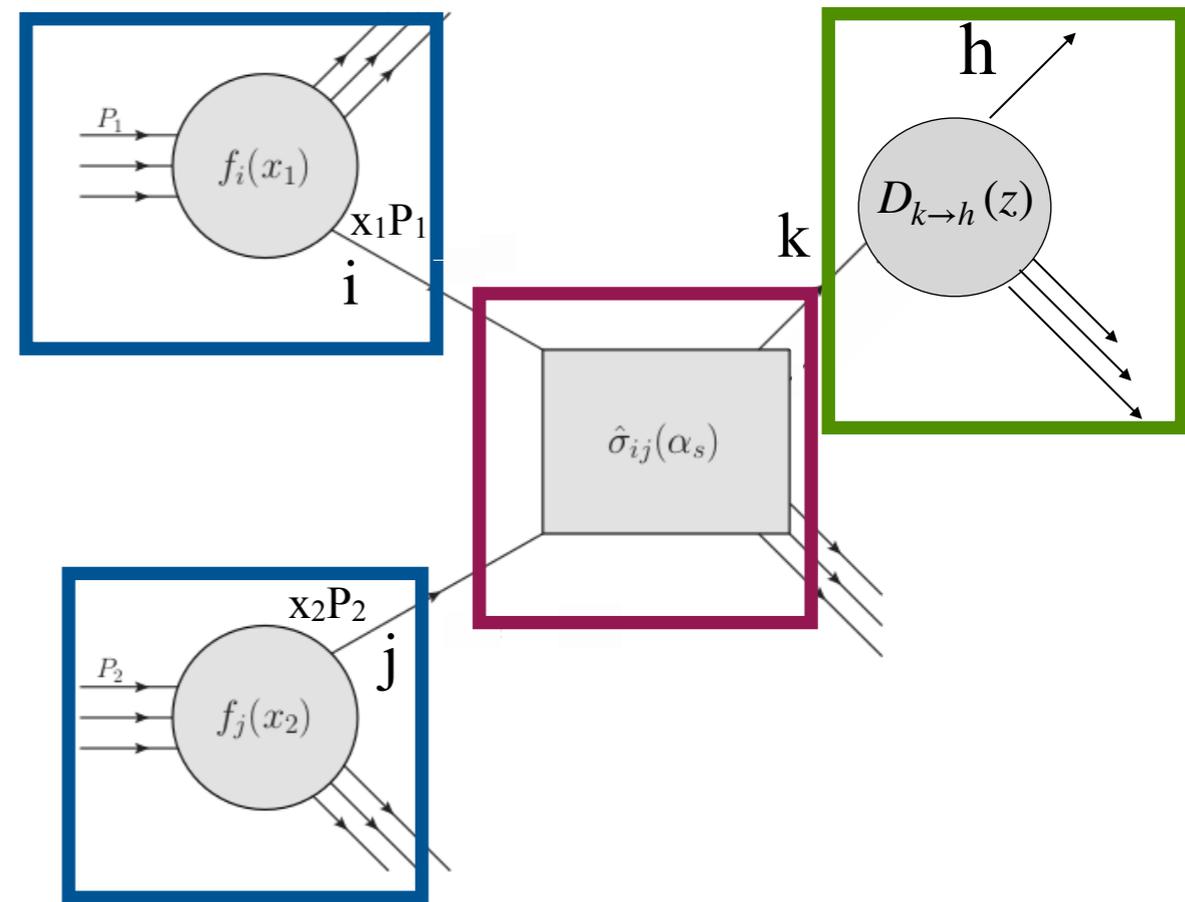
n PDFs      pQCD      Energy loss      FFs

- Non perturbative  
But universal

- Their evolution is perturbative  $\longrightarrow$  DGLAP

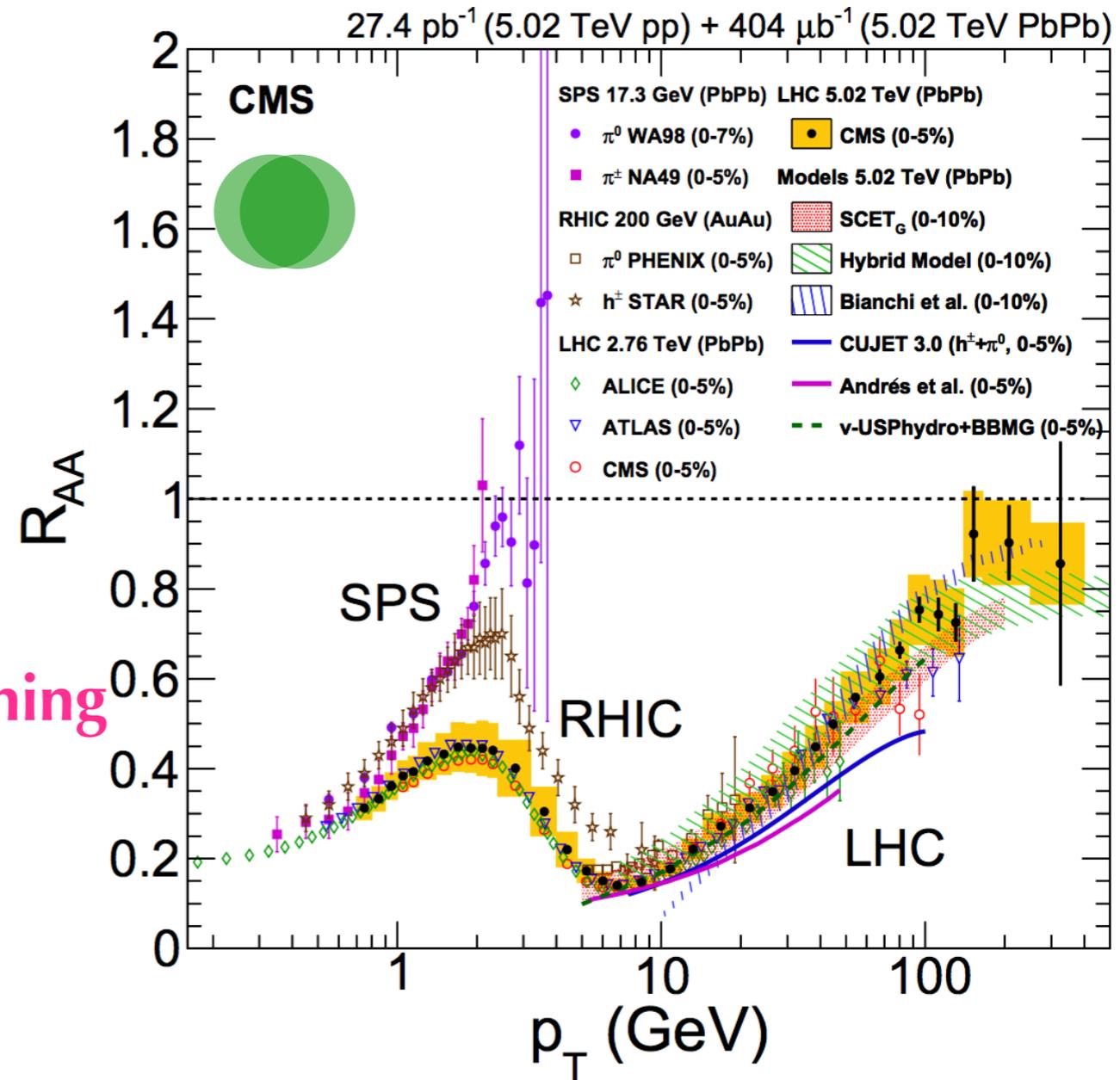
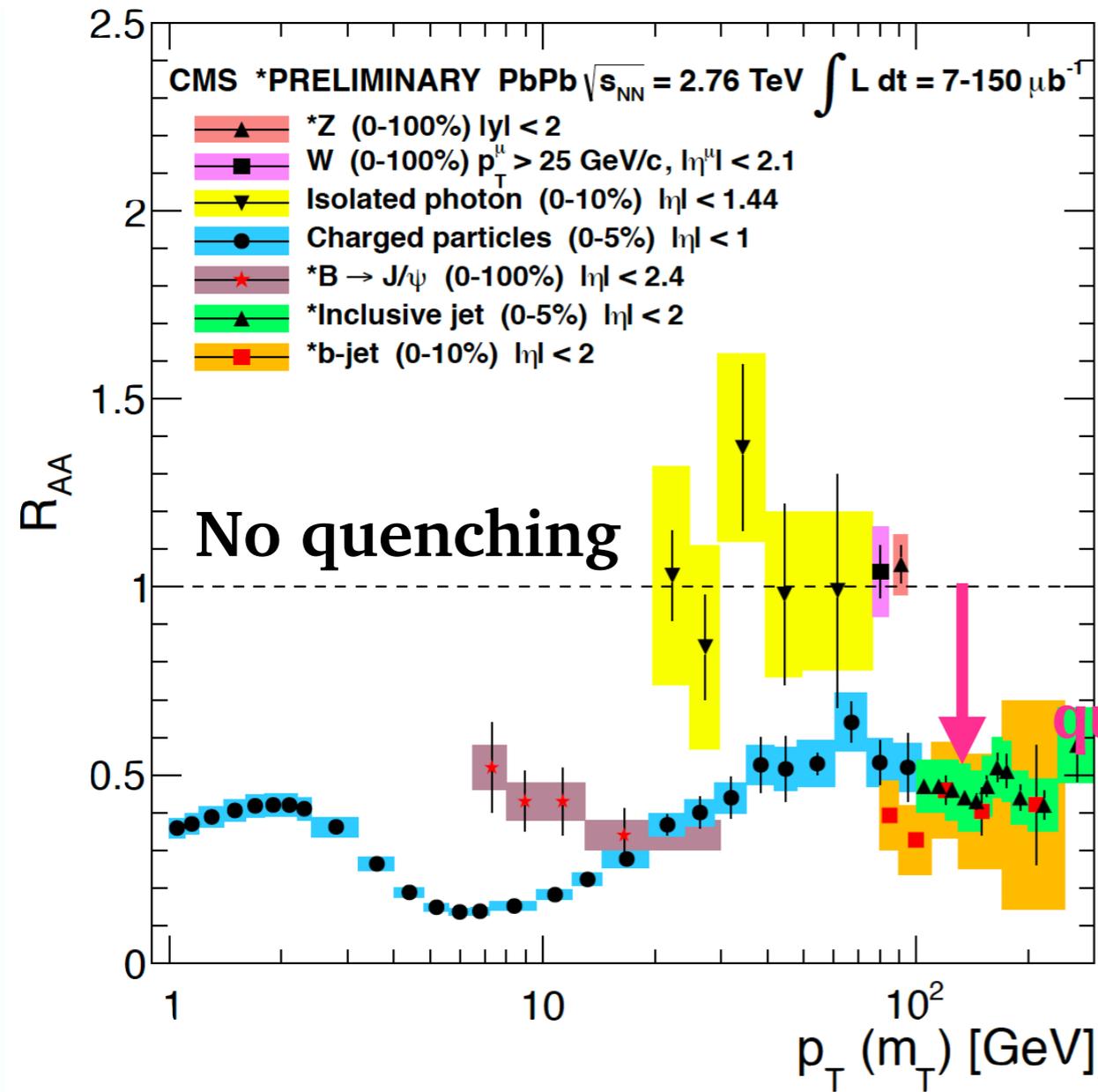
$$Q^2 \frac{\partial f_i(x, Q^2)}{\partial Q^2} = \sum_j \boxed{P_{ij}} \otimes f_j(x, Q^2)$$

Splitting functions (pQCD)



# Hadron suppression

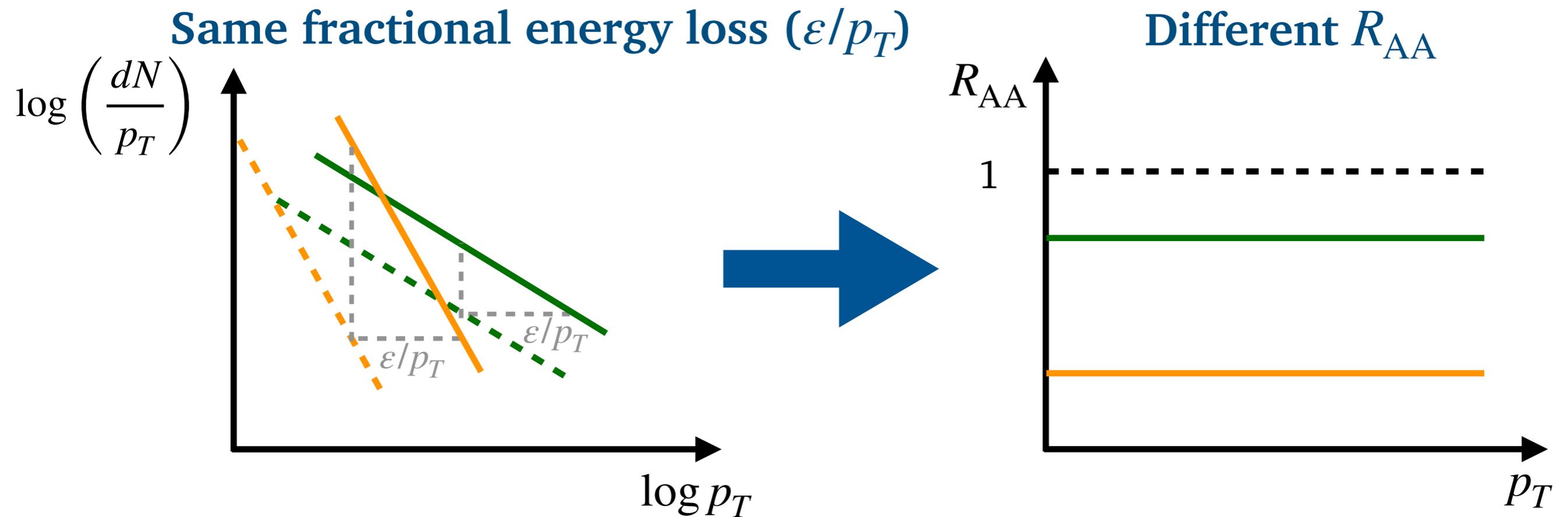
$$R_{AA} = \frac{\text{Pb-Pb} \text{ (two overlapping circles)}}{\text{scaled pp} \text{ (two overlapping circles with arrows)}}$$



CMS Collaboration, JHEP 04 039 (2017)

# Understanding the $R_{AA}$

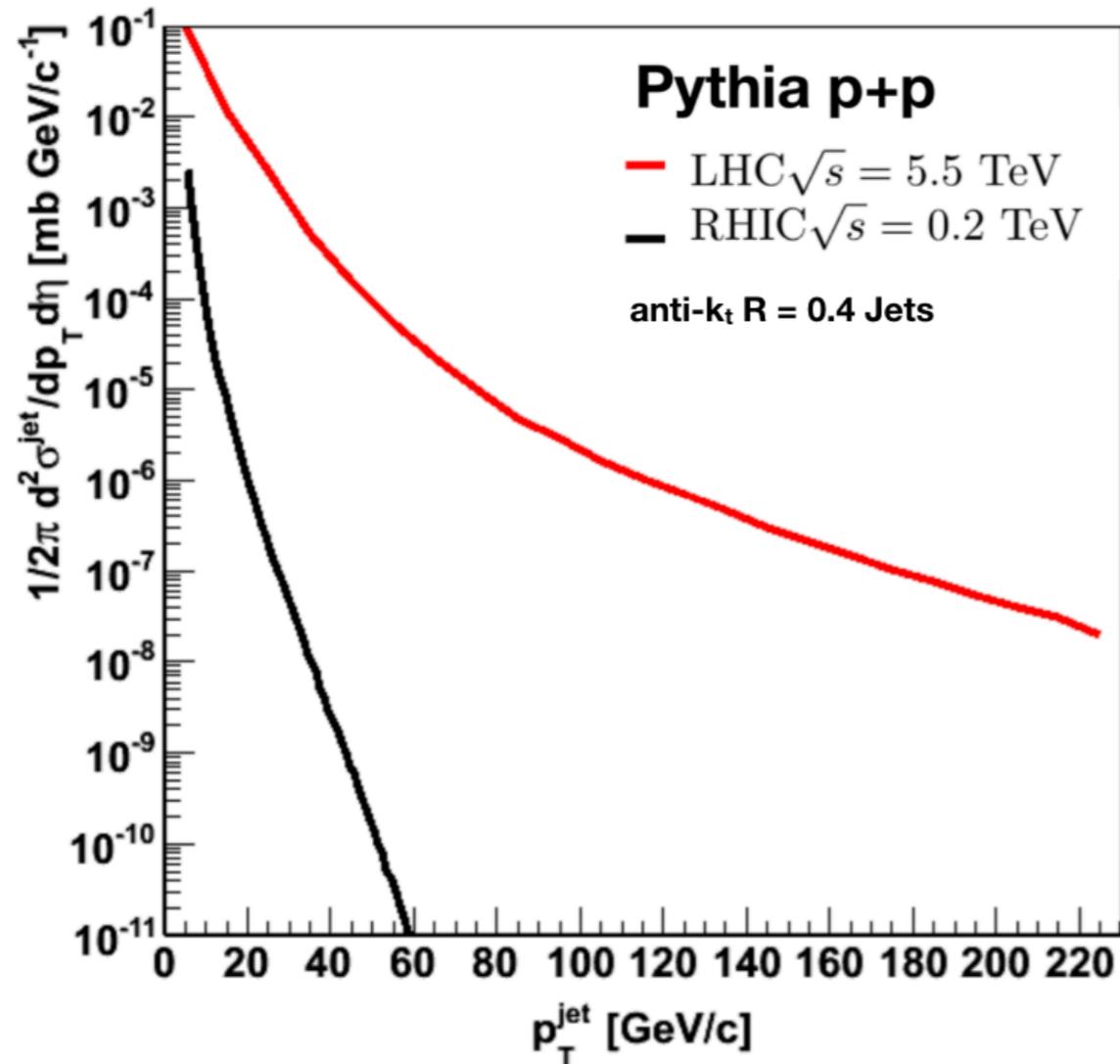
In p-p, the spectrum for high- $p_T$  particles behaves as  $\frac{dN^{pp}}{p_T} \propto \frac{1}{p_T^n}$



At parton level:

$$R_{AA} \sim \int d\epsilon P(\epsilon) \frac{d\sigma^{pp}(p_T + \epsilon)/dp_T}{d\sigma^{pp}(p_T)/dp_T} = \int d\epsilon P(\epsilon) \frac{p_T^n}{(p_T + \epsilon)^n} \approx \int d\epsilon P(\epsilon) \left( 1 - \frac{n\epsilon}{p_T} \right)$$

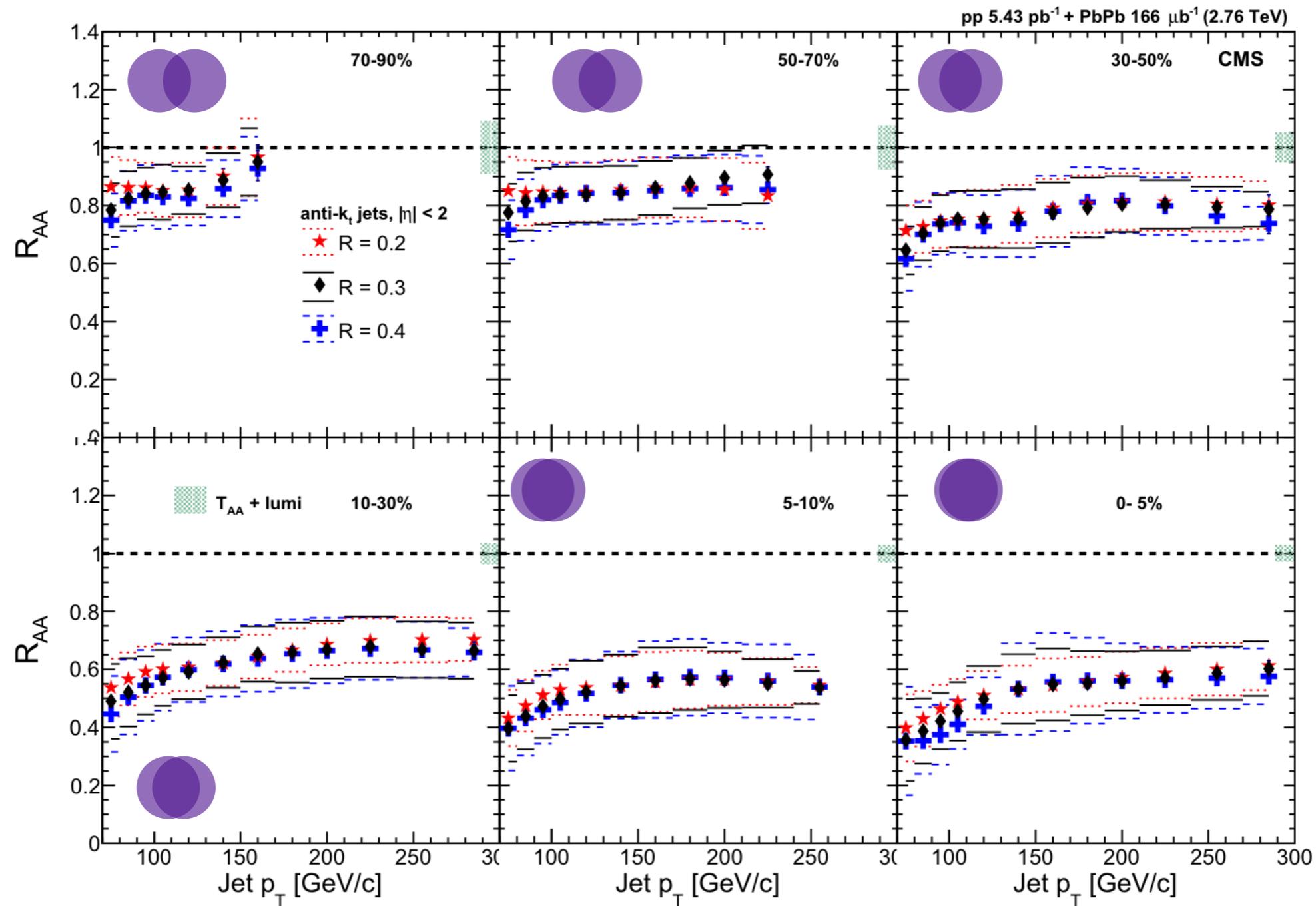
# Understanding the $R_{AA}$



- Jet p-p spectrum steeper at RHIC than at LHC
- $R_{AA}$  at RHIC similar to  $R_{AA}$  at LHC

At a given  $p_T$ , do jets at RHIC or at LHC lose more fractional energy?

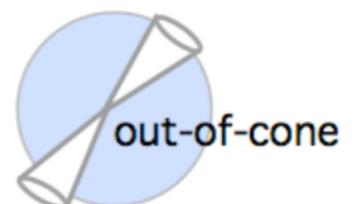
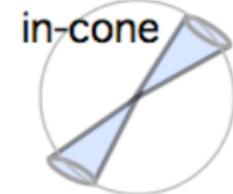
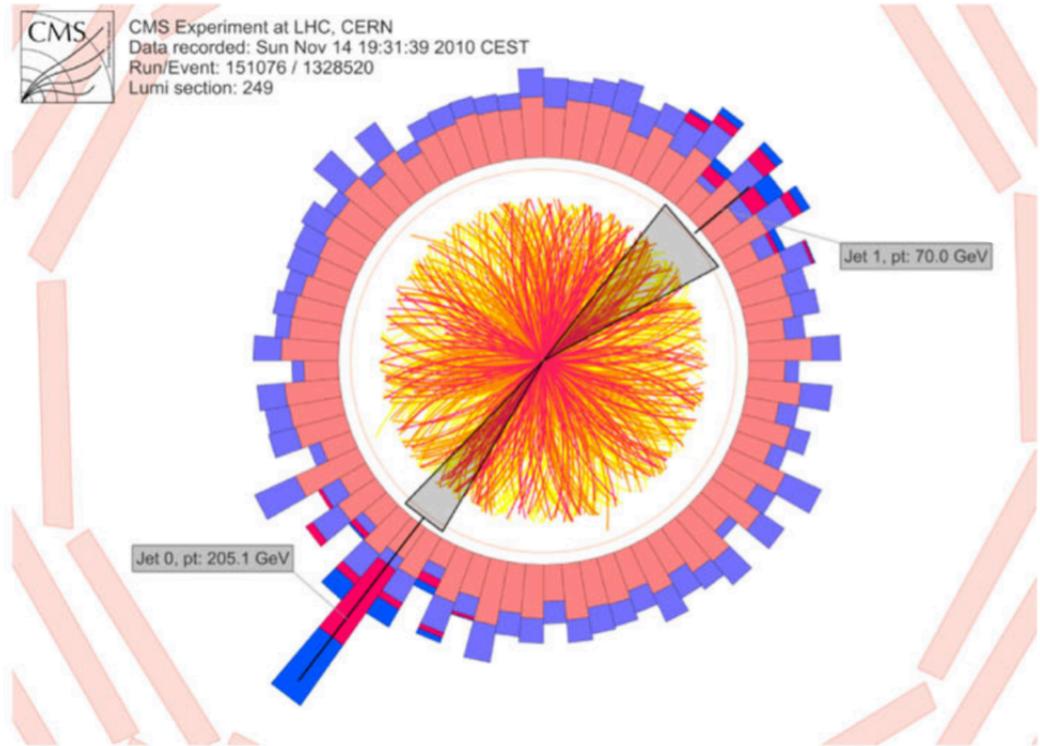
# Reconstructed jets



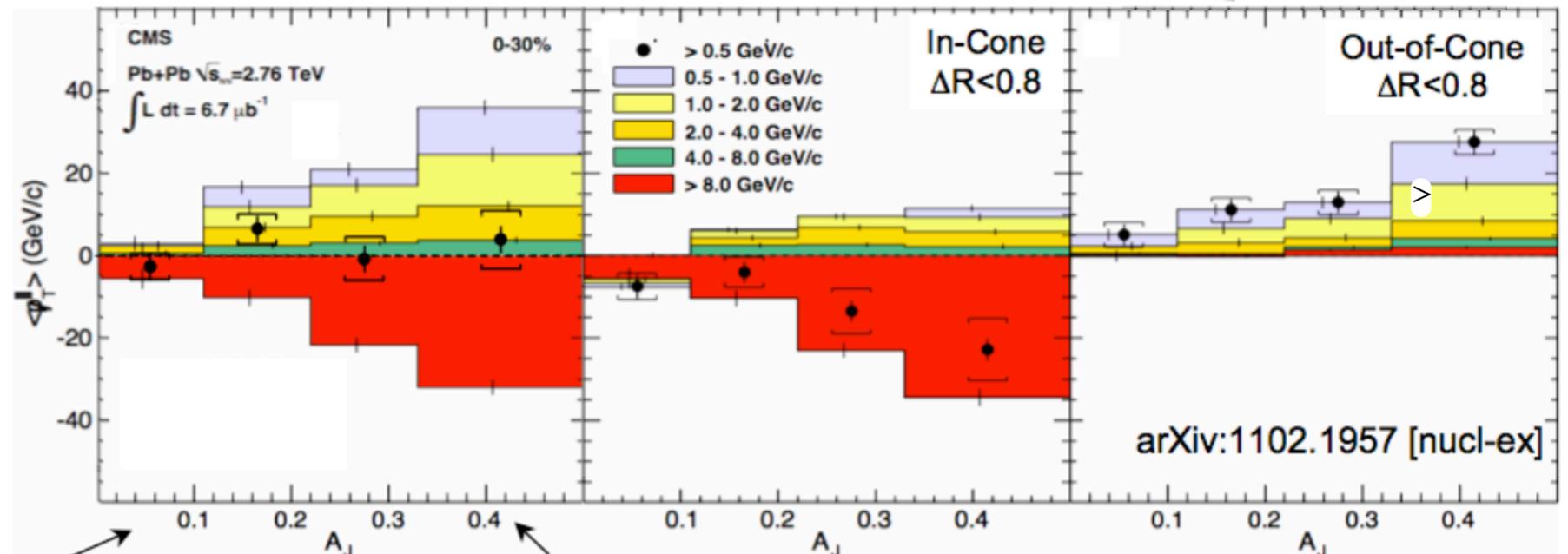
CMS, [Phys. Rev. C 96 \(2017\) 015202](#)

**Reconstructed jets are suppressed!**

# Where does the energy go?



$$A_J = \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}$$

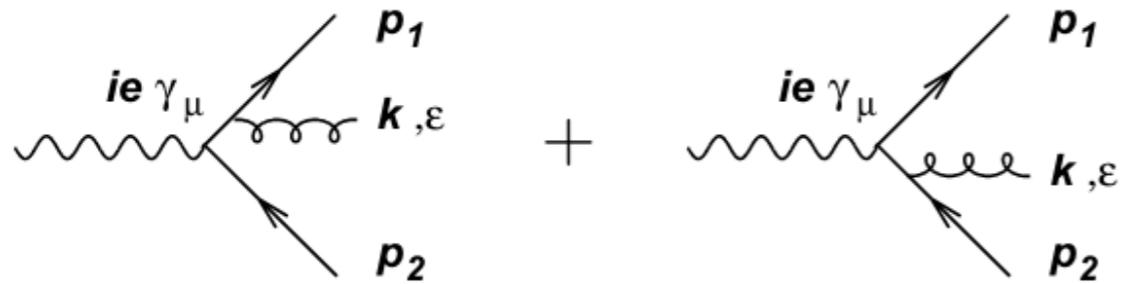


balanced jets

unbalanced jets

At large angles. Soft particles

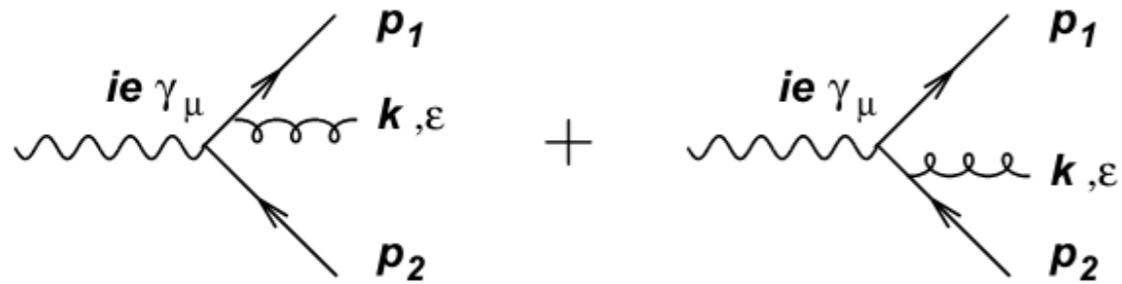
# Antenna in the vacuum



$$M_{q\bar{q}g} = \bar{u}(p_1) (-ig_s t^a) \not{\epsilon}(k) \frac{i(\not{p}_1 + \not{k})}{(p_1 + k)^2} (ie \gamma^\mu) v(p_2) \\ + \bar{u}(p_1) (ie \gamma^\mu) \frac{-i(\not{p}_2 + \not{k})}{(p_2 + k)^2} \not{\epsilon}(k) (-ig_s t^a) v(p_2)$$

$$M_{q\bar{q}g} \xrightarrow{k \ll p_1, p_2} \approx g_s t^a \left( \frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right) \bar{u}(p_1) ie \gamma^\mu v(p_2)$$

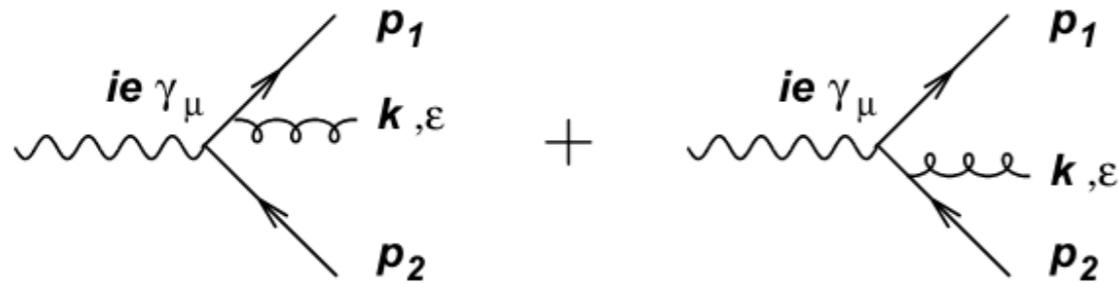
# Antenna in the vacuum



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$$M_{q\bar{q}g} \stackrel{k \ll p_1, p_2}{\approx} g_s t^a \left( \frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right) \bar{u}(p_1) ie\gamma^\mu v(p_2) \rightarrow M_{q\bar{q}}$$

# Antenna in the vacuum



$$M_{q\bar{q}g} = \bar{u}(p_1) (-ig_s t^a) \not{\epsilon}(k) \frac{i(p_1 + \not{k})}{(p_1 + k)^2} (ie\gamma^\mu) v(p_2) \\ + \bar{u}(p_1) (ie\gamma^\mu) \frac{-i(p_2 + \not{k})}{(p_2 + k)^2} \not{\epsilon}(k) (-ig_s t^a) v(p_2)$$

$k \ll p_1, p_2$

$$M_{q\bar{q}g} \approx g_s t^a \left( \frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right) \bar{u}(p_1) ie\gamma^\mu v(p_2)$$

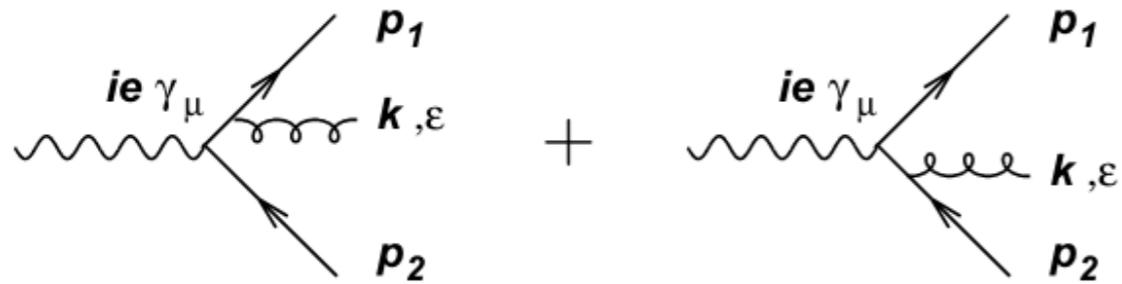
$\rightarrow M_{q\bar{q}}$

- Squaring the amplitude:

$$\sum_{\text{col}} \text{Tr}(t^a t^a) = C_F N_c \quad \sum_{\text{pol}} \epsilon_\mu(k) \epsilon_\nu^*(k) = -g_{\mu\nu} + \dots$$

$$|M_{q\bar{q}g}|^2 \approx \frac{1}{N_c} \sum_{\text{col, pol}} g_s^2 \text{Tr}(t^a t^a) \left( \frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right)^2 |M_{q\bar{q}}|^2 = g_s^2 C_F \frac{2 p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} |M_{q\bar{q}}|^2$$

# Antenna in the vacuum



$$M_{q\bar{q}g} = \bar{u}(p_1) (-ig_s t^a) \not{\epsilon}(k) \frac{i(p_1 + \not{k})}{(p_1 + k)^2} (ie\gamma^\mu) v(p_2) \\ + \bar{u}(p_1) (ie\gamma^\mu) \frac{-i(p_2 + \not{k})}{(p_2 + k)^2} \not{\epsilon}(k) (-ig_s t^a) v(p_2)$$

$k \ll p_1, p_2$

$$M_{q\bar{q}g} \approx g_s t^a \left( \frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right) \bar{u}(p_1) ie\gamma^\mu v(p_2) \rightarrow M_{q\bar{q}}$$

- Squaring the amplitude:

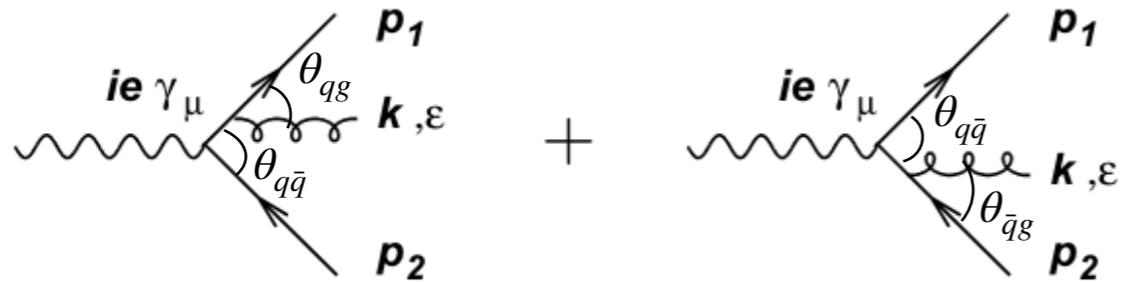
$$\sum_{\text{col}} \text{Tr}(t^a t^a) = C_F N_c \quad \sum_{\text{pol}} \epsilon_\mu(k) \epsilon_\nu^*(k) = -g_{\mu\nu} + \dots$$

$$|M_{q\bar{q}g}|^2 \approx \frac{1}{N_c} \sum_{\text{col, pol}} g_s^2 \text{Tr}(t^a t^a) \left( \frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right)^2 |M_{q\bar{q}}|^2 = g_s^2 C_F \frac{2 p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} |M_{q\bar{q}}|^2$$

↓ Including phase space factors  $\frac{d\omega d^2\mathbf{k}}{2\omega(2\pi)^3}$

$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{(2\pi)^2} \frac{2 p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}$$

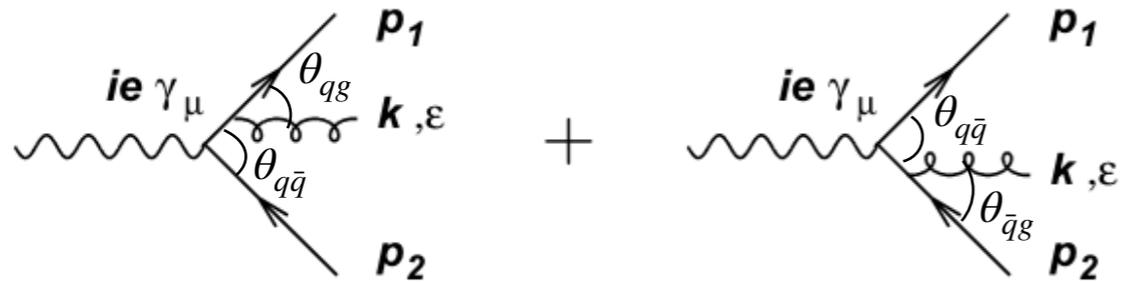
# Antenna in the vacuum II



$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} \frac{1 - \cos \theta_{q\bar{q}}}{(1 - \cos \theta_{qg})(1 - \cos \theta_{\bar{q}g})}$$

**Collinear and soft divergences!**

# Antenna in the vacuum II

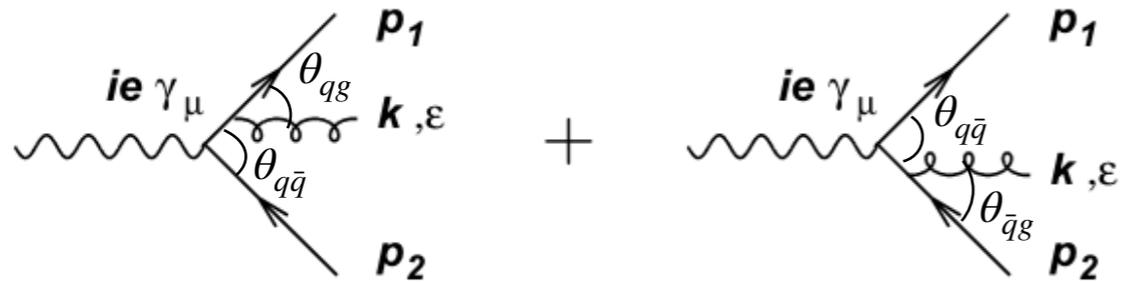


$$\omega \frac{dN}{d\omega d^2k} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} \frac{1 - \cos \theta_{q\bar{q}}}{(1 - \cos \theta_{qg})(1 - \cos \theta_{\bar{q}g})}$$

$\nearrow W_{q\bar{q}}$

**Collinear and soft divergences!**

# Antenna in the vacuum II



$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} \frac{1 - \cos \theta_{q\bar{q}}}{(1 - \cos \theta_{qg})(1 - \cos \theta_{\bar{q}g})}$$

$\nearrow W_{q\bar{q}}$

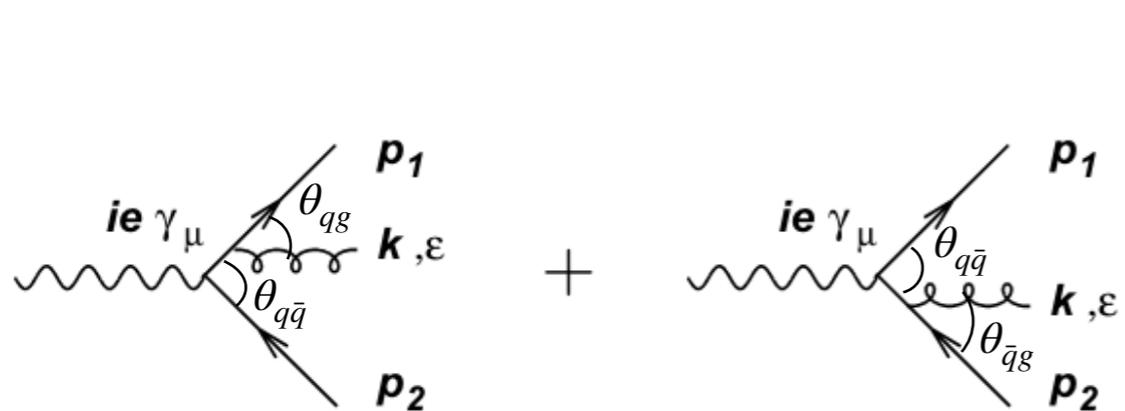
**Collinear and soft divergences!**

Defining:  $-2J = W_{q\bar{q}} - \left( \frac{1}{1 - \cos \theta_{qg}} + \frac{1}{1 - \cos \theta_{\bar{q}g}} \right)$

$\downarrow R_q \qquad \qquad \downarrow R_{\bar{q}}$

$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} \left( R_q + R_{\bar{q}} - 2J \right)$$

# Antenna in the vacuum II



$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} \frac{1 - \cos \theta_{q\bar{q}}}{(1 - \cos \theta_{qg})(1 - \cos \theta_{\bar{q}g})}$$

$\nearrow W_{q\bar{q}}$

**Collinear and soft divergences!**

Defining:  $-2J = W_{q\bar{q}} - \left( \frac{1}{1 - \cos \theta_{qg}} + \frac{1}{1 - \cos \theta_{\bar{q}g}} \right)$

$\downarrow R_q \qquad \qquad \downarrow R_{\bar{q}}$

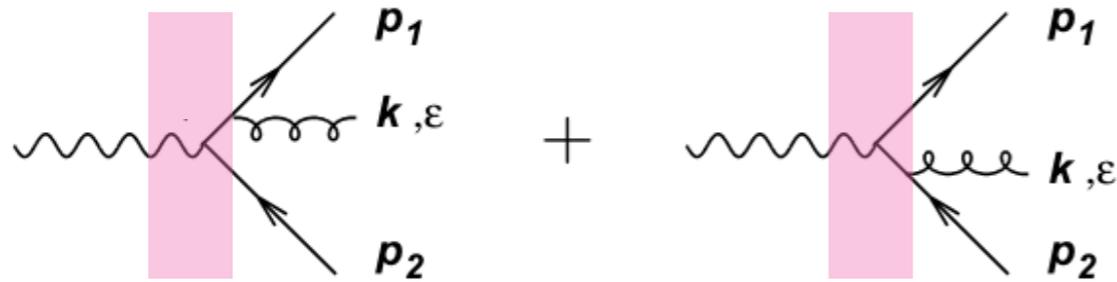
$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} (R_q + R_{\bar{q}} - 2J)$$

Integrating over the azimuthal angle: angular ordering

$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} \left( R_q - J + R_{\bar{q}} - J \right)$$

# Antenna in the medium

Mehtar-Tani, Salgado, Tywoniuk, *Phys. Rev. Lett.* 106 (2011) 122002,  
*Phys. Lett B* 707 (2012) 156, *JHEP* 10 (2012) 197,  
 J. Casalderrey-Solana and E. Iancu, *JHEP* 08 (2011) 015



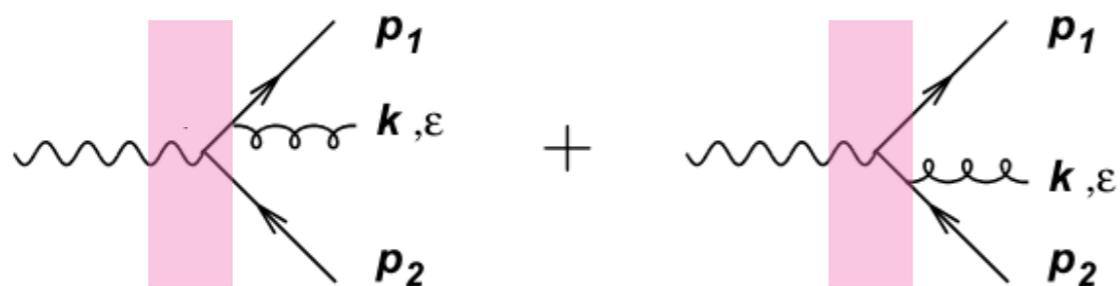
$$M_{q\bar{q}g} \sim M_{q\bar{q}} t^a g_s W(r_1) W^\dagger(r_2) \frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} + M_{q\bar{q}} W(r_1) W^\dagger(r_2) t^a g_s \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k}$$

$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} \left( R_q - S J + R_{\bar{q}} - S J \right)$$

If  $S \rightarrow 0$ : independent emissions (decoherence)

# Antenna in the medium

Mehtar-Tani, Salgado, Tywoniuk, *Phys. Rev. Lett.* 106 (2011) 122002,  
*Phys. Lett B* 707 (2012) 156, *JHEP* 10 (2012) 197,  
 J. Casalderrey-Solana and E. Iancu, *JHEP* 08 (2011) 015



$$M_{q\bar{q}g} \sim M_{q\bar{q}} t^a g_s W(r_1) W^\dagger(r_2) \frac{p_1 \cdot \varepsilon(k)}{p_1 \cdot k} + M_{q\bar{q}} W(r_1) W^\dagger(r_2) t^a g_s \frac{p_2 \cdot \varepsilon(k)}{p_2 \cdot k}$$

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If  $S \rightarrow 0$ : independent emissions (decoherence)

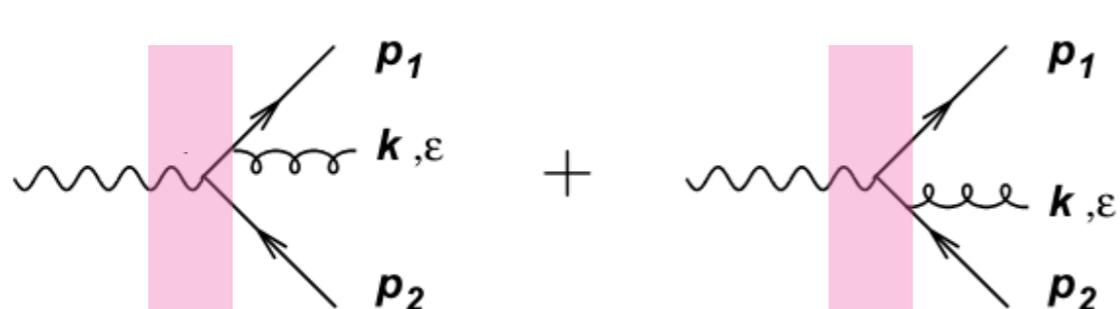
- What is  $S$ ?

$$S(\mathbf{r}_1, \mathbf{r}_2) \equiv \frac{1}{N_c^2 - 1} \text{Tr} \langle W_A(\mathbf{r}_1) W_A^\dagger(\mathbf{r}_2) \rangle = \exp \left\{ -\frac{1}{2} \int ds n(s) \sigma(\mathbf{r}_1 - \mathbf{r}_2) \right\}$$

$$S(\mathbf{r}_1, \mathbf{r}_2) \approx \exp \left\{ -\frac{1}{12} \hat{q} \theta_{q\bar{q}}^2 L^3 \right\} = \exp \{ -\theta_{q\bar{q}}^2 / \theta_c^2 \} \quad \theta_c = \sqrt{\frac{12}{\hat{q} L^3}}$$

Harmonic oscillator, static medium

# Antenna in the medium



$$M_{q\bar{q}g} \sim M_{q\bar{q}} t^a g_s W(r_1) W^\dagger(r_2) \frac{p_1 \cdot \varepsilon(k)}{p_1 \cdot k} + M_{q\bar{q}} W(r_1) W^\dagger(r_2) t^a g_s \frac{p_2 \cdot \varepsilon(k)}{p_2 \cdot k}$$

$$\omega \frac{dN}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2 \omega^2} \left( R_q - S J + R_{\bar{q}} - S J \right)$$

If  $S \rightarrow 0$ : independent emissions (decoherence)

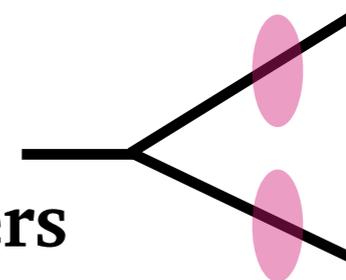
• What is S?

$$S(\mathbf{r}_1, \mathbf{r}_2) \equiv \frac{1}{N_c^2 - 1} \text{Tr} \langle W_A(\mathbf{r}_1) W_A^\dagger(\mathbf{r}_2) \rangle = \exp \left\{ -\frac{1}{2} \int ds n(s) \sigma(\mathbf{r}_1 - \mathbf{r}_2) \right\}$$

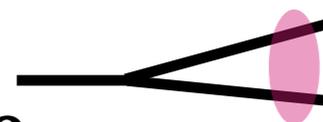
$$S(\mathbf{r}_1, \mathbf{r}_2) \approx \exp \left\{ -\frac{1}{12} \hat{q} \theta_{q\bar{q}}^2 L^3 \right\} = \exp \{ -\theta_{q\bar{q}}^2 / \theta_c^2 \} \quad \theta_c = \sqrt{\frac{12}{\hat{q} L^3}}$$

Harmonic oscillator, static medium

$\theta_{q\bar{q}} > \theta_c$  : The medium resolves the antenna.  
 Color coherence is broken: **two-independent emitters**

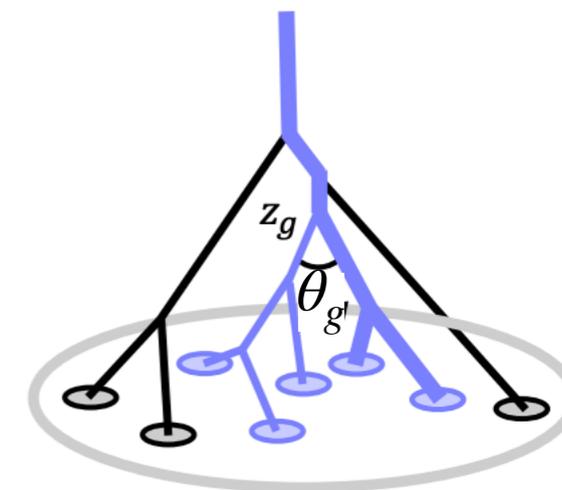


$\theta_{q\bar{q}} < \theta_c$  : The medium cannot resolve the antenna.  
 Color coherence maintained. **Vacuum-like**



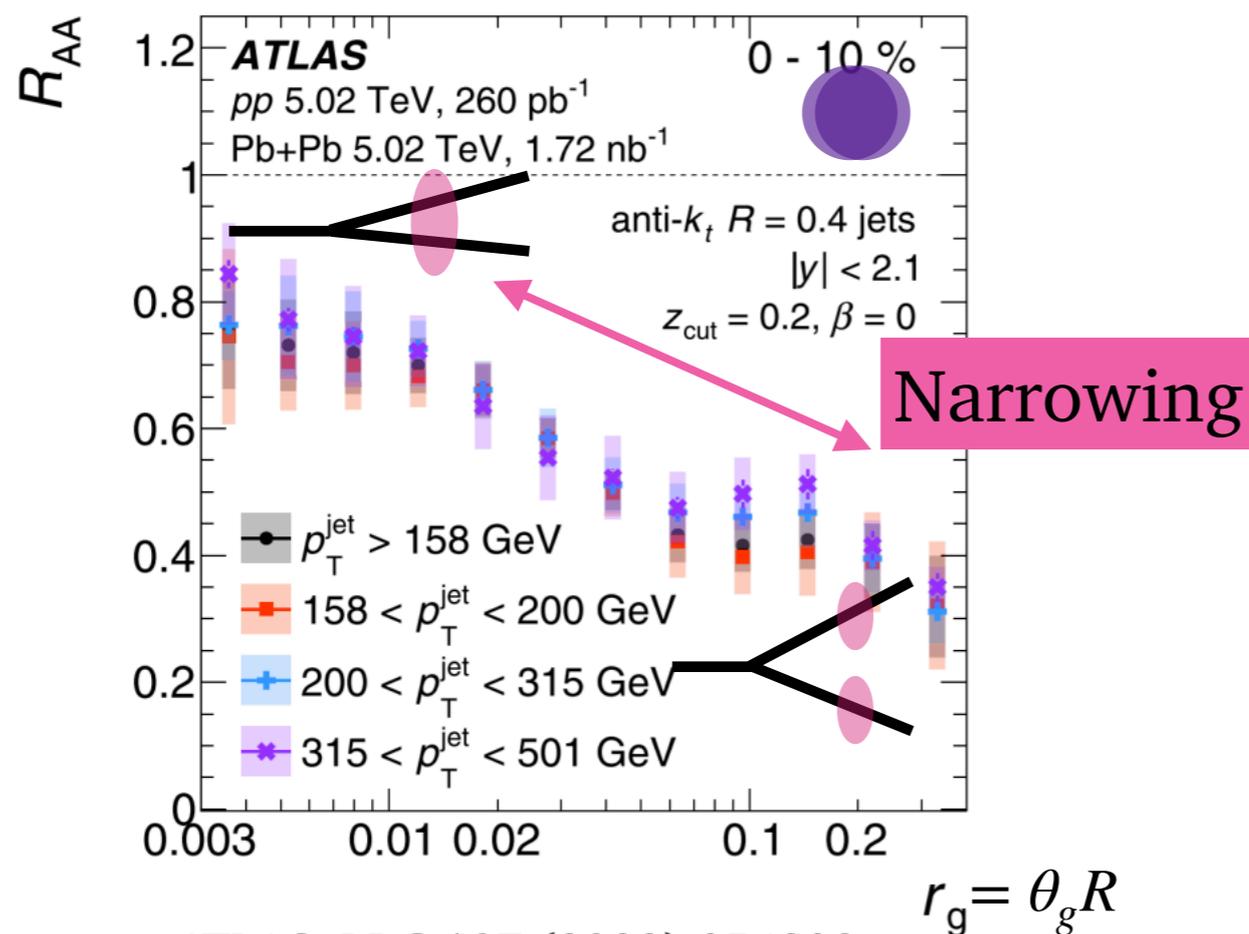
# Color (de)coherence

Jet constituents are re-clustered (through C/A) and soft/wide angle radiation is rejected in this process



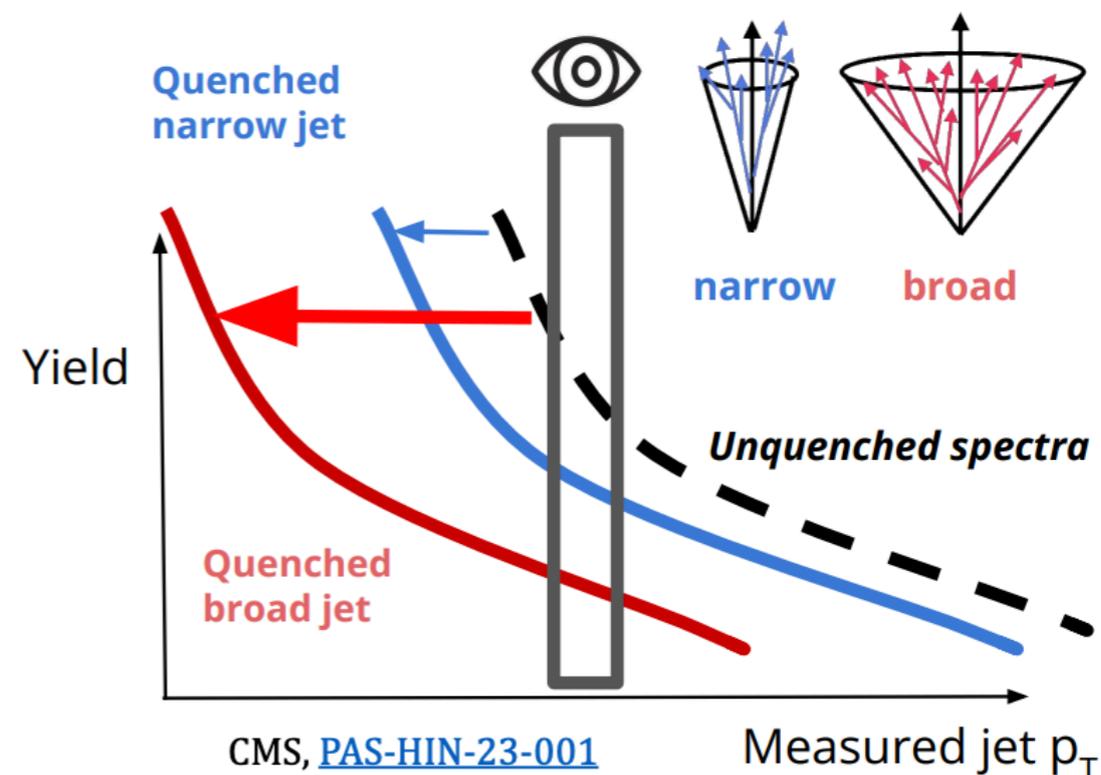
Groomed jet radius

$$R_{AA} = \frac{\text{Pb-Pb}}{\text{scaled pp}}$$



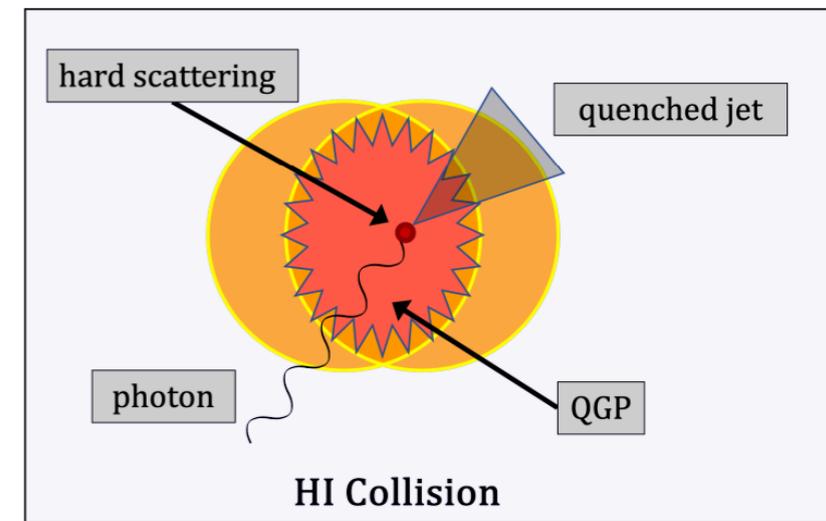
ATLAS, PRC 107 (2023) 054909

Jet quenching bias

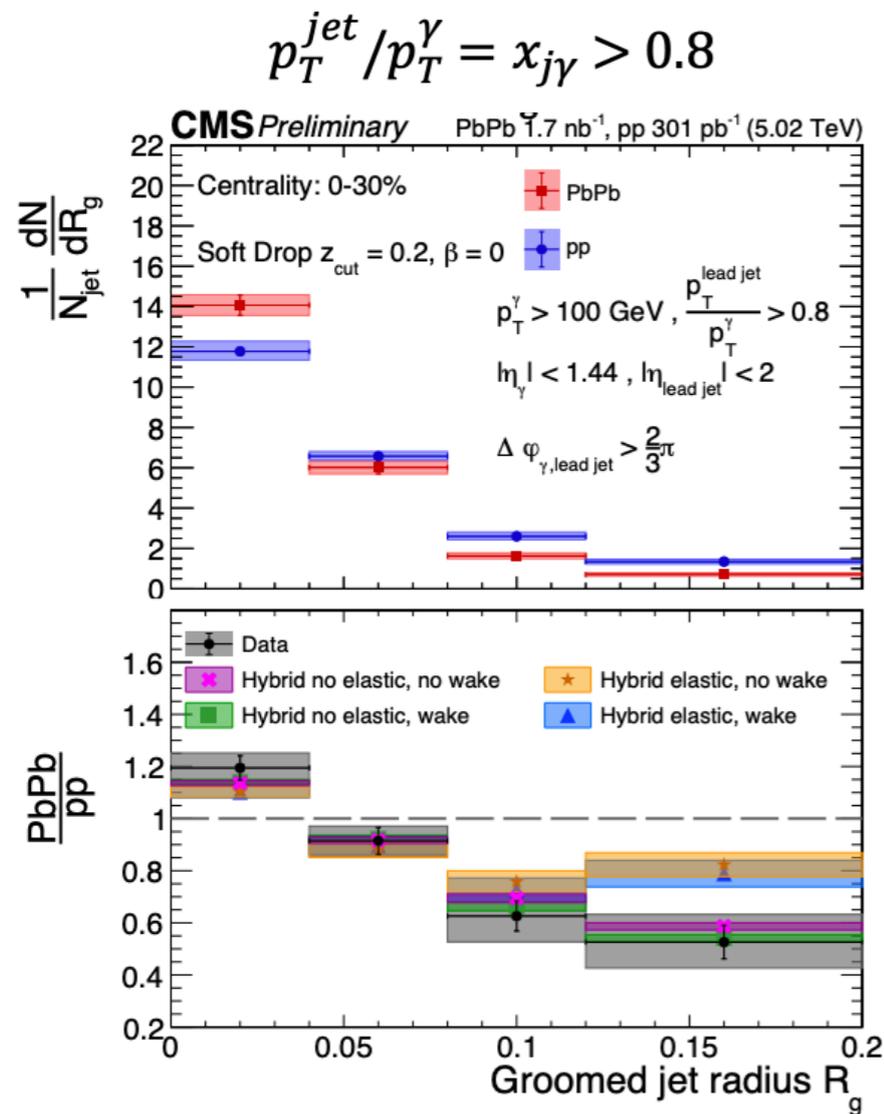


# Color (de)coherence

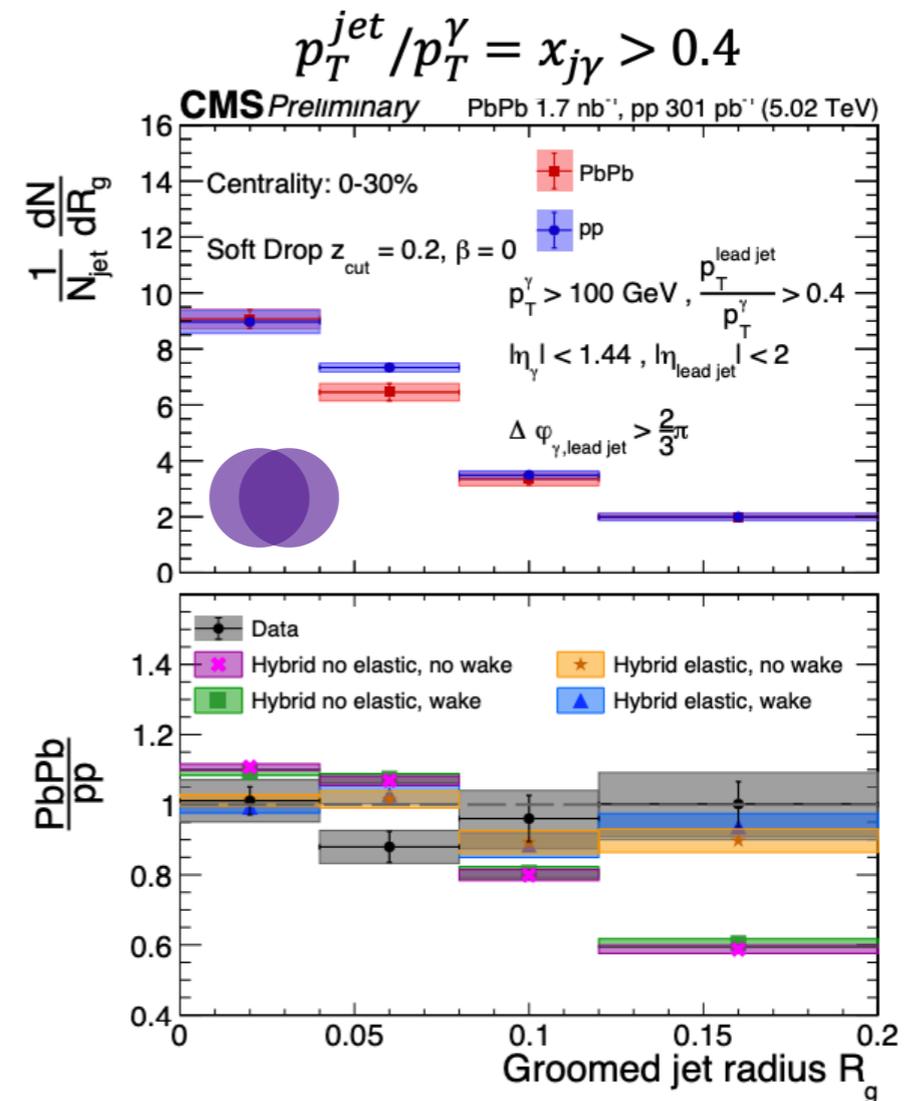
- Use photon-tagged jets



CMS-PAS-HIN-23-001



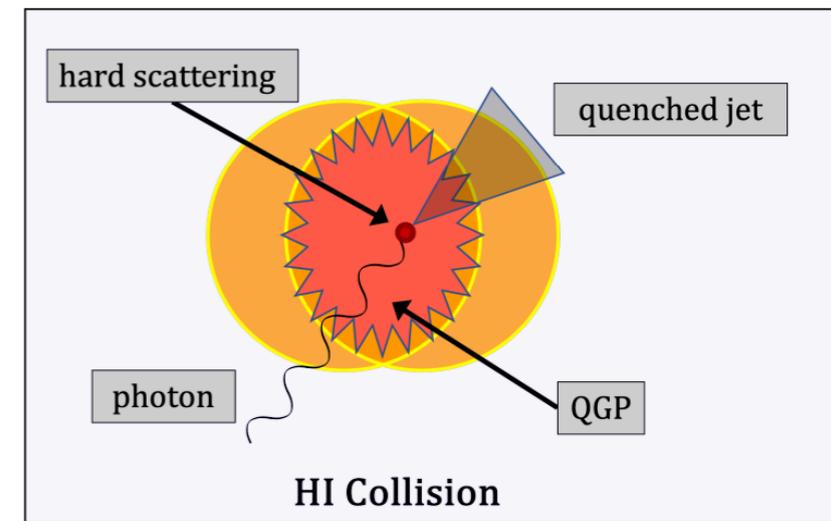
Less quenched jets  
Narrowing



Quenched and unquenched jets  
No narrowing

# Color (de)coherence

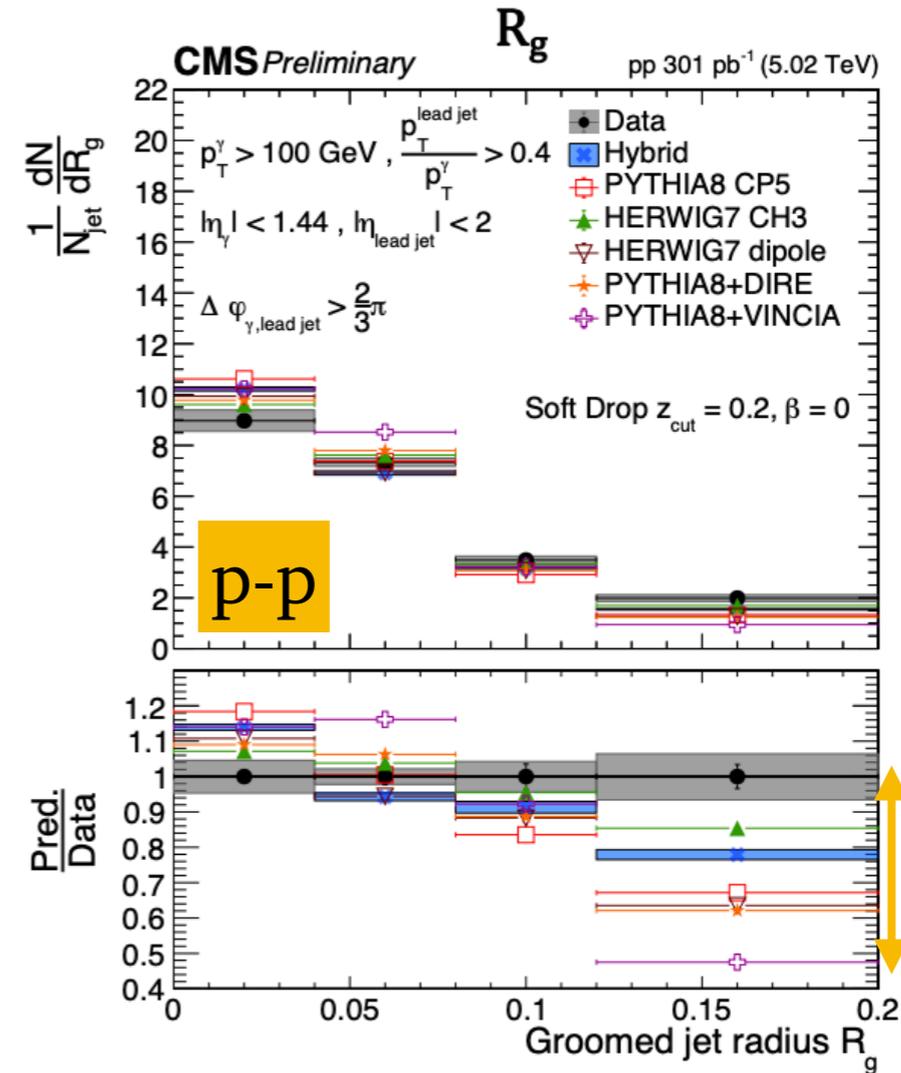
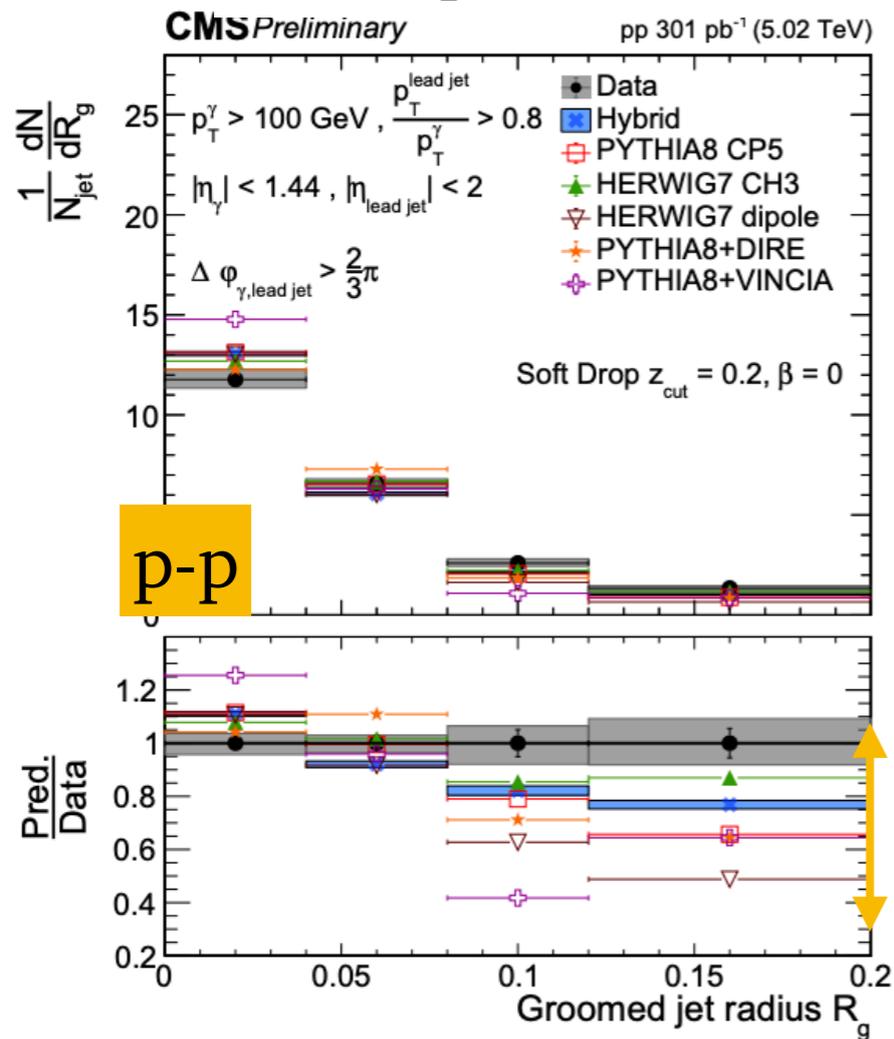
- Use photon-tagged jets



$$p_T^{jet} / p_T^\gamma = x_{j\gamma} > 0.8$$

CMS-PAS-HIN-23-001

$$p_T^{jet} / p_T^\gamma = x_{j\gamma} > 0.4$$



**p-p baseline not under control!**

Thank you!