Partonic structure and small x: TMD PDFs

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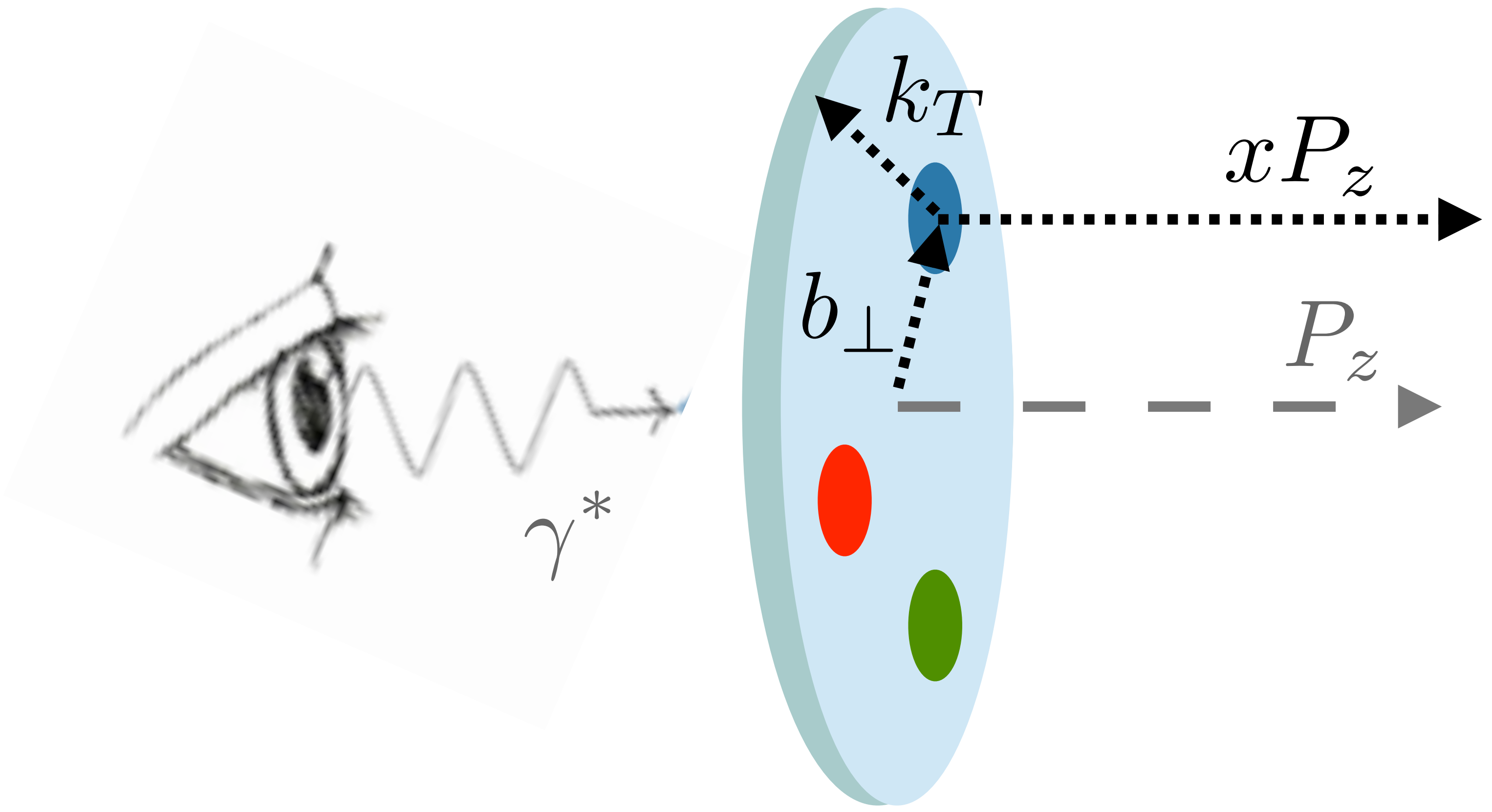
GDR QCD "From Hadronic Structure to Heavy Ion Collisions”
10-14 June 2024
IJCLab, Orsay, France

The various dimensions of the nucleon structure

Wigner distributions $W(x, \vec{k}_T, \vec{b}_\perp)$



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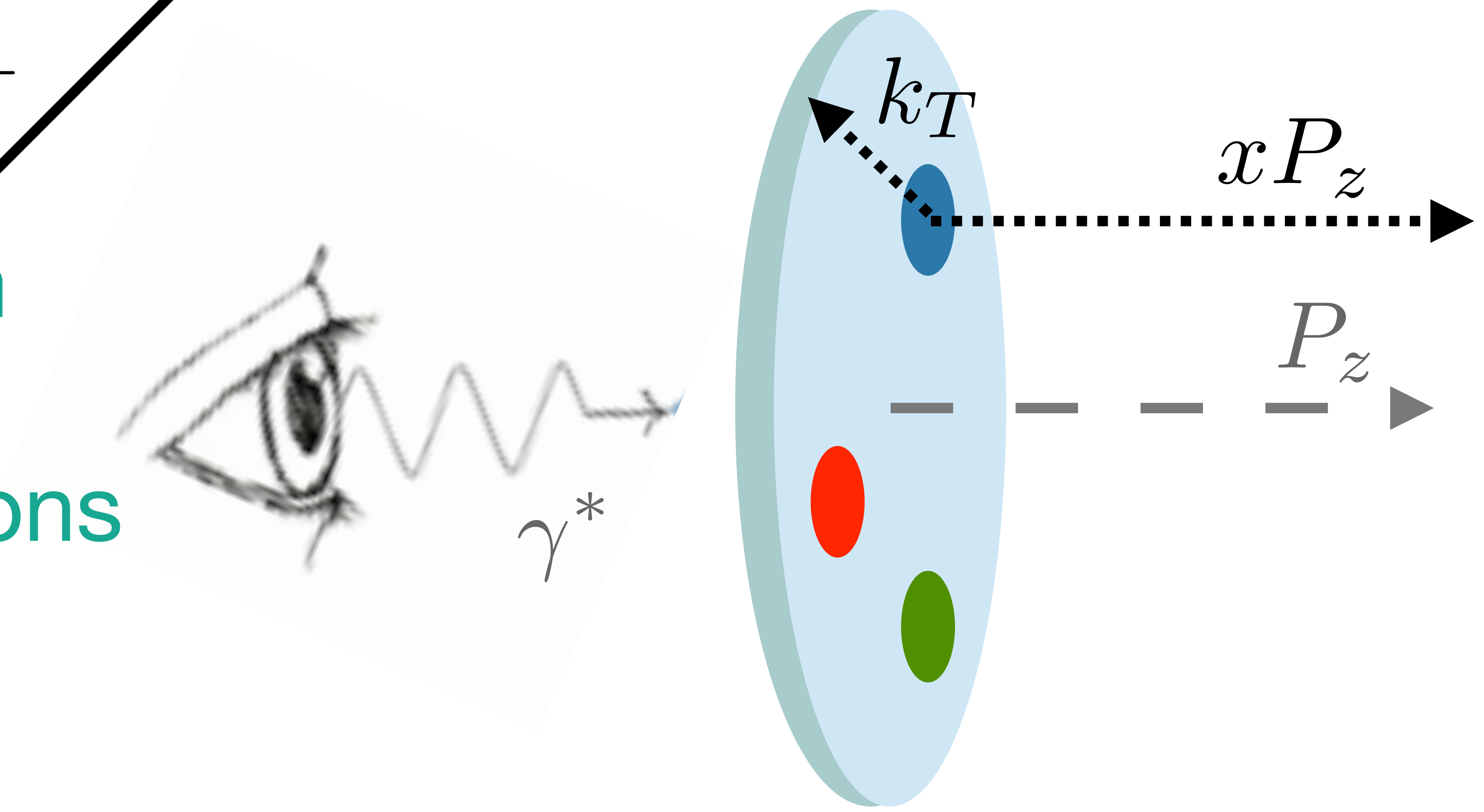
$\int d^2\vec{b}_\perp$

transverse-momentum dependent (TMD) parton distribution functions (PDFs)



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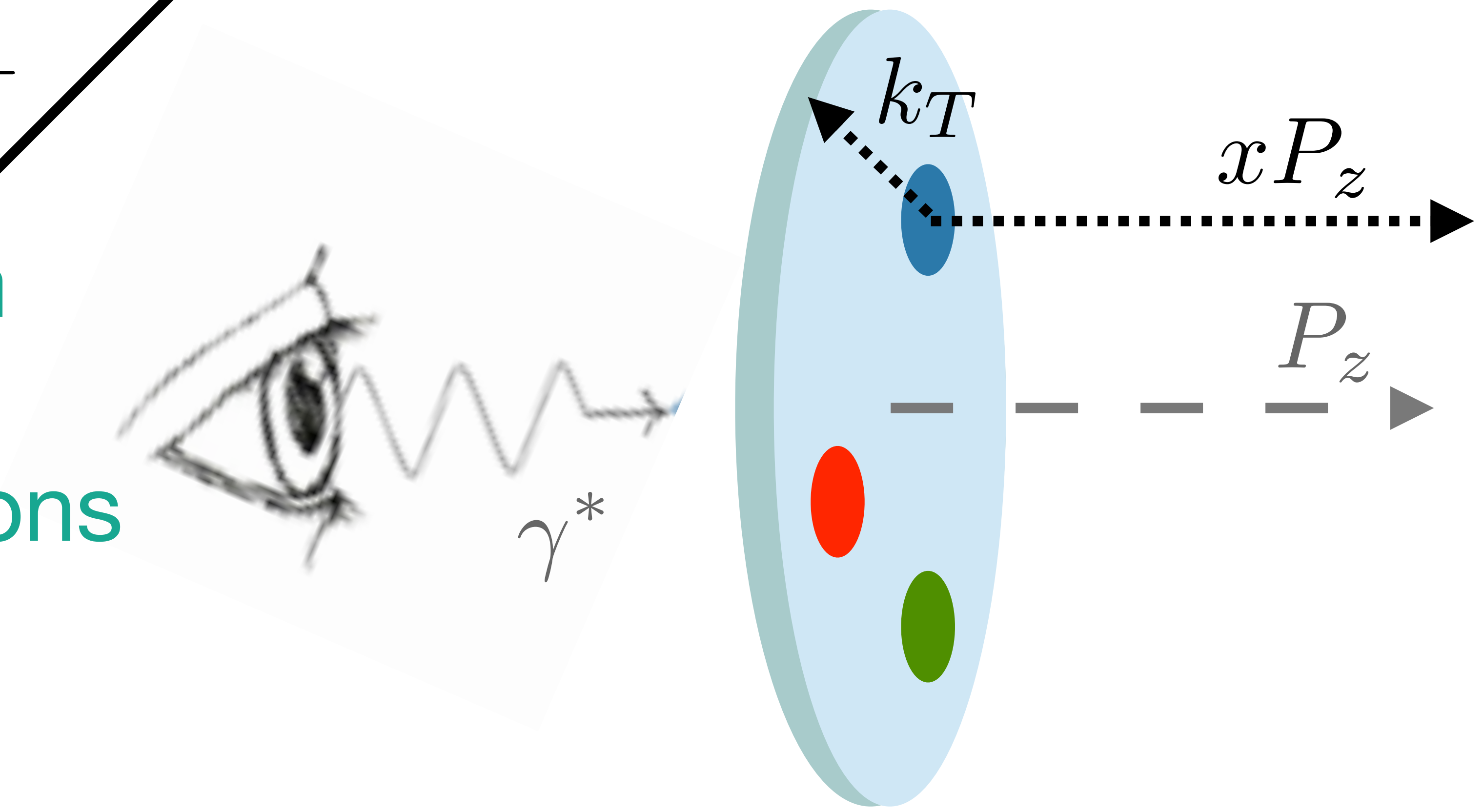
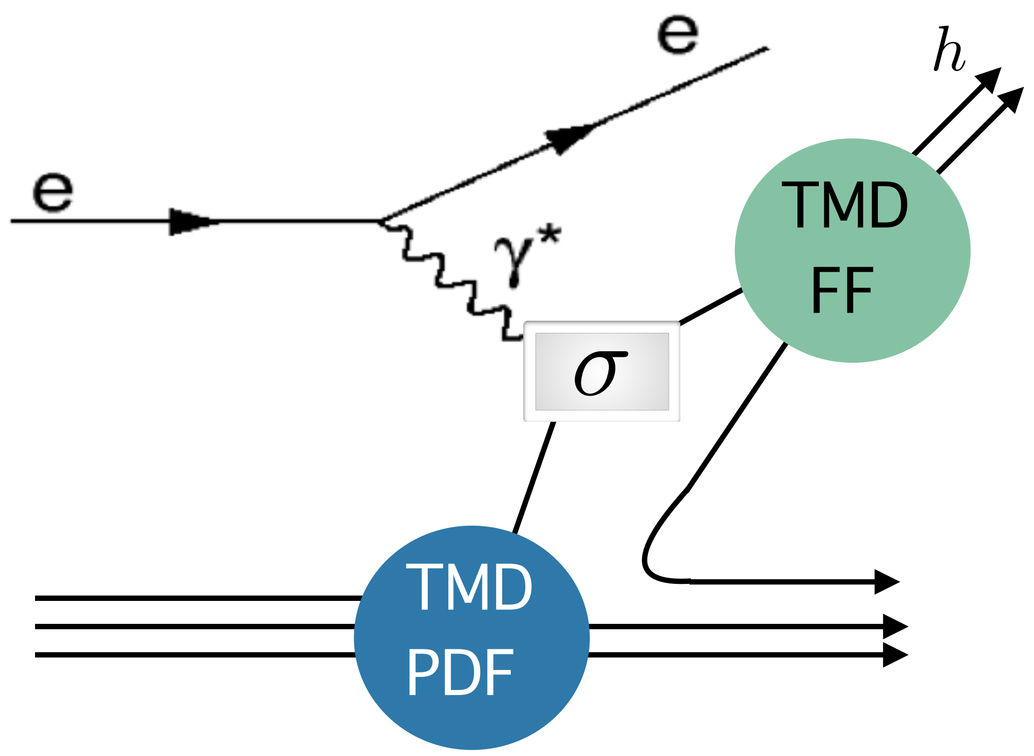


Semi-inclusive production



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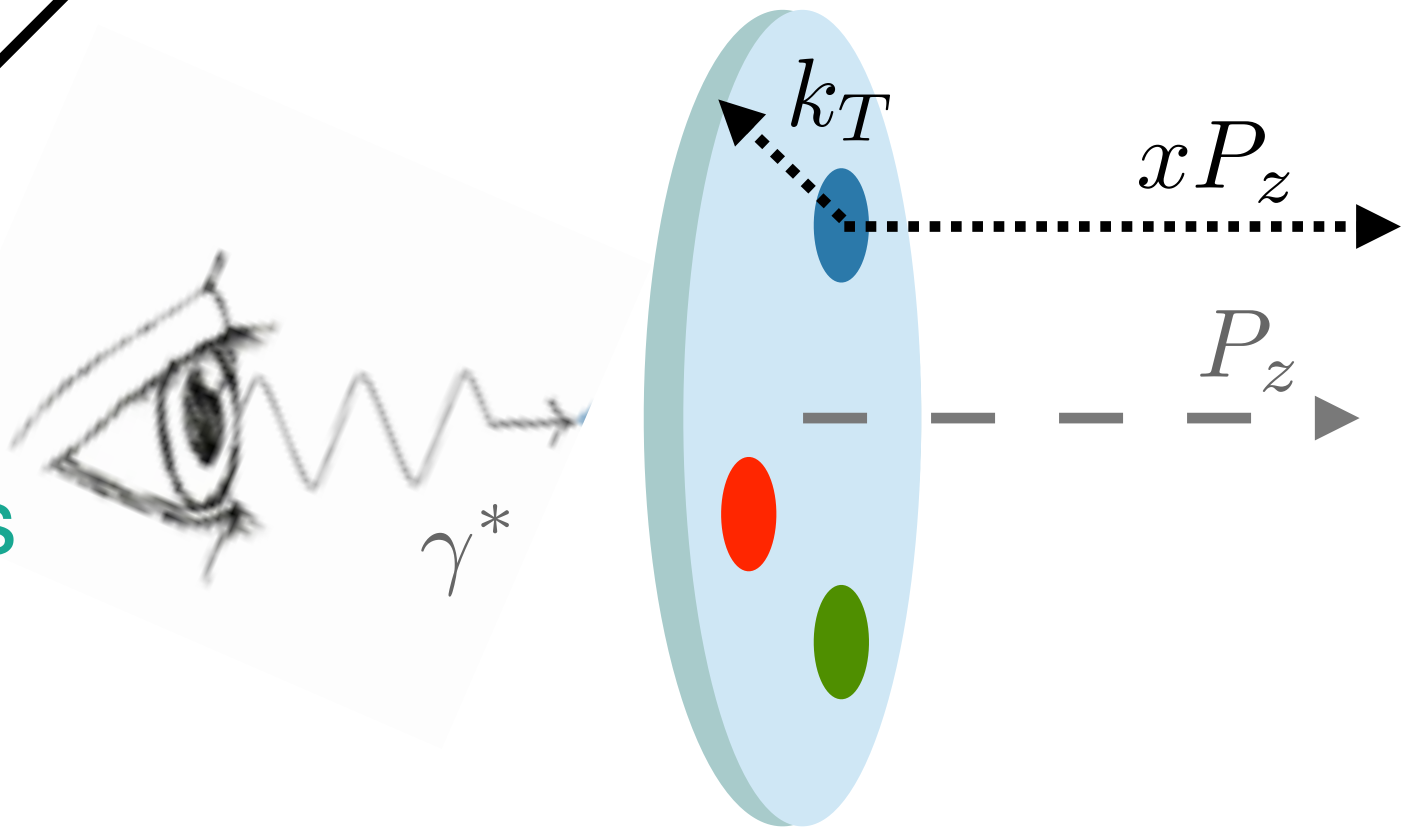

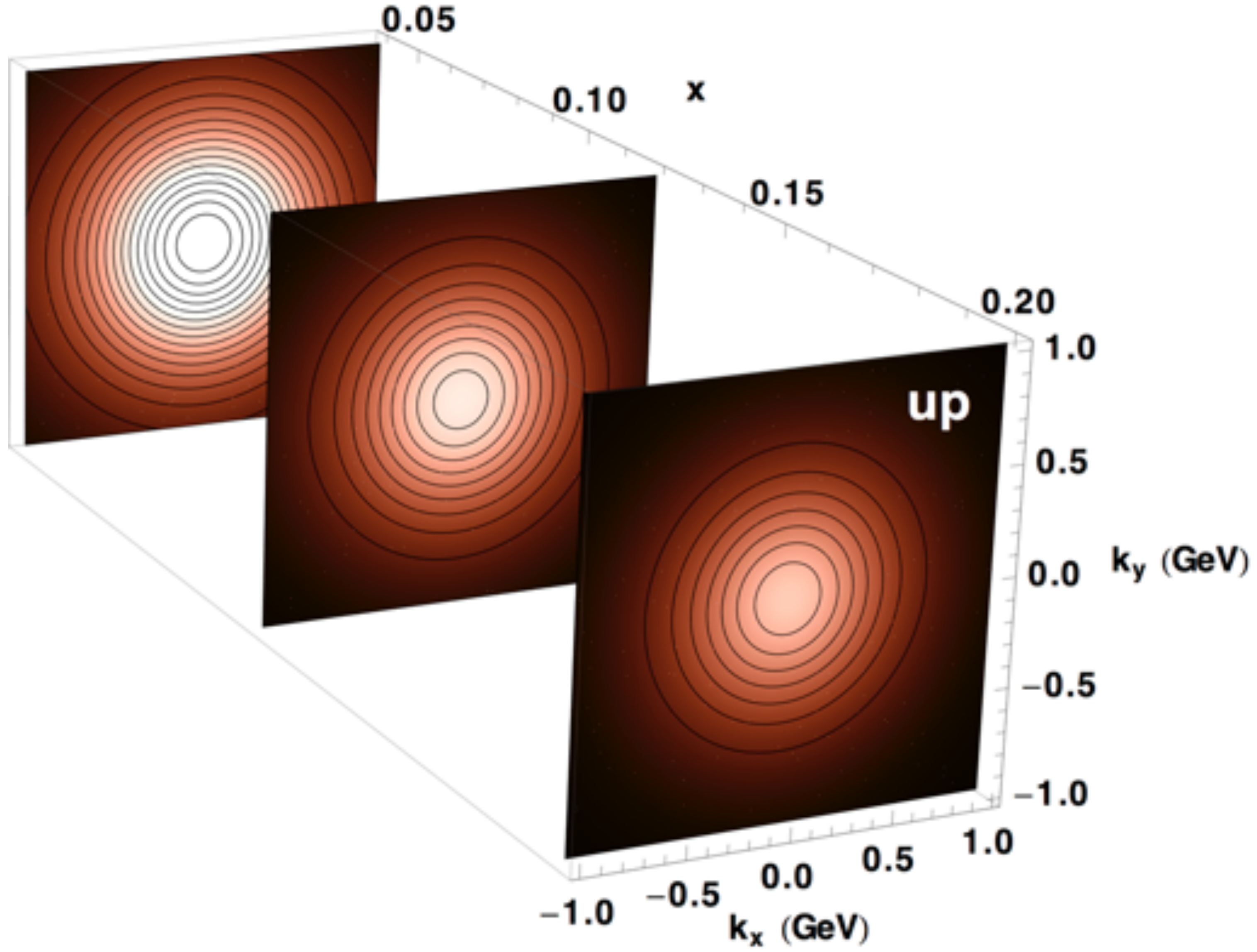

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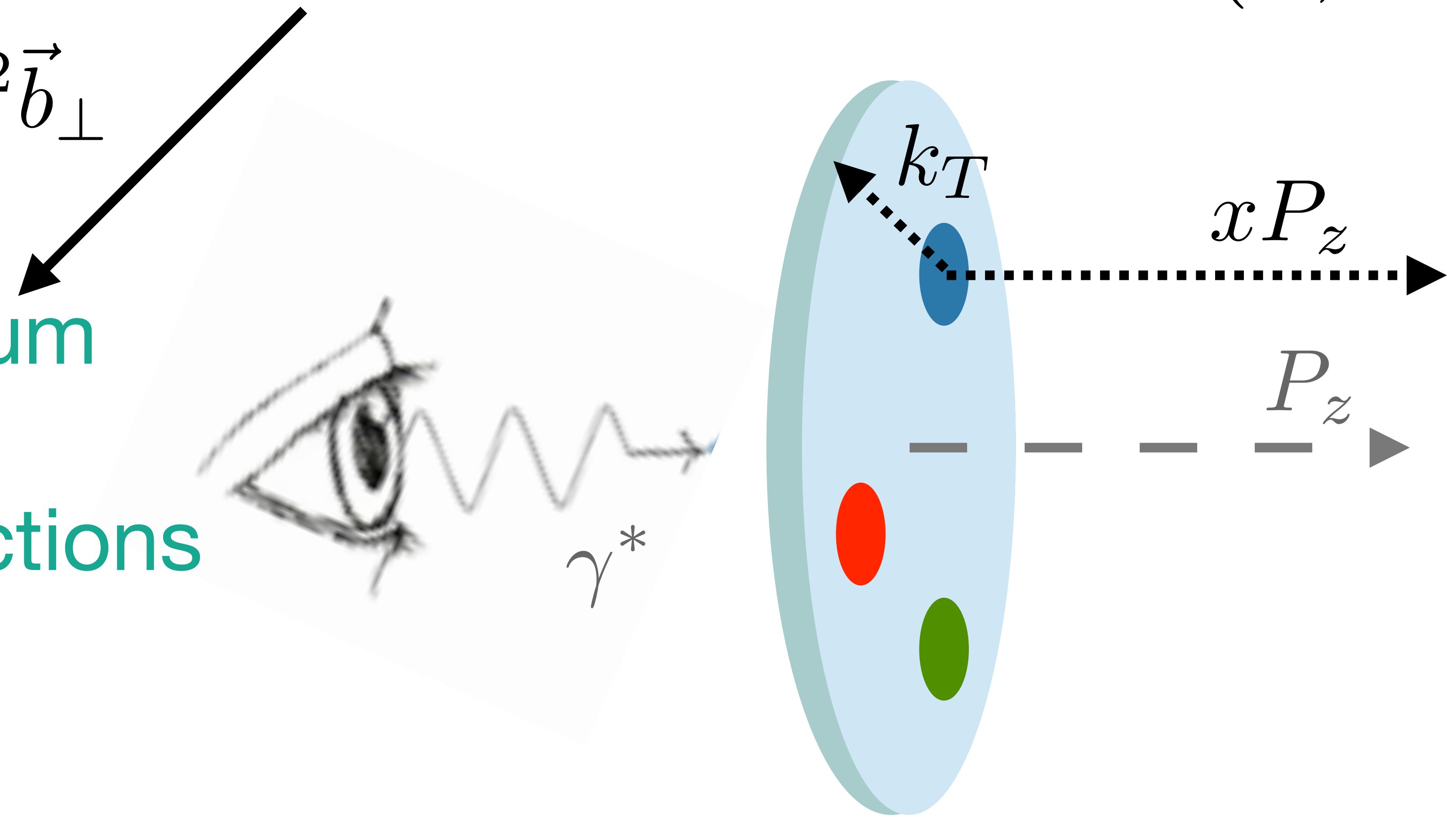

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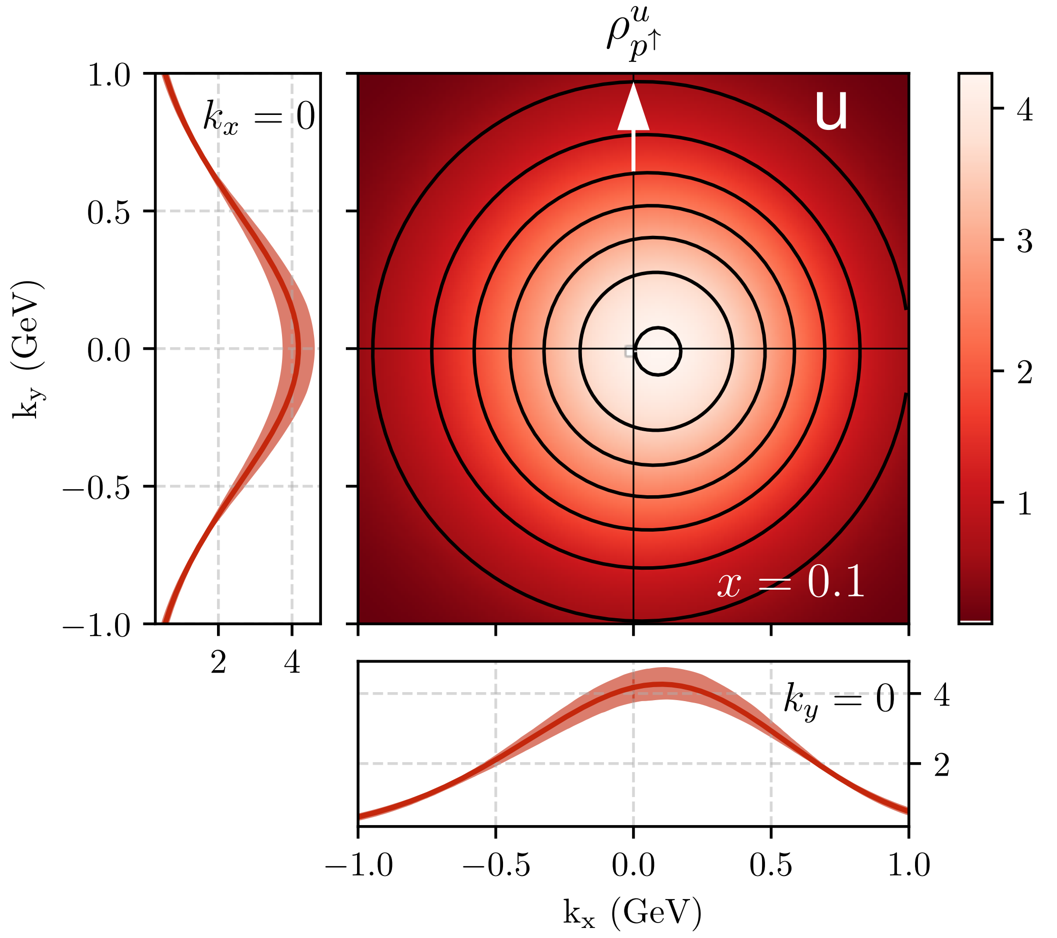

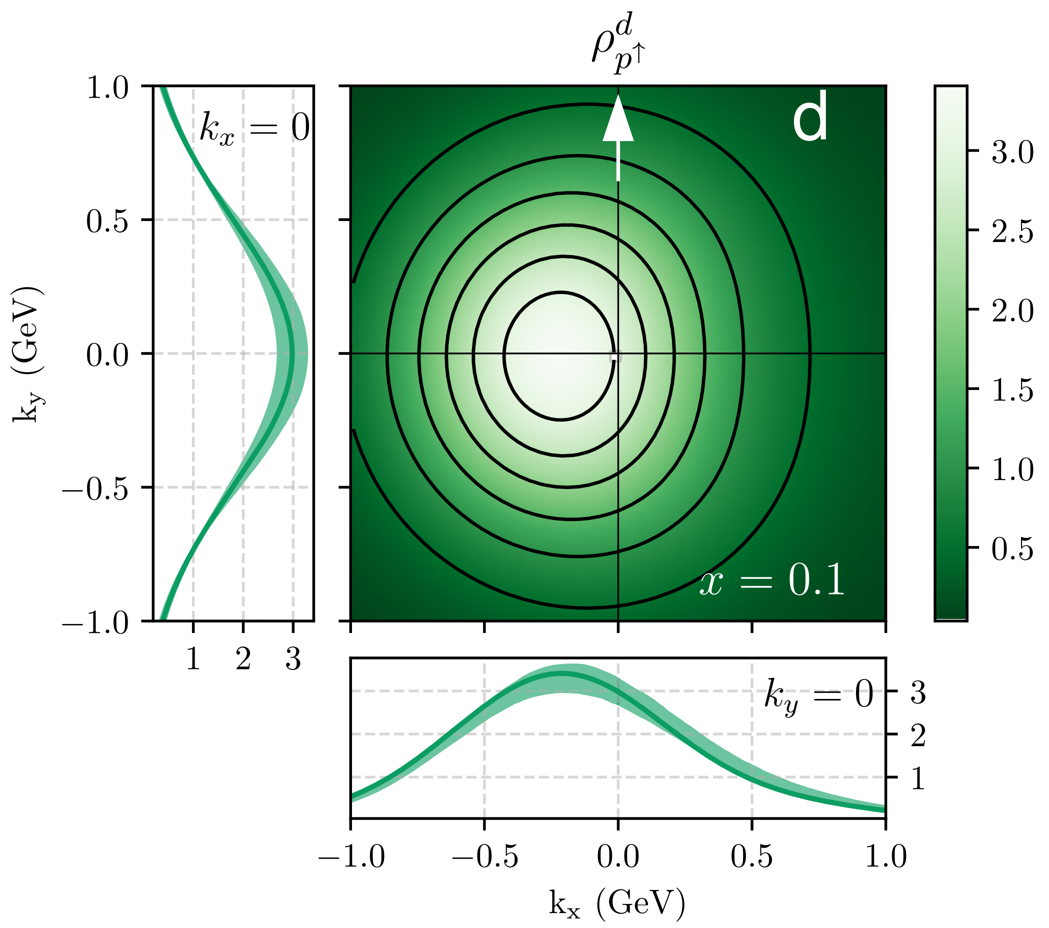

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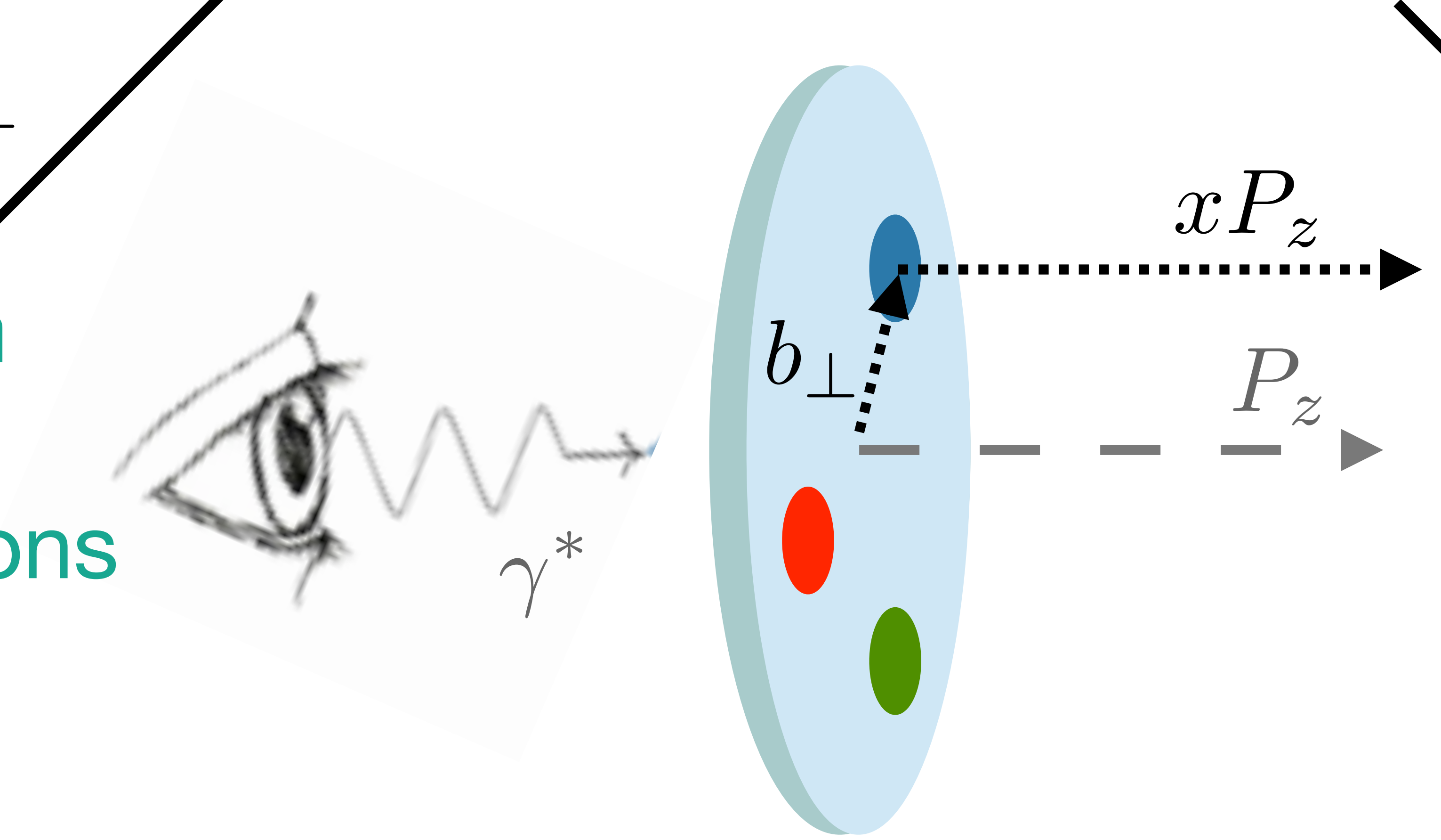


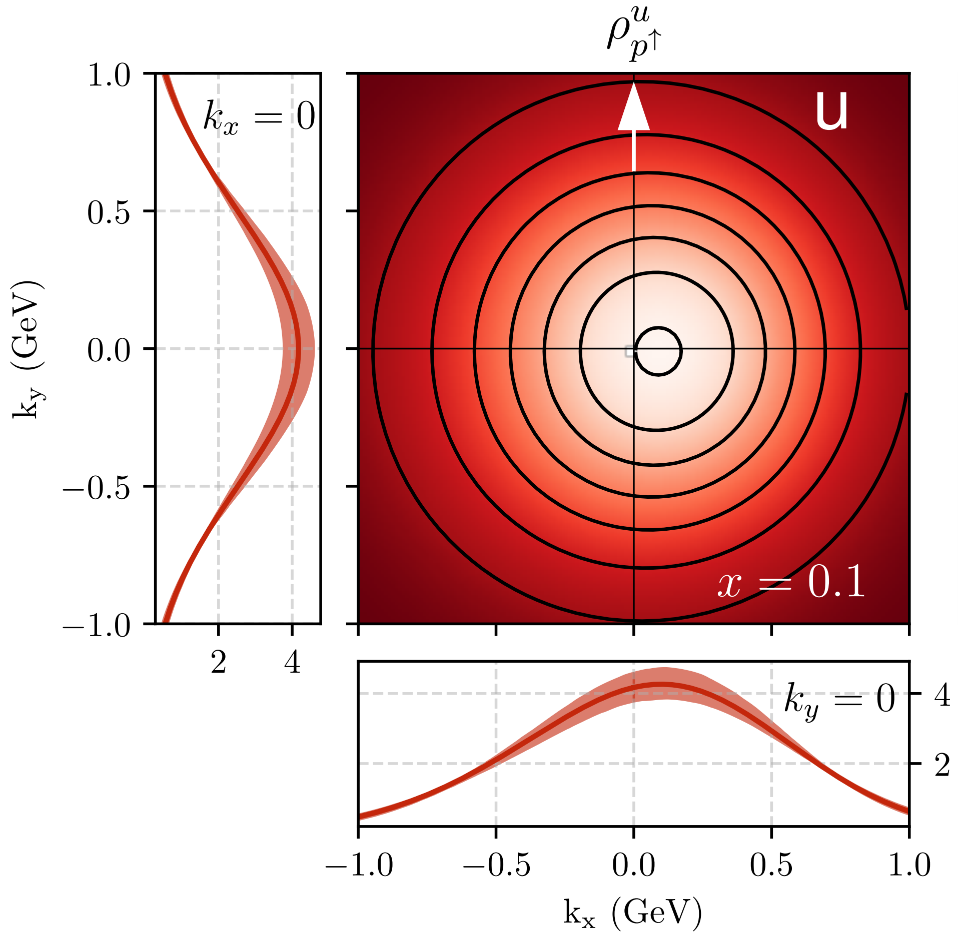
Wigner distributions $W(x, \vec{k}_T, \vec{b}_\perp)$

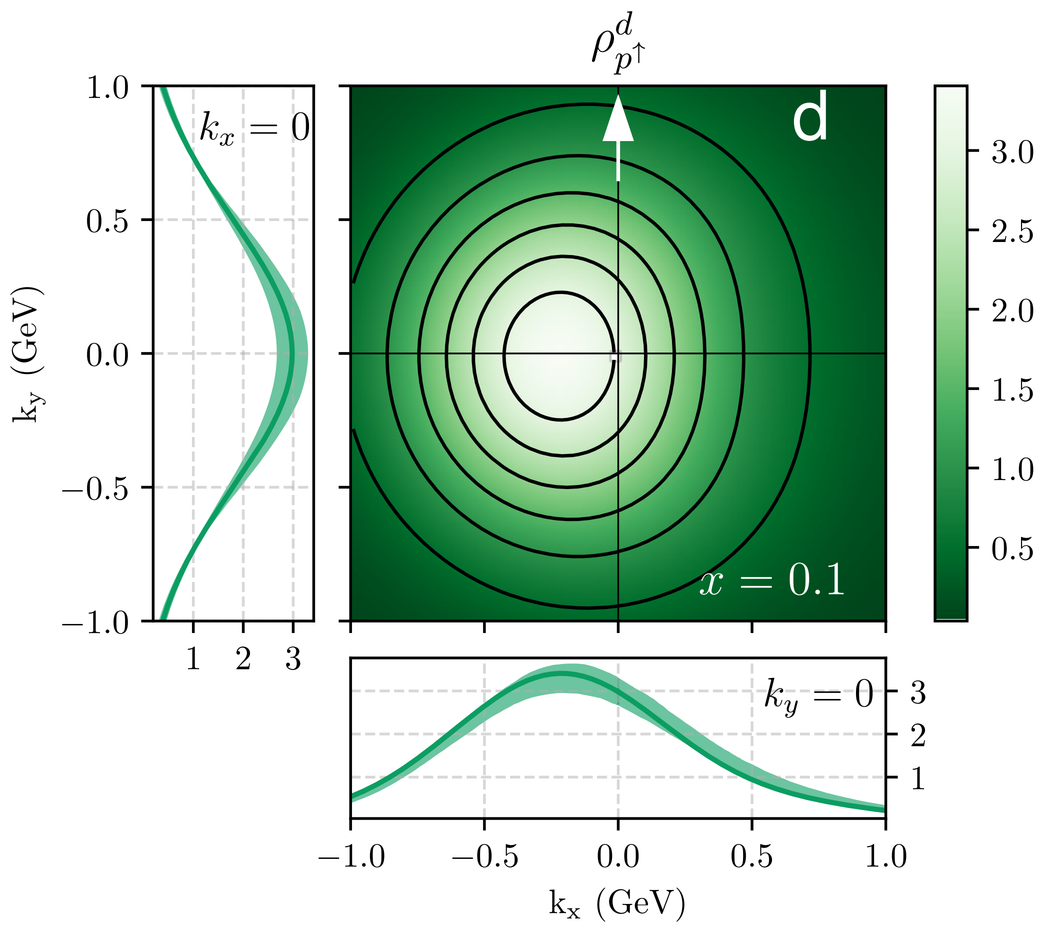
$\int d^2\vec{b}_\perp$

$\int d^2\vec{k}_T$

transverse-momentum dependent (TMD) parton distribution functions (PDFs)

impact-parameter dependent PDFs





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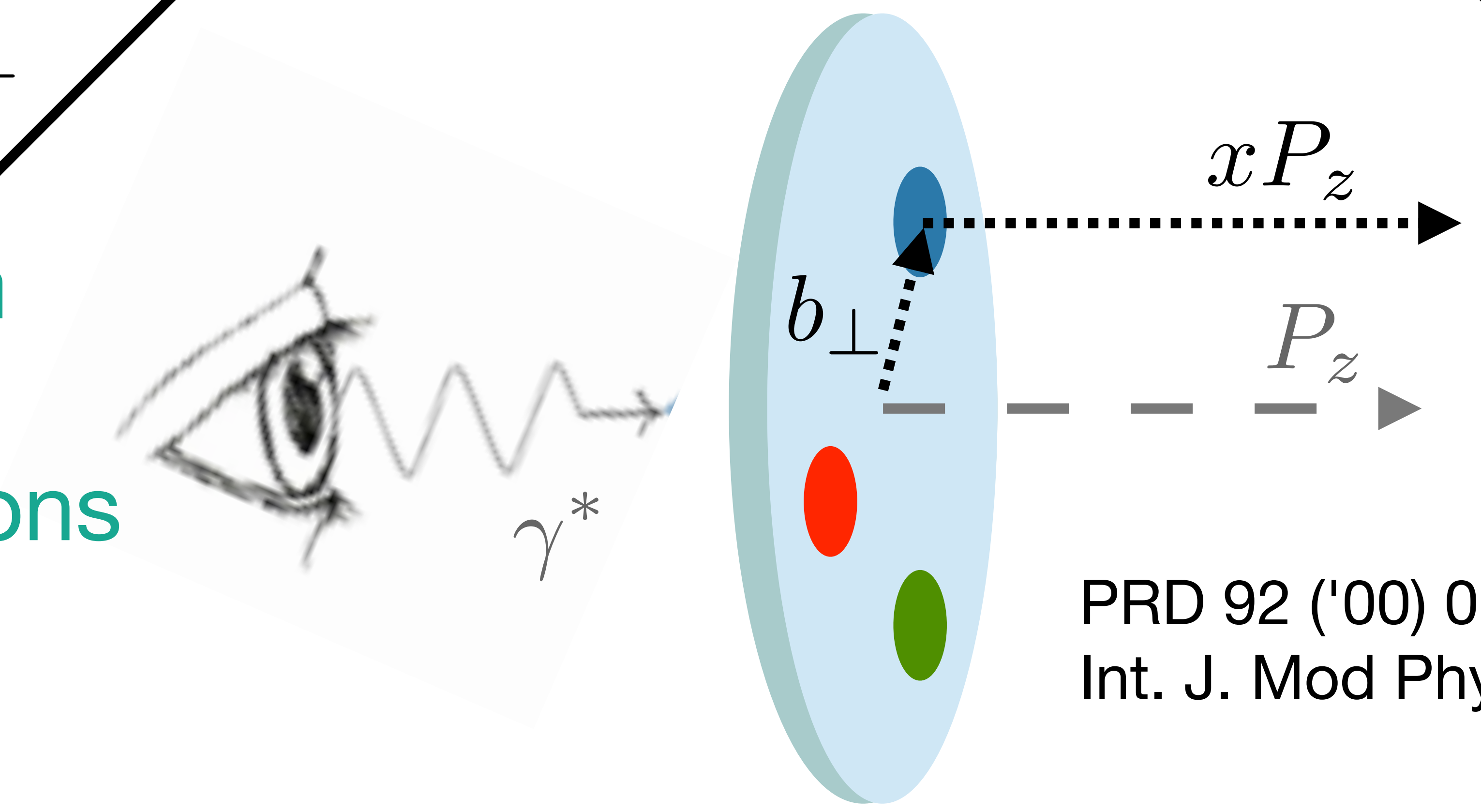

$\int d^2\vec{k}_T$

impact-parameter dependent PDFs

Fourier transform

generalised parton distributions (GPDs)

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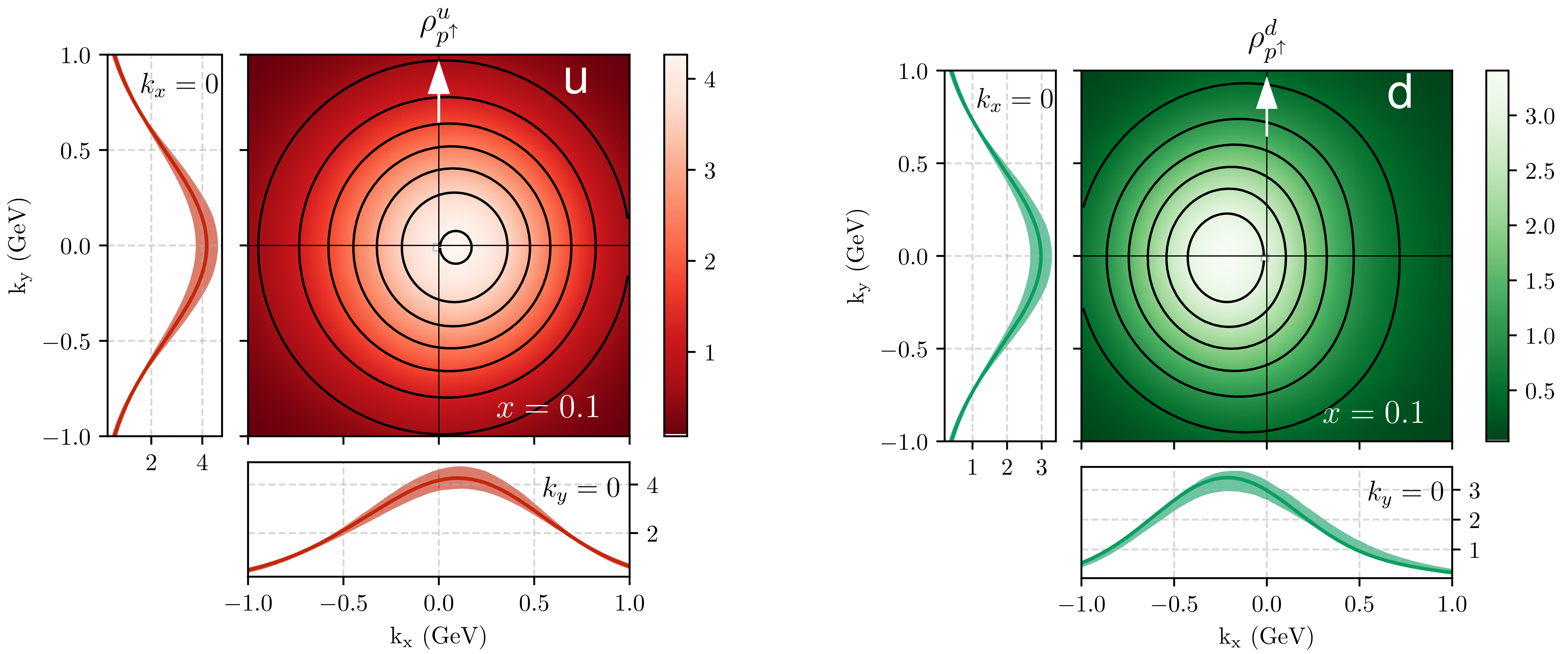

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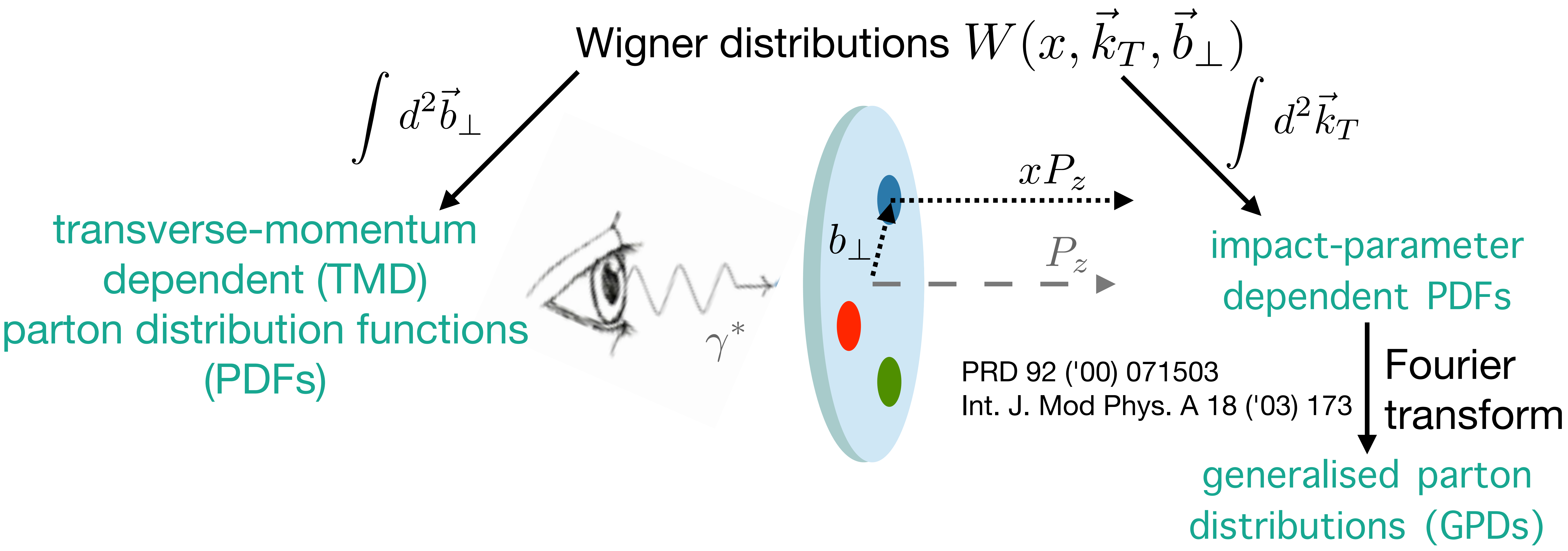

Wigner distributions $W(x, \vec{k}_T, \vec{b}_\perp)$

$\int d^2\vec{b}_\perp$

transverse-momentum dependent (TMD) parton distribution functions (PDFs)

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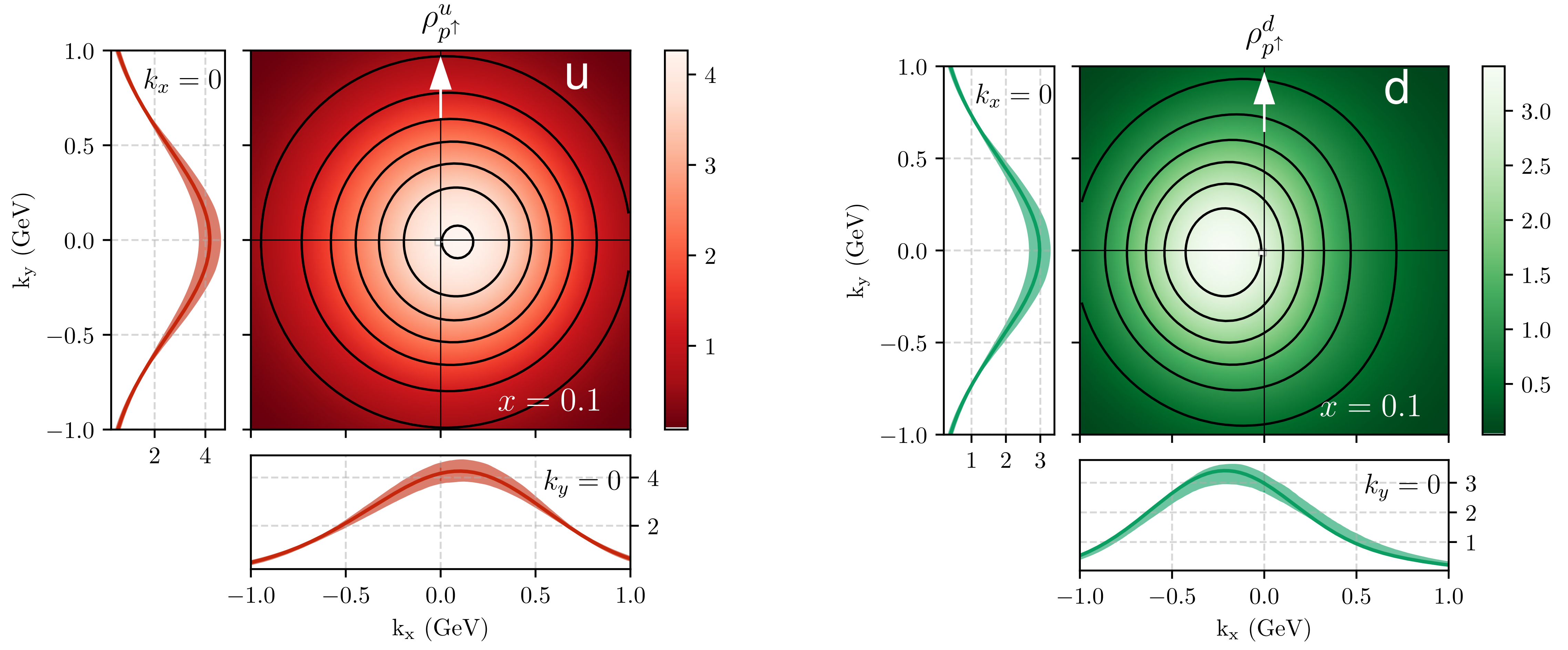
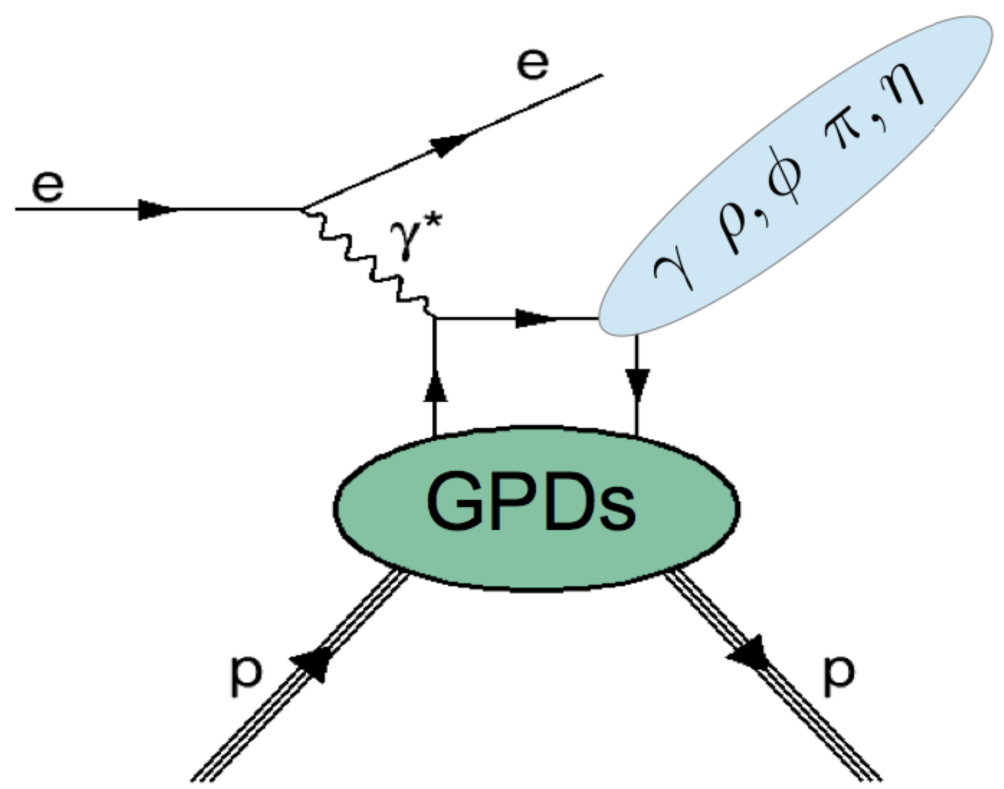


Exclusive production



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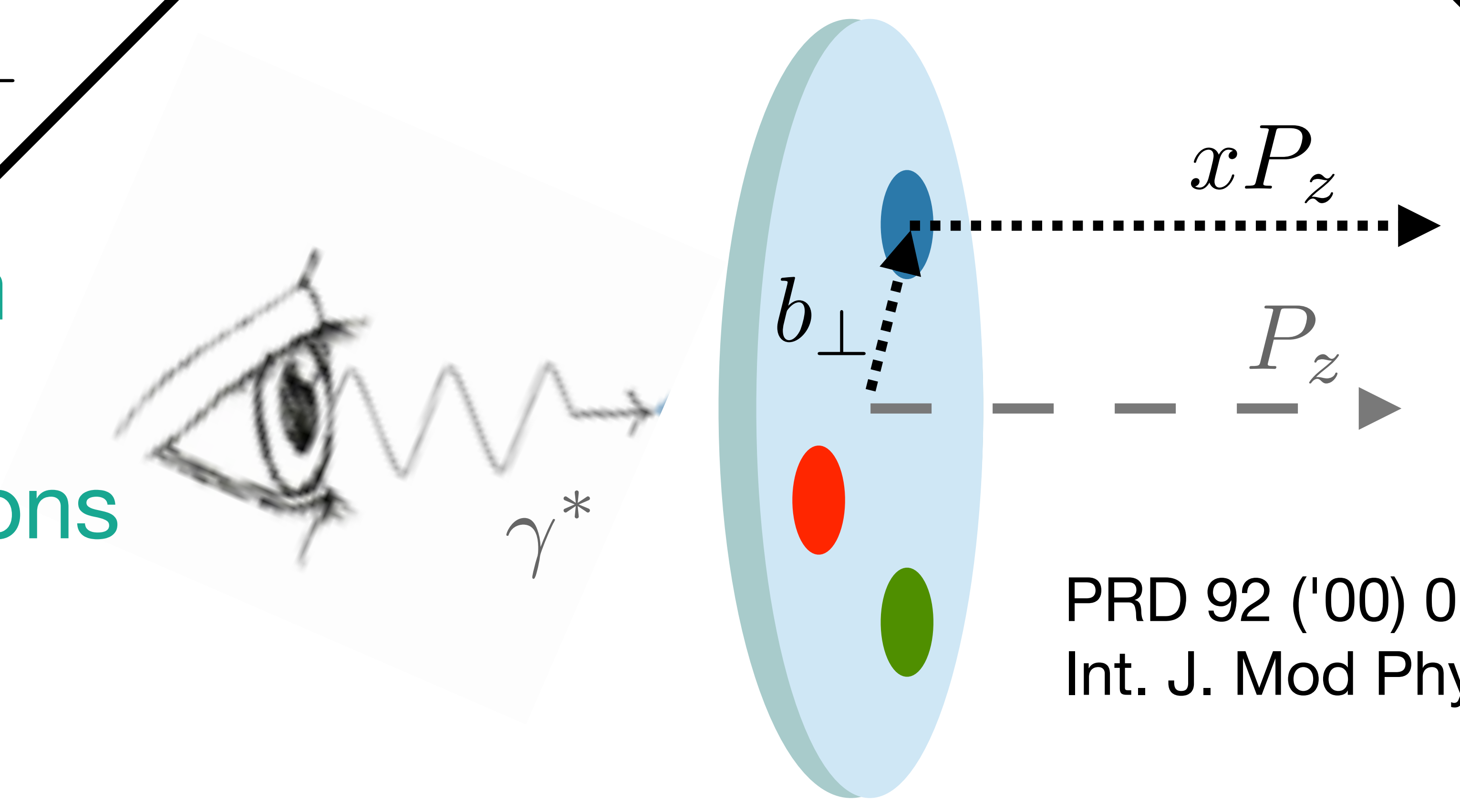

impact-parameter dependent PDFs

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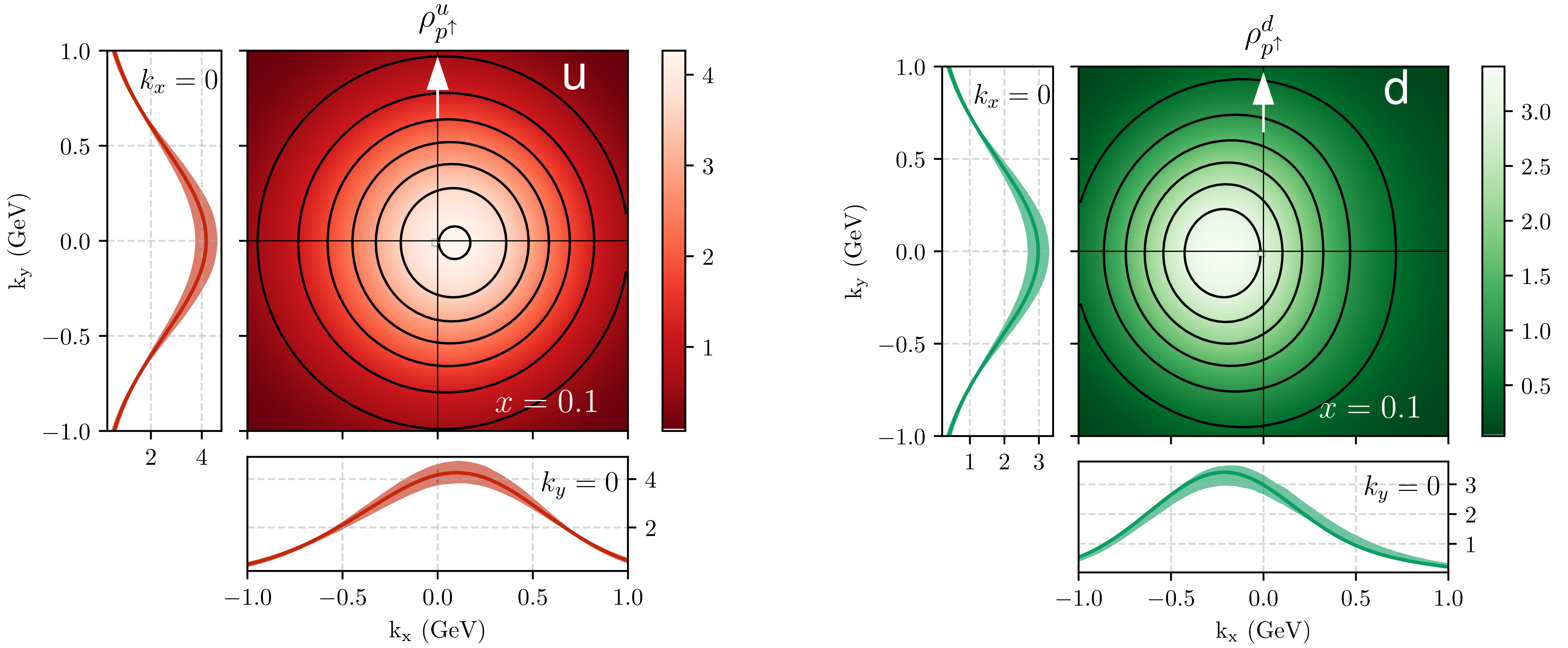

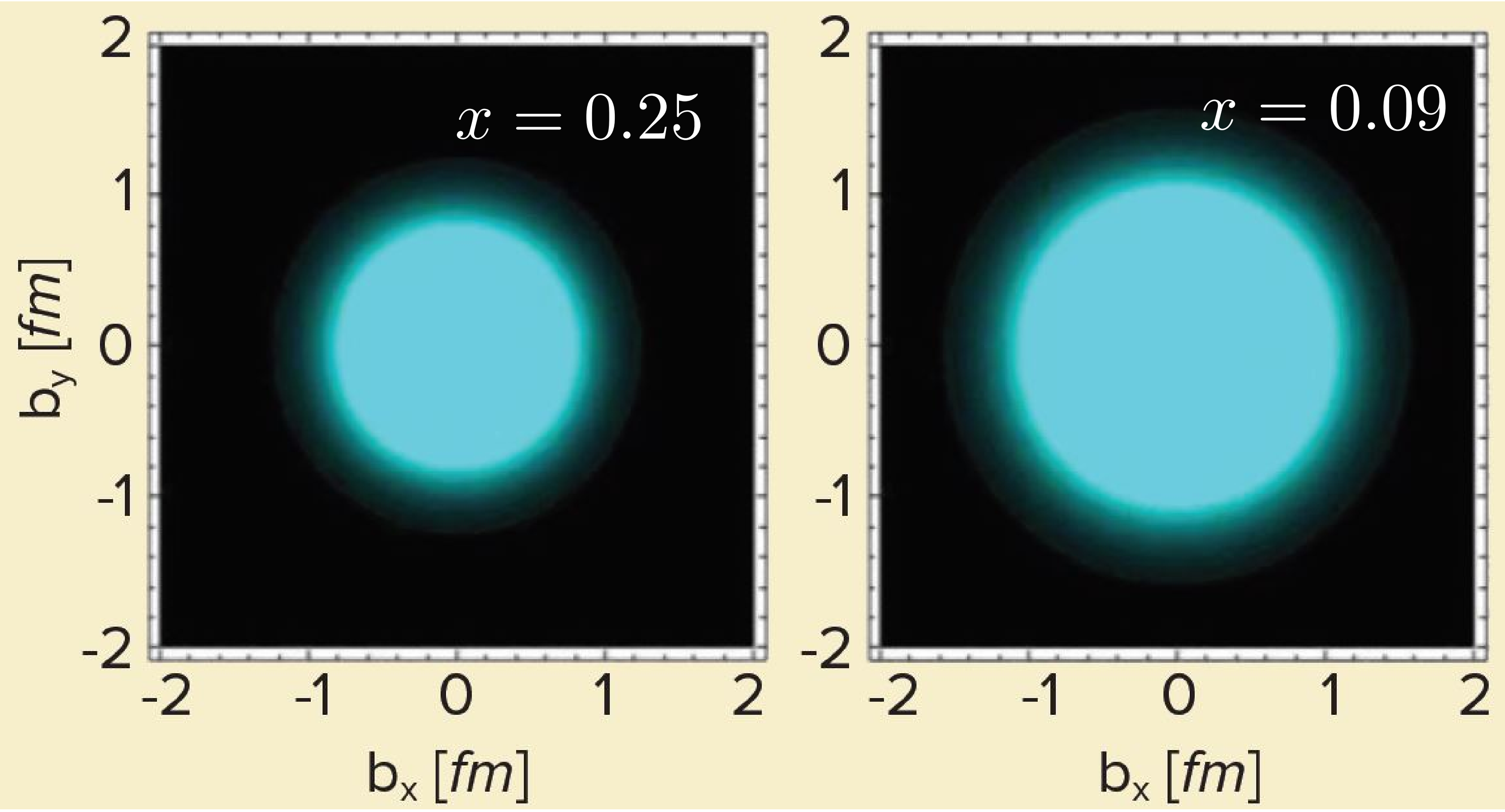

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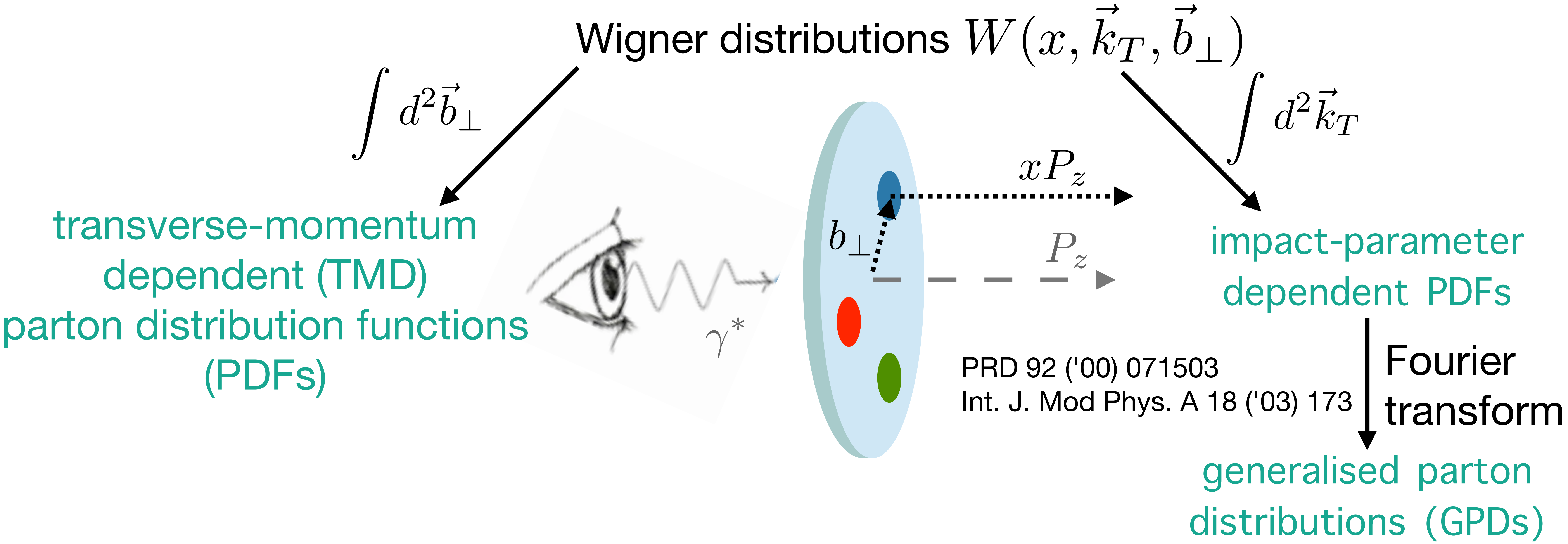

Wigner distributions $W(x, \vec{k}_T, \vec{b}_\perp)$

$\int d^2\vec{b}_\perp$

$\int d^2\vec{k}_T$

transverse-momentum dependent (TMD) parton distribution functions (PDFs)

impact-parameter dependent PDFs

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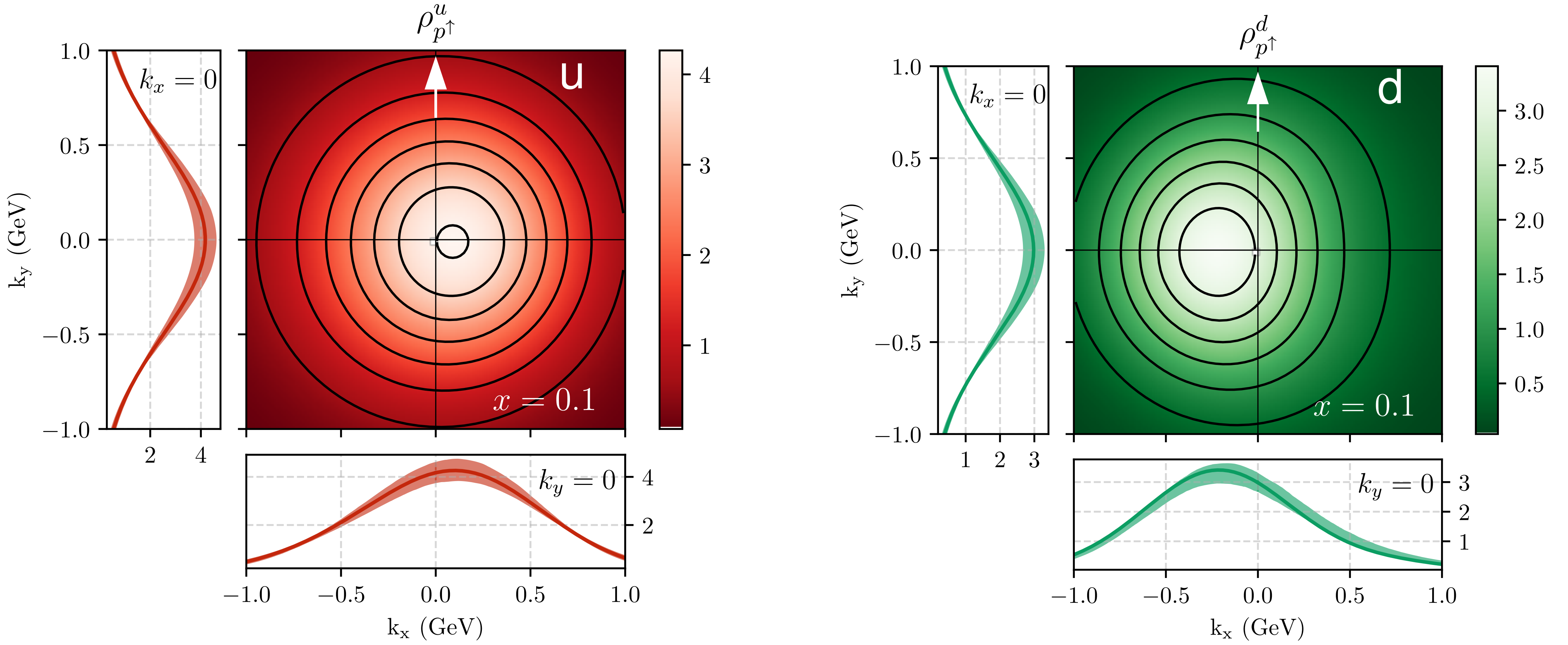

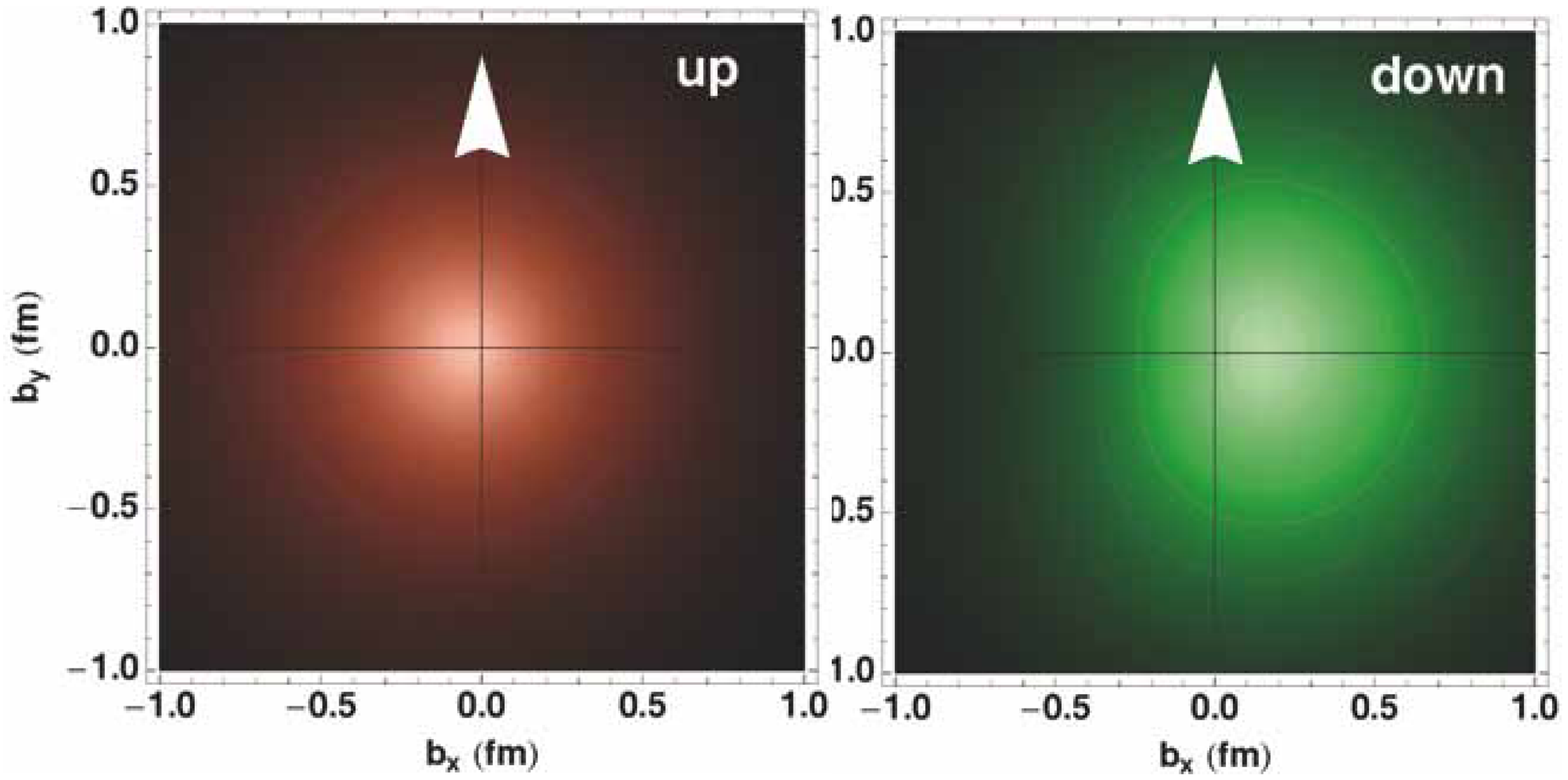

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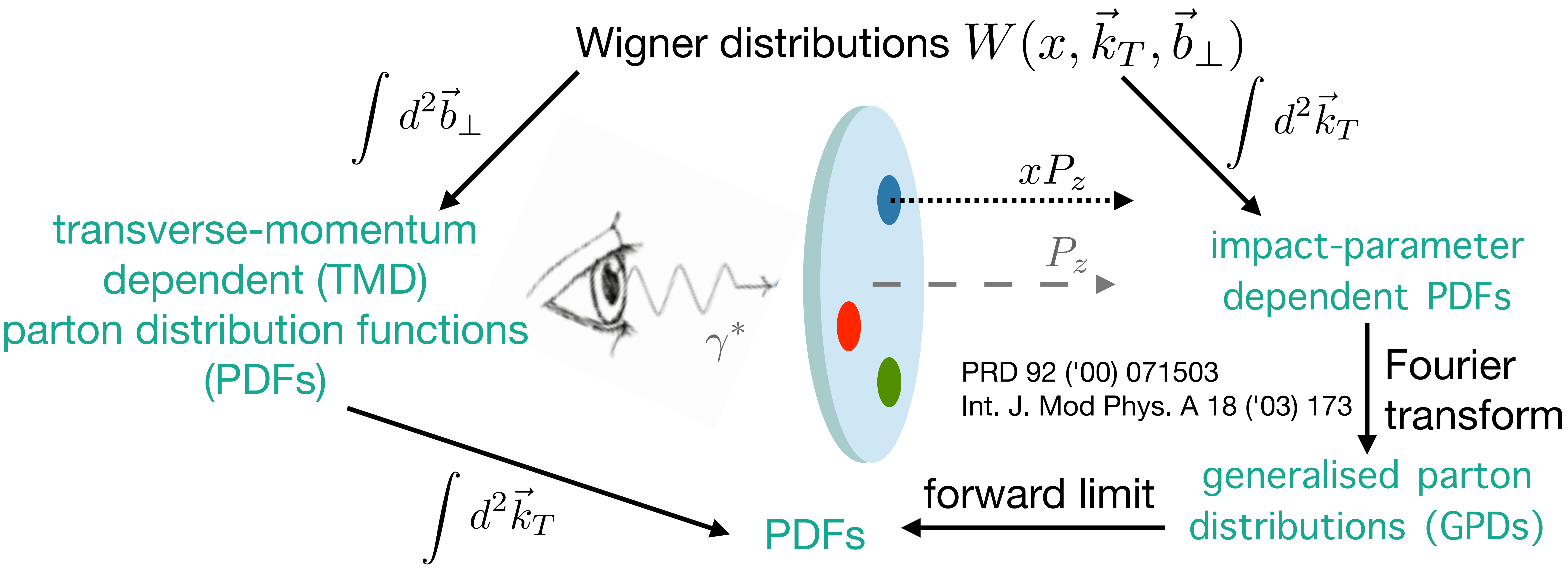

Wigner distributions $W(x, \vec{k}_T, \vec{b}_\perp)$

$\int d^2\vec{b}_\perp$

transverse-momentum dependent (TMD) parton distribution functions (PDFs)

$\int d^2\vec{k}_T$

impact-parameter dependent PDFs

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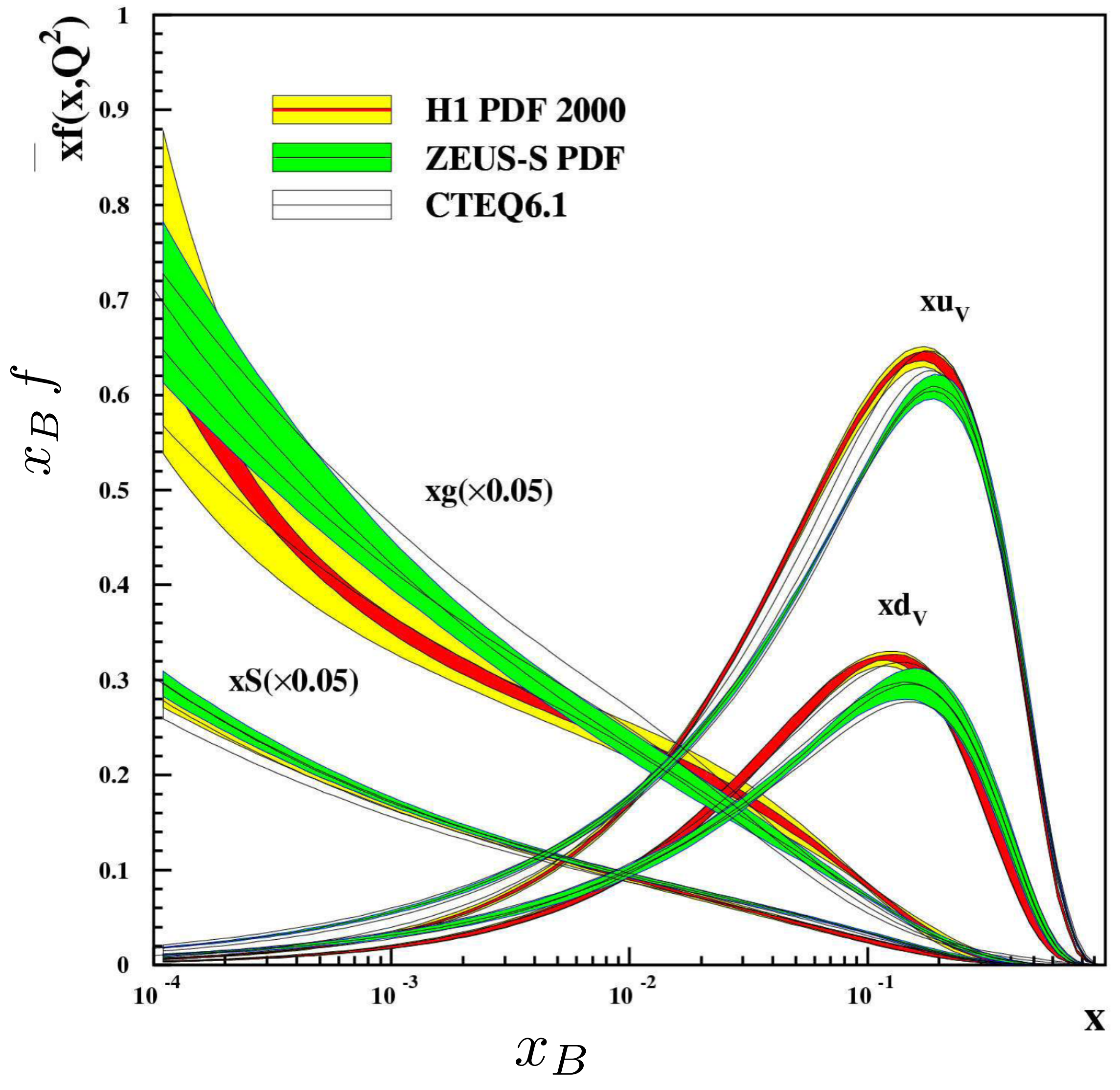
Fourier transform

generalised parton distributions (GPDs)

forward limit

$\int d^2\vec{k}_T$

PDFs



Transverse momentum dependent parton distribution functions

quark polarisation

nucleon polarisation

	U	L	T
U	f_1		
L		g_{1L}	
T			h_{1T}

survive integration of parton transverse momentum

Transverse momentum dependent parton distribution functions

quark polarisation

nucleon polarisation

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_{1T}\,h_{1T}^\perp$

Transverse momentum dependent parton distribution functions

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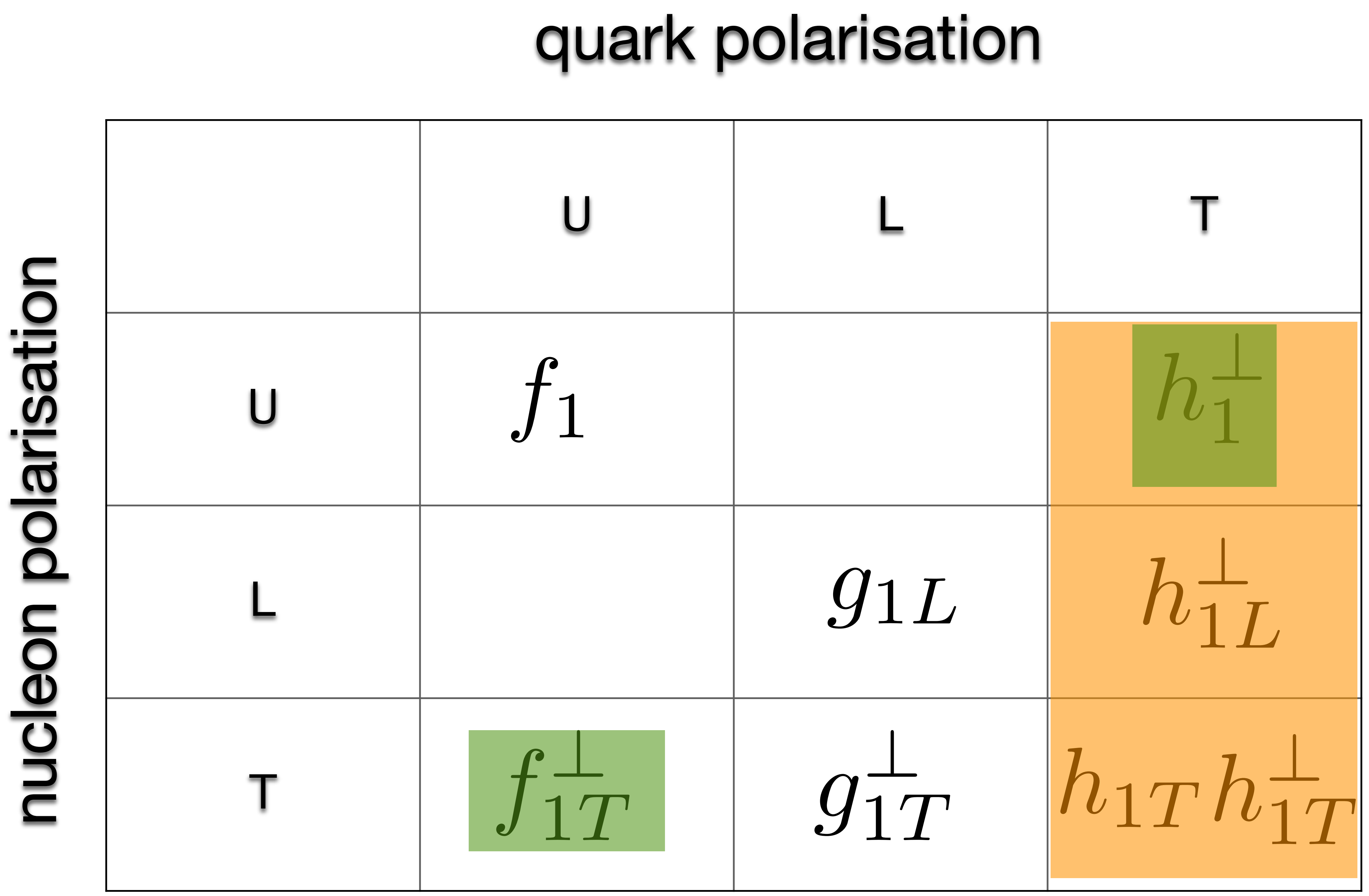
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_{1T}\, h_{1T}^\perp$

Chiral odd

Transverse momentum dependent parton distribution functions



quark polarisation

nucleon polarisation

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_{1T} $h_{1T}^\perp$

Chiral odd

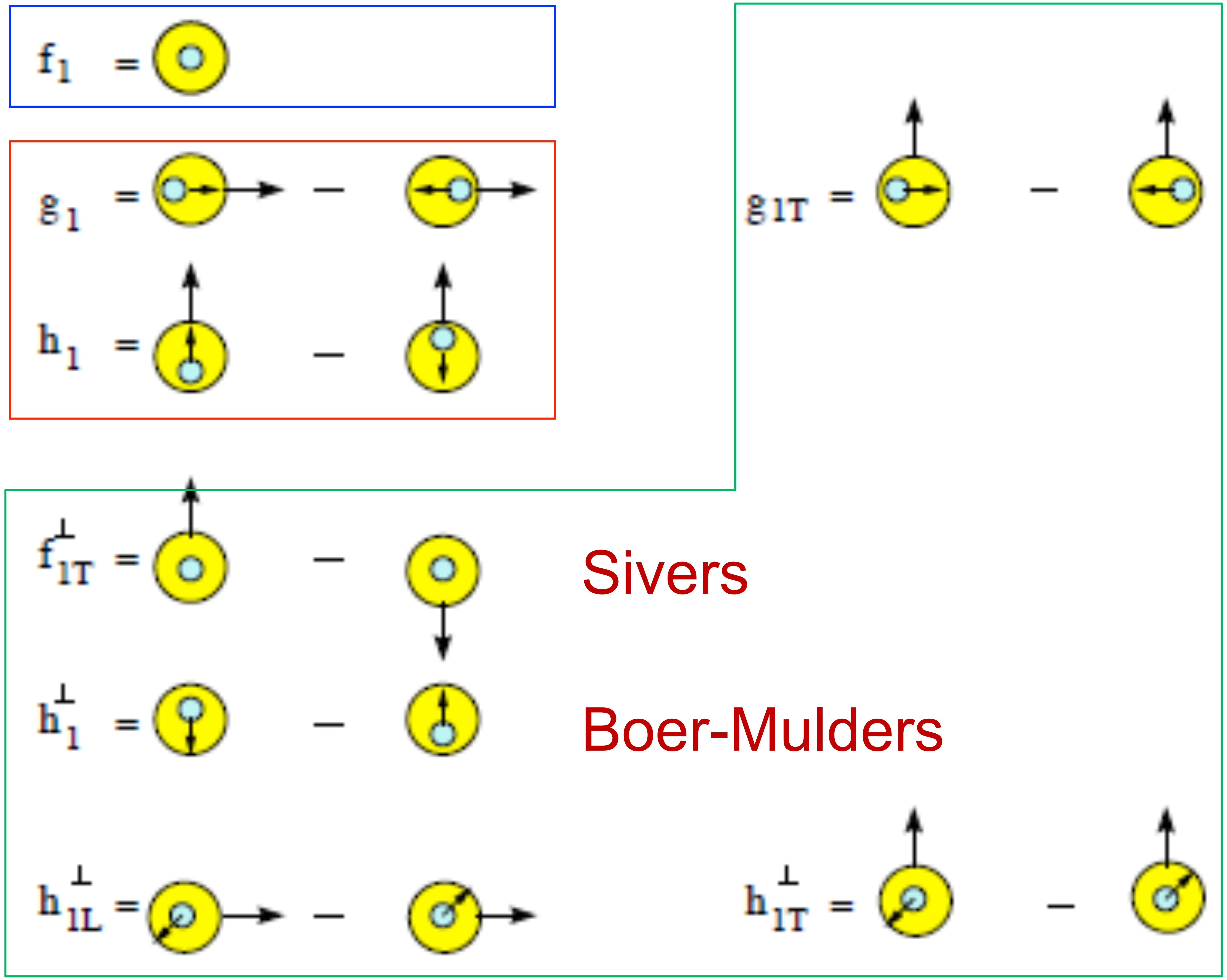
Naive T-odd

Transverse momentum dependent parton distribution functions

Unpolarized

Spin-spin correlations

Spin-momentum correlations



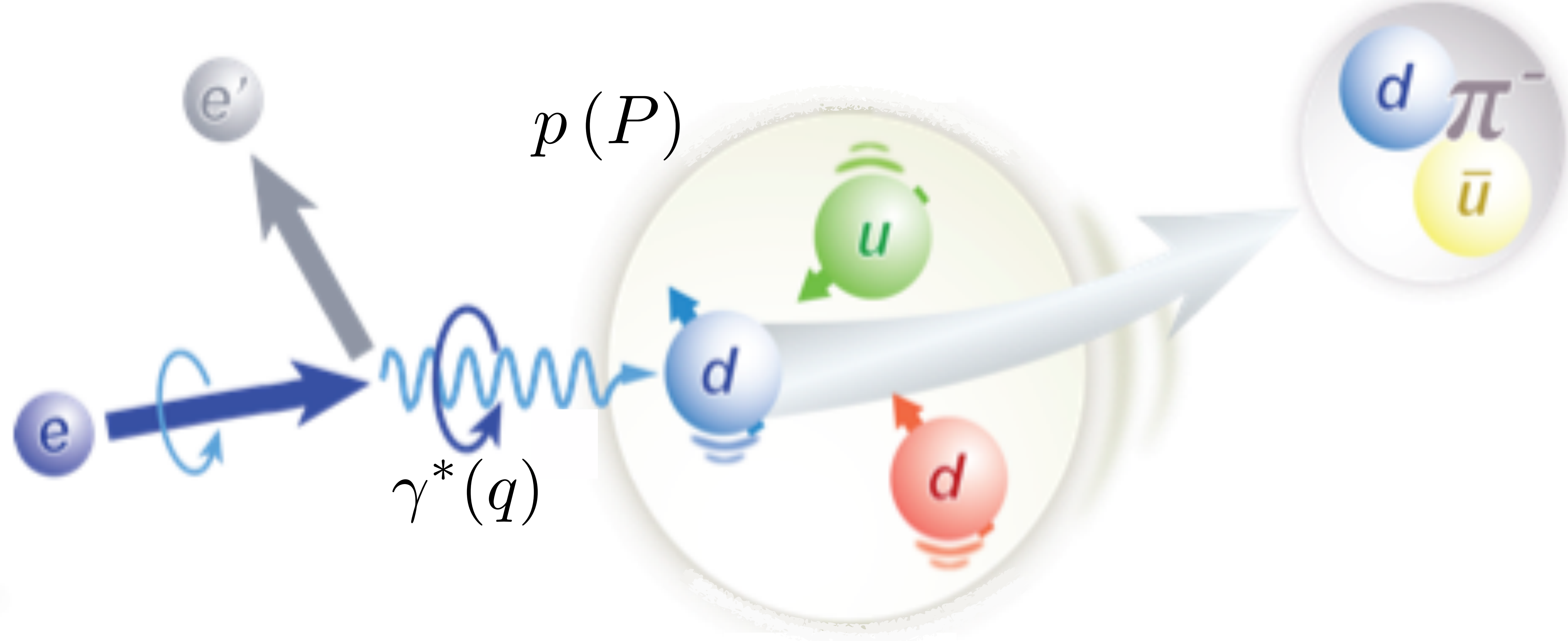
Sivers

Boer-Mulders

Single-hadron production in semi-inclusive DIS

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2P \cdot q}$$



Highly virtual photon:
$Q^2 \gg 1$ GeV^2
provides hard
scale of process

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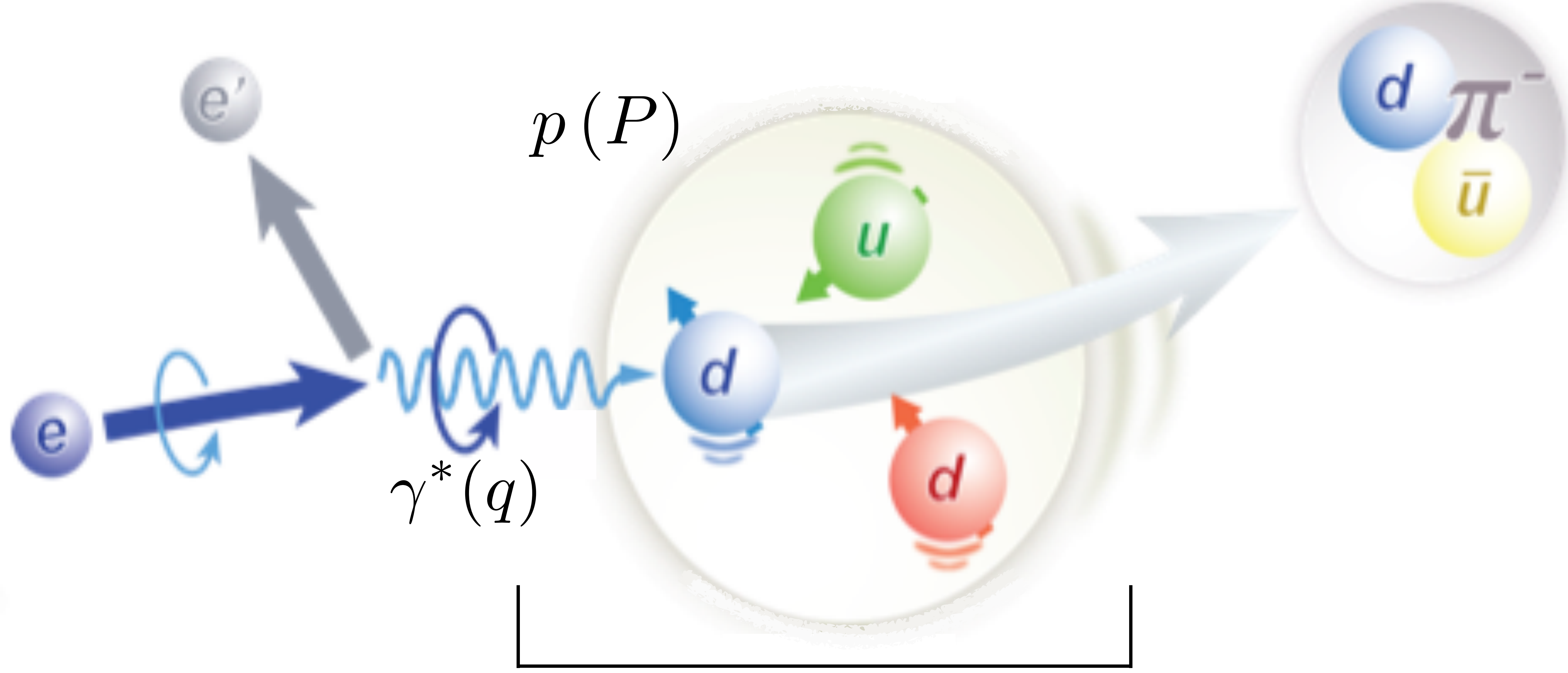

parton distribution function $PDF(x_B)$

Highly virtual photon:
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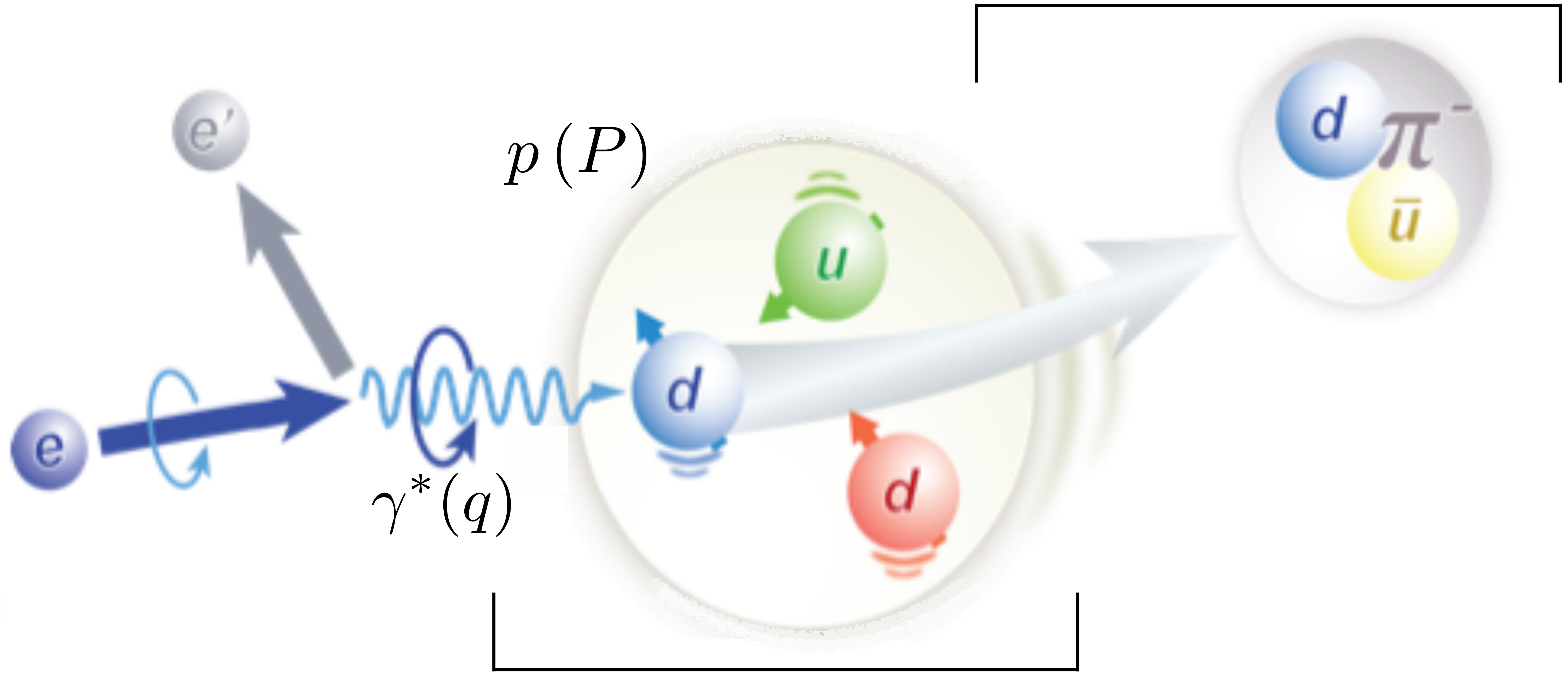
Single-hadron production in semi-inclusive DIS

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2P \cdot q}$$

$$z \overset{\text{lab}}{=} \frac{E_h}{E_{\gamma*}}$$

fragmentation function $FF(z)$



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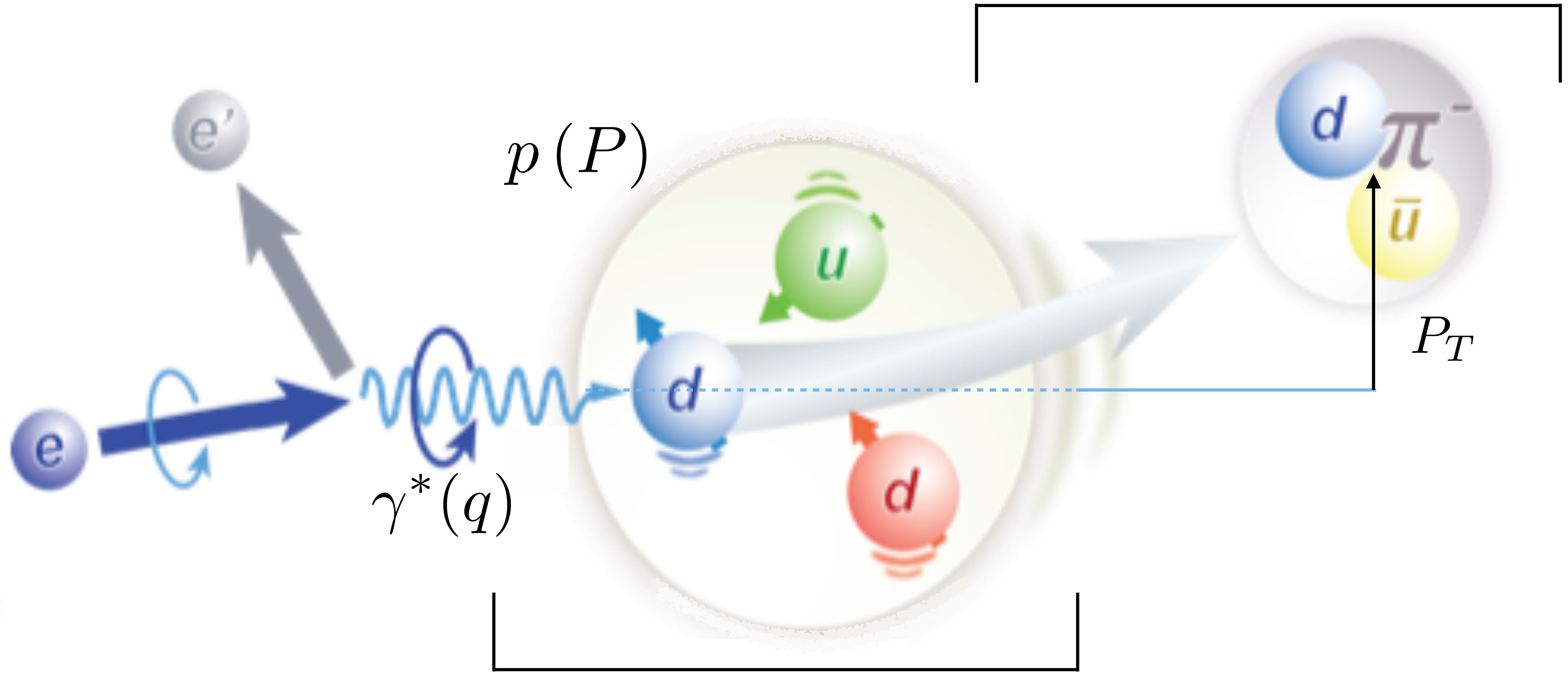
Transverse-momentum-dependent (TMD)
fragmentation function $FF(z, p_\perp)$



Transverse-momentum-dependent (TMD)
parton distribution function $PDF(x_B, k_\perp)$

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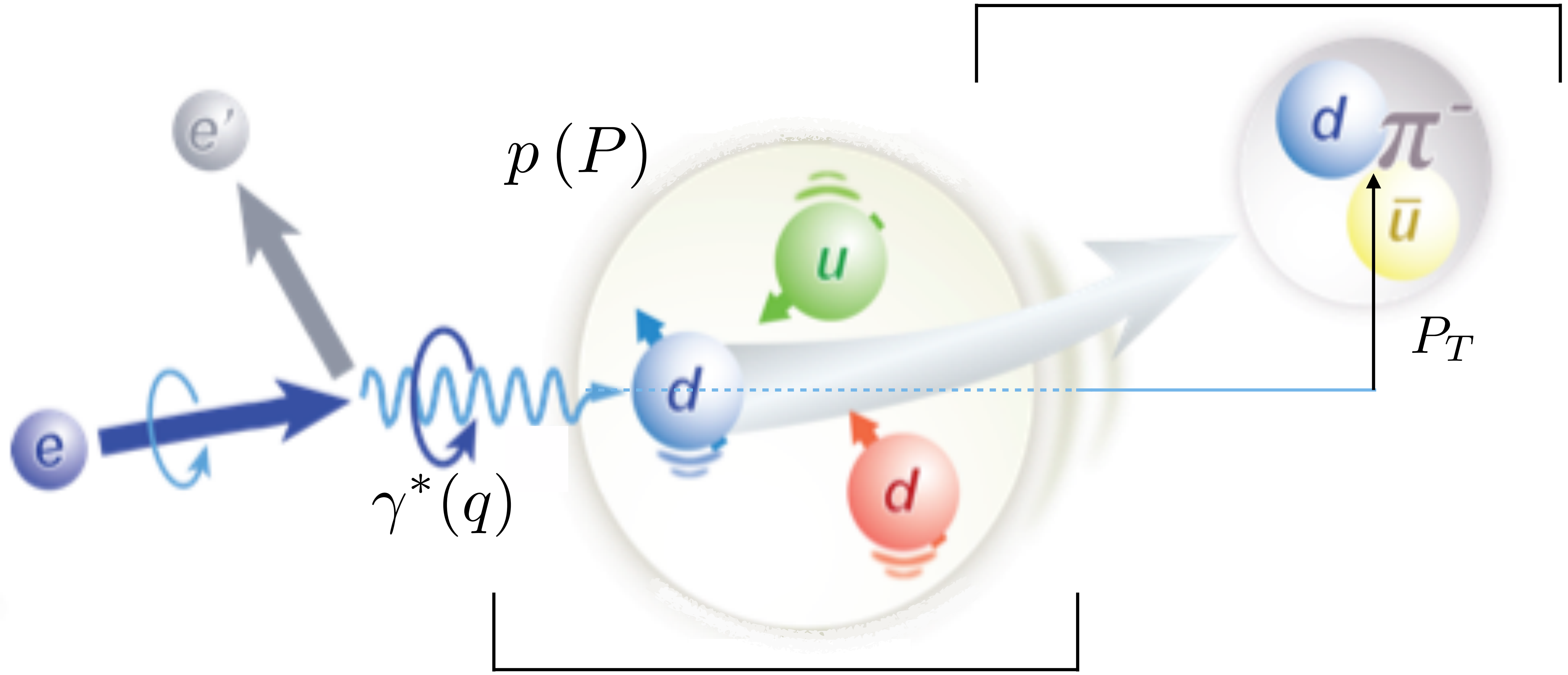


Transverse-momentum-dependent (TMD)
parton distribution function $PDF(x_B, k_\perp, Q^2)$

TMD evolution

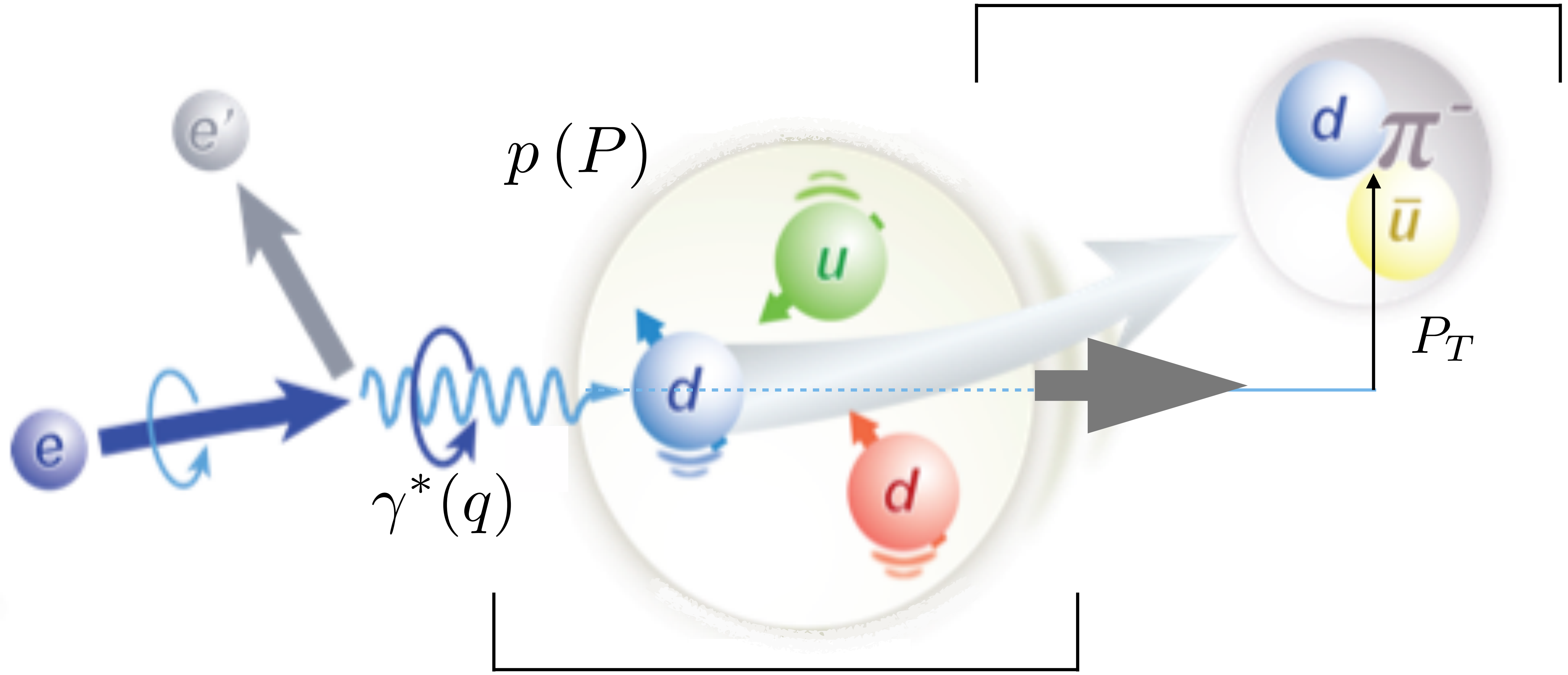
Highly virtual photon:
$Q^2 \gg 1$ GeV^2
provides hard
scale of process

Single-hadron production in semi-inclusive DIS

$Q^2 = -q^2$

$x_B = \frac{Q^2}{2P \cdot q}$

$z \overset{\text{lab}}{=} \frac{E_h}{E_{\gamma*}}$



Transverse-momentum-dependent (TMD) fragmentation function $FF(z, p_\perp, Q^2)$

Transverse-momentum-dependent (TMD) parton distribution function $PDF(x_B, k_\perp, Q^2)$

TMD evolution

Highly virtual photon:
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Single-hadron production in semi-inclusive DIS

$Q^2 = -q^2$

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Transverse-momentum-dependent (TMD)
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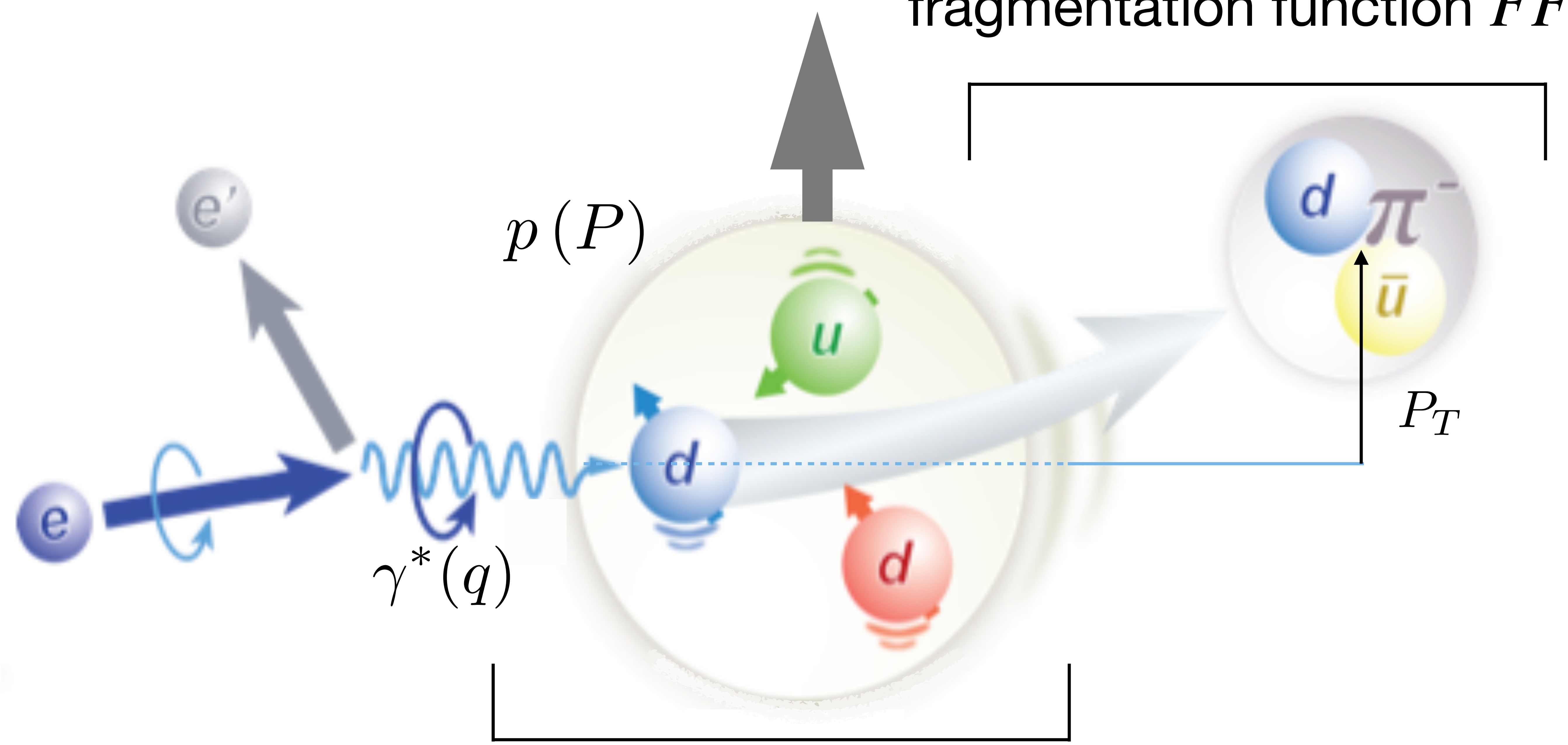

Transverse-momentum-dependent (TMD)
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TMD evolution

Highly virtual photon:
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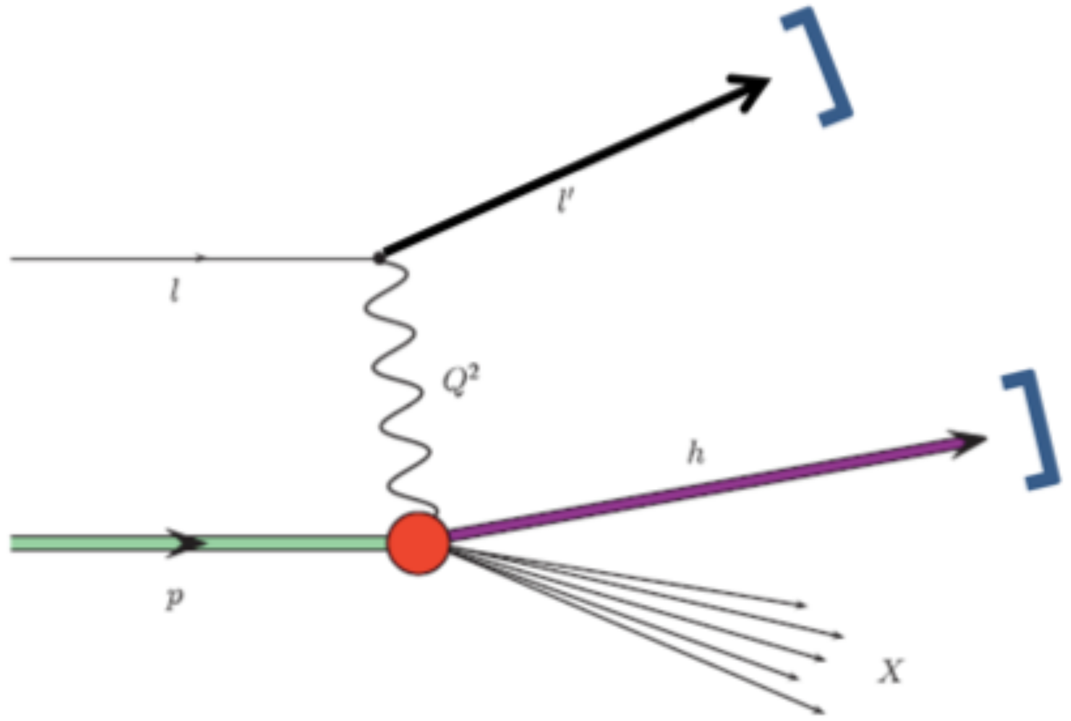
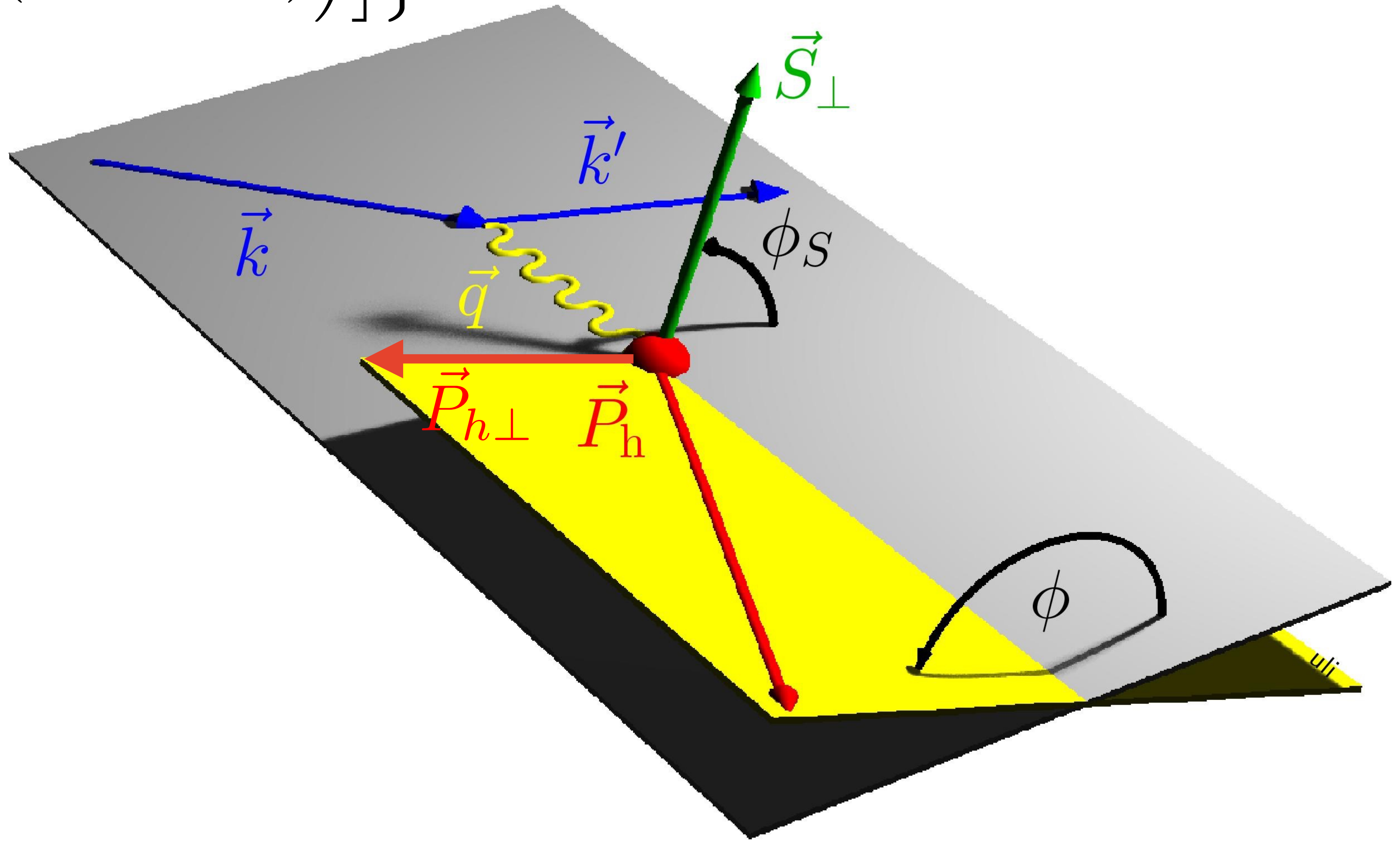
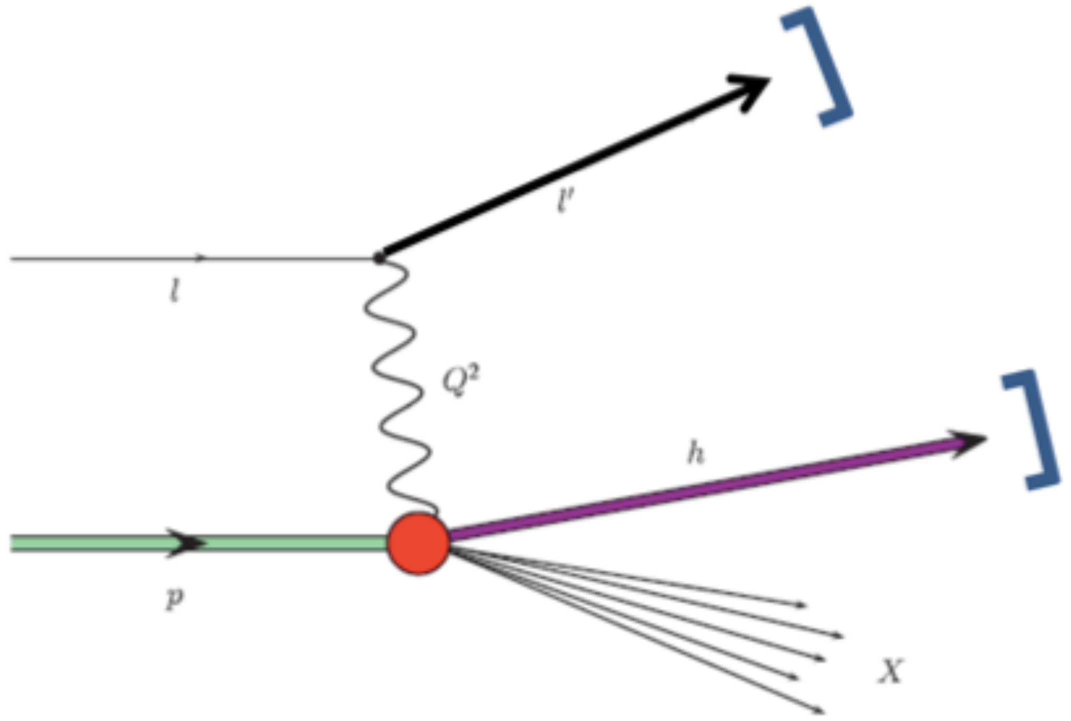
Semi-inclusive DIS cross section



$$
\begin{aligned}
\sigma^h(\phi,\phi_S) \;=\;& \sigma^h_{UU}\Big\{1 + 2\langle\cos(\phi)\rangle^h_{UU}\,\cos(\phi) + 2\langle\cos(2\phi)\rangle^h_{UU}\,\cos(2\phi)\\
+\;& \lambda_l\, 2\langle\sin(\phi)\rangle^h_{LU}\,\sin(\phi)\\
+\;& S_L\Big[2\langle\sin(\phi)\rangle^h_{UL}\,\sin(\phi) + 2\langle\sin(2\phi)\rangle^h_{UL}\,\sin(2\phi)\\
+\;& \lambda_l\Big(2\langle\cos(0\phi)\rangle^h_{LL}\,\cos(0\phi) + 2\langle\cos(\phi)\rangle^h_{LL}\,\cos(\phi)\Big)\Big]\\
+\;& S_T\Big[2\langle\sin(\phi-\phi_S)\rangle^h_{UT}\,\sin(\phi-\phi_S) + 2\langle\sin(\phi+\phi_S)\rangle^h_{UT}\,\sin(\phi+\phi_S)\\
+\;& 2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\,\sin(3\phi-\phi_S) + 2\langle\sin(\phi_S)\rangle^h_{UT}\,\sin(\phi_S)\\
+\;& 2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\,\sin(2\phi-\phi_S)\\
+\;& \lambda_l\Big(2\langle\cos(\phi-\phi_S)\rangle^h_{LT}\,\cos(\phi-\phi_S)\\
+\;& 2\langle\cos(\phi_S)\rangle^h_{LT}\,\cos(\phi_S) + 2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\,\cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$



Semi-inclusive DIS cross section



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& + \lambda_l\, 2\langle\sin(\phi)\rangle^h_{LU}\,\sin(\phi) \\
& + S_L\Big[2\langle\sin(\phi)\rangle^h_{UL}\,\sin(\phi) + 2\langle\sin(2\phi)\rangle^h_{UL}\,\sin(2\phi) \\
& + \lambda_l\Big(2\langle\cos(0\phi)\rangle^h_{LL}\,\cos(0\phi) + 2\langle\cos(\phi)\rangle^h_{LL}\,\cos(\phi)\Big)\Big] \\
& + S_T\Big[2\langle\sin(\phi-\phi_S)\rangle^h_{UT}\,\sin(\phi-\phi_S) + 2\langle\,\sin(\phi+\phi_S)\rangle^h_{UT}\,\sin(\phi+\phi_S) \\
& + 2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\,\sin(3\phi-\phi_S) + 2\langle\sin(\phi_S)\rangle^h_{UT}\,\sin(\phi_S) \\
& + 2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\,\sin(2\phi-\phi_S) \\
& + \lambda_l\Big(2\langle\cos(\phi-\phi_S)\rangle^h_{LT}\,\cos(\phi-\phi_S) \\
& + \;2\langle\cos(\phi_S)\rangle^h_{LT}\,\cos(\phi_S) + 2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\,\cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$

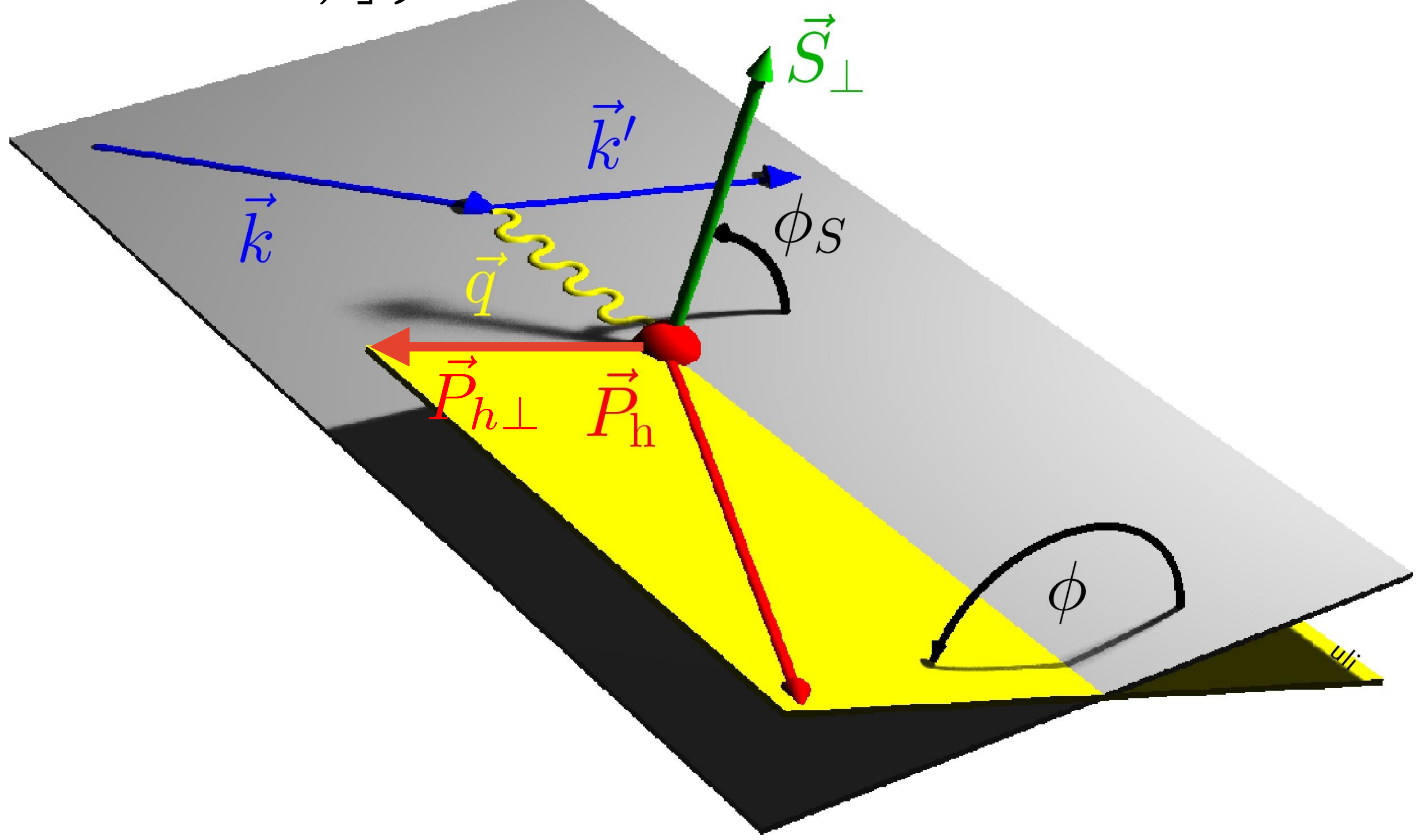
longitudinal target polarisation

transverse target polarisation

beam polarisation

beam polarisation

target polarisation



Semi-inclusive DIS cross section

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\begin{aligned}
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&+\lambda_l\,2\langle\sin(\phi)\rangle^h_{LU}\,\sin(\phi)\\
&+S_L\Big[2\langle\sin(\phi)\rangle^h_{UL}\,\sin(\phi)+\boxed{2\langle\sin(2\phi)\rangle^h_{UL}}\,\sin(2\phi)\\
&+\lambda_l\Big(\boxed{2\langle\cos(0\phi)\rangle^h_{LL}}\,\cos(0\phi)+2\langle\cos(\phi)\rangle^h_{LL}\,\cos(\phi)\Big)\Big]\\
&+S_T\Big[\boxed{2\langle\sin(\phi-\phi_S)\rangle^h_{UT}}\,\sin(\phi-\phi_S)+\boxed{2\langle\sin(\phi+\phi_S)\rangle^h_{UT}}\,\sin(\phi+\phi_S)\\
&+\boxed{2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}}\,\sin(3\phi-\phi_S)+2\langle\sin(\phi_S)\rangle^h_{UT}\,\sin(\phi_S)\\
&+2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\,\sin(2\phi-\phi_S)\\
&+\lambda_l\Big(\boxed{2\langle\cos(\phi-\phi_S)\rangle^h_{LT}}\,\cos(\phi-\phi_S)\\
&+2\langle\cos(\phi_S)\rangle^h_{LT}\,\cos(\phi_S)+2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\,\cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$

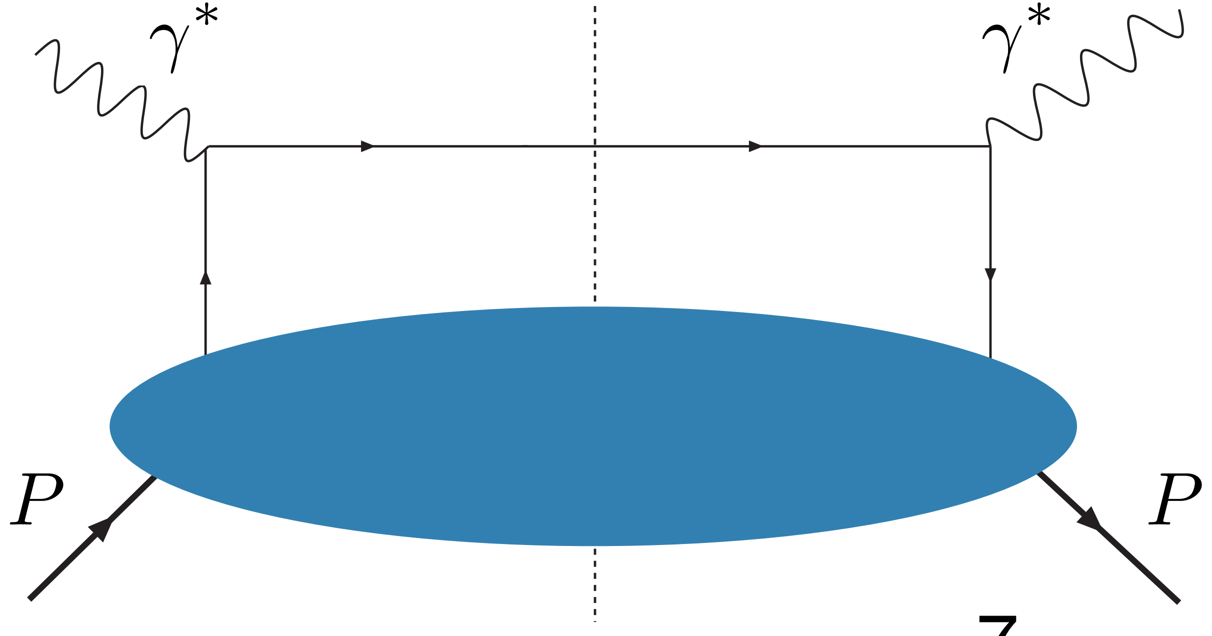
longitudinal target polarisation

transverse target polarisation

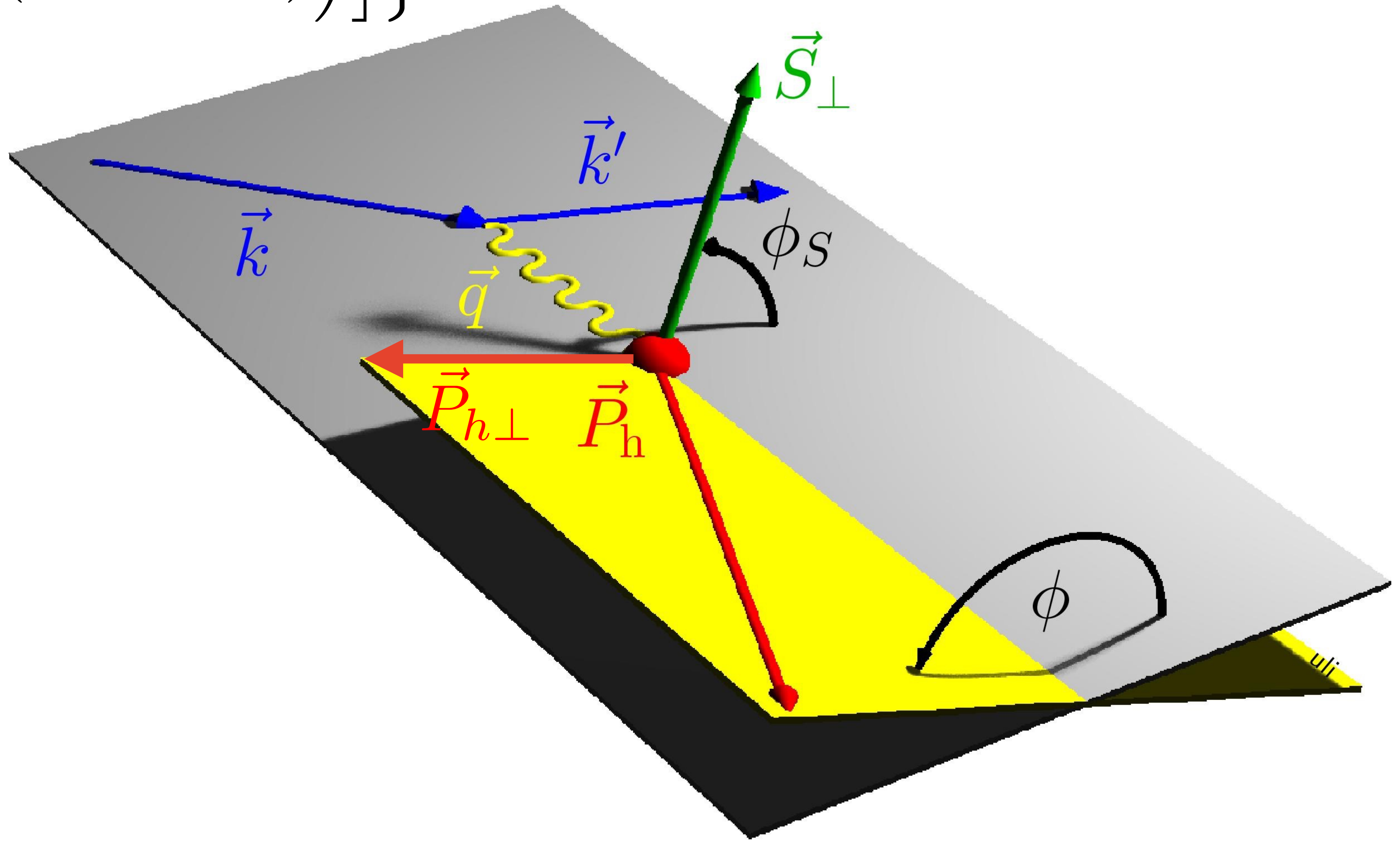
beam polarisation

beam polarisation

target polarisatic



leading twist



Semi-inclusive DIS cross section

$$
\begin{aligned}
\sigma^h(\phi,\phi_S) = \; & \sigma^h_{UU}\Big\{1 + 2\langle\cos(\phi)\rangle^h_{UU}\cos(\phi) + 2\langle\cos(2\phi)\rangle^h_{UU}\cos(2\phi) \\
& + \lambda_l\, 2\langle\sin(\phi)\rangle^h_{LU}\sin(\phi) \\
& + S_L\Big[2\langle\sin(\phi)\rangle^h_{UL}\sin(\phi) + 2\langle\sin(2\phi)\rangle^h_{UL}\sin(2\phi) \\
& + \lambda_l\Big(2\langle\cos(0\phi)\rangle^h_{LL}\cos(0\phi) + 2\langle\cos(\phi)\rangle^h_{LL}\cos(\phi)\Big)\Big] \\
& + S_T\Big[2\langle\sin(\phi-\phi_S)\rangle^h_{UT}\sin(\phi-\phi_S) + 2\langle\sin(\phi+\phi_S)\rangle^h_{UT}\sin(\phi+\phi_S) \\
& + 2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\sin(3\phi-\phi_S) + 2\langle\sin(\phi_S)\rangle^h_{UT}\sin(\phi_S) \\
& + 2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\sin(2\phi-\phi_S) \\
& + \lambda_l\Big(2\langle\cos(\phi-\phi_S)\rangle^h_{LT}\cos(\phi-\phi_S) \\
& + 2\langle\cos(\phi_S)\rangle^h_{LT}\cos(\phi_S) + 2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$

longitudinal target polarisation

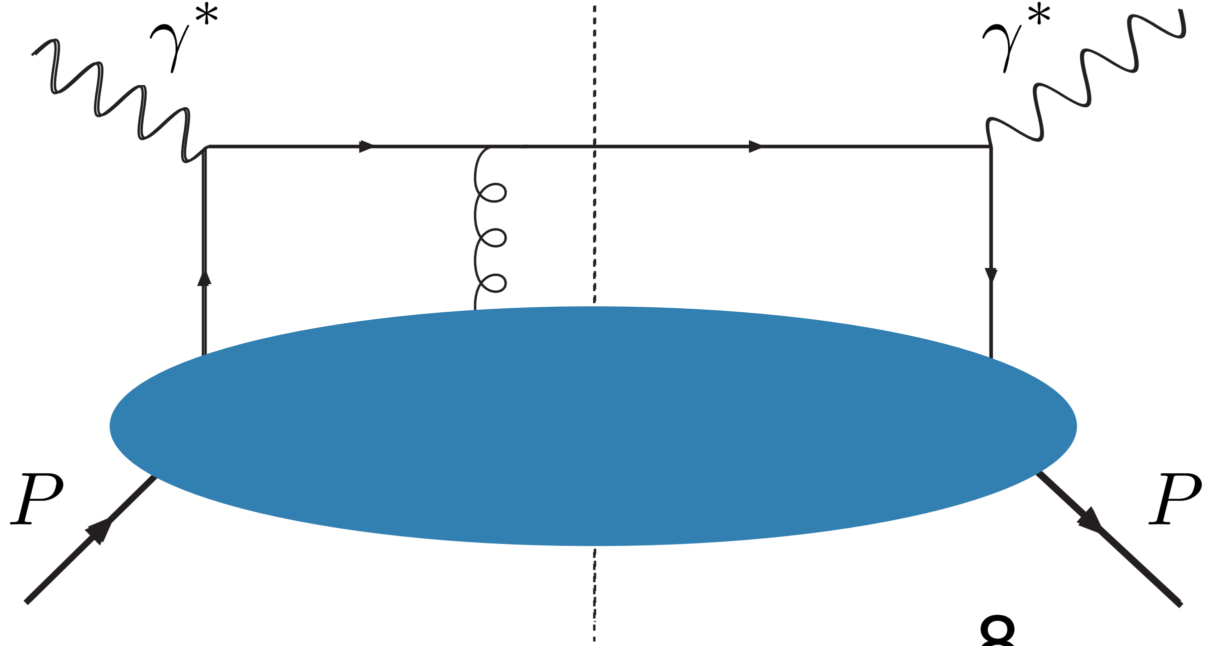
transverse target polarisation

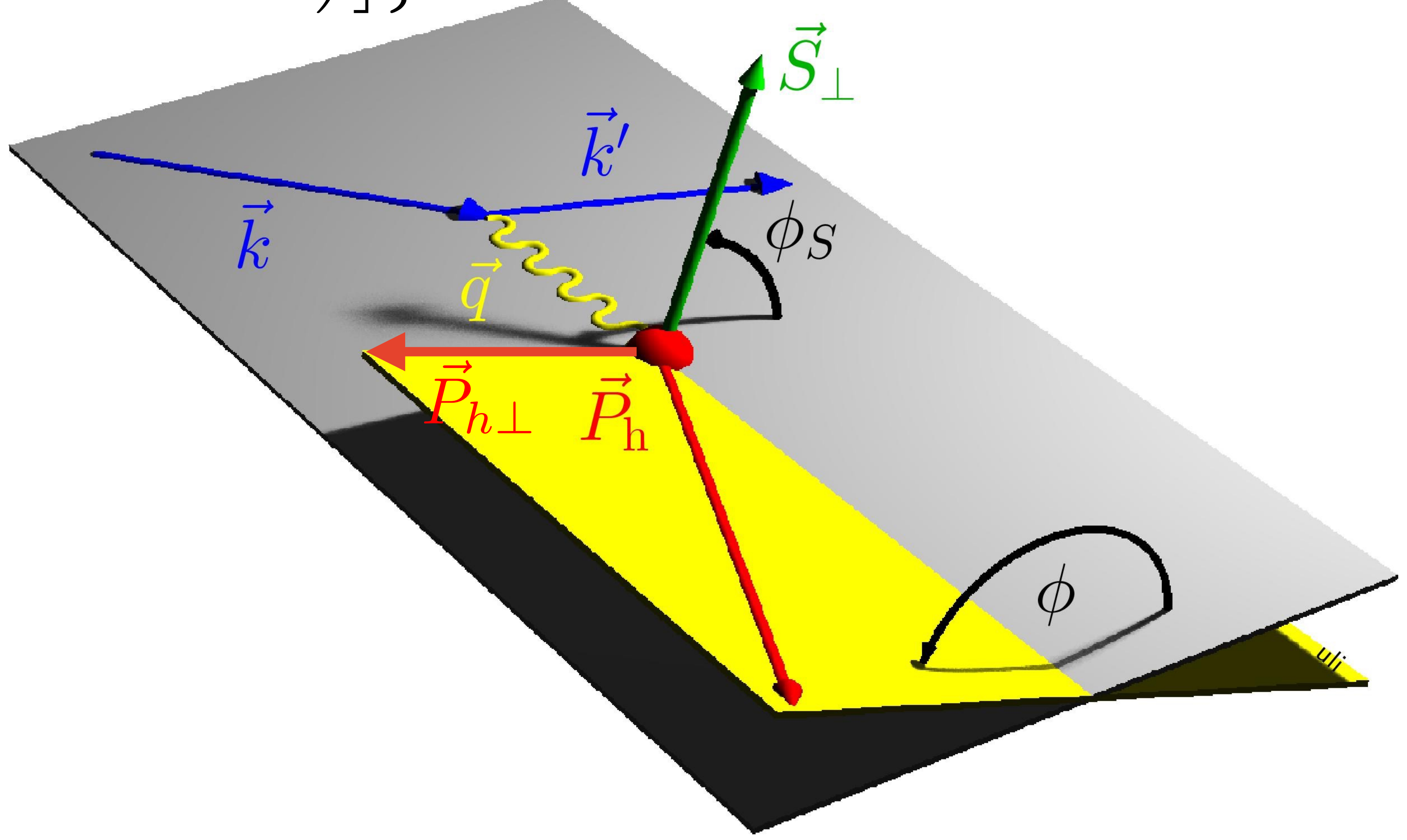
beam polarisation

beam polarisation

target polarisation

sub-leading twist





TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions F_{XY} :

$$2\langle\sin(\phi+\phi_S)\rangle^h_{UT} = \epsilon\, F_{UT}^{\sin(\phi+\phi_S)}$$

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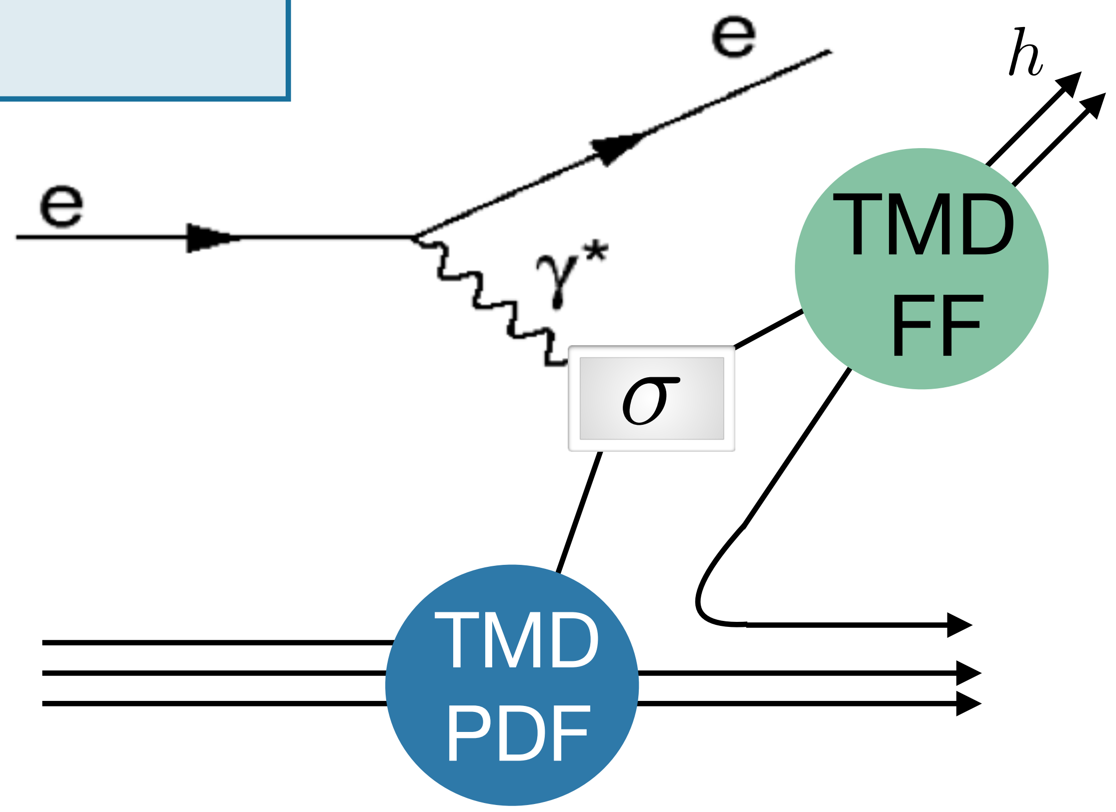
$$z \overset{\text{lab}}{=} \frac{E_h}{E_{\gamma *}}$$



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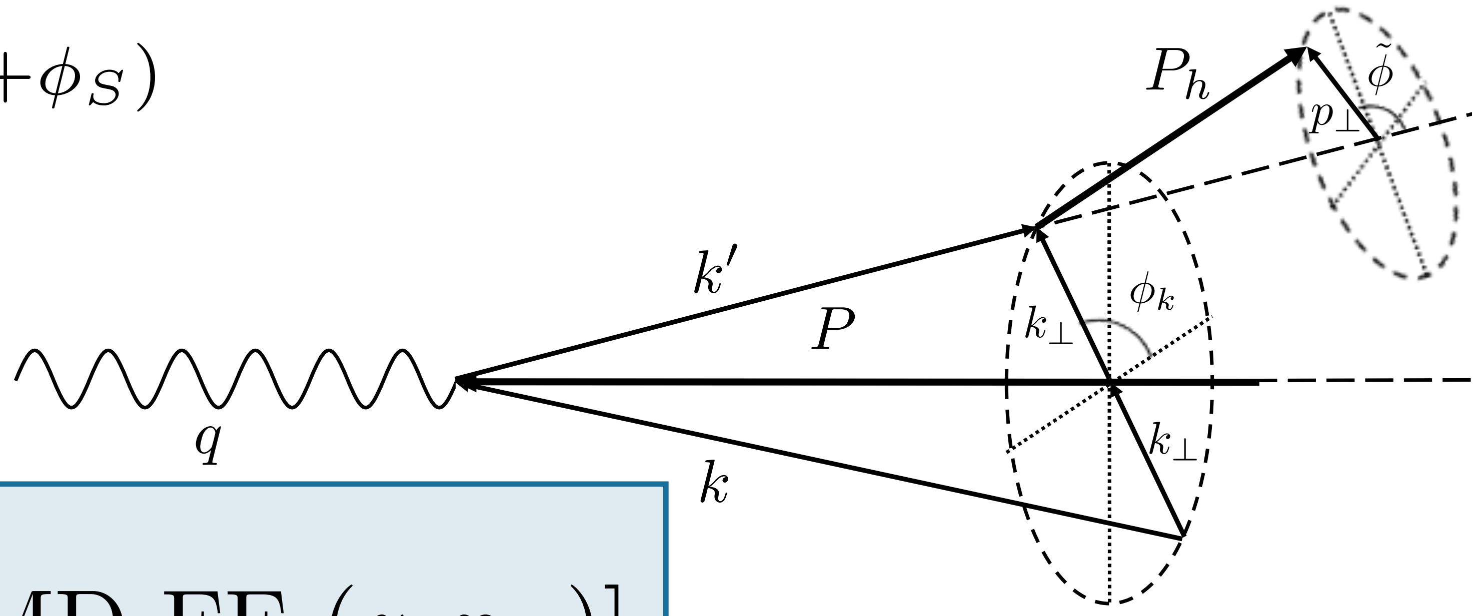

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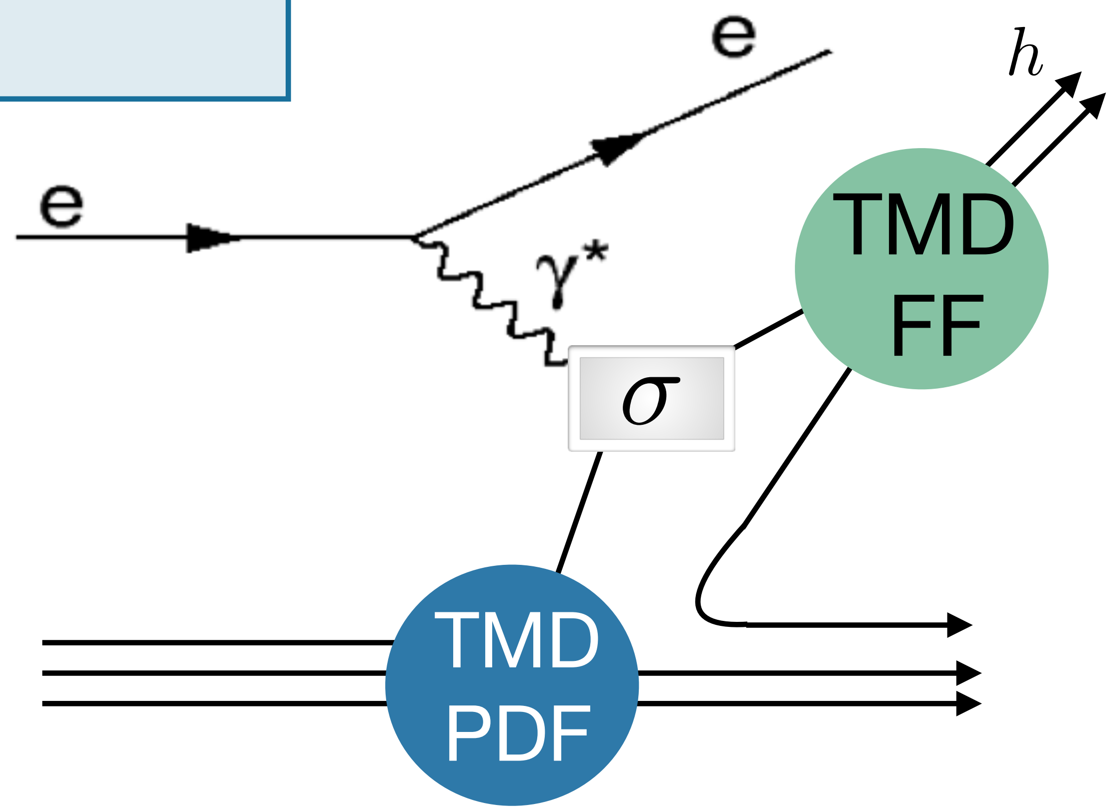

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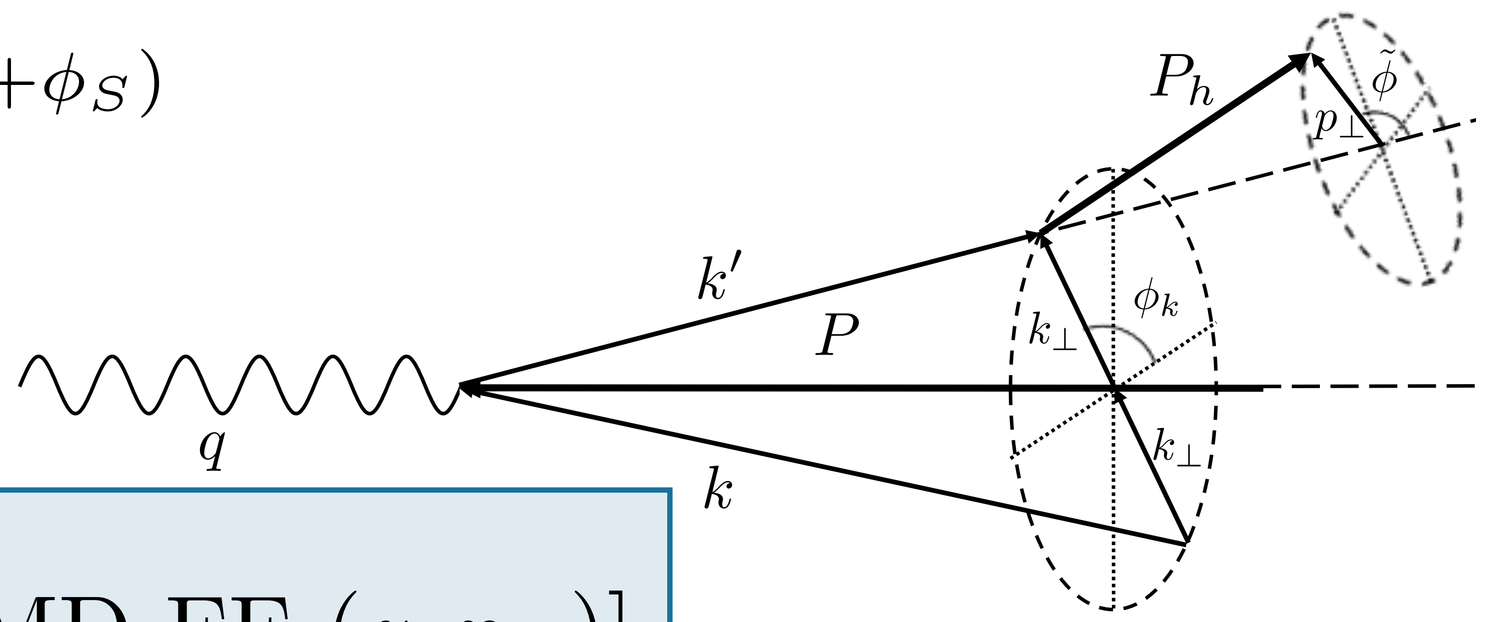

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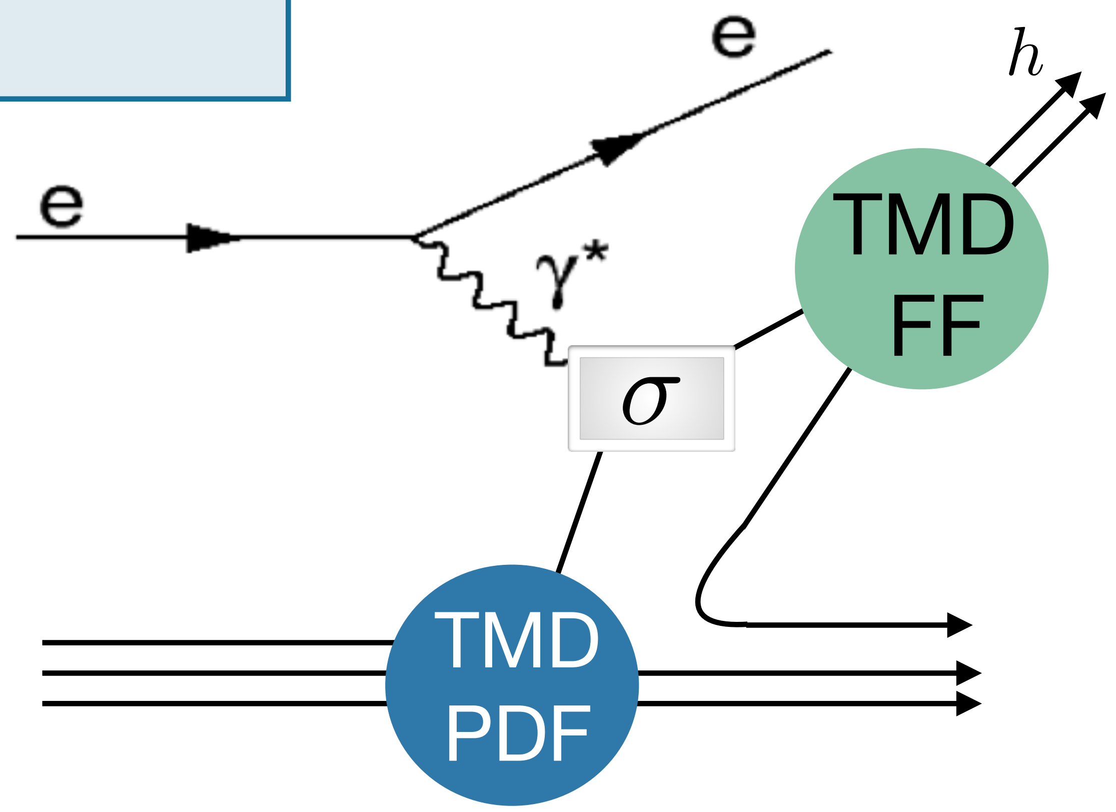
$$z \overset{\mathrm{lab}}{=} \frac{E_h}{E_{\gamma*}}$$

quark polarisation

nucleon polarisation

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_{1T}\, h_{1T}^\perp$



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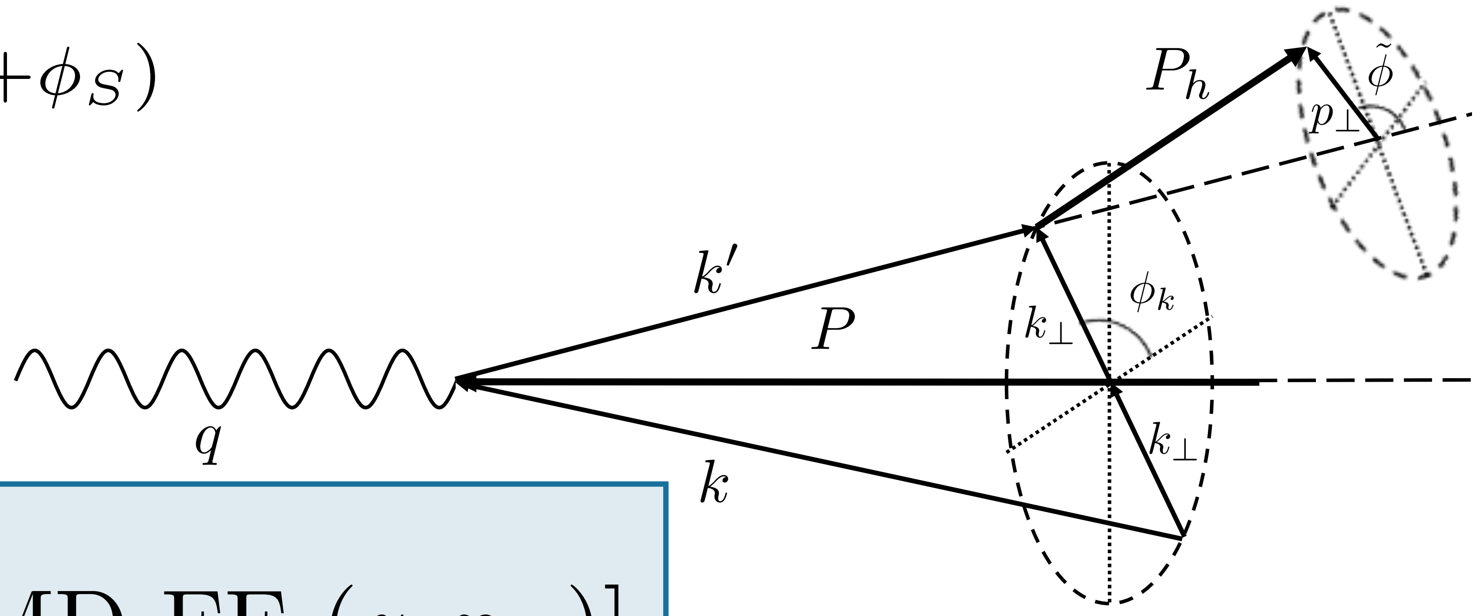

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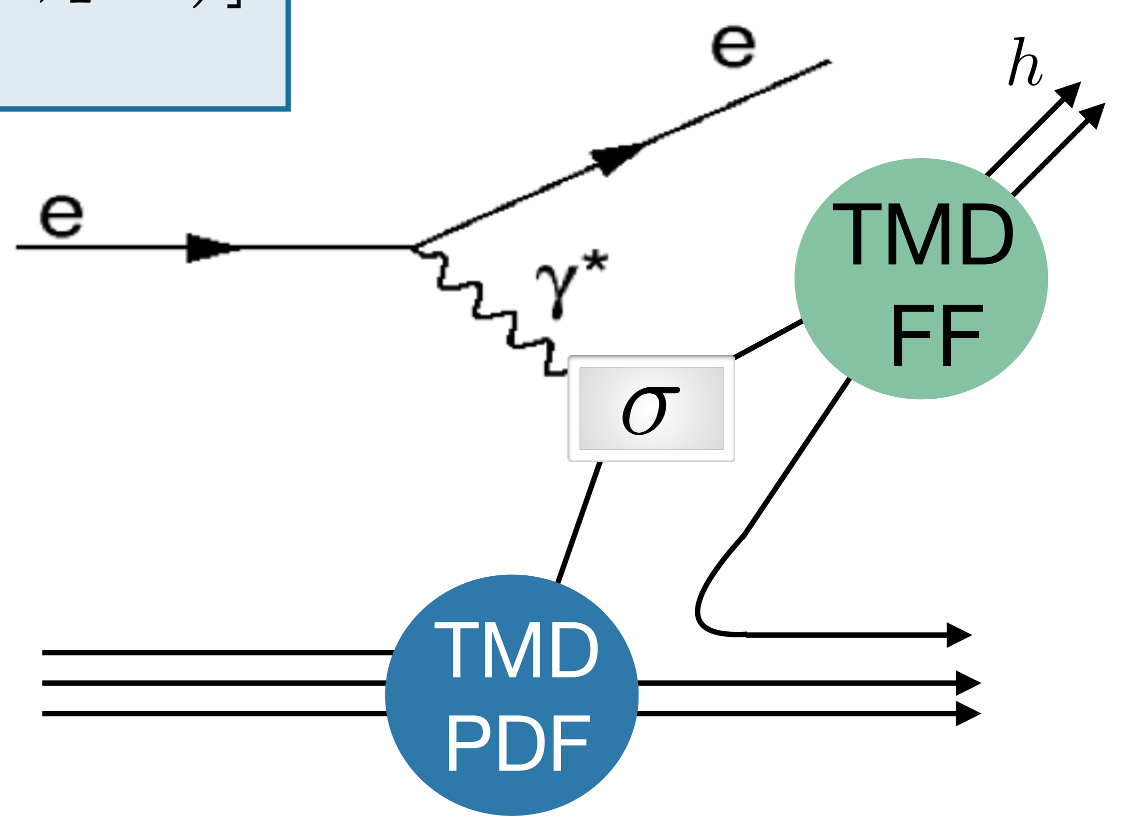

quark polarisation

nucleon polarisation

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	$h_{1T}\, h_{1T}^{\perp}$

quark polarisation

hadron polarisation

	U	L	T
U	D_1		$H_1^{\perp}$

TMD PDFs and fragmentation functions (FFs)

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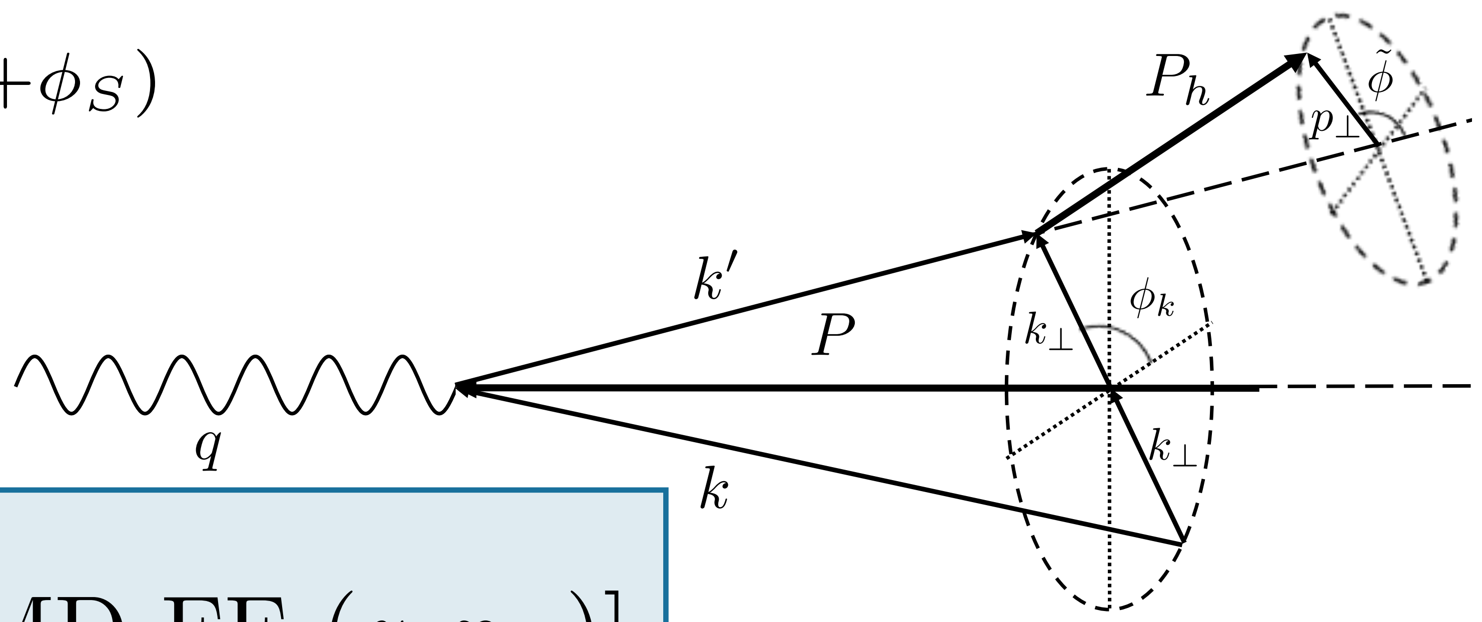

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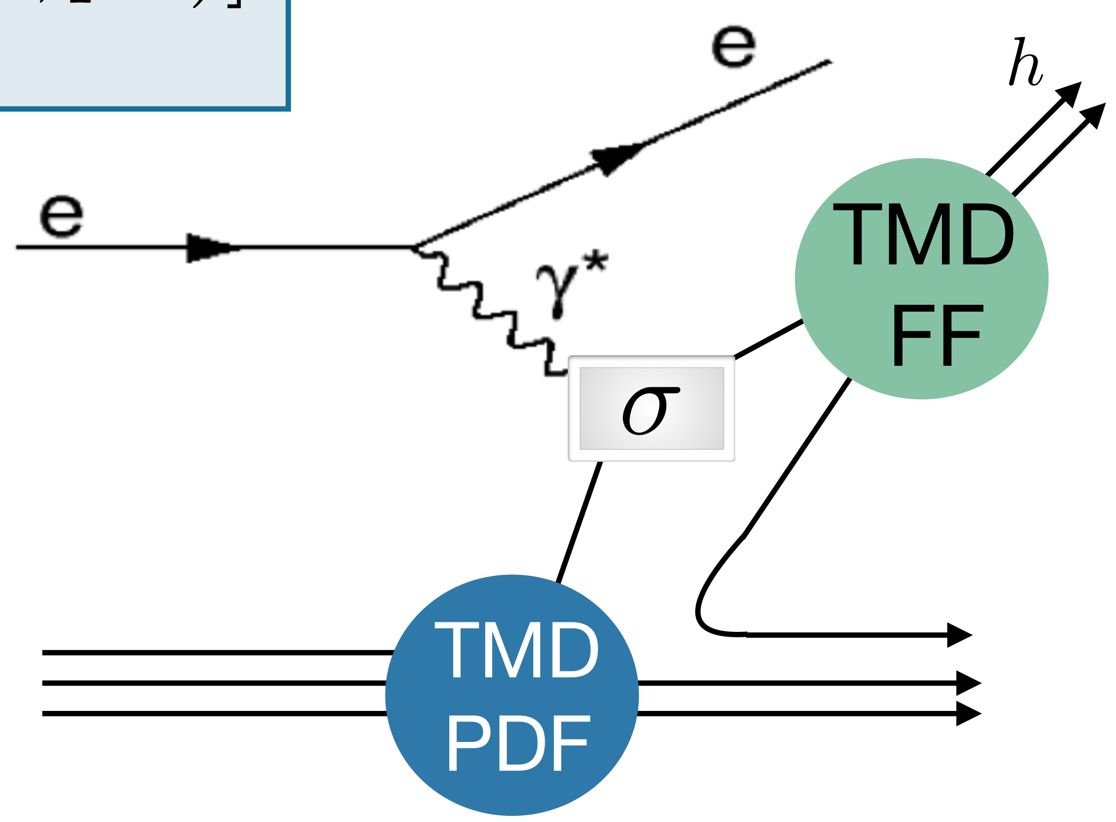

quark polarisation

nucleon polarisation

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quark polarisation

hadron polarisation

	U	L	T
U	D_1		$H_1^{\perp}$

Chiral odd

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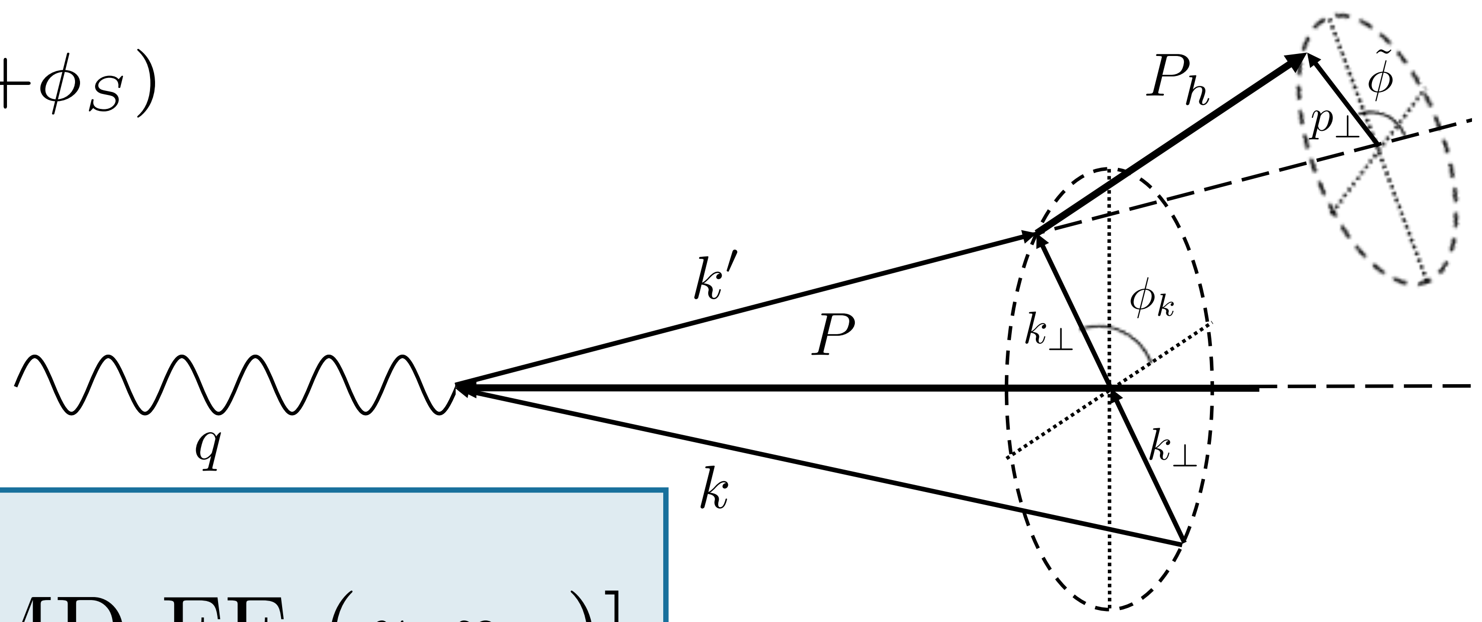

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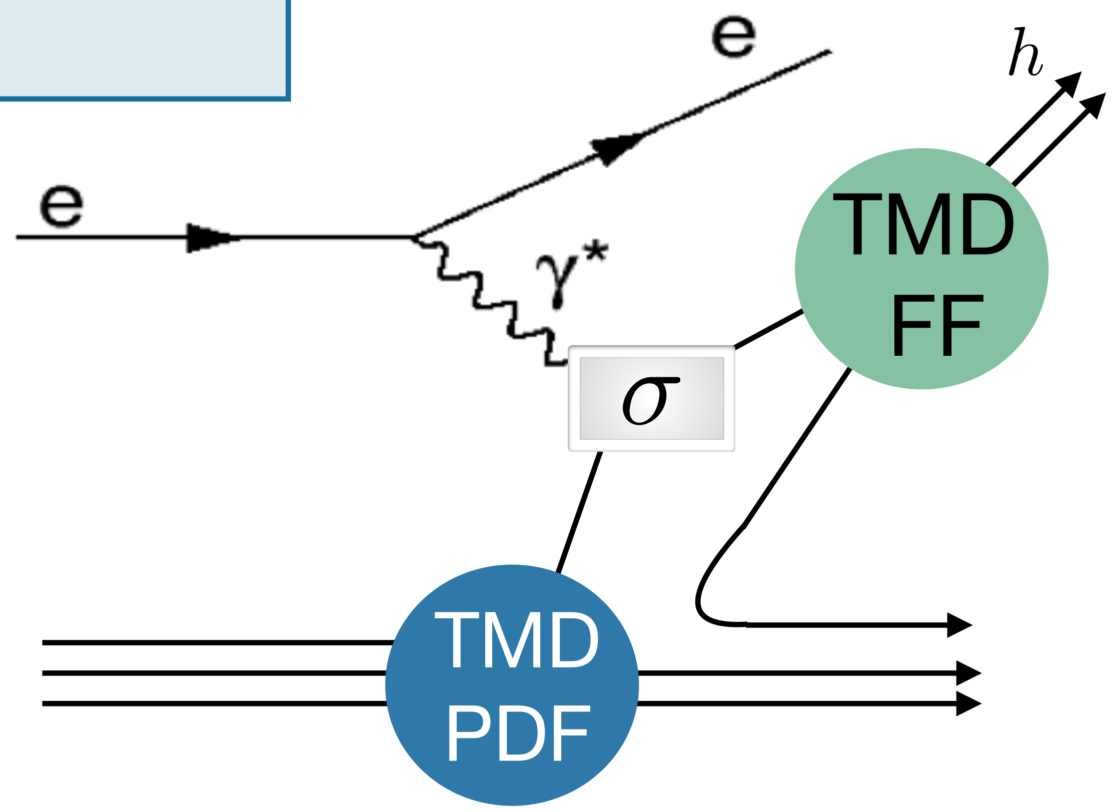
$$z^{\mathrm{lab}} = \frac{E_h}{E_{\gamma*}}$$



quark polarisation

nucleon polarisation

	U	L	T
U	f_1		🟧🟩 $h_1^\perp$
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quark polarisation

hadron polarisation

	U	L	T
U	D_1		🟧🟩 $H_1^\perp$

🟧 Chiral odd

🟩 Naive T-odd

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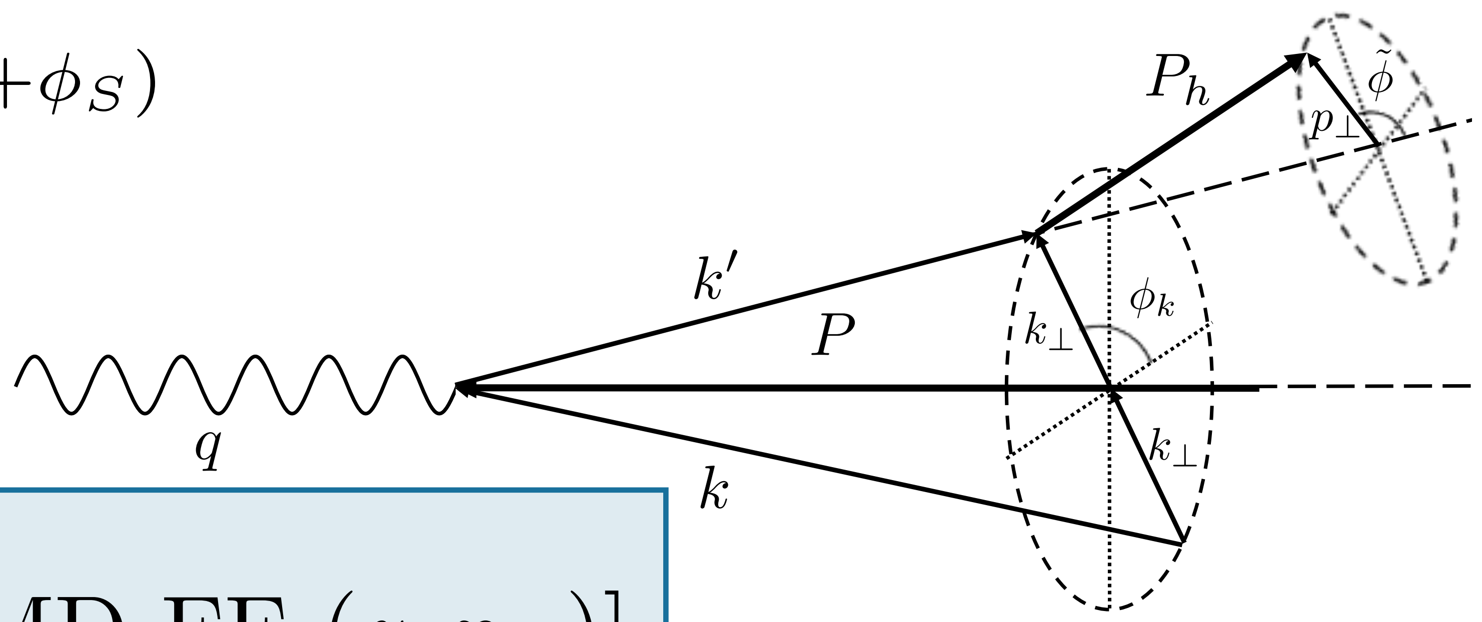

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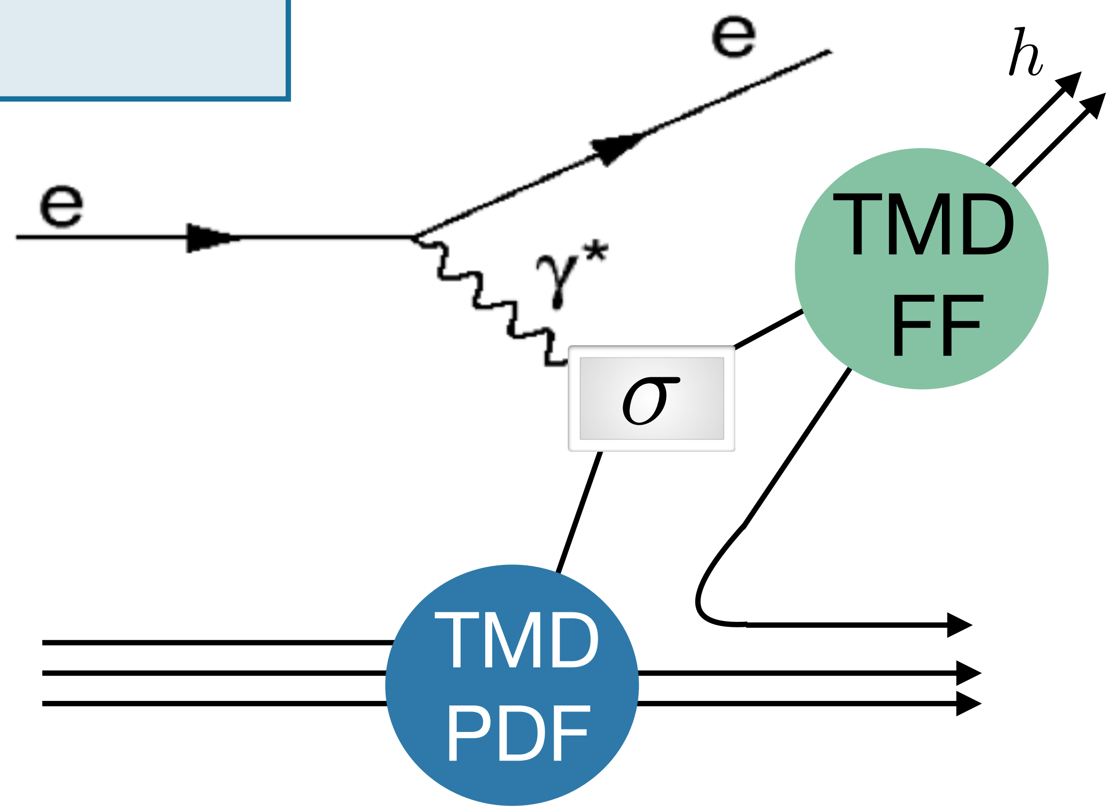

quark polarisation

nucleon polarisation

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U	f_1		$h_1^\perp$ 🟧🟩
L		g_{1L}	$h_{1L}^\perp$ 🟧
T	$f_{1T}^\perp$ 🟩	$g_{1T}^\perp$	$h_{1T}\, h_{1T}^\perp$ 🟧

quark polarisation

hadron polarisation

	U	L	T
U	D_1		$H_1^\perp$ 🟧🟩

🟧 Chiral odd

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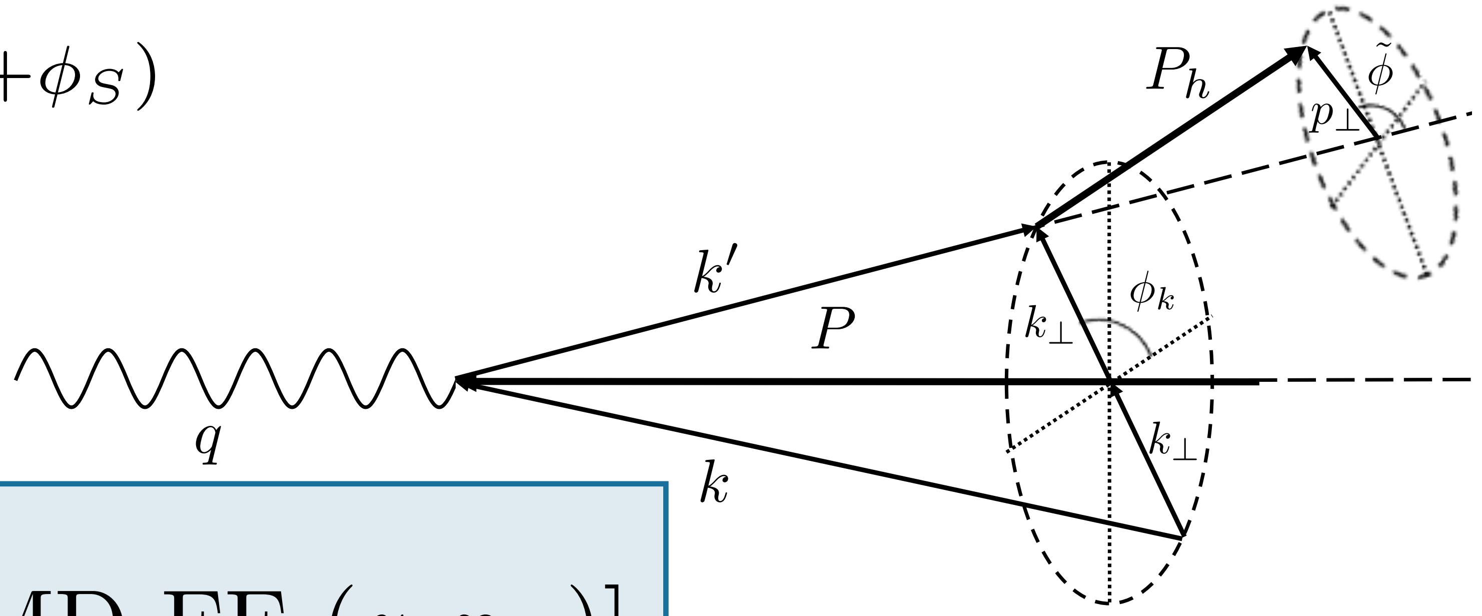

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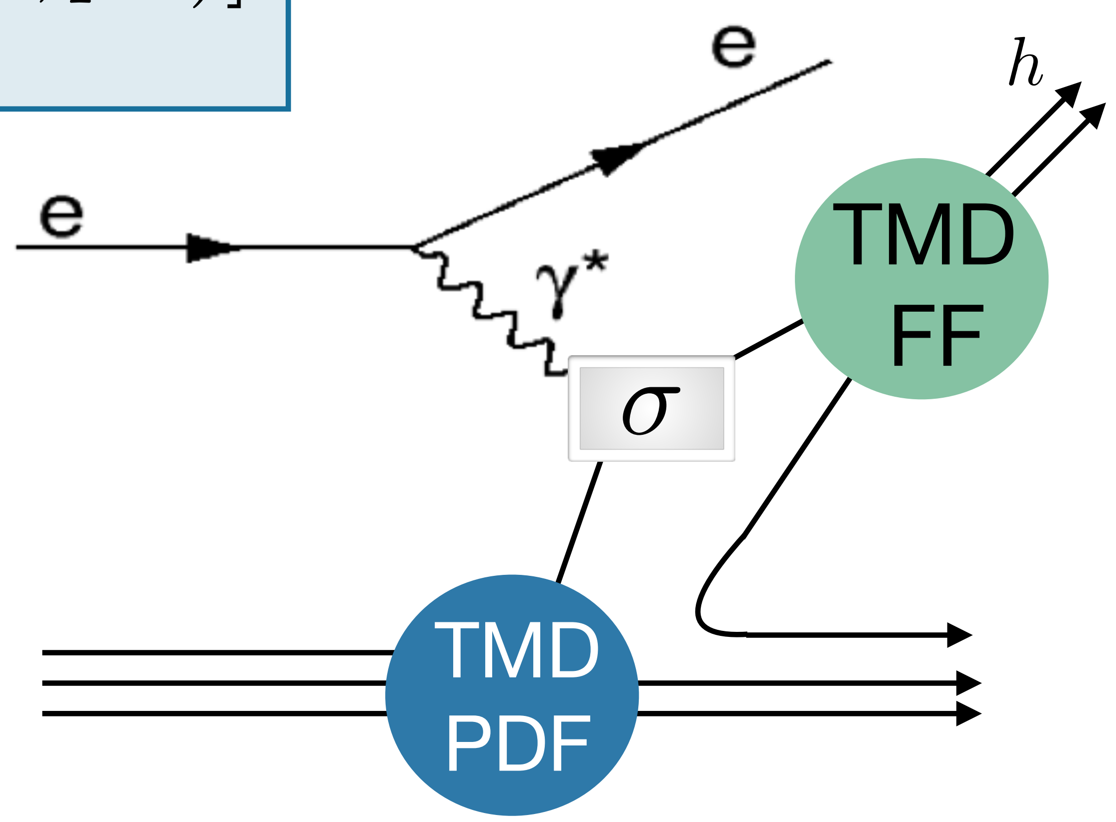


quark polarisation

nucleon polarisation

	U	L	T
U	f_1		$h_1^\perp$ 🟧🟩
L		g_{1L}	$h_{1L}^\perp$ 🟧
T	$f_{1T}^\perp$ 🟩	$g_{1T}^\perp$	$h_{1T}\, h_{1T}^\perp$ 🟧

quark polarisation

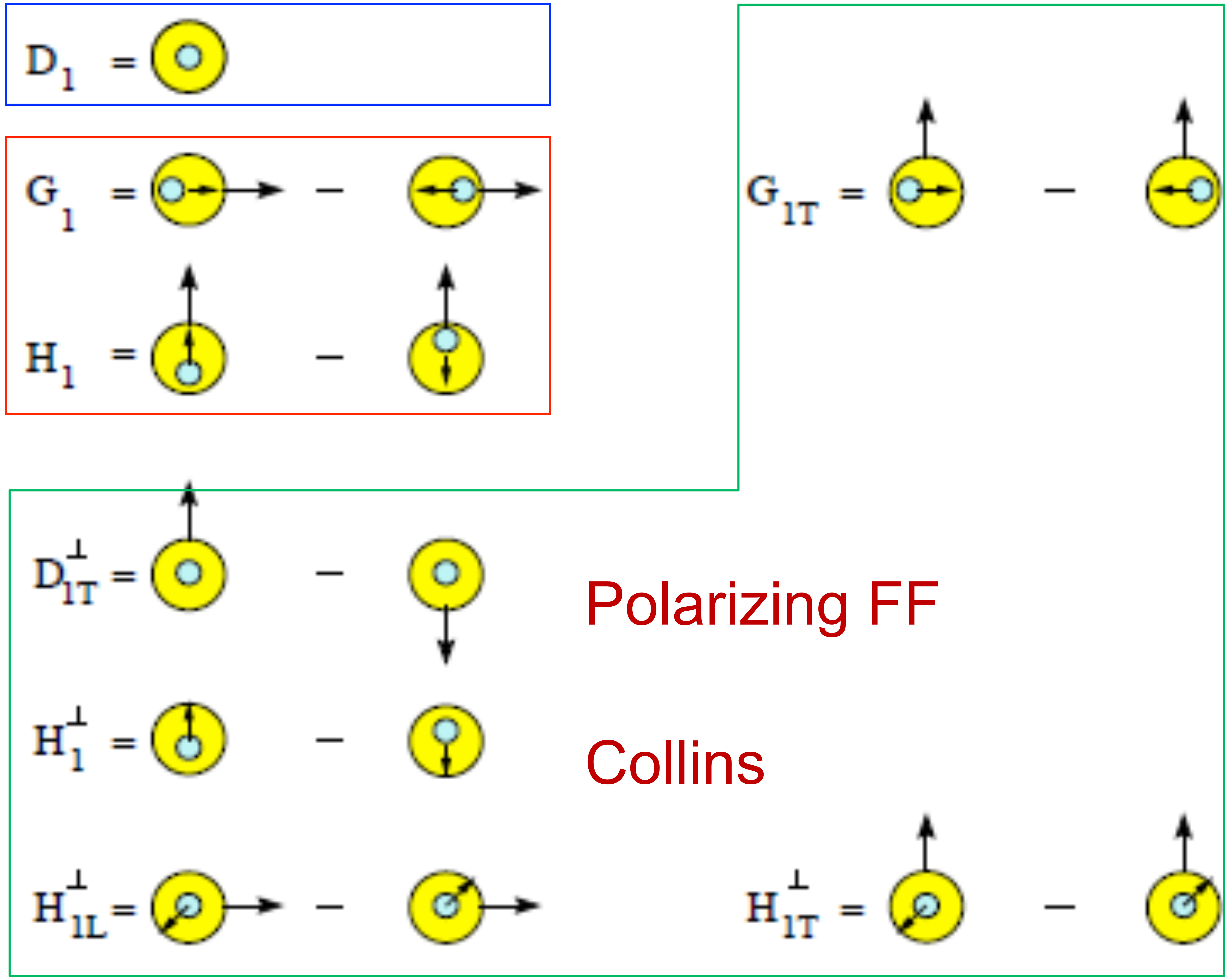
hadron polarisation

	U	L	T
U	D_1		$H_1^\perp$ 🟧🟩

🟧 Chiral odd

🟩 Naive T-odd

Transverse momentum dependent fragmentation functions



Unpolarized

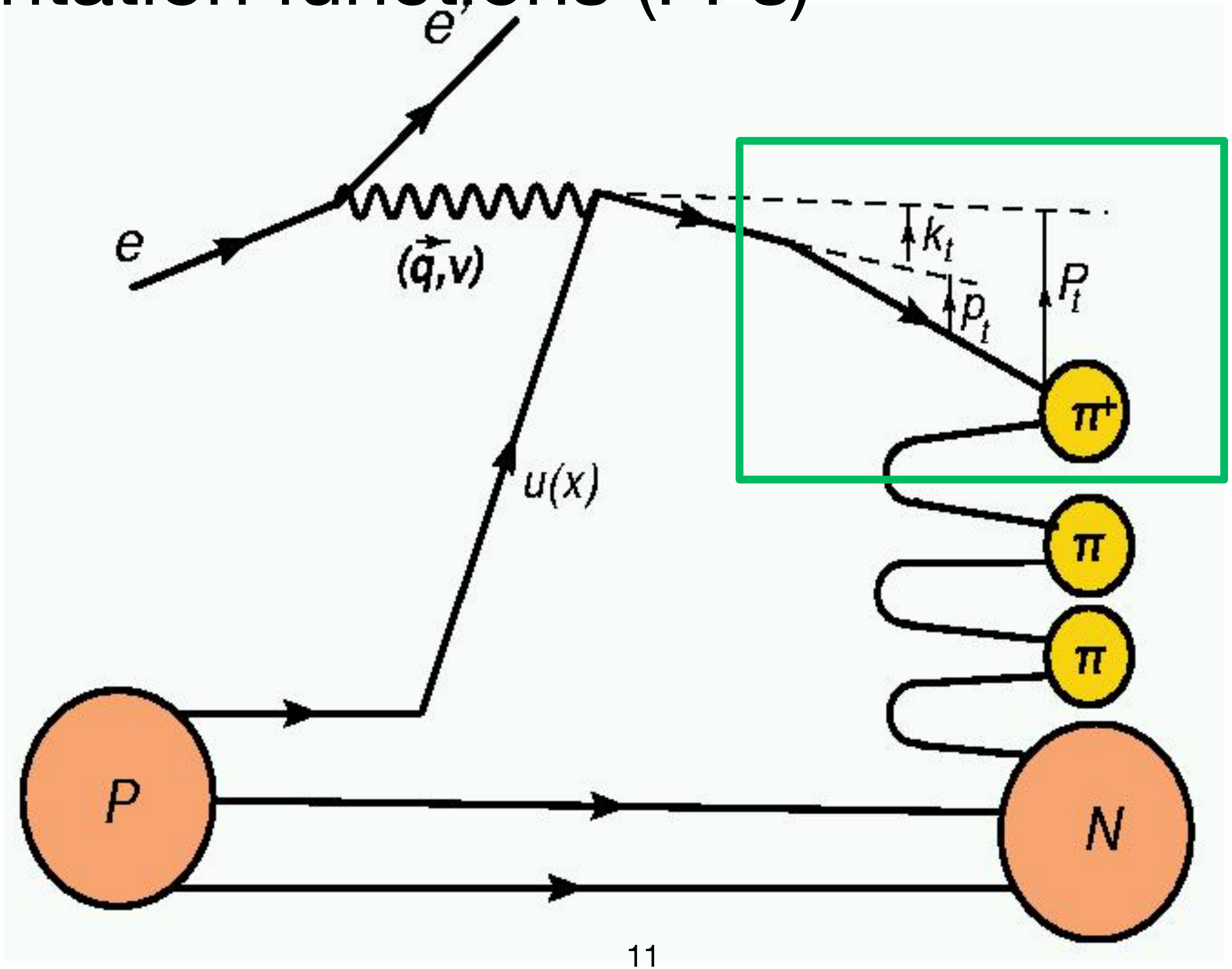
Spin-spin correlations

Spin-momentum correlations

Polarizing FF

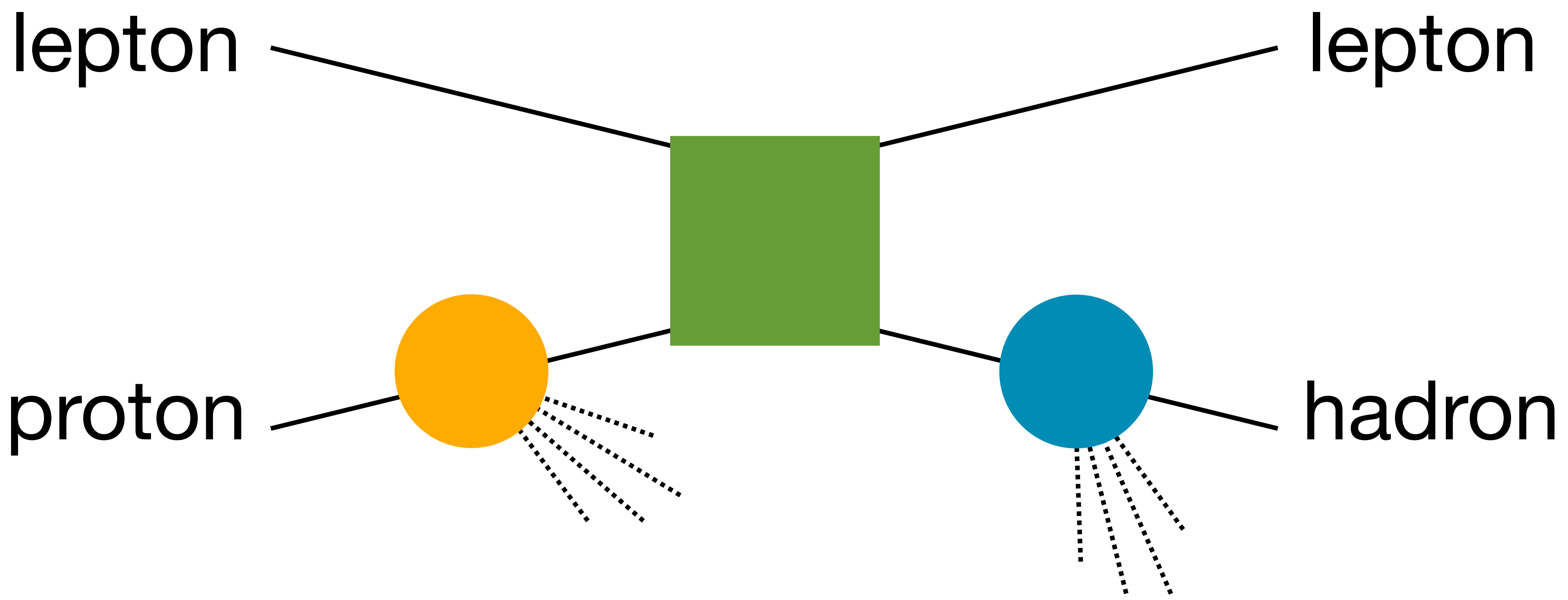
Collins

Fragmentation functions (FFs)



Fragmentation functions

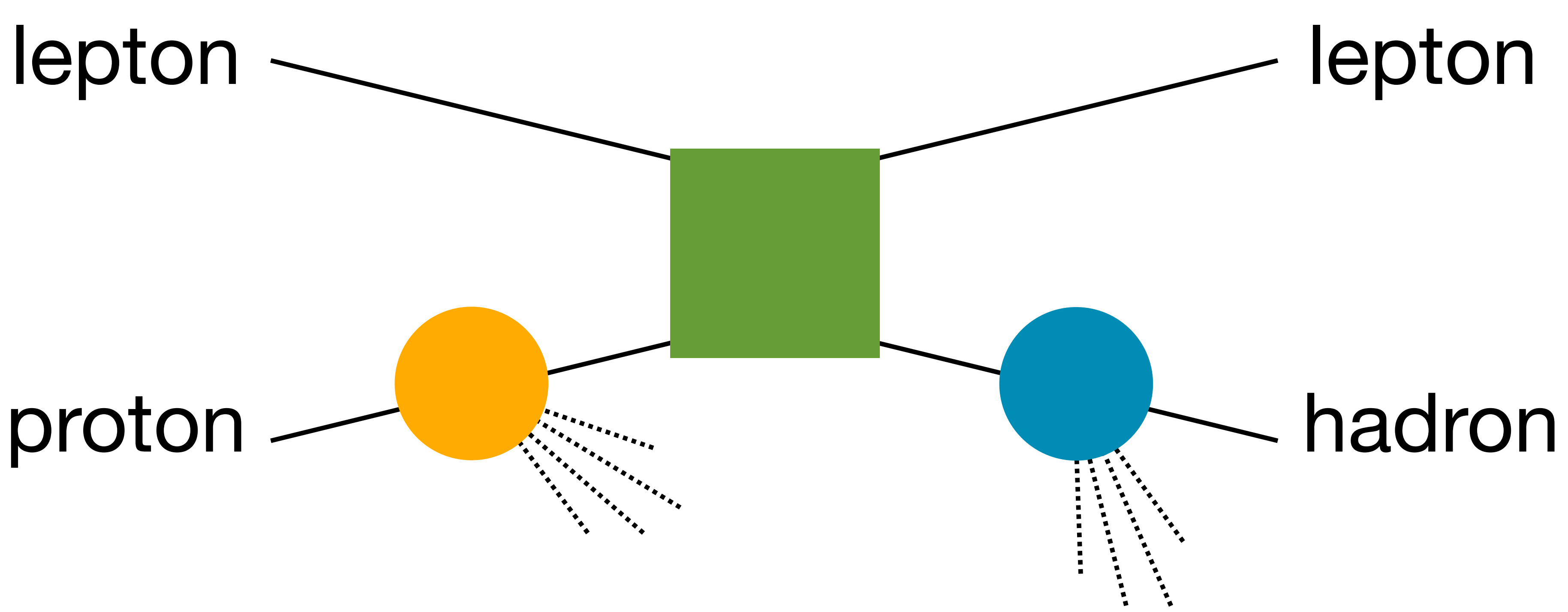
Factorisation and universality



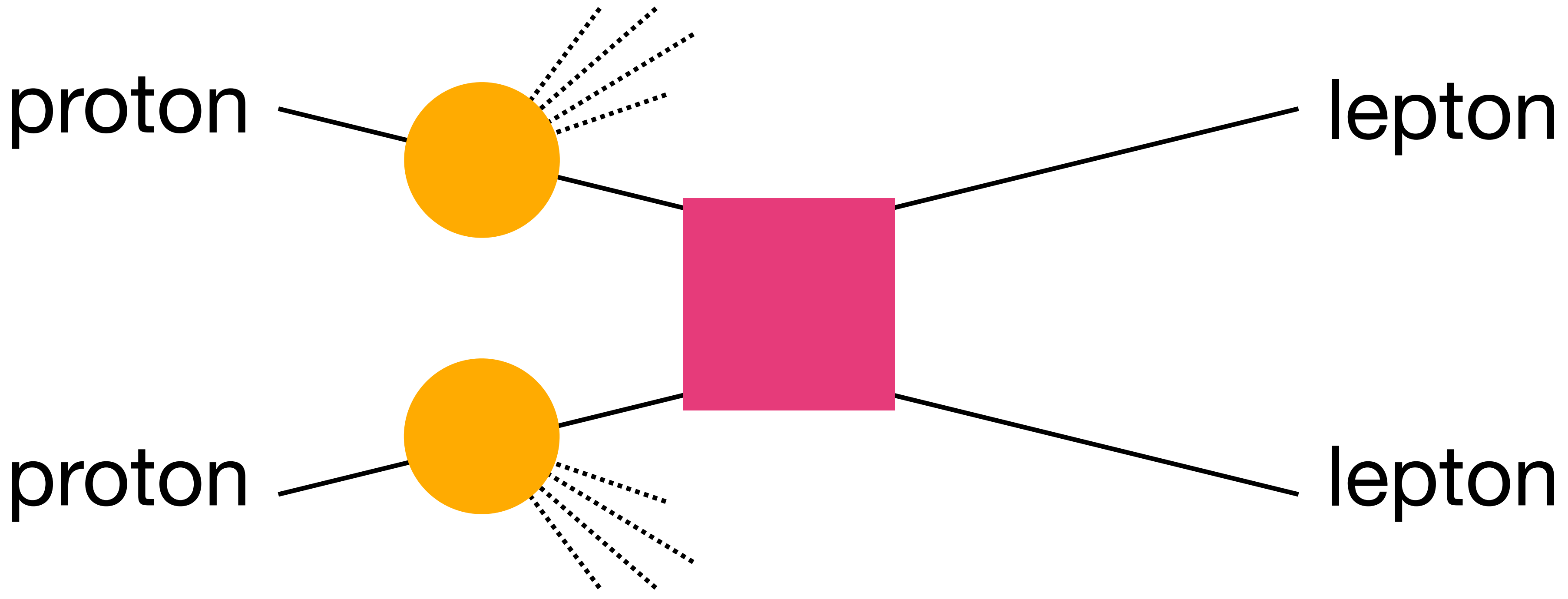
semi-inclusive DIS

Adapted from A. Bacchetta

Factorisation and universality



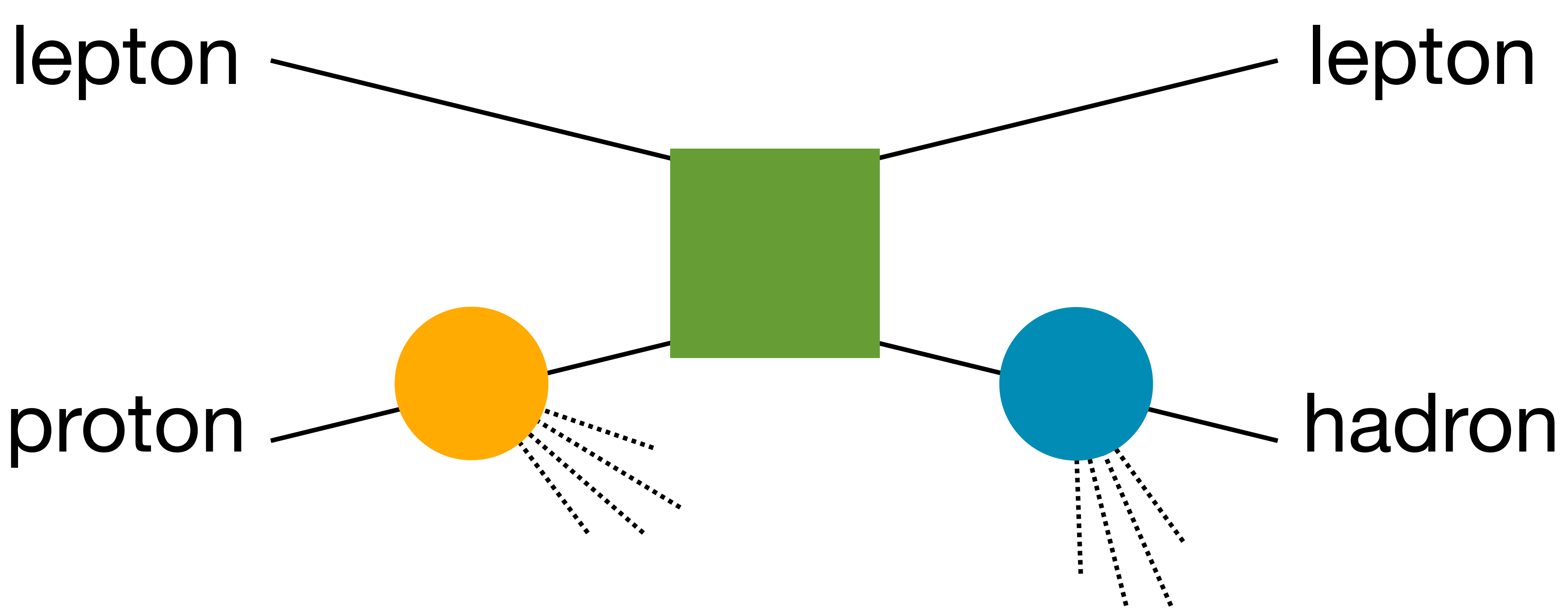
semi-inclusive DIS



Drell-Yan

Adapted from A. Bacchetta

Factorisation and universality



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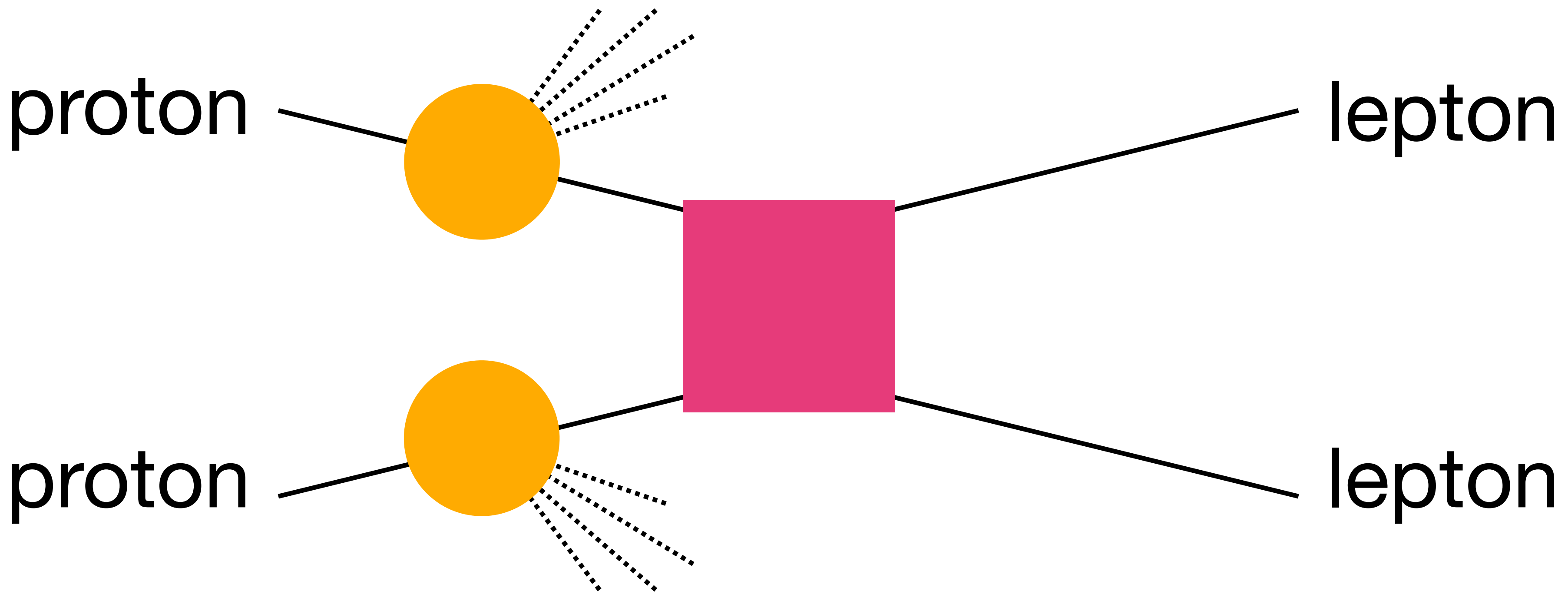
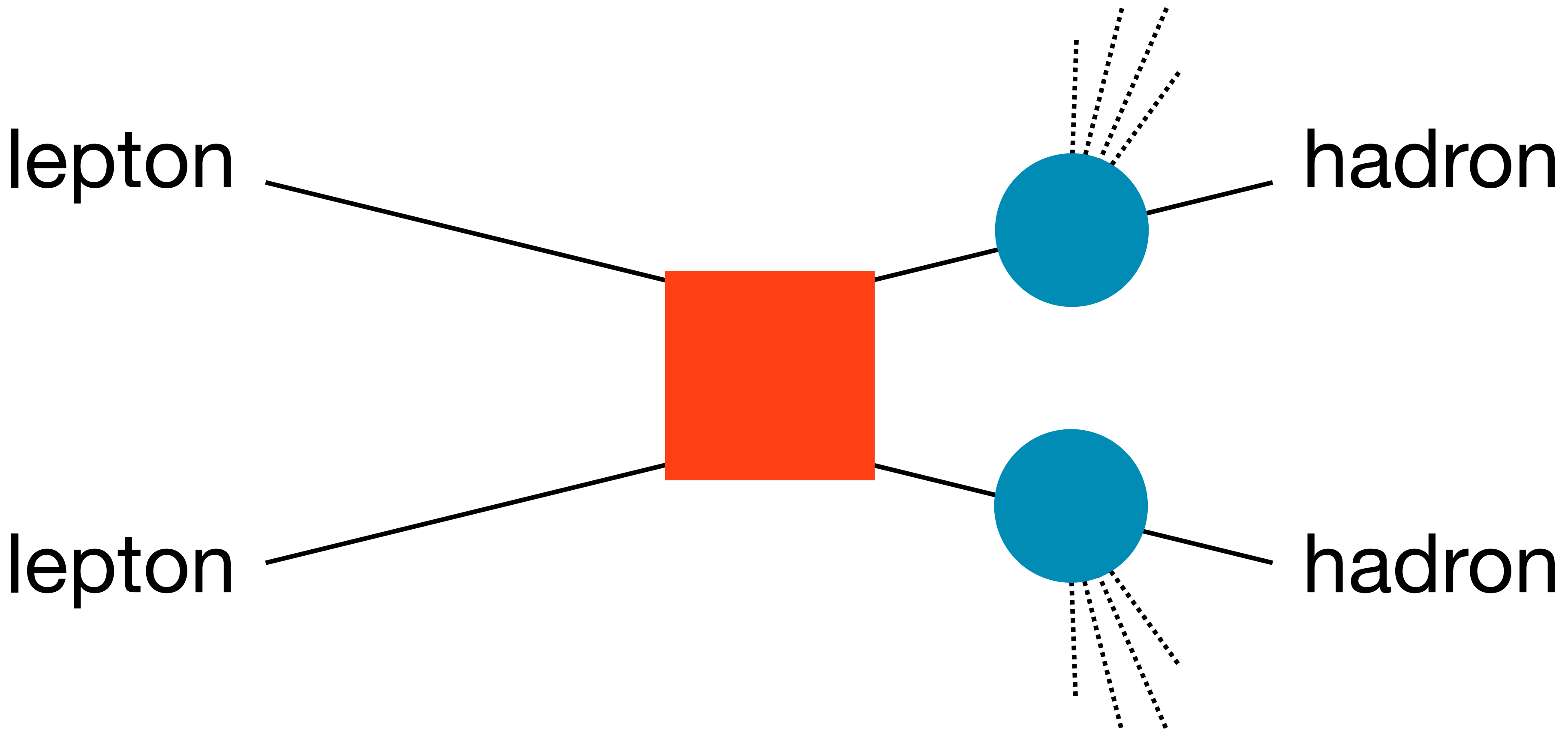


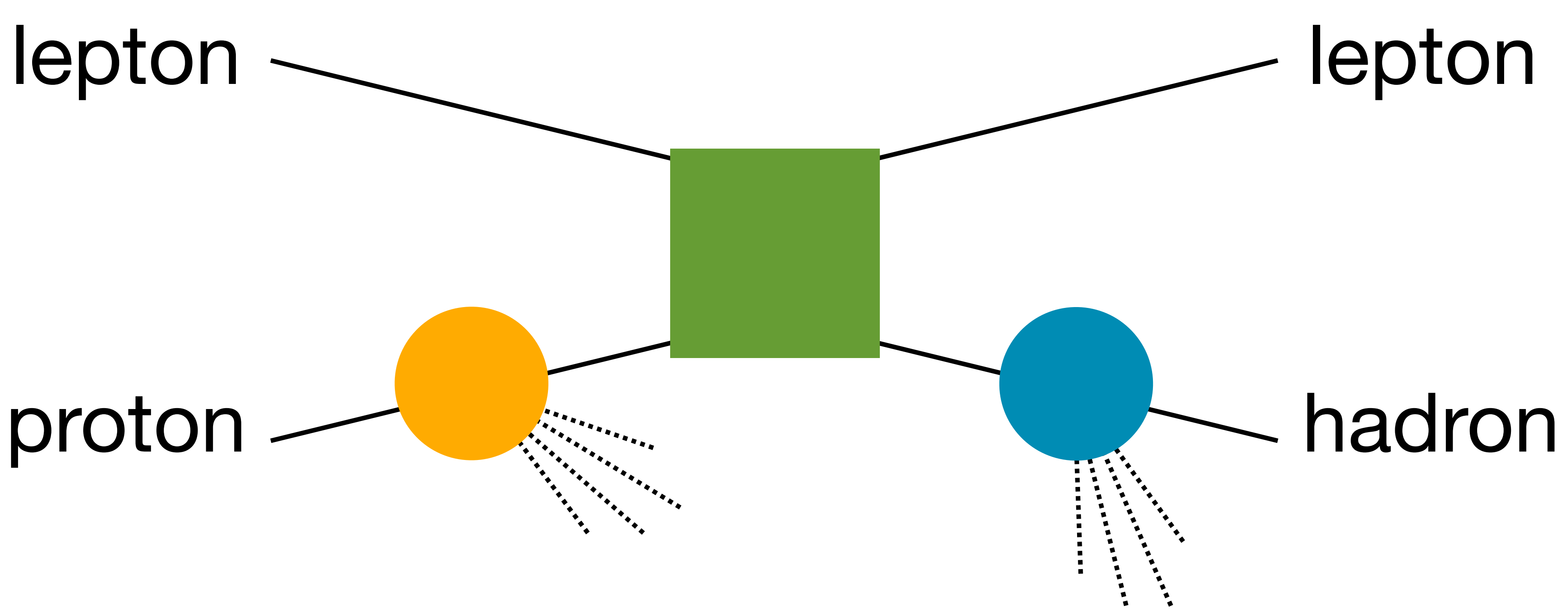
Drell-Yan



e^+e^- annihilation

Adapted from A. Bacchetta

Factorisation and universality



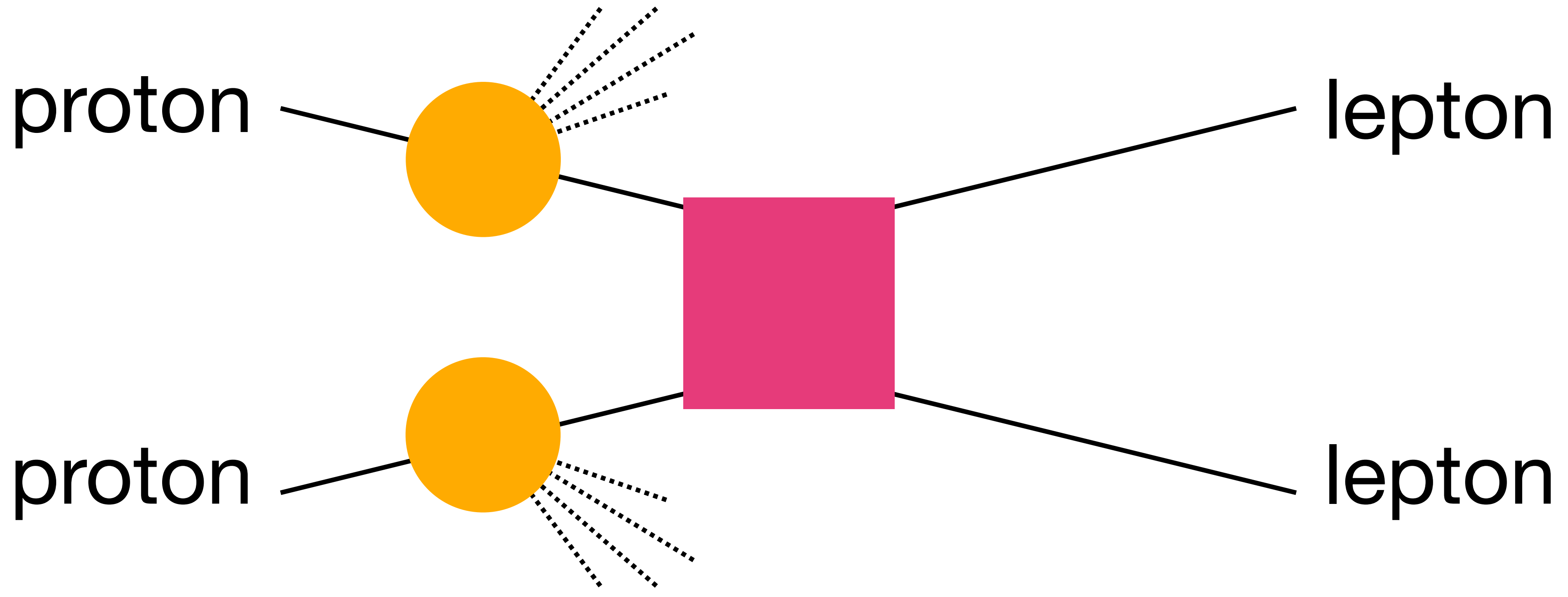
lepton

lepton

proton

hadron

semi-inclusive DIS



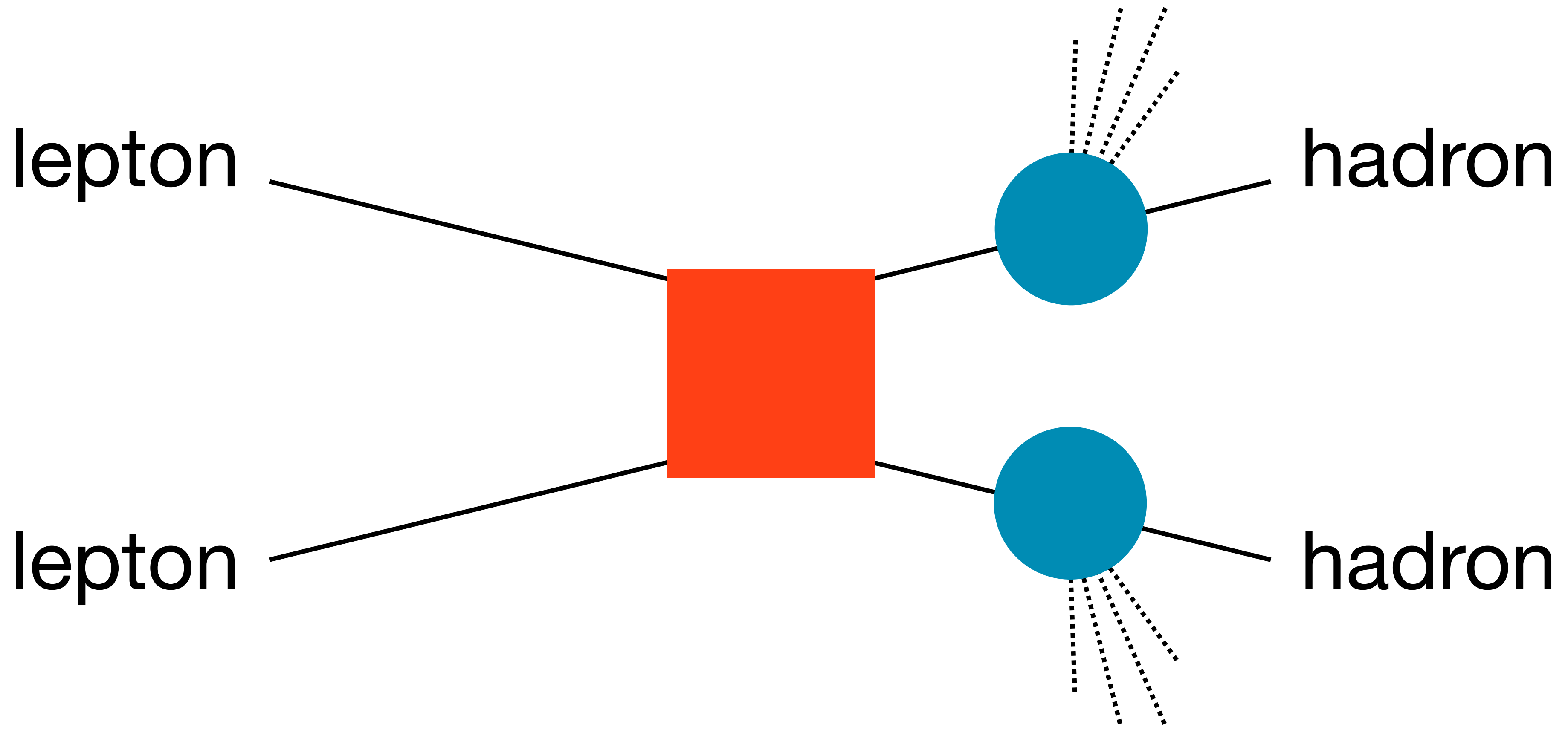
proton

lepton

proton

lepton

Drell-Yan



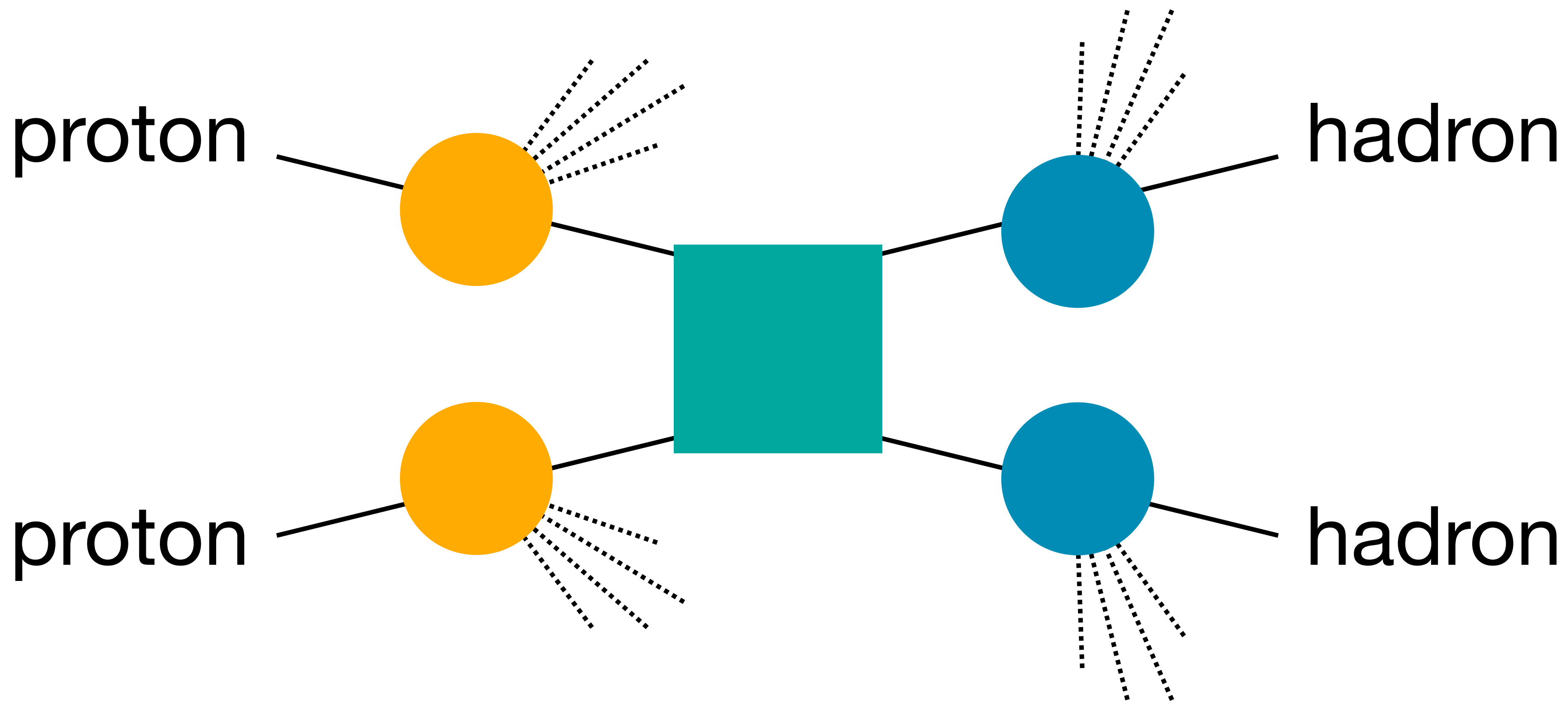
lepton

hadron

lepton

hadron

e^+e^- annihilation



proton

hadron

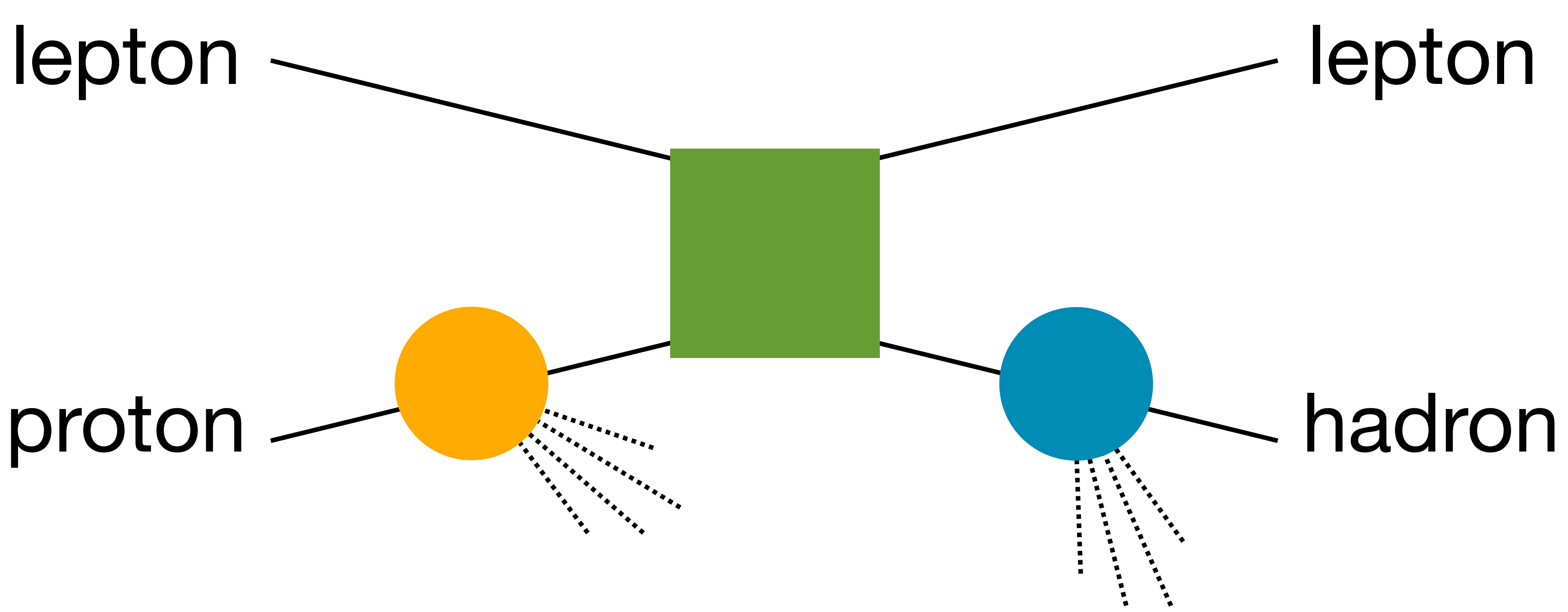
proton

hadron

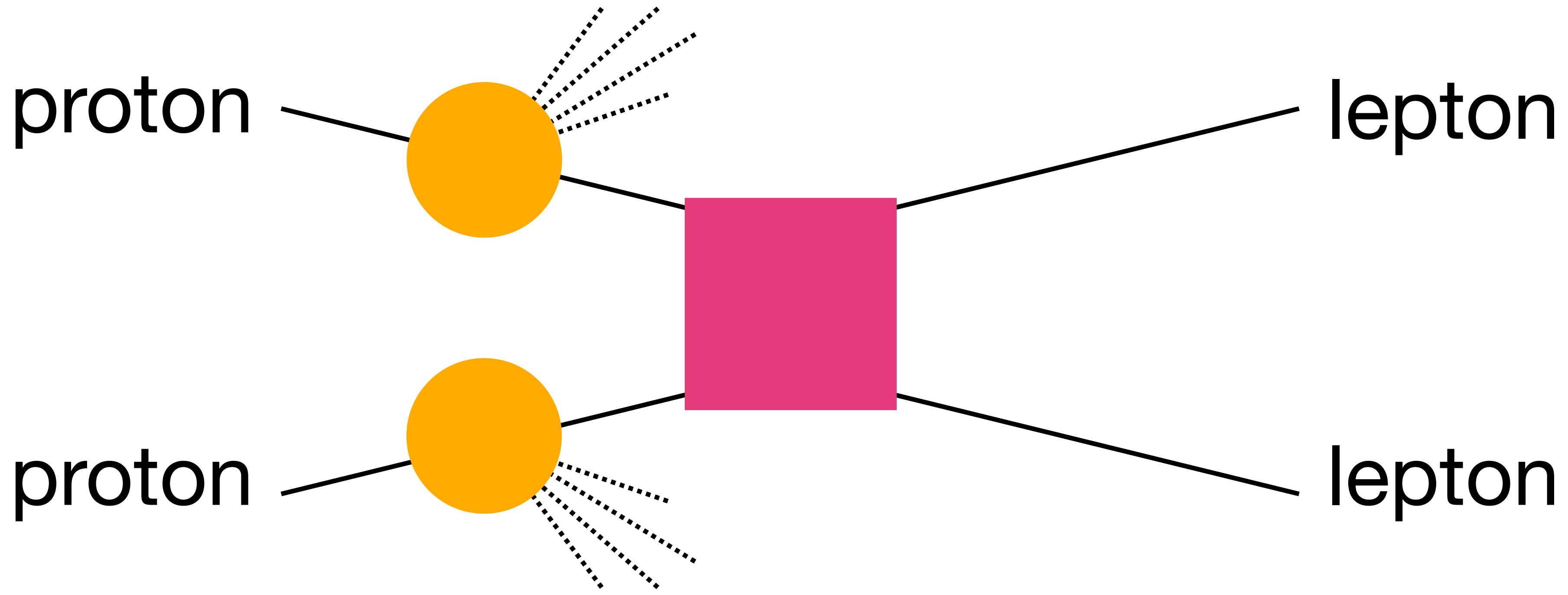
inclusive hadron production in pp collisions

Adapted from A. Bacchetta

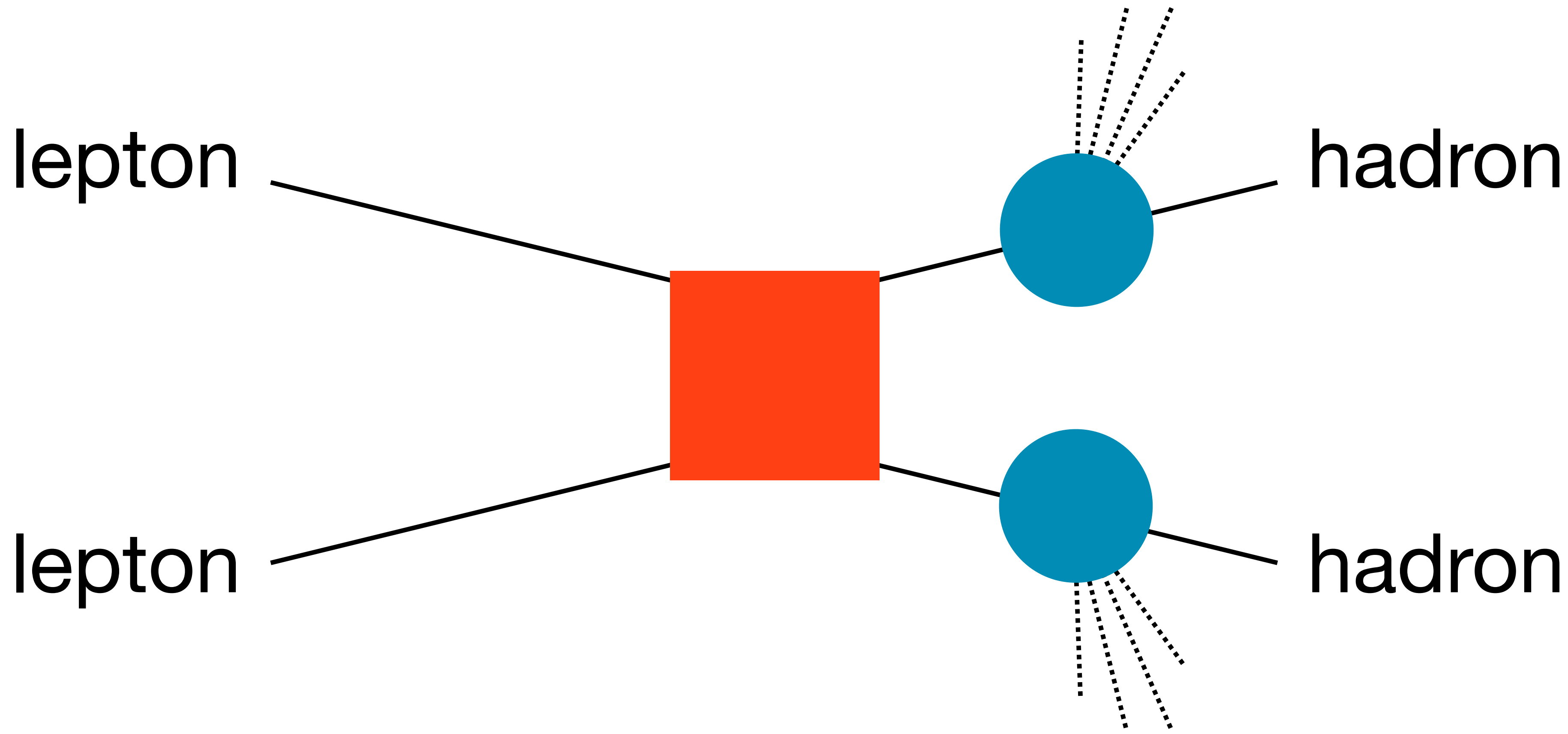
Factorisation and universality



semi-inclusive DIS



Drell-Yan



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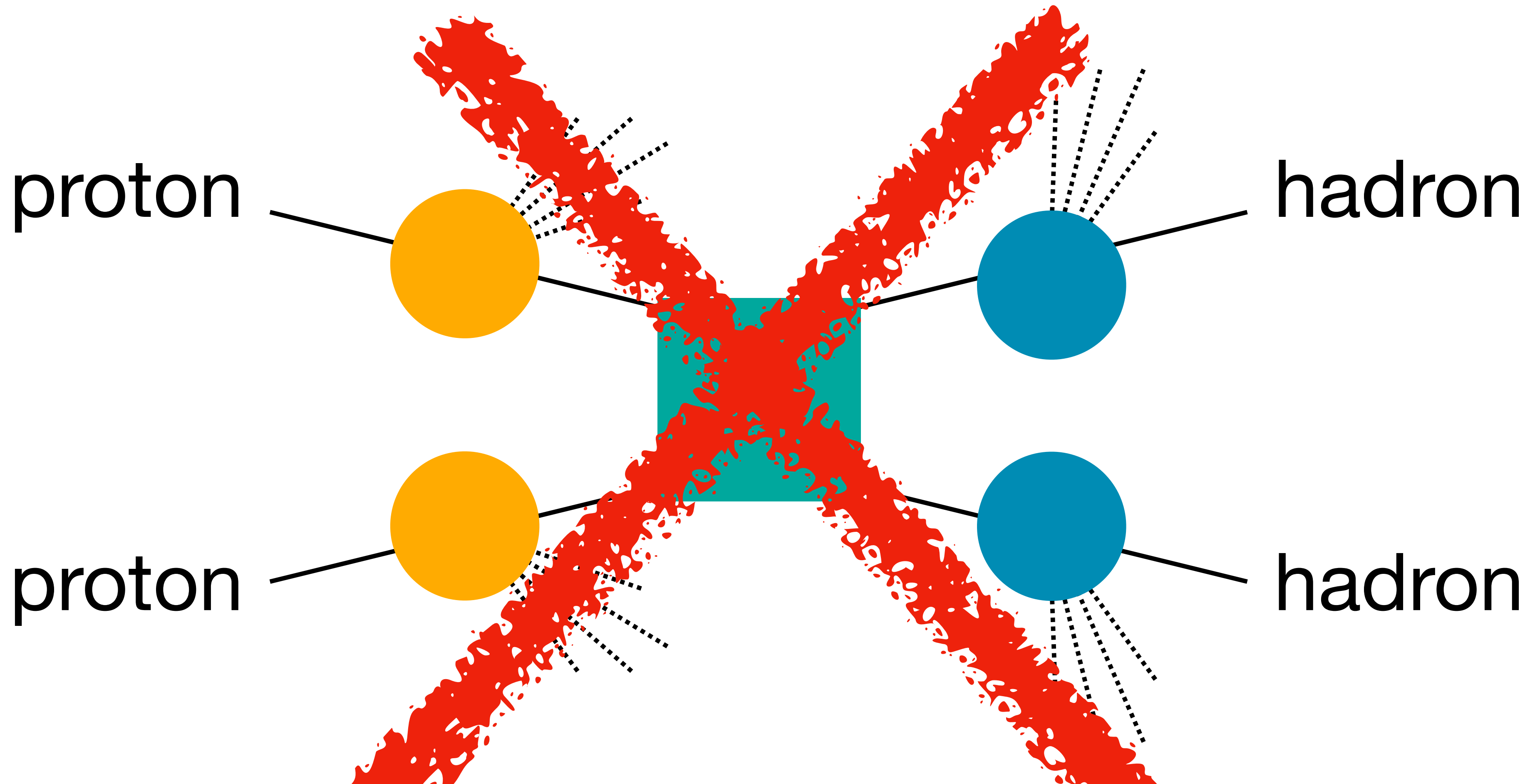
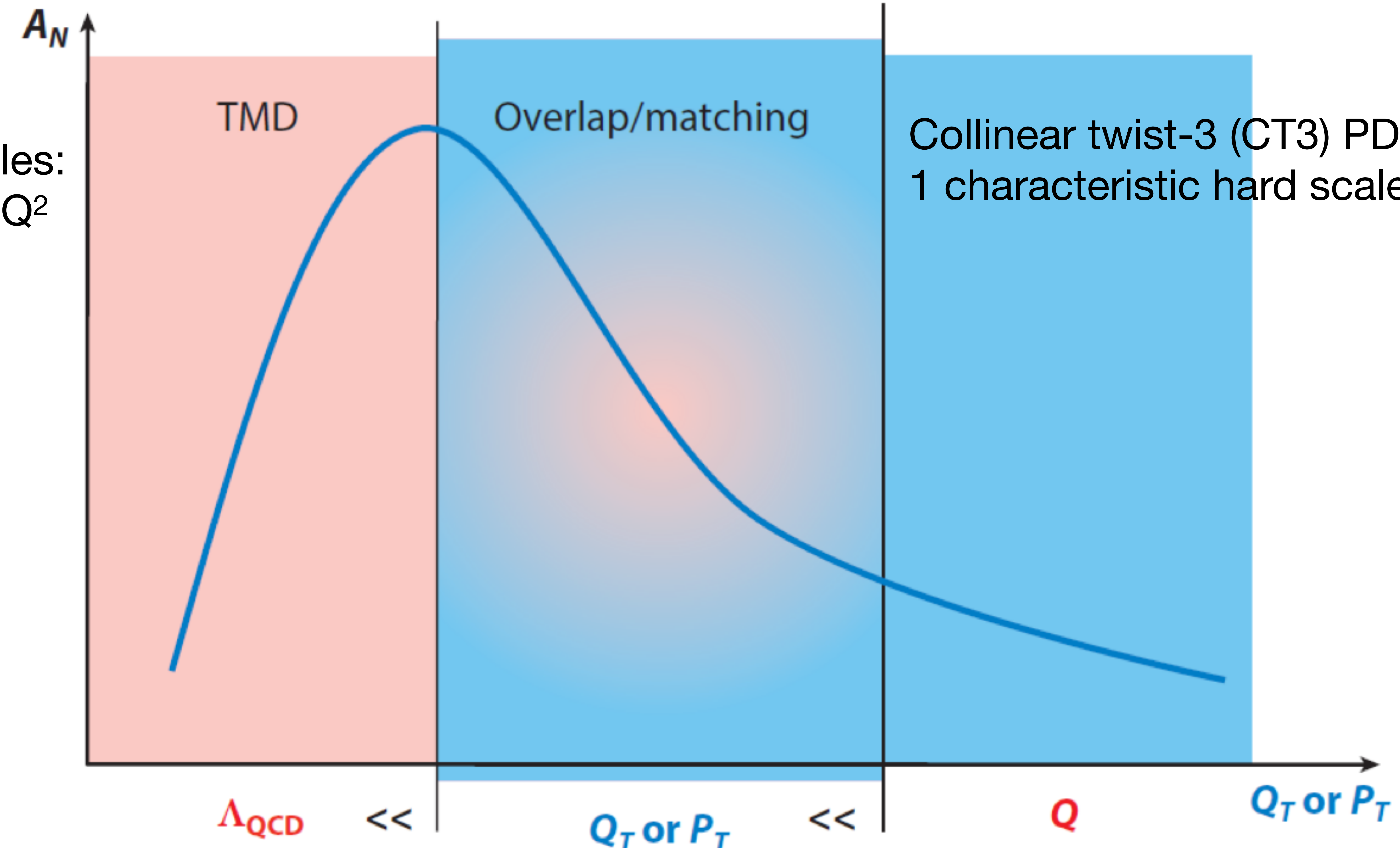


inclusive hadron production in pp collisions

Adapted from A. Bacchetta

Validity of TMD description

2 characteristic scales:
small P_{hT} and large Q^2



Collinear twist-3 (CT3) PDFs.
1 characteristic hard scale, e.g., P_{hT}

Consistent results for TMD
and CT3 in overlap region

Experiments investigating TMD PDFs and TMD FFs

$\vec{e} + \vec{p}, \vec{n}, \vec{d}$

$e^{+} + e^{-}$

$\vec{p}\vec{p}$ collider

$e^{+} + e^{-}$

COMPASS $\vec{\mu} + \vec{p}$ & $\pi + \vec{p}$

LHC $p + p$

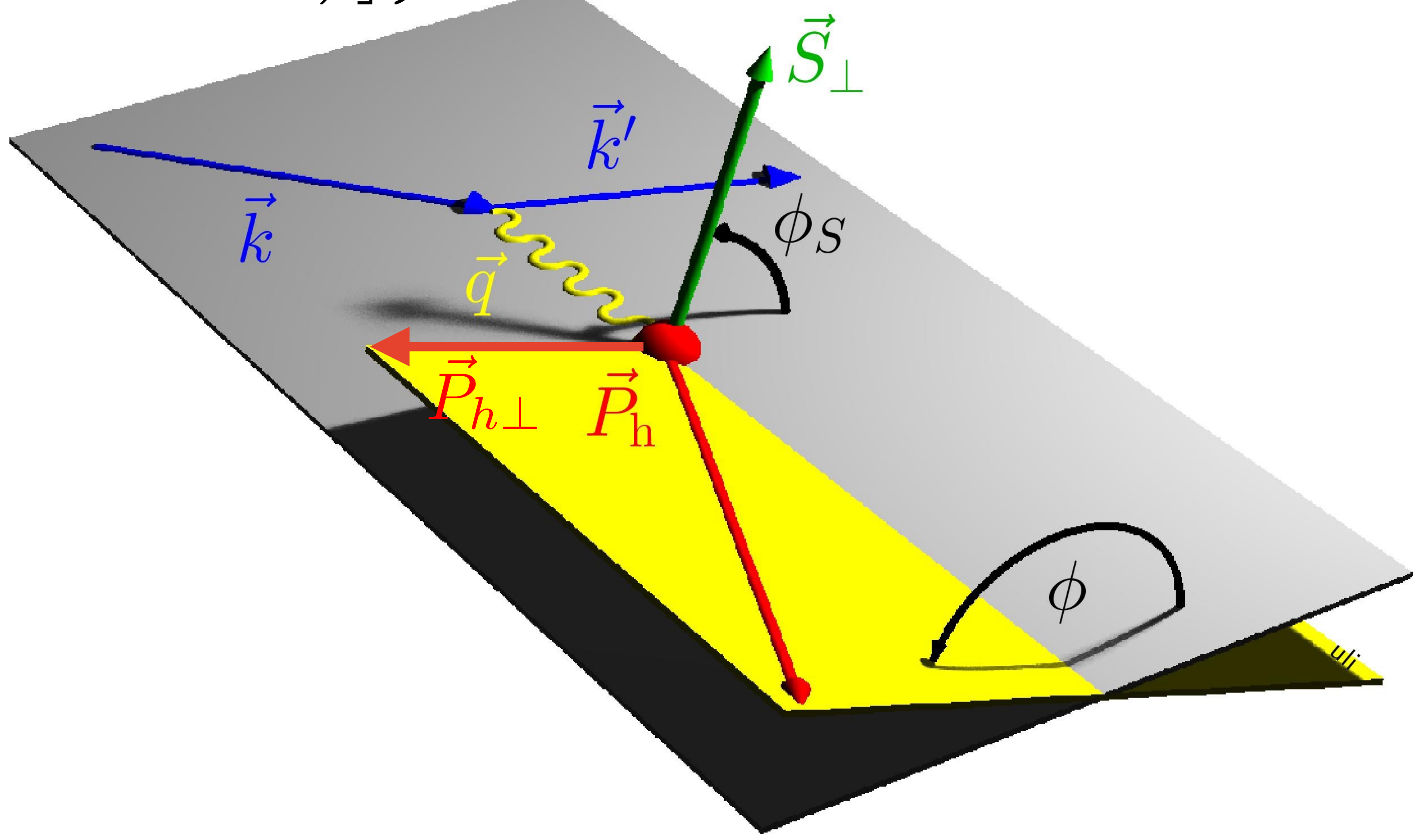
Tevatron: $p + \bar{p}$

$\vec{e} + \vec{p}, \vec{n}, \vec{d}$

Presented amplitudes

$$
\begin{aligned}
\sigma^h(\phi,\phi_S) &= \boxed{\sigma^h_{UU}}\Big\{1 + 2\langle\cos(\phi)\rangle^h_{UU}\ \cos(\phi) + \boxed{2\langle\cos(2\phi)\rangle^h_{UU}}\ \cos(2\phi) \\
&+ \lambda_l\, \boxed{2\langle\sin(\phi)\rangle^h_{LU}}\ \sin(\phi) \\
&+ S_L \Big[2\langle\sin(\phi)\rangle^h_{UL}\ \sin(\phi) + 2\langle\sin(2\phi)\rangle^h_{UL}\ \sin(2\phi) \\
&+ \lambda_l \Big(2\langle\cos(0\phi)\rangle^h_{LL}\ \cos(0\phi) + 2\langle\cos(\phi)\rangle^h_{LL}\ \cos(\phi)\Big)\Big] \\
&+ S_T \Big[\boxed{2\langle\sin(\phi-\phi_S)\rangle^h_{UT}}\ \sin(\phi-\phi_S) + \boxed{2\langle\ \sin(\phi+\phi_S)\rangle^h_{UT}}\ \sin(\phi+\phi_S) \\
&+ 2\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\ \sin(3\phi-\phi_S) + 2\langle\sin(\phi_S)\rangle^h_{UT}\ \sin(\phi_S) \\
&+ 2\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\ \sin(2\phi-\phi_S) \\
&+ \lambda_l \Big(2\langle\cos(\phi-\phi_S)\rangle^h_{LT}\ \cos(\phi-\phi_S) \\
&+ 2\langle\cos(\phi_S)\rangle^h_{LT}\ \cos(\phi_S) + 2\langle\cos(2\phi-\phi_S)\rangle^h_{LT}\ \cos(2\phi-\phi_S)\Big)\Big]\Big\}
\end{aligned}
$$

Presented here



Spin-independent TMD PDFs: global analysis

$e^+ + e^-$

$\vec{p}\,\vec{p}$ collider

Tevatron: $p + \bar{p}$

$\vec{e} + \vec{p}, \vec{n}, \vec{d}$

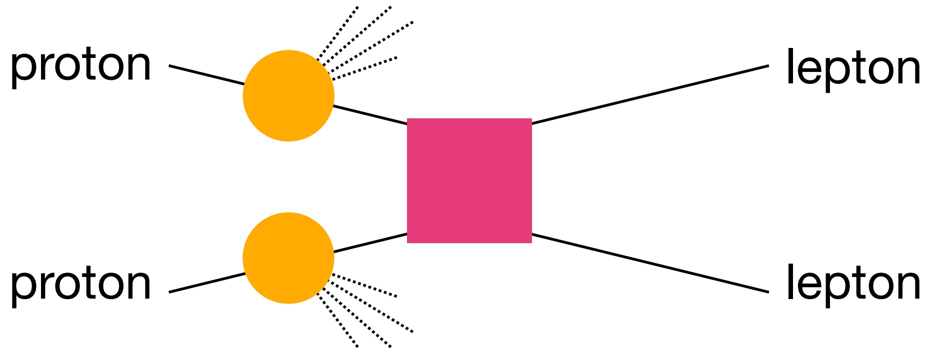
$\vec{e} + \vec{p}, \vec{n}, \vec{d}$

COMPASS $\vec{\mu} + \vec{p}$ & $\pi + \vec{p}$

LHC $p + p$

$e^+ + e^-$

Spin-independent TMD PDFs: global analysis



$e^+ + e^-$

SLAC NATIONAL ACCELERATOR LABORATORY

$\vec{p}\,\vec{p}$ collider

BROOKHAVEN NATIONAL LABORATORY

Jefferson Lab

$\vec{e} + \vec{p}, \vec{n}, \vec{d}$

Tevatron: $p + \bar{p}$

Fermilab

$\vec{e} + \vec{p}, \vec{n}, \vec{d}$

DESY

COMPASS $\vec{\mu} + \vec{p}$ & $\pi + \vec{p}$

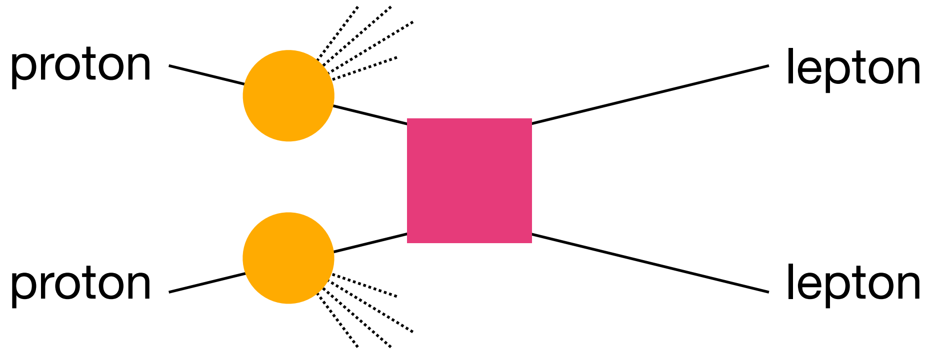
CERN

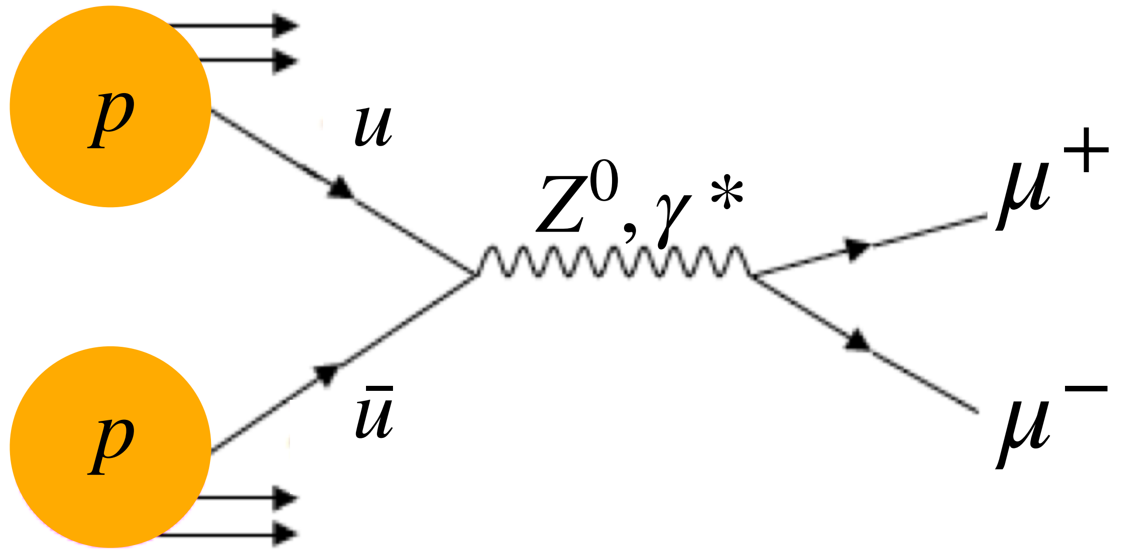
LHC $p + p$

KEK High Energy Accelerator Research Organization

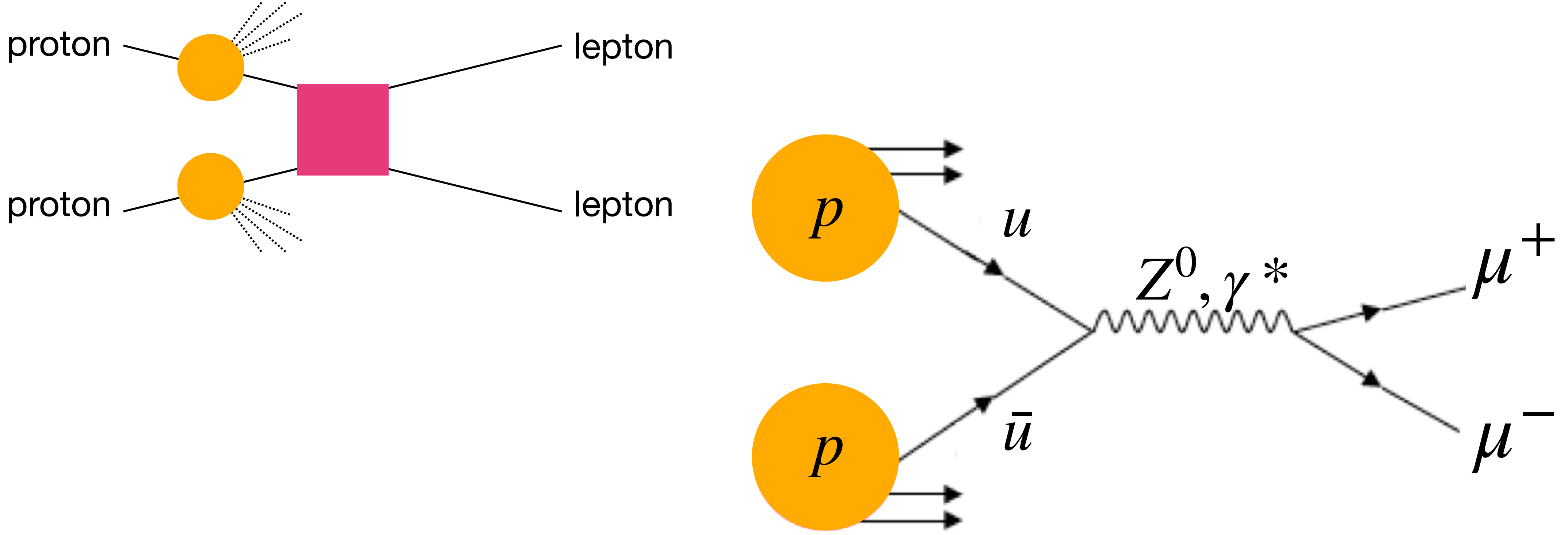
$e^+ + e^-$

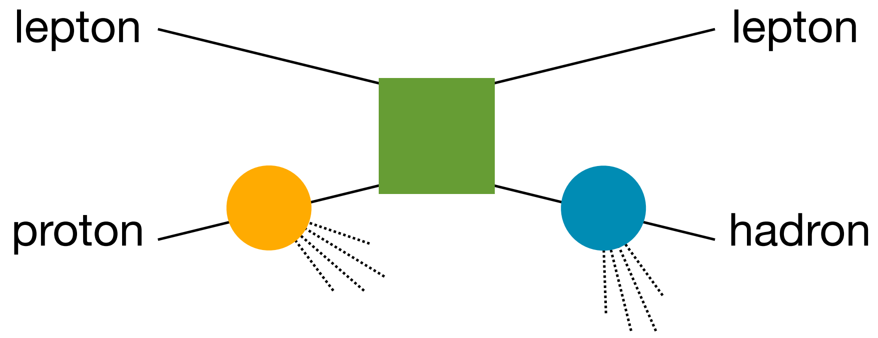
Spin-independent TMD PDFs: global analysis

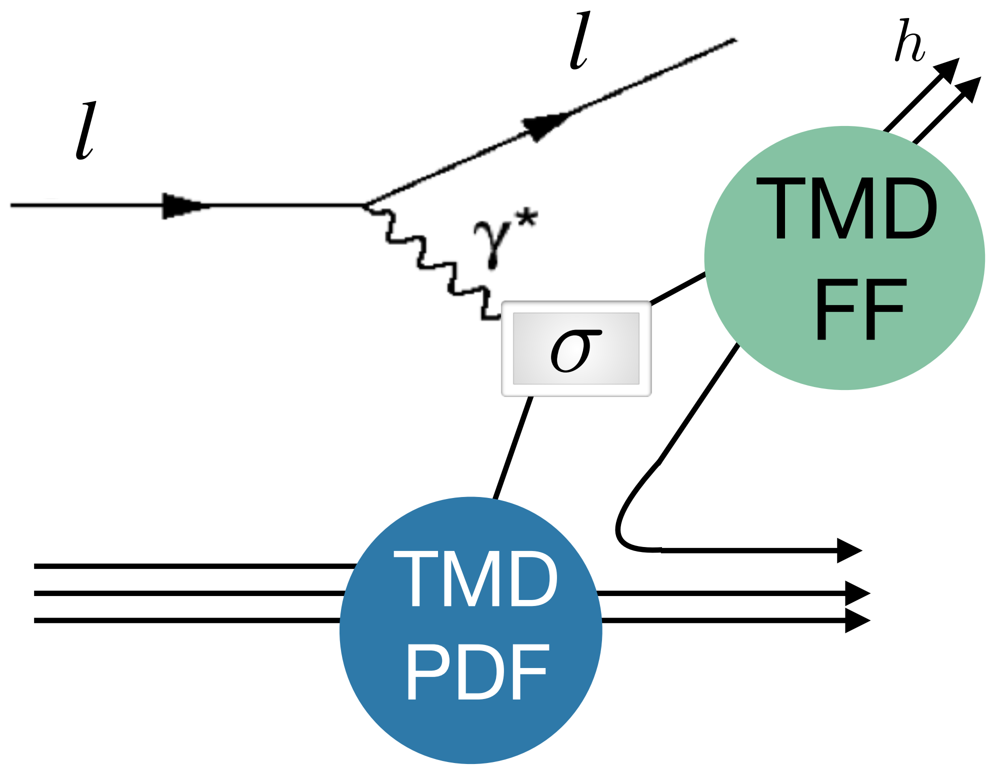




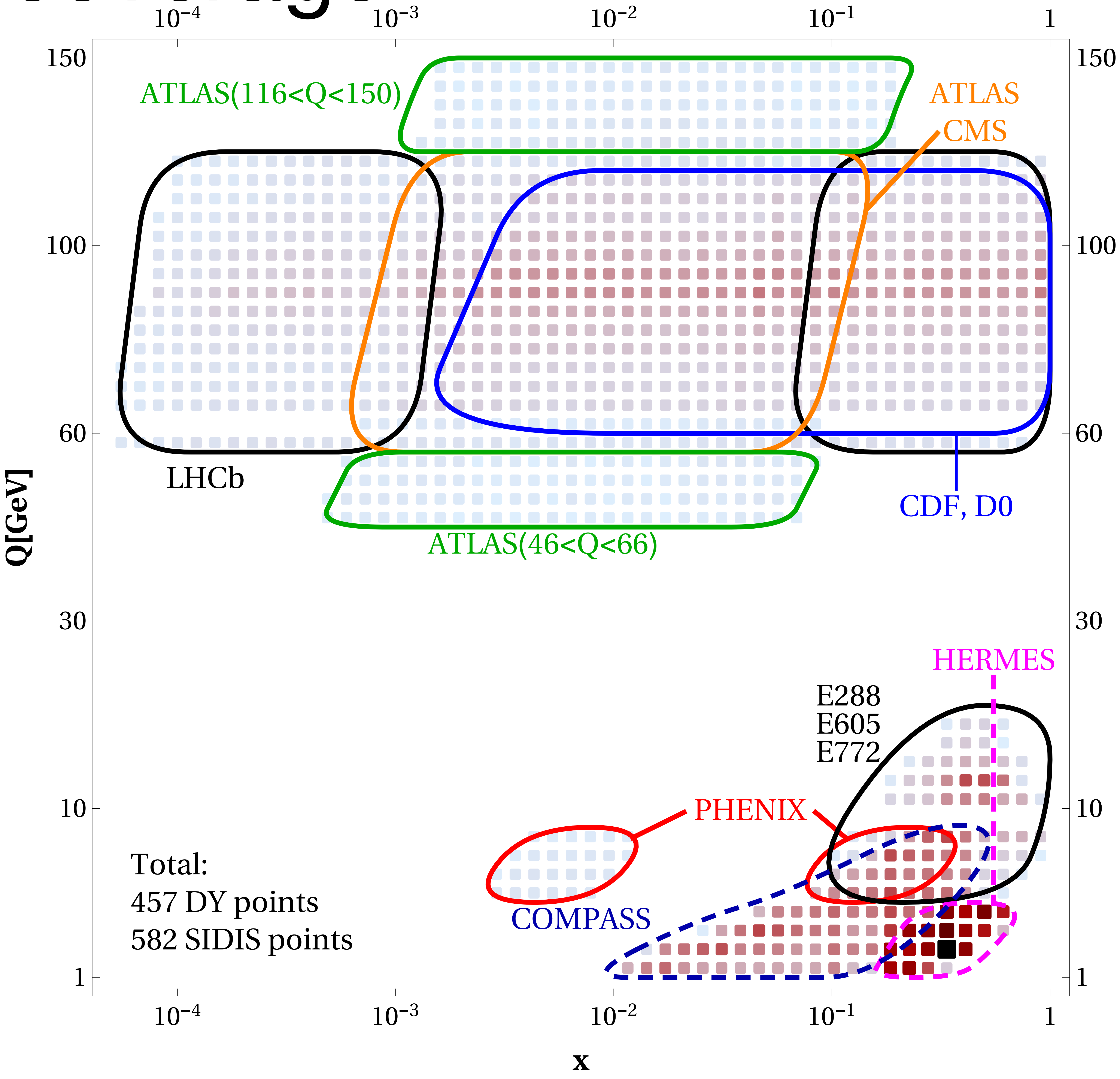
Spin-independent TMD PDFs: global analysis



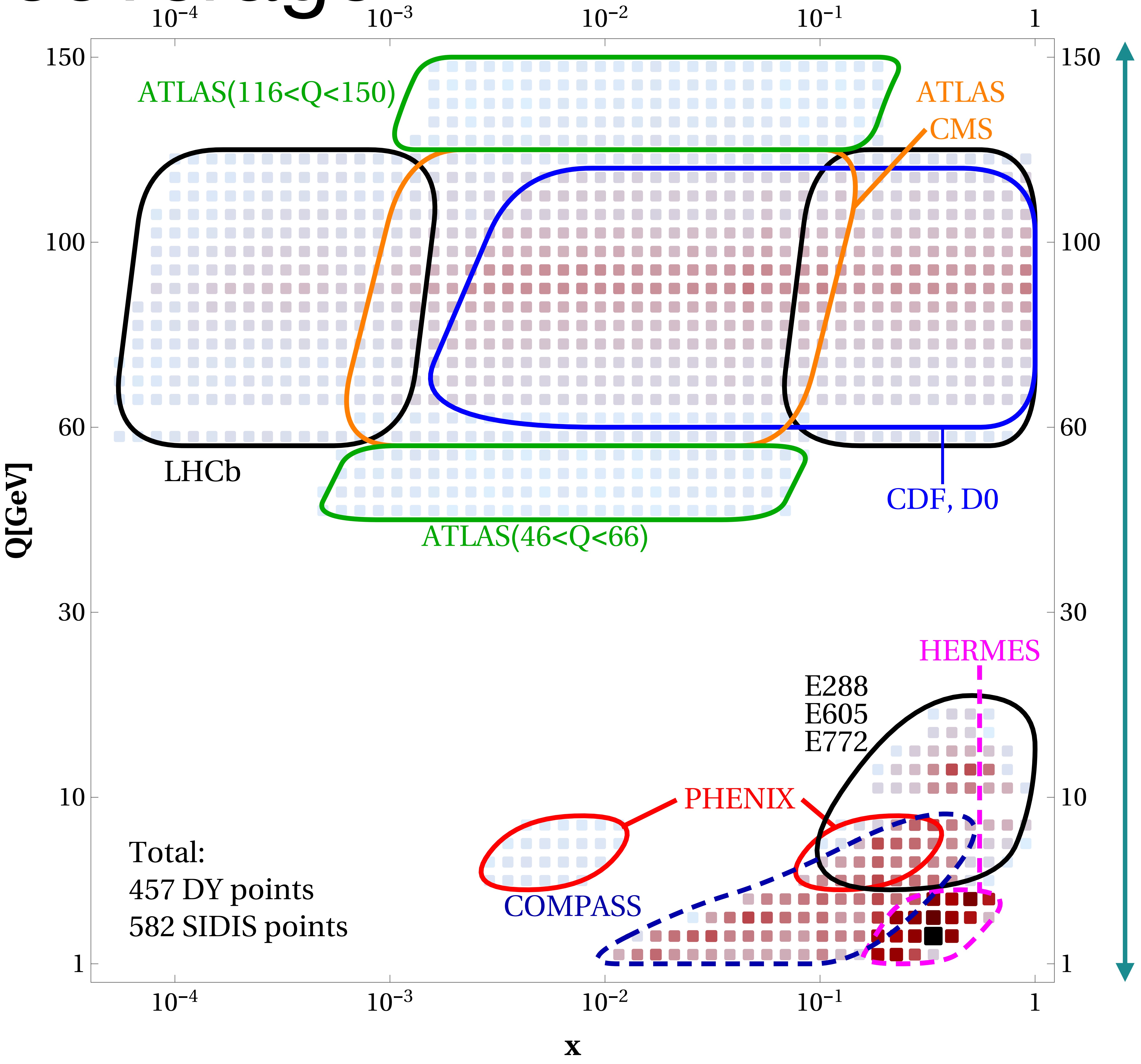




Kinematic coverage



Kinematic coverage

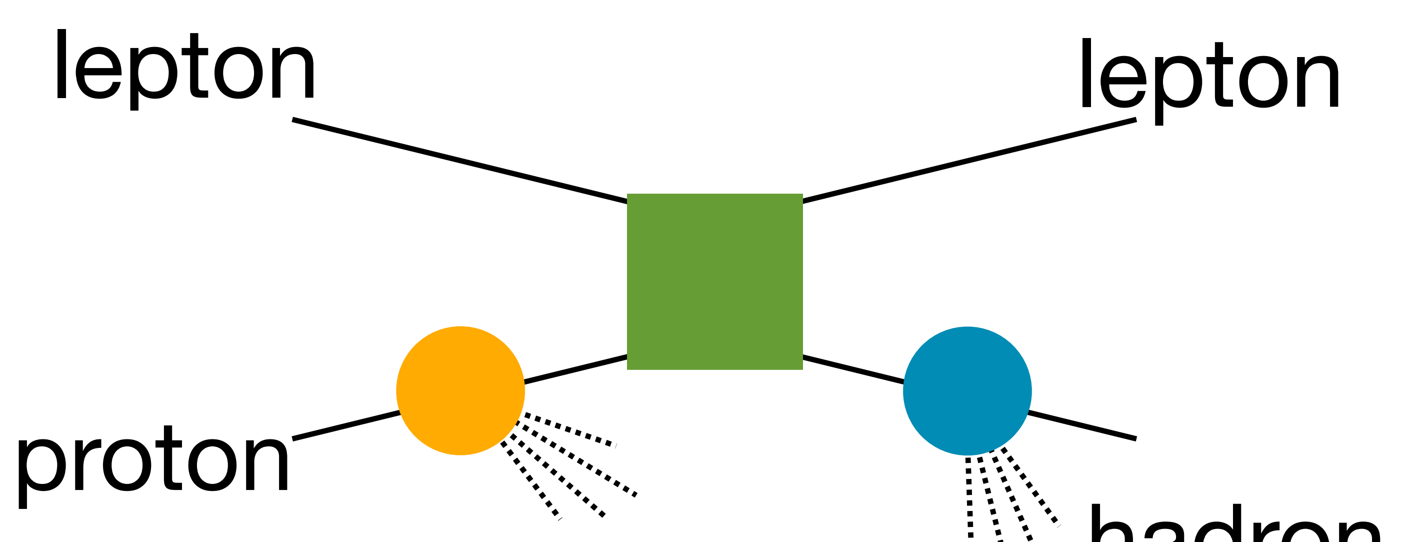


Large lever-arm in Q^2

→ Q^2 evolution of TMD PDF

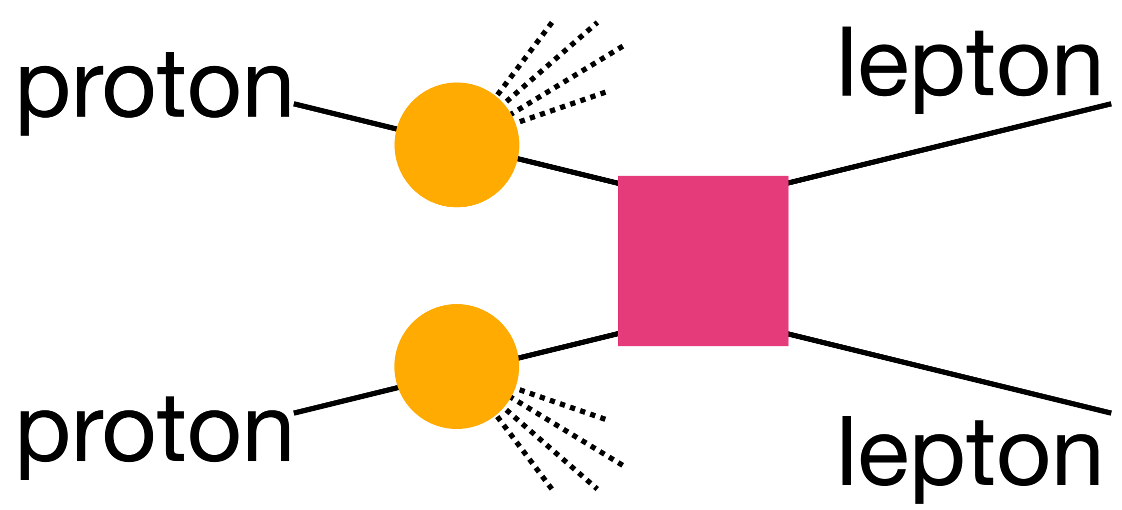
Spin-independent TMD PDFs: global analysis

I. Scimemi, A. Vladimirov JHEP **06** (2020)137



semi-inclusive DIS

Experiment	Reaction	ref.	Kinematics	N_{pt} after cuts
HERMES	$p \to \pi^+$	[67]	$0.023 < x < 0.6$ (6 bins) $0.2 < z < 0.8$ (6 bins) $1.0 < Q < \sqrt{20}\,\text{GeV}$ $W^2 > 10\text{GeV}^2$ $0.1 < y < 0.85$	24
	$p \to \pi^-$			24
	$p \to K^+$			24
	$p \to K^-$			24
	$D \to \pi^+$			24
	$D \to \pi^-$			24
	$D \to K^+$			24
	$D \to K^-$			24
COMPASS	$d \to h^+$	[68]	$0.003 < x < 0.4$ (8 bins) $0.2 < z < 0.8$ (4 bins) $1.0 < Q \simeq 9\,\text{GeV}$ (5 bins)	195
	$d \to h^-$			195
Total				582

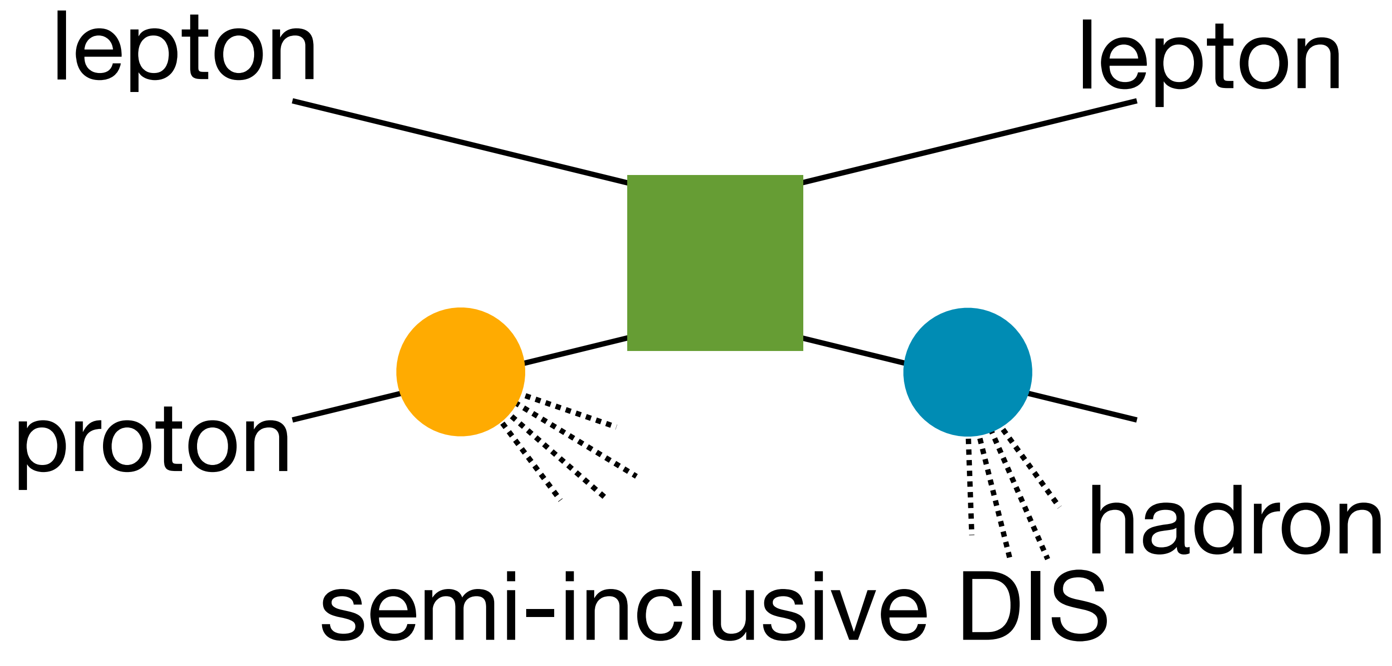


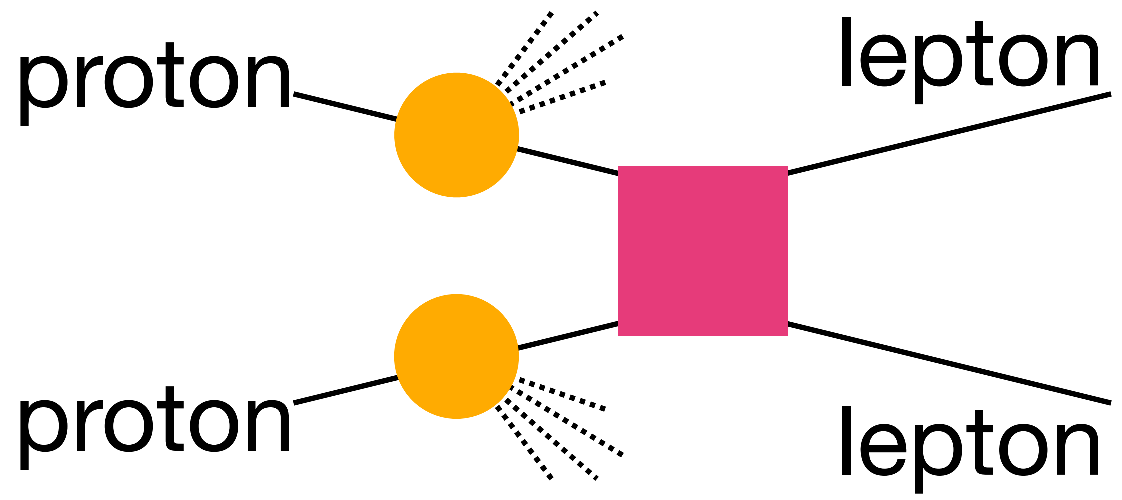
Drell-Yan

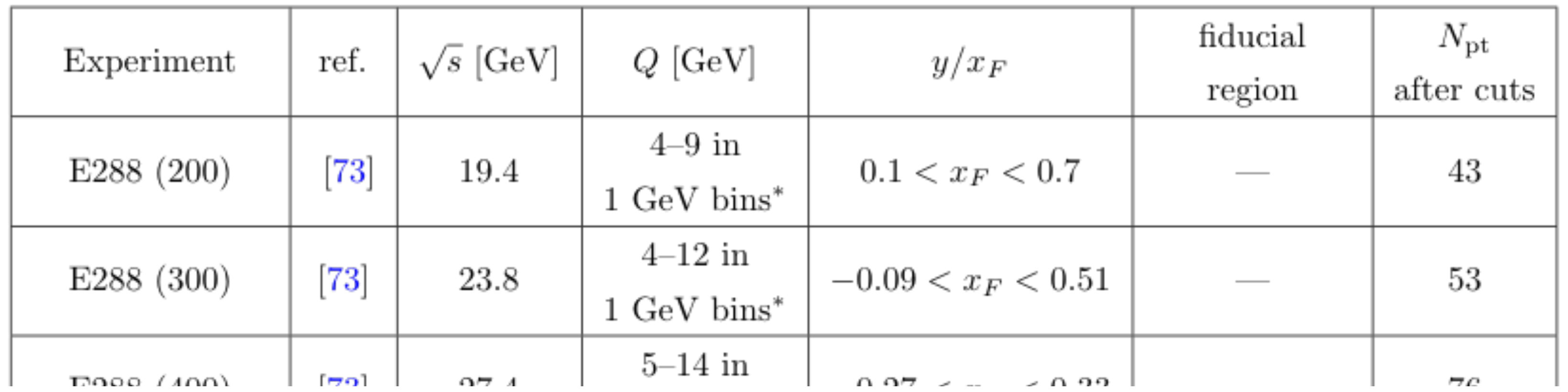
Experiment	ref.	$\sqrt{s}$ [GeV]	Q [GeV]	y/x_F	fiducial region	N_{pt} after cuts
E288 (200)	[73]	19.4	4–9 in 1 GeV bins*	$0.1 < x_F < 0.7$	—	43
E288 (300)	[73]	23.8	4–12 in 1 GeV bins*	$-0.09 < x_F < 0.51$	—	53
E288 (400)	[73]	27.4	5–14 in 1 GeV bins*	$-0.27 < x_F < 0.33$	—	76
E605	[74]	38.8	7–18 in 5 bins*	$-0.1 < x_F < 0.2$	—	53
E772	[75]	38.8	5–15 in 8 bins*	$0.1 < x_F < 0.3$	—	35
PHENIX	[76]	200	4.8–8.2	$1.2 < y < 2.2$	—	3
CDF (run1)	[77]	1800	66–116	—	—	33
CDF (run2)	[78]	1960	66–116	—	–	39
D0 (run1)	[79]	1800	75–105	—	—	16
D0 (run2)	[80]	1960	70–110	—	—	8
D0 (run2)$_\mu$	[81]	1960	65–115	$\lvert y\rvert < 1.7$	$p_T > 15$ GeV $\lvert\eta\rvert < 1.7$	3
ATLAS (7 TeV)	[47]	7000	66–116	$\lvert y\rvert < 1$ $1 < \lvert y\rvert < 2$ $2 < \lvert y\rvert < 2.4$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.4$	15
ATLAS (8 TeV)	[48]	8000	66–116	$\lvert y\rvert < 2.4$ in 6 bins	$p_T > 20$ GeV $\lvert\eta\rvert < 2.4$	30
ATLAS (8 TeV)	[48]	8000	46–66	$\lvert y\rvert < 2.4$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.4$	3
ATLAS (8 TeV)	[48]	8000	116–150	$\lvert y\rvert < 2.4$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.4$	7
CMS (7 TeV)	[49]	7000	60–120	$\lvert y\rvert < 2.1$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.1$	8
CMS (8 TeV)	[50]	8000	60–120	$\lvert y\rvert < 2.1$	$p_T > 20$ GeV $\lvert\eta\rvert < 2.1$	8
LHCb (7 TeV)	[82]	7000	60–120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	8
LHCb (8 TeV)	[83]	8000	60–120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	7
LHCb (13 TeV)	[84]	13000	60–120	$2 < y < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	9
Total						457

Spin-independent TMD PDFs: global analysis

I. Scimemi, A. Vladimirov JHEP **06** (2020)137

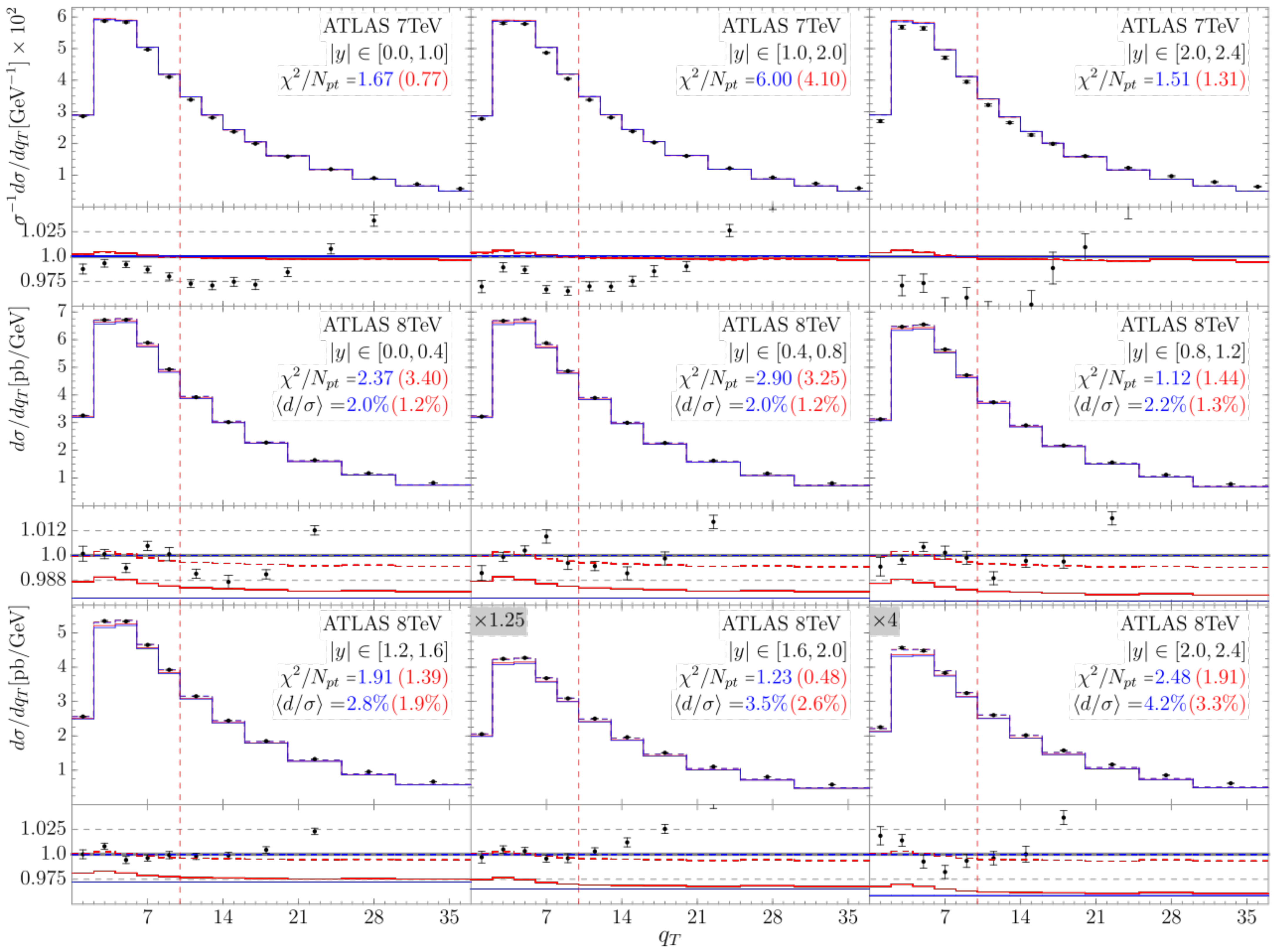




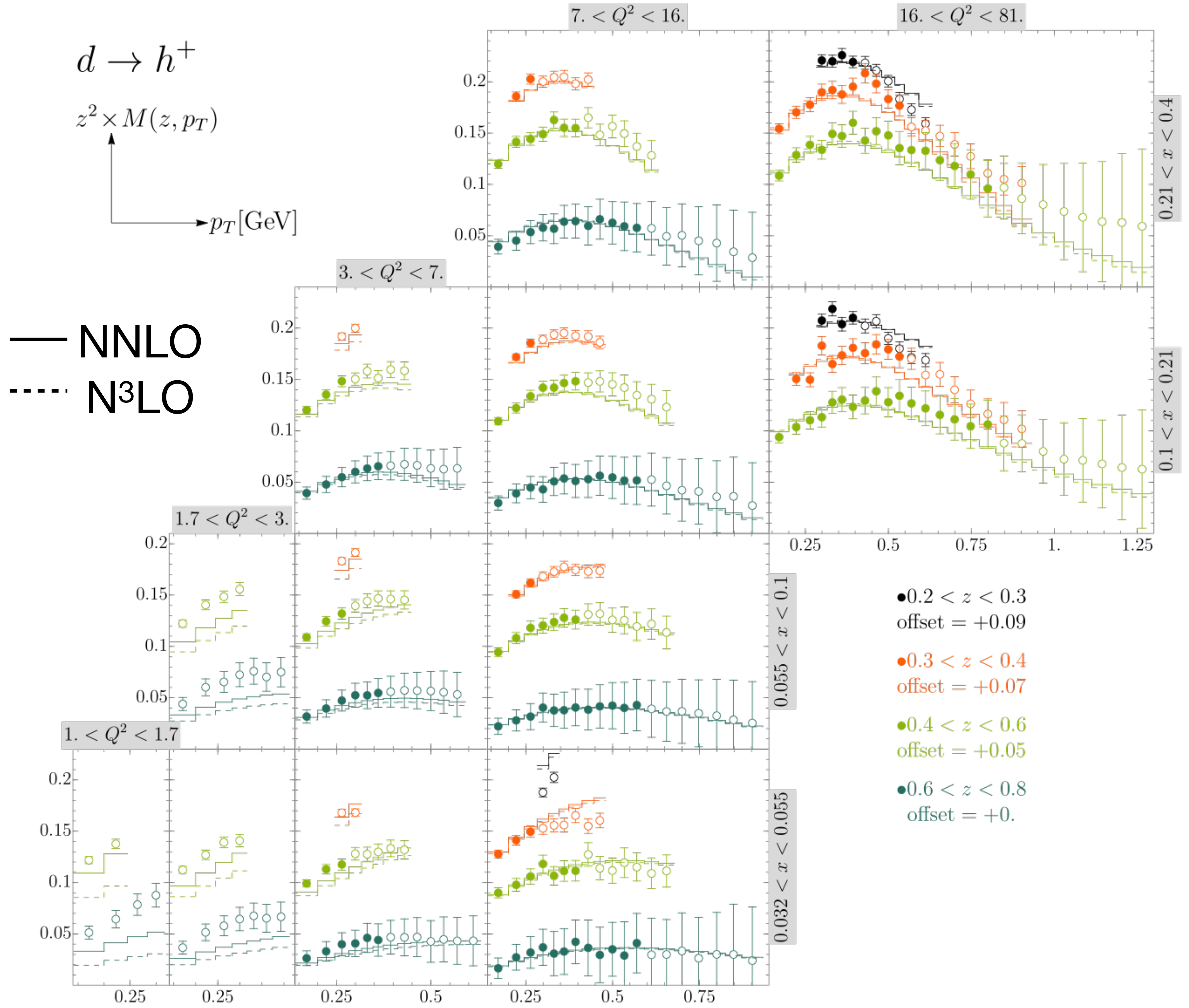


Experiment	ref.	$\sqrt{s}$ [GeV]	Q [GeV]	y/x_F	fiducial region	N_{pt} after cuts
E288 (200)	[73]	19.4	4–9 in 1 GeV bins*	$0.1 < x_F < 0.7$	—	43
E288 (300)	[73]	23.8	4–12 in 1 GeV bins*	$-0.09 < x_F < 0.51$	—	53
			5–14 in			
Total						457

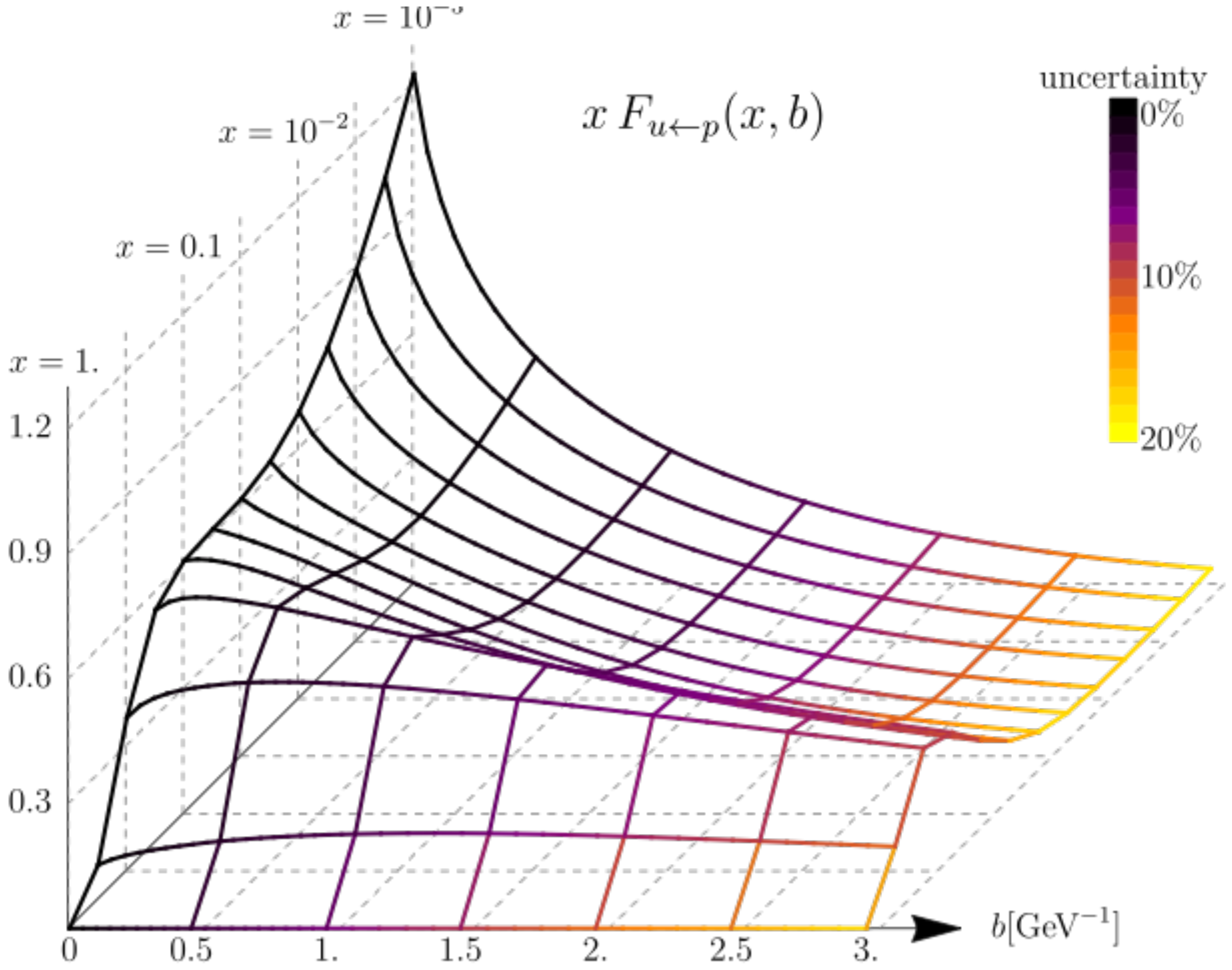
Description of the data

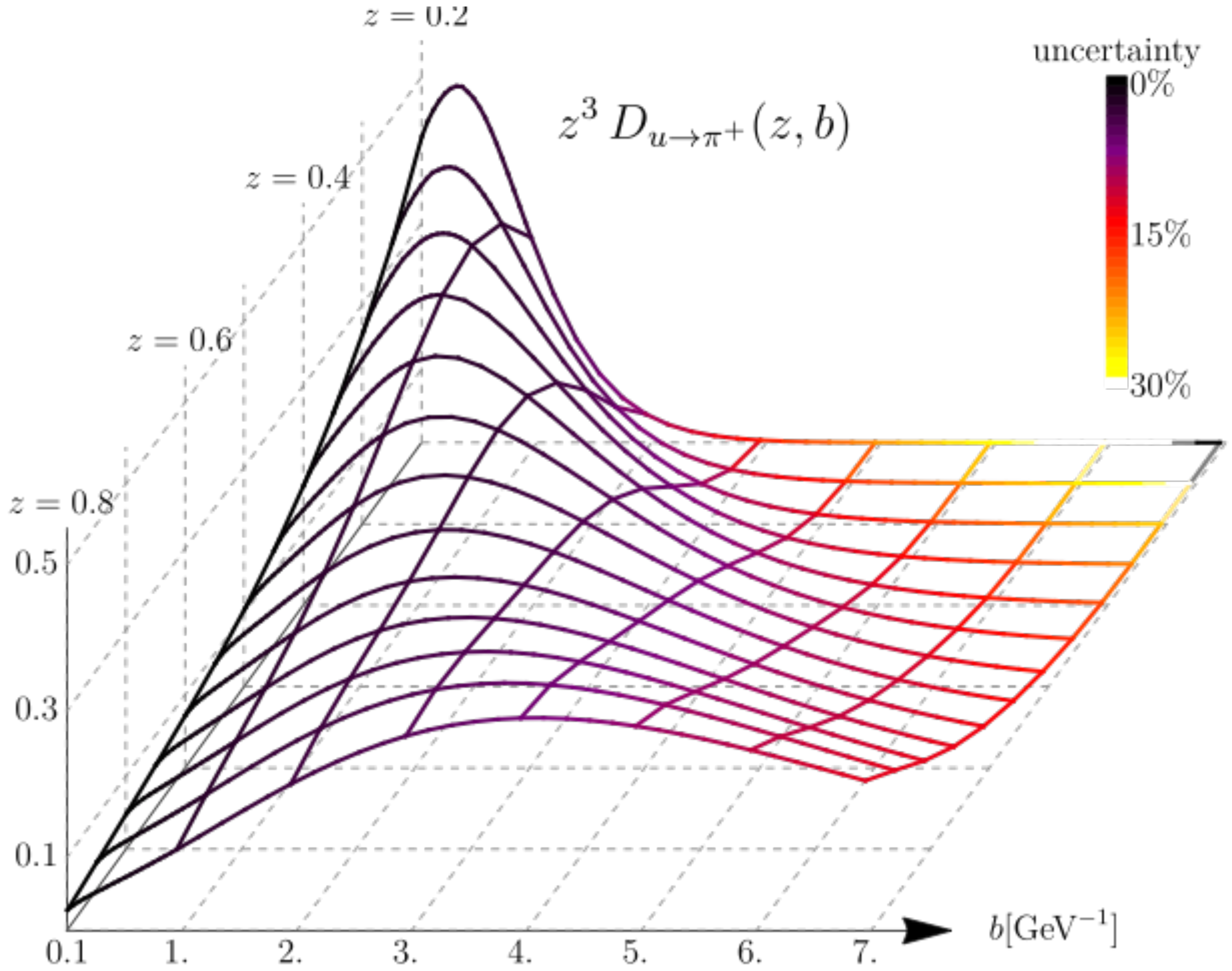


NNLO/N³LO

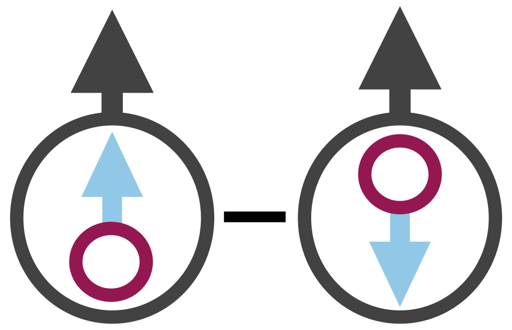


Spin-independent TMD PDFs: global analysis



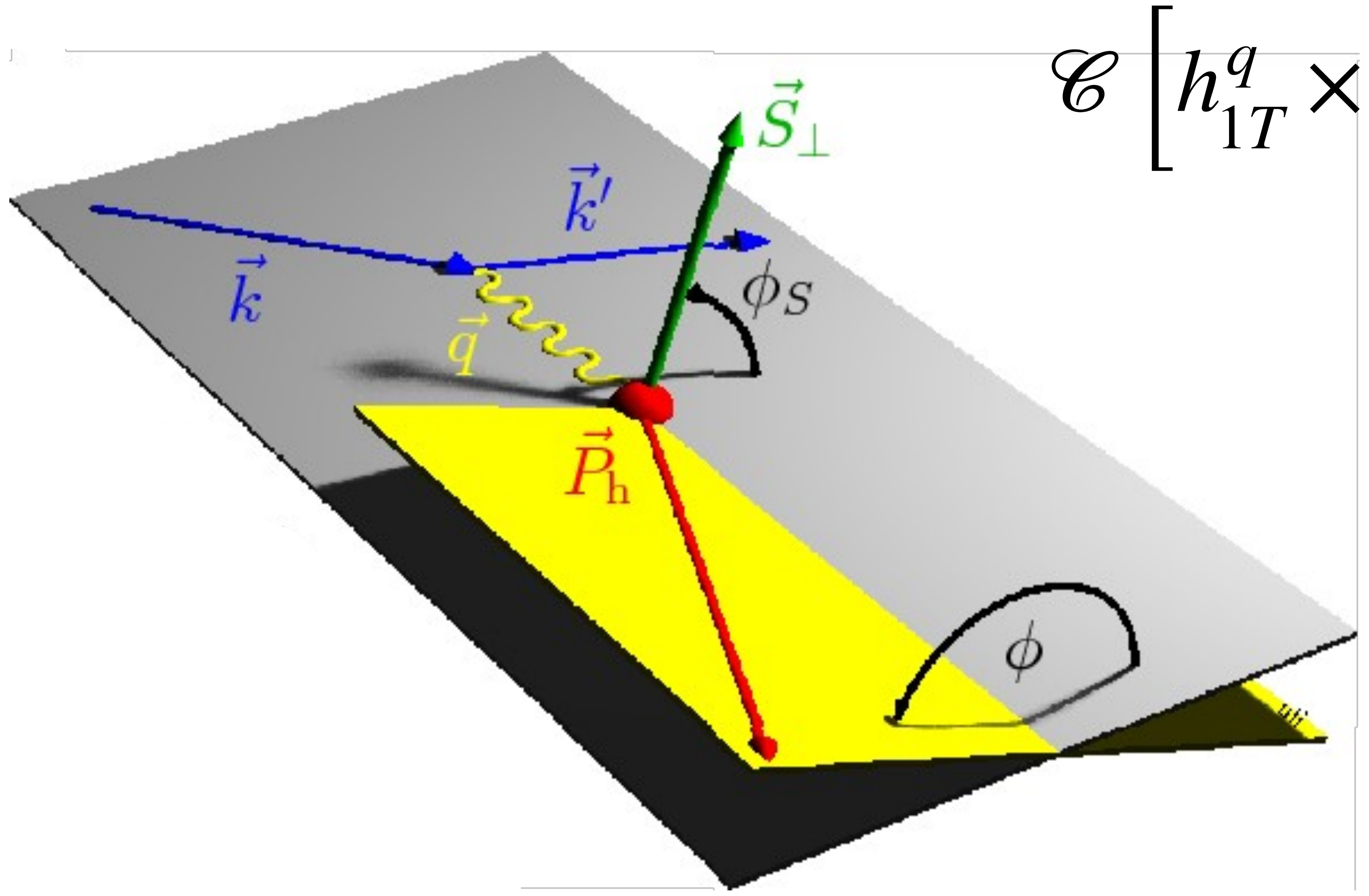


Collins amplitudes: measurement

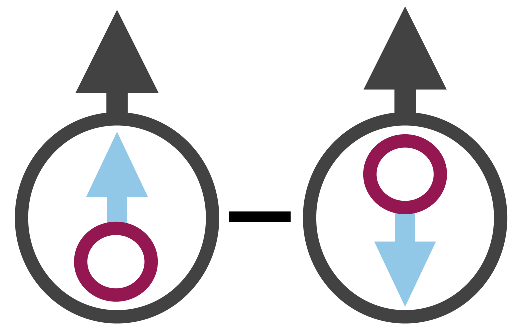


$\mathscr{C}\left[h_{1T}^{q} \times H_{1}^{\perp,q}\right]$

$$A_{UT} = \frac{1}{\langle|S_T|\rangle} \frac{N^{\uparrow}(\phi, \phi_S) - N^{\downarrow}(\phi, \phi_S)}{N^{\uparrow}(\phi, \phi_S) + N^{\downarrow}(\phi, \phi_S)}$$



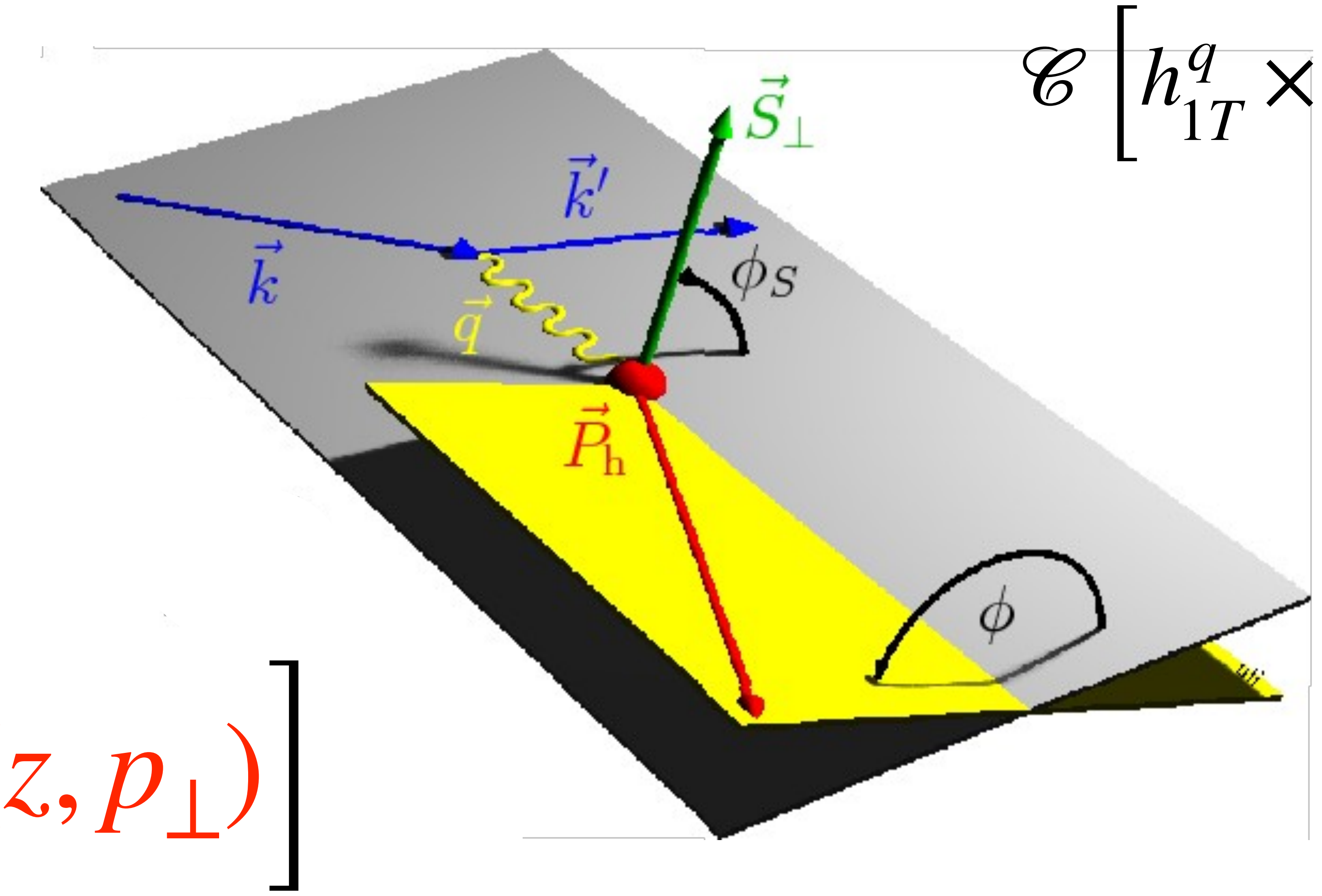
Collins amplitudes: measurement



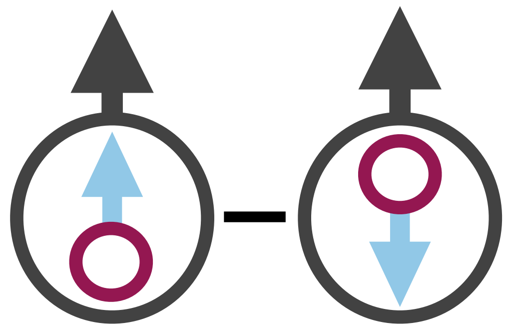
$\mathscr{C}\left[h_{1T}^{q} \times H_1^{\perp,q}\right]$

$$A_{UT} = \frac{1}{\langle|S_T|\rangle}\frac{N^{\uparrow}(\phi,\phi_S) - N^{\downarrow}(\phi,\phi_S)}{N^{\uparrow}(\phi,\phi_S) + N^{\downarrow}(\phi,\phi_S)}$$

$$\sim \sin(\phi + \phi_S)\sum_q e_q^2\, C\left[h_{1T}^{q}(x,k_\perp) \times H_1^{\perp,q}(z,p_\perp)\right]$$



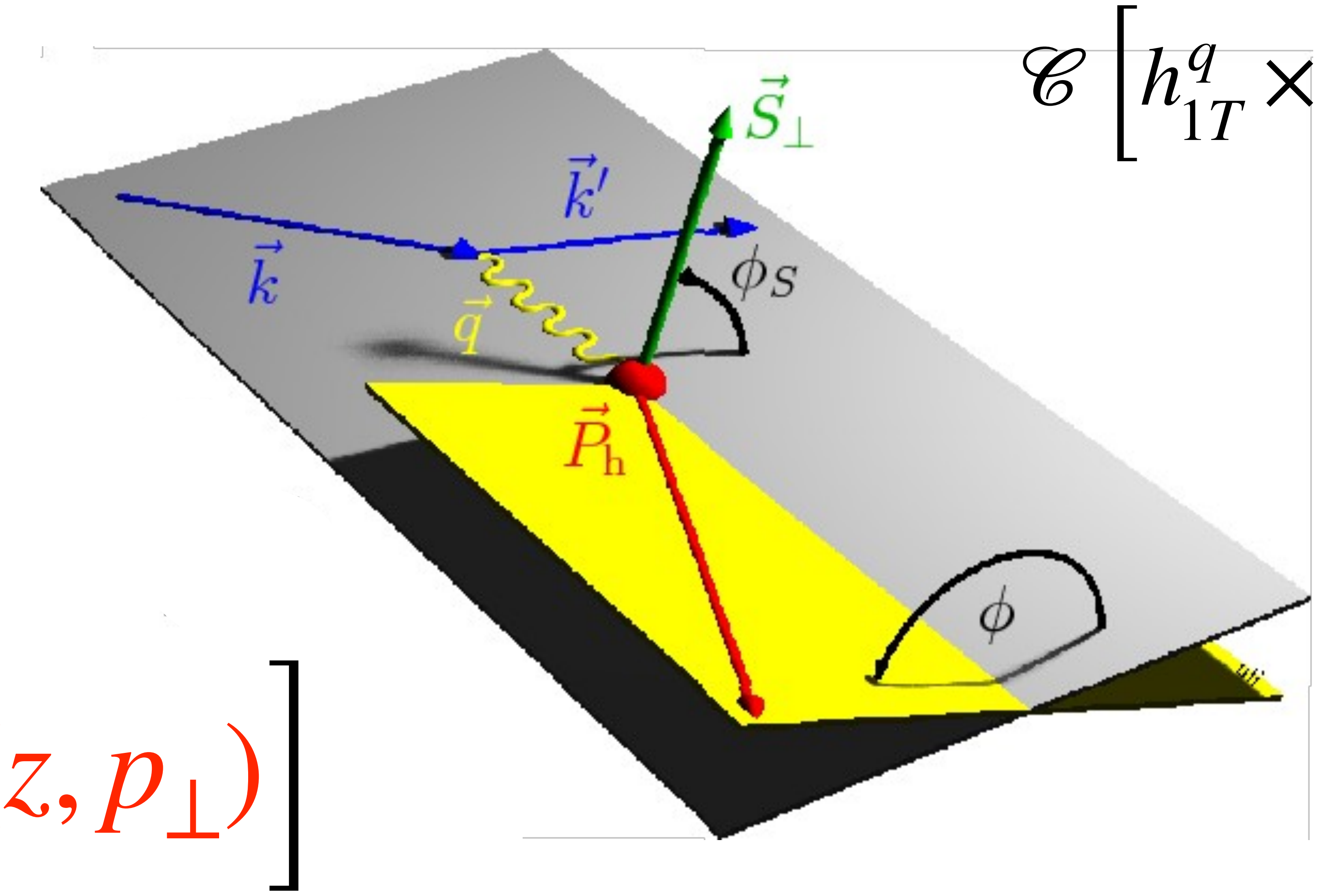
Collins amplitudes: measurement



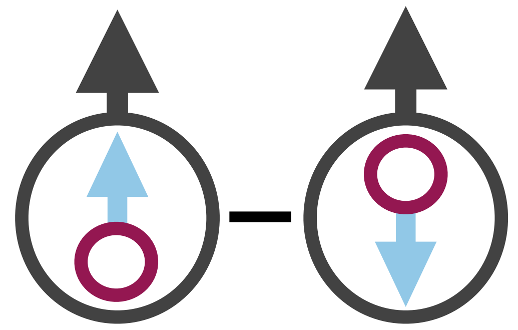
$\mathscr{C}\left[h_{1T}^{q} \times H_{1}^{\perp,q}\right]$

$$A_{UT} = \frac{1}{\langle|S_T|\rangle}\frac{N^{\uparrow}(\phi,\phi_S) - N^{\downarrow}(\phi,\phi_S)}{N^{\uparrow}(\phi,\phi_S) + N^{\downarrow}(\phi,\phi_S)}$$

$$\sim \sin(\phi + \phi_S)\sum_q e_q^2\, C\left[h_{1T}^{q}(x,k_\perp) \times H_1^{\perp,q}(z,p_\perp)\right]$$

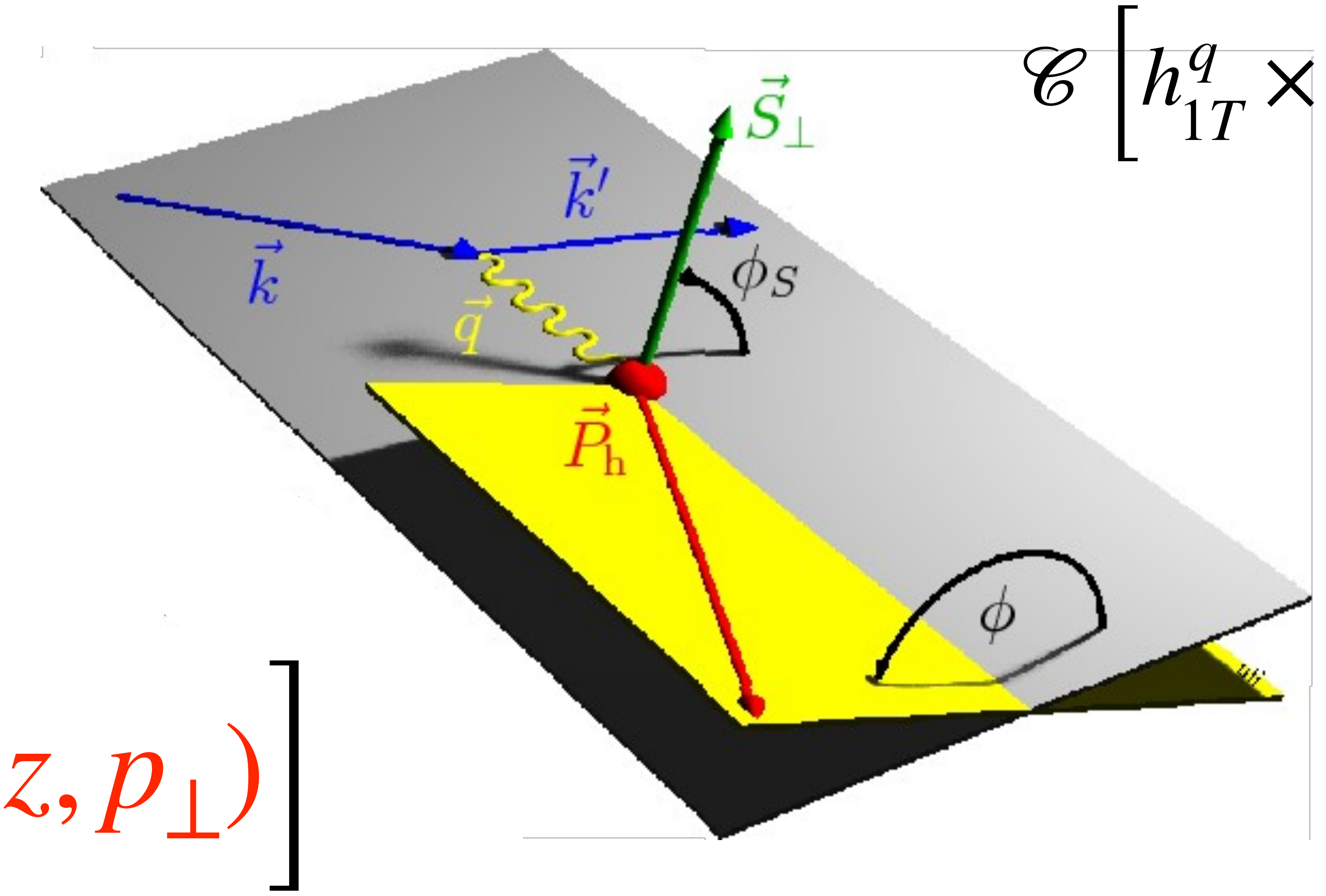


Collins amplitudes: measurement



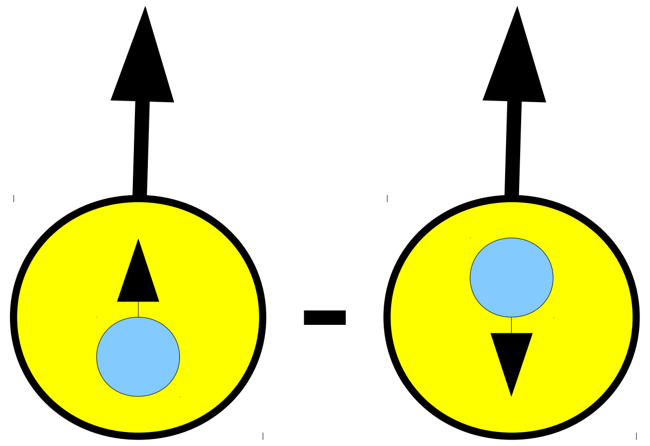
$$\mathscr{C}\left[h_{1T}^{q} \times H_{1}^{\perp,q}\right]$$

$$A_{UT} = \frac{1}{\langle|S_T|\rangle} \frac{N^{\uparrow}(\phi,\phi_S) - N^{\downarrow}(\phi,\phi_S)}{N^{\uparrow}(\phi,\phi_S) + N^{\downarrow}(\phi,\phi_S)}$$

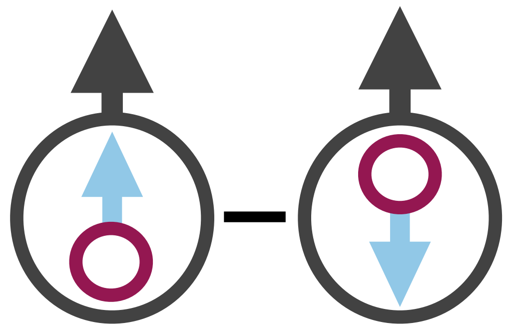


$$\sim \sin(\phi + \phi_S) \sum_q e_q^2 \, C\left[h_{1T}^{q}(x, k_\perp) \times H_1^{\perp,q}(z, p_\perp)\right]$$

$h_{1T}^{q}(x, k_\perp)$: transversity

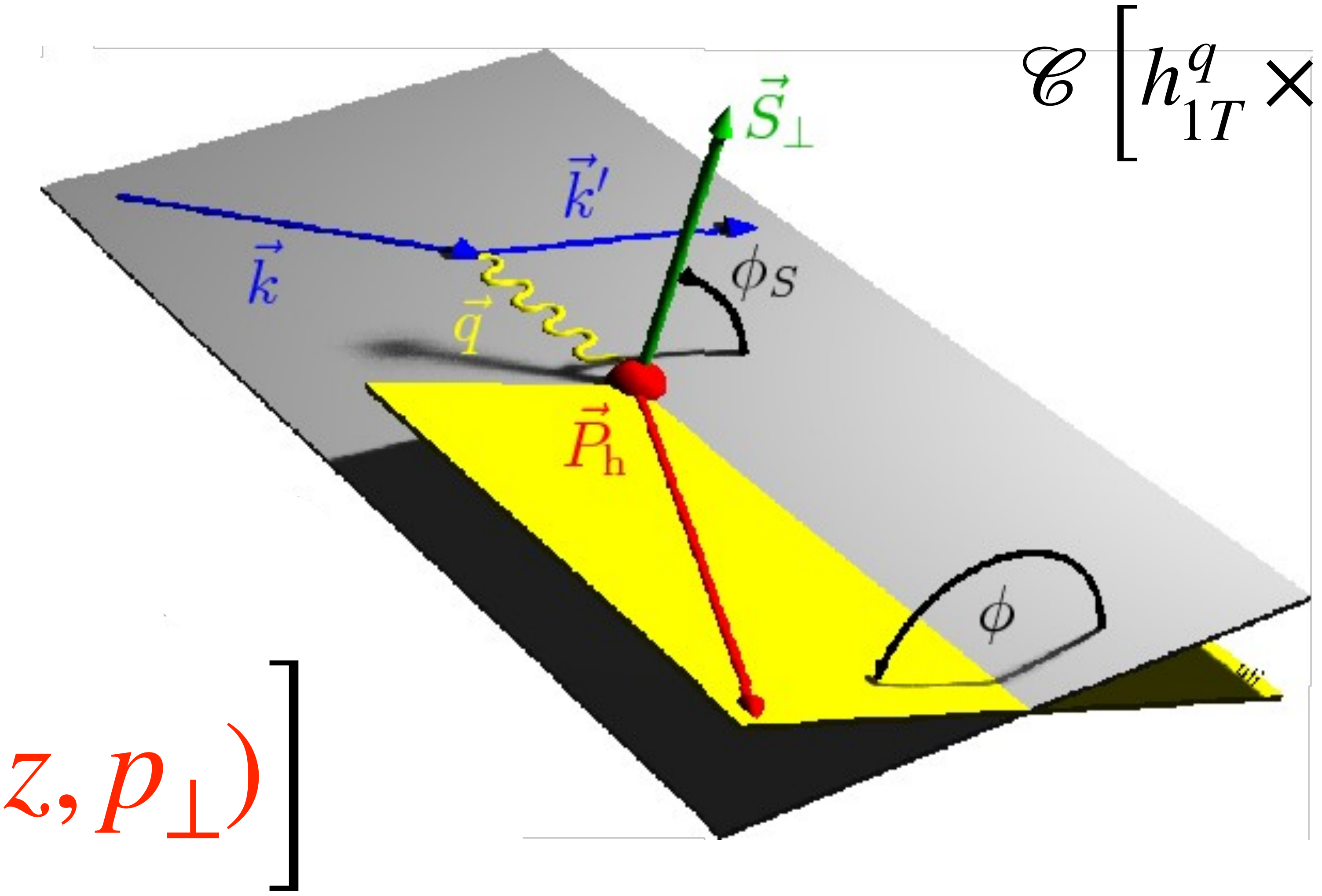


Collins amplitudes: measurement

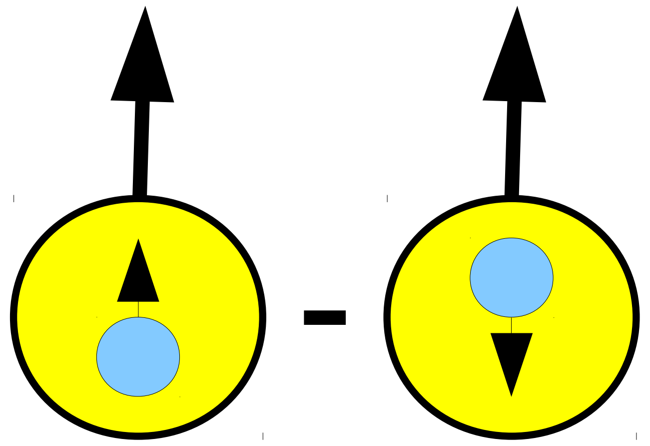


$\mathscr{C}\left[h_{1T}^{q} \times H_1^{\perp,q}\right]$

$$A_{UT} = \frac{1}{\langle|S_T|\rangle} \frac{N^{\uparrow}(\phi,\phi_S) - N^{\downarrow}(\phi,\phi_S)}{N^{\uparrow}(\phi,\phi_S) + N^{\downarrow}(\phi,\phi_S)}$$

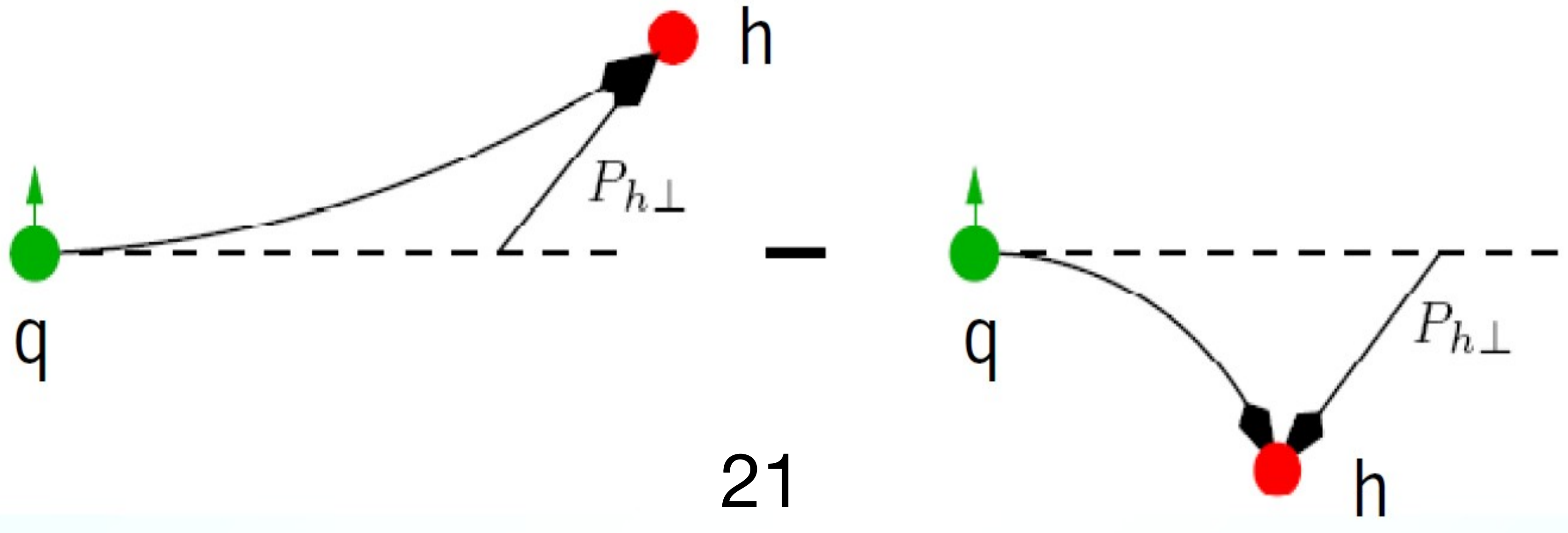


$$\sim \sin(\phi+\phi_S) \sum_q \boxed{e_q^2}\, C\left[h_{1T}^{q}(x,k_\perp) \times H_1^{\perp,q}(z,p_\perp)\right]$$

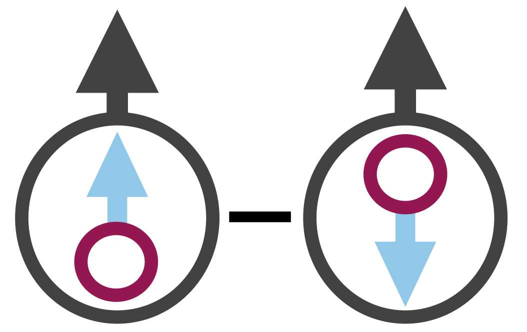


$h_{1T}^{q}(x,k_\perp)$: transversity

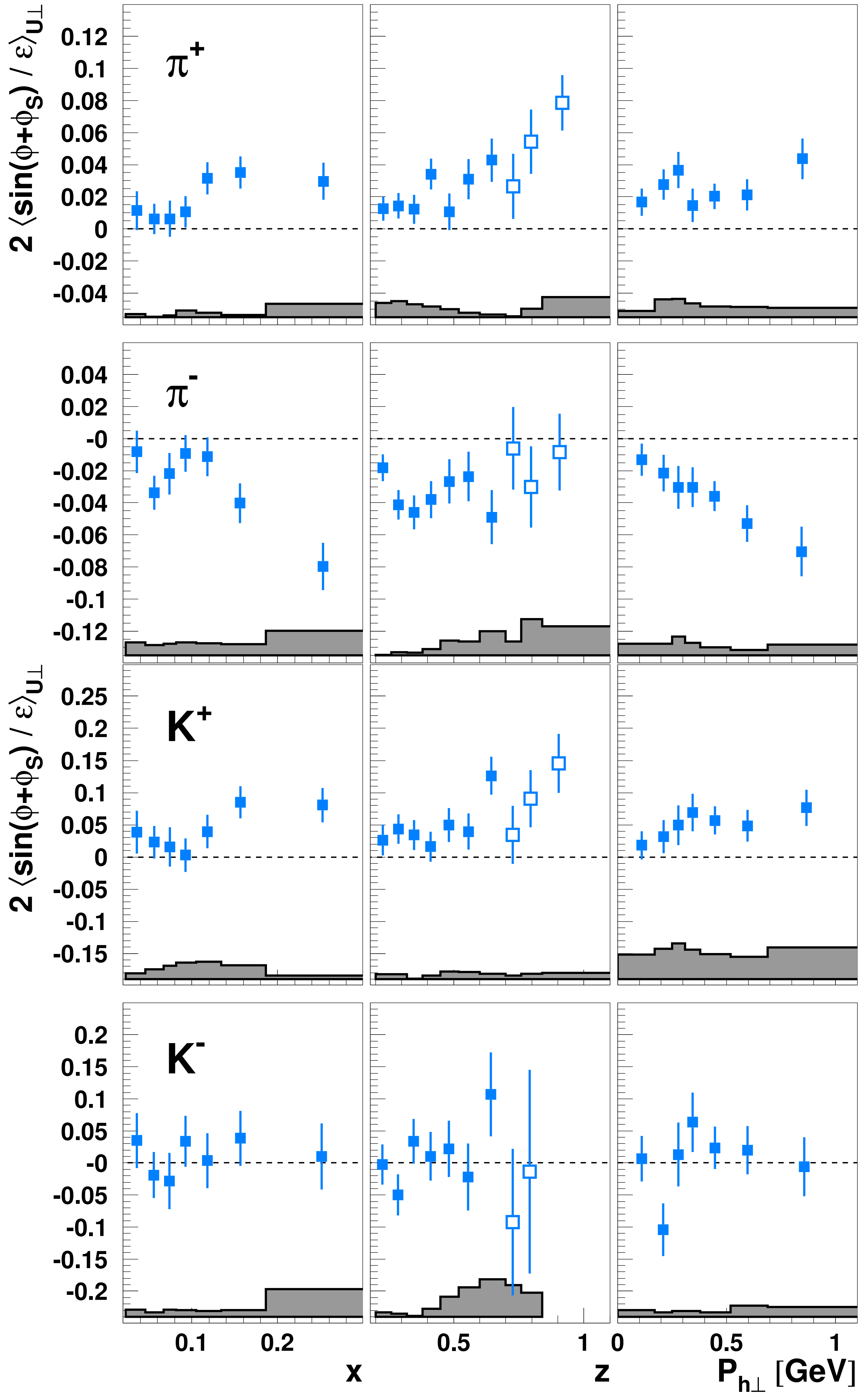
$H_1^{\perp,q}(z,p_\perp)$: Collins fragmentation function



Collins amplitudes



$$\mathscr{C}\left[h_{1T}^{q} \times H_{1}^{\perp,q}\right]$$



HERMES, JHEP **12**(2020)010

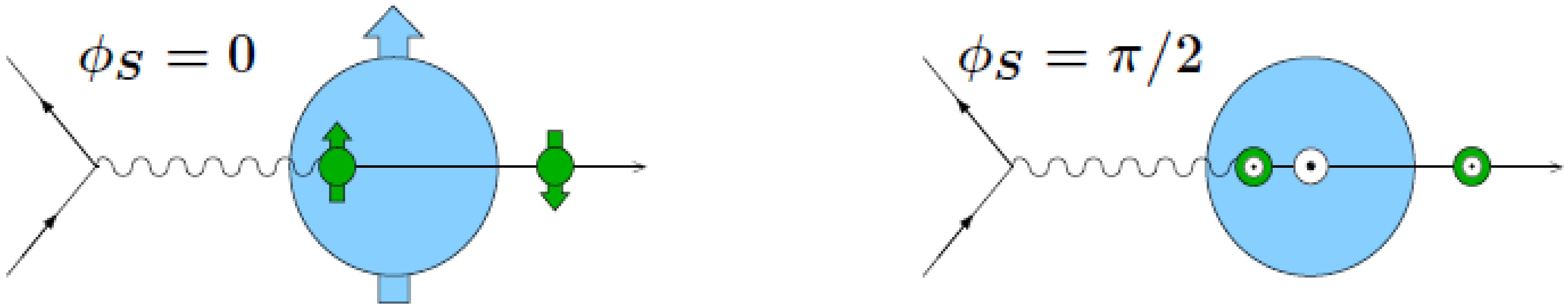
- Oppositely signed amplitudes for π+ and π-:

$$H_1^{\perp,u\to\pi^+} \approx -H_1^{\perp,u\to\pi^-}$$

Artru model

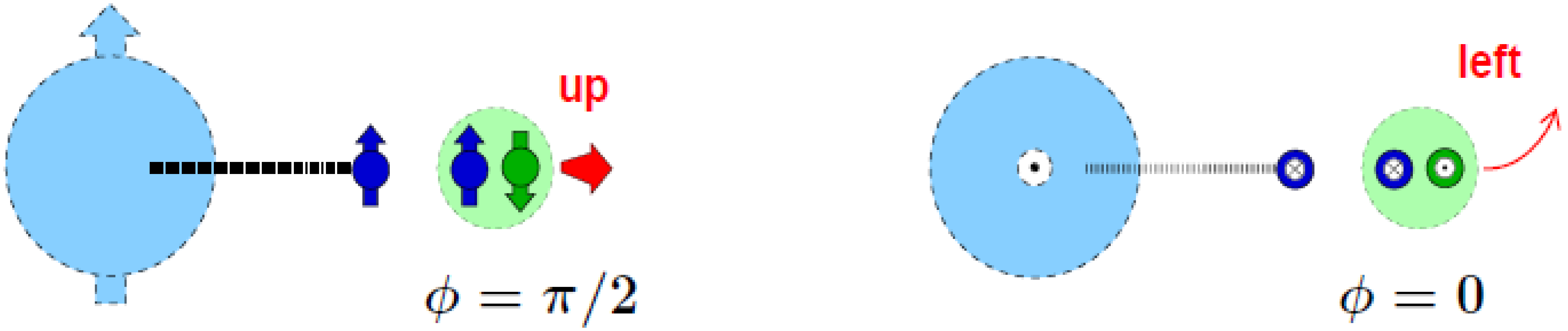
X. Artru et al., Z. Phys. C**73** (1997) 527

polarisation component in lepton scattering plane reversed by photoabsorption:



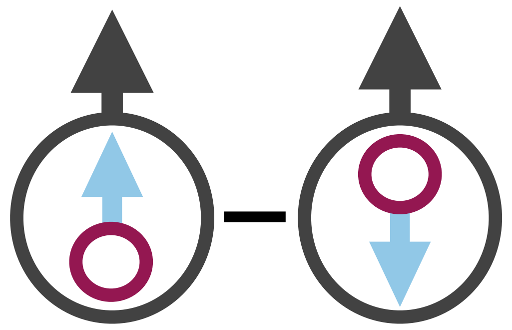
string break, quark-antiquark pair with vacuum numbers:

orbital angular momentum creates transverse momentum:



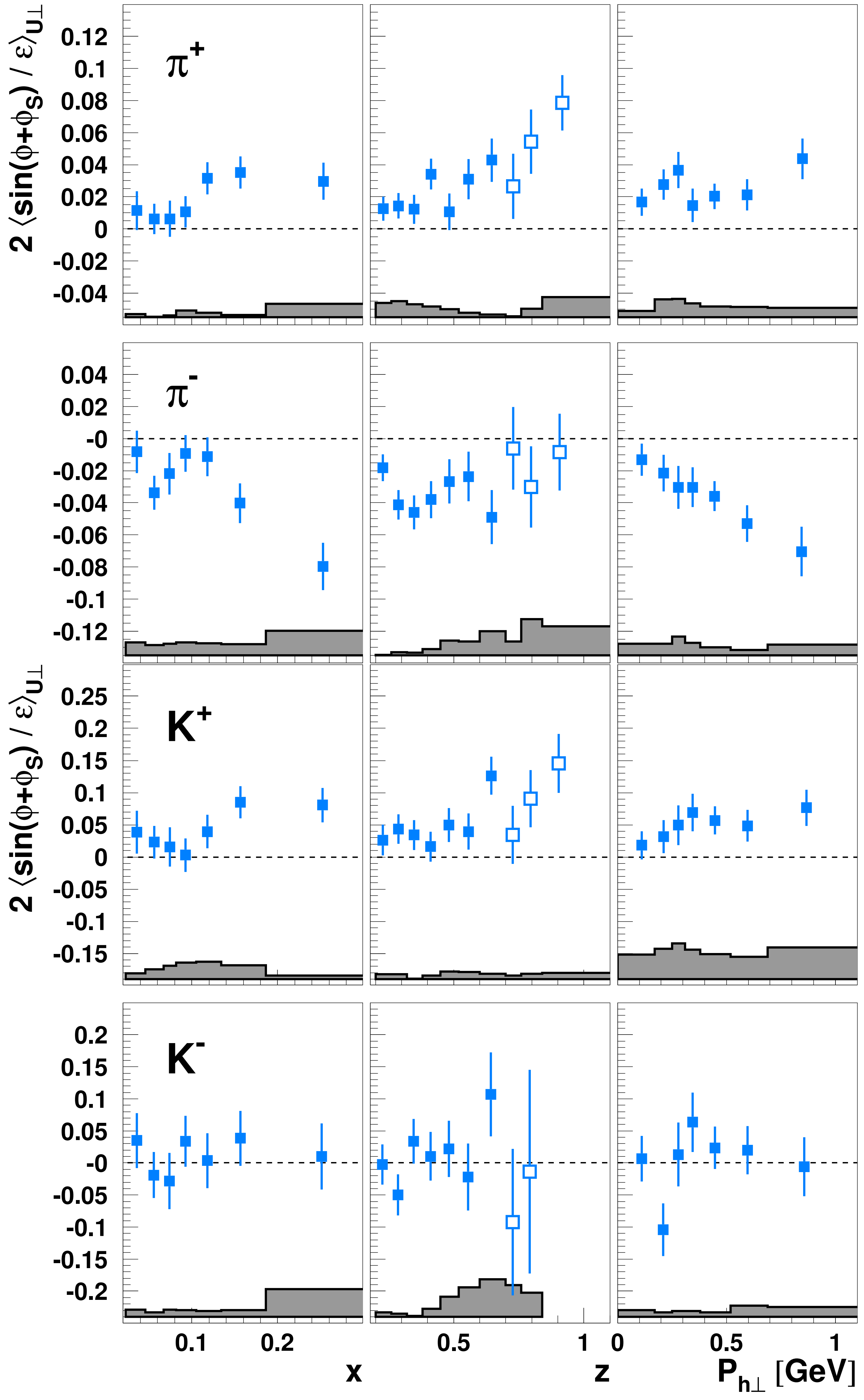
Courtesy U. Elschenbroich

Collins amplitudes



$\mathscr{C}\left[h_{1T}^{q} \times H_1^{\perp,q}\right]$

HERMES, JHEP **12**(2020)010



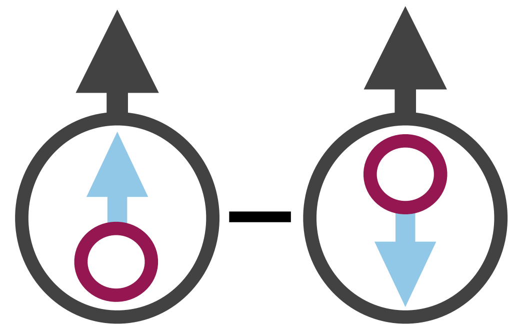
- Oppositely signed amplitudes for π+ and π-:

$$H_1^{\perp,u\to\pi^+} \approx -H_1^{\perp,u\to\pi^-}$$

- Amplitudes for K+ larger than for π+:

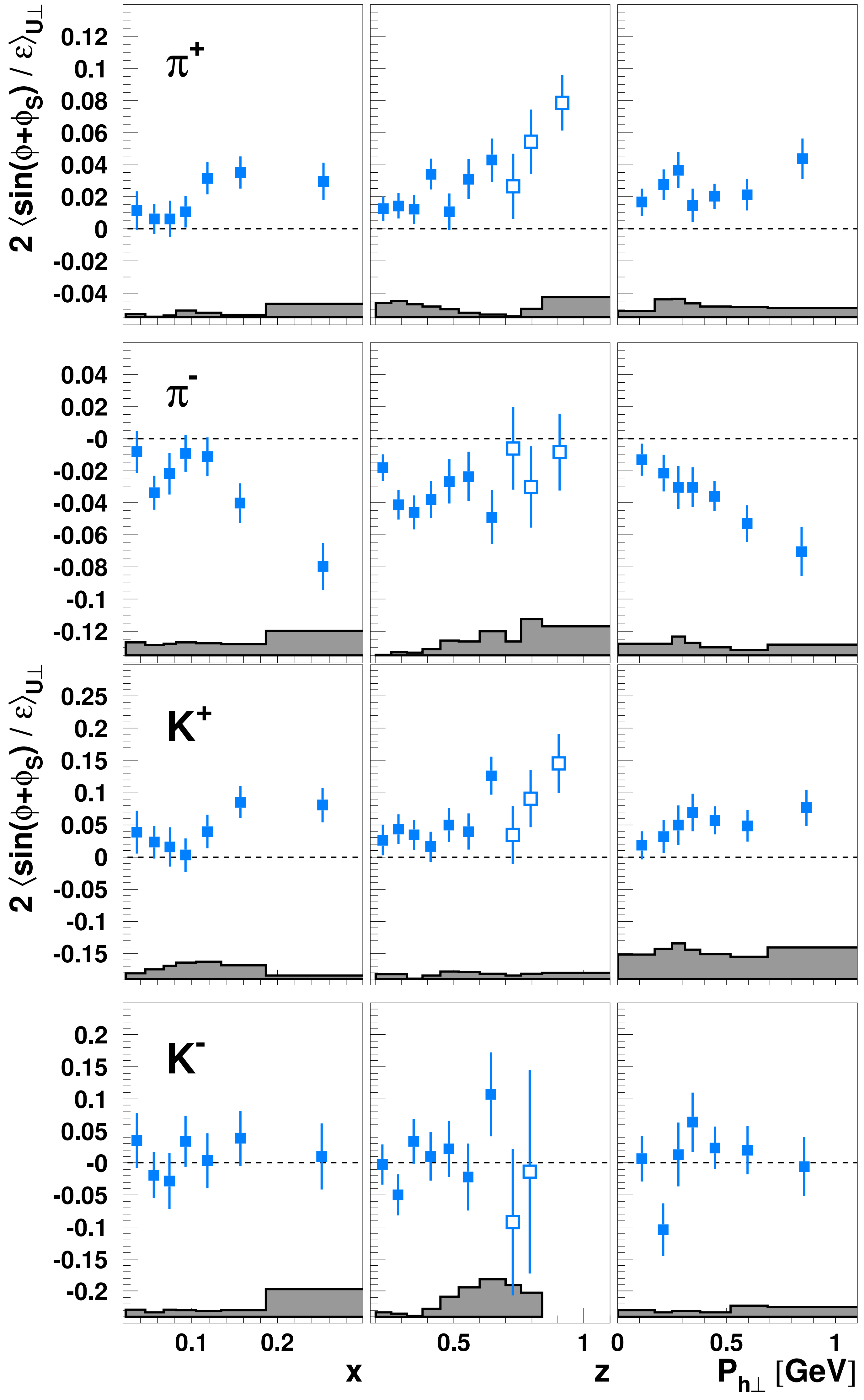
$$H_1^{\perp,u\to K^+} > H_1^{\perp,u\to\pi^+}$$

Collins amplitudes



$$\mathscr{C}\left[h_{1T}^{q} \times H_1^{\perp,q}\right]$$

HERMES, JHEP **12**(2020)010

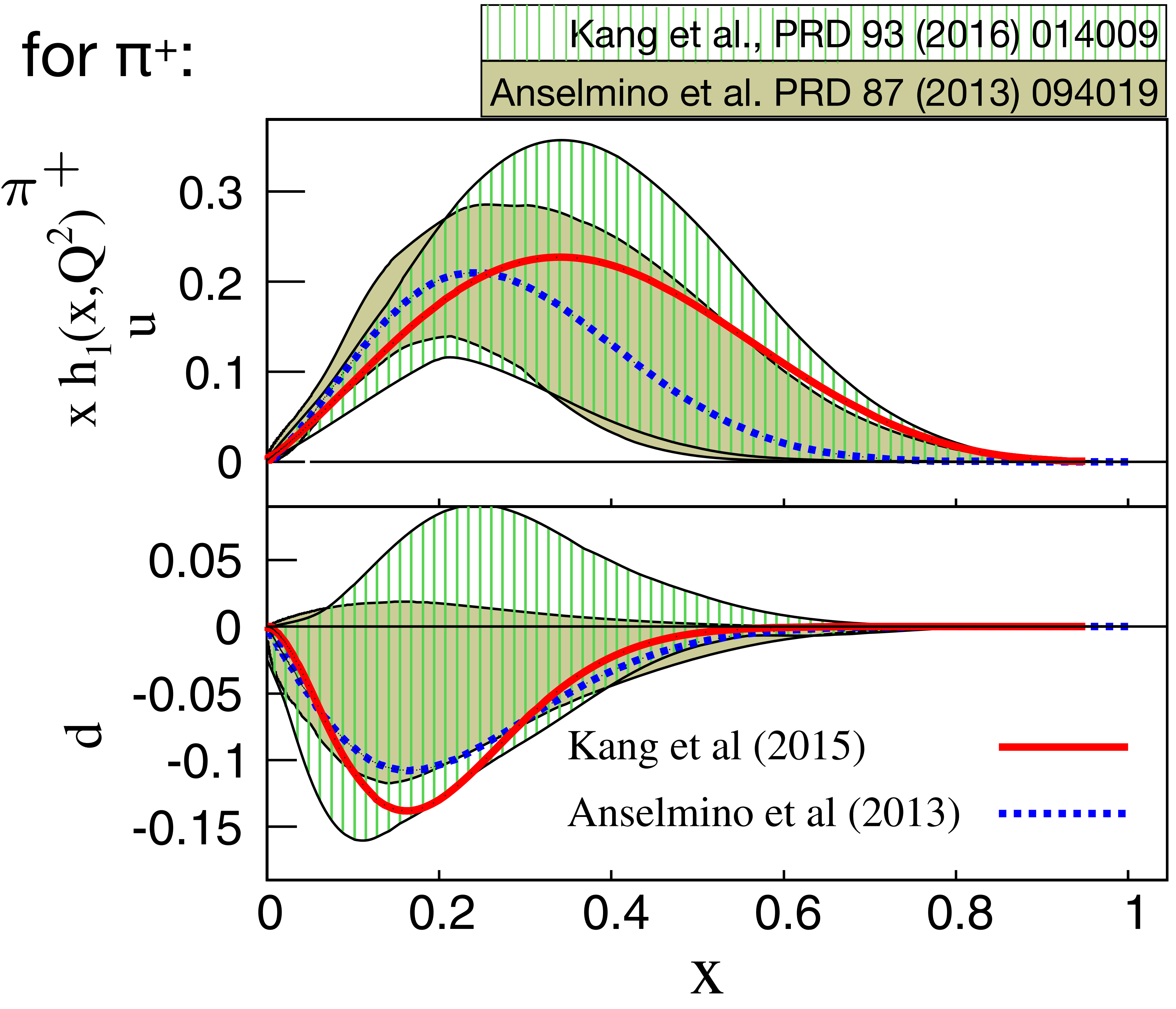


- Oppositely signed amplitudes for π⁺ and π⁻:

$$H_1^{\perp,u\to\pi^+} \approx -H_1^{\perp,u\to\pi^-}$$

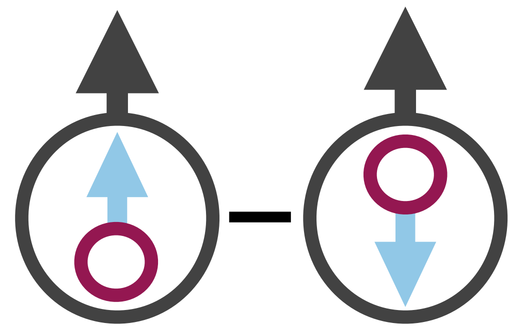
- Amplitudes for K⁺ larger than for π⁺:

$$H_1^{\perp,u\to K^+} > H_1^{\perp,u\to\pi^+}$$



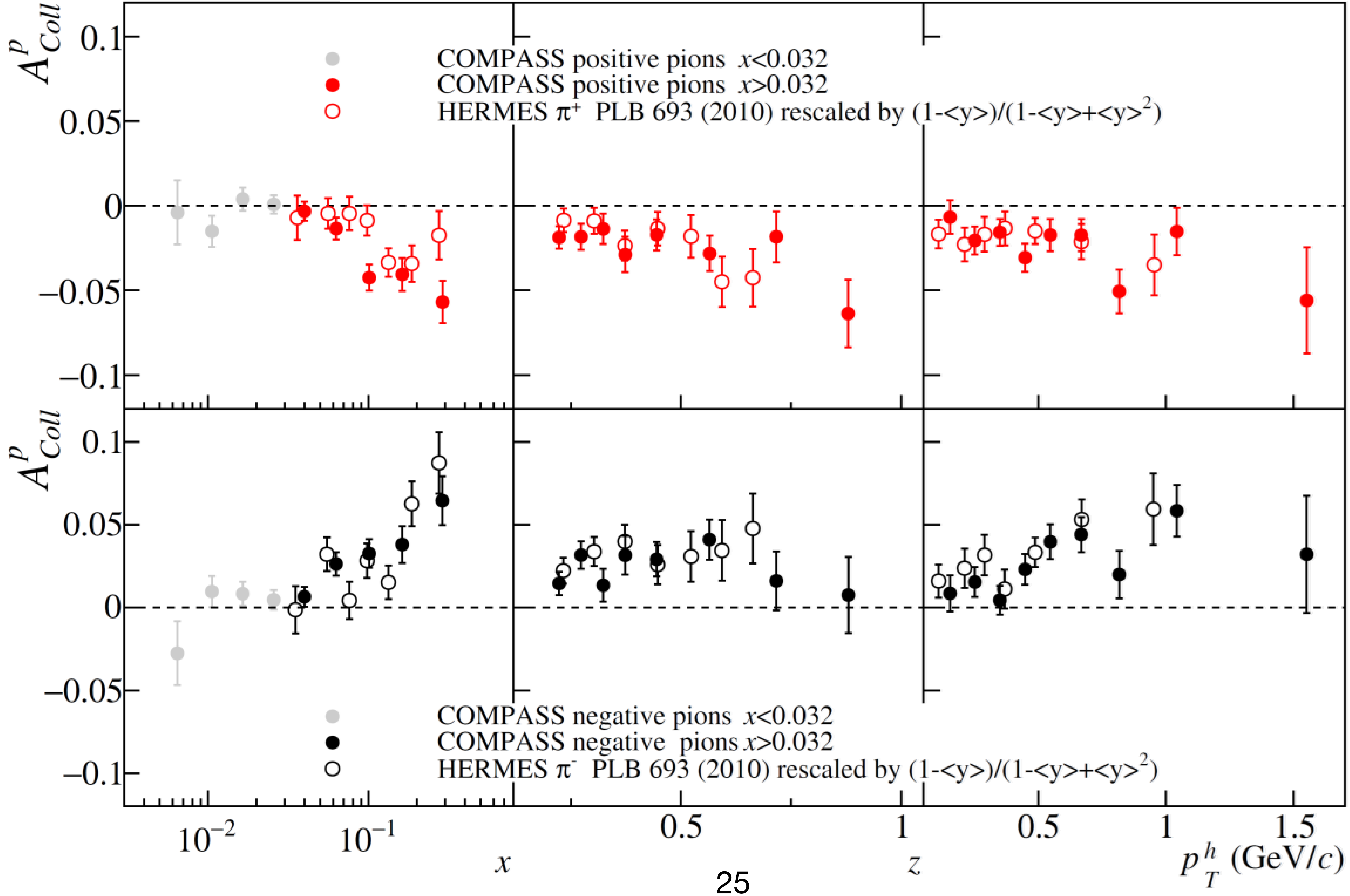
data from Belle, Babar, COMPASS, HERMES, Jefferson Lab Hall A

Collins amplitudes: QCD evolution

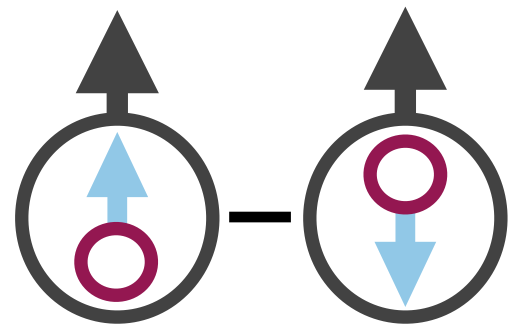


$\mathscr{C}\left[h_{1T}^{q} \times H_{1}^{\perp,q}\right]$

COMPASS, Phys. Lett. **B 744** (2015) 250

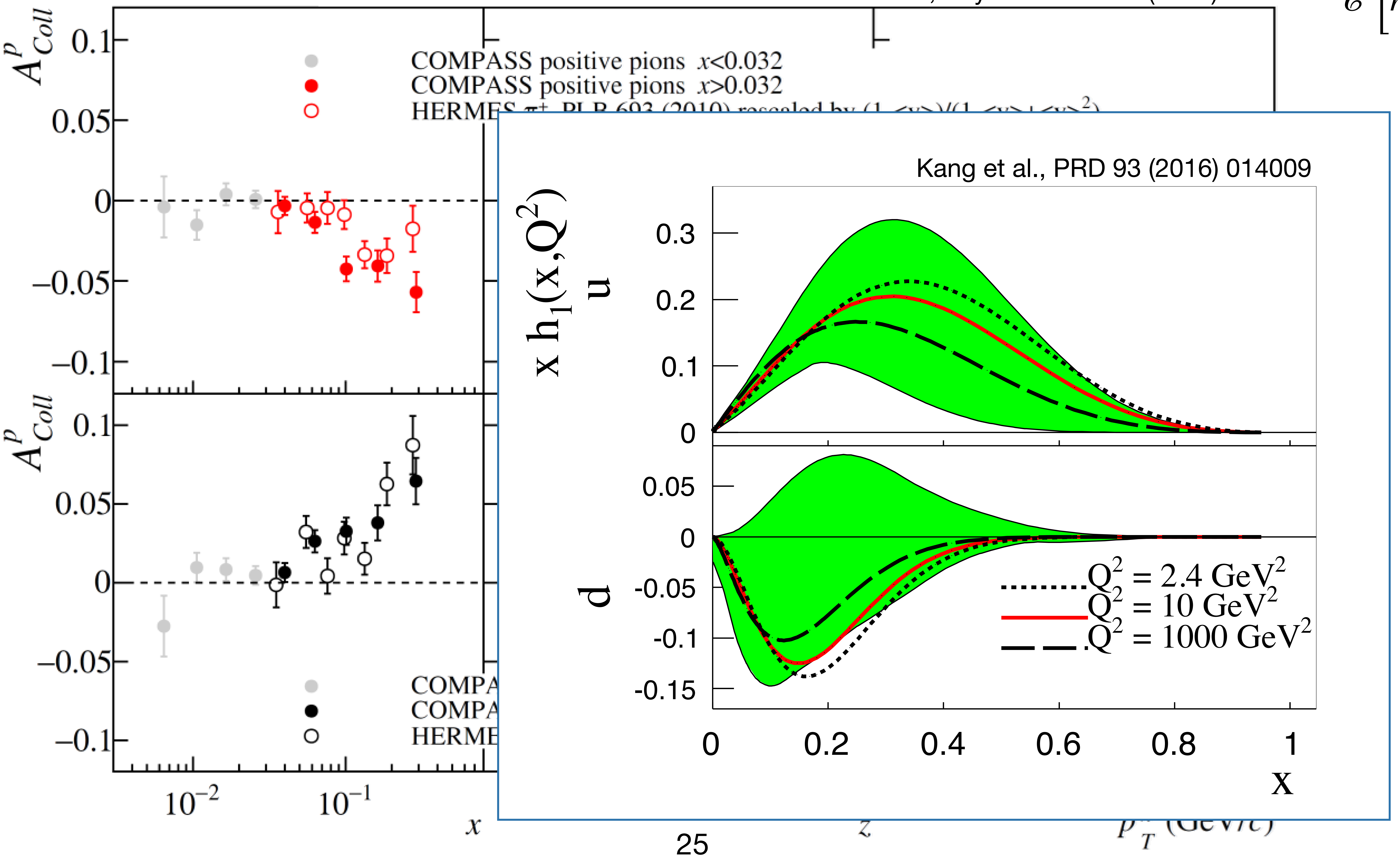


Collins amplitudes: QCD evolution



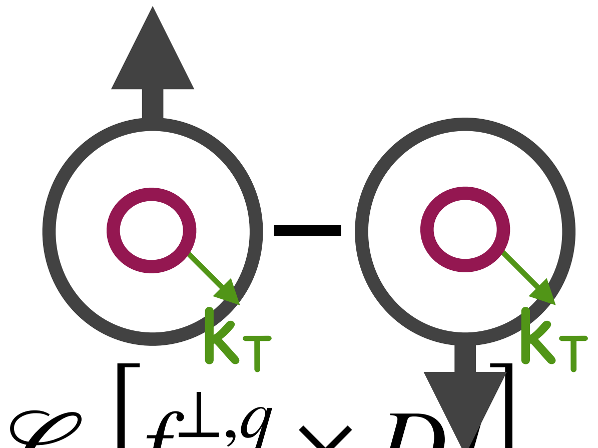
$\mathscr{C}\left[h_{1T}^{q} \times H_1^{\perp,q}\right]$

COMPASS, Phys. Lett. **B 744** (2015) 250



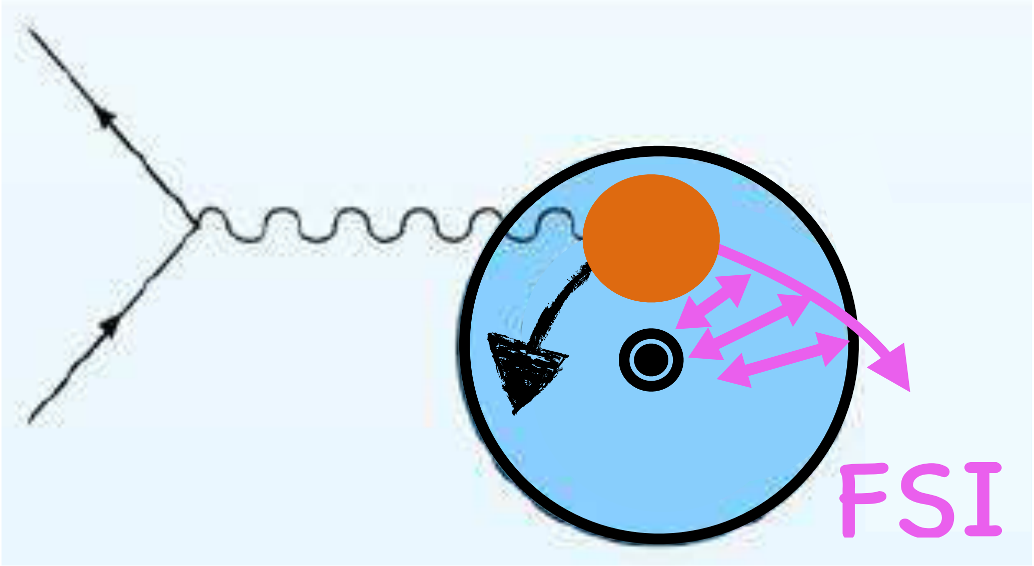
Kang et al., PRD 93 (2016) 014009

Sivers amplitudes

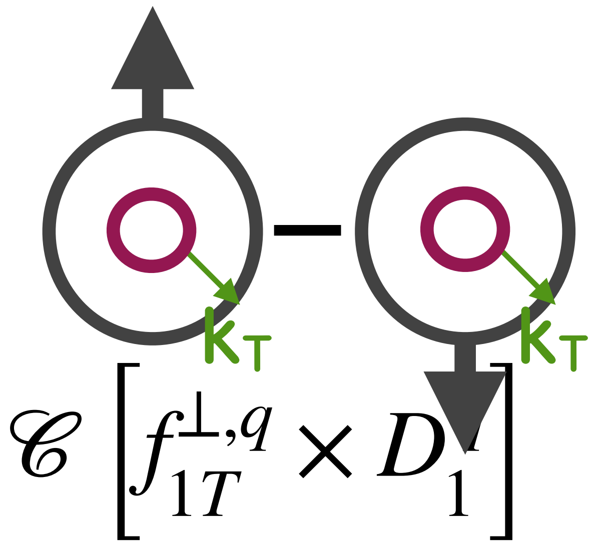


$\mathscr{C}\left[f_{1T}^{\perp,q}\times D_1\right]$

- Sivers function:
  - requires non-zero orbital angular momentum
  - final-state interactions ⟶ azimuthal asymmetries

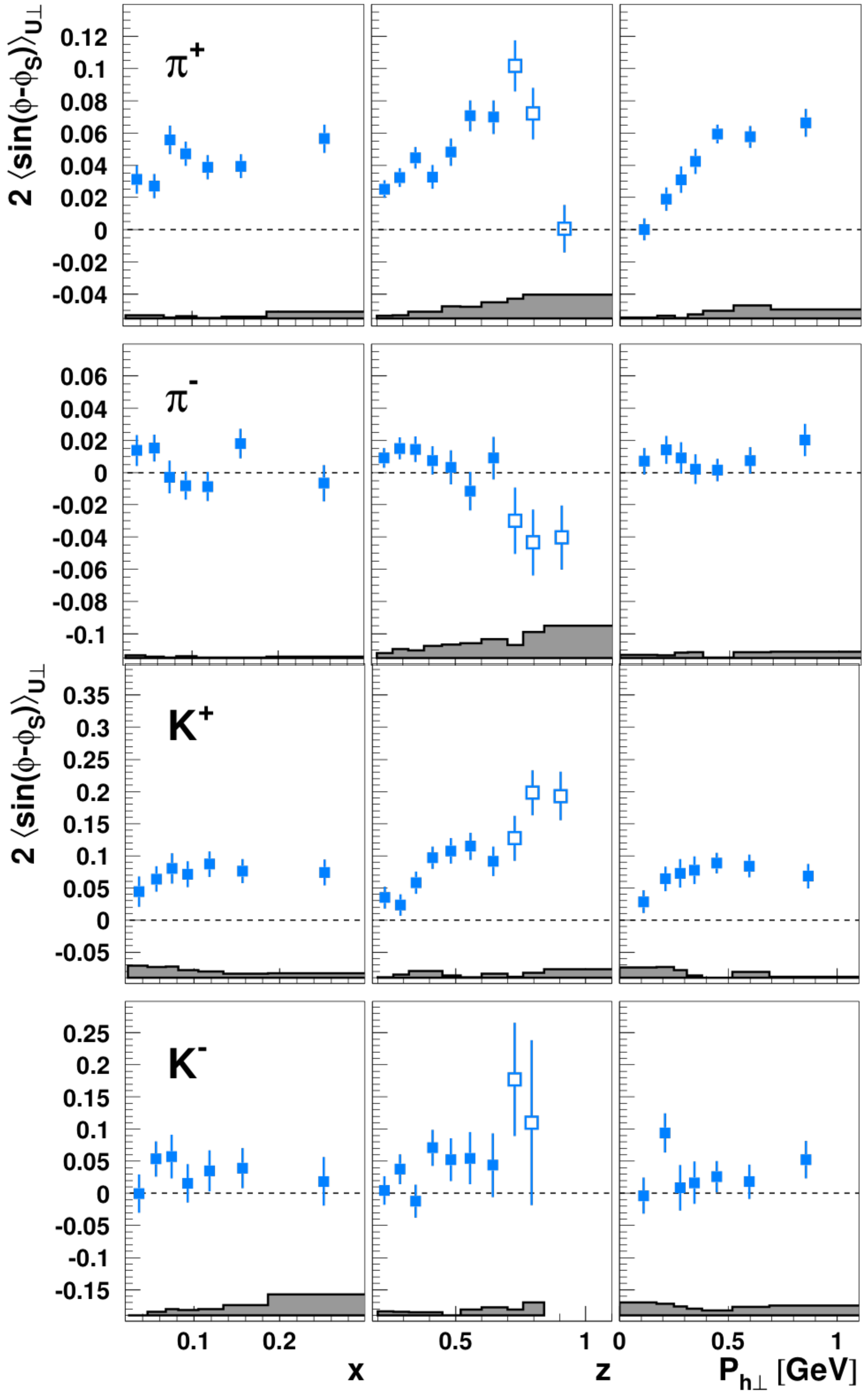


Sivers amplitudes



$\mathscr{C}\left[f_{1T}^{\perp,q} \times D_1\right]$

HERMES, JHEP **12**(2020)010



- Sivers function:
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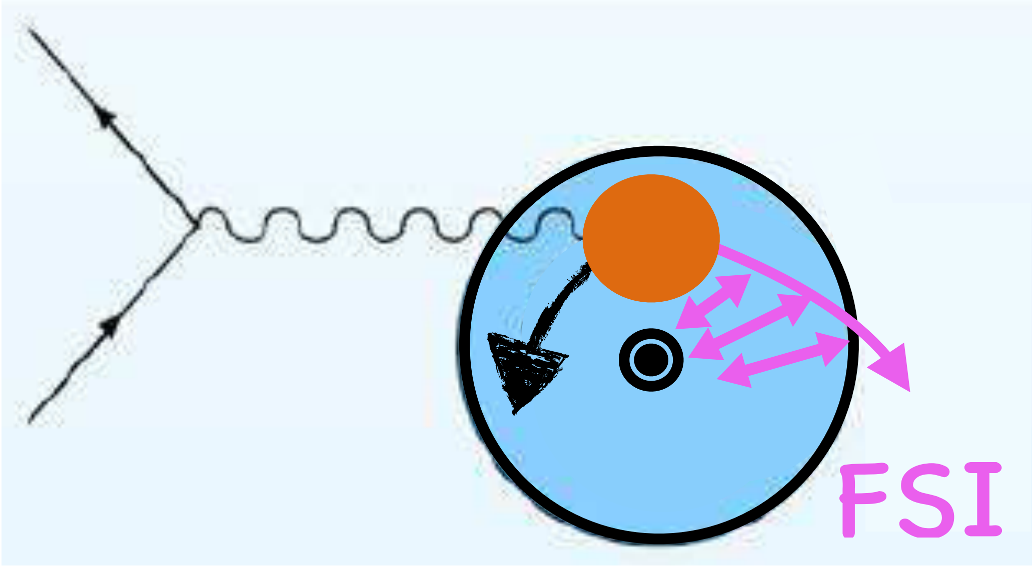
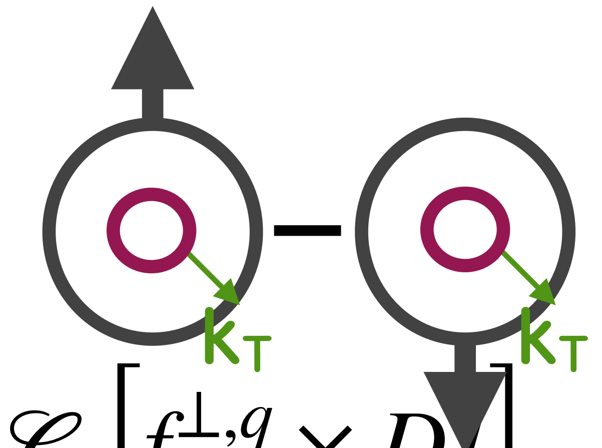
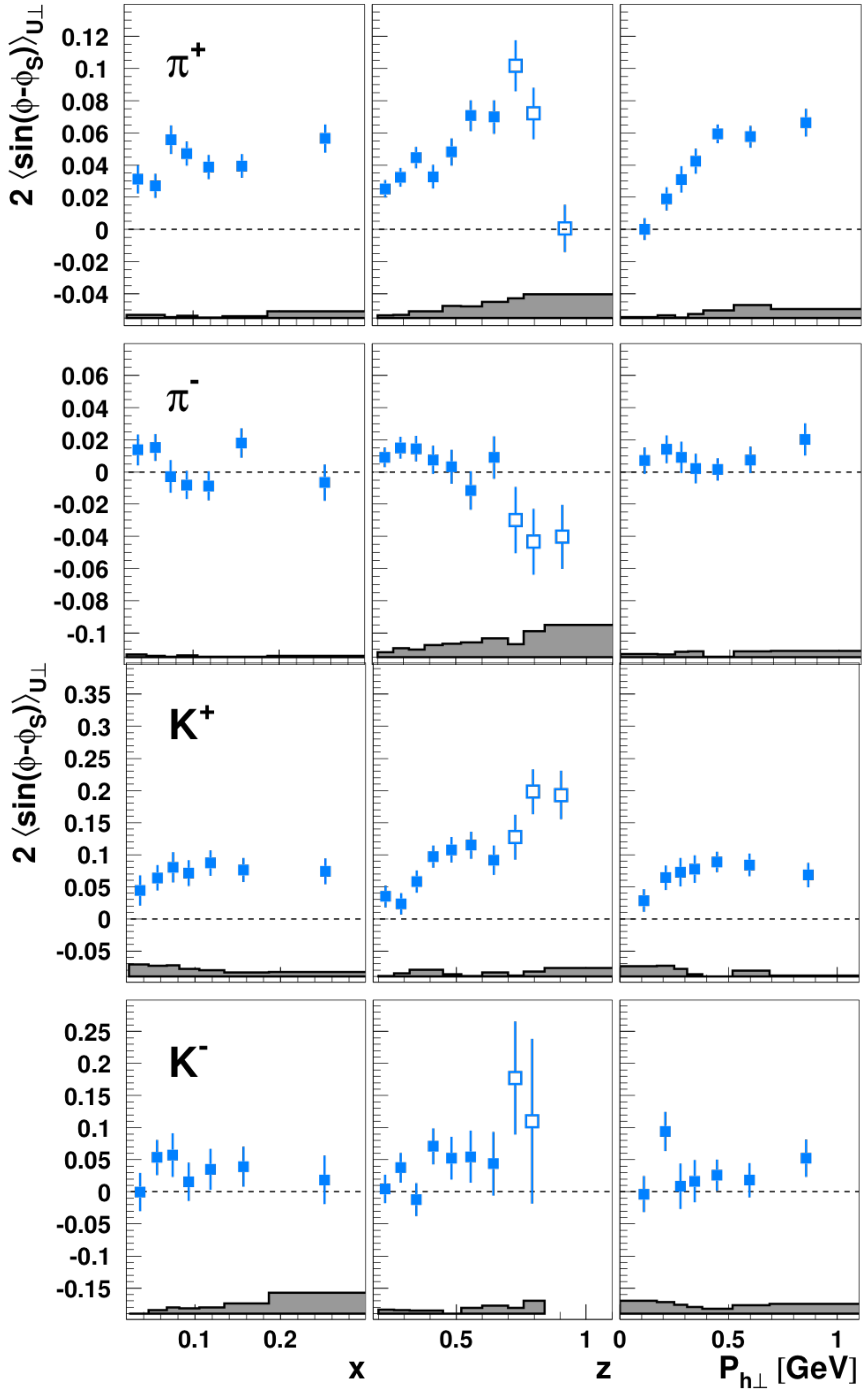


Sivers amplitudes

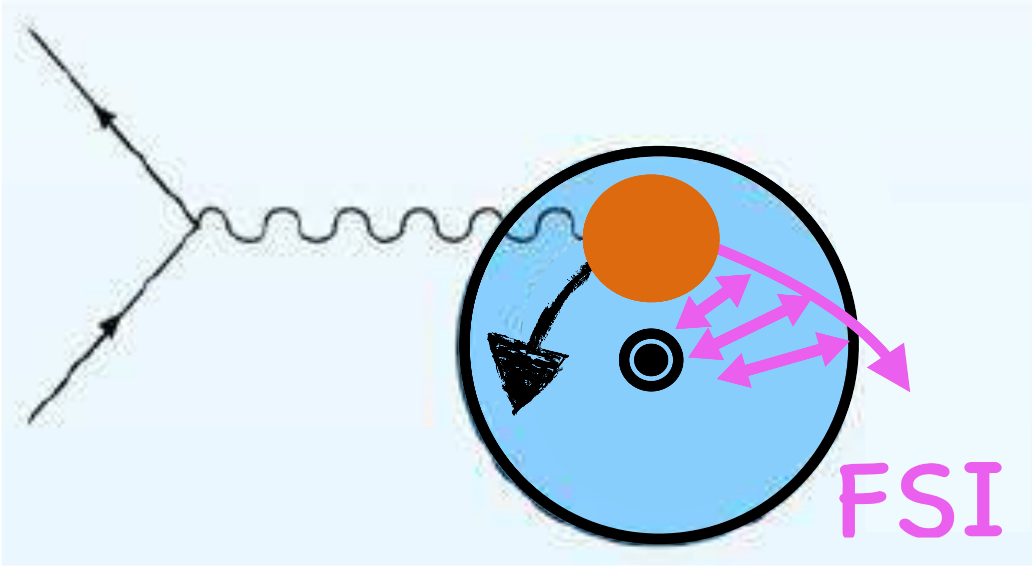


$\mathscr{C}\left[f_{1T}^{\perp,q}\times D_1\right]$

HERMES, JHEP **12**(2020)010



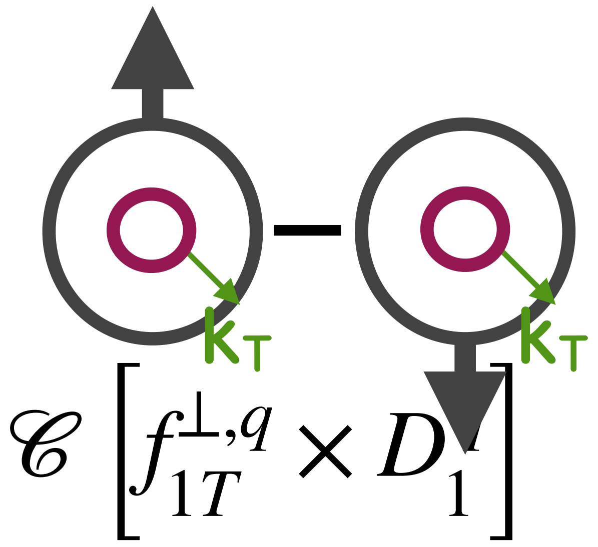
- Sivers function:
  - requires non-zero orbital angular momentum
  - final-state interactions ⟶ azimuthal asymmetries



- π^+:
  - positive -> non-zero orbital angular momentum

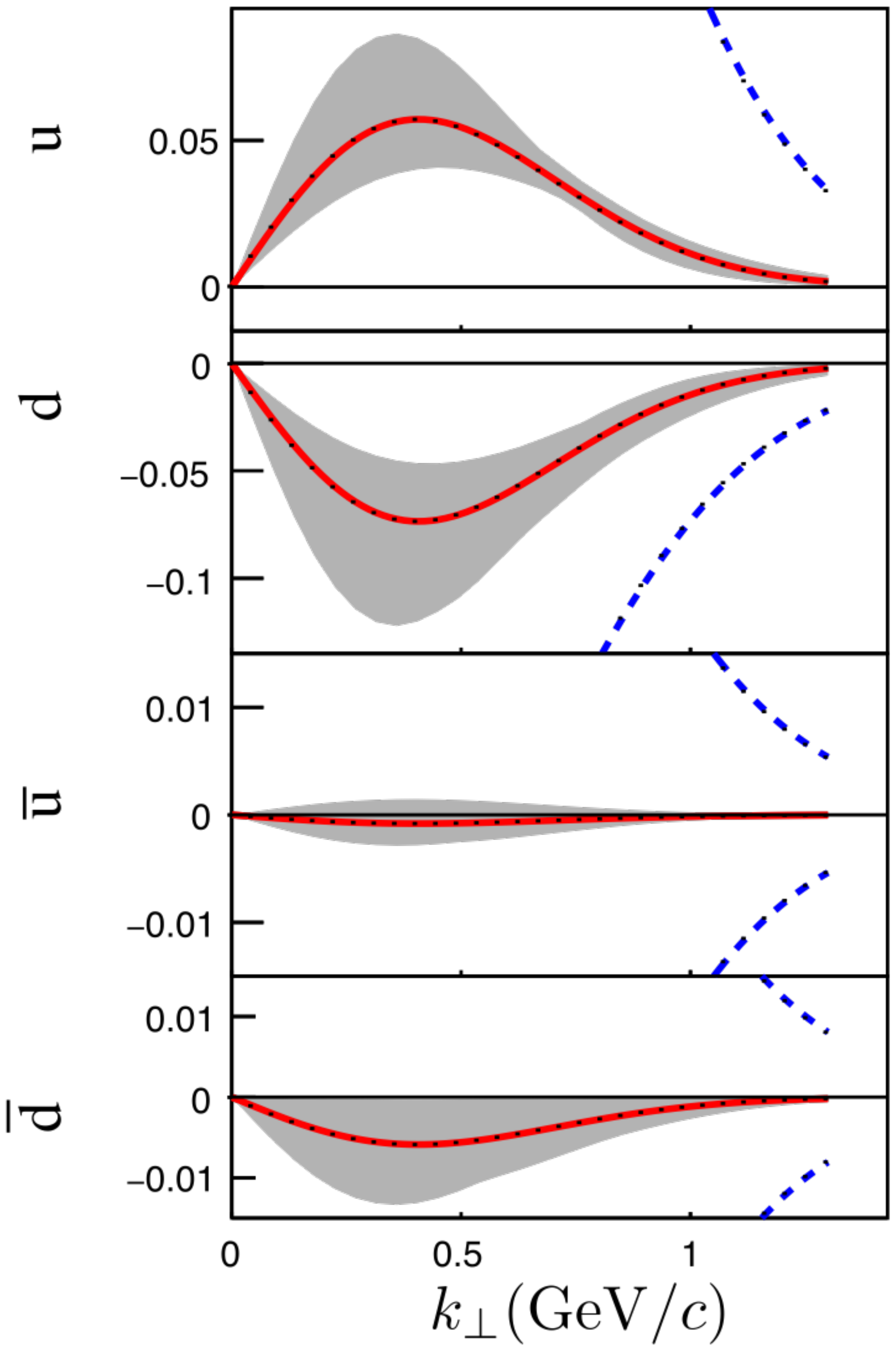
- π^-:
  - consistent with zero → u and d quark cancelation

Sivers function



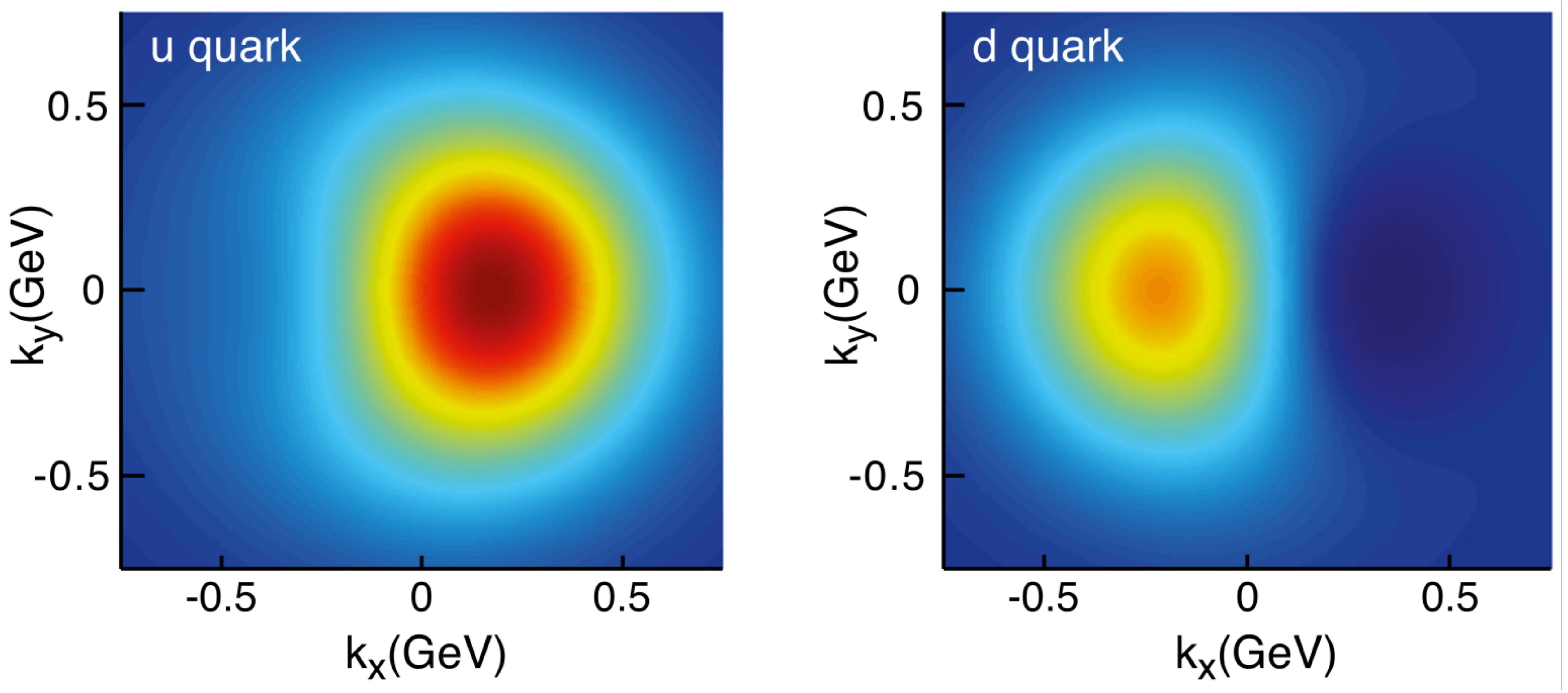
$\mathscr{C}\left[f_{1T}^{\perp,q} \times D_1\right]$

M. Anselmino et al., JHEP **04** (2017) 046



x f$_1$(x, k$_T$, S$_T$)

A. Accardi et al.,
Eur. Phys. J. A **52** (2016) 268

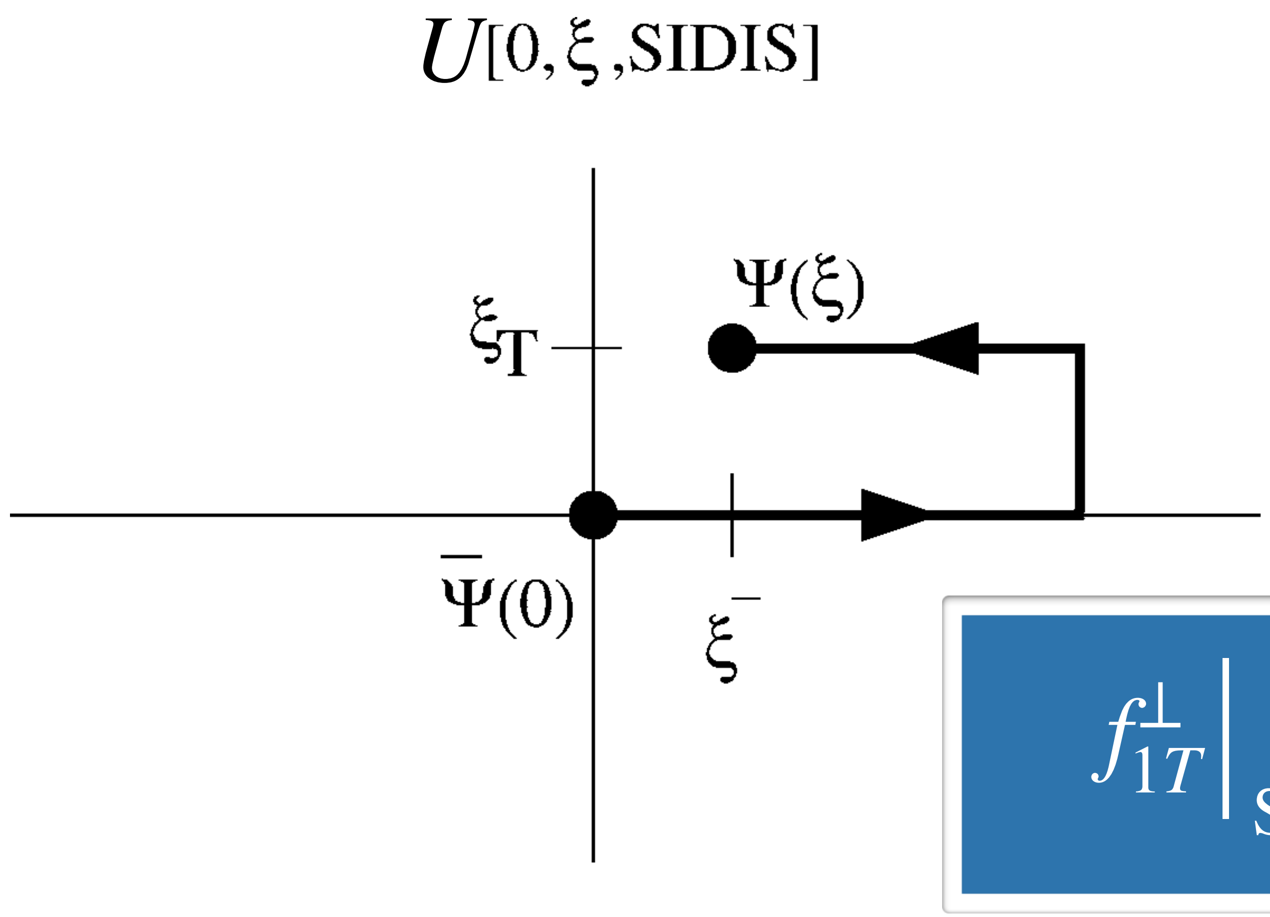


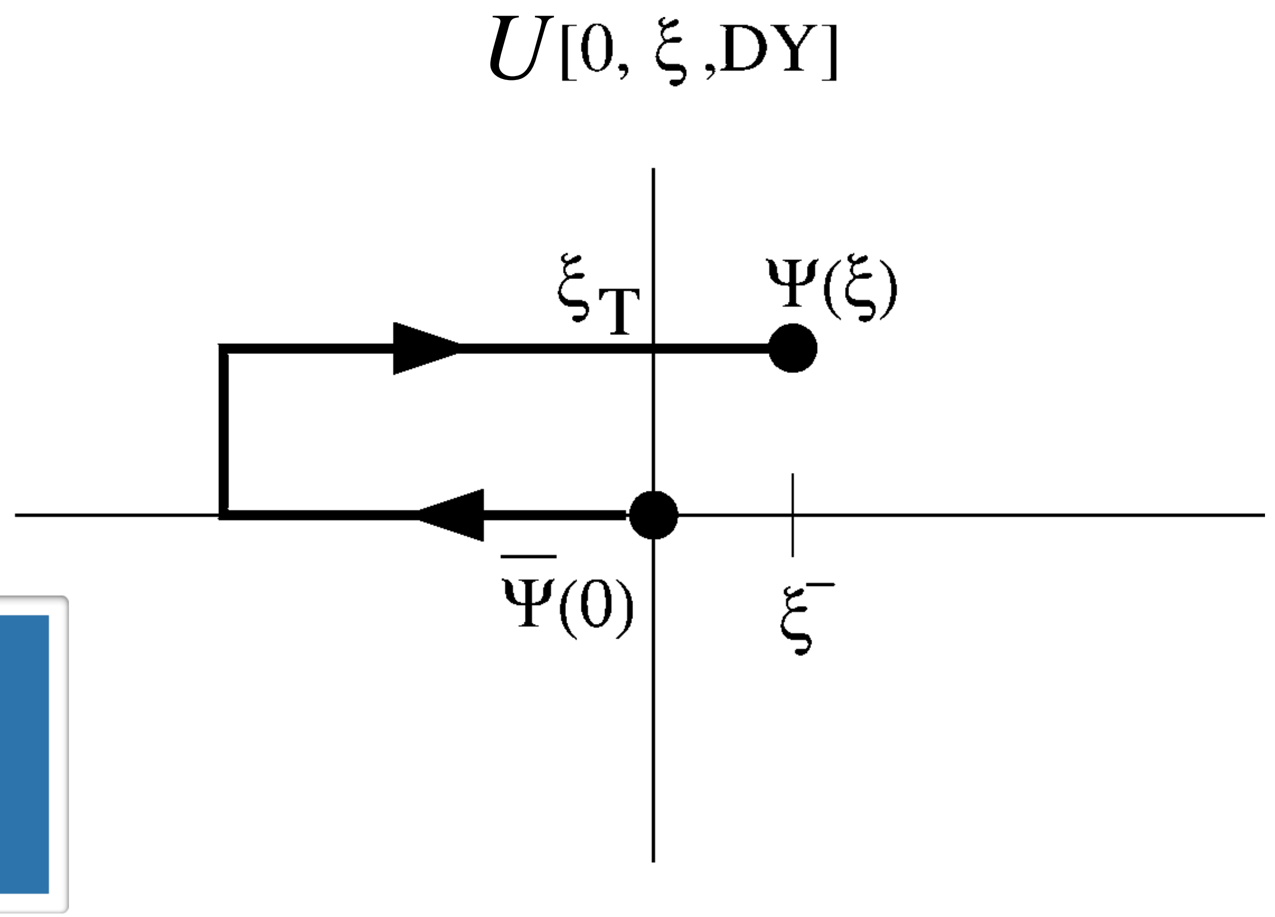
nucleon polarised along $\hat{y}$

Predicted Sivers sign change for SIDIS and Drell-Yan

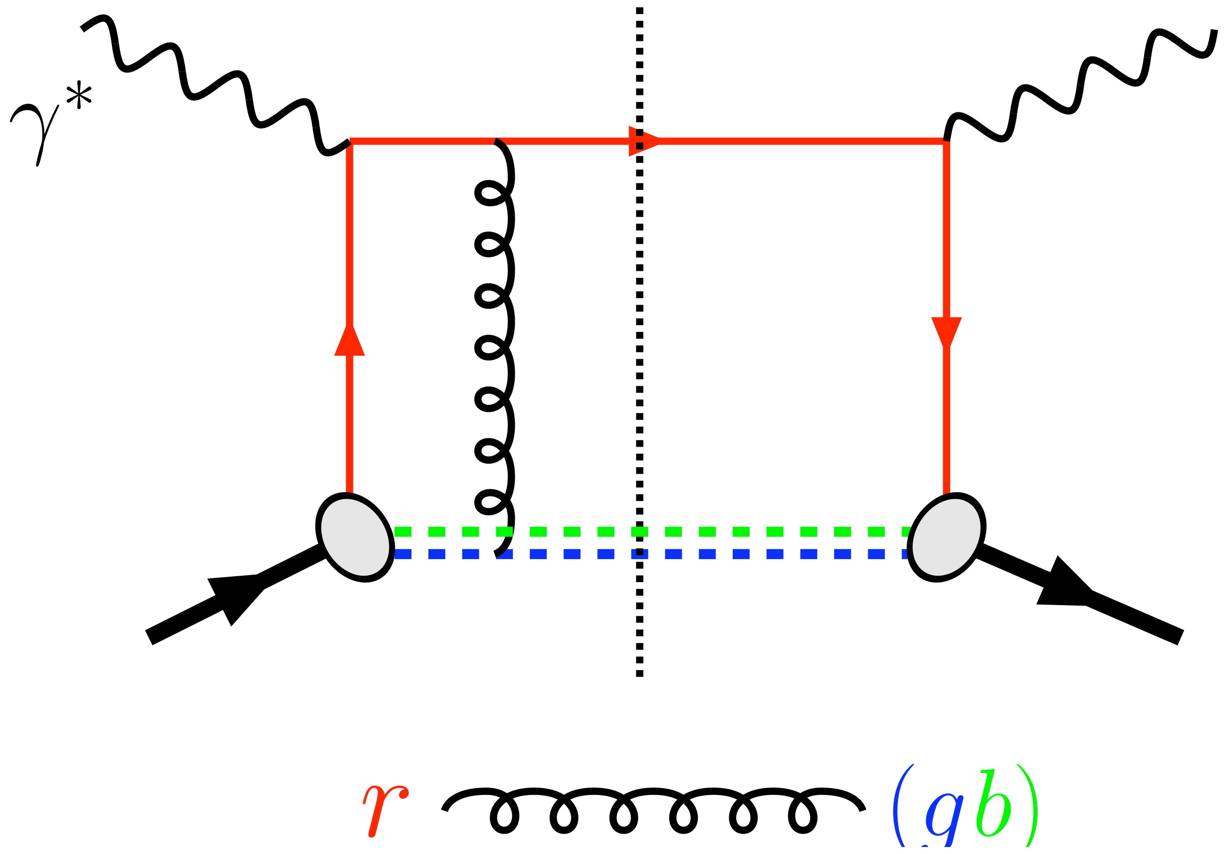
J. C. Collins, Phys. Lett. B 536 (2002) 43

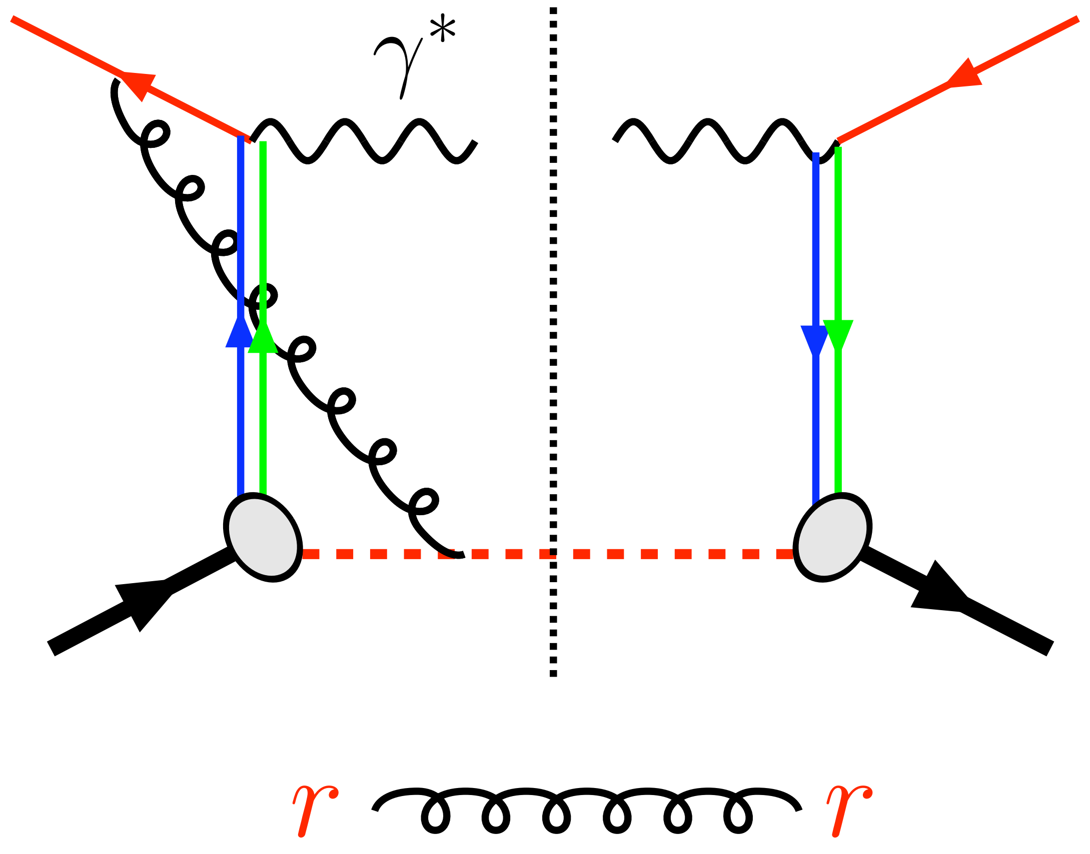
$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\langle P,S|\,\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,|P,S\rangle$$





$$f_{1T}^{\perp}\Big|_{\mathrm{SIDIS}} = -f_{1T}^{\perp}\Big|_{\mathrm{DY}}$$

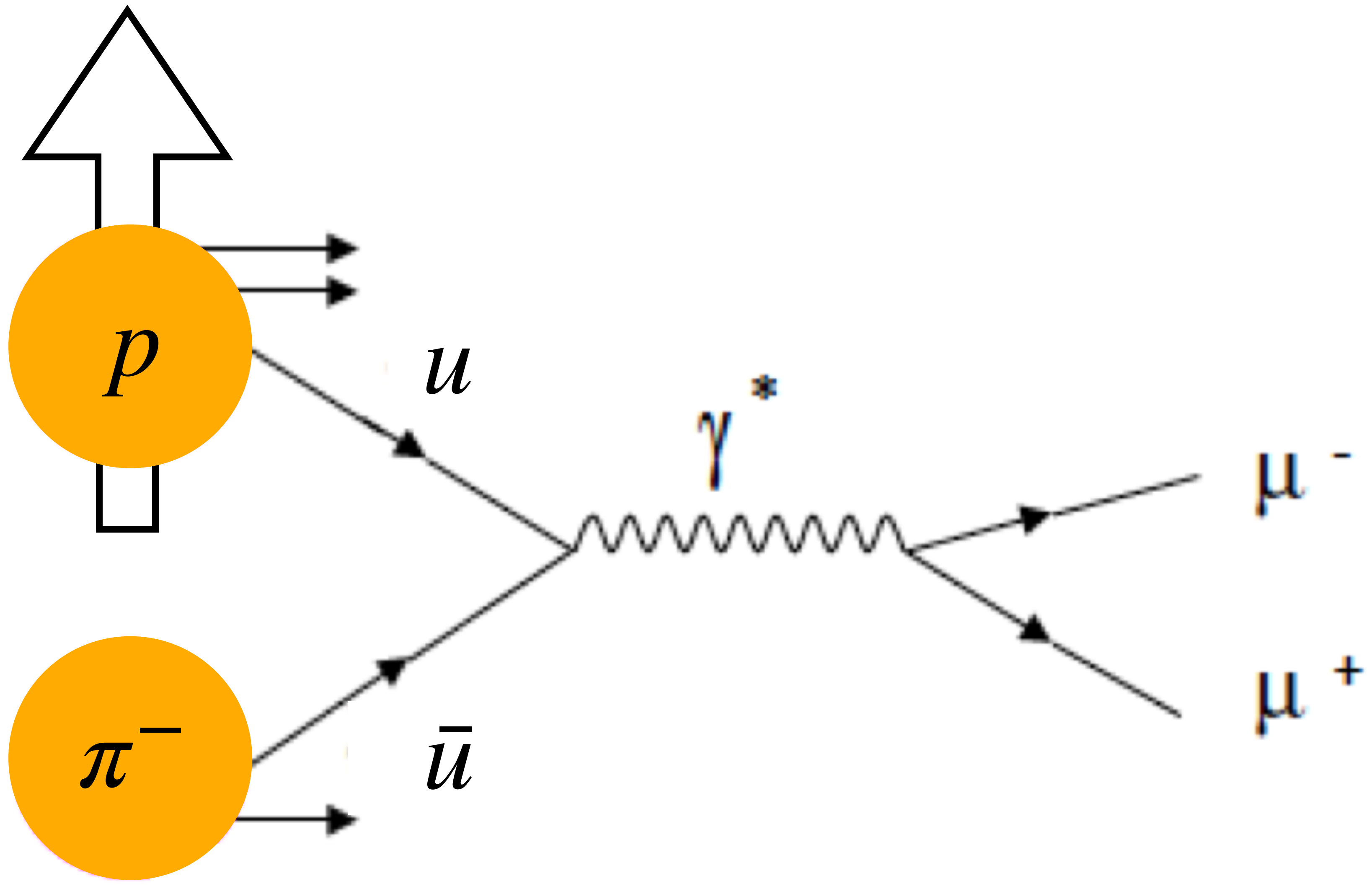




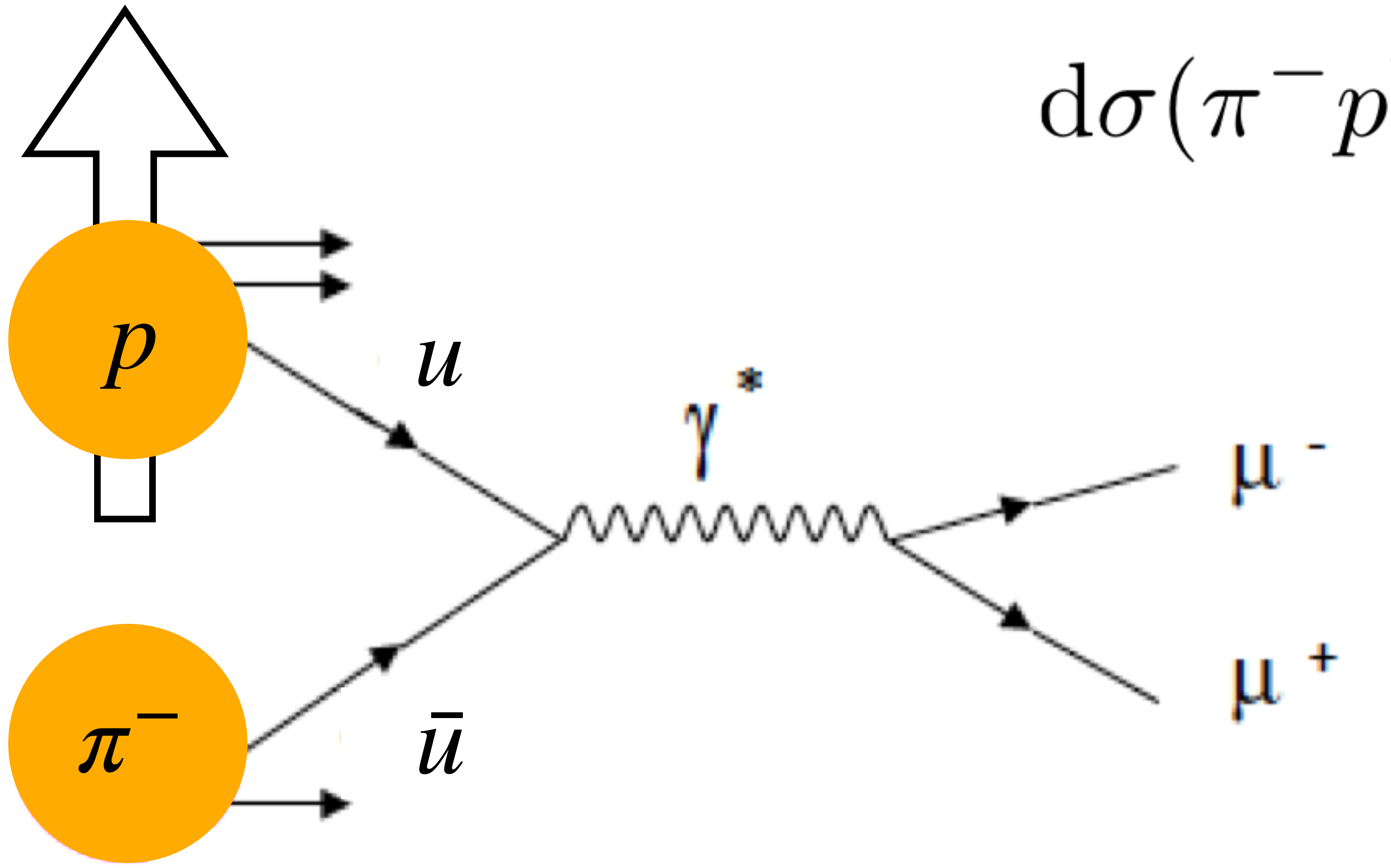
SIDIS

Drell-Yan

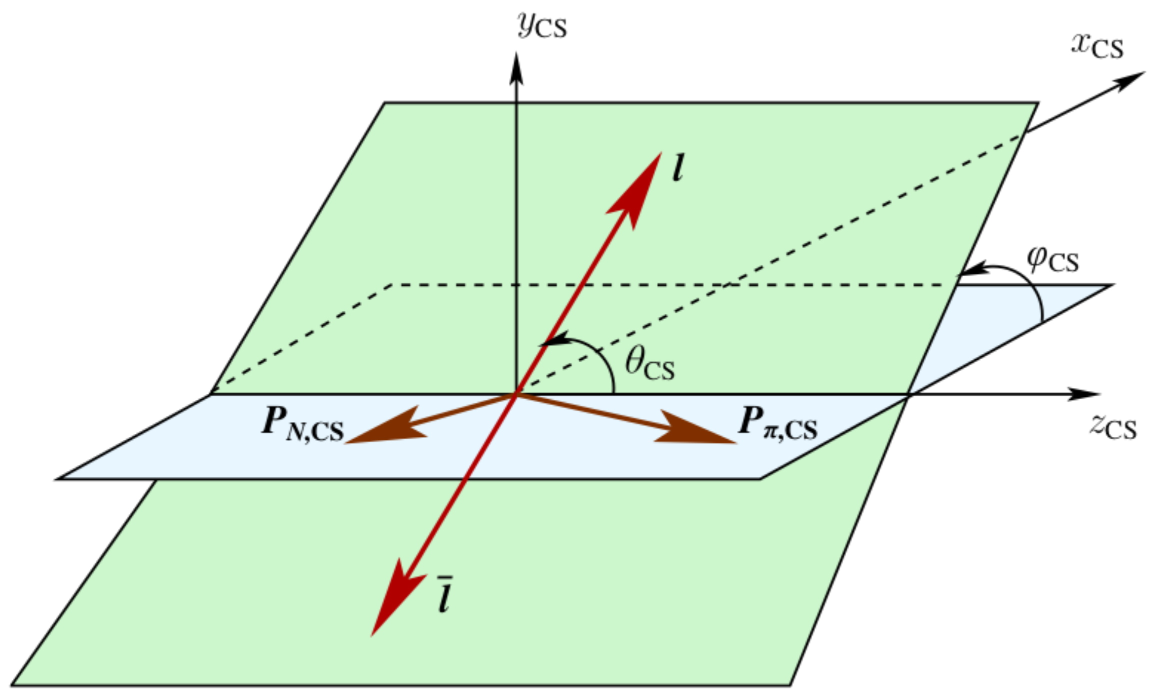
Experimental access to Sivers in Drell-Yan

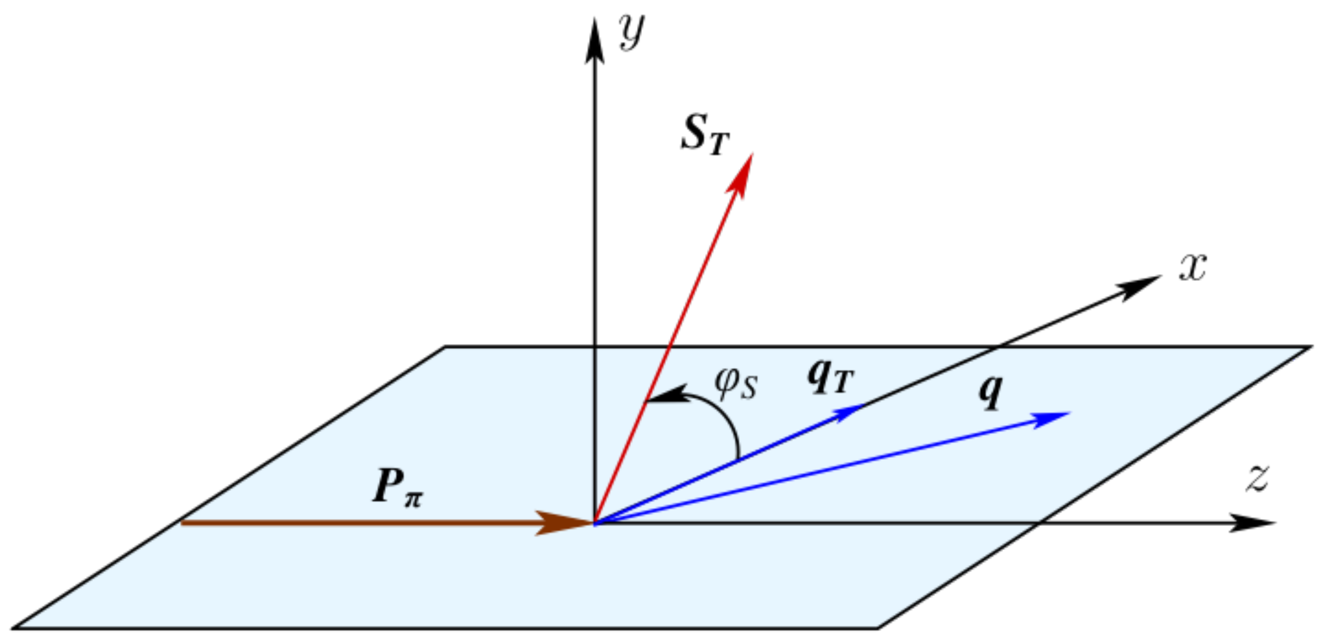


Experimental access to Sivers in Drell-Yan

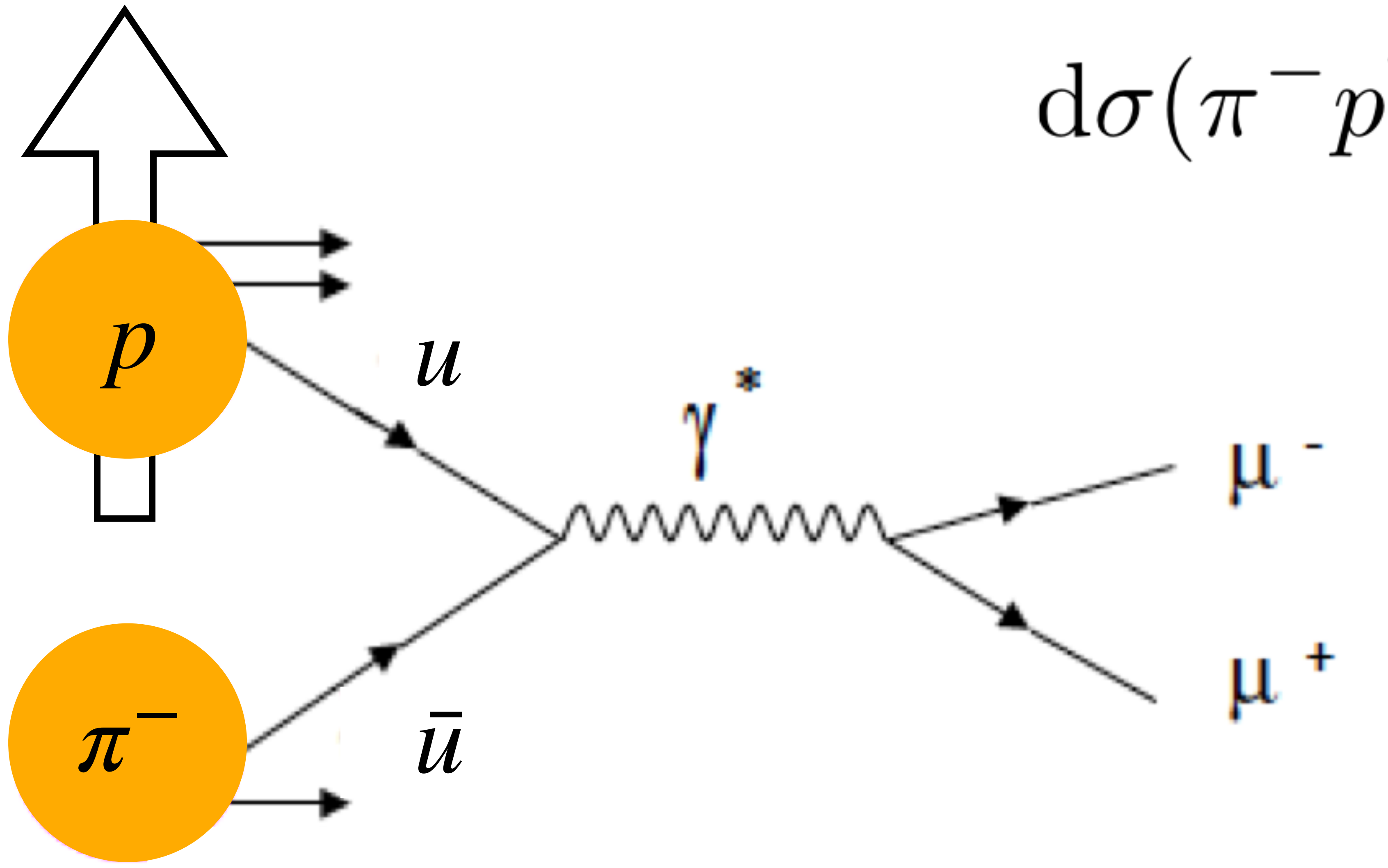


$$
\begin{aligned}
\mathrm{d}\sigma(\pi^- p^\uparrow \to \mu^+\mu^- X) \sim\ & 1 + \overline{h}_1^\perp \otimes h_1^\perp \cos(2\phi) \\
& + |S_T| \ \ \overline{f}_1 \otimes f_{1T}^\perp \sin\phi_S \\
& + |S_T| \ \ \overline{h}_1^\perp \otimes h_{1T}^\perp \sin(2\phi + \phi_S) \\
& + |S_T| \ \ \overline{h}_1^\perp \otimes h_{1T} \sin(2\phi - \phi_S)
\end{aligned}
$$





Experimental access to Sivers in Drell-Yan



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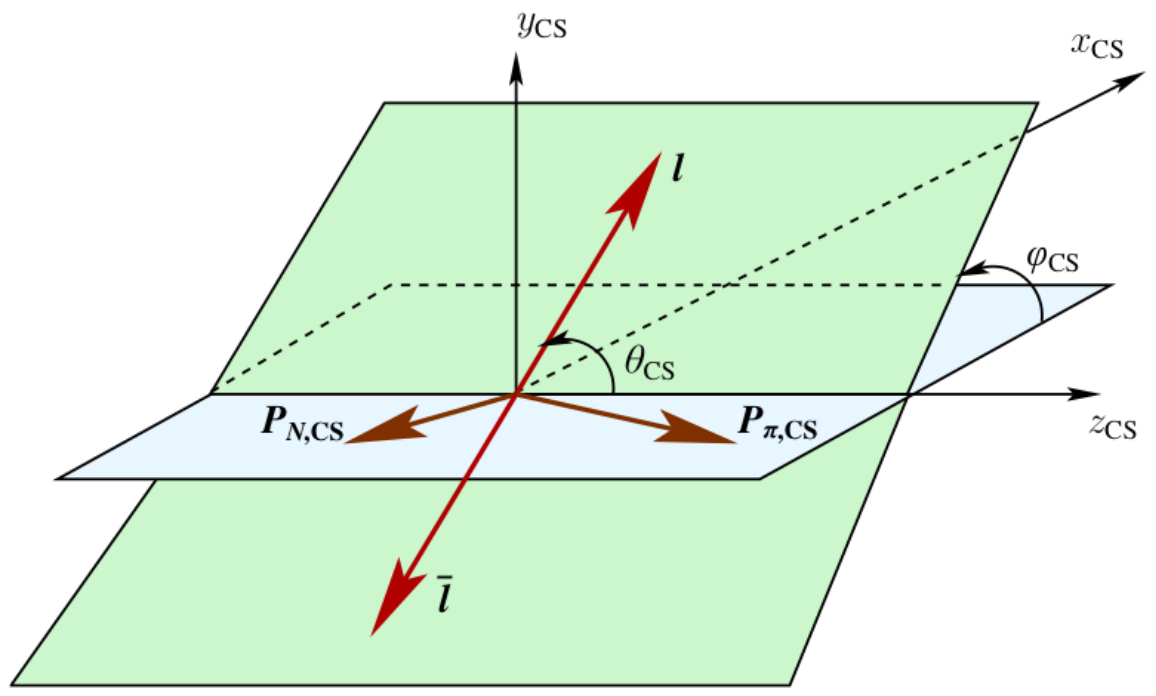
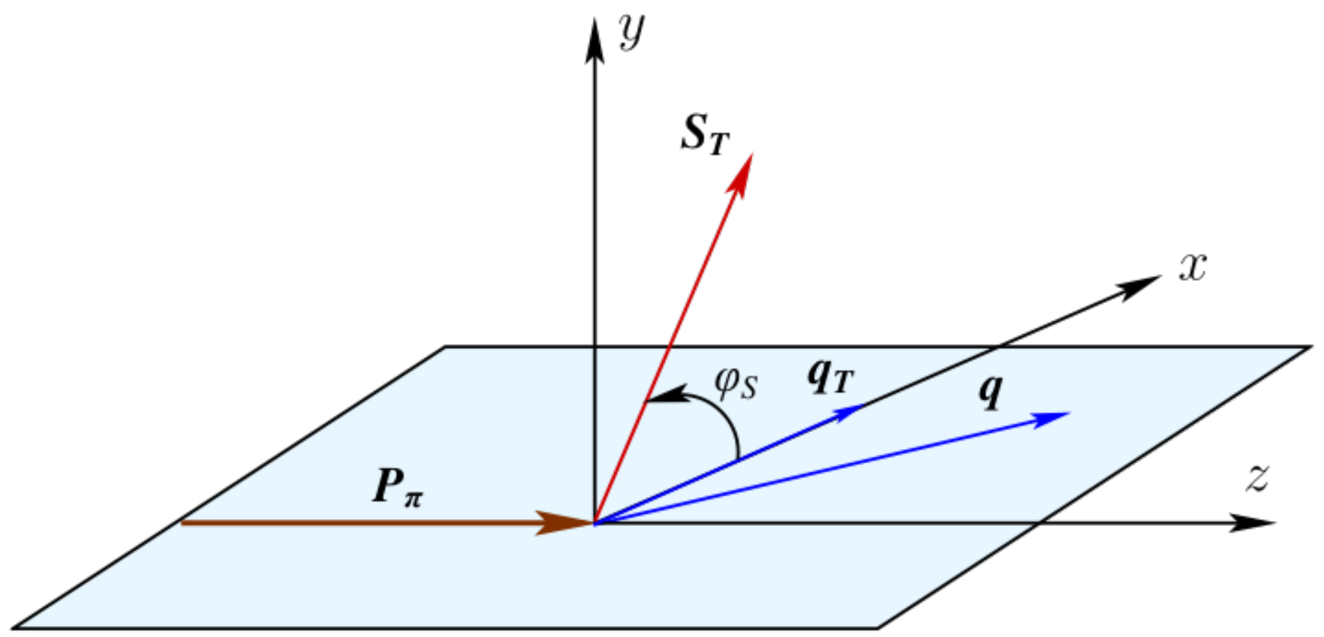
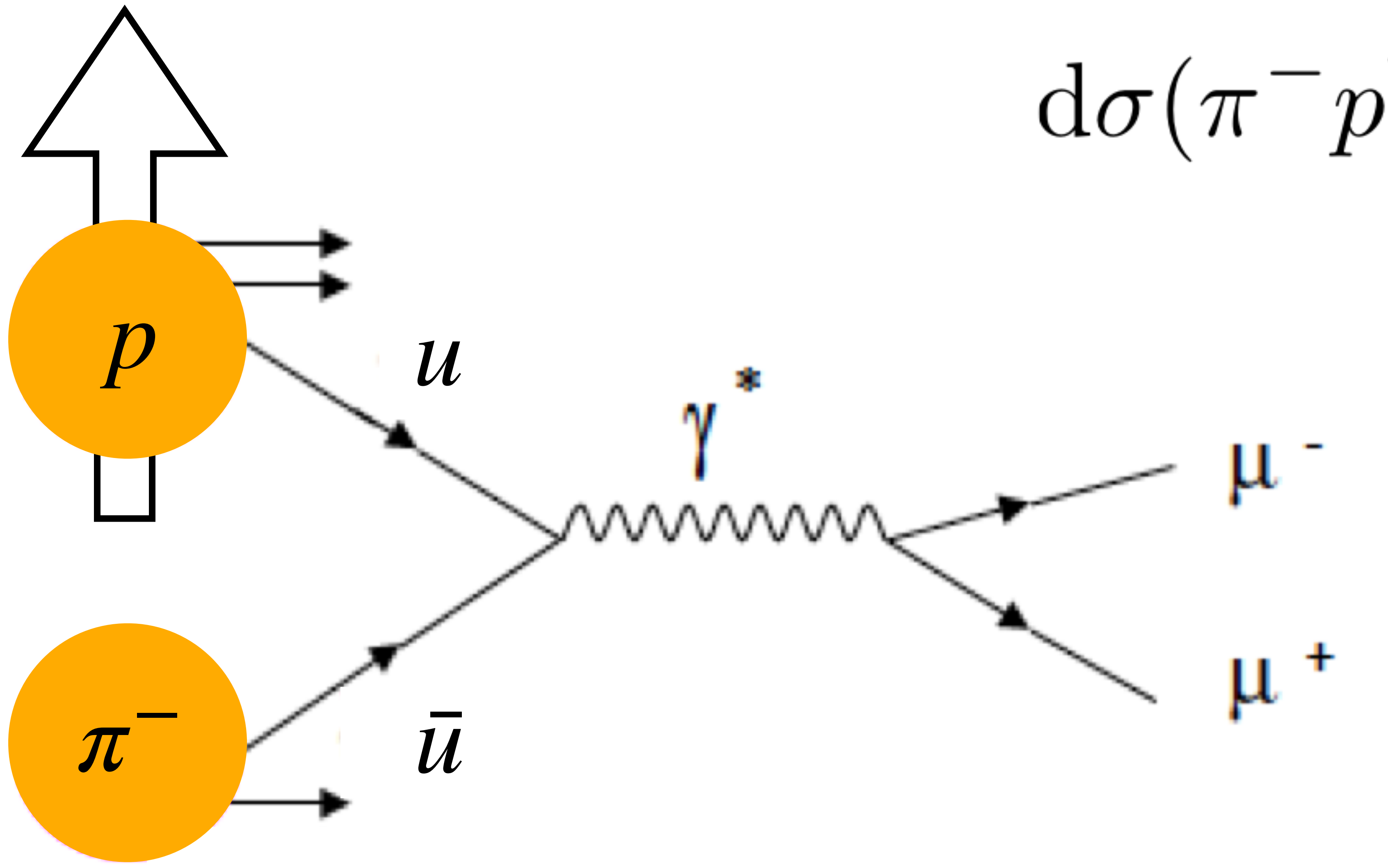
$$





Experimental access to Sivers in Drell-Yan



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\end{aligned}
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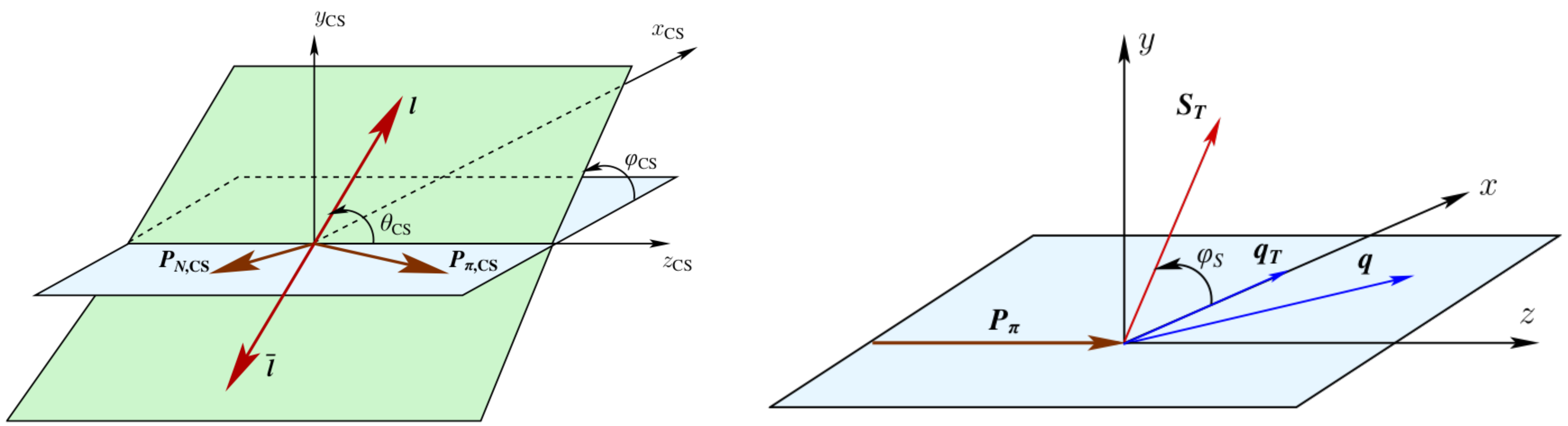


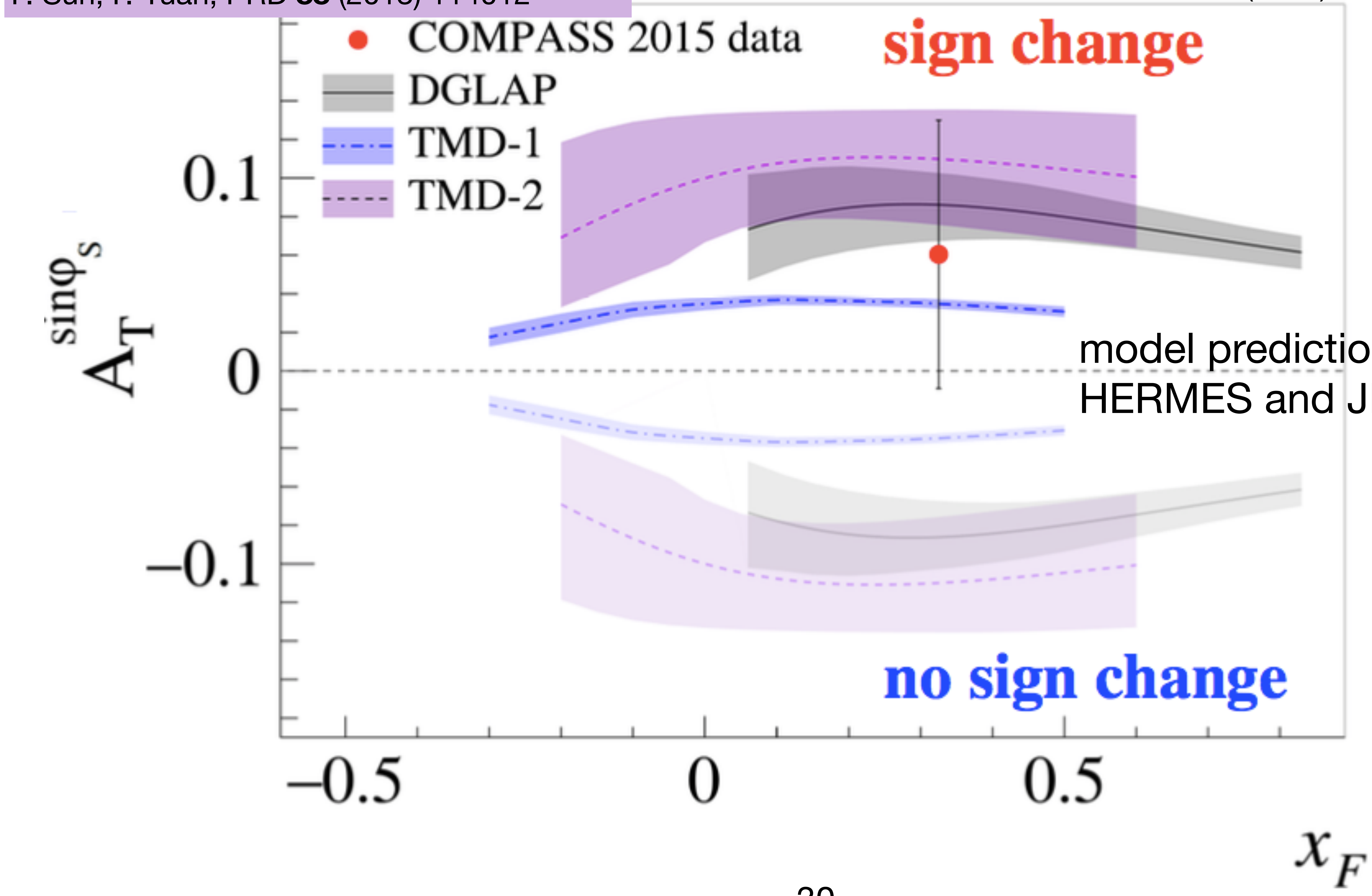
Investigation of the Sivers sign change in $p^{\uparrow}\pi^{-}$ collisions

M. Anselmino et al., JHEP **04** (2017) 046

M. G. Echevarria et al. PRD **89** (2014)074013

P. Sun, F. Yuan, PRD **88** (2013) 114012

COMPASS, PRL **119** (2017) 112002



model predictions based on COMPASS, HERMES and JLab semi-inclusive DIS data

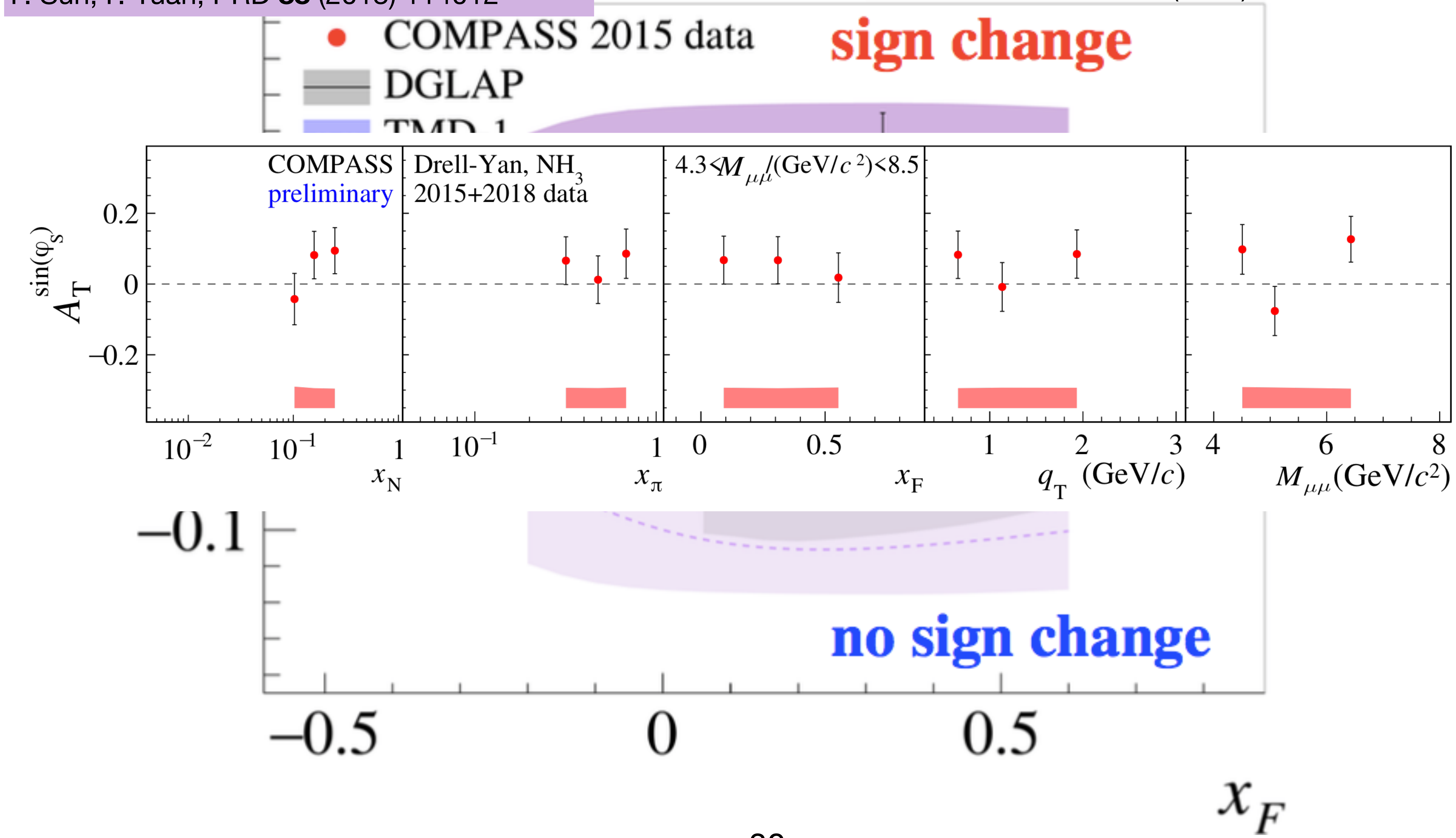
Investigation of the Sivers sign change in $p^{\uparrow}\pi^{-}$ collisions

M. Anselmino et al., JHEP **04** (2017) 046

M. G. Echevarria et al. PRD **89** (2014)074013

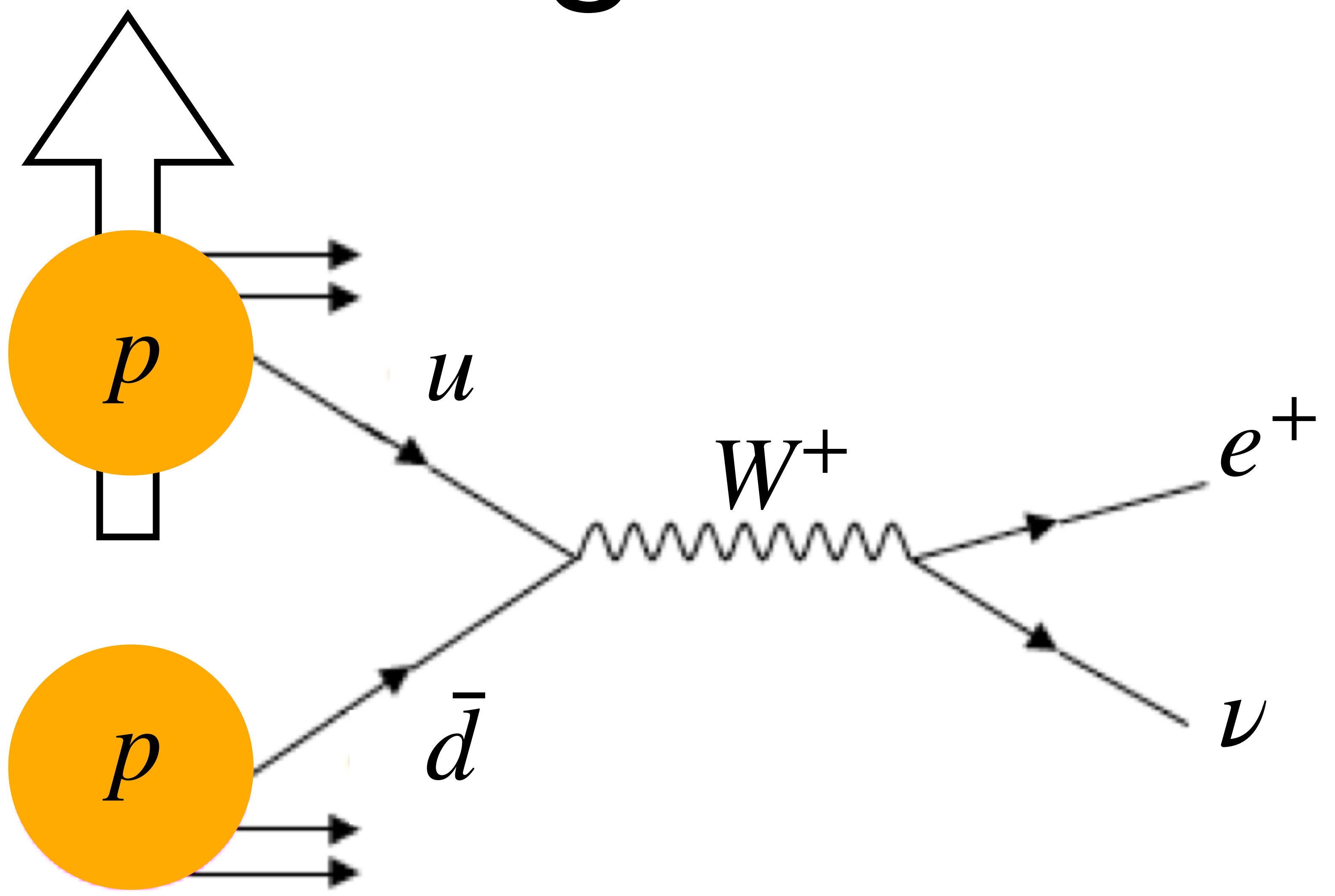
P. Sun, F. Yuan, PRD **88** (2013) 114012

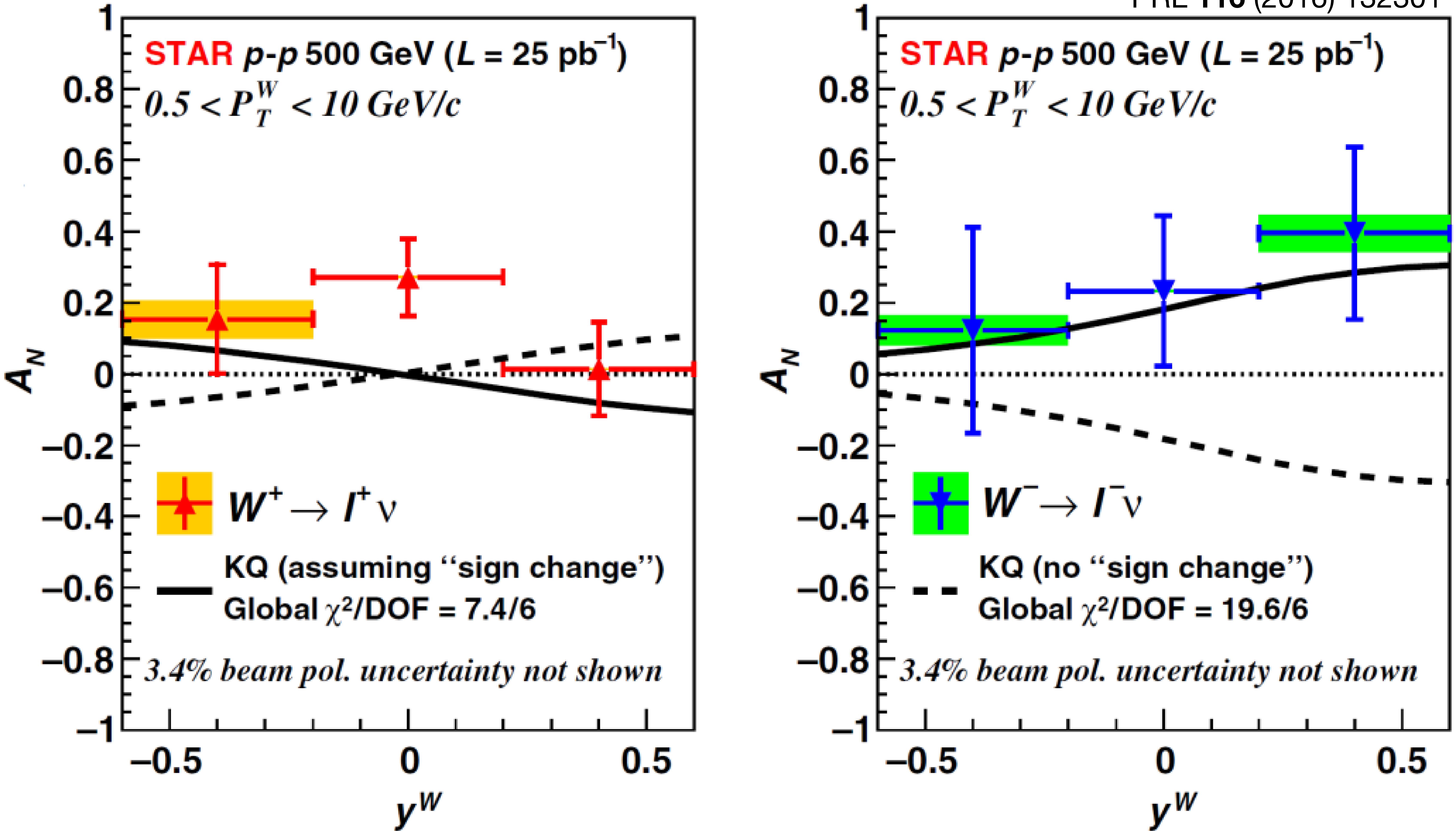
COMPASS, PRL **119** (2017) 112002



on COMPASS,
inclusive DIS data

Investigation of the Sivers sign change in $p^{\uparrow}p$ collisions

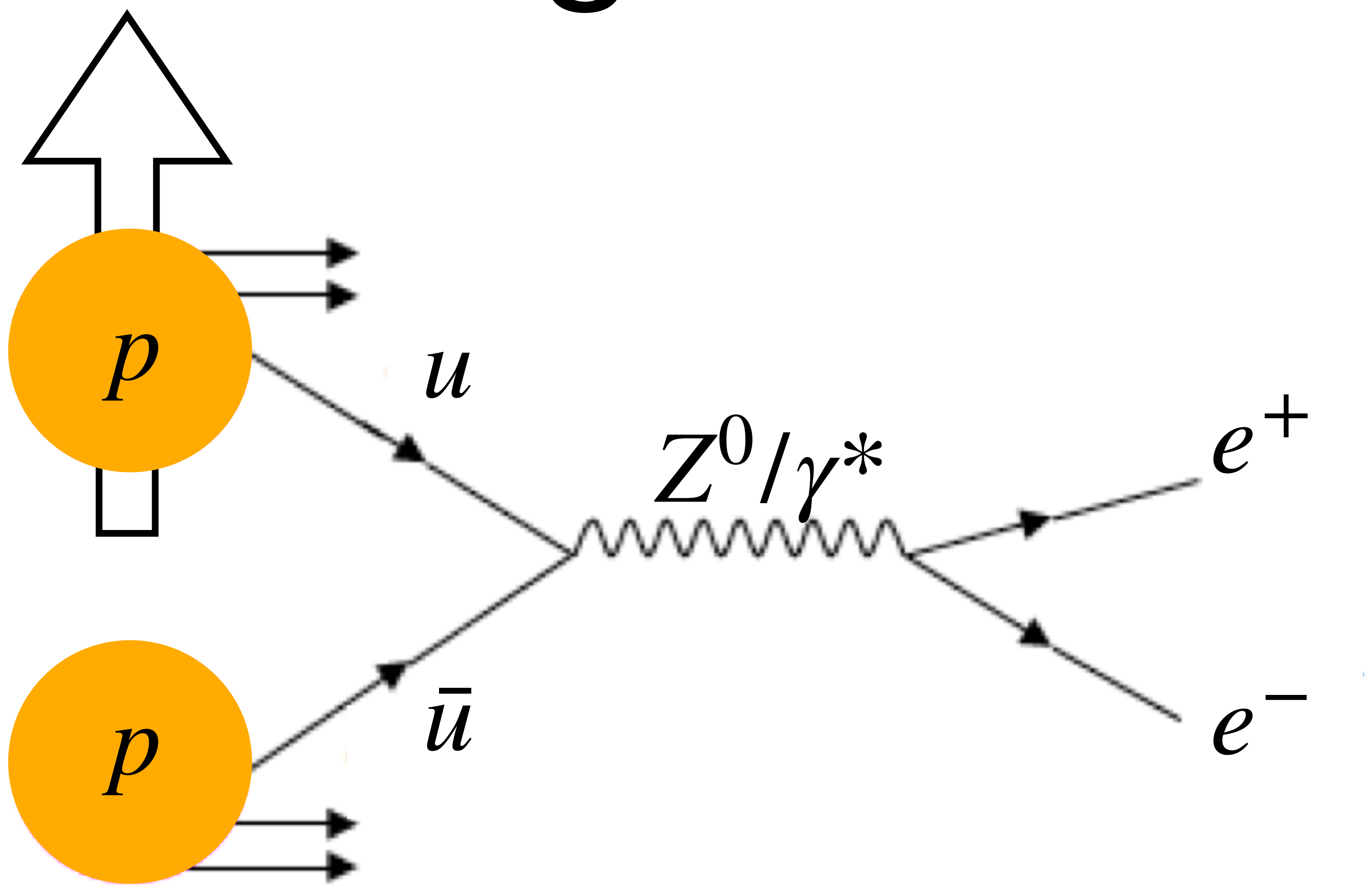




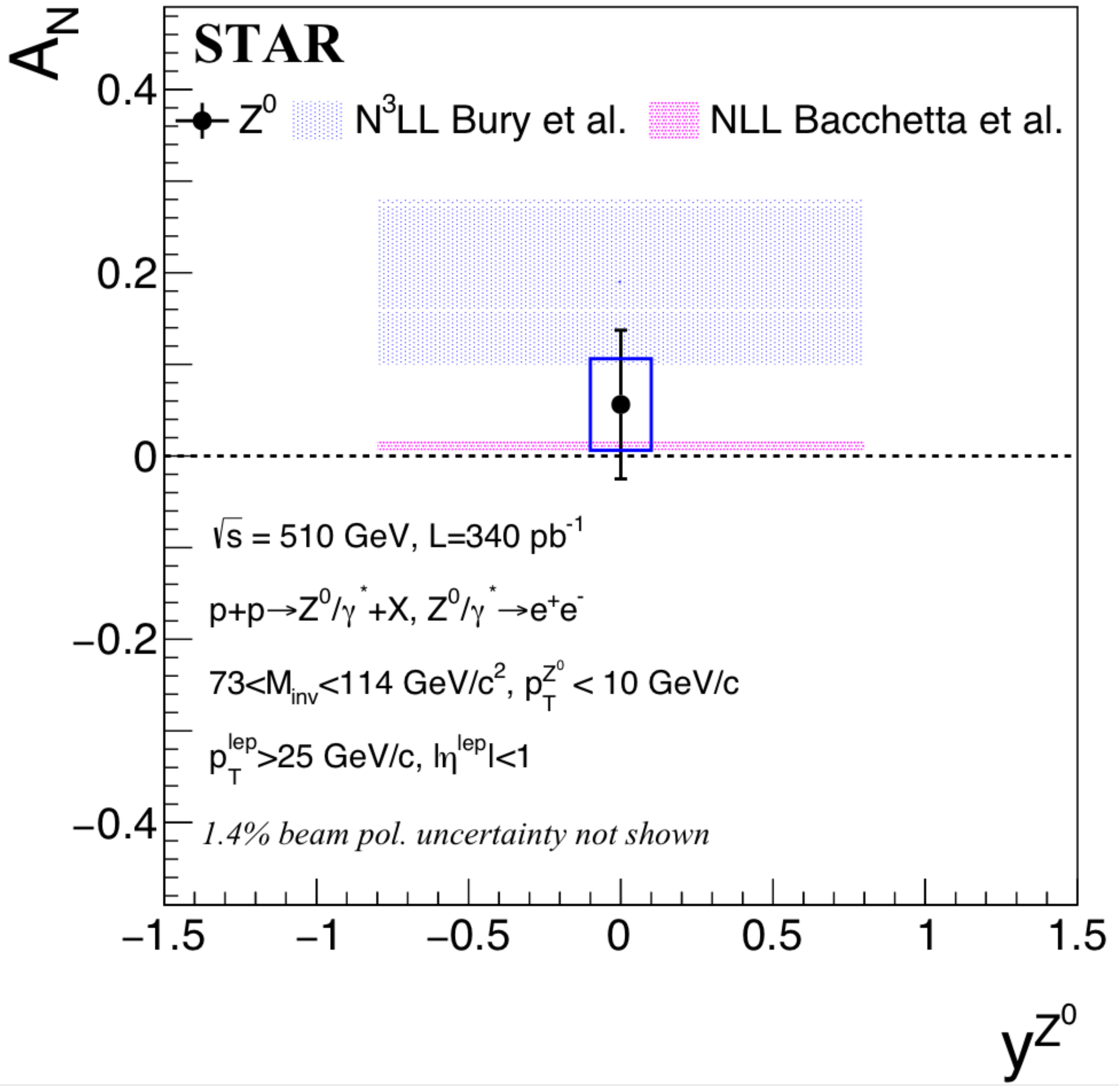
PRL **116** (2016) 132301

$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

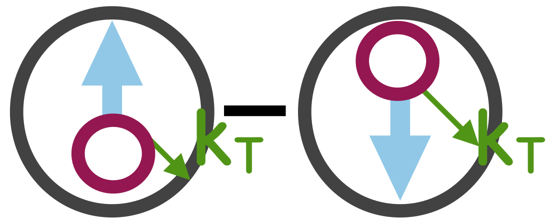
Investigation of the Sivers sign change in $p^{\uparrow}p$ collisions



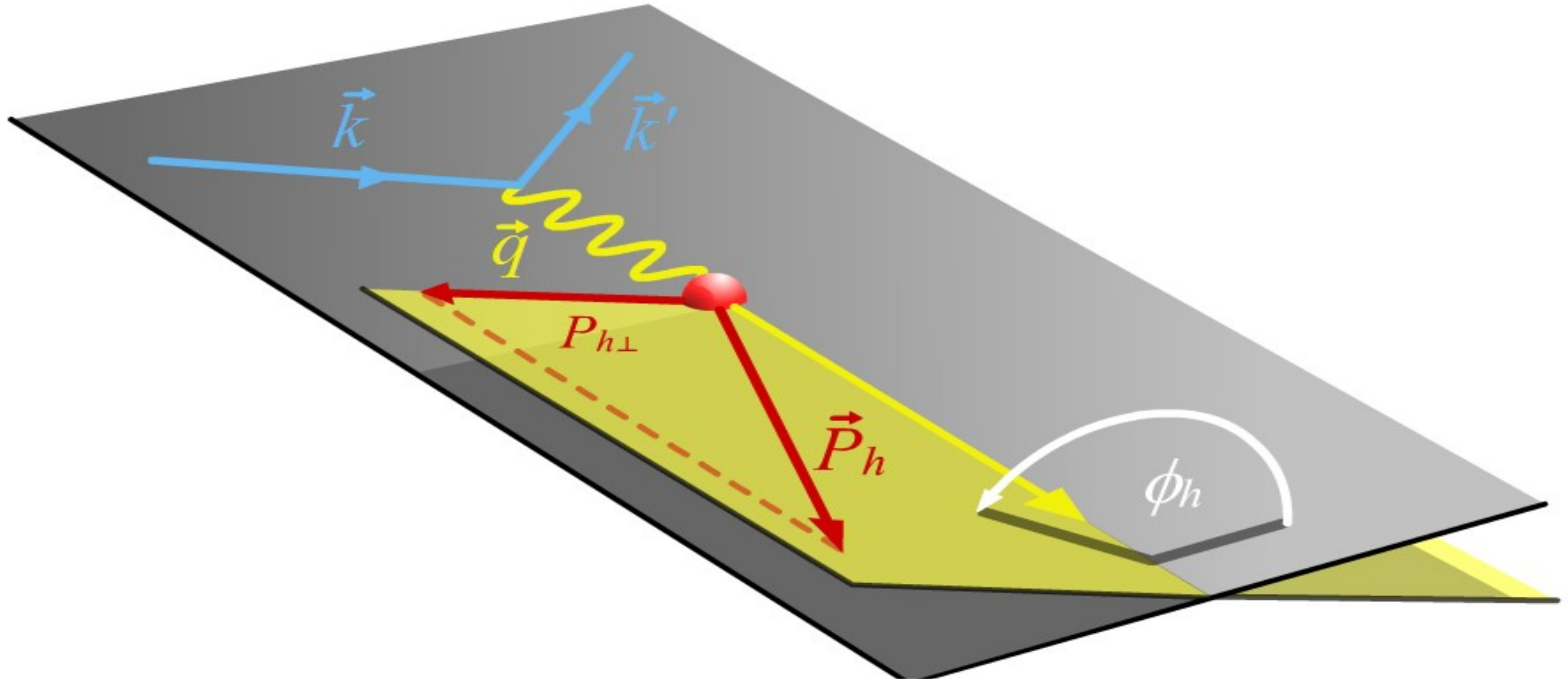
arXiv:2308.15496v1



Boer-Mulders modulation



$$\mathscr{C}\left[h_1^{\perp,q} \times H_1^{\perp,q}\right]$$



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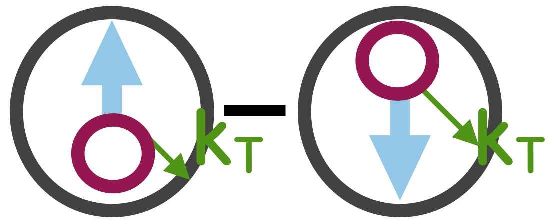

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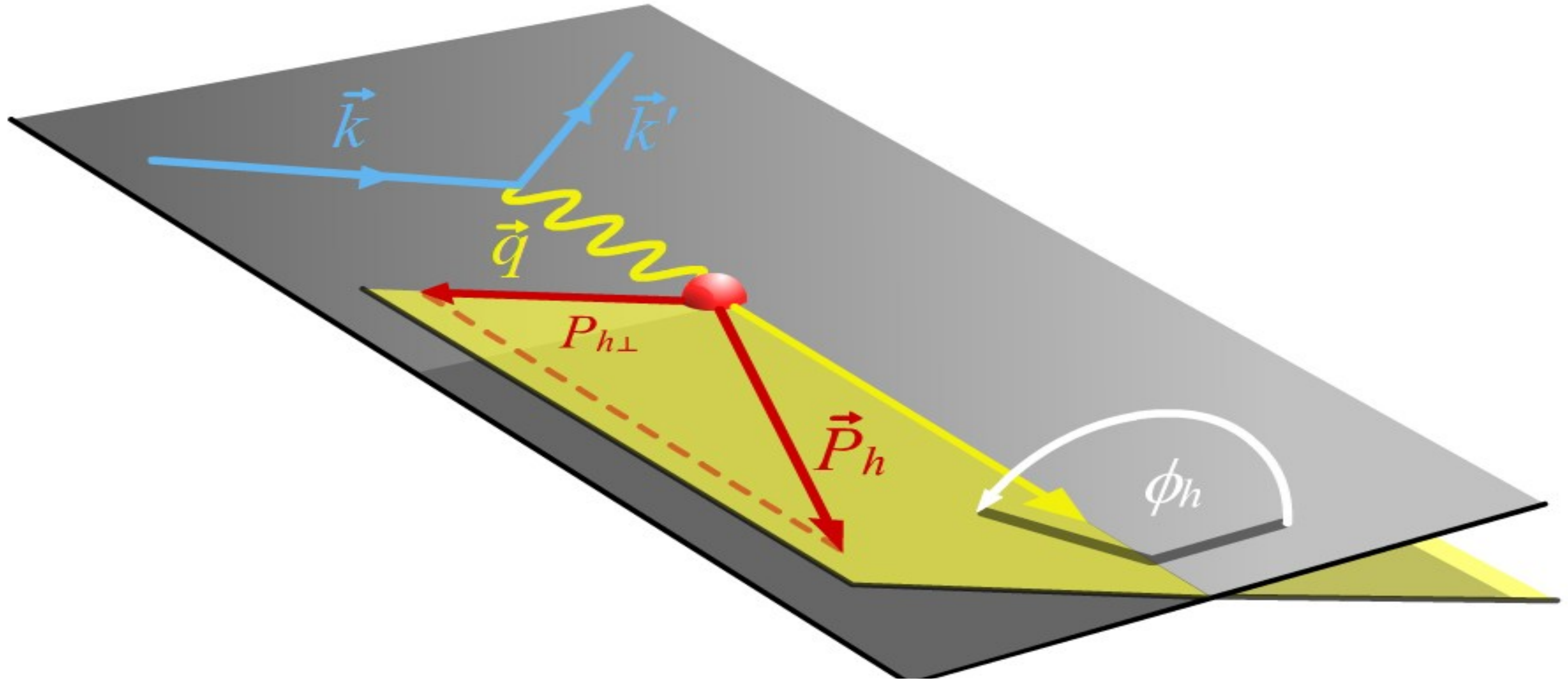
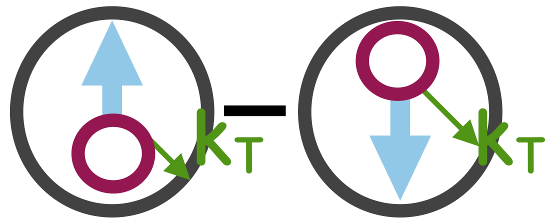


$$\cos(2\phi_h)\sum_q e_q^2\, C\left[h_1^{\perp,q}(x,k_\perp)\times H_1^{\perp,q}(z,p_\perp)\right]$$

Boer-Mulders modulation



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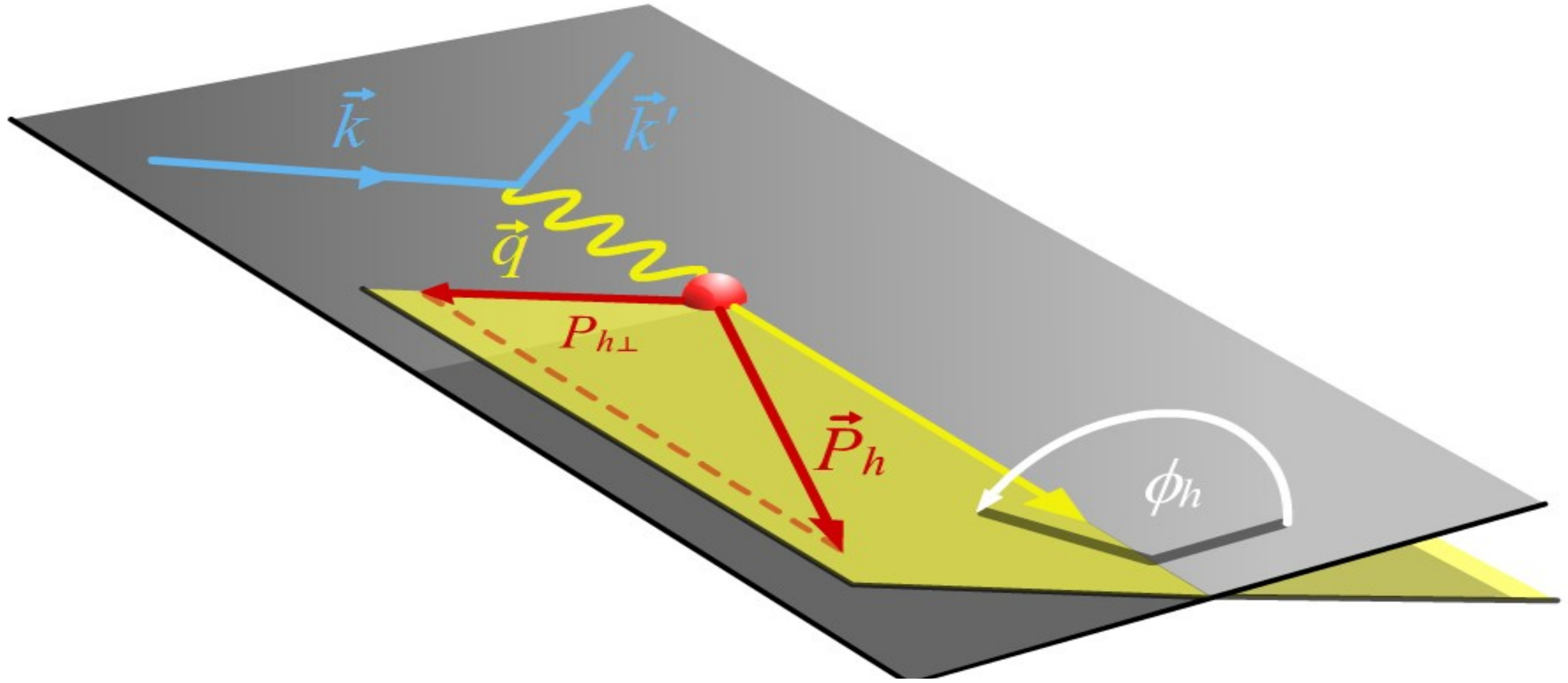

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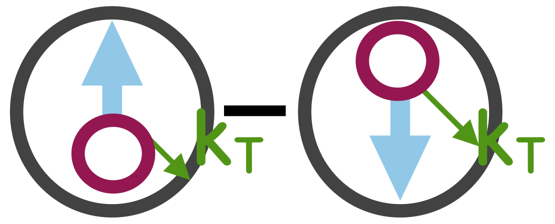
Spin-dependence with unpolarised hadrons!

Boer-Mulders modulation



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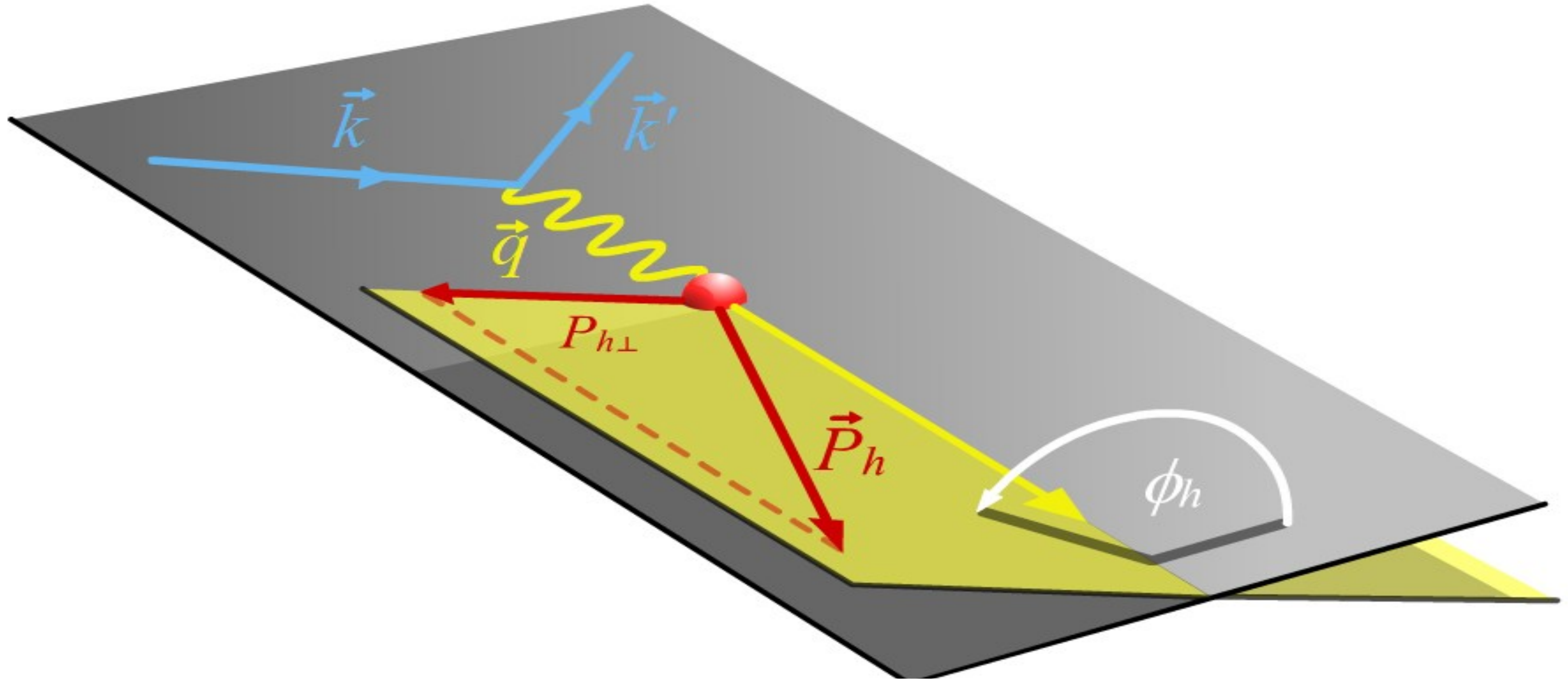
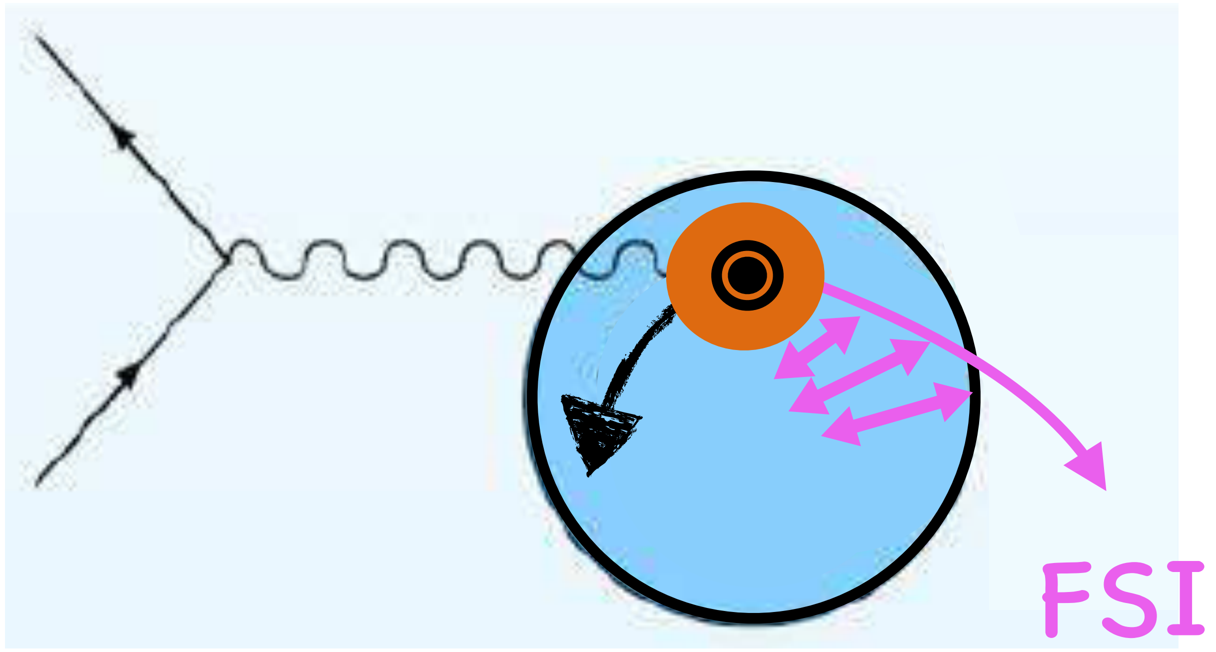


Boer-Mulders PDF



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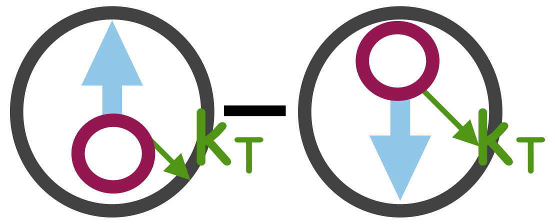
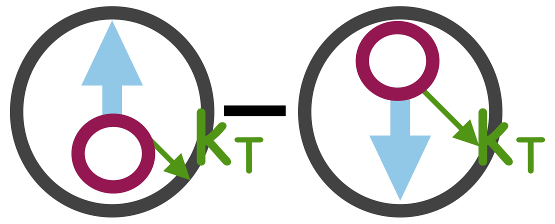


$\mathscr{C}\left[h_1^{\perp,q}\times H_1^{\perp,q}\right]$

Measurement in ep: $\langle\cos(2\phi_h)\rangle_{Born}(j)$ $\langle\cos(2\phi_h)\rangle_{meas}(i)$

Boer-Mulders modulation

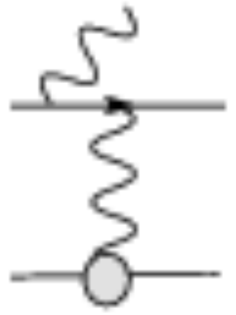


$\mathscr{C}\left[h_1^{\perp,q} \times H_1^{\perp,q}\right]$

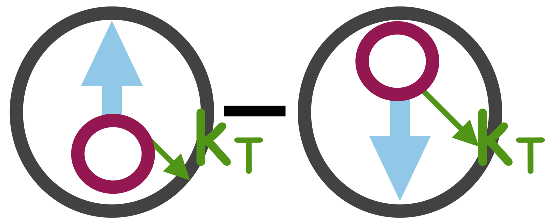
Measurement in ep:

$$\begin{tikzcd} \langle\cos(2\phi_h)\rangle_{Born}(j) & \langle\cos(2\phi_h)\rangle_{meas}(i) \arrow[l, bend left] \end{tikzcd}$$

- QED radiate effects



Boer-Mulders modulation



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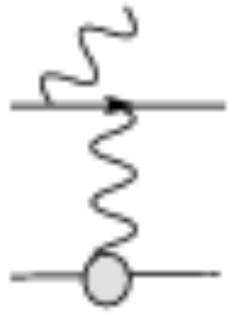
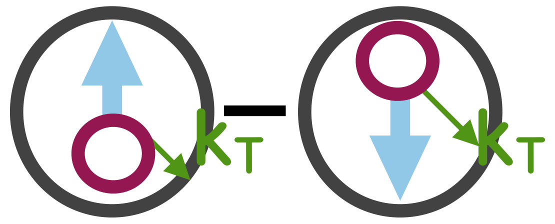


- limited geometric and kinematic acceptance of detector

Boer-Mulders modulation



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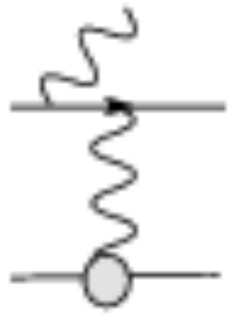
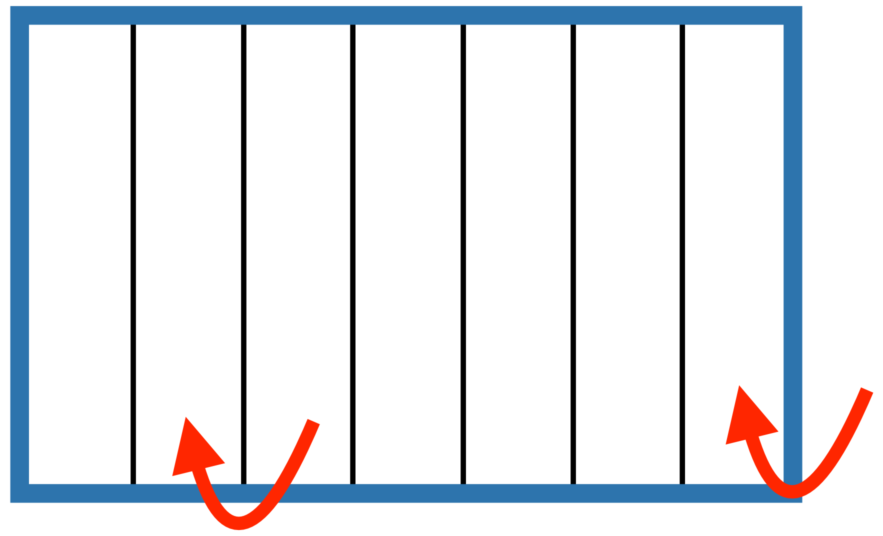
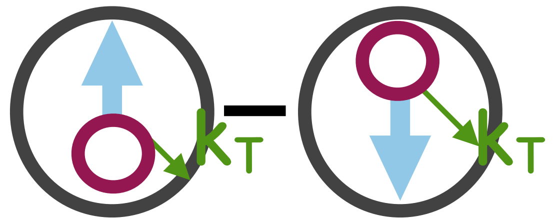


- limited geometric and kinematic acceptance of detector
- limited detector resolution



Boer-Mulders modulation



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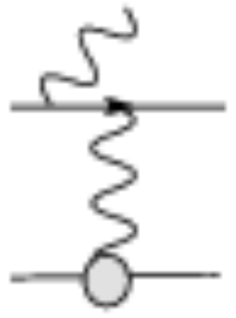

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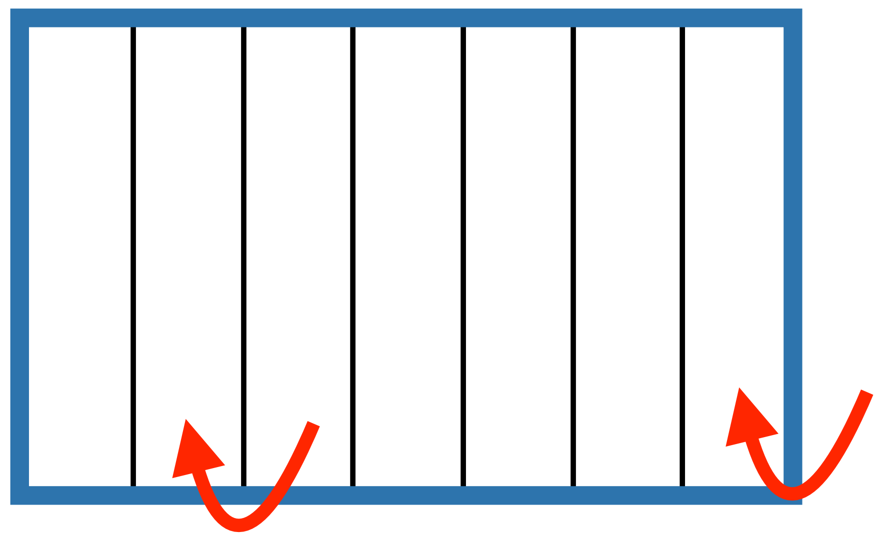
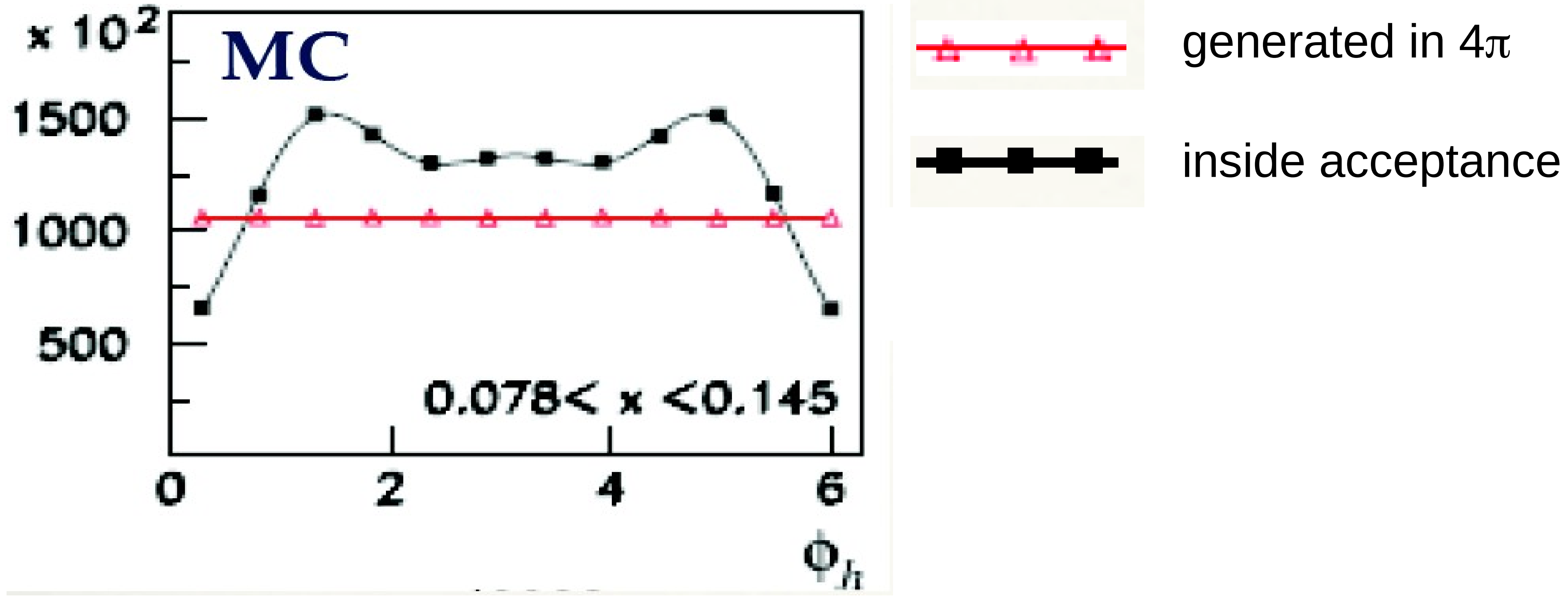
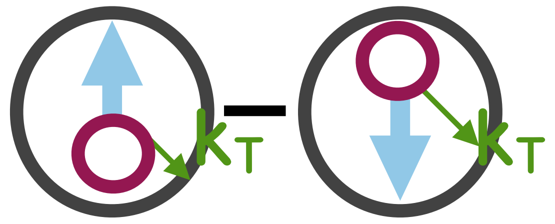




Boer-Mulders modulation



$\mathscr{C}\left[h_1^{\perp,q} \times H_1^{\perp,q}\right]$

Measurement in ep:

Fully differential analysis

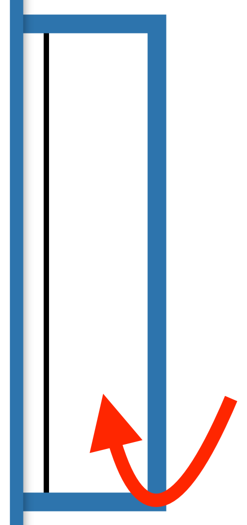
Unfolding in 400 x 12 bins

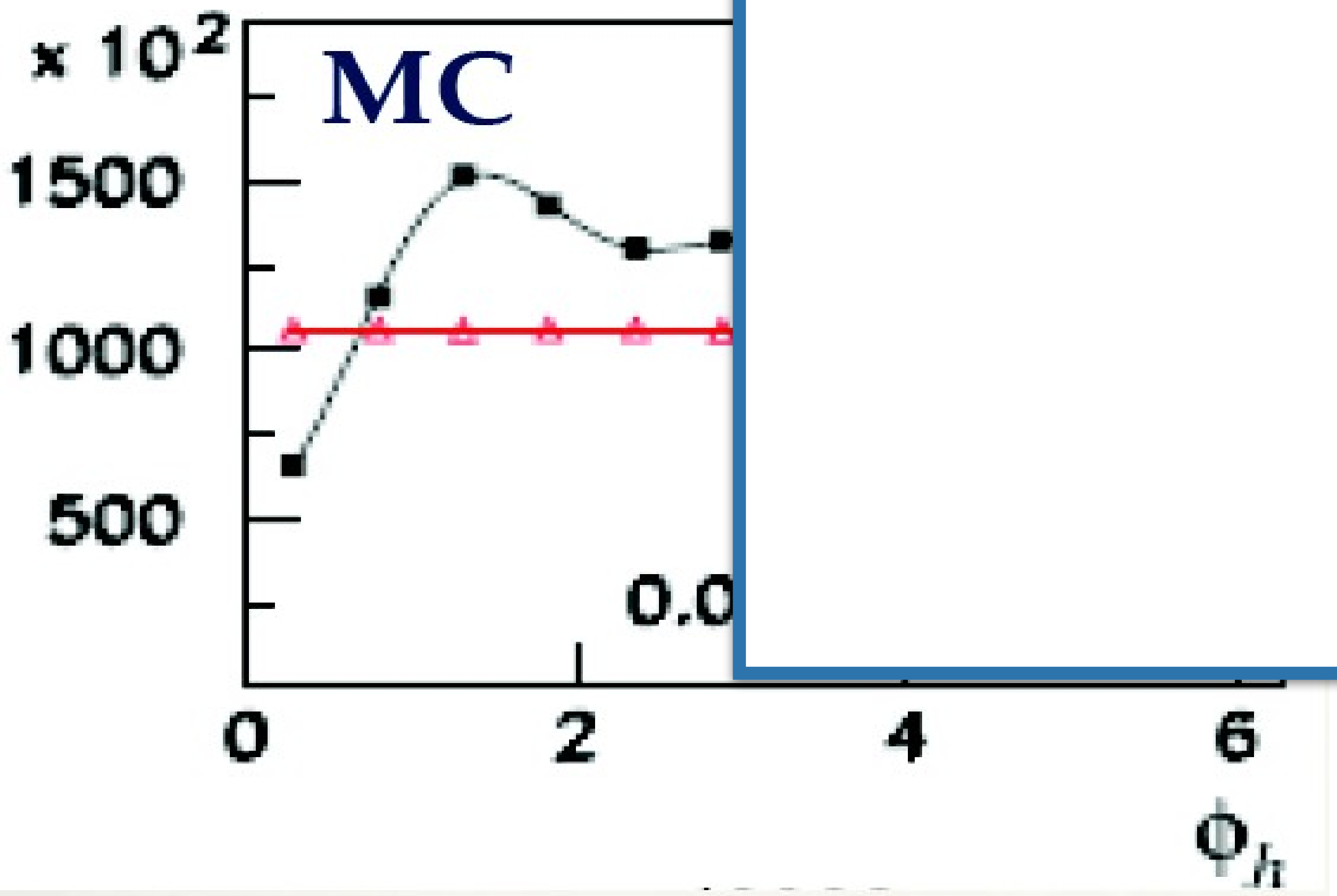
BINNING

400 kinematic bins x 12 ϕ-bins

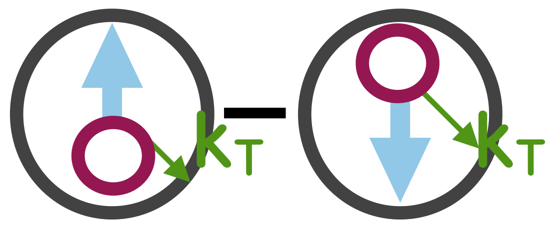
Variable	Bin limits						#
x	0.023	0.042	0.078	0.145	0.27	1	5
y	0.3	0.45	0.6	0.7	0.85		4
z	0.2	0.3	0.45	0.6	0.75	1	5
P_{hT}	0.05	0.2	0.35	0.5	0.75		4

tector



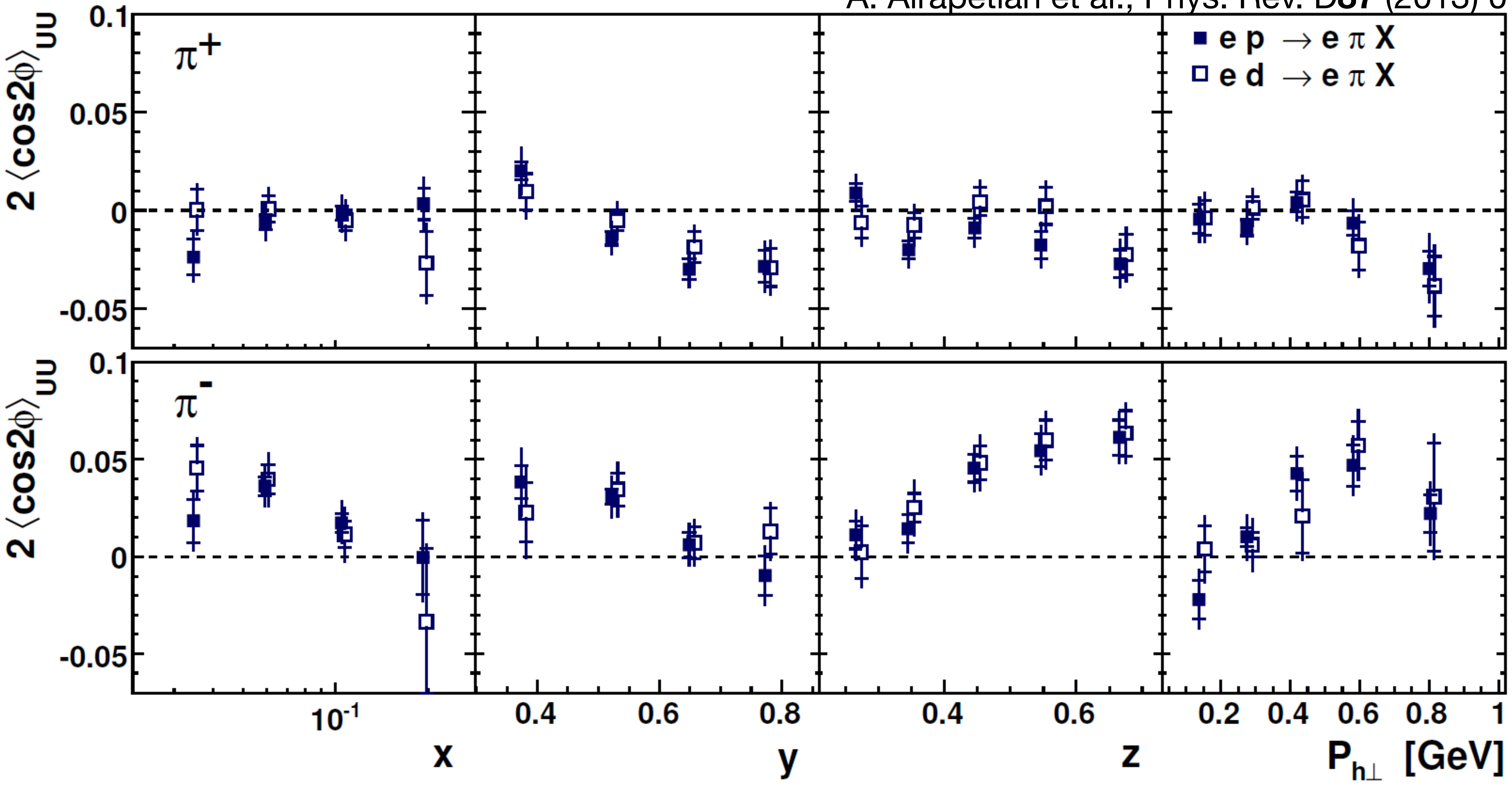


Boer-Mulders asymmetries



$\mathscr{C}\left[h_1^{\perp,q} \times H_1^{\perp,q}\right]$

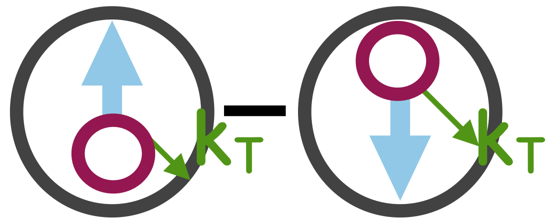
A. Airapetian et al., Phys. Rev. D**87** (2013) 012010



H–D comparison: $h_1^{\perp,u} \approx h_1^{\perp,d}$

Negative for π^+; positive for π^- $\rightarrow H_1^{\perp,fav} \approx -H_1^{\perp,disfav}$

Boer-Mulders asymmetries



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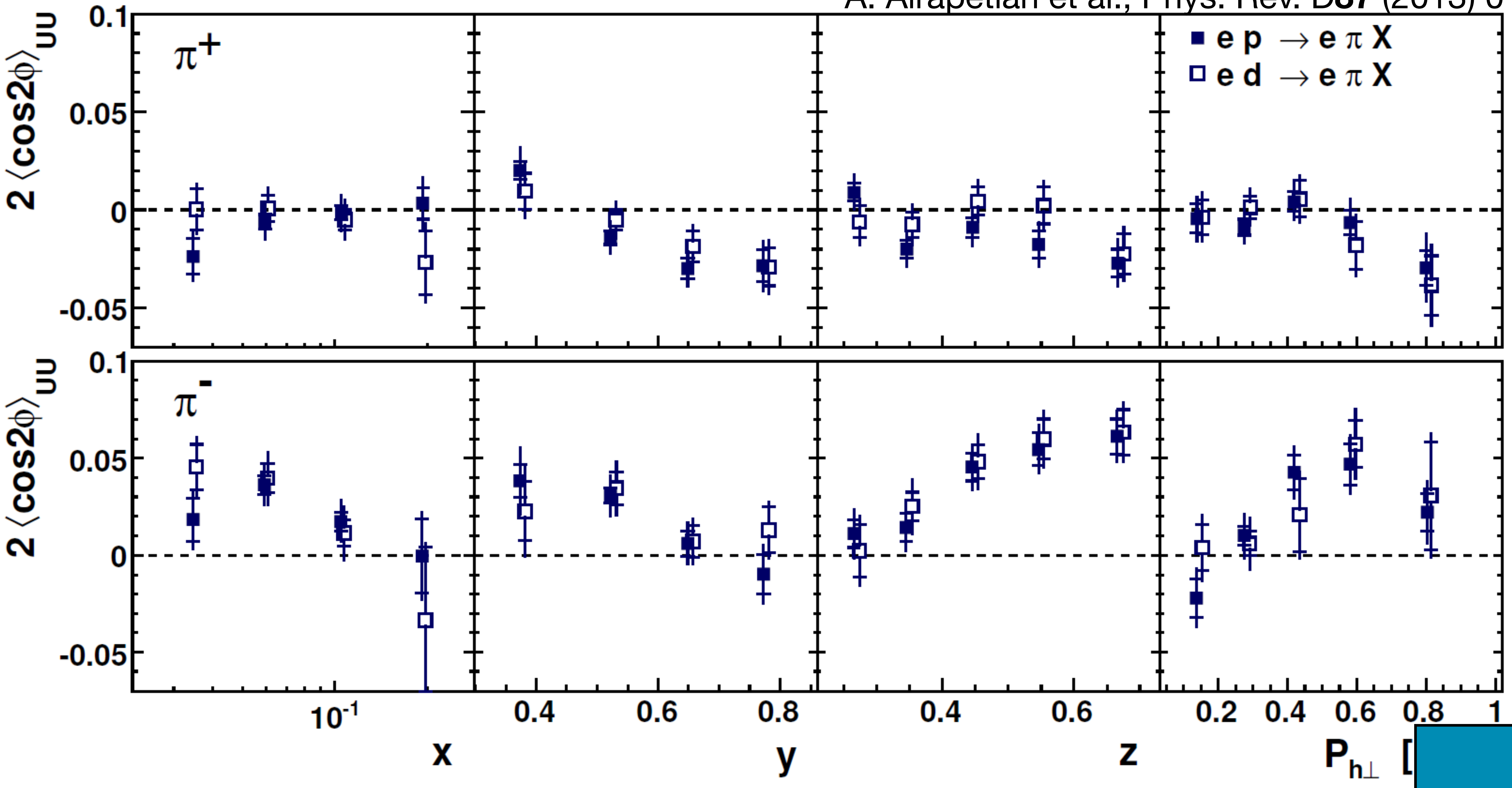
A. Airapetian et al., Phys. Rev. D**87** (2013) 012010

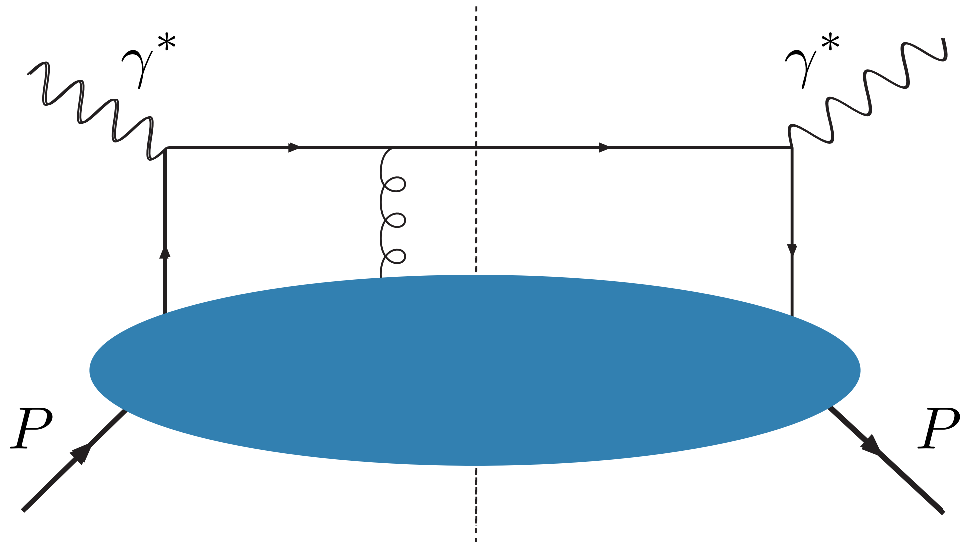


Measurement also possible in Drell Yan.

H–D comparison: $h_1^{\perp,u} \approx h_1^{\perp,d}$

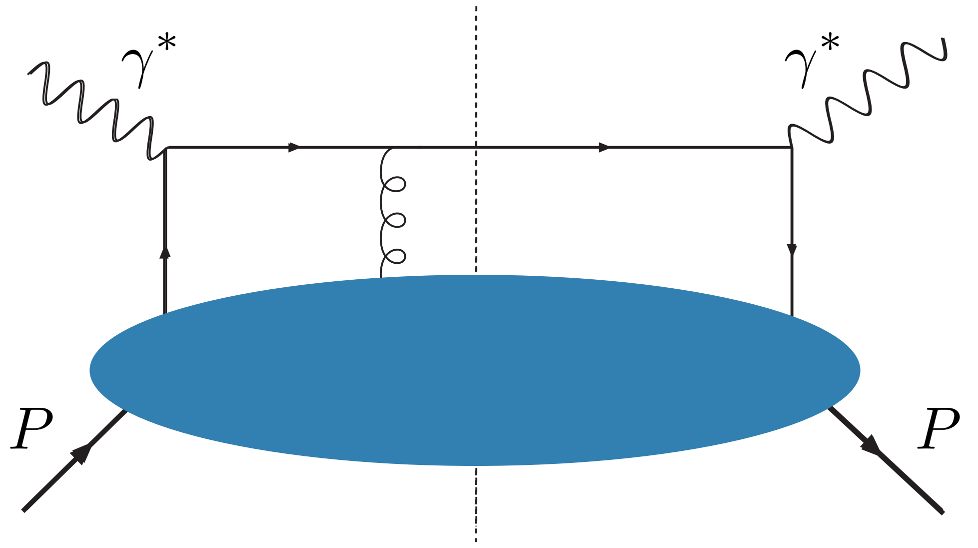
Negative for π^+; positive for π^- $\rightarrow H_1^{\perp,fav} \approx -H_1^{\perp,disfav}$

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$



$$\langle \sin(\phi) \rangle_{LU}^{h} \propto \mathcal{C}\left[h_1^{\perp} \times \tilde{E}, e \times H_1^{\perp}, g^{\perp} \times D_1, f_1 \times \tilde{G}^{\perp}\right]$$

Twist-3: $\langle\sin(\phi)\rangle^h_{LU}$



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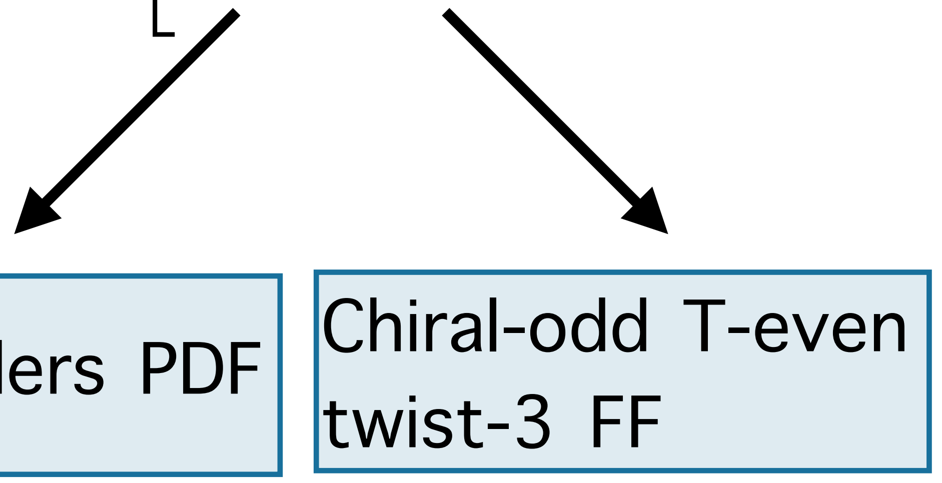
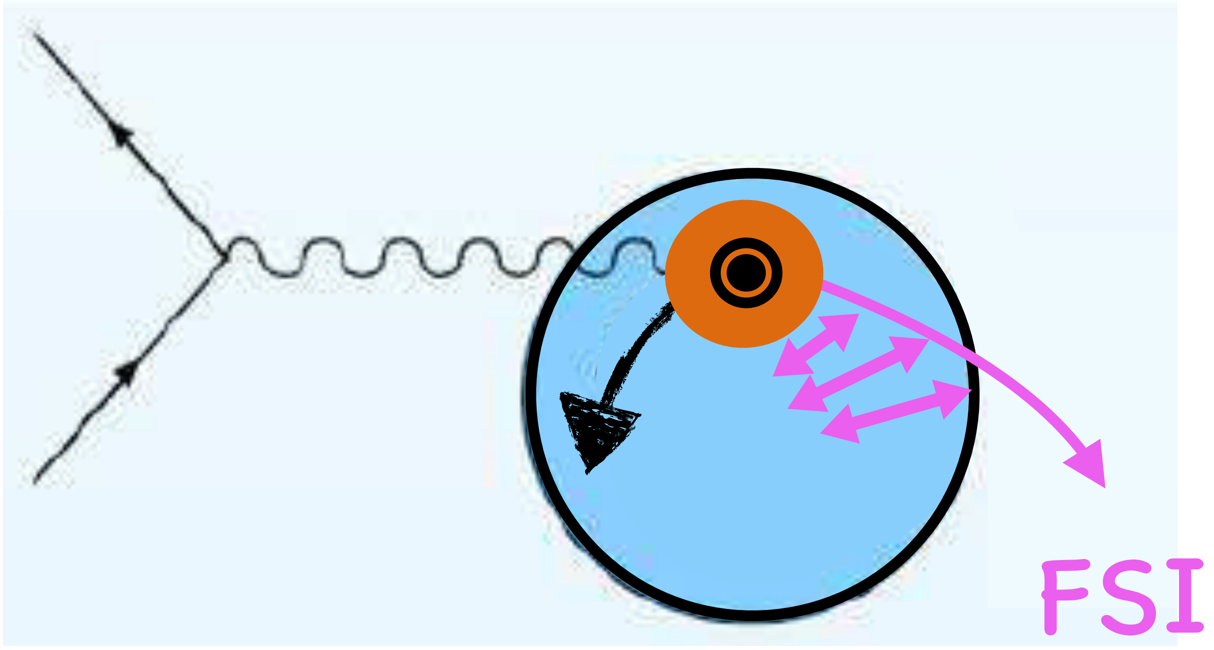


Boer-Mulders PDF

Chiral-odd T-even twist-3 FF

Boer-Mulders PDF



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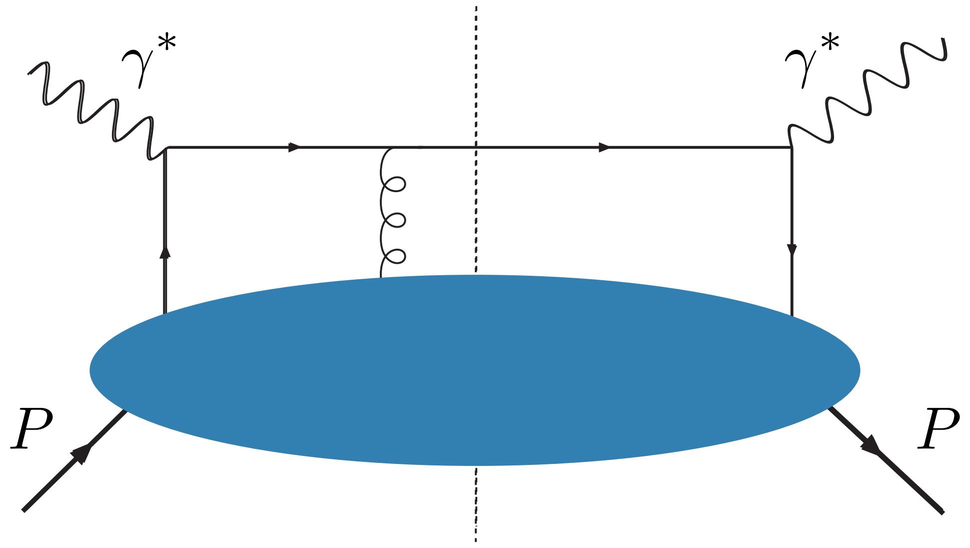

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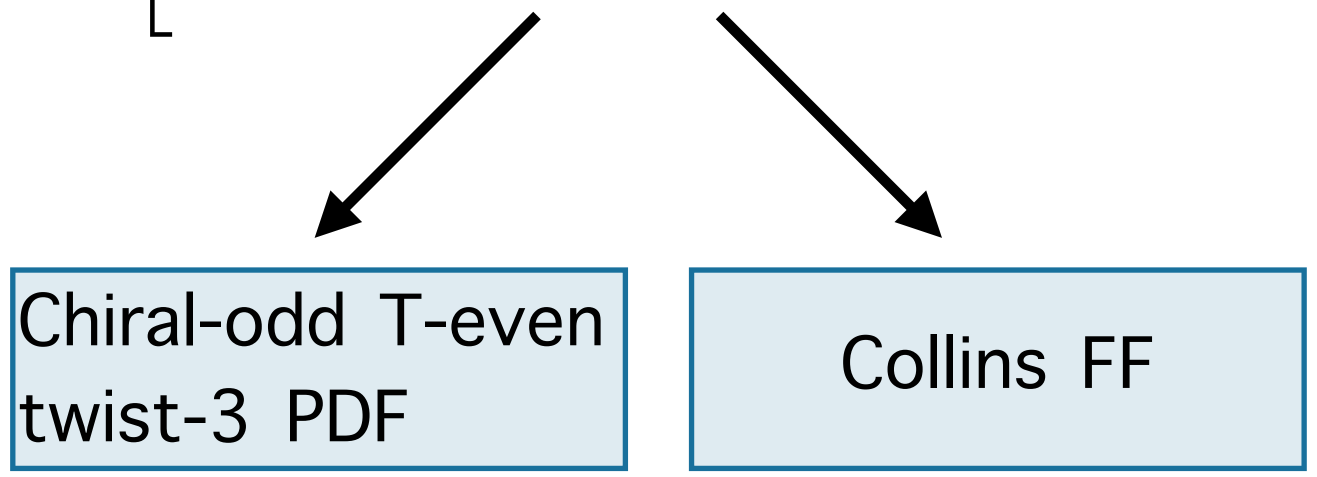

Chiral-odd T-even twist-3 PDF

Collins FF

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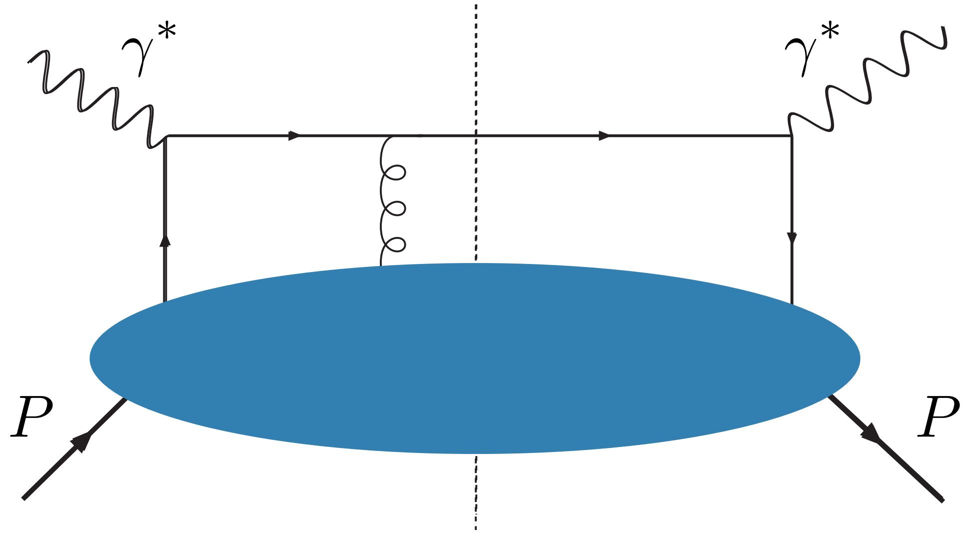

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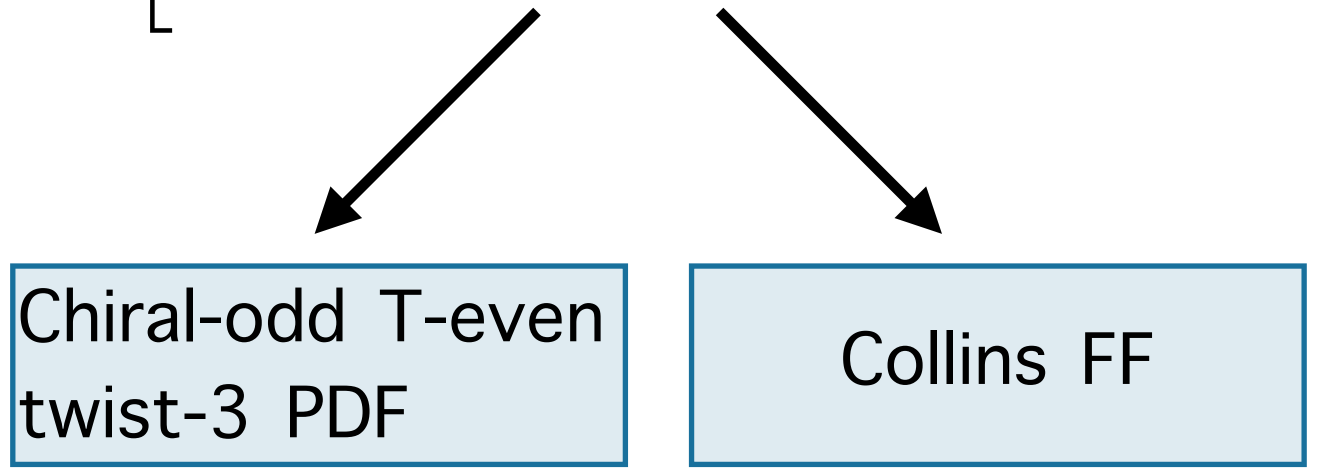
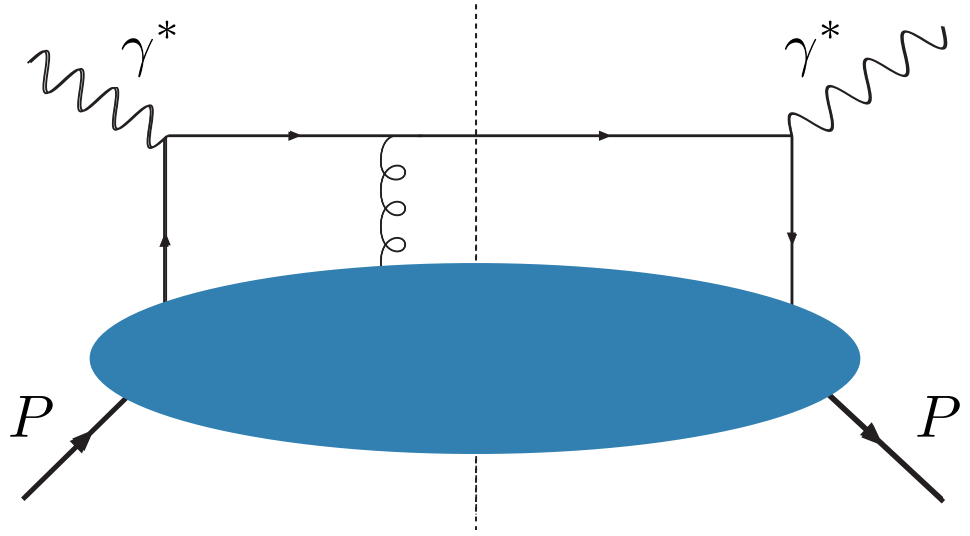


Chiral-odd T-even twist-3 PDF

Collins FF

$$e(x) = e^{\mathrm{WW}}(x) + \bar{e}(x)$$

Twist-3: $\langle\sin(\phi)\rangle^{h}_{LU}$



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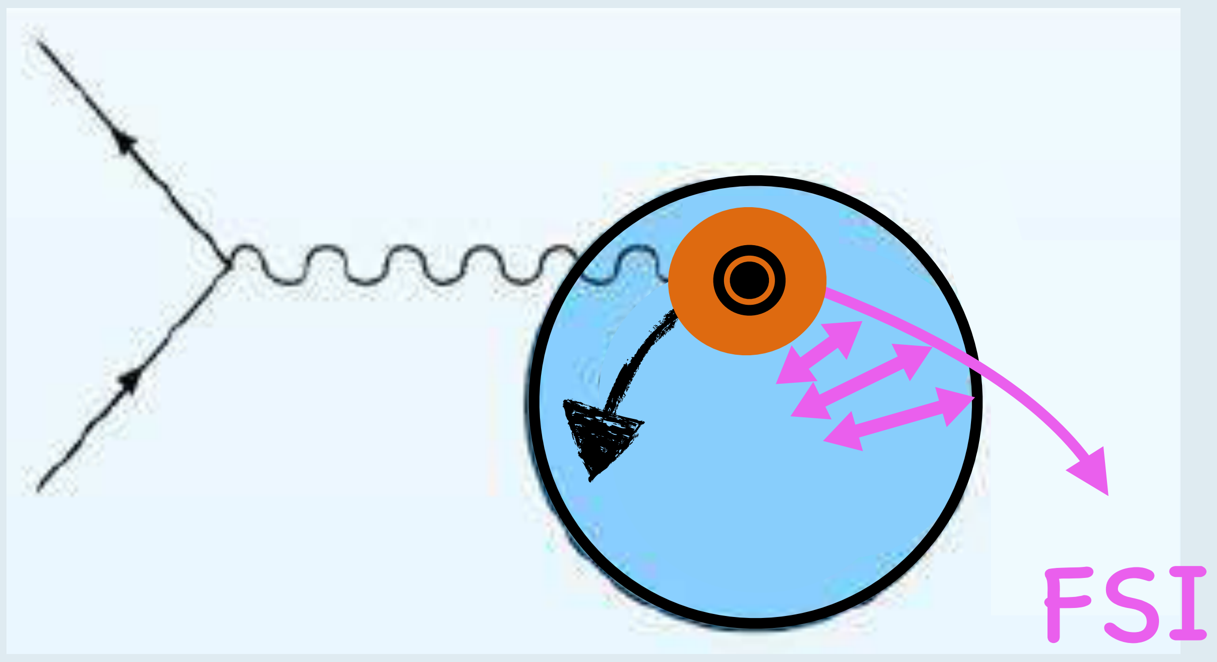
$$e(x) = e^{\mathrm{WW}}(x) + \bar{e}(x)$$

$$e_2 \equiv \int_0^1 dx\, x^2 \bar{e}(x)$$

→ force on struck quark at t=0

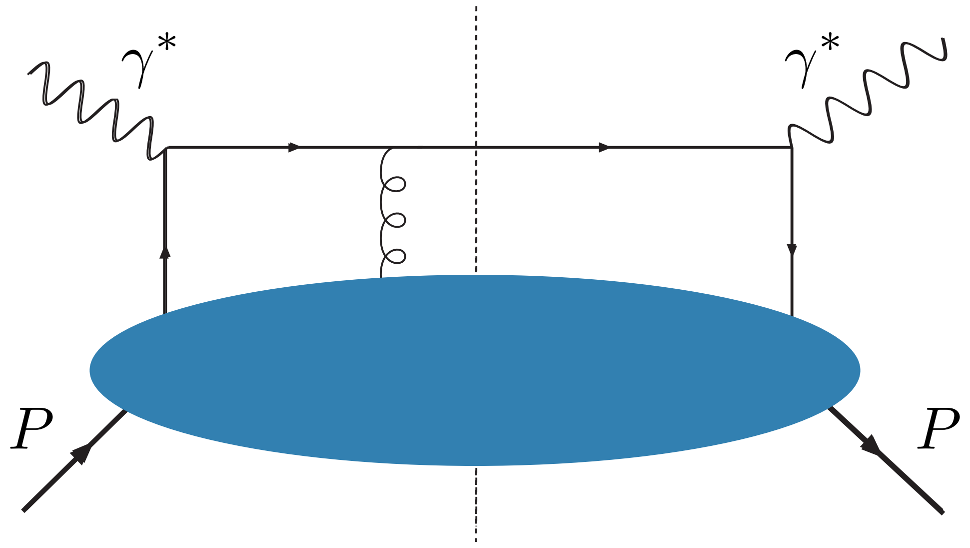
M. Burkardt, arXiv:0810.3589

Boer-Mulders PDF



FSI: t=0→∞

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$



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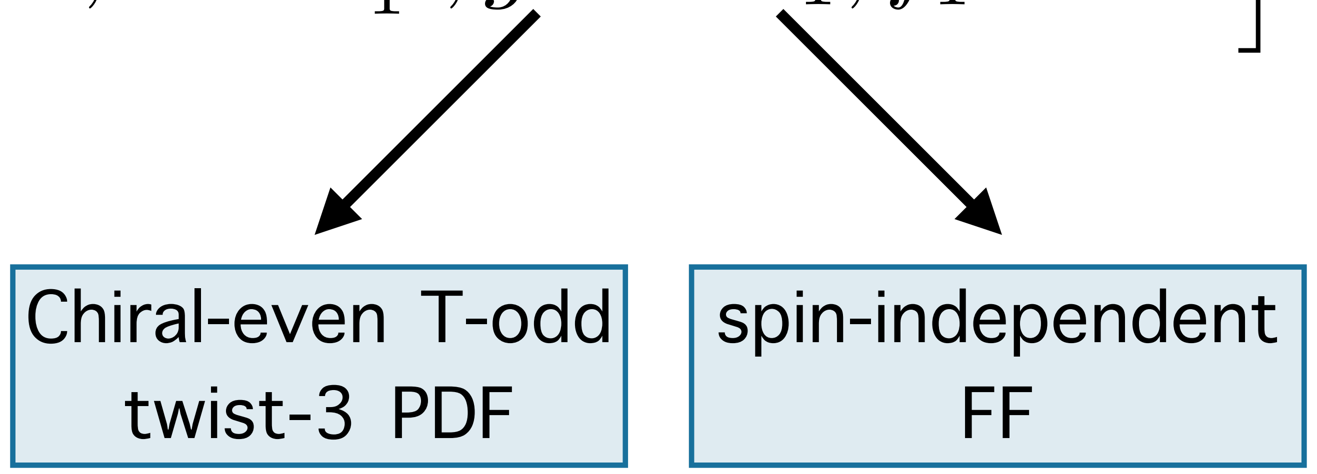
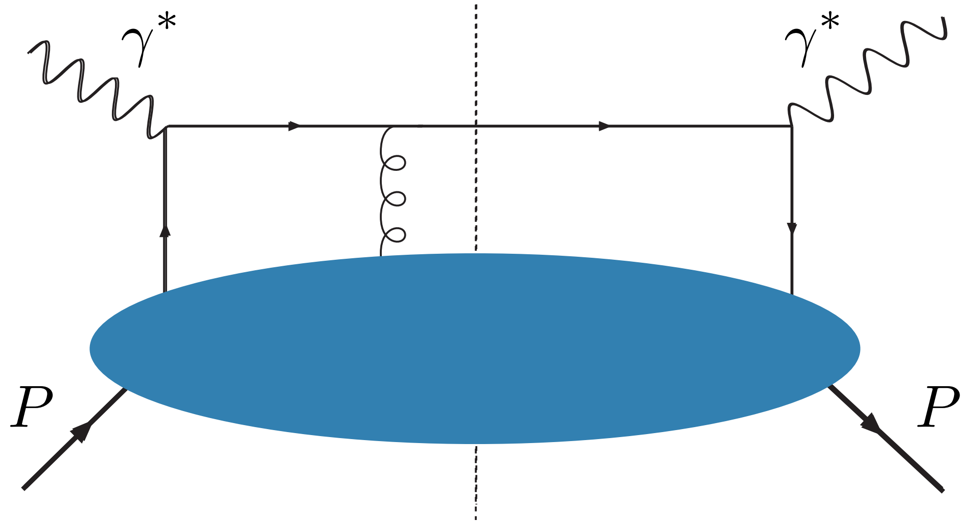


Chiral-even T-odd
twist-3 PDF

spin-independent
FF

Twist-3: $\langle\sin(\phi)\rangle_{LU}^{h}$



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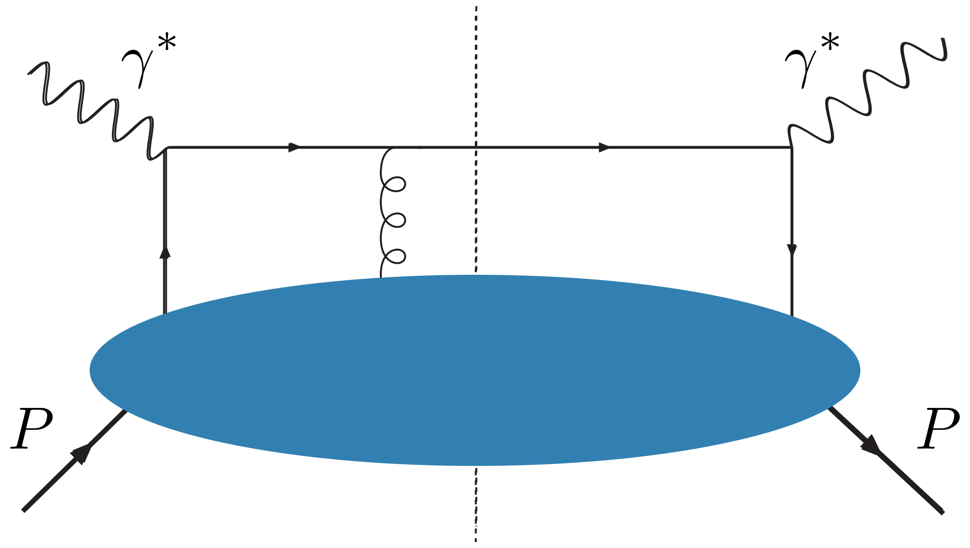
Chiral-even T-odd twist-3 PDF

spin-independent FF

Only term to survive in TMD single-jet inclusive DIS

$$e + p \to e' + \text{jet} + X$$

Twist-3: $\langle \sin(\phi) \rangle_{LU}^{h}$



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spin-independent PDF

chiral-even, T-odd twist-3 FF

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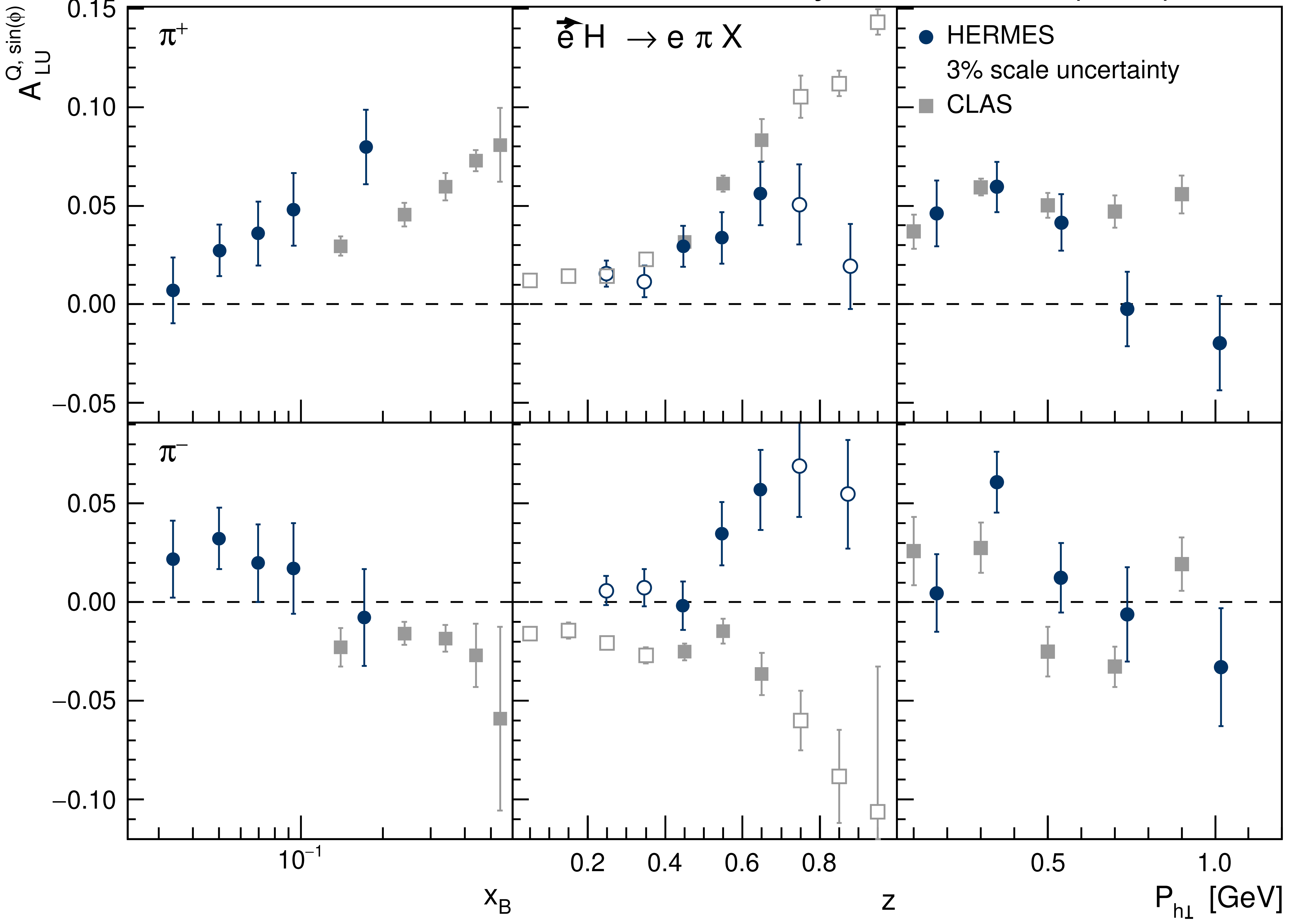
Phys. Lett. B **797** (2019) 134886



- Opposite behaviour for π⁻ z projection due to different x range probed
- CLAS probes higher x region: more sensitive to $e \times H_1^\perp$?

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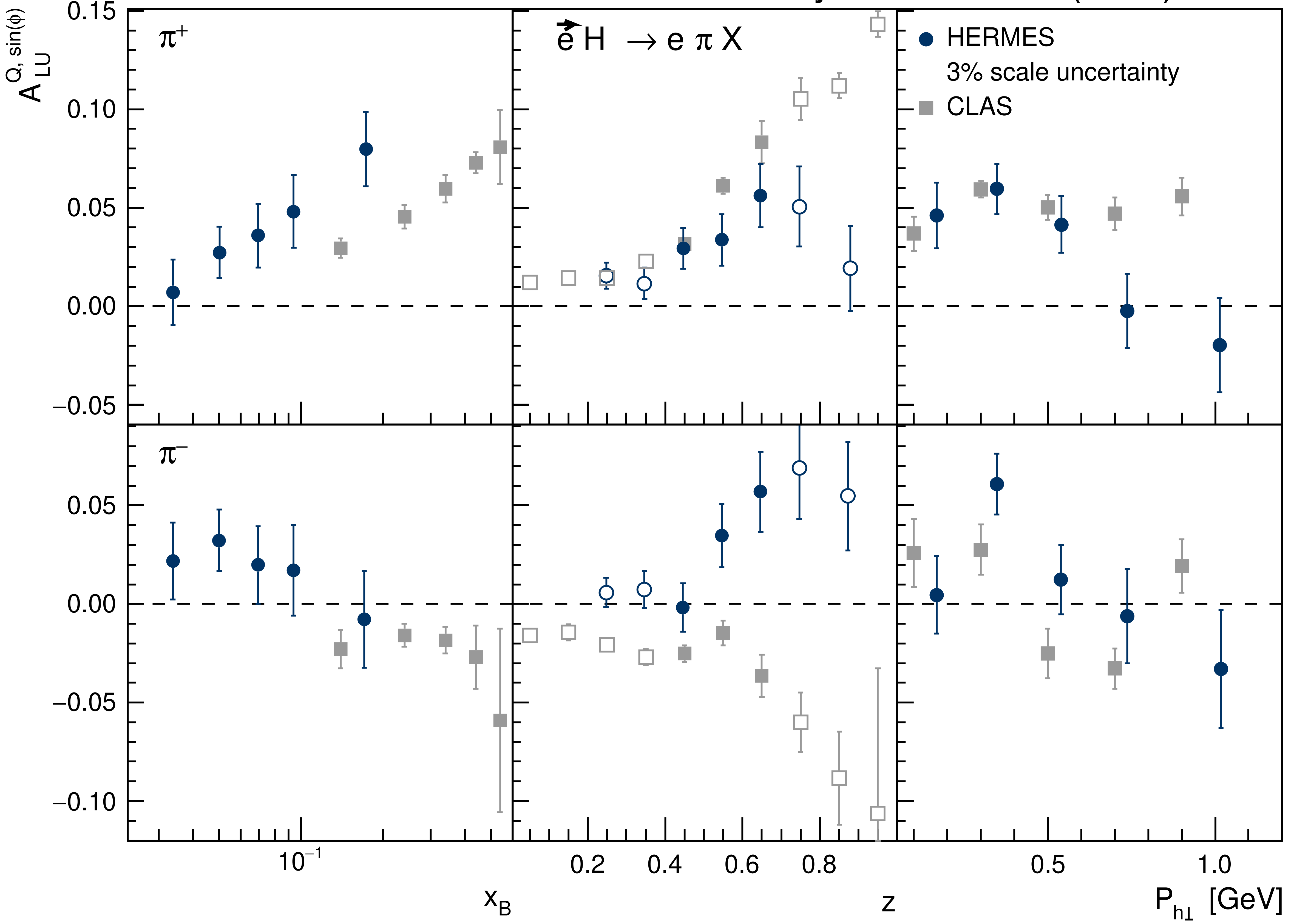
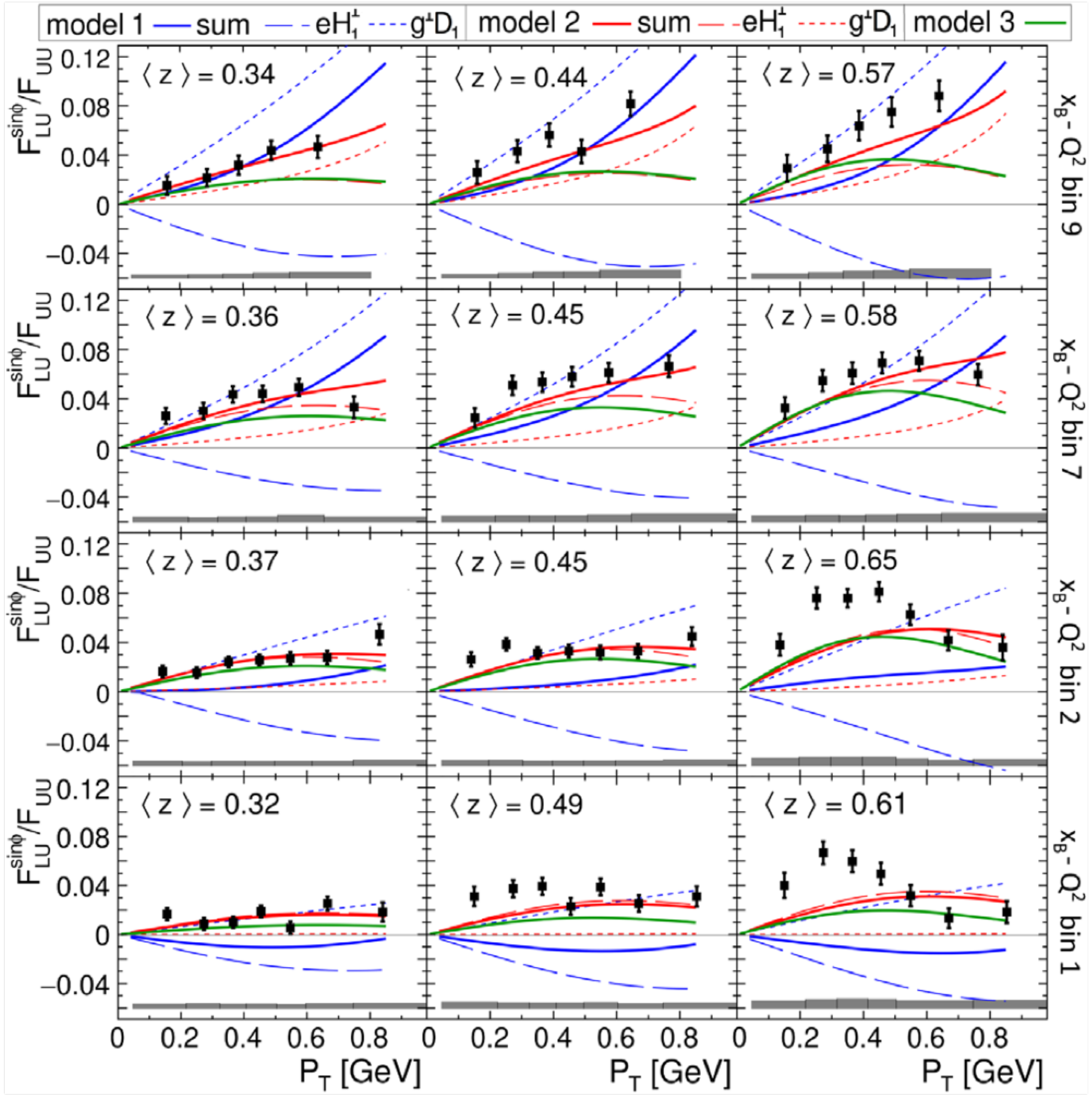
Phys. Lett. B **797** (2019) 134886



CLAS12, Phys. Rev. Lett. **128** (2022) 062005



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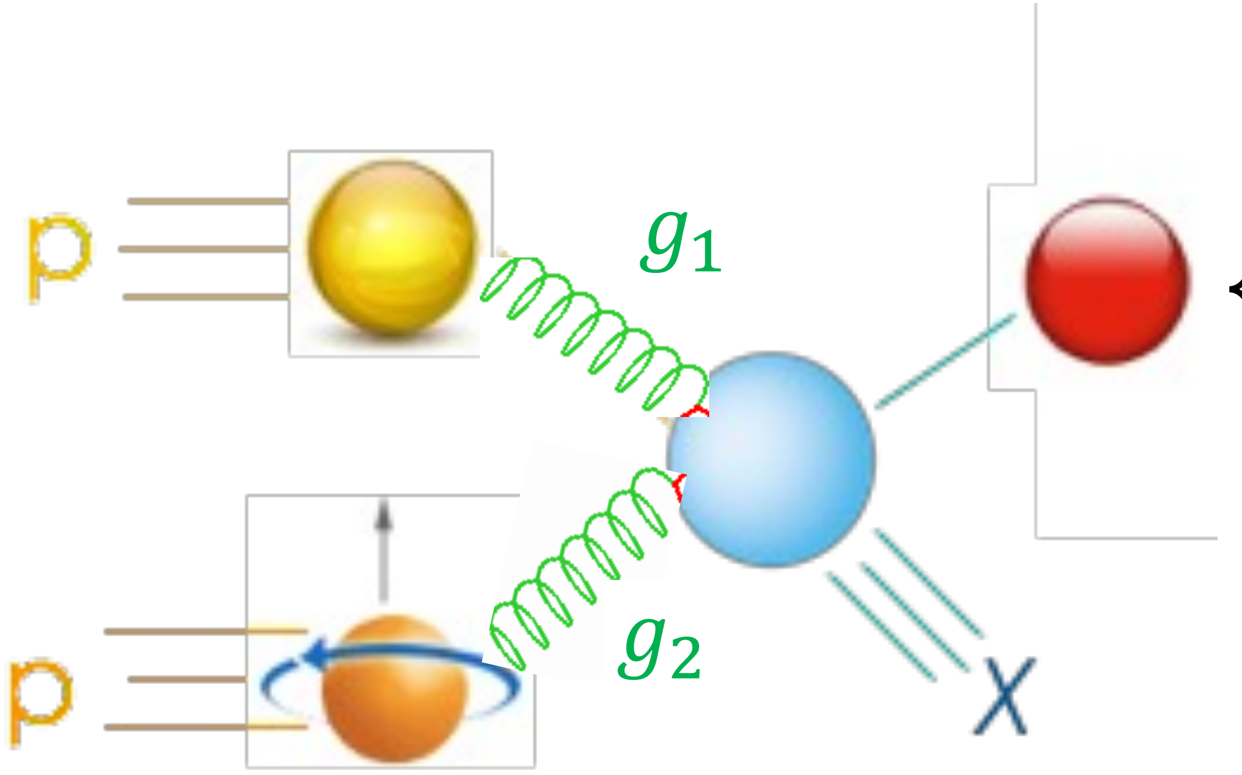
Gluon TMD PDFs

gluon polarisation

nucleon polarisation

	U	circular	linear
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$



$$\begin{cases} \eta_c, \chi_{c0}, \dots \\ \eta_b, \chi_{b0}, \dots \\ J/\psi + J/\psi \\ J/\psi + \gamma \\ \dots \end{cases}$$

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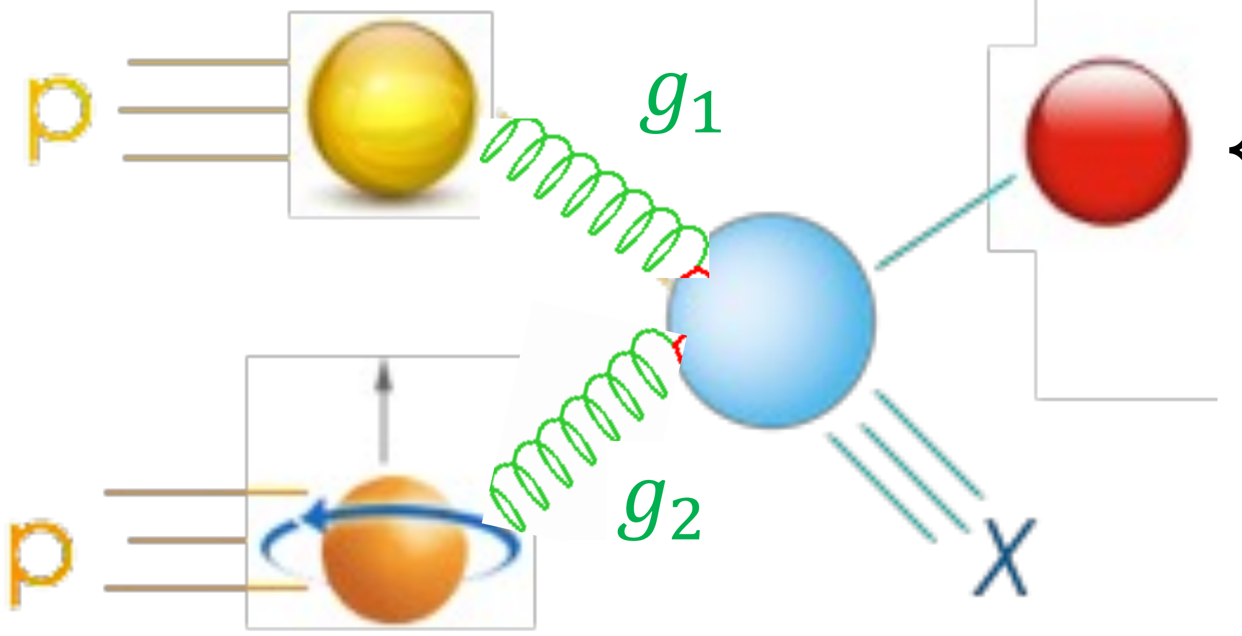

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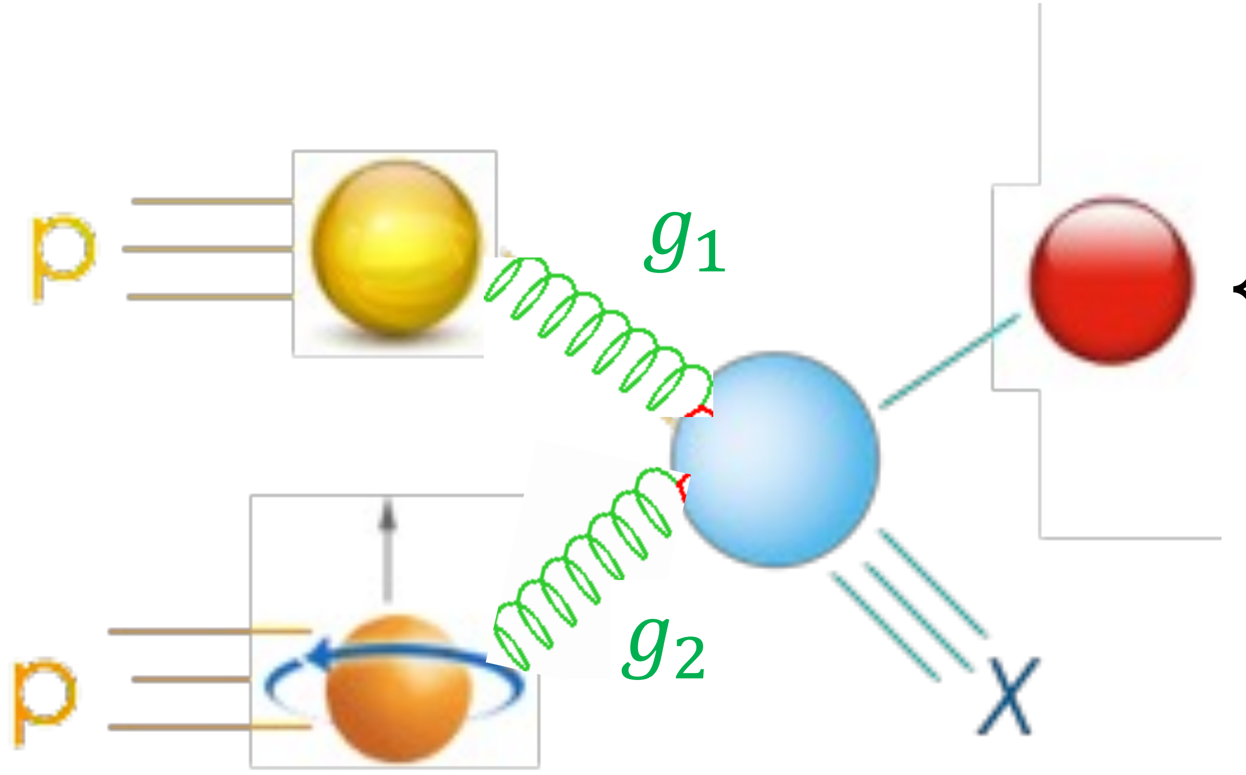
- Accessible through production of dijets, high-P$_T$ hadron pairs, quarkonia



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Gluon TMDs via $J/\psi J/\psi$ production

- $J/\psi J/\psi$ production largely dominated by gluon-induced processes

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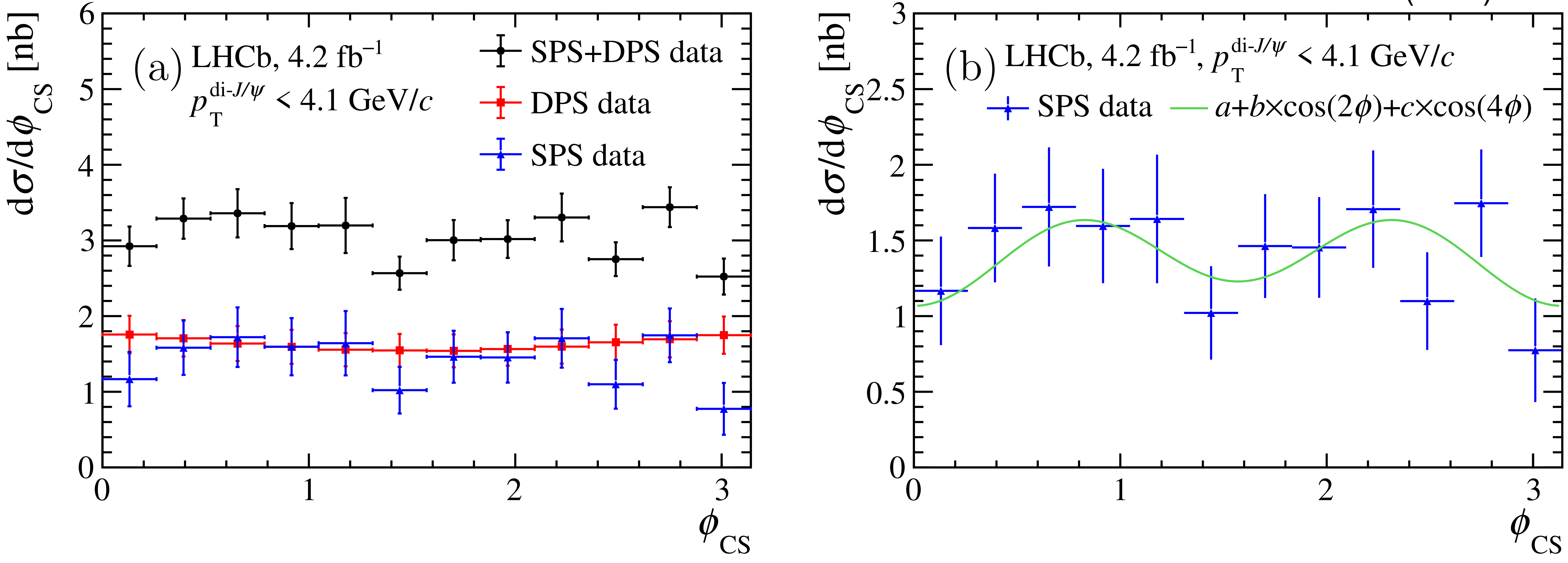
JHEP 03 (2024) 088



$p_T^{J/\psi J/\psi} < \dfrac{\langle M_{J/\psi J/\psi}\rangle}{2}$,

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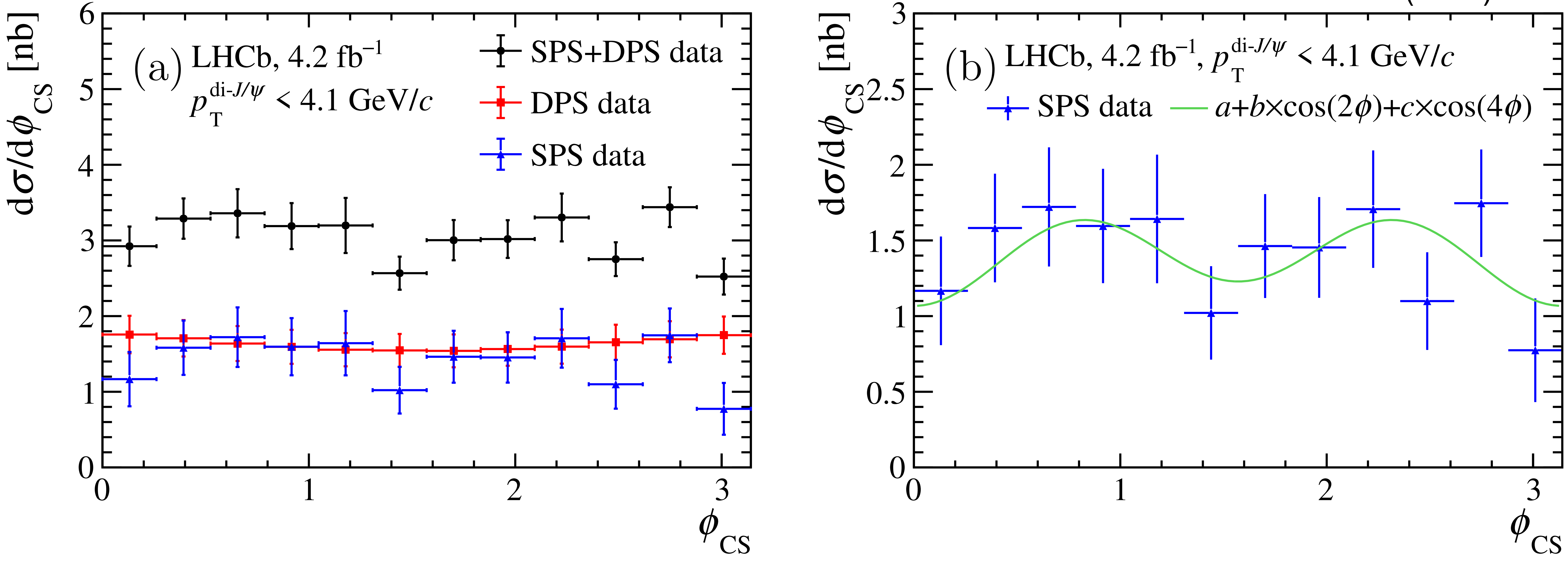

JHEP 03 (2024) 088

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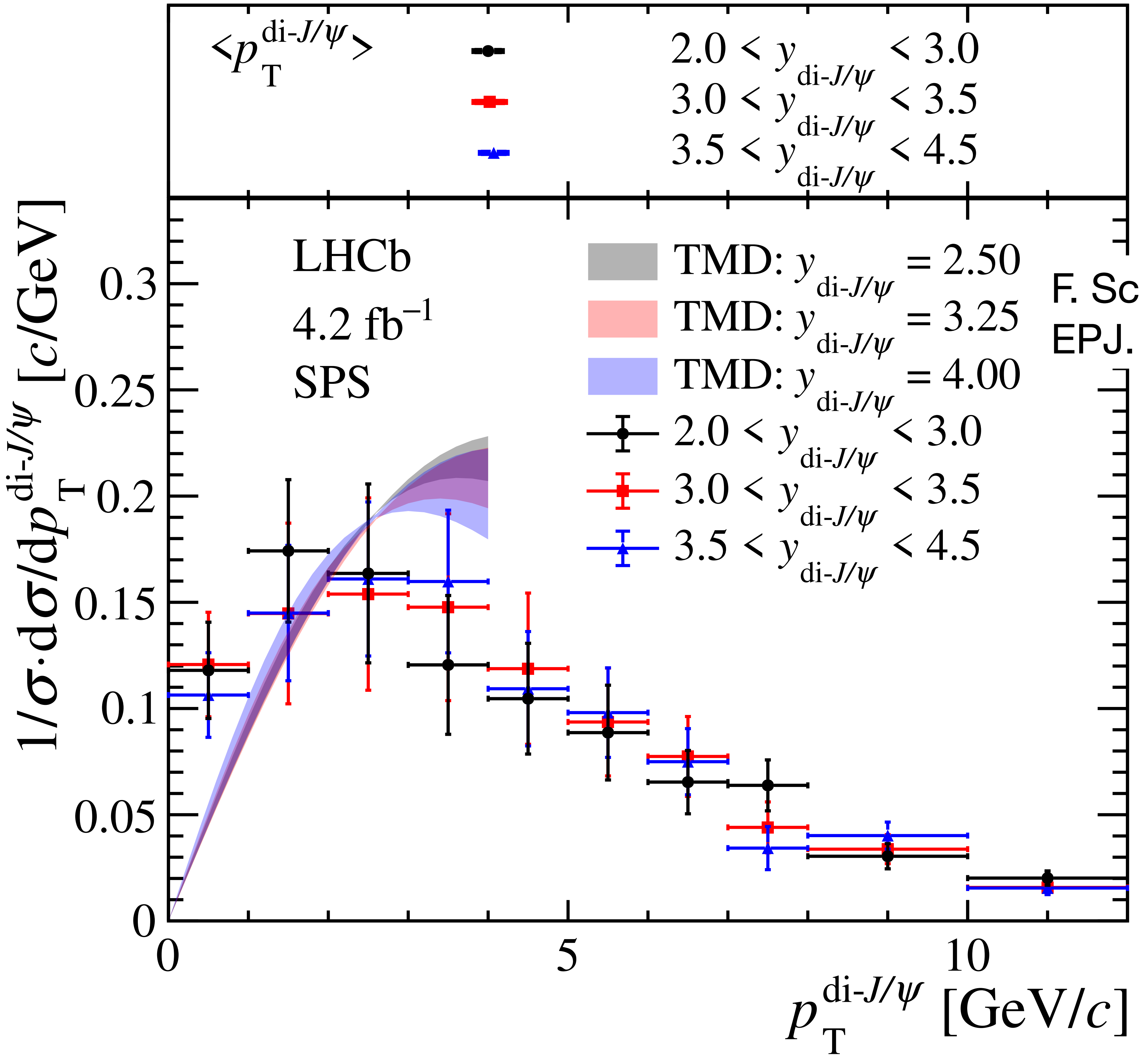
$$\langle\cos 2\phi_{\mathrm{CS}}\rangle = -0.029 \pm 0.050\ (\mathrm{stat}) \pm 0.009\ (\mathrm{syst})$$

$$\langle\cos 4\phi_{\mathrm{CS}}\rangle = -0.087 \pm 0.052\ (\mathrm{stat}) \pm 0.013\ (\mathrm{syst})$$

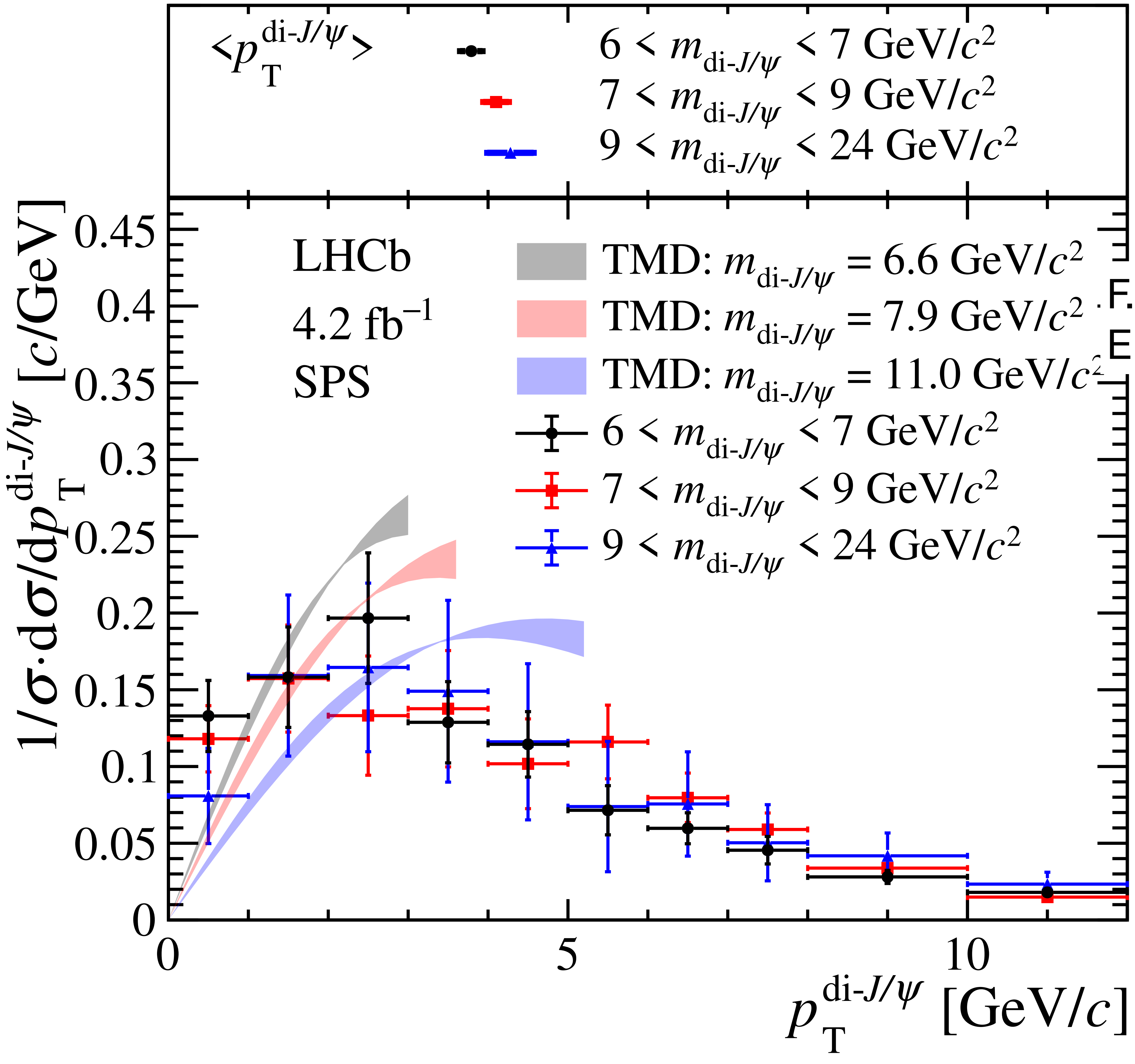
Spin-independent gluon TMDs via $J/\psi J/\psi$ production

JHEP 03 (2024) 088



F. Scarpa et al.,
EPJ. C **80** (2020) 87

JHEP 03 (2024) 088



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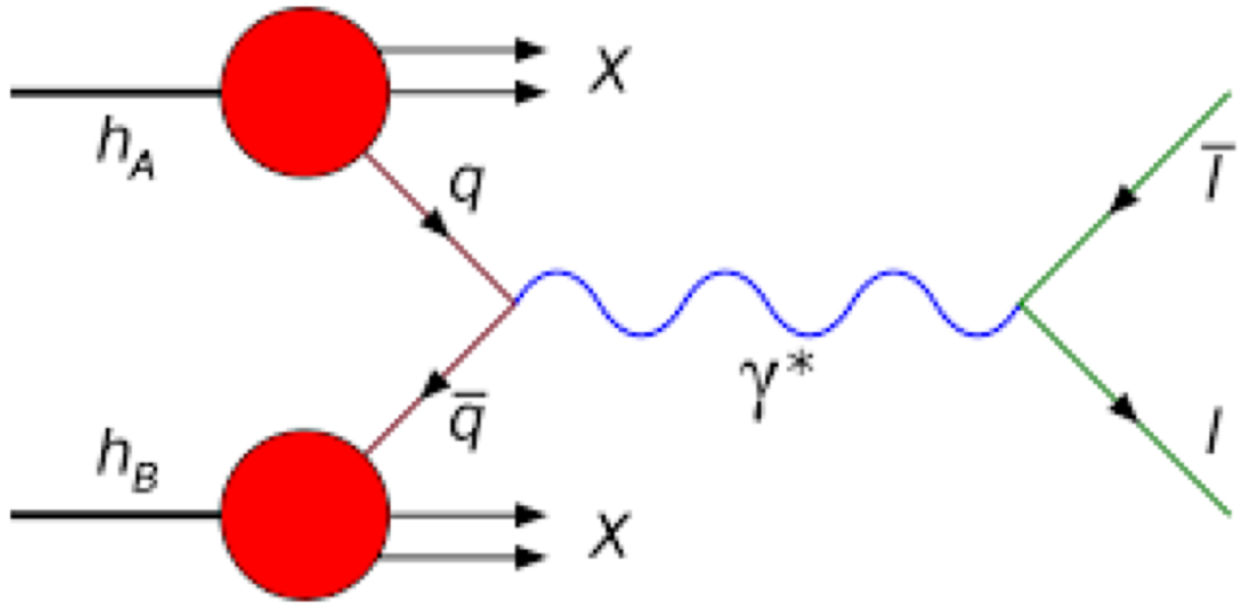
Upcoming

AMBER

Apparatus for Meson and Baryon
Experimental Research



Meson structure

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AMBER

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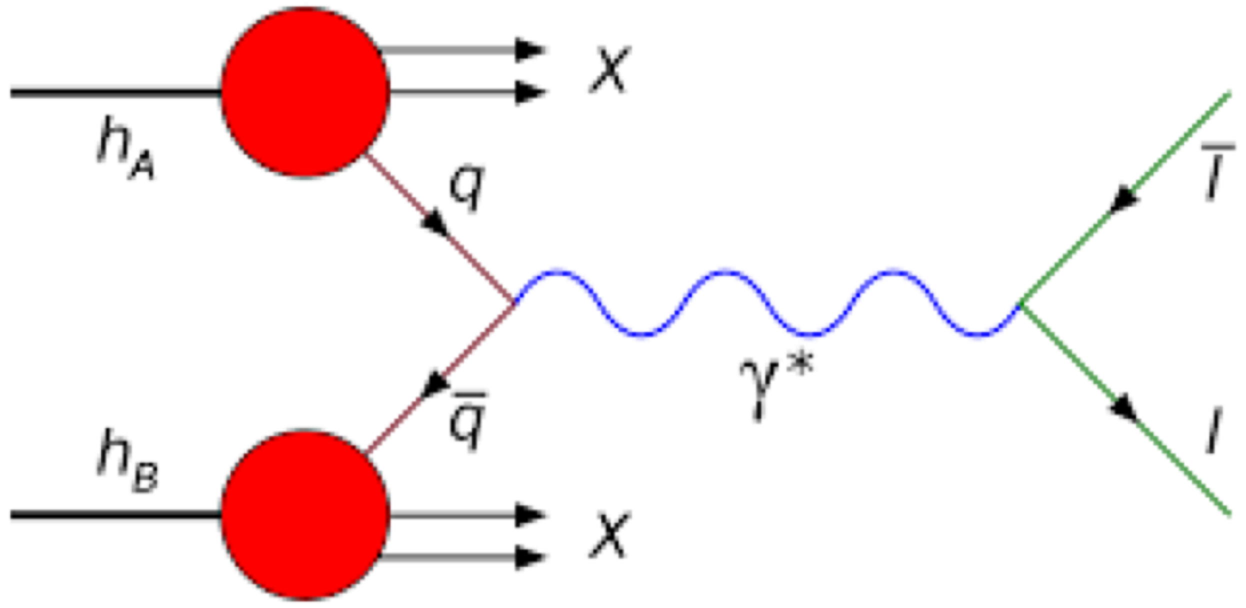
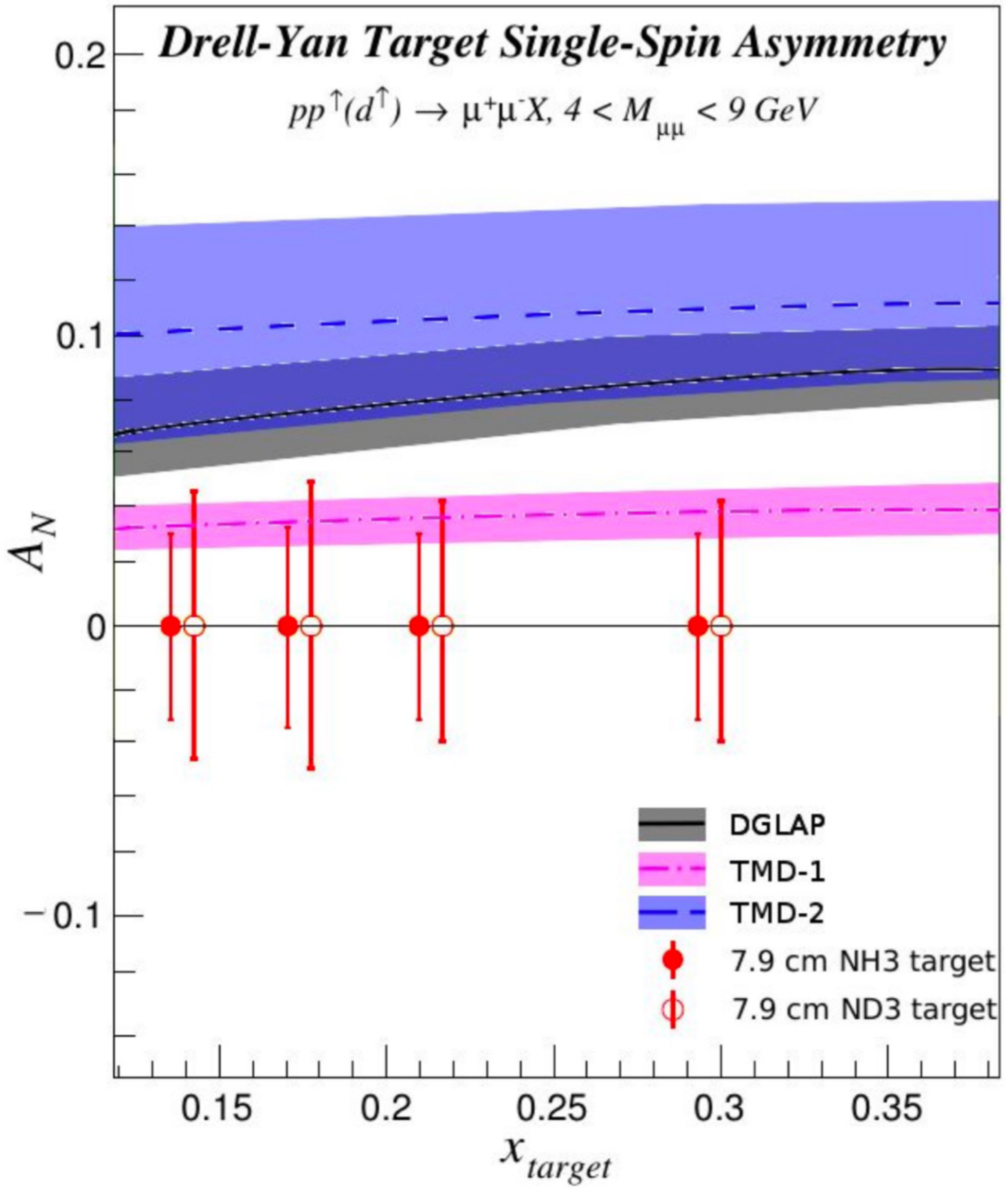


Meson structure

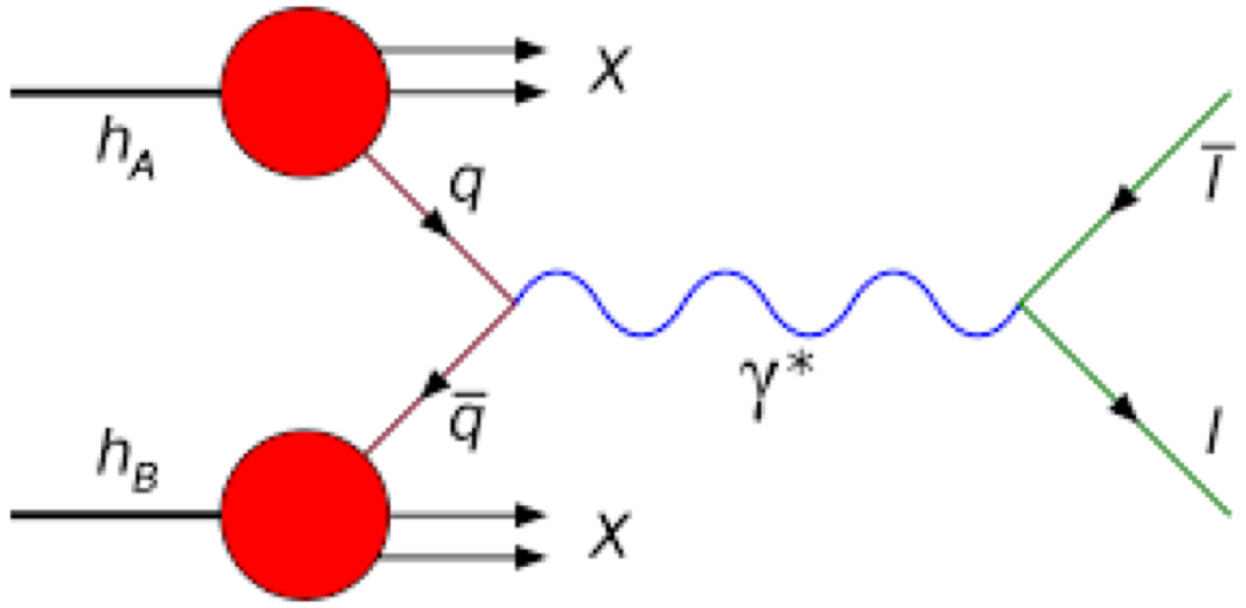
SpinQuest ⟶ Sivers function



Upcoming

AMBER

Apparatus for Meson and Baryon Experimental Research



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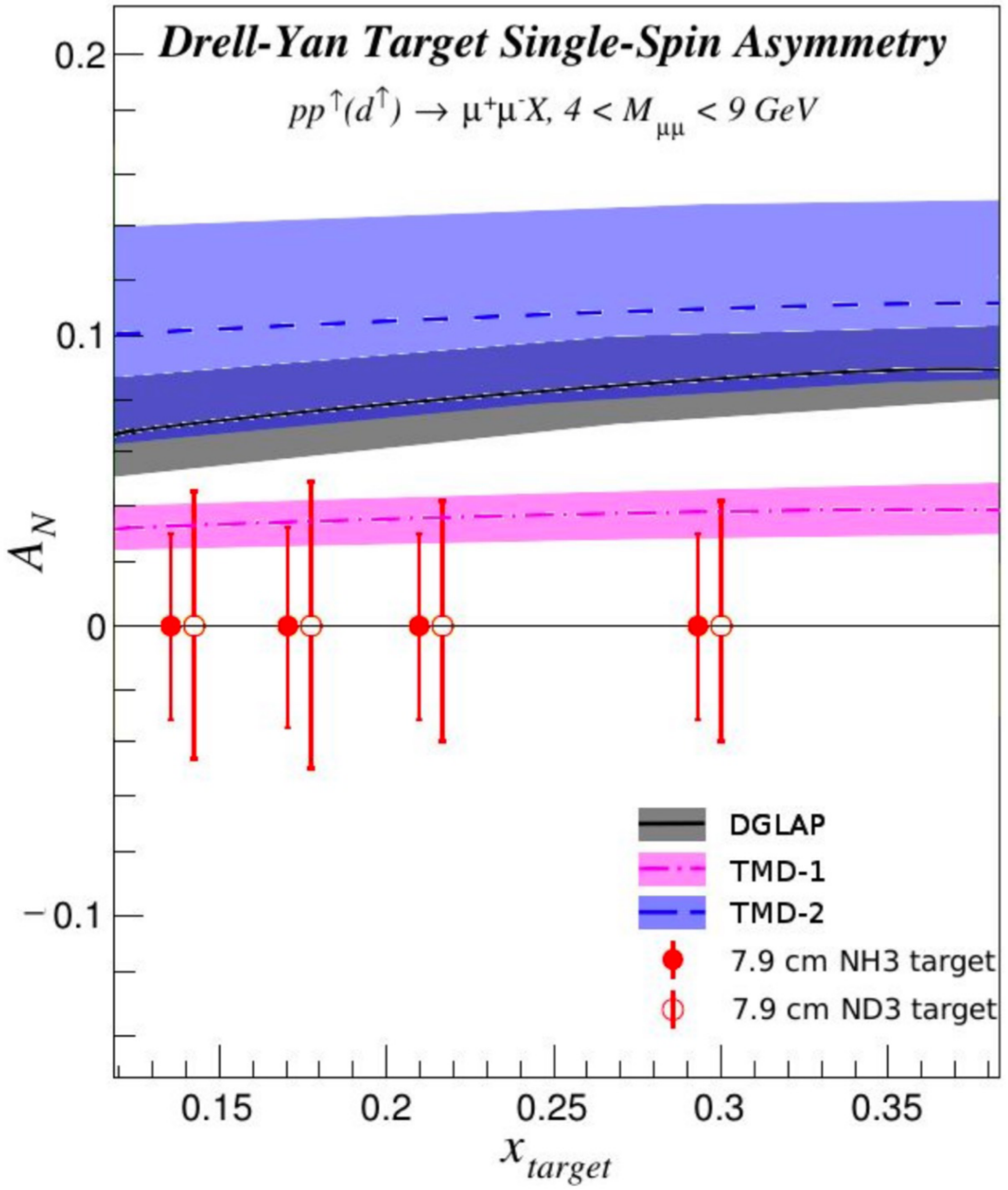
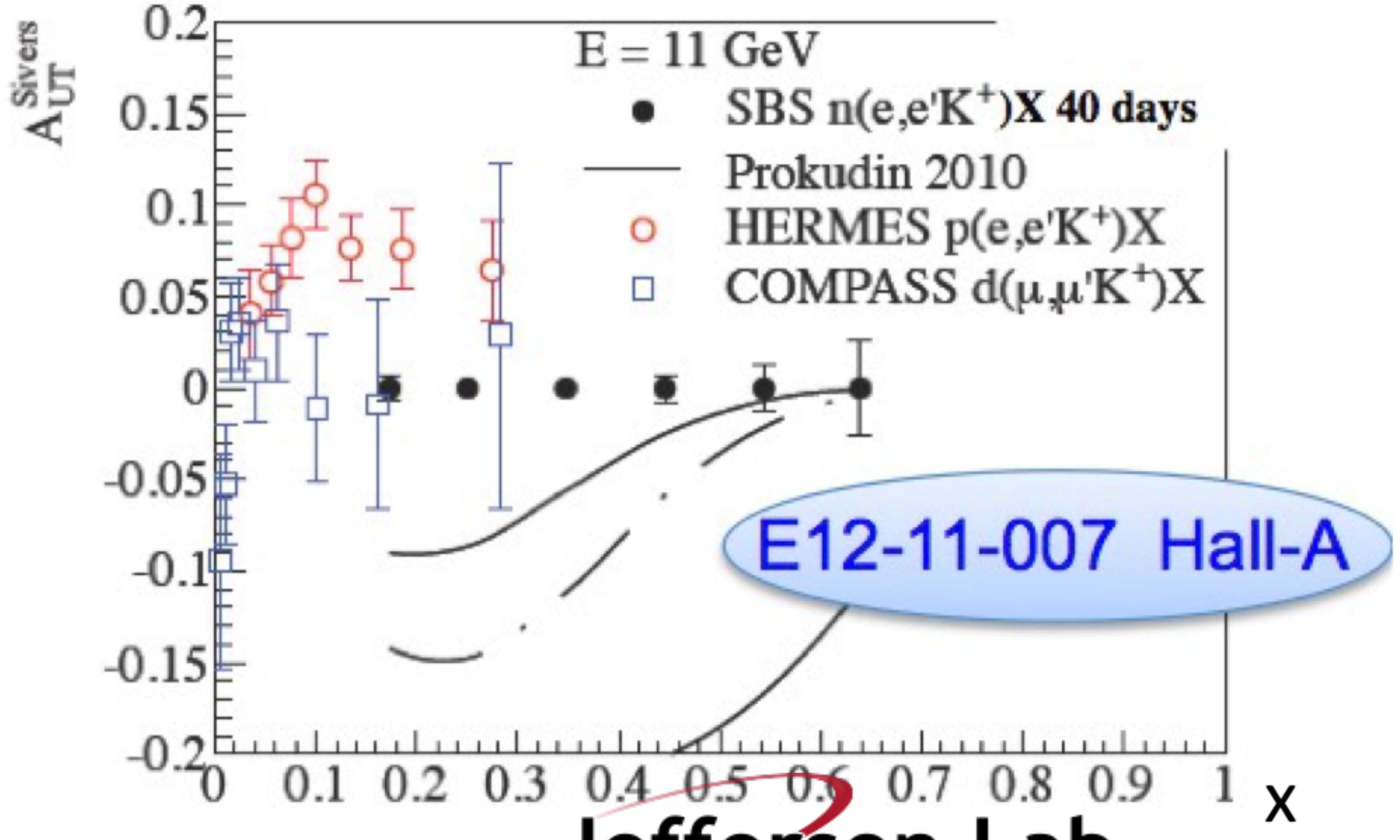
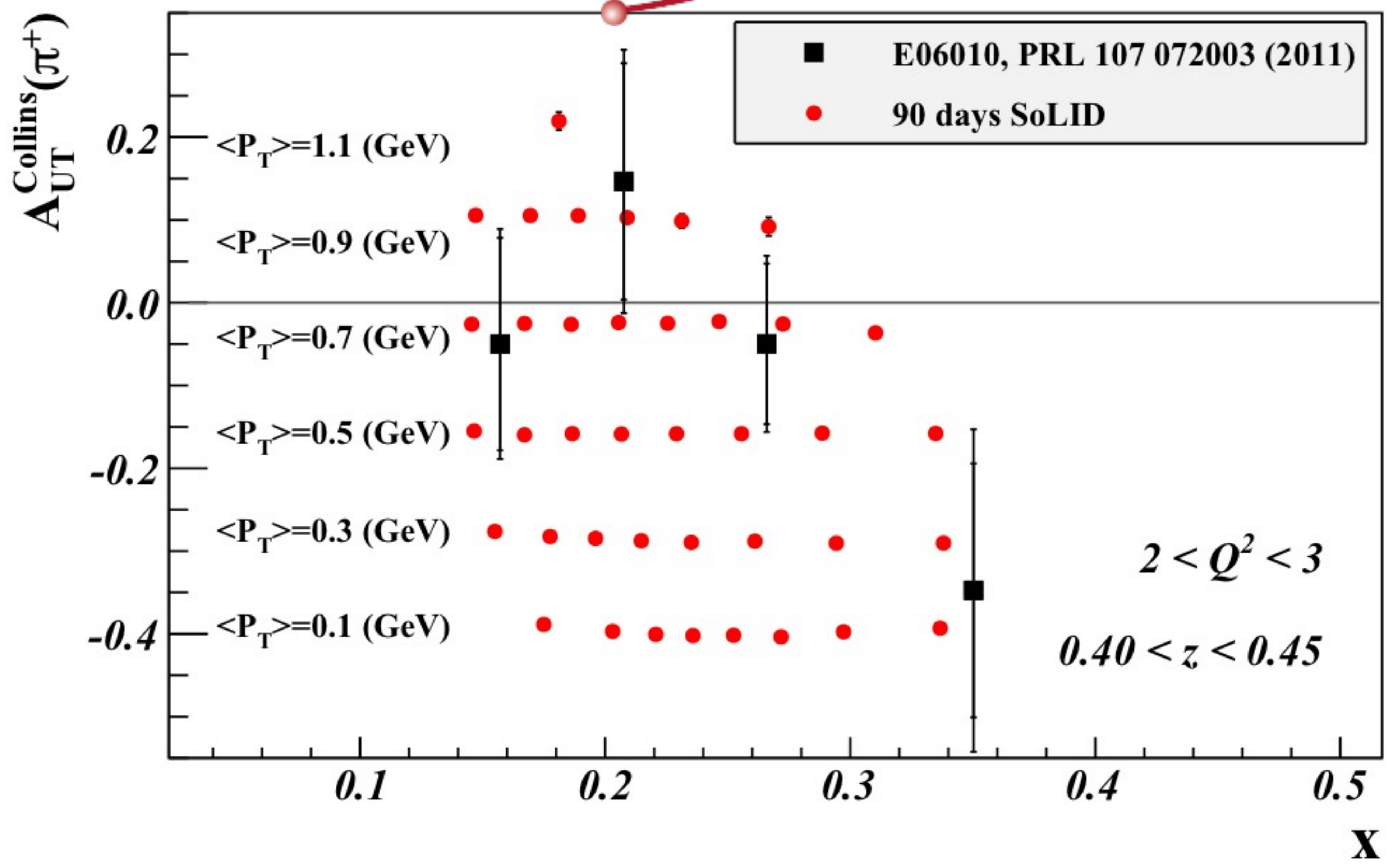


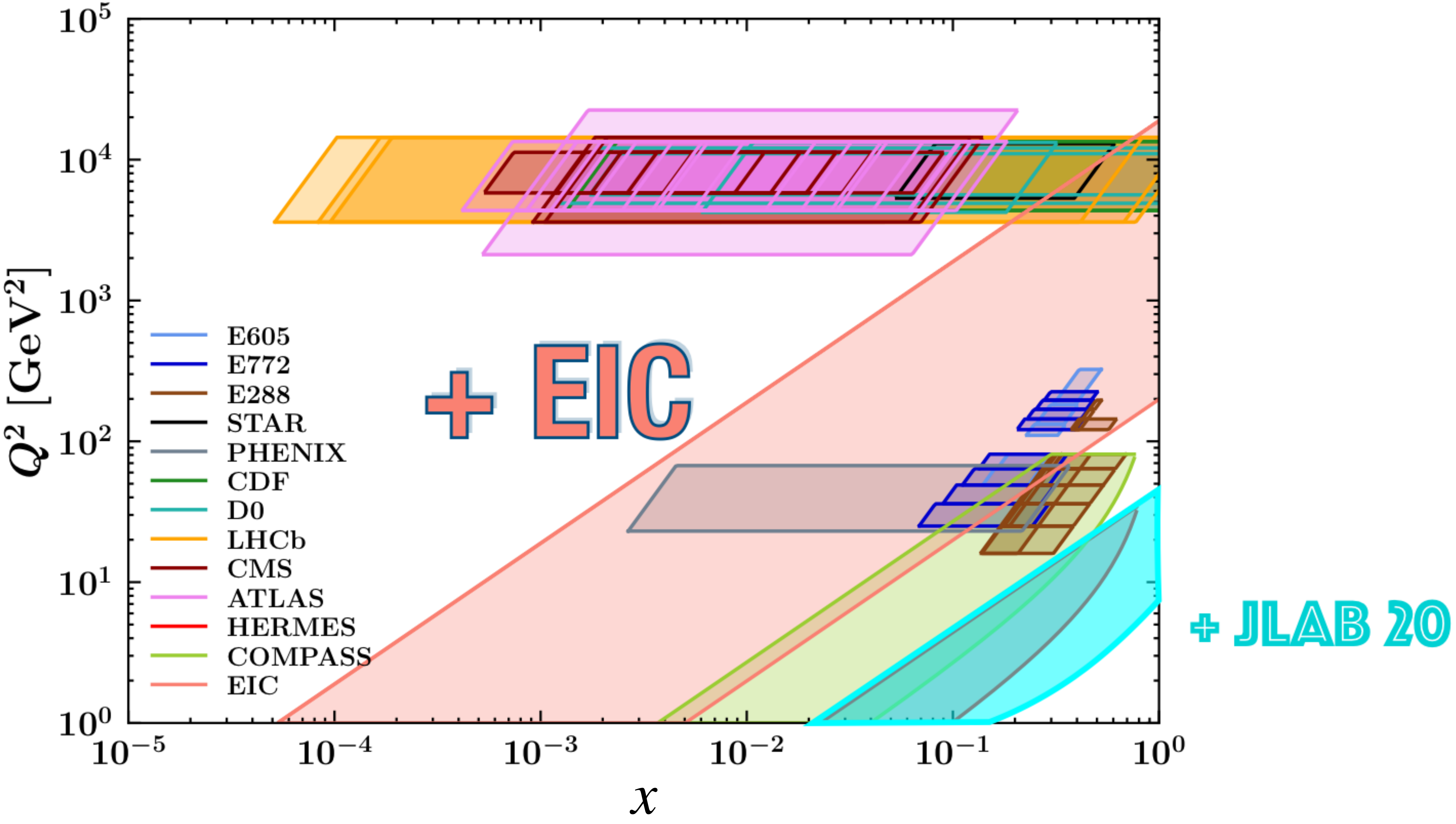


E12-11-007 Hall-A

Jefferson Lab

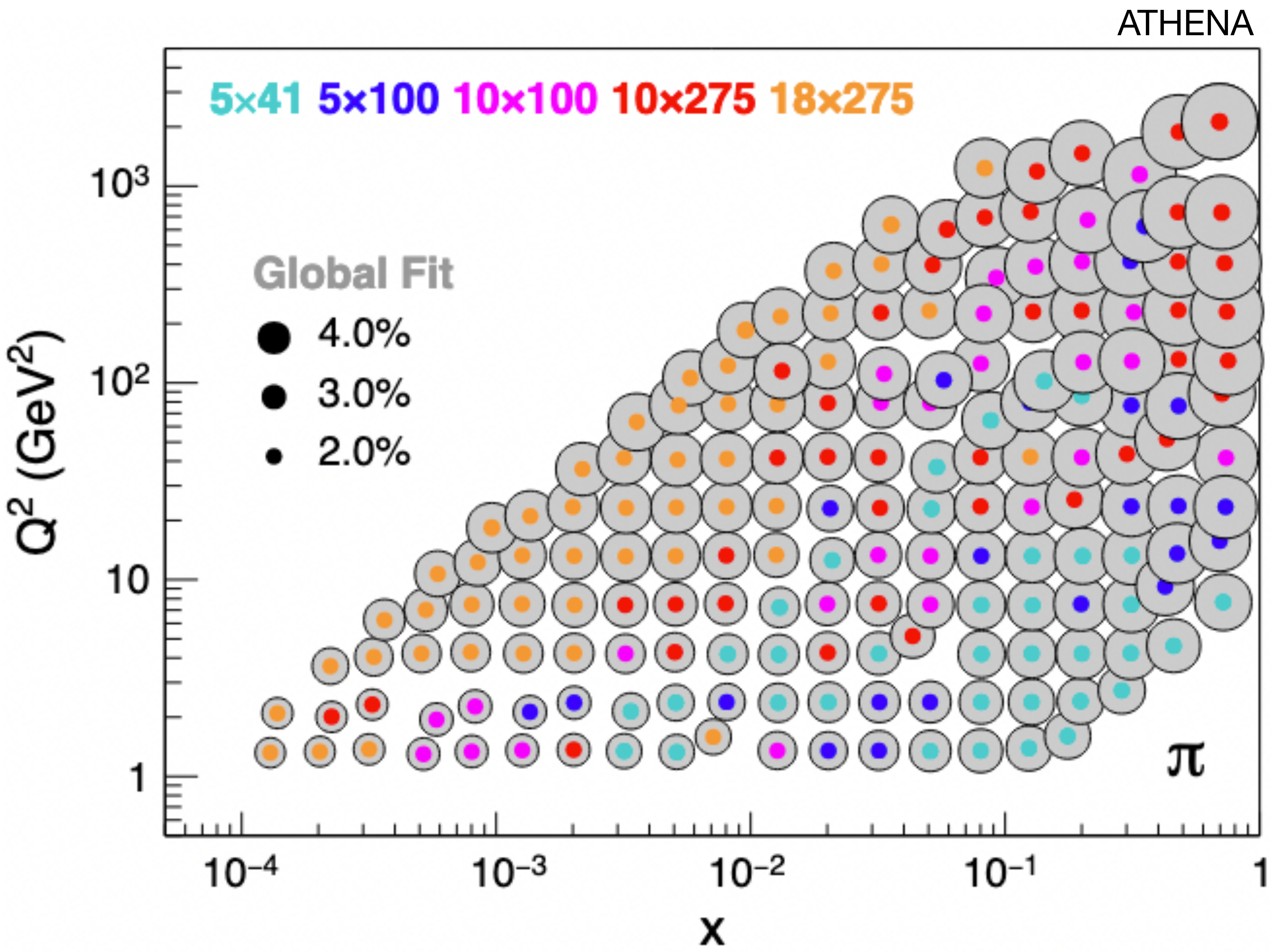


Future



10^5
10^4
10^3
10^2
10^1
10^0
Q^2 [GeV^2]
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
x
E605
E772
E288
STAR
PHENIX
CDF
D0
LHCb
CMS
ATLAS
HERMES
COMPASS
EIC
+ EIC
+ JLAB 20

Spin-independent TMD PDFs at EIC



Fit:
A. Bacchetta et al.,
JHEP 06 (2017) 081,
JHEP 06 (2019) 051 (erratum)

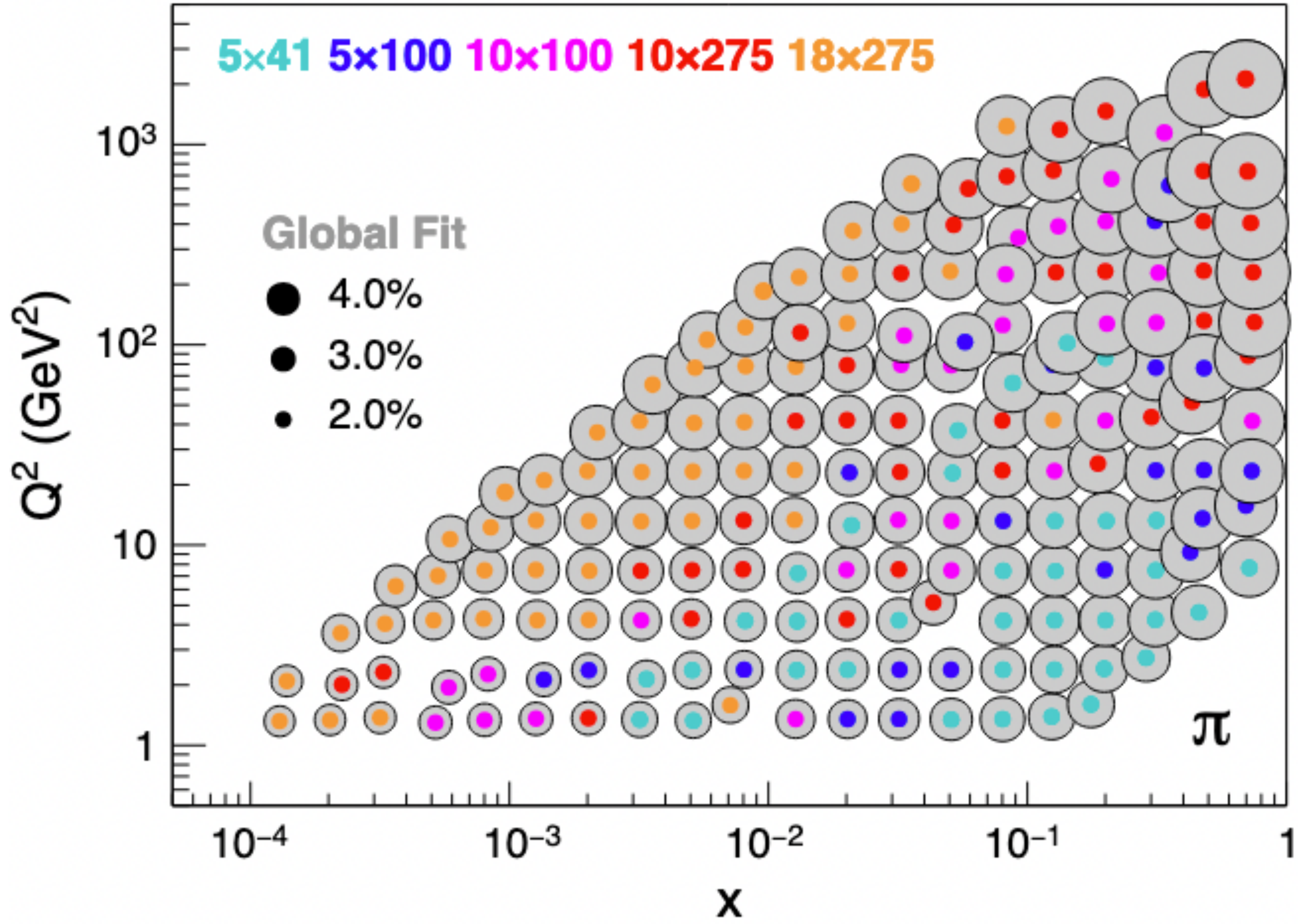
EIC uncertainties dominated by assumed
3% point-to-point uncorrelated uncertainty
3% scale uncertainty

Theory uncertainties dominated by TMD evolution.

Spin-independent TMD PDFs at EIC

Large lever-arm in Q^2 over large x range
→ Q^2 evolution of TMD PDF

ATHENA



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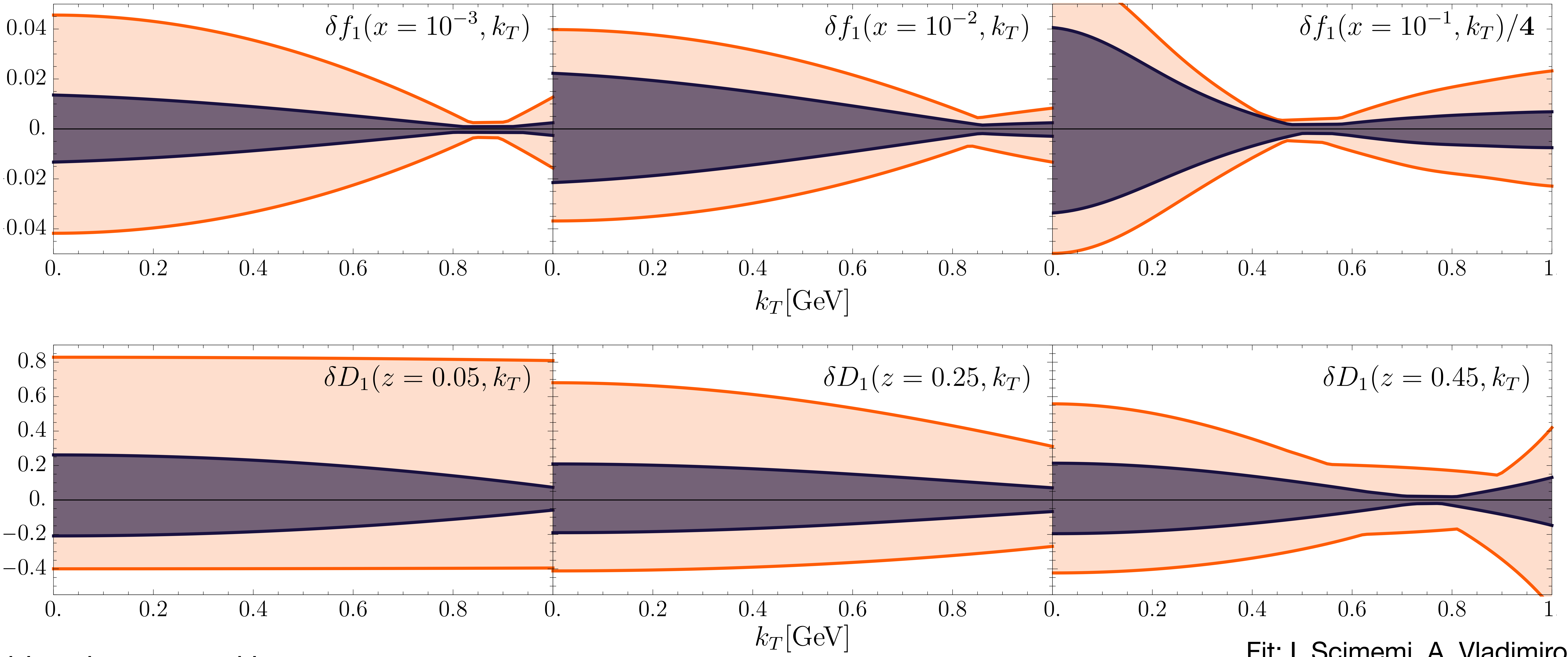
Spin-independent TMD PDF: impact of EIC

ECCE



DIS variables via scattered lepton

$Q^2 > 1\ \mathrm{GeV}^2$

$0.01 < y < 0.95$

$W^2 > 10\ \mathrm{GeV}^2$

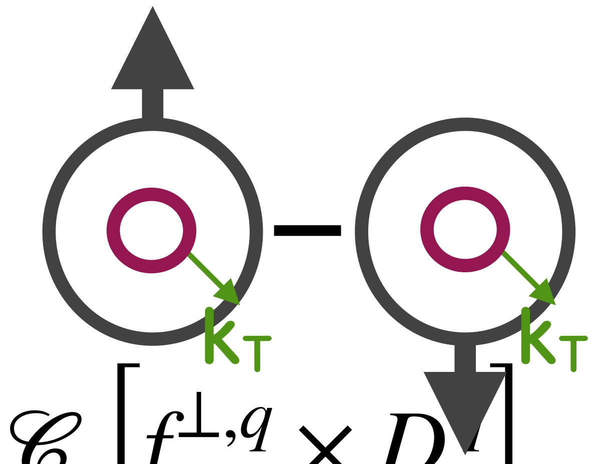
$5 \times 41\ \mathrm{GeV}^2$
$10 \times 100\ \mathrm{GeV}^2$
$18 \times 100\ \mathrm{GeV}^2$
$18 \times 275\ \mathrm{GeV}^2$

$\mathcal{L} = 10\ \mathrm{fb}^{-1}$ for each collision energy

Fit: I. Scimemi, A. Vladimirov
JHEP, 06:137, 2020

systematic uncertainty= |generated - reconstructed|

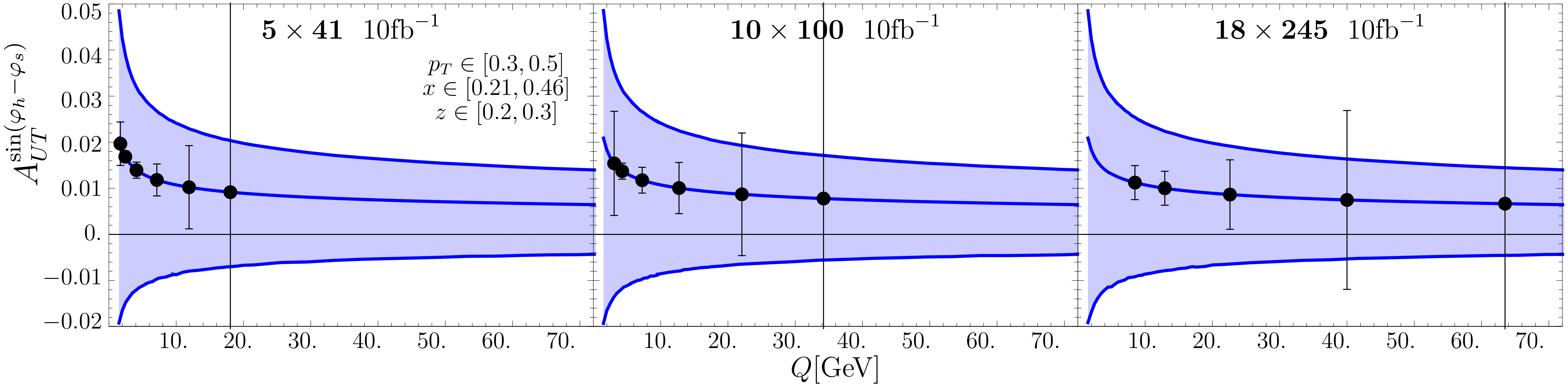
Sivers TMD PDF: TMD evolution



$\mathscr{C}\left[f_{1T}^{\perp,q} \times D_1\right]$

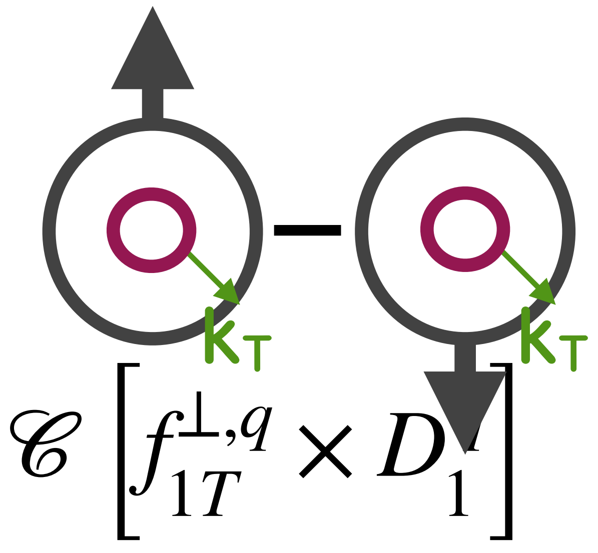
Sivers asymmetry

ECCE
Parametrisation: M. Bury et al., JHEP, 05:151, 2021



Decrease of asymmetry with increasing Q^2 → need high precision (<1%) to measure asymmetry at high Q^2

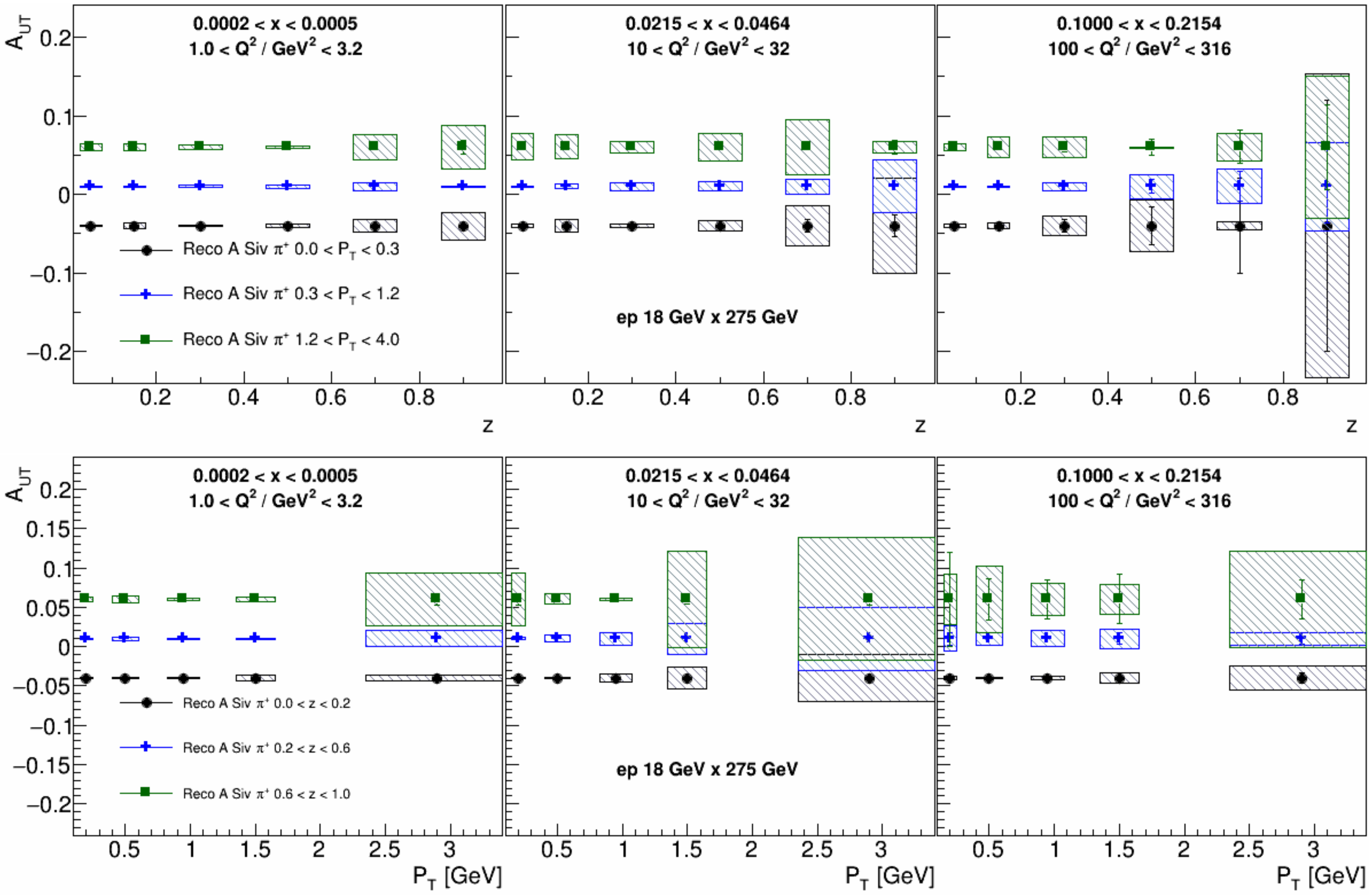
Uncertainties Sivers asymmetry at EIC



$\mathscr{C}\left[f_{1T}^{\perp,q} \times D_1\right]$

Sivers asymmetry

ECCE



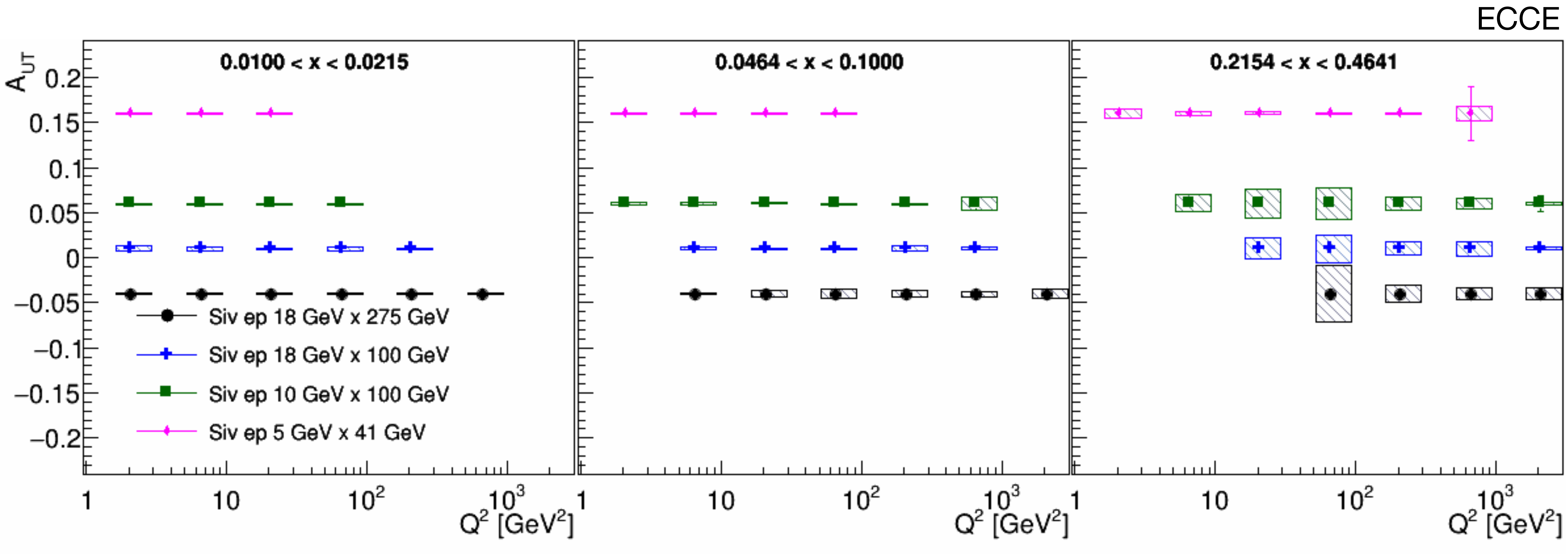
Beam polarisations assumed to be 70%.

systematic uncertainty= |generated - reconstructed|

Additionally: 3% scale uncertainty

- Low x and Q^2: small statistical uncertainty. High precision is needed since asymmetry at low x and Q^2 well below 1%.
- For not too large z and P_T, statistical uncertainty well below 1%.
- Systematic uncertainties increase with z and P_T: likely because of higher smearing effects.

Q^2 dependence of the Sivers asymmetry at EIC



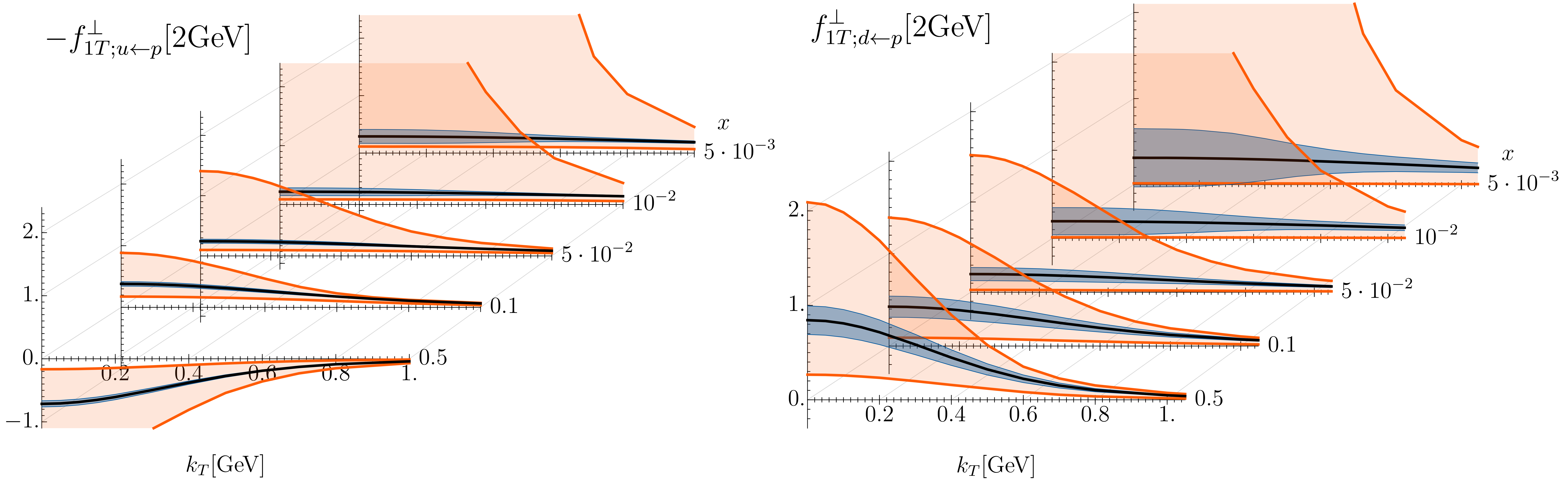
Intermediate and high x: good coverage in Q^2,
with complementarity in coverage at different COM energies.

Sivers TMD PDF: impact of EIC

Parametrisation from
M. Bury et al., JHEP, 05:151, 2021

Q=2 GeV

ECCE



DIS variables via scattered lepton

$$Q^2 > 1\ \text{GeV}^2$$
$$0.01 < y < 0.95$$
$$W^2 > 10\ \text{GeV}^2$$

$$5\times41\ \text{GeV}^2$$
$$10\times100\ \text{GeV}^2$$
$$18\times100\ \text{GeV}^2$$
$$18\times275\ \text{GeV}^2$$

$\mathcal{L} = 10\ \text{fb}^{-1}$ for each collision energy

Summary

- Transverse momentum dependent hadron structure and hadron formation: rich field of physics, with sensitivity to correlations between quark and hadron spin and transverse momentum.

- Pioneering fixed-target experiments at HERMES, COMPASS, JLab 6 GeV: quark distributions

- Entering era of precision measurements:
  - JLab 12 GeV: unique precision in the valence region
  - EIC: extending down to $x=10^{-4}$
  - LHC measurements can provide additional, invaluable high energy input
  - need to extend measurements with sensitivity to gluons