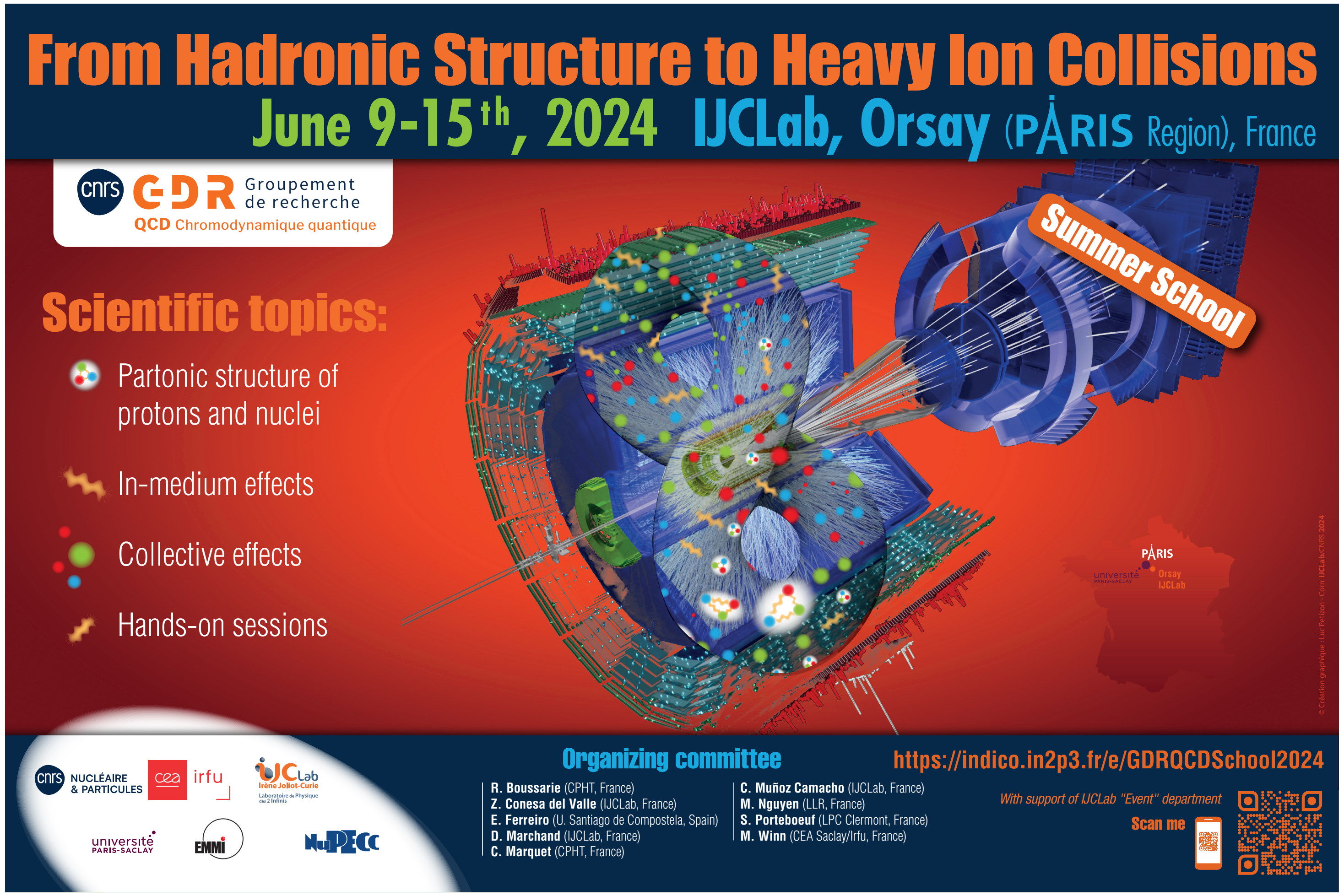
Partonic structure & small x: theory



From Hadronic Structure to Heavy Ion Collisions

June 9-15th, 2024 IJCLab, Orsay (PARIS Region), France

Scientific topics:

- Partonic structure of protons and nuclei
- In-medium effects
- Collective effects
- Hands-on sessions

Organizing committee

- **R. Boussarie** (CPHT, France)
- **Z. Conesa del Valle** (IJCLab, France)
- **E. Ferreiro** (U. Santiago de Compostela, Spain)
- **D. Marchand** (IJCLab, France)
- **C. Marquet** (CPHT, France)
- **C. Muñoz Camacho** (IJCLab, France)
- **M. Nguyen** (LLR, France)
- **S. Porteboeuf** (LPC Clermont, France)
- **M. Winn** (CEA Saclay/Irfu, France)

https://indico.in2p3.fr/e/GDRQCDSchool2024

With support of IJCLab "Event" department

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FONDO EUROPEO DE DESENVOLVEMENTO REXIONAL "Unha maneira de facer Europa"

Contents:

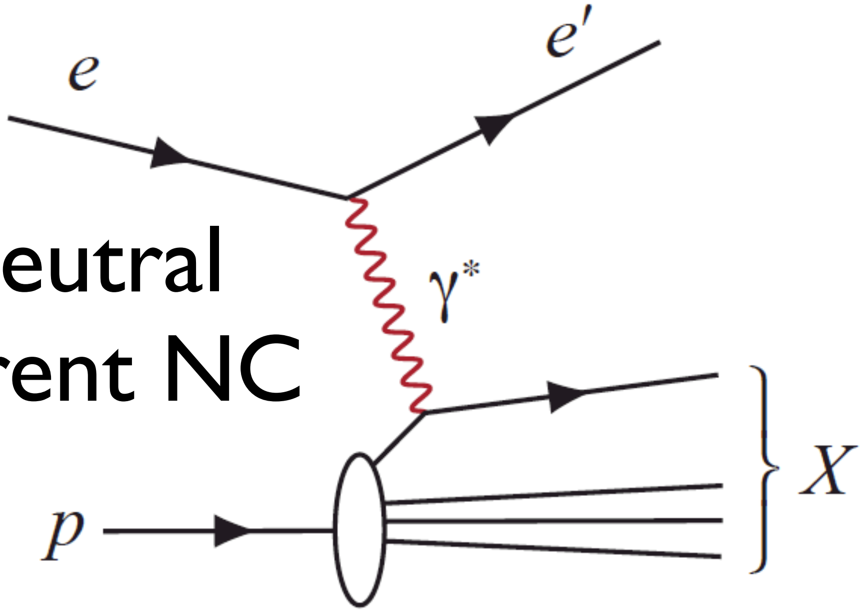
Some bibliography:

- R. Devenish and A. Cooper-Sarker, *Deep Inelastic Scattering*, Oxford University Press 2004.
- G. P. Salam, *Elements of QCD for hadron colliders*, CERN Yellow Rep. School Proc. 5 (2020) 1-56, https://inspirehep.net/literature/1820528.
- Yu. V. Kovchegov and E. Levin, *Quantum Chromodynamics at High Energy*, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 33 (2012) 1-350.

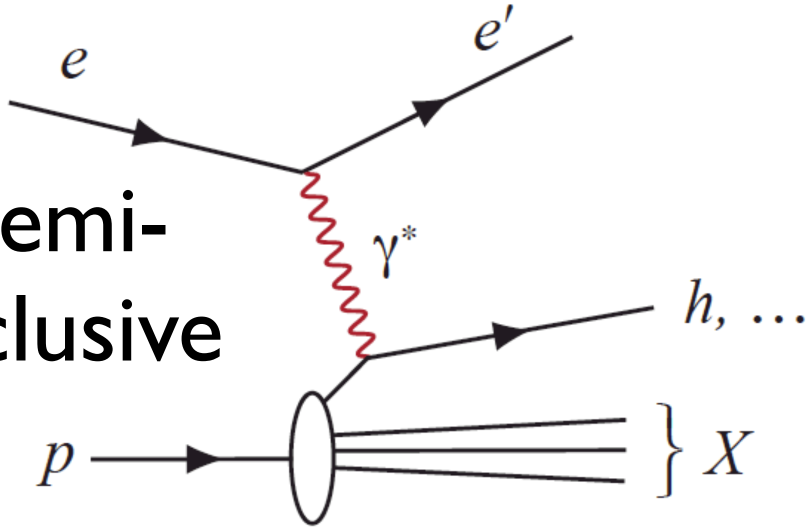
DIS: basics

➜ Consider the process of lepton (e, μ, ν) scattering on a proton (or neutron or nucleus): equivalent to the Rutherford experiment.

Neutral current NC



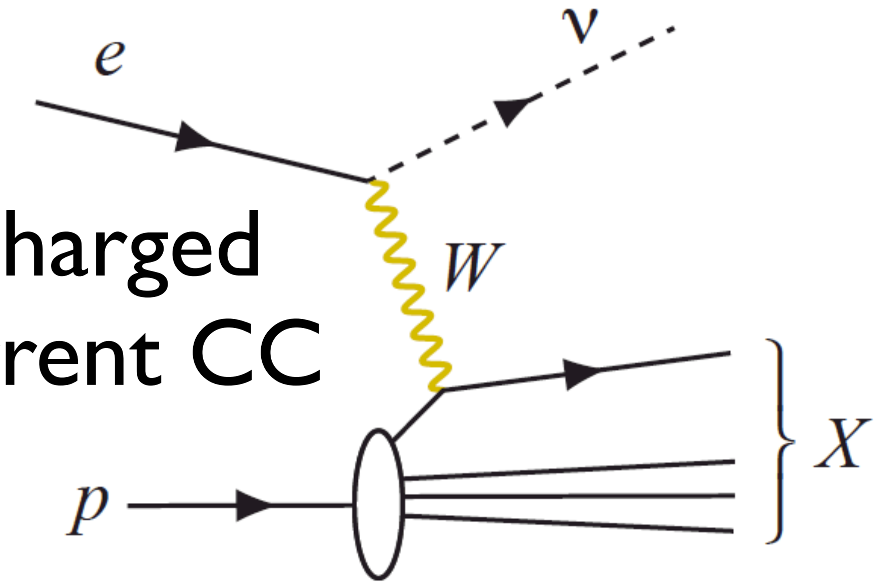
Semi-inclusive



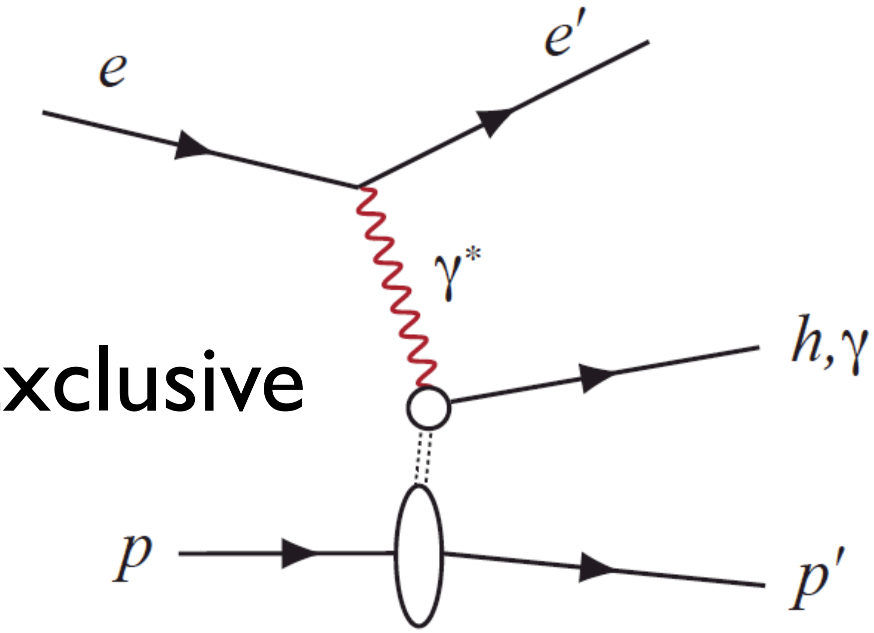
Totally inclusive DIS

Less inclusive DIS

Charged current CC



Exclusive



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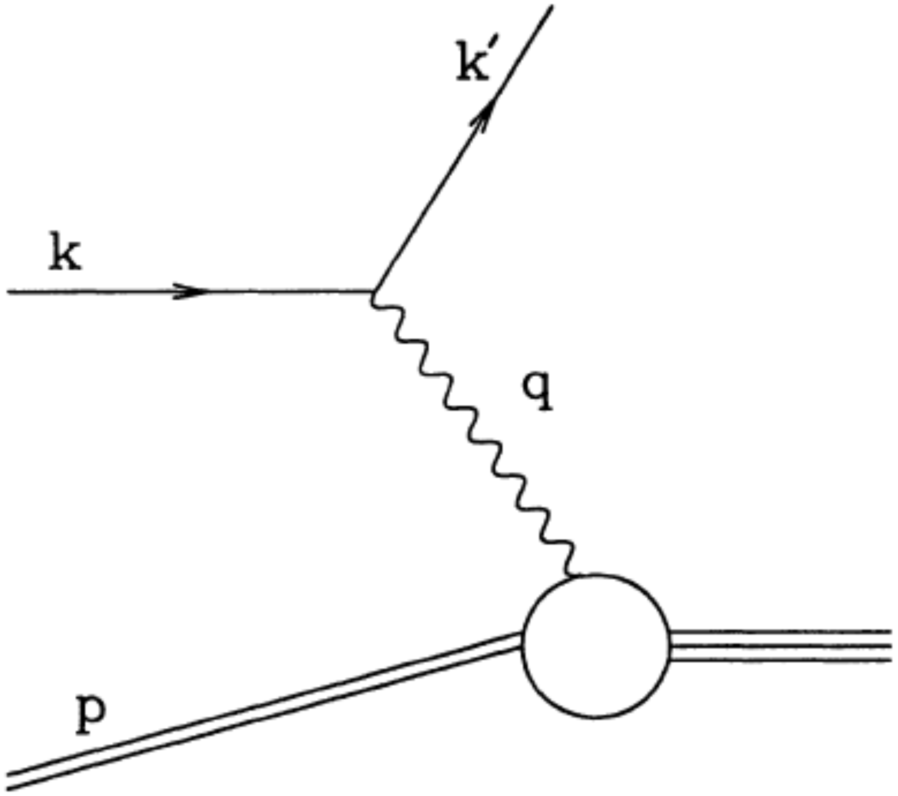

Standard DIS variables:

electron-proton cms energy squared:

$$s = (k+p)^2$$

photon-proton cms energy squared:

$$W^2 = (q+p)^2$$

energy transferred by the electron: $\nu = p \cdot q$

inelasticity

$$y = \frac{p \cdot q}{p \cdot k}$$

Bjorken x

$$x = \frac{-q^2}{2p \cdot q}$$

(minus) photon virtuality

$$Q^2 = -q^2$$

➜ For charged lepton scattering and neglecting Z exchange,

$$\frac{d^2\sigma}{dQ^2\,dx} = \frac{4\pi\alpha^2}{Q^4 x}\left[(1-y)F_2(x,Q^2) + xy^2F_1(x,Q^2)\right]$$

F_1, F_2: structure functions of the hadron

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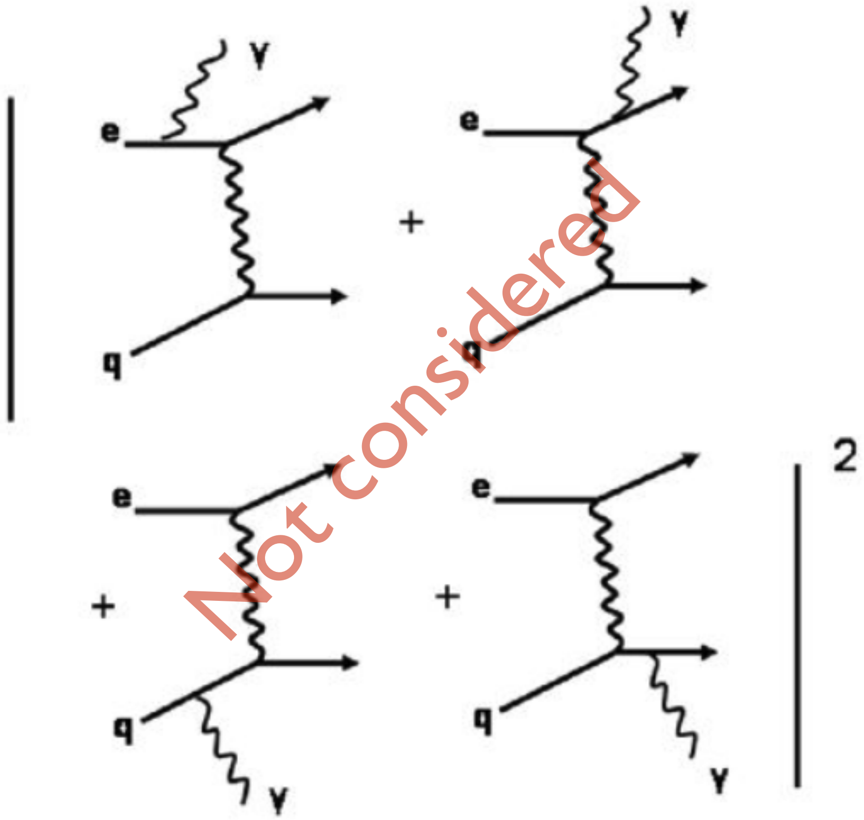

+ loop corrections

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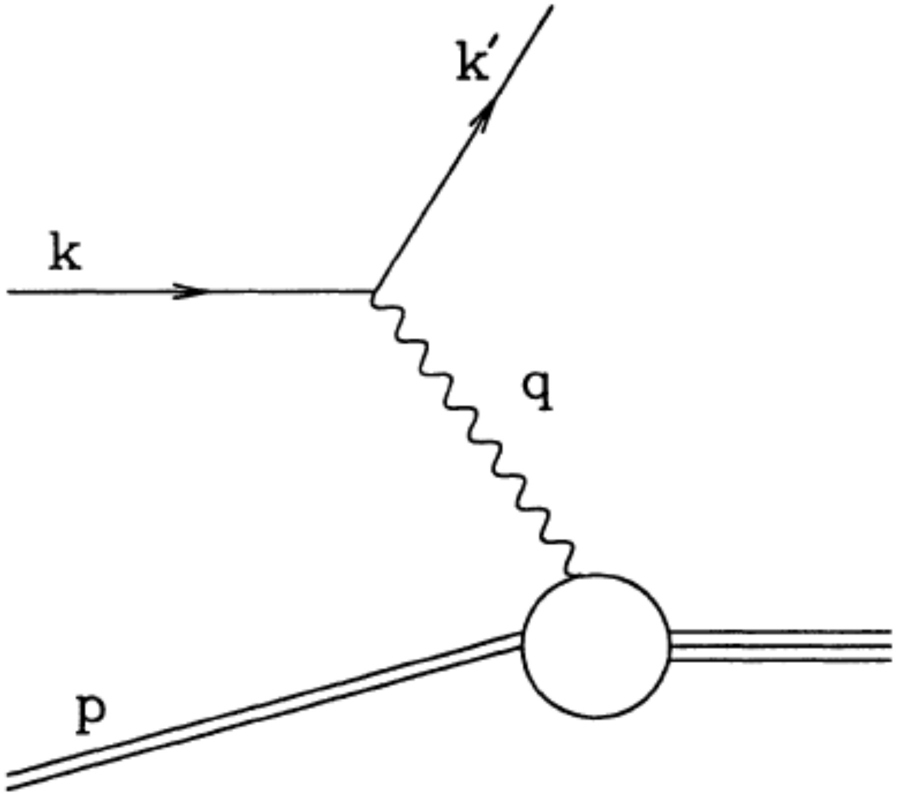

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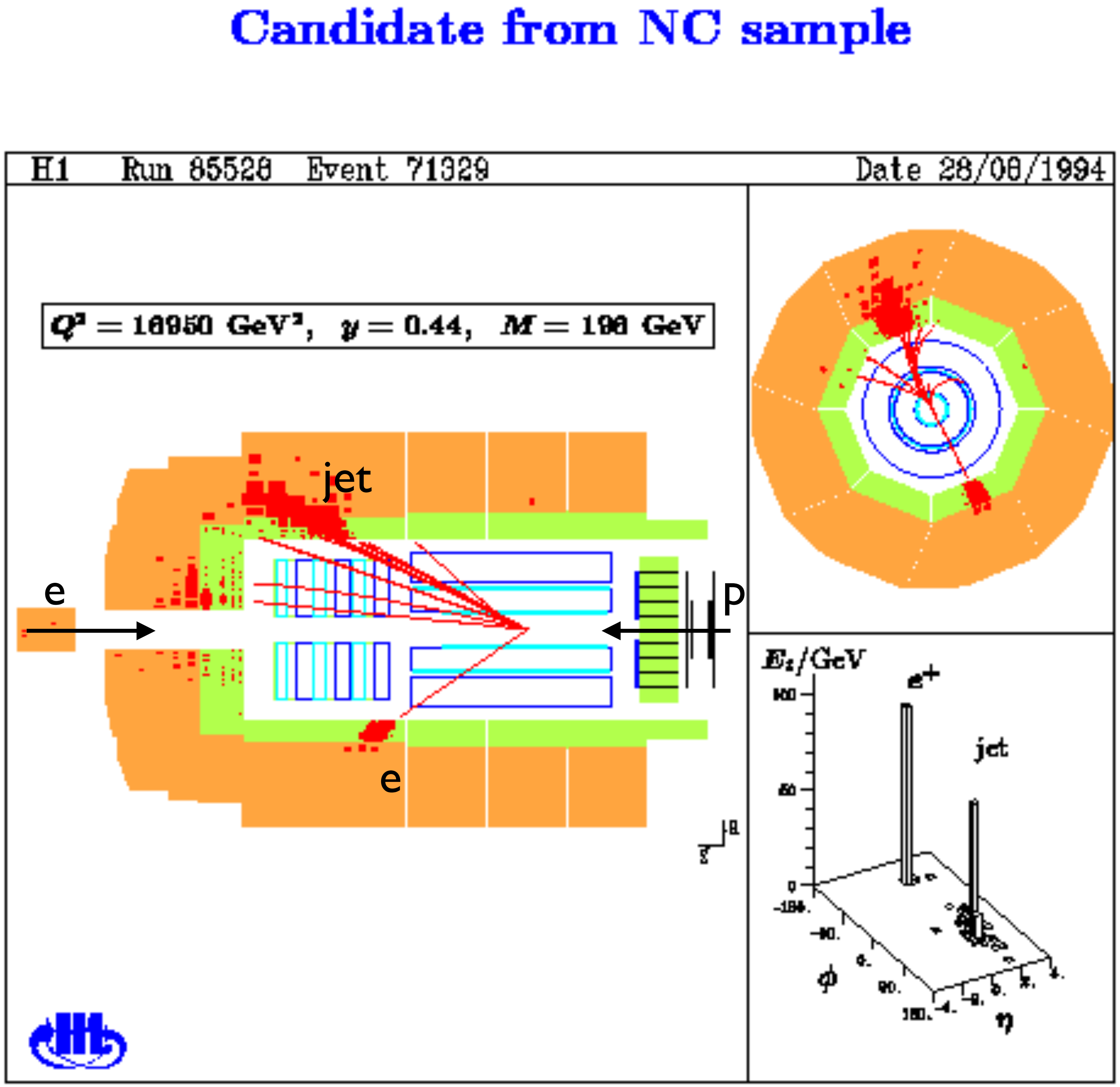
F_1, F_2: structure functions of the hadron

Experiment:



Lepton method

$$
\begin{aligned}
Q_e^2 &= 4E_eE'_e\cos^2(\frac{\theta_e}{2}) \\
y_e &= 1-\frac{E'_e}{E_e}\sin^2(\frac{\theta_e}{2})
\end{aligned}
$$

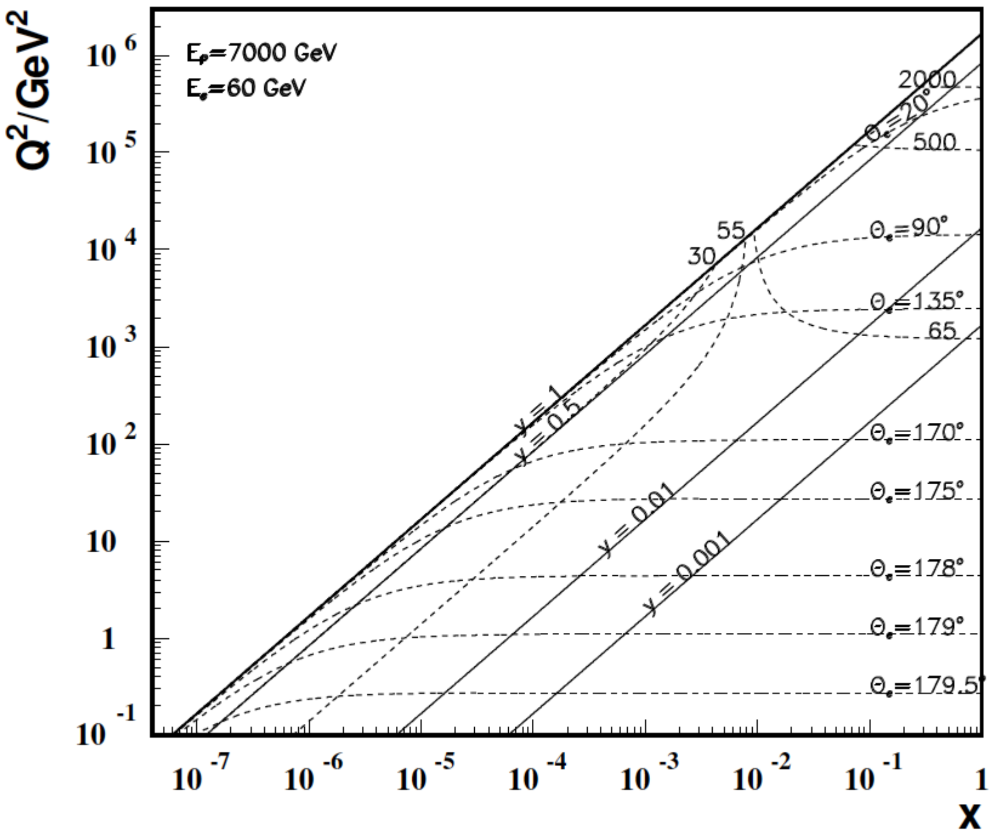
Hadron method

$$
\begin{aligned}
Q_h^2 &= \frac{1}{1-y_h}\cdot E_h^2\sin^2(\theta_h) \\
y_h &= \frac{E_h}{E_e}\sin^2(\frac{\theta_h}{2})
\end{aligned}
$$

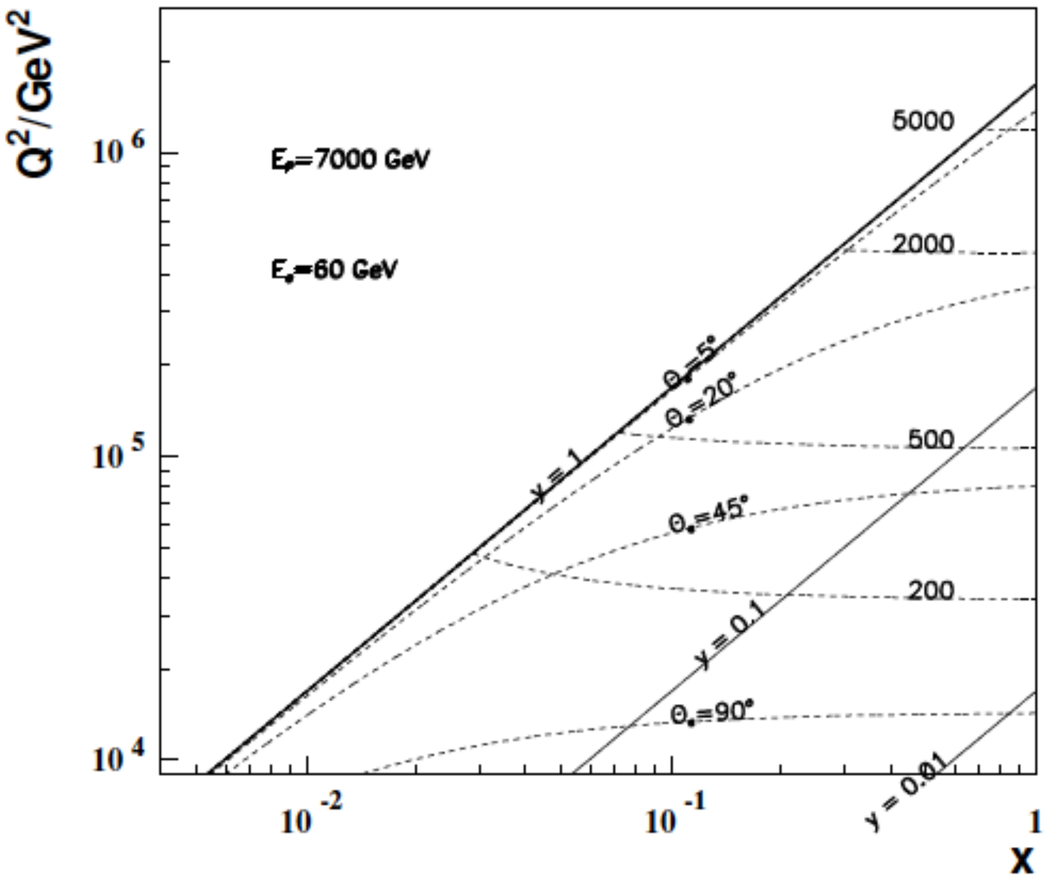
Note: angles measured with respect to the p direction (HERA convention).

HERA: $e^{\pm}(27.5) + p(920)$, $\sqrt{s}=318$ GeV

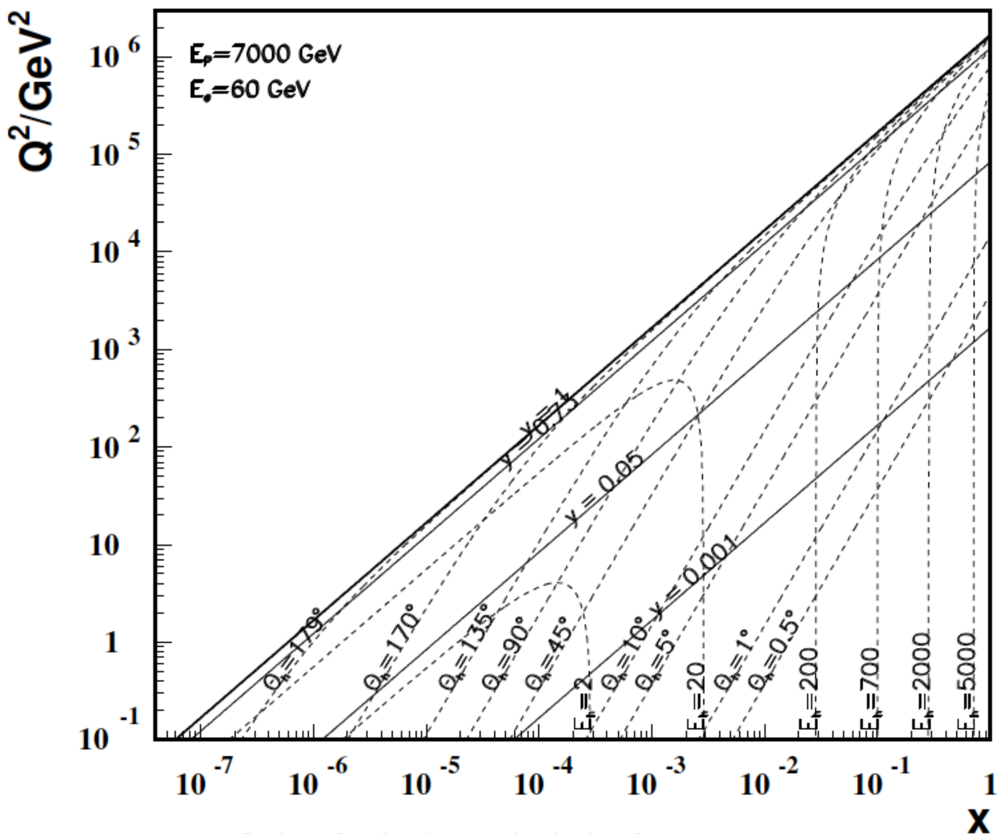
Kinematics:



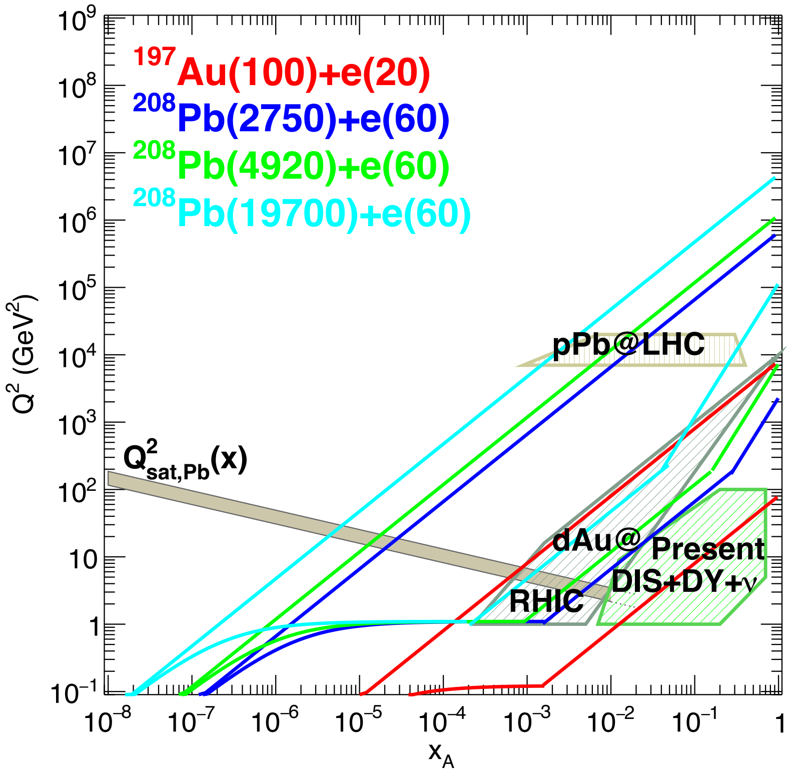
LHeC - electron kinematics



LHeC - hadronic final state kinematics



Large acceptance + excellent EM and HAD calorimetry required.



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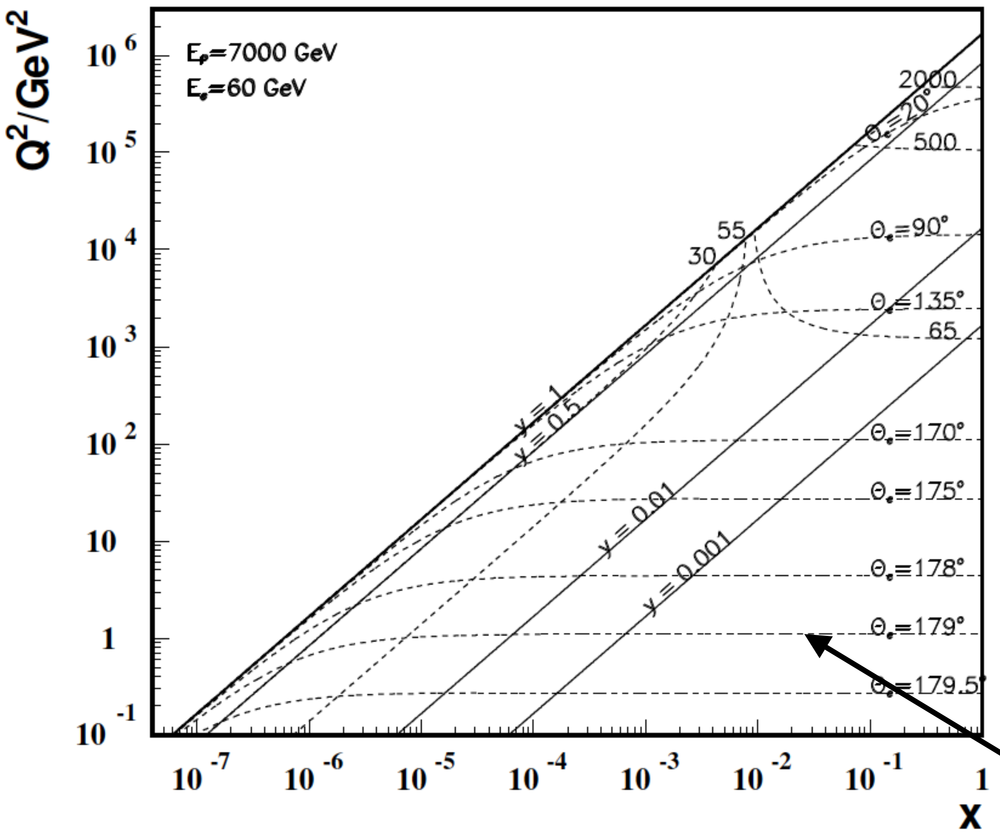
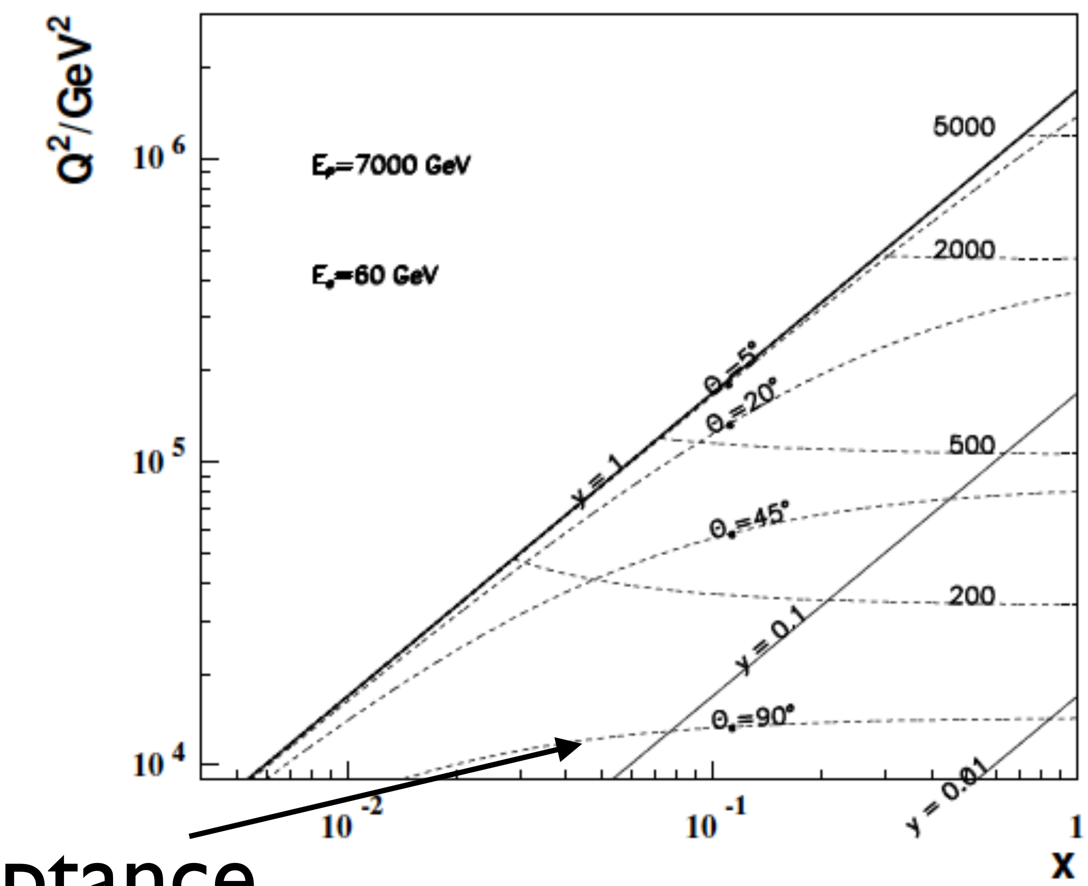
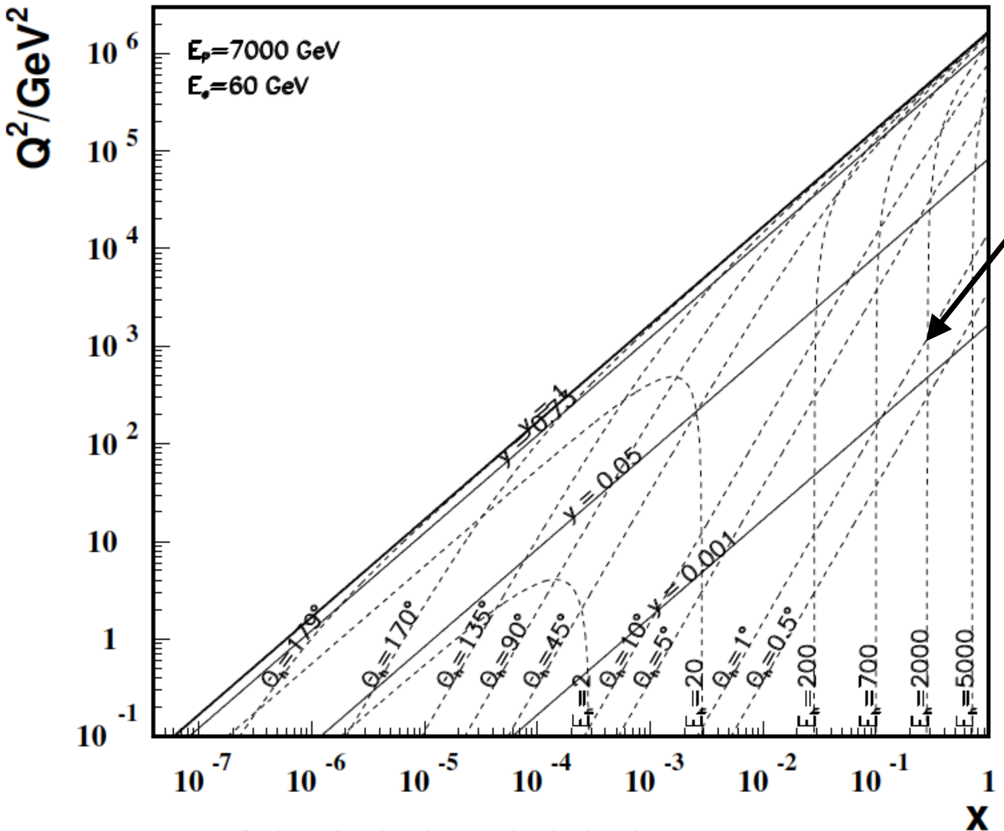


LHeC - electron kinematics



LHeC - hadronic final state kinematics



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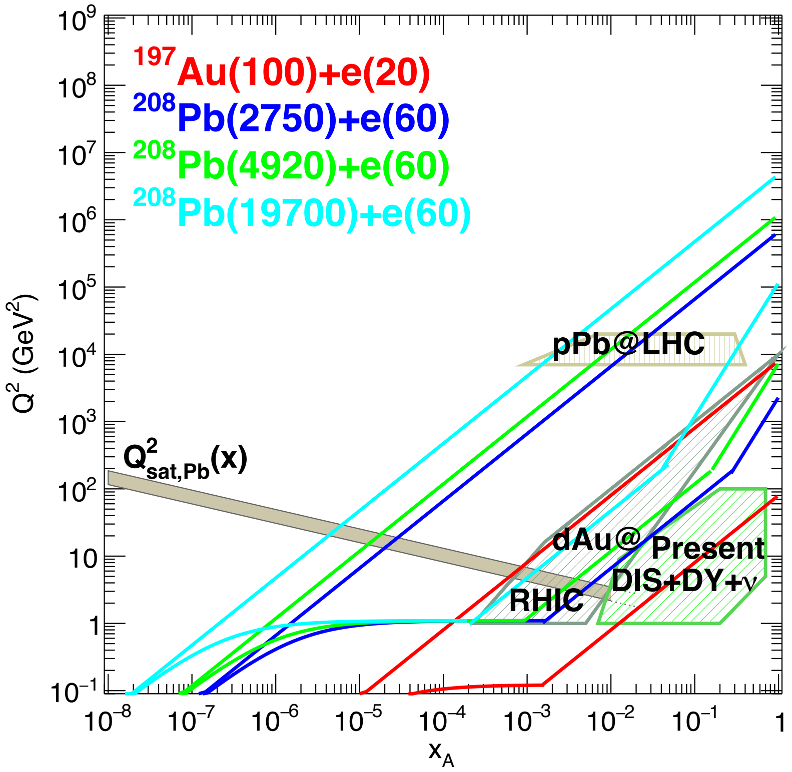
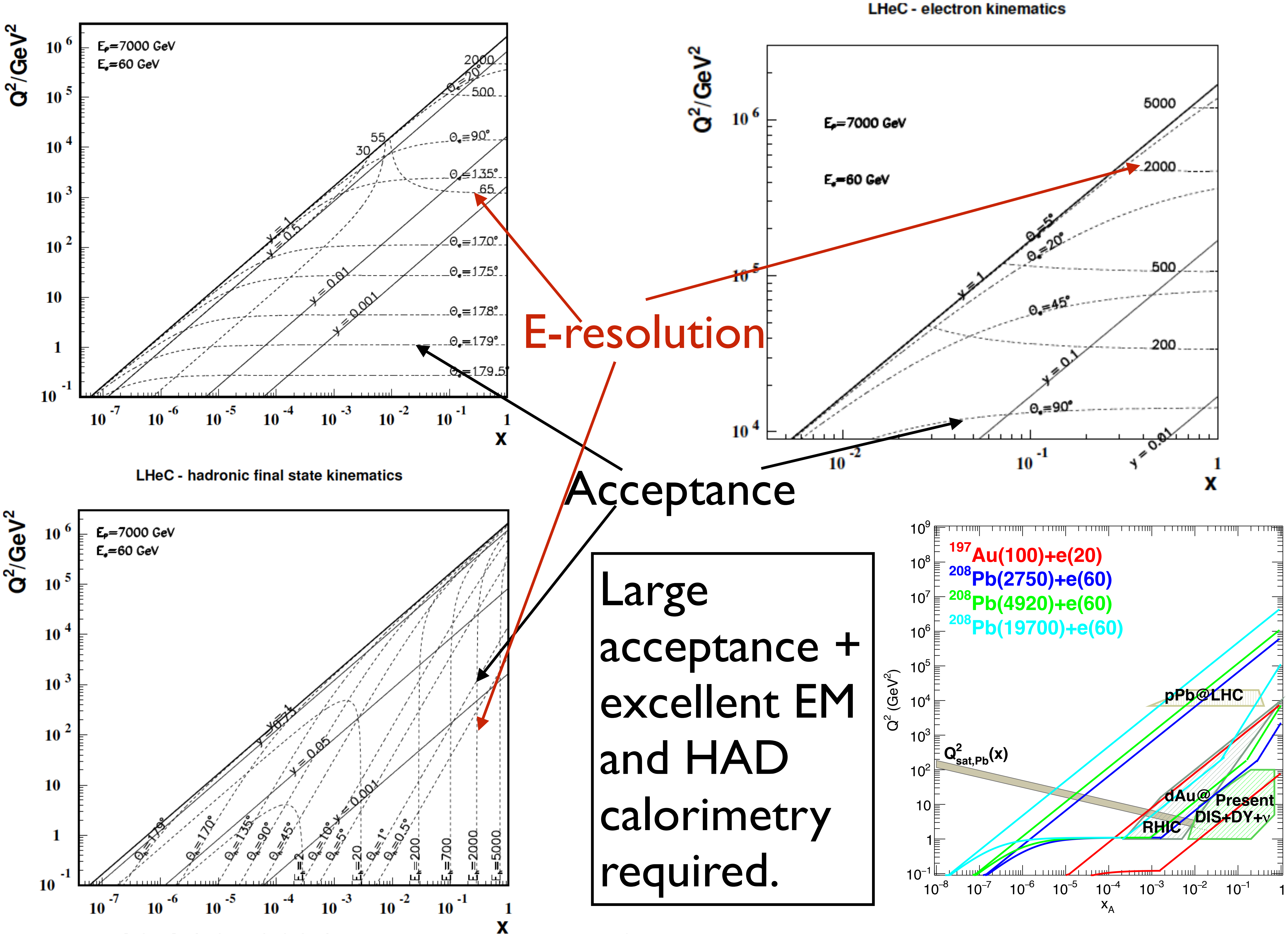


Kinematics:



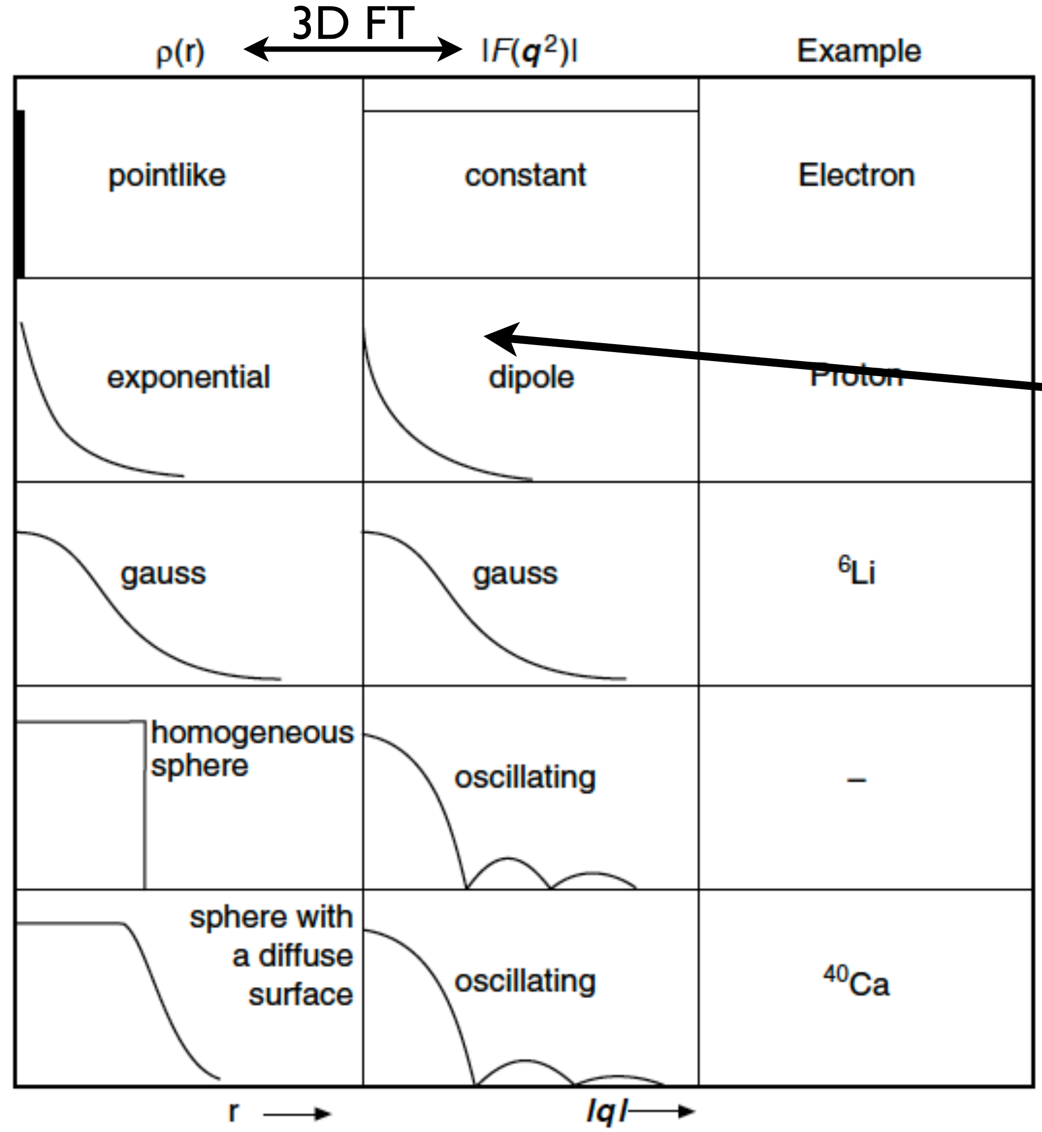
E-resolution

Acceptance

Large acceptance + excellent EM and HAD calorimetry required.

DIS: proton substructure

➜ Let us compare elastic scattering (x=1) on a pointlike s=1/2 particle with that on a proton and the inelastic one (for x~$\mathcal{O}$(1)):



$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point}\\\text{spin 1/2}}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau \tan^2\frac{\theta}{2}\right]$$

$$\tau = \frac{Q^2}{4m_N^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\right]$$

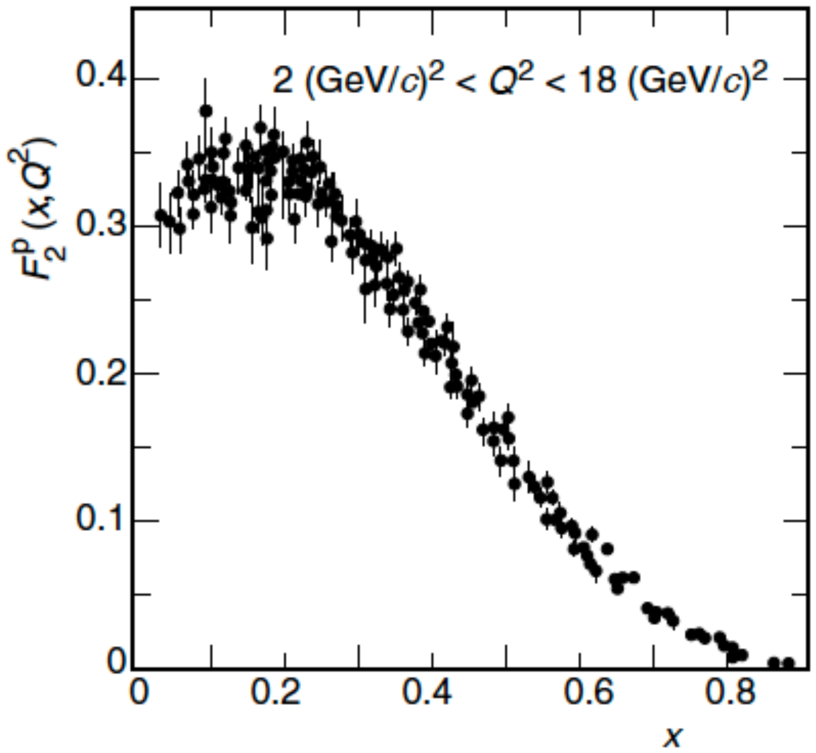
$$\frac{d^2\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right]$$

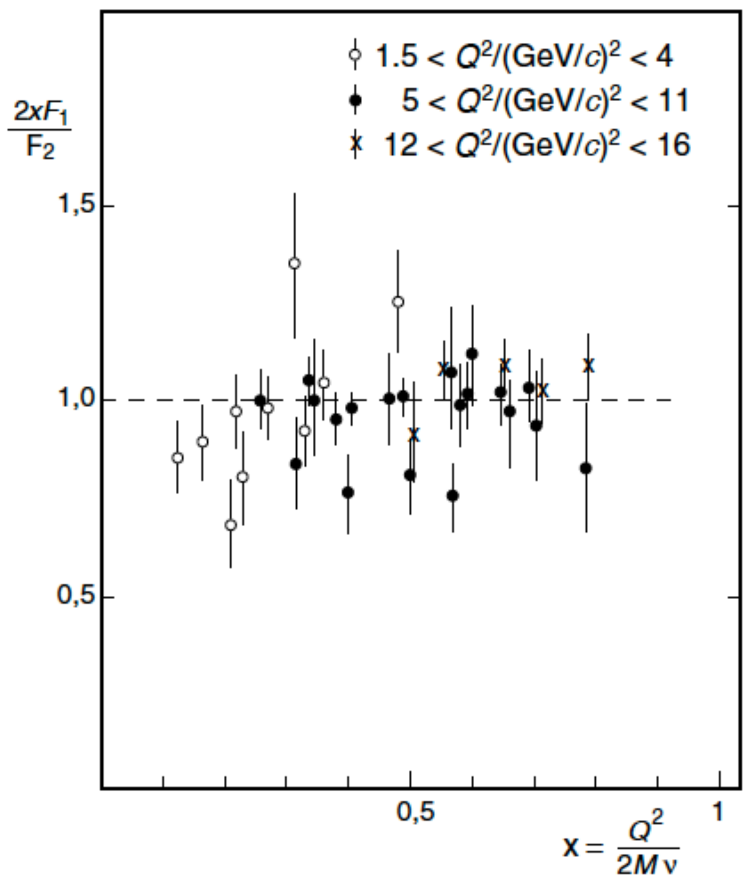
$$F_1(x,Q^2) = Mc^2\, W_1(Q^2,\nu)$$
$$F_2(x,Q^2) = \nu\, W_2(Q^2,\nu)\,.$$

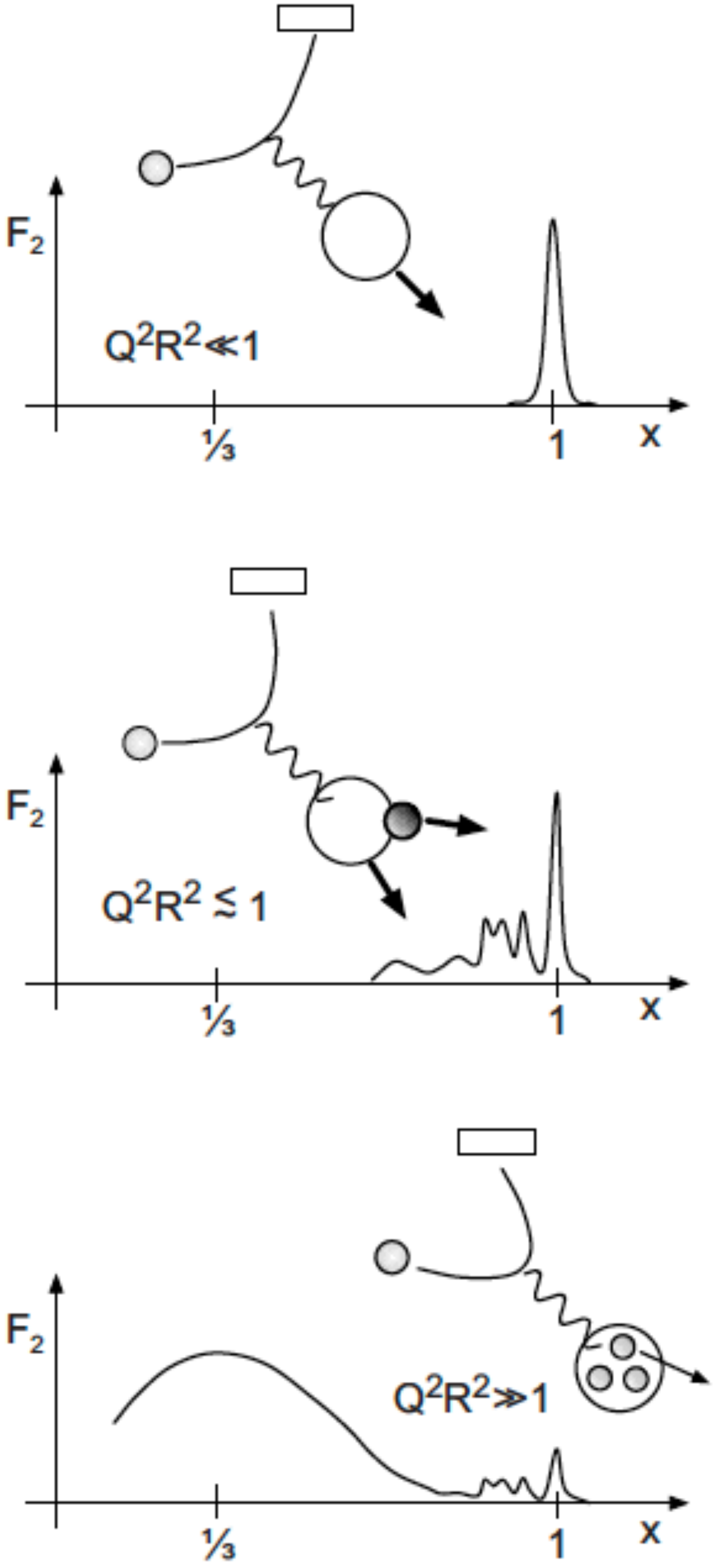
➜ For fixed x, $F_{1,2}$ roughly independent of Q (note $1/Q^4$ behaviour of proton form factors): Bjorken scaling, pointlike scatterers.

➜ $2xF_1=F_2$: Callan-Gross relation, spin-1/2 scatterers.

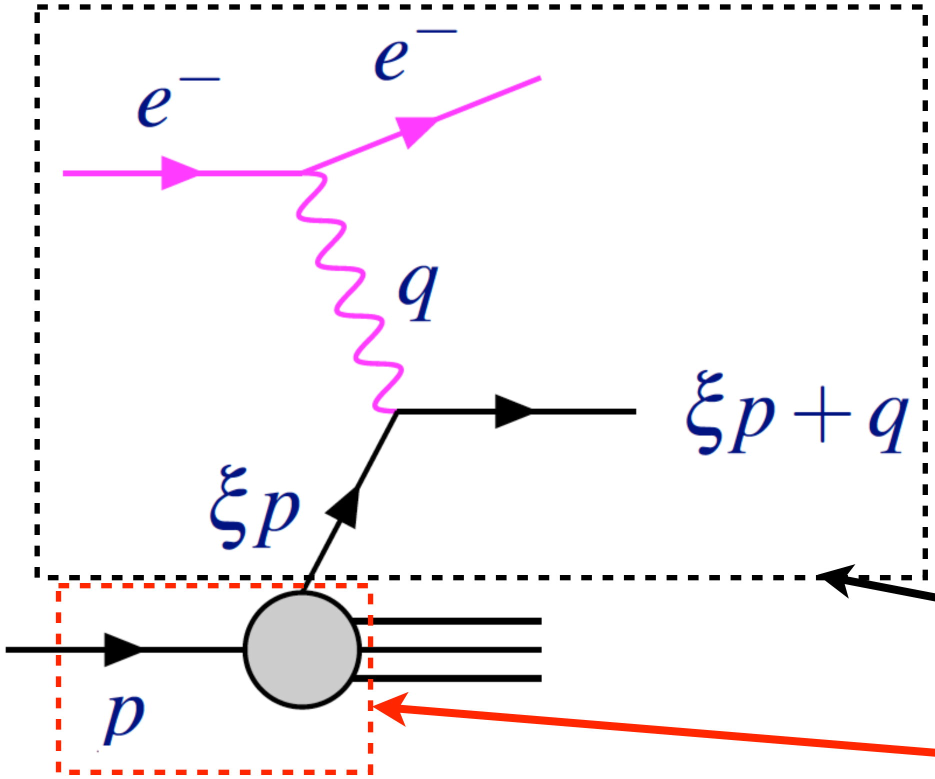
DIS: proton substructure







DIS: parton model



➜ For very large momentum (IMF), the hadron can be considered an incoherent superposition of quanta (partons) during the interaction (Q>>Λ_{QCD}): parton model (Feynman, Bjorken, Gribov).

$$\sigma(e^-(k)p(p) \to e^-(k')X) = \int_0^1 d\xi \sum_f \boxed{f_{q_f}(\xi)}\,\boxed{\sigma(e^-(k)q_f(\xi p) \to e^-(k')q_f(\xi p+q))}$$

$$\begin{aligned} F_2(x) = 2xF_1(x) &= \sum_{q,\bar{q}} \int_0^1 d\xi\, q(\xi)\, x e_q^2 \boxed{\delta(x-\xi)} \quad + \mathcal{O}(1/Q^2) \\ &= \sum_{q,\bar{q}} e_q^2\, x q(x)\,. \end{aligned}$$

➜ Relation between PDFs for valence and sea quarks and gluons:

$$\boxed{F_2^{eN}(x) = \frac{5}{18} F_2^{\nu N}(x)}$$

electric charges

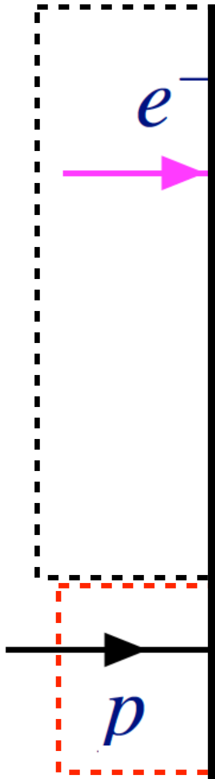
$$\boxed{F_2^{ep}\text{-}F_2^{en} = \frac{1}{3} x(u_v(x)\text{-}d_v(x))}$$

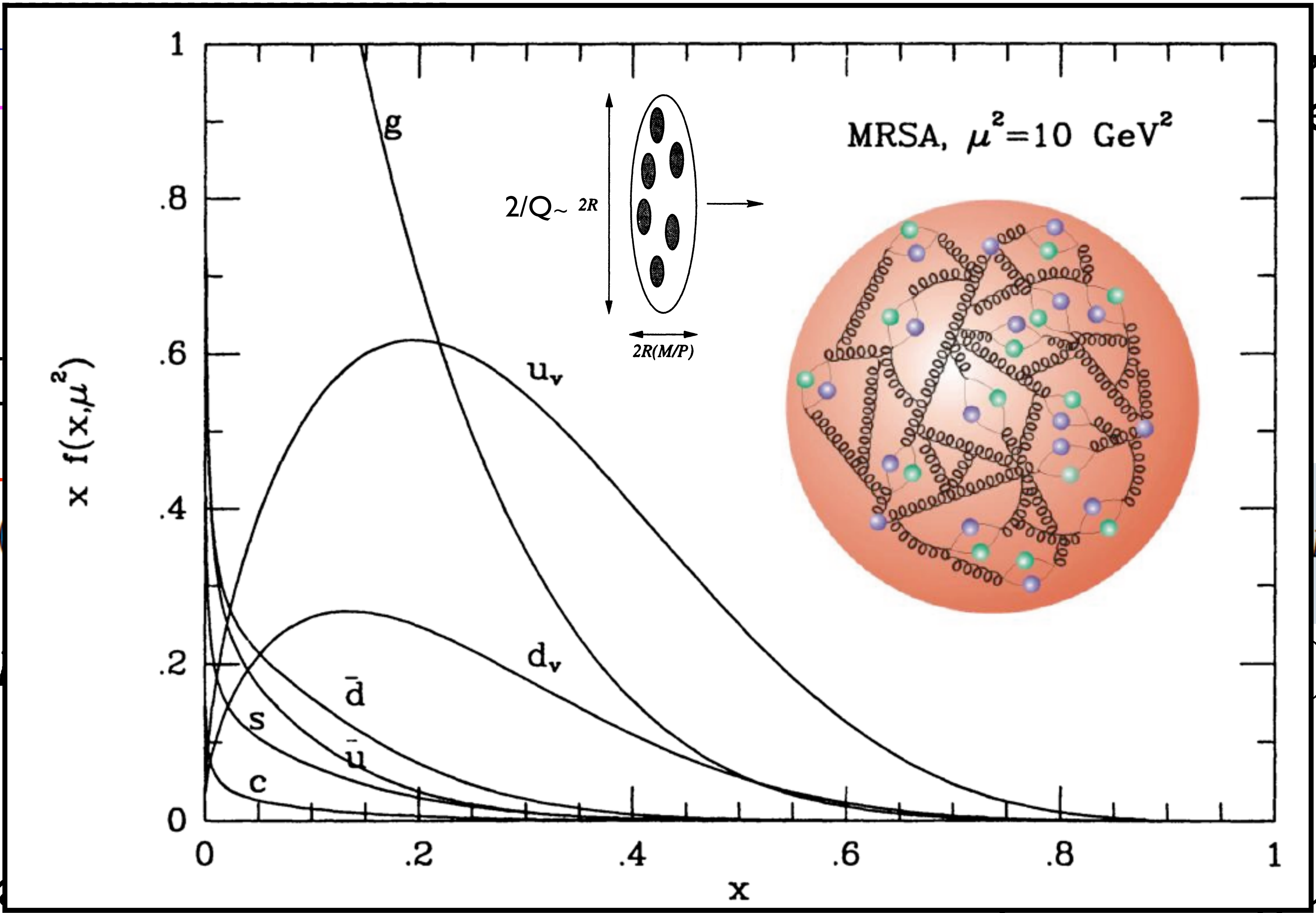
valence

$$\boxed{\int_0^1 F_2^{\nu N}(x)dx = \int_0^1 x(q(x)+\bar{q}(x))dx = 0.44}$$

gluons

DIS: parton model





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model

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➜ Rel

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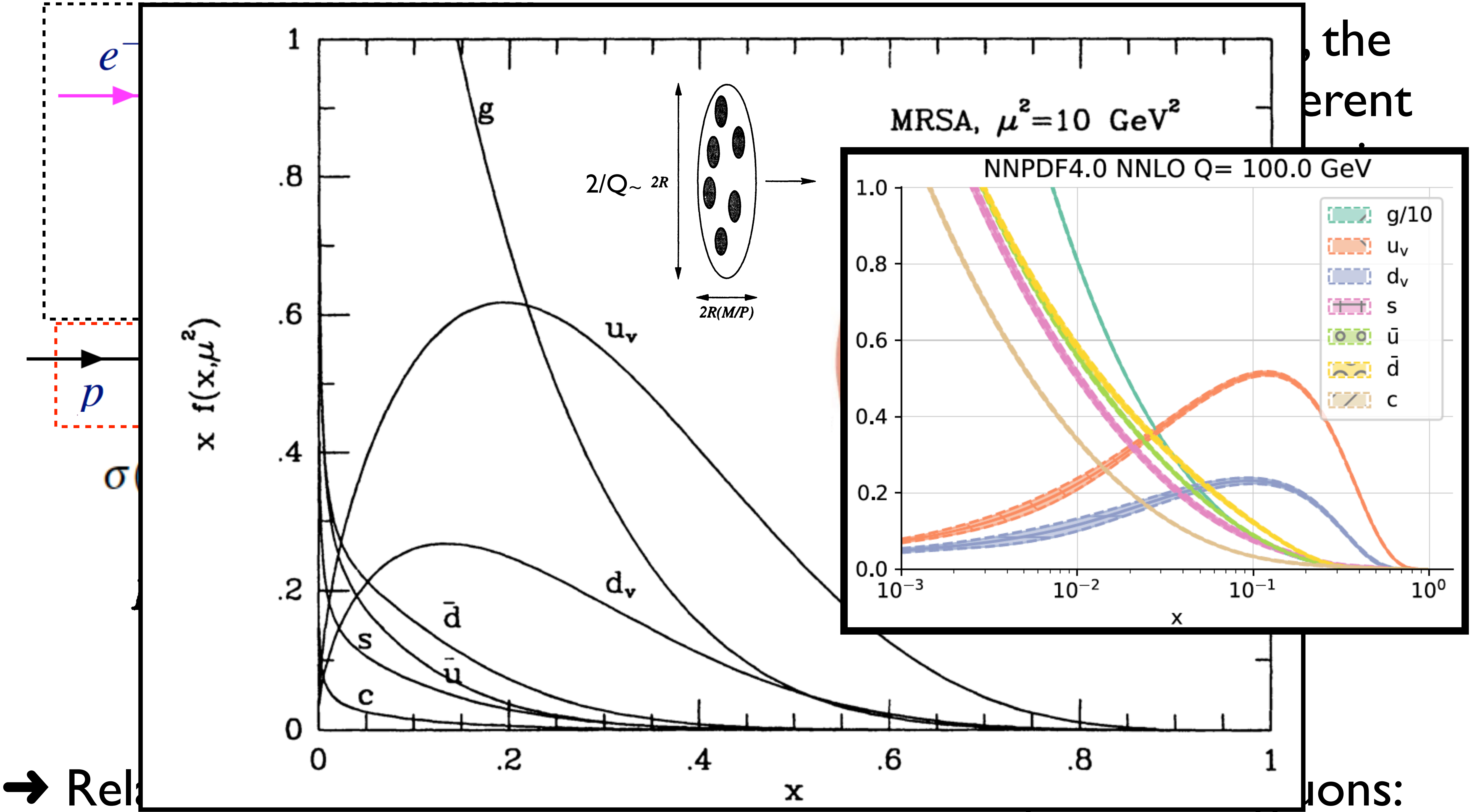

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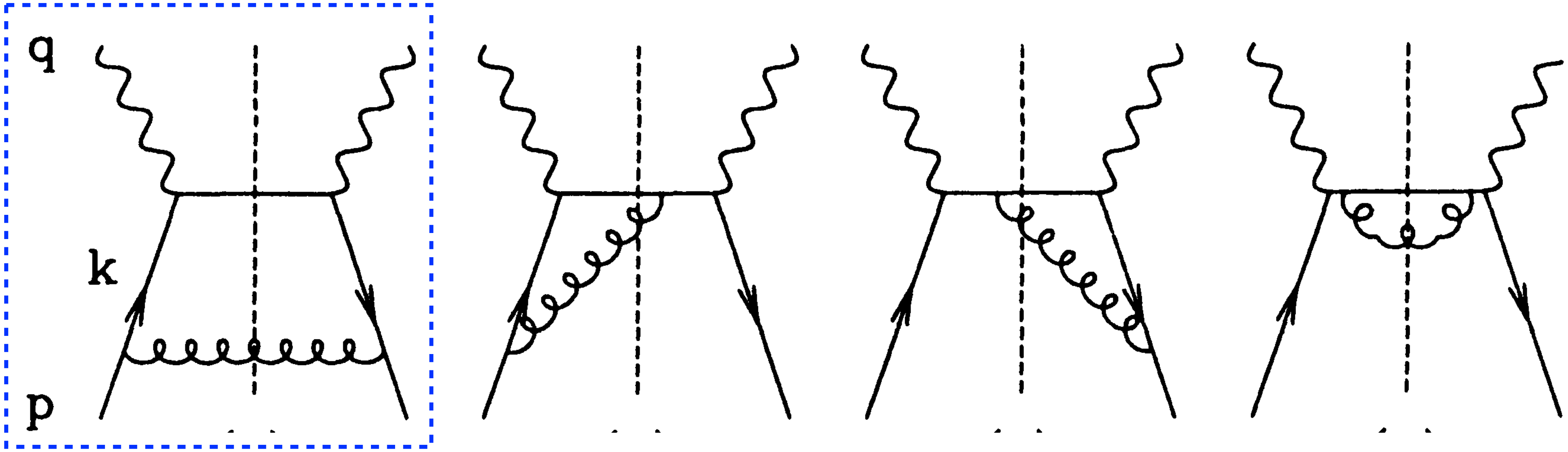
gluons

DIS: QCD corrections

➜ The parton model receives corrections from the fact that partons radiate: PDFs evolve with scale Q, DGLAP evolution equations.

only diagram that gives (logarithmic) divergencies (in LC gauge)



$$Q^2\partial_{Q^2}\begin{pmatrix} q_i(x,Q^2) \\ \bar{q}_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P_{q_iq_j}\left(\frac{x}{\xi}\right) & 0 & P_{q_ig}\left(\frac{x}{\xi}\right) \\ 0 & P_{q_iq_j}\left(\frac{x}{\xi}\right) & P_{q_ig}\left(\frac{x}{\xi}\right) \\ P_{gq}\left(\frac{x}{\xi}\right) & P_{gq}\left(\frac{x}{\xi}\right) & P_{gg}\left(\frac{x}{\xi}\right) \end{pmatrix}\begin{pmatrix} q_j(x,Q^2) \\ \bar{q}_j(x,Q^2) \\ g(x,Q^2) \end{pmatrix}$$

DGLAP@LO

➜ PDFs are unknown, non-perturbative quantities but we know its perturbative evolution (at leading logarithmic accuracy). They have to be extracted from data or lattice.

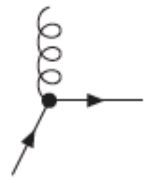
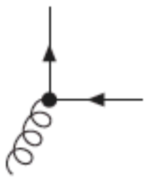
DIS: QCD corrections

➜ The parton mo ... t that partons radiate: PDFs evol ... quations.

only diagram that gives (logarithmic) divergencies (in LC gauge)

q

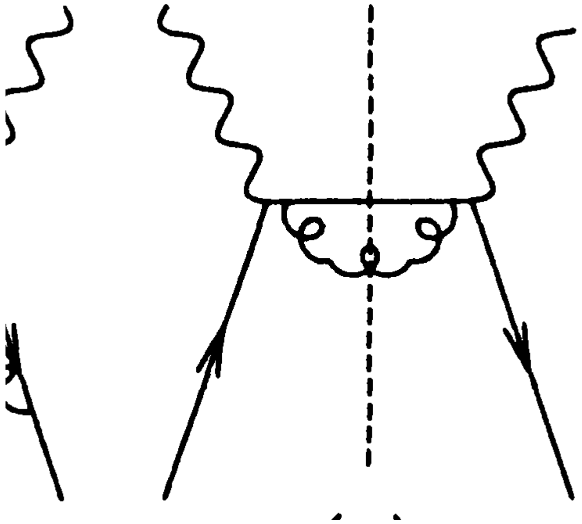
p

Diagram	Splitting
	$P_{qq} = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$
	$P_{gq} = C_F\left[\frac{1+(1-x)^2}{x}\right]$
	$P_{qg} = T_R\left[x^2+(1-x)^2\right]$
	$P_{gg} = 2C_A\left[\frac{x}{(1-x)_+} + (1-x)\left(x+\frac{1}{x}\right)\right] + \frac{11C_A-4n_fT_R}{6}\delta(1-x)$

T_R=1/2

C_F=(N^2-1)/(2N)=4/3

C_A=N=3



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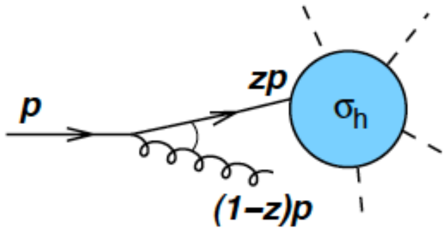
DGLAP@LO

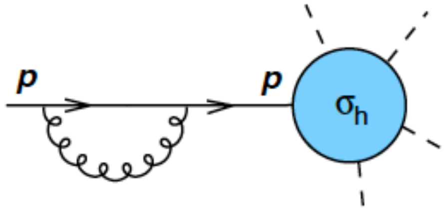
➜ PDFs are unknown, non-perturbative quantities but we know its perturbative evolution (at leading logarithmic accuracy). They have to be extracted from data or lattice.

DIS: virtual plus real

➜ When we consider radiation from initial state (before a hard scattering σσ_h), both real and virtual corrections appear:



$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

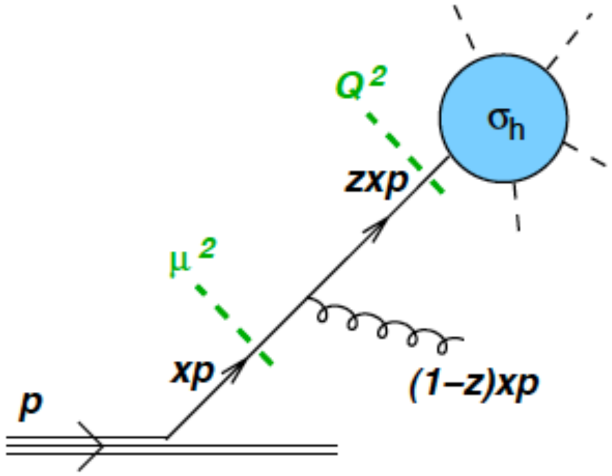


$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

➜ They combine into a IR finite but collinearly divergent cross section:

$$\sigma_{g+h}+\sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\underbrace{\int_0^{Q^2}\frac{dk_t^2}{k_t^2}}_{\text{infinite}}\underbrace{\int_0^1\frac{dz}{1-z}[\sigma_h(zp)-\sigma_h(p)]}_{\text{finite}}$$

➜ The collinear divergence is absorbed in a redefinition of the PDFs putting a cut-off: the independence of its choice leads to DGLAP.

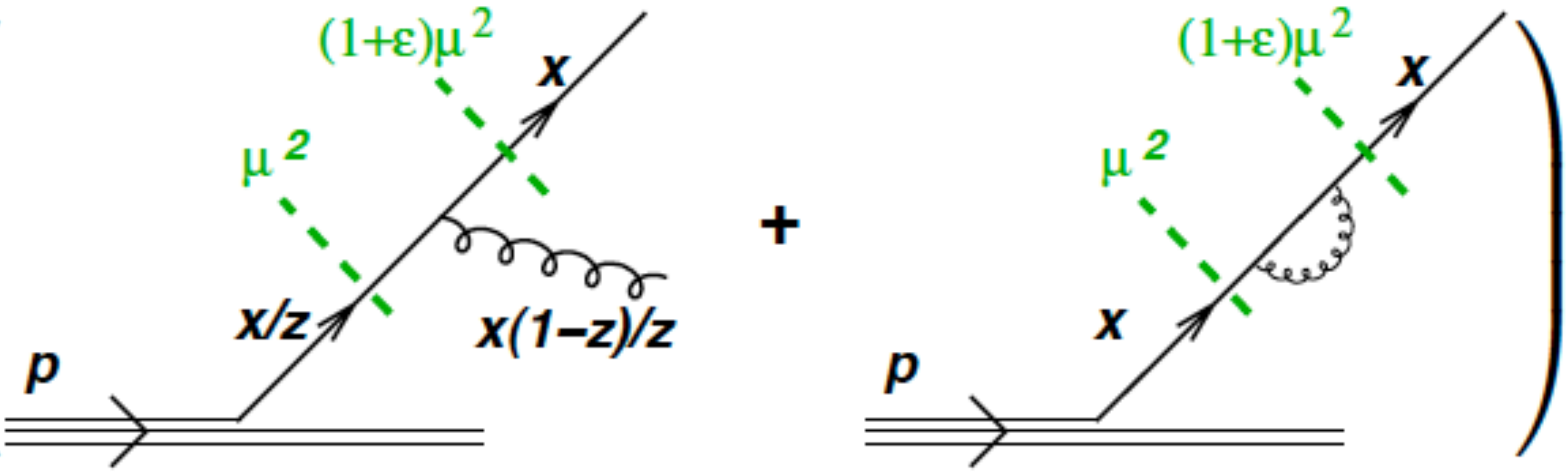


$$\sigma_0 = \int dx\ \sigma_h(xp)\ q(x,\mu_F^2)\,,$$

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi}\underbrace{\int_{\mu_F^2}^{Q^2}\frac{dk_t^2}{k_t^2}}_{\text{finite (large?)}}\underbrace{\int\frac{dx\,dz}{1-z}\,[\sigma_h(zxp)-\sigma_h(xp)]\ q(x,\mu_F^2)}_{\text{finite}}$$

DIS: virtual plus real

$$\frac{dq(x,\mu_{\rm F}^2)}{d\ln\mu_{\rm F}^2} = \frac{1}{\epsilon}\Bigg($$



$$\Bigg)$$

$$= \frac{\alpha_{\rm s}}{2\pi}\int_x^1 dz\, p_{qq}(z)\,\frac{q(x/z,\mu_{\rm F}^2)}{z} - \frac{\alpha_{\rm s}}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu_{\rm F}^2)$$

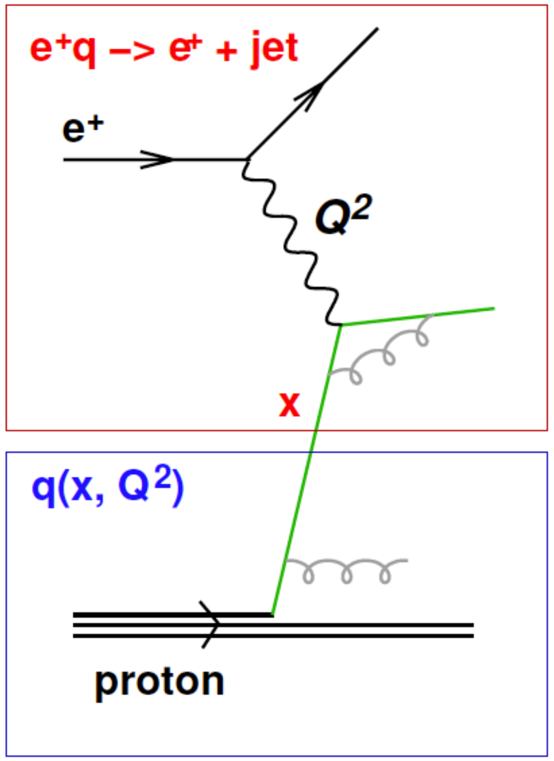
$$= \frac{\alpha_{\rm s}}{2\pi}\underbrace{\int_x^1 dz\, P_{qq}(z)\,\frac{q(x/z,\mu_{\rm F}^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

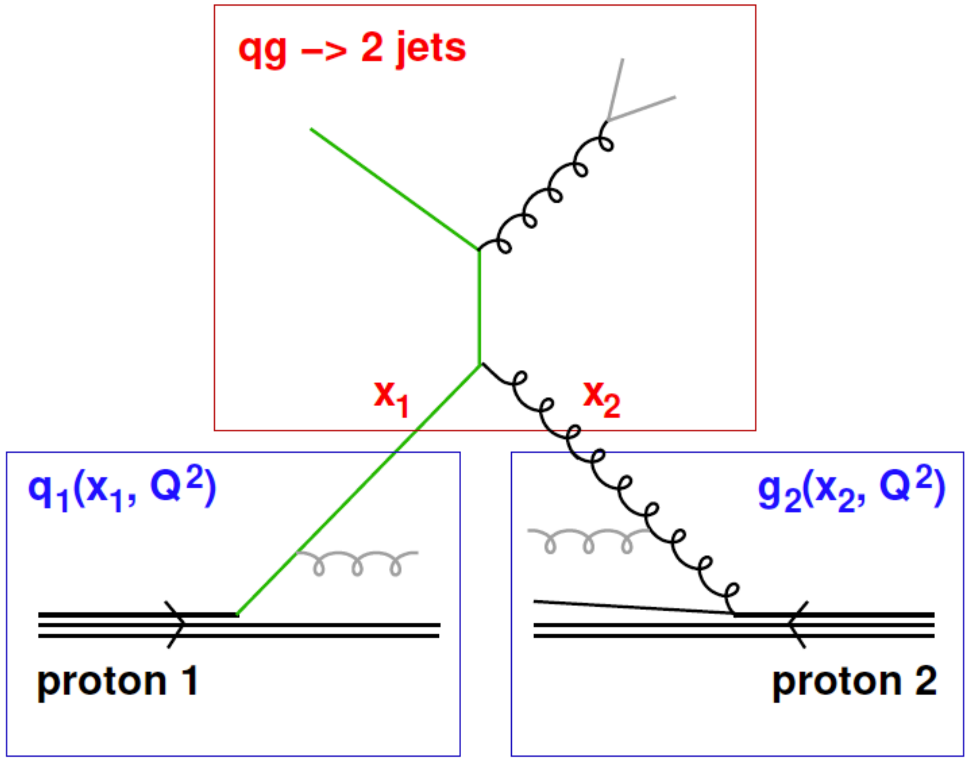
$$\int_x^1 dz\,[g(z)]_+\, f(z) = \int_x^1 dz\, g(z)\, f(z) - \int_0^1 dz\, g(z)\, f(1)$$

$$= \int_x^1 dz\, g(z)\,(f(z)-f(1)) - \int_0^x dz\, g(z)\, f(1)$$

Collinear factorisation:

- Factorisation is the statement of how a cross section can be computed:





$$d\sigma_{h_1+h_2\to H+X} = \sum_{i,j,k} \int f_{i/h_1}\left(x_1,\mu_F\right) \otimes f_{j/h_2}\left(x_2,\mu_F\right) \otimes d\hat{\sigma}_{i+j\to k+X}\left(sx_1x_2,\mu_F,p_k\right) \otimes D_{k\to H}\left(\frac{p_H}{p_k},\mu_F\right) + \mathcal{O}\left(\frac{\mu_F}{\Lambda_{QCD}}\right)$$

Parton density, universal: independent of h_2, j, k, H, non perturbative (DGLAP)

Hard scattering element, independent of h_1, h_2, H, computable $\hat{\sigma} = \sum_n \alpha^n C_n$

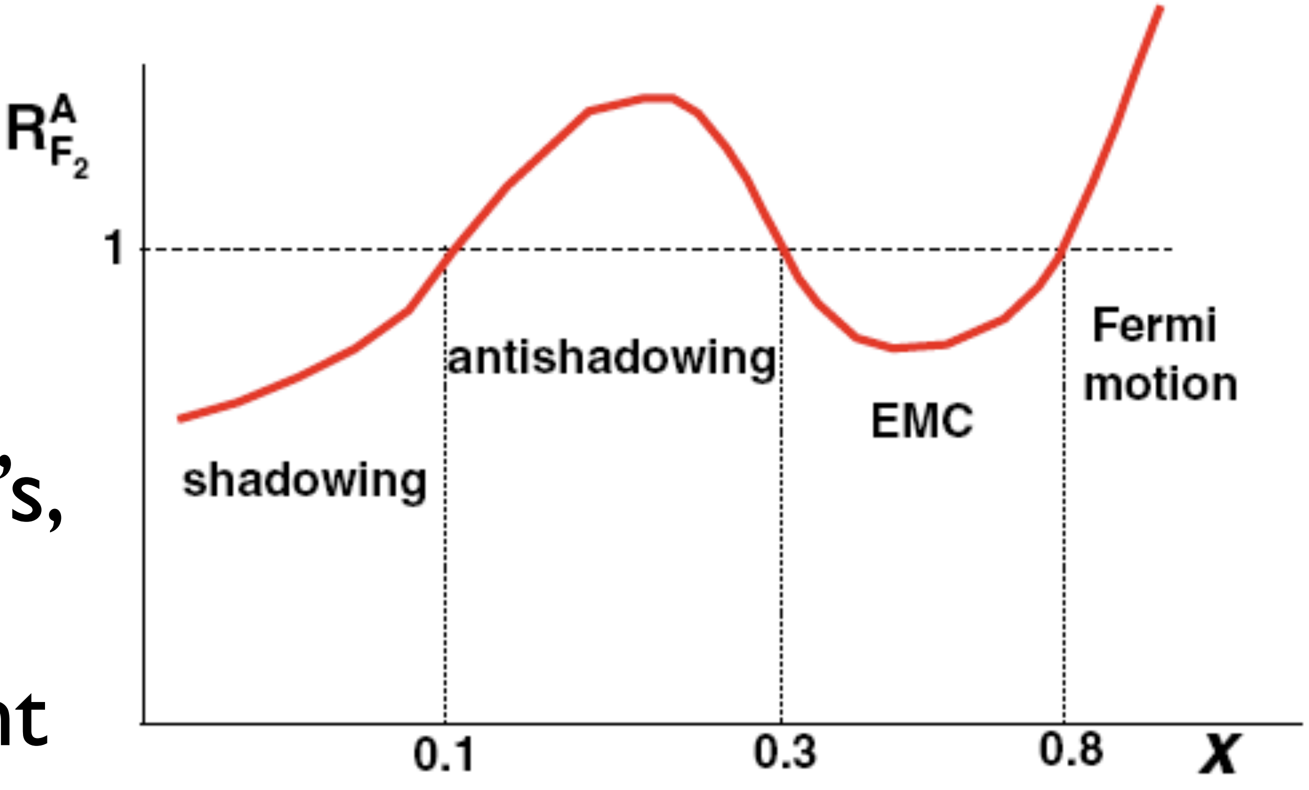
Fragmentation or jet function, if any, universal: independent of h_1, h_2, i, j, non perturbative (DGLAP)

- Standard collinear factorisation only proven in e^+e^-, DIS, DY (CSS 1980's): unitarity, gauge invariance, inclusivity essential; known cases where it has to be extended (TMD) or fails.

DIS on nuclei:

$$R^A_{F_2}(x,Q^2)=\frac{F_2^A(x,Q^2)}{AF_2^{\rm nucleon}(x,Q^2)}$$

- R=1 indicates the absence of nuclear effects.
- R≠1 discovered in the early 70's, significant beyond isospin effects.
- Each region demands a different explanation.



$$R^A_{F_2}(x,Q^2)=\frac{F_2^A(x,Q^2)}{AF_2^p(x,Q^2)}$$

Multiple scattering, saturation,...; high-energy QCD

How much does the structure of a hadron change when it is immersed in a nuclear medium?

Flavour dependence?; relation with shadowing and coherence

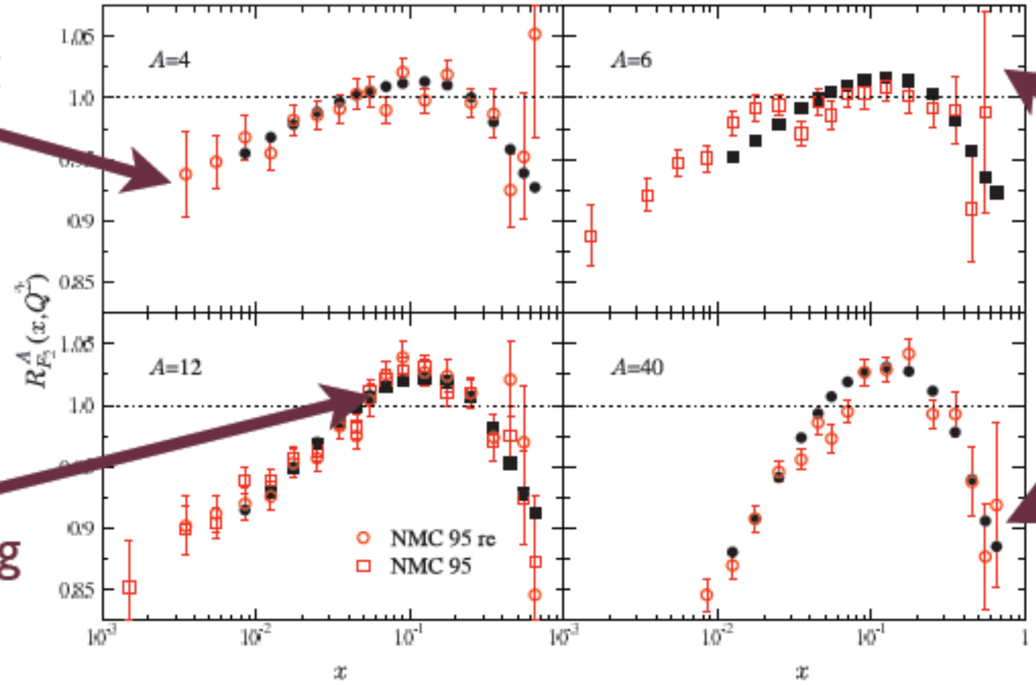
Short versus long range correlations, pion cloud, intrinsic charm,...

shadowing

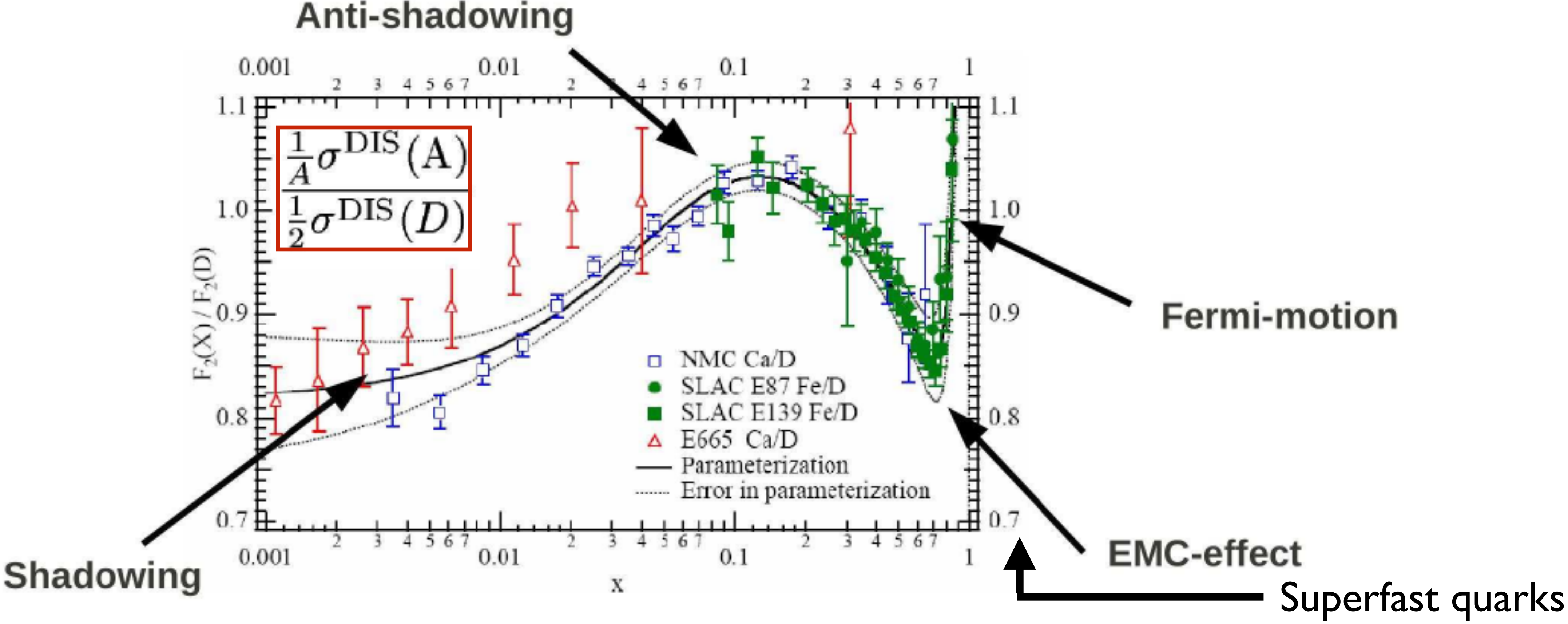
antishadowing

Fermi motion

EMC effect



Collinear approach for nuclei:



Anti-shadowing

Fermi-motion

EMC-effect

Superfast quarks

Shadowing

- Bound nucleon$\neq$free nucleon: search for process independent nPDFs that realise this condition, assuming collinear factorisation.

$$\sigma_{\rm DIS}^{\ell+A\to\ell+X} = \sum_{i=q,\overline{q},g} \underbrace{f_i^A(\mu^2)}_{\textbf{Nuclear PDFs, obeying the standard DGLAP}} \otimes \underbrace{\hat{\sigma}_{\rm DIS}^{\ell+i\to\ell+X}(\mu^2)}_{\textbf{Usual perturbative coefficient functions}}$$

$$f_i^{p,A}(x,Q^2) = \boxed{R_i^A(x,Q^2)} f_i^p(x,Q^2)$$

$$\boxed{R = \frac{f_{i/A}}{A f_{i/p}} \approx \frac{\text{measured}}{\text{expected if no nuclear effects}}}$$

Procedure of extraction:

PDFs, or nuclear effects on them, parametrised at initial scale $Q_0 \gg \Lambda_{QCD}$ employing sum rules (parametrisation biases)

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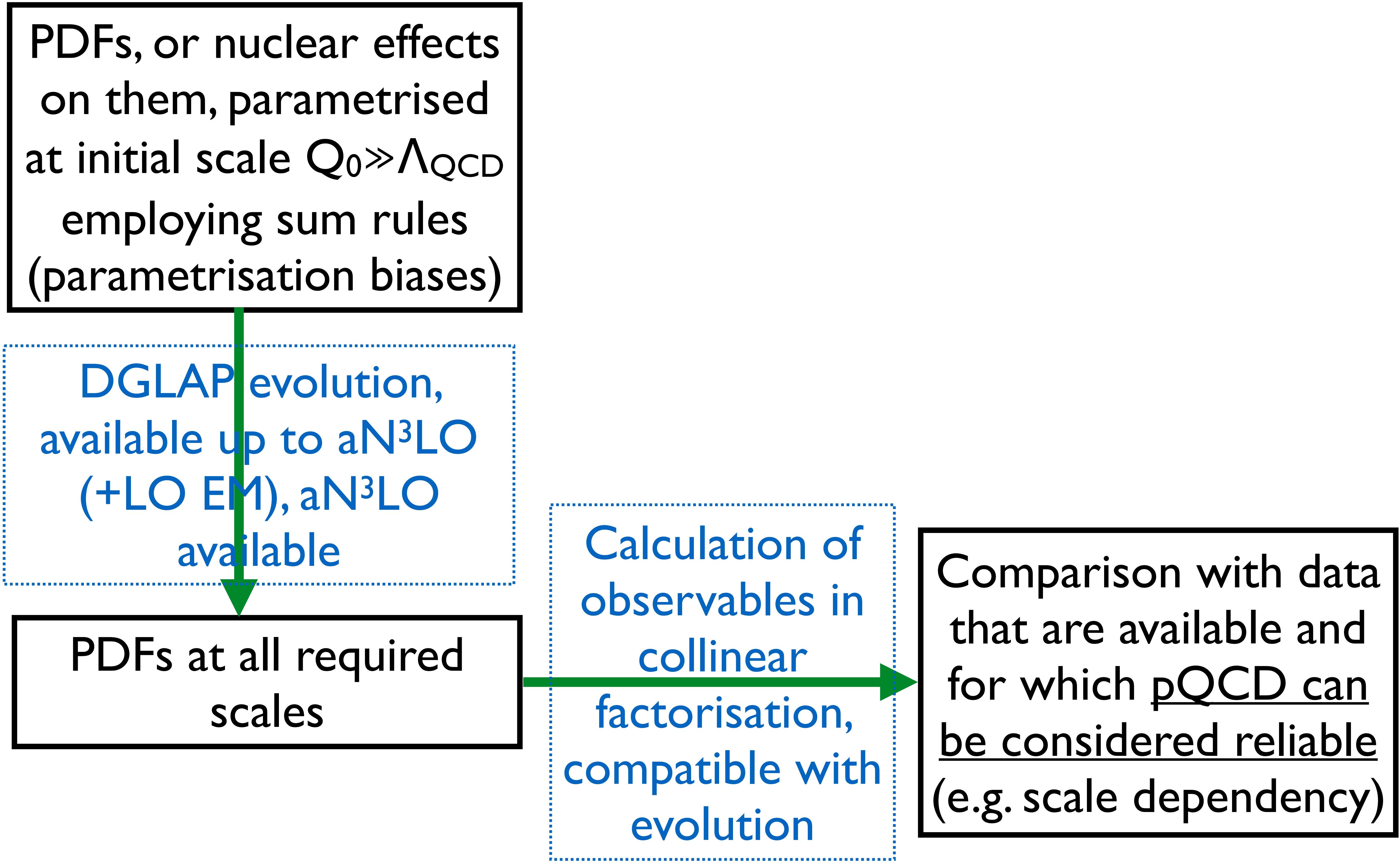
DGLAP evolution, available up to aN^{3}LO (+LO EM), aN^{3}LO available

PDFs at all required scales

Procedure of extraction:



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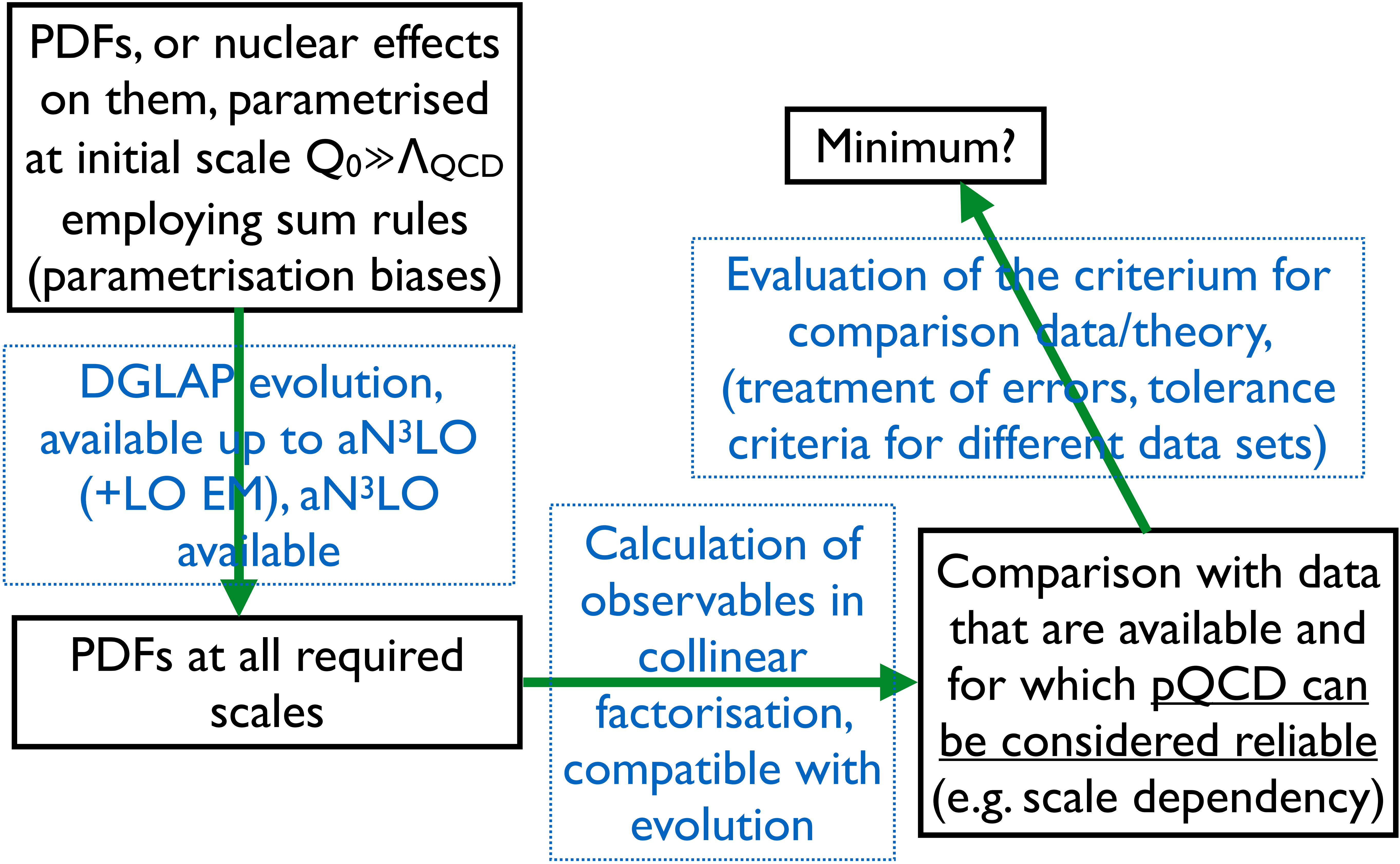
DGLAP evolution, available up to aN^{3}LO (+LO EM), aN^{3}LO available

PDFs at all required scales

Calculation of observables in collinear factorisation, compatible with evolution

Comparison with data that are available and for which <u>pQCD can be considered reliable</u> (e.g. scale dependency)

Procedure of extraction:



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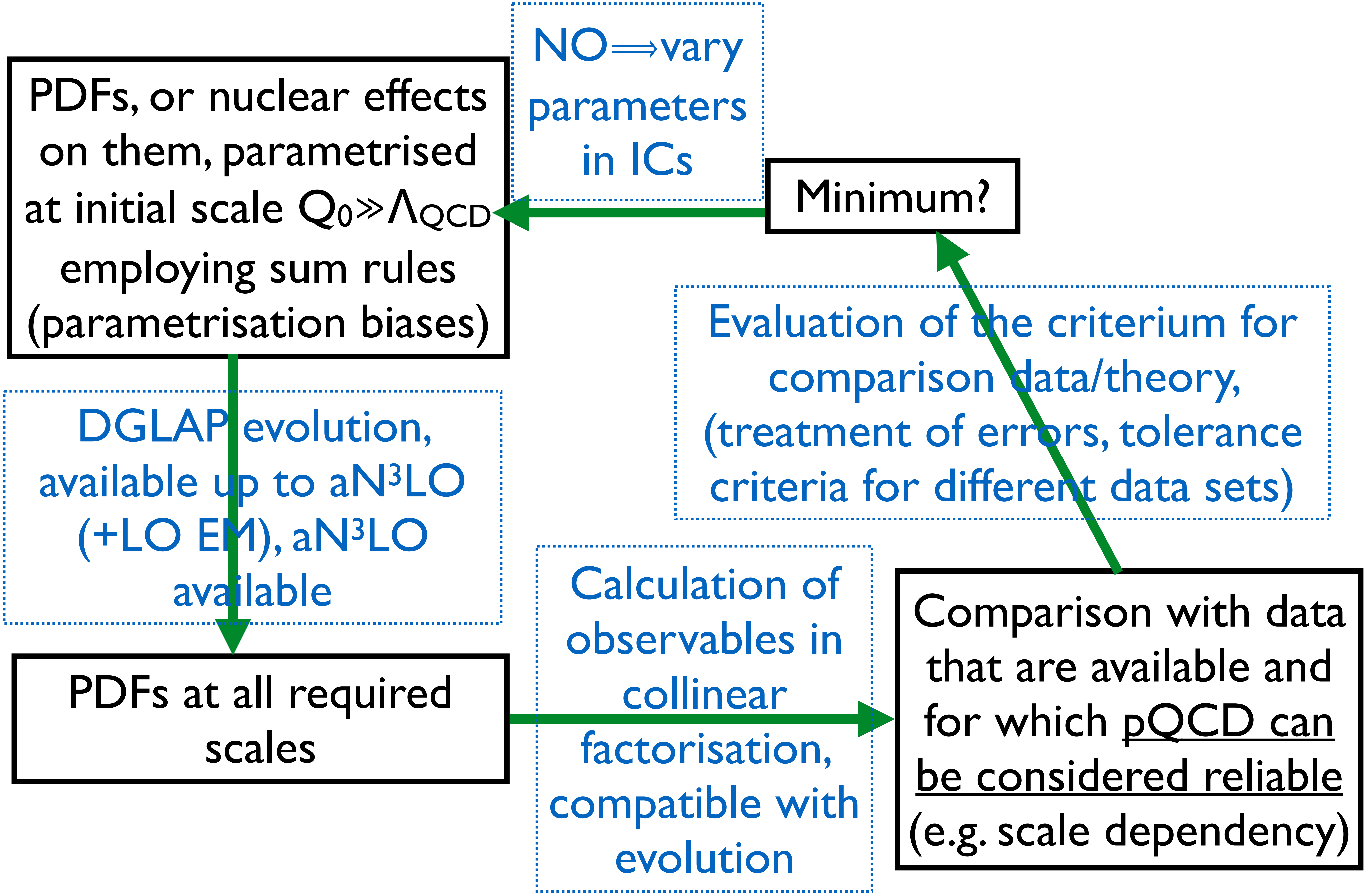
Comparison with data that are available and for which <u>pQCD can be considered reliable</u> (e.g. scale dependency)

Evaluation of the criterium for comparison data/theory, (treatment of errors, tolerance criteria for different data sets)

Minimum?

Procedure of extraction:



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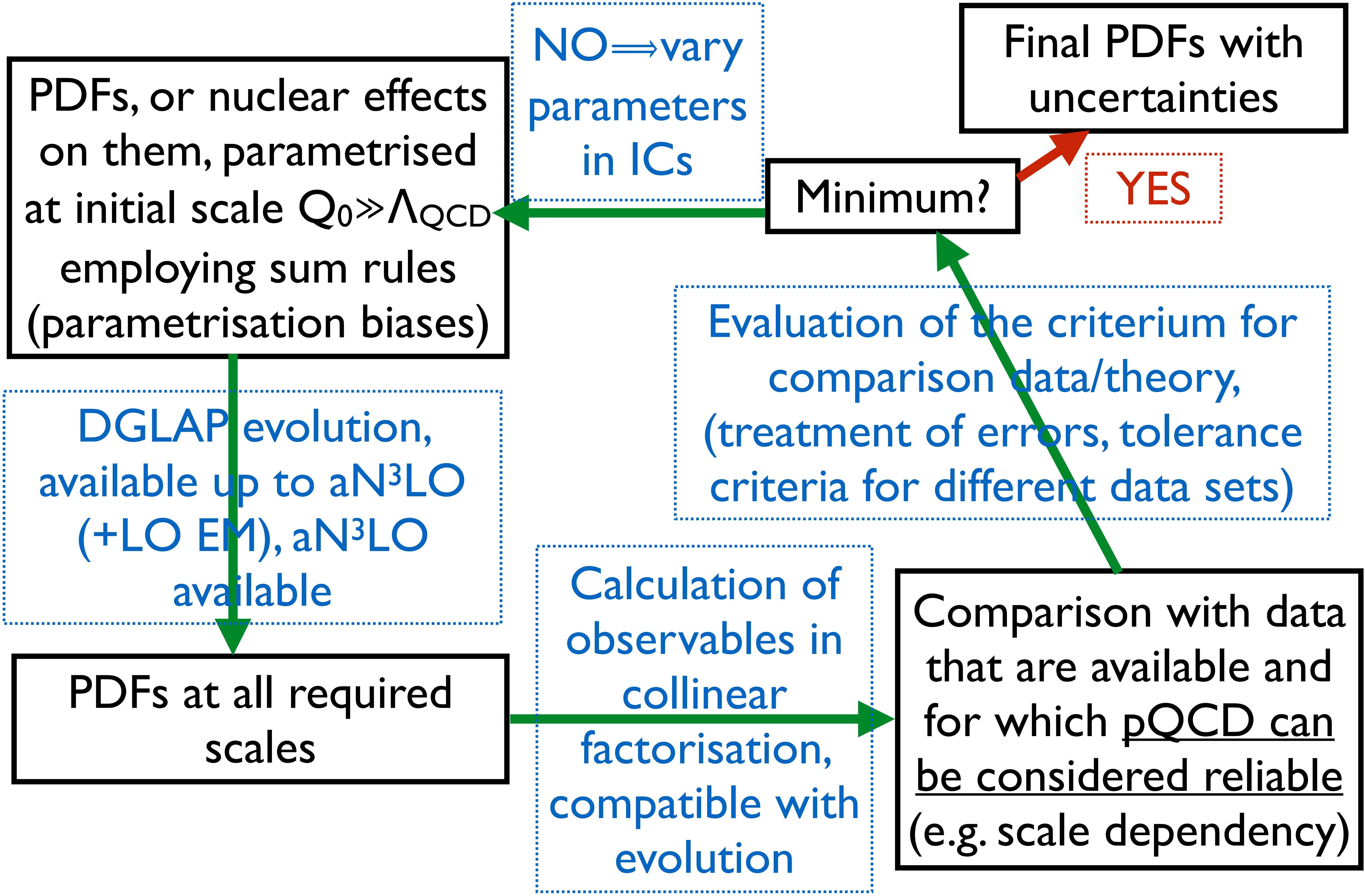
Minimum?

NO⟹vary parameters in ICs

Procedure of extraction:



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DGLAP evolution, available up to aN^{3}LO (+LO EM), aN^{3}LO available

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Evaluation of the criterium for comparison data/theory, (treatment of errors, tolerance criteria for different data sets)

Minimum?

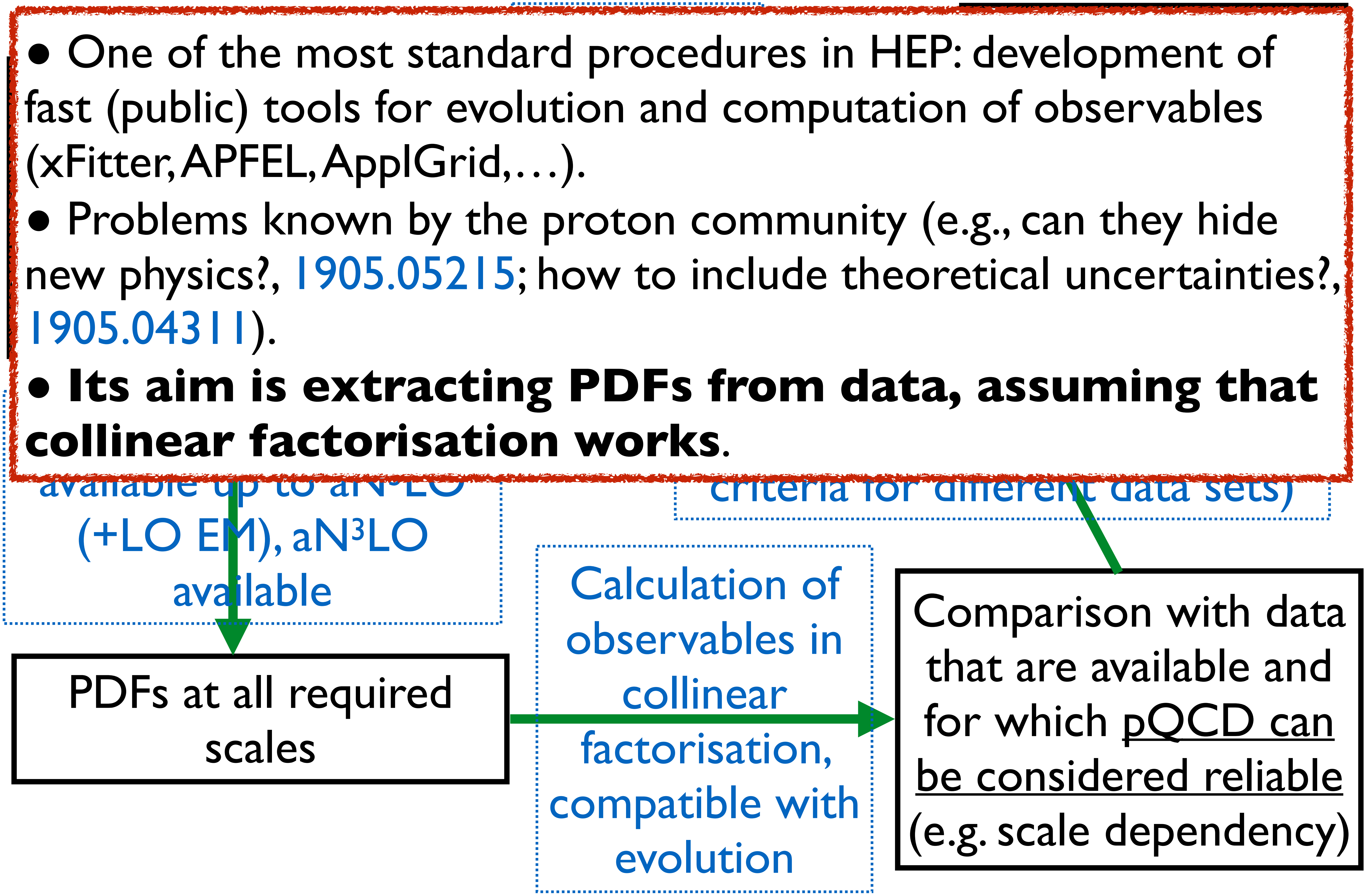
NO $\Longrightarrow$ vary parameters in ICs

YES

Final PDFs with uncertainties

Procedure of extraction:

- One of the most standard procedures in HEP: development of fast (public) tools for evolution and computation of observables (xFitter, APFEL, ApplGrid,…).
- Problems known by the proton community (e.g., can they hide new physics?, 1905.05215; how to include theoretical uncertainties?, 1905.04311).
- **Its aim is extracting PDFs from data, assuming that collinear factorisation works**.



available up to aN LO (+LO EM), aN³LO available

criteria for different data sets)

PDFs at all required scales

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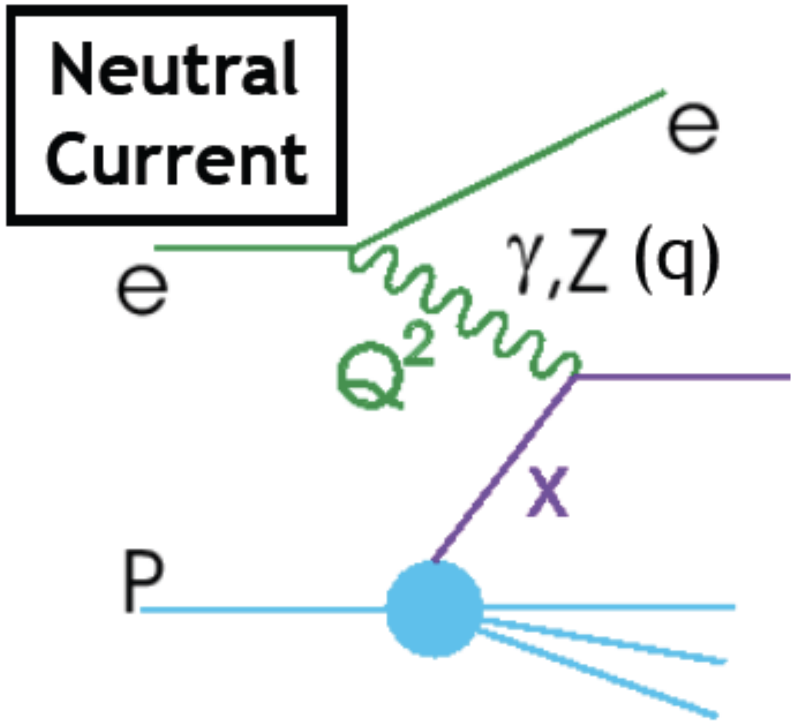
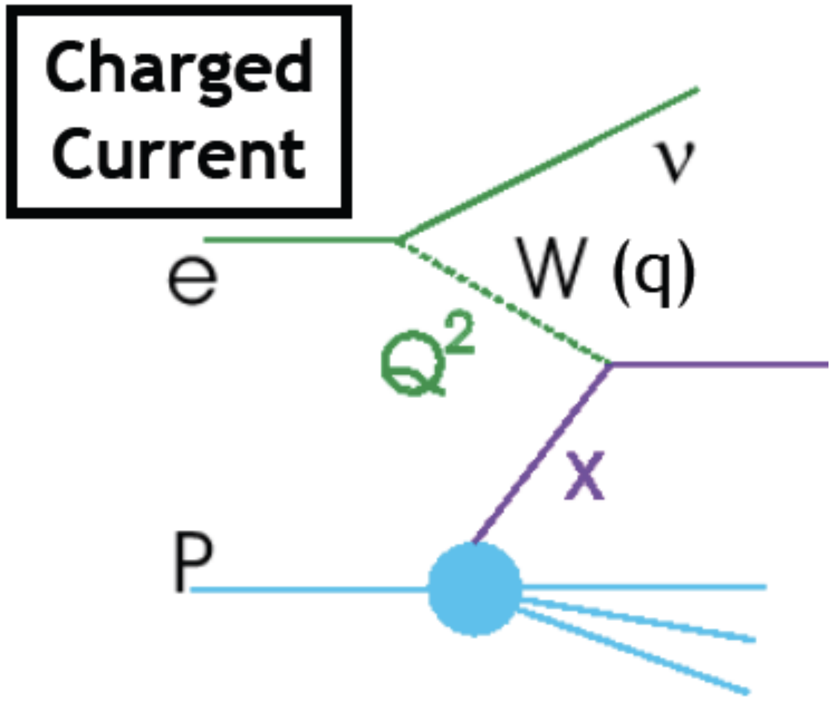
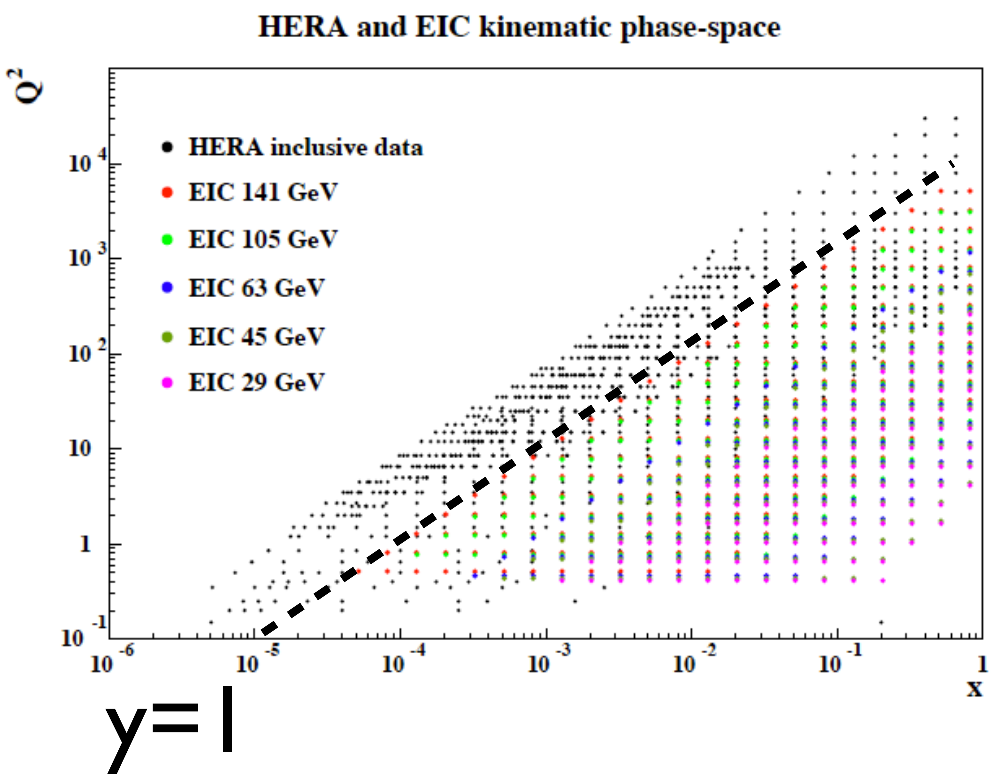
Comparison with data that are available and for which pQCD can be considered reliable (e.g. scale dependency)

Extraction of PDFs: DIS







- Method:

$F_2(x,Q^2) \propto \sum xq(x,Q^2)$: determines directly valence (large x) and sea (low x)

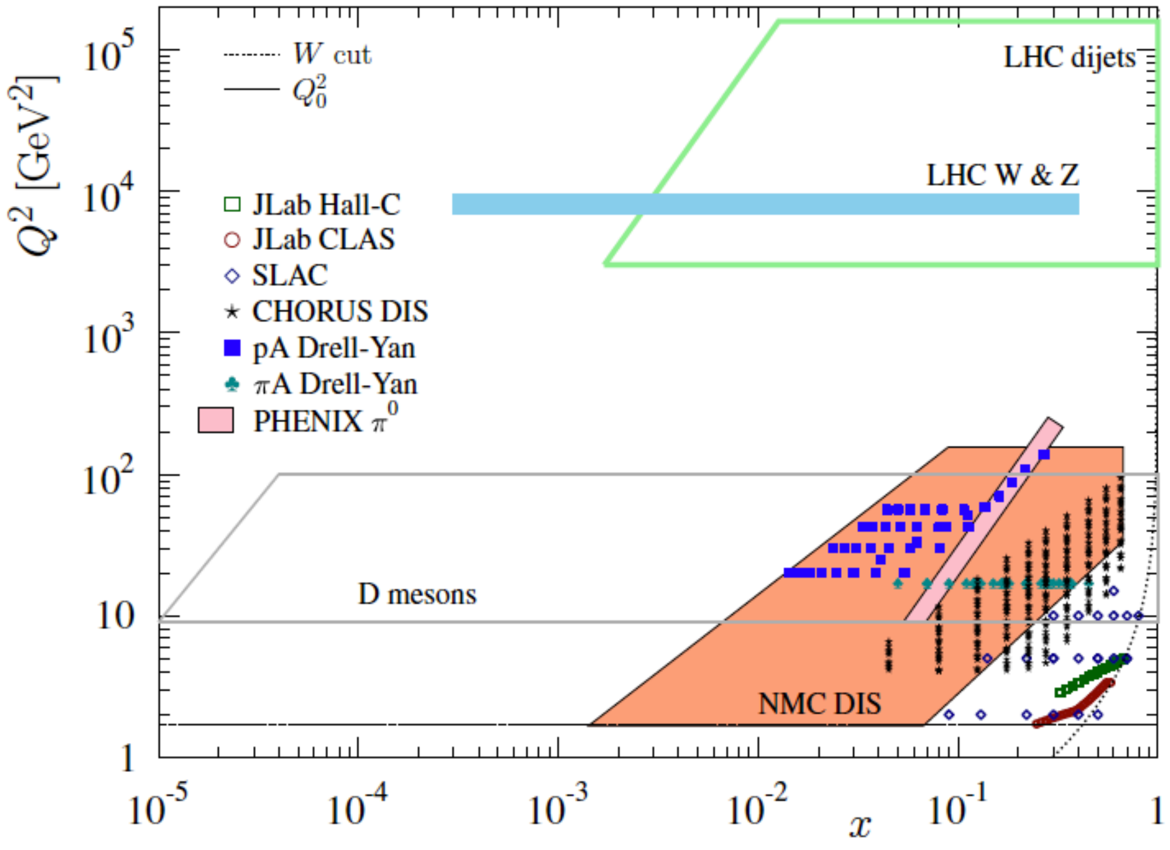
$\frac{\partial F_2(x,Q^2)}{\partial \log Q^2} \propto xg(x,Q^2)$: determines glue via DGLAP, $\mathcal{O}(\alpha_s)$: requires lever arm in Q^2.

$F_L(x,Q^2) \propto xg(x,Q^2) - F_2(x,Q^2)$: determines the glue via DGLAP, $\mathcal{O}(\alpha_s)$: requires lever arm in s (different y at fixed x,Q^2, use $\sigma_{\rm red}$).

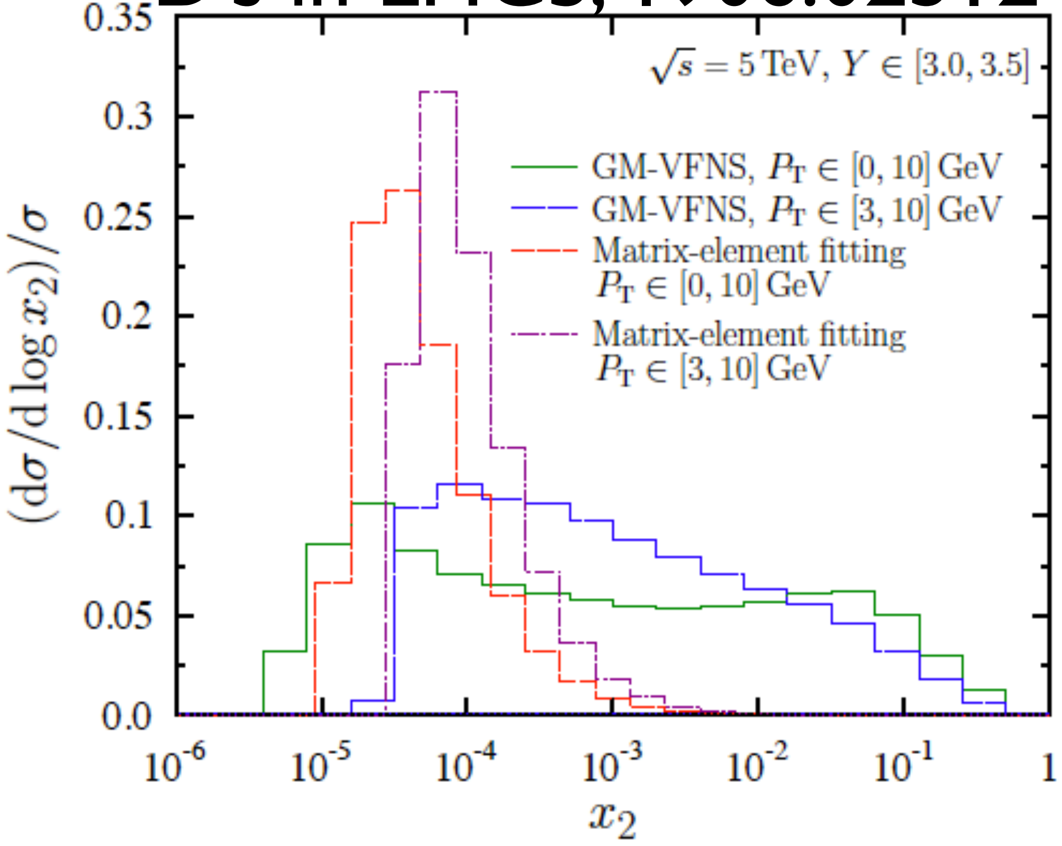
$F_2^{c,b,t}(x,Q^2)$: determines heavy flavour PDFs: requires HQ ID.

σ_r^{CC} : determines strange PDFs: requires HQ ID and measurement of missing energy.

Extraction of PDFs: hh



D's in LHCb, 1906.02512



- Kinematics at LO ($2 \to 1$, $2 \to 2, \ldots$) gives estimators:

$$x_{min,2\to 1} = x_T e^{\pm\eta}, \quad x_{min,2\to 2} = \frac{x_T e^{-\eta}}{2 - x_T e^{\eta}}, \quad x_T = \frac{2p_T}{\sqrt{s}}$$

- DY at large mass, W/Z sensitive to sea;
- DY at low mass, W/Z, γ, jets, heavy flavours sensitive to glue (either directly or through evolution).

Uncertainty estimation:

- Hessian method: first order expansion around minimum χ^2_0.

$$\chi^2 \approx \chi_0^2 + \sum_{ij} \delta a_i H_{ij} \delta a_j \quad H_{ij} \equiv \frac{1}{2} \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}\bigg|_{a=a^0} \quad \chi^2 \approx \chi_0^2 + \sum_i z_i^2$$

$$\Delta\chi^2 \equiv \sum_i \frac{\Delta\chi^2(z_i^+) + \Delta\chi^2(z_i^-)}{2N} \approx \sum_i \frac{(z_i^+)^2 + (z_i^-)^2}{2N}$$

$$\begin{aligned} S_0 &= (0,0,0,\ldots,0) \\ S_1^\pm &= \pm\delta z_1^\pm\,(1,0,0,\ldots,0) \\ S_2^\pm &= \pm\delta z_2^\pm\,(0,1,0,\ldots,0) \end{aligned}$$

$$(\Delta X)^2_{\text{extremum}} \approx \Delta\chi^2 \sum_j \left(\frac{\partial X}{\partial z_j}\right)^2$$

$$(\Delta X^+)^2 \approx \sum_k \left[\max\left\{X(S_k^+) - X(S^0), X(S_k^-) - X(S^0), 0\right\}\right]^2$$

$$(\Delta X^-)^2 \approx \sum_k \left[\max\left\{X(S^0) - X(S_k^+), X(S^0) - X(S_k^-), 0\right\}\right]^2$$

- MC method: repeated fits (NN) to many replicas of data.

- Any error analysis is linked to a functional form for the i.c. (NNPDF uses more flexibility, 4 times more paramaters, ~50 to ~200).

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Tolerance to reconcile data sets

$$(\Delta X)^2_{\text{extremum}} \approx \boxed{\Delta\chi^2} \sum_j \left(\frac{\partial X}{\partial z_j}\right)^2$$

$$(\Delta X^+)^2 \approx \sum_k \left[\max\left\{X(S_k^+) - X(S^0), X(S_k^-) - X(S^0), 0\right\}\right]^2$$

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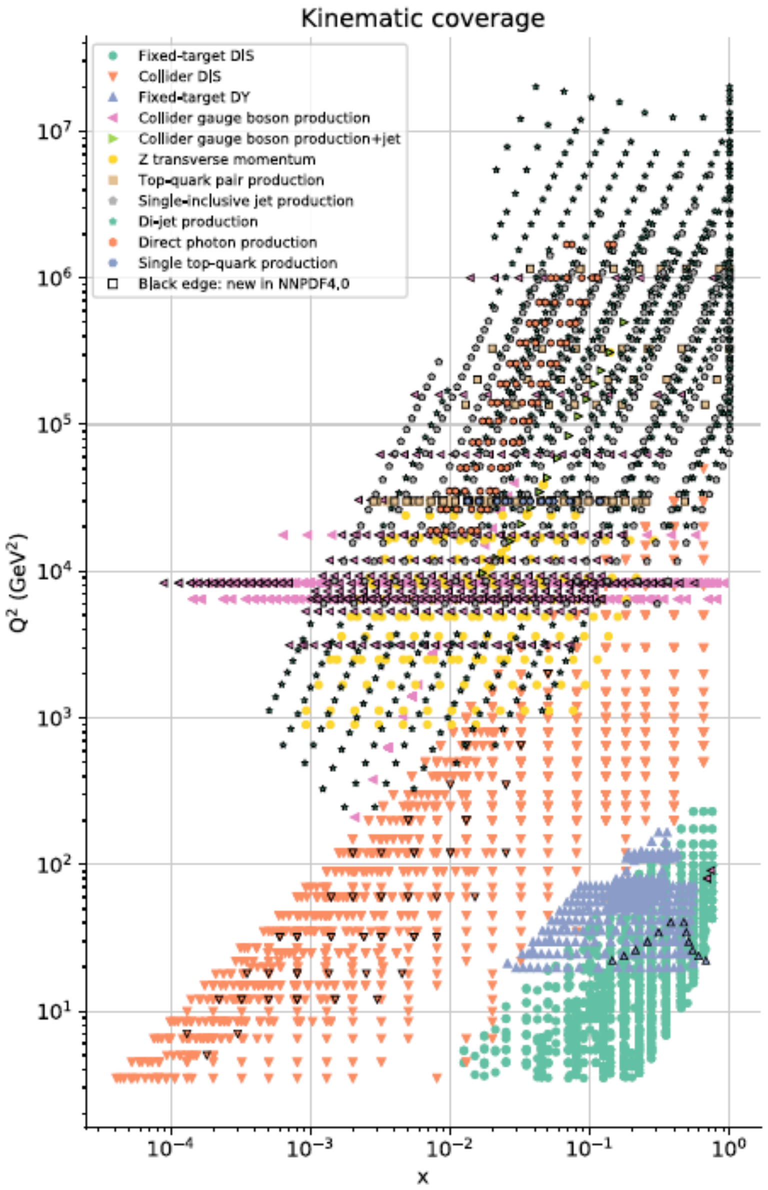
DIS: DGLAP global analysis

➜ Fit as many data as possible: DIS charged lepton and neutrino data, DY, jets, W/Z/γ,… ~4600, ~3100 from DIS (~1400 from A).

➜ Present accuracy: NNLO (aN^{3}LO) for evolution, NLO for all cross sections (NNLO jets start to be employed). Several groups: CT, MMHT, NNPDF, ABJM, HERAPDF,...



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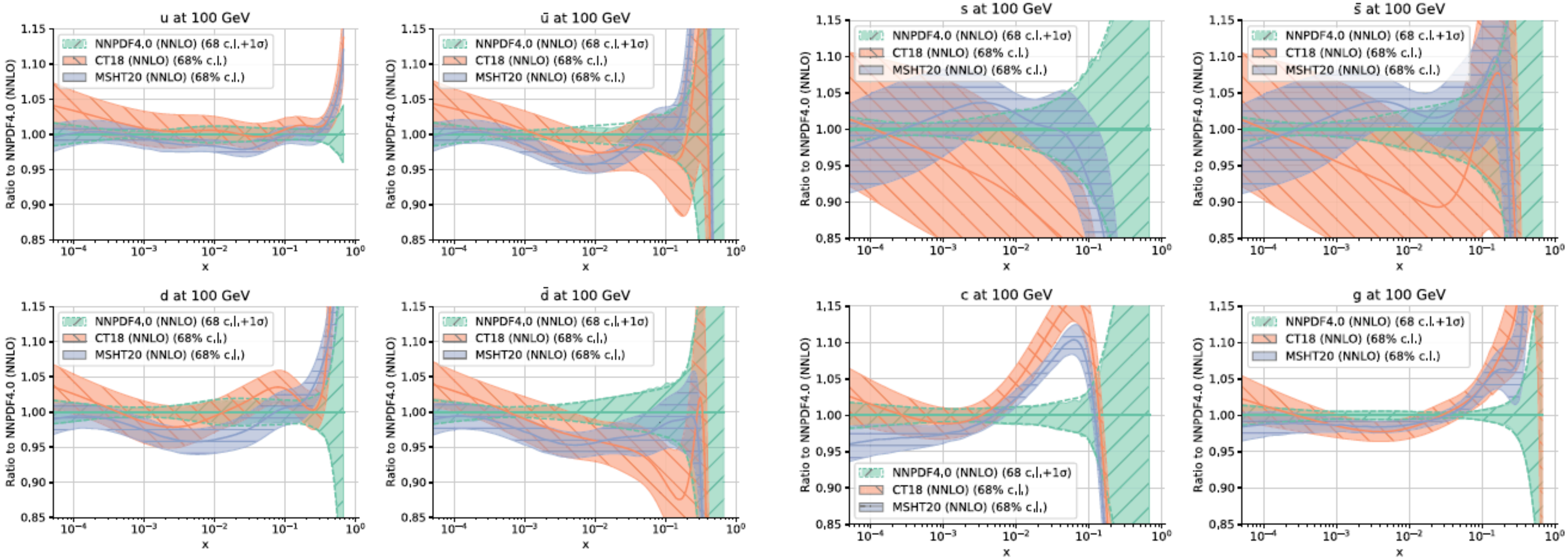

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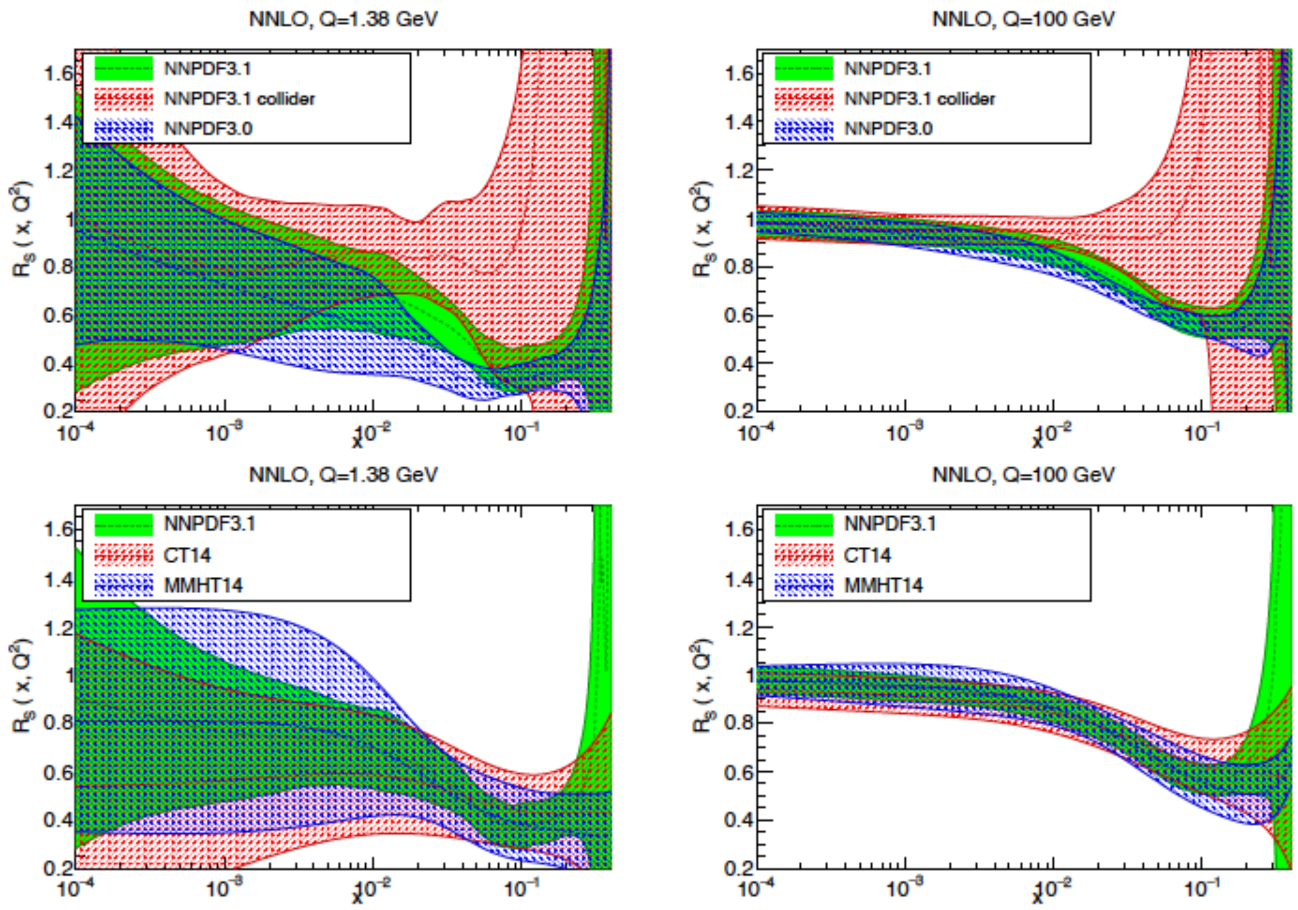
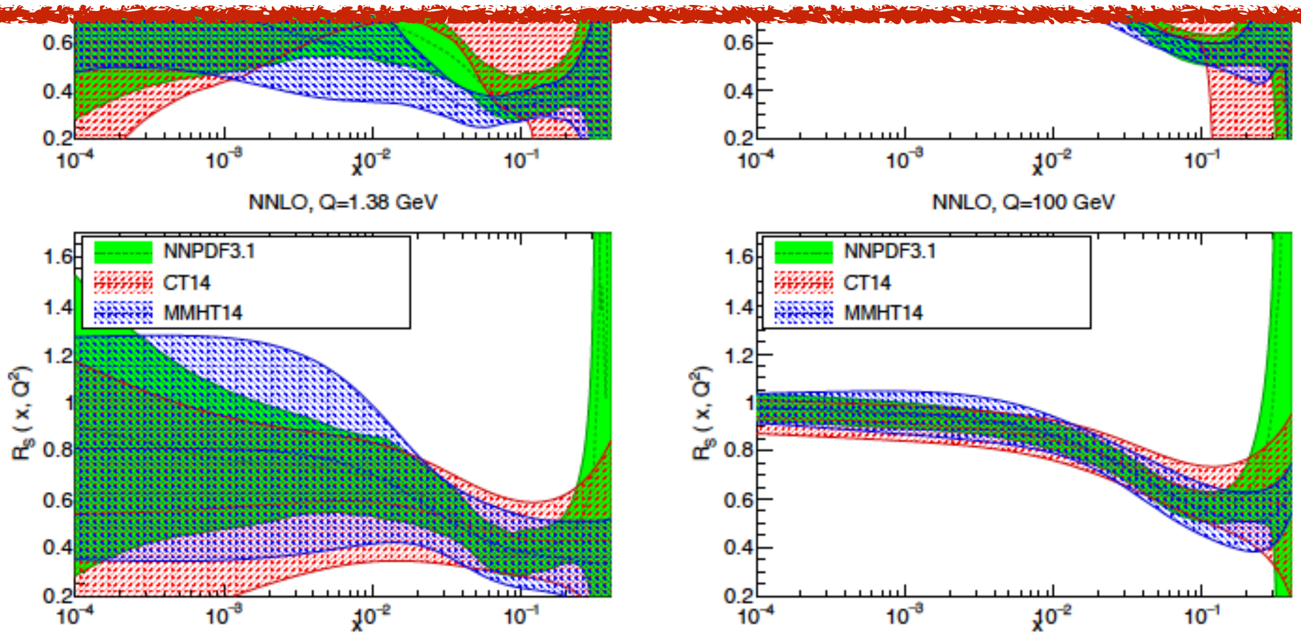
$$R_s(x,Q^2) = \frac{s(x,Q^2)+\bar{s}(x,Q^2)}{\bar{u}(x,Q^2)+\bar{d}(x,Q^2)}$$

PDF set	$R_s(0.023, 1.38\ \text{GeV})$	$R_s(0.023, M_Z)$
NNPDF3.0	0.45±0.09	0.71±0.04
NNPDF3.1	0.59±0.12	0.77±0.05
NNPDF3.1 collider-only	0.82±0.18	0.92±0.09
NNPDF3.1 HERA + ATLAS W, Z	1.03±0.38	1.05±0.240
`xFitter` HERA + ATLAS W, Z (Ref. [72])	$1.13\,^{+0.11}_{-0.11}$	-



DIS: DGLAP global analysis

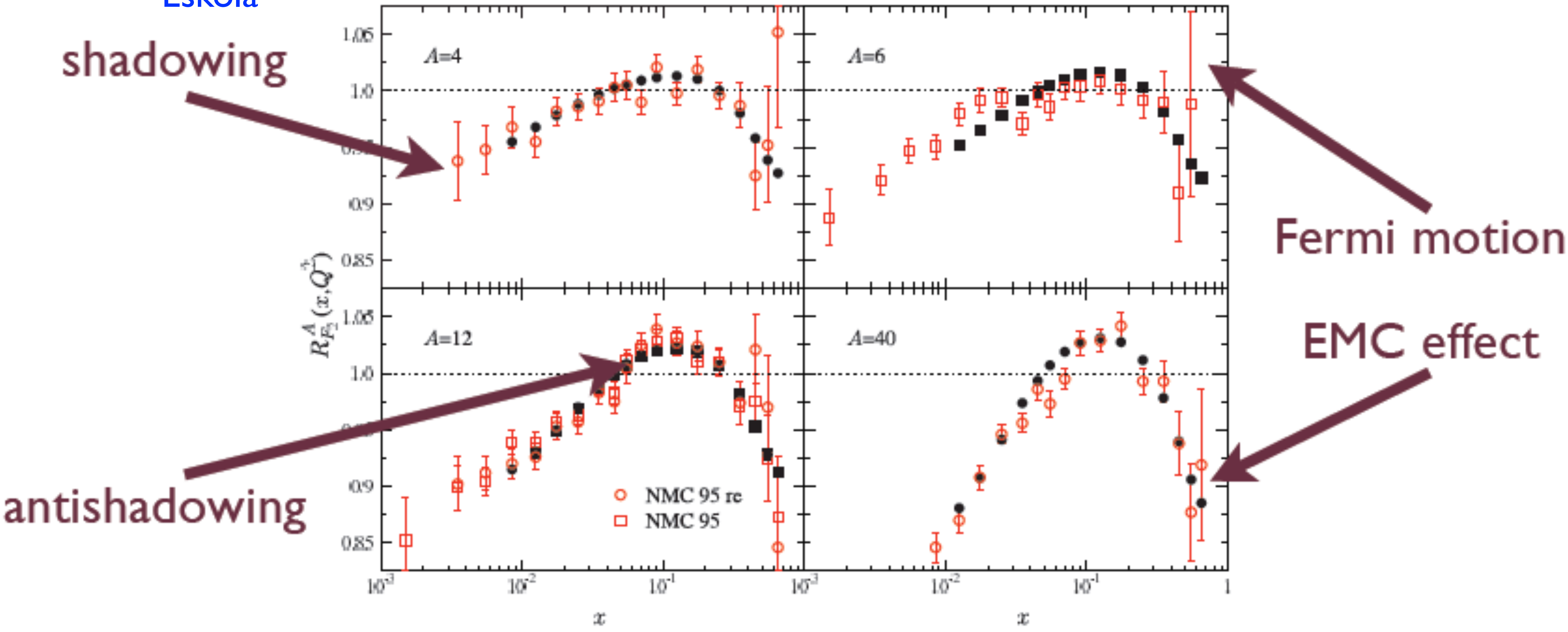
- Extremely sophisticated methods to address all types of uncertainties: statistical, systematics, theoretical,…
- Groups keep differences: combinations exist (PDF4LHC).
- Fits can be used to extract values of the strong coupling constant, heavy quark masses, even EW boson masses or $\sin\theta_W$.
- Analogous methods can be used to extract FFs, but in this case e^+e^- is the main source of precise data.
- Interplay between PDFs and new physics is currently a growing concern at the (HL-)LHC (usually addressed within SMEFT).



nPDFs:

$$R^A_{F_2}(x,Q^2) = \frac{F_2^A(x,Q^2)}{AF_2^p(x,Q^2)}$$

Eskola



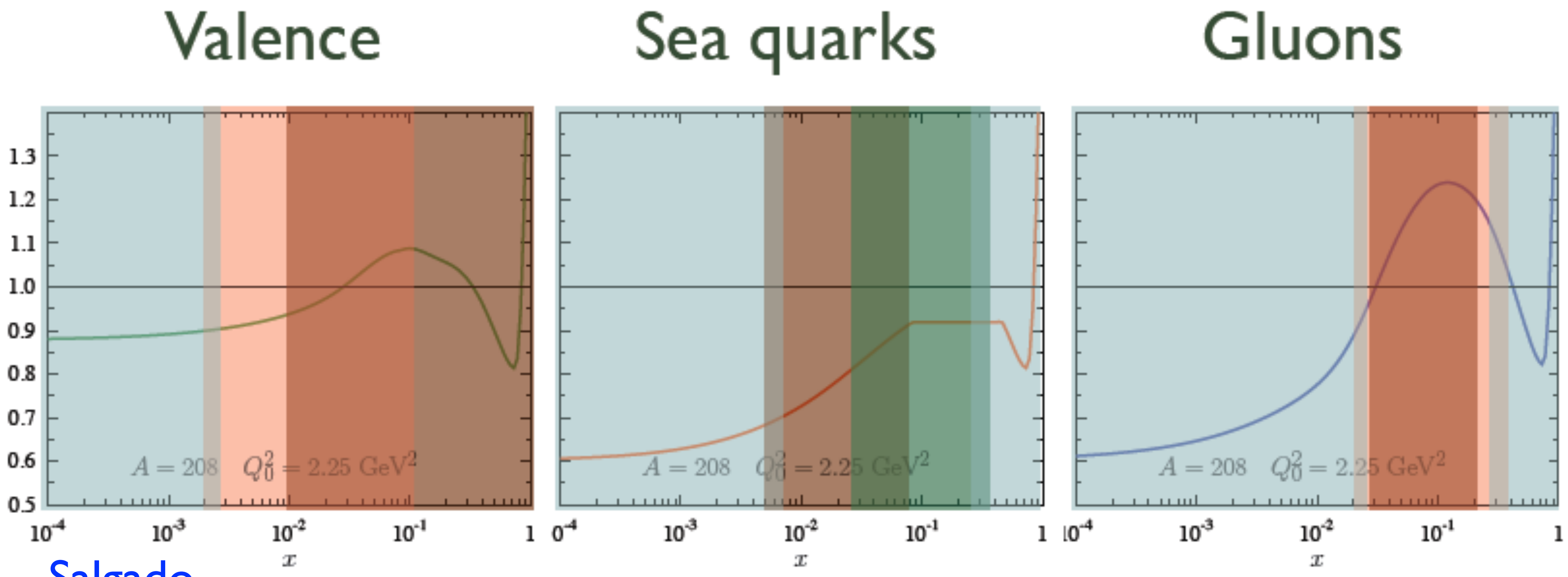
shadowing

Fermi motion

EMC effect

antishadowing

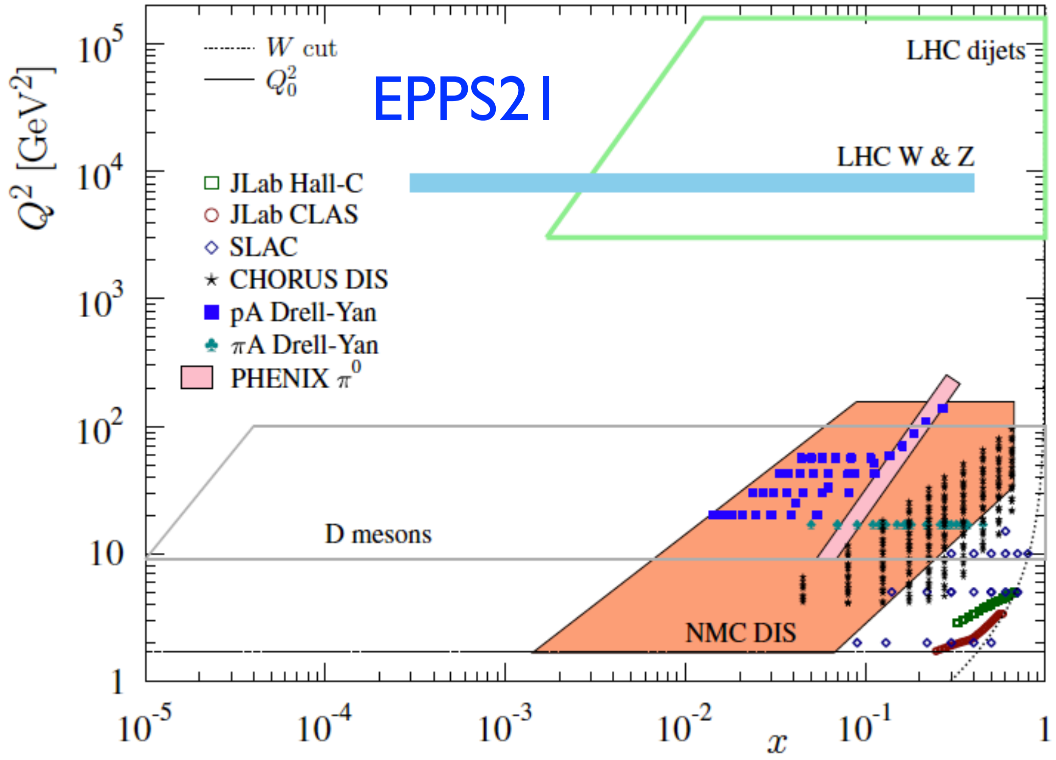
nPDFs:



Salgado

- 🟫 Constrained by DIS
- 🟩 Constrained by DY
- 🟧 Constrained by Sum rules
- 🟦 Assumptions

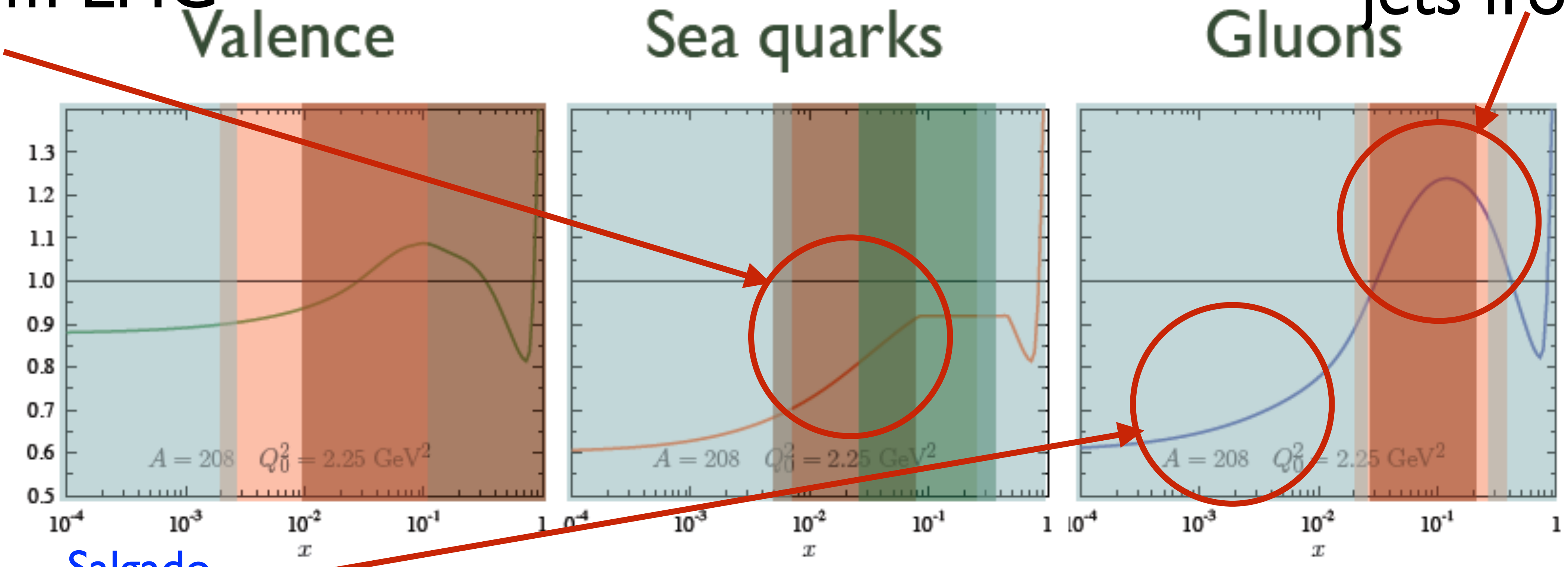
- Lack of experimental data makes the small-x region unconstrained ⇒ uncertainties on observables.



nPDFs:

W,Z from LHC

π from RHIC, jets from LHC



Salgado

D,B from LHC

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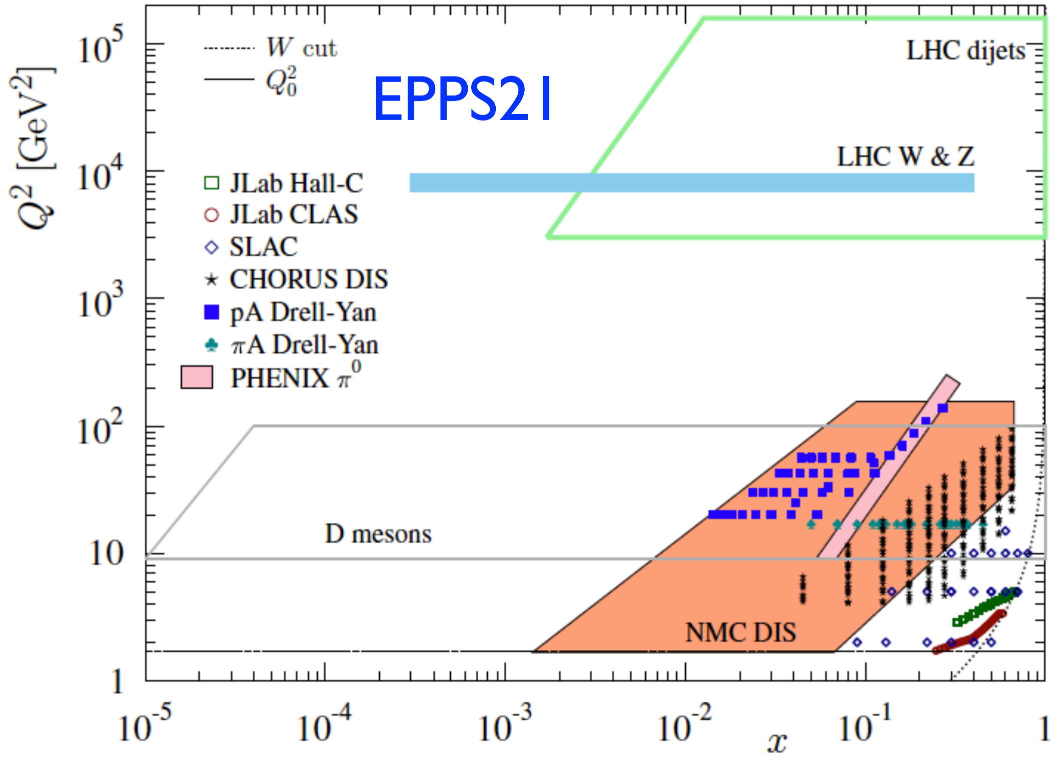
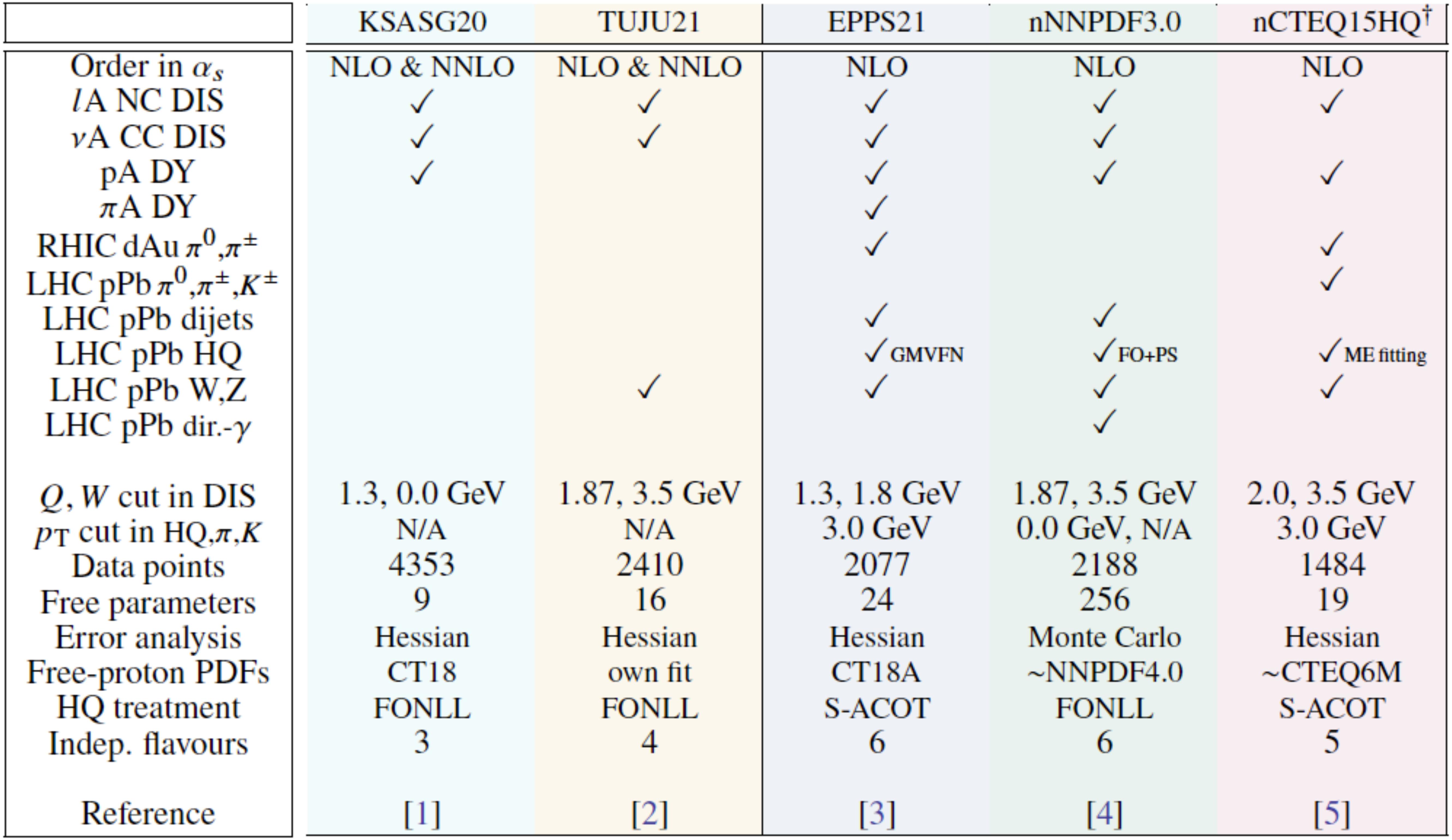


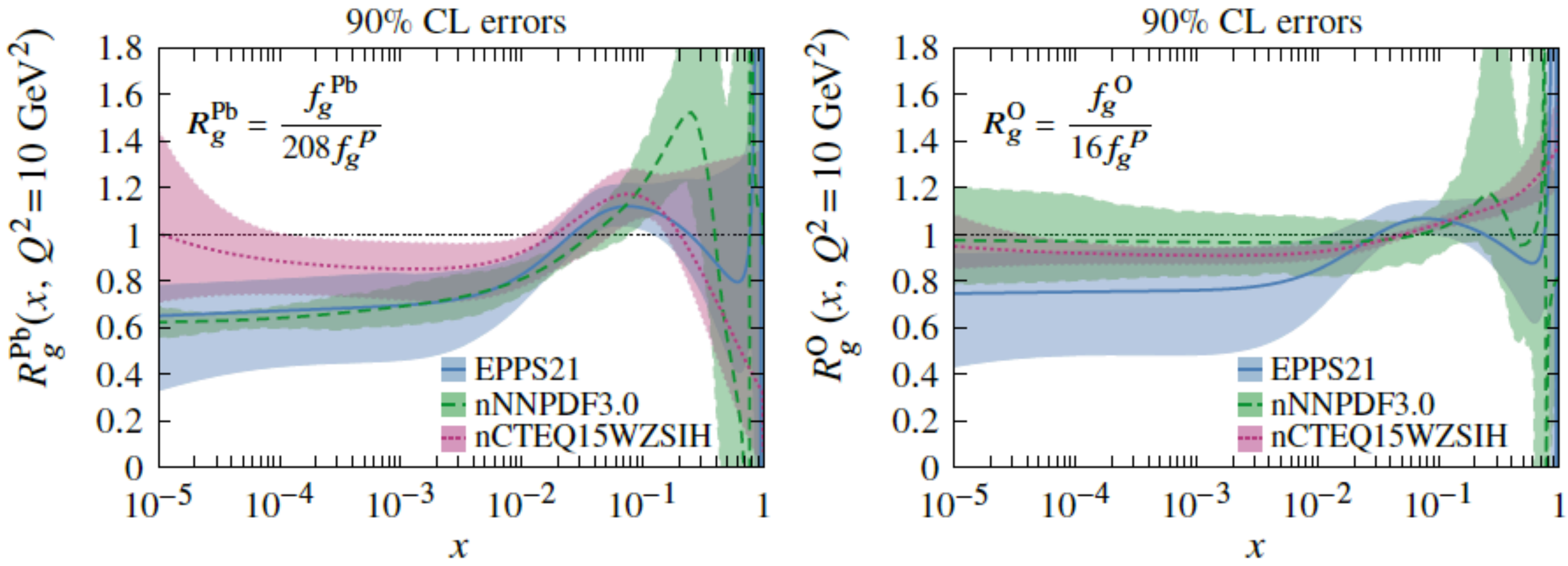
Available sets:



	KSASG20	TUJU21	EPPS21	nNNPDF3.0	nCTEQ15HQ†
Order in α_s	NLO & NNLO	NLO & NNLO	NLO	NLO	NLO
lA NC DIS	✓	✓	✓	✓	✓
νA CC DIS	✓	✓	✓	✓	
pA DY	✓		✓	✓	✓
πA DY			✓		
RHIC dAu π^0,$\pi^\pm$			✓		✓
LHC pPb π^0,$\pi^\pm$,$K^\pm$					✓
LHC pPb dijets			✓	✓	
LHC pPb HQ			✓ GMVFN	✓ FO+PS	✓ ME fitting
LHC pPb W,Z		✓	✓	✓	✓
LHC pPb dir.-γ				✓	
Q, W cut in DIS	1.3, 0.0 GeV	1.87, 3.5 GeV	1.3, 1.8 GeV	1.87, 3.5 GeV	2.0, 3.5 GeV
p_T cut in HQ,π,K	N/A	N/A	3.0 GeV	0.0 GeV, N/A	3.0 GeV
Data points	4353	2410	2077	2188	1484
Free parameters	9	16	24	256	19
Error analysis	Hessian	Hessian	Hessian	Monte Carlo	Hessian
Free-proton PDFs	CT18	own fit	CT18A	~NNPDF4.0	~CTEQ6M
HQ treatment	FONLL	FONLL	S-ACOT	FONLL	S-ACOT
Indep. flavours	3	4	6	6	5
Reference	[1]	[2]	[3]	[4]	[5]

P. Paakkinen, 2211.08906

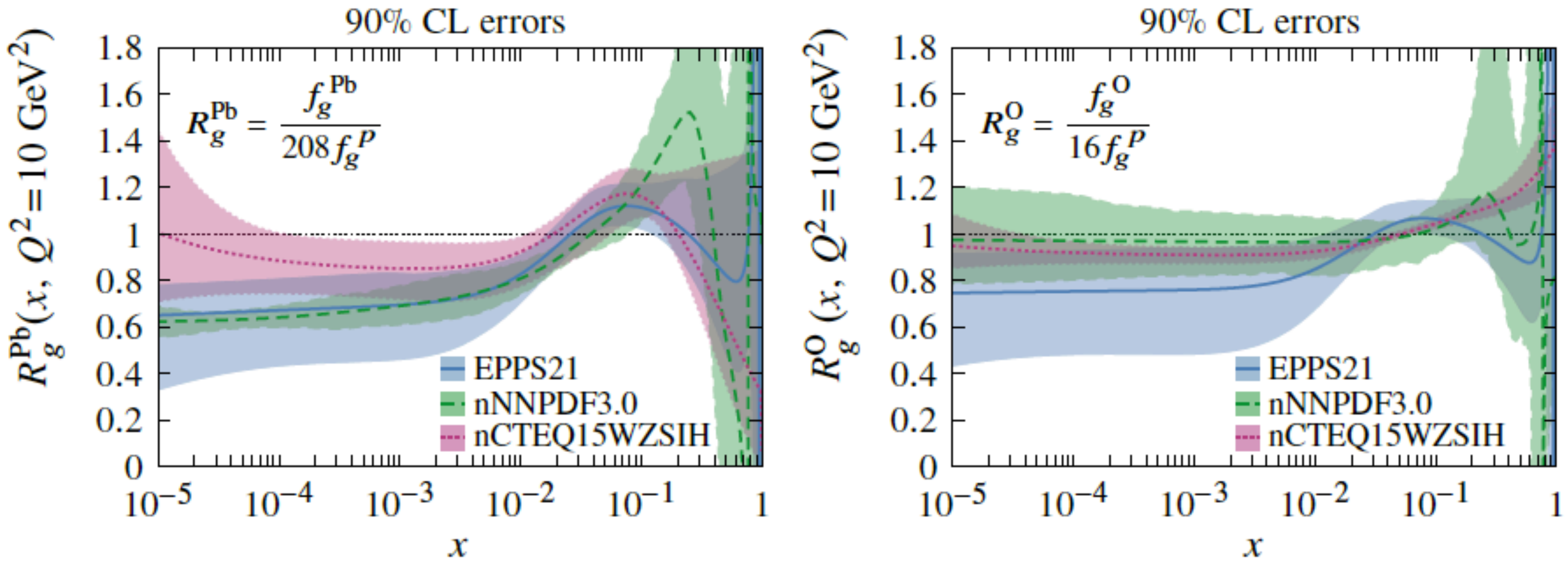
Available sets:



P. Paakkinen, 2211.08906

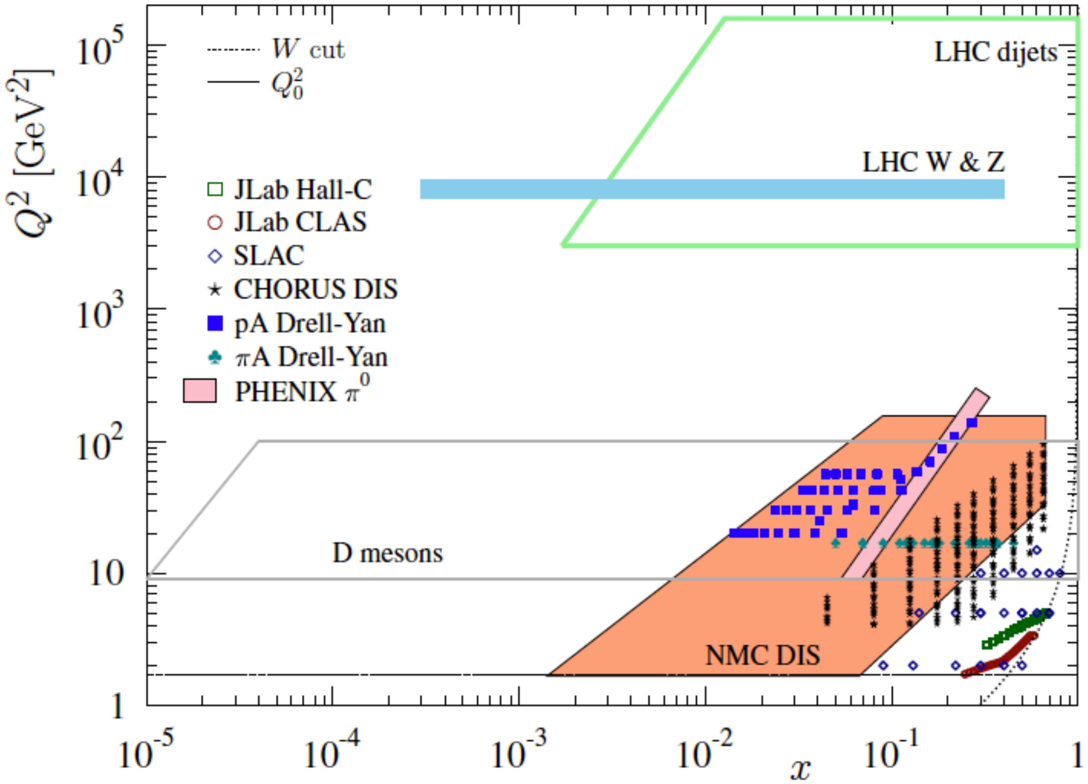
Available sets:

- Centrality dependence (EPS09s) not from data but from the A-dependence of the parameters.
- Several models provide it: Vogt et al., FGS, Ferreiro et al.,…



P. Paakkinen, 2211.08906

EPPS21:



- Most Pb data from CHORUS, 198 Pb points from pPb@LHC: fit for a single nucleus not possible.

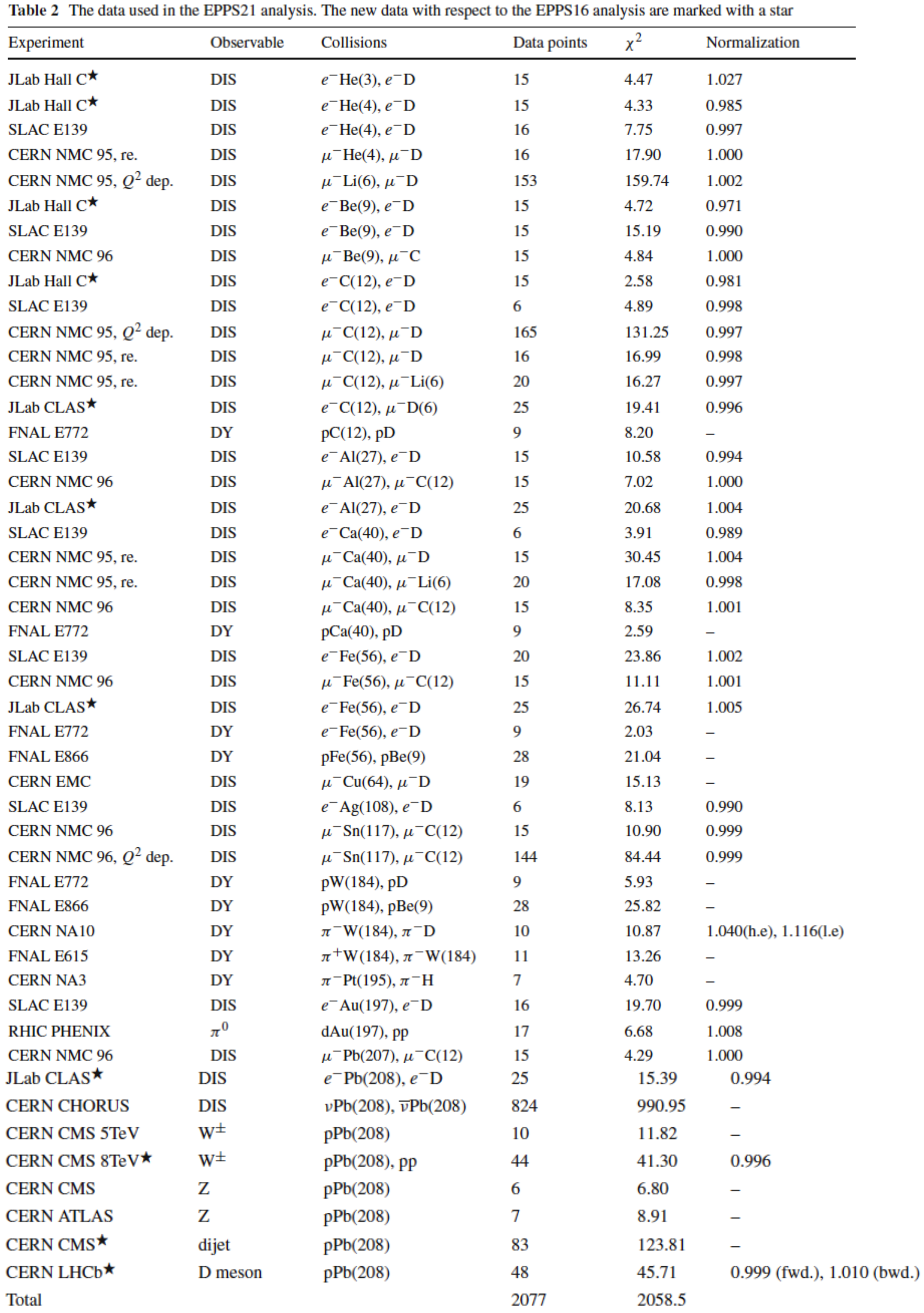


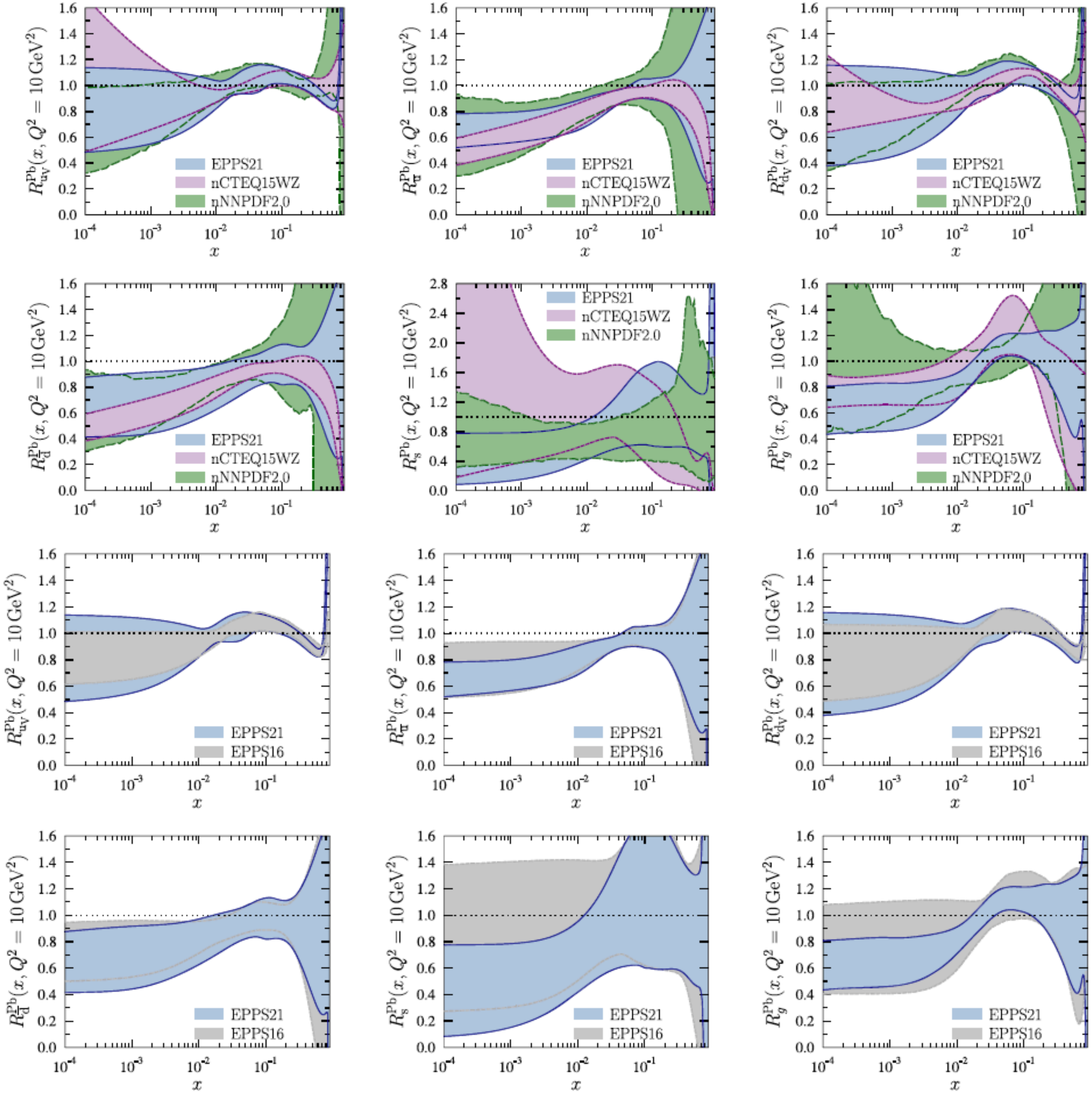
Table 2 The data used in the EPPS21 analysis. The new data with respect to the EPPS16 analysis are marked with a star

Experiment	Observable	Collisions	Data points	χ^2	Normalization
JLab Hall C★	DIS	e^-He(3), e^-D	15	4.47	1.027
JLab Hall C★	DIS	e^-He(4), e^-D	15	4.33	0.985
SLAC E139	DIS	e^-He(4), e^-D	16	7.75	0.997
CERN NMC 95, re.	DIS	μ^-He(4), μ^-D	16	17.90	1.000
CERN NMC 95, Q^2 dep.	DIS	μ^-Li(6), μ^-D	153	159.74	1.002
JLab Hall C★	DIS	e^-Be(9), e^-D	15	4.72	0.971
SLAC E139	DIS	e^-Be(9), e^-D	15	15.19	0.990
CERN NMC 96	DIS	μ^-Be(9), μ^-C	15	4.84	1.000
JLab Hall C★	DIS	e^-C(12), e^-D	15	2.58	0.981
SLAC E139	DIS	e^-C(12), e^-D	6	4.89	0.998
CERN NMC 95, Q^2 dep.	DIS	μ^-C(12), μ^-D	165	131.25	0.997
CERN NMC 95, re.	DIS	μ^-C(12), μ^-D	16	16.99	0.998
CERN NMC 95, re.	DIS	μ^-C(12), μ^-Li(6)	20	16.27	0.997
JLab CLAS★	DIS	e^-C(12), μ^-D(6)	25	19.41	0.996
FNAL E772	DY	pC(12), pD	9	8.20	–
SLAC E139	DIS	e^-Al(27), e^-D	15	10.58	0.994
CERN NMC 96	DIS	μ^-Al(27), μ^-C(12)	15	7.02	1.000
JLab CLAS★	DIS	e^-Al(27), e^-D	25	20.68	1.004
SLAC E139	DIS	e^-Ca(40), e^-D	6	3.91	0.989
CERN NMC 95, re.	DIS	μ^-Ca(40), μ^-D	15	30.45	1.004
CERN NMC 95, re.	DIS	μ^-Ca(40), μ^-Li(6)	20	17.08	0.998
CERN NMC 96	DIS	μ^-Ca(40), μ^-C(12)	15	8.35	1.001
FNAL E772	DY	pCa(40), pD	9	2.59	–
SLAC E139	DIS	e^-Fe(56), e^-D	20	23.86	1.002
CERN NMC 96	DIS	μ^-Fe(56), μ^-C(12)	15	11.11	1.001
JLab CLAS★	DIS	e^-Fe(56), e^-D	25	26.74	1.005
FNAL E772	DY	e^-Fe(56), e^-D	9	2.03	–
FNAL E866	DY	pFe(56), pBe(9)	28	21.04	–
CERN EMC	DIS	μ^-Cu(64), μ^-D	19	15.13	–
SLAC E139	DIS	e^-Ag(108), e^-D	6	8.13	0.990
CERN NMC 96	DIS	μ^-Sn(117), μ^-C(12)	15	10.90	0.999
CERN NMC 96, Q^2 dep.	DIS	μ^-Sn(117), μ^-C(12)	144	84.44	0.999
FNAL E772	DY	pW(184), pD	9	5.93	–
FNAL E866	DY	pW(184), pBe(9)	28	25.82	–
CERN NA10	DY	π^-W(184), π^-D	10	10.87	1.040(h.e), 1.116(l.e)
FNAL E615	DY	π^+W(184), π^-W(184)	11	13.26	–
CERN NA3	DY	π^-Pt(195), π^-H	7	4.70	–
SLAC E139	DIS	e^-Au(197), e^-D	16	19.70	0.999
RHIC PHENIX	π^0	dAu(197), pp	17	6.68	1.008
CERN NMC 96	DIS	μ^-Pb(207), μ^-C(12)	15	4.29	1.000
JLab CLAS★	DIS	e^-Pb(208), e^-D	25	15.39	0.994
CERN CHORUS	DIS	νPb(208), $\overline{\nu}$Pb(208)	824	990.95	–
CERN CMS 5TeV	W$^\pm$	pPb(208)	10	11.82	–
CERN CMS 8TeV★	W$^\pm$	pPb(208), pp	44	41.30	0.996
CERN CMS	Z	pPb(208)	6	6.80	–
CERN ATLAS	Z	pPb(208)	7	8.91	–
CERN CMS★	dijet	pPb(208)	83	123.81	–
CERN LHCb★	D meson	pPb(208)	48	45.71	0.999 (fwd.), 1.010 (bwd.)
Total			2077	2058.5	

EPPS21:

Q^2=10 GeV^2

- Note the parametrisation bias.
- Presently available LHC data start to largely influence fits.
- Influence of proton uncertainties.
- u/d decomposition challeging.



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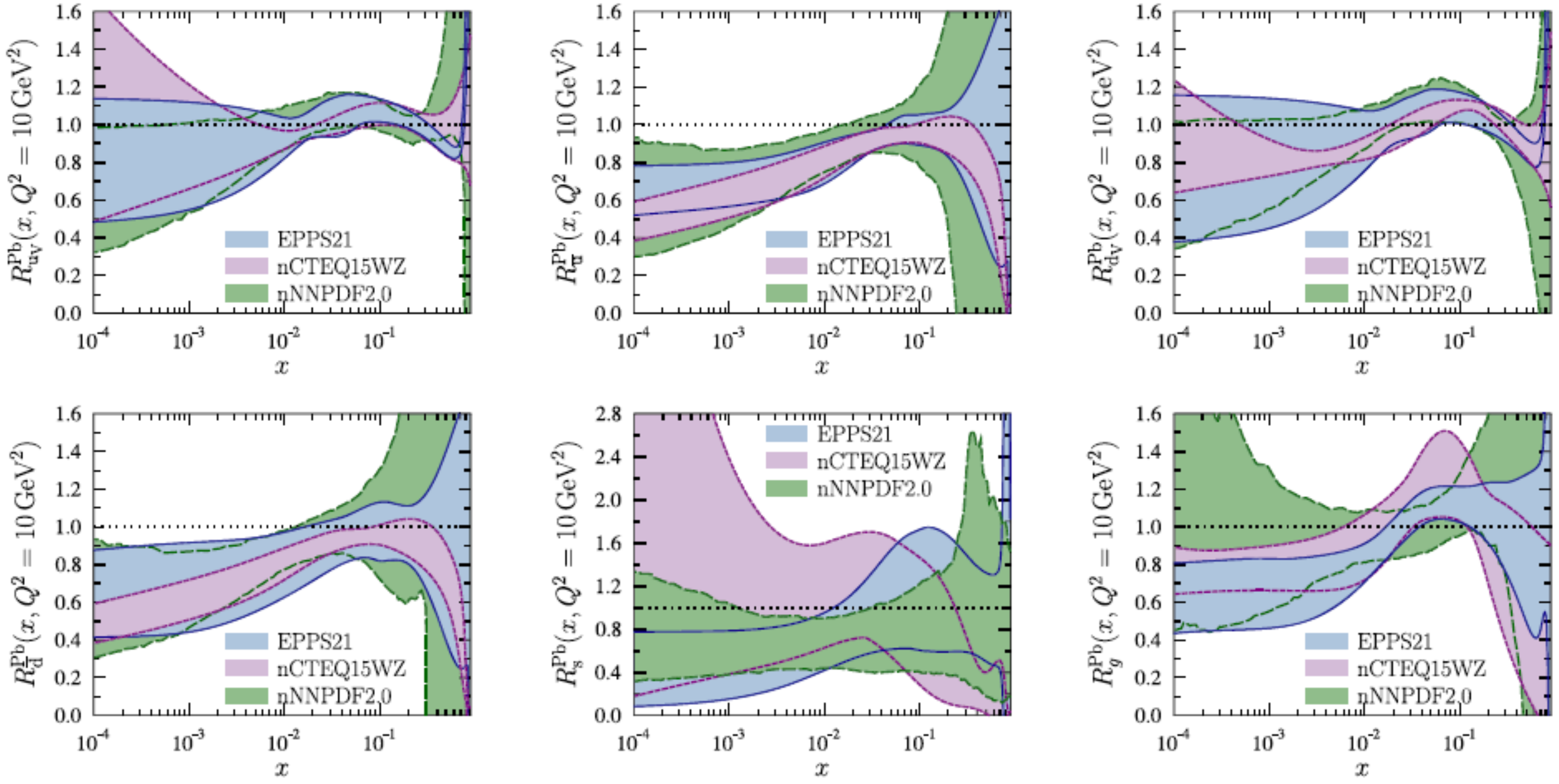
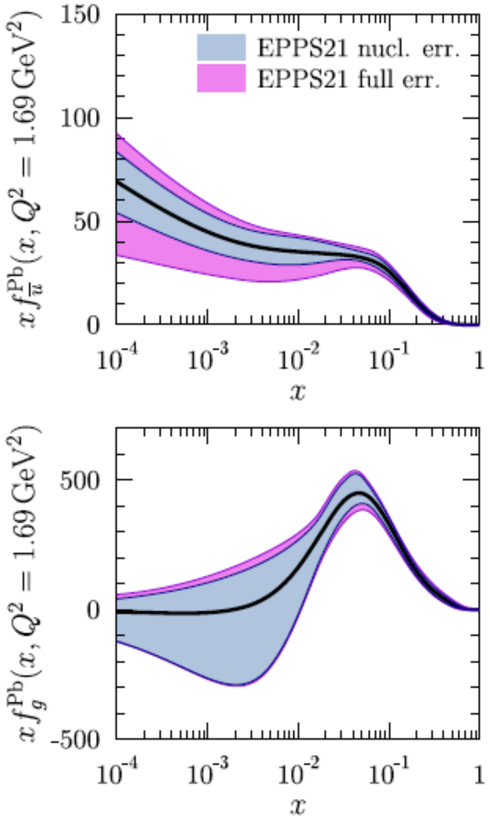
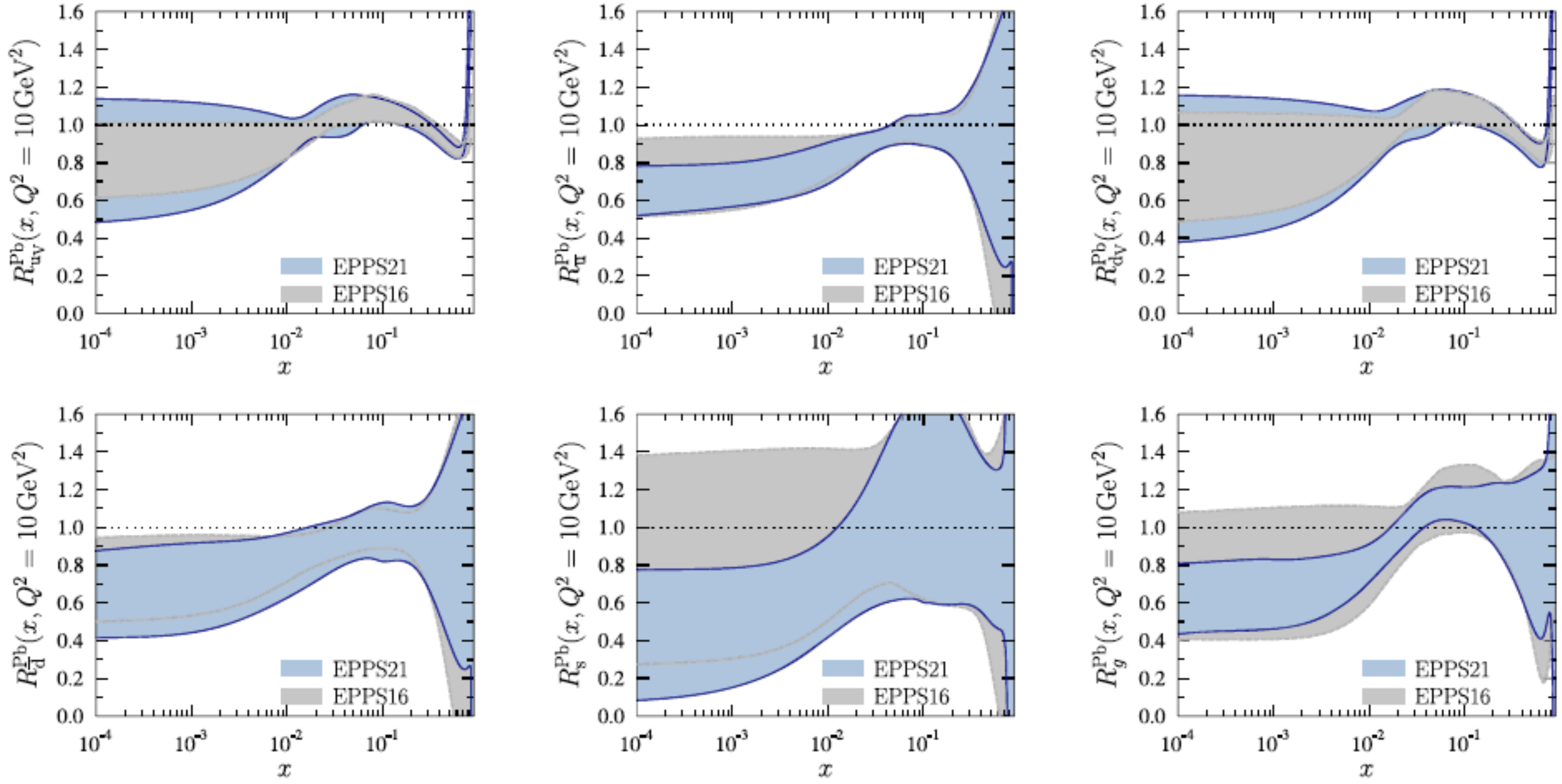
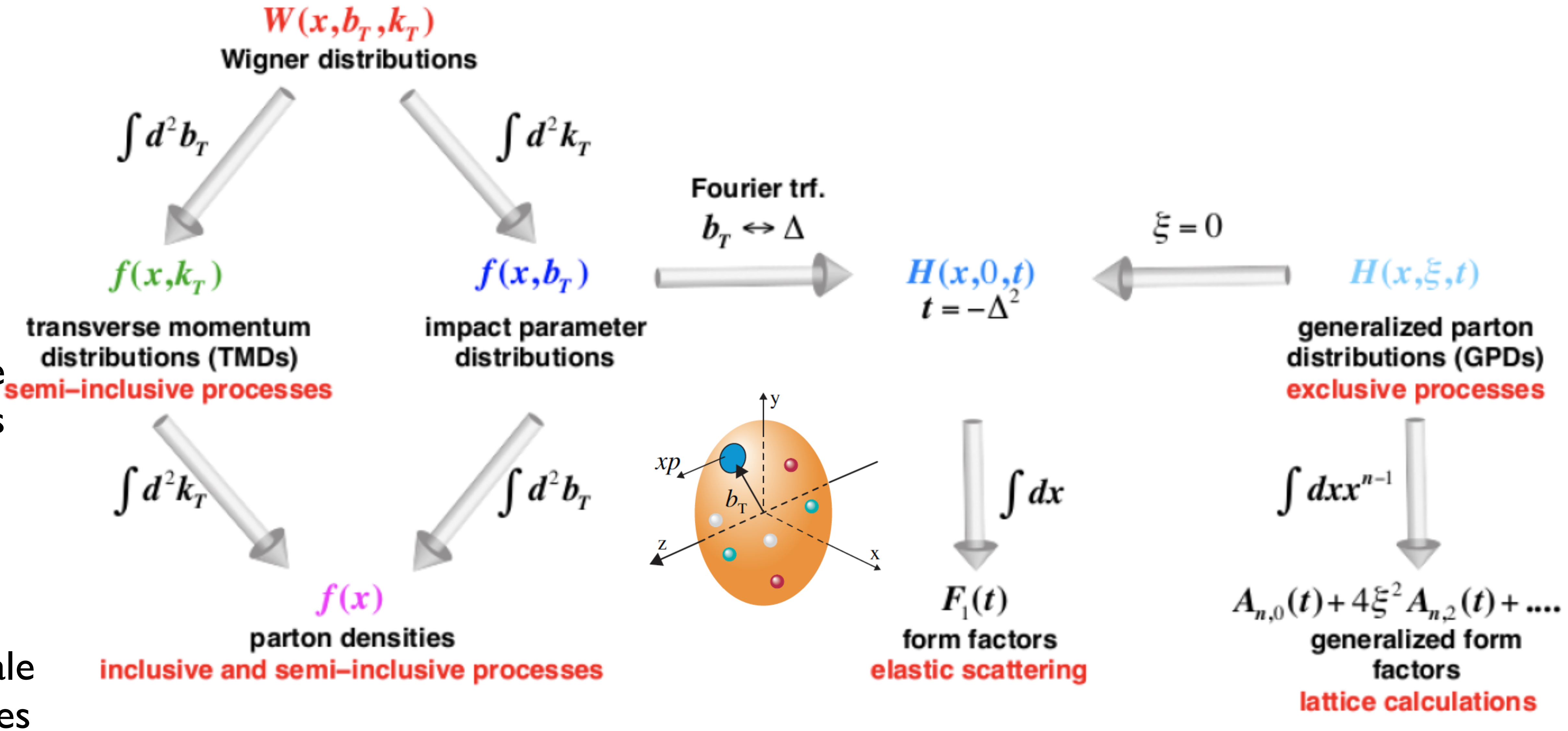






Hadron structure beyond collinear:

- Collinear distributions provide limited information on hadrons.
- More complete information requires new distributions and factorisations and evolution equations: TMD, GPD,…



$W(x,b_T,k_T)$
Wigner distributions

$\int d^2 b_T$

$\int d^2 k_T$

$f(x,k_T)$
transverse momentum distributions (TMDs)
semi–inclusive processes

$f(x,b_T)$
impact parameter distributions

Fourier trf.
$b_T \leftrightarrow \Delta$

$H(x,0,t)$
$t=-\Delta^2$

$\xi=0$

$H(x,\xi,t)$
generalized parton distributions (GPDs)
exclusive processes

$\int d^2 k_T$

$\int d^2 b_T$

$\int dx$

$\int dx x^{n-1}$

$f(x)$
parton densities
inclusive and semi–inclusive processes

$F_1(t)$
form factors
elastic scattering

$A_{n,0}(t)+4\xi^2 A_{n,2}(t)+....$
generalized form factors
lattice calculations

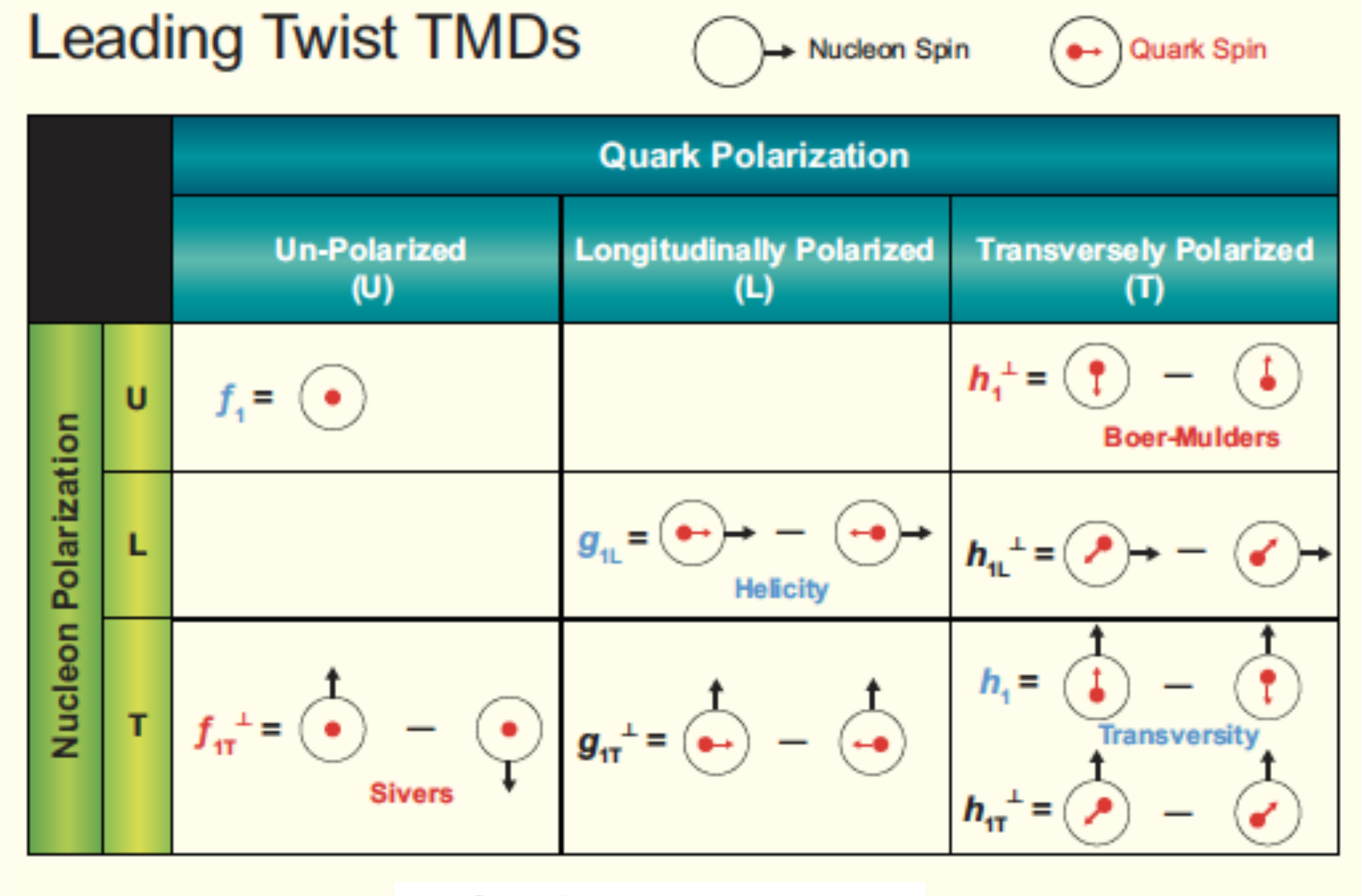
Two-scale processes

One-scale processes

- See e.g. J. Collins, *Foundations of Perturbative QCD*, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 32 (2011) 1-624; R. Boussarie et al., *TMD Handbook*, 2304.03302; M. Diehl, Phys. Rept. 388 (2003) 41-277.

Hadron structure:

- Several TMDs (both PDF and FF) to be determined.
- Beyond inclusive DIS, further possibilities are SIDIS (FFs required), CC, polarised proton collisions,…
- TMD factorisation can also be tested in non-polarised collisions: dijets, charm,… Two scales required!



Leading Twist TMDs

Nucleon Spin

Quark Spin

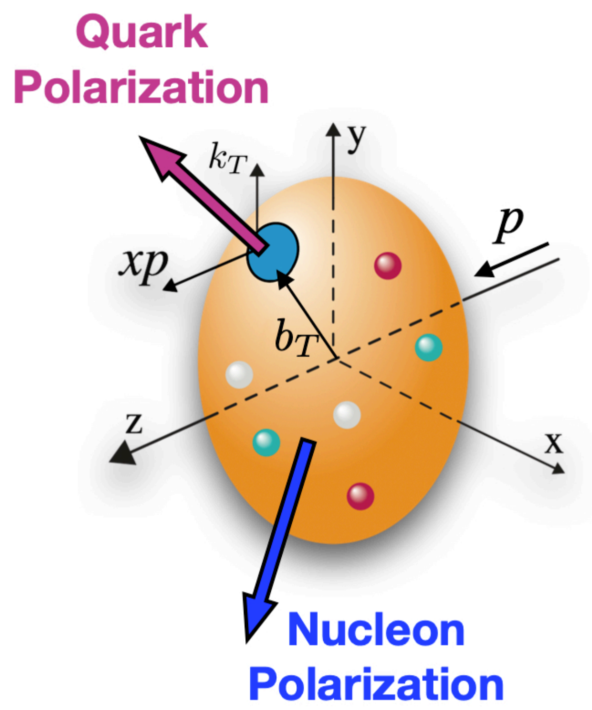
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 =		$h_1^\perp$ = − Boer-Mulders
	L		g_{1L} = − Helicity	$h_{1L}^\perp$ = −
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_1 = − Transversity $h_{1T}^\perp$ = −

$$f_i(x,\mu) = \int \frac{dy^-}{4\pi} \mathrm{e}^{-ixp^+y^-} \langle p|\, \bar{\psi}_i(0,y^-,\mathbf{0}_T) W[y,0] \gamma^+ \psi_i(0)\, |p\rangle_R$$

$$P_i(x,\boldsymbol{k}_T,\zeta,\mu) = \int \frac{dy^-\, d^2\boldsymbol{y}_T}{16\pi^3} \mathrm{e}^{-ixp^+y^- + i\boldsymbol{k}_T\cdot\boldsymbol{y}_T}\, \tilde{P}_i(y^-,\boldsymbol{y}_T,\zeta,\mu)$$

$$\tilde{P}_i(y^-,\boldsymbol{y}_T,\zeta,\mu) = \langle p|\, \bar{\psi}_i(0,y^-,\boldsymbol{y}_T)\, W_y(u)^\dagger\, I_{u;y,0}\, \gamma^+\, W_0(u)\, \psi_i(0)\, |p\rangle_R$$

$$W_y(u) = P \exp\left[-ig_{(0)} \int_0^\infty d\lambda\, u^\mu\, A_\mu^{(0)}(y+\lambda u)\right]$$



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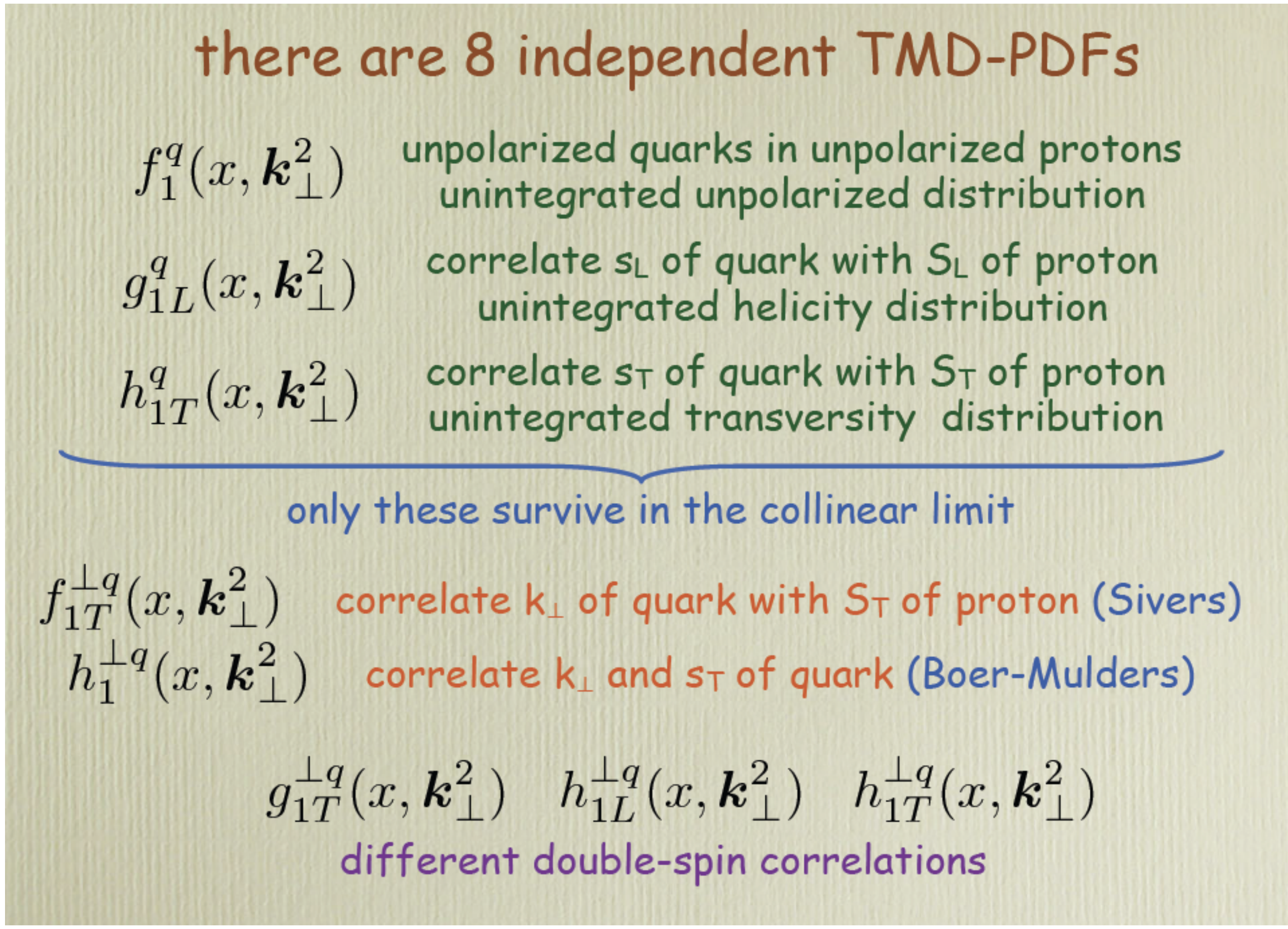


there are 8 independent TMD-PDFs

$f_1^q(x,\boldsymbol{k}_\perp^2)$	unpolarized quarks in unpolarized protons unintegrated unpolarized distribution
$g_{1L}^q(x,\boldsymbol{k}_\perp^2)$	correlate s_L of quark with S_L of proton unintegrated helicity distribution
$h_{1T}^q(x,\boldsymbol{k}_\perp^2)$	correlate s_T of quark with S_T of proton unintegrated transversity distribution

only these survive in the collinear limit

$f_{1T}^{\perp q}(x,\boldsymbol{k}_\perp^2)$	correlate $k_\perp$ of quark with S_T of proton (Sivers)
$h_1^{\perp q}(x,\boldsymbol{k}_\perp^2)$	correlate $k_\perp$ and s_T of quark (Boer-Mulders)

$g_{1T}^{\perp q}(x,\boldsymbol{k}_\perp^2)$ $h_{1L}^{\perp q}(x,\boldsymbol{k}_\perp^2)$ $h_{1T}^{\perp q}(x,\boldsymbol{k}_\perp^2)$

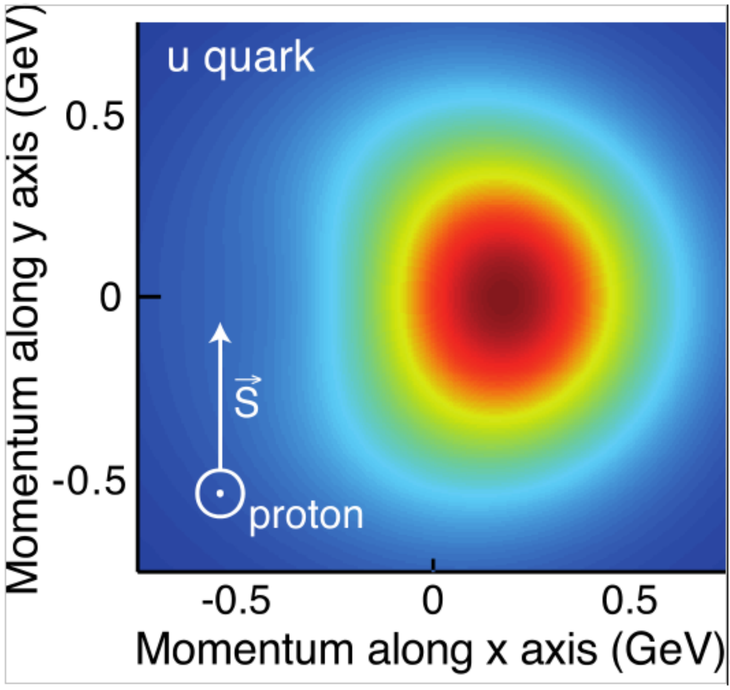
different double-spin correlations

$$f_i(x,\mu) = \int \frac{dy^-}{4\pi} \mathrm{e}^{-ixp^+y^-} \langle p|\,\bar\psi_i(0,y^-,\boldsymbol{0}_T) W[y,0]\gamma^+\psi_i(0)\,|p\rangle_R$$

$$P_i(x,\boldsymbol{k}_T,\zeta,\mu) = \int \frac{dy^-\,d^2\boldsymbol{y}_T}{16\pi^3} \mathrm{e}^{-ixp^+y^- + i\boldsymbol{k}_T\cdot\boldsymbol{y}_T}\,\tilde P_i(y^-,\boldsymbol{y}_T,\zeta,\mu)$$

$$\tilde P_i(y^-,\boldsymbol{y}_T,\zeta,\mu) = \langle p|\,\bar\psi_i(0,y^-,\boldsymbol{y}_T)\,W_y(u)^\dagger\,I_{u;y,0}\,\gamma^+\,W_0(u)\,\psi_i(0)\,|p\rangle_R$$

$$W_y(u) = P\exp\left[-ig_{(0)}\int_0^\infty d\lambda\,u^\mu\,A_\mu^{(0)}(y+\lambda u)\right]$$

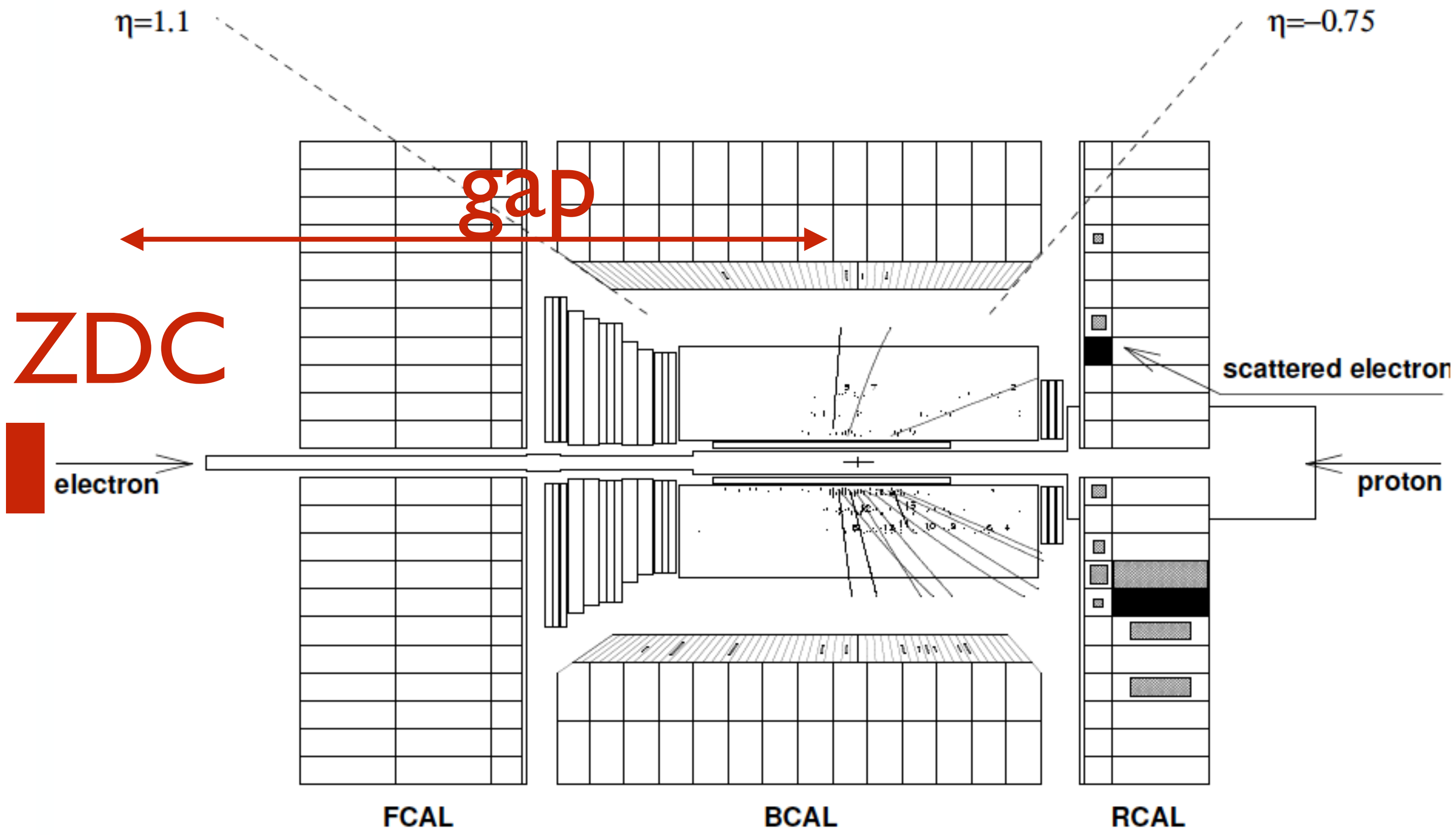


Diffraction:

- At HERA, ~10 % of the events have a pseudorapidity gap in hadronic activity (or intact detected proton): **diffractive**.

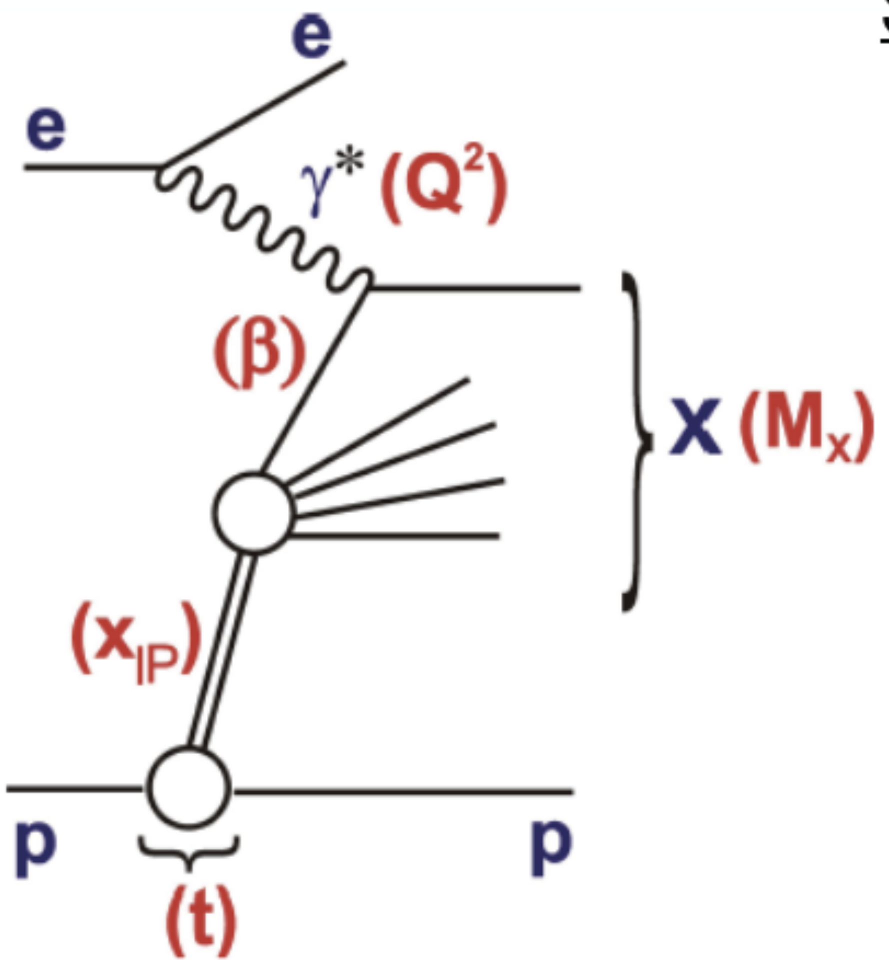
- They measure the probability of the proton to remain intact in the scattering, while producing some activity far from the proton: exchange of colourless object(s), called *Pomeron* and *Reggeon*.

LPS, ZDC



Diffractive event in ZEUS at HERA

Diffraction:



$$x_{Bj} = x_{IP}\beta$$

Standard DIS variables:

electron-proton
cms energy squared:

$$s = (k+p)^2$$

photon-proton
cms energy squared:

$$W^2 = (q+p)^2$$

inelasticity

$$y = \frac{p \cdot q}{p \cdot k}$$

Bjorken x

$$x = \frac{-q^2}{2p \cdot q}$$

(minus) photon virtuality

$$Q^2 = -q^2$$

Diffractive DIS variables:

$$\xi \equiv x_{IP} = \frac{Q^2 + M_X^2 - t}{Q^2 + W^2}$$

momentum fraction of
the Pomeron w.r.t hadron

$$\beta = \frac{Q^2}{Q^2 + M_X^2 - t}$$

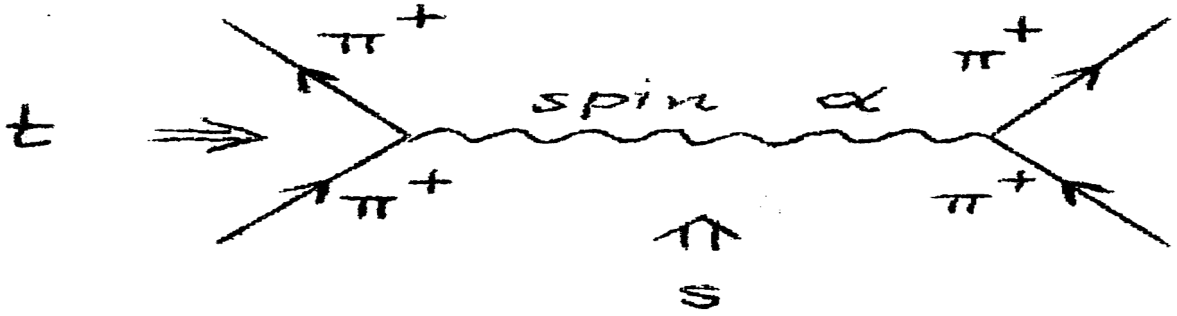
momentum fraction of
parton w.r.t Pomeron

$$t = (p - p')^2$$

4-momentum transfer squared

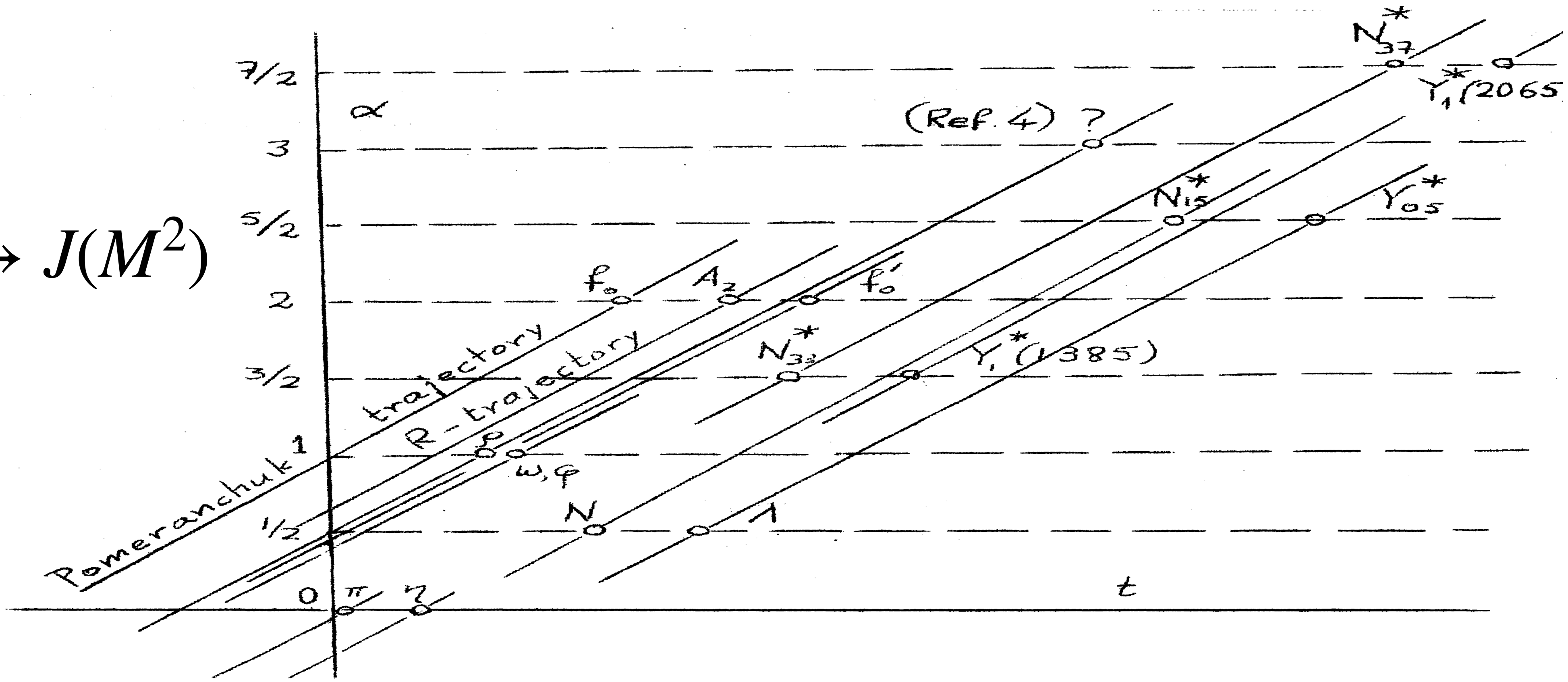
Regge poles:

- Regge theory: pre-QCD theory for the strong interaction, they tried to derive the theory of strong interaction from first principles of QFT: unitarity, analyticity, crossing symmetry, short range.
- Justifying it from QCD has been a major endeavour of the field: BFKL.

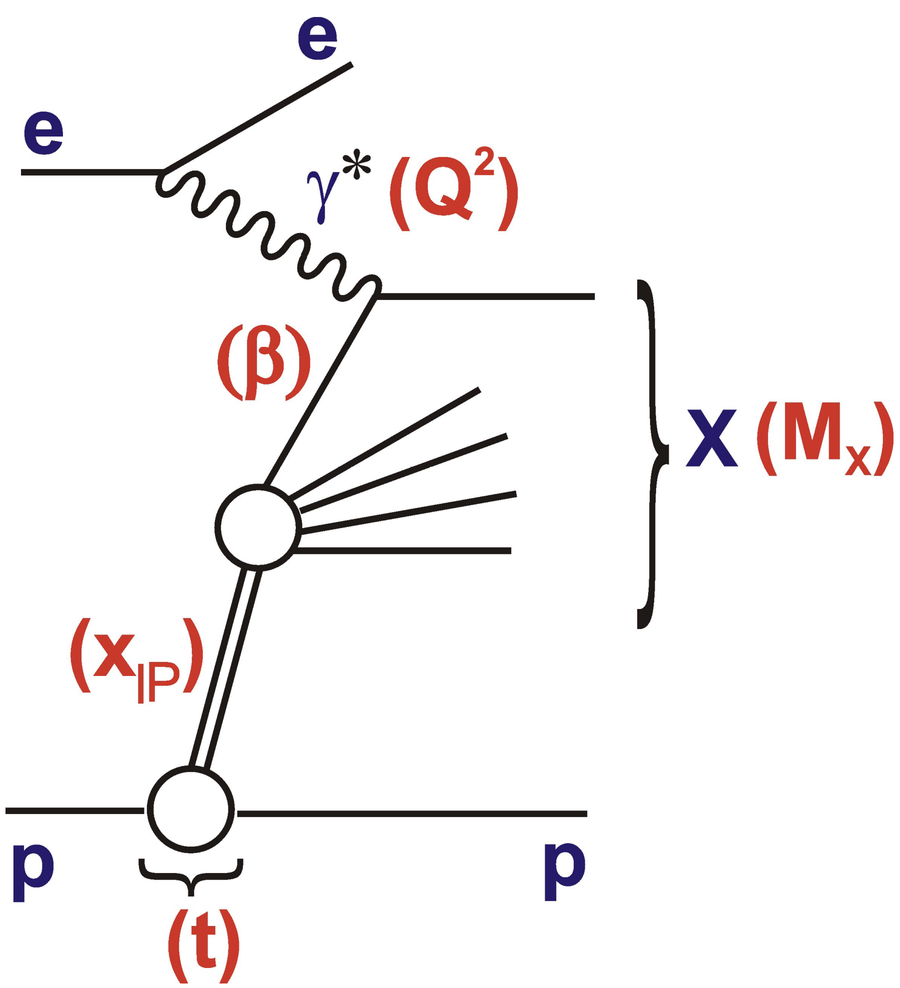


$$A(s,t) \sim \frac{s^{\alpha}}{t-m^2}$$

$$\alpha(t) = \alpha(0) + \alpha' t \longrightarrow J(M^2)$$



Diffractive SF and factorisation:



$$\frac{d^3\sigma^D}{dx_{IP}\,dx\,dQ^2}=\frac{2\pi\alpha_{\rm em}^2}{xQ^4}Y_+\,\sigma_r^{D(3)}(x_{IP},x,Q^2)$$

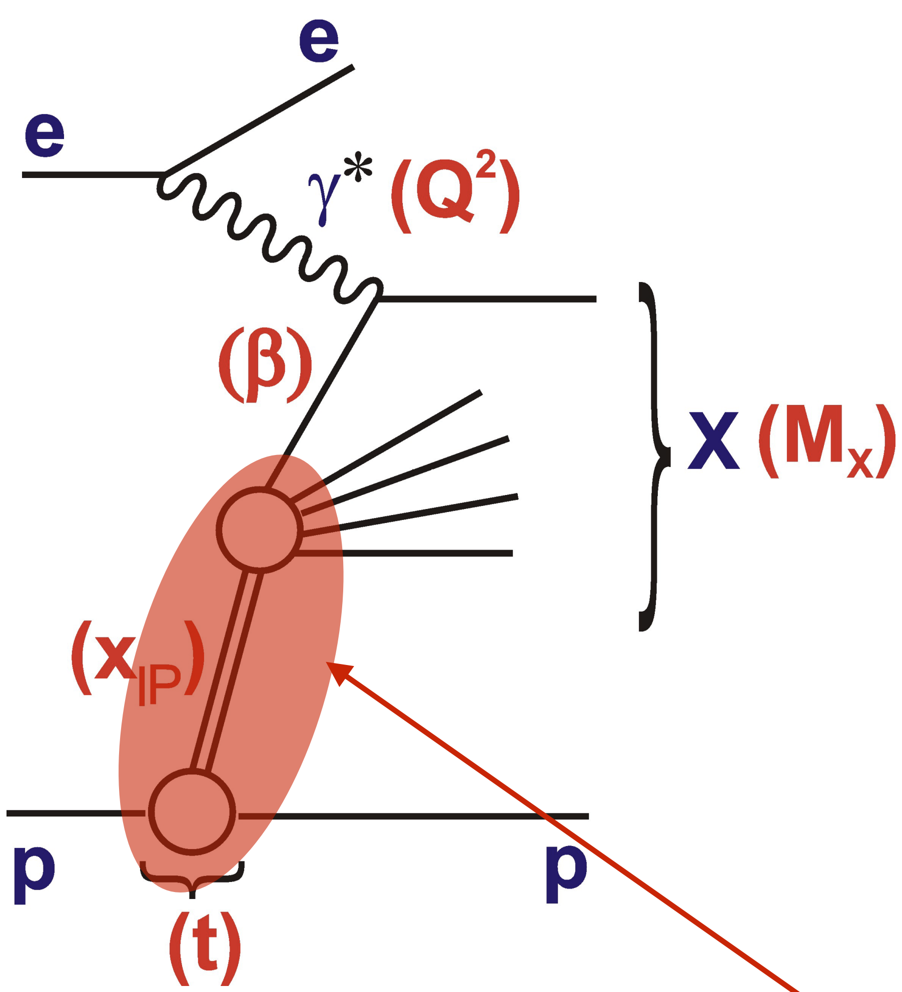
$$\sigma_r^{D(3)}=F_2^{D(3)}-\frac{y^2}{Y_+}F_L^{D(3)}$$

$$Y_+=1+(1-y)^2$$

$$F_{T,L}^{D(3)}(x,Q^2,x_{IP})=\int_{-\infty}^0 dt F_{T,L}^{D(4)}(x,Q^2,x_{IP},t)$$

$$F_2^{D(4)}=F_T^{D(4)}+F_L^{D(4)}$$

Diffractive SF and factorisation:



$$\frac{d^3\sigma^D}{dx_{IP}\,dx\,dQ^2} = \frac{2\pi\alpha^2_{\rm em}}{xQ^4} Y_+\,\sigma_r^{D(3)}(x_{IP},x,Q^2)$$

$$\sigma_r^{D(3)} = F_2^{D(3)} - \frac{y^2}{Y_+}F_L^{D(3)}$$

$$Y_+ = 1+(1-y)^2$$

$$F_{T,L}^{D(3)}(x,Q^2,x_{IP}) = \int_{-\infty}^{0} dt F_{T,L}^{D(4)}(x,Q^2,x_{IP},t)$$

$$F_2^{D(4)} = F_T^{D(4)} + F_L^{D(4)}$$

- For fixed t, x_P, collinear factorisation holds (Collins): diffractive PDFs expressing the conditional probability of finding a parton with momentum fraction β with the proton remaining intact.

$$d\sigma^{ep\to eXY}(x,Q^2,x_{IP},t) = \sum_i f_i^D \otimes d\hat{\sigma}^{ei} + \mathcal{O}(\Lambda^2/Q^2)$$

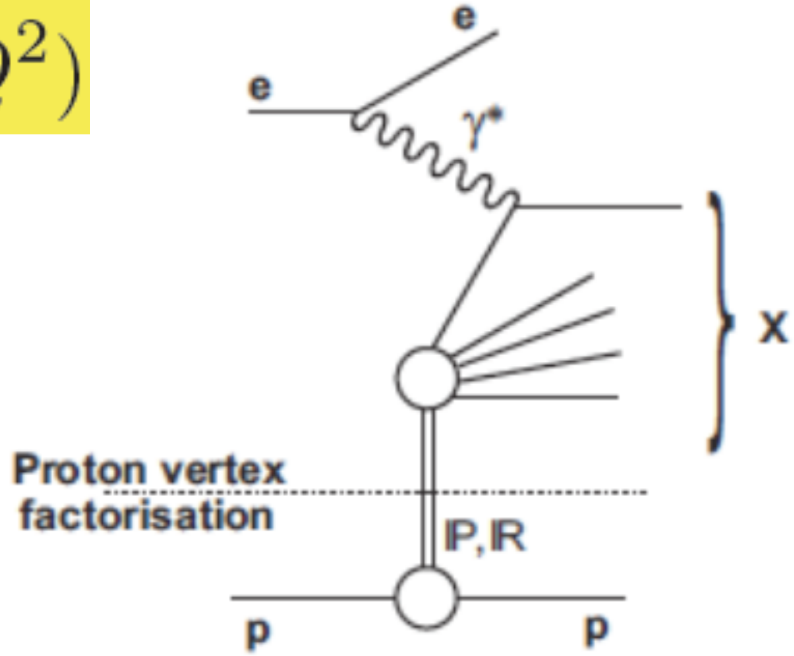
Diffractive PDFs:

- To extract DPDFs, an additional assumption is made: Regge factorisation for P and R that seems to work for not large too x_P.

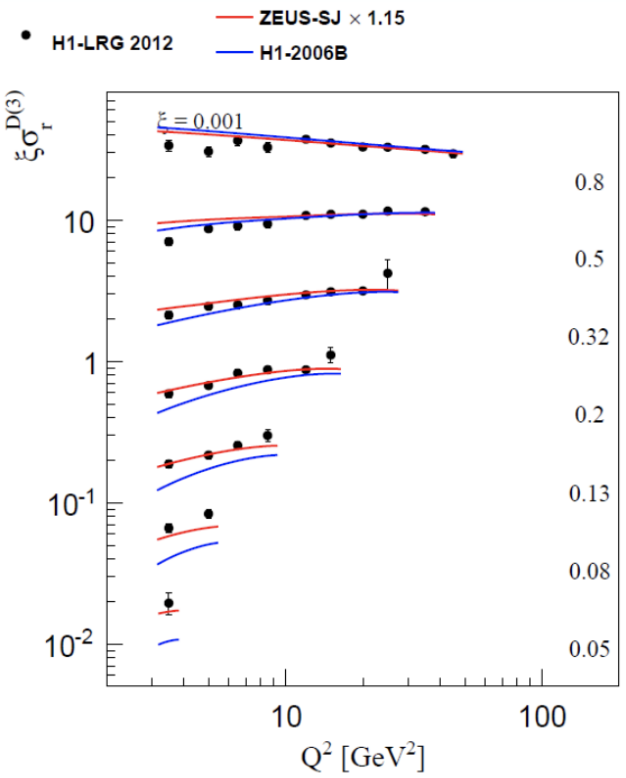
$$f_i^D(x,Q^2,x_{IP},t) = f_{IP/p}(x_{IP},t)\, f_i(\beta = x/x_{IP},Q^2)$$

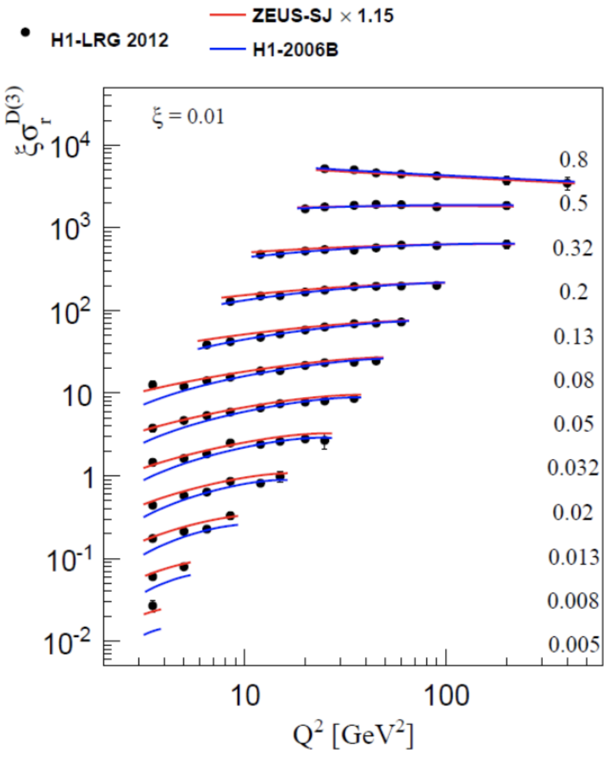
Pomeron flux

$$f_{IP/p}(x_{IP},t) = A_{IP}\frac{e^{B_{IP}t}}{x^{2\alpha_{IP}(t)-1}}$$



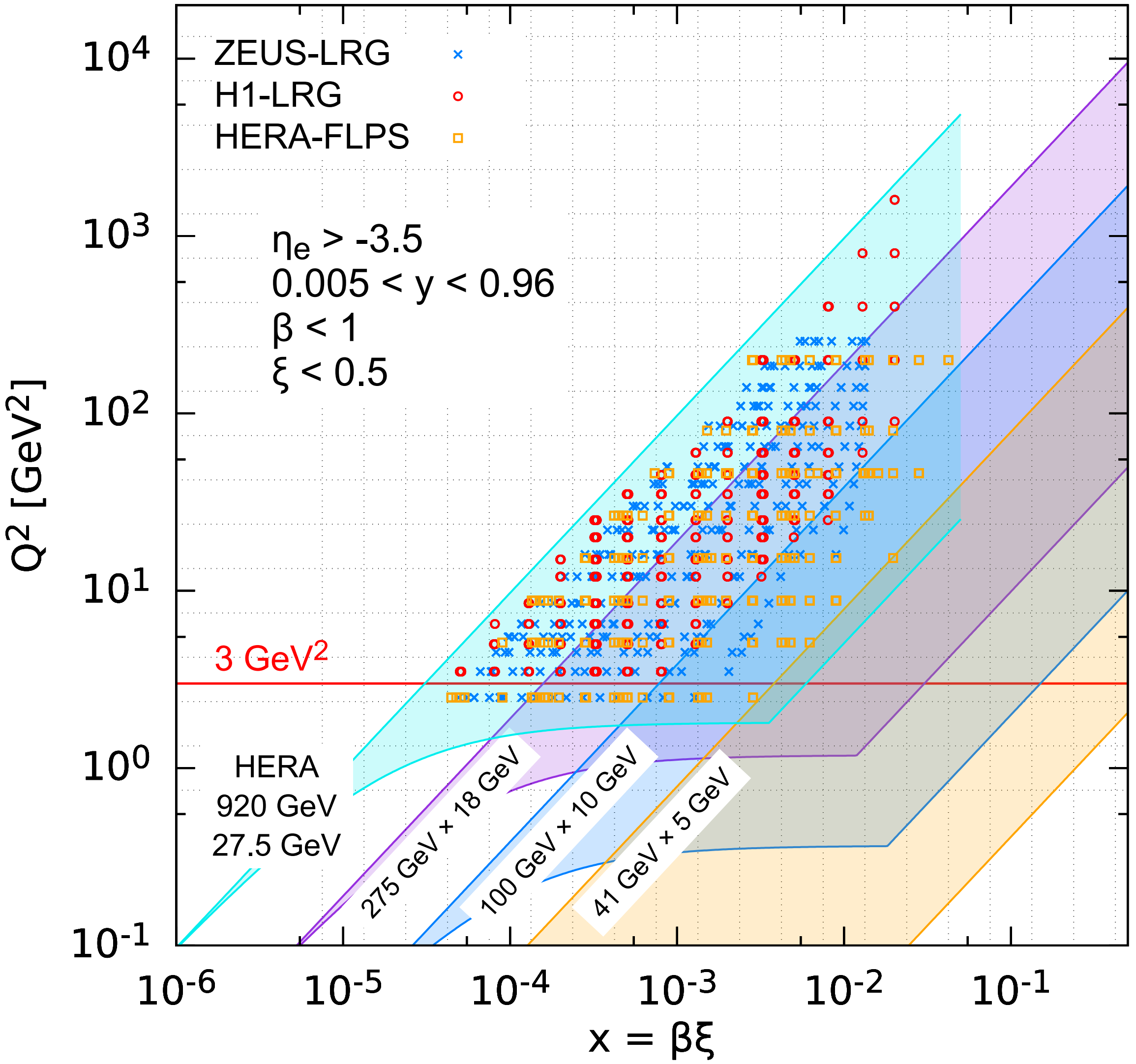
$f_i(\beta,Q^2)$ evolve with DGLAP evolution equations: fits to HERA data (additional contributions at large $x_P=\xi$ and small β).

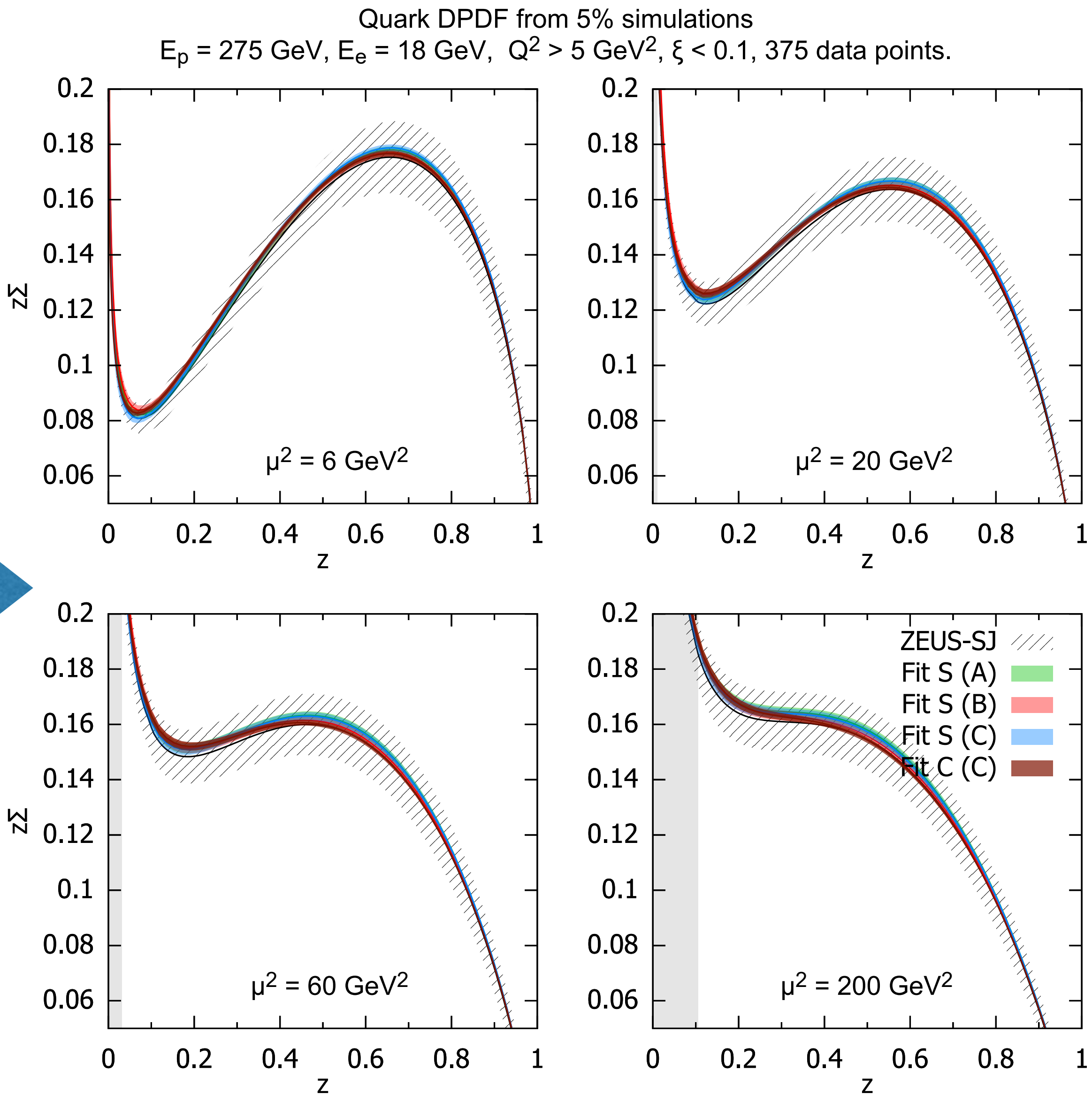




DPDFs at EICs:

- Limitations at HERA (check of Regge factorisation, P+R contributions with R modelled as π, size and shape of the diffractive glue, need to integrate over t) can be overcome with the EIC: determination of P and R! DPDFs, also in nuclei, t dependence, F_L^D,…




Quark DPDF from 5% simulations

E_p = 275 GeV, E_e = 18 GeV, $Q^2 > 5$ GeV^2, $\xi < 0.1$, 375 data points.

DPDFs at EICs:

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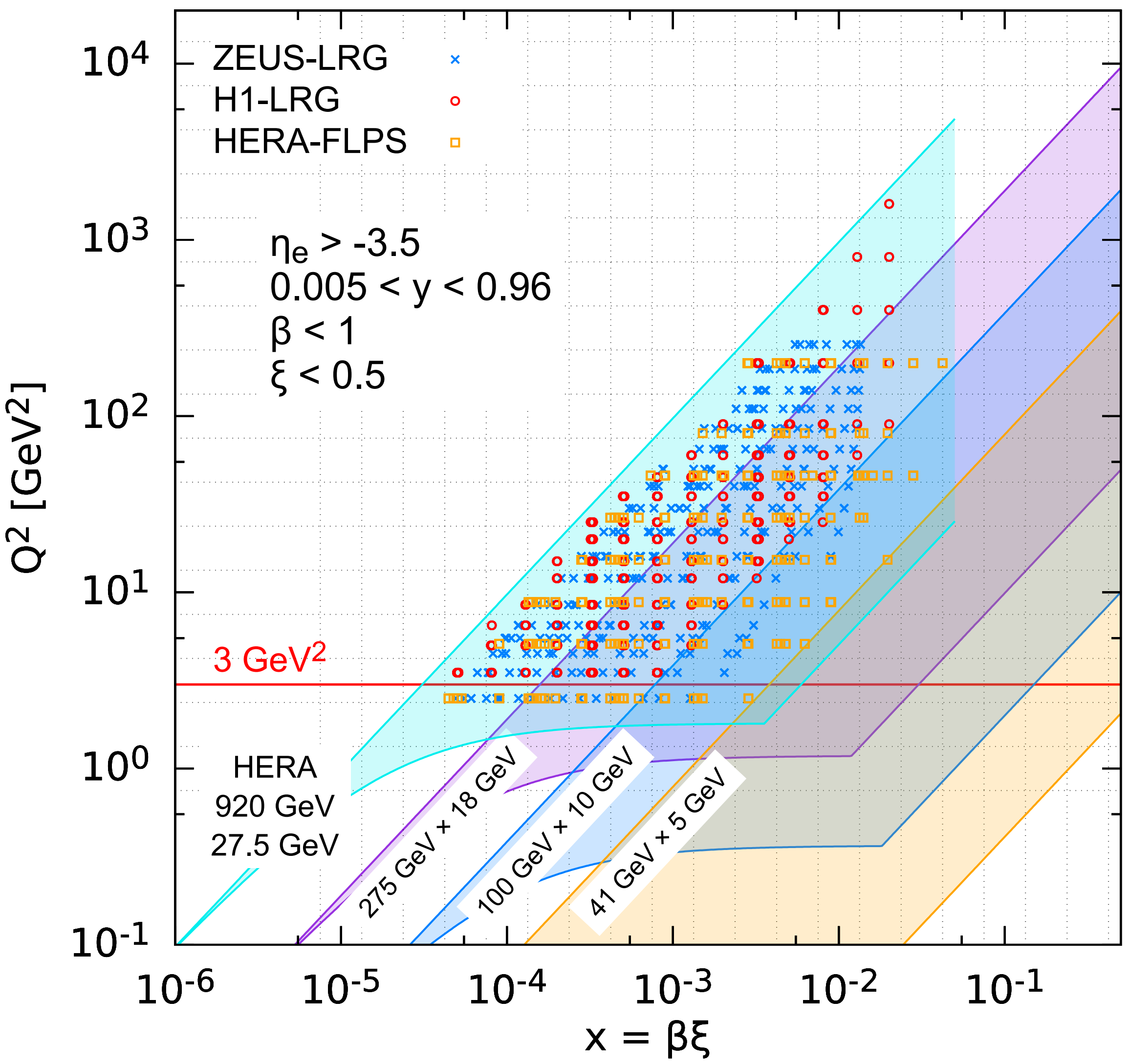
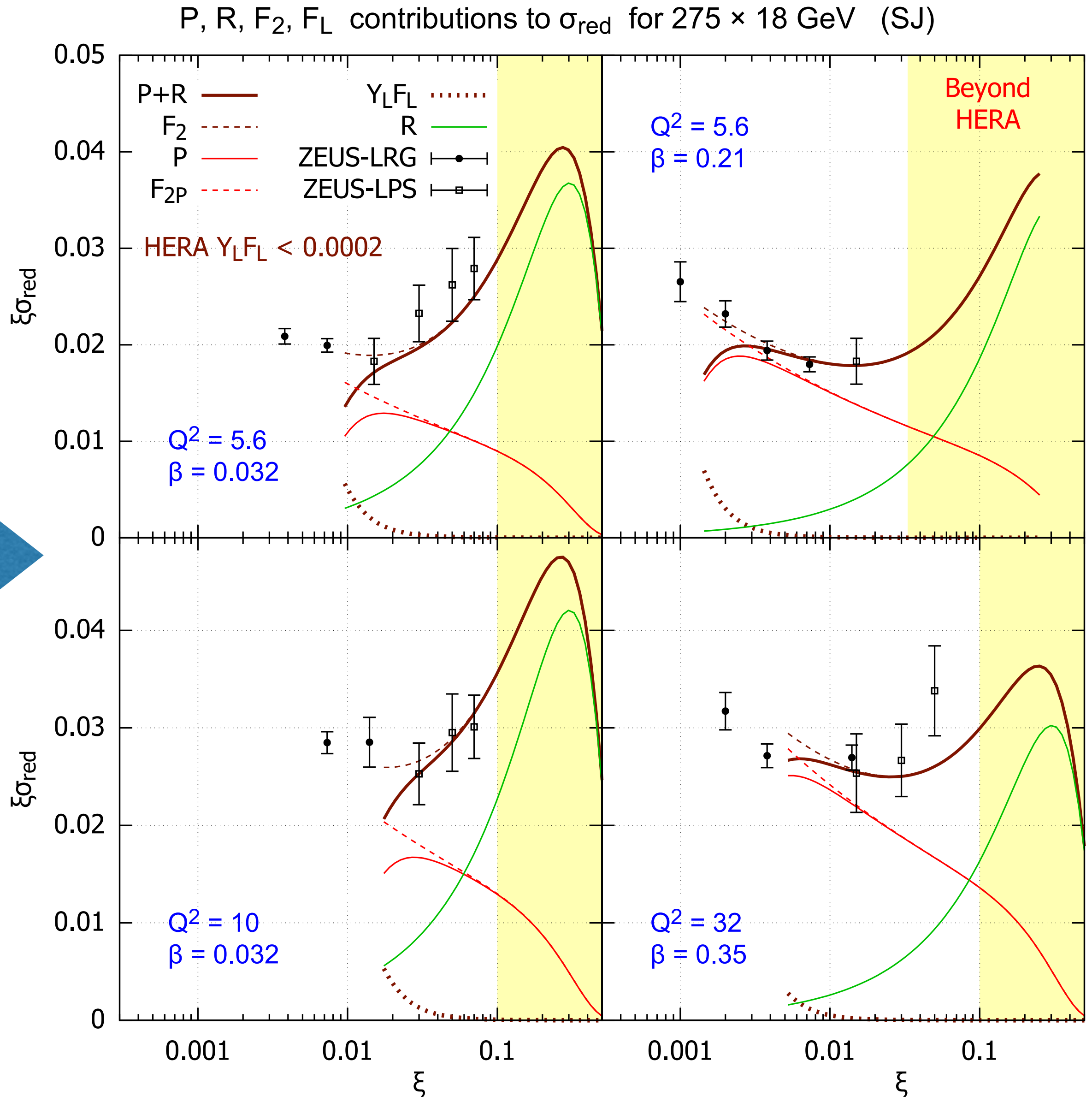
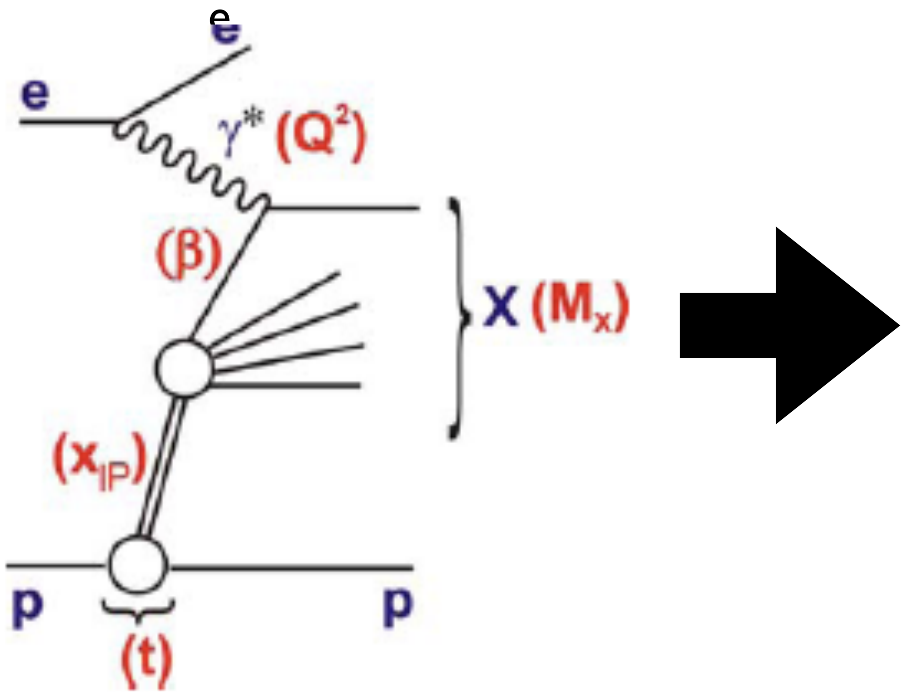
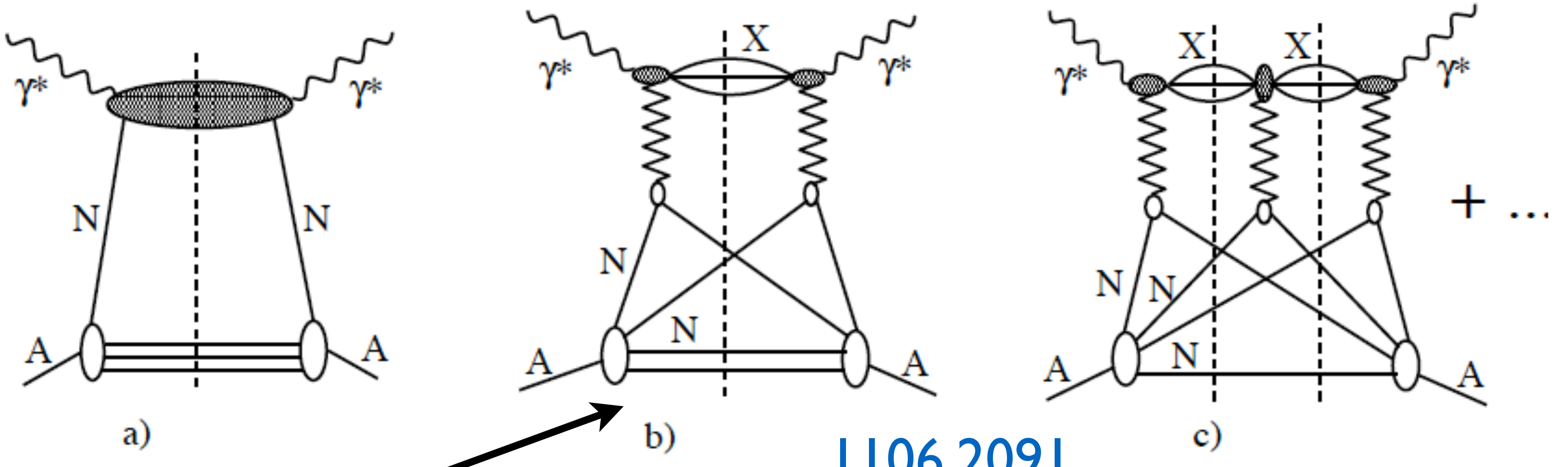




Diffraction in ep and shadowing:





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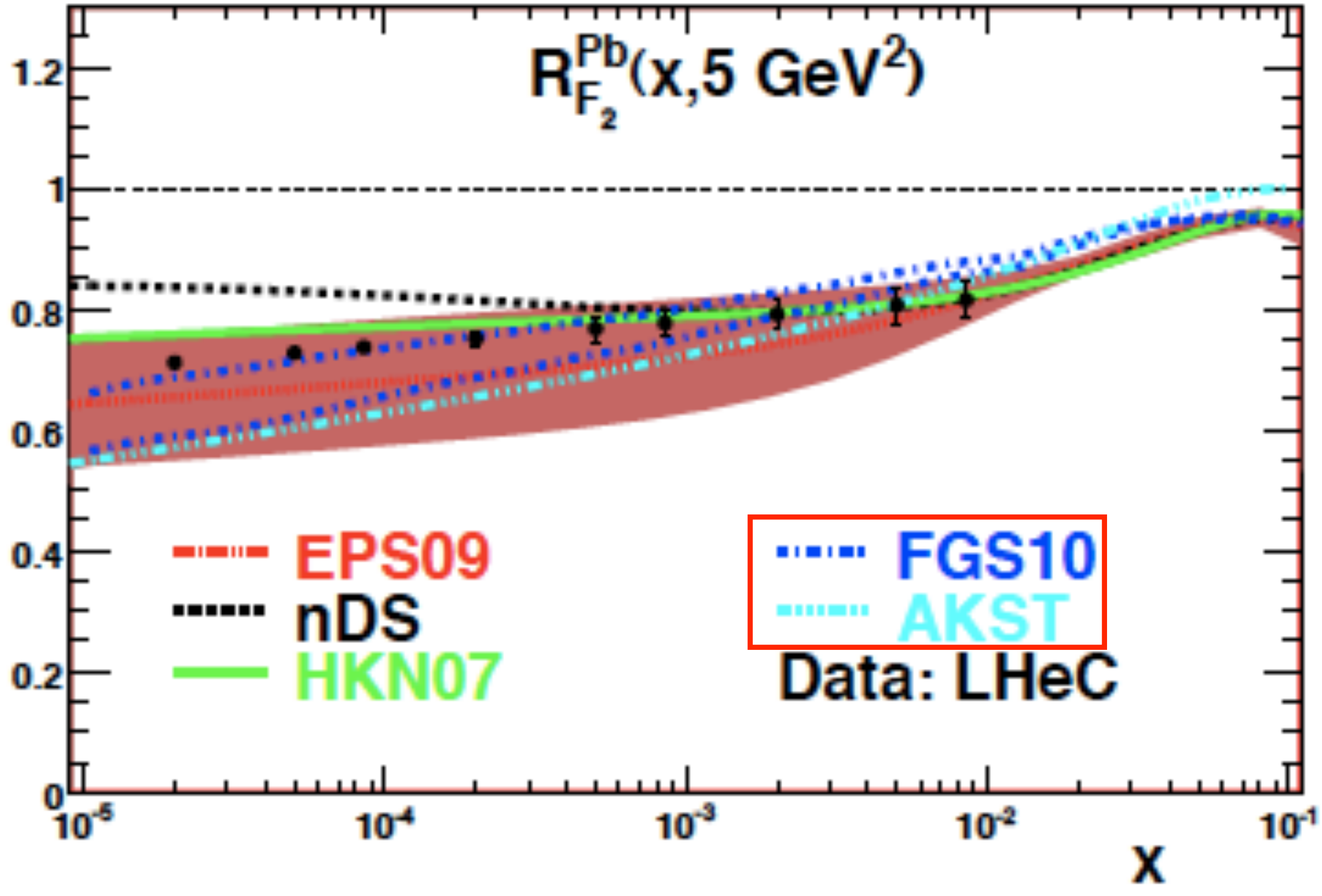
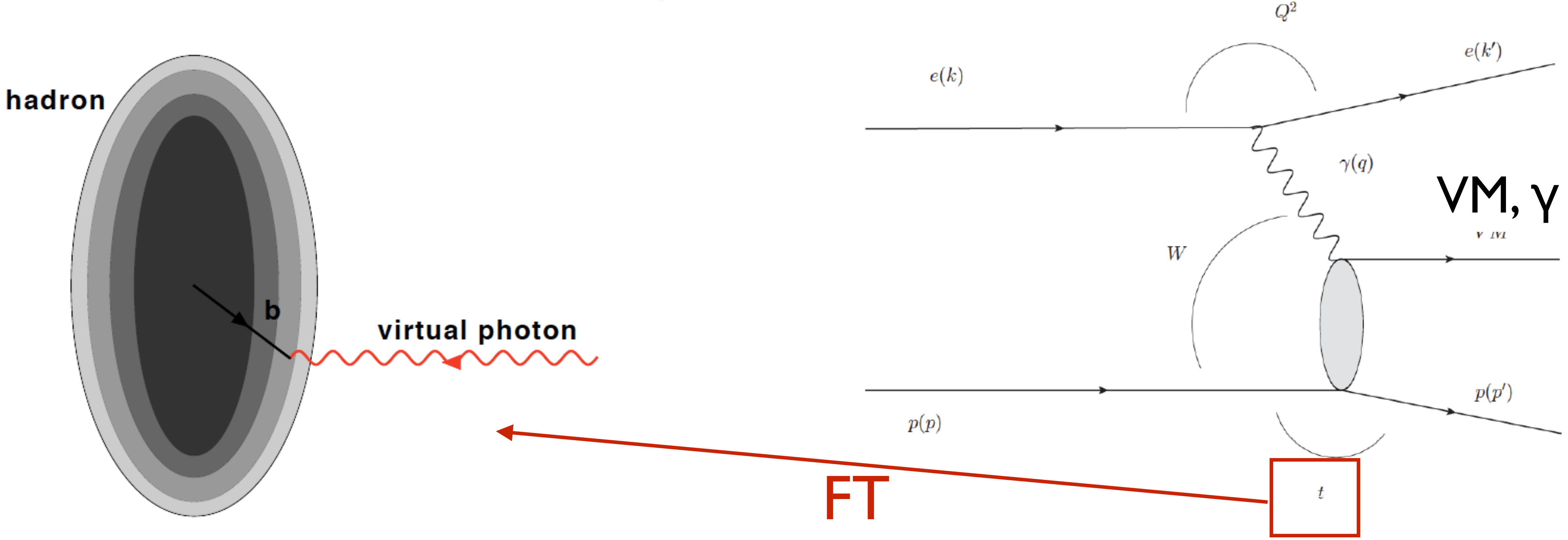
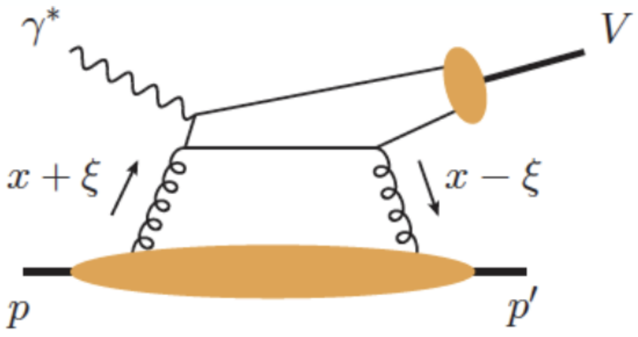
- Diffraction in ep is linked to nuclear shadowing through basic QFT (Gribov): eD to test and set the ‘benchmark’ for new effects.



Exclusive production:

- Exclusive production gives a 3D scan of the hadron/nucleus: gluon GPDs with vector mesons, quark GPDs with DVCS. It can be studied for Q=0 in UPCs, precision and Q>0 in EICs.



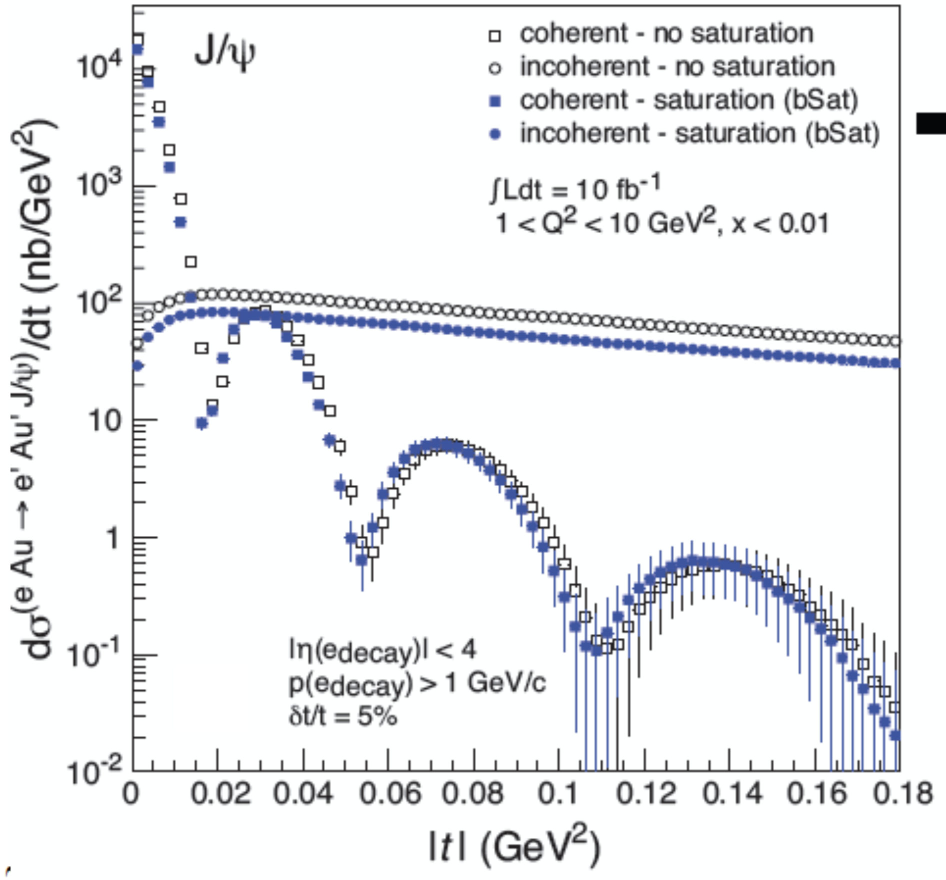


$$\int \frac{\mathrm{d}w^-}{2\pi} e^{-i\xi P^+ w^-} \left\langle P' \middle| T \overline{\psi}_j \left(0, \frac{1}{2}w^-, \mathbf{0}_\mathrm{T}\right) \frac{\gamma^+}{2} \psi_j \left(0, -\frac{1}{2}w^-, \mathbf{0}_\mathrm{T}\right) \middle| P \right\rangle_\mathrm{c}$$

Off-diagonal matrix elements, appear in amplitudes.

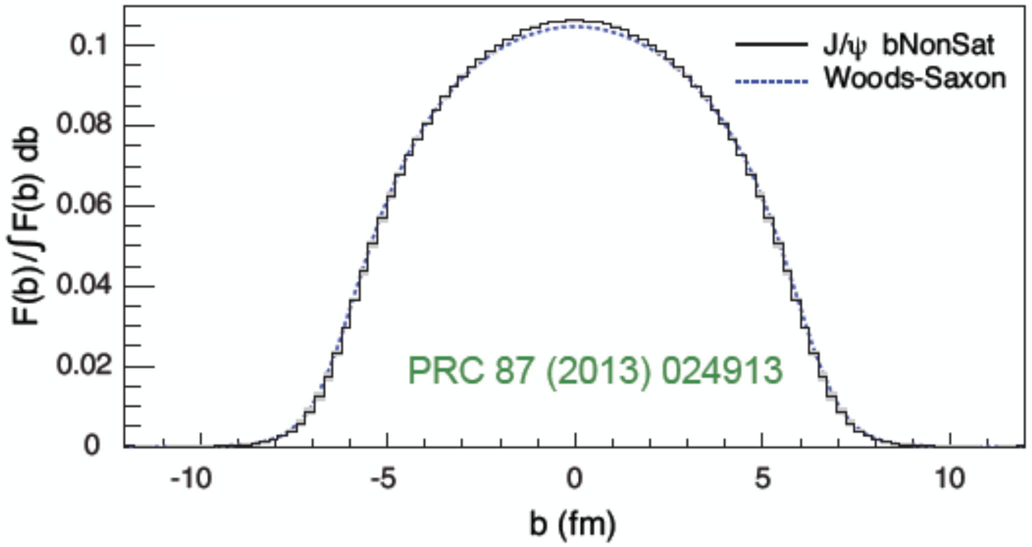
Elastic vector mesons (I):

$$e + Au \rightarrow e + J/\psi + Au^{(*)}$$



$$F(b) \sim \frac{1}{2\pi}\int_0^\infty d\Delta\, \Delta\, J_0(\Delta b)\sqrt{\frac{d\sigma}{dt}}$$

$t = \Delta^2/(1-x) \approx \Delta^2$



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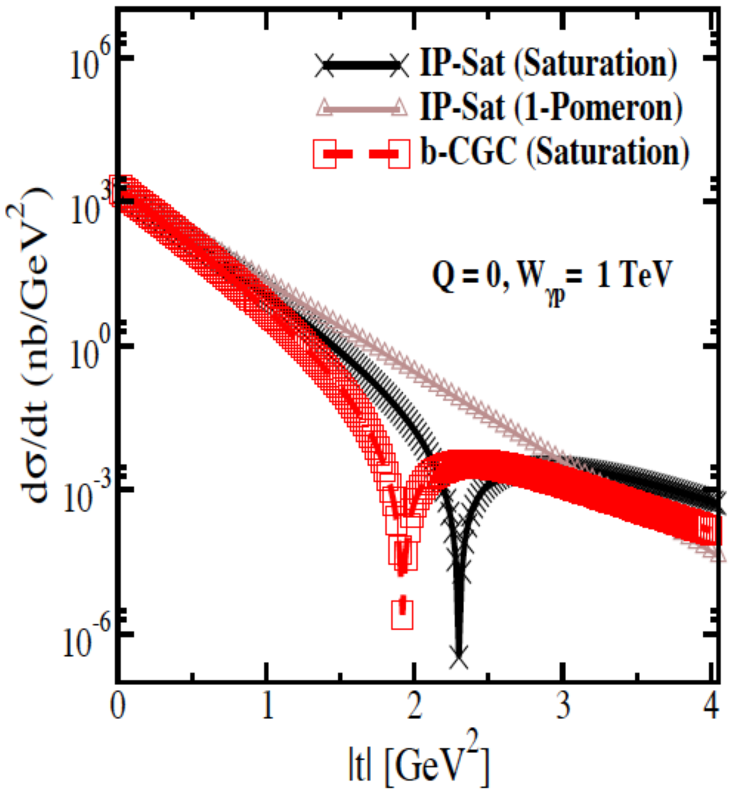

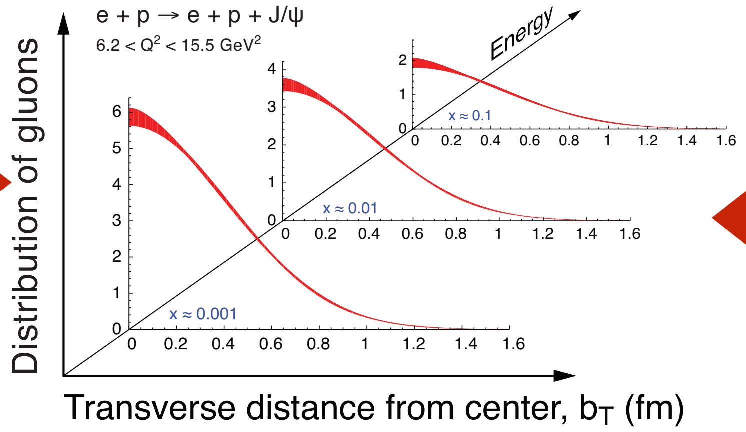

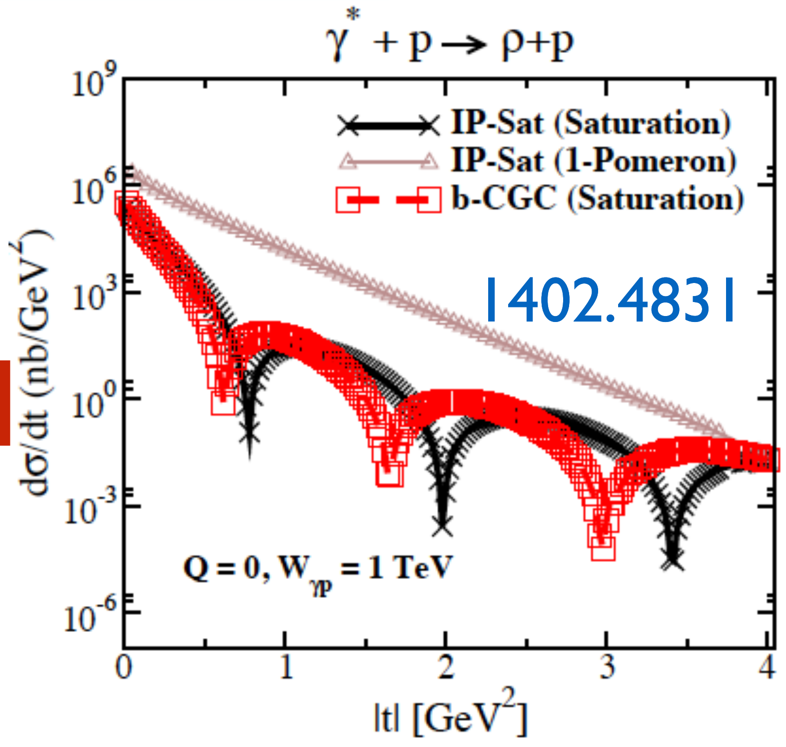

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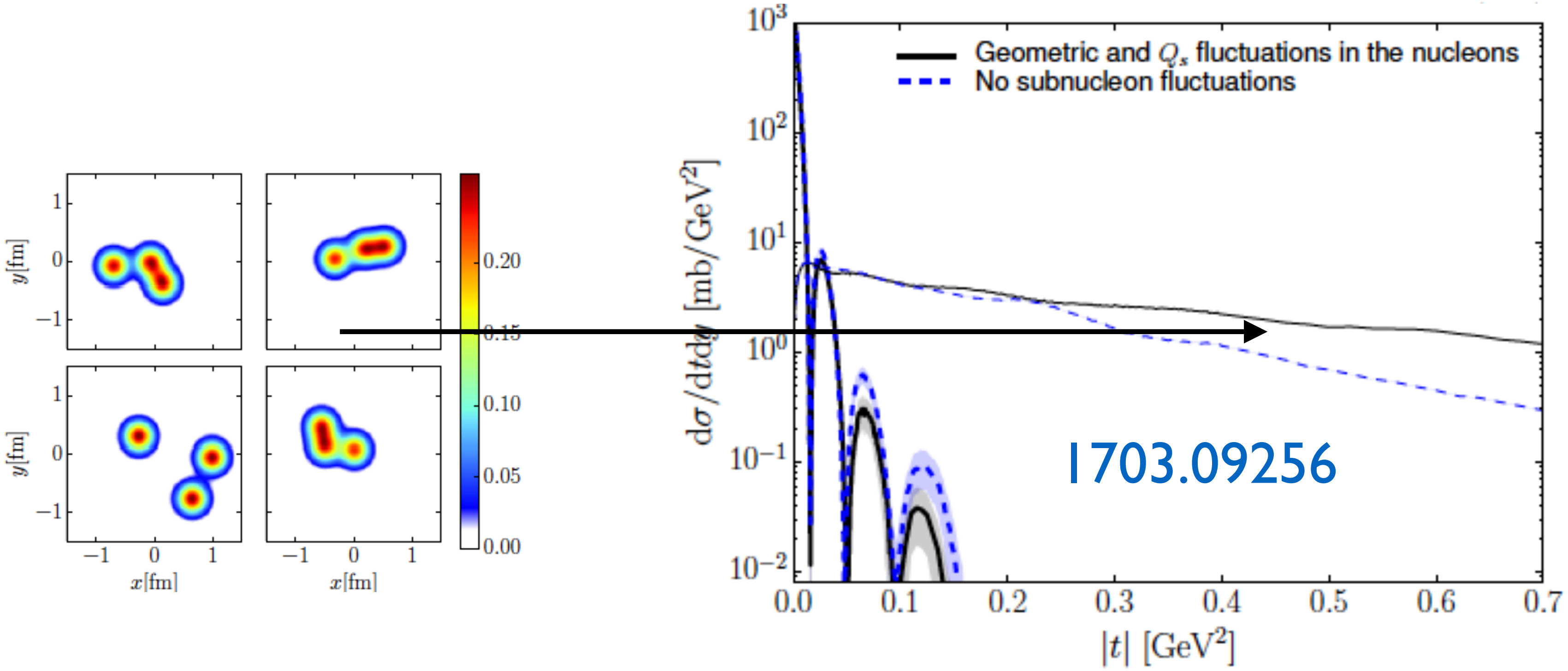
Elastic vector mesons (II):

- Incoherent diffraction sensitive to fluctuations: hot spots? that determine the initial stage of HIC, the distribution of MPIs,…

$$\left.\frac{d\sigma(\gamma p\rightarrow \mathrm{J}/\psi p)}{dt}\right|_{T,L} = \frac{(R_g^{T,L})^2}{16\pi}\left|\left\langle A(x,Q^2,\vec{\Delta})_{T,L}\right\rangle\right|^2$$

$$\left.\frac{d\sigma(\gamma p\rightarrow \mathrm{J}/\psi Y)}{dt}\right|_{T,L} = \frac{(R_g^{T,L})^2}{16\pi}\left(\left\langle\left|A(x,Q^2,\vec{\Delta})_{T,L}\right|^2\right\rangle - \left|\left\langle A(x,Q^2,\vec{\Delta})_{T,L}\right\rangle\right|^2\right)$$



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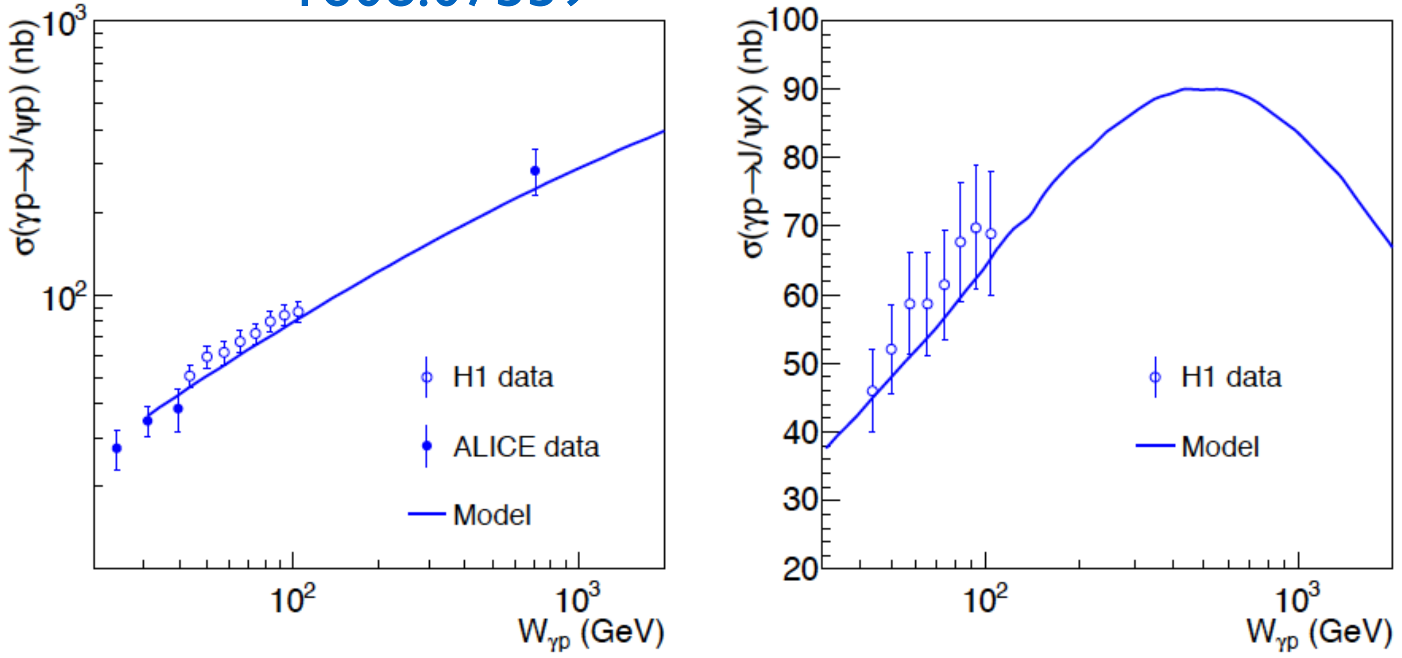
1608.07559



$$T(\vec{b}) = \frac{1}{N_{hs}}\sum_{i=1}^{N_{hs}} T_{hs}(\vec{b}-\vec{b_i})$$

$$T_{hs}(\vec{b}-\vec{b_i}) = \frac{1}{2\pi B_{hs}} e^{-\frac{(\vec{b}-\vec{b_i})^2}{2B_{hs}}}$$

$$N_{hs}(x) = p_0 x^{p_1}(1+p_2\sqrt{x})$$

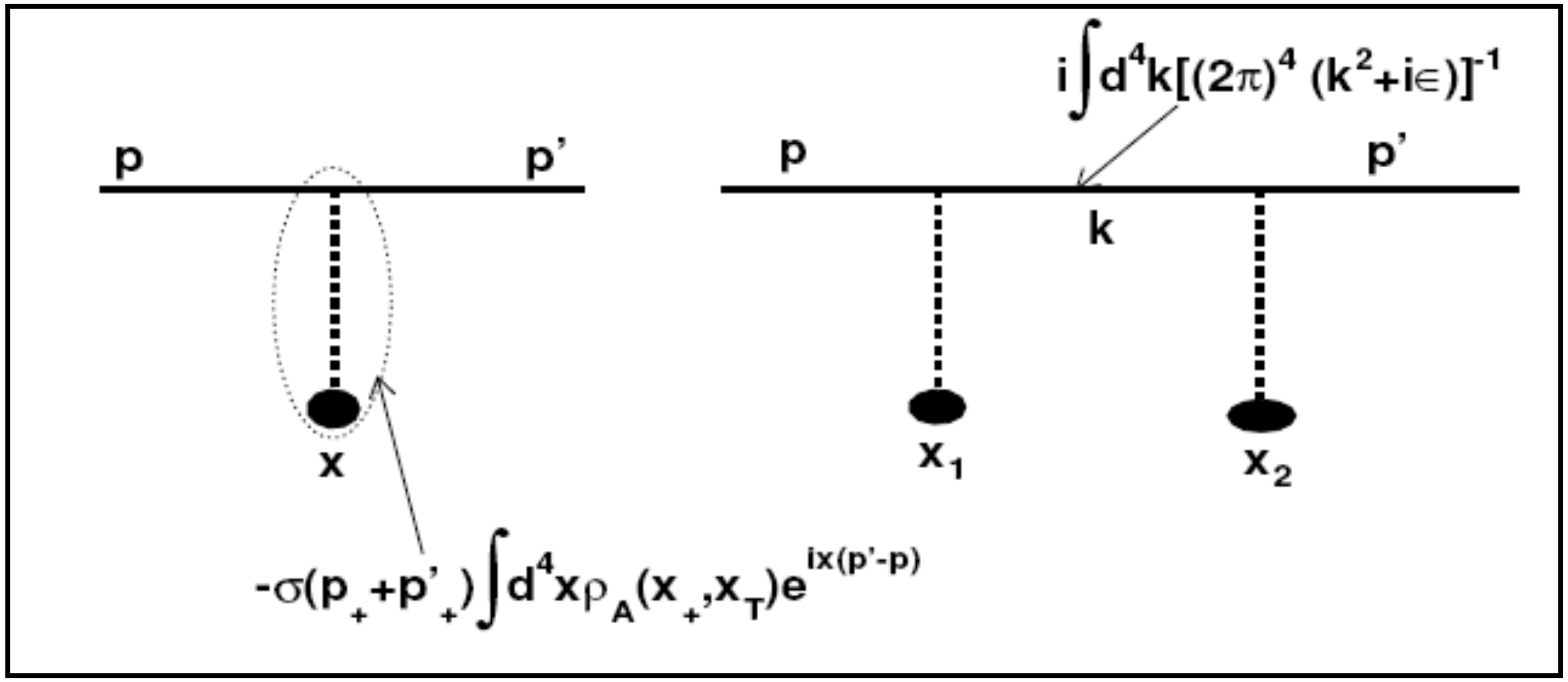
Two scattering case:

$$\mathrm{i}t\,(q=0) = \mathrm{i}t_{\mathrm{forw}} = -\sigma$$

$$\mathrm{i}\mathcal{T}_n(q=0) = -\sigma_A^n$$

$$q = p' - p$$

$$T_A(x_T) = \int_{-\infty}^{+\infty} \mathrm{d}x_+\,\rho_A(x_+, x_T)$$



$$c(p_+, p'_+)\mathrm{i}\mathcal{T}_1(q) = \mathrm{i}t_{\mathrm{forw}}c(p_+, p'_+)A\int \mathrm{d}^2x_T\,T_A(x_T)\,\mathrm{e}^{-\mathrm{i}x_T\cdot(p'_T-p_T)} \quad \Rightarrow \quad \sigma_A^1 = A\sigma$$

$$\begin{aligned} c(p_+, p'_+)\mathrm{i}\mathcal{T}_2(q) = {} & c(p_+, p'_+)A(A-1)(\mathrm{i}t_{\mathrm{forw}})^2 \\ & \times \int \frac{\mathrm{d}^2k_T}{(2\pi)^2}\,\mathrm{d}x_{1+}\,\mathrm{d}x_{2+}\,\mathrm{d}^2x_{1T}\,\mathrm{d}^2x_{2T}\,\exp\left(-\mathrm{i}k_T^2(x_{2+}-x_{1+})/(2p_+)\right) \\ & \times \exp(-\mathrm{i}[x_{1T}\cdot(k_T-p_T)+x_{2T}\cdot(p'_T-k_T)]\rho_A(x_{1+}, x_{1T}) \\ & \times \rho_A(x_{2+}, x_{2T})\theta(x_{2+}-x_{1+}) \end{aligned}$$

Two scattering case:

$$\mathrm{i}t\,(q=0)=\mathrm{i}t_{\mathrm{forw}}=-\sigma$$

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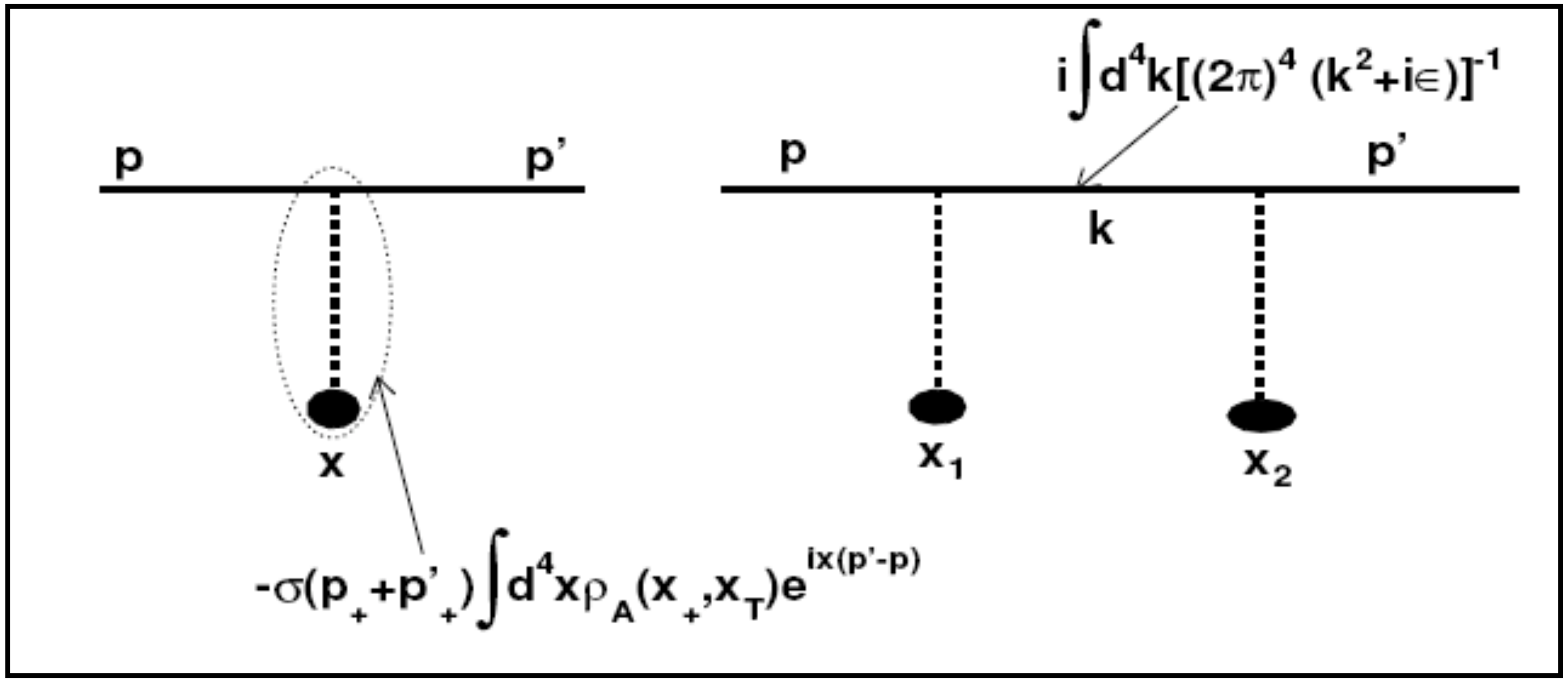

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Coherence and shadowing:

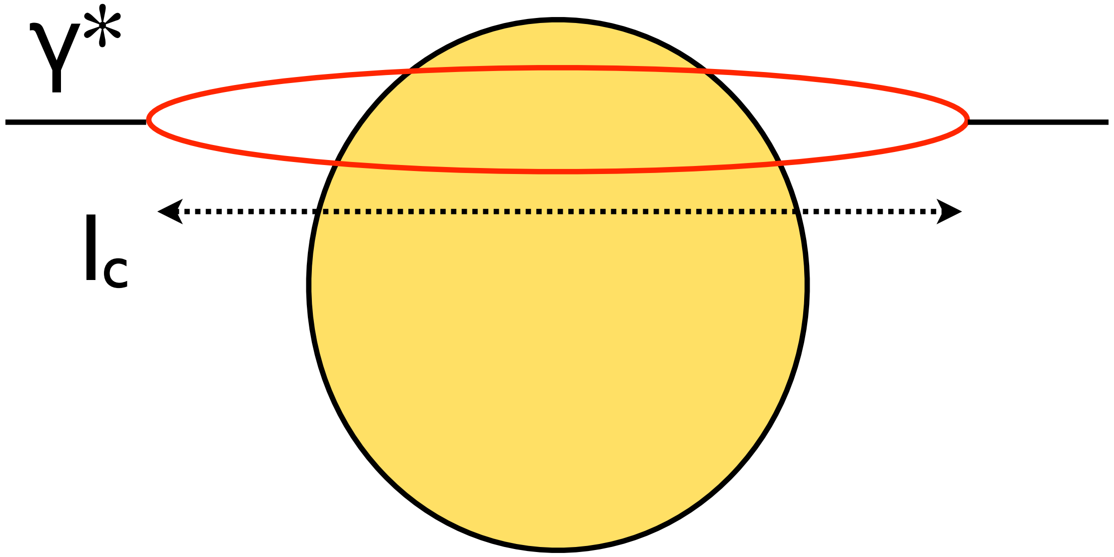
$$\boxed{\exp\left[-\mathrm{i}k_T^2(x_{2+}-x_{1+})/(2p_+)\right] = \exp\left[-\mathrm{i}(x_{2+}-x_{1+})/l_c\right], \text{ with } l_c = 2p_+/k_T^2}$$

A) $p_+ \to 0 \Rightarrow \mathrm{i}\mathcal{T}_2(q) \to 0$: incoherent, $\sigma_A = A\sigma^1$

B) $p_+ \to \infty, \exp\left[-\mathrm{i}(x_{2+}-x_{1+})/l_c\right] \to 1$

$$\mathrm{i}\mathcal{T}_2(q) = \frac{A(A-1)}{2}(\mathrm{i}t_{\mathrm{forw}})^2 \int \mathrm{d}^2x_T\, \mathrm{e}^{-\mathrm{i}x_T\cdot(p_T'-p_T)}\, T_A^2(x_T),$$

$$\sigma_A^2 \overset{\downarrow}{=} -\frac{A(A-1)}{2}\int \mathrm{d}^2x_T\,[T_A(x_T)\sigma]^2$$: coherent, $\sigma_A < A\sigma^1$



The lifetime of the qqbar fluctuation is $\geq R_A$ for $x \leq 0.1A^{-1/3}$: **small x**.

$$\tau \sim \frac{1}{Q}\times\frac{E_{\mathrm{lab}}}{Q} \simeq \frac{W^2}{2m_{\mathrm{nucleon}}Q^2} \simeq \frac{1}{2m_{\mathrm{nucleon}}x}$$

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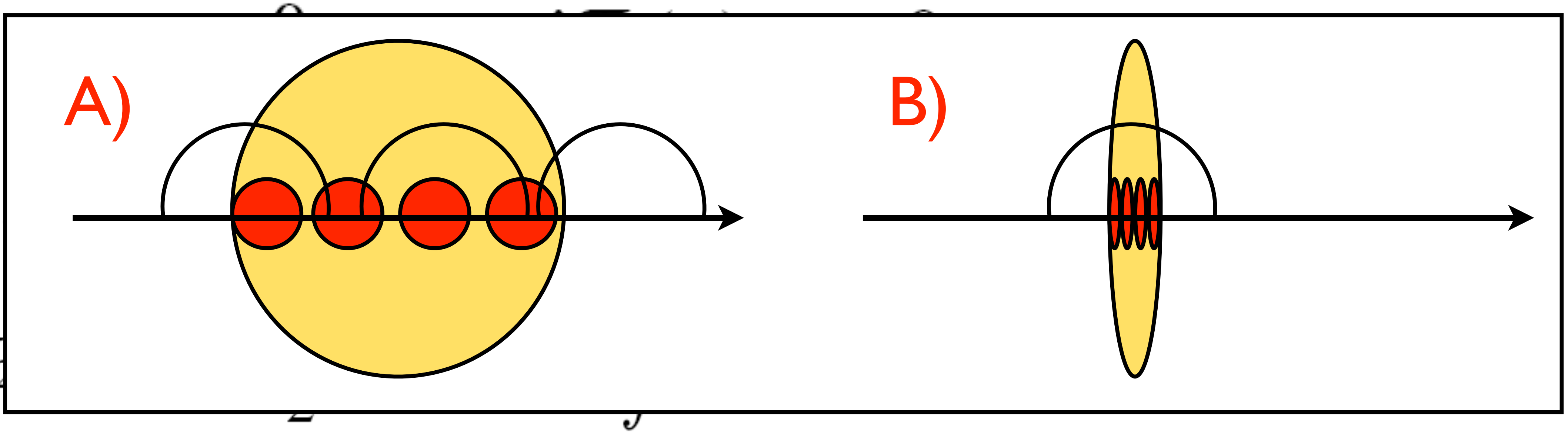
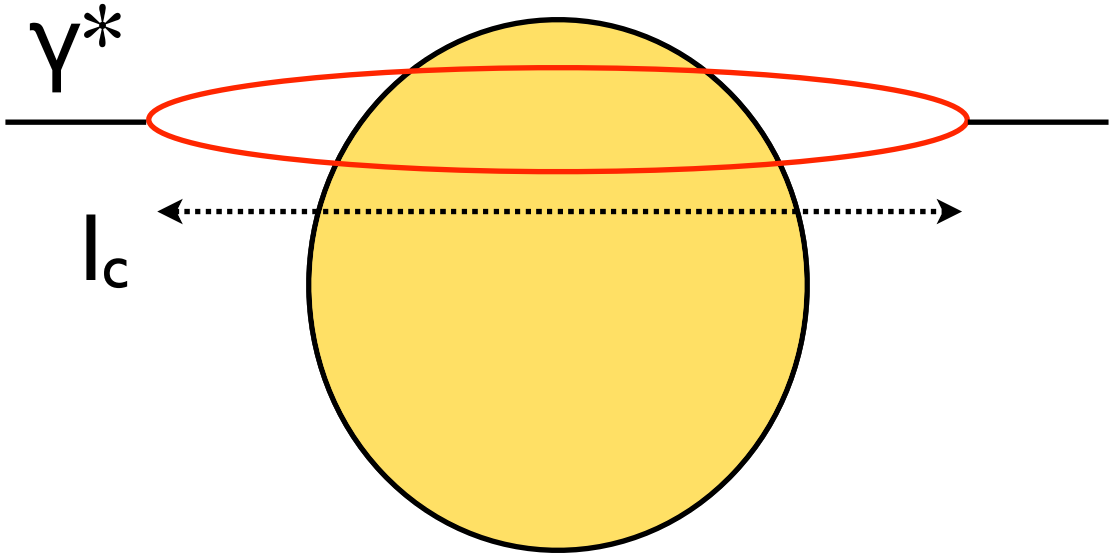
A)

B)



$$\sigma_A^2 \overset{\downarrow}{=} -\frac{A(A-1)}{2}\int d^2x_T\,[T_A(x_T)\sigma]^2$$

: coherent, $\sigma_A < A\sigma^1$



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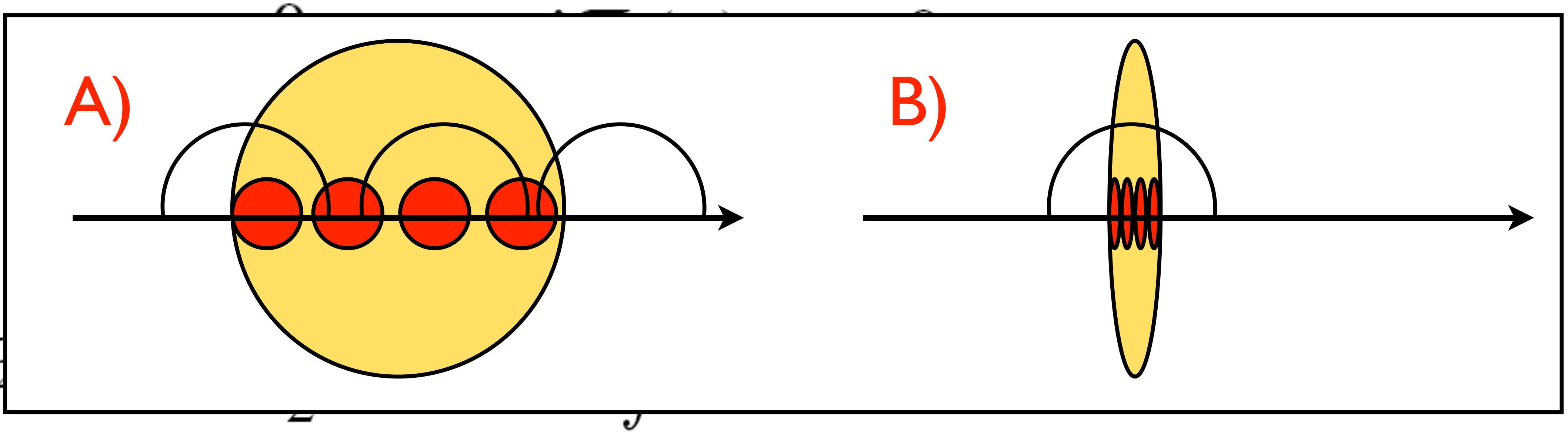
B)



$$\sigma^2 \downarrow \frac{A(A-1)}{} \int d^2x_T [T_A(x_T)\sigma]^2$$: coherent $\sigma_A < A\sigma$

- In the totally coherent limit and when resummed on the number of scatterings this model leads to Glauber-Gribov in pA, and to the Wilson line (eikonal approximation) in QCD.
- Connection with jet quenching: corrections to the totally coherent limit (non-eikonal) lead to the LPM effect.

$$l \sim \frac{}{Q} \times \frac{}{Q} = \frac{}{2m_{\rm nucleon} Q^2} = \frac{}{2m_{\rm nucleon} x}$$

Contents:

1. Part 1: partonic structure

➜ Basics of DIS and collinear factorisation.
➜ PDFs and their determination.
➜ Beyond collinear factorisation: TMDs, GPDs.
➜ Diffraction.

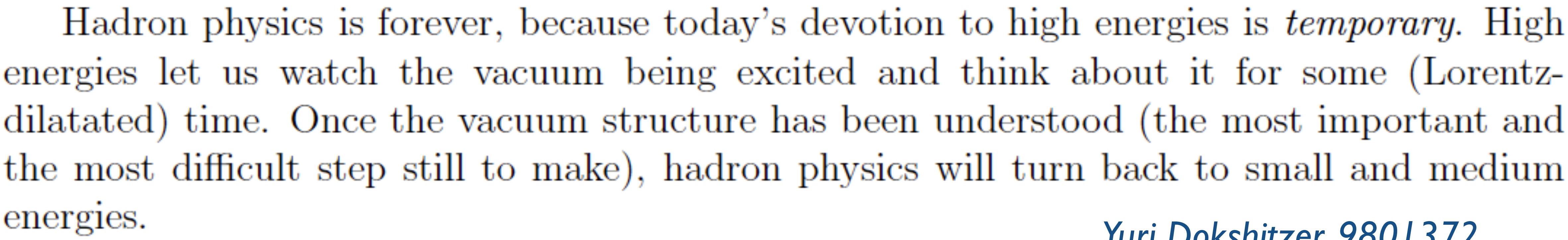
2. Part 2: small x

➜ High-energy QCD.
➜ Non-linear phenomena.
➜ The Color Glass Condensate approach: evolution equations.
➜ The dipole model.
➜ How to compute observables.
➜ Phenomenology in DIS: inclusive and exclusive observables.
➜ Phenomenology in hadronic collisions: single inclusive particle production, correlations.

Some bibliography:

- R. Devenish and A. Cooper-Sarker, *Deep Inelastic Scattering*, Oxford University Press 2004.
- G. P. Salam, *Elements of QCD for hadron colliders*, CERN Yellow Rep. School Proc. 5 (2020) 1-56, https://inspirehep.net/literature/1820528.
- Yu. V. Kovchegov and E. Levin, *Quantum Chromodynamics at High Energy*, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 33 (2012) 1-350.

High-energy QCD:



Hadron physics is forever, because today's devotion to high energies is *temporary*. High energies let us watch the vacuum being excited and think about it for some (Lorentz-dilatated) time. Once the vacuum structure has been understood (the most important and the most difficult step still to make), hadron physics will turn back to small and medium energies.

Yuri Dokshitzer, 9801372

- High energies provide:

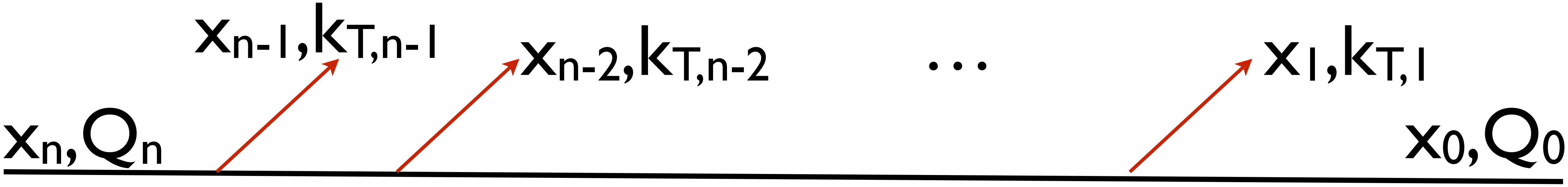
➜ Large scales for the use of perturbation theory.

➜ Large phase space for QCD radiation: large logarithms.

➜ Strong simplifications: eikonal approximation, neglection of spin exchanges,…

- Large energy is equivalent to probing the small x structure of hadrons (semihard scales).
- Dilation of scales: configurations become frozen between interaction times.
- Besides, we are interested in the high energy behaviour of gauge theories for the high-energy programme: QCD and, eventually, SM as background.

Radiation: DGLAP vs. BFKL



$$dP_i \propto \frac{dx_i}{x_i}\frac{d\theta_i}{\theta_i},\ \omega_i = x_i E,\ \theta_i \simeq \frac{k_{T,i}}{\omega_i} \qquad x_n < x_{n-1} < x_{n-2} < \ldots < x_1 < x_0$$

A) DGLAP, moderate x: $Q_n^2 \gg k_{T,n-1}^2 \gg k_{T,n-2}^2 \gg \ldots \gg k_{T,1}^2 \gg Q_0^2$

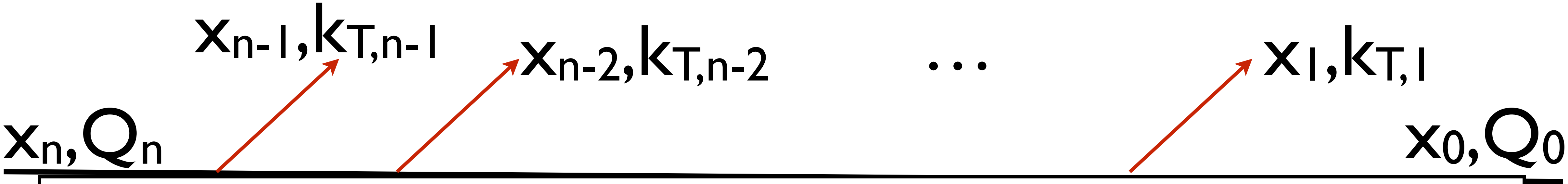
$$\int_{Q_0}^{Q_n} dP_{n-1} \int_{Q_0}^{k_{T,n-1}} dP_{n-2} \ldots \int_{Q_0}^{k_{T,2}} dP_1 \propto \left[\frac{\alpha_s N_c}{\pi} \ln\frac{Q_n}{Q_0}\right]^n$$

B) BFKL, small x: $\displaystyle\int_{x_n}^{x_0} dP_{n-1} \int_{x_{n-1}}^{x_0} dP_{n-2} \ldots \int_{x_2}^{x_0} dP_1 \propto \left[\frac{\alpha_s N_c}{\pi} \ln\frac{x_0}{x_n}\right]^n$

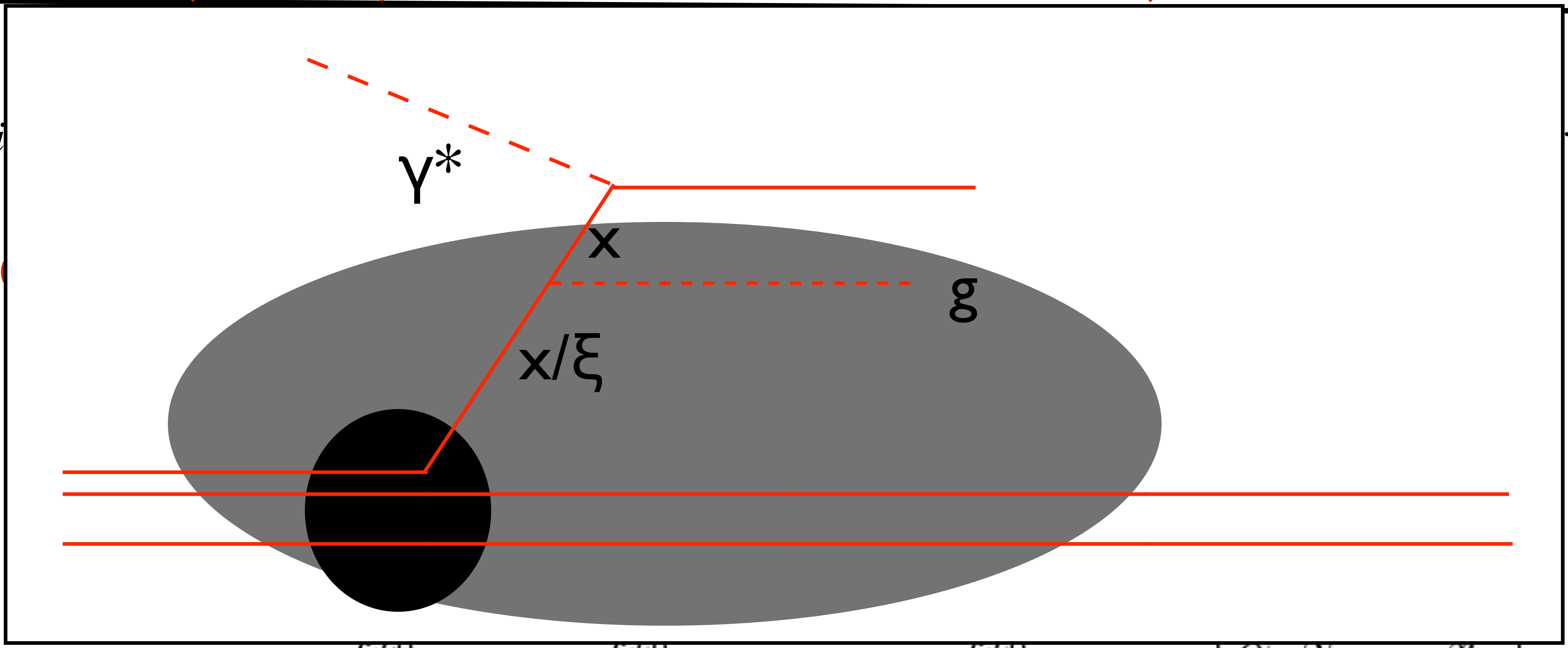
$$x_n \ll x_{n-1} \ll x_{n-2} \ll \ldots \ll x_1 \ll x_0$$

- Both of them lead to a gluon distribution at small x behaving like $xg(x,Q^2) \propto x^{-\lambda}$ at fixed Q^2, $\lambda \approx 0.2\text{-}0.3$ in data.

Radiation: DGLAP vs. BFKL



dP_i … x_0



A) D

B) BFKL, small x: $\int_{x_n}^{x_0} dP_{n-1} \int_{x_{n-1}}^{x_0} dP_{n-2} \dots \int_{x_2}^{x_0} dP_1 \propto \left[\frac{\alpha_s N_c}{\pi} \ln \frac{x_0}{x_n}\right]$

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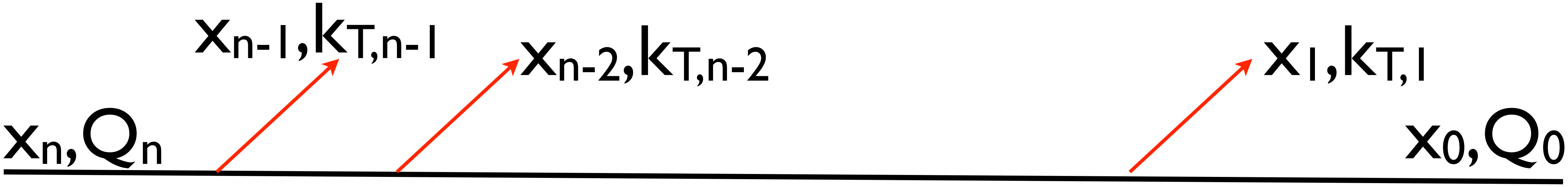
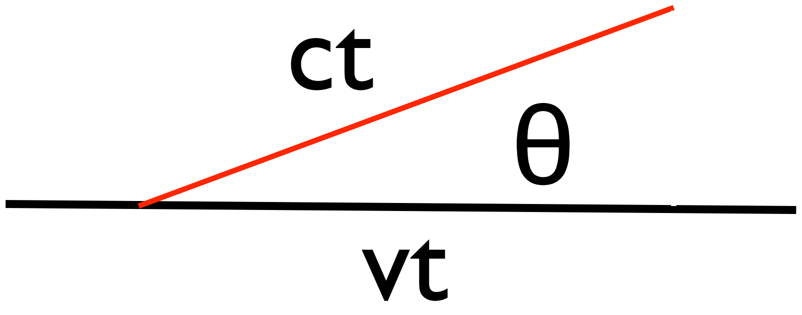
Radiation: dead cone, ang. ordering



$$dP_i = \frac{dx_i}{x_i} \frac{dk_{T,i}^2}{k_{T,i}^2}, \quad \omega_i = x_i E, \quad \theta_i^2 \simeq \frac{k_{T,i}^2}{\omega_i^2}$$

$$Q_n^2 \gg k_{T,n-1}^2 \gg k_{T,n-2}^2 \gg \ldots \gg k_{T,1}^2 \gg Q_0^2$$

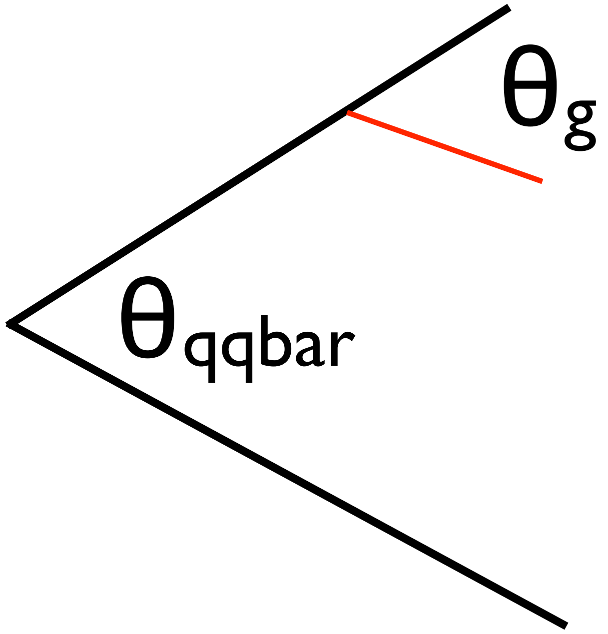
$$x_n \ll x_{n-1} \ll x_{n-2} \ll \ldots \ll x_1 \ll x_0$$



$$(vt)^2 + (ct)^2 \sin^2\theta_0 \simeq (vt)^2 + (ct)^2\theta_0^2 = (ct)^2$$

$$\Rightarrow \theta_0^2 \simeq 1 - (v/c)^2 = m^2/E^2, \quad \theta^2 \to \theta^2 + \theta_0^2$$

Infrared (soft) and collinear (mass) divergencies.



Angular ordering: |qqbarg> → |qqbar>+|g>

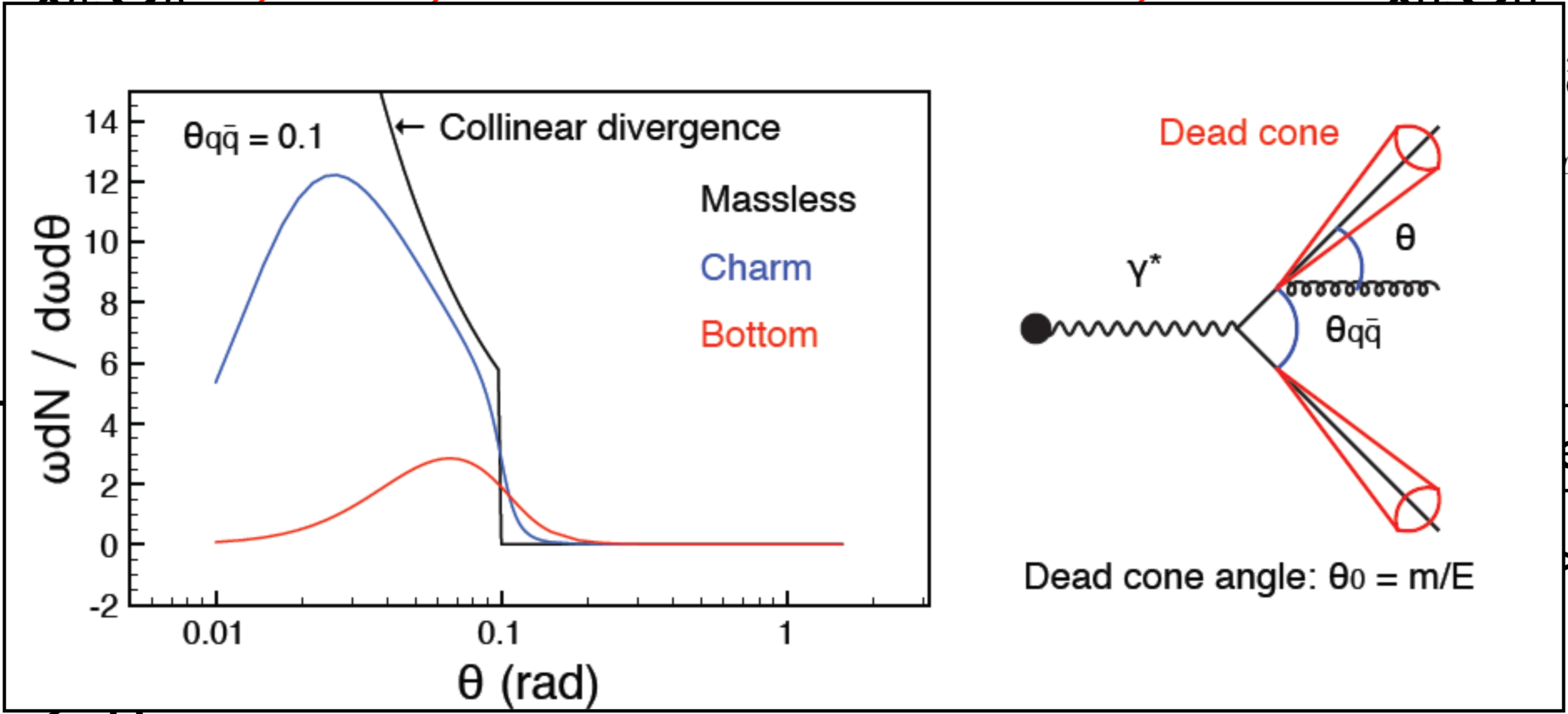
$$D_{q\bar{q}} = \theta_{q\bar{q}} t_{coh}, \quad t_{coh} \sim \omega/k_T^2, \quad D_g \sim 1/k_T$$

$$D_{q\bar{q}} = \frac{\theta_{q\bar{q}}}{k_T \theta_g} > D_g \Rightarrow \theta_g < \theta_{q\bar{q}}$$

Radiation: dead cone, ang. ordering

$x_{n-1},k_{T,n-1}$ $x_{n-2},k_{T,n-2}$ $x_1,k_{T,1}$

x_n,Q_n x_0,Q_0



es.

θ_{qqbar}

$$D_{q\bar{q}} = \frac{\theta_{q\bar{q}}}{k_T\theta_g} > D_g \Rightarrow \theta_g < \theta_{q\bar{q}}$$

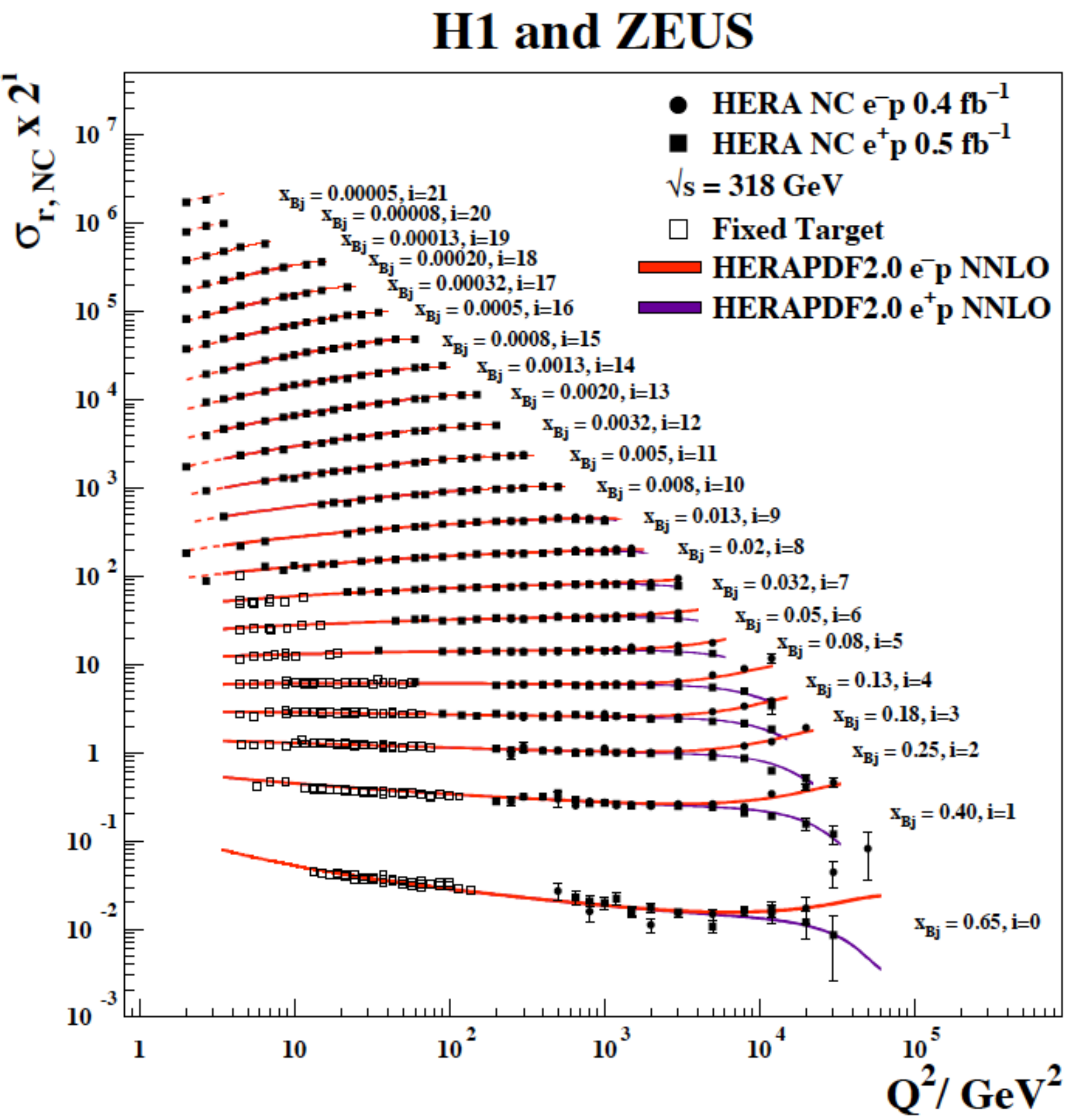
DIS: legacy from HERA

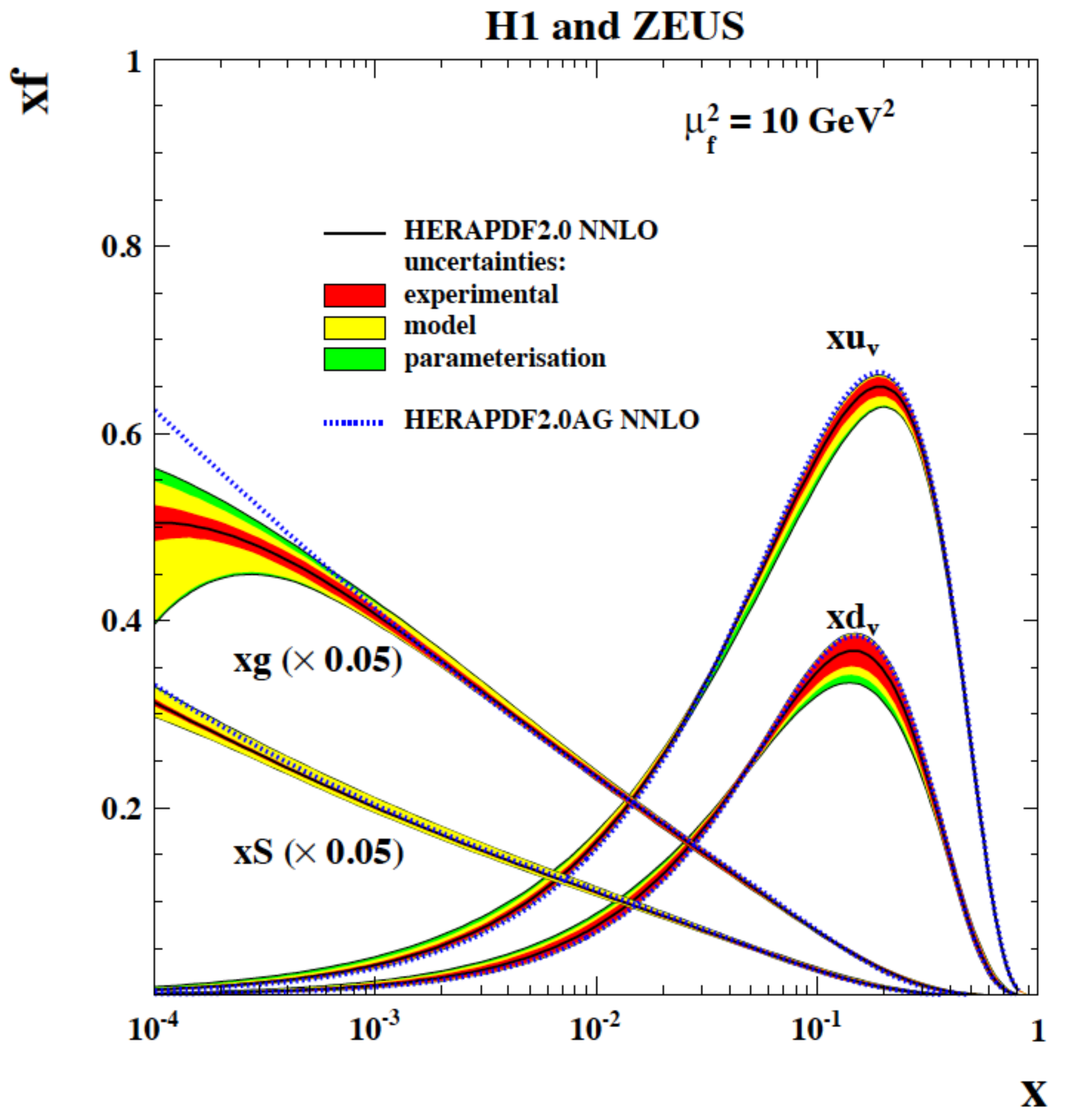
- Three pQCD-based alternatives to describe ep and eA data (differences at moderate $Q^2(>\Lambda^2_{QCD})$ and small x):

➜ DGLAP evolution (fixed order pQCD).

➜ Resummation schemes (of $[\alpha_s \ln(1/x)]^n$ terms).

➜ Non linear effects: saturation.





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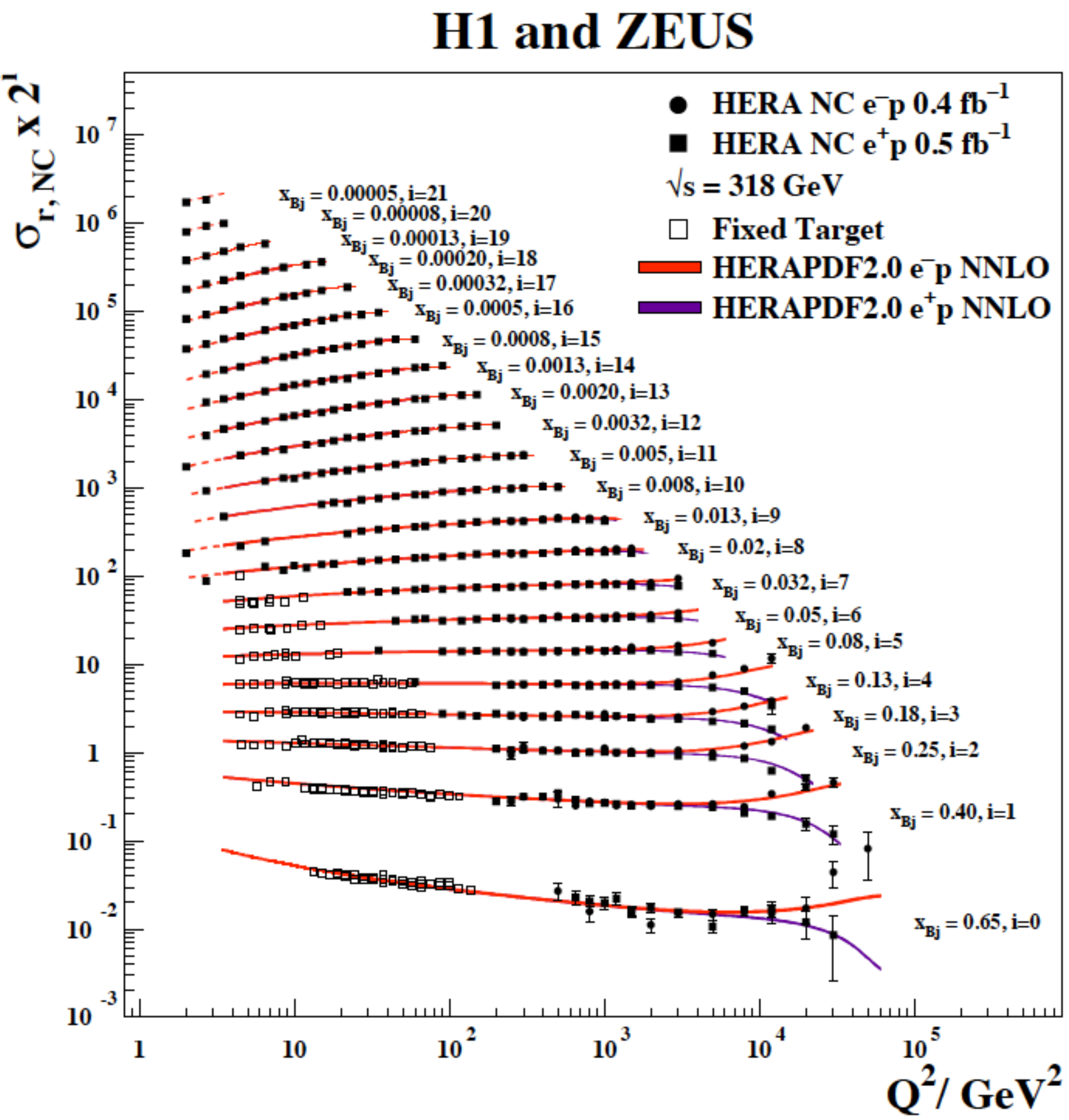
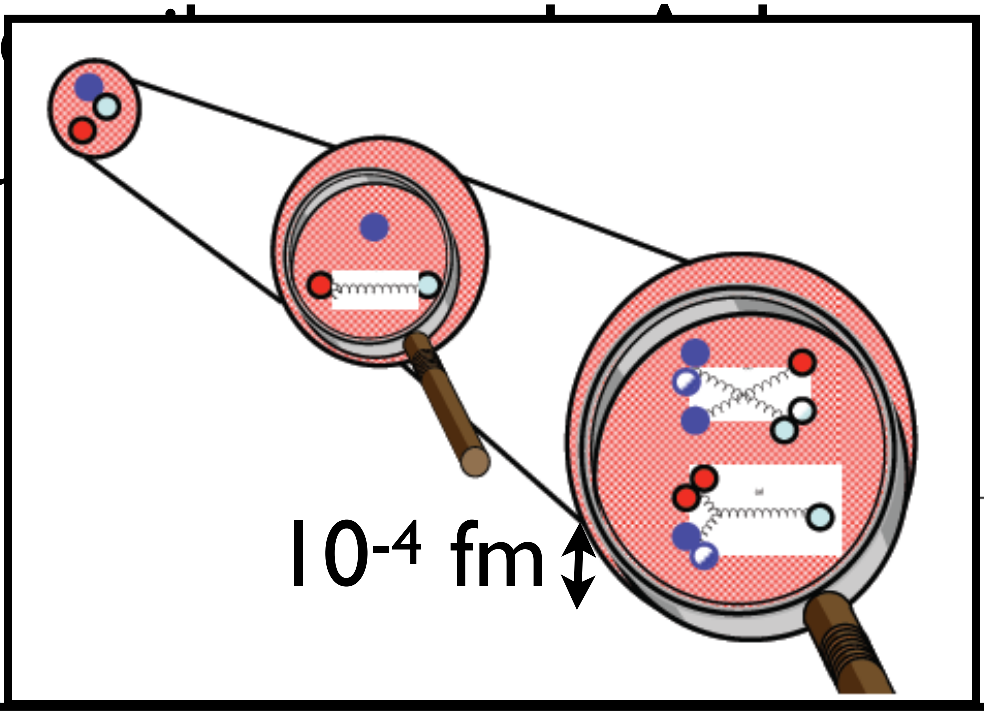
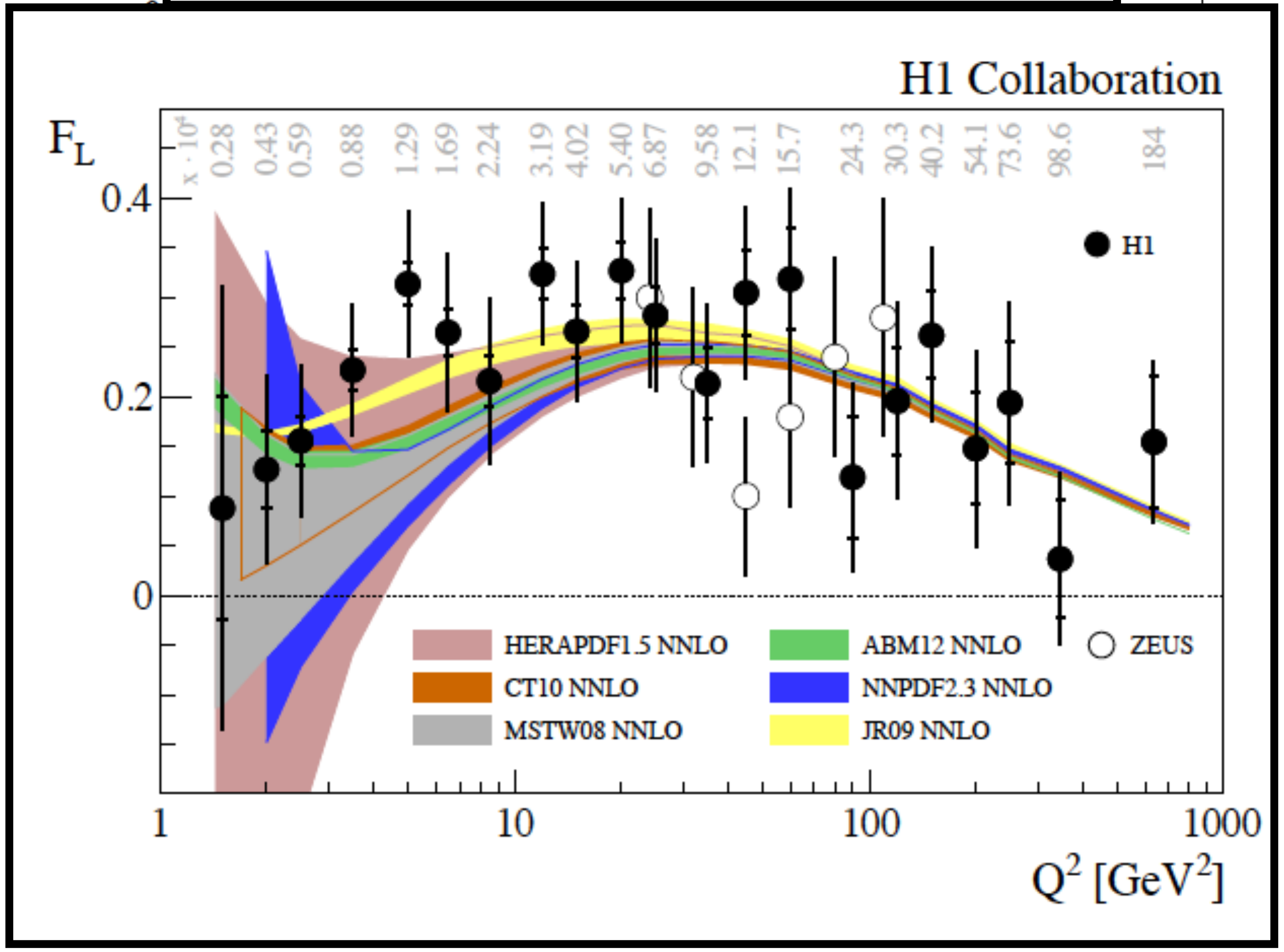
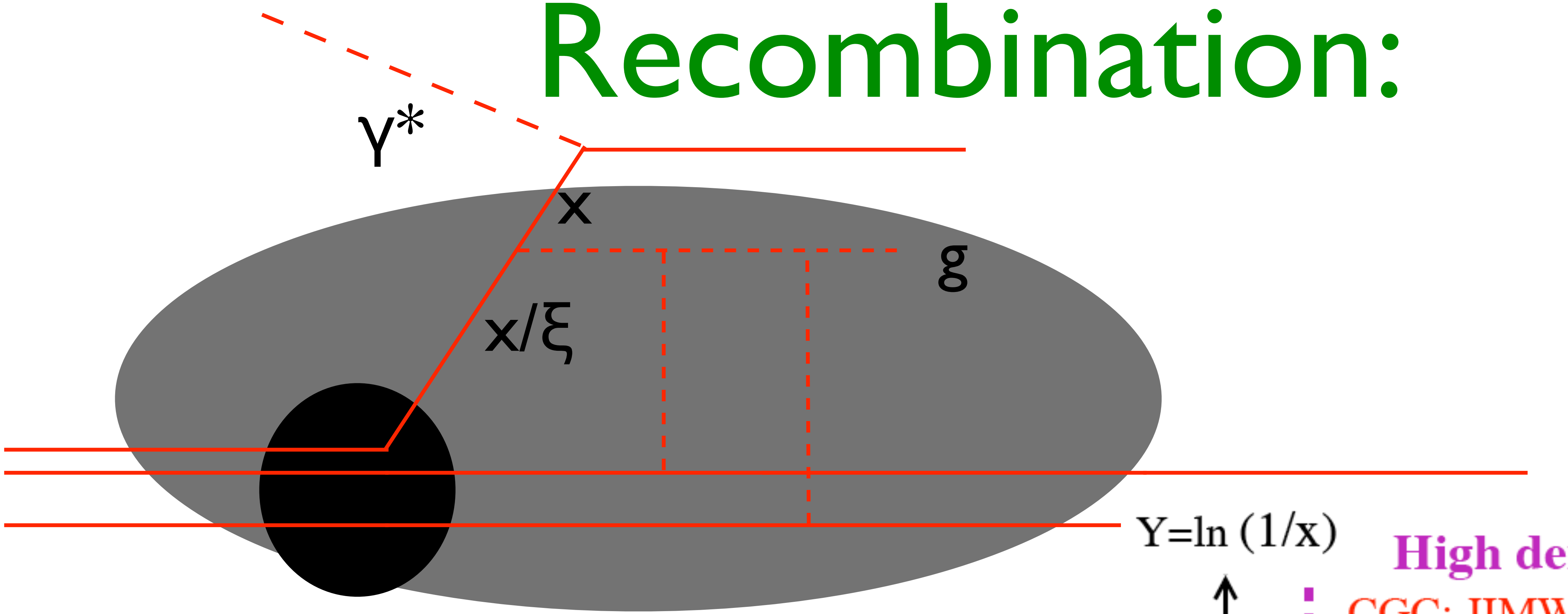






Recombination:

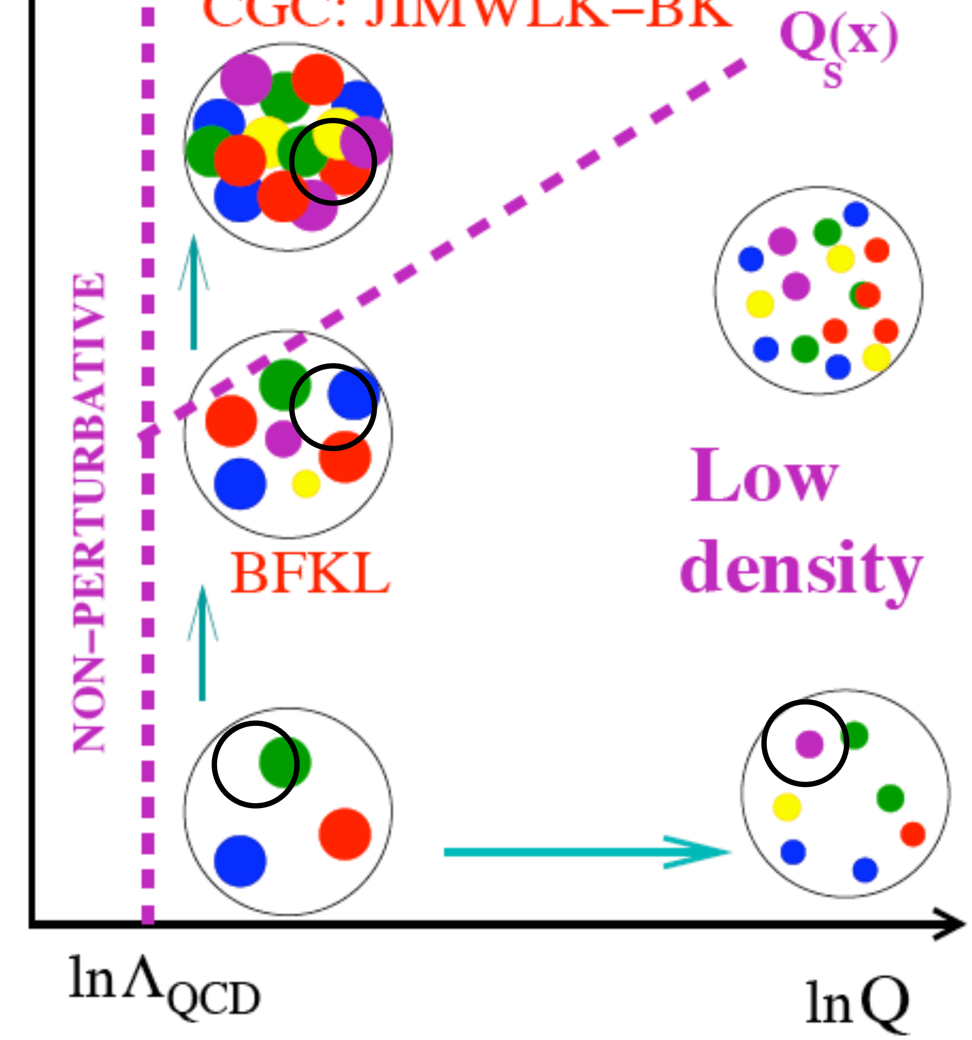


- At small x (gluon dominated), with the gluon increasing power-like with 1/x, we go from a linear regime:

$\Delta xg \propto K\otimes xg$,

to a non-linear, recombination one whose first correction reads:

$\Delta xg \propto K\otimes xg$ - $\mathbf{cA^{1/3}}(xg)^2$.



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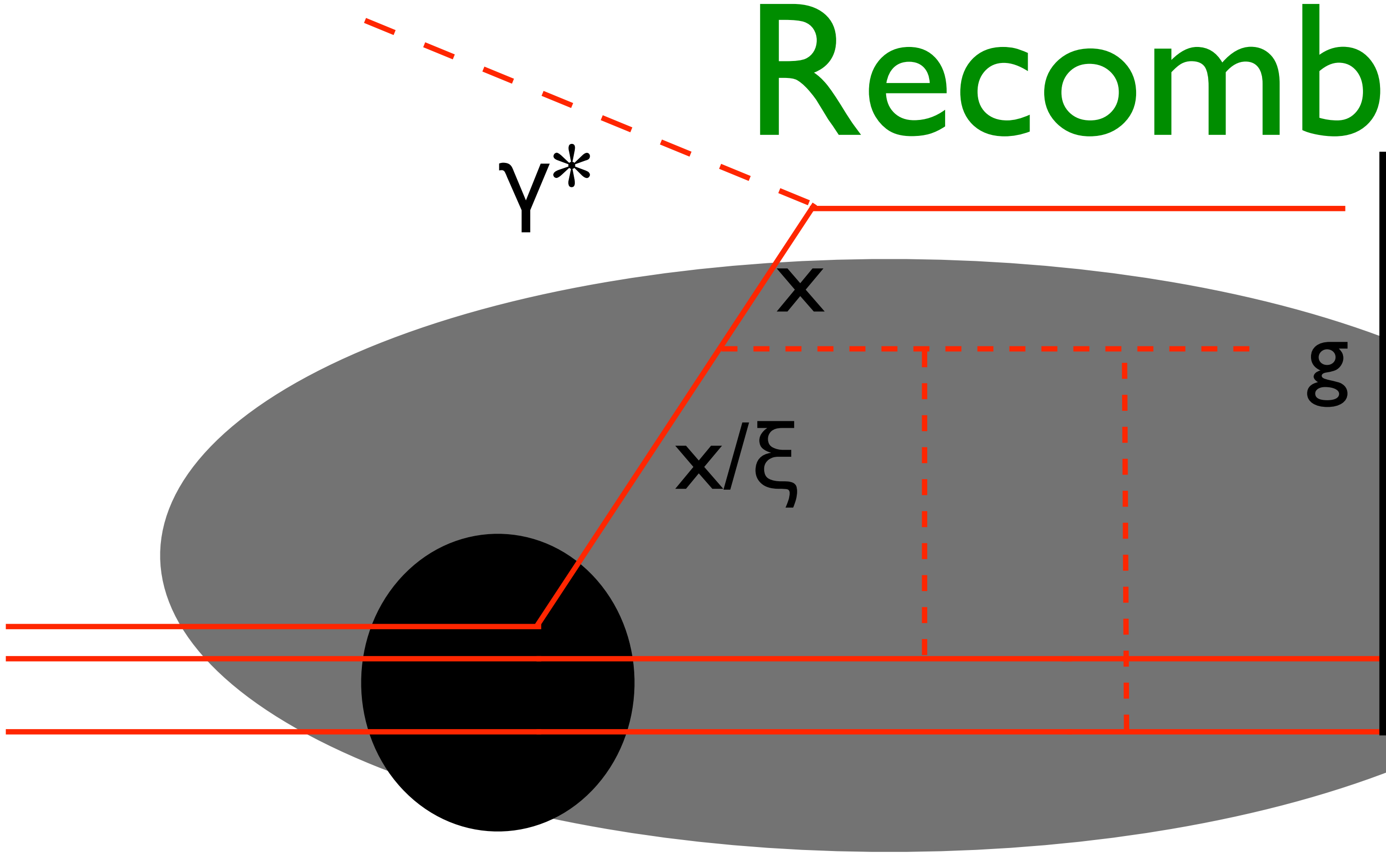

Non-linear saturation effects enhanced by increasing energy and nuclear size!!!

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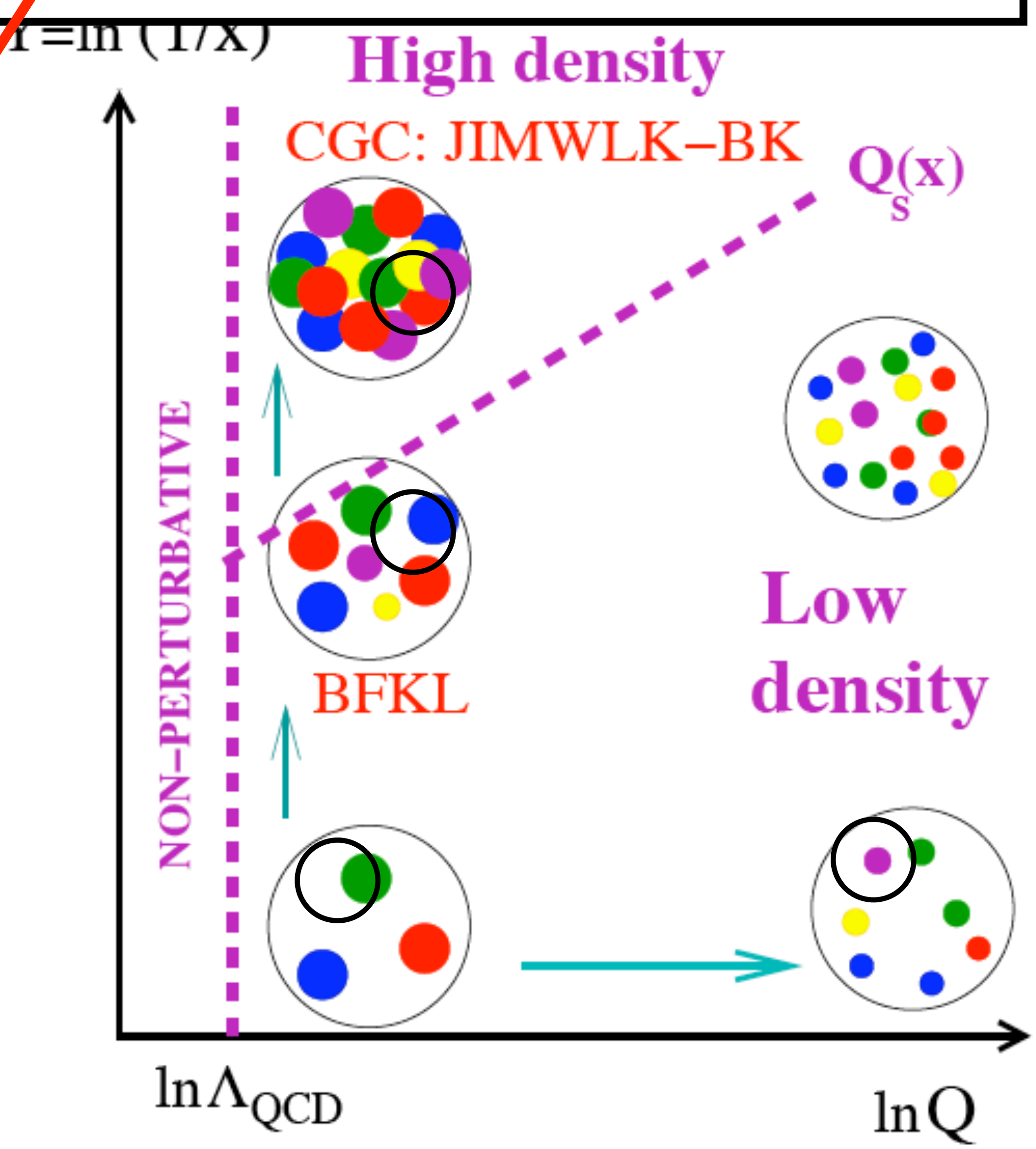
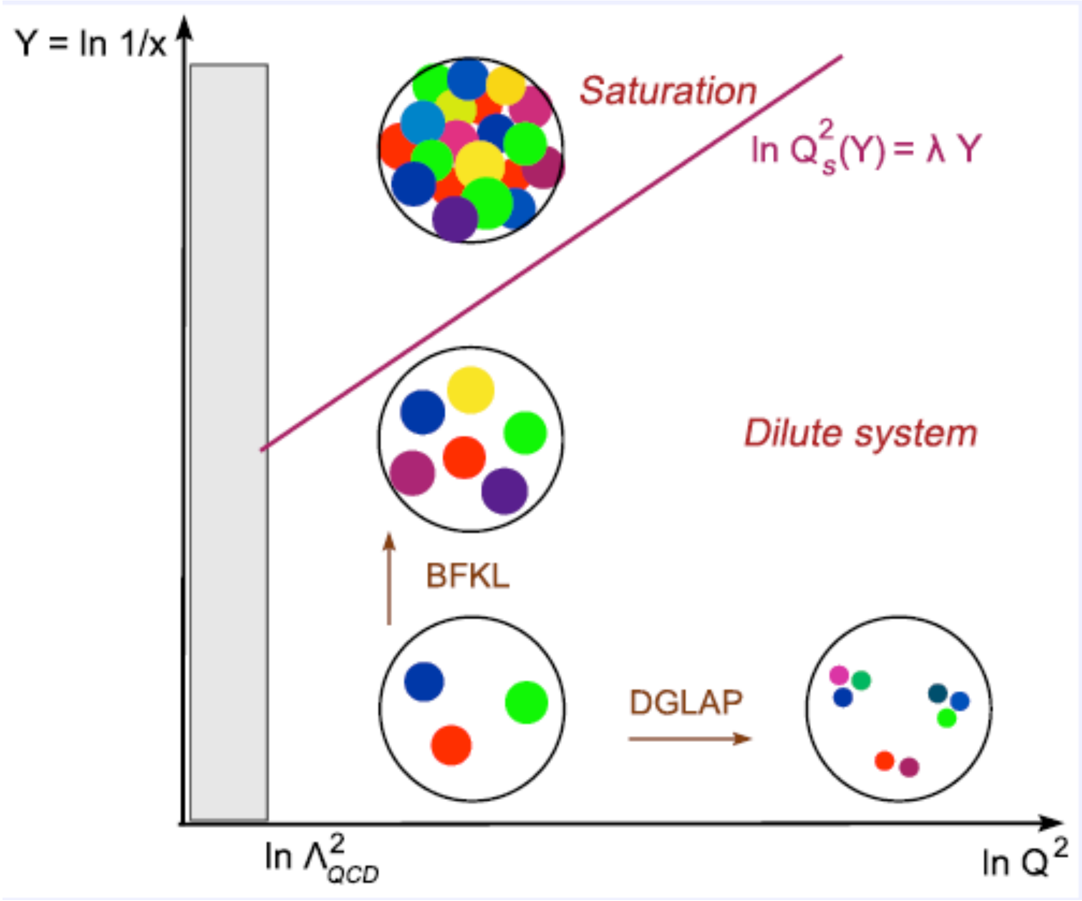


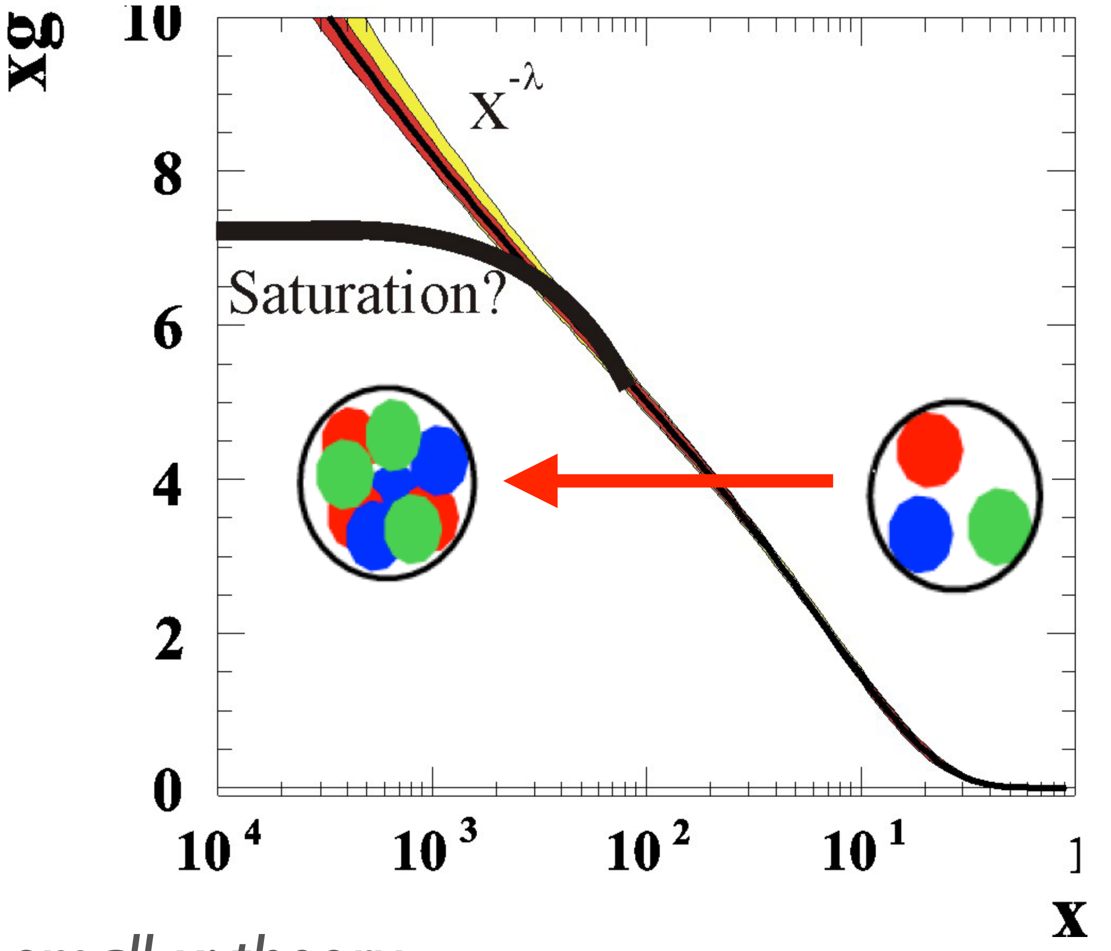
Saturation:

- Standard fixed-order perturbation theory (DGLAP, linear evolution) must eventually fail:

➜ Large logs e.g. $\alpha_s \ln(1/x) \sim 1$: resummation (BFKL,CCFM,ABF,CCSS).

➜ High density ⇒ linear evolution must not hold: saturation, either perturbative (CGC) or non-perturbative.





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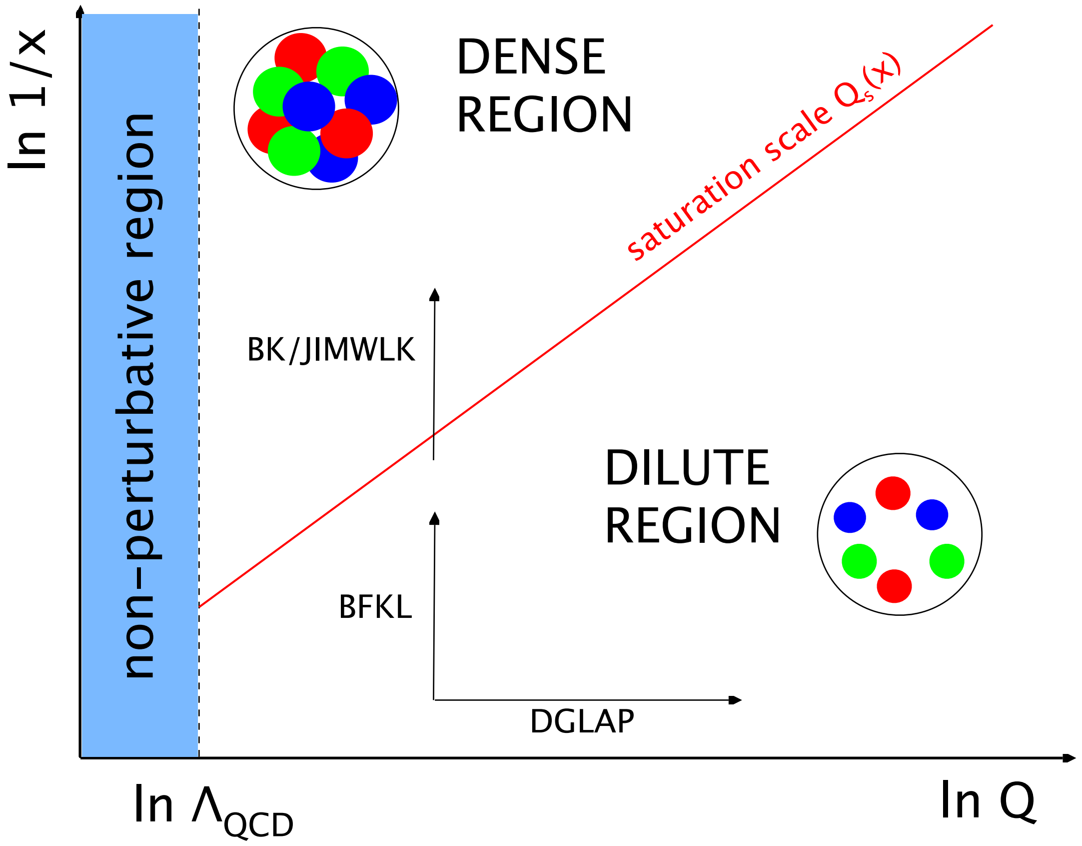
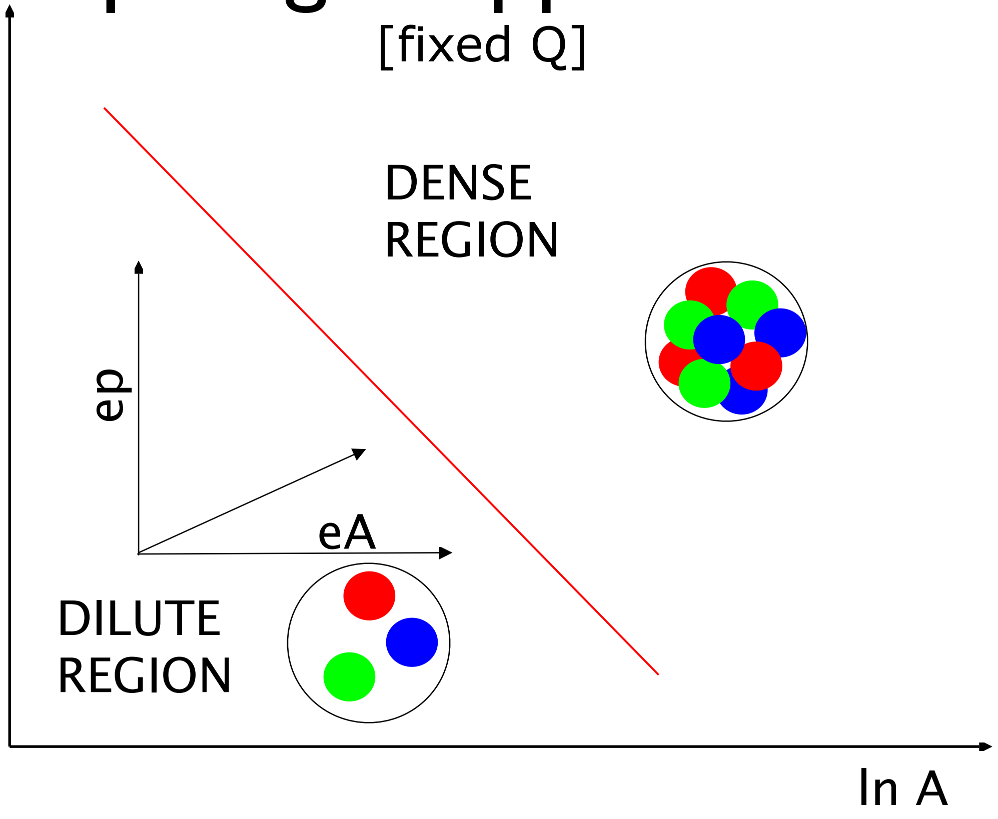
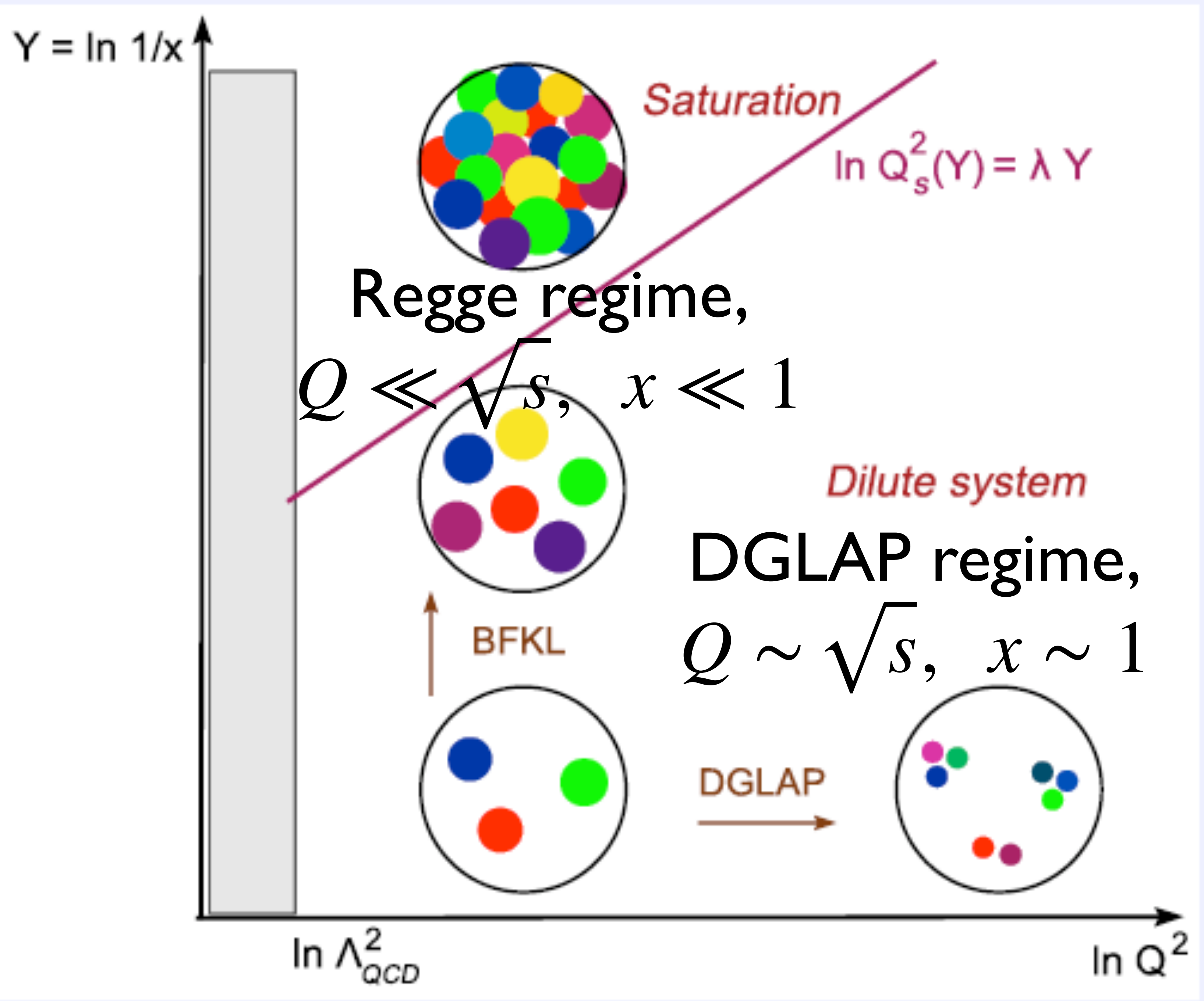
- Non-linear effects driven by density⇒2-pronged approach: ↓x/↑A.





Saturation:



Regge regime,
$Q \ll \sqrt{s}, \quad x \ll 1$

DGLAP regime,
$Q \sim \sqrt{s}, \quad x \sim 1$

Origin in the early 80's: Gribov-Levin-Ryskin, Mueller, McLerran-Venugopalan.

Our aims: understanding

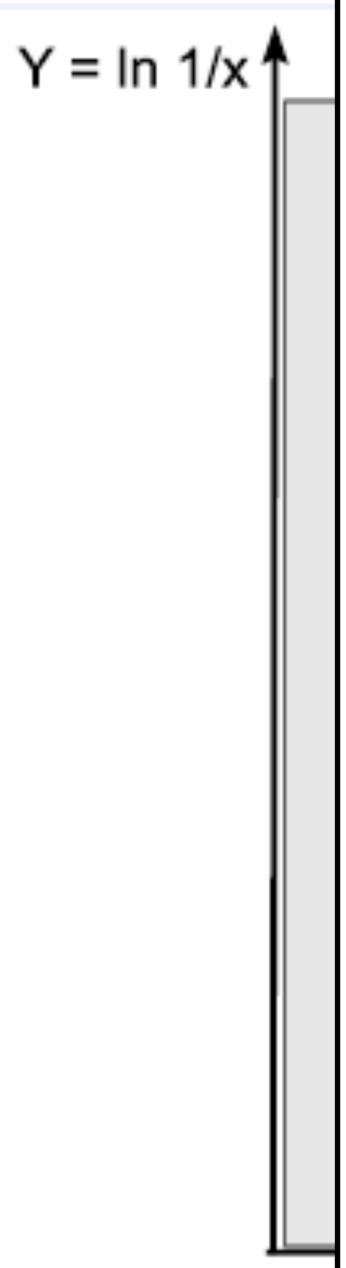
➜ The implications of unitarity in a QFT.

➜ The behavior of QCD at large energies.

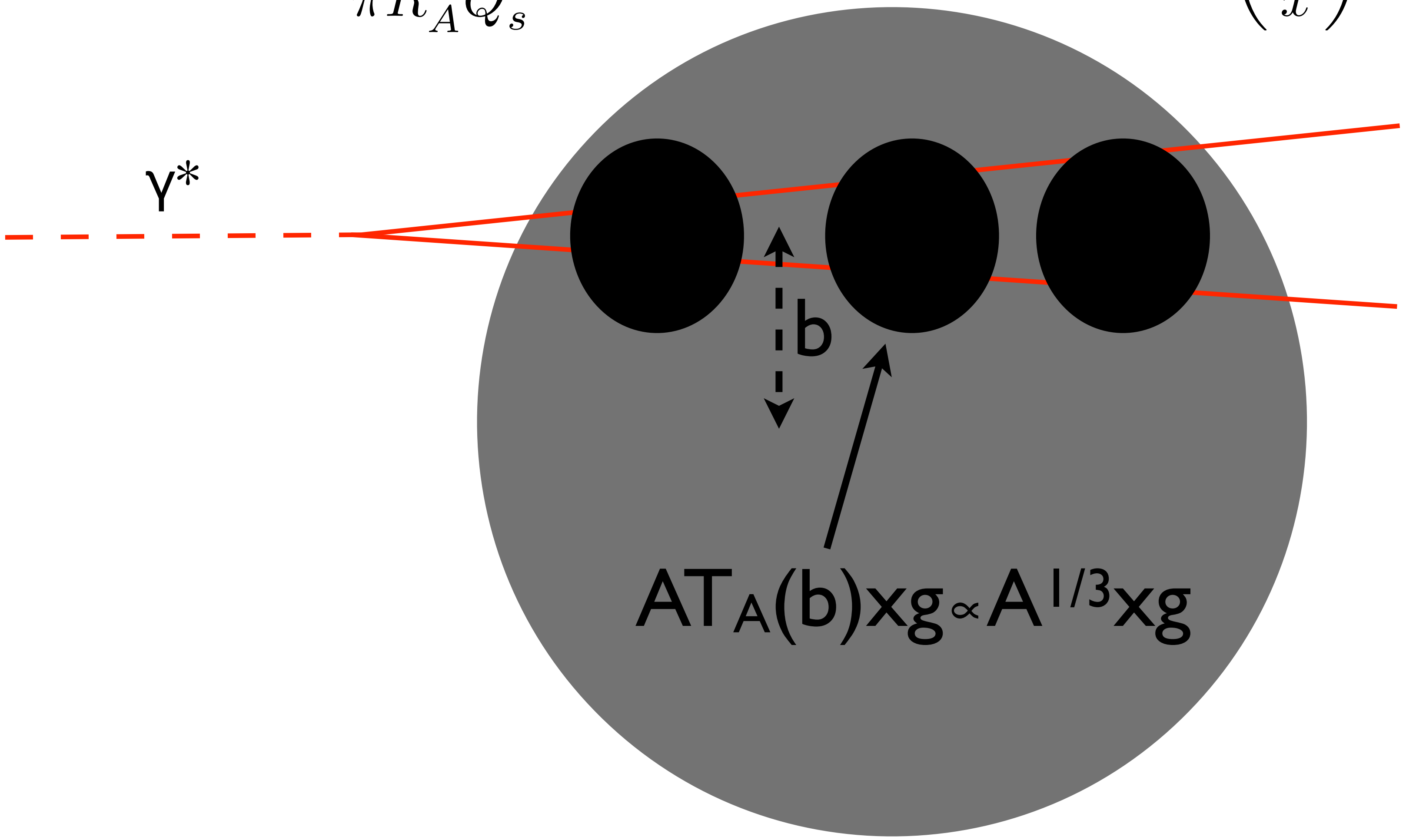
➜ The hadron wave function at small x.

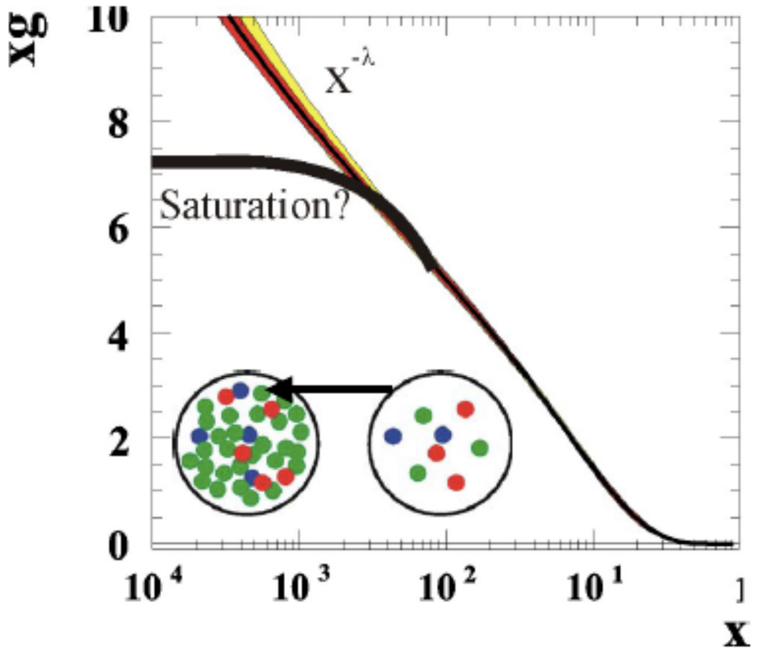
➜ The initial conditions for the creation of a dense medium in heavy-ion collisions.

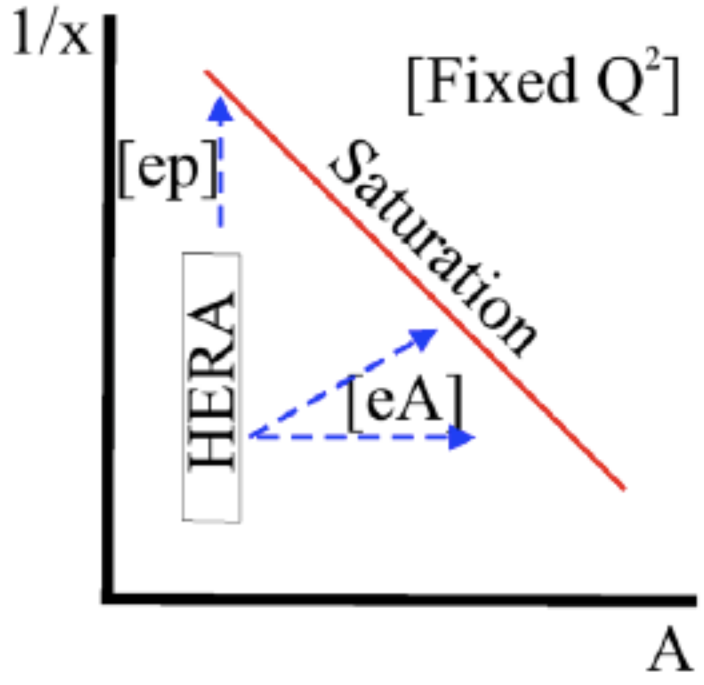
Saturation:



$$\sigma\rho \sim 1 \Longrightarrow \frac{Axg(x,Q_s^2)}{\pi R_A^2 Q_s^2} \sim 1 \Longrightarrow Q_s^2 \propto A^{1/3}x^{-\lambda} \sim \left(\frac{A}{x}\right)^{1/3}$$







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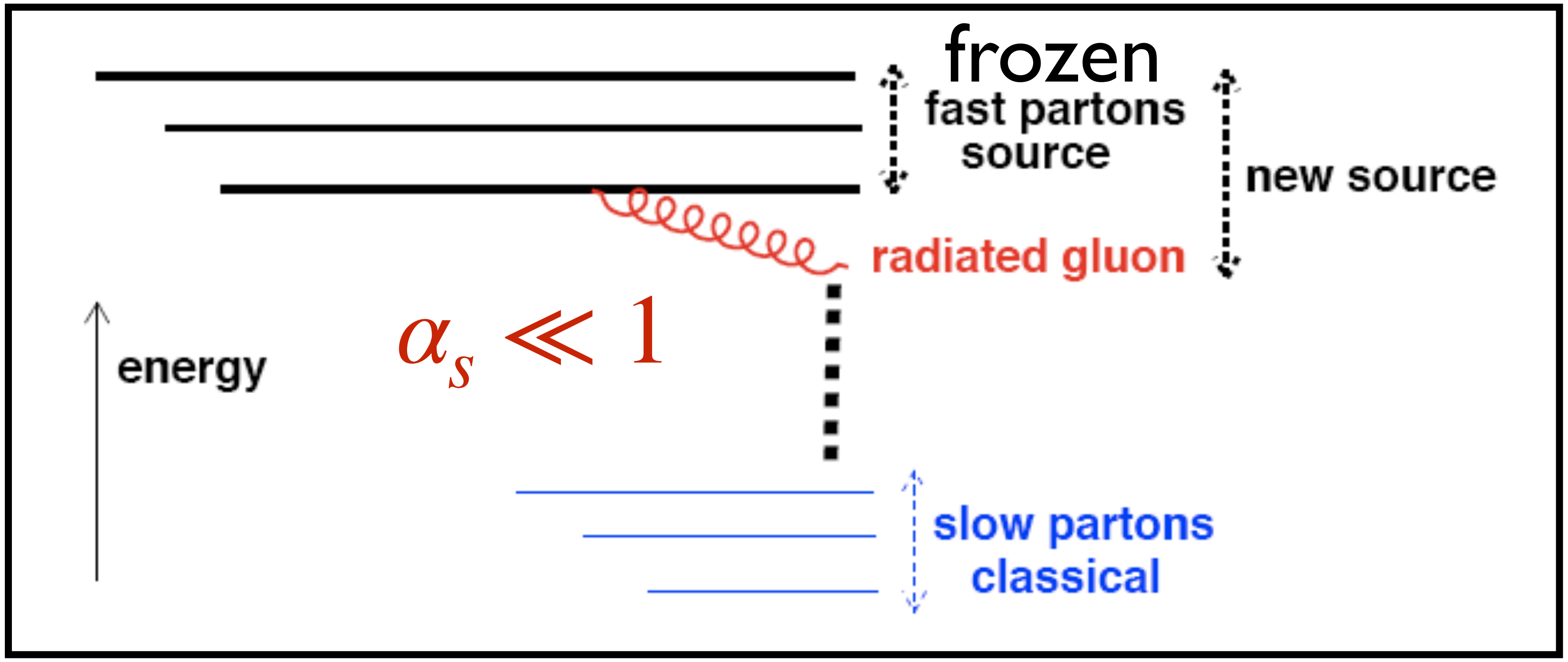
Color Glass Condensate:

- At **small enough x** for the projectile to interact coherently with the whole hadron, the CGC offers a (weak coupling but non-perturbative) description of the hadron wave function.

$$x \leq \frac{1}{2m_N R_A} \sim 0.1 A^{-1/3}$$

- The RG equation for the slow/fast separation (JIMWLK) was derived for scattering of a dilute projectile on a dense target. Gluon # becomes as high as it can be $O(\alpha_s^{-1})$ below Q_s^2.



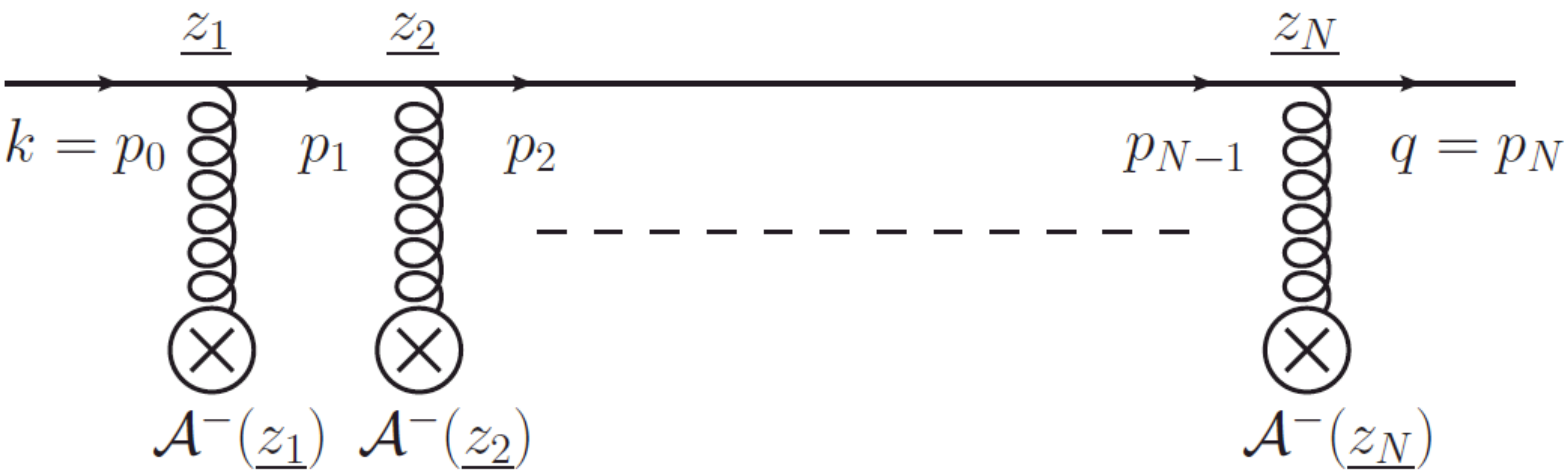
$$\rho_{proj} \simeq \mathcal{O}(1), \quad \rho_{tar} \simeq \mathcal{O}(1/\alpha_s)$$

- Its mean-field version (the Balitsky-Kovchegov equation) is the tool for phenomenology: numerically and analytically understood.

Wilson lines:

- The objects representing the hadron in the CGC are hadron (target) averages (color singlets) of Wilson lines (in the color representation of the particle traversing the field):

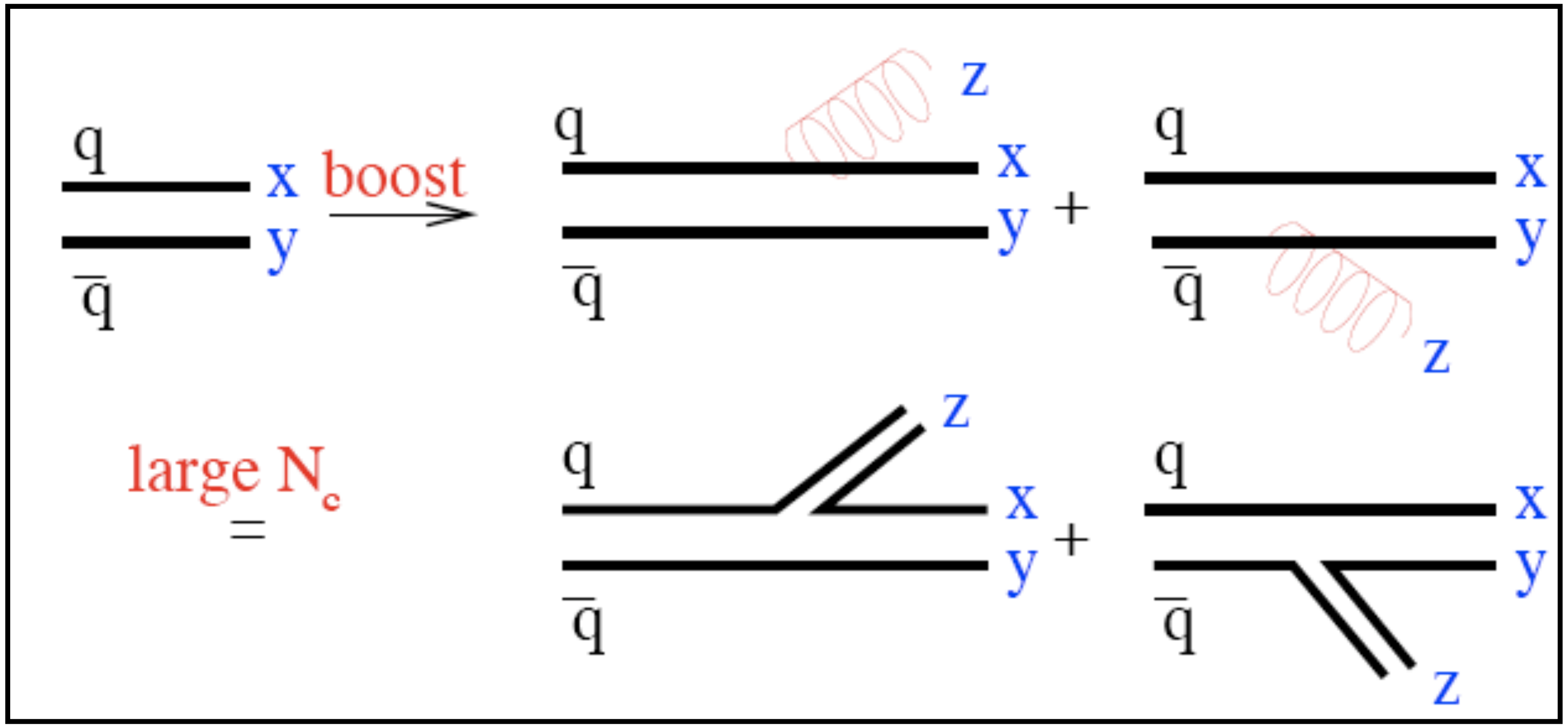
$$U(x^+, y^+, \mathbf{x}) \equiv \mathcal{P}^+ \exp\left\{ ig \int_{y^+}^{x^+} dz^+ A_T^{-,a}(z^+, \mathbf{x}) T^a \right\}$$



$$\langle U_{x_1} \cdots U_{x_n} \rangle_{\text{target}}, \quad U_x \equiv U(x^+, y^+, \mathbf{x})$$

- Wilson lines come through an eikonal approximation, and they resume multiple scatterings, they represent the color rotation of the particle traversing the field.

The BK equation:



- Neglecting correlations (expected O(1/N$_c$^{2}), actually smaller): **BK equation**.

$$N(r) = 1 - \langle U_r^{F\dagger} U_0^F \rangle_{\rm target}$$

$$\frac{\partial N(r,Y)}{\partial Y} = \int \frac{d^2z}{2\pi} K(\vec{r},\vec{r}_1,\vec{r}_2)\left[N(r_1,Y)+N(r_2,Y)-N(r,Y)\boxed{-N(r_1,Y)N(r_2,Y)}\right]$$

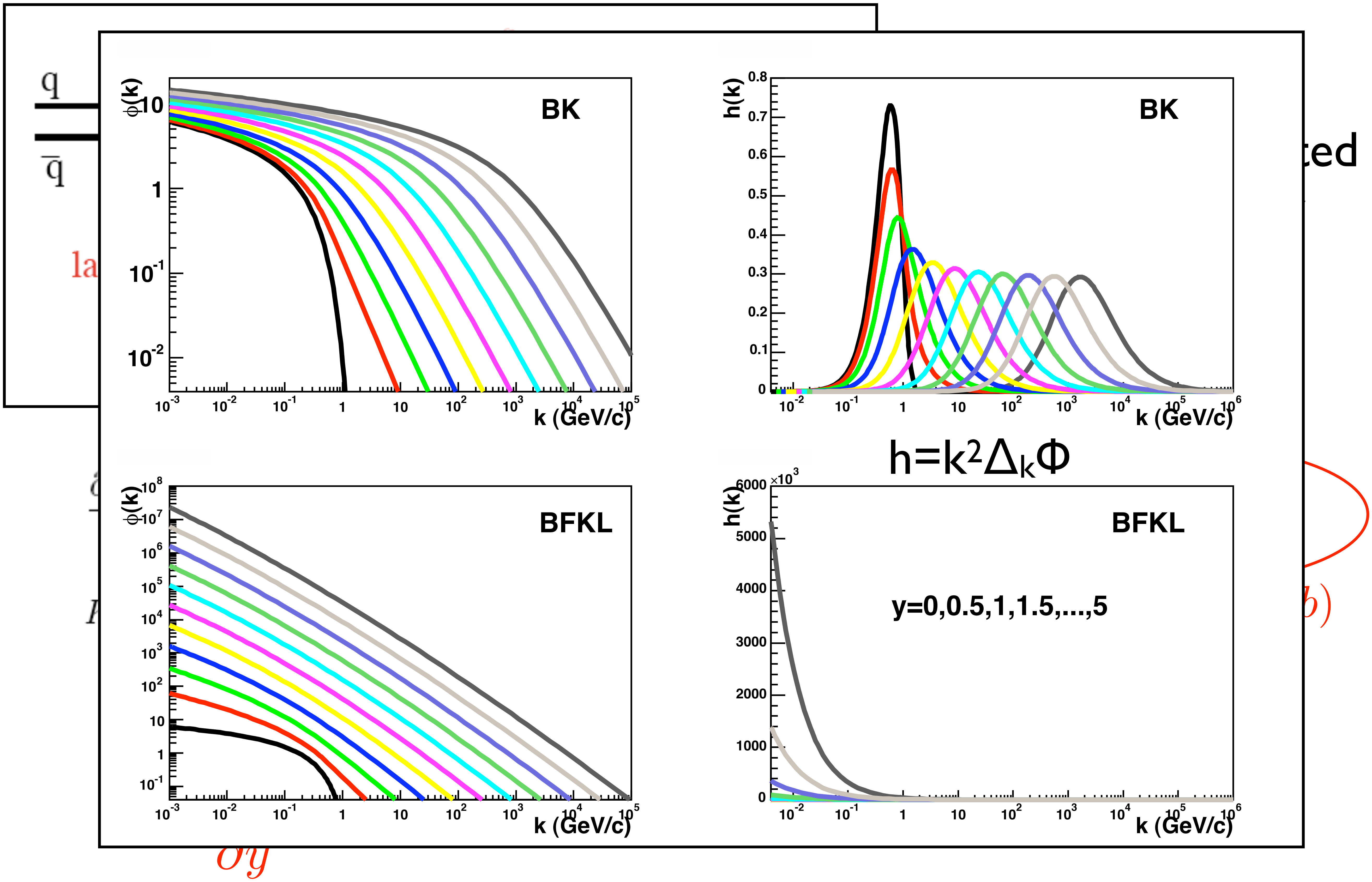
$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \bar{\alpha}_s \frac{r^2}{r_1^2 r_2^2} \quad , \quad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

$$\phi(Y,k,b) = \int \frac{d^2r}{2\pi r^2} e^{i\vec{k}\cdot\vec{r}} N(Y,r,b)$$

- Neglecting the dependence on impact parameter:

$$\frac{\partial \phi(y,k)}{\partial y} = H_{BFKL} \otimes \phi(y,k) - \phi^2(y,k), \quad y = \bar{\alpha}_s Y$$

The BK equation:



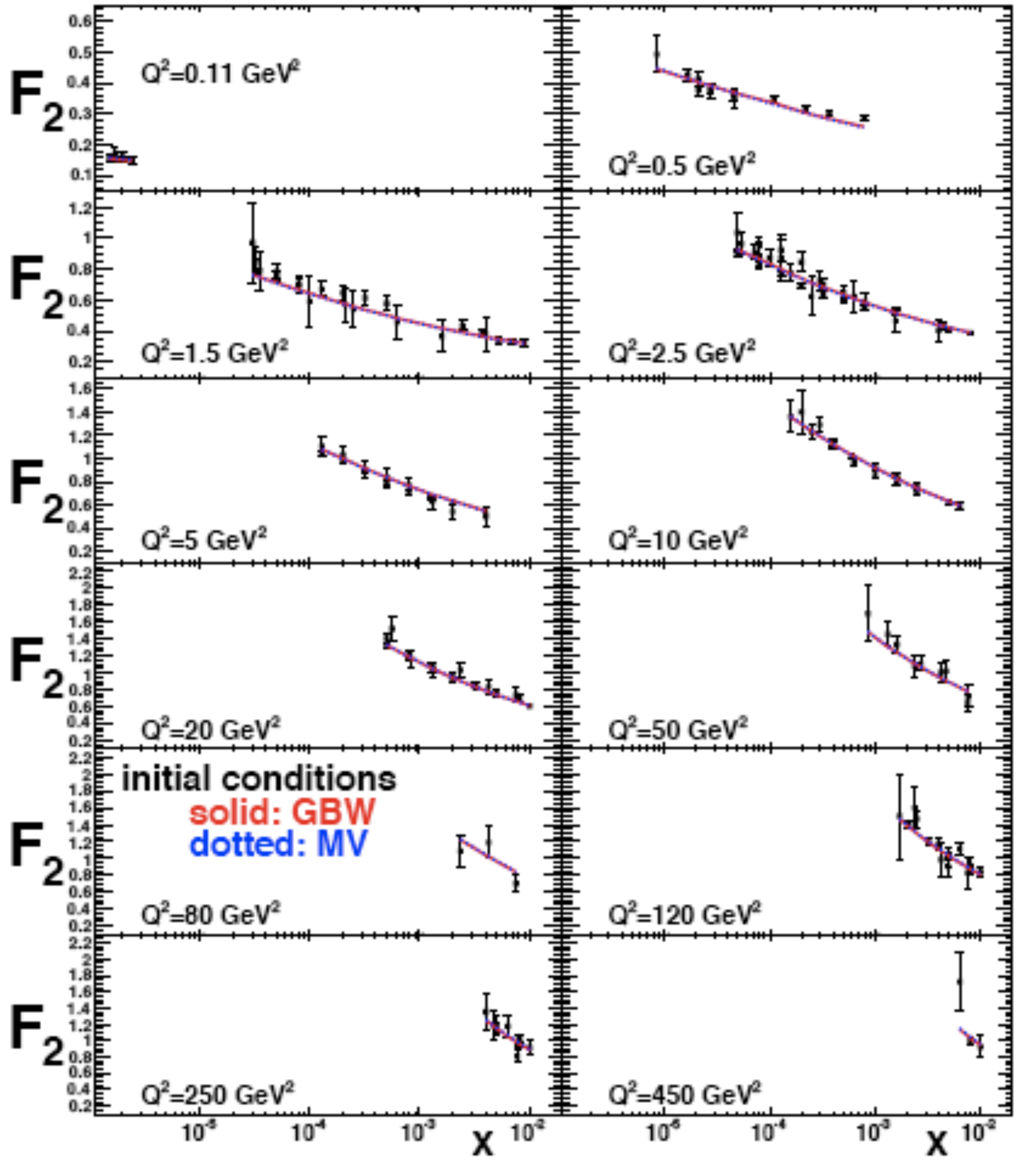
$h=k^2\Delta_k\Phi$

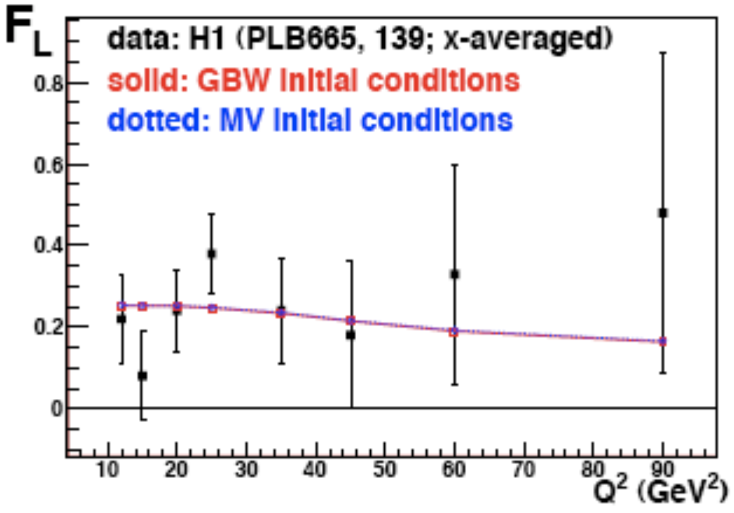
y=0,0.5,1,1.5,...,5

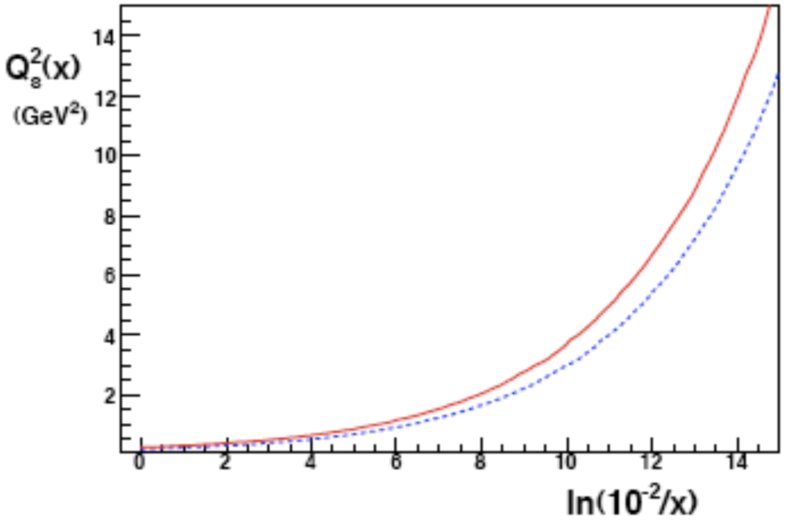
The BK equation:

The (NLL) BK equation is the actual tool for phenomenology:

- IR safe.
- Geometric scaling.
- Saturation scale $Q_s^2 \propto \exp(\lambda Y)$ [fc], $\exp(c\sqrt{Y})$ [rc], $Y=\ln(1/x)$.



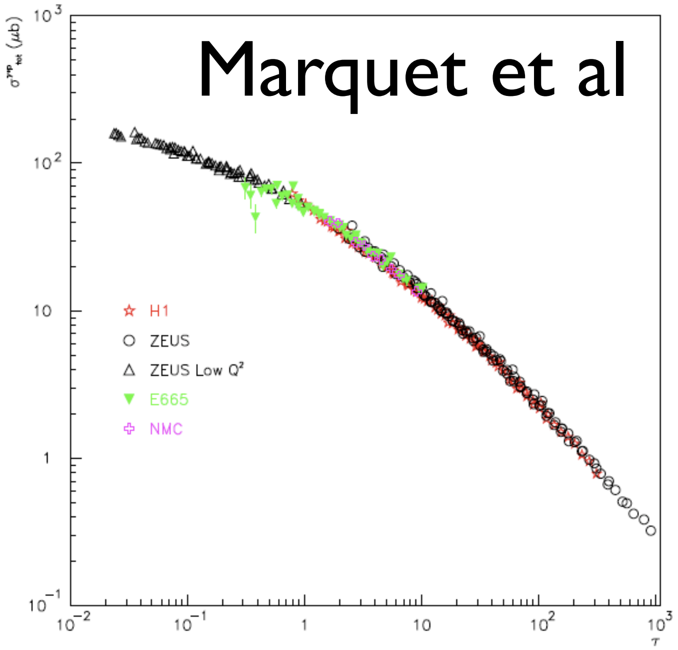


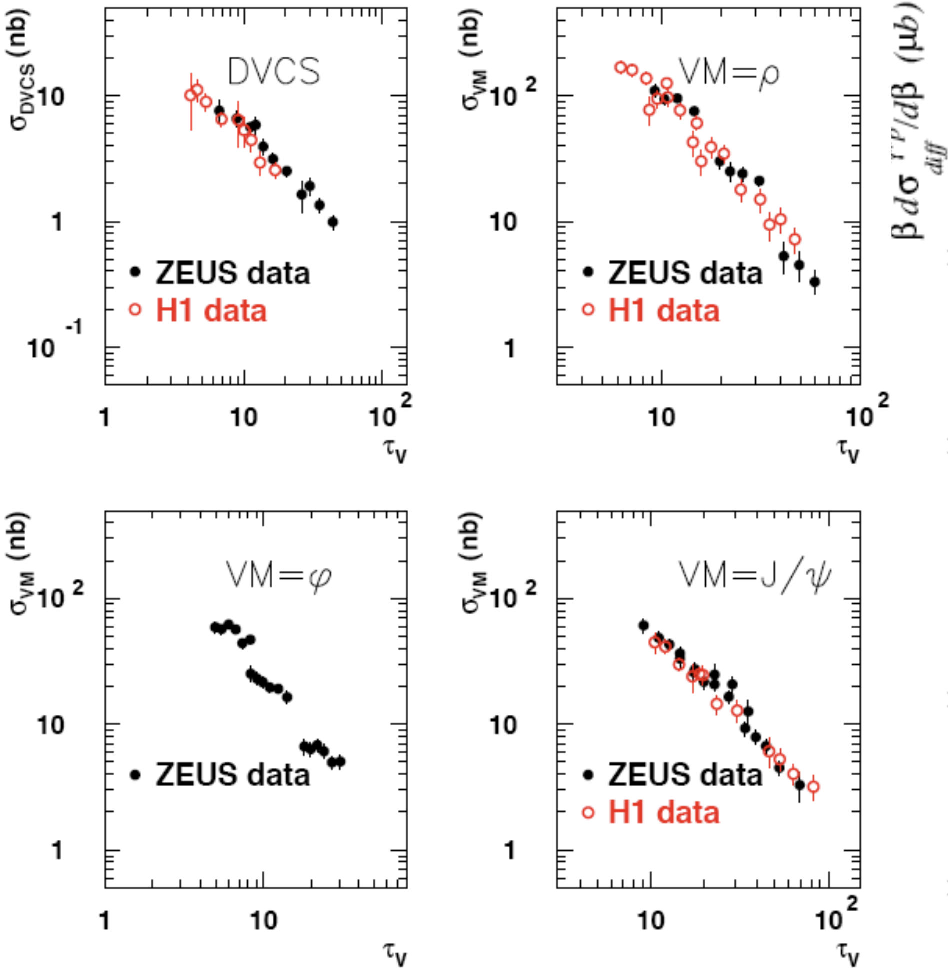


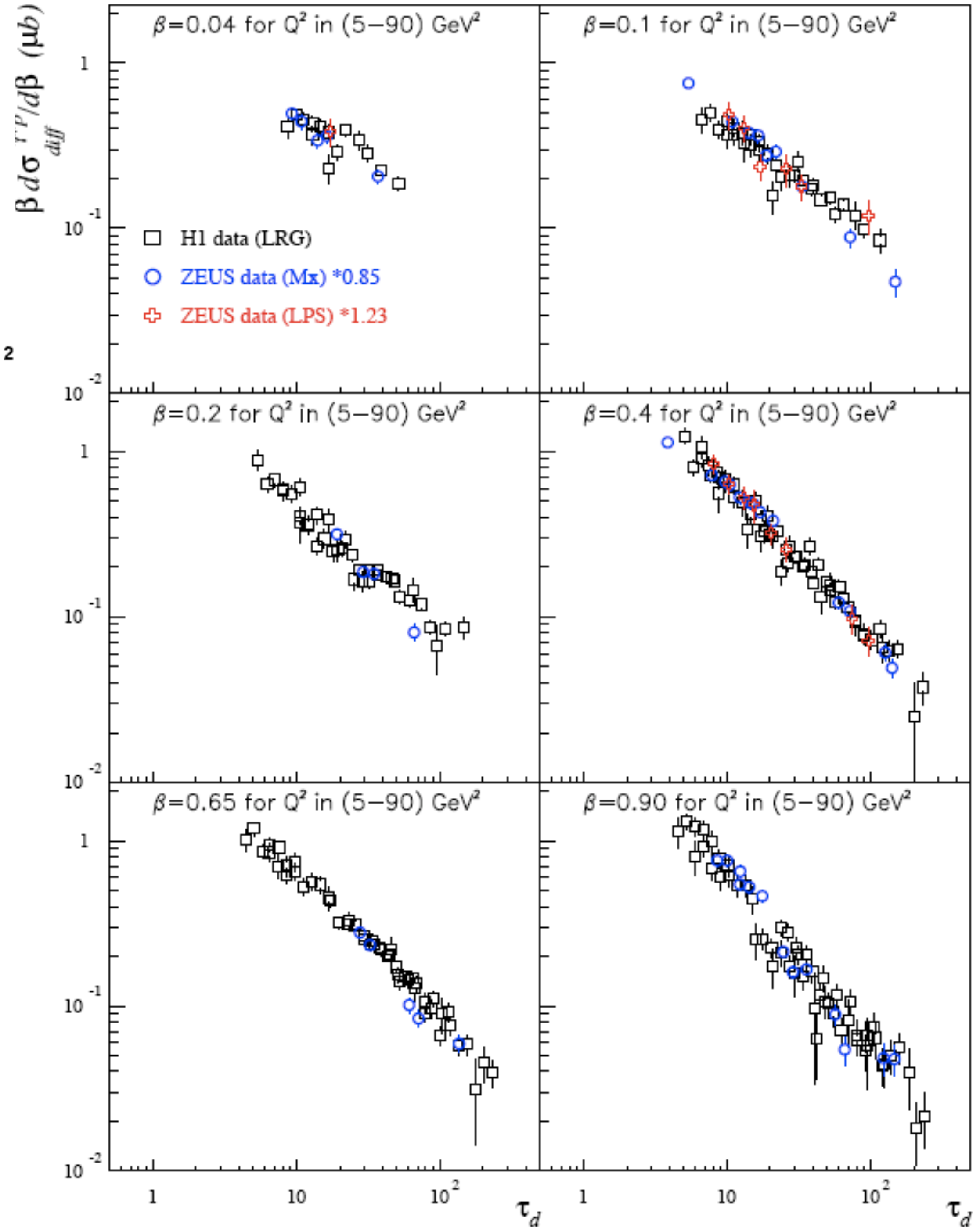
Geometric scaling:

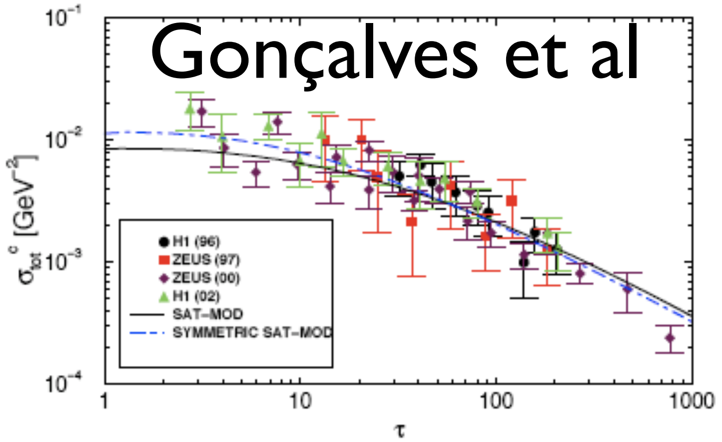
- The key feature in data is **geometric scaling** (Golec-Biernat et al).

$$\tau=\frac{Q^2}{Q_s^2(x)},\quad \tau_D=\frac{Q^2}{Q_s^2(x_P)}\ \ \text{at}\ \ \text{fixed}\ \ \beta,\quad \tau_V=\frac{Q^2+M_V^2}{Q_s^2(x_P)}$$


Marquet et al



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$$Q_s^2(x)=\left(\frac{x_0}{x}\right)^{\lambda}$$

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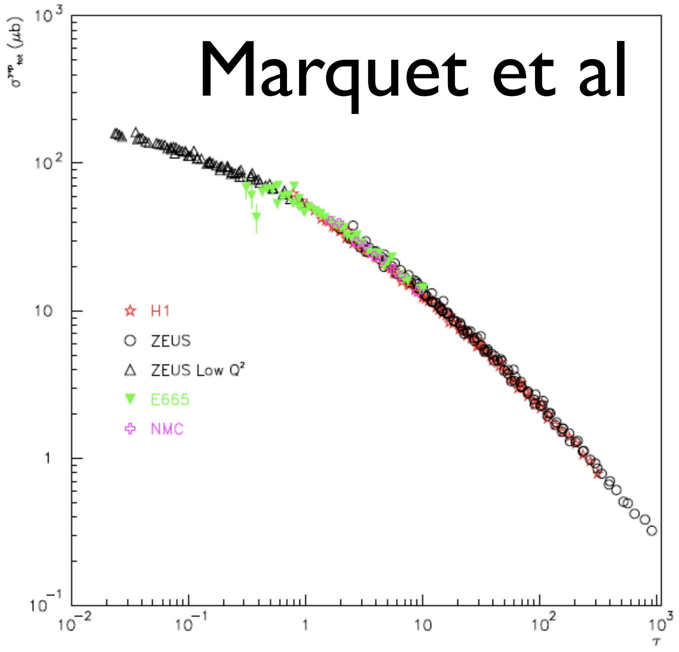
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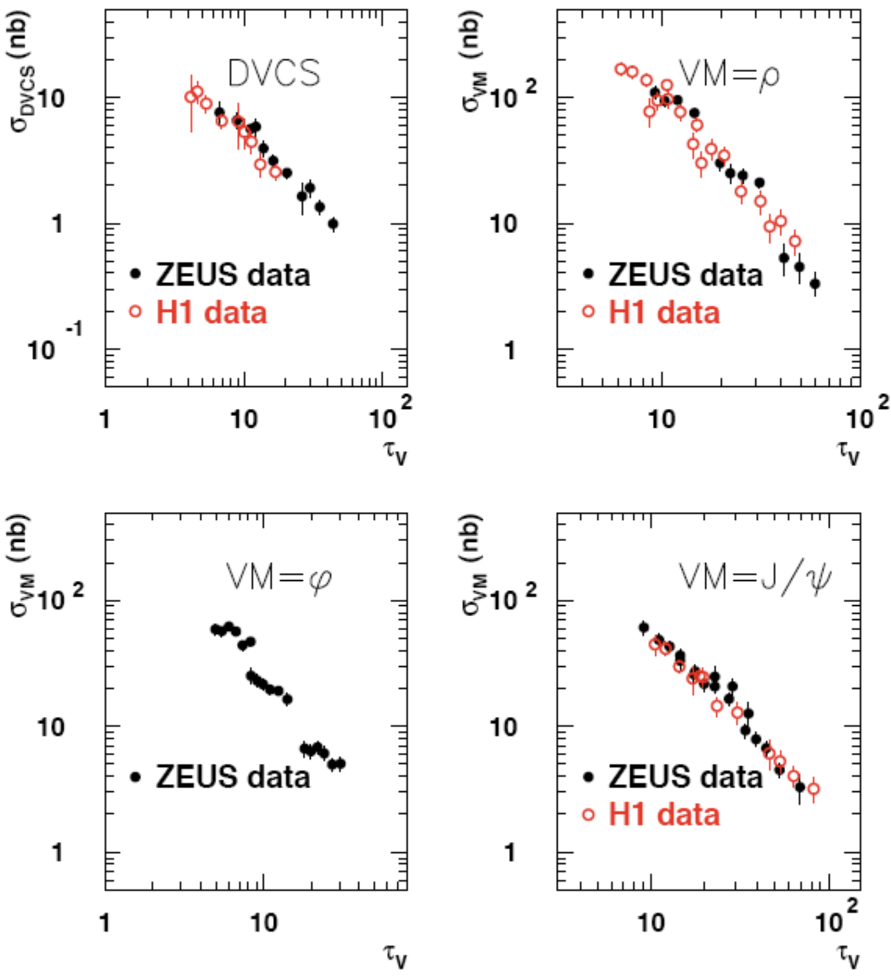

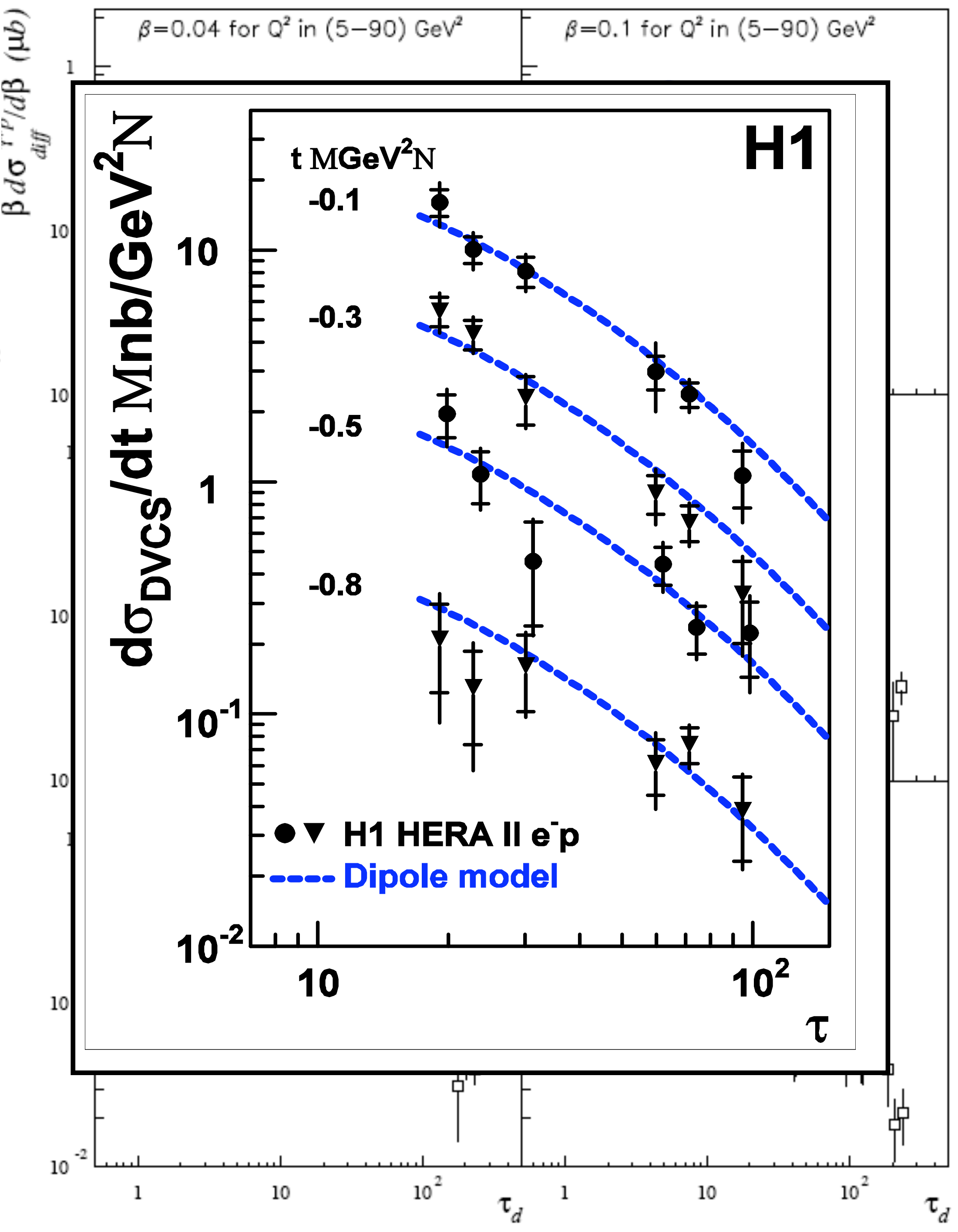

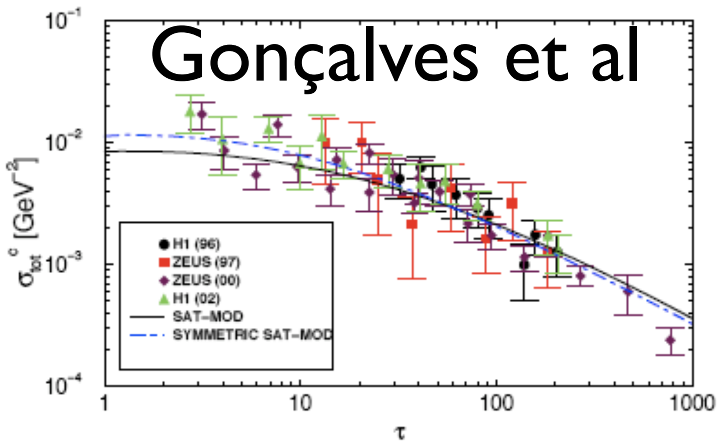
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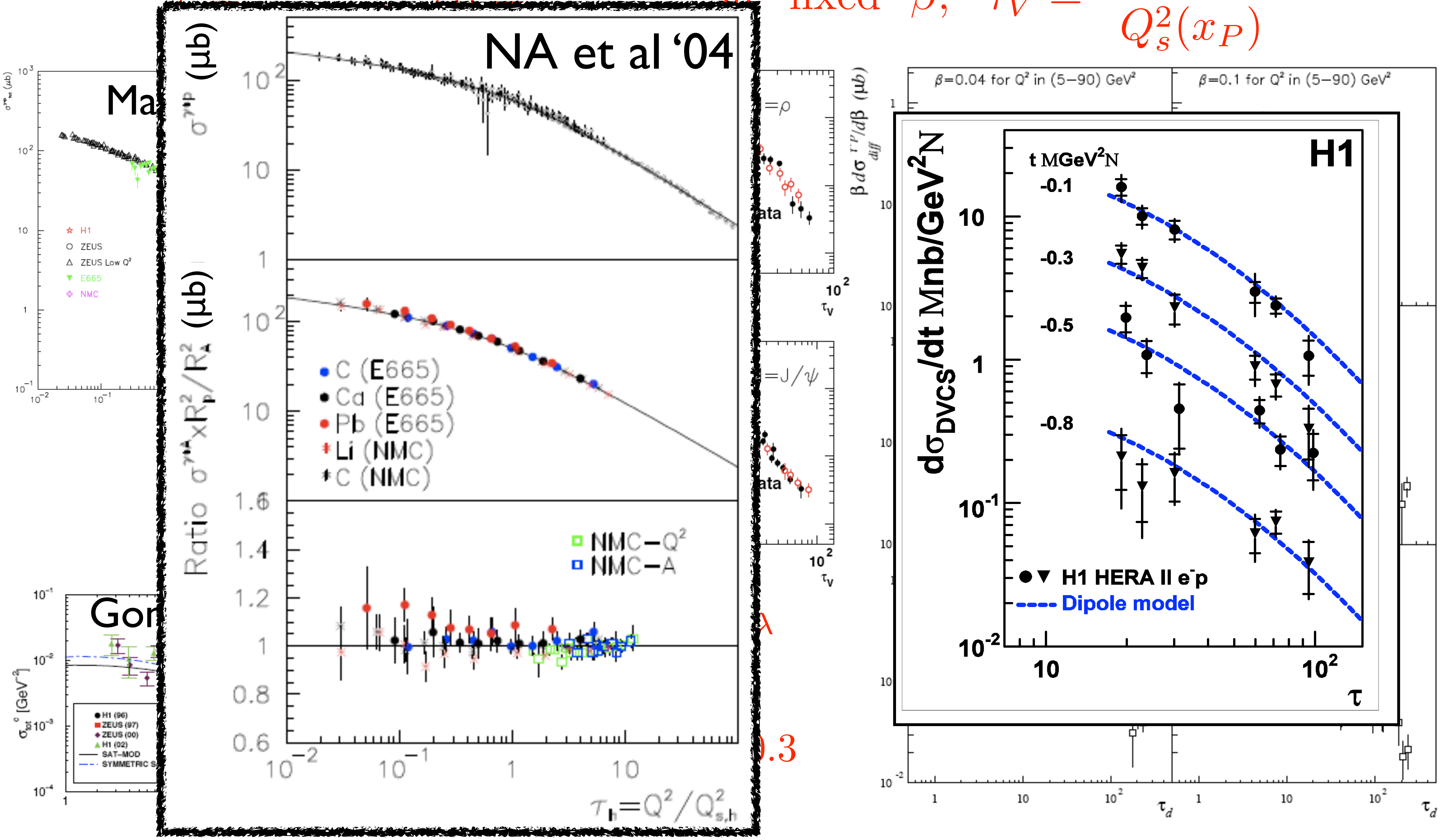


The dipole picture:

- Photon described by its hadronic fluctuations (long lived, $x<(m_N R)^{-1} \sim 0.1 A^{1/3}$)

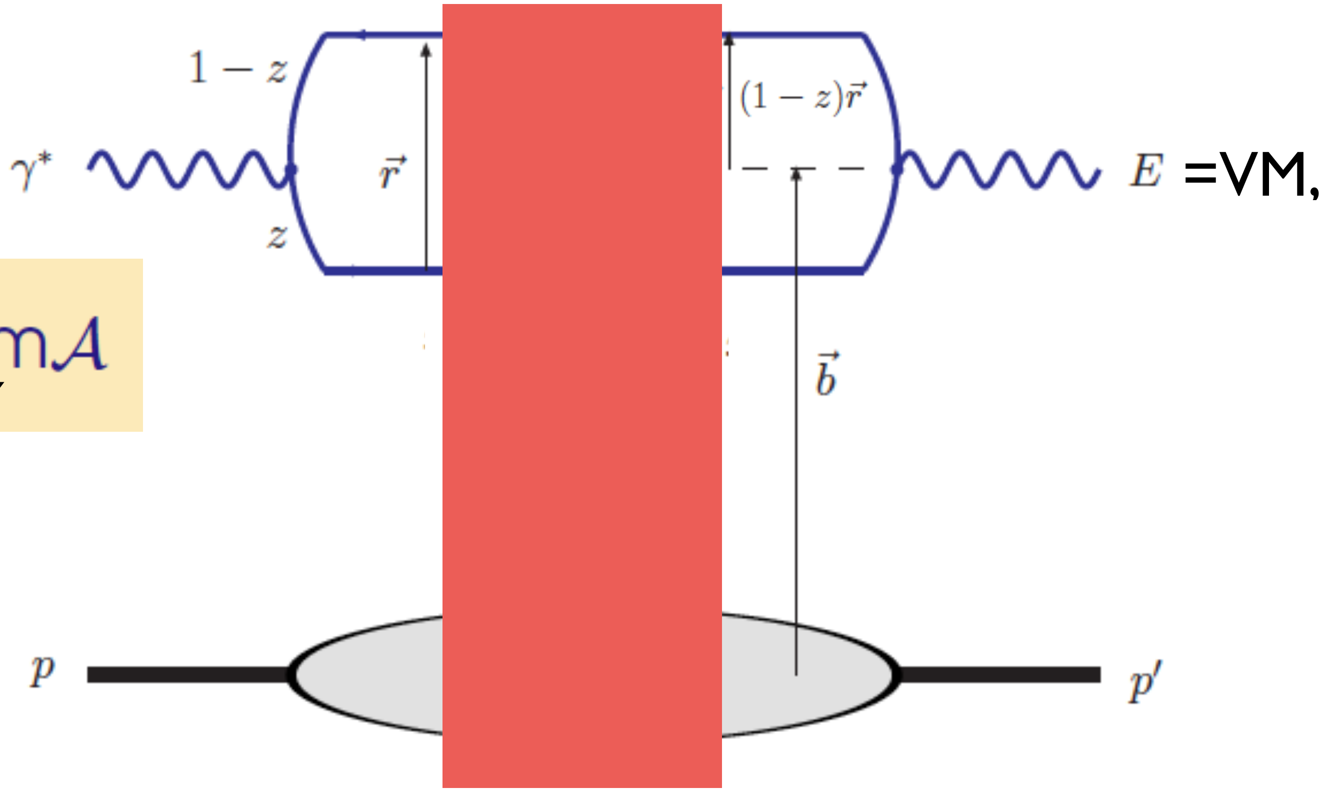
$$|\gamma^*\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}\gamma\rangle + |q\bar{q}gg\rangle + |q\bar{q}q\bar{q}\rangle \ldots$$

- Components at fixed transverse position during interaction (eikonal approximation).
- Picture useful for saturation (target at rest).

$$2Im\mathscr{A} = N(\mathbf{r}_\perp)$$

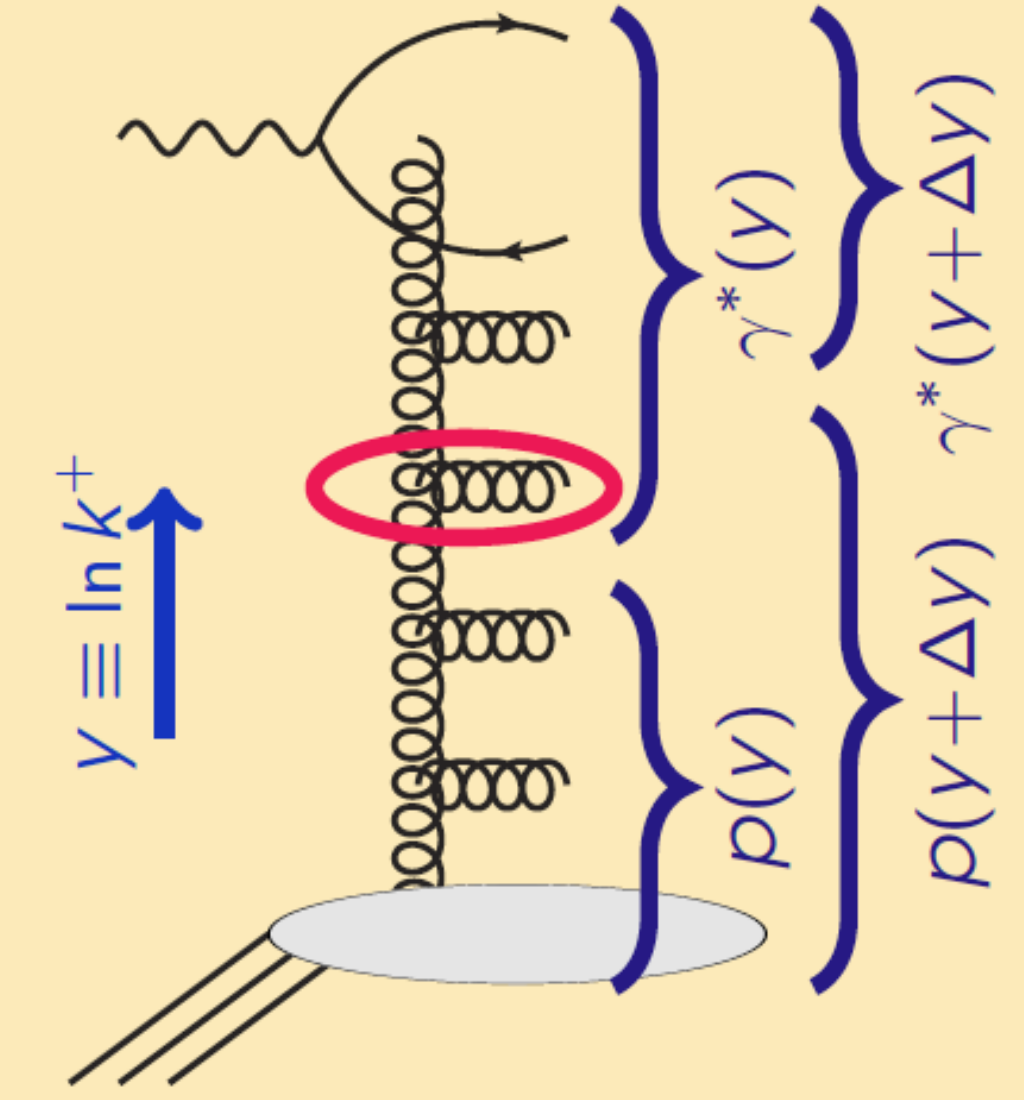
$$\sigma_{T,L}^{\gamma^* p} = \int d^2\mathbf{r}_\perp \, dz \left|\psi^{\gamma^*\to q\bar{q}}(\mathbf{r}_\perp, z)_{T,L}\right|^2 2\mathrm{Im}\mathcal{A}$$

Target, evolved,
dilute-dense picture



- Unified description of inclusive, diffractive and exclusive processes.
- Now $|q\bar{q}g\rangle$ included (NLO) in inclusive, diffractive and exclusive VM production.

Evolution in the dipole picture:

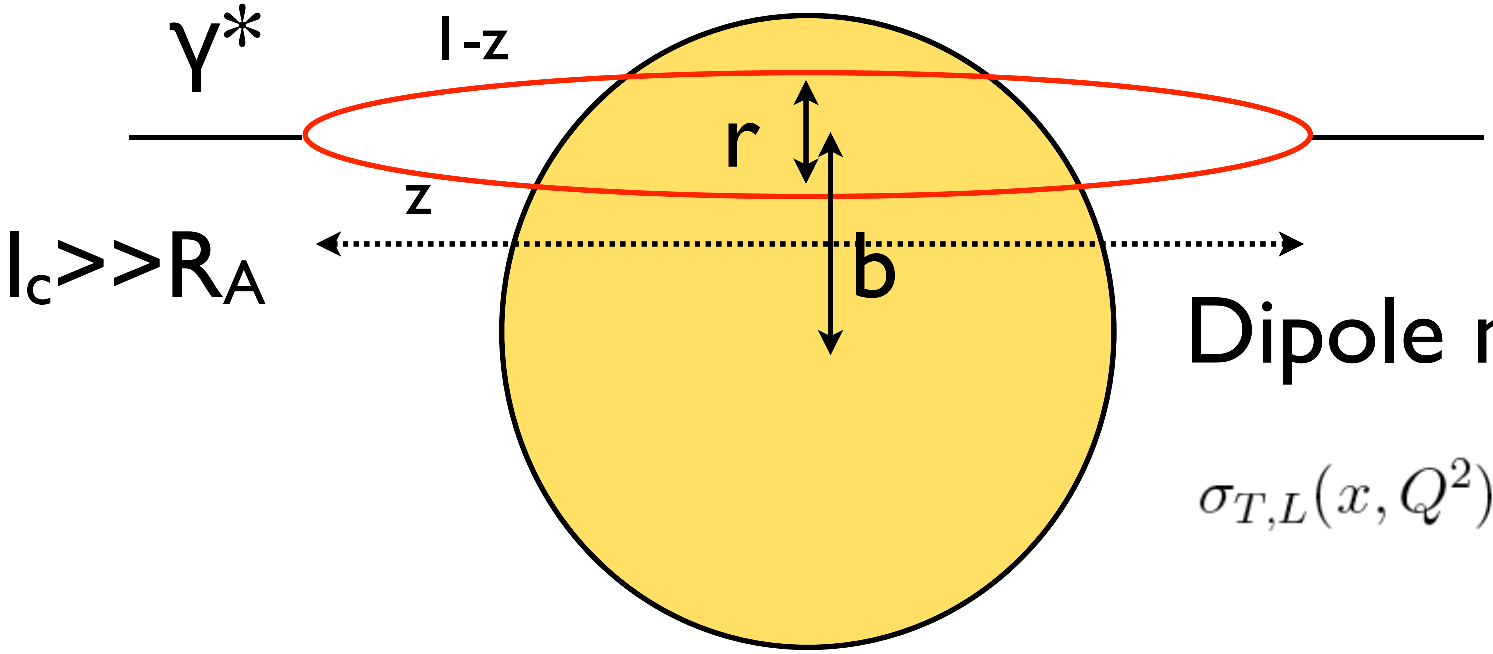


- Emitted gluons have momentum fraction $\in [x, z]$.
- Gluons are attributed either to the hadron or the dipole, but cross section is the same.
- Factors $\alpha_s \ln 1/x \sim 1$ come from each emission; such emissions must be resummed: NLO+NLL evolution equation in $y = \ln 1/x$.

$$\sigma^{\gamma^* p} = \overbrace{\left|\psi^{\gamma^* \to q\bar{q}}\right|^2_y \otimes 2\mathrm{Im}\mathcal{A}_y^{q\bar{q}p} + \left|\psi^{\gamma^* \to q\bar{q}g}\right|^2_y \otimes 2\mathrm{Im}\mathcal{A}_y^{q\bar{q}gp} + \ldots}^{\text{gluons up to } y \text{ are part of proton}}$$

$$= \underbrace{\left|\psi^{\gamma^* \to q\bar{q}}\right|^2_{y+\Delta y} \otimes 2\mathrm{Im}\mathcal{A}_{y+\Delta y}^{q\bar{q}p} + \left|\psi^{\gamma^* \to q\bar{q}g}\right|^2_{y+\Delta y} \otimes 2\mathrm{Im}\mathcal{A}_{y+\Delta y}^{q\bar{q}gp} + \ldots}_{\text{gluons up to } y+\Delta y \text{ are part of proton}}$$

Unitarity:



$$F_2(x,Q^2) = \frac{Q^2}{4\,\pi^2\,\alpha_{em}}\,(\sigma_T+\sigma_L)$$

Dipole model: valid for small x and Q^2.

$$\sigma_{T,L}(x,Q^2) = \int_0^1 dz \int d\mathbf{b}\, d\mathbf{r}\, |\Psi_{T,L}(z,Q^2,\mathbf{r})|^2\, \mathcal{N}(\mathbf{b},\mathbf{r},x)$$

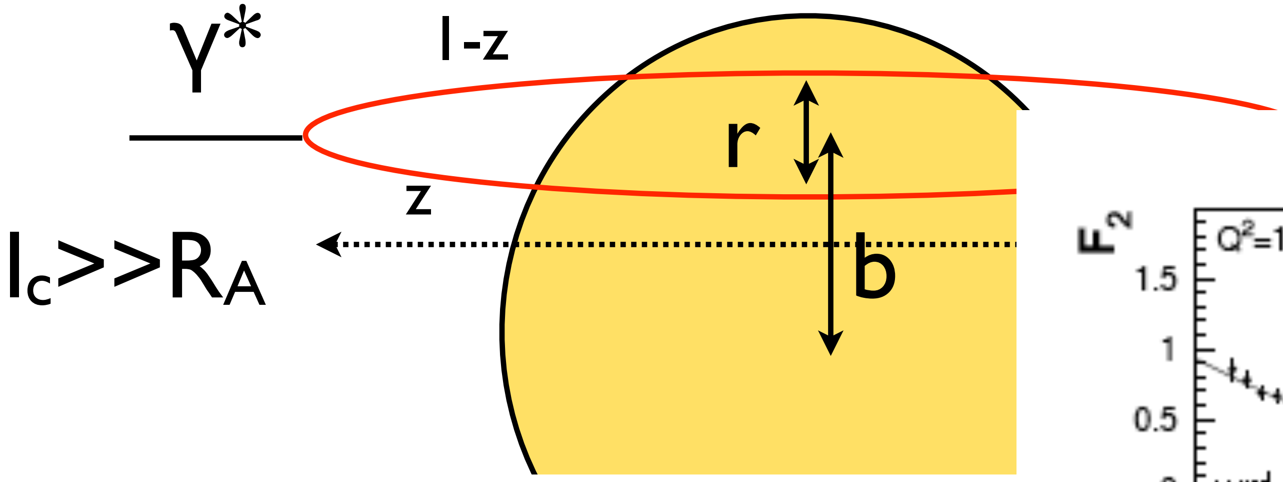
- Unitarity (probability conservation in QM) implies that the (Img forward) scattering amplitude N≤1 (optical theorem ⇒ $\sigma \propto N$). But

$$xg(x,Q^2) \propto \int^{Q^2} dk^2 \phi(x,k^2), \quad \phi(x,k^2) \propto \int \frac{d^2r}{r^2}\, e^{ik\cdot r} N(x,r)$$

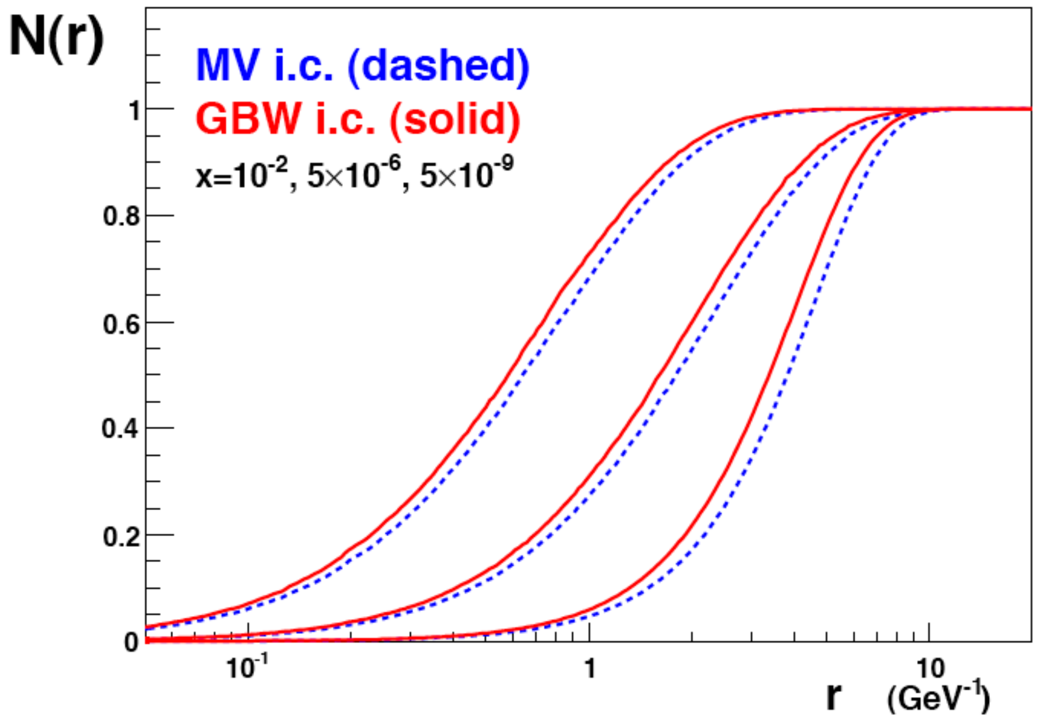
so $xg(x,Q^2) \propto x^{-\lambda}$ at fixed Q^2 is not compatible with unitarity. The most celebrated dipole model is GBW, $Q_s^2 \propto x^{-\lambda}$.

$$\mathcal{N}^{GBW}(r, Y=0) = 1 - \exp\left[-\left(\frac{r^2\,Q_{s\,0}^2}{4}\right)^{\gamma}\right]$$

Unitarity:



Q^2



so $xg(x,Q)\propto x$ at fixed

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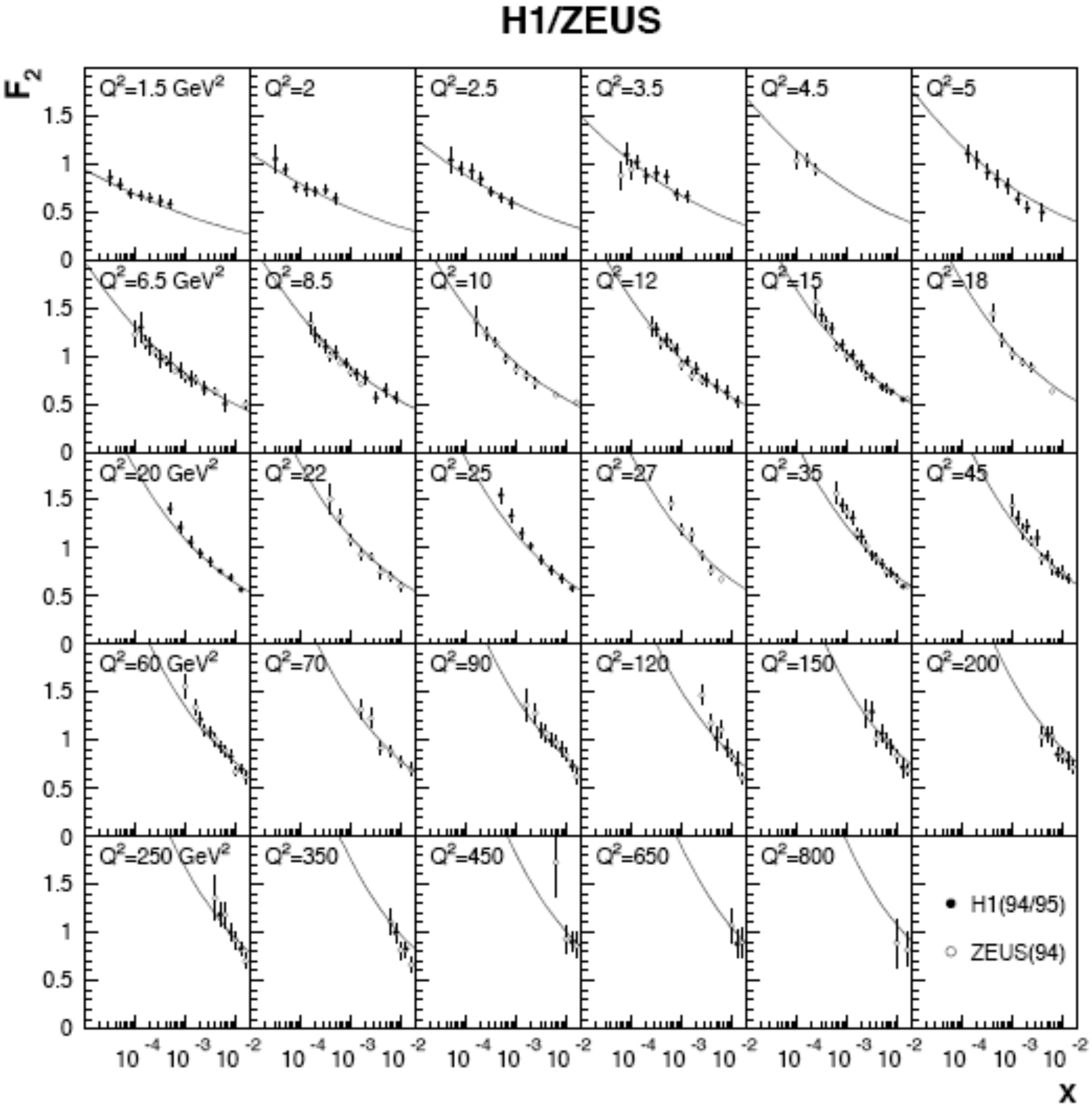
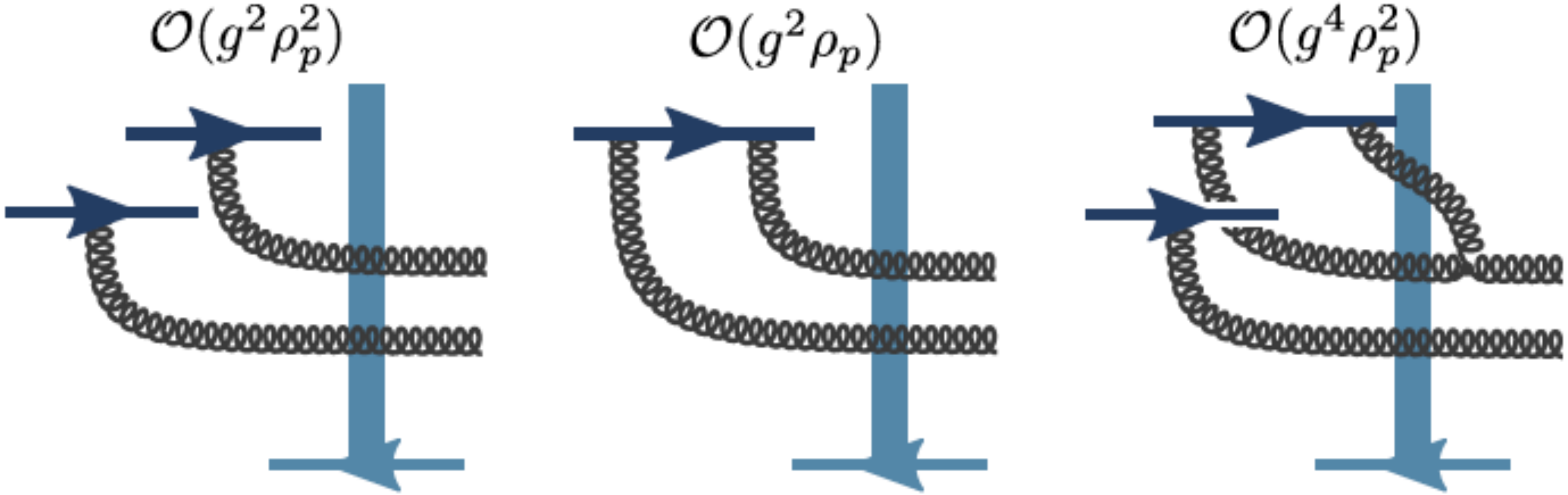


Dilute-dense:

- Analytical calculations at a classical level only available in a dilute-dense situation: dilute projectile on dense target.
- Dense-dense is addressed through numerics of classical QCD.

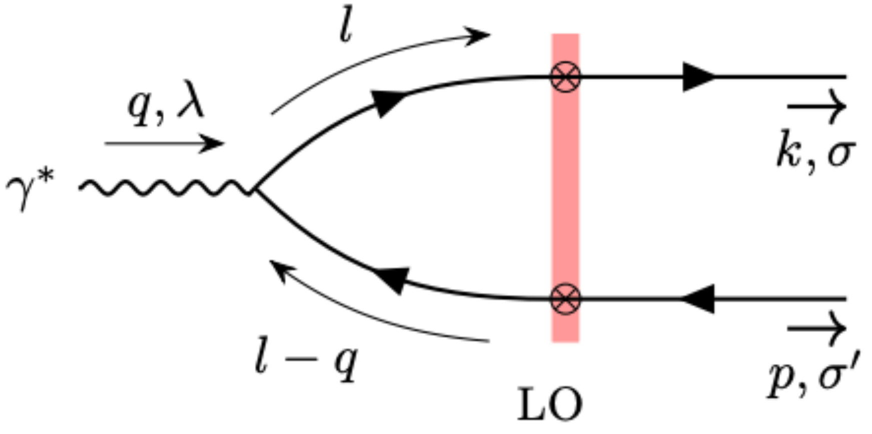


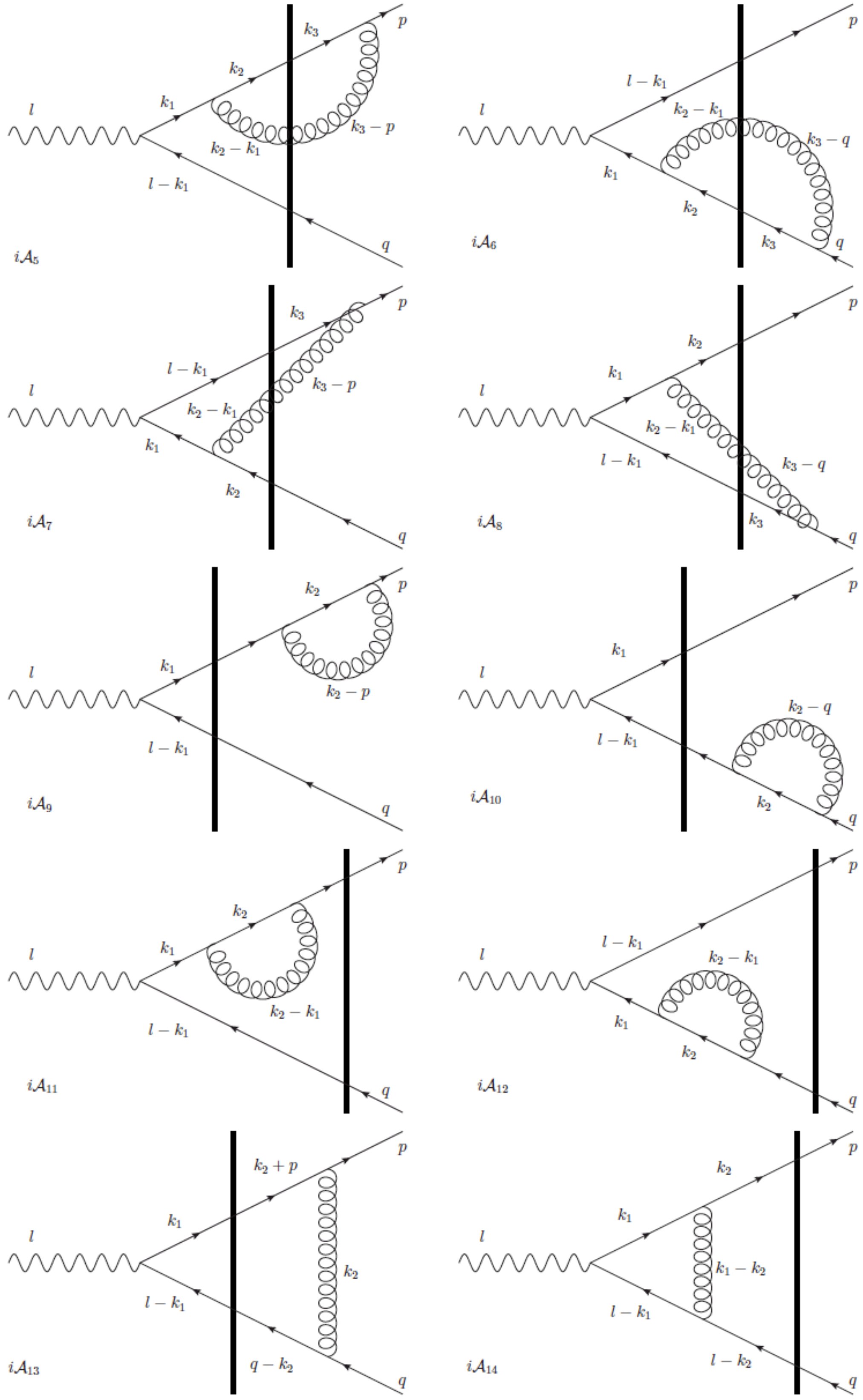
- Dilute-dilute should go to the linear regime (BFKL) that cannot not be extended to too large energies (unitarity, Froissart bound).
- Evolution equations (JIMWLK/BK) only available for a dilute projectile on a dense target (*jargon: no BFKL Pomeron loops*).

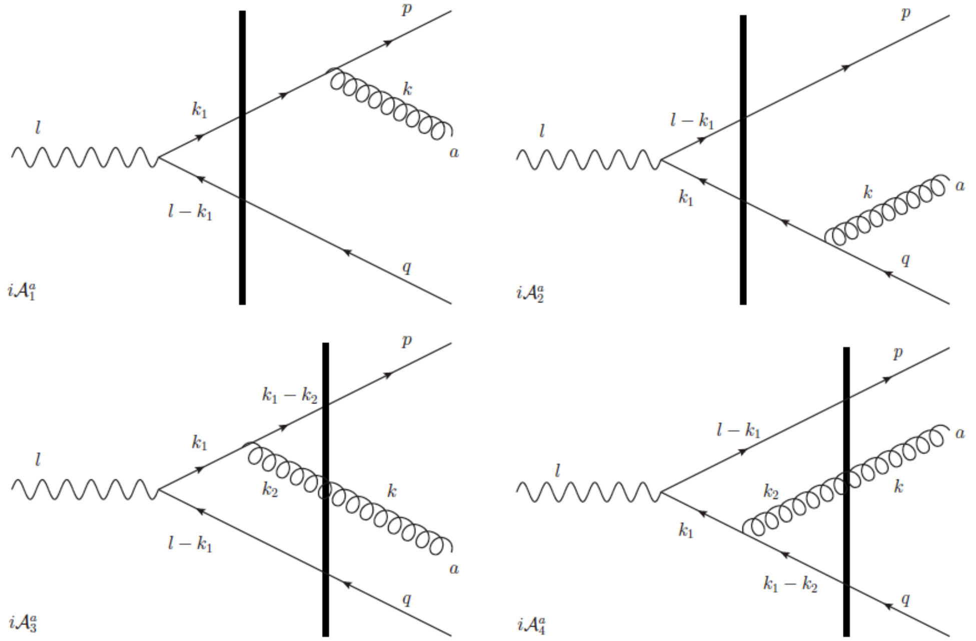
The path to precision:

- LO calculations: they show qualitative agreement with experimental data but lack precision to estimate uncertainties and establish clearly the existence of saturation.
- **NLO calculations**: burst of activity in recent years.
  - ➜ Evolution equations: massive quarks in DIS.
  - ➜ eA: dijet, dihadron and single hadron.
  - ➜ Forward pA: single hadron and jet production in hybrid factorization.
- Relation with TMDs (t, p) and TMD factorization.
- Further: production at central rapidities, diffraction, exclusive processes, particle correlations, non-eikonal corrections, models for averages,…

The path to precision:







Non-eikonal corrections:

- CGC calculations are usually done in the eikonal approximation, which amounts to neglecting terms subleading in energy.
- Subeikonal effects are key for those observables that are subleading, like spin; they also produce odd harmonics.
- Terms subleading in energy may be important at the EIC.

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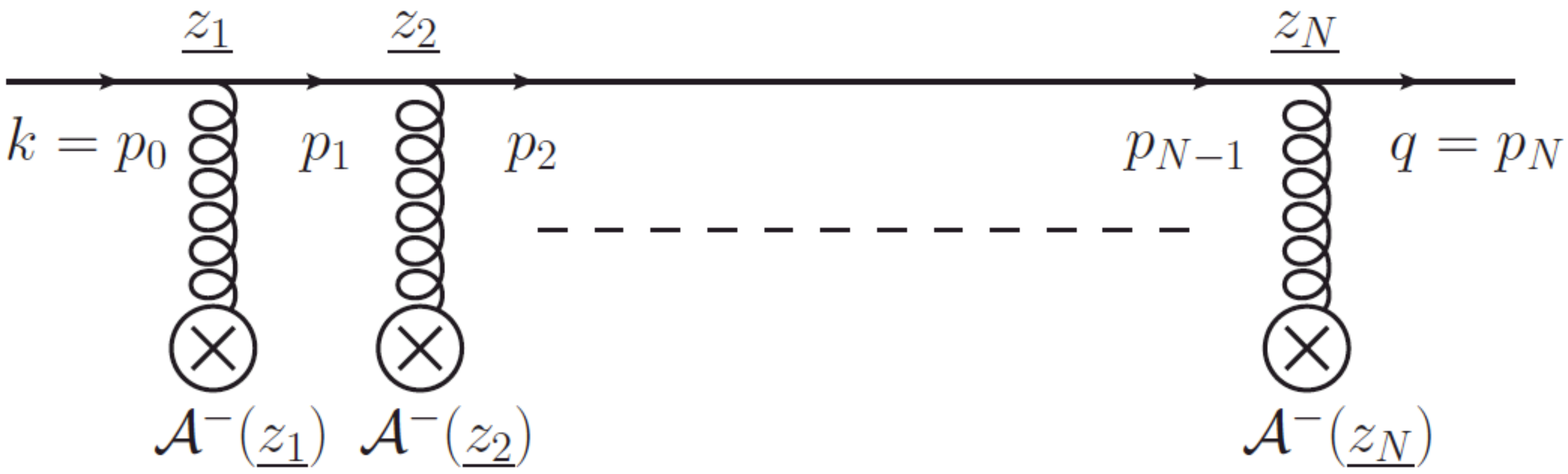

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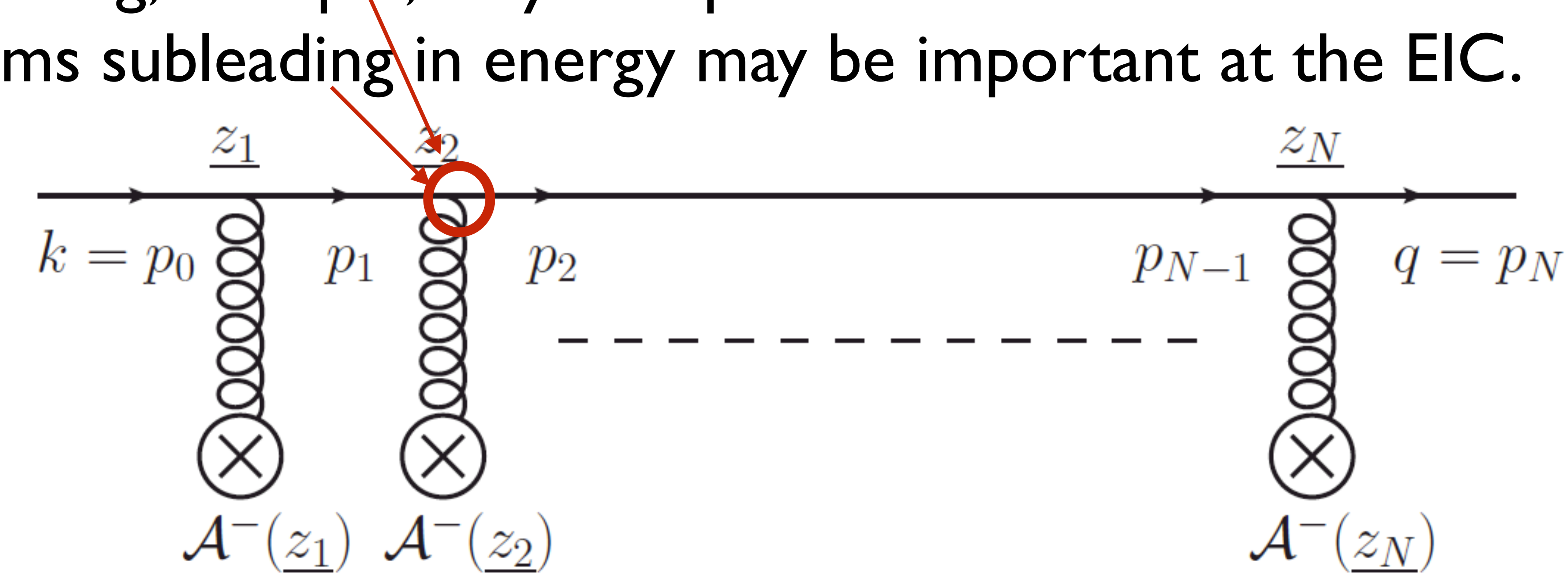

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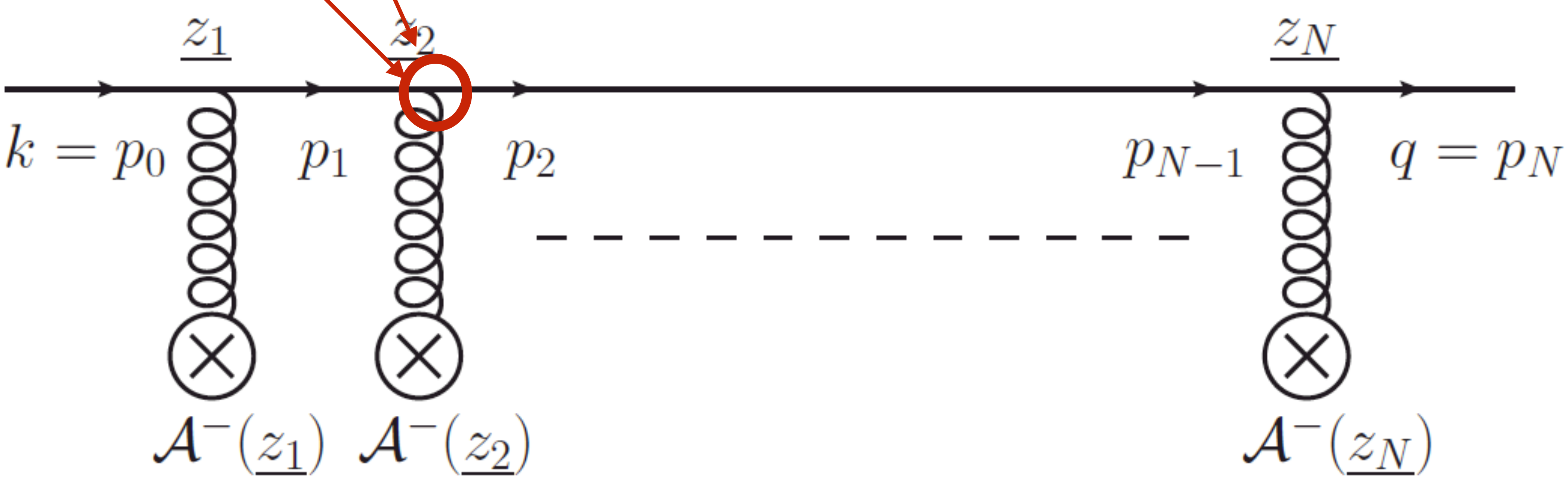

$$\mathscr{A}^\mu(x^+, x^-, \mathbf{x}) \approx \delta^{\mu -}\mathscr{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$$

Longitudinal momentum exchange, collisional eloss

Other components of the background field, new averages

Shockwave approximation, finite size of the target

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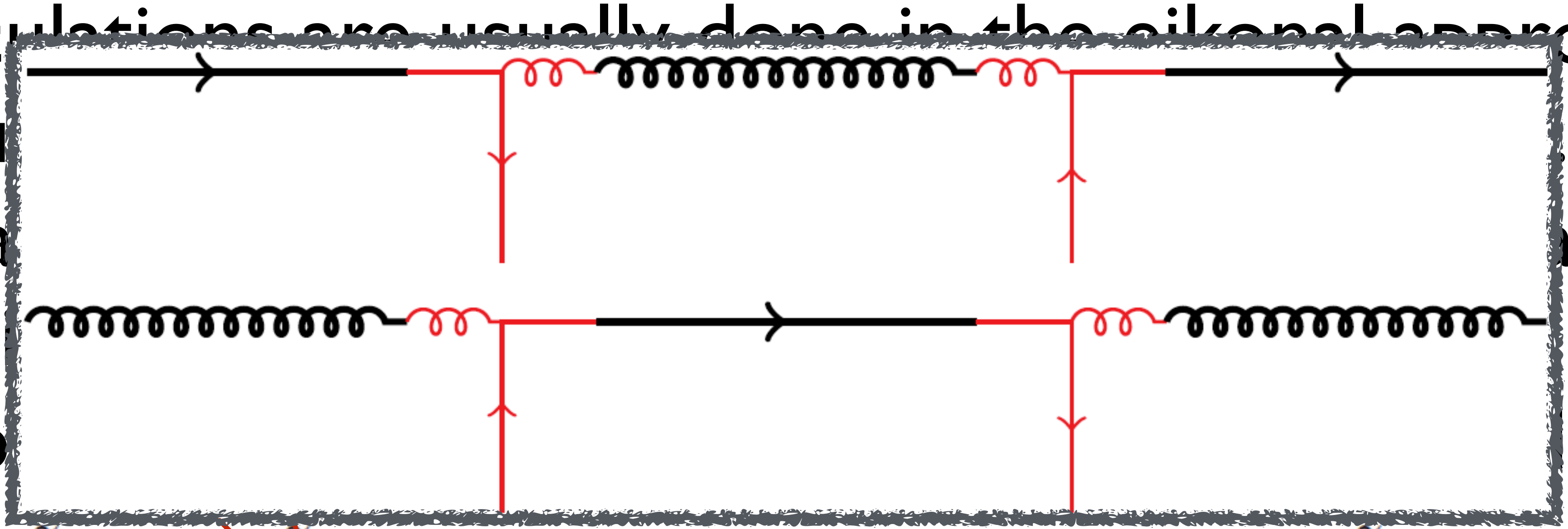

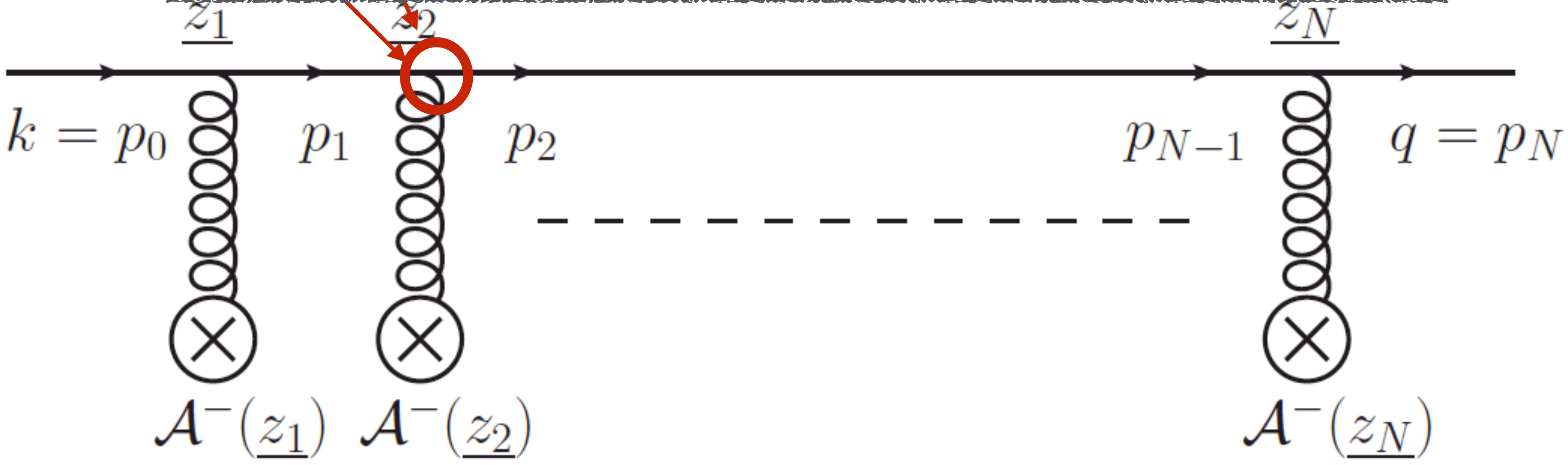

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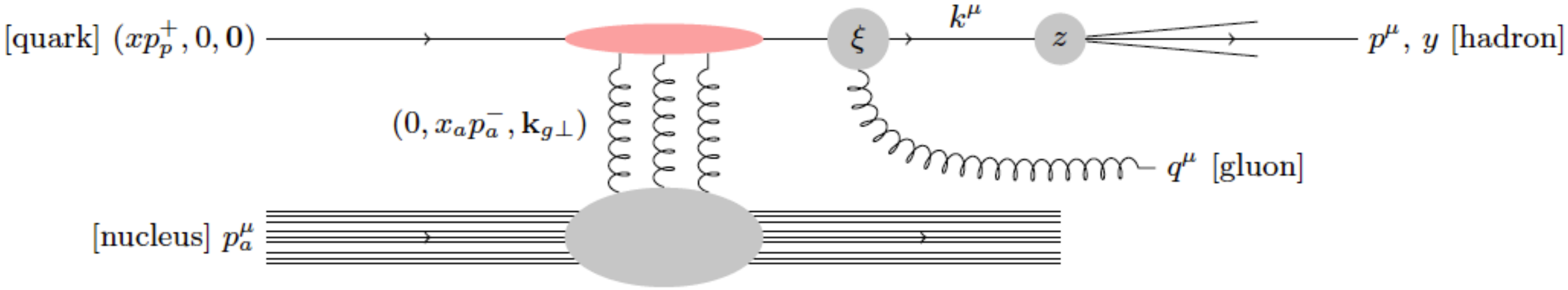
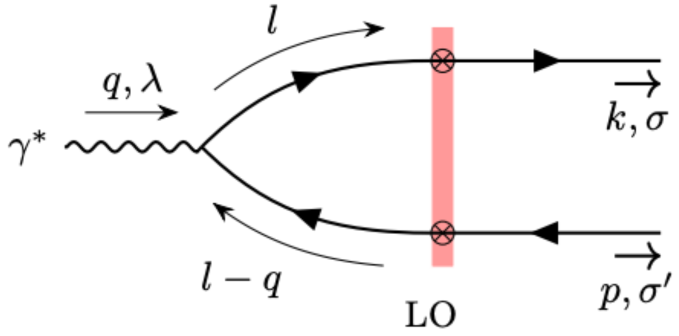
Observables:

- Compute the contributions relevant for the process from the projectile point of view (using equal or light-front quantization, covariant or light-cone gauges, Feynman diagrams or wave functions in Light Cone PT,…).

- Partons in the different contributions interact with the target through Wilson lines (usually at fixed transverse positions, eikonal approximation), that in the cross section appear as ensembles $\langle W \cdots W \rangle_T$.

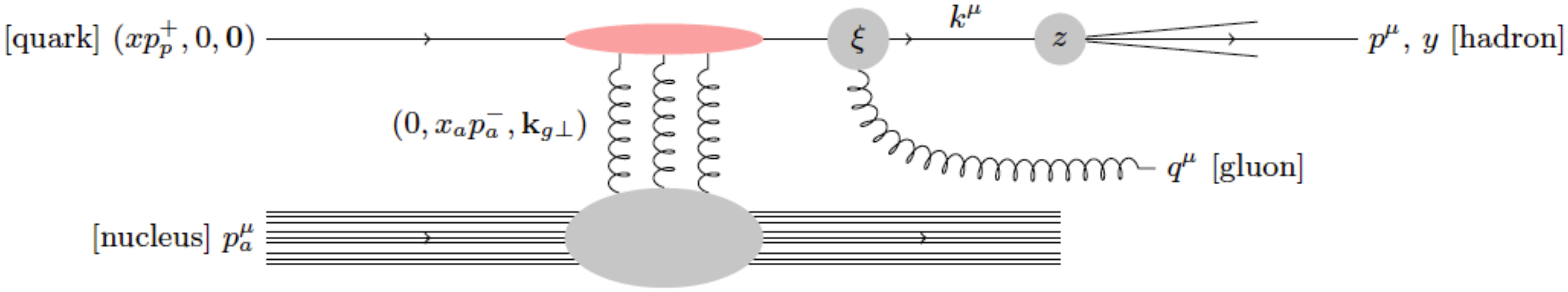


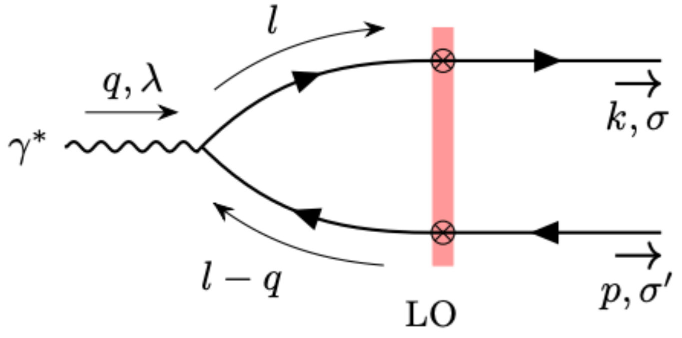


$$W(x_\perp) = P exp \left[-ig \int A \cdot dl \right]$$

Observables:

- At NLO, collinear and soft divergencies appear, which must be shown to be absorbed in DGLAP-type evolution (of PDFs, FFs, jet functions,…) and JIMWLK-type evolution of $\langle W \cdots W \rangle_T$, respectively; additional large logs may appear.
- Models for the non-perturbative input of objects later evolved: PDFs, FFs, jet functions, $\langle W \cdots W \rangle_T$ (MV), Wigner functions,…





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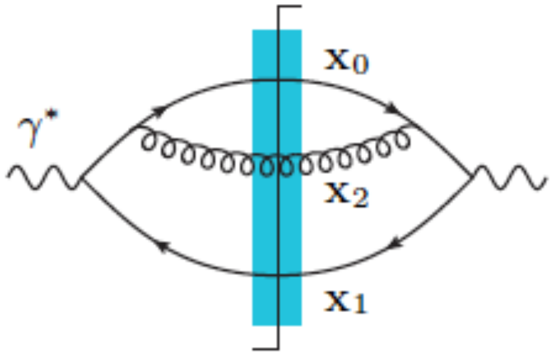
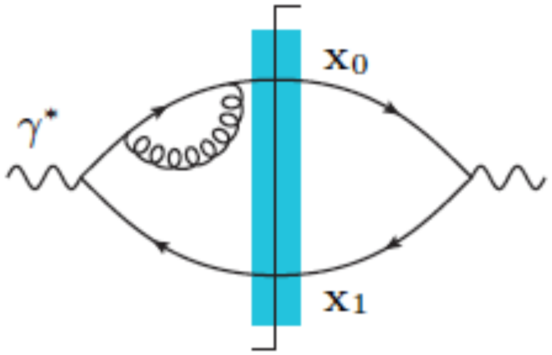
Inclusive DIS:

- Description of small x HERA data including heavy quarks: NLO BK.



(a) $q\bar{q}g$



(b) $q\bar{q}$

$$\sigma_{T,L}^{\gamma A\to X}(x_{Bj},Q^2) \propto \int_{\mathbf{x}_0,\mathbf{x}_1}\int_0^1 dz_1 \Phi_{T,L}^{q\bar{q},LO}(\mathbf{x}_{01},z_1,Q^2)\left[1-\langle s_{01}\rangle\right]$$

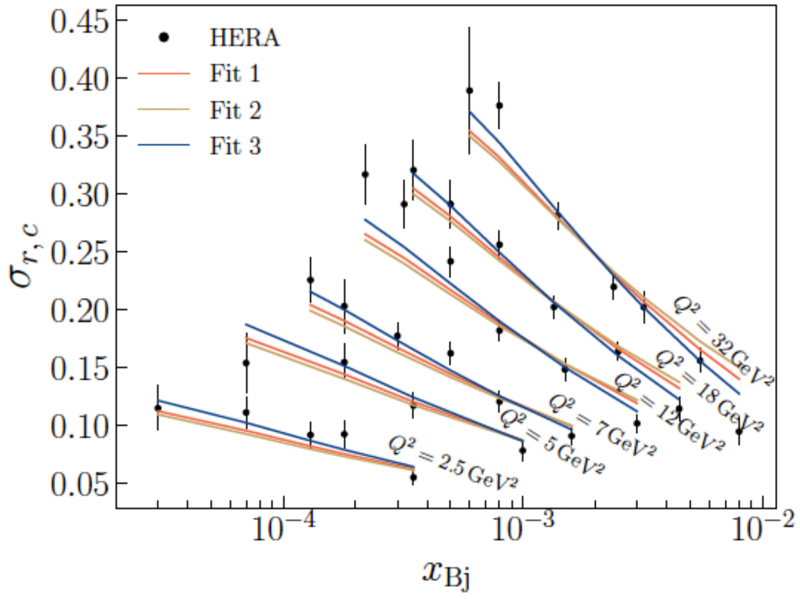
$$\sigma_{T,L}(x_{Bj},Q^2) = \sum_{q\bar{q}\ st.}\left|\Psi_{q\bar{q}}^{\gamma^*_{T,L}}\right|^2\left[1-\langle s_{01}\rangle_0\right] + \sum_{q\bar{q}g\ st.}\left|\Psi_{q\bar{q}g}^{\gamma^*_{T,L}}\right|^2\left[1-\langle s_{012}\rangle_0\right]$$

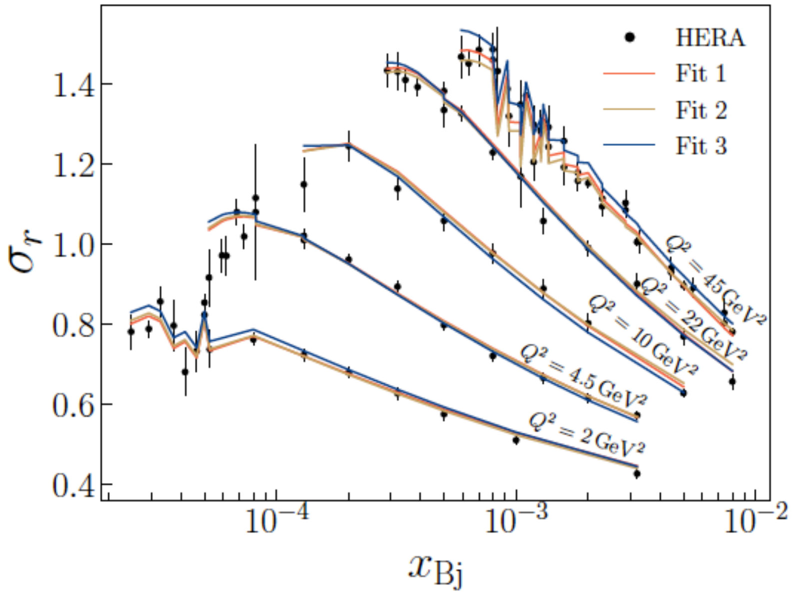
$$S_{01} = \frac{1}{N_c}\left\langle \mathrm{Tr}\{V(\mathbf{x}_0)V^\dagger(\mathbf{x}_1)\}\right\rangle,$$

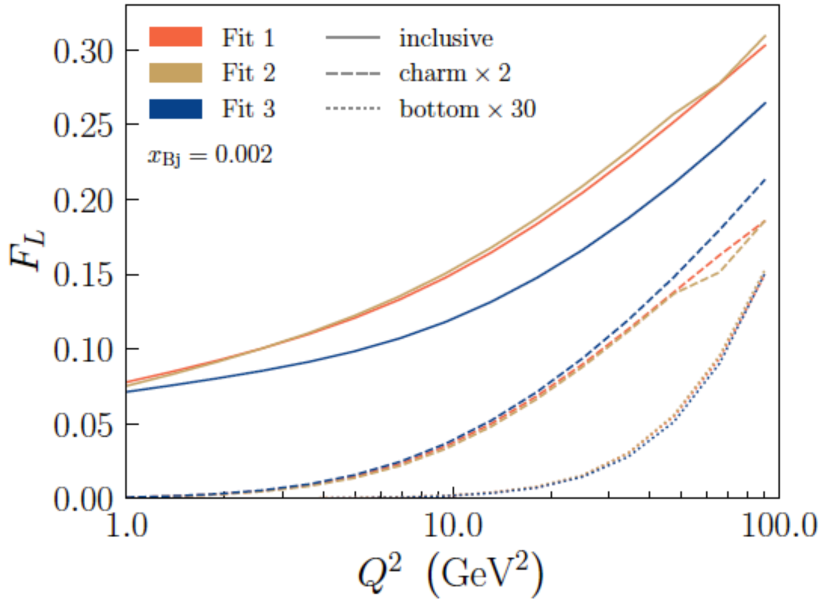
$$S_{012} = \frac{N_c}{2C_F}\left(S_{02}S_{21} - \frac{1}{N_c^2}S_{01}\right)$$

$$\langle s_{01}\rangle_{y=0} = \exp\left[-\left(\frac{Q_s^2 x_{01}^2}{4}\right)^{\gamma}\ln\left(\frac{1}{x_{01}\Lambda_{QCD}}+e\right)\right]$$

$$\sigma_r(y,x,Q^2) = F_2\left(x,Q^2\right) - \frac{y^2}{1+(1-y)^2}F_L\left(x,Q^2\right)$$

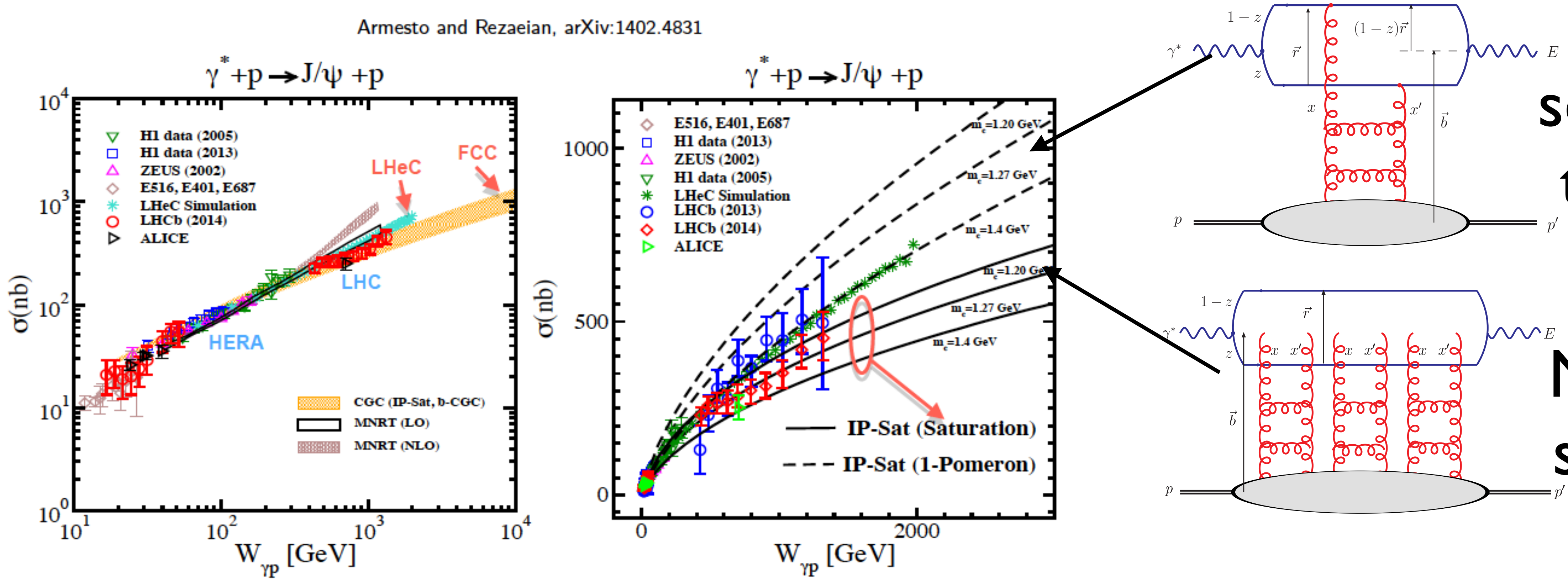






DIS and UPCs: diffraction

- Diffraction is a promising observable for saturation.



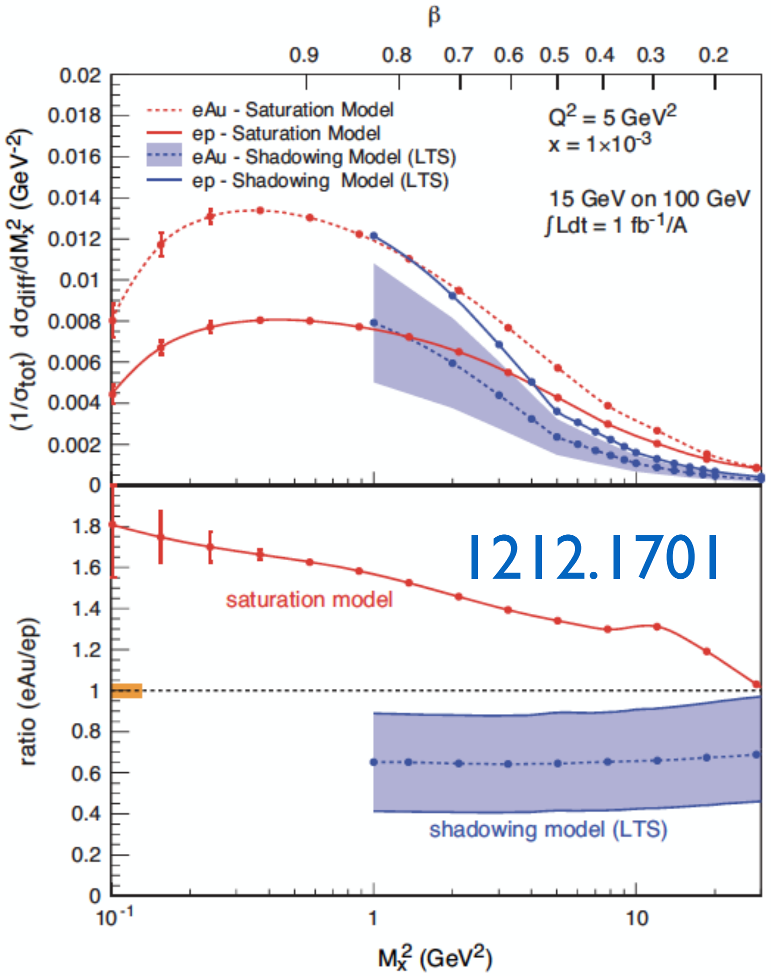
Linear, sensitivity to $(xg)^2$.

Non-linear, saturation.

DIS and UPCs: diffraction

- Diffraction is a promising observable for saturation.

  - Present saturation models lead to a blackening of the hadron (shrinking of the diffractive peak) and a larger total diffractive cross section in eA: interplay between non-linear phenomena and survival probability.



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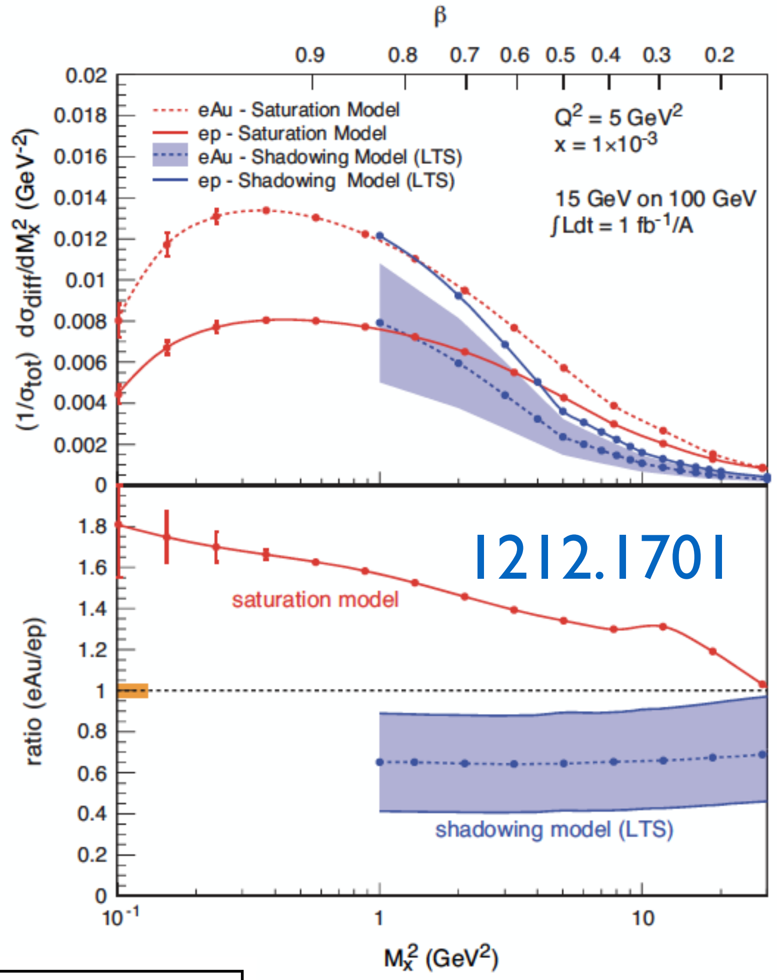
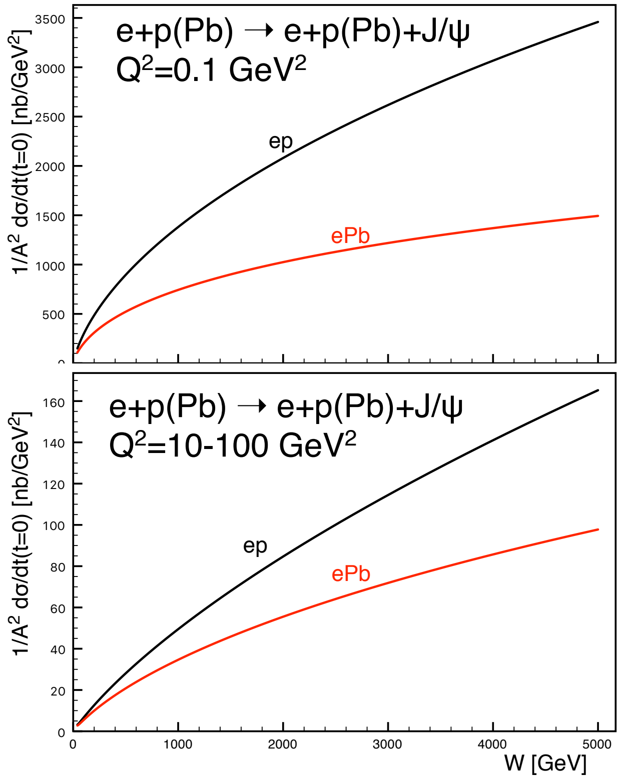
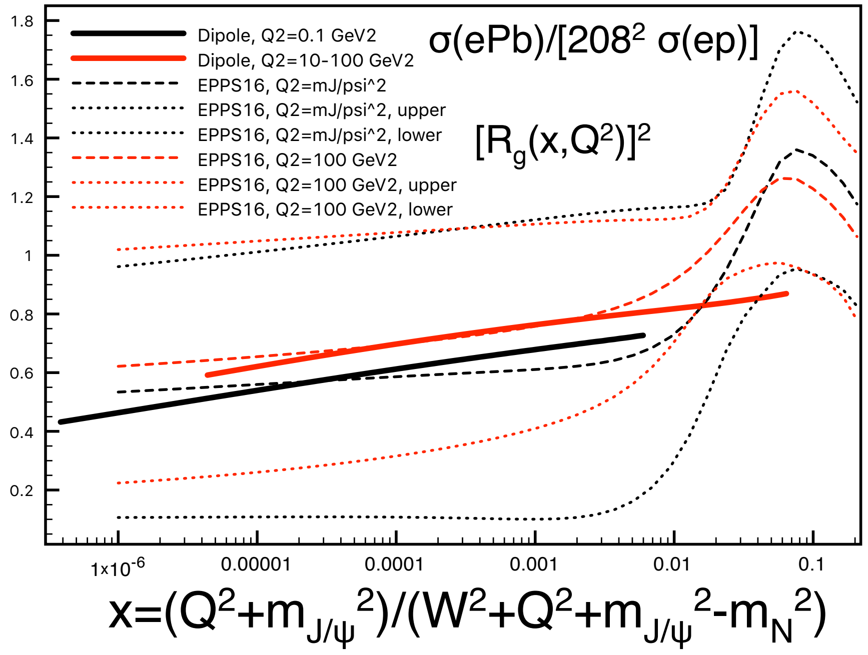


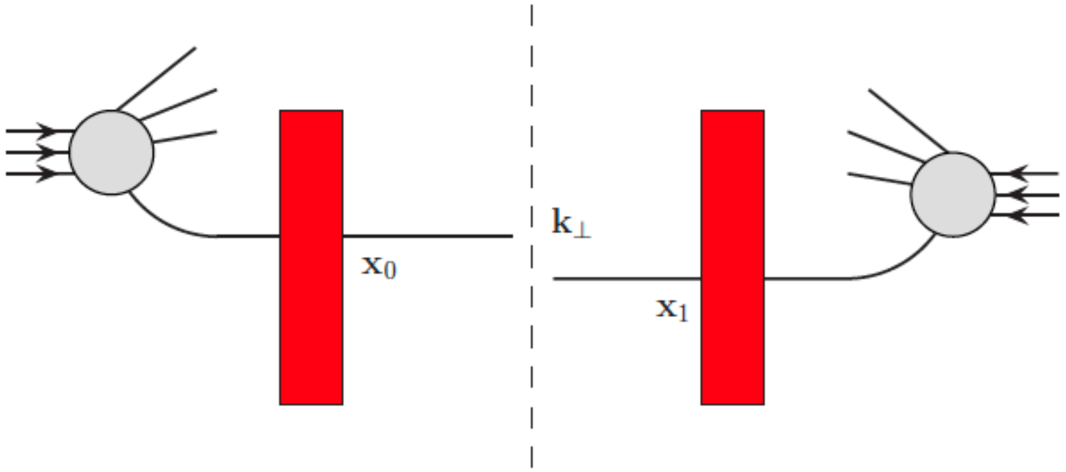




Mantysaari, Paukkunen

pA: single inclusive

- State of the art for forward particle production in pA collisions: hybrid model, proposed at LO in 2005 (hep-ph/0506308).

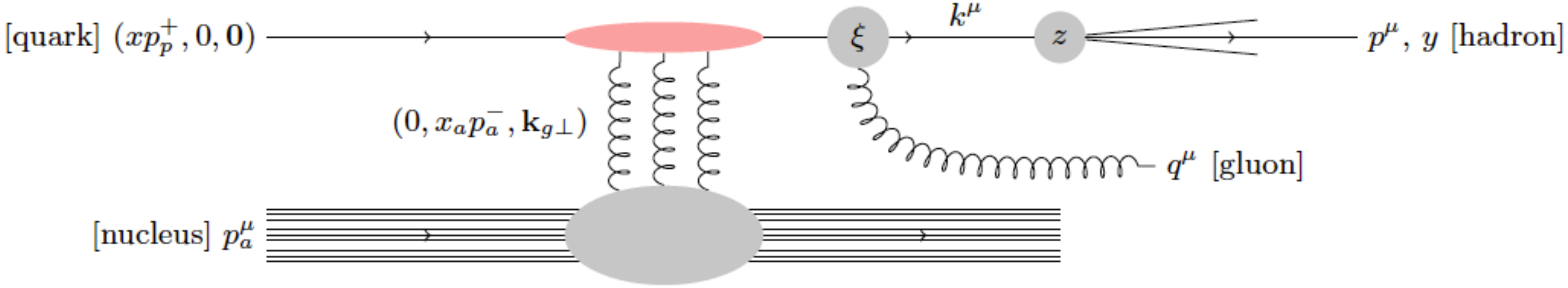


$$\frac{d\sigma^{q\to H}}{d^2k\, d\eta} = \int_{x_F}^{1} \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) \int e^{ik(x_0-x_1)} \langle s(x_0,x_1)\rangle$$

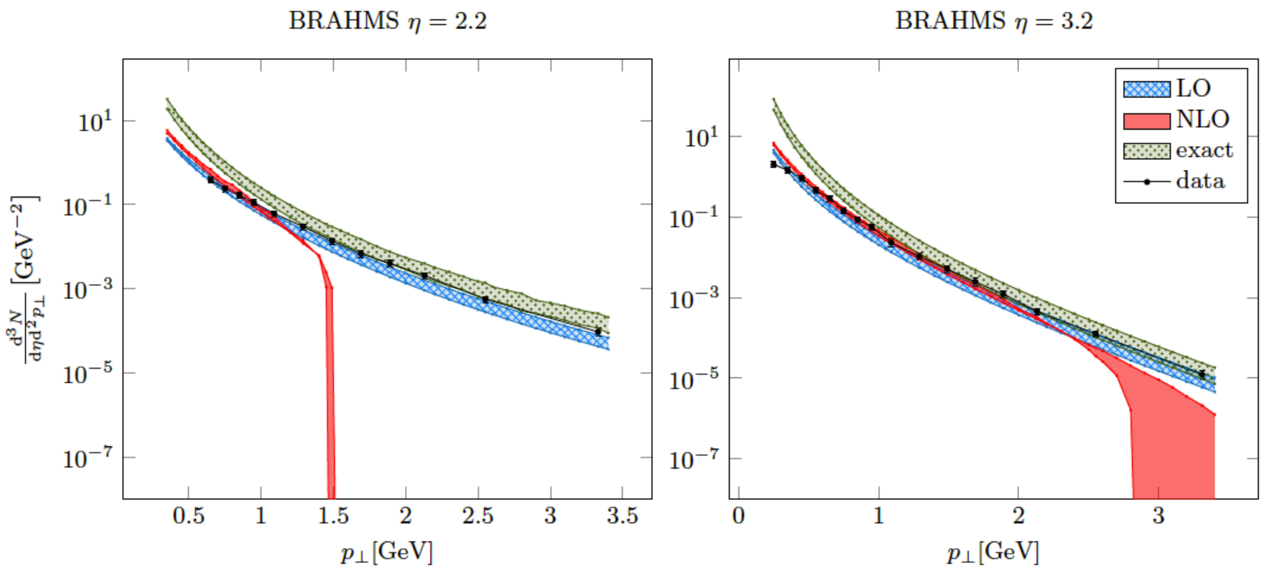
- Wave function of the projectile proton treated in the spirit of collinear factorization (incoming parton with negligible transverse momentum).
- Perturbative corrections to this wave function given by usual QCD (+QED for photons) perturbative processes.
- CGC treatment of the target: collection of strong color fields that transfer transverse momentum to the rescattering partons.
- At LO, transverse momentum gained solely from rescattering.

pA: single inclusive

- Full NLO corrections in 2011 (1112.1061, 1203.6139): collinear divergencies absorbed in the DGLAP evolution of PDFs and FFs, rapidity divergencies in the BK evolution of $\langle W \cdots W \rangle_T$.



- Numerical analysis (1405.6311): cross sections turned out to be negative at large transverse momentum.
- Recent solutions require additional resummations.



pA: single inclusive

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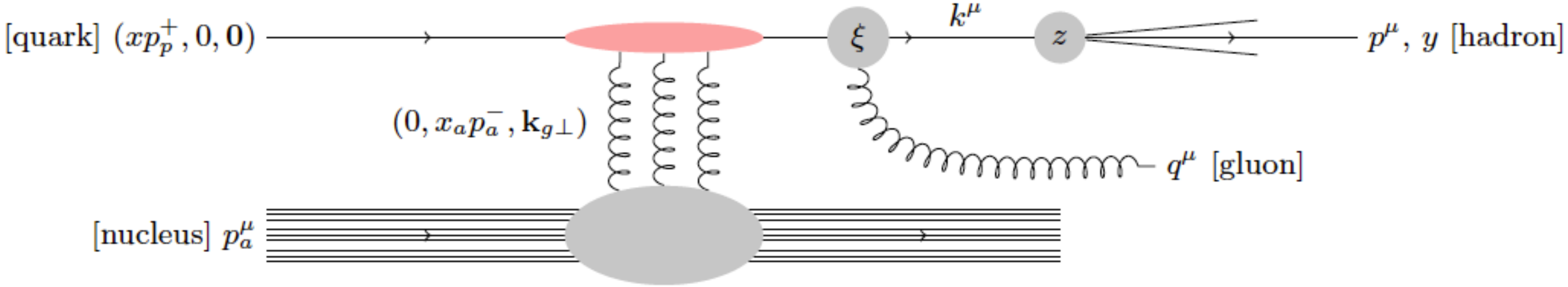

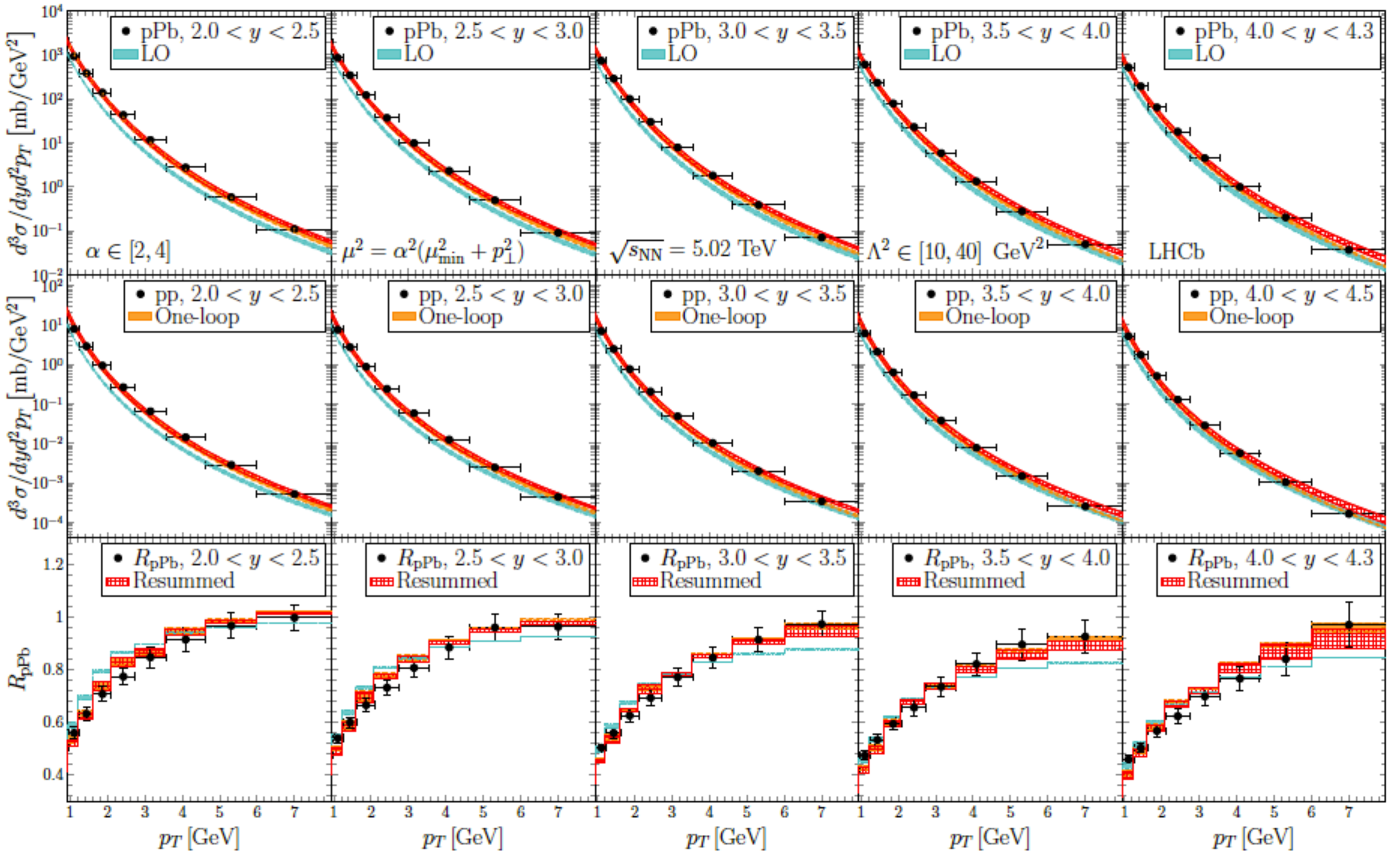

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- Recent solutions require additional resummations.

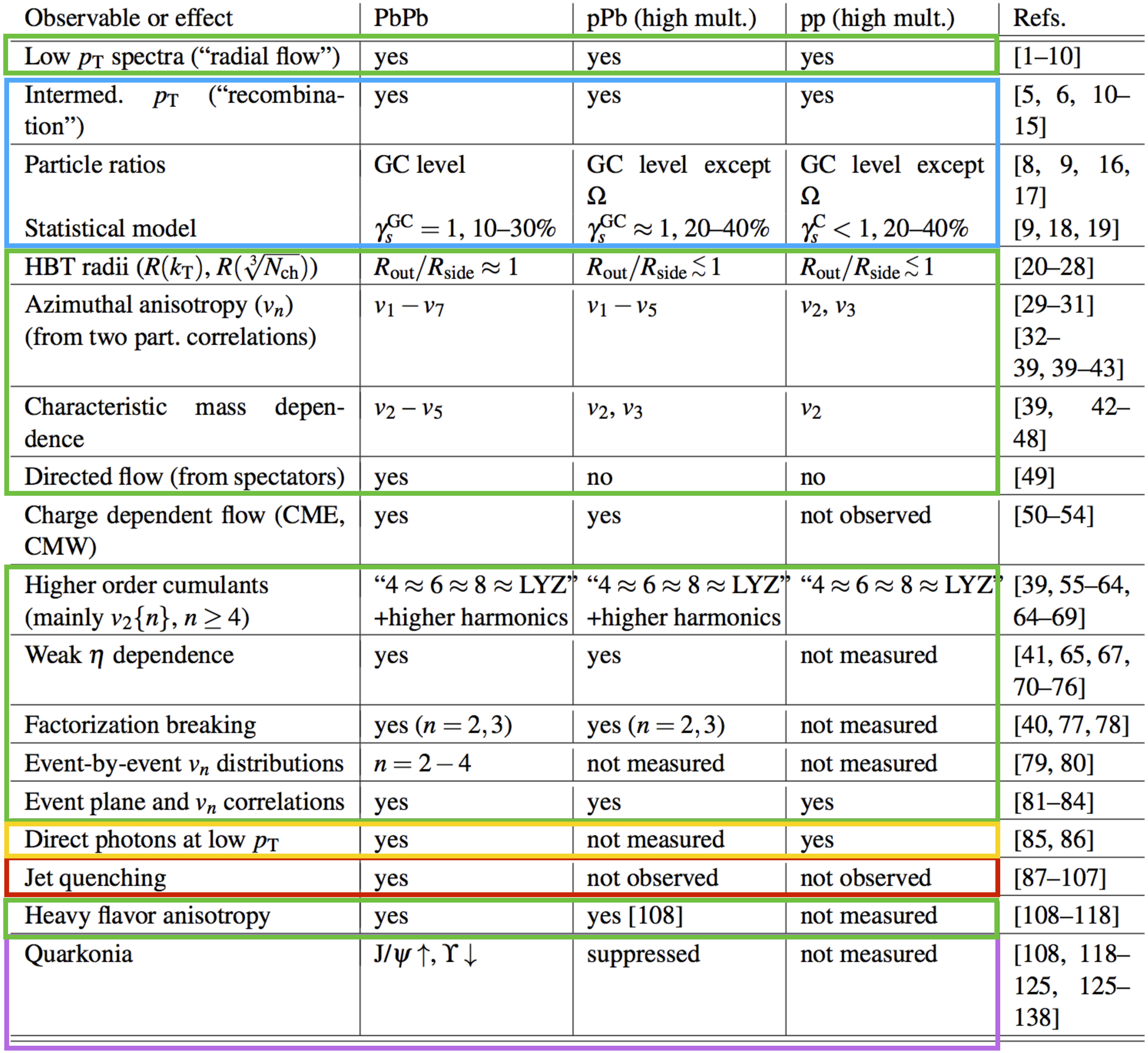
The small system puzzle:

Collective hadronisation

Collective expansion (hydro-like)

Direct photons

Final state interactions (non-hydro)



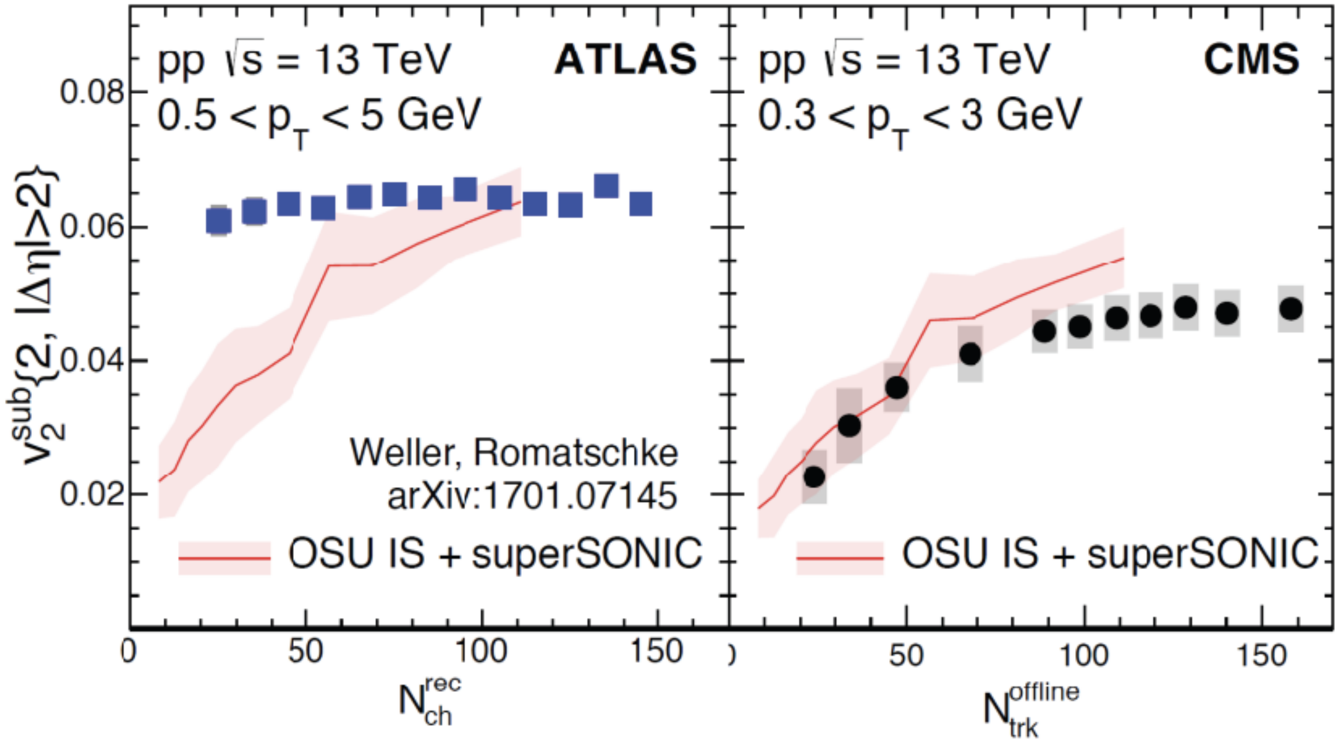
Observable or effect	PbPb	pPb (high mult.)	pp (high mult.)	Refs.
Low p_T spectra (“radial flow”)	yes	yes	yes	[1–10]
Intermed. p_T (“recombination”)	yes	yes	yes	[5, 6, 10–15]
Particle ratios	GC level	GC level except Ω	GC level except Ω	[8, 9, 16, 17]
Statistical model	$\gamma_s^{GC} = 1$, 10–30%	$\gamma_s^{GC} \approx 1$, 20–40%	$\gamma_s^{C} < 1$, 20–40%	[9, 18, 19]
HBT radii ($R(k_T)$, $R(\sqrt[3]{N_{ch}})$)	$R_{out}/R_{side} \approx 1$	$R_{out}/R_{side} \lesssim 1$	$R_{out}/R_{side} \lesssim 1$	[20–28]
Azimuthal anisotropy (v_n) (from two part. correlations)	$v_1 - v_7$	$v_1 - v_5$	v_2, v_3	[29–31] [32–39, 39–43]
Characteristic mass dependence	$v_2 - v_5$	v_2, v_3	v_2	[39, 42–48]
Directed flow (from spectators)	yes	no	no	[49]
Charge dependent flow (CME, CMW)	yes	yes	not observed	[50–54]
Higher order cumulants (mainly $v_2\{n\}$, $n \geq 4$)	“$4 \approx 6 \approx 8 \approx$ LYZ” +higher harmonics	“$4 \approx 6 \approx 8 \approx$ LYZ” +higher harmonics	“$4 \approx 6 \approx 8 \approx$ LYZ”	[39, 55–64, 64–69]
Weak η dependence	yes	yes	not measured	[41, 65, 67, 70–76]
Factorization breaking	yes ($n = 2, 3$)	yes ($n = 2, 3$)	not measured	[40, 77, 78]
Event-by-event v_n distributions	$n = 2 - 4$	not measured	not measured	[79, 80]
Event plane and v_n correlations	yes	yes	yes	[81–84]
Direct photons at low p_T	yes	not measured	yes	[85, 86]
Jet quenching	yes	not observed	not observed	[87–107]
Heavy flavor anisotropy	yes	yes [108]	not measured	[108–118]
Quarkonia	$J/\psi\uparrow$, $\Upsilon\downarrow$	suppressed	not measured	[108, 118–125, 125–138]

The small system puzzle:

- Azimuthal correlations extended in η (the ridge) are found in all systems from almost minimum bias pp (10) to central AA (2000) and are describable by viscous relativistic hydro (with suitable ICs):

  ➜ Final state interactions, so QGP-like physics in all systems?

  ➜ Correlations already present in the hadron or nucleus wave functions, as in CGC calculations?



- One way to proceed: go to even smaller systems, ep/eA, down to a point where final state interactions cannot be justified.

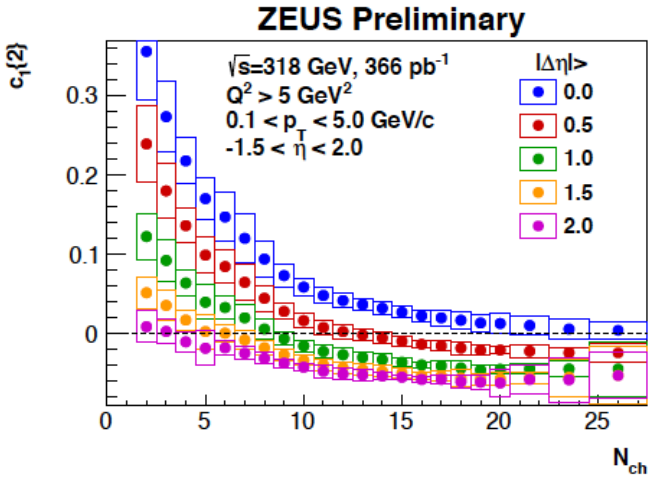
  ➜ Correlations appear (e.g. in eA, CGC): evidence of initial state effects?

  ➜ No correlations: evidence of final state interactions?

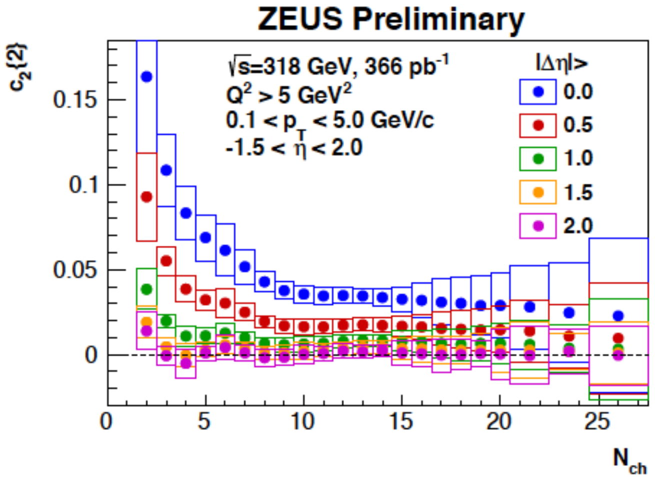
- Note: ZEUS and ALEPH put strong limits on azimuthal 2-particle correlations in ep at HERA and e^+e^- at LEP.

The small system puzzle:

Multiplicity-dependent $c_1\{2\}$ and $c_2\{2\}$ with increasing η-separation



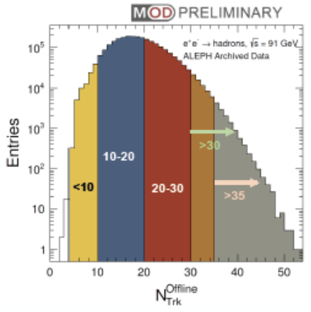
$|\Delta\eta| > 2.0$: $c_1\{2\}$ changes sign $\rightarrow$ consistent with momentum conservation.



$|\Delta\eta| > 2.0$: $c_2\{2\}$ consistent with zero.

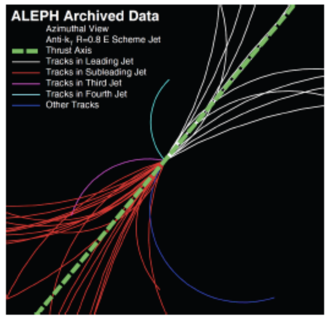
Switching off the flow: e+e-

Talk: J-Y Lee



High-multiplicity events

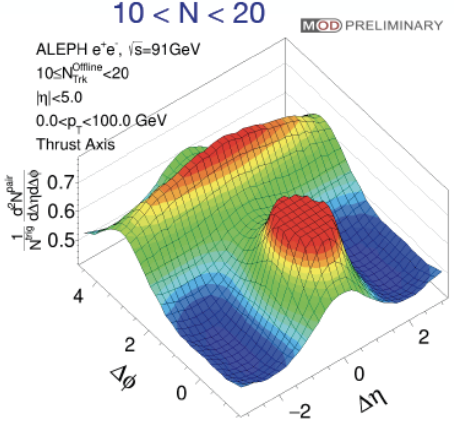
Low T; 'multi-jet'



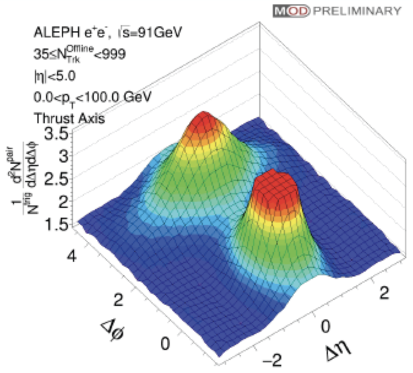
High T; 'di-jet'

ALEPH e+e- thrust axis

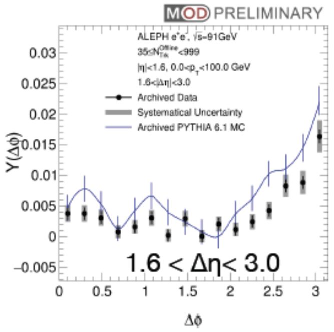
10 < N < 20



N ≥ 35

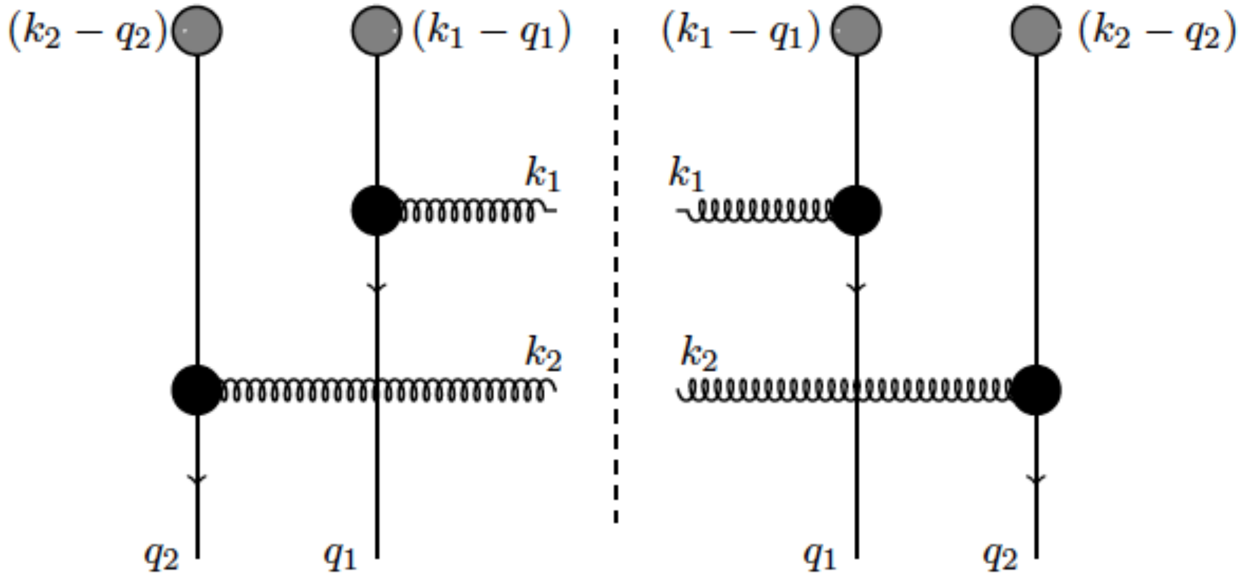


No evidence of long-range correlations beyond Pythia expectation



pA: correlations

- Several explanations in the CGC, that use/assume that:
  - ➜ the final state carries the imprint of initial-state correlations,
  - ➜ the CGC wave function is rapidity invariant over $Y \propto 1/\alpha_s$,
  - ➜ the projectile is a dilute object (proton).



Projectile: sources

Black blob: rescatterings

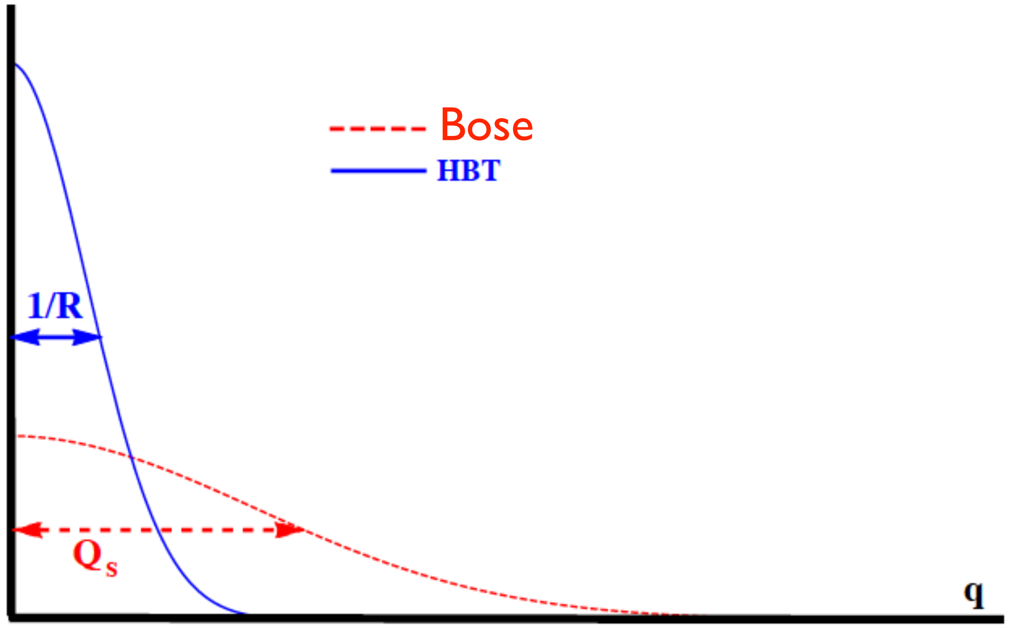
Target: classical colour field

1) Bose enhancement of gluons in the projectile wave function.

$$\propto \delta^{(2)}[k_1 - q_1 - (k_2 - q_2)] + \delta^{(2)}[k_1 - q_1 + (k_2 - q_2)]$$

2) HBT of gluons separated in rapidity.

$$\propto \delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2)$$



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$$2^n(2\pi)^{3n}\frac{d^nN}{d^2\mathbf{k}_1\cdots d^2\mathbf{k}_n} = \left\langle \left|\mathcal{M}^{a_1}_{i_1}(\mathbf{k}_1)\cdots\mathcal{M}^{a_n}_{i_n}(\mathbf{k}_n)\right|^2\right\rangle_{p,T} \qquad \mathcal{M}^a_i(\mathbf{k}) = 2g\int_{\mathbf{q}} L^i(\mathbf{k},\mathbf{q})\rho^b_p(\mathbf{q})U^{ba}(\mathbf{k}-\mathbf{q})$$

$$\begin{aligned}\frac{d^2N}{d^2\mathbf{k}_1 d^2\mathbf{k}_2} = \frac{g^4}{(2\pi)^6}\int_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mathbf{q}_4} & L^i(\mathbf{k}_1,\mathbf{q}_1)L^i(\mathbf{k}_1,\mathbf{q}_2)L^j(\mathbf{k}_2,\mathbf{q}_3)L^j(\mathbf{k}_2,\mathbf{q}_4)\\ & \times\left\langle \rho^{b_1}_p(\mathbf{q}_1)\rho^{*b_2}_p(\mathbf{q}_2)\rho^{b_3}_p(\mathbf{q}_3)\rho^{*b_4}_p(\mathbf{q}_4)\right\rangle_p\\ & \times\left\langle U^{a_1b_1}(\mathbf{k}_1-\mathbf{q}_1)U^{\dagger b_2a_1}(\mathbf{k}_1-\mathbf{q}_2)U^{a_2b_3}(\mathbf{k}_2-\mathbf{q}_3)U^{\dagger b_4a_2}(\mathbf{k}_2-\mathbf{q}_4)\right\rangle_T\end{aligned}$$

1) Bose enhancement of gluons in the projectile wave function.

$$\propto \delta^{(2)}[k_1-q_1-(k_2-q_2)]+\delta^{(2)}[k_1-q_1+(k_2-q_2)]$$

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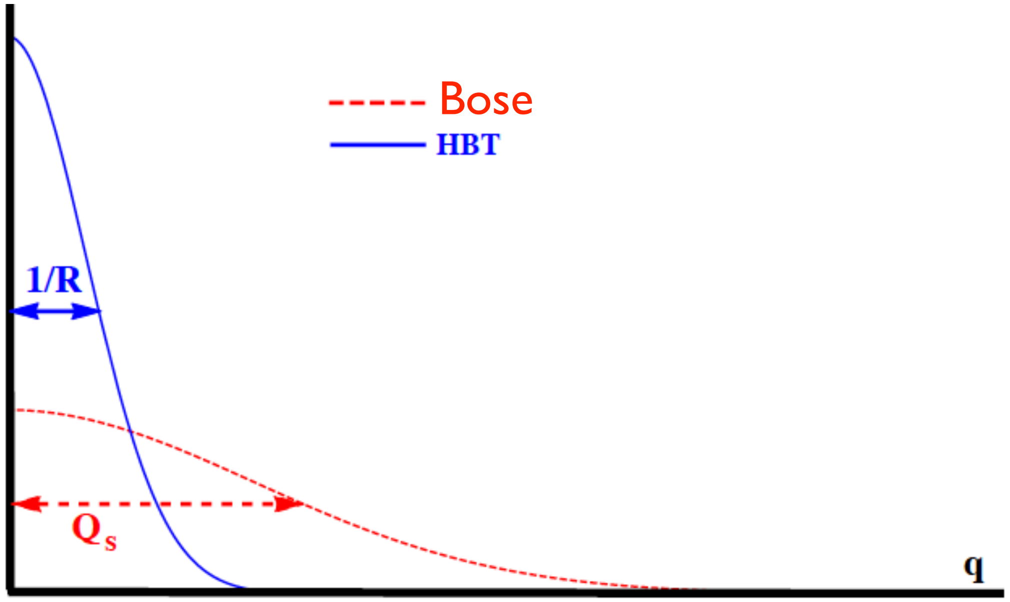


To conclude:

- At high energies, we hope that QCD fields and occupation numbers become large ⇒ classical dynamics, while scales allow a perturbative treatment, $\alpha_s \ll 1$: domain of the CGC, where dynamics becomes non linear and parton densities saturate.

- We still demand non-perturbative input, both for initial conditions, target averages, treatment of tails of hadron profiles (both for nuclei and proton),…. ⇒ experimental input required, coming from RHIC, LHC, JLab, and in the future from the EIC.

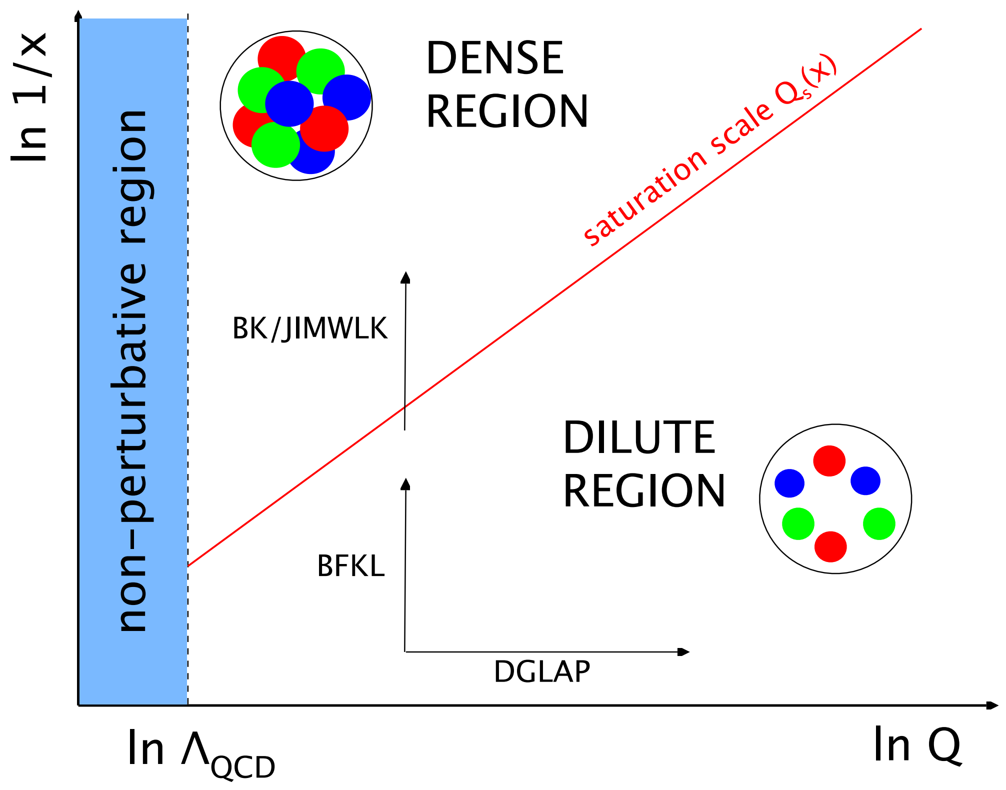
- Efforts ongoing to obtain evolution equations linking with DGLAP, non-eikonal and higher-order corrections, etc., to increase precision and estimate the uncertainties.

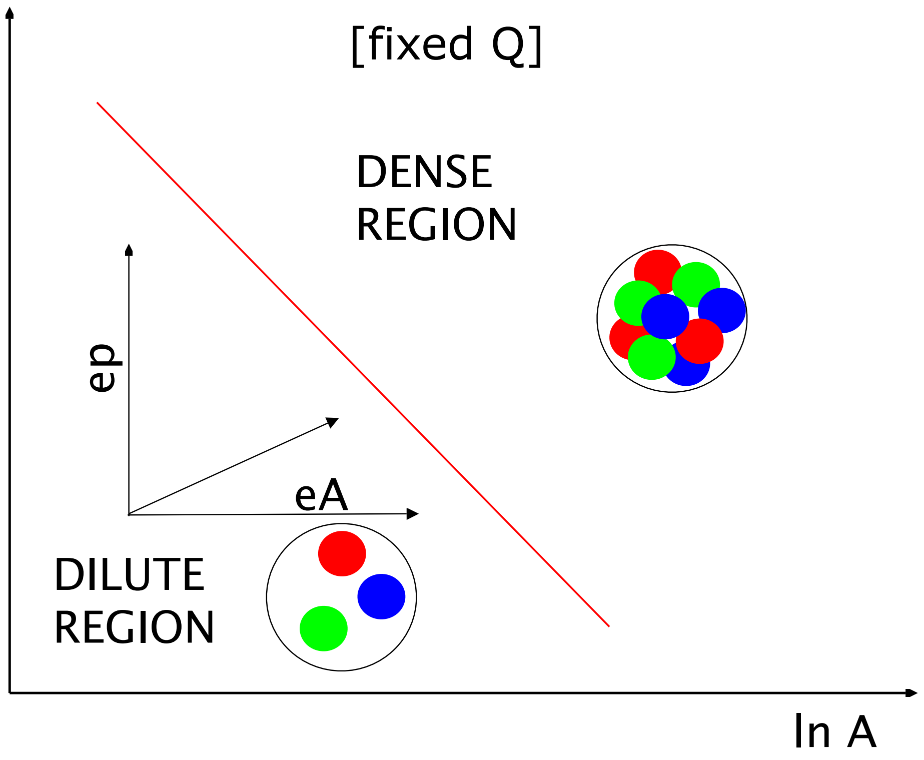
To conclude:

- To clearly determine whether this new regime of QCD is there (there is no phase transition!), we expect different dependencies of the cross sections. We need lever arms **at small x** in:

  ➜ A (pp to pA, ep to eA): different in linear and non-linear dynamics, but may be hidden by the initial conditions and uncertainties in modelling, e.g., diffraction.

  ➜ $Q^2 \in [Q_s^2/C, CQ_s^2]$, $C \sim 2-10$, $\Lambda_{QCD}^2 \ll Q_s^2$: stronger in non-linear dynamics (power-like) that in linear case (logarithmic).





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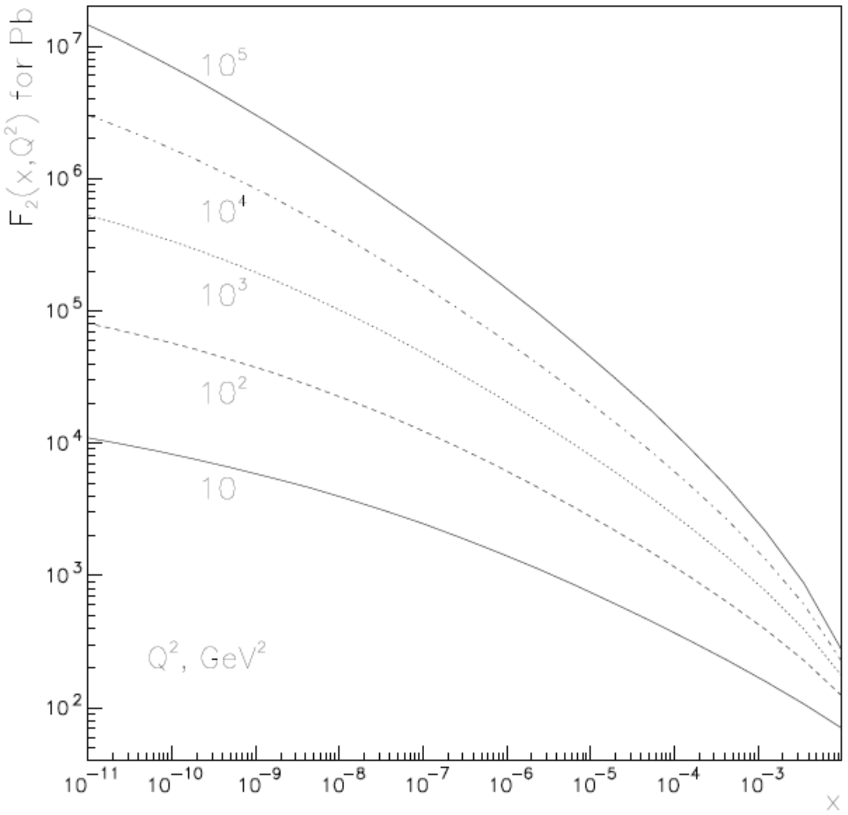
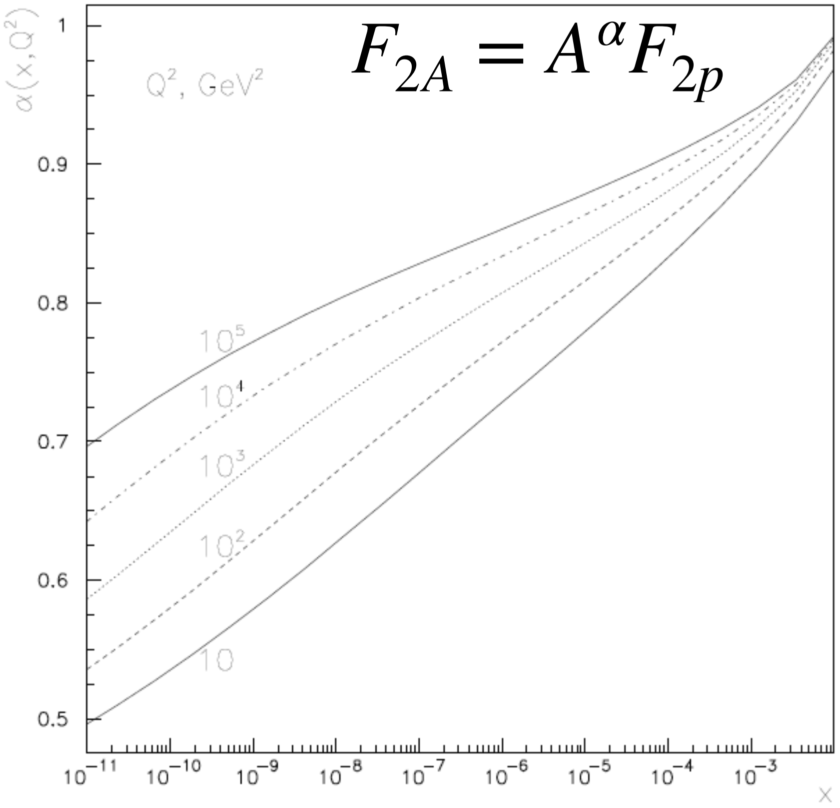
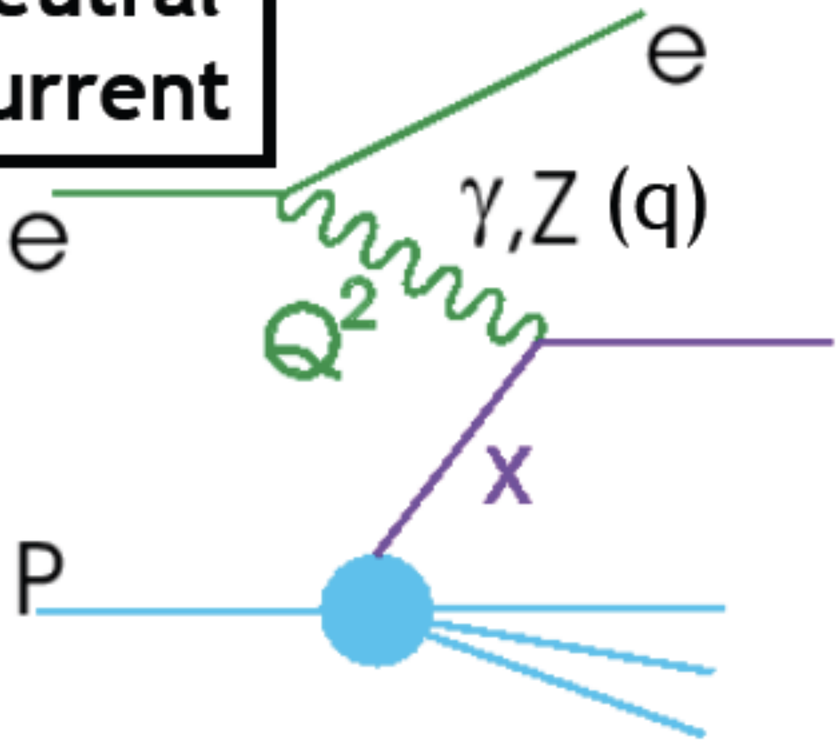




Backup:

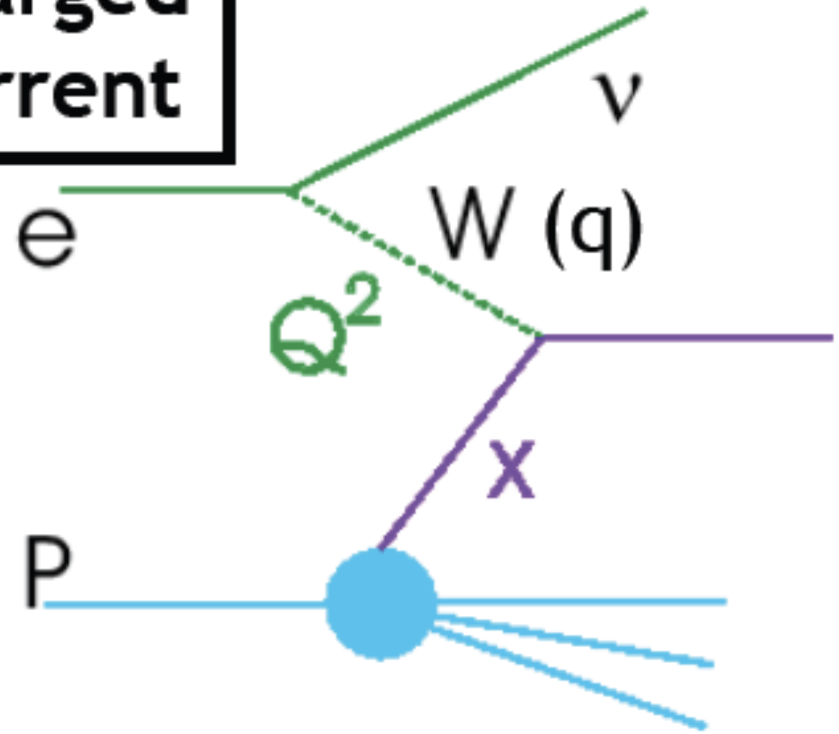
In more detail:

Neutral Current



Sensitivity to EW physics through CC and γZ interference.

Charged Current



$$\frac{d^2\sigma_{NC}}{dxdQ^2} = \frac{2\pi\alpha^2 Y_+}{Q^4 x}\cdot\sigma_{r,NC}$$

$$\frac{d^2\sigma^{\pm}_{CC}}{dxdQ^2} = \frac{1\pm P}{2}\cdot\frac{G_F^2}{2\pi x}\cdot\left[\frac{M_W^2}{M_W^2+Q^2}\right]^2 Y_+\cdot\sigma_{r,CC}$$

$$\sigma_{r,NC} = \mathbf{F_2} + \frac{Y_-}{Y_+}\mathbf{xF_3} - \frac{y^2}{Y_+}\mathbf{F_L},$$

$Y_\pm = 1 \pm (1-y)^2$

$$\sigma^{\pm}_{r,CC} = W_2^{\pm} \mp \frac{Y_-}{Y_+}xW_3^{\pm} - \frac{y^2}{Y_+}W_L^{\pm}$$

$$\begin{aligned}\mathbf{F_2^{\pm}} &= F_2 + \kappa_Z(-v_e \mp P a_e)\cdot F_2^{\gamma Z} + \kappa_Z^2(v_e^2 + a_e^2 \pm 2Pv_e a_e)\cdot F_2^Z \\ \mathbf{xF_3^{\pm}} &= \kappa_Z(\pm a_e + P v_e)\cdot xF_3^{\gamma Z} + \kappa_Z^2(\mp 2 v_e a_e - P(v_e^2 + a_e^2))\cdot xF_3^Z\end{aligned}$$

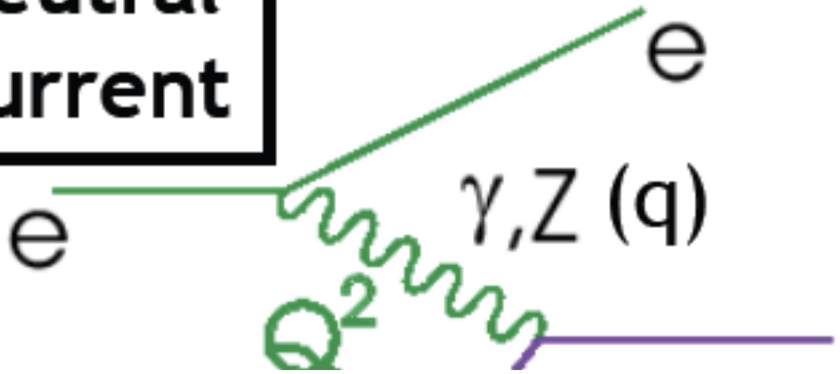
v_e, a_e: vector/axial couplings

$$\kappa_Z(Q^2) = \frac{Q^2}{(Q^2+m_Z^2)4\sin^2\theta_W\,\cos^2\theta_W}$$

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

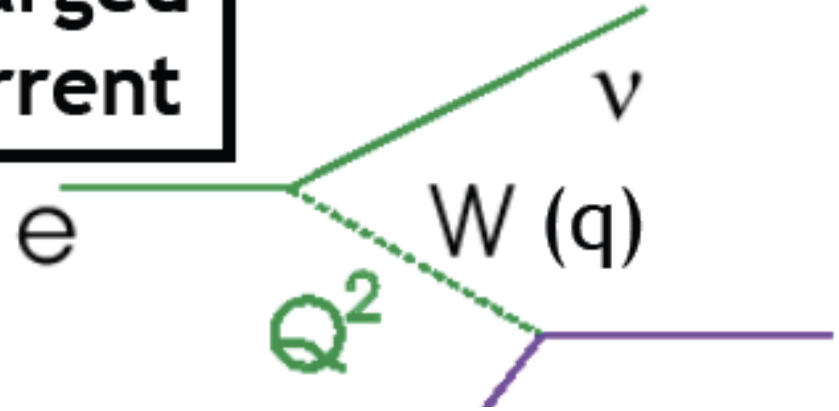
In more detail:

Neutral Current



Sensitivity to EW physics

Charged Current



$$(F_2, F_2^{\gamma Z}, F_2^Z) \approx [(e_u^2, 2e_u v_u, v_u^2 + a_u^2)(xU + x\bar{U}) + (e_d^2, 2e_d v_d, v_d^2 + a_d^2)(xD + x\bar{D})]$$

$$(xF_3^{\gamma Z}, xF_3^Z) \approx 2[(e_u a_u, v_u a_u)(xU - x\bar{U}) + (e_d a_d, v_d a_d)(xD - x\bar{D})] \,,$$

$$xU = xu + xc \,, \qquad x\bar{U} = x\bar{u} + x\bar{c} \,, \qquad xD = xd + xs \,, \qquad x\bar{D} = x\bar{d} + x\bar{s}$$

$$\boxed{\frac{d^2\sigma_{NC}}{dxdQ^2} = \frac{2\pi\alpha^2 Y_+}{Q^4 x} \cdot \sigma_{r,NC}} \qquad \boxed{\frac{\mathrm{d}^2\sigma^{\pm}_{CC}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{1 \pm P}{2} \cdot \frac{G_F^2}{2\pi x} \cdot \left[\frac{M_W^2}{M_W^2 + Q^2}\right]^2 Y_+ \cdot \sigma_{r,CC}}$$

$$\sigma_{r,NC} = \mathbf{F_2} + \frac{Y_-}{Y_+}\mathbf{xF_3} - \frac{y^2}{Y_+}\mathbf{F_L}, \quad Y_\pm = 1 \pm (1-y)^2 \qquad \sigma^{\pm}_{r,CC} = W_2^{\pm} \mp \frac{Y_-}{Y_+} x W_3^{\pm} - \frac{y^2}{Y_+} W_L^{\pm}$$

$$\mathbf{F_2^{\pm}} = F_2 + \kappa_Z(-v_e \mp P a_e) \cdot F_2^{\gamma Z} + \kappa_Z^2(v_e^2 + a_e^2 \pm 2P v_e a_e) \cdot F_2^Z$$

$$\mathbf{xF_3^{\pm}} = \kappa_Z(\pm a_e + P v_e) \cdot xF_3^{\gamma Z} + \kappa_Z^2(\mp 2 v_e a_e - P(v_e^2 + a_e^2)) \cdot xF_3^Z$$

v_e, a_e: vector/axial couplings $\quad \kappa_Z(Q^2) = \frac{Q^2}{(Q^2 + m_Z^2) 4 \sin^2\theta_W \cos^2\theta_W} \qquad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$

Massive quarks:

- Massive quarks do not have a collinear divergence (dead cone effect).
- The treatment of DGLAP evolution including massive quarks is an open issue, see e.g. 1510.02491.

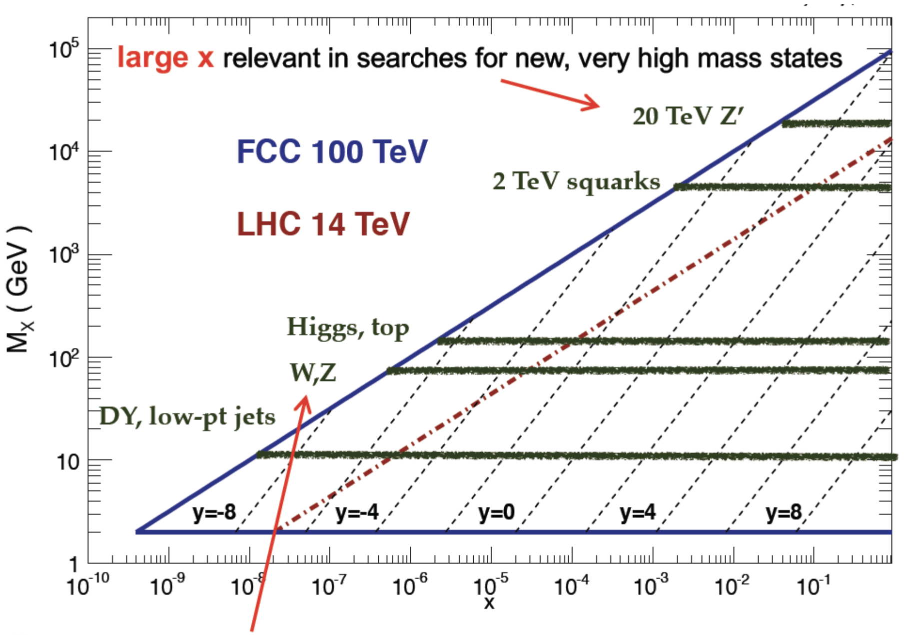
- FFNS: fixed number of massless species in evolution, HQ generated radiatively, good close to mass threshold, misses $\ln^n(Q^2/m_{HQ}^2)$.

- ZM-VFNS: variable number of massless species in evolution when increasing Q^2, captures $\ln^n(Q^2/m_{HQ}^2)$, bad at threshold.

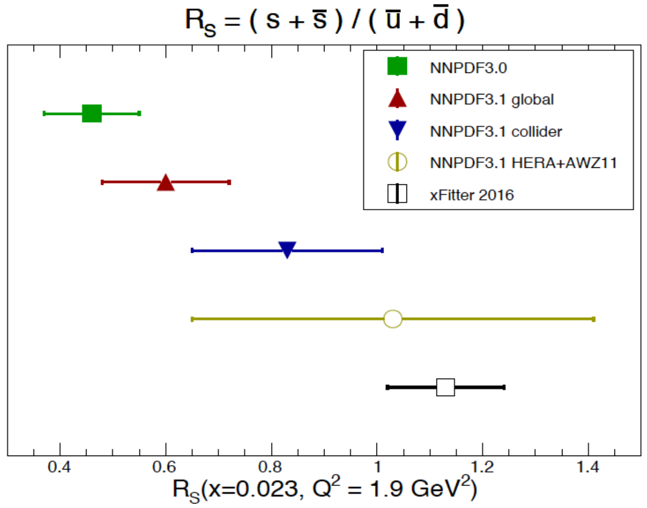
- Matching of both schemes: GM-VFNS, requires matching between parts that are exactly computed (massive matrix elements) and the massless evolution, several recipes.

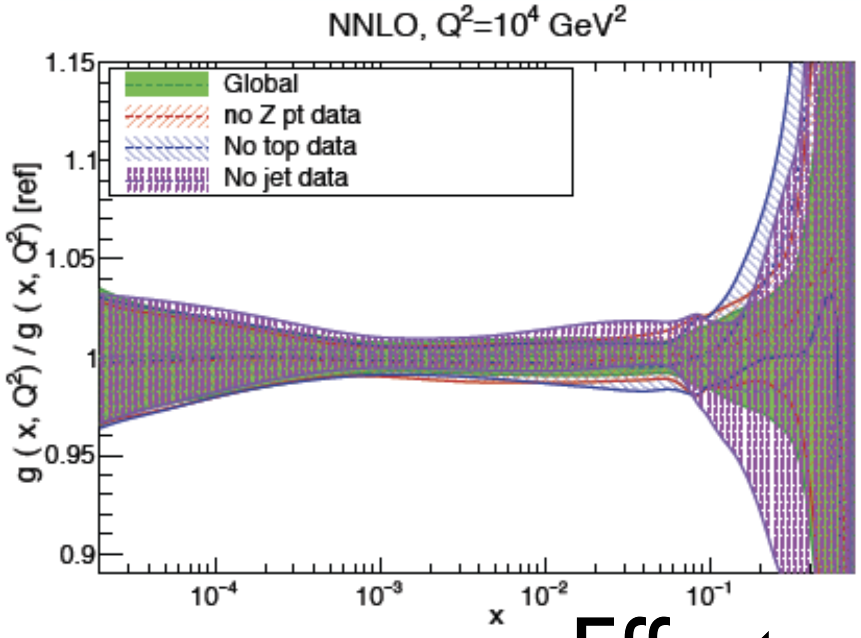
Proton PDFs at EICs:

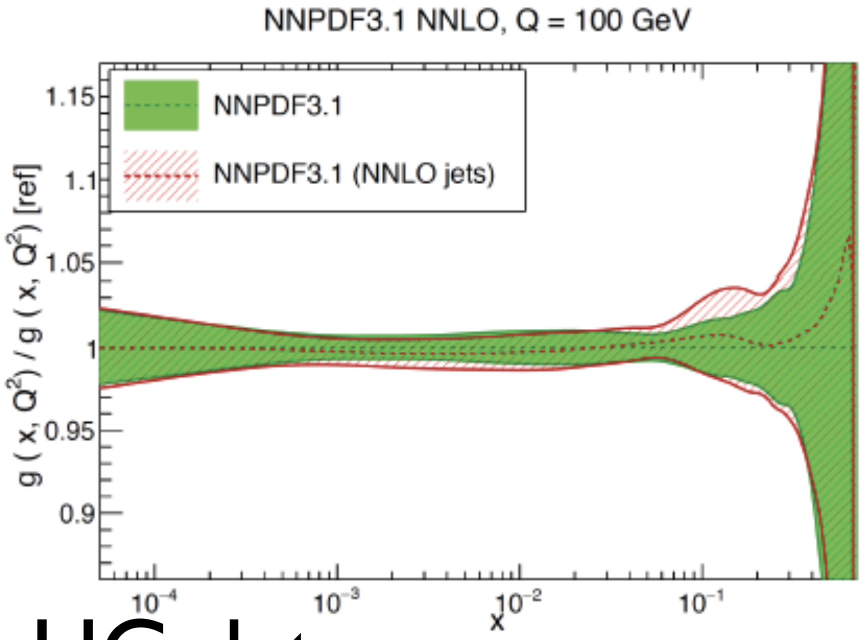
- Unpolarised proton PDFs show large uncertainties in regions of interest for HL-LHC and future hadron colliders.



small x becomes relevant even for “common” physics (EG. W, Z, H, t)







Effect of LHC data

Proton PDFs at EICs:

- Unpolarised proton PDFs show large uncertainties in regions of interest for HL-LHC and future hadron colliders.
- Inclusive measurements in ep largely improve the situation, plus new possibilities: full flavour decomposition, top, intrinsic charm,…
- The EIC adds precision in regions complementary to HERA.
- Note: factorization in pp/pA/AA is taken for granted, may not be so precise or maybe we hide other physics in PDFs.

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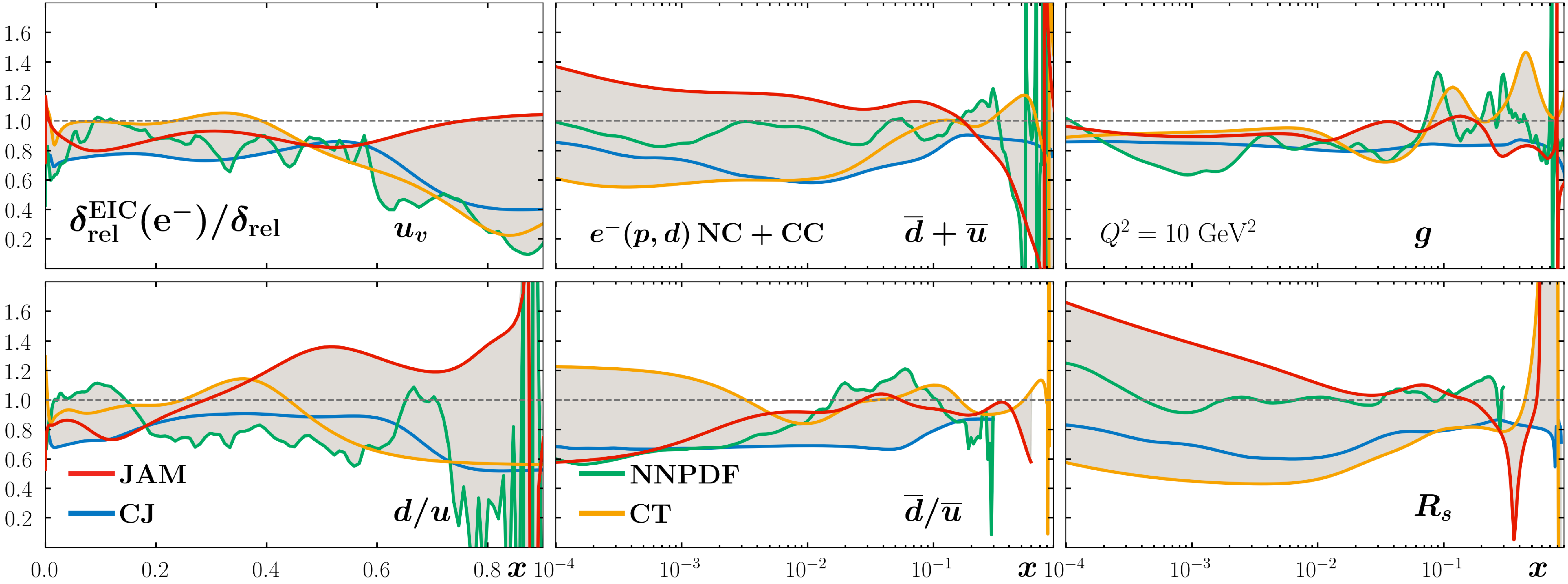

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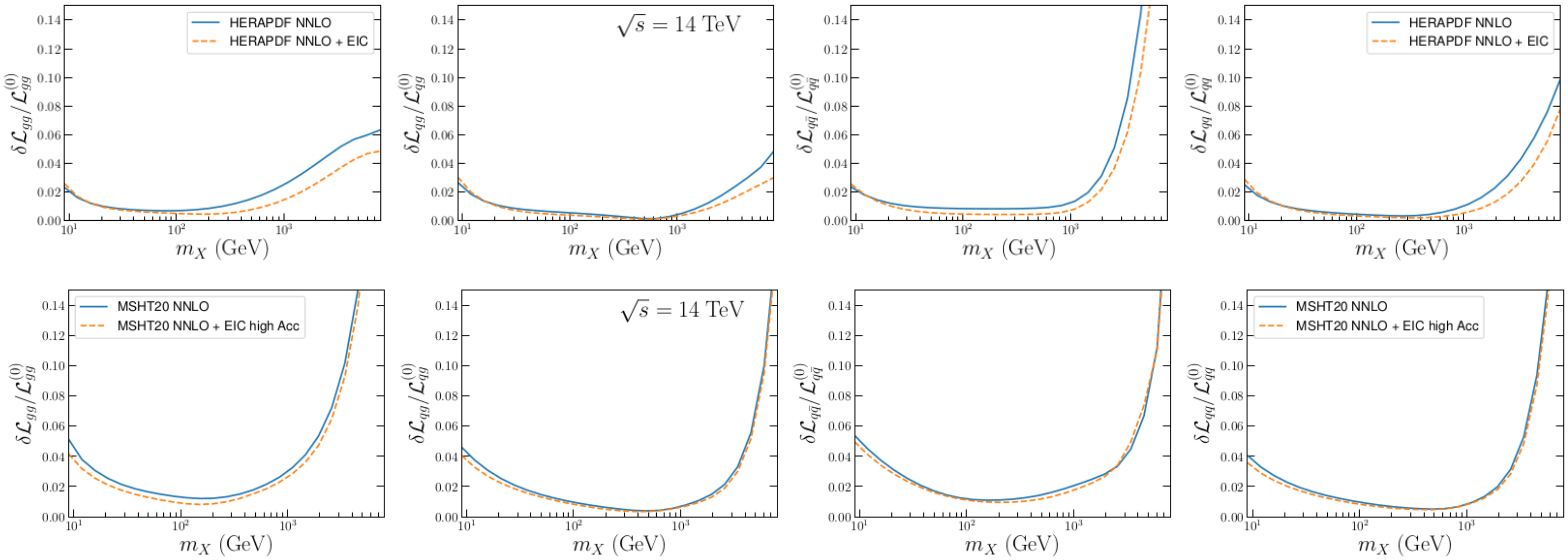
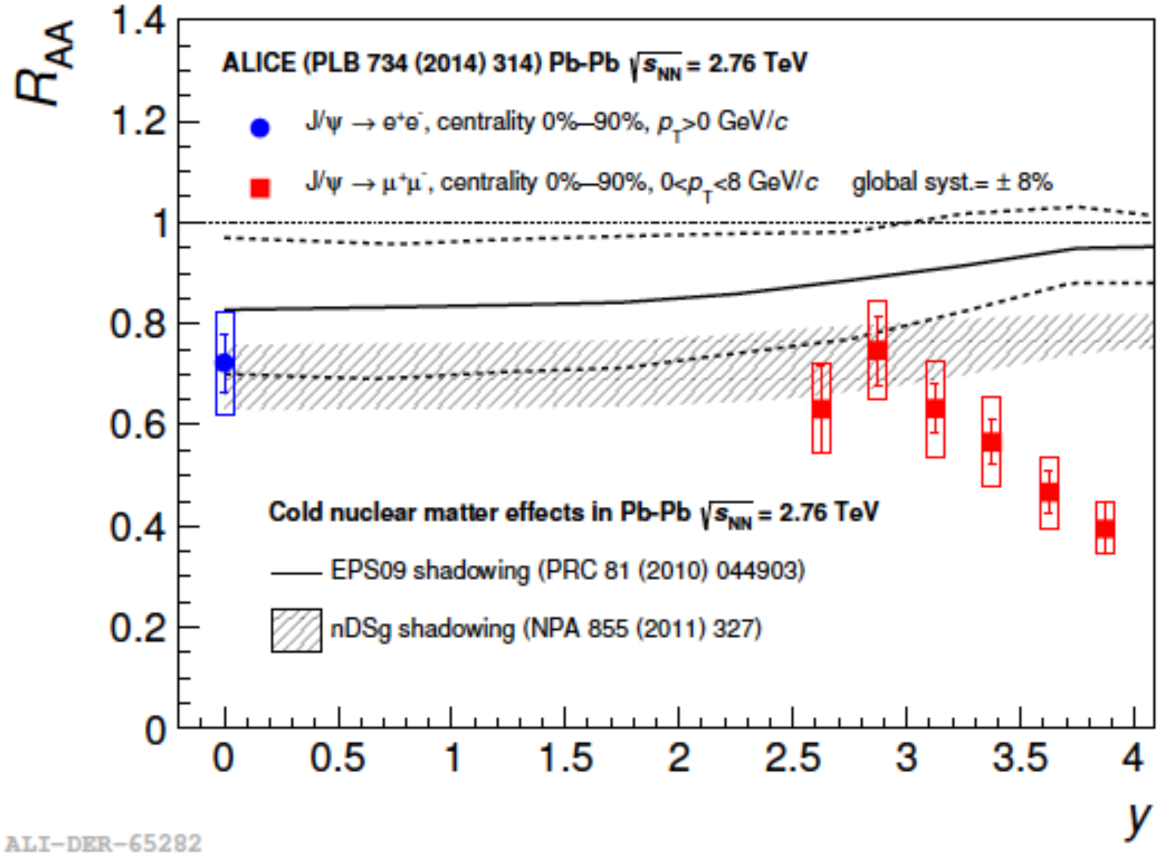
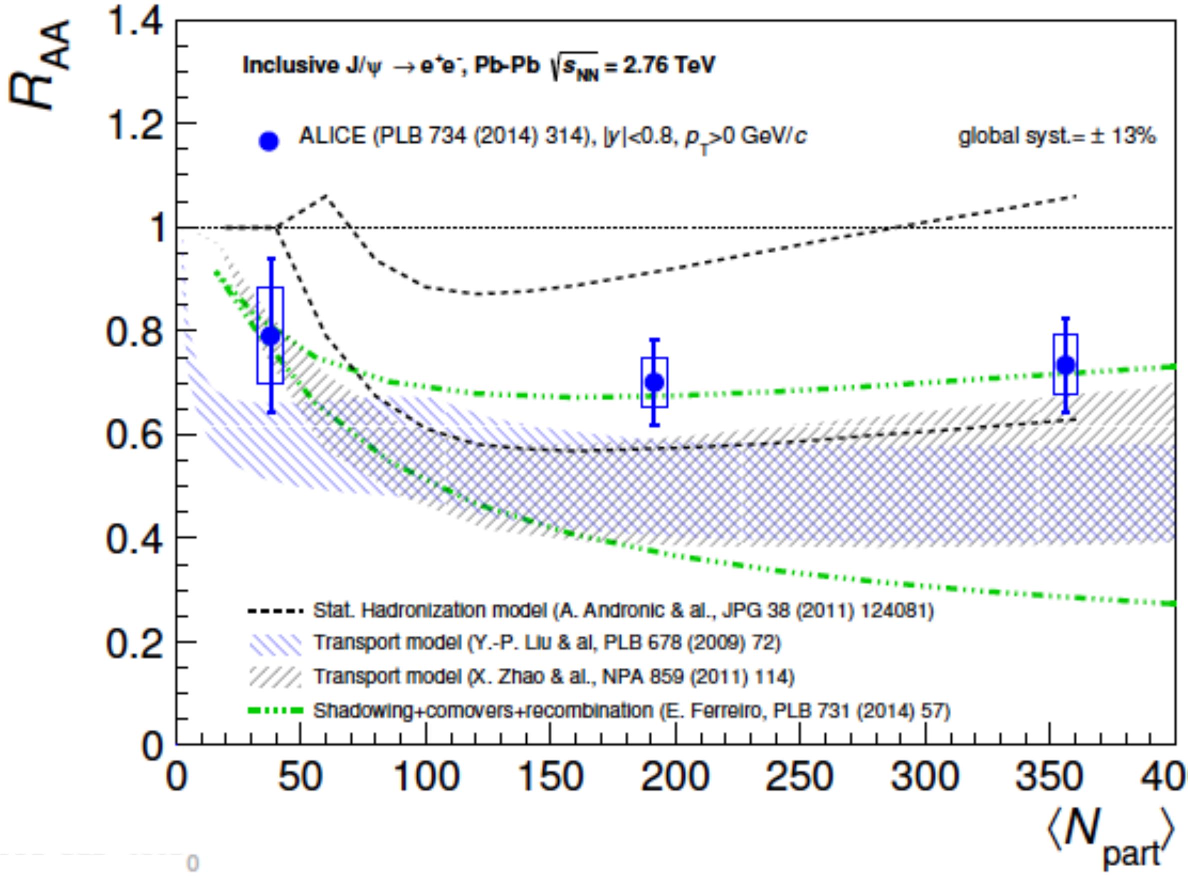


nPDFs for HIC:

$$R = \frac{f_{i/A}}{A f_{i/p}} \approx \frac{\text{measured}}{\text{expected if no nuclear effects}}$$

- Lack of data ⇒ large uncertainties for the nuclear glue at small scales and x: problem for benchmarking in HIC in order to extract ‘medium’ parameters.



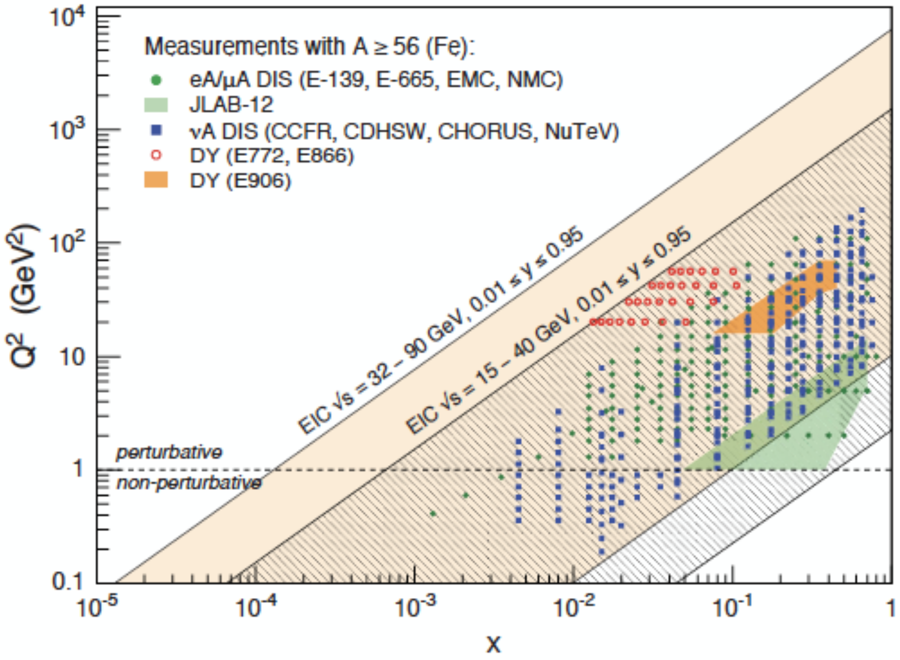


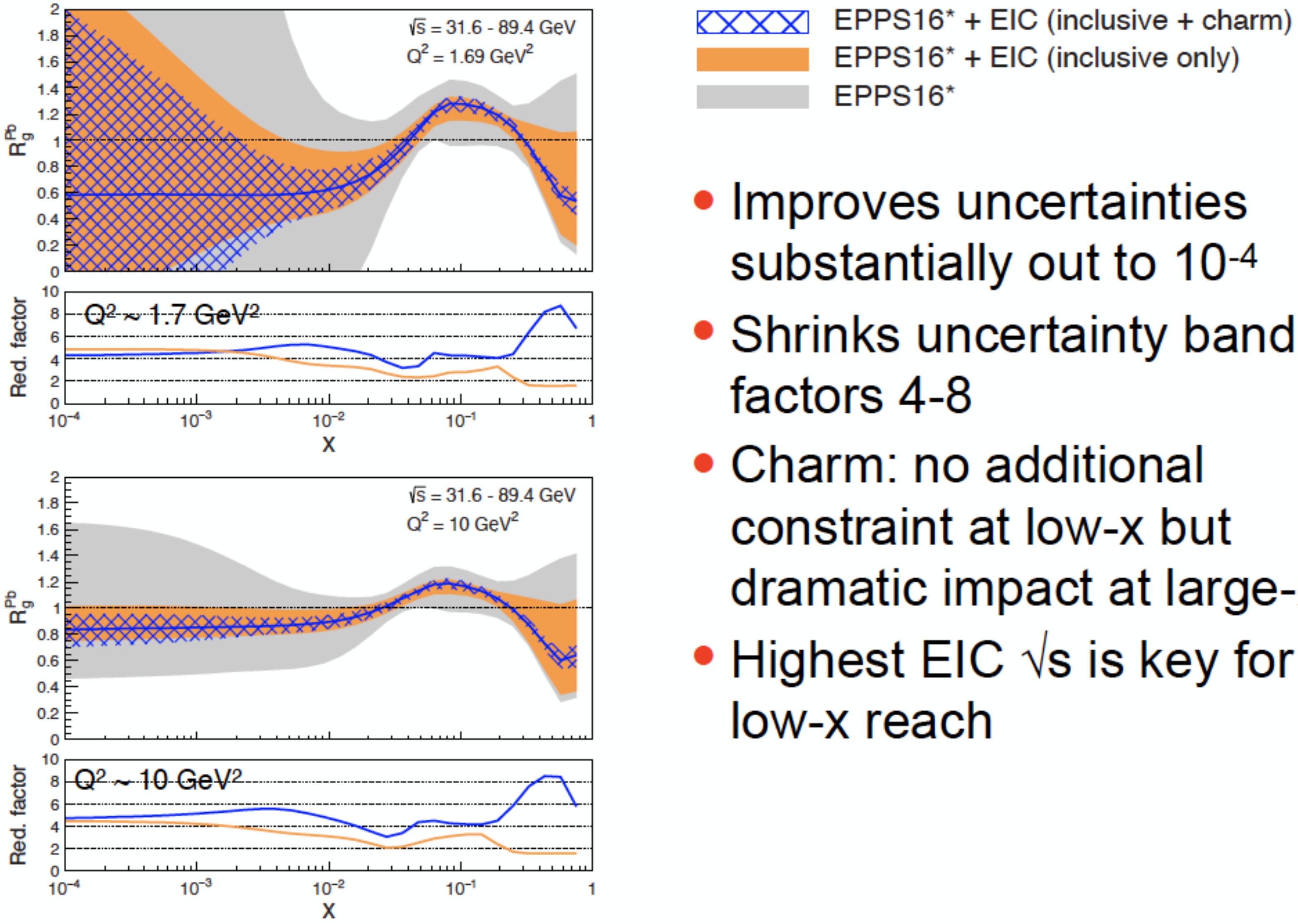
1506.03981

Nuclear PDFs at the EIC:

- Unpolarised nPDFs are very poorly known, particularly for $x<10^{-2}$.
- Inclusive measurements in eA largely improve the situation, plus new possibilities: flavour decomposition (but u-d challenging), fits for a single nucleus, release assumptions in unknown regions,…
- Fit to a single nucleus possible for the first time: no A-dependent initial conditions.

1708.05654





- Improves uncertainties substantially out to 10^{-4}
- Shrinks uncertainty band by factors 4-8
- Charm: no additional constraint at low-x but dramatic impact at large-x
- Highest EIC √s is key for low-x reach

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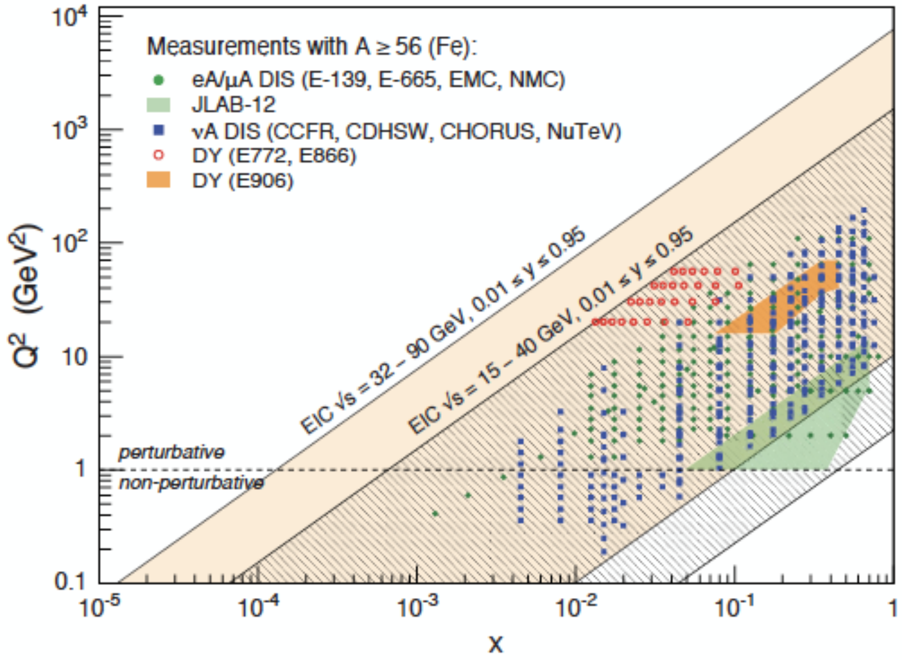
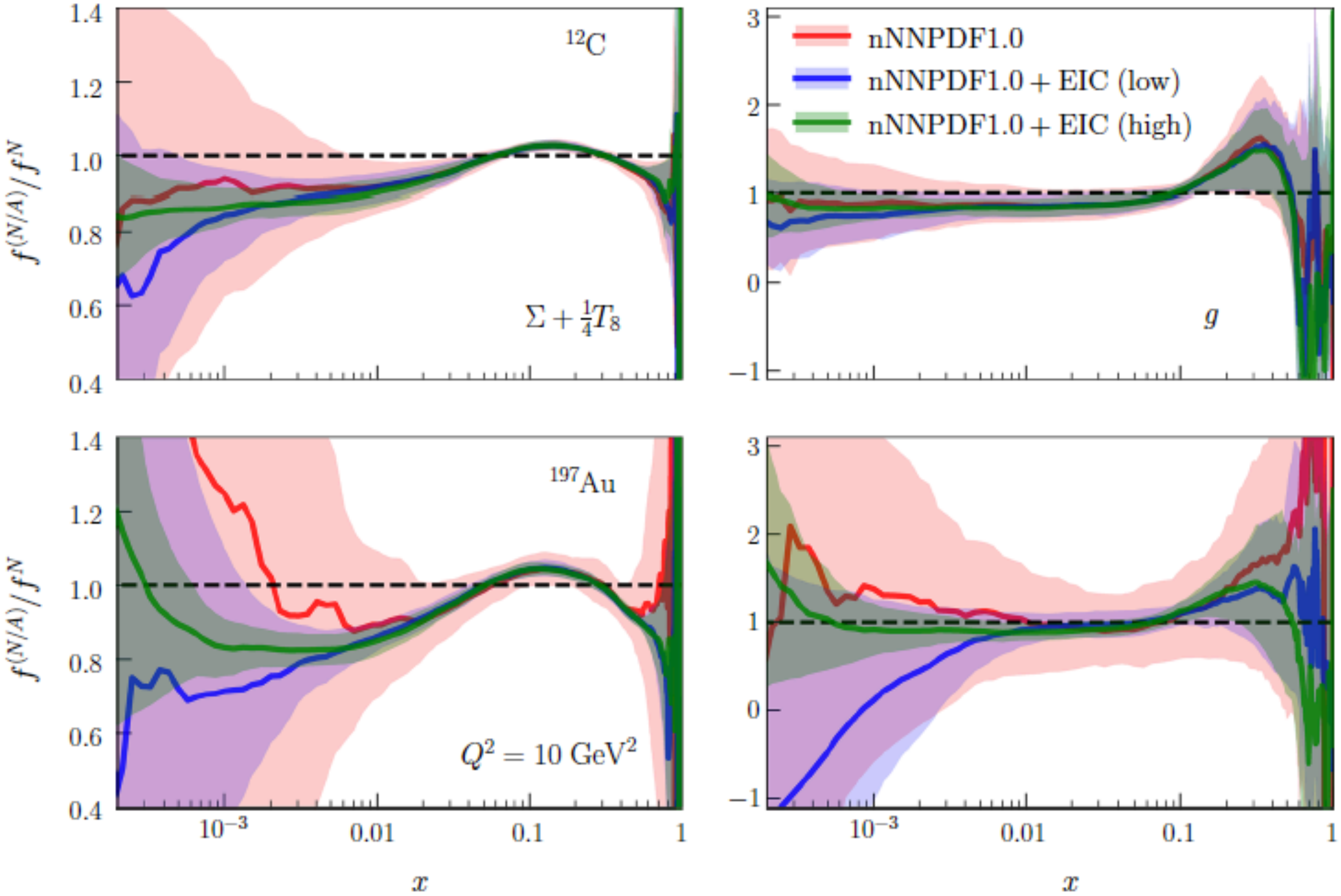
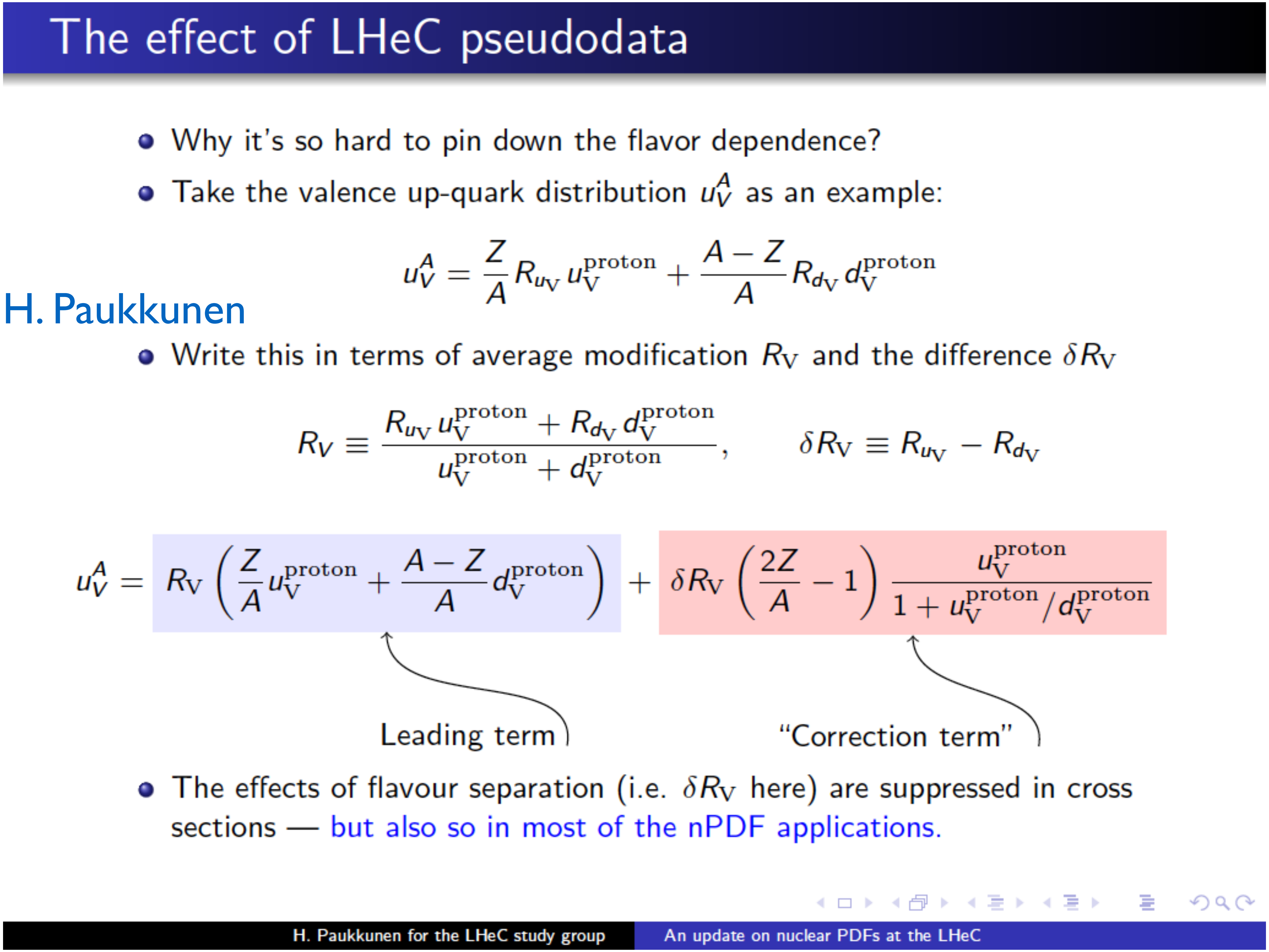
1904.00018





u-d separation in eA:

H. Paukkunen



The effect of LHeC pseudodata

- Why it's so hard to pin down the flavor dependence?
- Take the valence up-quark distribution u_V^A as an example:

$$u_V^A = \frac{Z}{A} R_{u_V} \, u_V^{\rm proton} + \frac{A-Z}{A} R_{d_V} \, d_V^{\rm proton}$$

- Write this in terms of average modification R_V and the difference δR_V

$$R_V \equiv \frac{R_{u_V} \, u_V^{\rm proton} + R_{d_V} \, d_V^{\rm proton}}{u_V^{\rm proton} + d_V^{\rm proton}}, \qquad \delta R_V \equiv R_{u_V} - R_{d_V}$$

$$u_V^A = \underbrace{R_V \left(\frac{Z}{A} u_V^{\rm proton} + \frac{A-Z}{A} d_V^{\rm proton} \right)}_{\text{Leading term}} + \underbrace{\delta R_V \left(\frac{2Z}{A} - 1 \right) \frac{u_V^{\rm proton}}{1 + u_V^{\rm proton} / d_V^{\rm proton}}}_{\text{“Correction term”}}$$

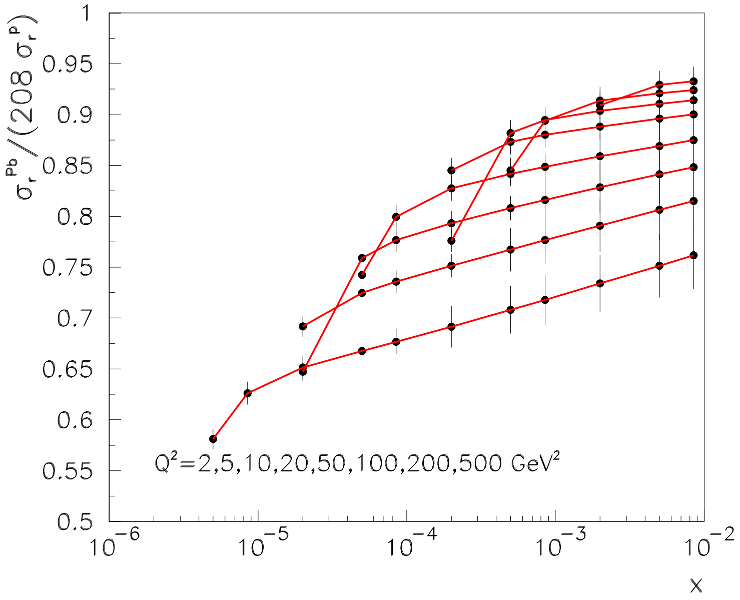
- The effects of flavour separation (i.e. δR_V here) are suppressed in cross sections — but also so in most of the nPDF applications.

H. Paukkunen for the LHeC study group | An update on nuclear PDFs at the LHeC

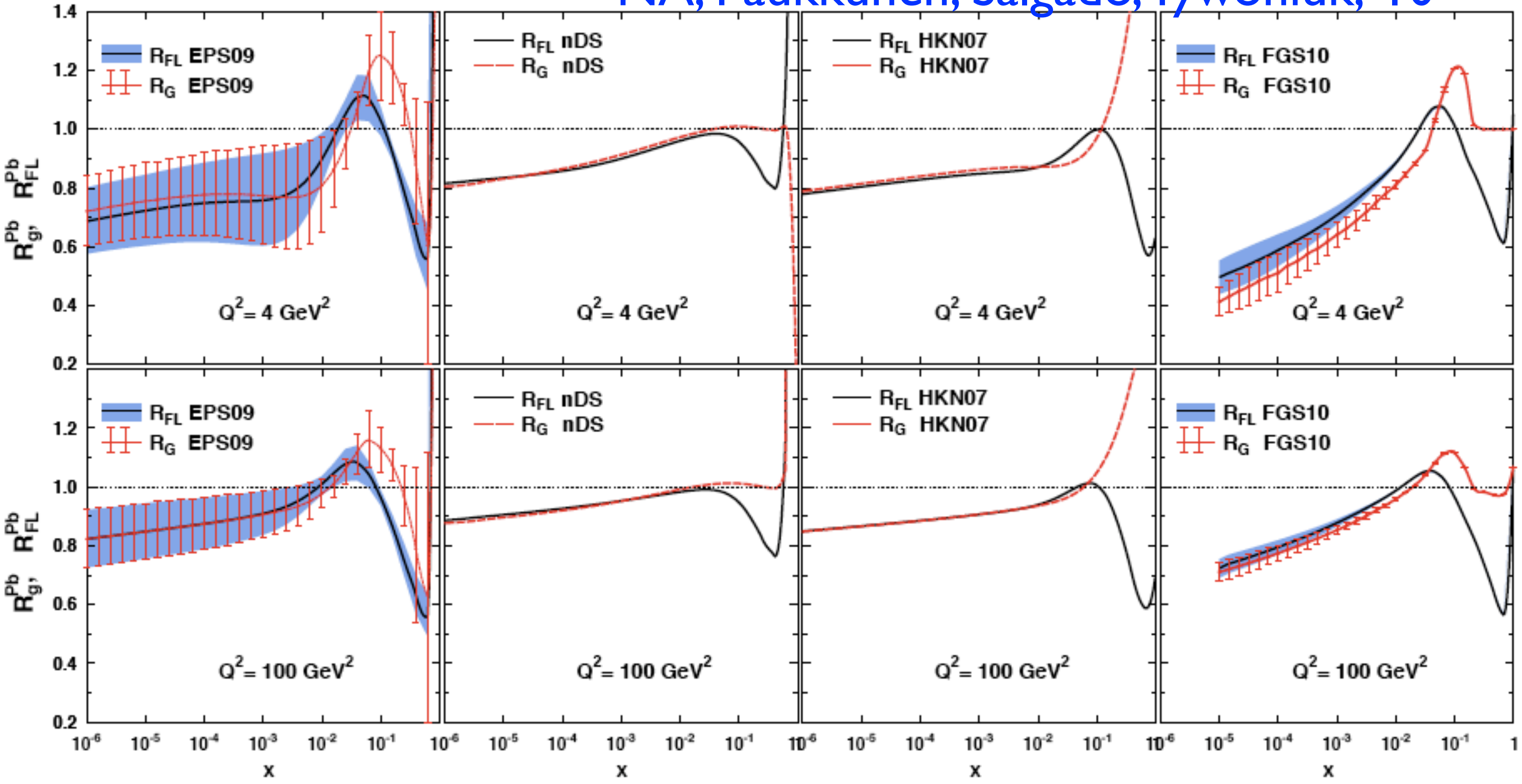
F_L in eA:

$$\sigma_r^{NC} = \frac{Q^4 x}{2\pi\alpha^2 Y_+}\frac{d^2\sigma^{NC}}{dxdQ^2} = F_2\left[1-\frac{y^2}{Y_+}\frac{F_L}{F_2}\right], \qquad Y_+ = 1+(1-y)^2$$

- F_L traces the nuclear effects on the glue (Cazarotto et al ’08): most sensitive to deviations wrt fixed order perturbation theory.
- Uncertainties in the extraction of F_2 due to the unknown nuclear effects on F_L of order 5 % (> stat.+syst.) ⇒ either measure F_L or use the reduced cross section (but then ratios at two energies...).



NA, Paukkunen, Salgado, Tywoniuk, ‘10

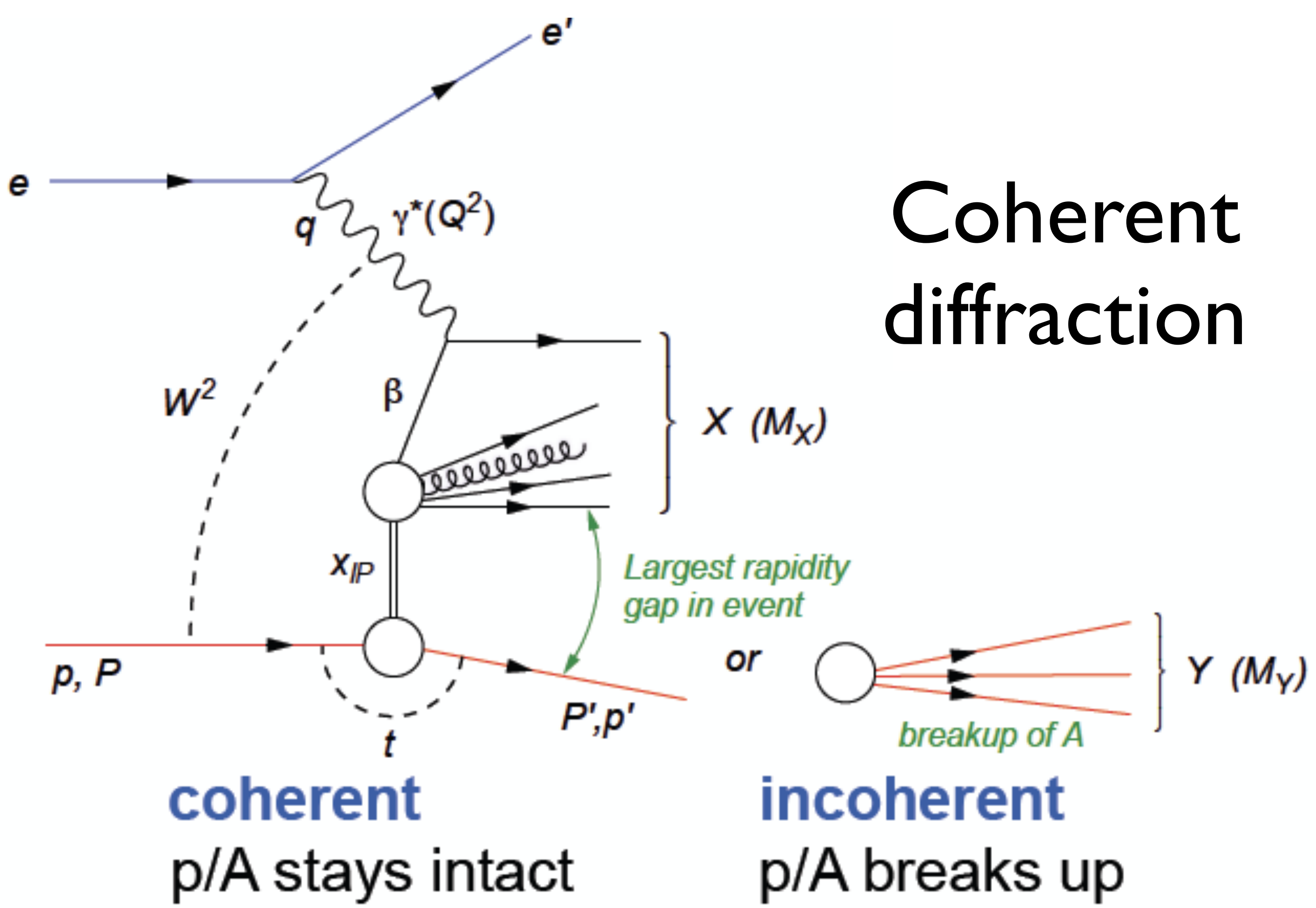


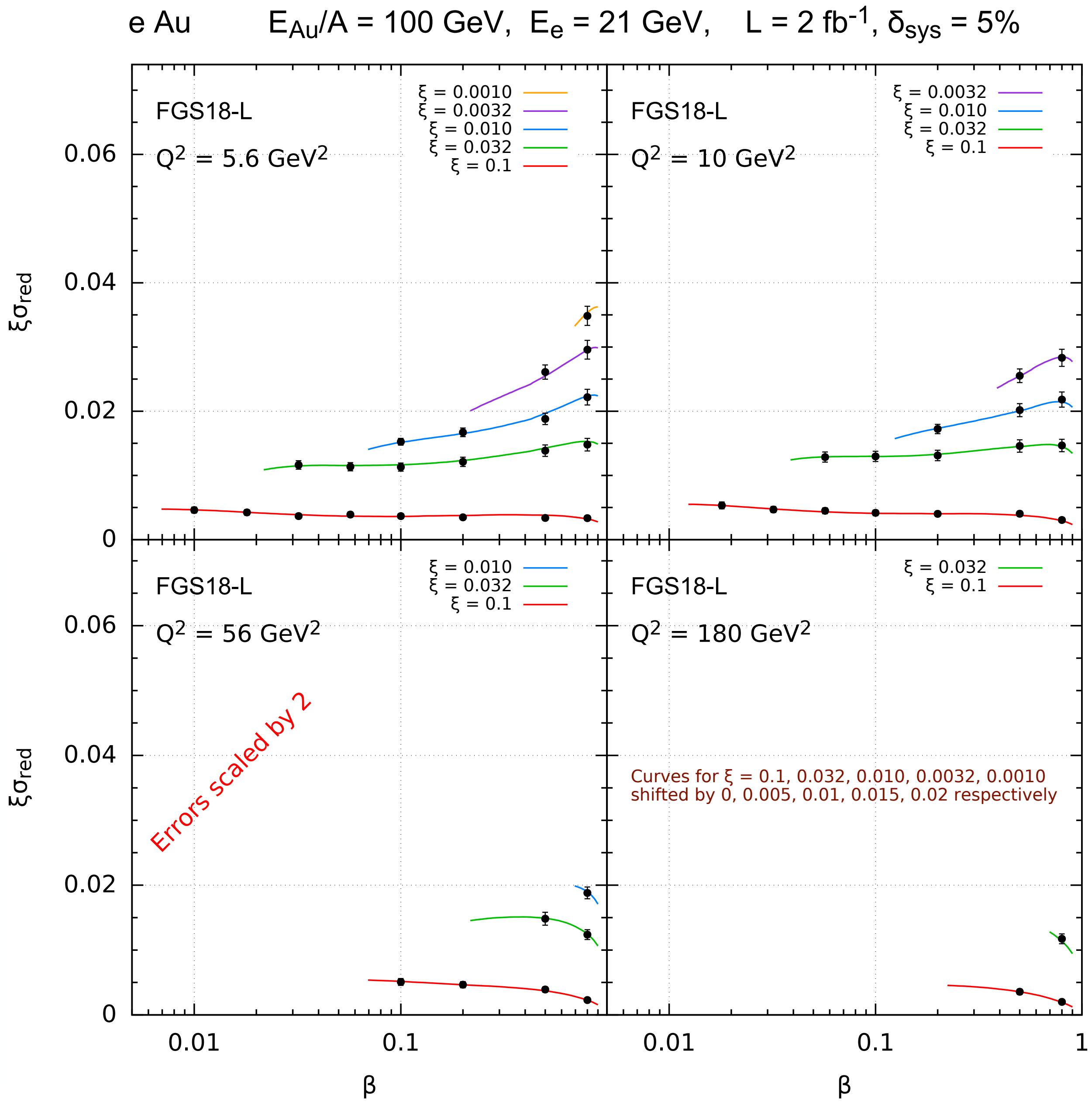
nDPDFs at EICs:

- Diffractive PDFs have never been measured in nuclei, where incoherent diffraction becomes dominant at relatively small -t: interplay between shadowing and gap survival probability.

- Challenging experimental problem (LPS + ZDC?).

Coherent diffraction





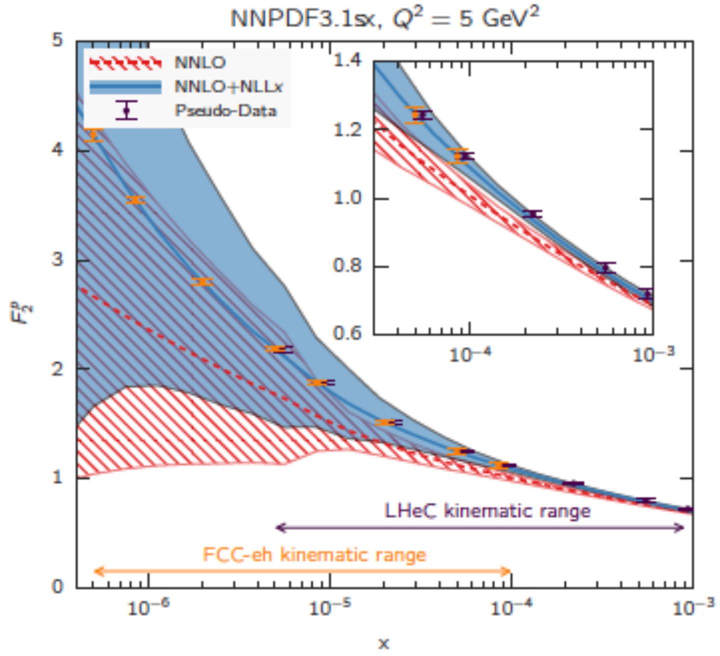
Resummation:

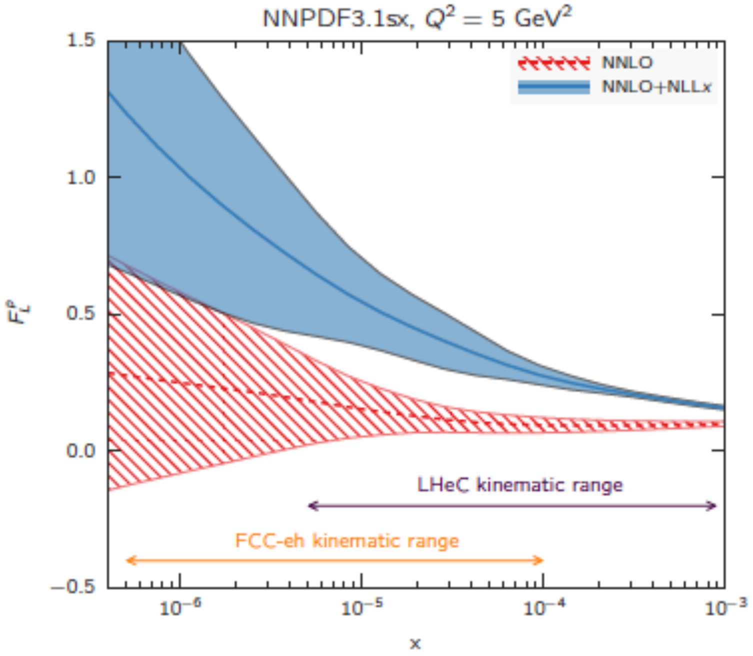
- Resummation has been suggested (1710.05935) to cure the problem seen in HERA data of a worsening of the PDF fit quality with decreasing x and Q^2: the problem lies in F_L (i.e., in the glue).

$$P_{ij}^{\mathrm{N}^k\mathrm{LO}+\mathrm{N}^h\mathrm{LL}x}(x) = P_{ij}^{\mathrm{N}^k\mathrm{LO}}(x) + \Delta_k P_{ij}^{\mathrm{N}^h\mathrm{LL}x}(x)$$

$$k = 0{,}1{,}2,\ h = 0{,}1\ \text{at present}$$

- This approach, and saturation, can be checked at smaller x through the tension between observables: F_2, F_L, σ_r^{HQ}.





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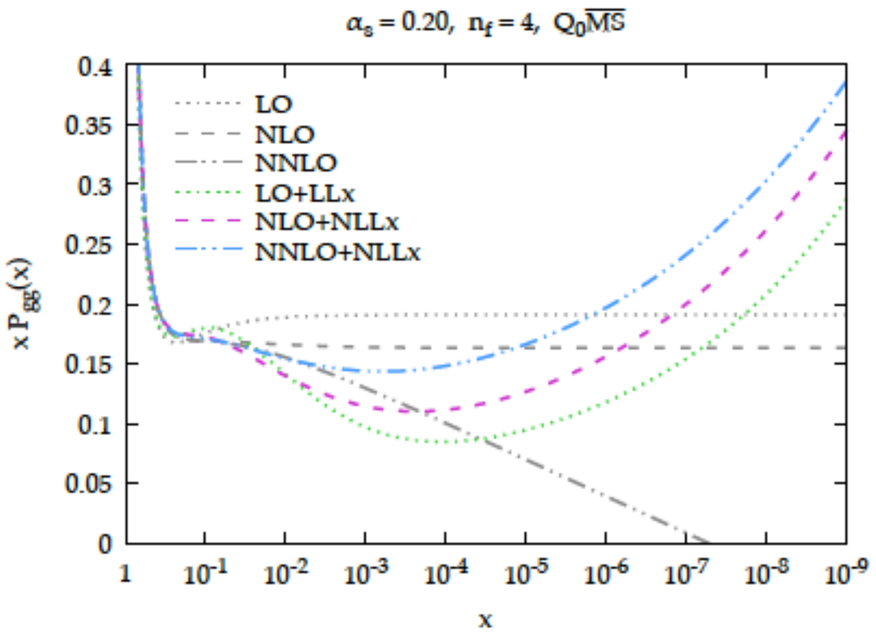
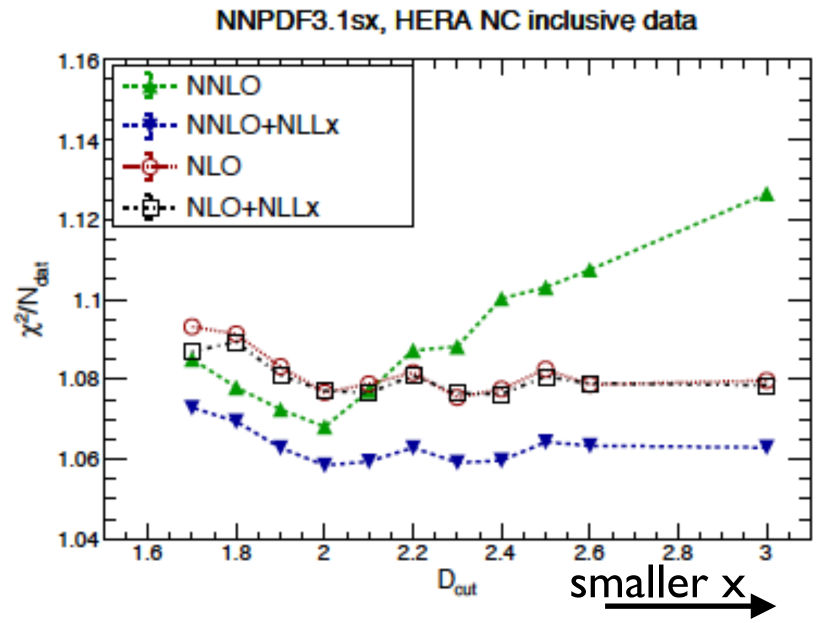
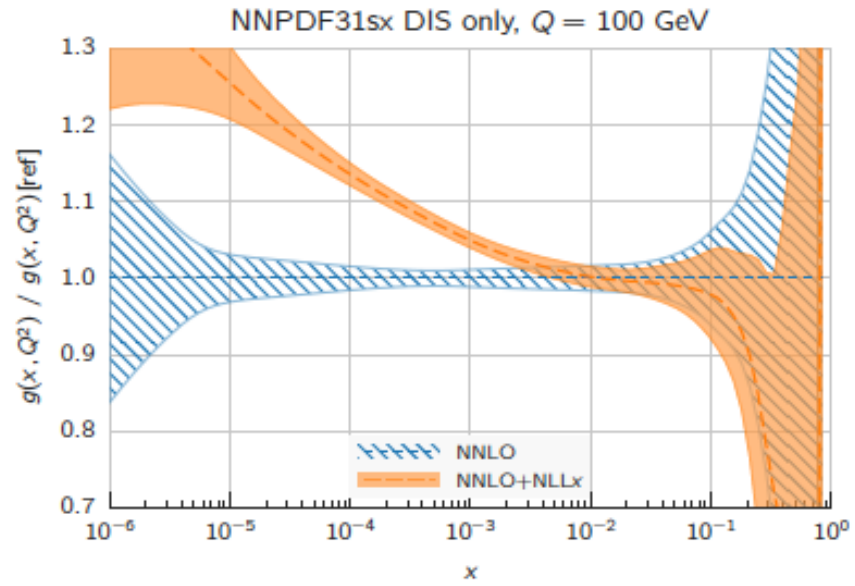




smaller x



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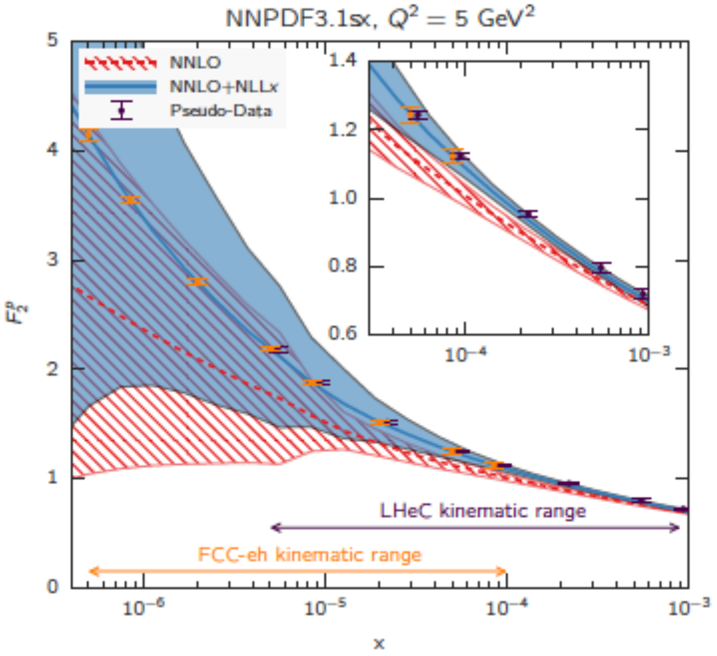

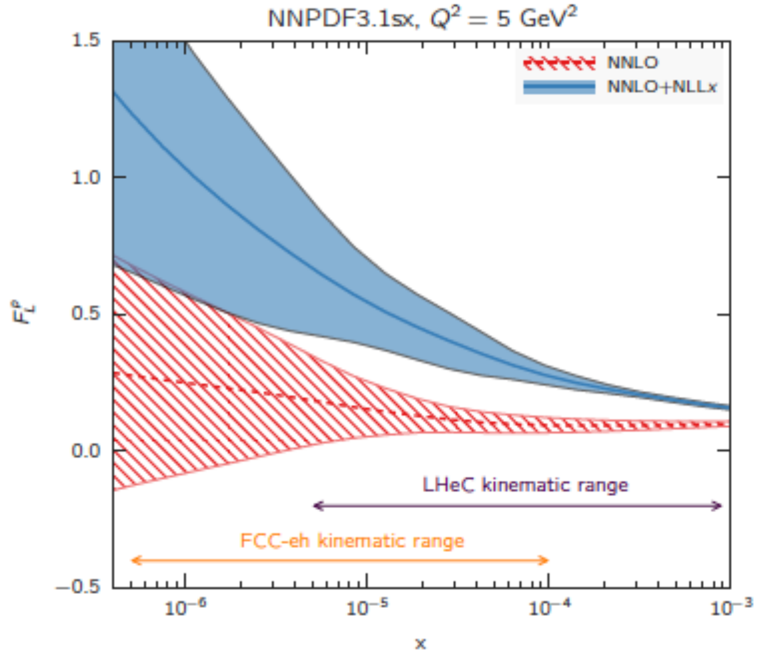

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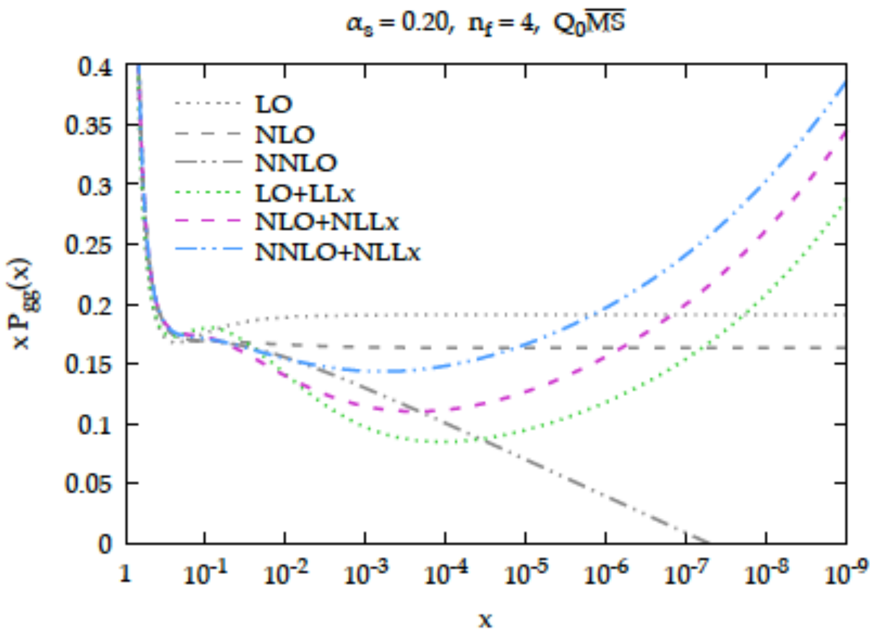

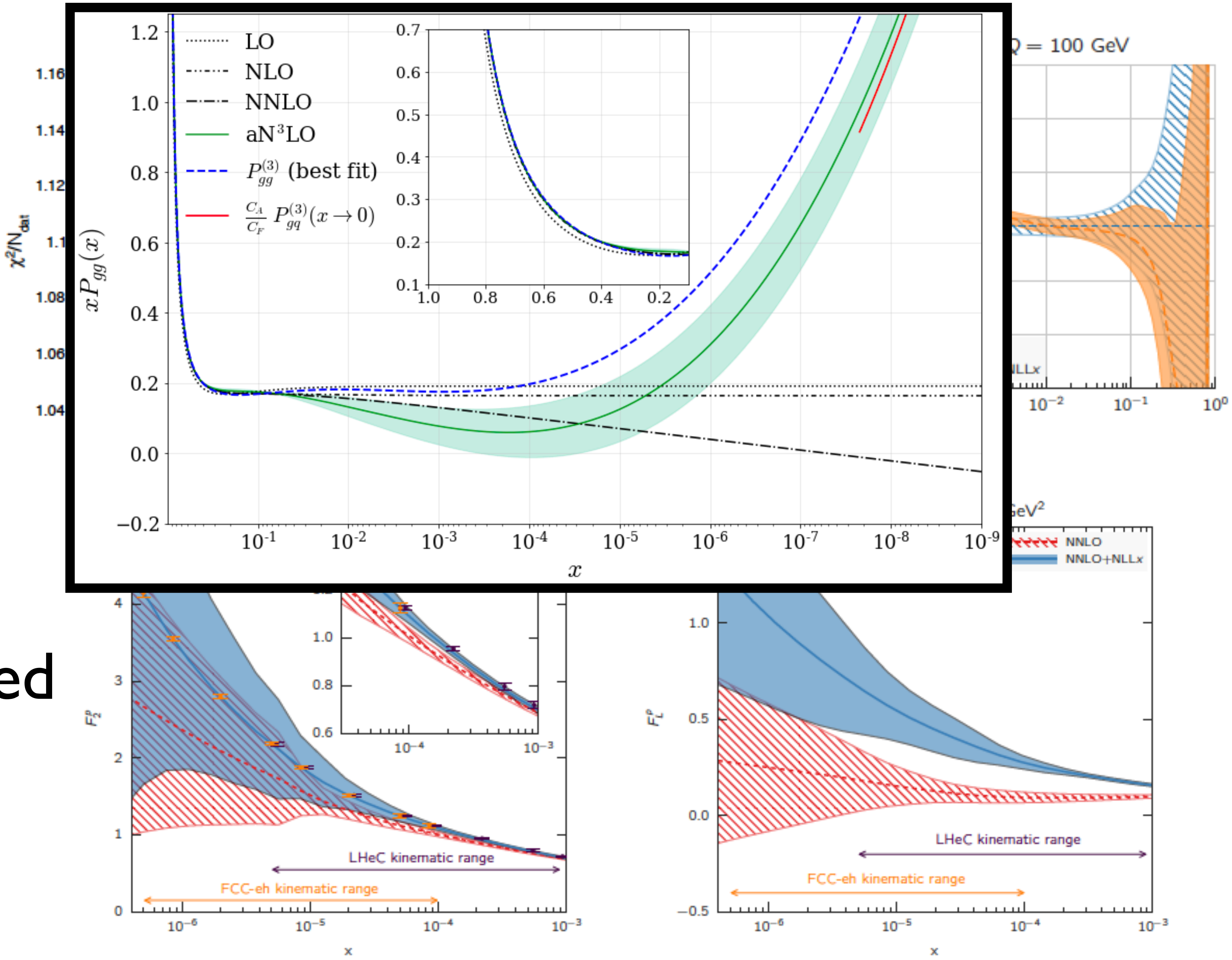

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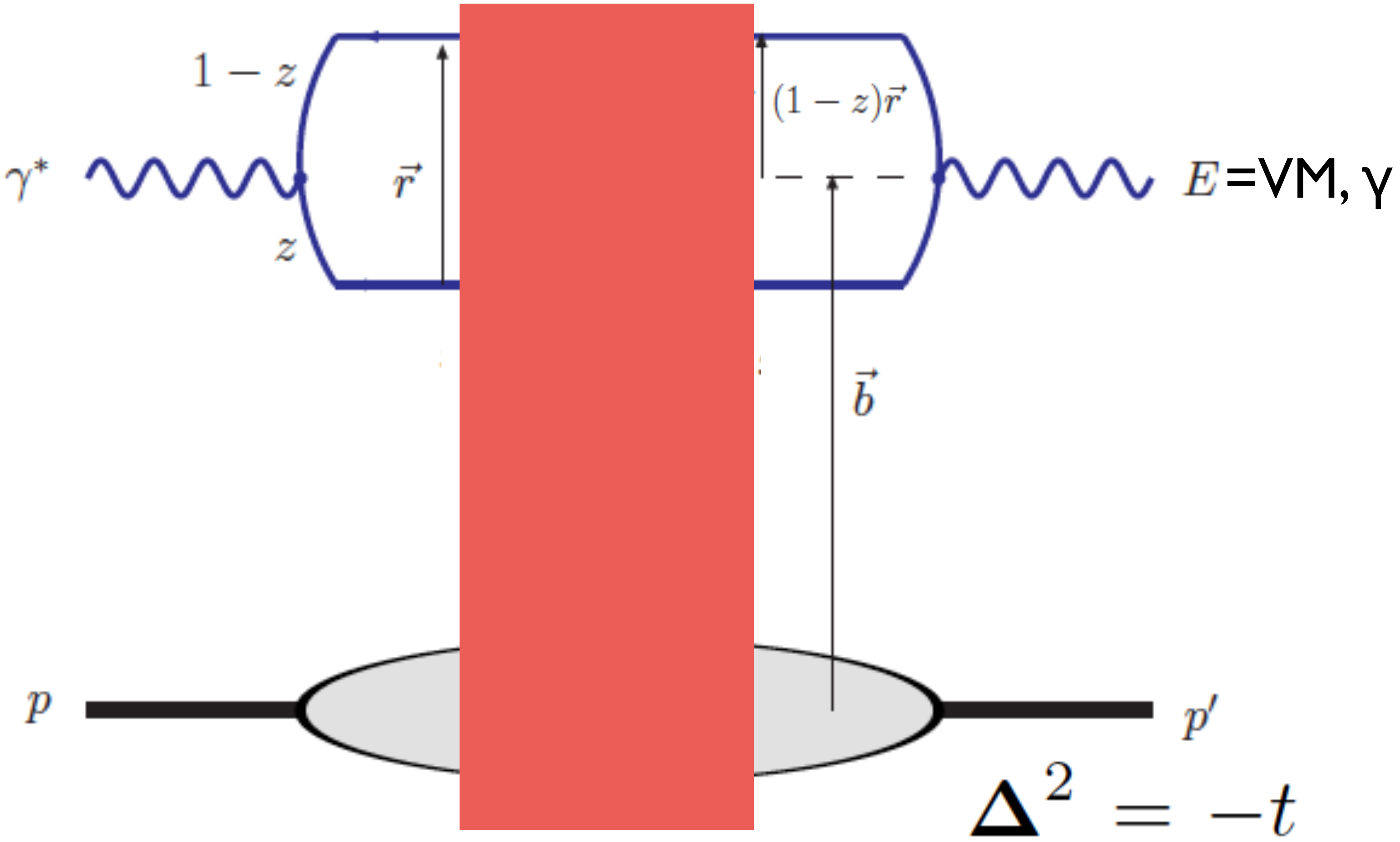
The dipole picture:

- Long-lived (virtual) photon fluctuation, $x<(m_N R)^{-1}\sim 0.1A^{1/3}$.

- Unified description of inclusive, diffractive and exclusive processes.



[redacted]

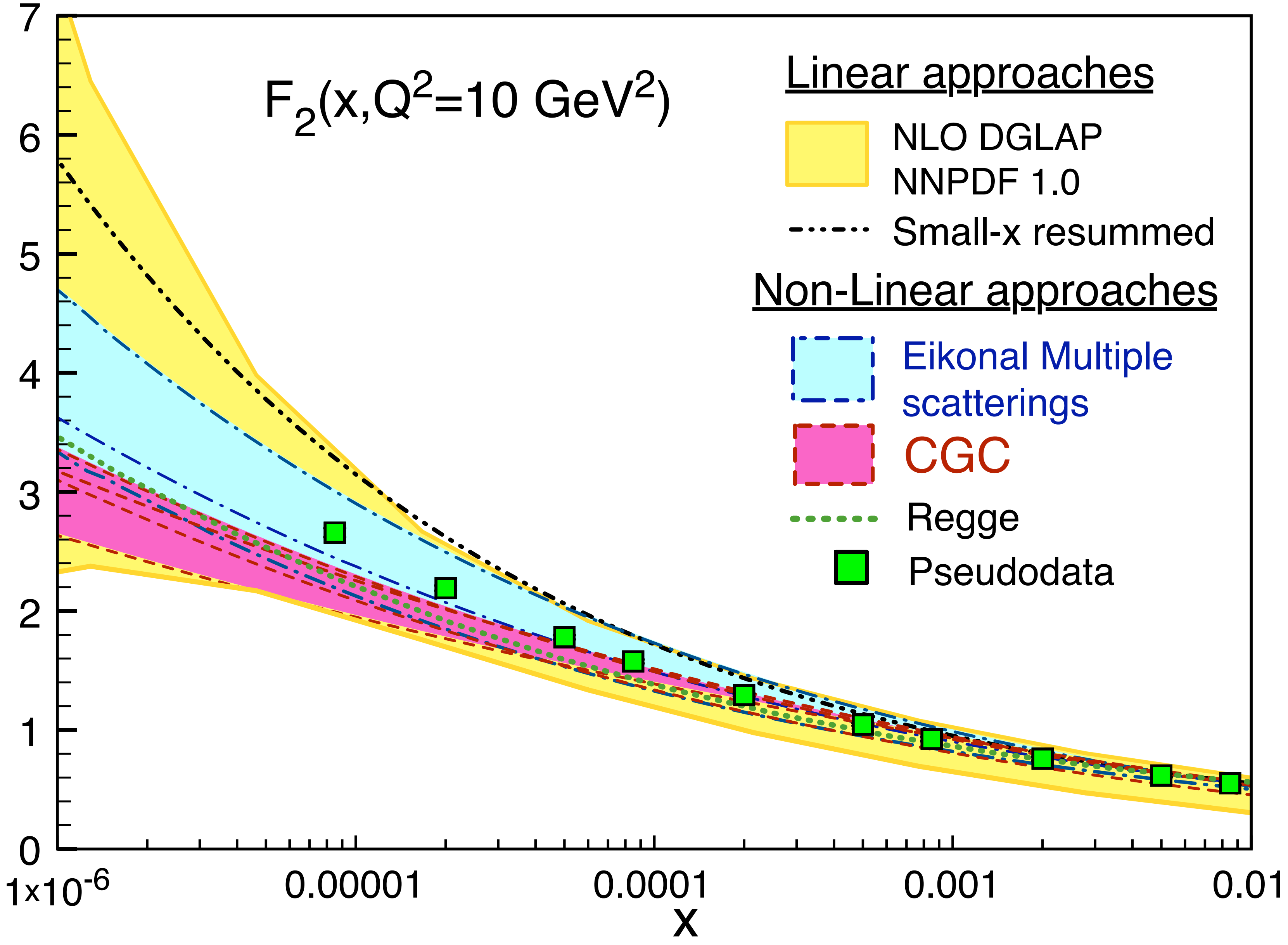
$$\frac{\mathrm{d}\sigma_{T,L}^{\gamma^* p\to Ep}}{\mathrm{d}t}=\frac{1}{16\pi}\left|\mathcal{A}_{T,L}^{\gamma^* p\to Ep}\right|^2\,(1+\beta^2)R_g^2\qquad \beta=\tan\left(\frac{\pi\lambda}{2}\right),\;\lambda\equiv\frac{\partial\ln\left(\mathcal{A}_{T,L}^{\gamma^* p\to Ep}\right)}{\partial\ln(1/x)}$$

$$\mathcal{A}_{T,L}^{\gamma^* p\to Ep}=2\mathrm{i}\int\mathrm{d}^2\boldsymbol{r}\int_0^1\mathrm{d}z\int\mathrm{d}^2\boldsymbol{b}\,(\Psi_E^*\Psi)_{T,L}\,\mathrm{e}^{-\mathrm{i}[\boldsymbol{b}-(1-z)\boldsymbol{r}]\cdot\boldsymbol{\Delta}}\mathcal{N}\,(x,r,b)$$

- Correction to non-diagonal gluon PDF (skewedness) introduced.
- Boosted Gaussian VM WF fitted to leptonic decays.
- qqbarg component in diffraction, not yet in exclusive VM.

Small x: inclusive observables

- Simultaneous description of different inclusive observables (with different sensitivities to the gluon and the sea) in DGLAP may show tensions e.g. F_2 and F_L or σ_r^{HQ} if enough lever arm in Q^2 is available.



N. Armesto, 12-14.06.2024 - Partonic structure & small x: theory.

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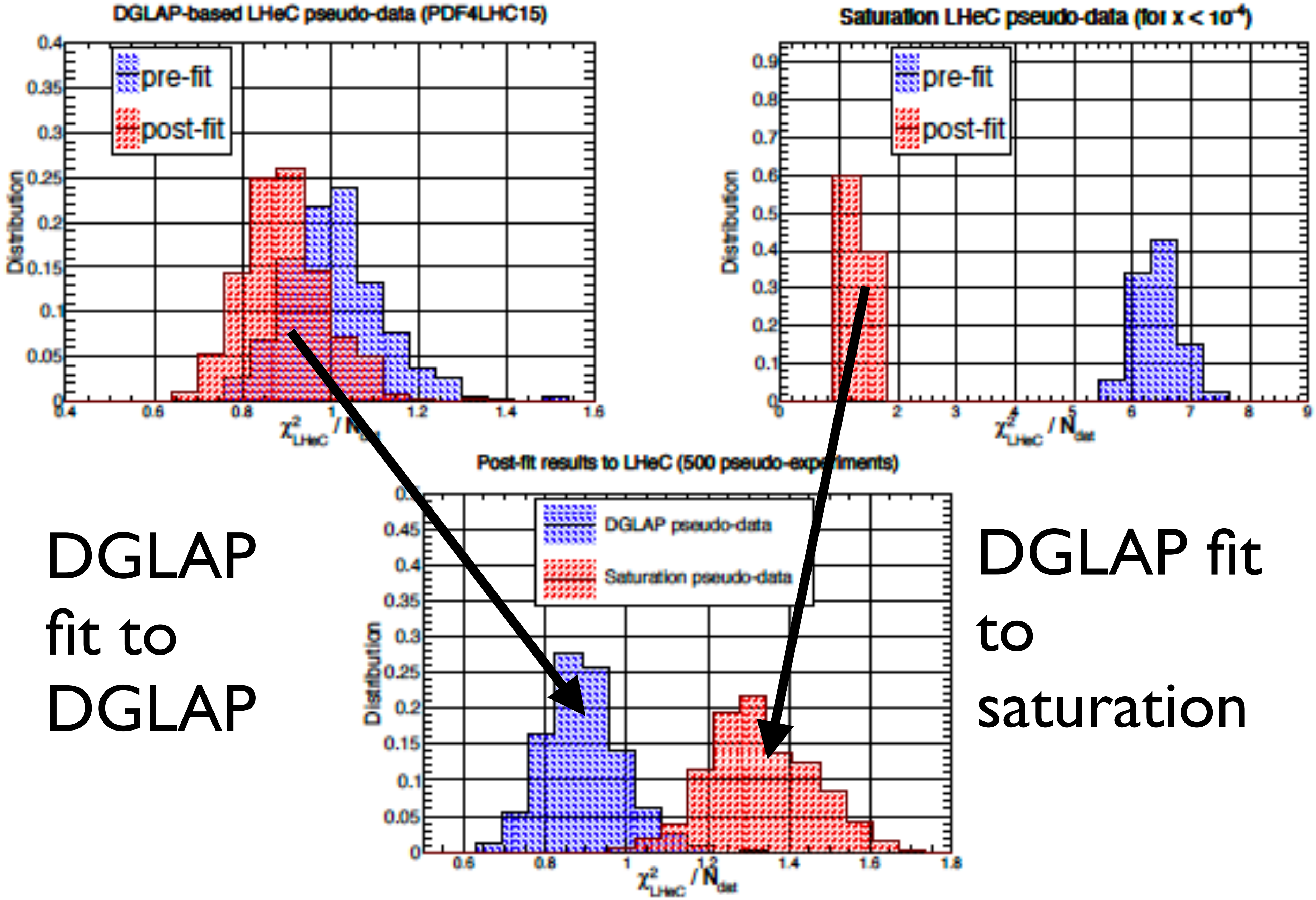

DGLAP fit to DGLAP

DGLAP fit to saturation

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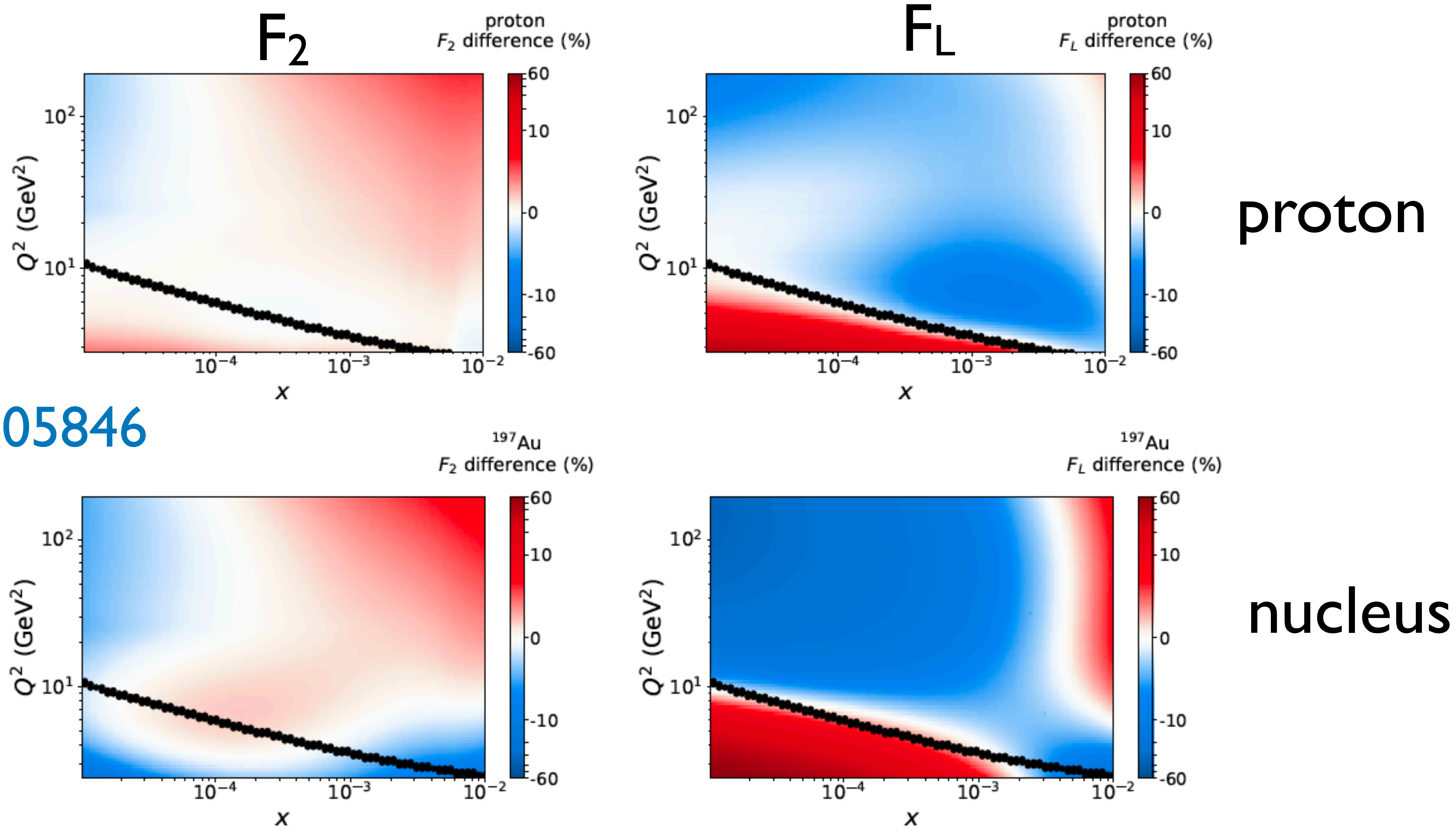
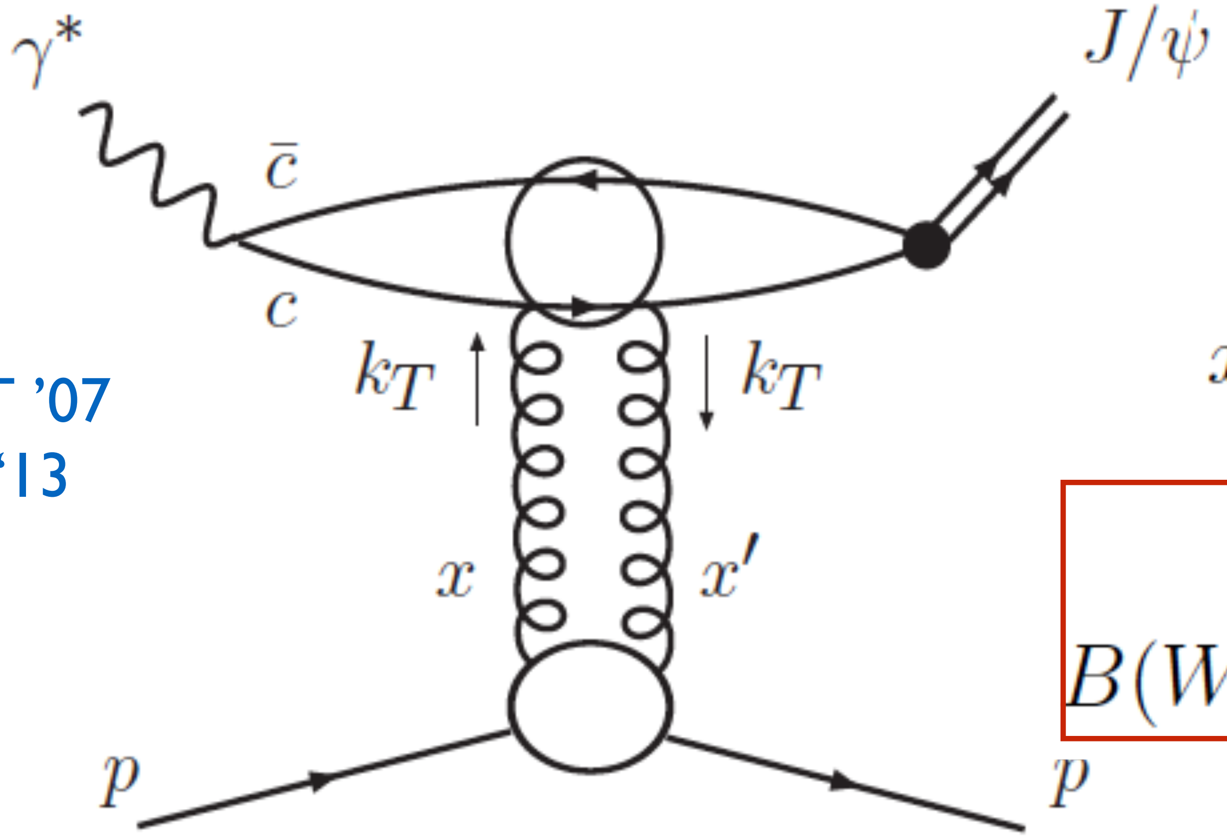


2203.05846

pQCD for ep→eJ/ψp:



MNRT '07
JMRT '13

$$\bar{Q}^2 = (Q^2 + M^2_{J/\psi})/4$$

$$x = (Q^2 + M^2_{J/\psi})/(W^2 + Q^2)$$

$$\boxed{\begin{array}{c}\sigma \sim \exp(-Bt) \\ B(W) = (4.9 + 4\alpha' \ln(W/W_0)) \;\; \mathrm{GeV}^{-2}\end{array}}$$

by hand (Regge)

LO

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(\gamma^* p \to J/\psi\ p)\Big|_{t=0} = \frac{\boxed{\Gamma_{ee}} M^3_{J/\psi}\pi^3}{48\alpha} \boxed{\left[\frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} xg(x, \bar{Q}^2)\right]^2} \left(1 + \frac{Q^2}{M^2_{J/\psi}}\right)$$

NR WF

- It should not be the gluon PDF but the GPD:

$$R_g = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + \frac{5}{2})}{\Gamma(\lambda + 4)}$$

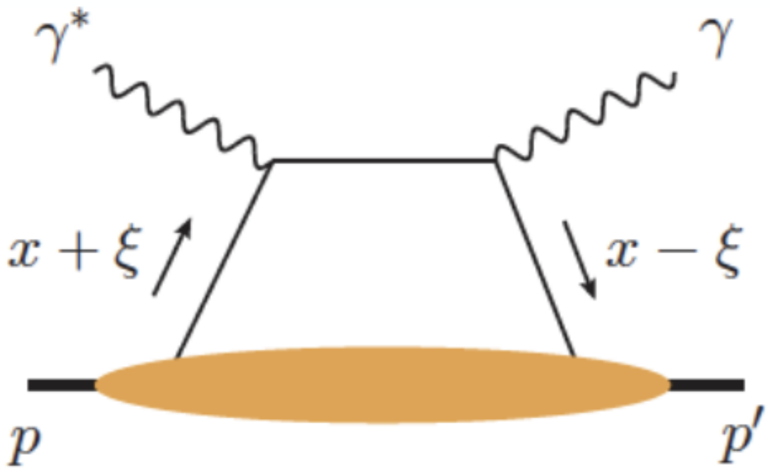
$$\lambda(Q^2) = \partial \left[\ln(xg)\right] / \partial \ln(1/x)$$

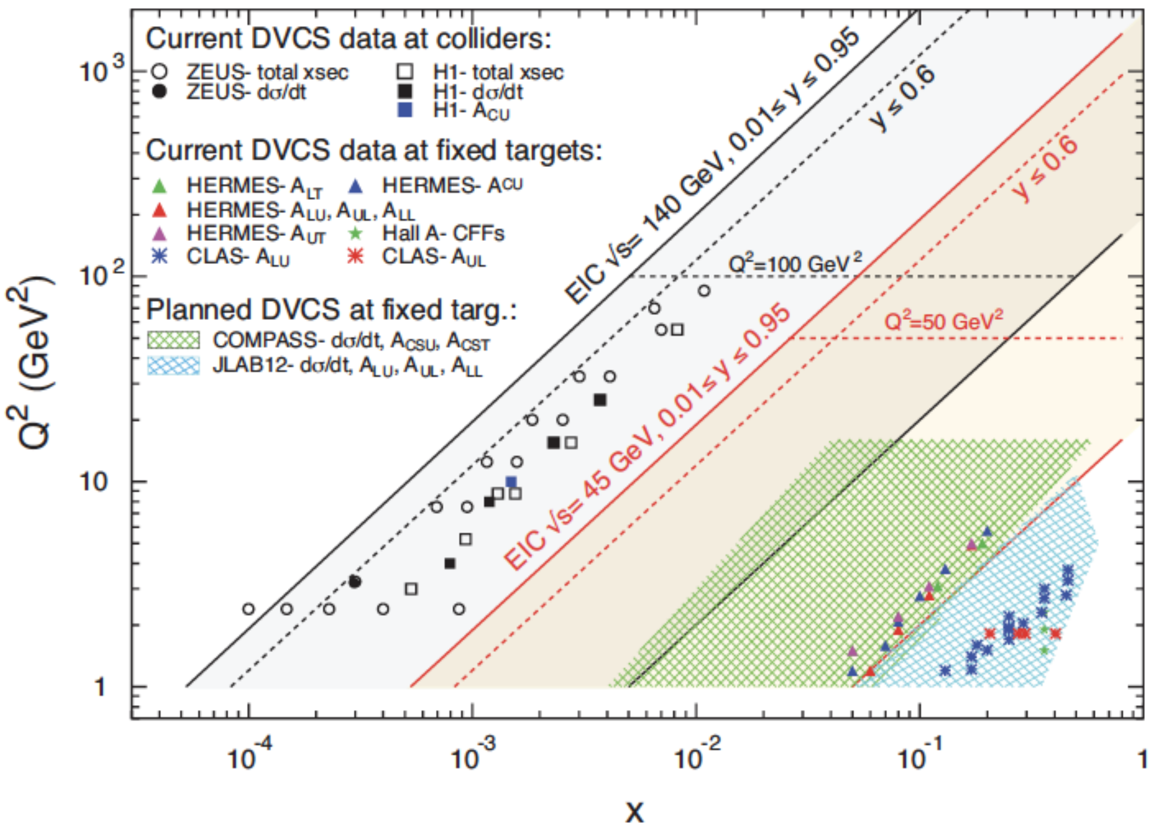
- NLO estimated, not complete.
- Real part via dispersion relations:

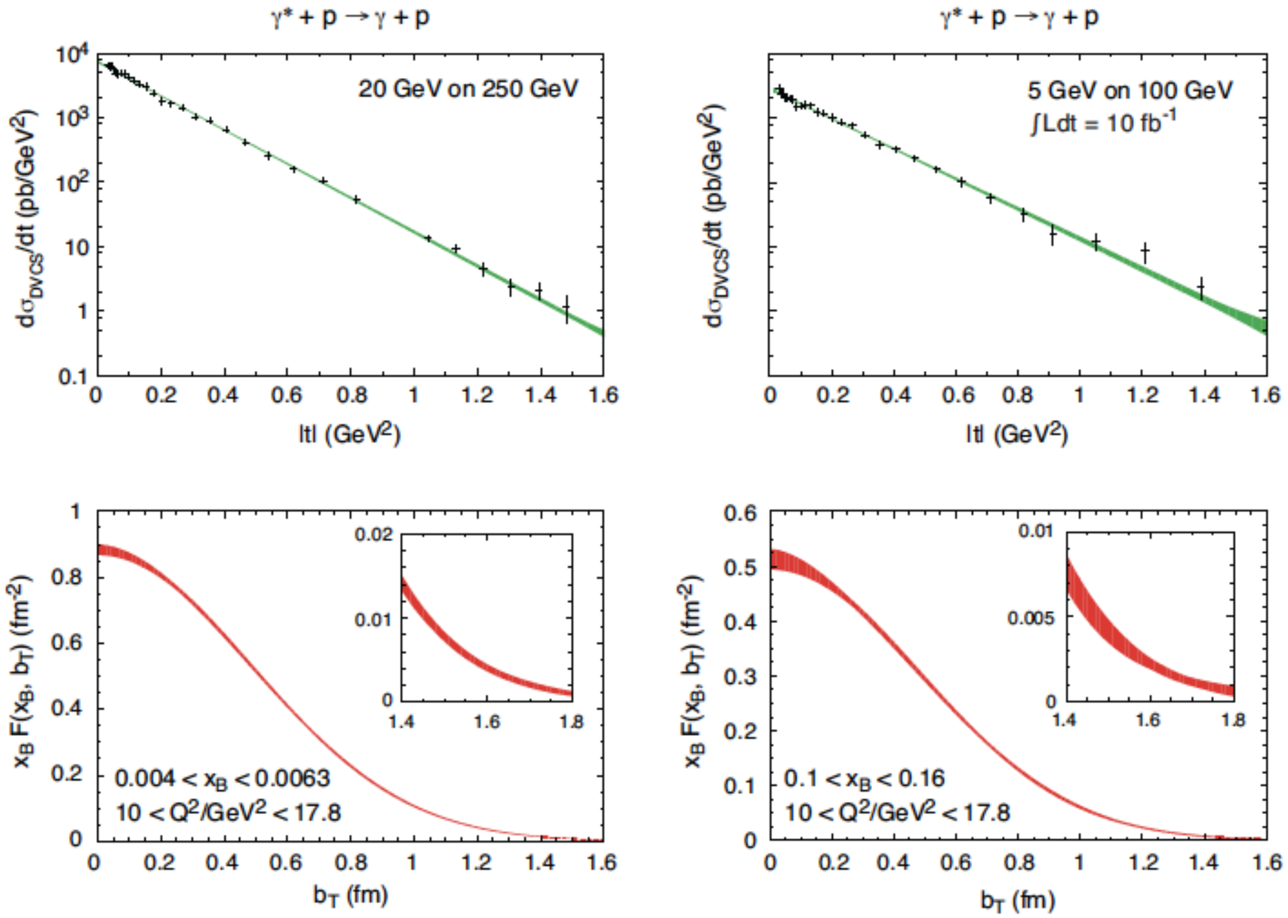
$$\frac{\mathrm{Re}A}{\mathrm{Im}A} \simeq \frac{\pi}{2}\lambda$$

DVCS:

- Quark GPDs can be studied in DVCS.





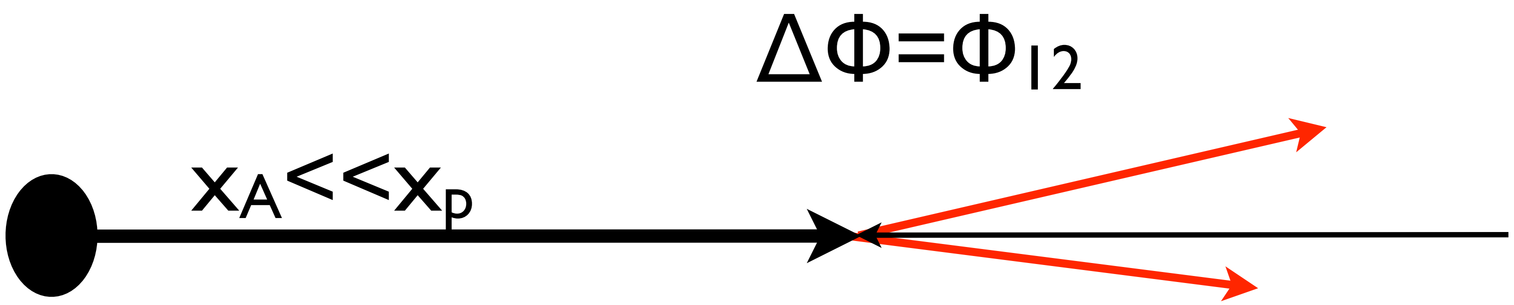


- Different evolution equations for TMDs and GPDs.

Small-x: correlations

- Dihadron azimuthal decorrelation: currently discussed at RHIC as suggestive of saturation.
- To be studied at LHeC/FCC-eh far from kinematical limits.

- **Nuclear and saturation effects on usual BFKL signals** (e.g. dijet azimuthal decorrelation, Mueller-Navelet jets) has not been extensively addressed: A-dependence contrary to linear resummation?



$$C(\phi_{12}) = \frac{1}{\frac{d\sigma(\gamma^* N \to h_1 X)}{dz_{h1}}} \frac{d\sigma^{\gamma^* N \to h_1 h_2 + X}}{dz_{h1} dz_{h2} d\phi_{12}}$$