GDR Summer School: “From hadronic structure to heavy-ion collisions”

Heavy Flavour and Quarkonia: Theory

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Disclaimer: This brief STUDENT lecture…

- … is just meant to be an introduction to the topic for fresh students so that they can get an overall picture and better appreciate the talks in the future
- => NOT intended to enter in any technical aspect
- => NOT intended to resolve ambiguities or express my personal opinion (just a bit)
- => NOT intended to cover all the fascinating topics

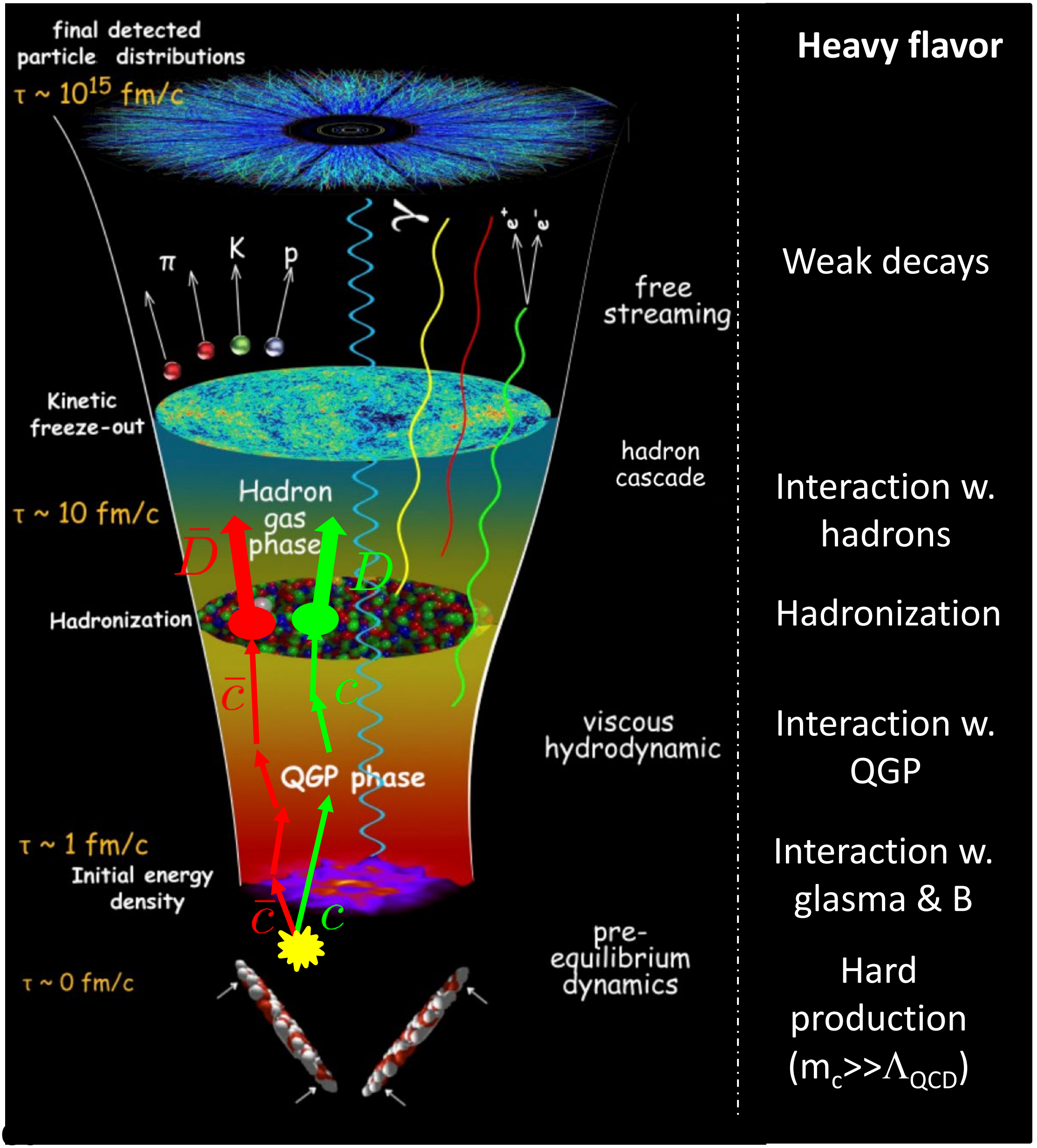
Subatech and Pays de la Loire IMT Atlantique Bretagne-Pays de la Loire École Mines-Télécom cnrs IN2P3 Nantes Université STRONG 2020

The Menu

- Heavy Quarks in the QGP and Open Heavy Flavors
- Quarkonia (Φ) production in AA

Effect through $Q + \bar{Q} \to \Phi$

Standard model of URHIC

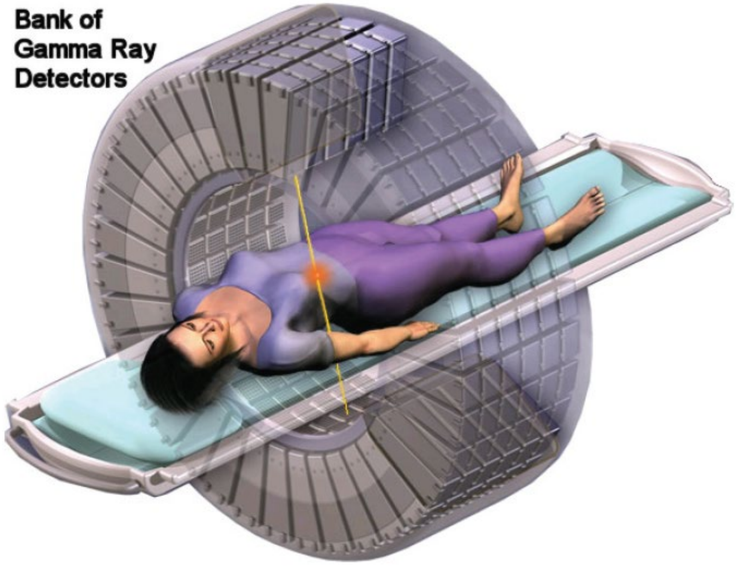


Established before 2010

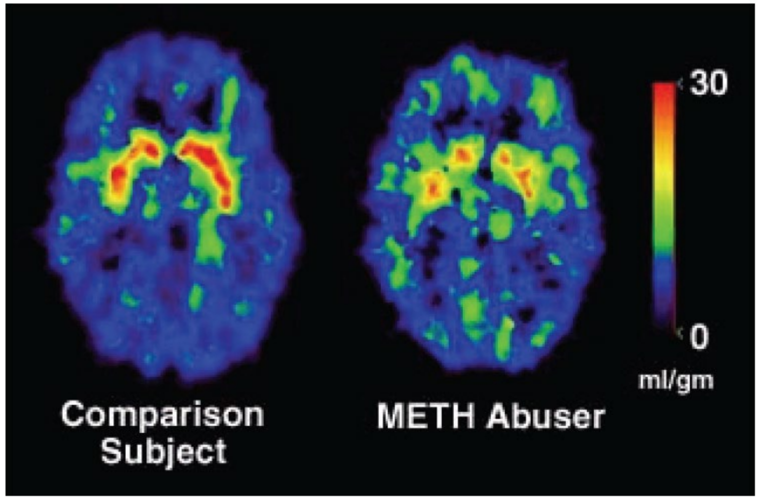
Since ≈ 2010

- Produced early ($t \approx 1/m_Q$)
  - => No further $Q\bar{Q}$ generation in ensuing QGP
  - Initial production well controlled (advantage of $m_Q >> \Lambda_{QCD}$)
  - But early phase might not be so innocent (magnetic field, CGC-glasma,…)
- **Experience the full deconfined phase + hadronic phase**
  - probes the QGP on harder scales than the other hadronic observables *while not fully thermalized* ($t_{relax} \alpha\ m_Q/T^2$)
  - accumulates several effects => need to compare different systems to better differentiate them
- Produced over a wide range of rapidities and p_T
  - increased richness in scrutinizing the interaction of HQ with medium…
  - but also sets more challenges (interactions for $p_T << m_Q$, $p_T \approx m_Q$, $p_T >> m_Q$, appropriate transport theory ?).
- **Nowadays turning into precision physics thanks to abundance of RHIC and LHC results !!!**

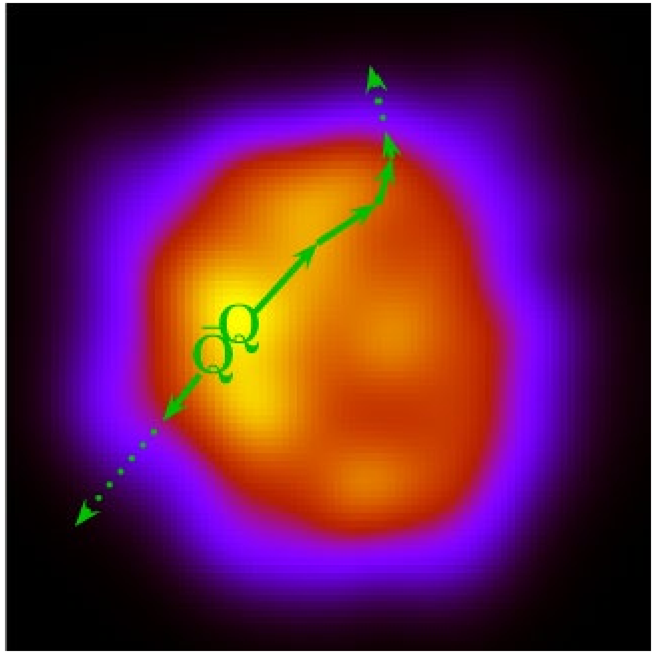
Extracting density profiles with HF Tomography ?



Schematic diagram of a PET scanner



PET scan showing abnormal brain function of a METH user

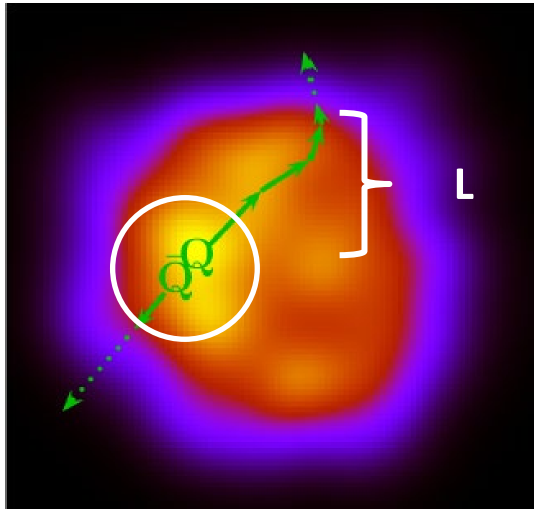


QGP tomography with Q-Qbar pairs

Seems pretty attractive concept…

← Well formulated inverse problem.

Hard probes: Nuclear modification factor (R_{AA})



Naive assumption:

$$\frac{dp}{dl} = \rho \times f(p,\dots)$$

$$\Downarrow$$

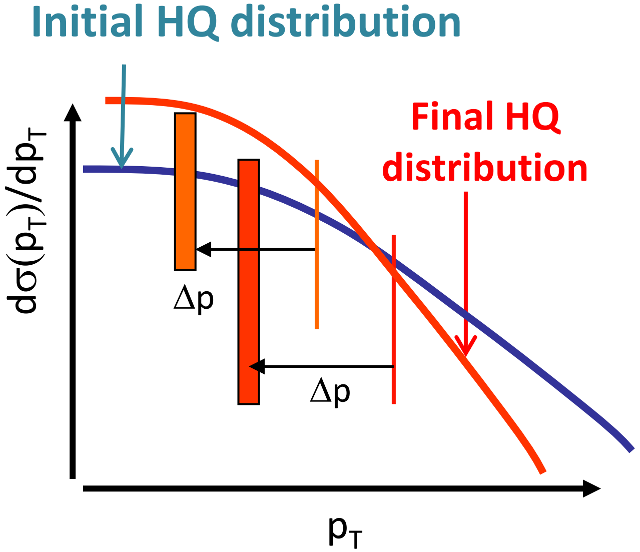
$$\Delta p(p,\dots) = \int_0^L dl\,\frac{dp}{dl} = f(p,\dots) \times \boxed{\int_0^L dl\,\rho}$$

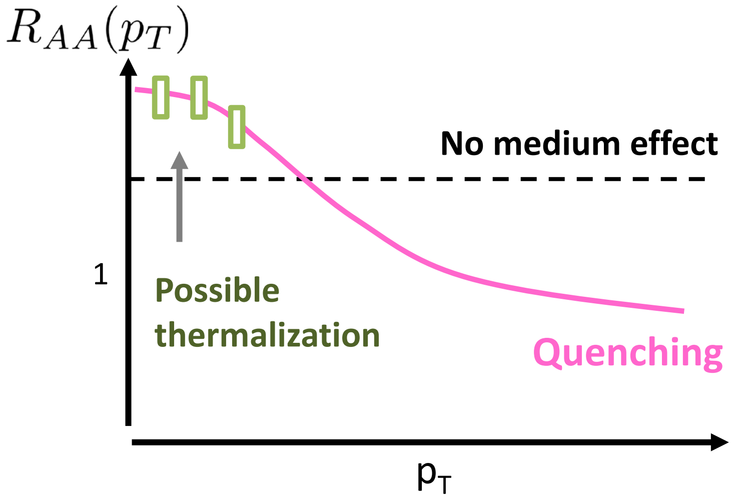
tomography

Experimental observable

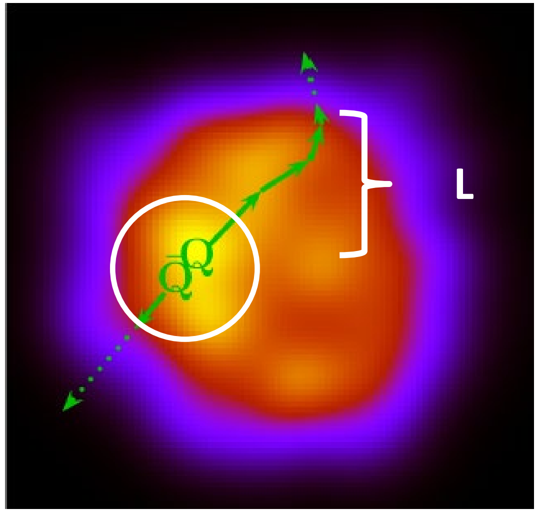
$$\boxed{R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle N_{\mathrm{col}}\rangle \times \frac{dN_{pp}}{dp_T}}} \approx \boxed{\frac{\frac{d\sigma_{pp}}{dp_T}(p_T + \Delta p(p_T))}{\frac{d\sigma_{pp}}{dp_T}(p_T)}}$$

Model





Hard probes: Nuclear modification factor (R_{AA})



Naive assumption: $\frac{dp}{dl} = \rho \times f(p, \ldots)$

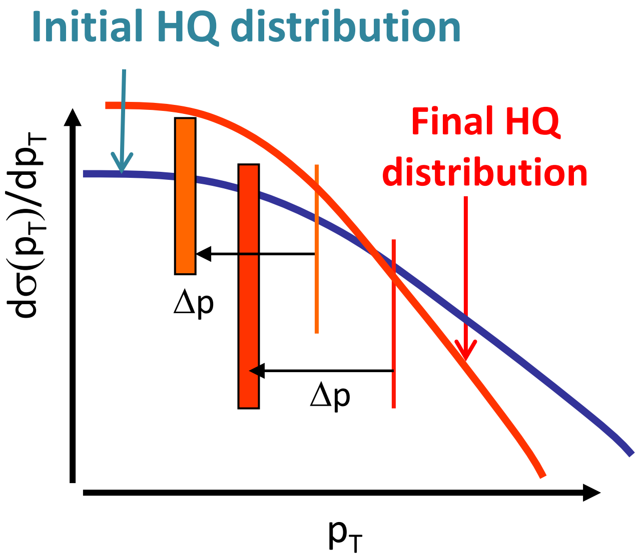
$$\Delta p(p, \ldots) = \int_0^L dl \, \frac{dp}{dl} = f(p, \ldots) \times \boxed{\int_0^L dl \, \rho}$$

tomography

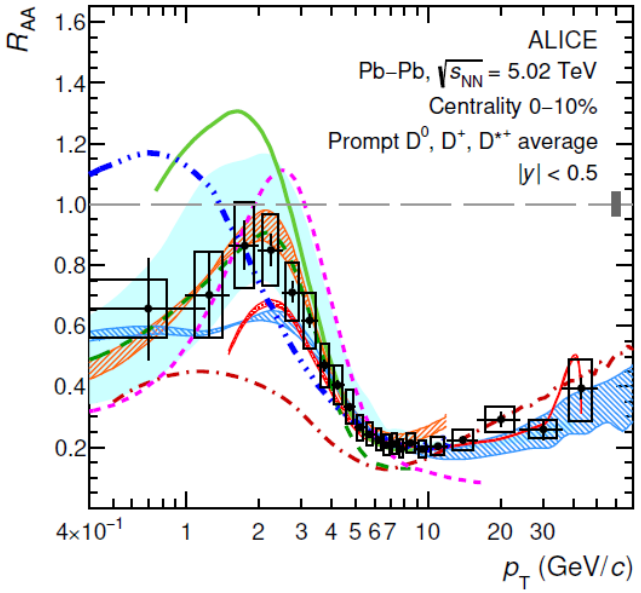
Experimental observable

$$\boxed{R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle N_{\rm col} \rangle \times \frac{dN_{pp}}{dp_T}}} \approx \boxed{\frac{\frac{d\sigma_{pp}}{dp_T}(p_T + \Delta p(p_T))}{\frac{d\sigma_{pp}}{dp_T}(p_T)}}$$

Model



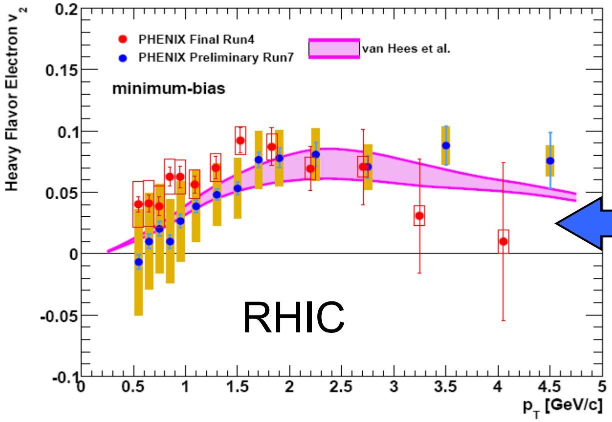
ALICE, JHEP 01 (2022) 174

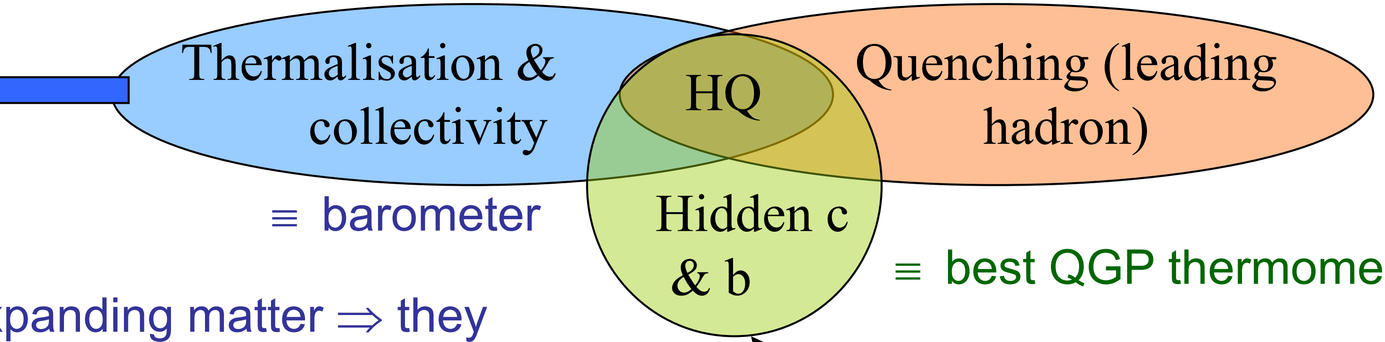


Why heavy flavors in A-A ?

- The Swiss knife of QGP hard probing !!!

The Trilogy:





≡ densimeter

Thermalisation & collectivity

HQ

Quenching (leading hadron)

≡ barometer

Hidden c & b

≡ best QGP thermometer ???

HQ are imbedded in expanding matter ⇒ they participate to collective motion and gain elliptic flow (v_2:azimutal asymmetry) at finite impact parameter… with additional inertia effect

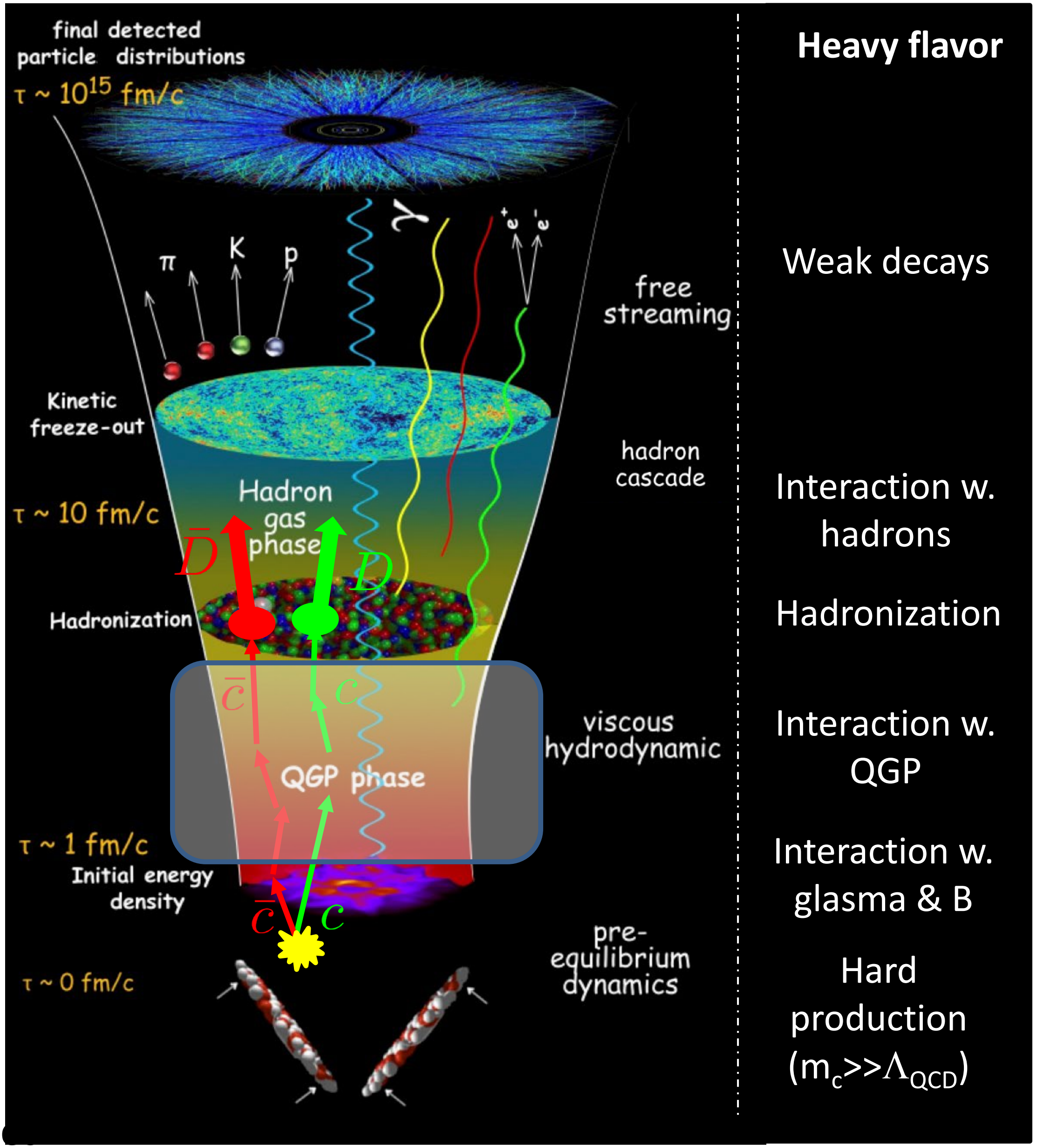
Quarkonia suppression and Dimuons product; no heavy Q thermal production

- Mandatory to understand HQ evolution in QGP ($\frac{dp}{dl} = \rho \times f(p, \ldots)$) & quarkonia production

Standard model of URHIC

Established before 2010

Since ≈ 2010



Challenge:

Description of HQ E-loss / equilibration from fundamental theory.

In fact we are at the same time probing the system but also using the results to better understand our probe (and the coupling to QGP) at the same time !
=> useful to rely on other methods to constrain the bulk.

HQ Eloss: some models…

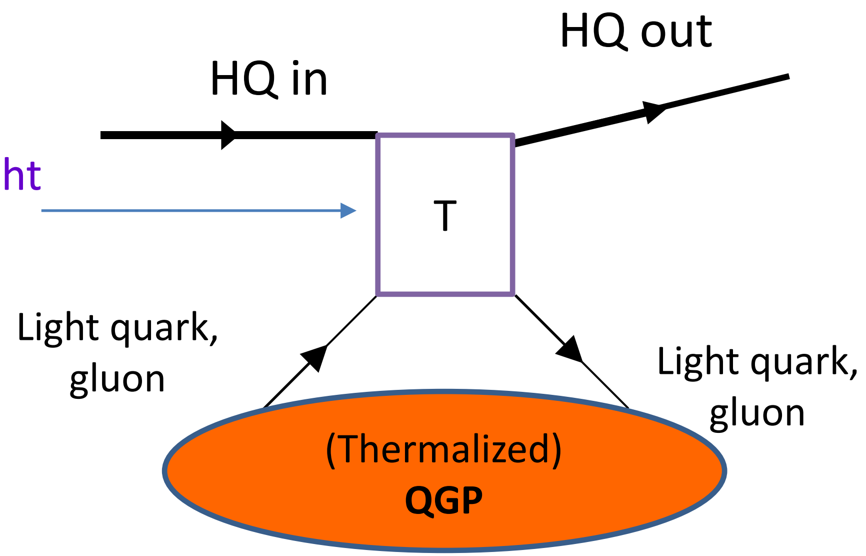
- Only a few !!! Sorry for this

- Not necessarily by historical appearance

HQ Eloss: Collisional (elastic) vs Radiative

Strictly speaking: Both terms apply to pQCD processes (small and moderate coupling), or to pQCD-inspired processes for which quasi-particles still exists

Collisional (elastic): 2 → 2



HQ in

HQ out

The heavy-light interaction

T

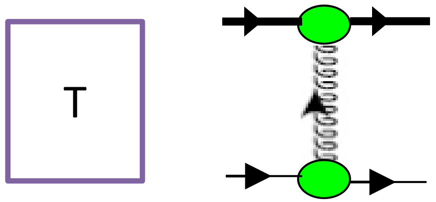
Light quark, gluon

Light quark, gluon

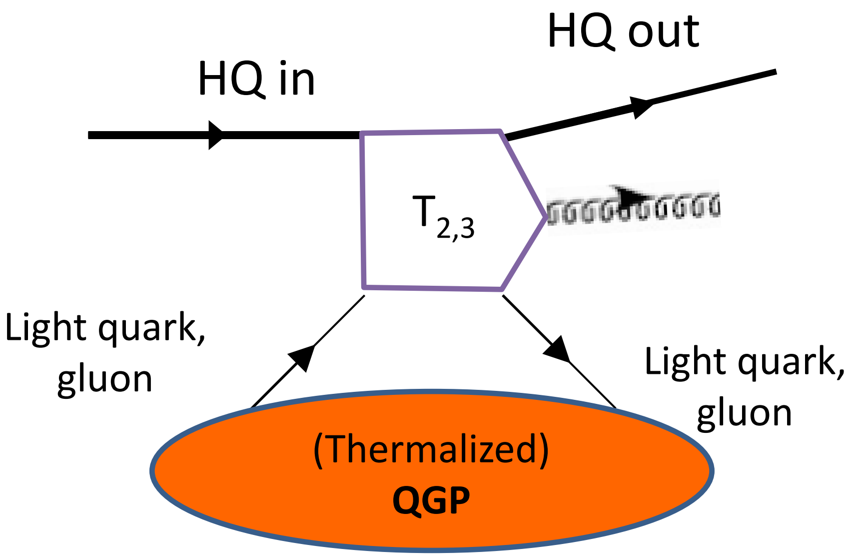
(Thermalized) **QGP**

Energy flows from HQ ↔ medium

e.g. :



Radiative : 2 → 3



HQ in

HQ out

$T_{2,3}$

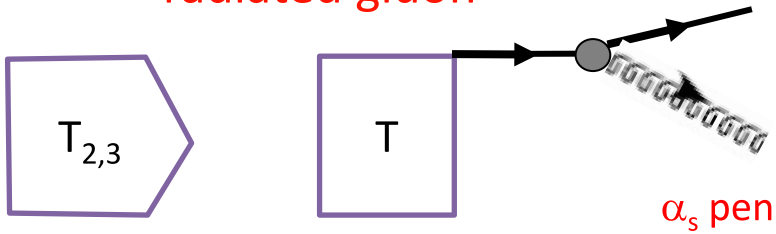
Light quark, gluon

Light quark, gluon

(Thermalized) **QGP**

Most of energy flows from HE HQ → radiated gluon

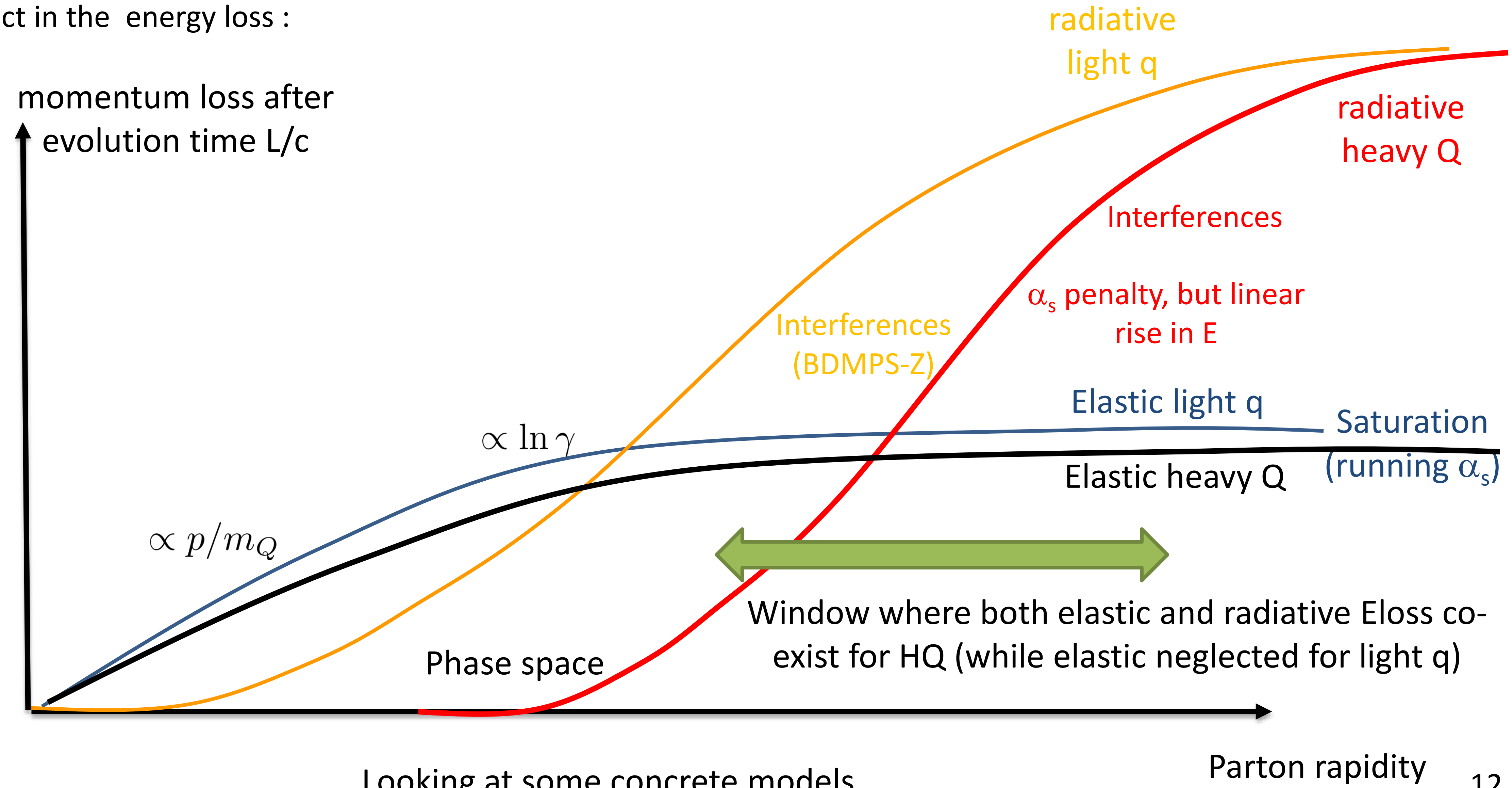
e.g. :



α_s penalty

Collisional (elastic) vs Radiative

Mass effect in the energy loss :



Window where both elastic and radiative Eloss co-exist for HQ (while elastic neglected for light q)

See as well lecture by Carlota

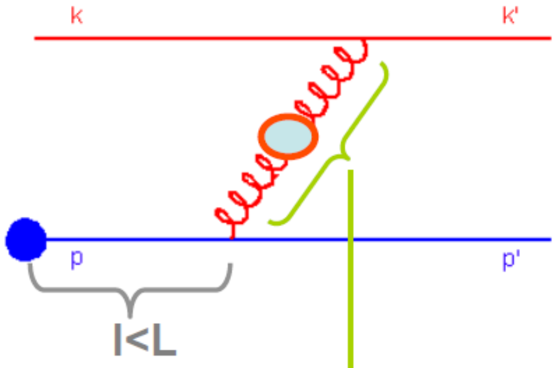
Looking at some concrete models…

The dynamical energy loss formalism (Djordjevic)

M. D., Phys.Rev.C74:064907,2006

Collisional energy loss+

Dynamical QGP parton (as opposed to fixed scattering centers in other Eloss models)



L: path length in the QGP

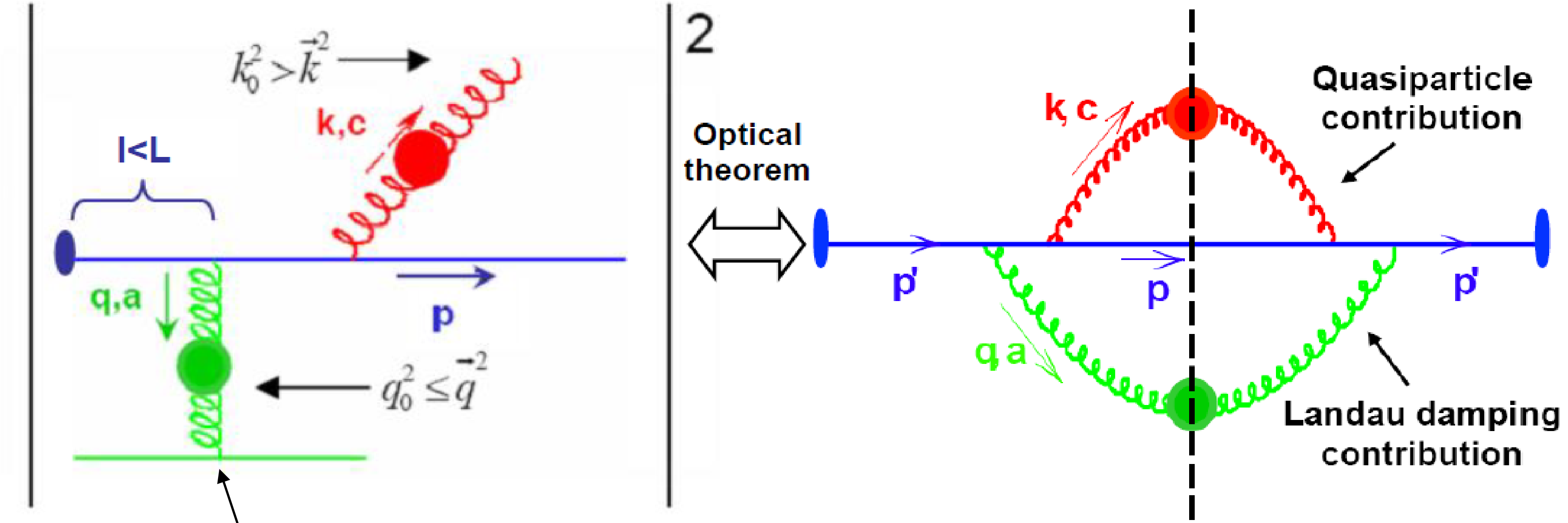
The effective gluon propagator:

$$D^{\mu\nu}(\omega,\vec{\mathrm{q}}) = -P^{\mu\nu}\Delta_T(\omega,\vec{\mathrm{q}}) - Q^{\mu\nu}\Delta_L(\omega,\vec{\mathrm{q}})$$

Transverse and longitudinal gluon propagators in Hard Thermal Loop approximation

The dynamical energy loss formalism (Djordjevic)

Radiative energy loss: radiation of one gluon induced by one collisional interaction with the medium.



first (lowest) order in number of scattering centers: first opacity expansion

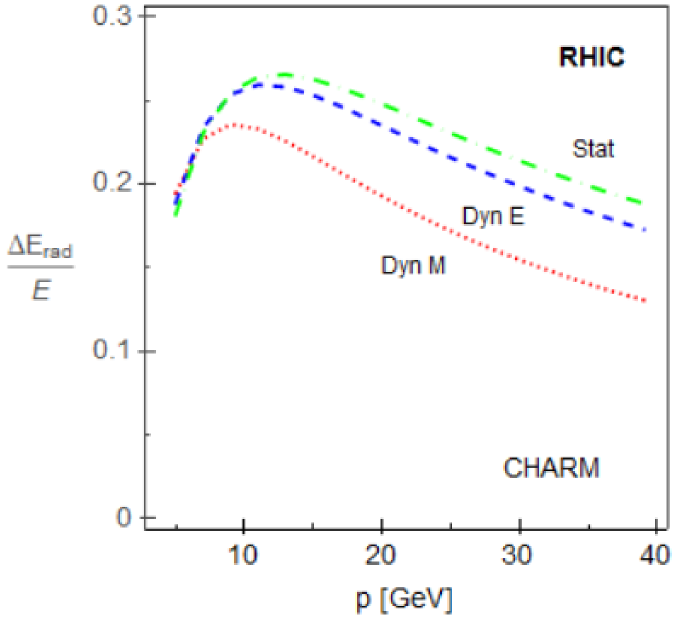
Effect of chromo-electric and chromo-magnetic contributions

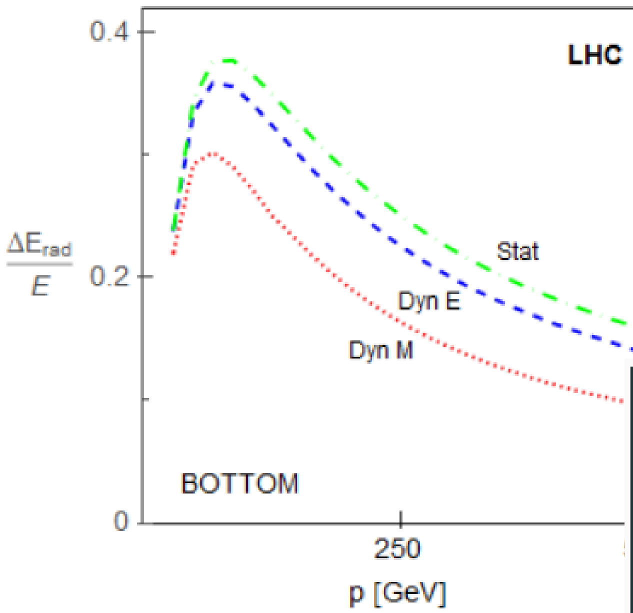
$$\frac{\Delta E_{\rm dyn}}{E} = \frac{C_R\alpha_s}{\pi}\frac{L}{\lambda_{\rm dyn}}\int dx\,\frac{d^2\mathbf{k}}{\pi}\frac{d^2\mathbf{q}}{\pi}\boxed{\frac{\mu^2}{\mathbf{q}^2(\mathbf{q}^2+\mu^2)}}\boxed{\left(1-\frac{\sin\frac{(\mathbf{k}+\mathbf{q})^2+\chi}{xE^+}L}{\frac{(\mathbf{k}+\mathbf{q})^2+\chi}{xE^+}L}\right)} \times 2\frac{(\mathbf{k}+\mathbf{q})}{(\mathbf{k}+\mathbf{q})^2+\chi}\left(\frac{(\mathbf{k}+\mathbf{q})}{(\mathbf{k}+\mathbf{q})^2+\chi}-\frac{\mathbf{k}}{\mathbf{k}^2+\chi}\right),$$

Interference of various contributions along the path length **(strong reduction at short path length)**

The dynamical energy loss formalism (Djordjevic)

Radiative energy loss

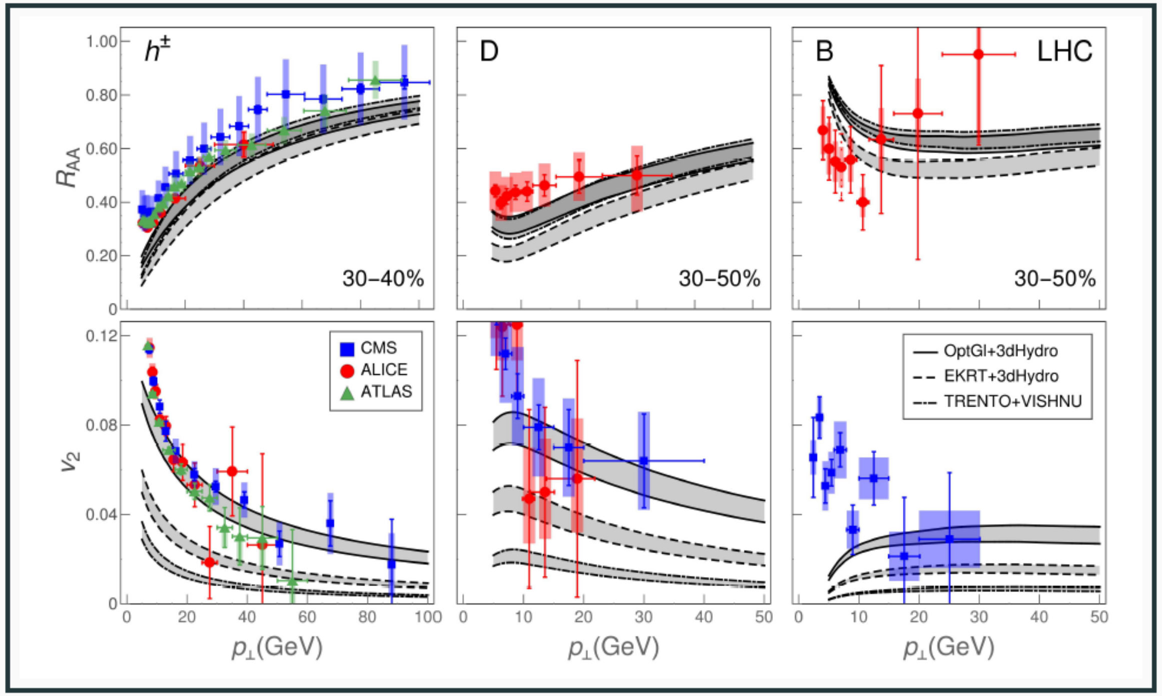




M.D.,J.Phys.G39:045007,2012

Dyn = Dyn M + Dyn E > Stat => **larger Energy Loss with the dynamical formalism**

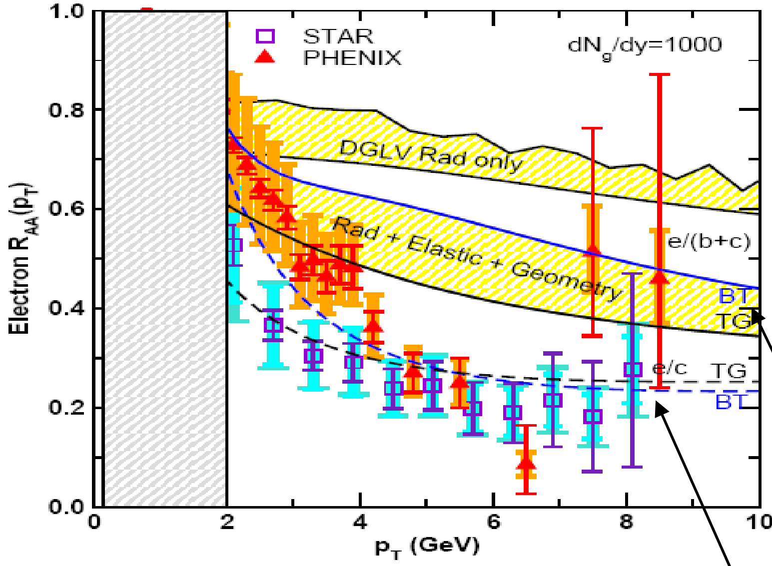
Evolved towards the DREENA framework

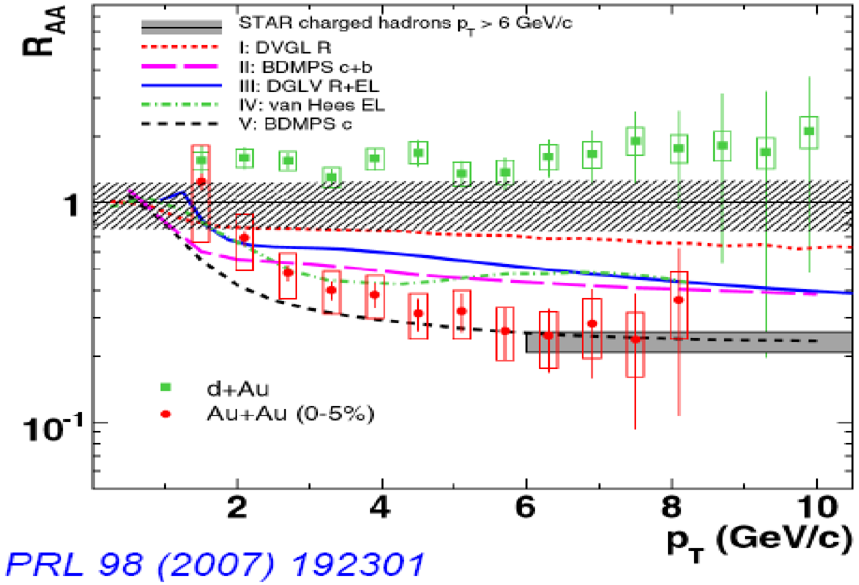


DREENA-A

ebe-DREENA framework as a QGP tomography tool, Dusan Zigic (Explore QGP, Belgrade, 2023)

Back to the future (2005)… more Eloss than expected from pQCD, even adding elastic part (often neglected up to then)

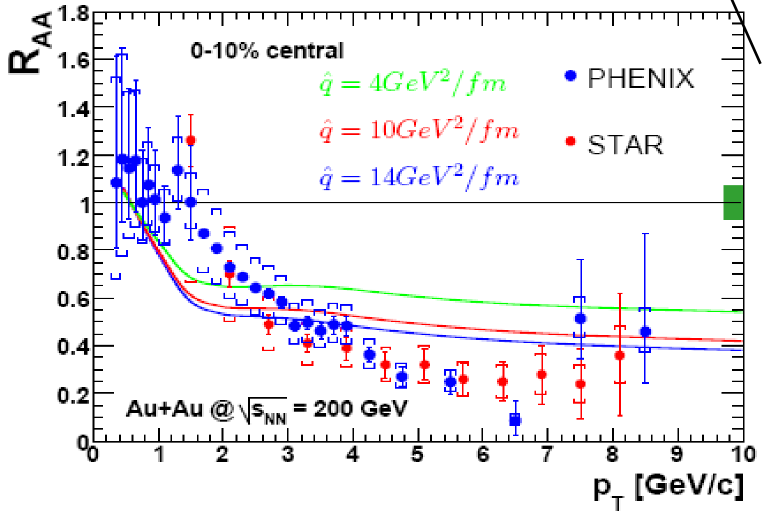


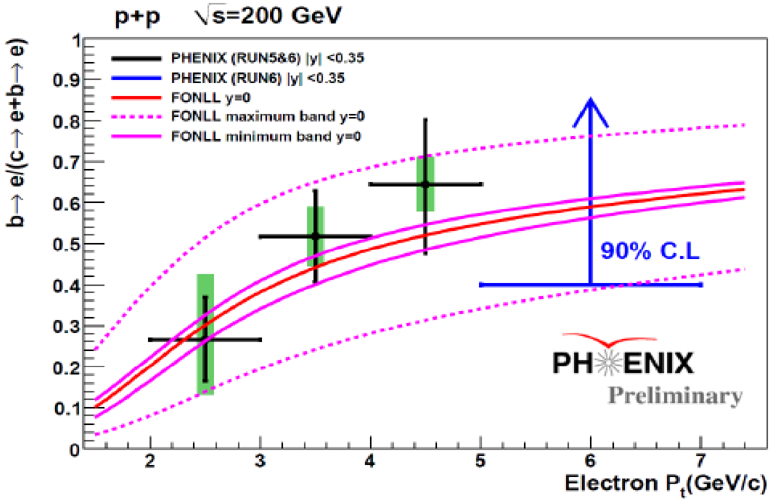


PRL 98 (2007) 192301

Elastic Eloss strikes back (Mustafa 05, Dutt Mazumber 05)

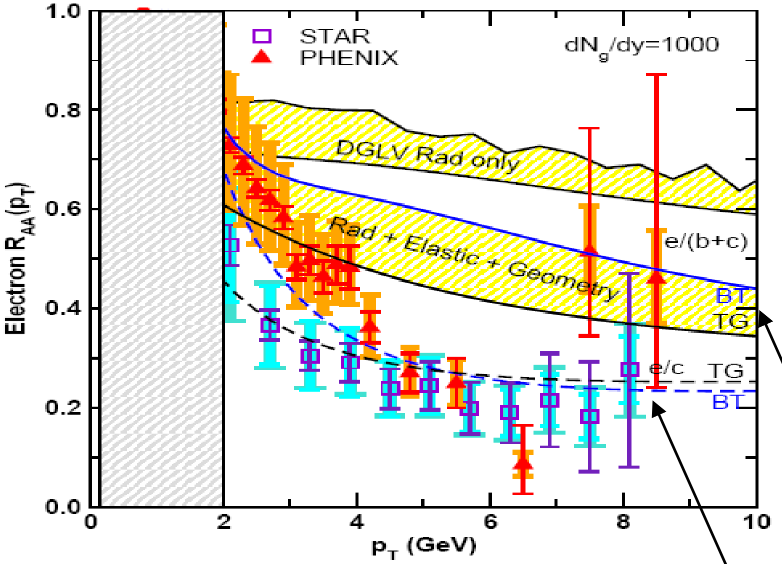
b quark is the puzzle and is definitively there:





Meanwhile: dN/dy(y≈0) scales like N$_{\text{bin}}$

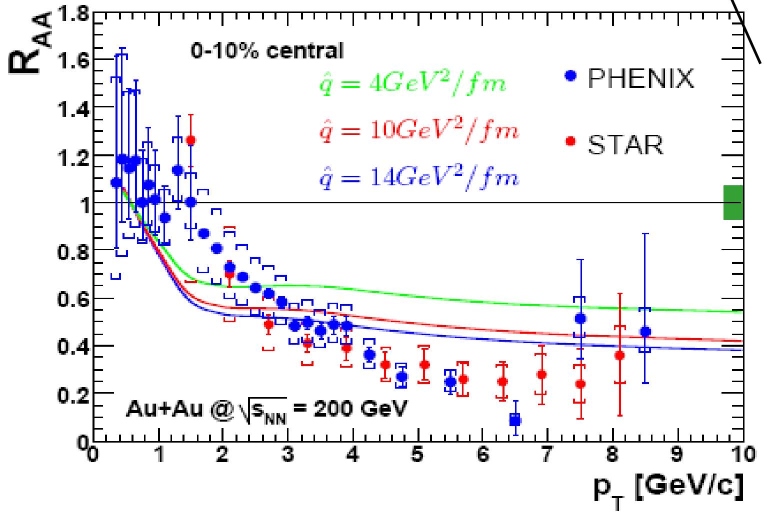
… more Eloss than expected from pQCD, even adding elastic part (often neglected up to then)



In the meanwhile, several improvements in the theoretical framework => the DGLV approach based on pQCD can reproduce the “high pT” data (DREENA)

See as well : Coleridge Faraday @ SQM 2024 for new developments

Elastic Eloss strikes back (Mustafa 05, Dutt Mazumber 05)



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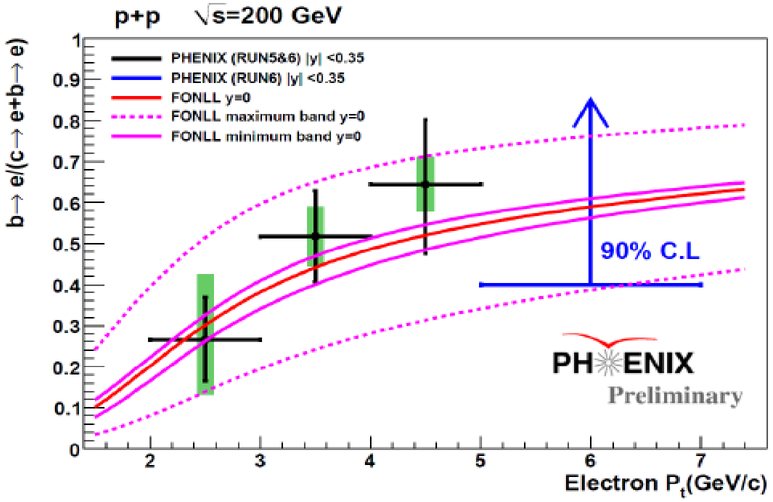


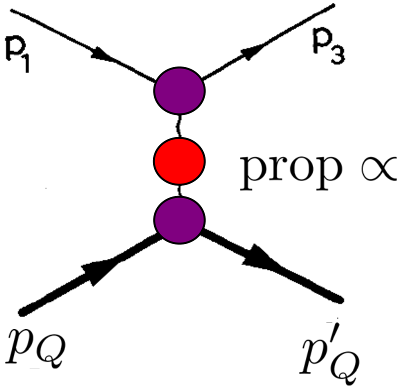
Meanwhile: dN/dy(y≈0) scales like N_{bin}

pQCD inspired models (f.i. Nantes, 2008)

Colisional component

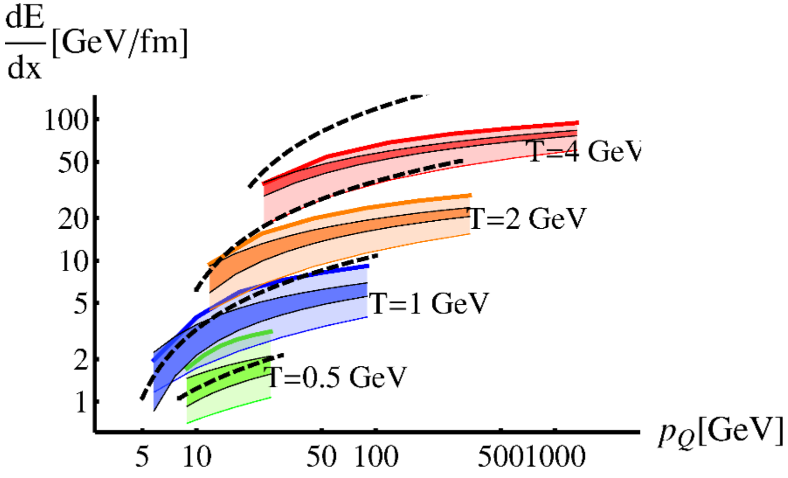
- One-gluon exchange model: reduced IR regulator κ m^2_{Dself} in the hard propagator, fixed on HTL Energy loss at large momentum (maximal insensitivity of dE/dx on q*)
- Running coupling α_{eff} (t)
- self consistent Debye mass

$$m_{Dself}^2\,(T) = (1+n_f/6)\; 4\pi\alpha_{eff}(m_{Dself}^2)T^2$$



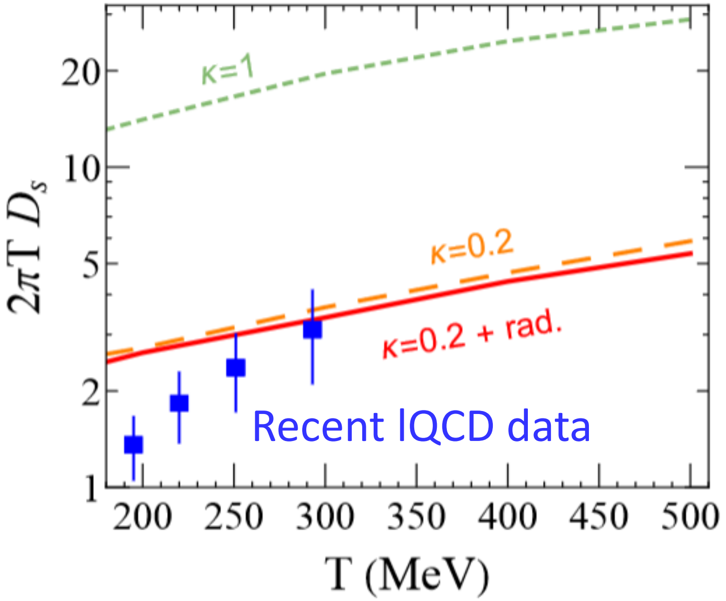
$$\text{prop} \propto \frac{1}{q^2-\kappa(p_Q)m^2_{\text{Dself}}(T)}$$

+ u and s channels



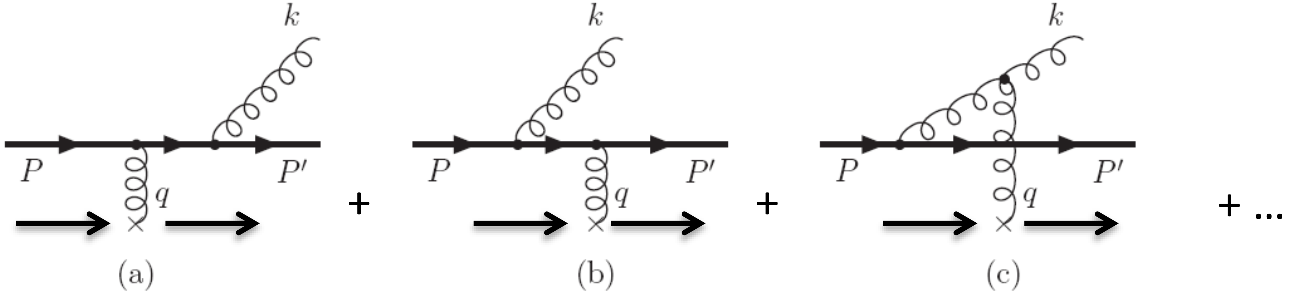
Comparison with Peigné-Peshier (2008) at finite momentum

$$D_s = \lim_{p_Q\to 0} T/(M_Q\eta_D)$$ η_D : drag coefficient



pQCD inspired models (f.i. Nantes, 2010)

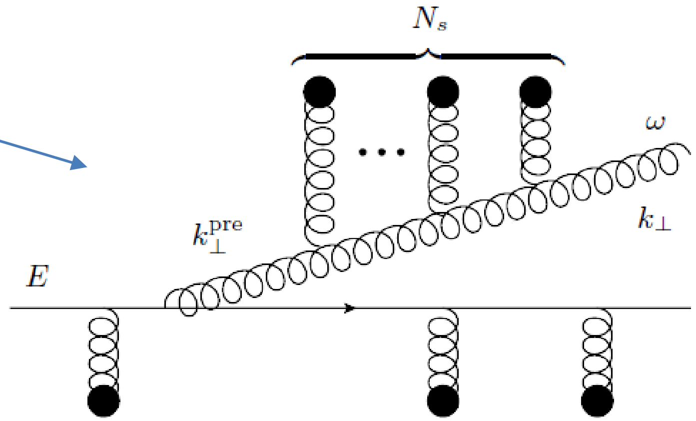
Radiative component



- Extension of Gunion-Bertsch approximation beyond mid-rapidity and to finite mass m_Q) distribution of induced gluon radiation per collision (ΔE_{rad} α E L):

$$P_g(x, \mathbf{k}_\perp, \mathbf{q}_\perp, m_Q) = \frac{3\alpha_s}{\pi^2} \frac{1-x}{x} \left(\frac{\mathbf{k}_\perp}{\mathbf{k}_\perp^2 + x m_Q^2} - \frac{\mathbf{k}_\perp - \mathbf{q}_\perp}{(\mathbf{k}_\perp - \mathbf{q}_\perp)^2 + x m_Q^2} \right)^2$$

- LPM / BDMPS-Z effect for intermediate HQ-energy



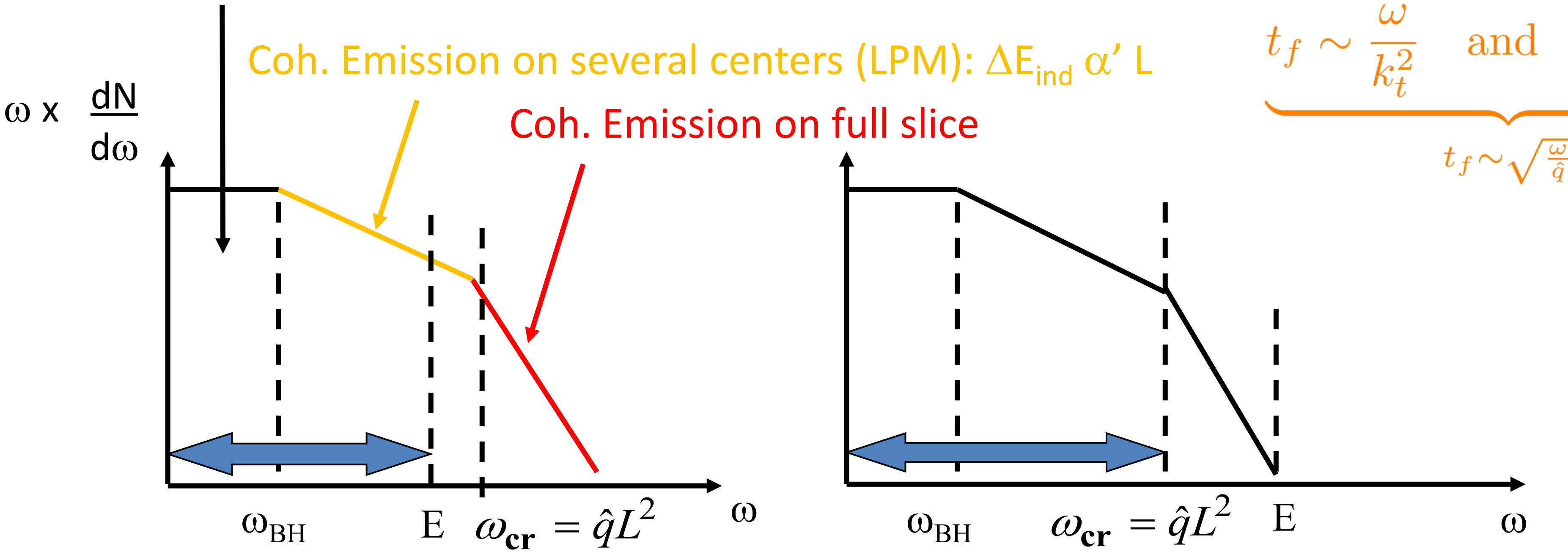
Implemented in EPOSn-HQ through Boltzmann transport

Some open heavy flavor history

01 (DK): contrarily to previous, HQ induced radiation is suppressed w.r.t. light quark: DEAD CONE EFFECT

gluon radiation from ***massless* parton**

Independent Emission on individual centers (BH): $\Delta E_{\text{ind}} \propto E \times L$



Coh. Emission on several centers (LPM): $\Delta E_{\text{ind}} \propto' L$

Coh. Emission on full slice

$$\underbrace{t_f \sim \frac{\omega}{k_t^2} \quad \text{and} \quad k_t^2 = \hat{q} t_f}_{t_f \sim \sqrt{\frac{\omega}{\hat{q}}}}$$

Intermediate E: $\Delta E_{\text{ind}} \propto \sqrt{E\hat{q}} \times L$

Large E: $\Delta E_{\text{ind}} \propto \hat{q} \times L^2$

Some open heavy flavor history

Soft gluon emission in hard process

$$\frac{d\theta}{\theta} \to \frac{d\theta}{\theta} \times \frac{1}{\left(1+\left(\frac{M}{E\,\theta}\right)^2\right)^2}$$

gluon radiation from *massive* parton (initial Bremsstrahlung)

Suppression factor DC(θ)

taken at $\theta^2 = \left\langle \theta^2 \right\rangle_{\mathrm{BDMPS}} = \sqrt{\frac{\hat{q}}{\omega^3}}$ for $t_f \sim \sqrt{\frac{\omega}{\hat{q}}}$

Suppression factor DC(ω)

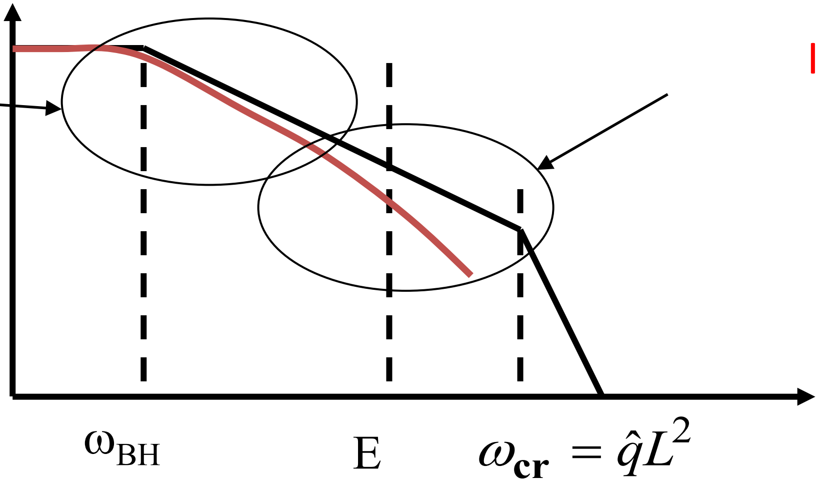
$$\mathbf{DC}(\omega) = \frac{1}{\left(1+\left(\frac{\omega}{\omega_{\mathbf{DC}}}\right)^{3/2}\right)^2}$$

New scale $\omega_{\mathbf{DC}} = \left(\frac{E^4 \hat{q}}{M^4}\right)^{1/3}$

$$\frac{dN}{d\omega} \longrightarrow \frac{dN}{d\omega} \times \mathrm{DC}(\omega)$$

Moderate for quenching

Important for ΔE



So-called “mass effect”

Quasi particle models (f.i DQPM)

- Nonperturbative effects near Tc are captured by $\alpha_s(T)$, leading to thermal masses/widths, determined from fits to lQCD EoS.

A. Peshier et al. PLB 337 (1994), PRD 70 (2004); M. Bluhm et al. EPJC 49 (2007); W. Cassing et al. NPA 795 (2007)

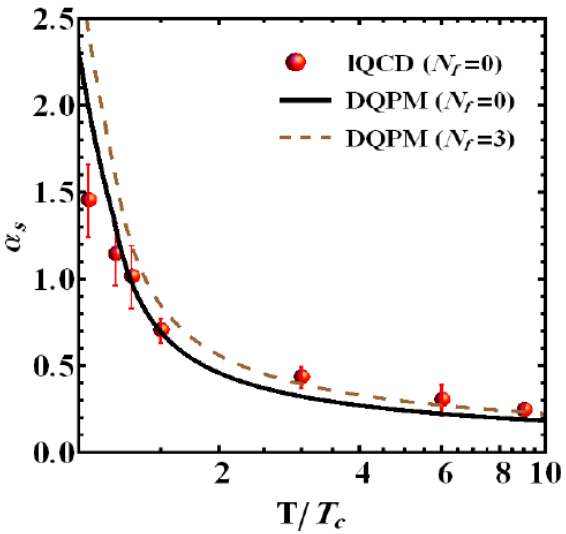
- Coupling between the effective DOF is then taken as $\alpha_s(T)$=> Relaxation rates larger then in pQCD for all T relevant for QGP, slightly smaller than the ones from TAMU

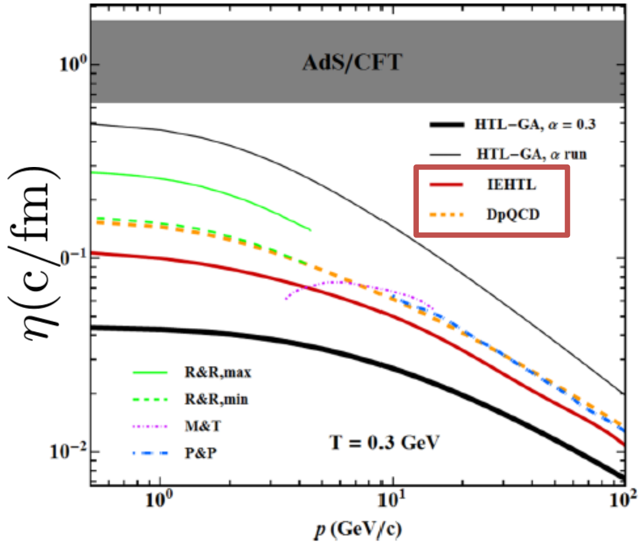
H. Berrehrah et al, PHYSICAL REVIEW C 90, 064906 (2014)

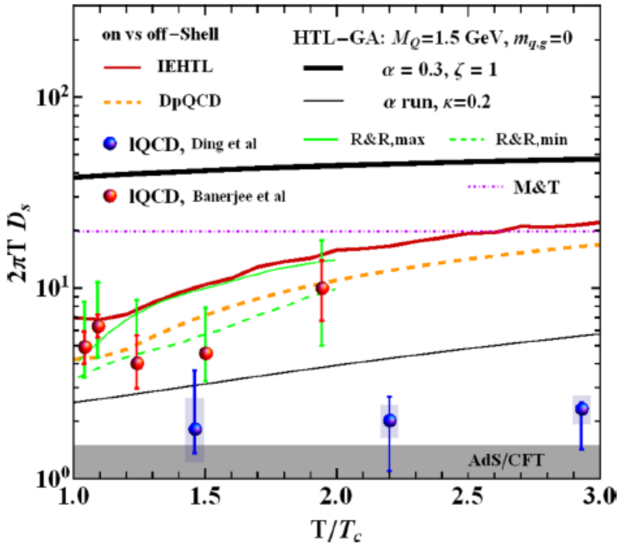
- Implemented for HF dynamics in e.g. PHSD (full off-shell, off-equilibrium transport).

T. Song et al. PRC 92 (2015), PRC 93 (2016)

See also CATANIA

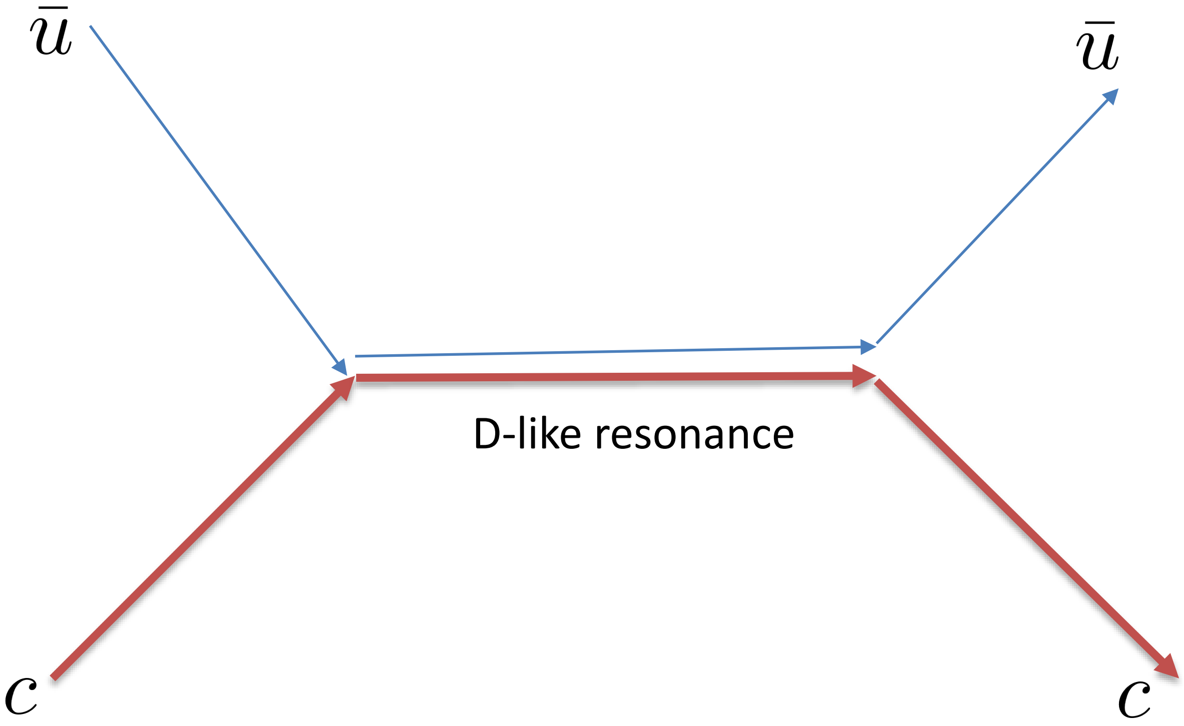






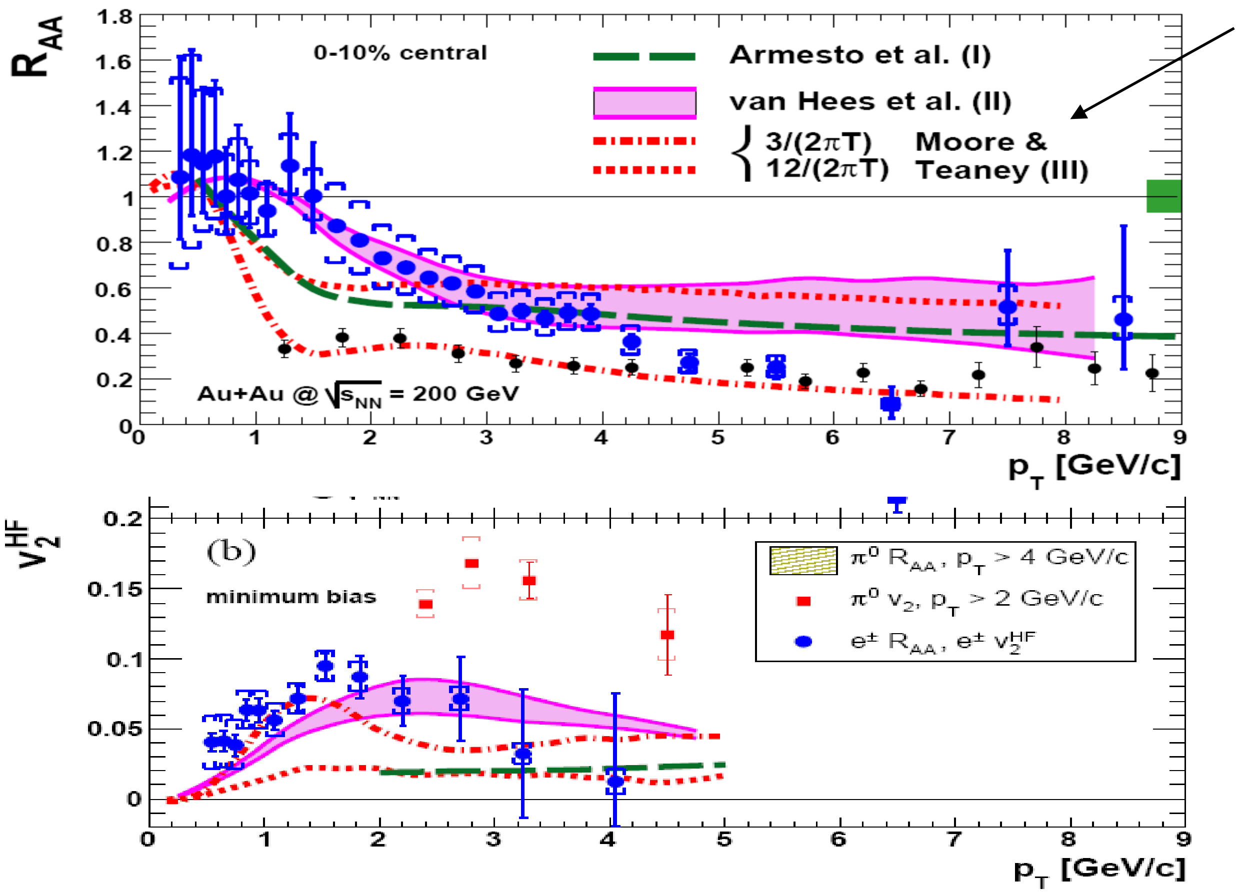
more thermalisation than expected from pQCD, some ways out

- Rapp and Van Hees (2004 ->): pQCD collisional + additional « strength » from quasi-bound D-like states, resorting to Langevin Dynamics



more thermalisation than expected from pQCD, some ways out

RHIC, Au+Au @ 200 GeV/p (≈2005)



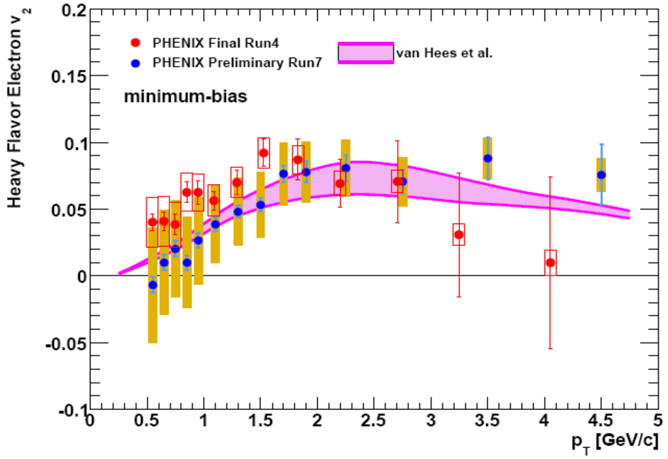
D_s: spatial diffus coeff ($fm^2/fm/c$)

pQCD:

$$D \approx \frac{6}{2\pi T}\left(\frac{0.5}{\alpha_s}\right)^2$$

$\approx 24/(2\pi T)$

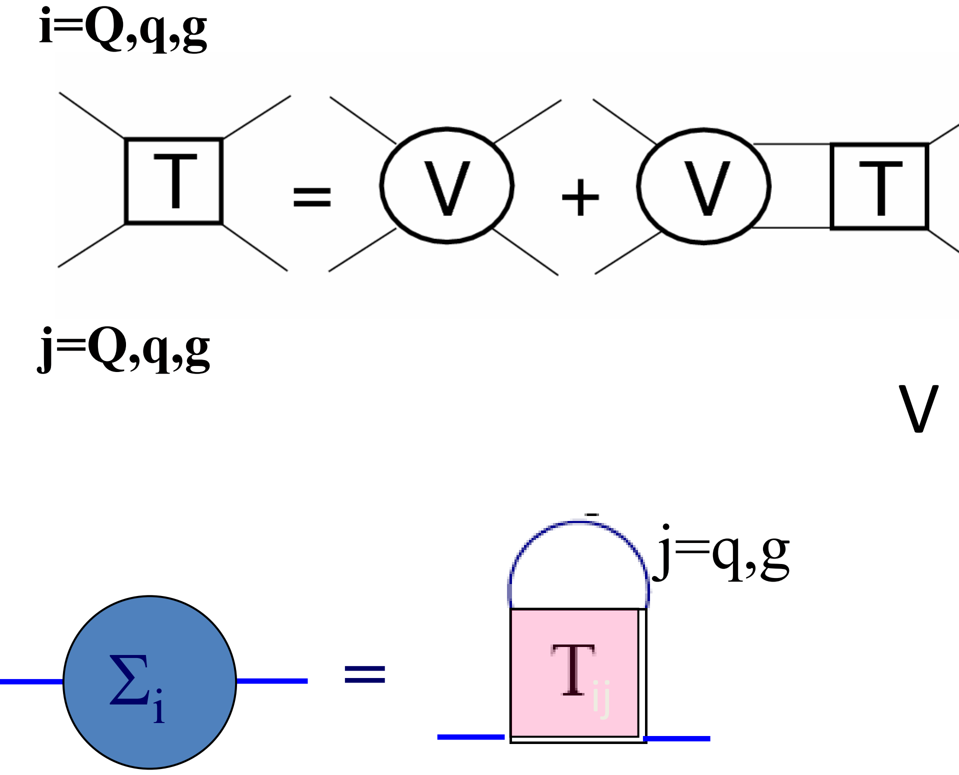
Charm flows !!!

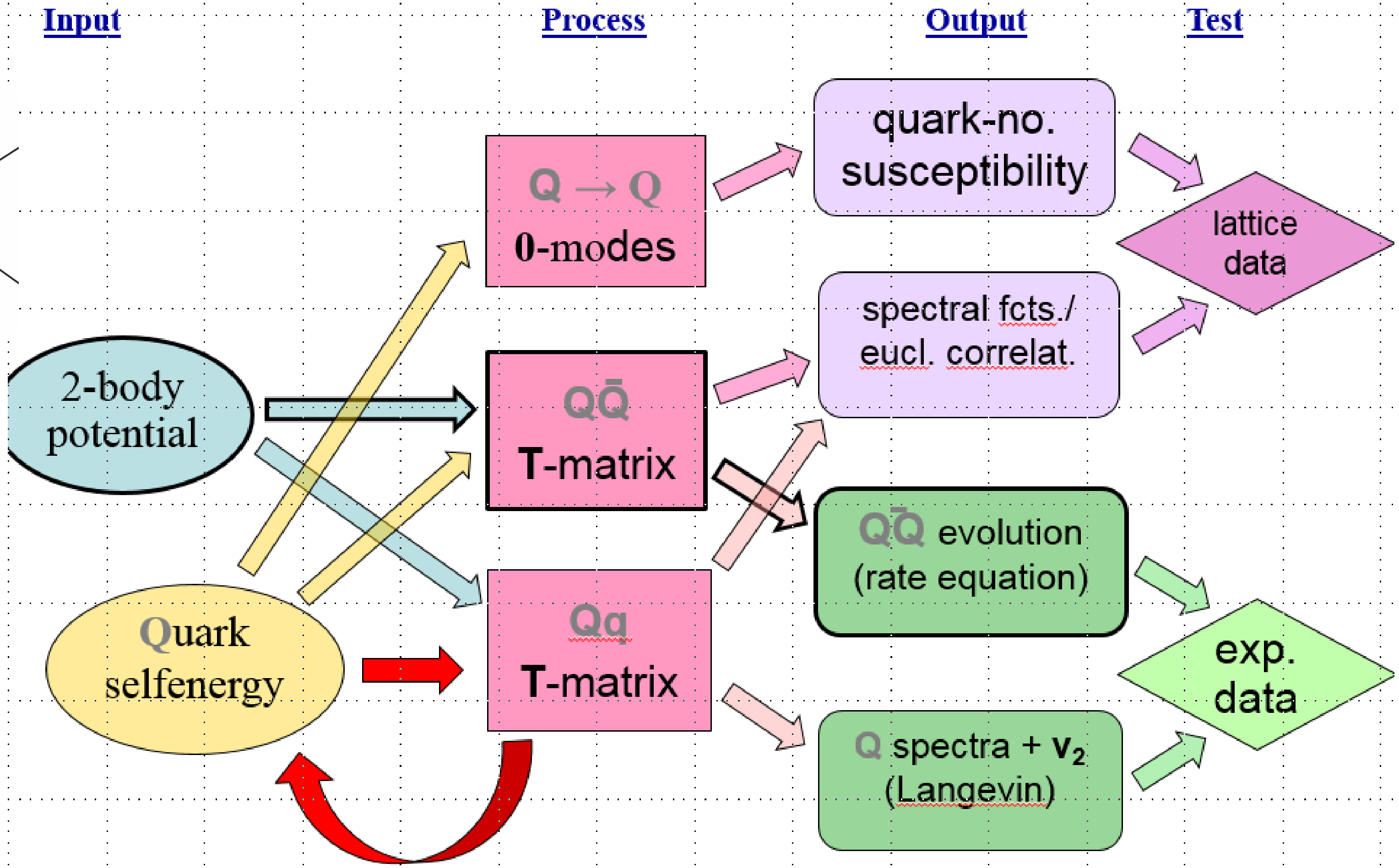


3 representative approach of HQ modelling : pQCD, non perturbative model, data-driven

more thermalisation than expected from pQCD, some ways out

- Rapp and Van Hees (2004 ->): pQCD collisional + additional « strength » from quasi-bound D-like states; then (2008) systematically developped using the T-matrix resummation of a bona-fide 2 body potential including non-perturbative contributions





Input

Process

Output

Test

2-body potential

Quark selfenergy

$Q \to Q$ 0-modes

$Q\bar{Q}$ T-matrix

Qq T-matrix

quark-no. susceptibility

spectral fcts./ eucl. correlat.

$Q\bar{Q}$ evolution (rate equation)

Q spectra + v_2 (Langevin)

lattice data

exp. data

Potential models (TAMU)

- Thermodynamic T-matrix approach, T = V +VGT, given by a two-body driving kernel V, estimated from the lQCD internal/free energy for a static Q-Qbar pair; increase of coupling with QGP at small momentum

D. Cabrera, R. Rapp PRD 76 (2007); H. van Hees, M. Mannarelli, V. Greco, R. Rapp PRL 100 (2008)

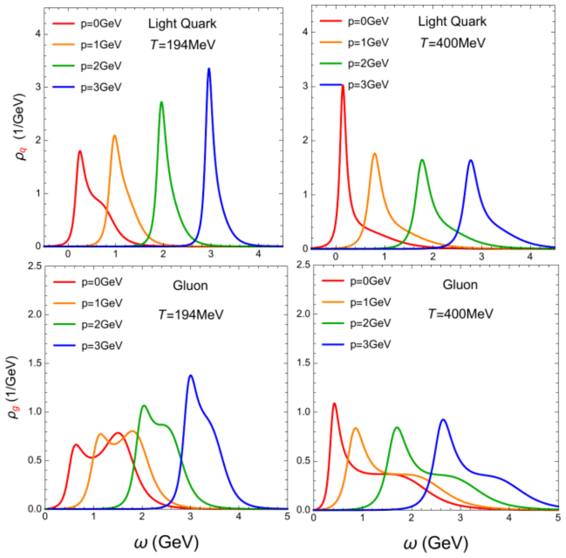
- Comprehensive sQGP approach for the EoS, light quark & gluon spectral functions, quarkonium correlators and HQ diffusion.

F. Riek, R. Rapp PRC 82 (2010); S. Liu, R. Rapp arxiv:1612.09138

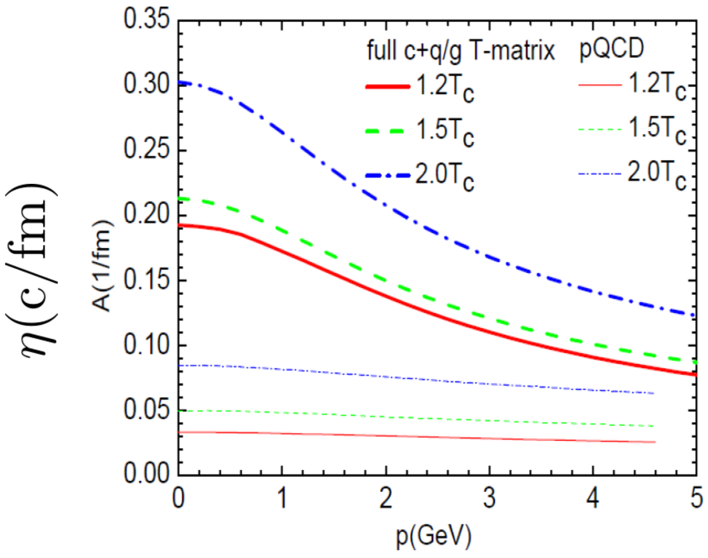
- Resonance correlations in the T-matrix naturally lead to recombination (resonance recombination model) near T_c from the same underlying interactions!

M. He, R. Fries, R. Rapp PRC 82 (2010), PRC 86 (2012)

- Implementation through Langevin dynamics in hydro evolution or in URQMD also corresponds to the disappearance of well defined quasi particles (for which Boltzmann breaks down while Langevin still holds)

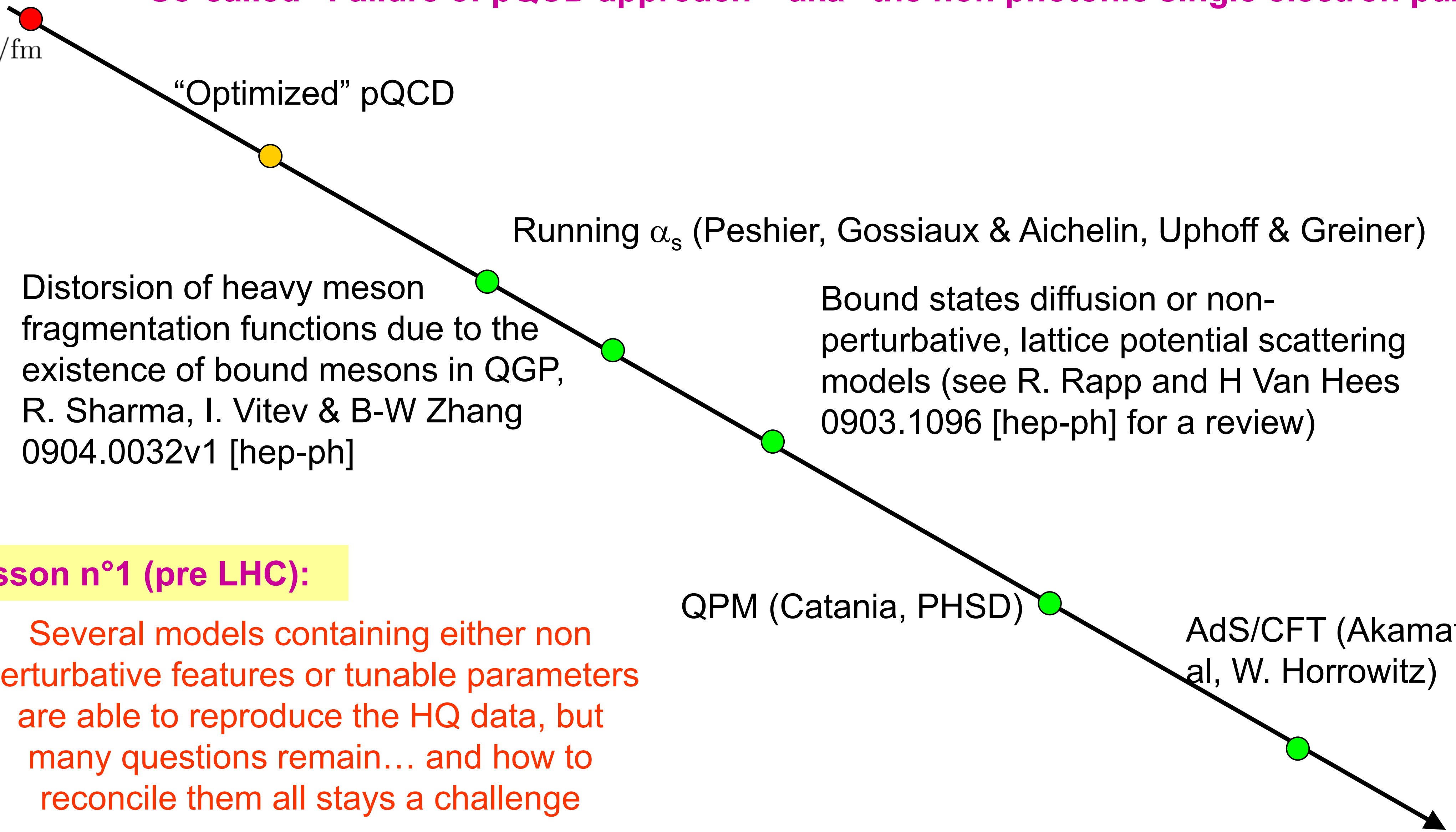


No good q-particle at low p



Large coupling at small p_Q

The weak to strong axis for HQ



Bona fide pQCD
(WHDG, ASW,…)

So-called "Failure of pQCD approach" aka "the non photonic single electron puzzle"

$\hat{q} \approx 1\ \mathrm{GeV}^2/\mathrm{fm}$

"Optimized" pQCD

Running α_s (Peshier, Gossiaux & Aichelin, Uphoff & Greiner)

Distorsion of heavy meson fragmentation functions due to the existence of bound mesons in QGP, R. Sharma, I. Vitev & B-W Zhang 0904.0032v1 [hep-ph]

Bound states diffusion or non-perturbative, lattice potential scattering models (see R. Rapp and H Van Hees 0903.1096 [hep-ph] for a review)

Lesson n°1 (pre LHC):

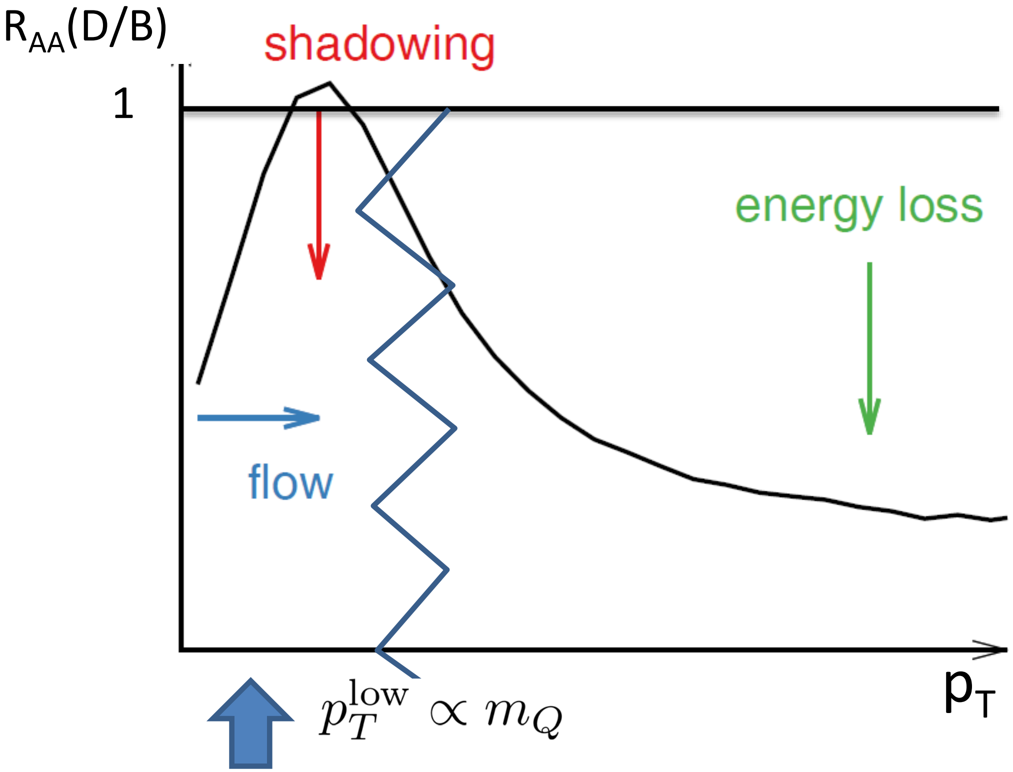
QPM (Catania, PHSD)

AdS/CFT (Akamatsu et al, W. Horrowitz)

Several models containing either non perturbative features or tunable parameters are able to reproduce the HQ data, but many questions remain… and how to reconcile them all stays a challenge

Basic Consequences of HQ interaction with QGP for the R_{AA}

The pattern seen in the data



- Dominated by elastic interactions
- $m_Q >> T$ => needs « many » collisions to equilibrate
- Physics close to « Langevin »

The acknowledged effects

Flow bump: due to

- *(radial) flow of the medium* and coupling at small p_T
- *recombination with light quarks*

shadowing: due to *initial state nuclear effects*

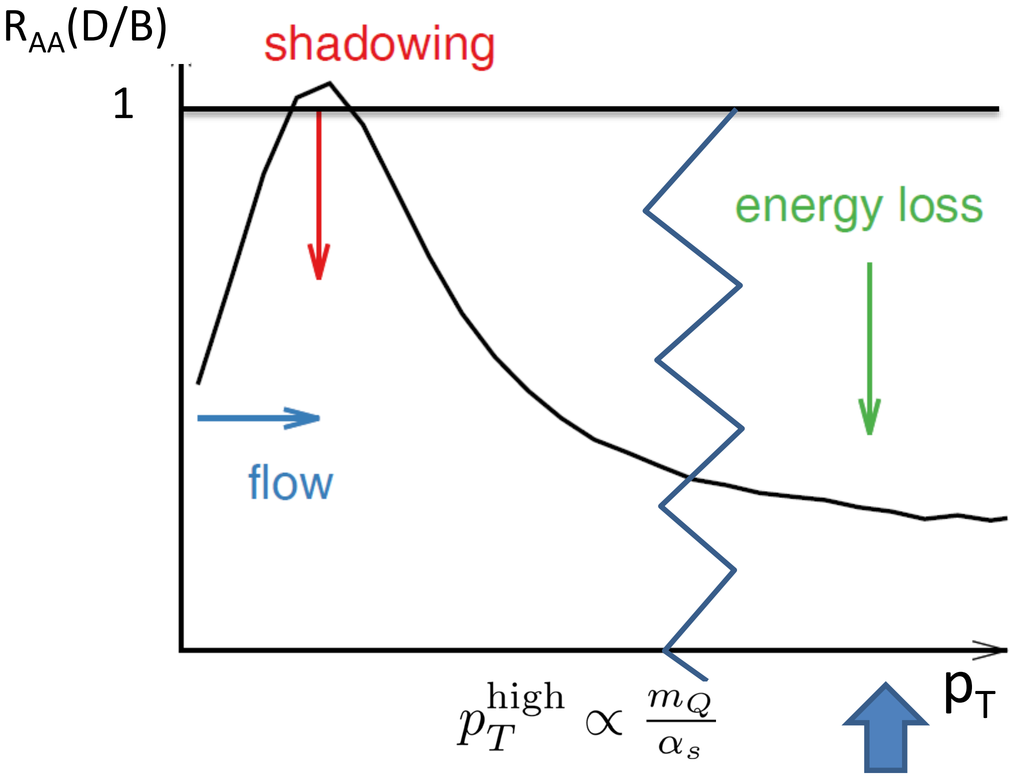
Quenching & energy loss: due to

- elastic and *inelastic* scatterings
- *opacity of the medium*

Italic: extrinsinc to the HF coupling with QGP AKA « energy loss model»

Basic Consequences of HQ interaction with QGP for the R_{AA}

The pattern seen in the data



The acknowledged effects

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- *(radial) flow of the medium* and coupling at small p_T
- *recombination with light quarks*

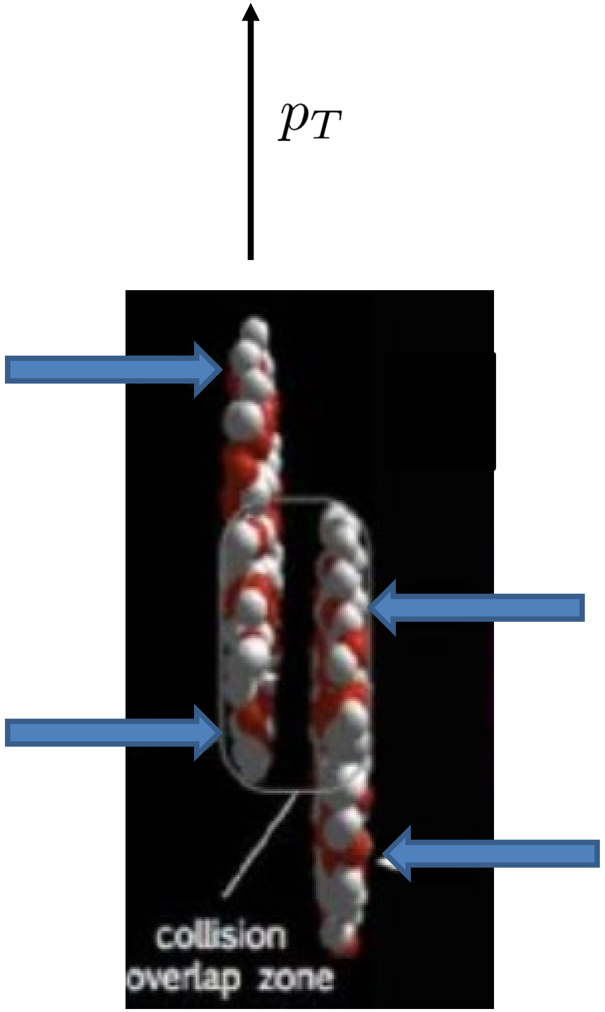
shadowing: due to *initial state nuclear effects*

Quenching & energy loss: due to

- elastic and *inelastic* scatterings
- *opacity of the medium*

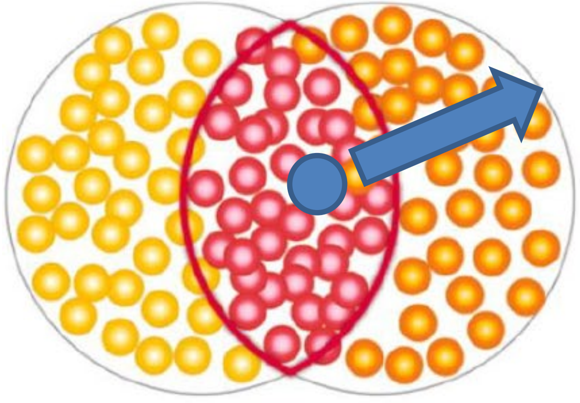
- Dominated by radiative energy loss (with important coherence effects: $\Delta E_{\text{rad}} \propto C_A \hat{q} L^2$)
- Eikonal regime (propagation along straight lines)
- 1 single transport coefficient dominates the whole physics: $\hat{q} \propto \kappa_T$
- HQ do not equilibrate with the medium
- m_Q becomes a subscale of the physics ($m_Q << p_T$)

Basic Consequences of HQ interaction with QGP for the R_{AA}

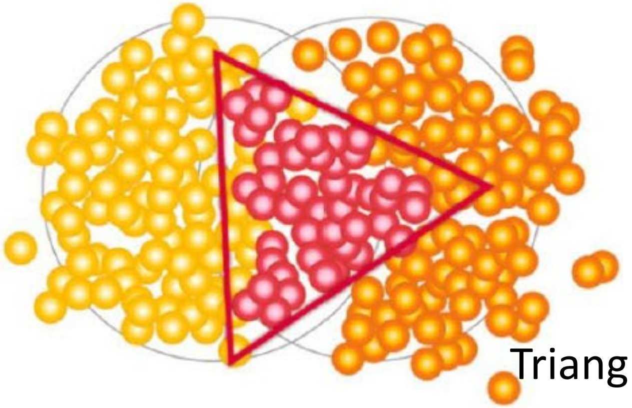


Beam axis
(Oz)

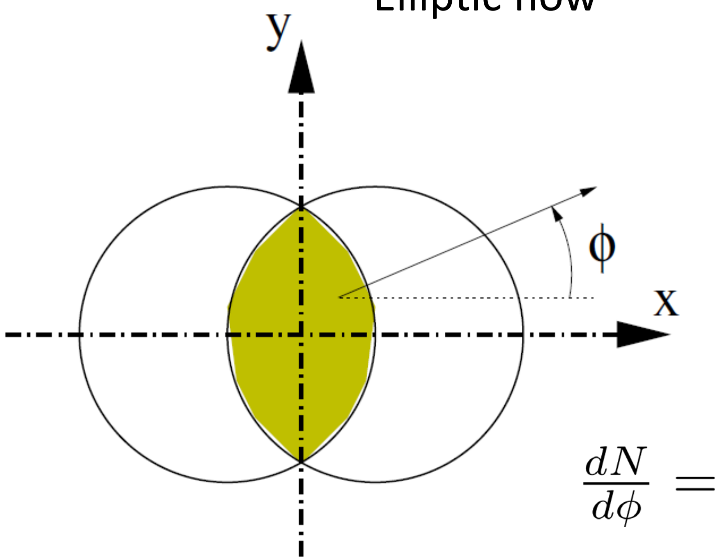
Initial stage of the collisions seen in the transverse plane: Non spherical initial spatial distribution due to eccentricity + fluctuations



Elliptic flow



Triangular flow



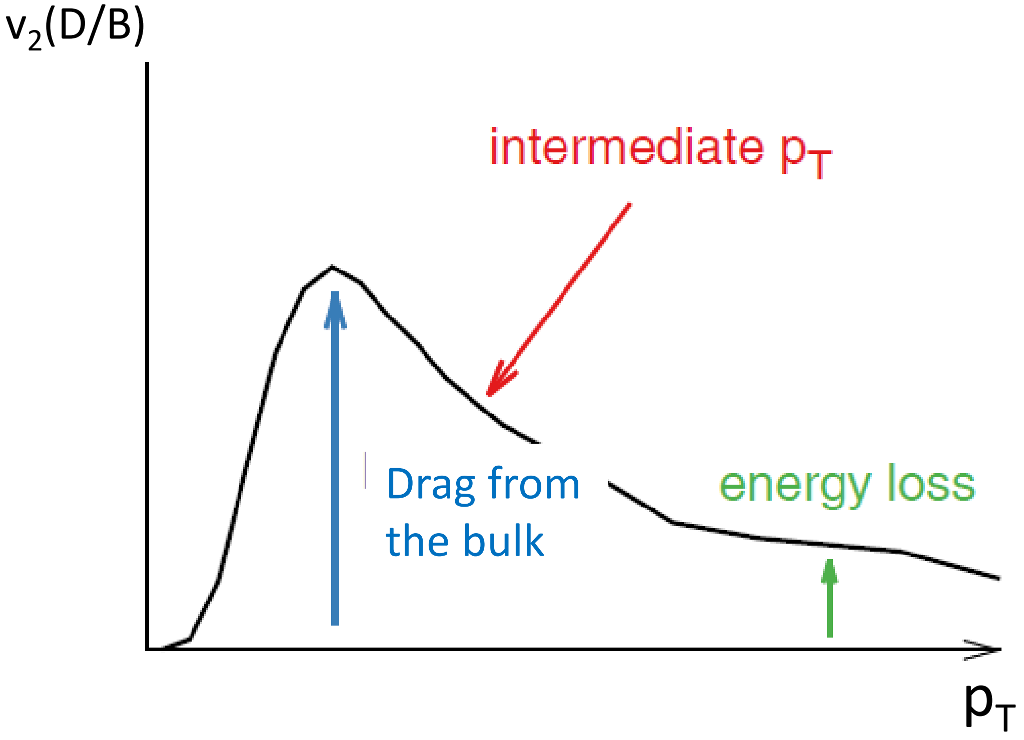
… later on converted in anisotropies due to the fluid dynamics evolution.

anisotropies in the final hadrons azimuthal distributions (Fourier series)

$$\frac{dN}{d\phi} = \frac{N_0}{2\pi}\left(1 + 2v_2\cos[2(\phi - \psi_{\mathrm{RP}})] + \cdots\right)$$

$$v_2 = \langle\cos[2(\phi - \psi_{\mathrm{RP}})]\rangle$$

Basic Consequences of HQ interaction with QGP for the v_2



Small pT: height of v2 at low p_T sensitive to:

- *Bulk anisotropy, mostly at the late times*
- The drag force acting locally on HF

high p_T non-0 v_2 is due to anisotropic Eloss (same ingredients as for the R_{AA} + *geometrical anisotropy of initial distribution of matter*)

intermediate p_T : onset and offset of many competing effects.

!!! Alternative pointed out recently within transport model (AMPT & MPC) study: so-called « escape mechanim » characterized by a large v_2 component stemming from $N_{coll} \approx 1$

L. He et al, Physics Letters B753 (2016) 506

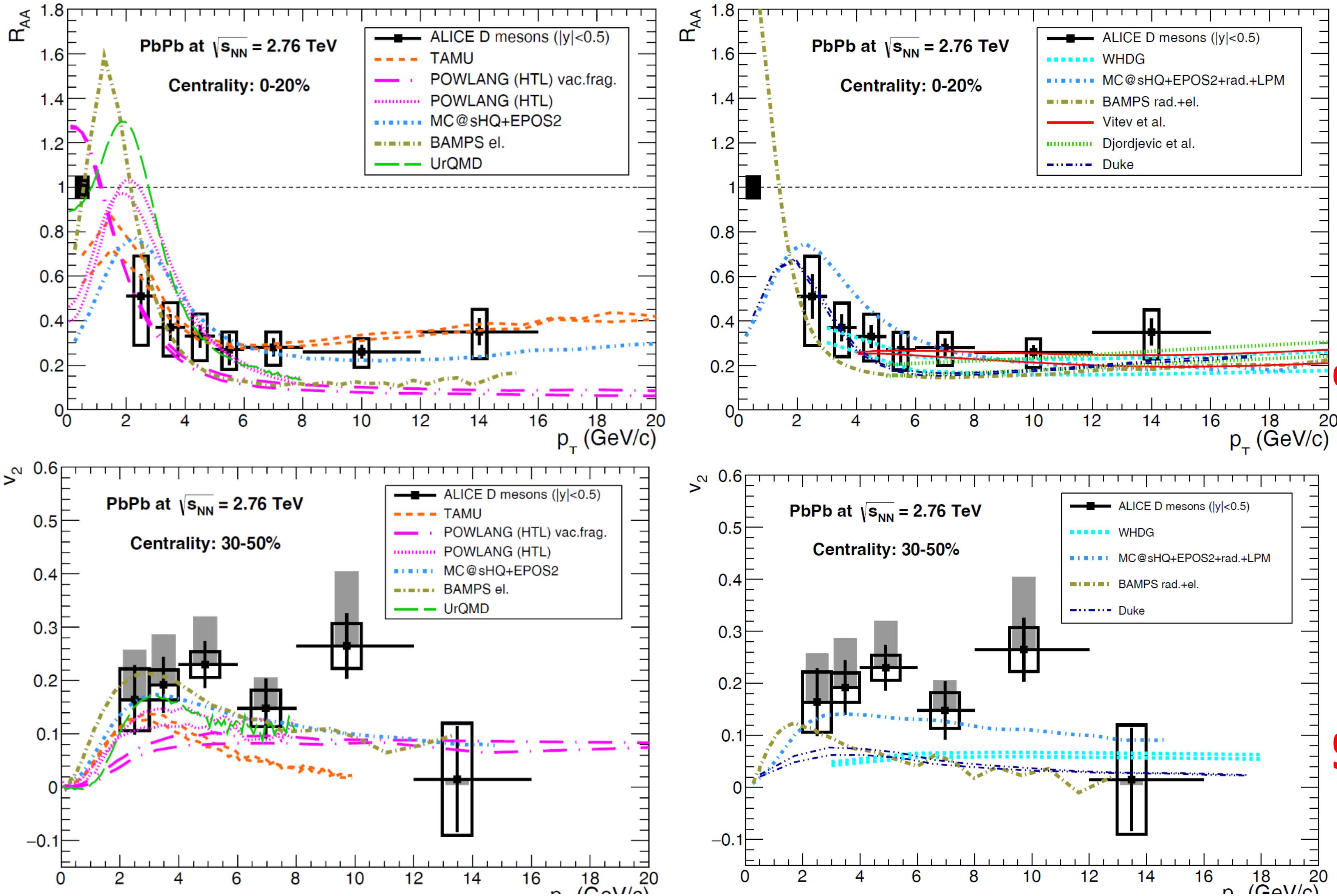
2 Important remarks:

- Any energy loss model, even the roughest one, will generate these typical structures in the R_{AA} and the v_2. Getting a correct **quantitative** agreement is much more involved.
- Quantitative predictions also depends on some « extra ingredients » (hydro, initial conditions,…)

Models vs DATA at LHC (Sapore Gravis Report compilation)

Andronic et al., Eur.Phys.J.C 76 (2016) 3, 107

Purely elastic scatterings



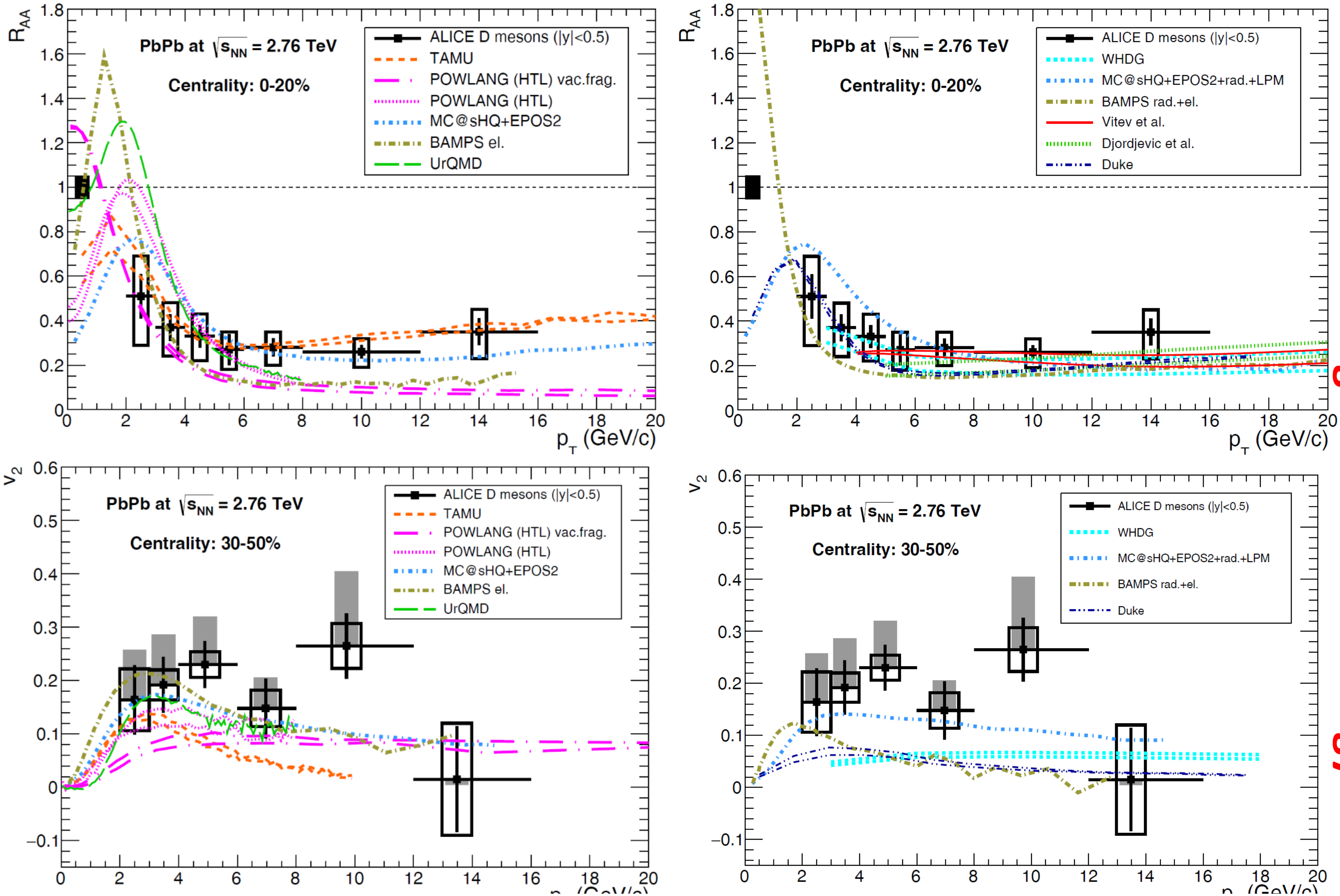
Elastic scatterings + radiative energy loss

Despite various prescriptions for Energy loss, a lot of models can cope with the data

Models vs DATA at LHC (Sapore Gravis Report compilation)

Andronic et al., Eur.Phys.J.C 76 (2016) 3, 107

Purely elastic scatterings



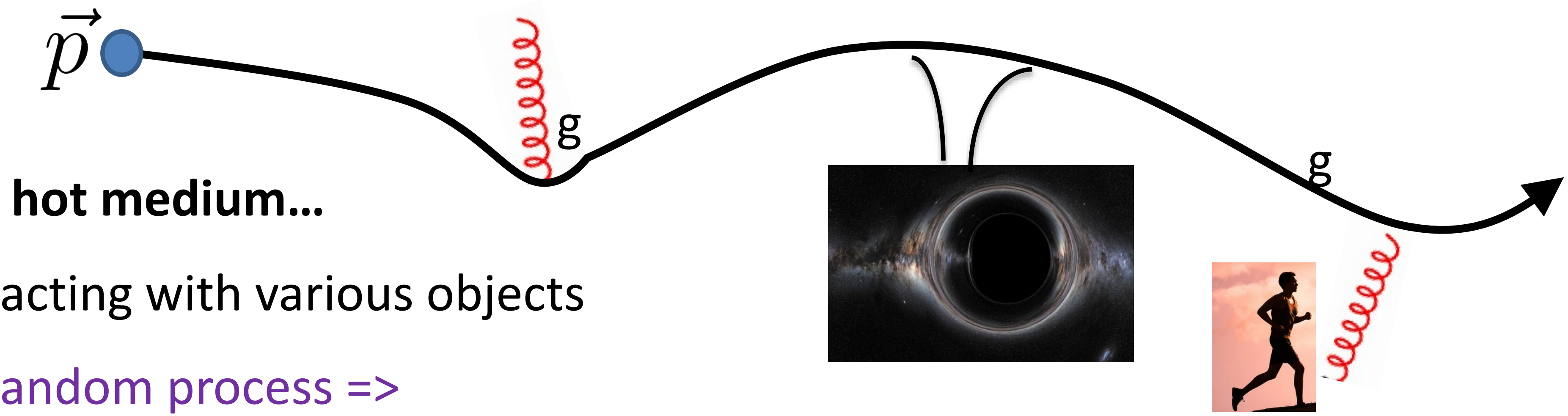
Elastic scatterings + radiative energy loss

Some historical advocated tension between R_{AA} and v_2

Going a bit deeper…

- How can we compare the energy loss models ?
- Are there any energy loss calculation stemming from fundamental principles…
- …and that does not rely on any assumption/vision of the QGP in terms of (effective) degrees of freedom ?

Transport coefficients



HQ in hot medium…

… interacting with various objects

Quasi random process =>

$$-\frac{d}{dt}\langle\vec{p}\rangle = \vec{A}(\vec{p},T) = \eta_D(\vec{p},T)\times\vec{p} \qquad \eta_D[\mathrm{fm}^{-1}]:$$ Relaxation rate

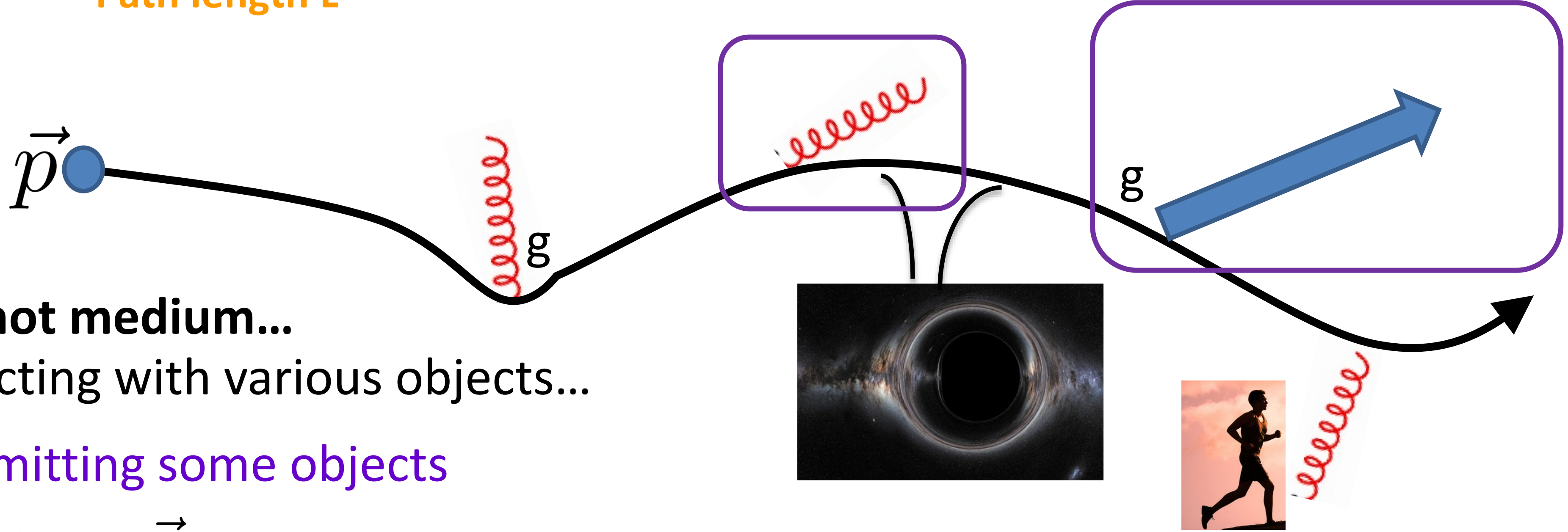
$$\frac{d}{dt}\langle\vec{p}_{T,i}\vec{p}_{T,j}\rangle = \kappa_T(\vec{p},T)\delta_{i,j} \qquad \kappa_T[\mathrm{GeV}^2\mathrm{fm}^{-1}]:$$ Transverse diffusion coef. (p space) ; $\boxed{\hat{q} = 2\kappa_T = 4B_0}$

Similar in longitudinal direction $\quad \kappa_L[\mathrm{GeV}^2\mathrm{fm}^{-1}]:$ Longitudinal diffusion coef.

In general, no relation between these coefficients except $\kappa_T = \kappa_L$ for p=0.

Transport coefficients and inelastic processes

Path length L



HQ in hot medium…

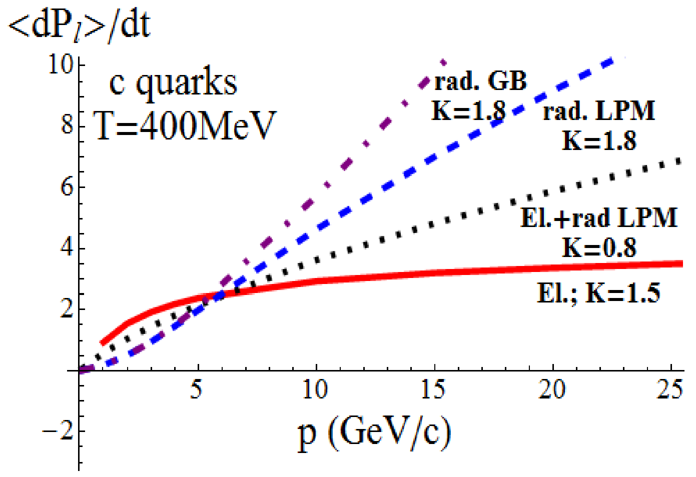
… interacting with various objects…

… and emitting some objects

$$\Delta\langle\vec{p}\rangle = \vec{A}(\vec{p},T)\times L + \underbrace{(\Delta\vec{p})_{\mathrm{rad}}}_{}$$

- contribution from « radiated » part
- In most of existing schemes: $(\Delta\vec{p})_{\mathrm{rad}} = \mathcal{F}(\underbrace{\eta_D,\kappa_T,\kappa_L}_{},p,L)$

Seeked ransport coeff.



!!! In this case, the relaxation rate < (<<) η_D

Transport coefficients at low momentum p≈m$_Q$

Langevin regime => Einstein relation: $\kappa(0) = 2TE_Q\eta_D(0)$

$$\langle r^2(t)\rangle = 2dD_s t$$

For historical reasons, physics displayed as a function of 2πT x the spatial diffusion coefficient

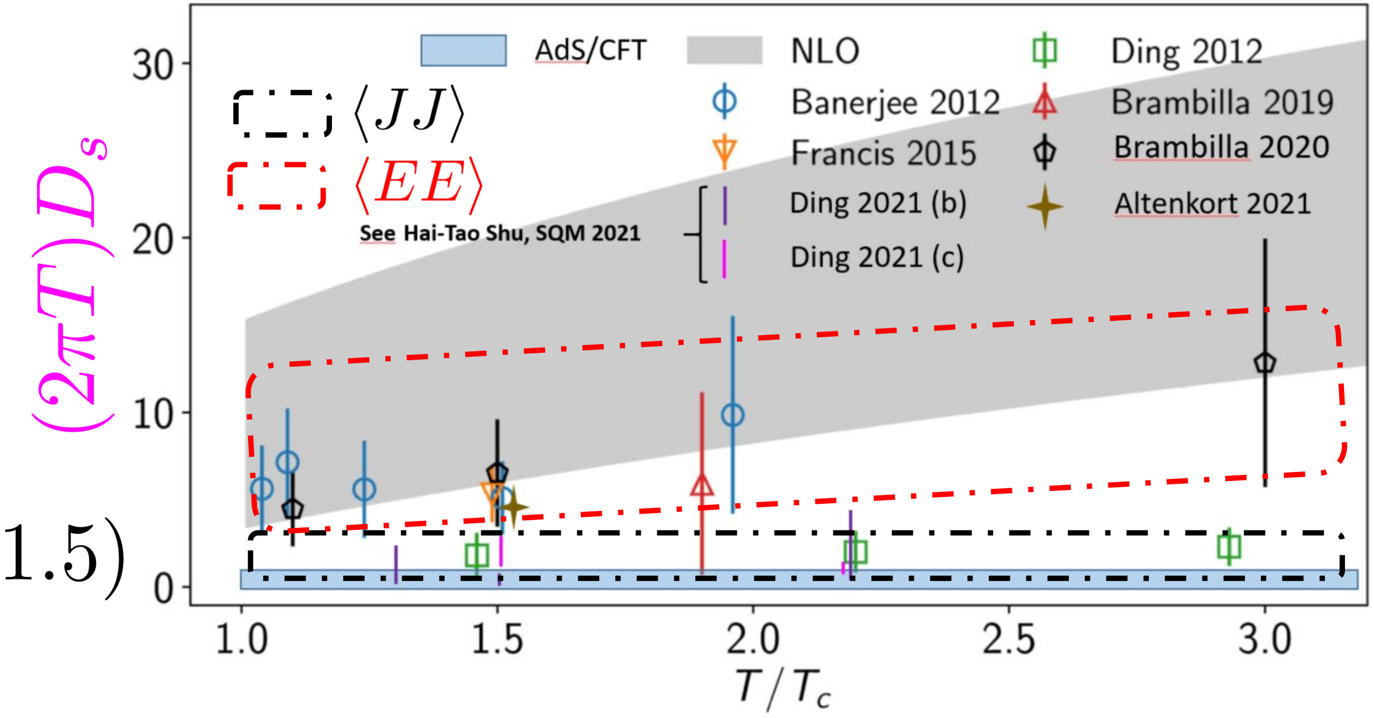
$$\underbrace{(2\pi T)D_s} = \frac{4\pi T^3}{\kappa} = \frac{2\pi T^2}{E_Q\eta_D} \Rightarrow \tau_{\rm relax} = \eta_D^{-1} = (2\pi T)D_s \times \frac{E_Q}{2\pi T^2}$$

Gauge for the coupling strength

lQCD results

The sole direct rigorous calculation of the transport coeff to my knowledge…

but no dependence on the momentum



$$\tau_{\rm relax}(T_c) \approx m_Q[{\rm GeV}] \times (3 \pm 1.5)$$

For b: Indeed a hard probe !

lQCD Calculation of D_s

- Lattice QCD at finite T is performed in Euclidean space notoriously difficult to calculate dynamical quantities.
- Up to 2014, D_s was evaluated directly through the (narrow) diffusion peak of the spectral function evaluated from current – current correlator (hard)
- From 2014: Use of the field – field correlator in order to obtain a better shaped spectral function:

$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re\,Tr}\left[U(\beta;\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau;0)\,gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}\left[U(\beta;0)\right]\right\rangle}$$

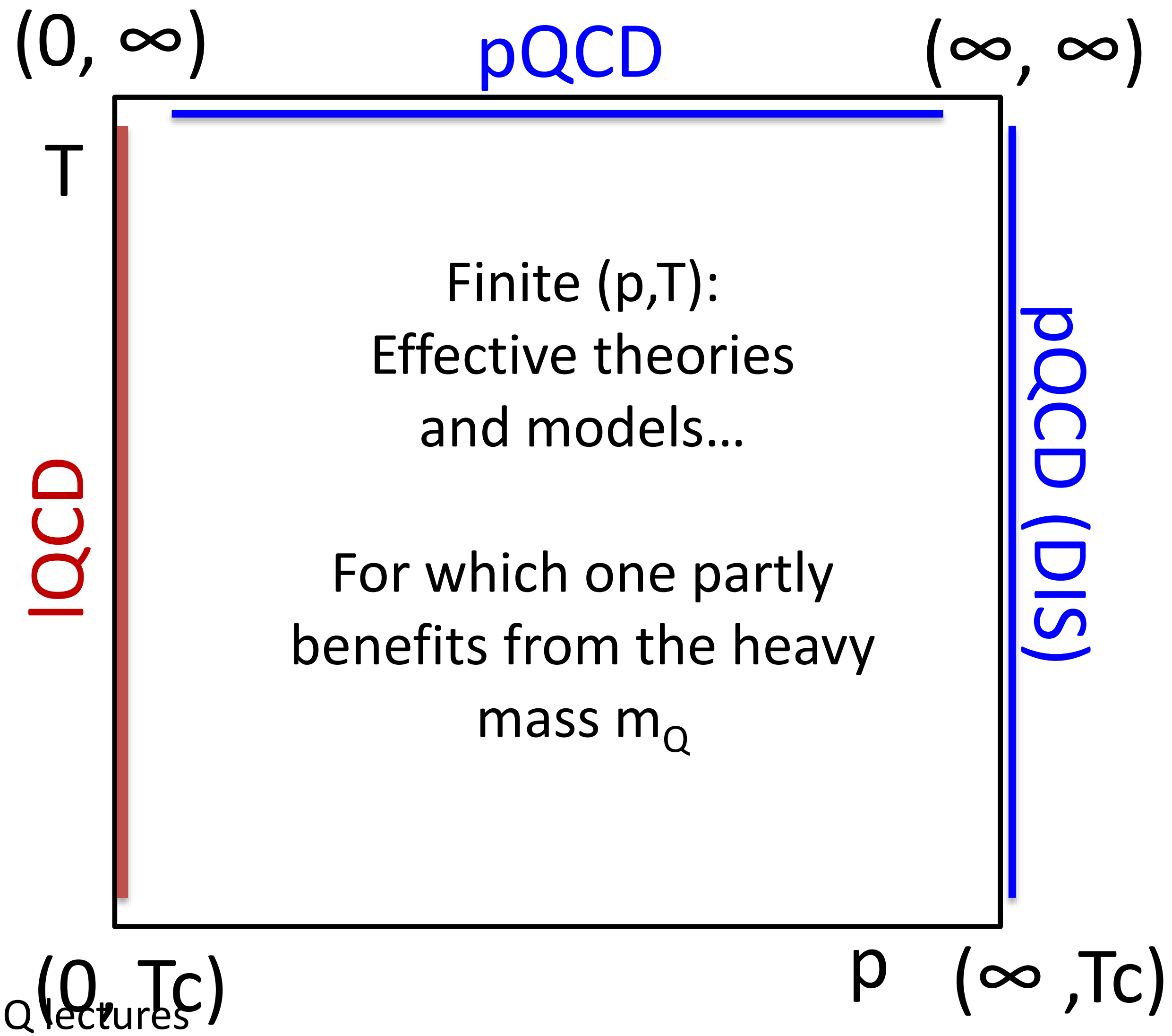
- Then obtain the variance κ of stochastic forces (a transport coefficient; κ = 2 x B) from the slope of spectral function ρ_E at $\omega = 0$:

$$\kappa \equiv \lim_{\omega\to 0}\frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$ with ρ_E extracted from $$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\,\rho_{\mathrm{E}}(\omega)\,\frac{\cosh[\omega(\frac{\beta}{2}-\tau)]}{\sinh[\frac{\omega\beta}{2}]}$$

- Main result : $\kappa/T^3 = 1.8\ldots 3.4$ then convert to D_s

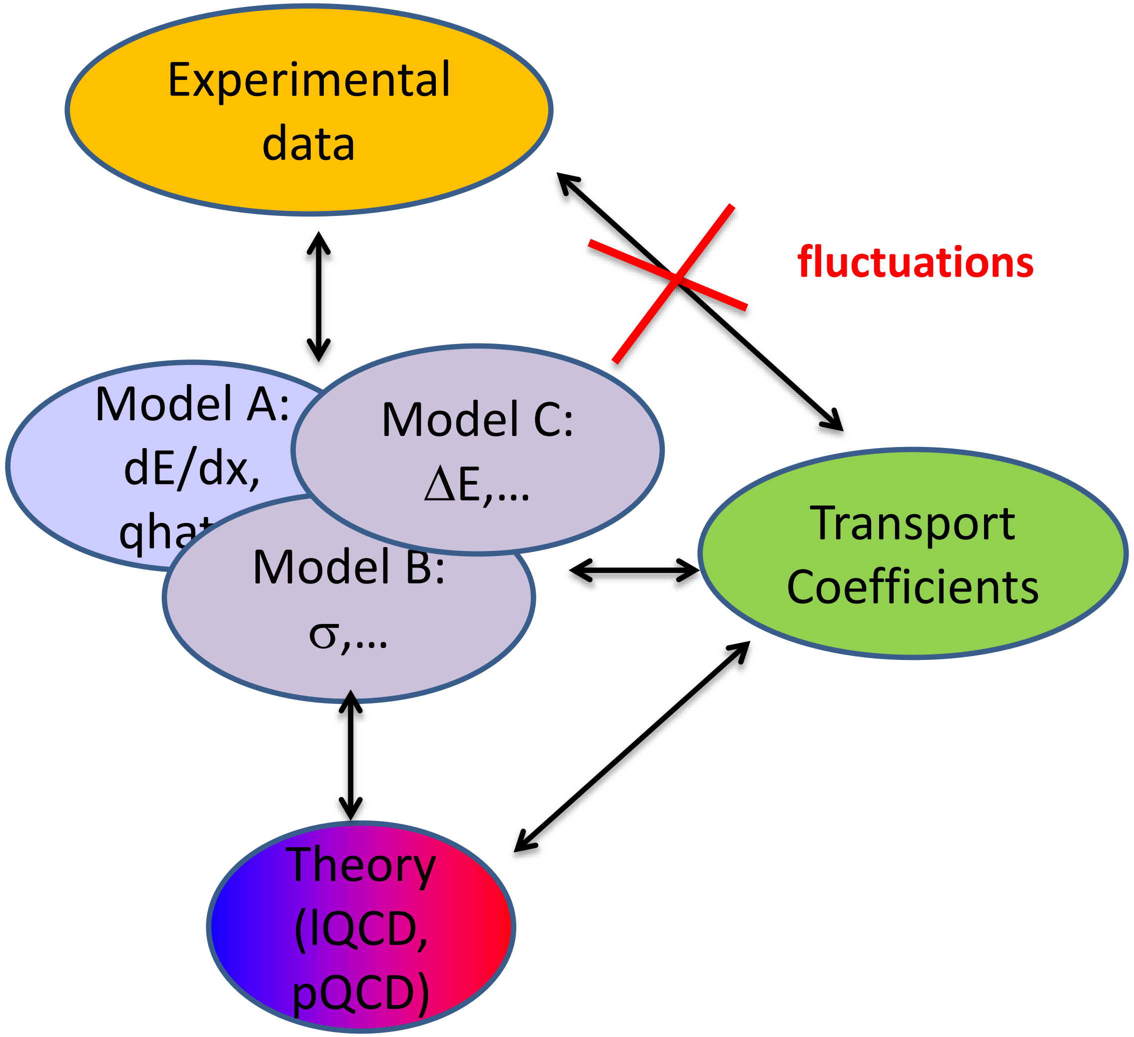
Lanscape of HF theory and modeling in URHIC

Transp. coeff. η_D, κ_T, κ_L



Finite (p,T):
Effective theories
and models…

For which one partly
benefits from the heavy
mass m_Q



Experimental data

Model A: dE/dx, qhat

Model C: ΔE,…

Model B: σ,…

Transport Coefficients

Theory (lQCD, pQCD)

fluctuations

Transport coefficients: theory vs models

Langevin regime => Einstein relation: $\kappa = 2TE_Q\eta_D$

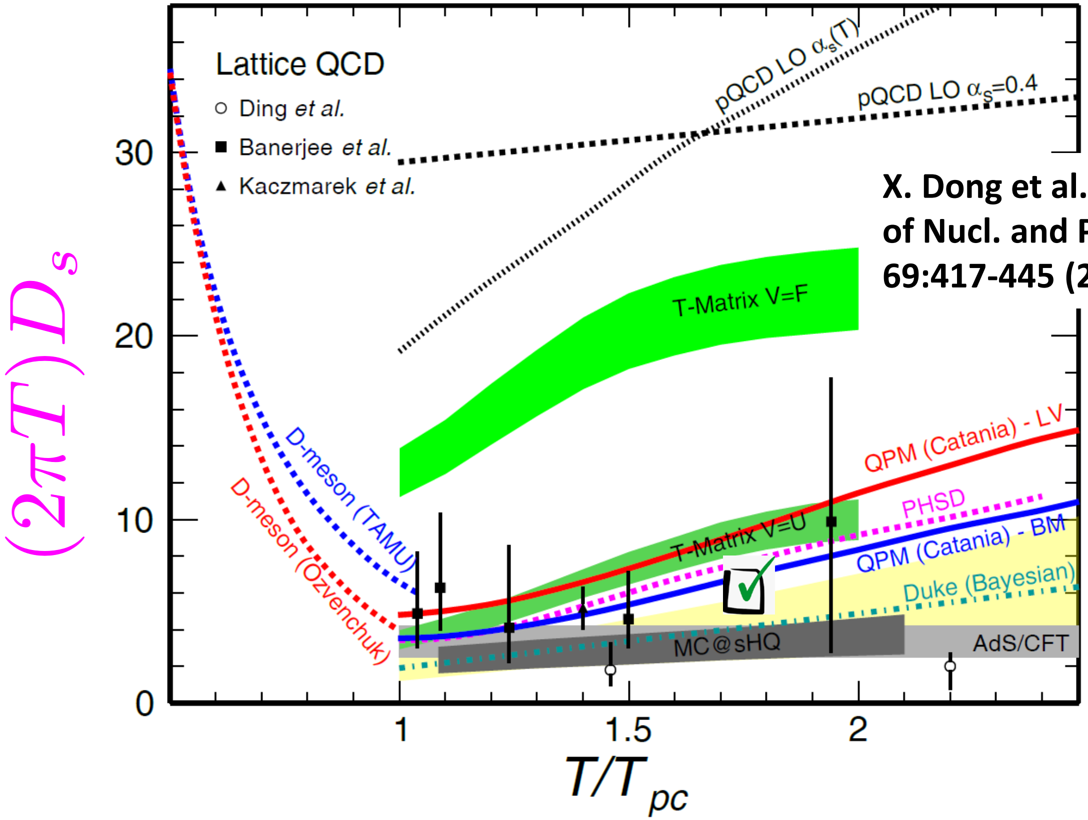
$$\langle r^2(t)\rangle = 2dD_st$$

For historical reasons, physics displayed as a function of 2πT x the spatial diffusion coefficient

$$\underbrace{(2\pi T)D_s}_{\text{Gauge for the coupling strength}} = \frac{4\pi T^3}{\kappa} = \frac{2\pi T^2}{E_Q\eta_D} \Rightarrow \tau_{\text{relax}} = \eta_D^{-1} = (2\pi T)D_s \times \frac{E_Q}{2\pi T^2}$$

Most of the models which reproduce exp. observables (☑) are compatible with lQCD constrains…

… But once again : this is just p=0 physics.



X. Dong et al. Annual Review of Nucl. and Part. Science 69:417-445 (2019)

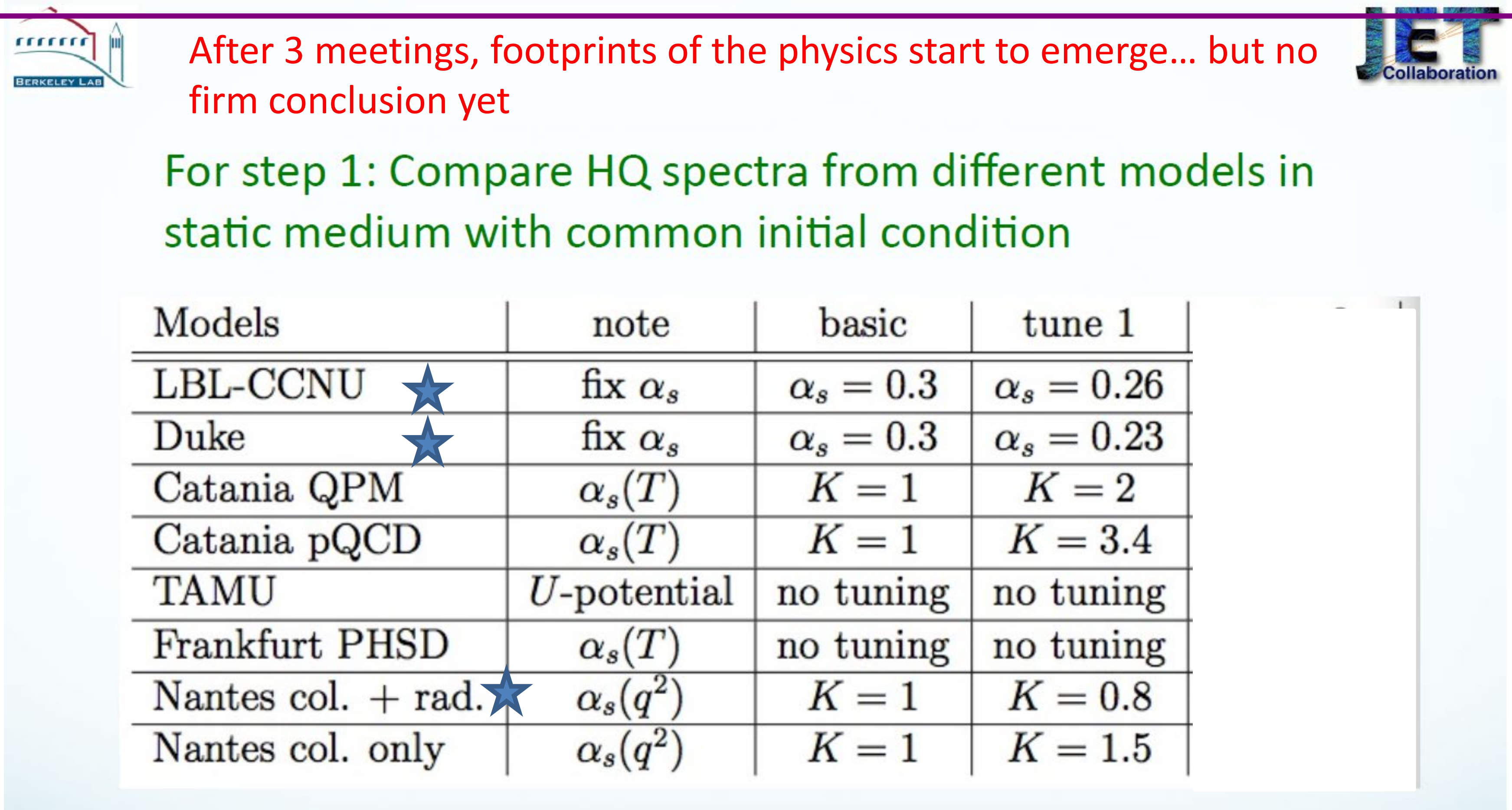
Recent Collective actions beyond Sapore Gravis

- **Heavy Quark – Working Group** (convener: X-N Wang); in the spirit of the Jet Collaboration, the goal is, in a first stage, to :
  - Collect and compare the transport coefficients from various models,
  - Measure and understand their consequences by first studying a simpler brick problem
  - Estimate some systematics + uncertainties

- LBL-CCNU (XN Wang, S. Cao)
- Duke (S. Bass , S. Cao, M. Nahrgang, Y. Xu)
- Catania (V. Greco, S. Das, S. Plumari, F. Scardina)
- TAMU (R. Rapp, M. He)
- Frankfurt pHSD (E. Bratkovskaya, T. Song, H. Berrehrah)
- Nantes (J. Aichelin, PB Gossiaux, M. Nahrgang)

Phys.Rev.C 99 (2019) 5, 054907

Heavy-Quark Working Group



After 3 meetings, footprints of the physics start to emerge… but no firm conclusion yet

For step 1: Compare HQ spectra from different models in static medium with common initial condition

Models	note	basic	tune 1
LBL-CCNU ★	fix α_s	$\alpha_s = 0.3$	$\alpha_s = 0.26$
Duke ★	fix α_s	$\alpha_s = 0.3$	$\alpha_s = 0.23$
Catania QPM	$\alpha_s(T)$	$K = 1$	$K = 2$
Catania pQCD	$\alpha_s(T)$	$K = 1$	$K = 3.4$
TAMU	U-potential	no tuning	no tuning
Frankfurt PHSD	$\alpha_s(T)$	no tuning	no tuning
Nantes col. + rad. ★	$\alpha_s(q^2)$	$K = 1$	$K = 0.8$
Nantes col. only	$\alpha_s(q^2)$	$K = 1$	$K = 1.5$

★: Radiative included

Basic: original model

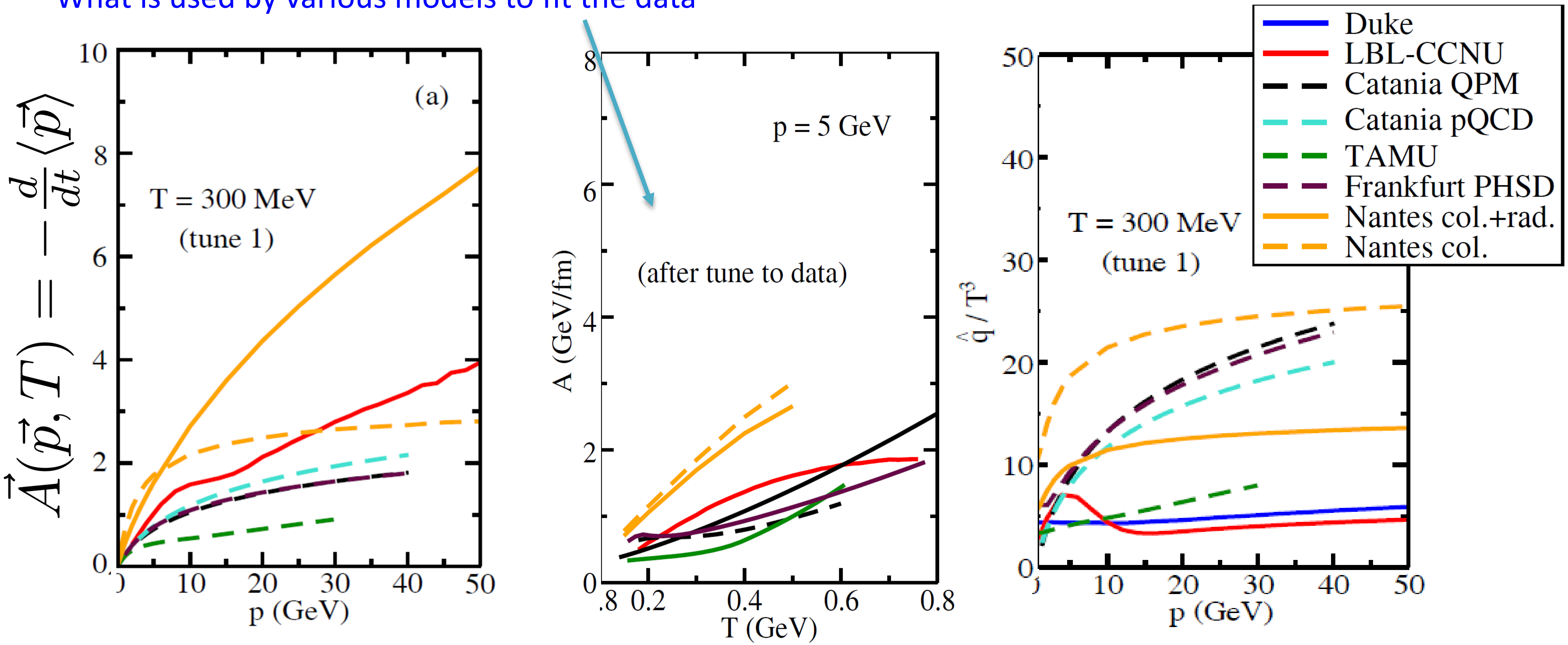
Tune 1: favorite tuning of each group in order to describe *D* meson data with their own ingredients (background, hadronization,…) ; K = rate multiplyer

HQ Working Group

c-quarks

- Collect and compare the transport coefficients from various models:

What is used by various models to fit the data

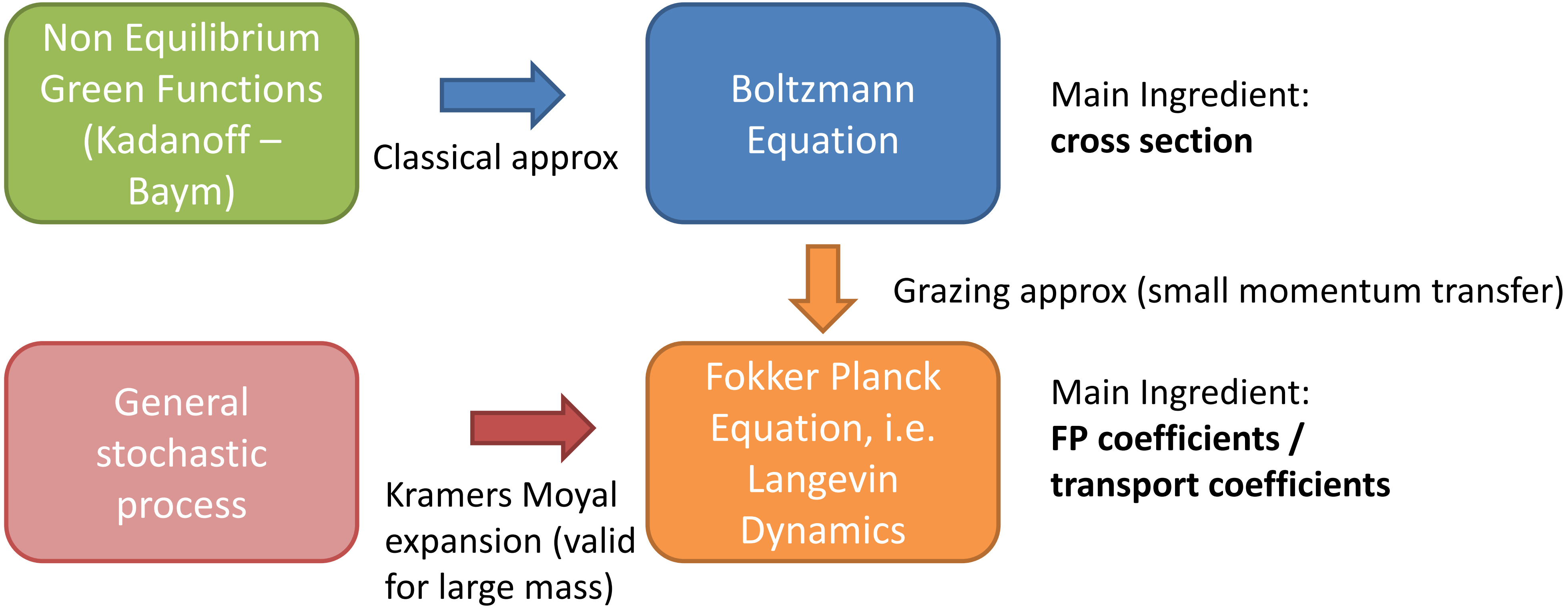


- Obviously not satisfying (from many perspectives) !
- Larger dispersion than the predictions for concrete observables… WHY ?
- Because of « extra ingredients », chosen differently in each model !!!

Various approaches to transport

Bottom-up schemes (microscopic -> mesoscopic):

- Assume (effective) degrees of freedom and (effective) interactions
- Take insights and constrains from the fundamental QCD theory, but often inholds some free parameter
- Rely on more or less sophisticated realizations of the transport theory



Non Equilibrium Green Functions (Kadanoff – Baym)

Classical approx

Boltzmann Equation

Main Ingredient:
cross section

Grazing approx (small momentum transfer)

General stochastic process

Kramers Moyal expansion (valid for large mass)

Fokker Planck Equation, i.e. Langevin Dynamics

Main Ingredient:
FP coefficients / transport coefficients

Why Fokker – Planck (AKA Langevin forces) ?

Bona fide answer: because HQ are heavy => long relaxation times => accumulate many collisions before thermalization => the “details” are averaged (central limit theorem) .

μ-model

$$R(\mathbf{p},t) = \frac{1}{2E_{\mathbf{p}}}\int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_{\mathbf{q}}}\int \frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{\mathbf{q}'}}\int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}}$$

$$\times \frac{1}{\gamma_c}\sum |\boxed{\mathcal{M}|^2}(2\pi)^4\delta^4\left(p+q-p'-q'\right)$$

$$\times \left[f(\mathbf{p}')g(\mathbf{q}')\tilde{g}(\mathbf{q}) - f(\mathbf{p})g(\mathbf{q})\tilde{g}(\mathbf{q}')\right],$$

$$\begin{cases} A_i = \langle\langle (p-p')_i \rangle\rangle \\ \kappa_{i,j} = \langle\langle (p-p')_i (p-p')_j \rangle\rangle \end{cases}$$

mesoscopic model (FP equation)

$$\boxed{\frac{\partial f}{\partial t} = \vec{\nabla}_p \cdot \boxed{\left[\vec{A} f + \frac{1}{2}\vec{\nabla}_p(\hat{\kappa} f)\right]}}$$

distribution f in phase space… which fulfills

$$\frac{d}{dt}\langle\vec{p}\rangle_f = -\langle\vec{A}(T)\rangle_f$$

$$\frac{d}{dt}\langle\vec{p}_i\vec{p}_j\rangle_f = \langle\kappa_{ij}(T)\rangle_f$$

Recovers the averages from the μ-model

… also because it is much easier to solve than sampling the rate !

MC simulation then writes:

$$\Delta\vec{p} = -\vec{A}\Delta t + \underbrace{\vec{\xi}}_{\text{Random force (fluctuations)}} \qquad \text{for each } \Delta t$$

Langevin vs Boltzmann Dynamics for HQ at intermediate p_T

😀 I am the most faithful to lQCD

😀 I allow to grasp the main aspects of the physics with a limited set of transport coefficients

☹ "Hard" tails in the distributions are not well taken into account by the first moments

☹ Transport coefficients derived from differential microscopic rates do not systematically satisfy Einstein relations => need a correction "by hand"

😀 I am the most faithful to pQCD for hard transfers

☹ I rely on a quasi-particle picture that may not apply in view of the widths in the spectral distributions

😀 I can describe the hard tails in the distributions

😀 I naturally drive the HQ distributions towards the genuine equilibrium Boltzmann distribution

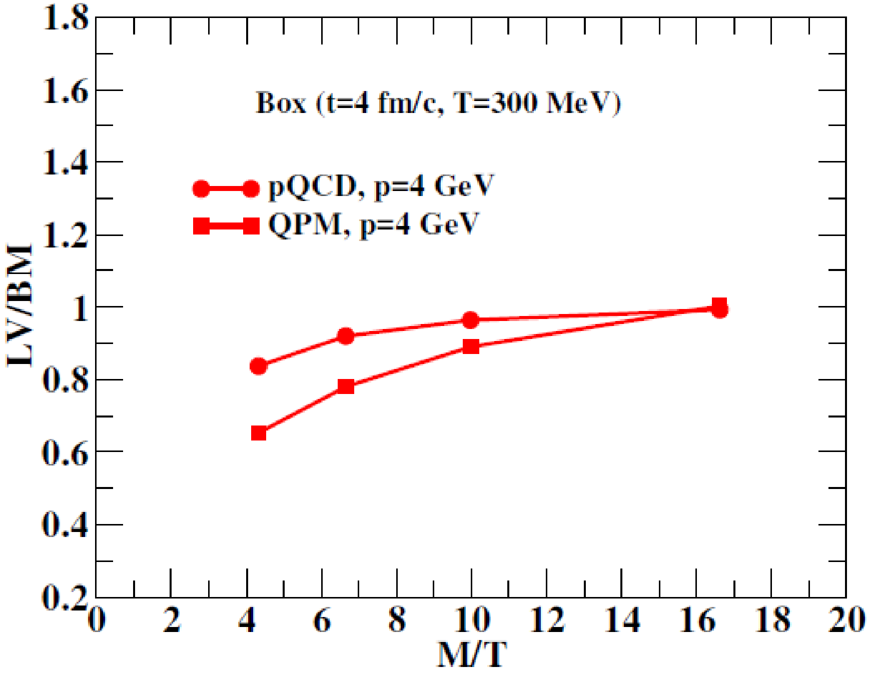
Fokker Planck

Boltzmann with μ model

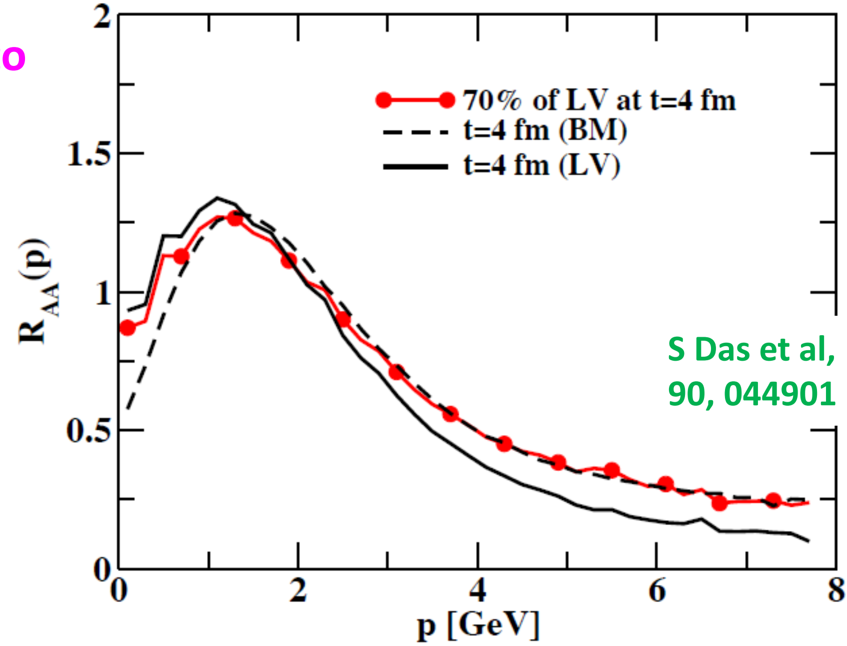
Boltzmann vs Langevin Dynamics

Langevin from Boltzmann view point:

- Lesson: For coarse grained observables like the R_{AA} and the v_2, the agreement between the 2 transport schemes essentially depends on the isotropization strength of the cross section (i.e., the Debye mass of the gluon propagator)



Differences up to 40% found in QPM model



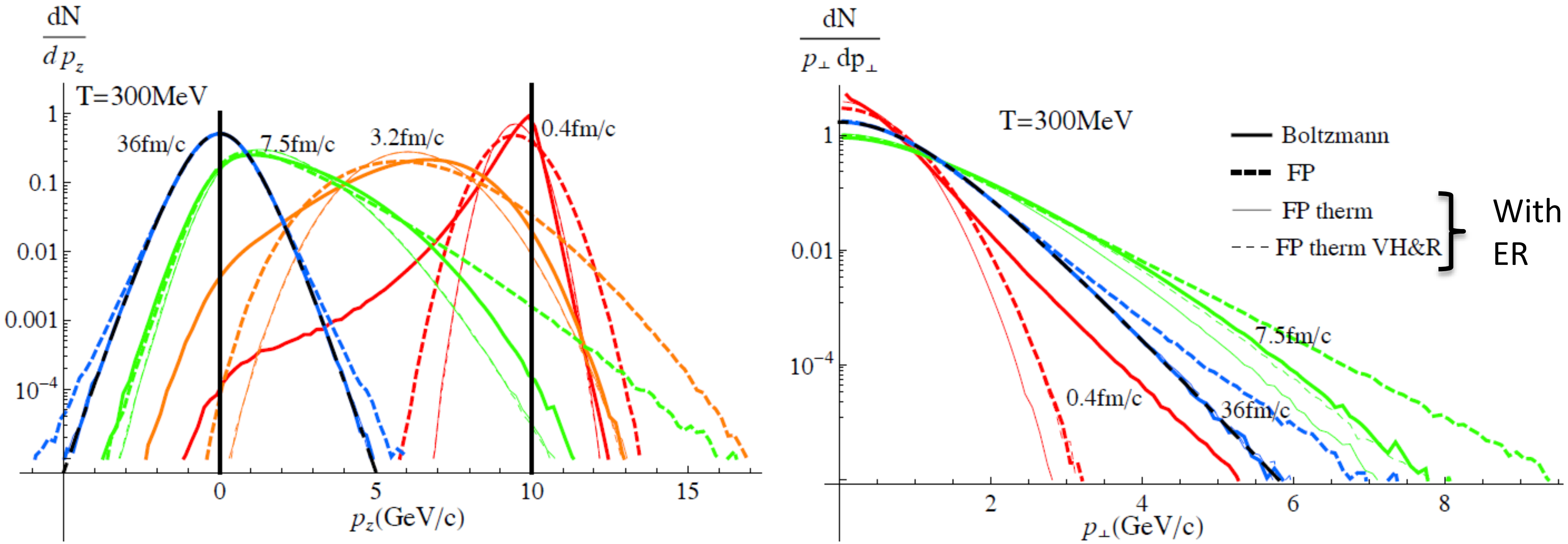
S Das et al, Phys. Rev. C 90, 044901 (2014)

- For $m_D = g\,T \approx 2\,T$ found f.i. in the Quasi Particle Model, extra coupling is found for the R_{AA} using LV, which can be suppressed by reducing the FP coefficients by ≈ 30 %

Boltzmann vs Langevin Dynamics

Langevin from Boltzmann view point:

- For « exclusive process », momentum distributions differ significantly, even after imposing Einstein relation (ER):



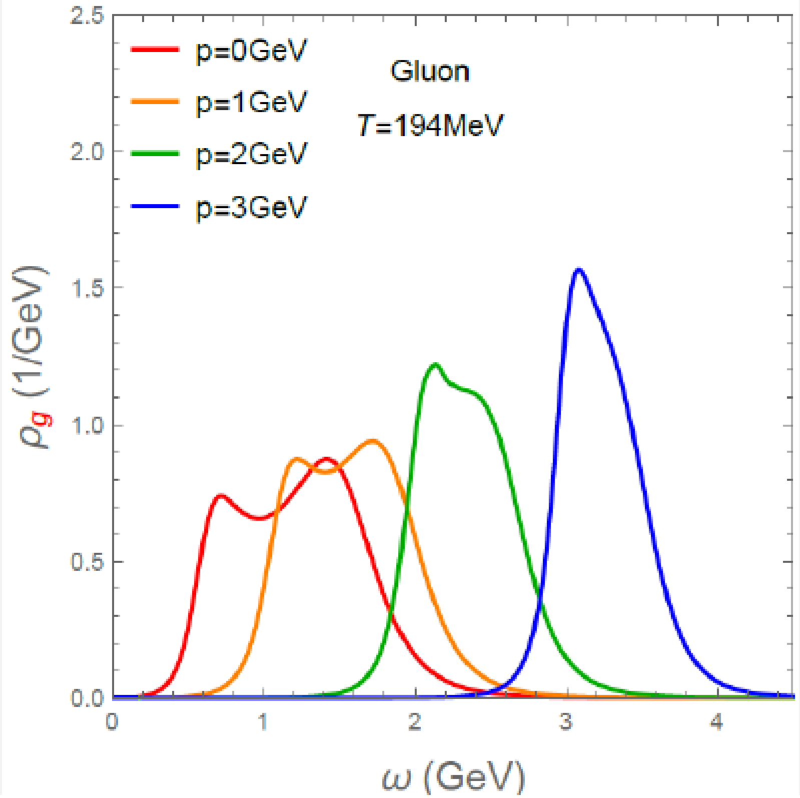
With ER

- These differences should me seen in observables like γ-HQ correlations

Boltzmann vs Langevin Dynamics

Boltzmann from Langevin view point:

- There are a lot of situations where Langevin dynamics applies, but not Boltzmann, thanks to the large mass of the particles.
- It is even a result proven for dynamical systems (conditions on the velocity applies as well)
- In a dense strongly coupled system, this is likely to be the case !

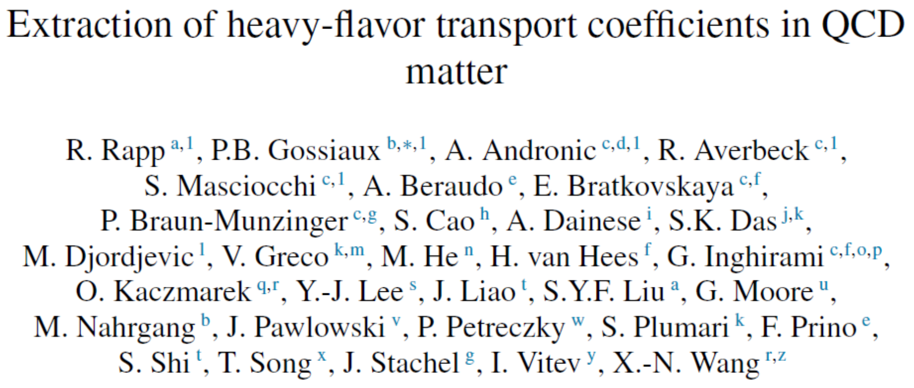


Gluon spectral functions (Liu & Rapp 2016)

Boltzmann vs Langevin Dynamics

Take home message for concrete 1-body observables:

R. Rapp et al. / Nuclear Physics A 979 (2018) 21–86



Extraction of heavy-flavor transport coefficients in QCD matter

R. Rapp [a,1], P.B. Gossiaux [b,*,1], A. Andronic [c,d,1], R. Averbeck [c,1], S. Masciocchi [c,1], A. Beraudo [e], E. Bratkovskaya [c,f], P. Braun-Munzinger [c,g], S. Cao [h], A. Dainese [i], S.K. Das [j,k], M. Djordjevic [l], V. Greco [k,m], M. He [n], H. van Hees [f], G. Inghirami [c,f,o,p], O. Kaczmarek [q,r], Y.-J. Lee [s], J. Liao [t], S.Y.F. Liu [a], G. Moore [u], M. Nahrgang [b], J. Pawlowski [v], P. Petreczky [w], S. Plumari [k], F. Prino [e], S. Shi [t], T. Song [x], J. Stachel [g], I. Vitev [y], X.-N. Wang [r,z]

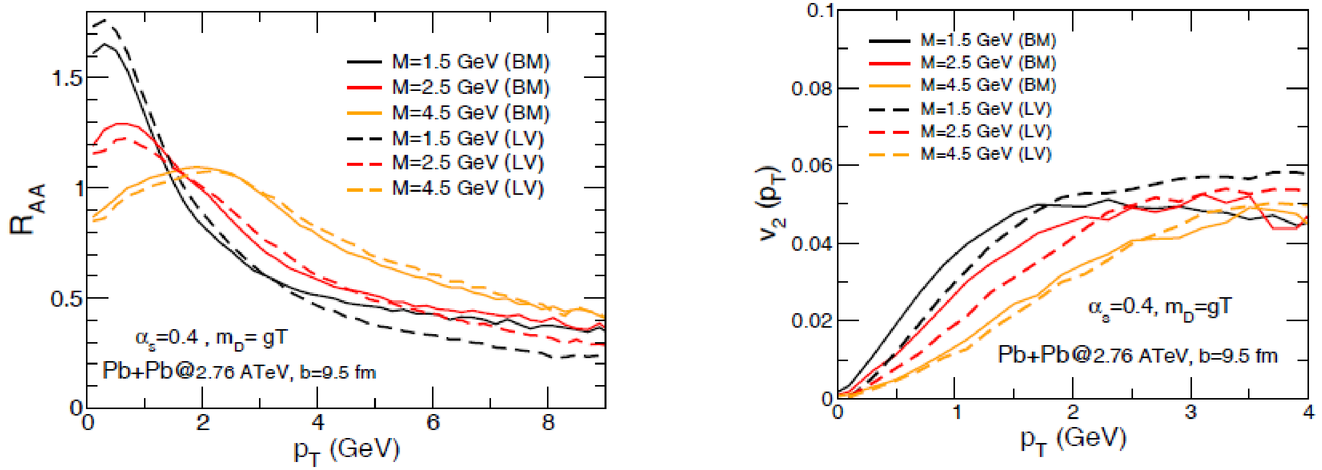


Figure 24: Nuclear modification factor (left panel) and elliptic flow (right panel) for heavy quarks in semi-central Pb+Pb($\sqrt{s_{NN}} = 2.76$ TeV) collisions (at $b = 9.5$ fm) for different values of the HQ mass, M_Q (indicated by the different line colors), in a Boltzmann (solid lines) and in a Langevin approach (dashed lines).

The adopted transport scheme mostly affects the c-quarks, Langevin leading to a reduced v_2

Recent Collective actions beyond Sapore Gravis

- **EMMI Rapid Reaction Task Force** (organizers: R. Rapp, PB Gossiaux, A. Andronic, R. Averbeck, S. Maschiocchi):
  - Global strategy to extract the diffusion coefficient from the intercomparison between models and data
  - Collect and analyse all ingredients from various models
  - Identify constrains from lQCD
  - Initiate discussions to assess the limitations of some existing models.

R. Rapp[*1], P.B. Gossiaux[*2], A. Andronic[*3,4], R. Averbeck[*3], S. Masciocchi[*3], A. Beraudo[5],
E. Bratkovskaya[3,6], P. Braun-Munzinger[3,7], S. Cao[8], A. Dainese[9], S.K. Das[10,11],
M. Djordjevic[12], V. Greco[11,13], M. He[14], H. van Hees[6], G. Inghirami[3,6,15,16], O. Kaczmarek[17,18]
Y.-J. Lee[19], J. Liao[20], S.Y.F. Liu[1], G. Moore[21], M. Nahrgang[2], J. Pawlowski[22], P. Petreczky[23]
S. Plumari[11], F. Prino[5], S. Shi[20], T. Song[24], J. Stachel[7], I. Vitev[25], and X.-N. Wang[26,18]

Goal to attack the problem with a broad view right from the beginning…

R. Rapp et al, arXiv: 1803.03824
Nucl.Phys.A 979 (2018) 21-86

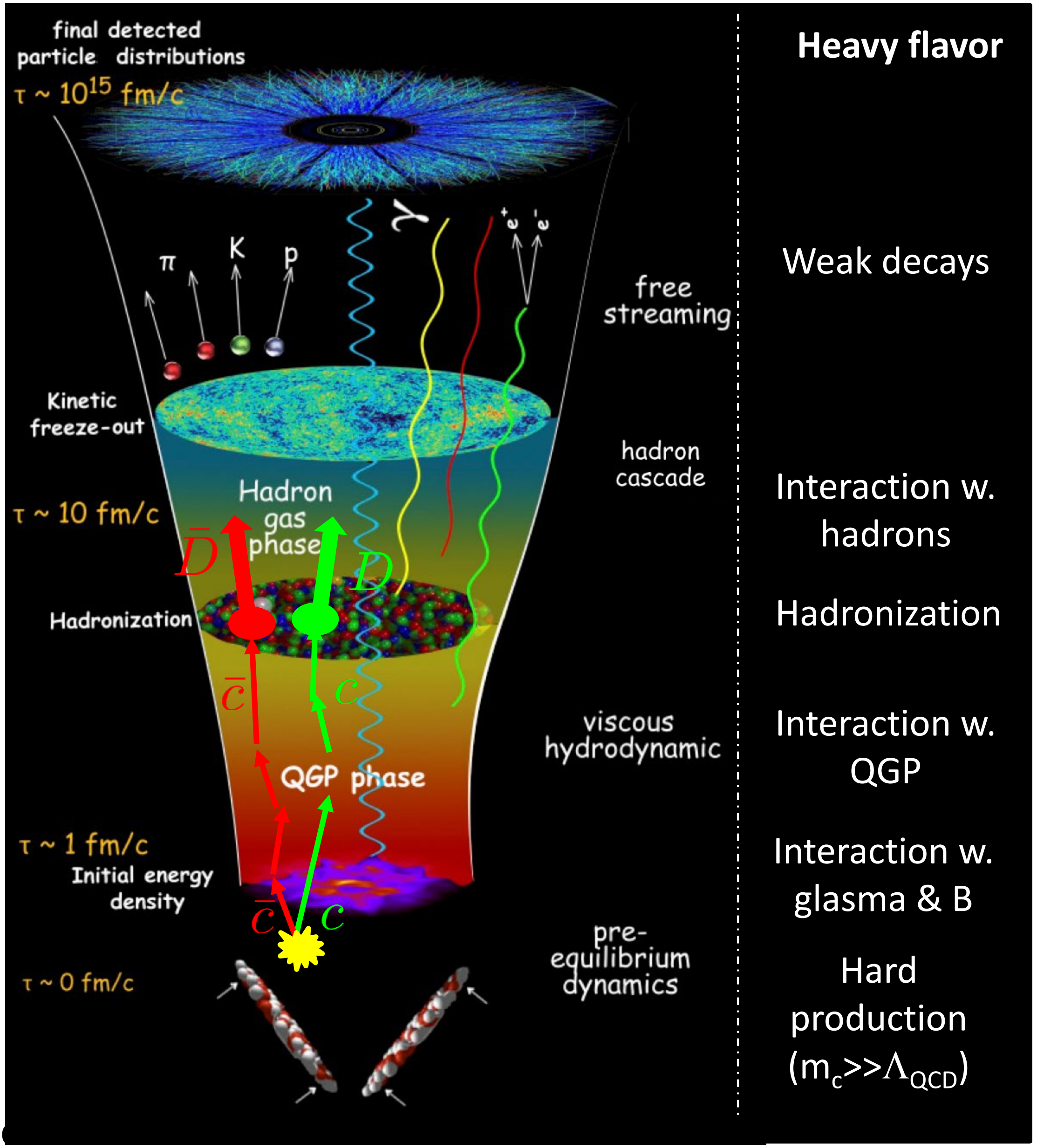
(20 monthes since first meeting)

But also :
S. Cao et al, Phys.Rev.C 99 (2019), 054907 ;
T. Song et al, Phys. Rev. C 101 (2020) , 044903

Standard model of URHIC

Established before 2010

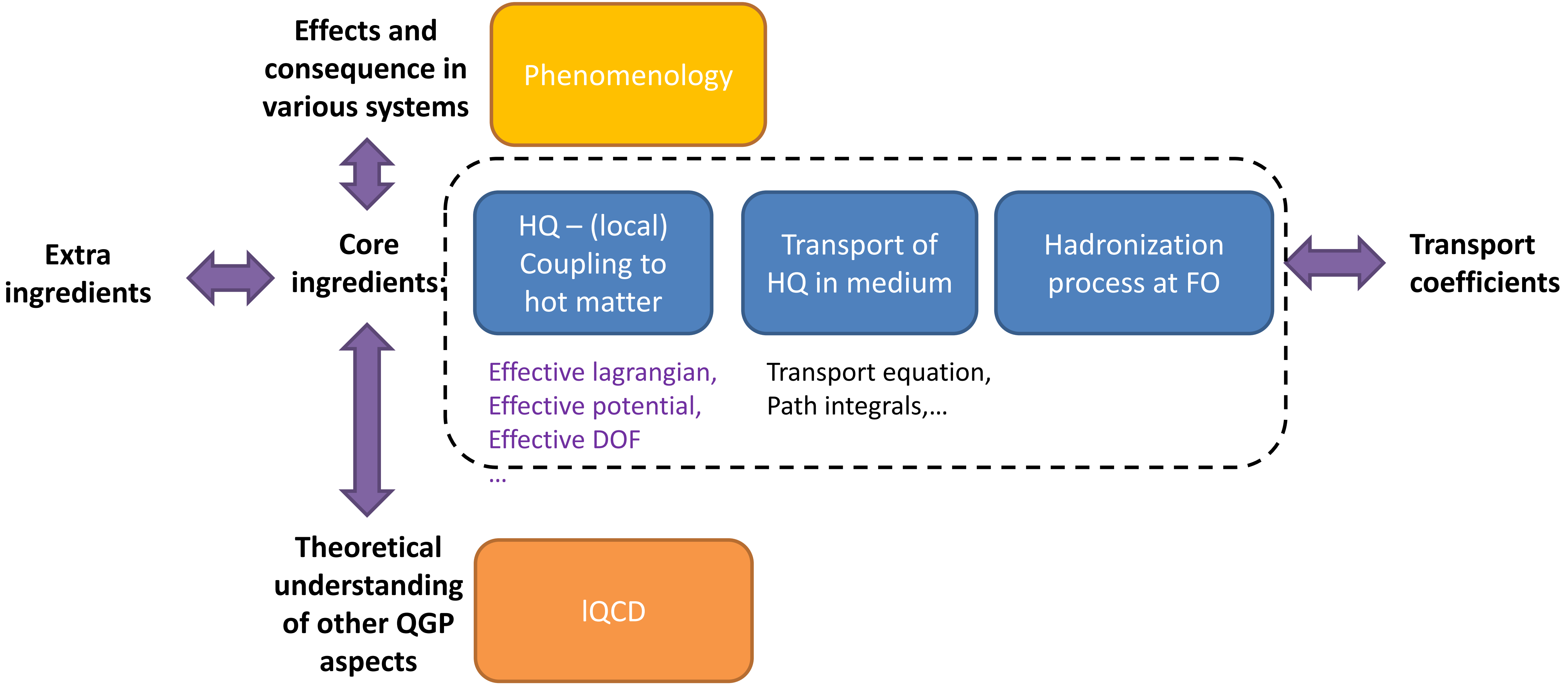
Since ≈ 2010



- Produced early (t≈ $1/m_Q$)
  - => No further $Q\bar{Q}$ generation in ensuing QGP
  - Initial production well controlled (advantage of m_Q>> Λ_{QCD})
  - But early phase might not be so innocent (magnetic field, CGC-glasma,…)
- **Experience the full deconfined phase + hadronic phase**
  - probes the QGP on harder scales than the other hadronic observables *while not fully thermalized* (t_{relax} α m_Q/T^2)
  - accumulates several effects => need to compare different systems to better differentiate them
- Produced over a wide range of rapidities and p_T
  - increased richness in scrutinizing the interaction of HQ with medium…
  - but also sets more challenges (interactions for p_T<<m_Q, p_T≈m_Q, p_T >>m_Q, appropriate transport theory ?).
- **Nowadays turning into precision physics thanks to abundance of RHIC and LHC results !!!**

A bit of structure

- HQ propagation in QM & URHIC…



Effects and consequence in various systems

Phenomenology

Extra ingredients

Core ingredients

HQ – (local) Coupling to hot matter

Transport of HQ in medium

Hadronization process at FO

Transport coefficients

Effective lagrangian,
Effective potential,
Effective DOF
…

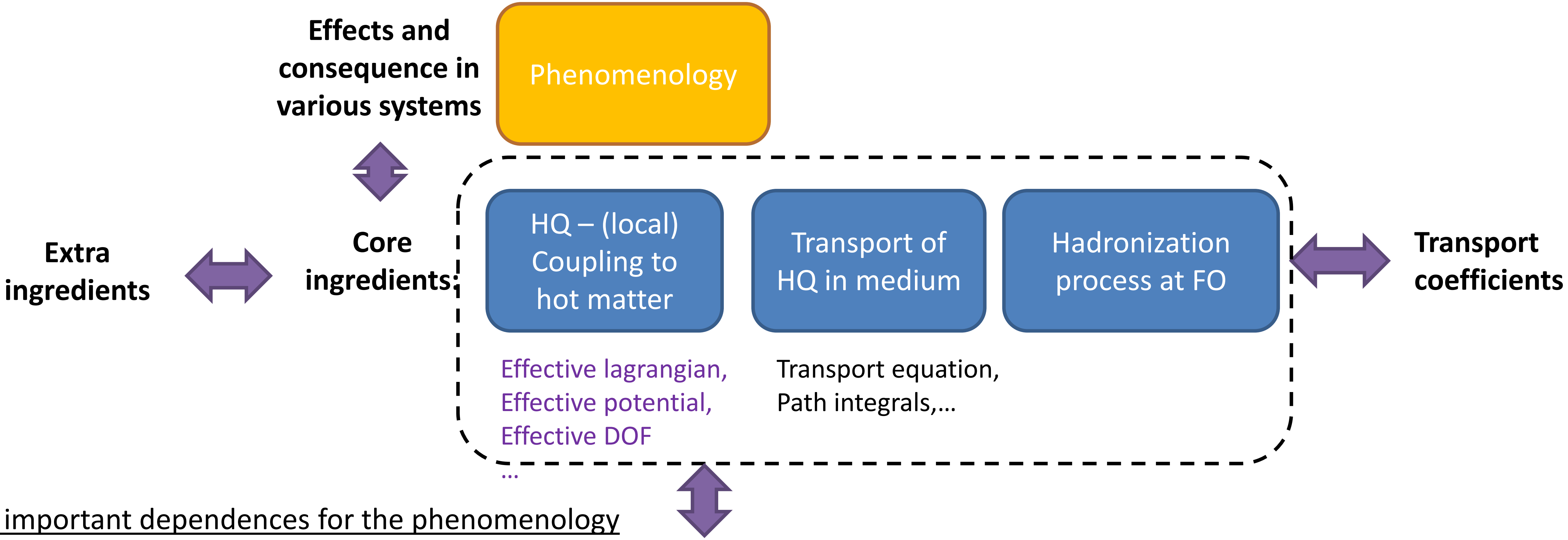
Transport equation,
Path integrals,…

Theoretical understanding of other QGP aspects

lQCD

A bit of structure

- HQ propagation in QM & URHIC…



Effects and consequence in various systems

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Effective potential,
Effective DOF
…

Transport equation,
Path integrals,…

<u>4 important dependences for the phenomenology</u>

- Energy dependence : the saturation at large E explain the restoration -> 1 at large p_T
- Mass dependence => less thermalization for b quarks
- T dependence weigths differently the initial stage and the late evolution (for which flows have developped)
- Path length dependence => makes it more transparent to the radiative in small systems ($\Delta E_{rad} \alpha L^2$)

Models & Effective Theories

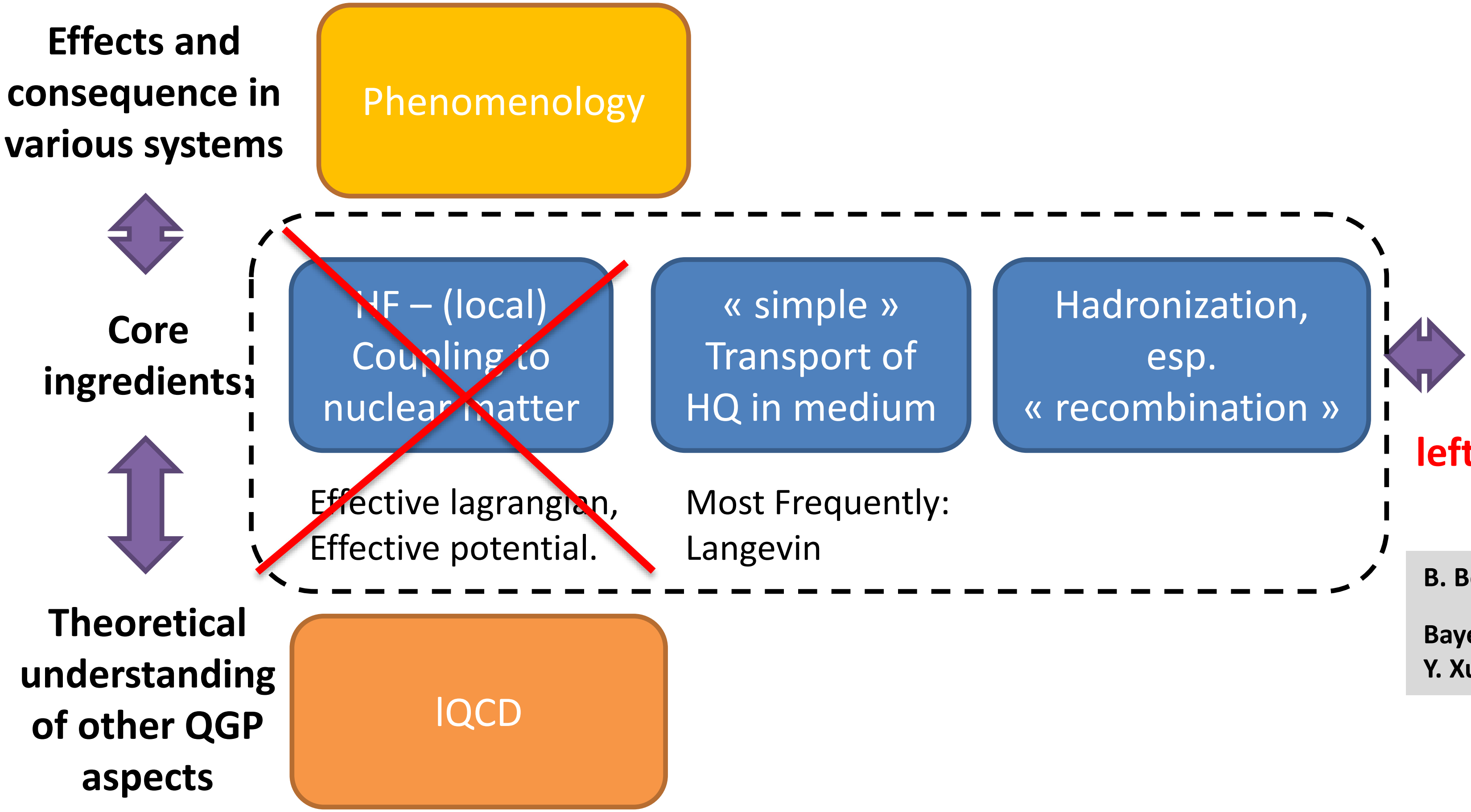
	elastic	Elastic + radiative	radiative	Other
Transport coefficient based (LV,…)	TAMU POWLANG HTL Catania LV	Duke, TAMU w rad.	ASW	ADS/CFT POWLANG lQCD *DABMOD* *S. Li et al, arXiv:1803.01508*
Cross section (or$\|M\|^2$) based (Boltzmann,…)	AMPT MC@sHQ el URQMD PHSD Catania BM	DREENA MC@sHQ el + rad BAMPS CUJET3 HYDJET++ Abir and Mustafa LBL-CCNU VNI/BMS LIDO	$\mathrm{SCET}_{G,M}$	

Red: Transport models

Disclaimer : If your model does not appear here, please forgive me and contact me for completion

A bit of structure

- No Model approach



Effects and consequence in various systems

Phenomenology

Core ingredients:

~~HF – (local) Coupling to nuclear matter~~

~~Effective lagrangian, Effective potential.~~

« simple » Transport of HQ in medium

Most Frequently: Langevin

Hadronization, esp. « recombination »

Transport coefficients

left as free parameters

Theoretical understanding of other QGP aspects

lQCD

B. Betz et al, JHEP 1408 (2014) 090 (for jets)

Bayesian approach
Y. Xu, et al , Phys.Rev. C97 (2018), 014907

See as well “Flavor hierarchy of parton energy loss in quark-gluon plasma from a Bayesian analysis” by Wen-Jing Xing at SQM 2024

Data driven extraction of transport coefficients

Y. Xu et al
arXiv:1710.00807v1

« Minimal model approach » : Bayesian analysis by the Duke group

$$\frac{d\vec{p}}{dt} = \underbrace{-\eta_D(p)\vec{p} + \vec{\xi}}_{} + \vec{f}_g \leftarrow \frac{dN_g}{dx\,dk_\perp^2\,dt} = \frac{2\alpha_s P(x)\hat{q}_g}{\pi k_\perp^4}\sin^2\left(\frac{t-t_i}{2\tau_f}\right)\left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^4$$

Higher twist

Usual Langevin $\langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$ with $\kappa = \frac{2T^2}{D_s}$

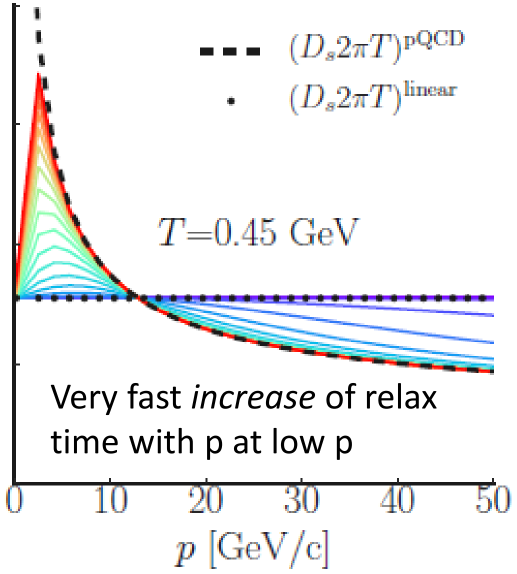
+ coal / frag hadronization and hadronic rescattering

$$D_s(T,p) = \frac{1}{1+(\gamma^2 p)^2}(D_s 2\pi T)^{\rm lin}(T;\alpha,\beta) + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2}(D_s 2\pi T)^{\rm pQCD}(T,p)$$

$$(D_s 2\pi T)^{\rm pQCD} = 8\pi/(\hat{q}/T^3)$$

$$(D_s 2\pi T)^{\rm linear} = \alpha\cdot(1+\beta(T/T_c-1))$$

Encodes possible Non Perturbative Effects around T_c through parameters α (magnitude), β (slope) and γ (inverse momentum range of NP effects)



Very fast *increase* of relax time with p at low p

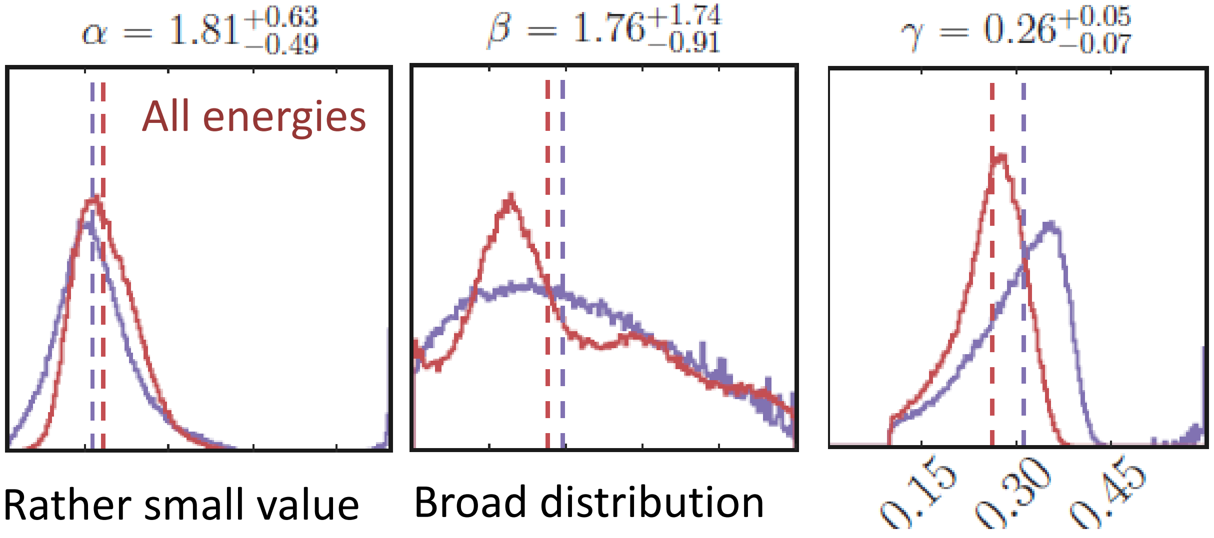
Duke “Bayesian approach”

- Choice of 60 « prior » for which the physical observabes are calculated
- Gaussian emulator to build a fast surrogate of physics
- Random walk throughout parameter space, with acceptance and rejection according to likelihood (with all uncertainties assumed to be uncorrelated).

Let the data speak

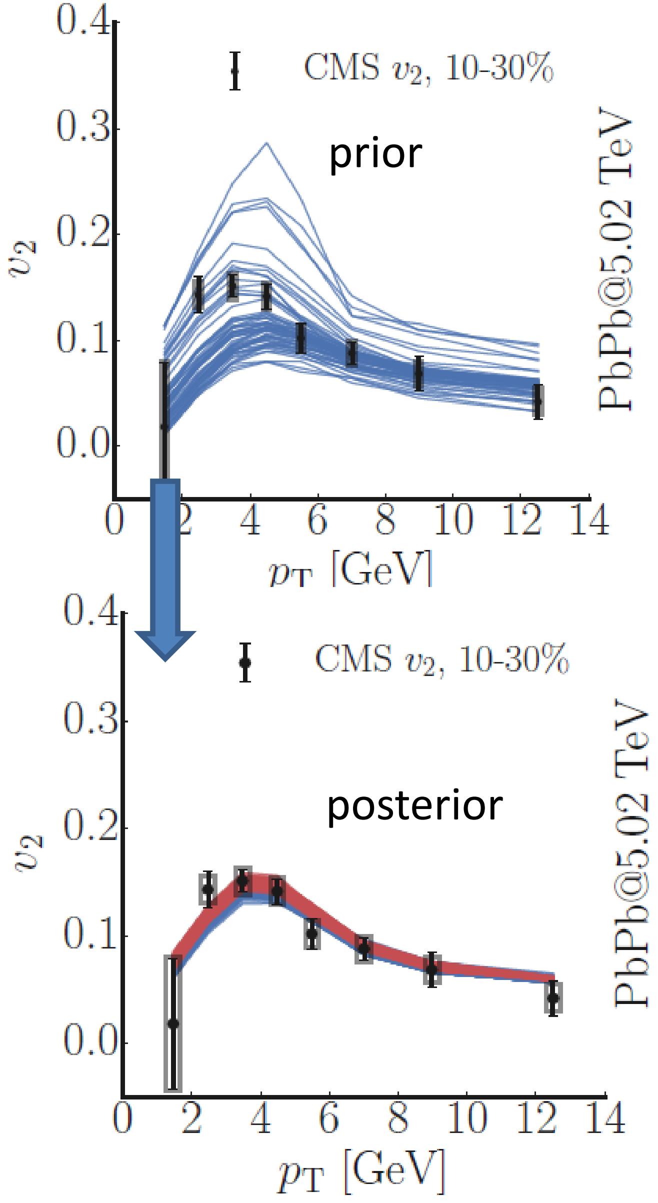
$$(D_s 2\pi T)^{\text{linear}} = \alpha \cdot (1 + \beta (T/T_c - 1))$$

RESULTS

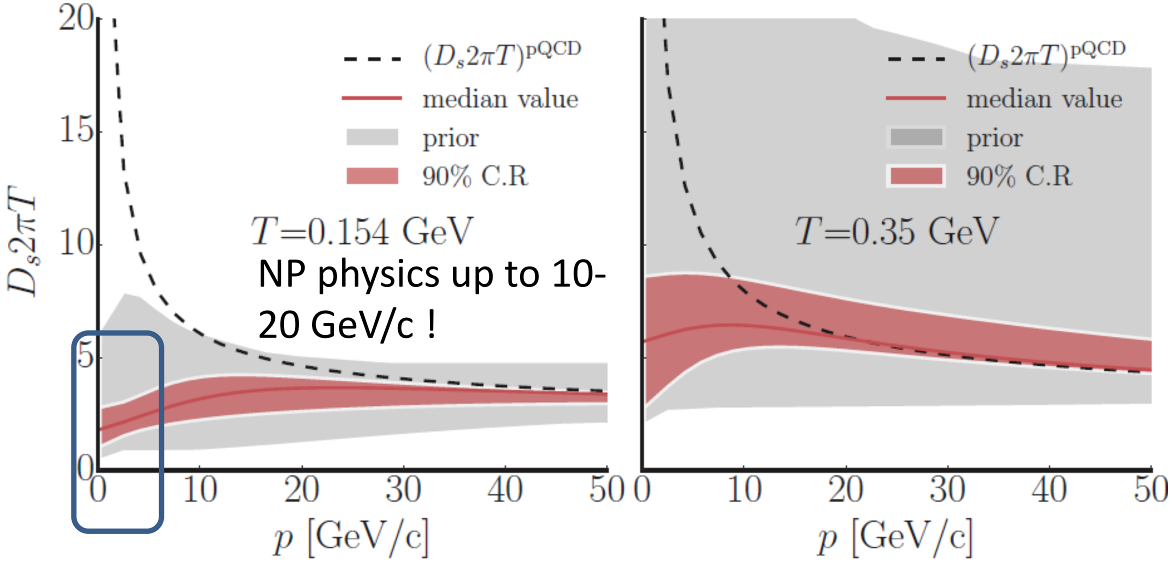


Rather small value
=> strong coupling !

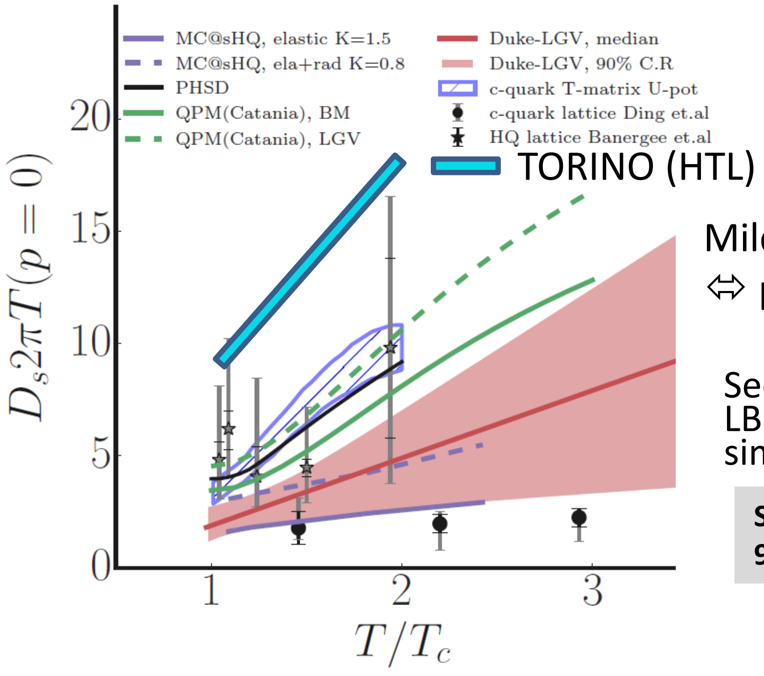
Broad distribution



Duke "Bayesian approach" vs models



NP physics up to 10-20 GeV/c !



TORINO (HTL)

Mild lin. increase of $2\pi D_s T$…

⇔ physics beyond pQCD.

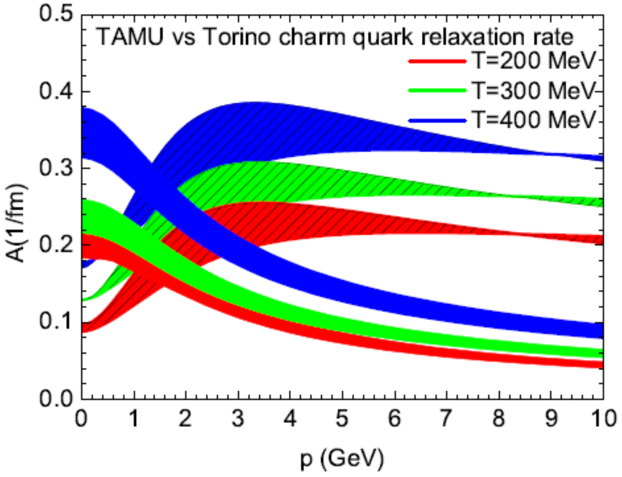
See as well analysis in the LBL-CCNU model with similar conclusions

S. Cao et al, Phys. Rev. C 94, 014909 (2016)

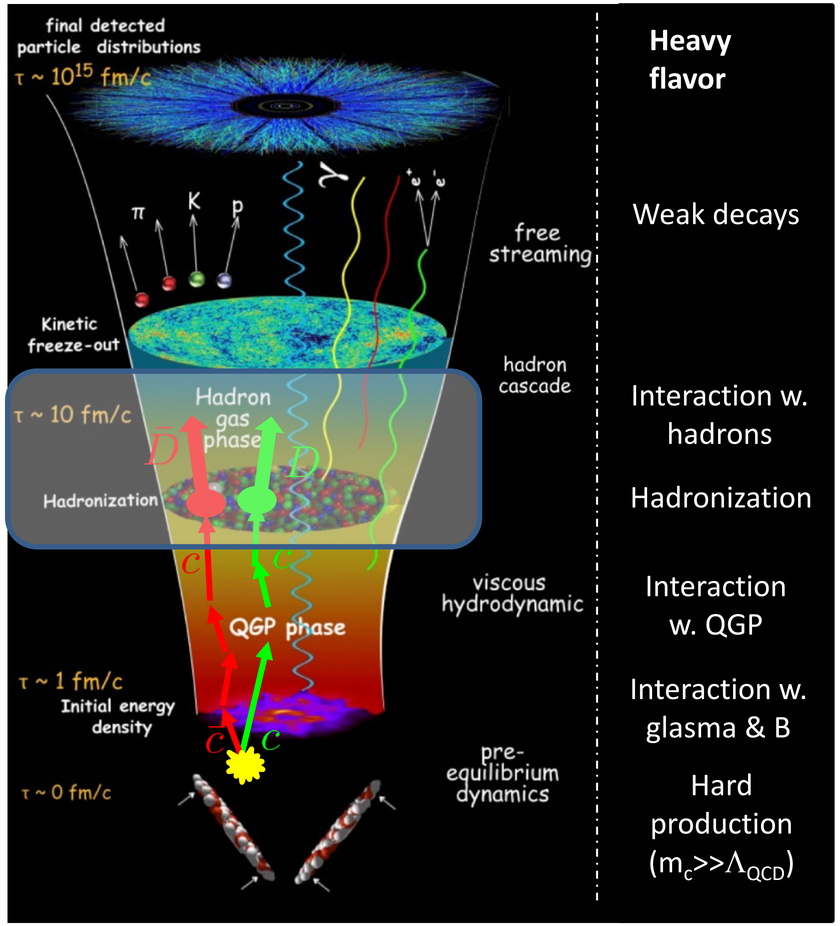
All together (lQCD, Bayesian analysis and most recent models) make a strong case for NP physics « around T_c » and at «low» p_T … needs to be precised in the future

- Does not mean that all models inhold the same physics…
- D_s (p=0) does not represent the full physics (different momentum dependences)
- D_s (finite p) in Duke's el + rad approach should not be compared to the same quantity in purely elastic models (additional contribution to energy loss due to the rad. part)

Prino and Rapp, J.Phys. G43 (2016), 093002



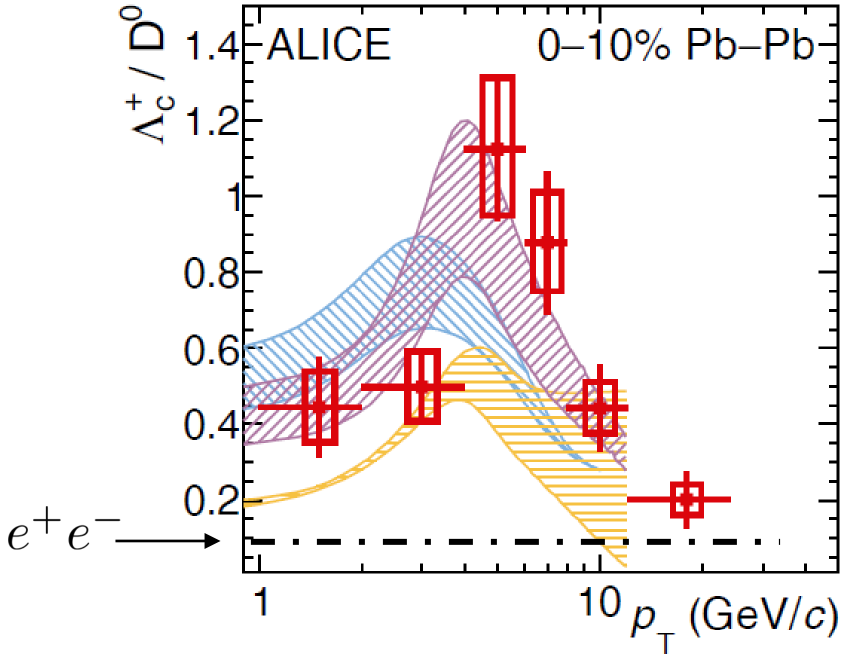
Heavy quarks as ideal hard probes:



- Evolution in the QGP DOES NOT modify the yield of initial Q and Qbar: Negligible annihilation rate !
- **It only impacts their distribution in momentum**
  - Hence, for usual observables like RAA and v2 … only the initial "1 body" distribution matters

Mostly like in elementary pp collisions…

- **… However hadronization is affected by the QGP**
  - Other mechanism wrt usual fragmentation of HQ in elementary collisions : **coalescence / recombination**



The recombination of heavy quark with some *existing* light quark(s) from the QGP is an essential mechanism at "low" $p_T < 5\text{-}10$ GeV/c…

Mandatory to understand the Λc/D^0 ratio

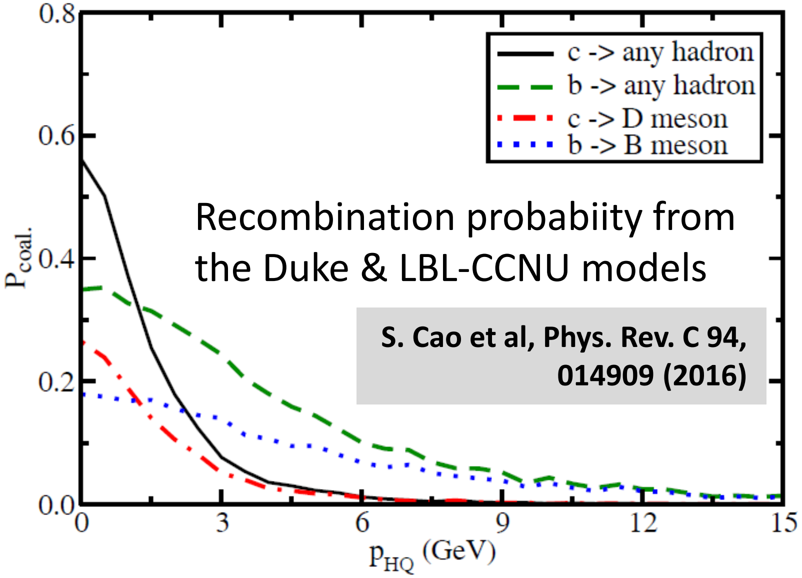
HQ-Hadronization (would deserve a full talk)

Acknowledged:

- towards the end of QGP, hadronization of (of equilibrium) HQ can proceed through a dual mechanism:

Low p_T :

- The quark partner(s) are already present in the hot cooling medium
- New specific recombination mechanism; no obvious calibration
- The footprint of reconfinment (?!)
- Crucial to explain the flow bump in $R_{AA}(D)$ and sizable $v_2(D)$ => large impact.



Recombination probabiity from the Duke & LBL-CCNU models

S. Cao et al, Phys. Rev. C 94, 014909 (2016)

High p_T :

- The quark partner(s) needed to create the HF-hadron have to be generated from the vacuum
- « usual » fragmentation calibrated on p+p and e^++e^- data (Petersen,…)

Uncertain (and not disputed enough):

- Genuine physical recombination process !

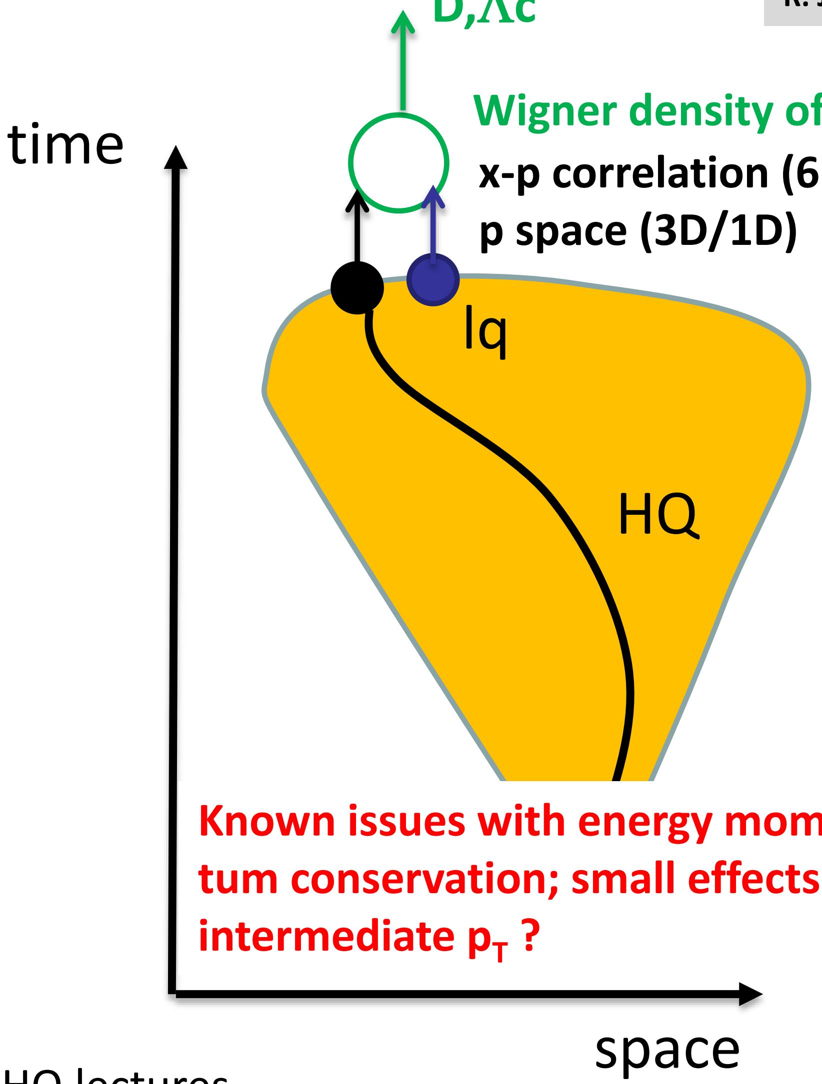
See Vincenzo Greco's plenary talk at SQM 2024 and Salvatore Plumari // talk

HQ - Recombination

Instantaneous coalescence:

Greco, Ko & Levai. Phys. Rev. C 68 (2003) 034904
V. Greco, C. Ko, and R. Rapp, Phys. Lett. B 595 (2004) 202
Y. Oh et al, Phys. Rev. C79 (2009) 044905
R. J. Fries, V. Greco, and P. Sorensen, Ann. Nucl. Part. Sci. 58 (2008) 177

S Plumari et al. arXiv:1712.00730



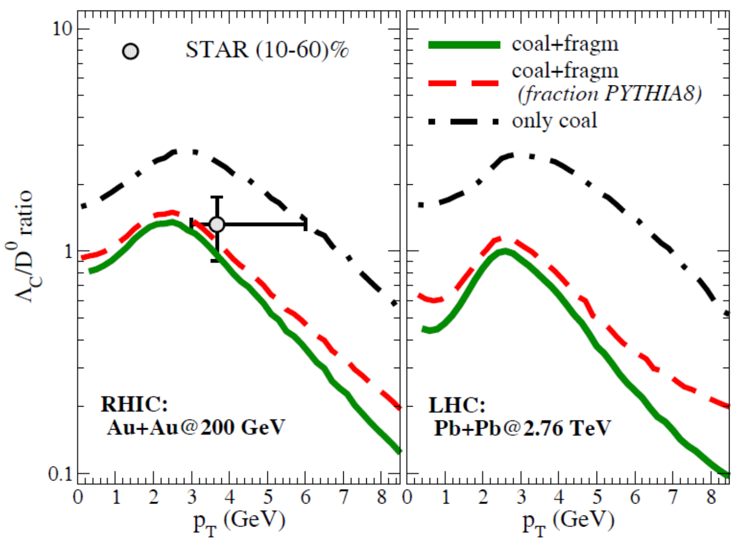
Wigner density of hadronic states

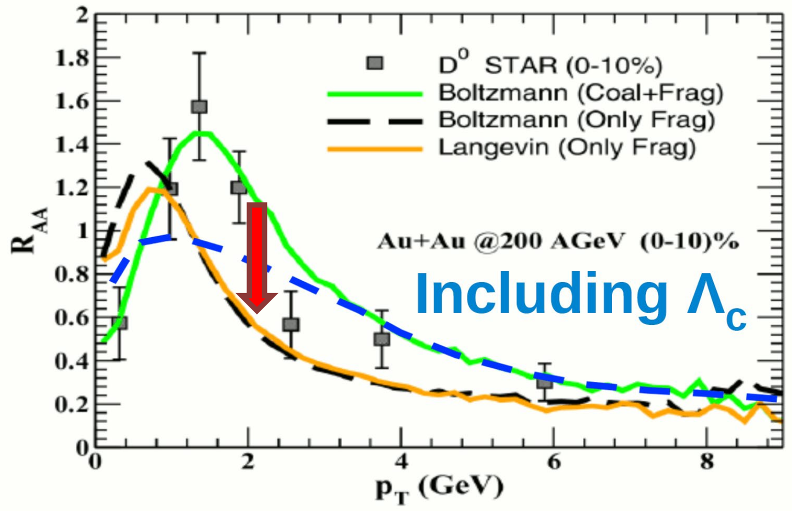
x-p correlation (6D) or « just » in p space (3D/1D)

Known issues with energy momentum conservation; small effects at intermediate p_T ?

Latest Catania's coalesence model:

- Full 6D coalescence
- New normalization to impose P_{coal} -> 1 for p_T->0
- Resonance decay
- Mini jet contribution
- Inclusion of Λ_c baryonic states
    - => reduction of R_{AA}(D) at small p_T
    - => increase of Λ_c/D^0 wrt pp and pPb.



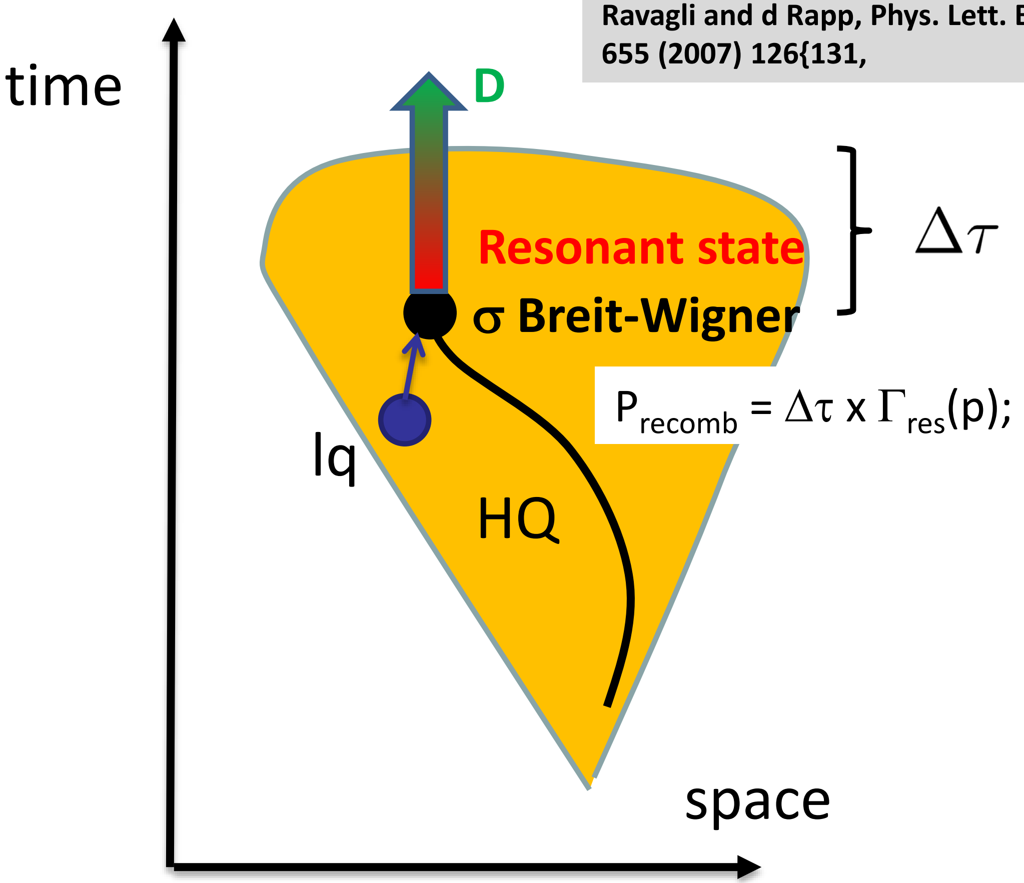


HQ - Recombination

Resonance Recombination Model:

$$\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)F_M(t,\vec{x},\vec{p}) = -\frac{\Gamma}{\gamma_p}F_M(t,\vec{x},\vec{p})+\beta(\vec{x},\vec{p})$$

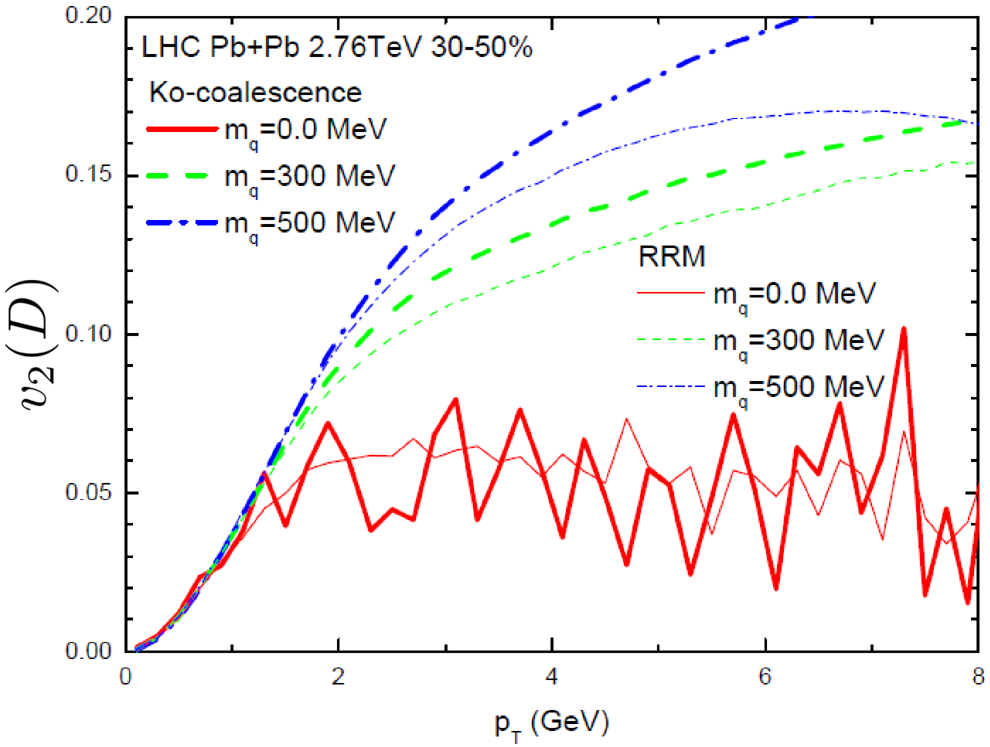
Ravagli and d Rapp, Phys. Lett. B 655 (2007) 126{131,



$P_{recomb} = \Delta\tau \times \Gamma_{res}(p);$

- Dynamical 2->1 process, implemented in the asymptotic limit of the kinetic equation
- A possible way to solve energy-momentum conservation
- Process governed by the interaction of HQ with QGP around T_c => natural link with the energy loss model.

In EMMI RRTF, comparison between Instant. Coal. & RRM



starting from the same bulk and from the same c spectrum

Significant differences found both for D meson p_T spectrum and v_2.

R. Rapp et al, arXiv: 1803.03824

EMMI RRTF : Consequences from various **Hadronization Mechanisms**

We define and display the H_{AA} quantity

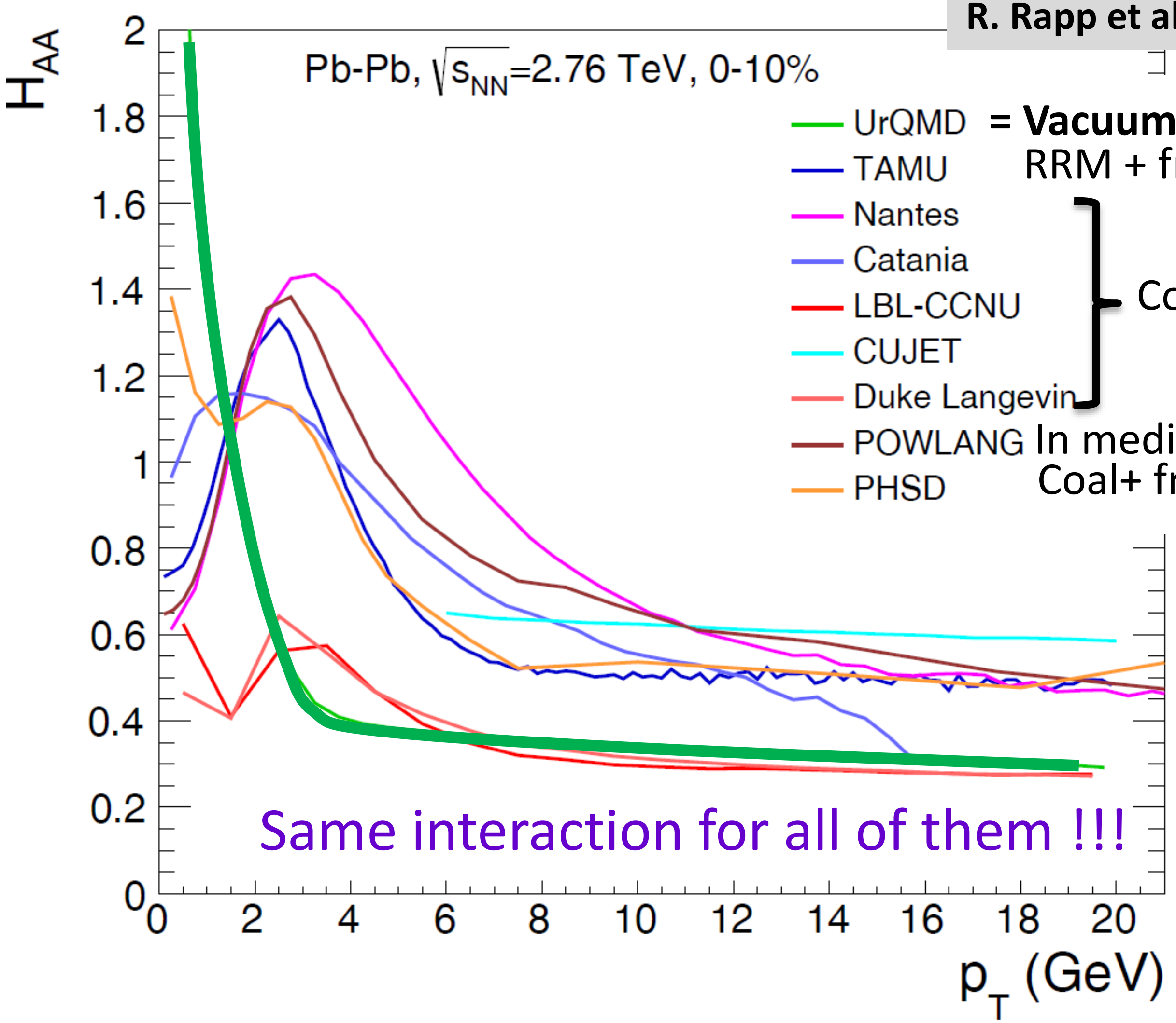
$$H_{AA} = \frac{\frac{dN_D}{dp_T}}{\frac{dN_{c\ \mathrm{final}}}{dp_T}}$$

…which exhibits at best the specific effects of hadronization :

Significant uncertainties !

=> Yes, one can for sure put more constrains with D_s and Λ_c, but probably one has also to converge on more robust schemes for « basic » D mesons

R. Rapp et al, arXiv: 1803.03824



= Vacuum = Fragmentation

RRM + frag

Coal + frag

In medium frag

Coal+ frag

Same interaction for all of them !!!

Recombination

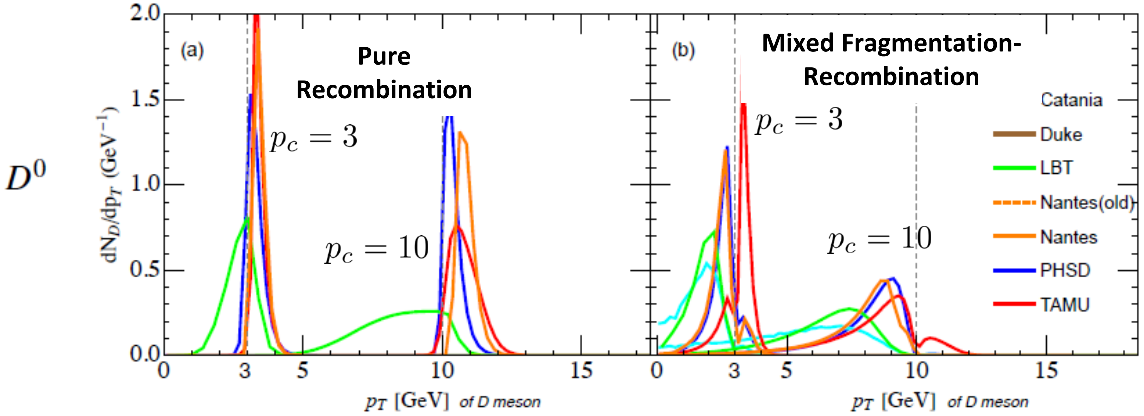
Fragmentation

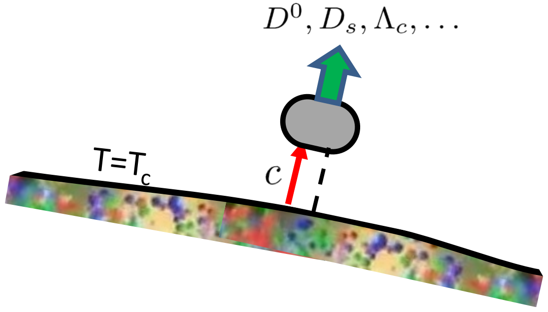
Hadronization of heavy quarks

- **Recent effort of theorists to compare their hadronization schemes at the end of the QGP**

dN_D/dp_T of the direct D^0 meson produced by a c-quark with p_T = 3 GeV *and* 10 GeV

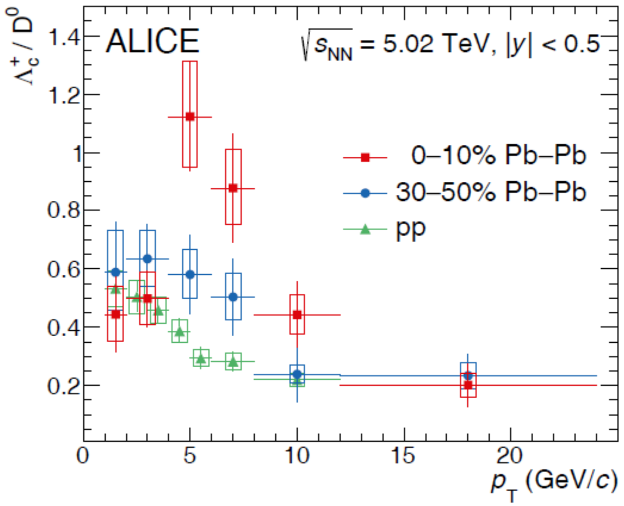
Jiaxing Zhao et al., 2311.10621

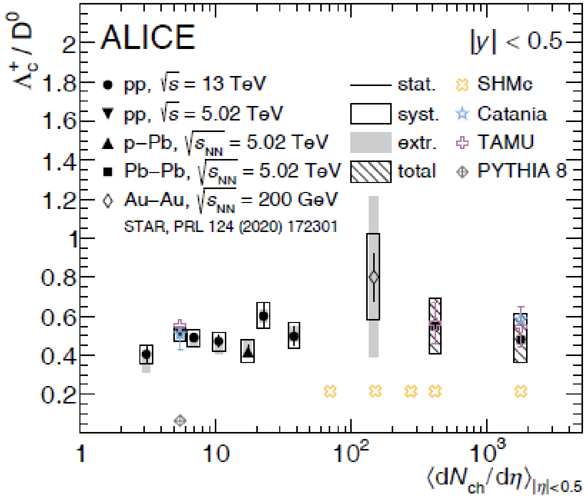


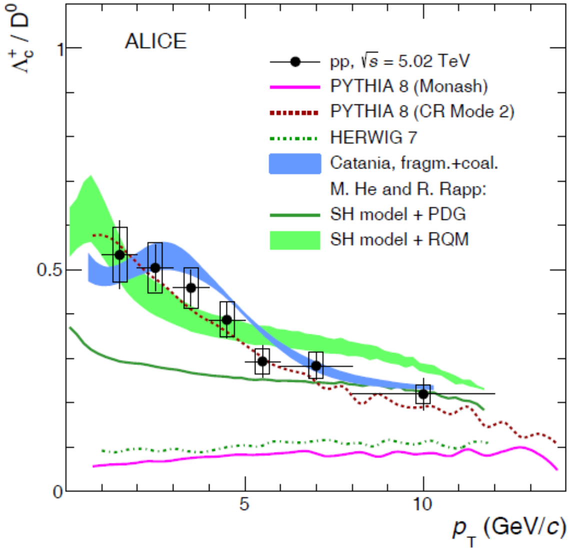


- Diversity => things to learn ! … Hadrochemistry of Heavy Flavor will be a major subject of investigation for ALICE 3.

- But also in small systems like pp (many signs of **collectivity in small systems => QGP ?**)





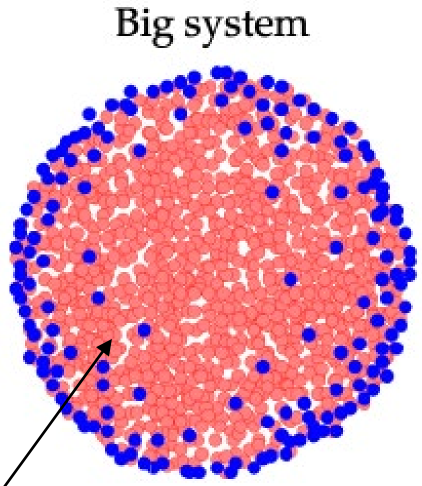


Small systems: The EPOS-HQ approach

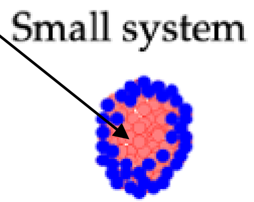
Heavy flavor as a probe of hot QCD matter produced in proton-proton collisions

Jiaxing Zhao, Joerg Aichelin, Pol Bernard Gossiaux, and Klaus Werner
Phys. Rev. D **109**, 054011 – Published 6 March 2024

EPOS4 : Simple but efficient initial-stage: Core - Corona picture

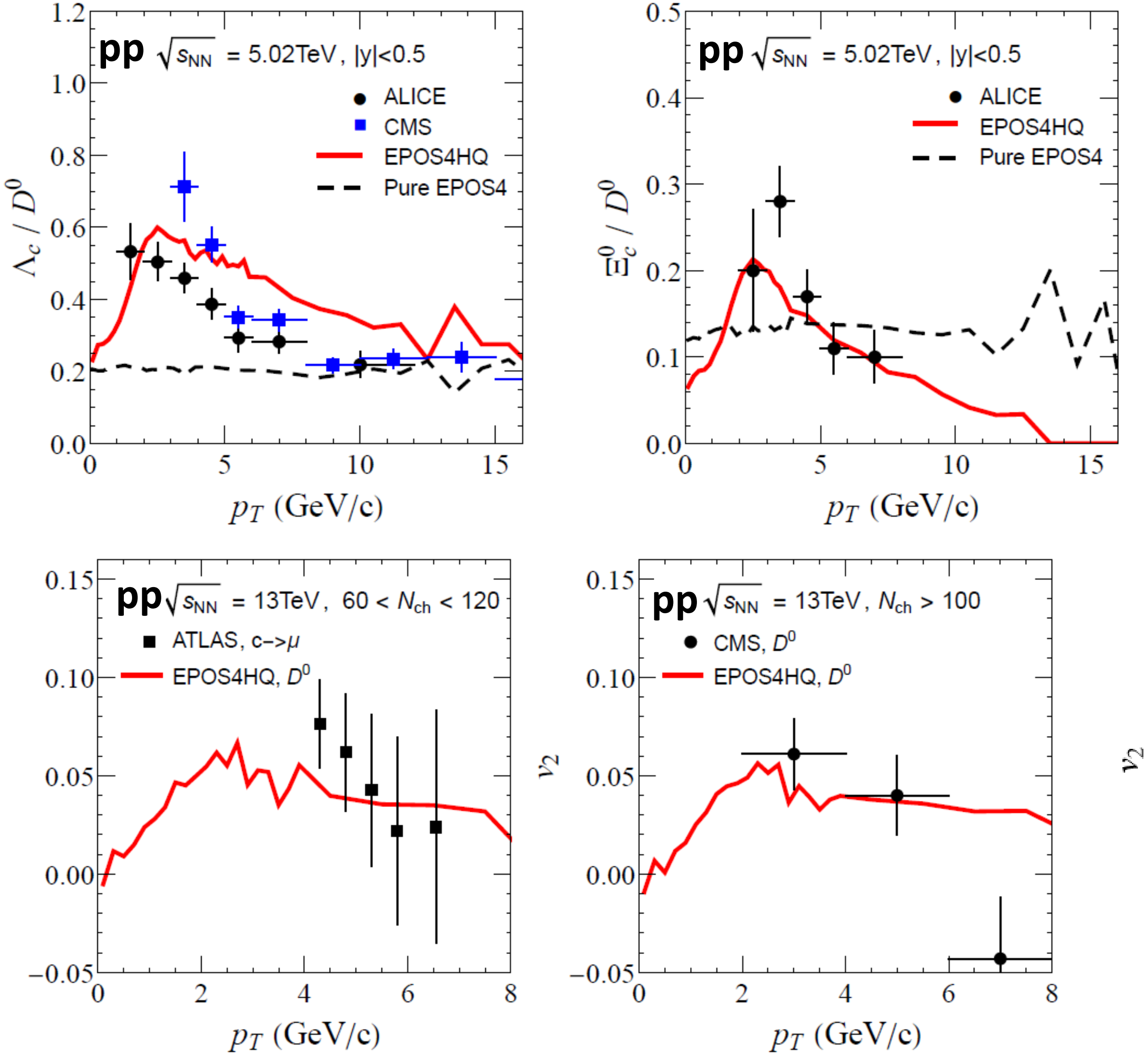


Core (red): further evolves according to fluid dynamics



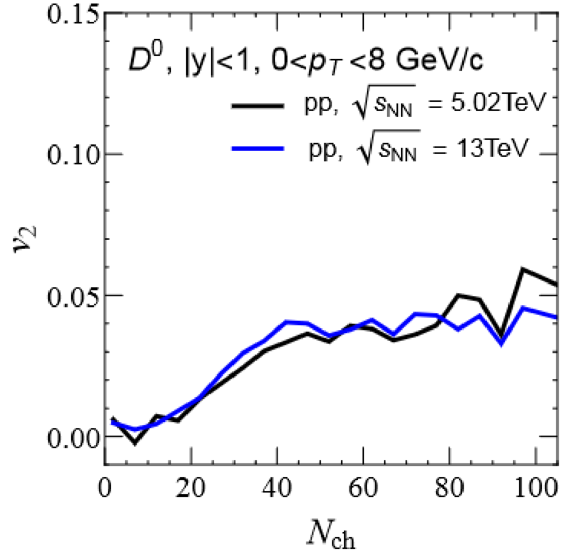
Corona (blue): further evolves as in vacuum

Propagating HQ in the fluid medium of EPOS (see talk PBG @ SQM 2024)



The coalescence + fragmentation hadronization is successful in describing the yield ratio between charmed baryon to meson !

EPOS4-HQ describes well the elliptic flow of D meson ! Sign of momentum redistribution during the short evolution: **collectivity**



See lecture by K Werner on Friday

Alternate observables

Key message:

Usual “1 body” observables like R_{AA} and v_2 do not allow a full discrimination between the models

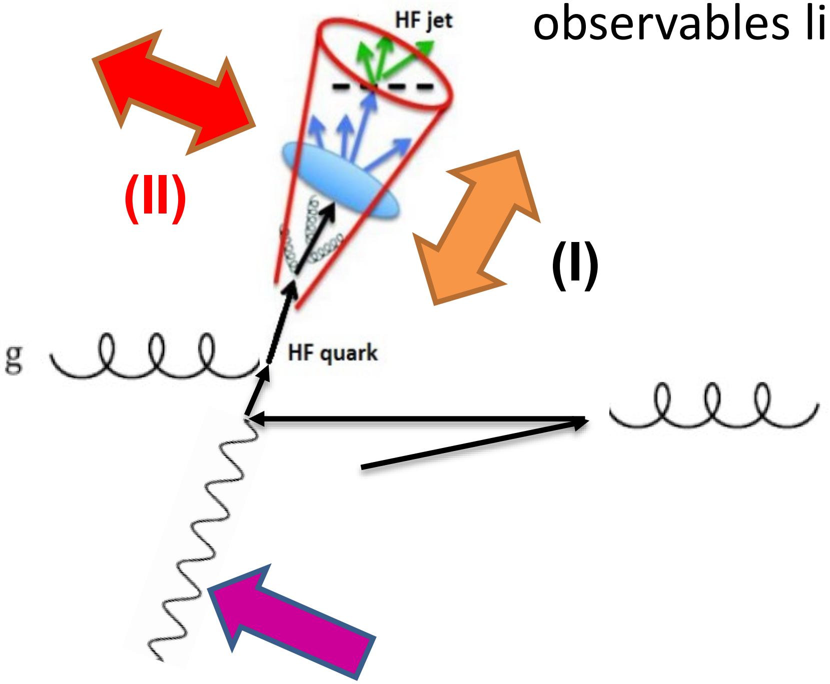
Many models can reproduce these observables, sometimes at the price of some tuning or extra ingredient

One needs to turn to **more exclusive observables that will to better allow discriminating the various aspects of the HQ “energy loss”.**

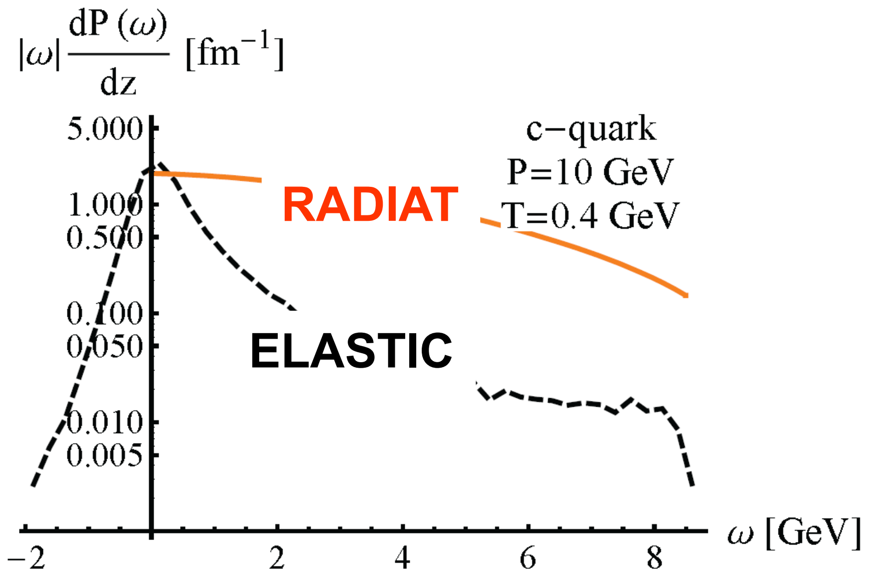
γ - b/c jet: Best HF Correlation ever ?

- γ – D/B/c jet /b jet:

In QGP: **Longitudinal and transverse (qhat) fluctuations** of the HQ, which crucially depend on the Eloss mechanism and cannot be measured in usual observables like R_{AA} or v_2



No E loss => perfect probe of initial $\vec{P}_{HQ}$

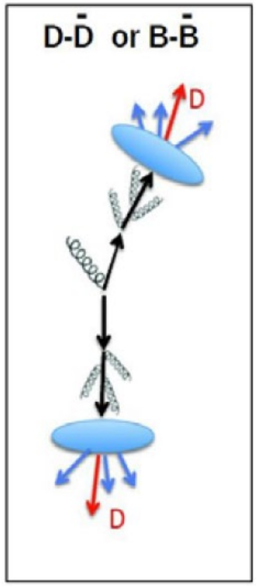


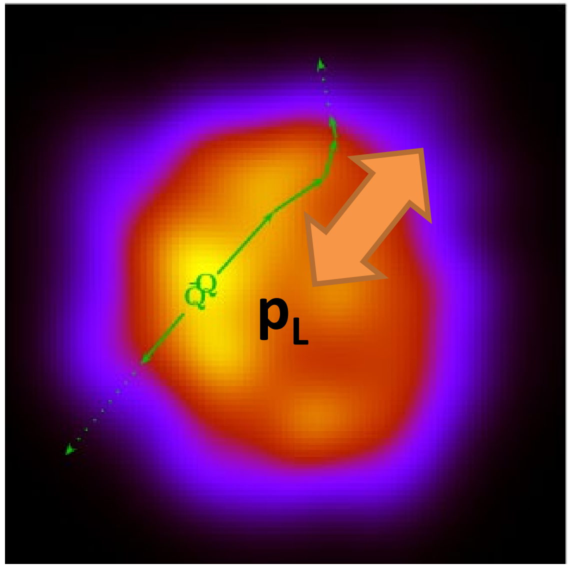
differential probability to loose energy ω per unit time

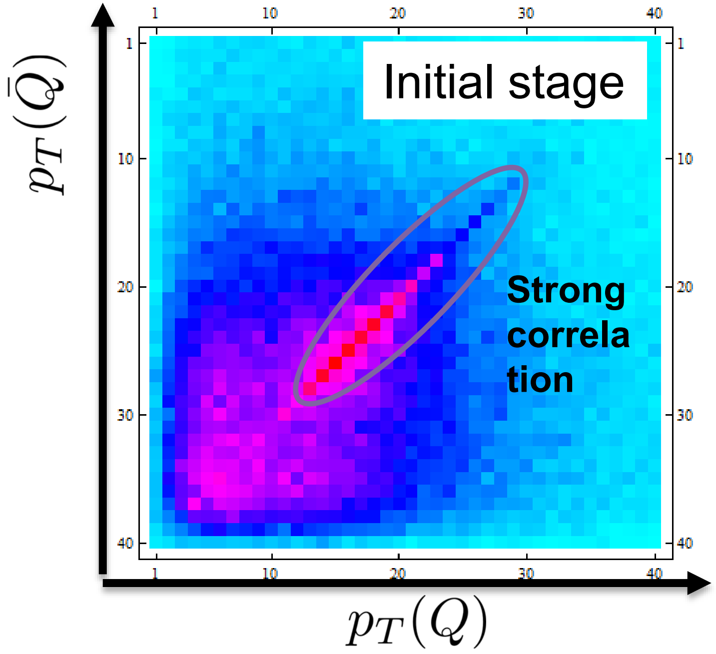
- Of course: NLO effect in the production mechanisms makes it not so trivial (not to speak about exp. Issues… RUN3 ? RUN4 ?)

Next best thing: HF-HF correlations

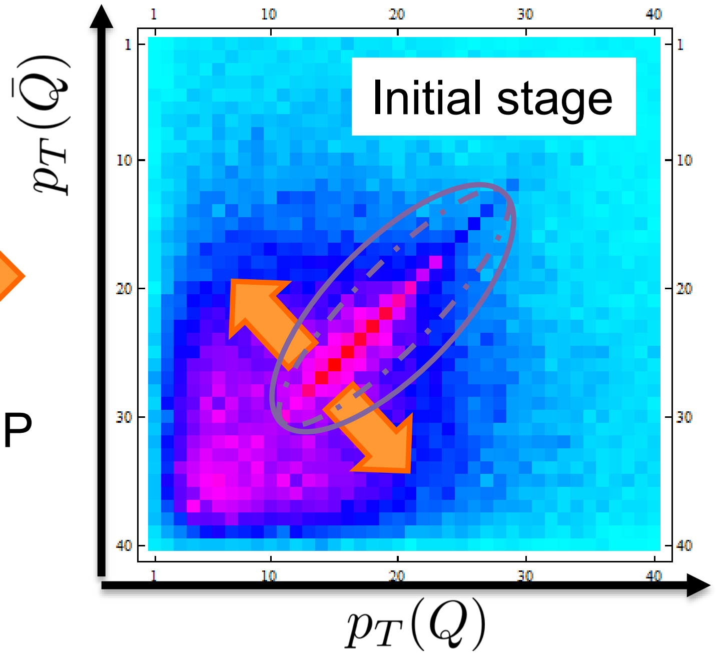
- Back to back D/Dbar or B/Bbar: As compared to γ-D/B: “triggering” itself is affected but symmetry between both particles could limitate the various effects:
- Large number of c-cbar from various NN collisions => large uncorrelated background
- Competing effects due to energy loss: …







Evolution in hot QGP medium

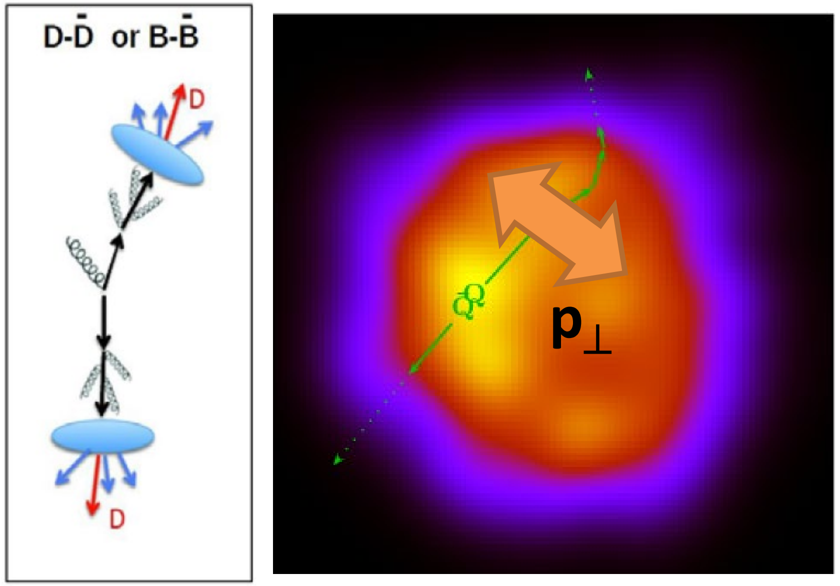


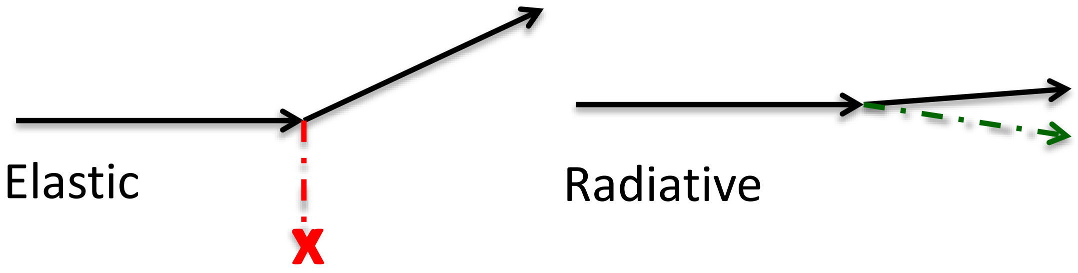
- decorrelation due to various path lengths + fluctuations: reduction

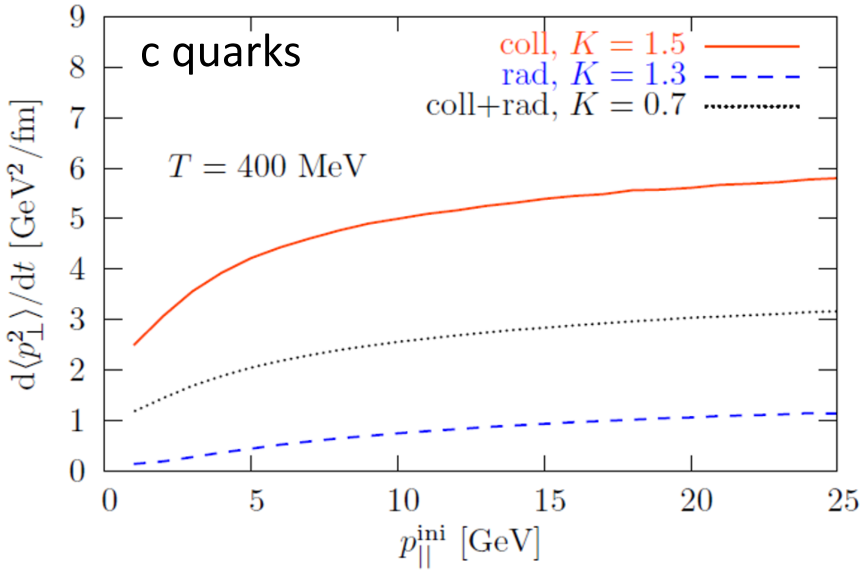
Next best thing: HF-HF correlations

- Back to back D/Dbar or B/Bbar: As compared to γ-D/B: “trigger” itself is affected but symmetry between both particles limitates the various effects.

- Elastic Eloss vs radiative Eloss: **The purely collisional scatterings lead to a larger average $\langle p_\perp{}^2 \rangle$ then the radiative “corrections”** (need for large scattering to be efficient)… although both types can give correct agreement with the data at intermediate p_T.







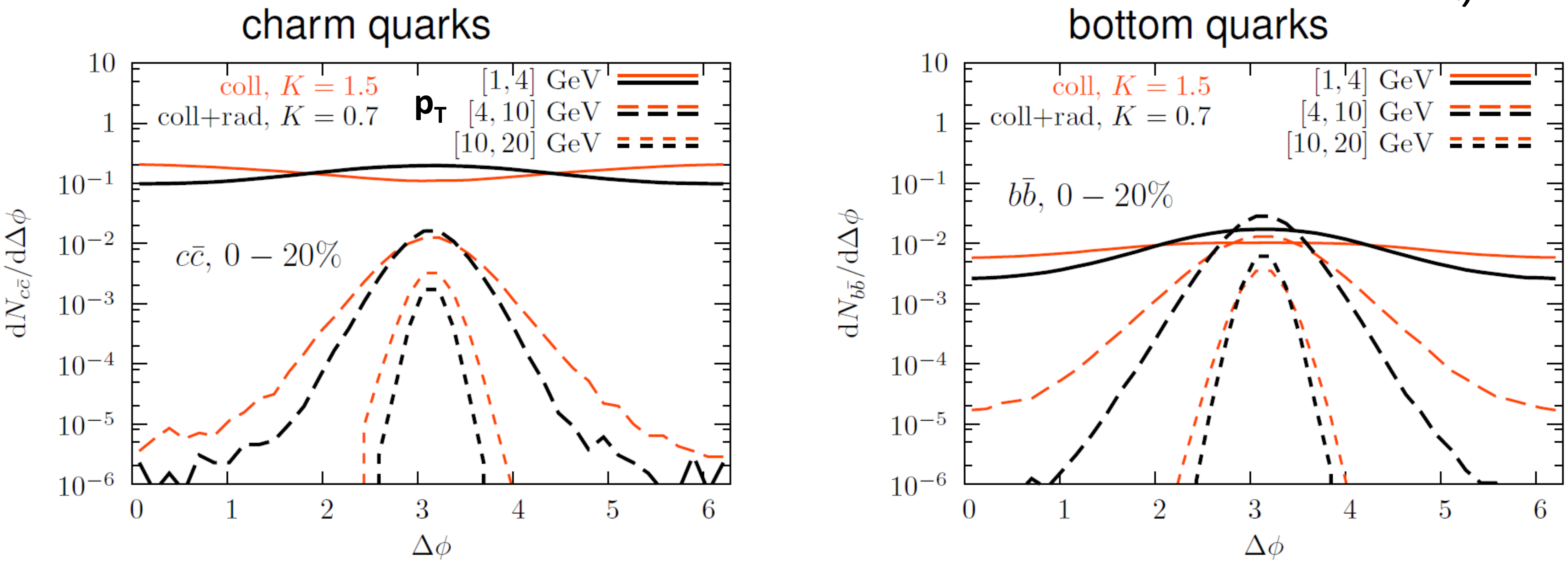
Tuned to reproduce the R_{AA}

- Expected consequences for azimuthal correlations (probe of B_T: good: **complimentary** to usual RAA and v2)

Next best thing: azimuthal correlations

- Assumption of back 2 back emission of initial QQbar (naïve LO…)

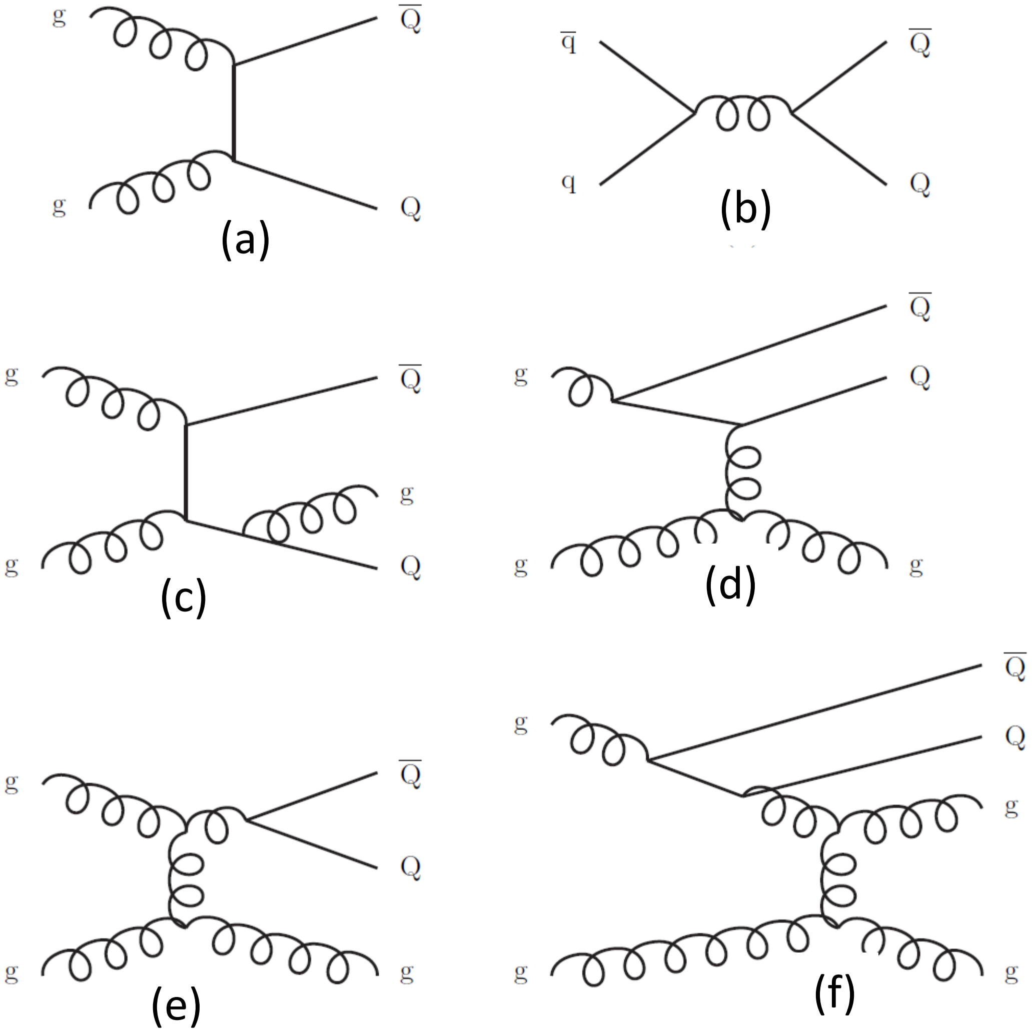
Nahrgang et al. Phys. Rev. C 90, 024907 (2014)



- Indeed, rather large differences found for both b and c, and all kind of p_T cuts (… but good to see there is an effect though,…)
- For the smallest p_T bin and elastic energy loss, we even find an inversion of the correlation (“hot partonic wind” push; v_0 bulk => v_1 correl; underlying event)

Next best thing: azimuthal correlations

- …but higher orders can have a significant impact:



- LO; (a): back to back peak

- NLO;

(c): “blurring” of B2B peak

(d): “flavor excitation”: no strong azimuthal correlation expected

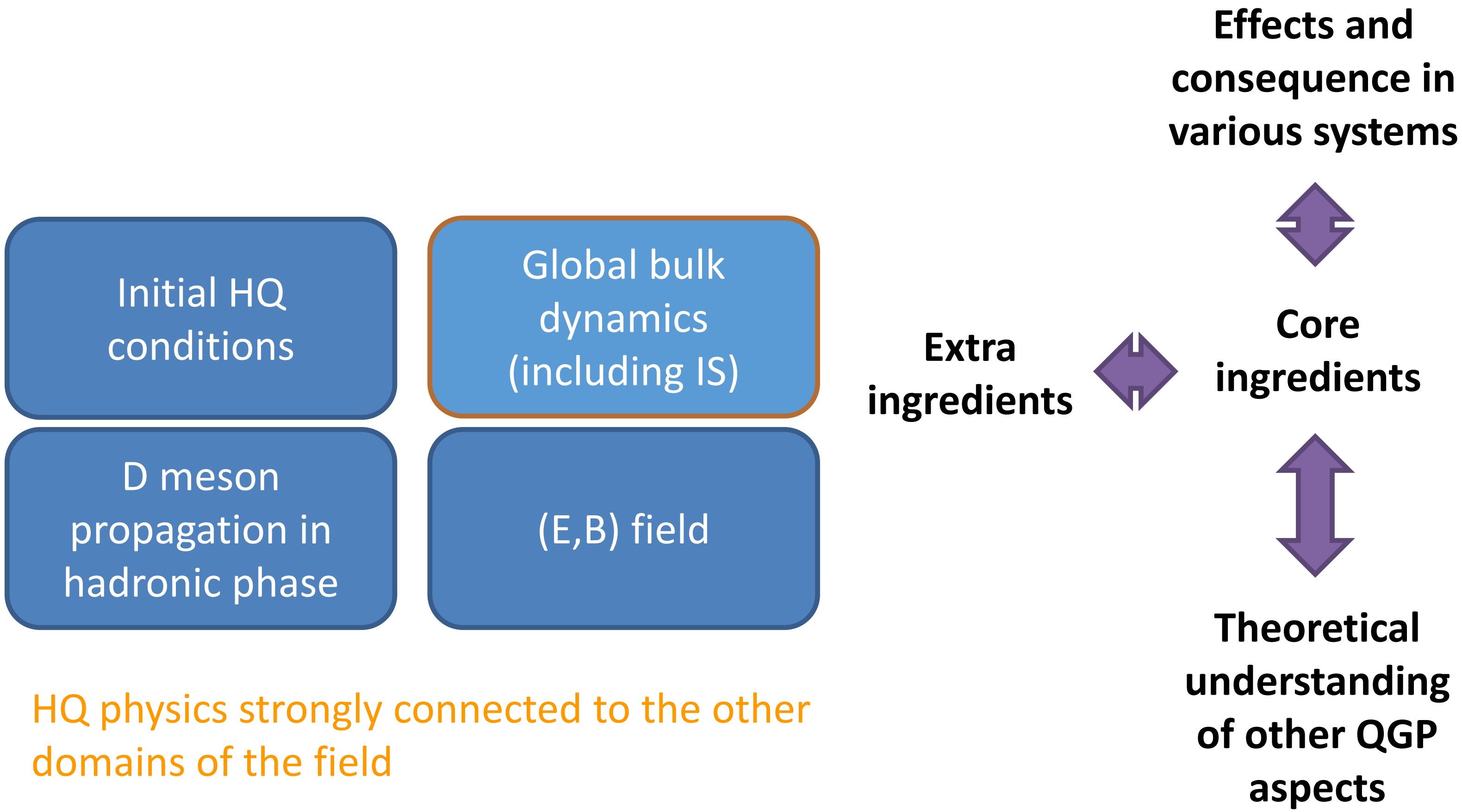
(e): gluon splitting: strong peak around $\Delta\phi=0$

(f): higher order FE; both Q and Qbar in the “remnant” region

Evolution of 2-body correlation in the QGP is a better probe of the HQ – QGP coupling !!!

A bit of structure

- HQ propagation in QM & URHIC…

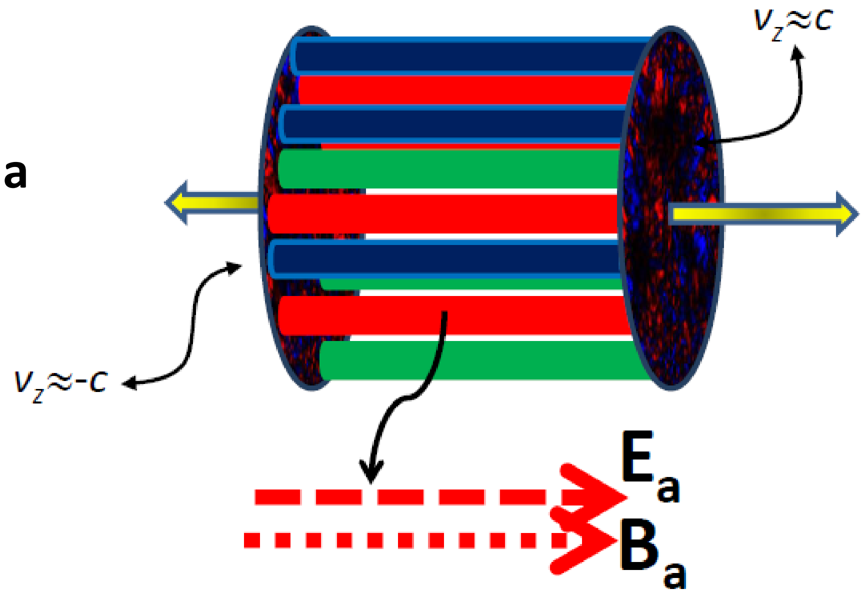


HQ physics strongly connected to the other domains of the field

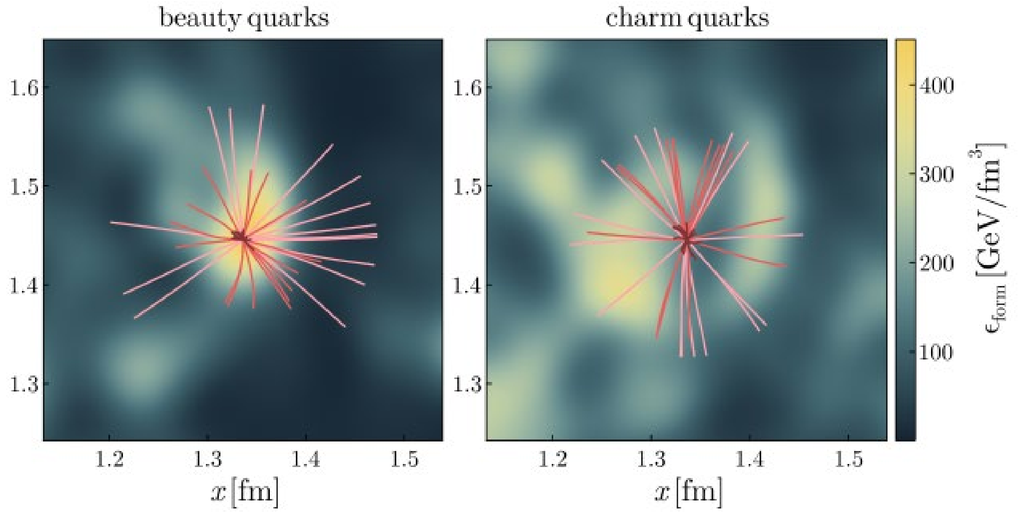
Early stage evolution of HQ in the Glasma phase

- Diffusion of heavy quarks in the early stages of high energy nuclear collisions

Early stage : **Glasma**



D. Avramescu et al, Phys. Rev. D 107 (2023), 114021



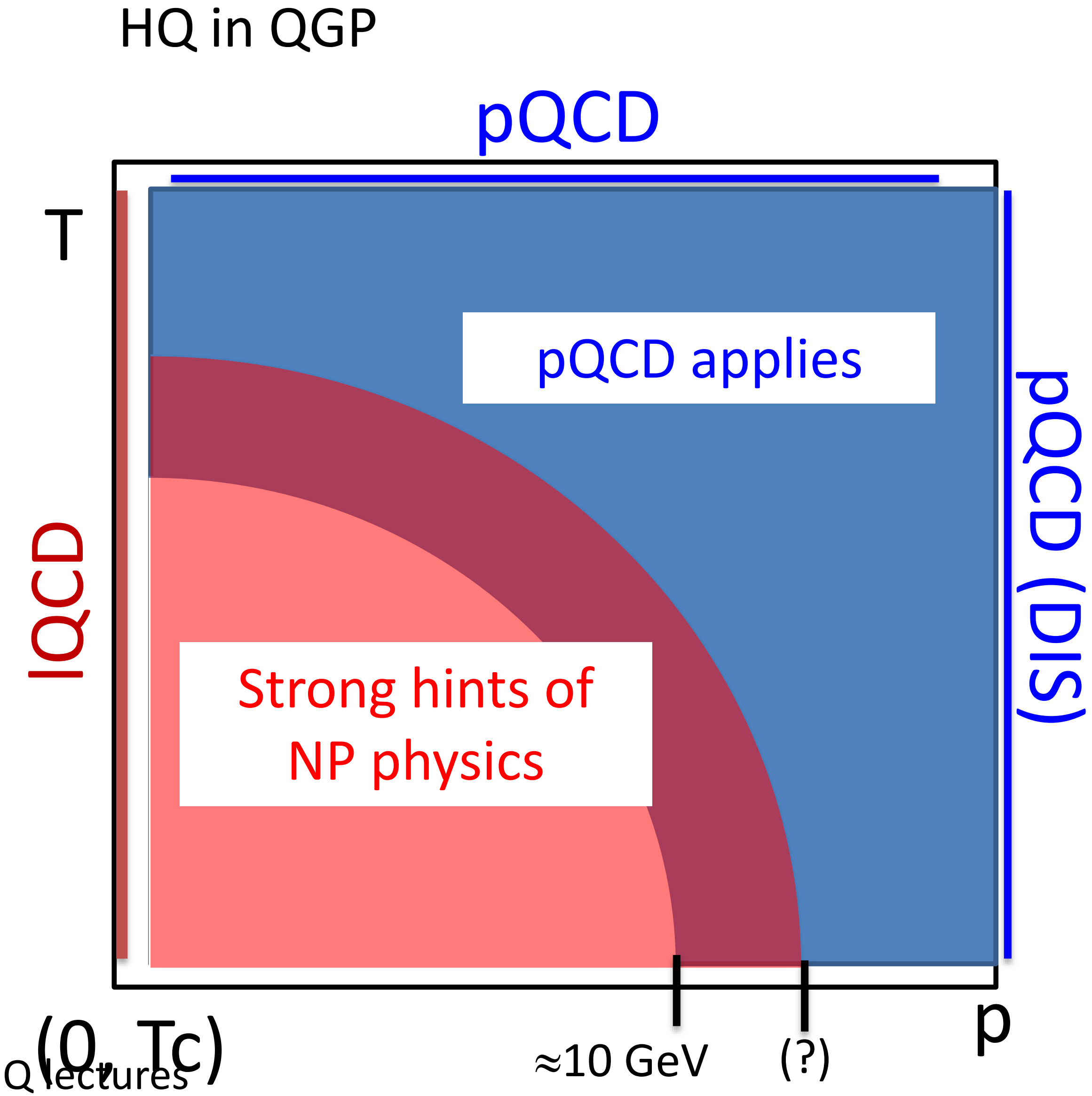
trajectories of heavy quarks propagating in a single Glasma flux tube

- Diffusion of HQs in the early stage of high energy collisions is affected by the strong fields: coherence memory effects are substantial.

See as well: “Simulating Charm Quarks in IP-Glasma Initial Stage and Quark-Gluon Plasma: A **Hybrid** Approach for charm quark phenomenology”, Manu Kurian @ SQM2024

- But Magnetic fields could influence the production as well : “Heavy flavor production under a strong magnetic field”, Shile Chen @ SQM 2024

Conclusions for Open Heavy Flavors



- Existing models offer the possibility to describe most of the OHF experimental AA data while being compatible with existing theory constrains…
- … however with unequal precision and no consensus on the physical NP content
- Improvements and quantitative understanding is on their way, but it will still take some time and a lot of efforts => need for ressources, bright people and collective work.
- Open Heavy Flavors are maybe not an ideal probe of QGP yet, but they are quite fascinating and offer bright future for the field, with multiple interconnections.

Quarkonia…

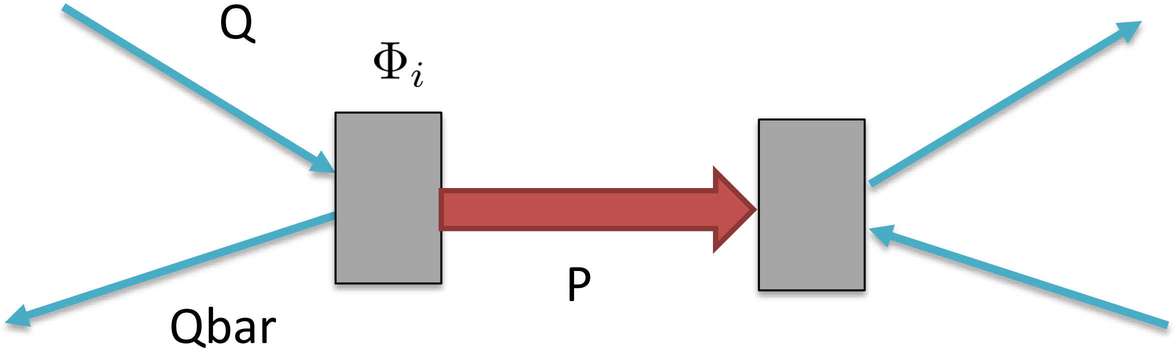
- From one to 2-body problem (2 HQ) =>
- Richer, more complex,…

What is a quarkonia ?

What is a quarkonium ? Bound state of Q and Qbar quarks

Bound states in QFT ? $\hat{H}|\Psi\rangle = E\Psi\rangle\,?$

More convenient : analyse the Q-Qbar scattering amplitude S_2 for a given total 4-momentum P



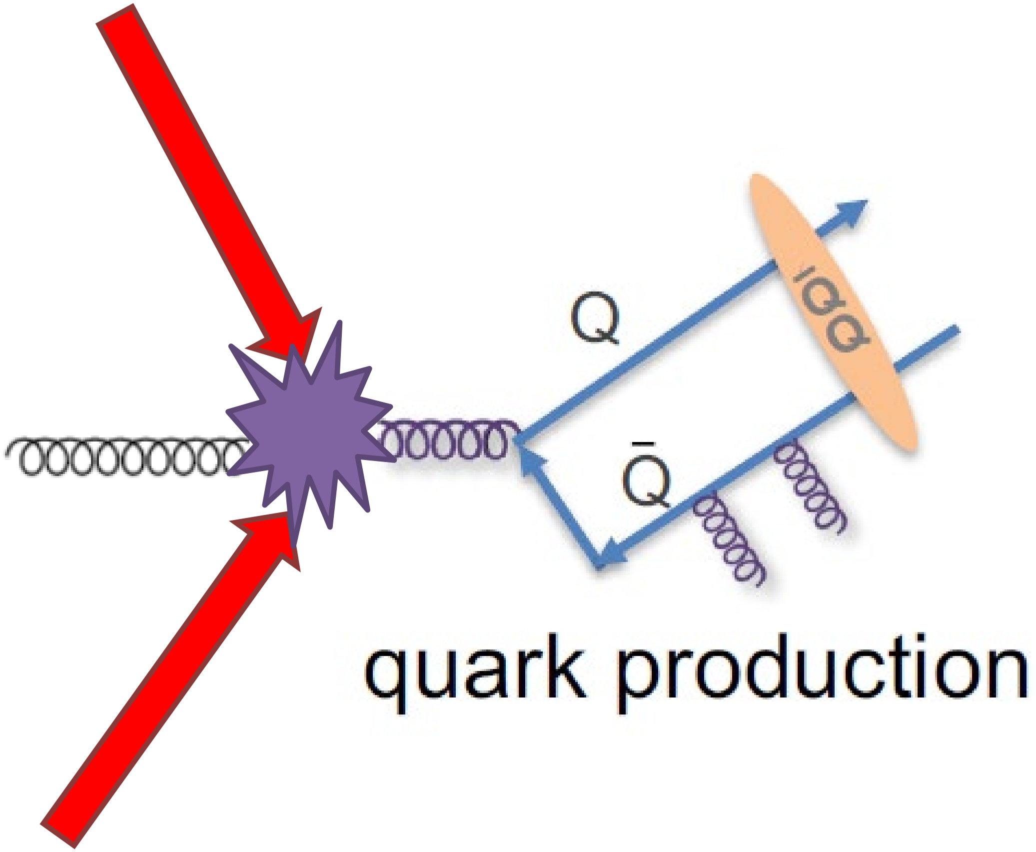
And decompose it as

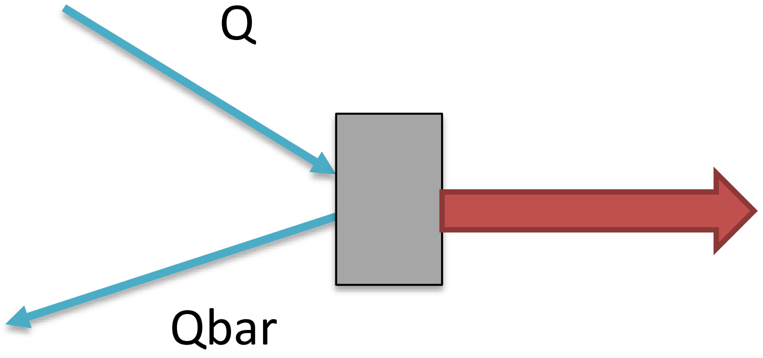
$$S_2 = \sum_i |\Phi_i\rangle \frac{1}{P^2 - M_i^2} \langle\Phi_i| + \underbrace{\cdots}_{\text{continuum}}$$

Then :
- M_i : bound state mass
- Φ_i (Bethe – Salpether) amplitude for Q-Qbar -> bound state transition

What is a quarkonia ?

This amplitude can thus be used to evaluate S – matrix and cross section for bound state formation in collisions





What is a quarkonia ?

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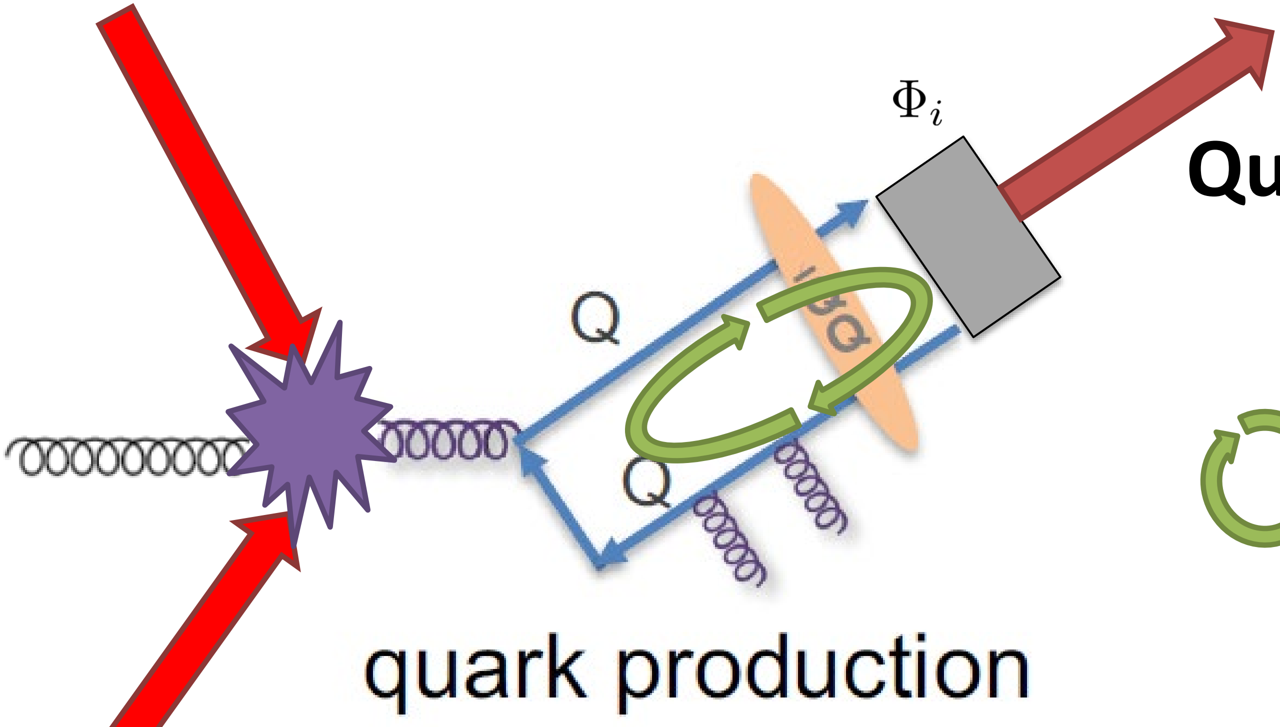


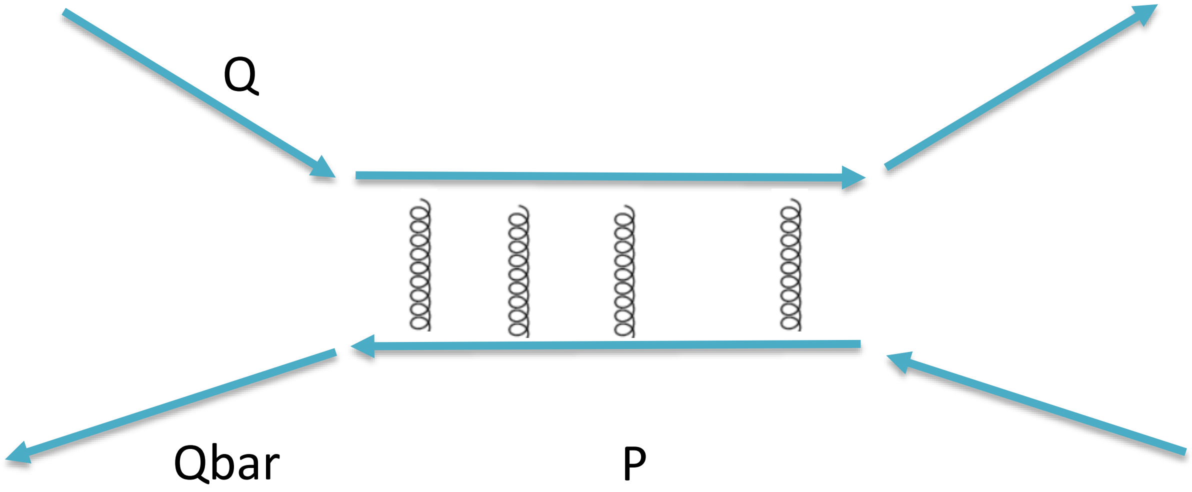
Quarkonium production

↻ Loop integral on the 4-relative momentum

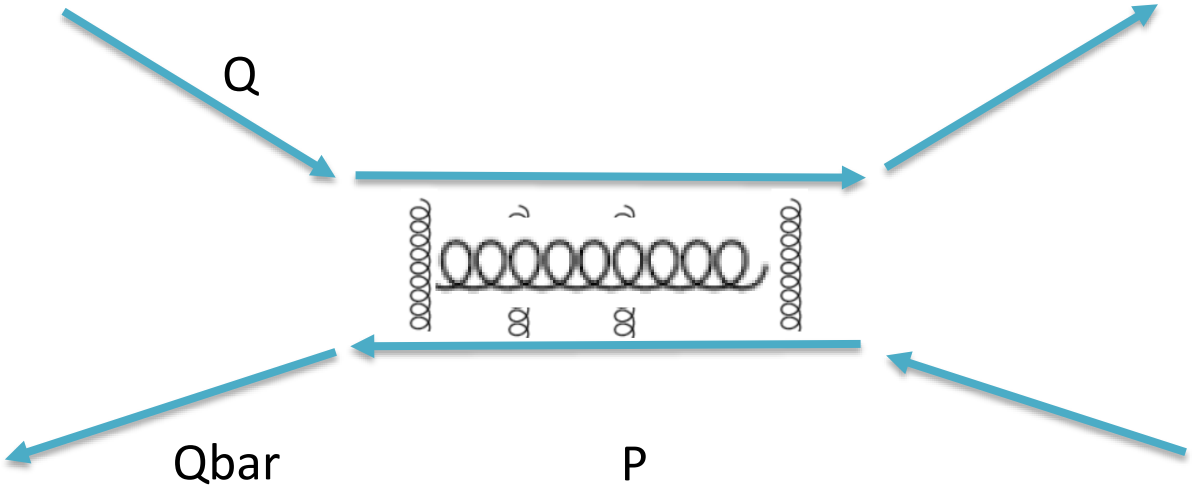
$$\approx \langle \Phi_i | Q\bar{Q} \rangle$$

What is a quarkonia ?

Evaluating the scattering amplitude from first principles:



« Easy »; potential like



« complicated »

One has to pay the price for QFT: the number of quanta at given « time » is not fixed:

$$|\Psi\rangle = |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + |Q\bar{Q}q\bar{q}\rangle \cdots \Rightarrow$$

Possible strategies :
- ressumation
- effective theories
- models

What is a quarkonia ?

$|\Psi\rangle = |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + |Q\bar{Q}q\bar{q}\rangle \cdots$ ⇨ Possible strategies :

- ressumation
- effective theories NRQCD
- models

Benefit from $m_Q >> \Lambda_{QCD}$: small velocities

$$m_Q \gg \Lambda_{QCD} \;\; \& \;\; m_Q \gg m_Q v \sim |\mathbf{p}_{rel}| \gg m_Q v^2 \sim E_{bind}$$

1. Integrate out the momentum scales above mQ => Non relativistic description => no more g -> Q+Qbar vertex in diagrams (but contact terms) => NRQCD

2. Integrate scales ≈ m_Q v => eliminates on shell gluon propagation => potential like

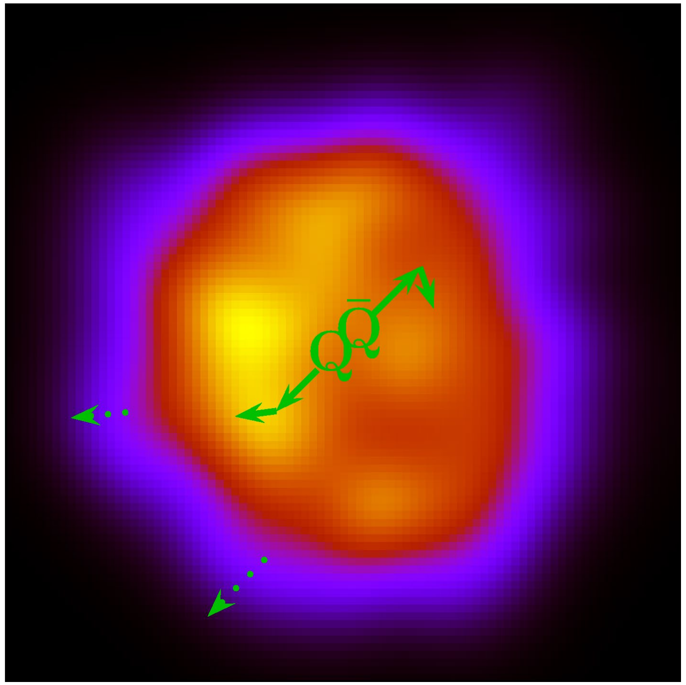
pNRQCD

$$L_{\mathrm{pNRQCD}} = L^{\mathrm{US}}_{\mathrm{NRQCD}} + L_{\mathrm{pot}}$$

Projection on the Q-Qbar sector => singlet and octet states

$$L_{\mathrm{pot}} = -\int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \; \psi^\dagger(t,\mathbf{x}_1)\chi(t,\mathbf{x}_2) \; V(\mathbf{r},\mathbf{p}_1,\mathbf{p}_2,\mathbf{S}_1,\mathbf{S}_2) \; \chi^\dagger(t,\mathbf{x}_2)\psi(t,\mathbf{x}_1)$$

What is a quarkonia… in a hot QGP medium ?



Answer may vary depending on how hot is the QGP, and how long you observe

Not to high T, not too long : Same as in vacuum + some external perturbation

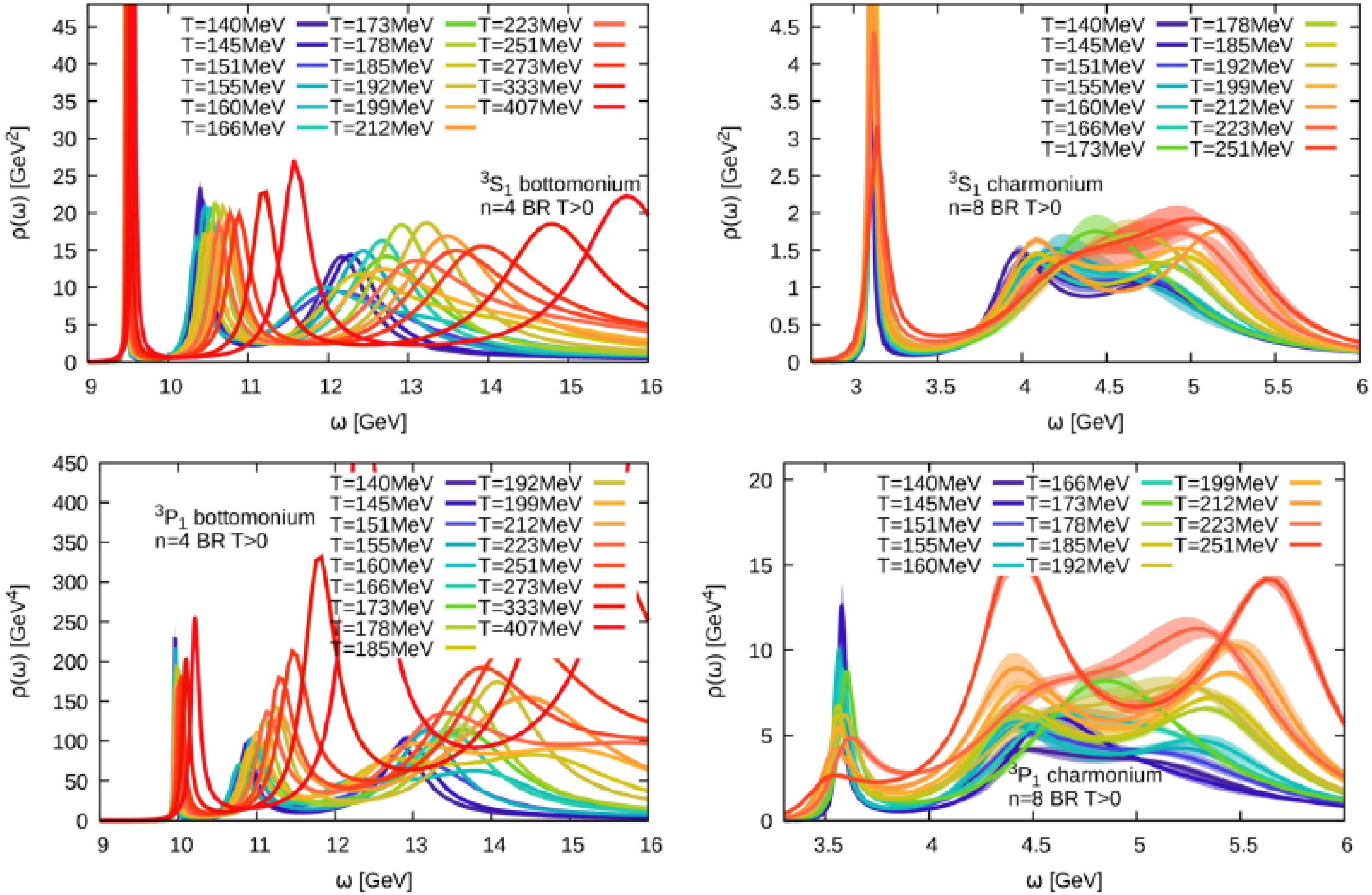
If not : probably better to speak a $Q\bar{Q}$ pair

When is it legitimate to speak of a bound state ?... And deal with it as such in the transport theory. Answer may vary depending on the fundamental ingredients

See as well: plenary talk by Jiaxing Zhao @SQM 2024

lQCD perspective : spectral function

Kim et al, JHEP11(2018)088



Many such kind of results in the literature

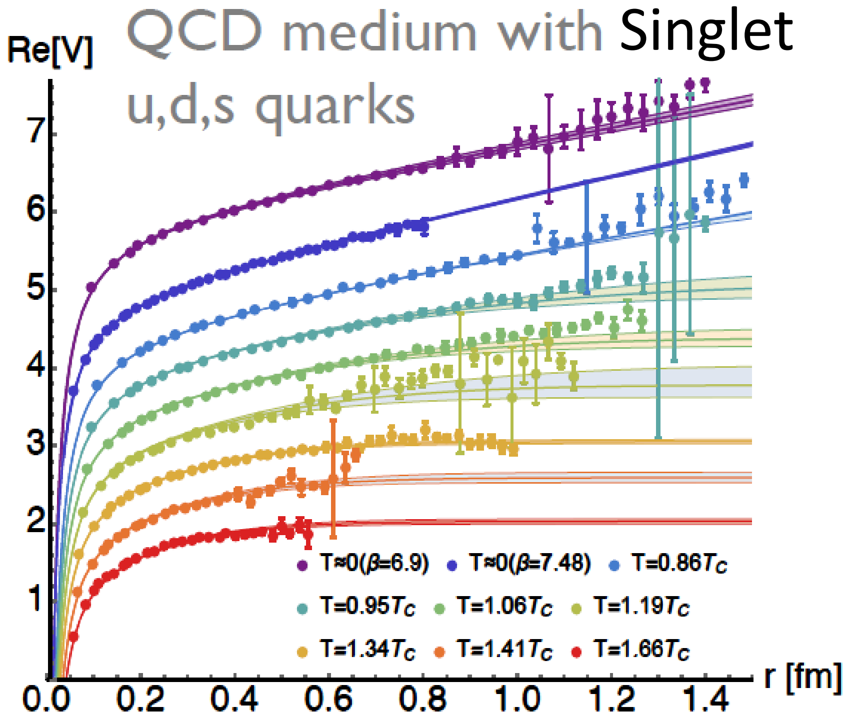
Rich structure : broadening and mass shift. What are the underlying “ingredients’” ?

The 3 pillars of quarkonia production in AA

Implicitly in the pNRQD EFT.

Screening of the real potential

Potential (recent lQCD calculations)



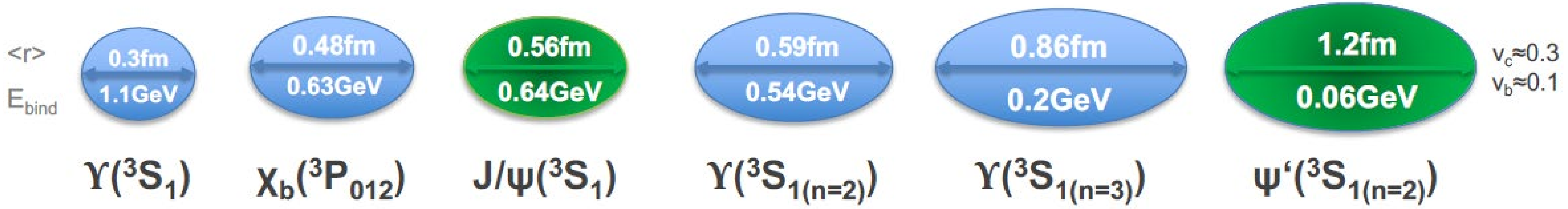
At T=0, well described by the Cornell shape:

$$V(r) = -\frac{\alpha}{r} + Kr$$

<u>Quarkonia scales</u>

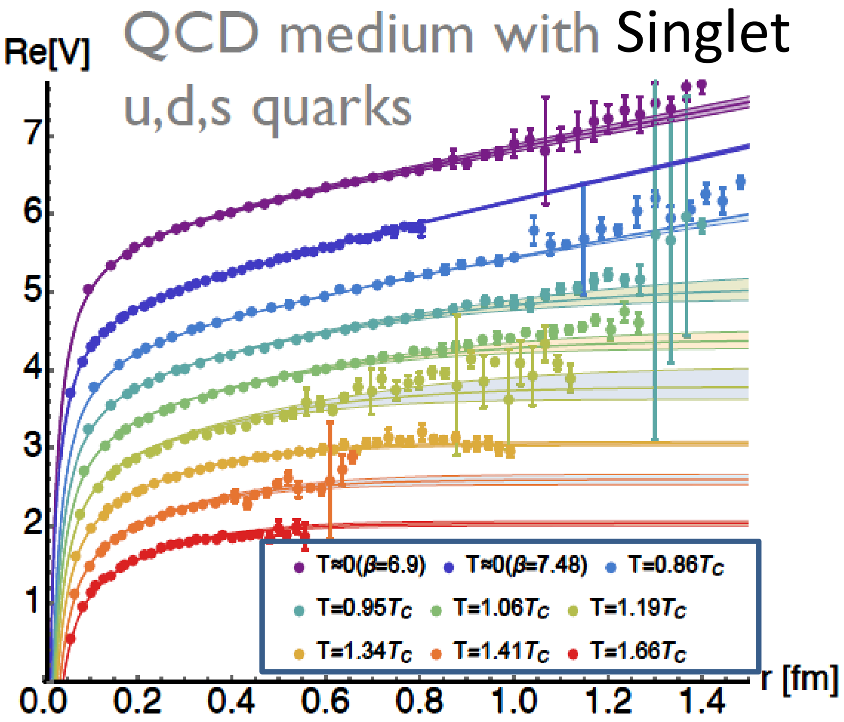
- m_Q
- In vacuum: Binding energy / separation energy btwn levels: ΔE α m_Q g^4 (Coulomb part) => v α g^2
- Radius : $(m_Q\ g\text{^}2)^{-1}$
- For a linear potental $\hbar\omega_0 = \left(\frac{\hbar^2 K_l^2}{m_b/2}\right)^{\frac{1}{3}} \approx 0.504\ \text{GeV}$

$$\Rightarrow\ v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$



Screening of the real potential

Protential (recent lQCD calculations)



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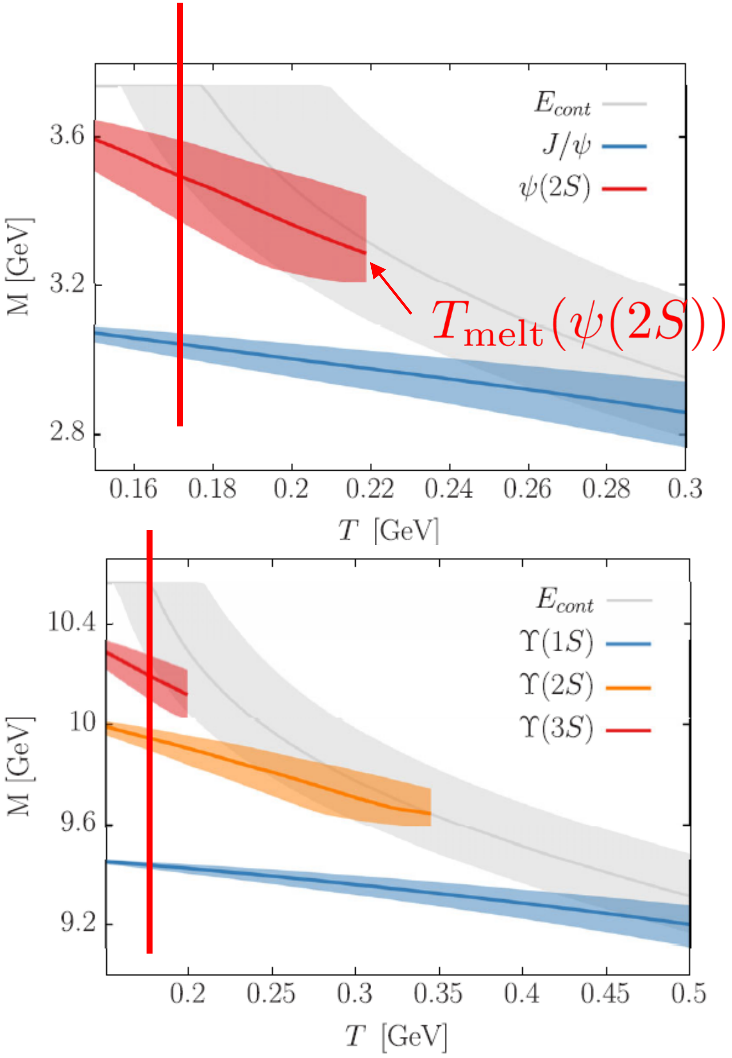
$$v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$

Compact and tightly bound states (at least for the lowest ones) => could survive QGP at low/mid T as well as to interactions with hadronic matter.

Screening of the real potential

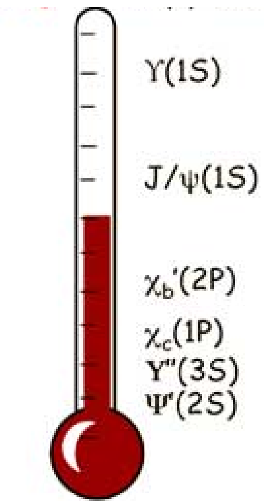
Recent In-medium spectrum (Lafferty and Rothkopf 2020)



« all or nothing scenario»:

- If $T_{\text{early QGP}} > T_{\text{melt}}$ => the state is not produced
- If $T_{\text{early QGP}} < T_{\text{melt}}$ => the state is produced like in pp

=> *SEQUENTIAL SUPPRESSION; Quarkonia as early QGP thermometer*



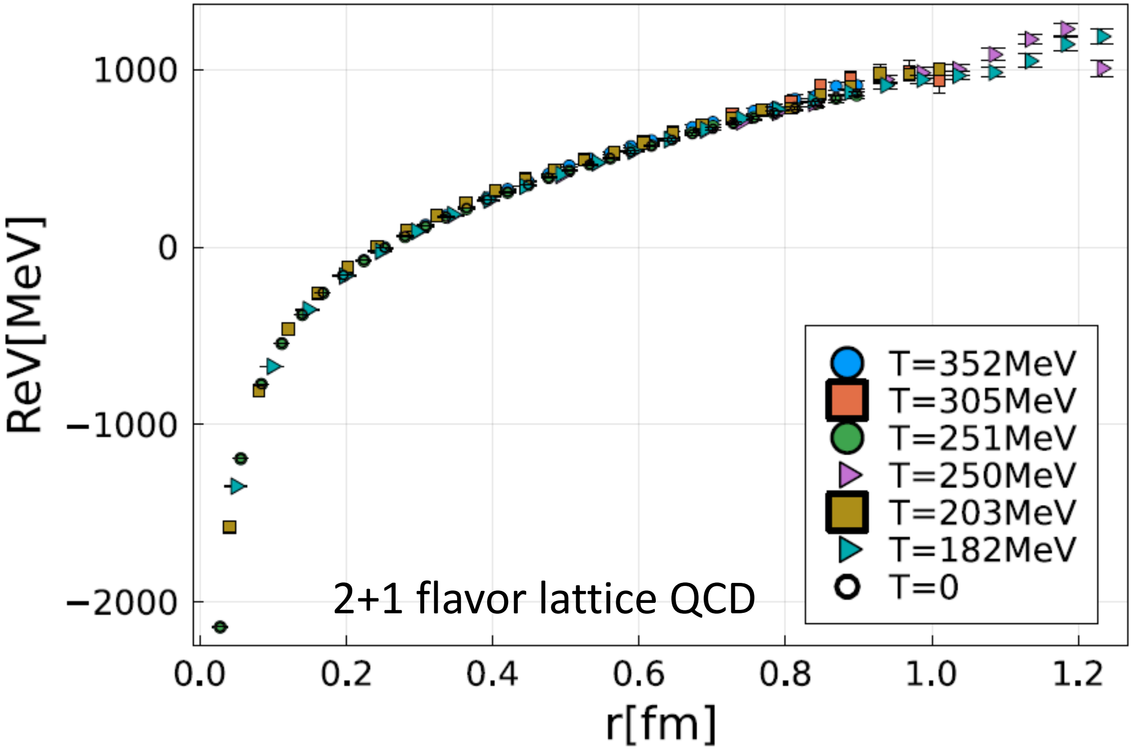
Most prominently : probing new state of matter in AA collision: Original idea by Matsui and Satz (86)…

… and advertized as a motivation in hundreds of talks (and papers) since then

David Lafferty, Alexander Rothkopf , Phys. Rev. D 101, 056010 (2020)

Screening of the real potential

Recent news : the real potential is not screened at temperatures reached in AA collisions !!!



Bazazov et al 2023 (Hot QCD collaboration)

How to define properly a “potential” on the lattice ?

<u>Historically</u> : thermodynamical potential like the free energy (in presence of a static dipole) or the total internal energy.

Modern approach : evaluate the Wilson loop and connect it to the r-dependent spectral density

$$W(\tau,r,T)=\int_{-\infty}^{+\infty}d\omega e^{-\omega\tau}\rho_r(\omega,T)$$

A “peak” contribution in the spectral density modelled as

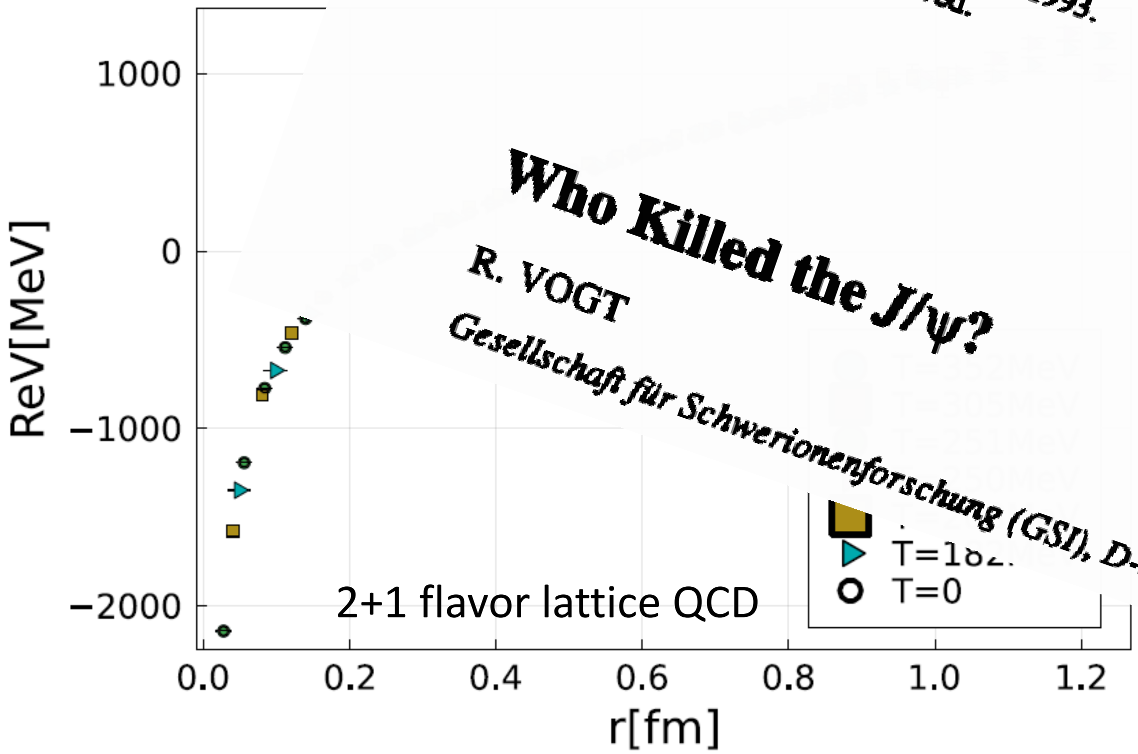
$$\rho_r^{\rm peak}(\omega,T)=\frac{1}{\pi}{\rm Im}\frac{A_r(T)}{\omega-{\rm Re}V(r,T)-i\Gamma(\omega,r,T)}$$

=> Lattice data then unfolded with this Ansatz.

Does not seems quite intuitive, may not be the end of the story

Screening of the real potential

Recent news : the real potential is screened at temperatures reached in AA collisions !!!



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=> Lattice data then deconvoluted with this Ansatz.

Prog. Part. Nucl. Phys., Vol. 30, pp. 405–406, 1993.
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0146–6410/93 $24.00
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Who Killed the J/ψ?

R. VOGT

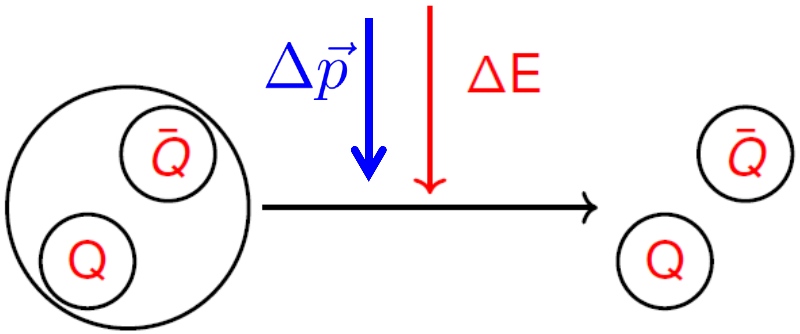
Gesellschaft für Schwerionenforschung (GSI), D-6100 Darmstadt 11, Germany

Does not seems quite intuitive, may not be the end of the story

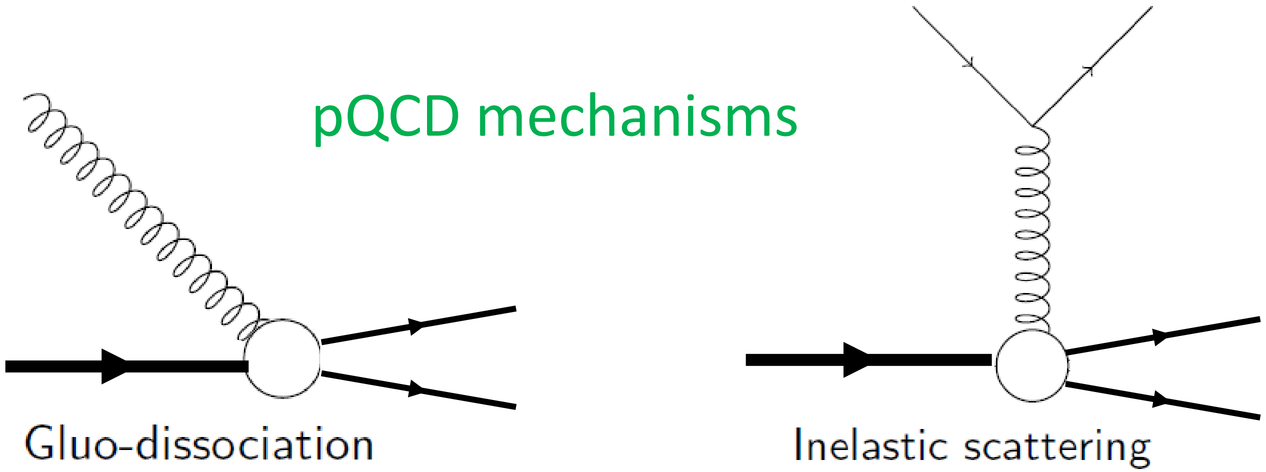
Collisions with the QGP

- Besides arguments based on the Debye mass / screening, it was pointed out already in the 90's that interactions with partons in the QGP could lead to dissociation of bound states (whose spectral function thus acquire some width Γ corresponding to the dissociation rate)

- Energy-momentum exchange with the QGP (gluo-dissociation, q – quarkonia quasi elastic scattering)



pQCD mechanisms


Gluo-dissociation

Inelastic scattering

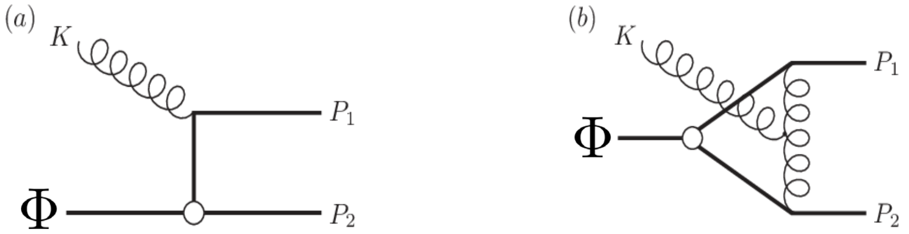
- => pair dissociation => **Suppression**

- ⇔ loss of probability of the quarkonia … Often described by some imaginary potential W in modern approaches

A central quantity: the decay rate Γ

Many approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)



$$\Phi + g \to Q + \bar{Q}$$

Dissociation cross section σ

$$\Gamma_\Phi(T) = \langle \sigma n_g \rangle_T$$

Other mechanisms : $x + \Phi \to x + Q + \bar{Q}$

QFT/Lattice QCD

Time correlator

$$\mathcal{C}_>(t,\vec{r}) \approx \langle \psi(t,\tfrac{\vec{r}}{2})\bar{\psi}(t,-\tfrac{\vec{r}}{2})\psi(0,0)\bar{\psi}(0,0)\rangle$$

Satisfies Schroedinger equation with complex potential V+iW . Breakthrough by Laine et al. (2006)

$$\Gamma_\Phi(T) = -2\langle \Phi | W | \Phi \rangle$$

Concept better suited at it genuinely encodes the "in medium" propagation

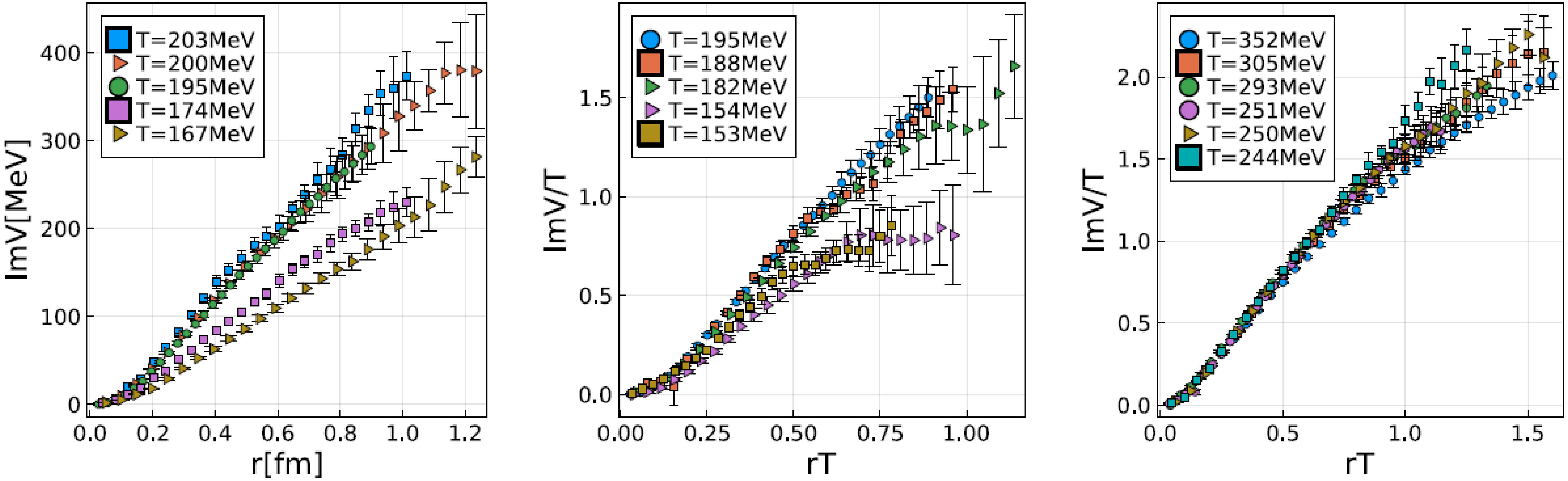
=> Simple decay law : Prob survival $= \exp\left(-\int_{t_0}^{t_{\rm fin}} \Gamma(T(t))dt\right)$

A central quantity: the decay rate Γ

Recent lQCD calculations of W(r) = Im(V(r)) (at ω=0)

$$\rho_r^{\mathrm{peak}}(\omega,T) = \frac{1}{\pi}\mathrm{Im}\frac{A_r(T)}{\omega - \mathrm{Re}V(r,T) - i\Gamma(\omega,r,T)}$$

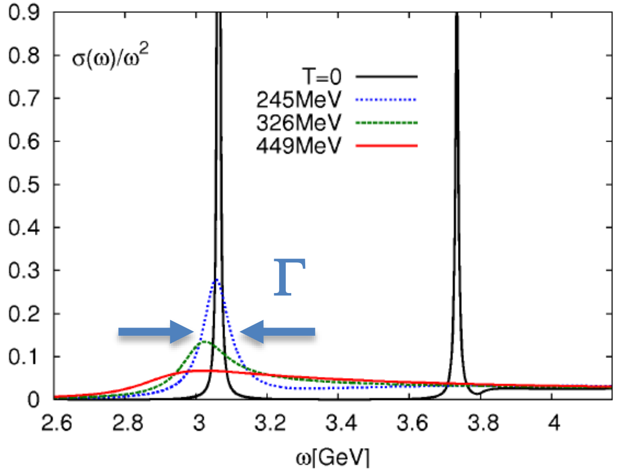
Bazazov et al 2023 (Hot QCD collaboration)



- ➢ Nice r T scaling
- ➢ Dipole structure at small r, no saturation seen at “large” r

Quarkonia at finite T

- Pheno: Yet, this picture of « no screening – large dissociaton » might still be compatible with the notion of sequential dynamical « suppression »…
- However, this notion has to be made more precise : (LQCD) spectral function lQCD



$$\rho(\omega,p,T)=\frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty}dte^{i\omega t}\int d^3xe^{ipx}\langle[J(x,t),J(0,0)]\rangle_T$$

At T=245 MeV, ψ' has disappeared but J/ψ still surviving for $\approx 1/\Gamma \approx$ a couple of fm/c … which needs to be compared with the local QGP cooling time τ_{cool} : Γ x τ_{cool} > 1 ⇔ suppressed

- N.B.: The opposite phenomenon might also be relevant: some state above the « melting » temperature can survive (for a short while < $1/\Gamma$) before getting lost definitively.

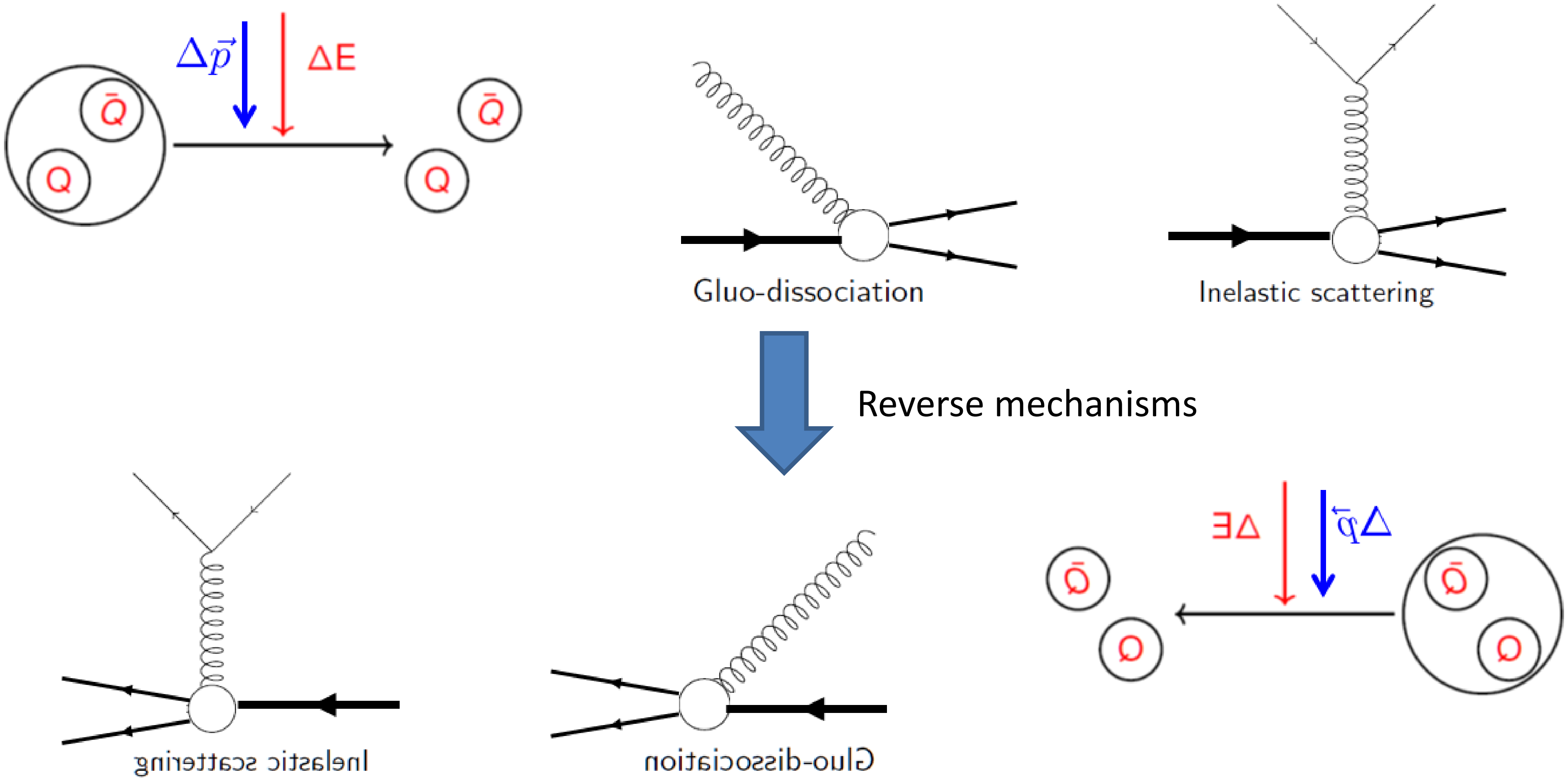
Will it melt
(even party) ?

- Key question : do the quarkonia states (chemically) equilibrate with the QGP ?

Modern era

If life was not complicated enough…: Regeneration

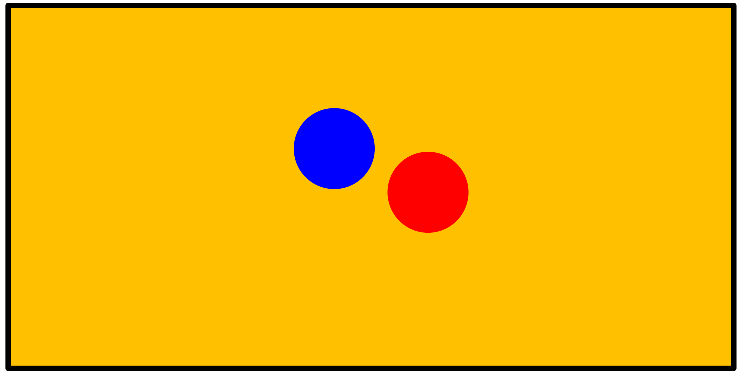
Detailed balance :



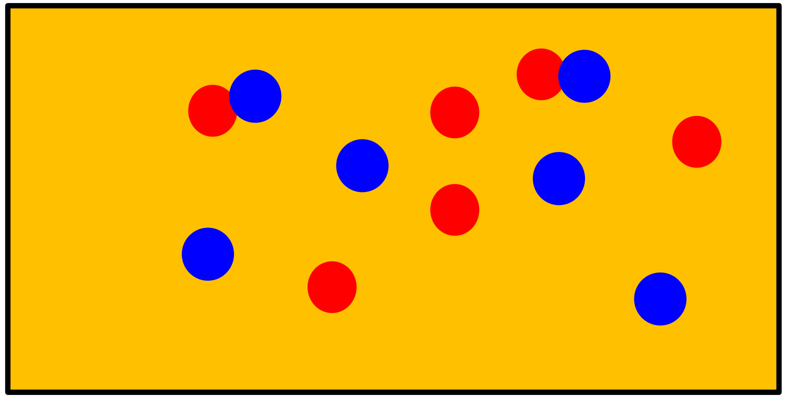
Reverse mechanisms

Regeneration: Dilute vs Dense

Bottomia

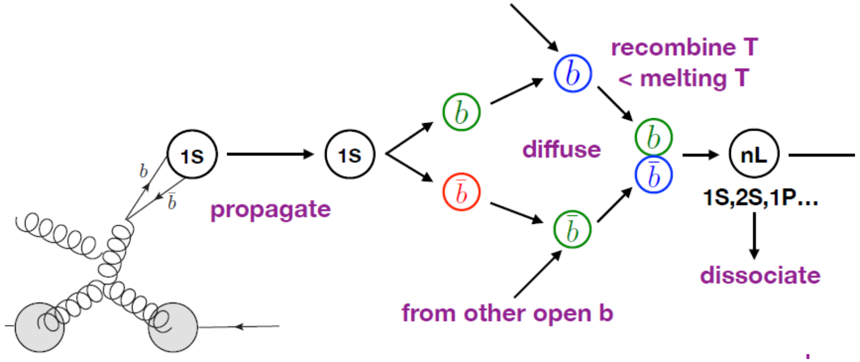


Charmonia



No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states

In some SC formalisms : intermediate regeneration

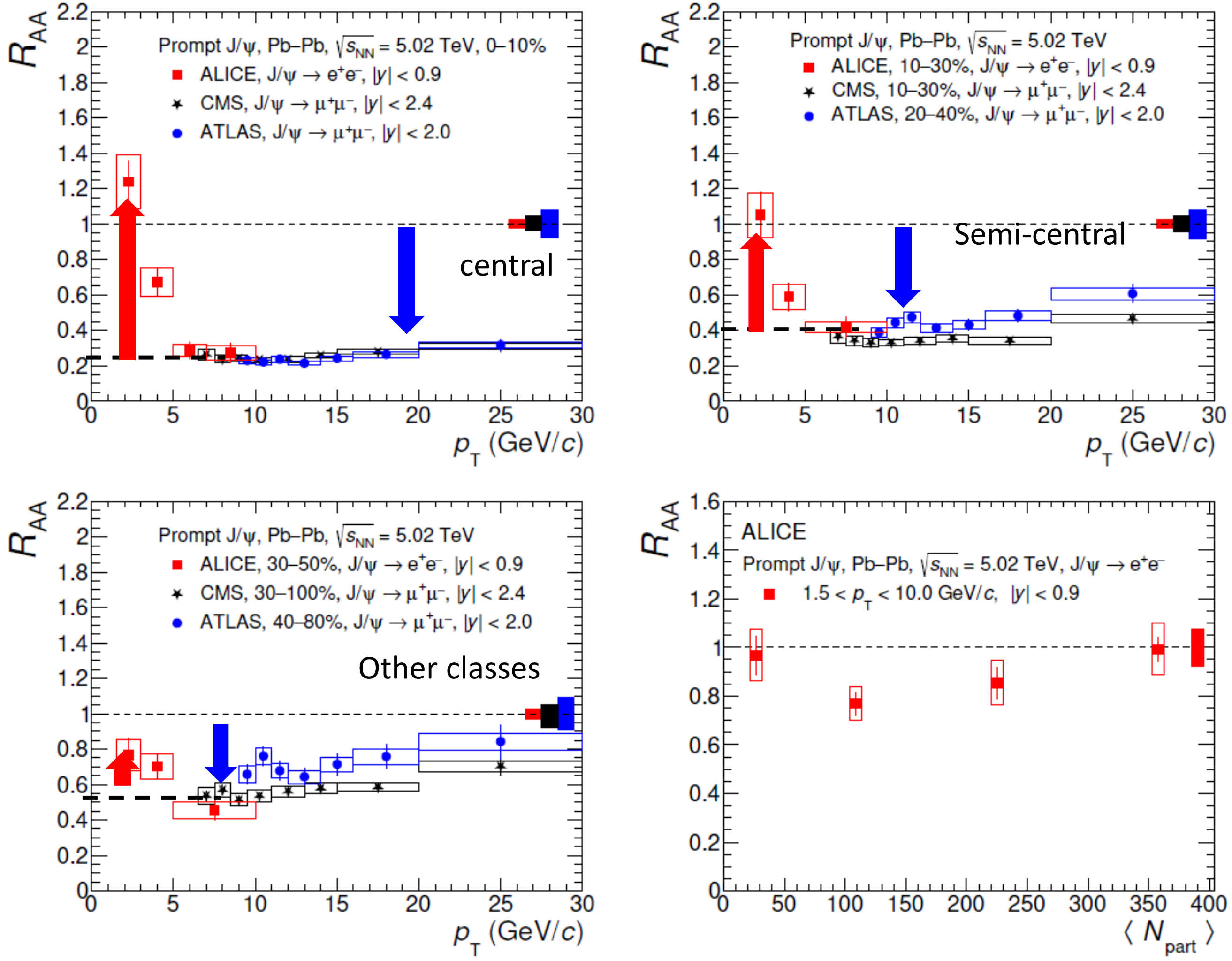


Yao, Mehen, Müller (2019)

Exogenous recombination : c & cbar initially far from each other may recombine and emerge as charmonia states

- ➢ No full quantum treatment => semi-classical approximation
- ➢ Key questions : when does the recombination (dominantly) happen ? Crucial role of the binding force.
- ➢ Are the c and cbar distributions equilibrated at this time ?
- ➢ One extreme viewpoint : regeneration happens at the end of the QGP (Statistical Hadronization Model: ask if you want)

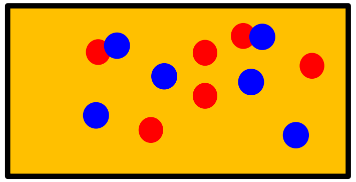
What experiment tells us



ALICE Collab. JHEP02 (2024) 066

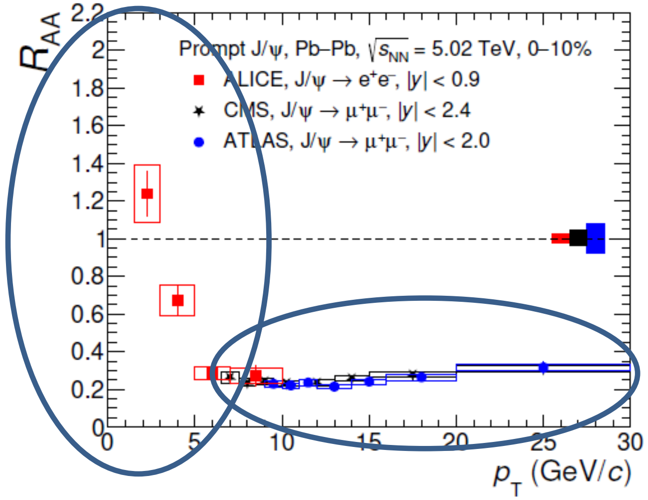
- Quarkonia production in AA strongly affected by the presence of the QGP => good probe of the QGP properties on small scales ($1/M_Q$)
- Increasing suppression with centrality at intermediate and high p_T
- Increasing yield with centrality at low p_T
- Increasing experimental precision => need for the models to gain in accuracy

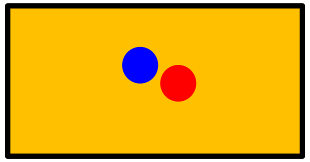
What experiment tells us



Dense (in phase space)
=> recombination

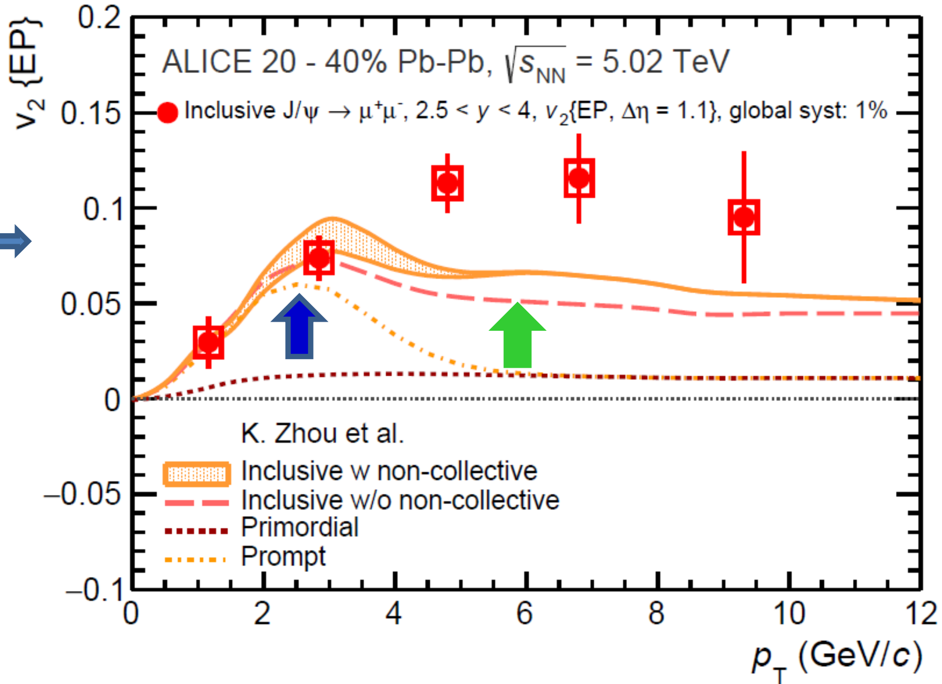
ALICE Collab. JHEP02 (2024)



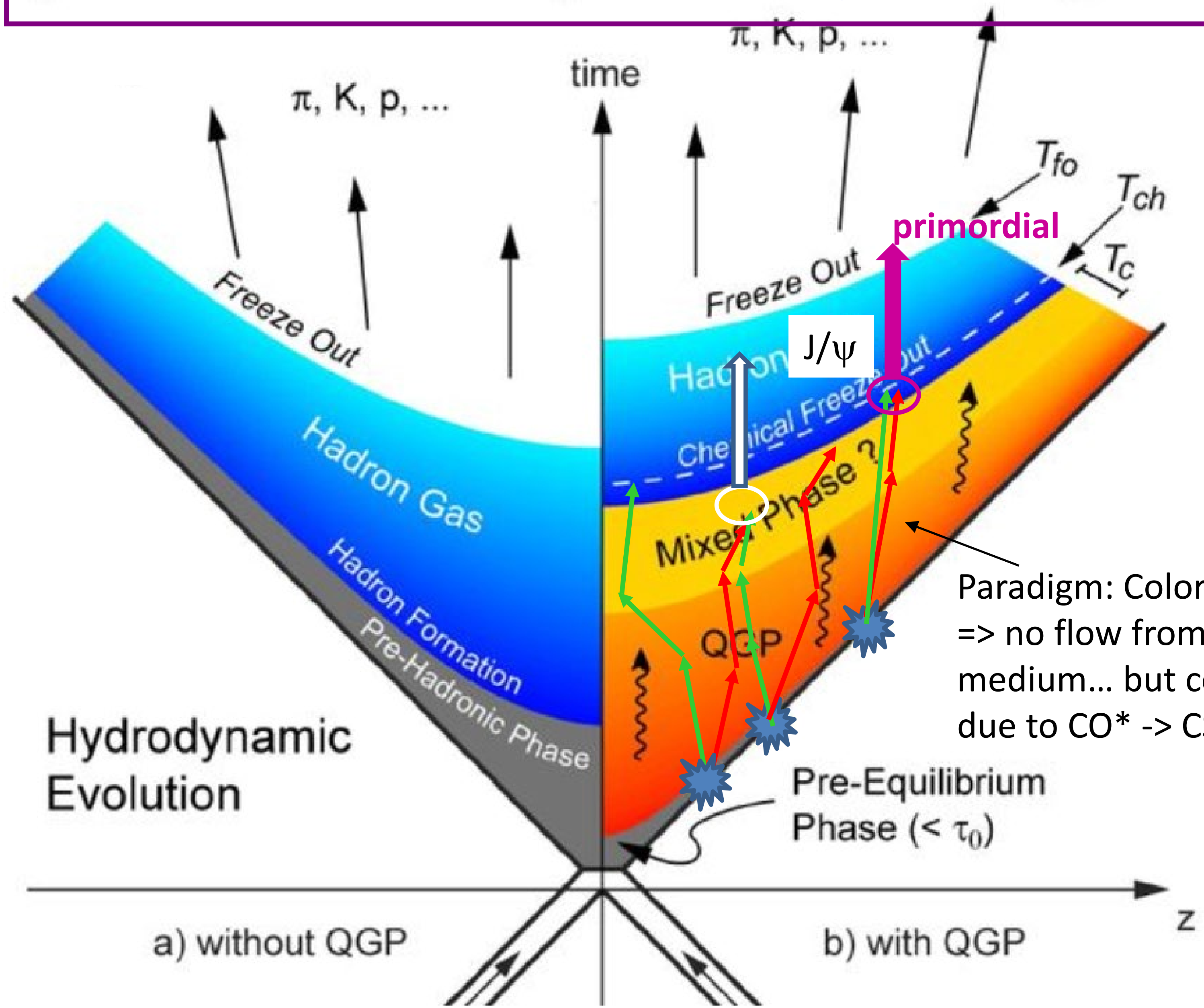


Dilute (in phase space)

Alternate possible explanation : p_T-dependent absorption cross section : not excluded, but not favored by the finite v_2 observed for J/ψ by ALICE



Quarkonia in transport models

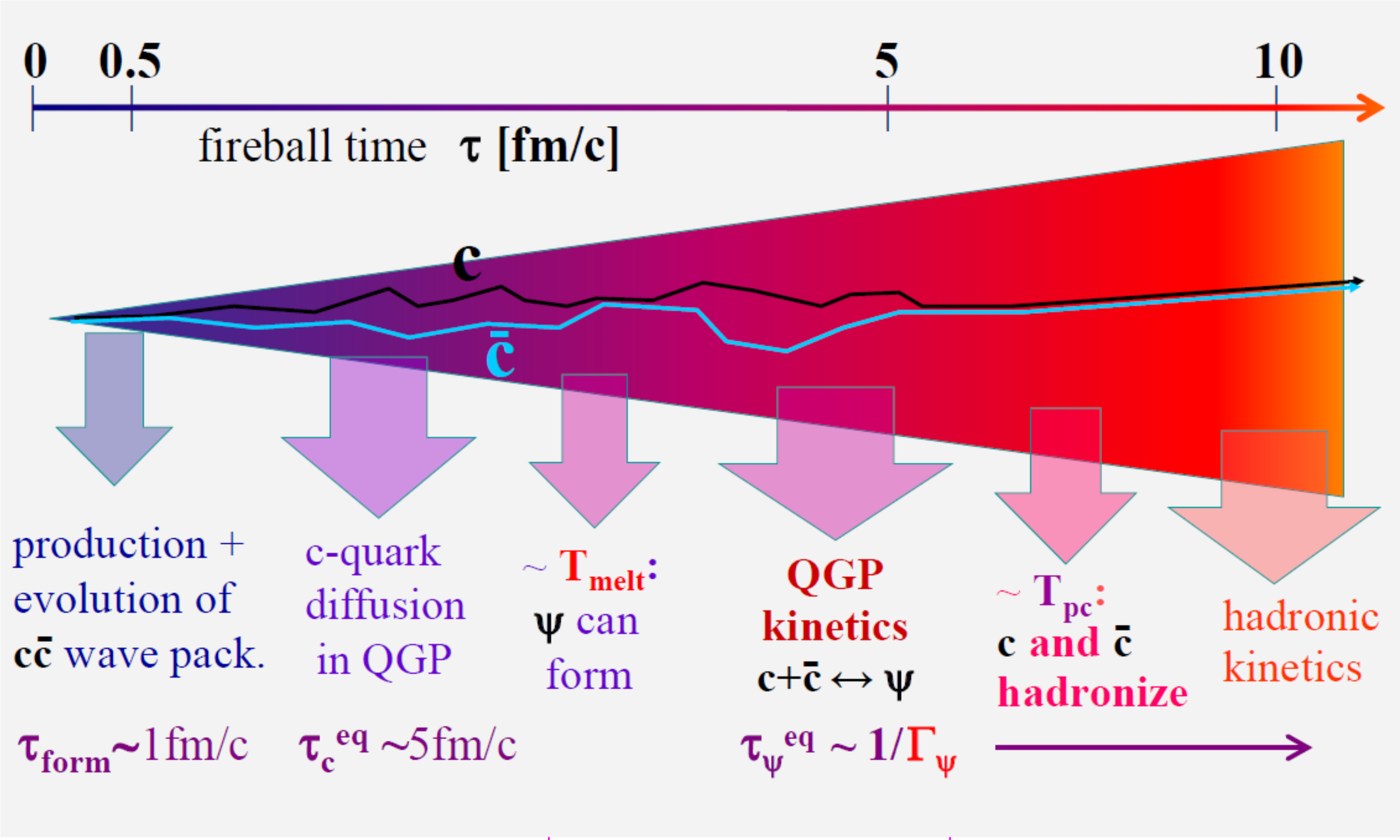


Paradigm: Color singlet => no flow from the medium… but could be due to CO* -> CS

The working horse of most concrete predictions for quarkonia production in AA, especially for charmonia

Quarkonia in transport models

Rapp and Du *Nucl.Phys.A* 967 (2017) 216-224



“in medium” quarkonia (bound states of possibly screened potential)

Reaction rate approach by TAMU

Rate equation for quarkonia (main dof) :

(the gain and loss are not evaluated locally in phase space)

$$\frac{dN(t)}{dt} = -\underset{\text{Loss}}{\Gamma(T(t))}\,(N(t) - \underset{\text{Gain}}{N^{\mathrm{eq}}(T(t))})$$

Automatically takes care of the recombination at time t assuming HQ are in thermal equilibrium

Can again be split into 2 components :

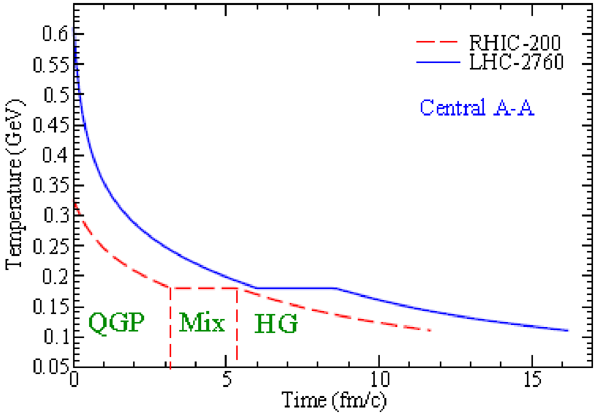
$$\frac{dN^{\mathrm{prim}}(t)}{dt} = -\Gamma(T(t))\,N^{\mathrm{prim}}(t) \quad \text{with} \quad N^{\mathrm{prim}}(t_0) = N_0$$

$$\frac{dN^{\mathrm{regen}}(t)}{dt} = -\Gamma(T(t))\,[\,N^{\mathrm{regen}}(t) - N^{\mathrm{eq}}(T)] \quad \text{with} \quad N^{\mathrm{regen}}(t_0) = 0$$

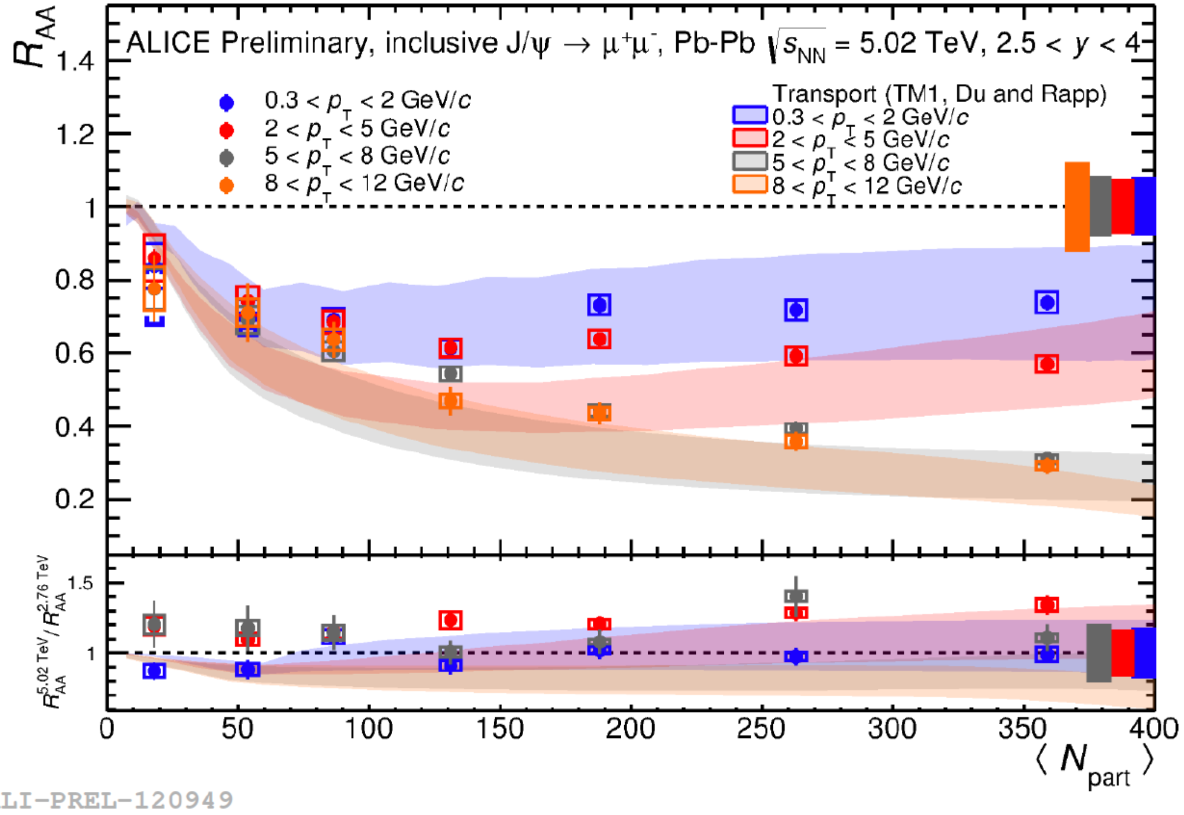
- Naturally interpolates between simple suppression and regeneration
- Dissociation Rate from usual cross sections with quasi particle masses compatible with lQCD EOS

$$\Gamma_\Psi(T) = \int \frac{d^3p}{(2\pi)^3} v_{i\Psi}\sigma_{i\Psi\to X}\,\underset{\text{Parton Density}}{f_i(m_i,T)}$$

- Mostly Implemented in fireball model
- Including contribution from hadron gas



Reaction rate approach by TAMU



- In transport theory, primordial component is mandatory to reproduce the absolute production as a function of centrality & p_T class

Not simple statistical hadronization at the end of the QGP

A consistent picture emerging in the charmonia sector

Good overall consistency of the following facts:

- Increase of J/ψ production from RHIC -> LHC
- Mostly at low p_T where regeneration is expected
- Finite v_2(J/ψ) observed for the first time with 5σ confidence
- Washing out of the spectral function (only J/ψ survive for T < 0.25 GeV)
- Statistical ratio achieved for ψ' / J/ψ at LHC for central and semi-central

Not paying too much attention at CNM effects:

With the interpretation that a large fraction of direct quarkonia are produced through recombination (also see in transport models)

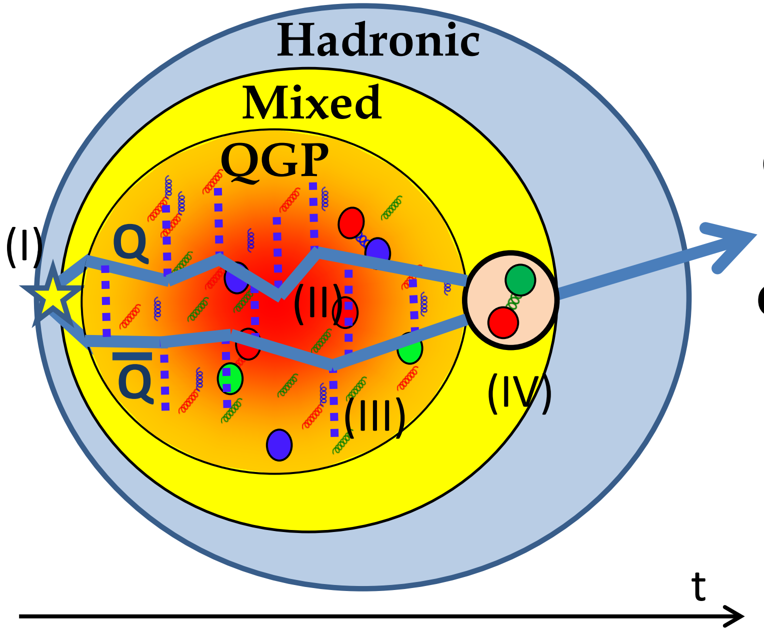
N.B.: if state not dissociated / tightly bound, then little recomb. as well (small Γ) => cannot benefit from the v_2(c), except it some significant elastic scattering (no sign for this)

$$\frac{dN_\Phi}{dt} = -\Gamma(T(t))\,(N_\Phi - N_{\Phi,\mathrm{eq}}(T(t)))$$

Remaining challenges:

- v_2 at finite (5-10 GeV) p_T (A lot of effects can destroy the Onium, but how to give it v_2 ?)
- role of the magnetic field (not discussed)

The full scheme for microscopic approaches



Quarkonium

?

Open mesons

Strictly speaking, only resolved at the end of the evolution

Beware of quantum coherences during the whole evolution !

Especially at early time…

Complicated QFT problem (also due to the evolving nature of the QGP that mixes several scales)… only started to be addressed at face value recently

How to proceed ?

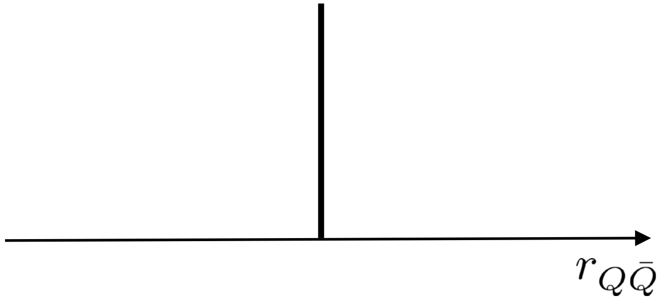
1) Initial state
2) (Possibly Screened) interaction between both HQ
3) Interactions with surrounding QGP partons
4) Projection on the final quarkonia

In practice, what counts is the so-called decoherence time, not the “Heisenberg time”

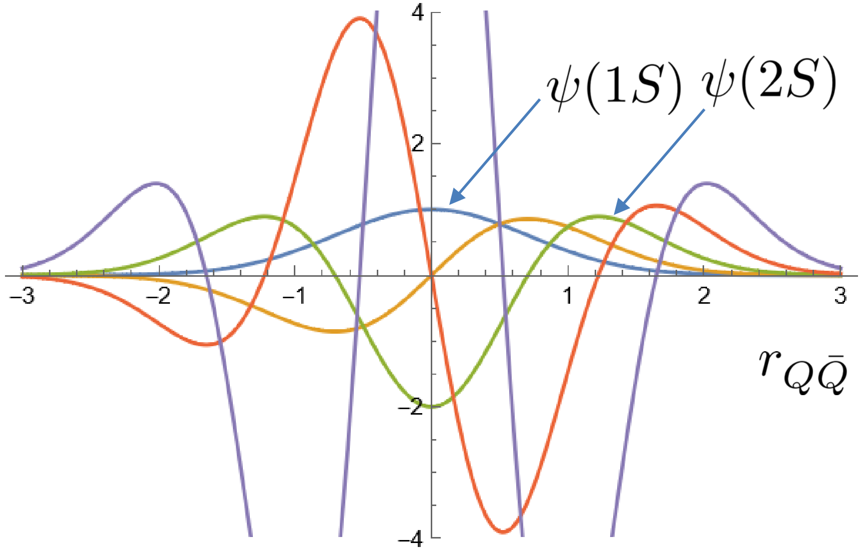
First incomplete QM treatments dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90’s

Quantum coherence at early time

Assume : $\psi_{\text{in}}(\vec{r}_{Q\bar{Q}}) \propto \delta^{(3)}(\vec{r}_{Q\bar{Q}})$



$= \Sigma$



Initial compact state : δ

Dissociation rate: $\Gamma(r_{Q\bar{Q}}) \propto \alpha_S T \times \Phi(m_D r_{Q\bar{Q}}) \sim \alpha_S^2 T^3 \times r_{Q\bar{Q}}^2$

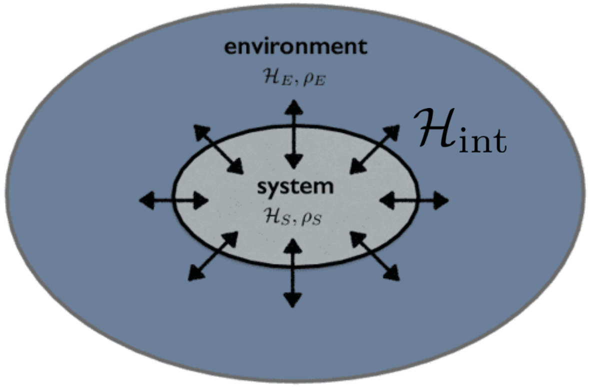
Including coherence

Neglecting coherence

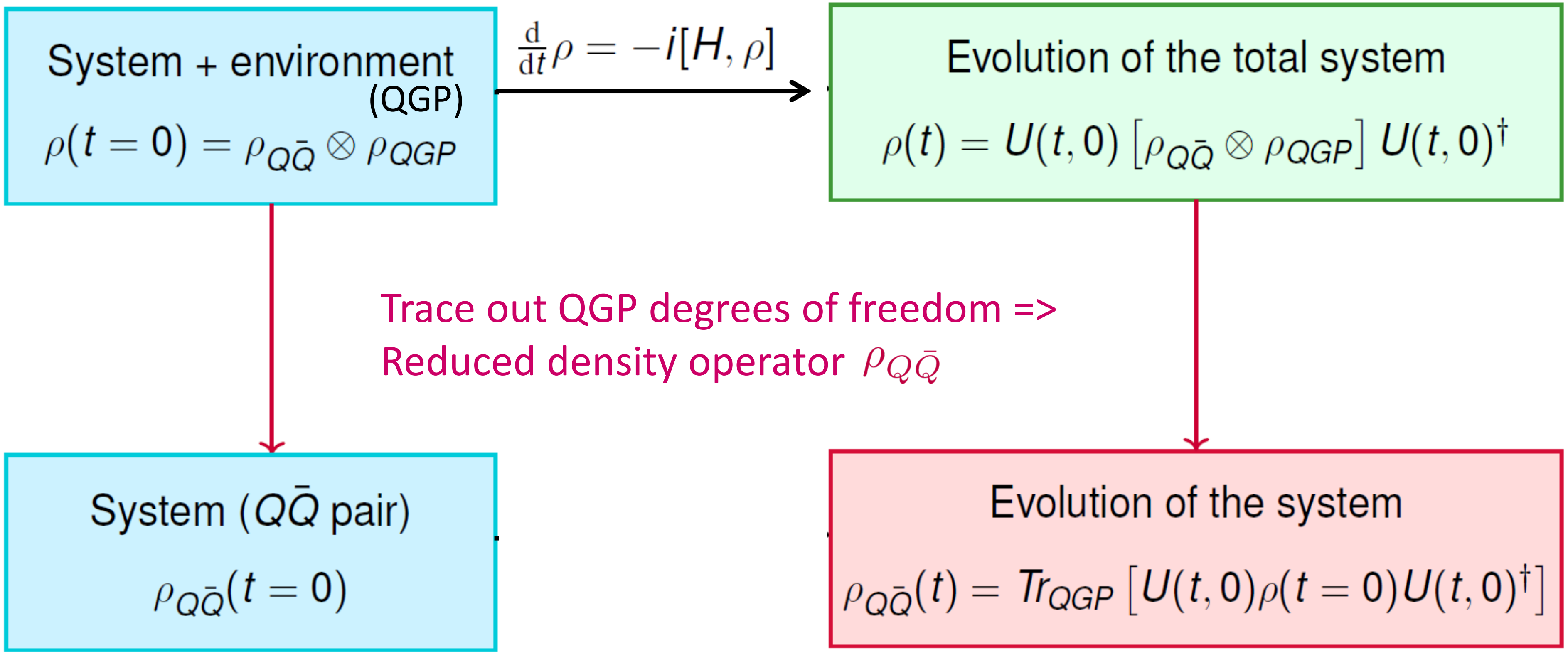
☺ $\Gamma(r_{Q\bar{Q}}) \approx 0 \propto \sum c_j^\star c_i \langle\psi_j|r^2|\psi_i\rangle \longrightarrow \Gamma \propto \sum_i |c_i|^2 \langle\psi_i|r^2|\psi_i\rangle \approx \sum_i |c_i|^2 \Gamma_i \neq 0$ ☹

Crucial to include coherence !

Open Quantum Systems & Quantum Master Equations



Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}^{(0)}_{Q\bar{Q}} + \hat{H}_E + \hat{H}_{\rm int}$ with $\hat{H}_E = \hat{H}_{\rm QGP}$



Von Neumann equation for the total density operator ρ

System + environment (QGP)

$\rho(t=0) = \rho_{Q\bar{Q}} \otimes \rho_{QGP}$

$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H, \rho]$

Evolution of the total system

$\rho(t) = U(t,0)\left[\rho_{Q\bar{Q}} \otimes \rho_{QGP}\right] U(t,0)^\dagger$

Trace out QGP degrees of freedom =>
Reduced density operator $\rho_{Q\bar{Q}}$

System ($Q\bar{Q}$ pair)

$\rho_{Q\bar{Q}}(t=0)$

Evolution of the system

$\rho_{Q\bar{Q}}(t) = Tr_{QGP}\left[U(t,0)\rho(t=0)U(t,0)^\dagger\right]$

Can be formulated differentially ./. time :

$$\frac{\mathrm{d}\rho_{Q\bar{Q}}}{\mathrm{d}t} = \mathcal{L}[\rho_{Q\bar{Q}}]$$

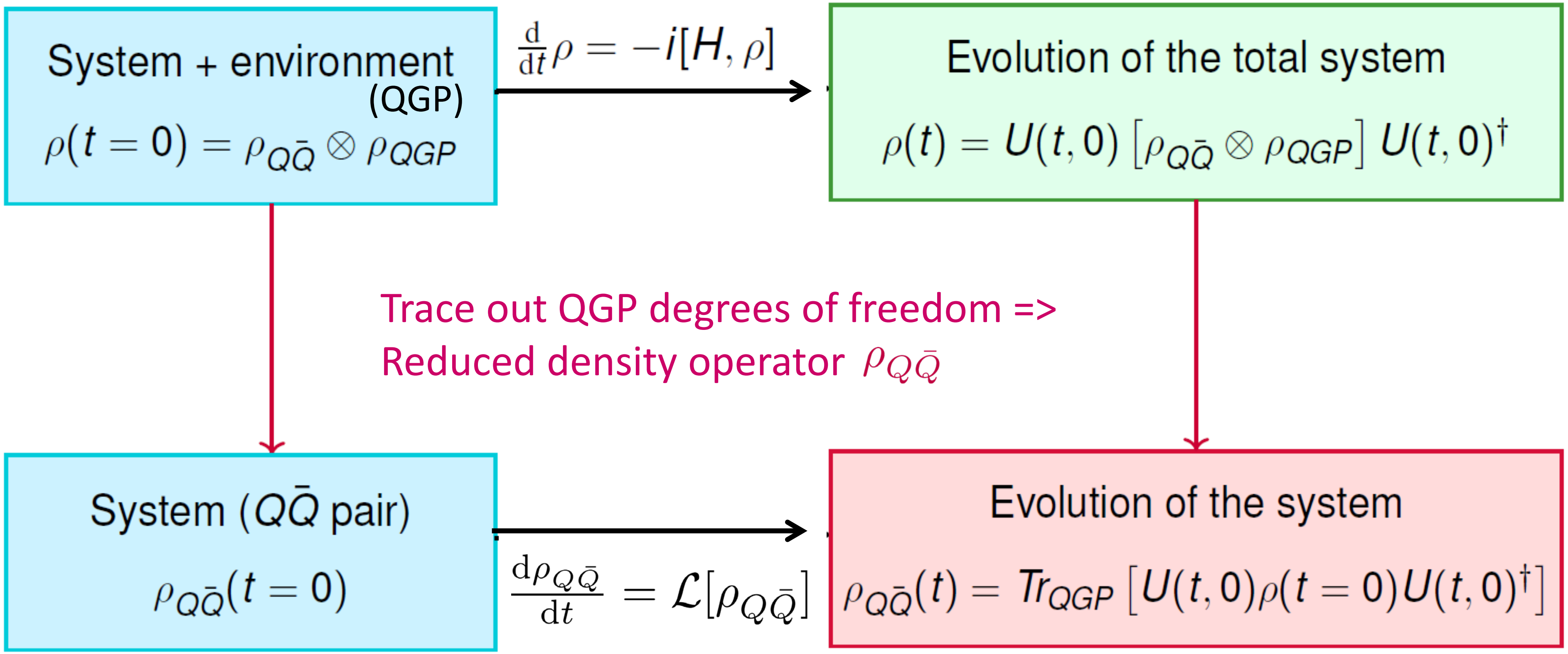
Definition of $\mathcal{L}[\cdot]$

Open Quantum Systems & Quantum Master Equations

$$\hat{\rho}_{Q\bar{Q}} = \sum_{\alpha,\beta} d_{\alpha,\beta} |\alpha\rangle\langle\beta|$$

QME deal with the (coupled) evolution of probabilities ($d_{\alpha,\alpha}$) and coherences ($d_{\alpha,\beta\neq\alpha}$)

Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}^{(0)}_{Q\bar{Q}} + \hat{H}_E + \hat{H}_{\rm int}$ with $\hat{H}_E = \hat{H}_{\rm QGP}$



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Reduced density operator $\rho_{Q\bar{Q}}$

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$\rho_{Q\bar{Q}}(t=0)$

$\frac{\mathrm{d}\rho_{Q\bar{Q}}}{\mathrm{d}t} = \mathcal{L}[\rho_{Q\bar{Q}}]$

Evolution of the system

$\rho_{Q\bar{Q}}(t) = Tr_{QGP}\left[U(t,0)\rho(t=0)U(t,0)^{\dagger}\right]$

However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time (linear map)

A special QME: The Lindblad Equation

There are many different QME… a special one :

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \boxed{\gamma_i}\left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

$H_{Q\bar{Q}}$: $\{Q, \bar{Q}\}$ $\underbrace{\text{kinetics + Vacuum potential V}}_{\hat{H}^{(0)}_{Q\bar{Q}}}$ + Lamb shift / screening

(every unitary term that is generated by tracing out the environment)

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}$ (Hermiticity)

$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$ (Norm)

$\langle\varphi|\rho_{Q\bar{Q}}|\varphi\rangle > 0, \forall|\varphi\rangle$ (Positivity)

… but in general, non unitary !!! (relaxation)

Nice feature : Can be brought to the form of a stochastic Schroedinger equation (quantum jump method : QTRAJ)

A special QME: The Lindblad Equation

Non unitary / dissipative evolution ≡ decoherence

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}},\rho_{Q\bar{Q}}(t)\right] + \boxed{\sum_i \gamma_i\left[L_i\rho_{Q\bar{Q}}(t)L_i^\dagger - \frac{1}{2}\left\{L_iL_i^\dagger,\rho_{Q\bar{Q}}(t)\right\}\right]}$$

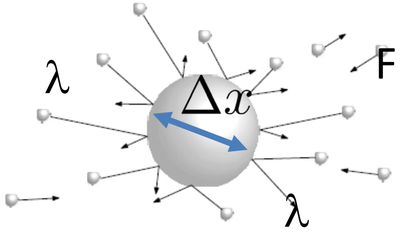
Genuine transitions :
- ✓ Singlet <-> octet
- ✓ Octet <-> octet

Can be reshuffled into non Hermitic effective hamiltonian

$$\hat{H}_{Q\bar{Q},\mathrm{eff}} = \hat{H}_{Q\bar{Q}} - i\boxed{\sum_j \gamma_j \frac{L_jL_j^\dagger}{2}}$$

≡ Dissociation width

For **infinitely massive single Q** and environment wave length λ >> wave packet size Δx:



Fluctuations from env. ⟺

$$\frac{\partial\rho_Q(x_Q,x'_Q)}{\partial t} = -F(x_Q - x'_Q)\rho_Q(x_Q,x'_Q)$$

Decoherence factor: $F \approx \kappa\,(x_Q - x'_Q)^2$

HQ momentum diffusion coefficient (adjoint)

In Q world: smaller objects live longer !

At 1rst order in 1/m$_Q$: recoil corrections ⟺ friction / dissipation

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

Pictorial summary

τ_E: environment autocorrelation time **τ_S: system intrinsic time scale** **τ_R: system relaxation time**

Subsystem + environment: von Neumann equation

↓ **Trace out environment**

Subsystem: non-unitary, time-irreversible evolution

↓ **Weak syst-environment coupling** **+ Markovian limit** $\tau_E \ll \tau_R$

Redfield equation

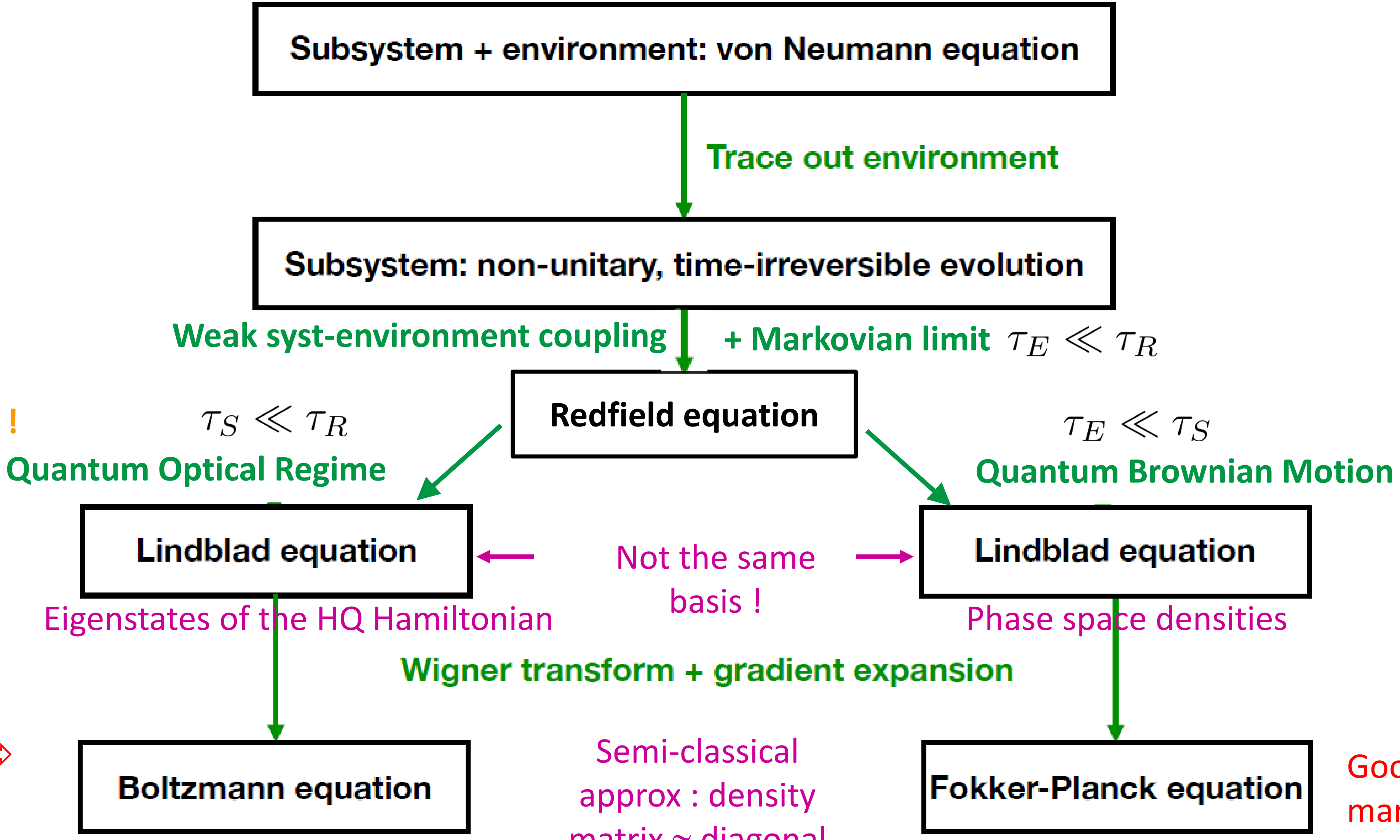
$$\frac{\partial}{\partial t}\rho_{\mathrm{I}}(t) = -\frac{1}{\hbar^2}\sum_{m,n}\int_0^\infty d\tau \Big(C_{mn}(\tau)\Big[S_{m,\mathrm{I}}(t), S_{n,\mathrm{I}}(t-\tau)\rho_{\mathrm{I}}(t)\Big] - C^*_{mn}(\tau)\Big[S_{m,\mathrm{I}}(t), \rho_{\mathrm{I}}(t)S_{n,\mathrm{I}}(t-\tau)\Big]\Big)$$

Similar structure to the Linblad equation but with time delay effects

Pictorial summary

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

τ_E: environment autocorrelation time τ_S: system intrinsic time scale τ_R: system relaxation time



Subsystem + environment: von Neumann equation

Trace out environment

Subsystem: non-unitary, time-irreversible evolution

Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

$\tau_S \ll \tau_R$

Quantum Optical Regime

Lindblad equation

Eigenstates of the HQ Hamiltonian

$\tau_E \ll \tau_S$

Quantum Brownian Motion

Lindblad equation

Phase space densities

Not the same basis !

Wigner transform + gradient expansion

Boltzmann equation

Semi-classical approx : density matrix ≈ diagonal

Fokker-Planck equation

Smallest time scales wins it all !

Rate equations: ⇔ transport models

Good method for many $c\bar{c}$ pairs

QCD time scales

τ$_E$: environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} \approx \frac{1}{CT} \approx \frac{1}{T}$$ (C taken as close to unity)

τ$_S$: system intrinsic time scale

$$\tau_S \approx \frac{1}{\underbrace{\Delta E}_{\text{Difference btwn energy levels}}} \approx \frac{1}{m_Q v^2} \text{ with } v \approx \alpha_S$$

… at the beginning of the evolution

τ$_R$: system relaxation time

$$\Gamma = \tau_R^{-1} \sim 2\langle\psi|W\psi\rangle \approx \alpha_s T \times \Phi(m_D r) \approx \alpha_s T \times \Phi(\frac{CT}{m_Q \alpha_s})$$

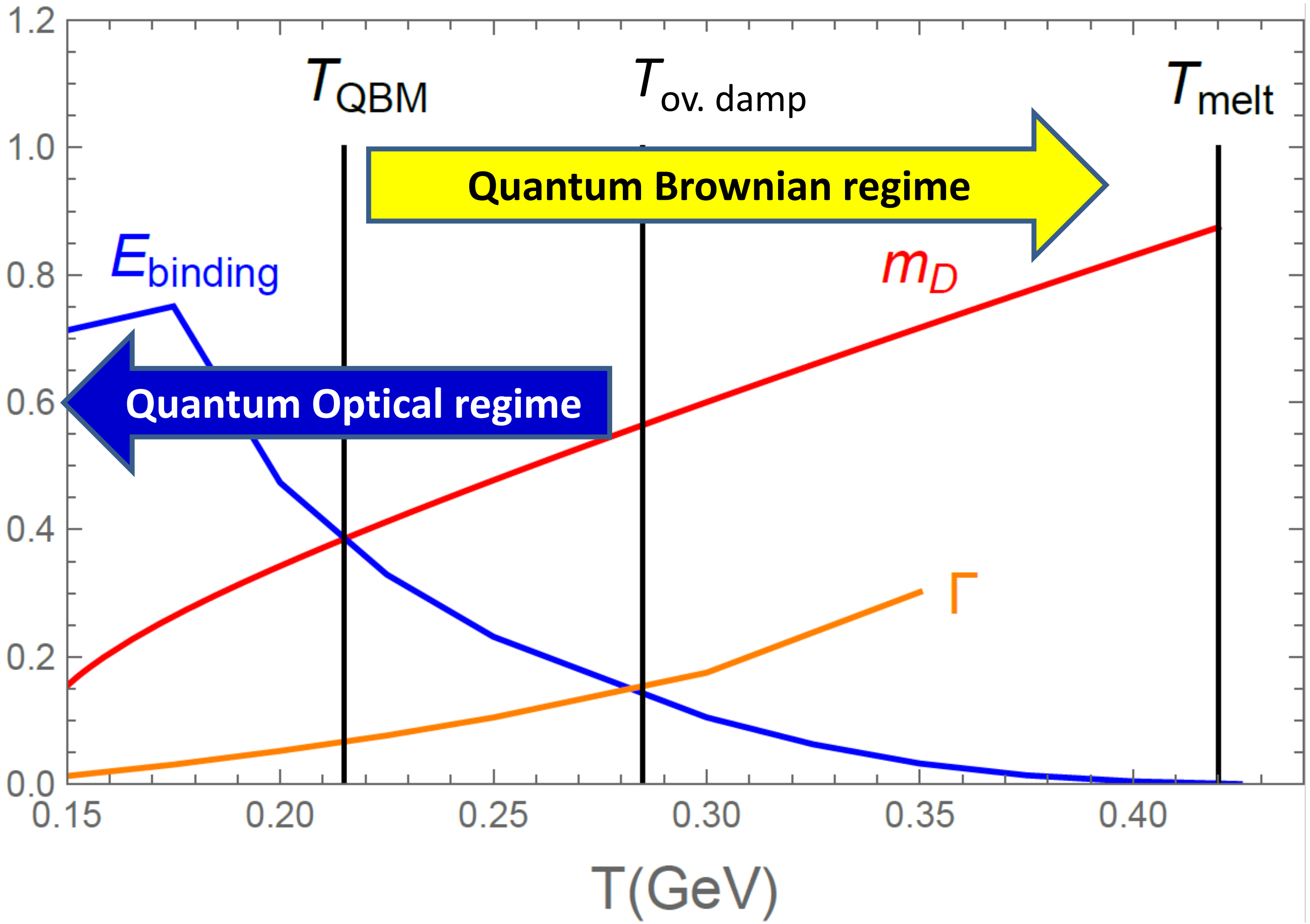
At “small” T $\left(T \lesssim \frac{m_Q \alpha_S}{C}\right)$: dipole approximation : $\Gamma = \tau_R^{-1} \approx \frac{C^2 T^3}{\alpha_s m_Q^2}$

$$\boxed{\frac{\tau_R}{\tau_E} = \frac{\alpha_s m_Q^2}{CT^2} \gg 1}$$

And $\frac{\tau_R}{\tau_S} = \frac{\alpha_s^3 m_Q^3}{C^2 T^3} \gg 1$ for $T \lesssim m_Q \frac{\alpha_S}{C^{2/3}}$

Fine with the Markovian assumption

Two types of dynamical modelling



$c\bar{c}$ pair

Numbers extracted from a specific potential model : Katz et al, Phys. Rev. D 101, 056010 (2020)

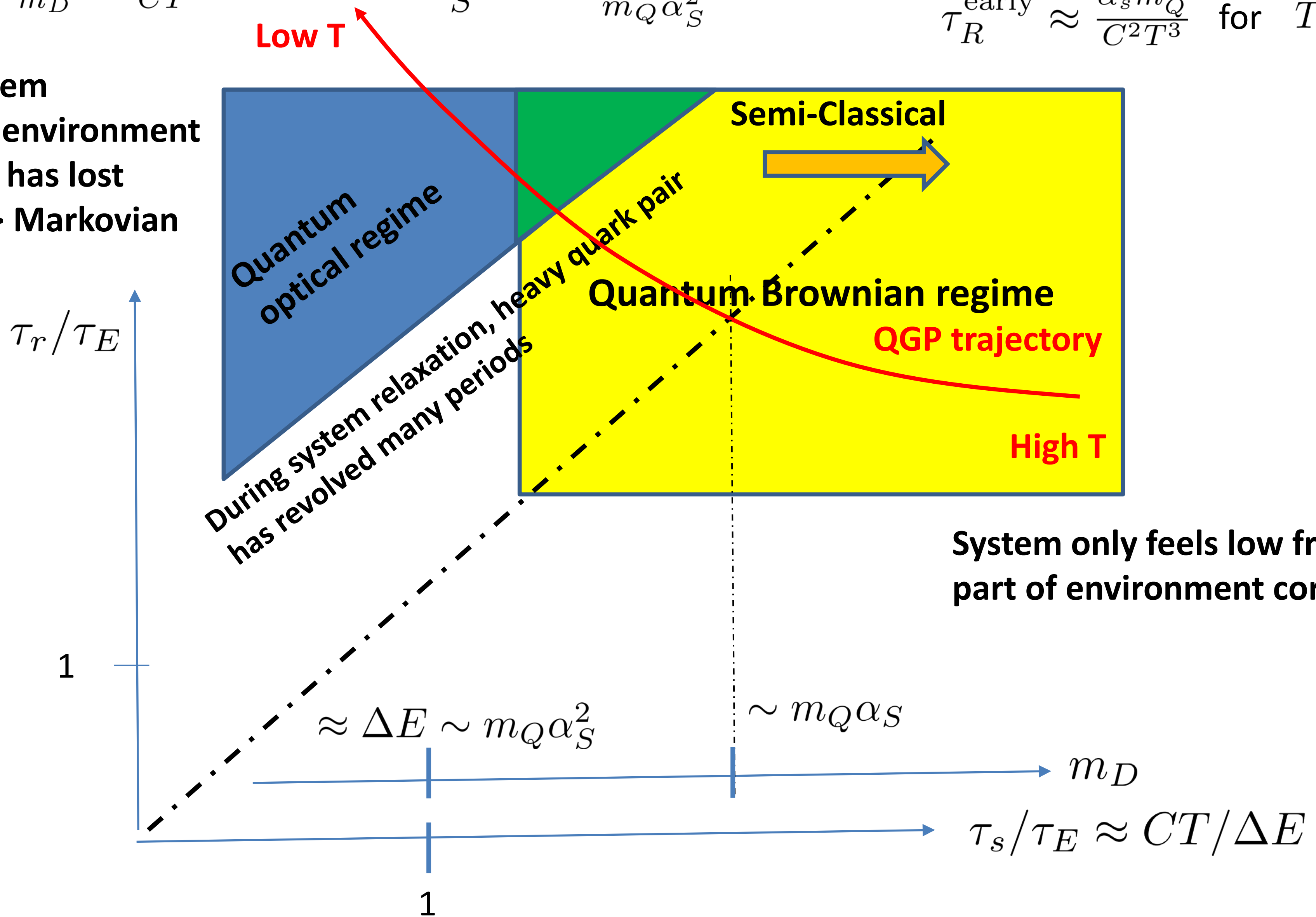
QCD time scales

$\tau_E \approx \frac{1}{m_D} = \frac{1}{CT}$

$\tau_S^{\text{early}} \approx \frac{1}{m_Q \alpha_S^2}$

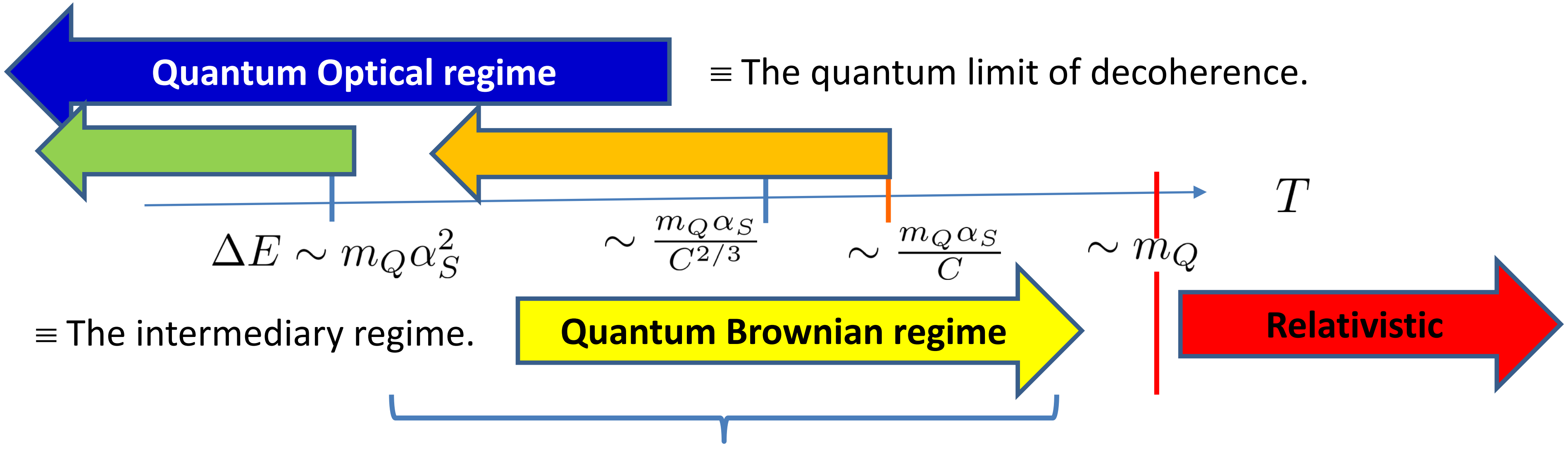
$\tau_R^{\text{early}} \approx \frac{\alpha_s m_Q^2}{C^2 T^3}$ for $T \lesssim \frac{m_Q \alpha_S}{C}$

During system relaxation, environment correlation has lost memory => Markovian process



System only feels low frequency part of environment correlation

QCD Temperature scales



Quantum Optical regime

≡ The quantum limit of decoherence.

T

$\Delta E \sim m_Q \alpha_S^2$

$\sim \frac{m_Q \alpha_S}{C^{2/3}}$

$\sim \frac{m_Q \alpha_S}{C}$

$\sim m_Q$

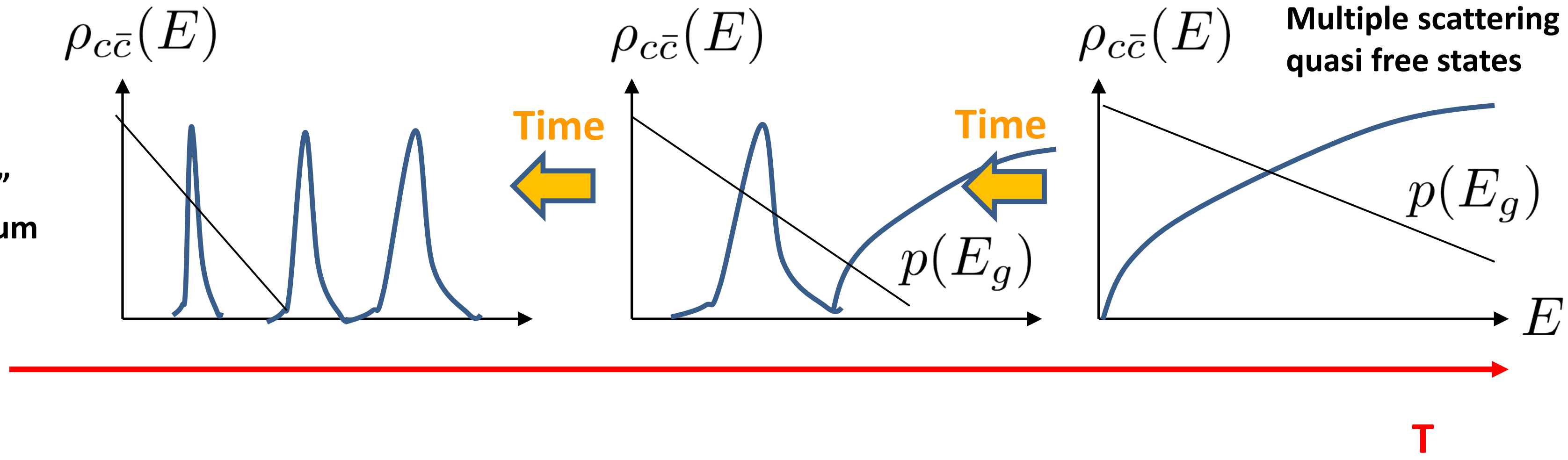
≡ The intermediary regime.

Quantum Brownian regime

Relativistic

For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential => larger distance => larger decoherence ….

dissociation of well identified levels by scarce "high-energy" modes (dilute medium => cross section ok)



Multiple scattering on quasi free states

In // : continuous evolution of the $Q\bar{Q}$ spectral function

Two types of dynamical modelling

$m_D \ll E_{\text{bind}}$

Quantum Optical Regime

- **Well identified resonances**
- Time long enough wrt quantum decoherence time (once we reach this regime)

Good description with transport models (TAMU, Tsinghua, Duke)

Central quantities :
2->2 and 2->3 Cross sections,
decay rates

Equilibrium : exp(-E_n/T) (theorem)

SC Approx: rate equations

$m_D \sim E_{\text{bind}}$

?

$m_D \gg E_{\text{bind}}$

Quantum Brownian Motion

- Correlations growing with cooling QGP
- **Best described in position-momentum space**
- Time short wrt quantum decoherence time ?

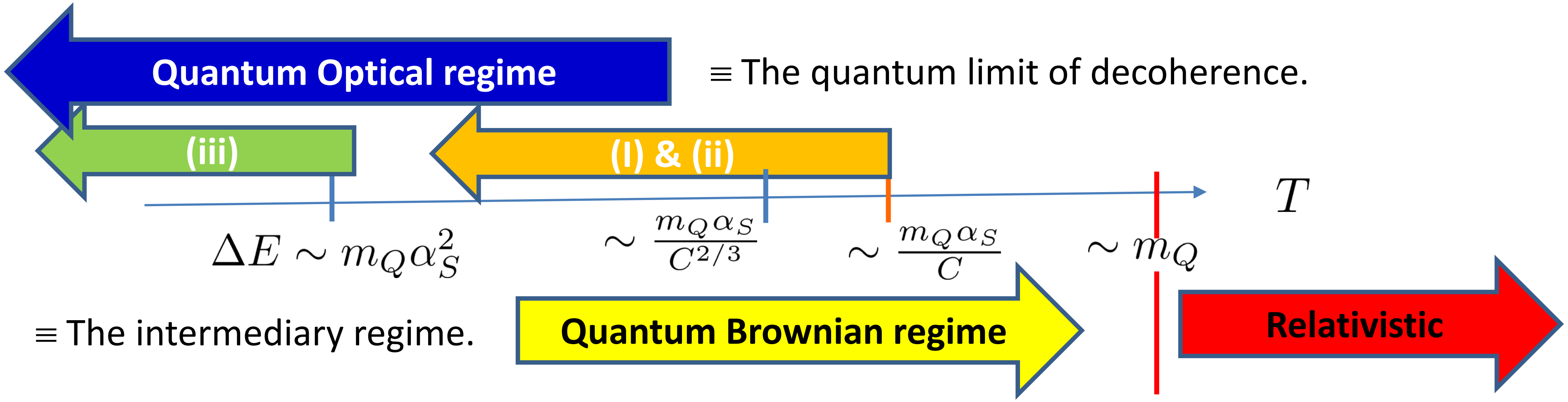
Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt* : some limiting cases

SC Approx: Fokker-Planck equations in position-momentum space

* Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these models is an important prerequisite !!!

QCD Temperature scales



Quantum Optical regime ≡ The quantum limit of decoherence.

(iii)

(I) & (ii)

T

$\Delta E \sim m_Q \alpha_S^2$ $\quad \sim \frac{m_Q \alpha_S}{C^{2/3}}$ $\quad \sim \frac{m_Q \alpha_S}{C}$ $\quad \sim m_Q$

≡ The intermediary regime. **Quantum Brownian regime**

Relativistic

Refined subregimes when playing with the scales of NRQCD / pNRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo, M Strickland et al, Yao, Müller and Mehen,…)

NRQCD: $Mv, \Lambda_{\rm QCD}, T \ll \mu_{\rm NR} \ll M$: most general scheme for markovian OQS !

pNRQCD:

(Singlet and octet quarkonium fields)

- $\boxed{\text{(i)}\ 1/r \gg T \sim m_D \gg E}$: « strongly coupled » QME same as small dipole limit of NRQCD (applies for small time evolution) :
- $\text{(ii)}\ 1/r \gg T \gg E \gg m_D$: « weakly coupled » : g T << T : essential contribution is gluo – dissociation from hard mode T : does not apply in QCD
- $\boxed{\text{(iii)}\ 1/r \gg T \sim E \gg m_D}$: Quantum optical regime

Recent OQS implementations (single $Q\bar{Q}$ pair)

regime	SU3 ?	Dissipation ?	3D / 1D	Num method	year	remark	ref
NRQCD ⇔ QBM	No	No	1D	Stoch potential	2018		Kajimotoet al. , Phys. Rev. D 97, 014003 (2018), 1705.03365
	Yes	No	3D	Stoch potential	2020	Small dipole	R. Sharma et al Phys. Rev. D 101, 074004 (2020), 1912.07036
	Yes	No	3D	Stoch potential	2021		Y. Akamatsu, M. Asakawa, S. Kajimoto (2021), 2108.06921
	No	Yes	1D	Quantum state diffusion	2020		T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293
	Yes ✓	Yes ✓	1D	Quantum state diffusion	**2021**		Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402
	No	Yes	1D	Direct resolution	**2021**		O. Ålund, Y. Akamatsu et al, Comput. Phys. 425, 109917 (2021), 2004.04406
	Yes ✓	Yes ✓	1D	Direct resolution	**2022**		S Delorme et al, https://inspirehep.net /literature/ 2026925
pNRQCD (i)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D96, 034021 (2017), 1612.07248
(i) Et (ii)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515
(i)	Yes	No	Yes	Quantum jump	**2021**	See SQM 2021	N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222
(i)	Yes ✓	Yes ✓	Yes ✓	Quantum jump	**2022**		N. Brambilla et al. 2205.10289
(iii)	Yes ✓	Yes ✓	Yes ✓	Boltzmann (?)	2019		Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027
NRQCD & « pNRQCD »	Yes	Yes	1D	Quantum state diffusion	**2022**		Miura et al. http://arxiv.org/abs/2205.15551v1
Other	No	Yes	1D	Stochastic Langevin Eq.	2016	Quadratic W	Katz and Gossiaux

(Year > 2015)

Not exhaustive

See as well table in 2111.15402v1

…

Quantum Brownian Motion : The Blaizot-Escobedo QME

See S. Delorme's talk @ SQM

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'} \mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

External "ingredients" : complex potential V + I W

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms** :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

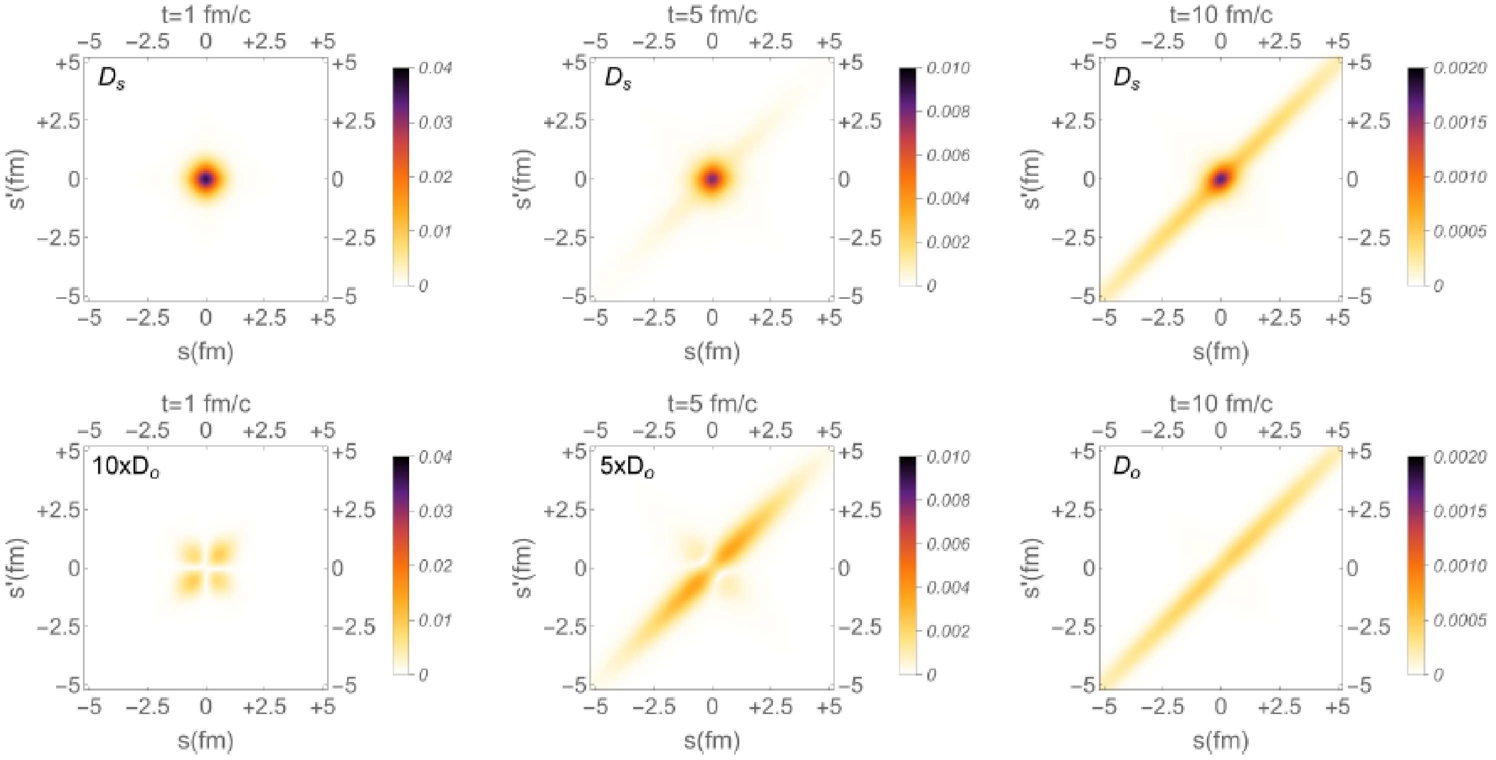
$\mathcal{L}_4$

Application to QED-like and QCD for both cases of 1 body and 2 body densities…

Quantum Brownian Motion : The Blaizot-Escobedo QME

See S. Delorme's talk @ SQM

$c\bar{c}$ evolution at fixed temperature



- Initial singlet in-medium 1S state at T = 300 MeV
- Octet populated via dipolar transitions
- Repulsive octet potential $\Rightarrow$ delocalization
- Delocalization in singlet channel via transitions
- Surviving central peak in singlet channel
- Non-diagonal elements (width equal to $\lambda_{th} = \frac{1}{\sqrt{MT}}$)

Quantum Brownian Motion : The Blaizot-Escobedo QME

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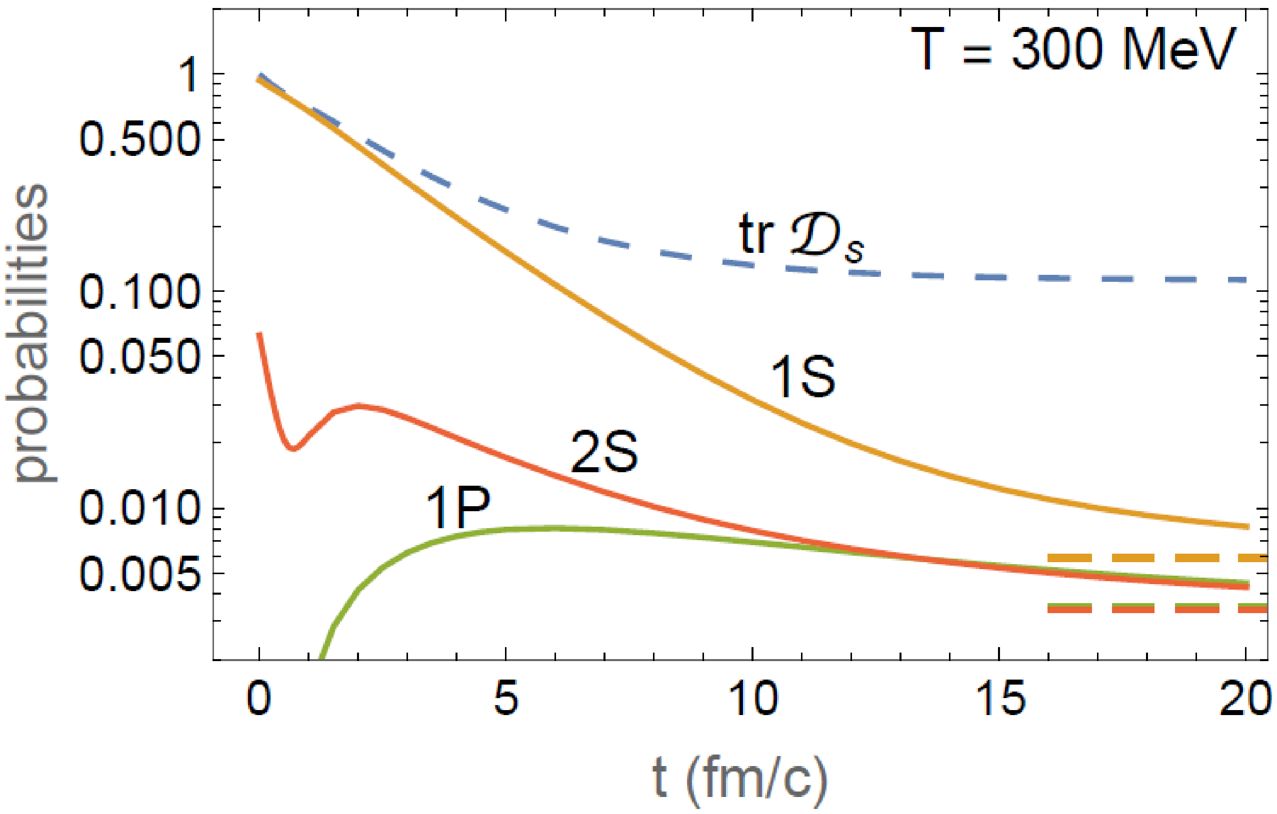
$c\bar{c}$ evolution at fixed temperature



- Instantaneous projections on vacuum eigenstates
- In-medium 1S state very close to vacuum ($p_{1S,\mathrm{v}}(0) \approx 0.95$)
- Complex evolution of p_{2S} (coupling to other states + decay to continuum)
- Delayed appearence of 1P states (chain of transitions at 3rd order in perturbation theory)
- Global evolution towards asymptotic values (dashed horizontal lines)

A consistent picture emerging in the bottomia sector

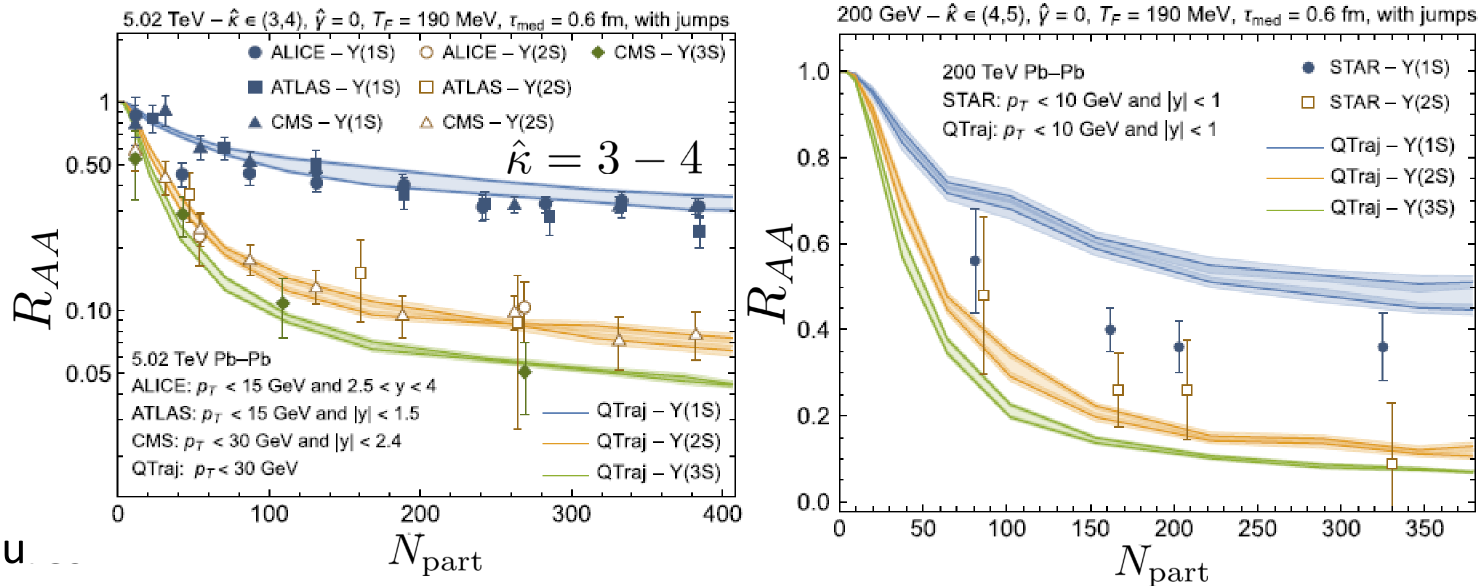
Beauty sector: good overall consistency of the following facts:

- Similar production of Υ(1S) from RHIC -> LHC
- Higher states strongly suppressed
- Washing out of the spectral function (but the Υ(1S) which survive up to T = 0.45 GeV)

Not paying too much attention at CNM effects:

With the interpretation that higher states (which contribute to the prompt Υ(1S)) are suppressed both at RHIC and LHC in the QGP, while the gound state Υ(1S) survive and is thus a genuine hard QGP probe; higher states could be produced (partly) through recombination. Especially true for Υ(3S)

N.B.: No precise v_2(Υ) measured up to now. One would expect very small v_2(Υ(1S)) and slightly larger v_2(Υ(2S))… but will be hard to measure.



M. Strickland & S. Thapa, Phys. Rev. D 108, 014031 (2023)

Good agreement with suppression at LHC but not at RHIC

Other implementations : Osaka, Saclay, Nantes, Duke,…

Back on the charmonia side

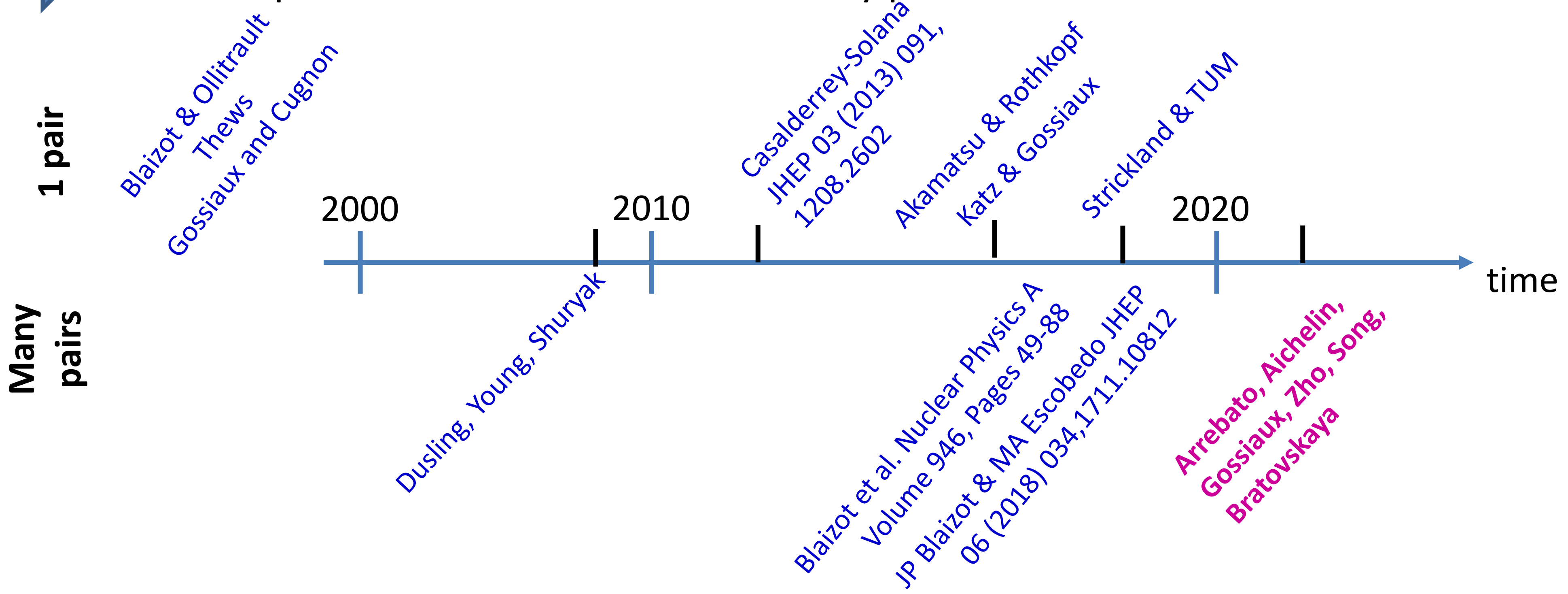
- More on the T >> M v^2 side and even T >> Mv side => pNRQCD is not right theory
- Besides, r* T may be >> 1 => not weak coupling to the QGP either

NRQCD should be privileged over pNRQD… or inspired models

Go microscopic in c and $\bar{c}$

Extra complication: For RHIC and LHC : many pairs !

Probes another aspect of
Quarkonium production in QGP !



1 pair

Blaizot & Ollitrault
Thews
Gossiaux and Cugnon

Casalderrey-Solana
JHEP 03 (2013) 091,
1208.2602

Akamatsu & Rothkopf
Katz & Gossiaux

Strickland & TUM

2000
2010
2020
time

Many pairs

Dusling, Young, Shuryak

Blaizot et al. Nuclear Physics A
Volume 946, Pages 49-88

JP Blaizot & MA Escobedo JHEP
06 (2018) 034,1711.10812

Arrebato, Aichelin,
Gossiaux, Zho, Song,
Bratovskaya

Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs (NRQCD) => mixed Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

Back on the charmonia side

- **Arrebato et al.** (2206.01308) : new microscopic model inspired by OQS principles and Remler method

$$\mathrm{prob}^{\Psi}(t) = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\hat{\rho}_N(t))\right]$$

E.A. Remler, ANNALS OF PHYSICS 136, 293-316 (1981)

Single quarkonia density operator $\hat{\rho}^{\Psi_i}_{Q\bar{Q}} = \sum_i |\Psi^i_{Q\bar{Q}}\rangle\langle\Psi^i_{Q\bar{Q}}|$

N-body density matrix (bulk partons + many c and many cbar)

Reformulation :

$$\mathrm{prob}^{\Psi}(t) = \mathrm{prob}^{\mathrm{prim}}(t_0) + \int_{t_0}^{t} \Gamma^{\Psi}(t^{'})dt^{'}$$

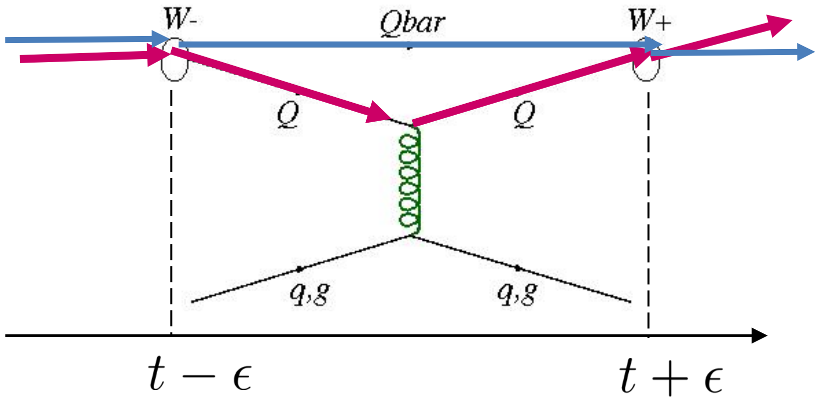
Von Neumann eq.

With rate of creation/destruction: $\Gamma^{\psi}(t) = \frac{d\mathrm{prob}^{\Psi}(t)}{dt} = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\frac{d\hat{\rho}_N(t)}{dt}\right]$

See talk T. Song @ SQM 2024

Passing to Wigner representation and using semi-classical trajectories for **Q** :

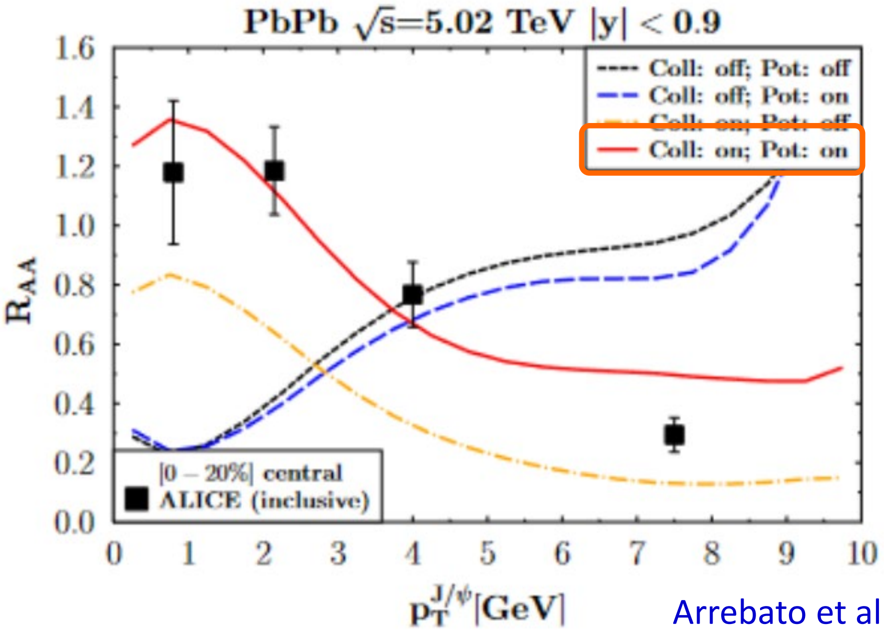
$$\Gamma^{\Psi}(t) = \sum_{i=1,2}\sum_{j\geq 3}\delta(t-t_{ij})\int \frac{d^3p_i d^3x_i}{h^3} W^{\Psi}_{Q\bar{Q}}(p_1,x_1;p_2,x_2)\left[W_N(t+\epsilon) - W_N(t-\epsilon)\right]$$

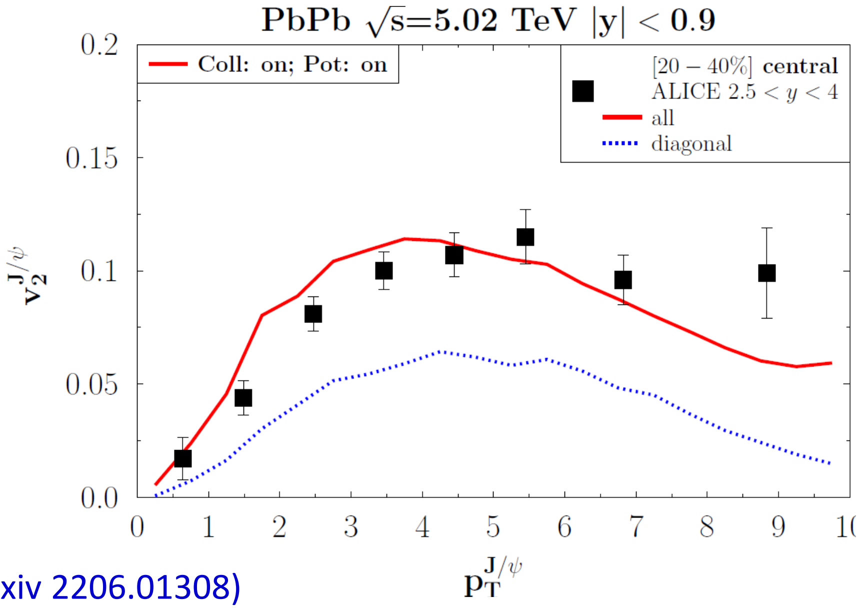


- Quarkonia production in the model is a 3-body process, the c & cbar interact only by collision !
- The “details” of H_{int} between Q and bulk partons are incorporated into the evolution of W_N after each collision / time step (good for the MC simulation)
- Dissociation and recombination treated in the same scheme

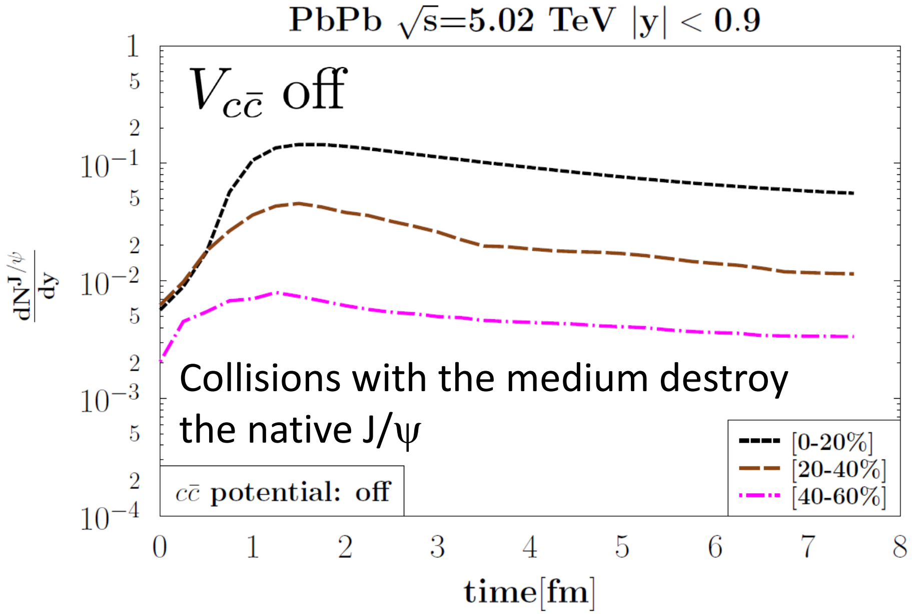
+ state of the art c-cbar potential D. Lafferty and A. Rothkopf, PHYS. REV. D 101, 056010 (2020)

Some results from one recent microscopic model

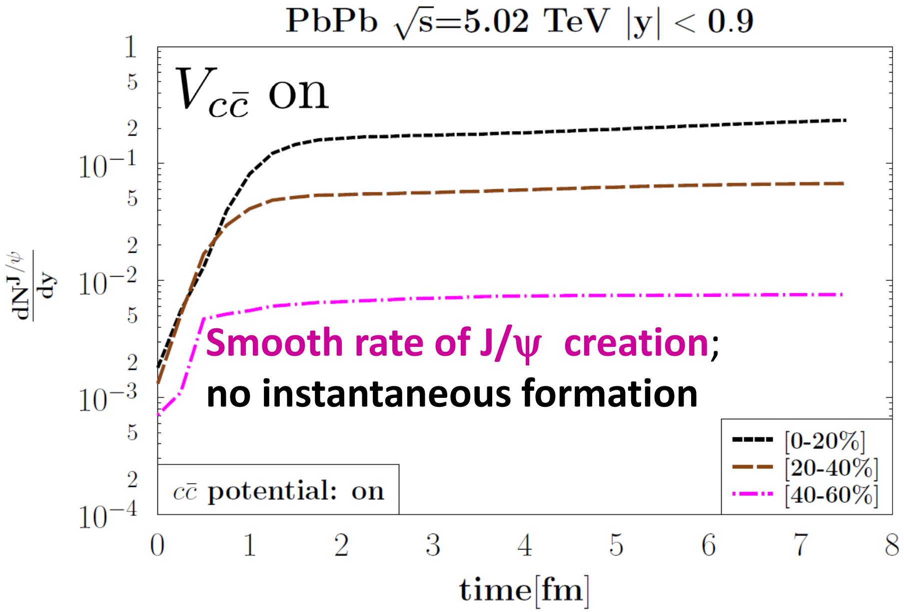




Arrebato et al. (arxiv 2206.01308)



Collisions with the medium destroy the native J/ψ



Smooth rate of J/ψ creation; **no instantaneous formation**

The present challenges for Quarkonium modelling in URHIC

Meet the higher and higher precision of experimental data (already beyond the present model uncertainties)

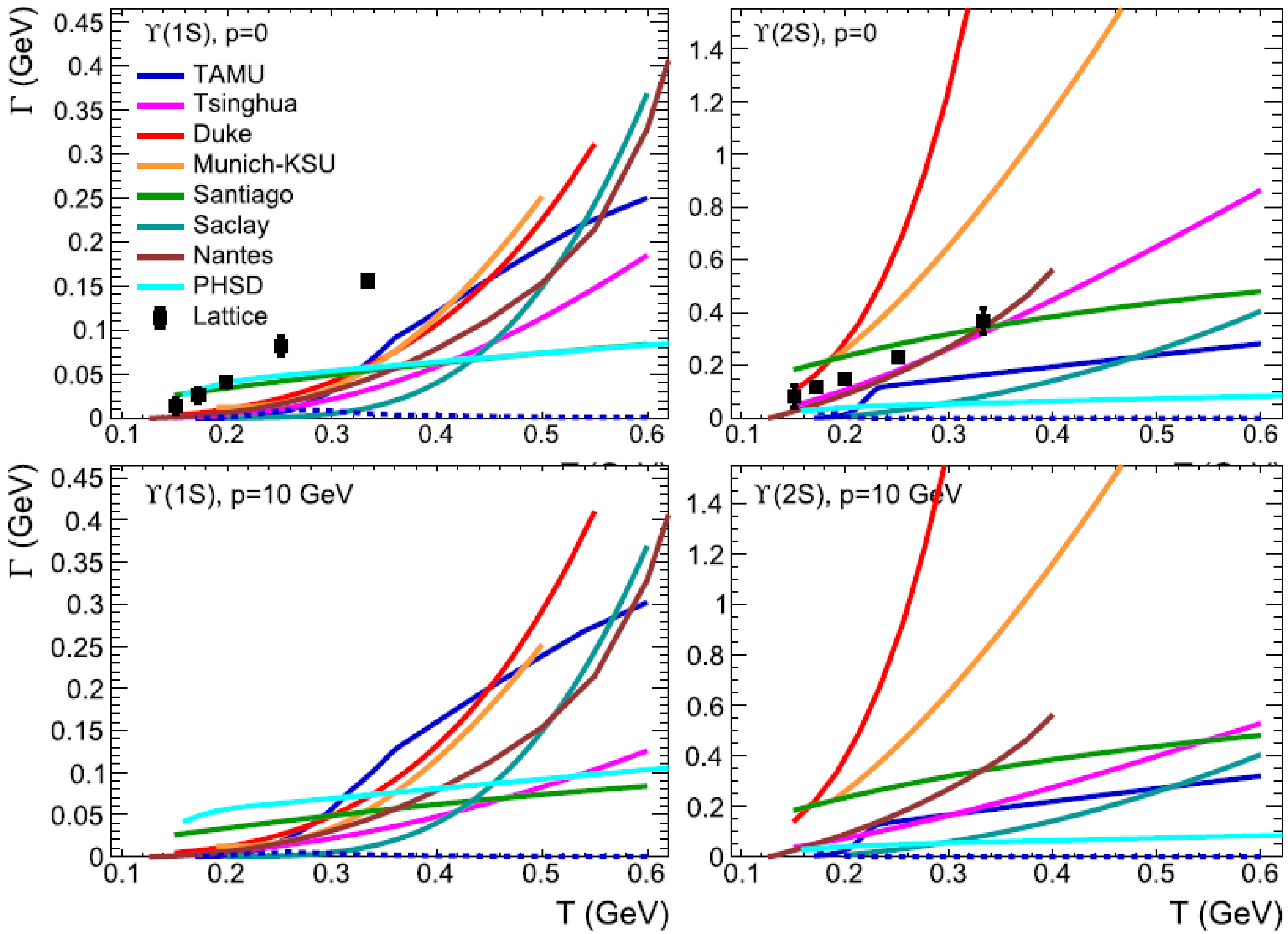
Unravel the Q-Qbar interactions under the influence of the surrounding QGP and with the QGP

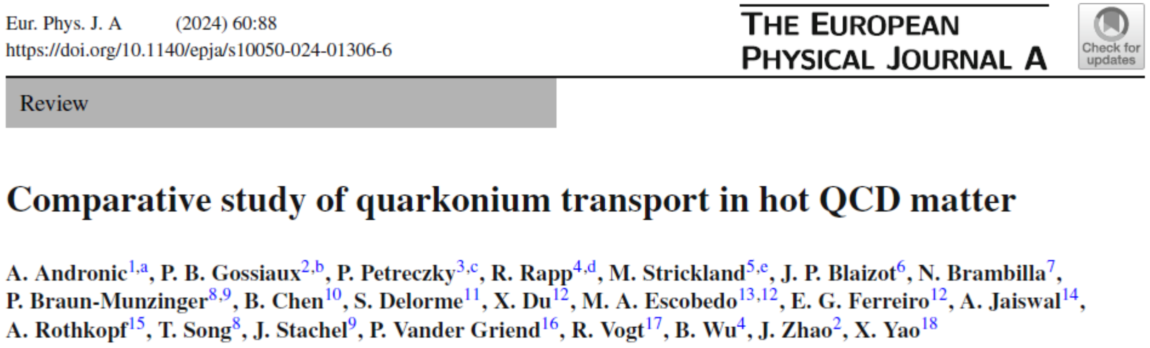
Develop a scheme able to deal with the evolution of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all fundamental principles (quantum features, gauge invariance, equilibration,…)

Need for lQCD constraints / inputs

Ultimately, go beyond the "one team – one model" paradigm

Collective work on quarkonia in AA





Eur. Phys. J. A (2024) 60:88
https://doi.org/10.1140/epja/s10050-024-01306-6

THE EUROPEAN PHYSICAL JOURNAL A

Check for updates

Review

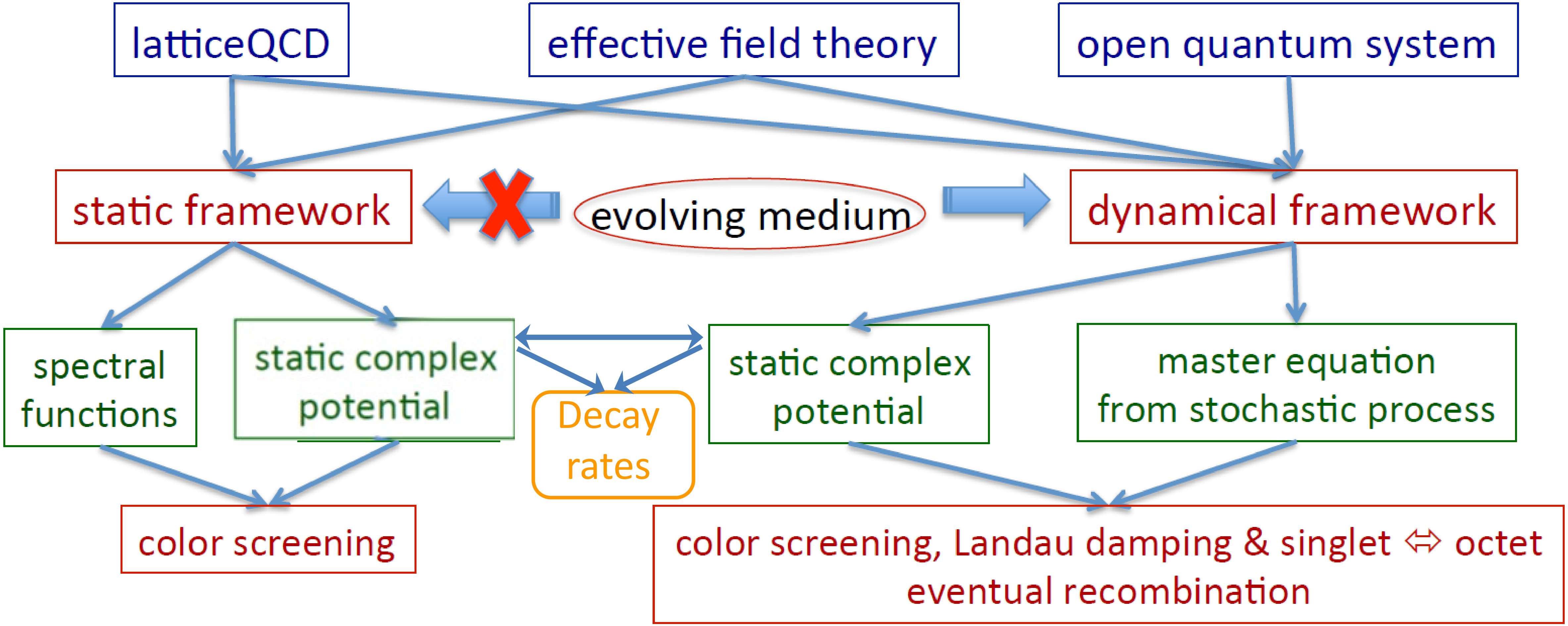
Comparative study of quarkonium transport in hot QCD matter

A. Andronic[1,a], P. B. Gossiaux[2,b], P. Petreczky[3,c], R. Rapp[4,d], M. Strickland[5,e], J. P. Blaizot[6], N. Brambilla[7], P. Braun-Munzinger[8,9], B. Chen[10], S. Delorme[11], X. Du[12], M. A. Escobedo[13,12], E. G. Ferreiro[12], A. Jaiswal[14], A. Rothkopf[15], T. Song[8], J. Stachel[9], P. Vander Griend[16], R. Vogt[17], B. Wu[4], J. Zhao[2], X. Yao[18]

- ➢ A lot of diversity in the models… including those which are compatible with the experimental data
  - Underlying binding force between Q & Qbar
  - Binding energy
  - Whether, on the top of dissociation, some « melting » is allowed
  - ….
- ➢ Larger diversity for finite momentum.
- ➢ Some tension with the lattice calculations (R. Larsen et al., Phys. Lett. B **800**, 135119 (2020), arXiv:1910.07374 [hep-lat])

Global picture (slightly adapted from E. Ferreiro; QM 2018)

Caveat I: we need firm theoretical understanding of quarkonium production in pp collisions



latticeQCD

effective field theory

open quantum system

static framework

evolving medium

dynamical framework

spectral functions

static complex potential

Decay rates

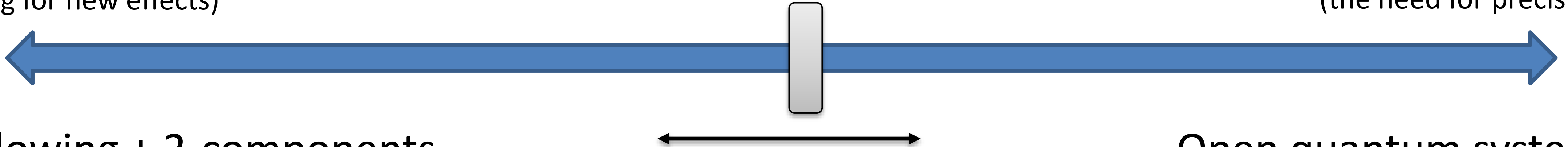
static complex potential

master equation from stochastic process

color screening

color screening, Landau damping & singlet ⇔ octet
eventual recombination

Global picture for quarkonia: the 2 faces of “theory”



Simple

(searching for new effects)

Sophisticated

(the need for precision physics)

Shadowing + 2-components model (comovers + statistical generation at FO)

Open quantum system with $H_{\text{interaction}}$ tuned on lQCD constrains

Where should we position the « cursor » in order to claim hard probing QGP with quarkonia ?