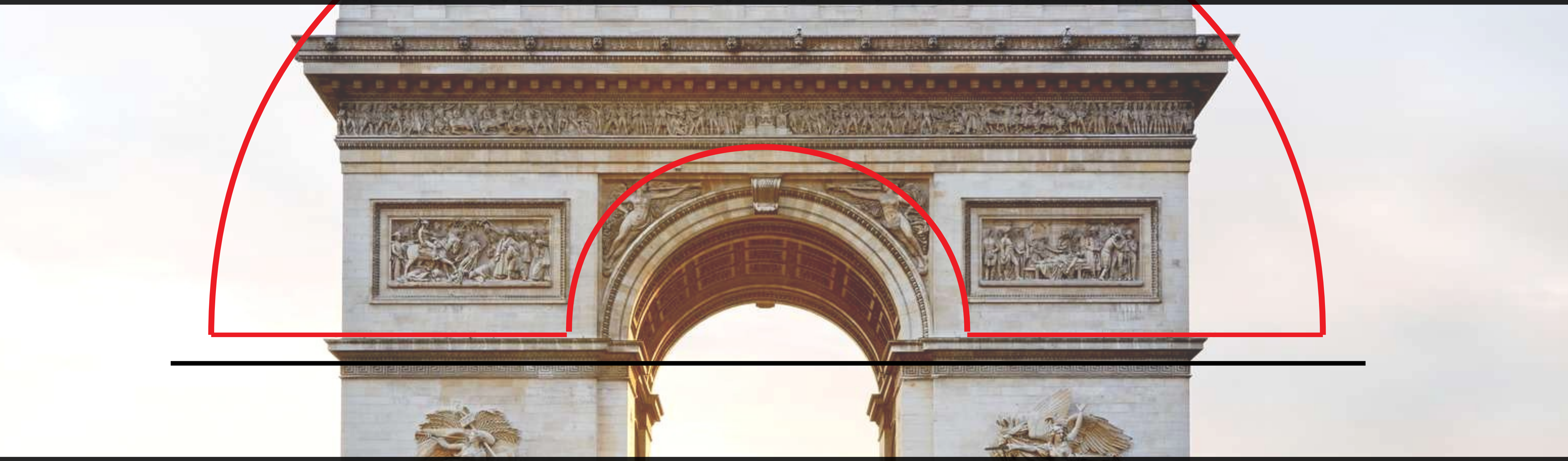
Positiveness and Collider Phenomenology



Marc Riembau
CERN

Rencontres de Physique des Particules, Paris, 25th January 2024

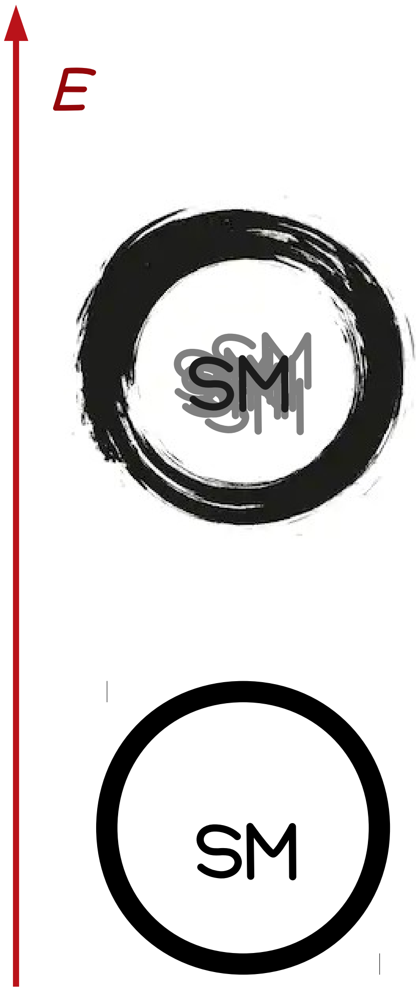
$$\mathcal{L}\,?$$

E

SM

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}}$$

$$\mathcal{L}\,?$$



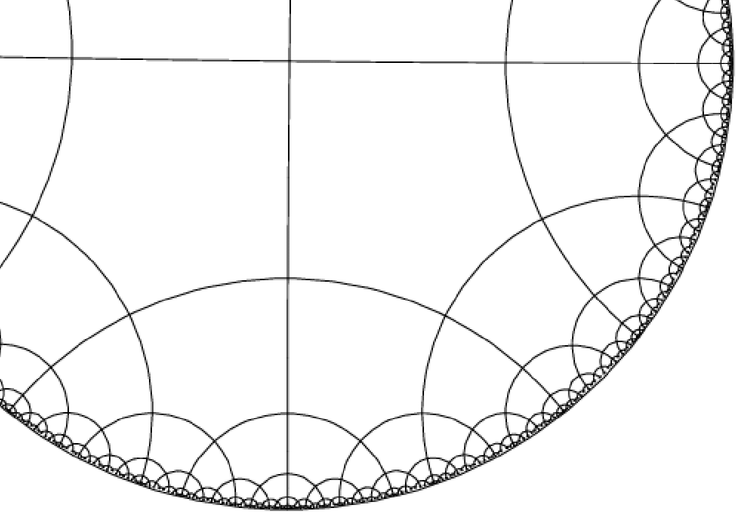
$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i}{\Lambda}\mathcal{O}_i$$

EFT operators encode information about the heavy dynamics, and tells us in which way the SM is deformed.

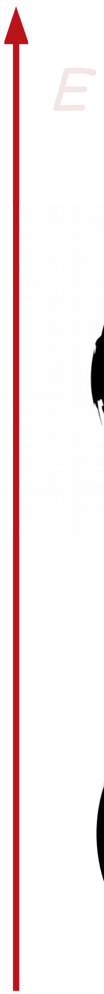
What is the space of allowed deformations?

How to chart this space and map it to microscopic dynamics?

$$\mathcal{L} = \mathcal{L}_{\rm SM}$$



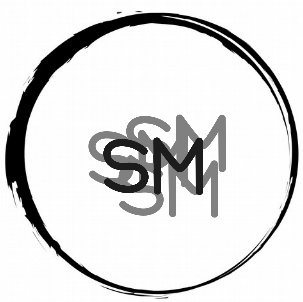
$\mathcal{L}\,?$



We provide a complete answer for a subclass of UV dynamics:

for Universal Theories

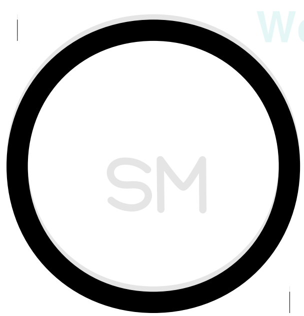
EFT operators encode information about the heavy dynamics, and tells us in which way the SM is deformed.



What is the space of allowed deformations?

How to chart this space and map it to microscopic dynamics?

We will provide a complete answer for a subclass of UV dynamics: for Universal Theories



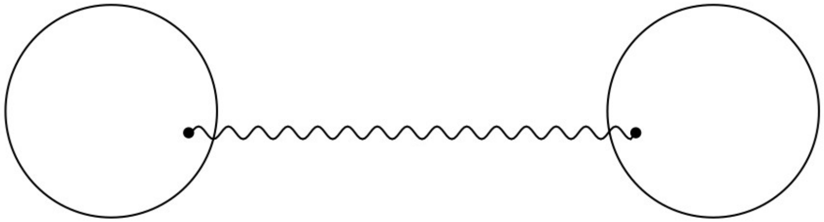
$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}}$$

Universal theories

Universal theories

McCullough, Ricci, MR ‘23

Universal theory: two sectors connected by gauging a common global symmetry



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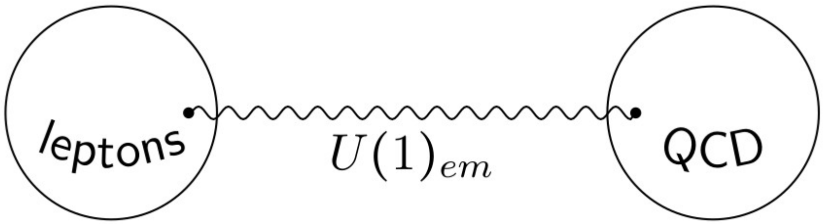

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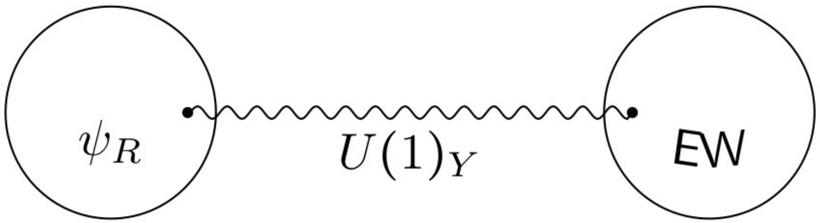

Universal theories

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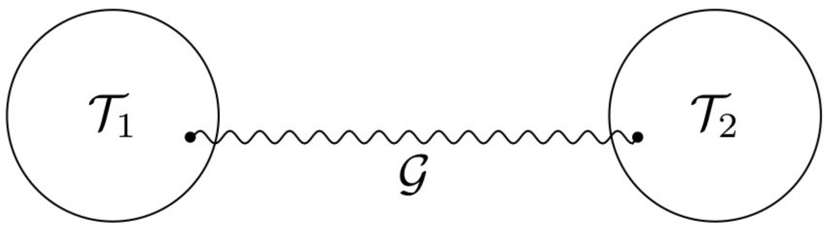
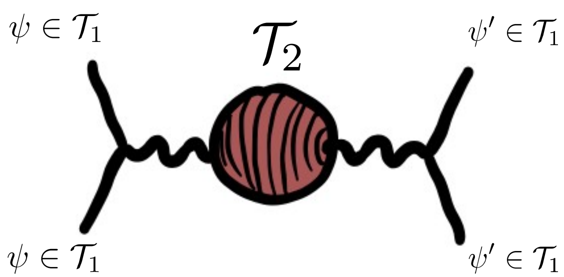
Universal theory: two sectors connected by gauging a common global symmetry



In this talk, T1 is a gapless weakly coupled theory, while T2 is a gapped, possibly strongly coupled sector

“Universal” because existence of T2 encoded in vector selfenergies:



Universal theories

- Below the T2 threshold, vector 2-point function modified by local operators:

$$\Pi_{ij}(q^2) = i\int d^4x e^{iqx}\langle 0|T J_i^\mu(x) J_j^\nu(0)|0\rangle$$
$$= \int_0^\infty d\mu^2 \frac{q^\mu q^\nu - q^2\eta^{\mu\nu}}{q^2-\mu^2+i\epsilon}\rho_{ij}^T(\mu^2)$$

$$\mathcal{L} \supset \frac{c_{2F}}{\Lambda^2}(D_\mu F_{\mu\nu})^2$$

$$\rho_{ij} \sim \int_\alpha \langle 0|J_i|\alpha\rangle\langle\alpha|J_j|0\rangle \succ 0$$

Spectral density is a
positive matrix in *flavour* space

$$c_n \sim \Pi^{(n)} = \frac{1}{\pi}\int_{m_\star^2}^\infty \frac{ds}{s}\frac{\mathrm{Im}\Pi(s)}{s^n}$$

Coefficients of local operators related to the moments of a positive definite matrix distribution

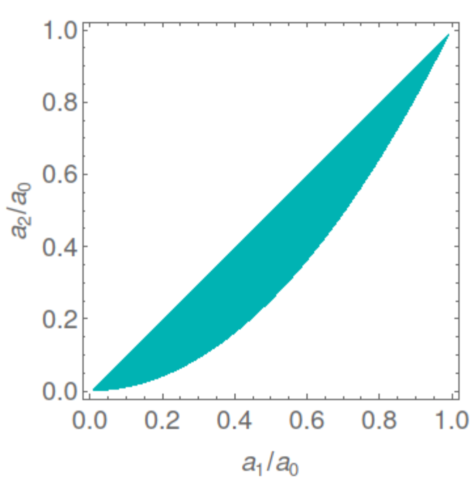
The sequence of matrices $\{\Pi^{(1)}, \Pi^{(2)}, \dots, \Pi^{(n)}\}$ can be identified as moments of a positive definite measure if and only if

$$H^1 \succ 0\,, \quad H^2 \succ 0\,, \quad H^1 - H^2 \succ 0\,, \quad H^2 - H^3 \succ 0$$

where H^k is the Hankel matrix of moments, $(H^k)_{ij} = \Pi^{(i+j+k-1)}$.

$$a_n = \int_0^1 d\mu(x) x^n\,, \quad d\mu(x) > 0$$

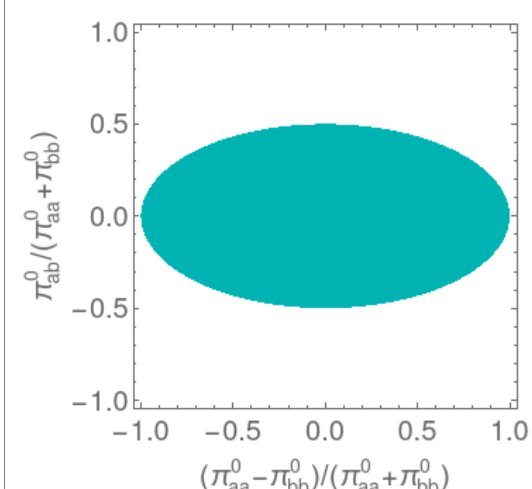
- $a_0 > 0$
- $a_0 - a_1 > 0$
- $\begin{pmatrix} a_0 & a_1 \\ a_1 & a_2 \end{pmatrix} \succ 0$



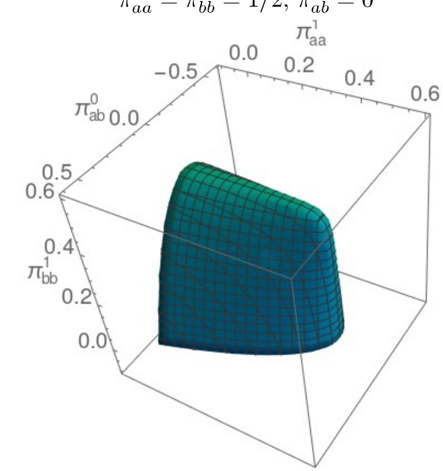
$$\Pi_n = \int_0^1 dM(x) x^n\,, \quad dM(x) \succ 0$$

Ex: M is 2x2

- $\Pi_0 = \begin{pmatrix} \pi^0_{aa} & \pi^0_{ab} \\ \pi^0_{ab} & \pi^0_{bb} \end{pmatrix} \succ 0$
- $\Pi_0 - \Pi_1 = \begin{pmatrix} \pi^0_{aa} - \pi^1_{aa} & \pi^0_{ab} - \pi^1_{ab} \\ \pi^0_{ab} - \pi^1_{ab} & \pi^0_{bb} - \pi^1_{bb} \end{pmatrix} \succ 0$
- $\begin{pmatrix} \Pi_0 & \Pi_1 \\ \Pi_1 & \Pi_2 \end{pmatrix} = \begin{pmatrix} \pi^0_{aa} & \pi^0_{ab} & \pi^1_{aa} & \pi^1_{ab} \\ \pi^0_{ab} & \pi^0_{bb} & \pi^1_{ab} & \pi^1_{bb} \\ \pi^1_{aa} & \pi^1_{ab} & \pi^2_{aa} & \pi^2_{ab} \\ \pi^1_{ab} & \pi^1_{bb} & \pi^2_{ab} & \pi^2_{bb} \end{pmatrix} \succ 0$



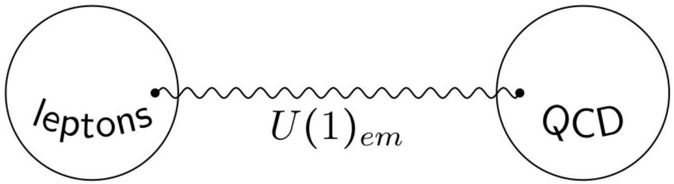
$\pi^0_{aa} = \pi^0_{bb} = 1/2,\ \pi^1_{ab} = 0$



Within the Standard Model

Within the SM (I): QCD

- QCD is an instance of a universal strongly coupled sector



Positivity follows from the textbook discussion of the vacuum polarization

$$\mathcal{L} \supset -\frac{c_{2F}}{\Lambda^2} D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}$$

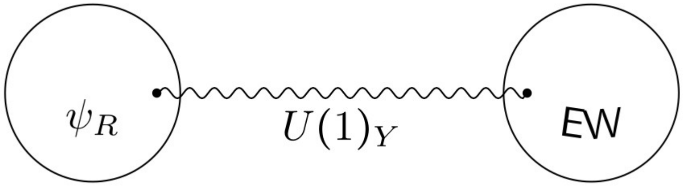
$$\mathrm{Im}\Pi(s) = s\sigma(e^+e^- \to \mathrm{hadrons})$$

$$c_{2F} \sim \int_{4m_\pi^2}^{\infty} \frac{ds}{s} \frac{\mathrm{Im}\Pi(s)}{s} > 0$$

Moreover, the moment problem guarantees that this is the largest Wilson coefficient and dominates the effects at low energies

Within the SM (II): EW, universal case

- Assume now only right-handed fermions, coupled to EW via hypercharge.



$$\mathcal{L} \supset (Y_{e_R} J^{\mu}_{e_R} + Y_{q_R} J^{\mu}_{q_R})(g' c_W A_\mu - g' s_W Z_\mu)$$

$$A_\mu \to A_\mu + t_W Z_\mu$$

$$\mathcal{L} \supset (Y_{e_R} J^{\mu}_{e_R} + Y_{q_R} J^{\mu}_{q_R}) g' c_W A_\mu \, + \, t_W D_\mu F_{\mu\nu} Z_\nu$$

A field redefinition makes clear that the theory is universal, i.e. the right handed fermions do not couple directly to the *strong sector*, only through a mixing of the vectors.

Within the SM (II): EW, universal case

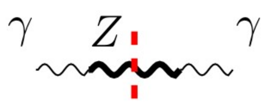
- The mixing induces a manifestly *positive* coefficient

$$\to \quad \frac{e^2}{g^2 c_W^2} m_Z^2 \delta(q^2 - m_Z^2)$$

$$\mathcal{L} \supset -\frac{1}{2m_Z^2}\frac{e^2}{g^2 c_W^2} D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}$$

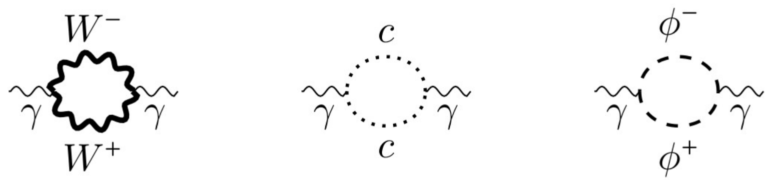
Within the SM (II): EW, universal case

- The mixing induces a manifestly *positive* coefficient



$$\rightarrow \frac{e^2}{g^2 c_W^2} m_Z^2 \delta(q^2 - m_Z^2) \qquad \Rightarrow \qquad \mathcal{L} \supset -\frac{1}{2m_Z^2}\frac{e^2}{g^2 c_W^2} D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}$$

- At one loop,



$$\mathrm{Im}\Pi = -\frac{1}{3}\frac{e^2}{64\pi}\sqrt{1-y}\,3(7+y)\,, \quad y = 4m_W^2/q^2 \leq 1$$

$$c_{2F} \propto \frac{1}{\pi}\int_0^1 dy \frac{\sqrt{1-y}}{64\pi}(7+y) = -\frac{1}{(4\pi)^2}\frac{37}{30}$$

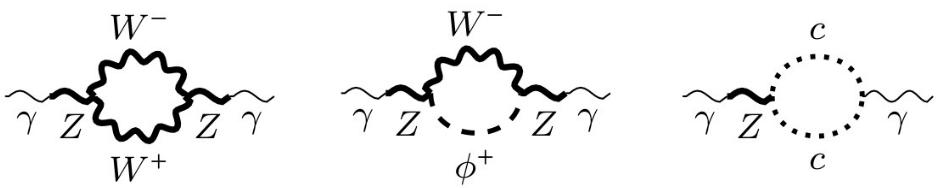
In agreement with `[Henning et.al. 1412.1837]` and `[Quevillon et.al. 1810.06994]`,

BUT NEGATIVE!!

Within the SM (II): EW, universal case

- Resolution in the *universal frame:*

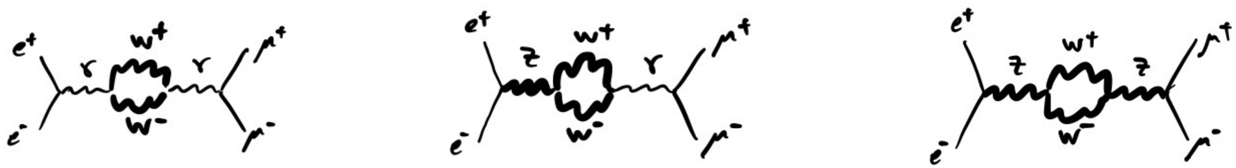
  the mixing requires new contributions at the same order in the expansion:



$$\mathrm{Im}\Pi = \frac{e^2}{3}\frac{\sqrt{1-y}}{64\pi}\frac{(1-y)(4+20y+3y^2)}{(4c_W^2-y)^2} \sim \sigma(e_R^+ e_R^- \to W^+W^-)$$

- Resolution in the standard *frame:*

  When computing physical quantities, like an amplitude, one gets a contribution from several operators:



$D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}$ $\quad$ $D_\mu F_{\mu\nu}\bar\psi\gamma^\nu\psi$ $\quad$ $\bar\psi\gamma_\mu\psi\,\bar\psi\gamma_\mu\psi$

  Only a combination enters in the physical amplitude, corresponding to the previous *positive* contribution.

- This gives an explicitly positive coefficient:

$$c_{2F} \sim \mathrm{Im}\Pi = \frac{e^2}{16\pi^2}\int_0^1 dy \frac{(1-y)^{3/2}(4+20y+3y^2)}{(4-y)^2} = \frac{e^2}{16\pi^2}\frac{8983-1650\sqrt{3}\pi}{15} \simeq \frac{e^2}{16\pi^2}\,0.31\ldots$$

Notice however the bizzarre cancelation between the rational and transcendental terms!

Within the SM (III): EW, non-universal case

- Now consider left-handed fermions.

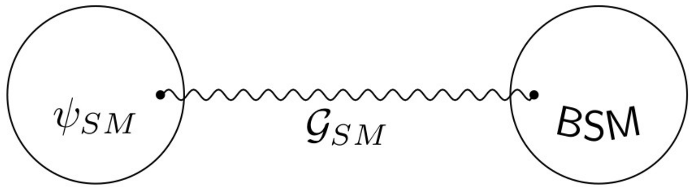
$$\mathcal{L} \supset eJ_e^\mu A_\mu + \frac{g}{2c_W}(J_e^\mu - J_\nu^\mu)Z_\mu + \frac{g}{\sqrt{2}}J_{e\nu}^\mu W_\mu^+ + h.c.$$

It is clear that this has not the form of a universal theory, so none of the discussion applies.

Beyond the Standard Model

Beyond the SM

Consider the microscopic description of the electroweak sector to be given by a universal theory:



The low energy dynamics are encoded in the so-called electroweak parameters:

Parameter	Form Factor	Operator
S	Δ'_{3B}	$\mathcal{O}_{WB} = H^\dagger \tau^a H W^a_{\mu\nu} B_{\mu\nu}$
X	Δ''_{3B}	$\mathcal{O}_X = H^\dagger \tau^a H D_\mu W^a_{\nu\rho} D_\mu B_{\nu\rho}$
W	Δ''_{33}	$\mathcal{O}_{WW} = -\frac{1}{2} D_\mu W^a_{\mu\nu} D_\rho W^a_{\rho\sigma}$
W^8	Δ'''_{33}	${}^{(8)}\mathcal{O}_{WW}$
Y	Δ''_{BB}	$\mathcal{O}_{BB} = -\frac{1}{2} D_\mu B_{\nu\rho} D_\mu B_{\nu\rho}$
Y^8	Δ'''_{BB}	${}^{(8)}\mathcal{O}_{BB}$
T	$\Delta_{33} - \Delta_{+-}$	$\mathcal{O}_T = \lvert H^\dagger D_\mu H\rvert^2$
U	$\Delta'_{33} - \Delta'_{+-}$	$\mathcal{O}_{T'} = \lvert H^\dagger D_\mu D_\nu H\rvert^2$
V	$\Delta''_{33} - \Delta''_{+-}$	$\mathcal{O}_{T''} = \lvert H^\dagger D_\mu D_\nu D_\rho H\rvert^2$
Z	Δ''_{gg}	$\mathcal{O}_{GG} = D_\mu G^A_{\nu\rho} D_\mu G^A_{\nu\rho}$
Z^8	Δ'''_{gg}	${}^{(8)}\mathcal{O}_{GG}$
ξ	Δ'_{HH}	$\mathcal{O}_H$ & $\mathcal{O}_r$
H	Δ''_{HH}	$\mathcal{O}_\Box = \Box H^\dagger \Box H$
H^8	Δ''_{HH}	${}^{(8)}\mathcal{O}_\Box$

Beyond the SM

- The simplest and most important consequence is the positivity of W and Y:

$$\mathcal{L} \supset -\frac{W}{2m_W^2} D_\mu W^a_{\mu\nu} D_\rho W^a_{\rho\nu} - \frac{Y}{2m_W^2} D_\mu B_{\mu\nu} D_\rho B_{\rho\nu}$$

$$W = \frac{g^2 m_W^2}{2\pi} \int_{m_\star^2}^{\infty} \frac{ds}{s} \frac{\mathrm{Im}\Pi_{33}(s)}{s}$$

$$W > 0 \qquad Y > 0$$

- Moreover, due to the convergence of the dispersion relation, they **must** dominate low energy dynamics

$$W - m_\star^2 W^{(8)} > 0\,, \mathrm{etc}\ldots$$

Beyond the SM

- The X parameter is given by

$$X = \frac{1}{2} g g' m_W^2 \Delta''_{3B}(0)$$

and is constrained by unitarity

$$\begin{pmatrix} W & X \\ X & Y \end{pmatrix} \succ 0 \quad \Rightarrow \quad W > 0\,, \quad Y > 0\,, \quad WY - X^2 > 0$$

Such constrain is automatically guaranteed in theories obeying a SILH-like power counting:

$$W = c_{2W} \frac{g^2 m_W^2}{g_\star^2 m_\star^2} \qquad Y = c_{2B} \frac{g'^2 m_W^2}{g_\star^2 m_\star^2}$$

$$X = c_{HWB} \frac{g g'}{m_\star^4} m_W^2 v^2 \simeq c_{HWB} \frac{g g'}{g_\star^2} \frac{m_W^2}{m_\star^2} \frac{v^2}{f^2}$$

$$\Rightarrow \quad c_{2W} c_{2B} > c_{HWB}^2 \left(\frac{v^2}{f^2} \right)^2$$

But it is possible to saturate it, see e.g. H-C Cheng, X-H Jiang, L Li, E Salvioni [2401.08785]

When positivity fails

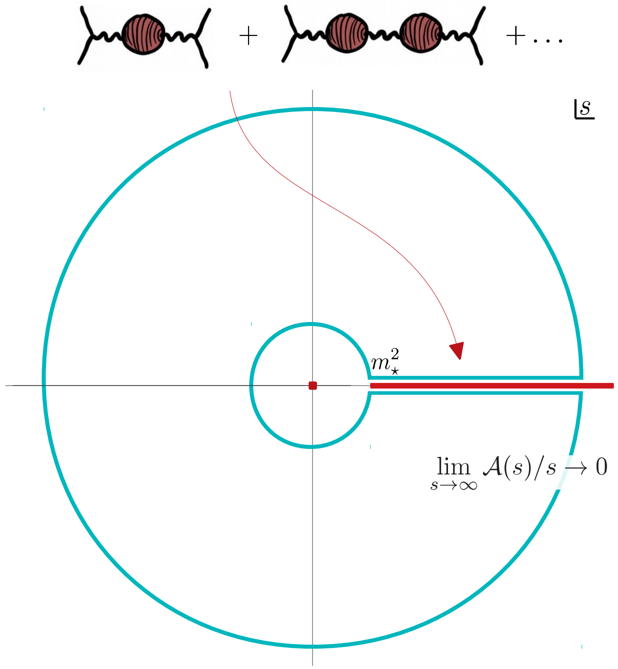
- So far we have assumed

  i) That the vector couples to a conserved current
  ii) That the gaugeless limit is a good limit

i) is crucial for convergence of the dispersion relation, while ii) is needed for gauge invariance

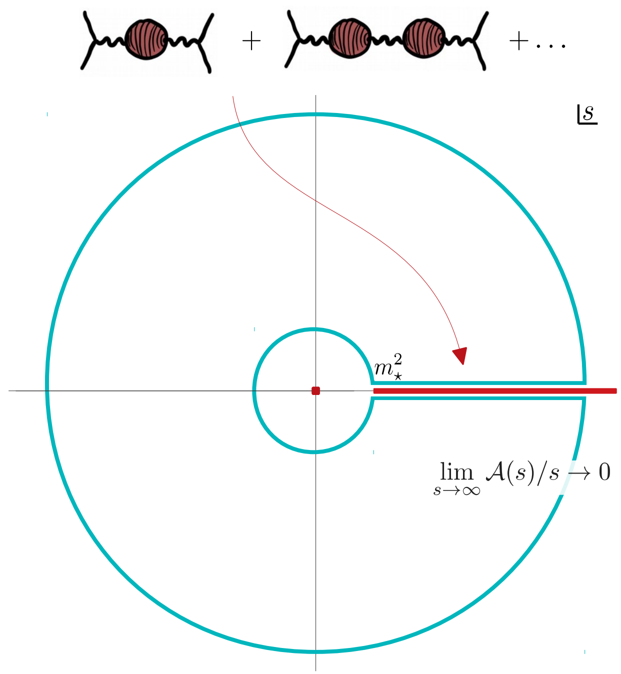
What if we lift such assumptions?

To make progress, we shall consider a physical quantity, like the scattering amplitude



When positivity fails

- As long as the vector exchange dominates, the scattering goes through a single partial wave:

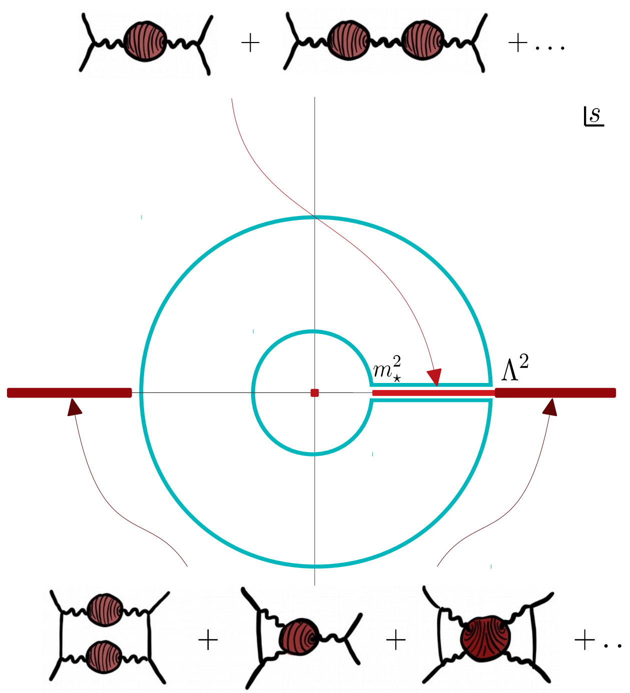


$$\mathcal{A}(s) = g^2 \frac{s}{s-m_W^2}\left(C + W\frac{s-m_W^2}{m_W^2} + \dots\right) \quad \Rightarrow \quad W = \frac{1}{g^2}\frac{m_W^2}{2\pi i}\int_{\mathcal{C}} \frac{dz}{z}\frac{\mathcal{A}(z)}{z} = \frac{1}{g^2}\frac{m_W^2}{\pi}\int_{m_\star^2}^{\infty}\frac{dz}{z}\frac{\mathrm{Im}\mathcal{A}(z)}{z}$$

Assumptions i) and ii) have the secret role of allowing us to neglect higher partial waves

When positivity fails

- Lift weak coupling requirement:



If EW coupling runs into strong coupling, higher partial waves might dominate.

$$W - \eta \frac{16\pi^2}{g^2}\frac{m_W^2}{\Lambda^2} = \frac{1}{g^2}\frac{m_W^2}{\pi}\int_{m_\star^2}^{\Lambda^2}\frac{dz}{z}\frac{\mathrm{Im}\mathcal{A}(z)}{z} \qquad \mathrm{Sign}(\eta) = \pm 1$$

So W is positive as long as contribution from large arc is smaller.

When positivity fails

$$W \sim \frac{m_W^2}{m_\star^2} \quad \eta \sim \mathcal{O}(1) \quad \Rightarrow \quad \Lambda > m_\star \frac{4\pi}{g} \to W > 0$$

- W parameter negative requires EW coupling to run into strong coupling not too far away from m_\star.

- A refinement including a sum over $\frac{\Lambda}{m_\star} \log \frac{\Lambda^2}{m_\star^2}$ partial waves leads to $\eta \sim \left(\frac{g^2(\Lambda)}{16\pi^2}\right)^2 \frac{\Lambda}{m_\star} \log \frac{\Lambda^2}{m_\star^2}$

  and both arcs become comparable at

$$\Lambda \sim m_\star \frac{g^2(\Lambda)}{16\pi^2} \frac{g^2(\Lambda)}{g^2}$$

It doesn’t modify the rough idea that negative W requires EW coupling to run into strong coupling relatively fast

Universal signatures below threshold

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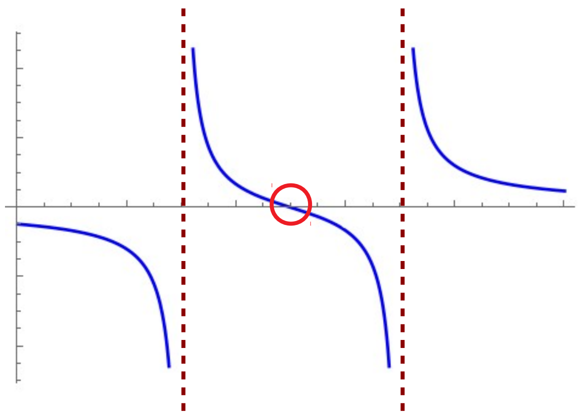
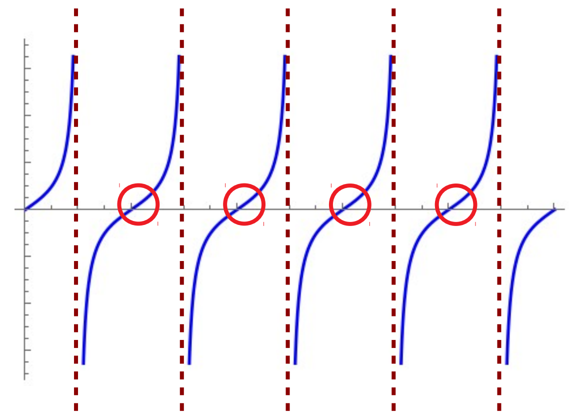
Universal signatures below threshold

- The particular structure of universal theories leads to further properties:
- Consecutive resonances are separated by a zero in the amplitude,

$$\mathcal{A} \sim \frac{g_1^2}{s - m_1^2} + \frac{g_2^2}{s - m_2^2}$$



$$\mathcal{A} \sim s\tan(s\pi/2)$$



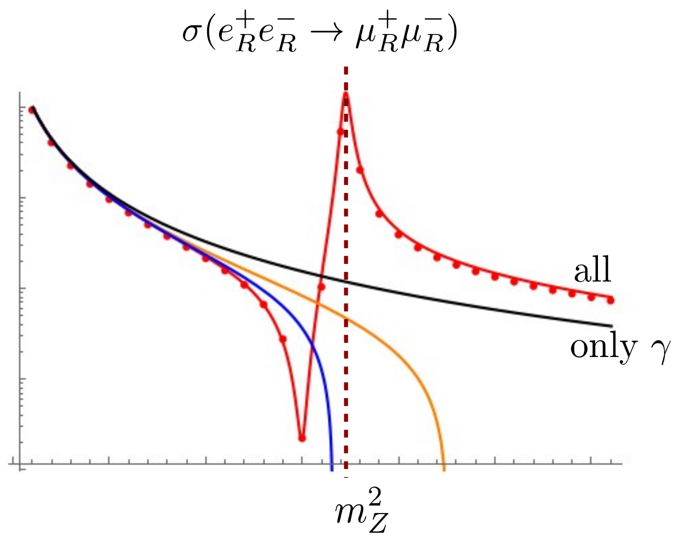
$$s_0 = \frac{g_1^2}{g_1^2 + g_2^2} m_2^2 + \frac{g_2^2}{g_1^2 + g_2^2} m_1^2$$

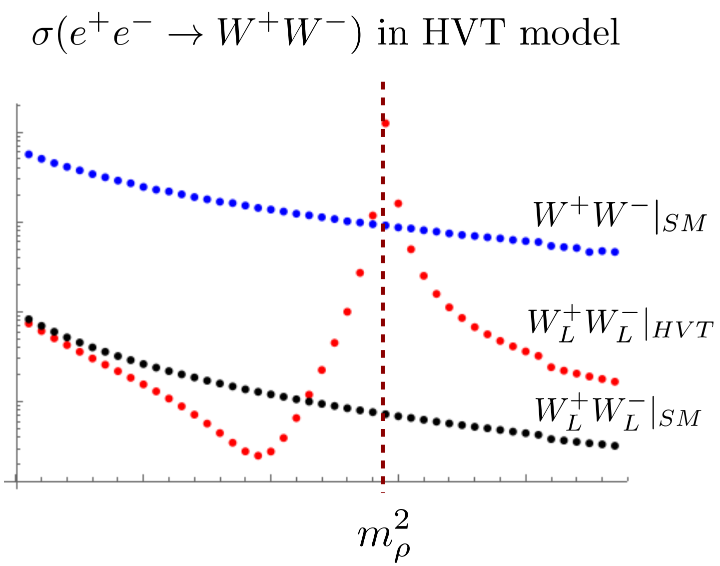
The zero is a convex sum (because the residues are positive!) of the masses, so always in between.

For $g_2^2 \gg g_1^2$ the zero is close to m_1^2.

Universal signatures below threshold

- The *same* dispersion relation used for the positivity can be used to conclude that universal theories have a zero below threshold.



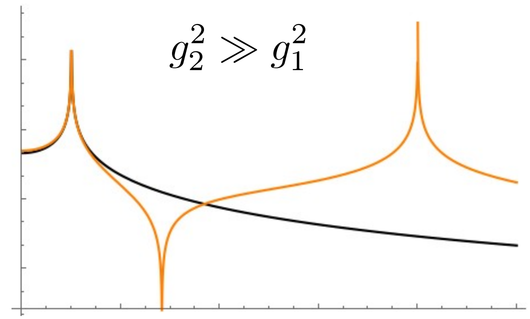


- Positivity of the W parameter implies *less* events in the tails of distributions, ending at a zero of the amplitude.

  Phenomenologically, a zero in the amplitude is an *antiresonance* or *resonance of Nothing*, and perhaps should be looked for, like usual resonances.

  It is a feature of universal theories,
  independent of basis/EFT validity/operators, etc.

  It can show up at a parametrically smaller scale
  than the actual BSM resonance if sufficiently strongly coupled.



Universal signatures below threshold

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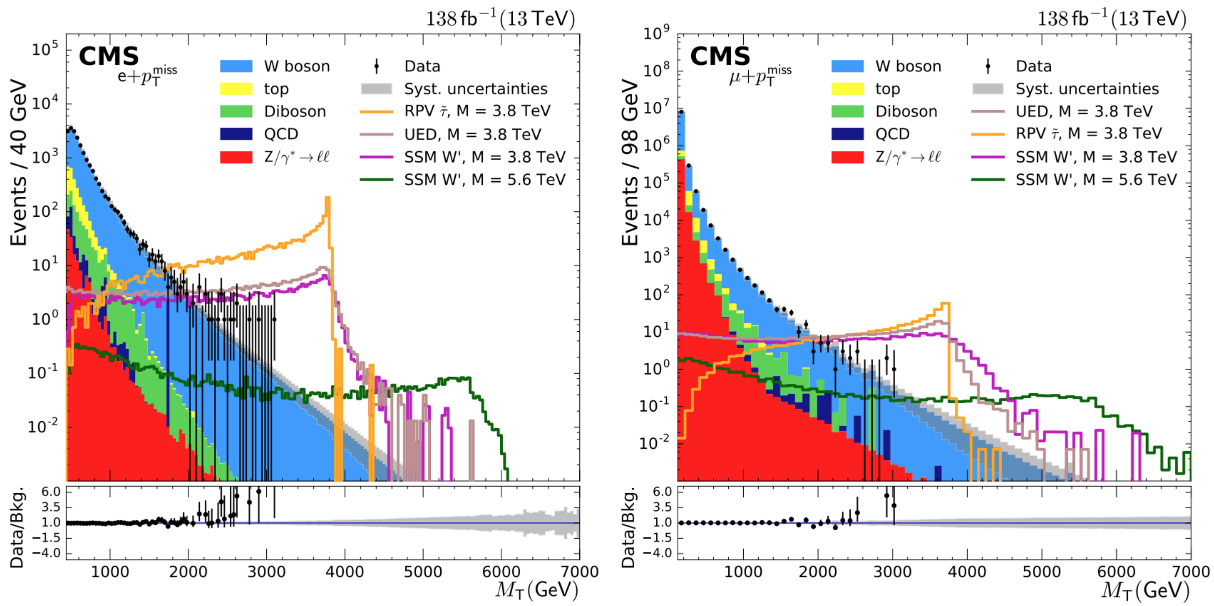
Phenomenological consequences

Current data and unitarity

- Best determination of W parameter is from CMS, **[CMS collab. 2202.06075]**



W parameter modifies the high energy lepton + MET events at LHC

Current data and unitarity

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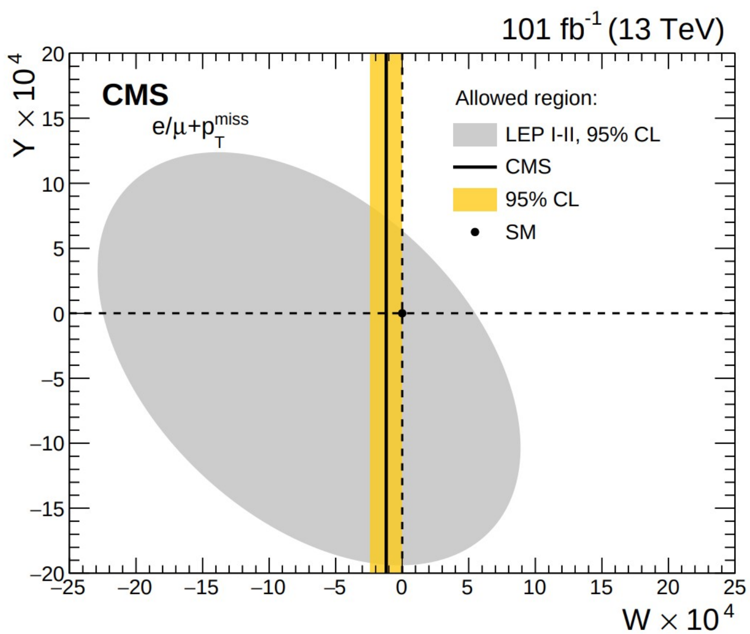


$$W = -1.2^{+0.5}_{-0.6} \times 10^{-4}$$!!

Two sigma tension, in both electron and muon channels, leads to a preference for

negative W!!

*Same channel in ATLAS shows some tension, but it is not interpreted in terms of the W parameter [ATLAS collab. 1906.05609]

Interpretation

$$W = -1.2^{+0.5}_{-0.6} \times 10^{-4}$$

- **A)** Universal theories.

Include prior $W > 0$ in the fit

$$W \in [0, 0.31] \times 10^{-4} \quad \text{at } 95\% CL$$

This implies a much stronger constraint in new physics. From

$$m_\star > (g/g_\star) 5.4 \mathrm{TeV}$$

to

$$m_\star > (g/g_\star) 14.3 \mathrm{TeV}$$

Interpretation

$$W = -1.2^{+0.5}_{-0.6} \times 10^{-4}$$

- **B)** Non-universal theories.

In non-universal theories not only the W parameter might be non-positive, but it is ill-defined.
The search must be interpreted in terms of the 4-fermion operator

$$\mathcal{L} \supset -c^{(3)}_{\ell q} \frac{g^2}{2m_W^2} \bar{\ell}_L \gamma^\mu \sigma^I \ell_L \, \bar{q}_L \gamma_\mu \sigma^I q_L$$

Current data can be accomodated by a tree-level contribution from a ~ 3TeV leptoquark

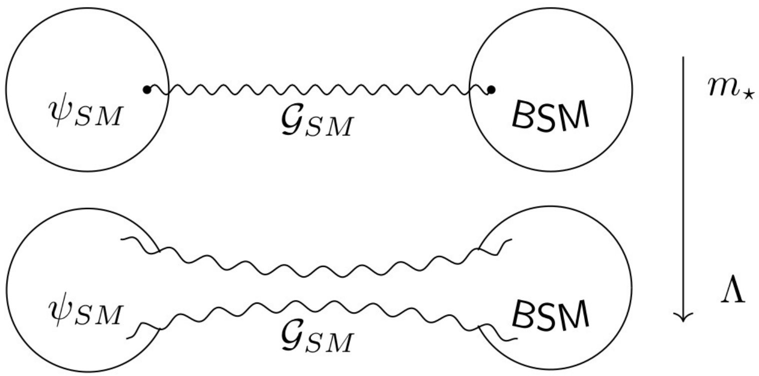
$$\begin{array}{l} \text{scalar in } (3,1)_{-1/3} \text{ or } (3,3)_{-1/3} \\ \text{vector in } (3,1)_{2/3} \text{ or } (3,3)_{2/3} \end{array} \Rightarrow \quad c^{(3)}_{\ell q} < 0$$

$$c^{(3)}_{\ell q} \sim \frac{y^2}{g^2} \frac{m_W^2}{m_\star^2} \;\Rightarrow\; y \sim g \,, \;\; m_\star \sim 3 \text{ TeV}$$

Interpretation

$$W = -1.2^{+0.5}_{-0.6} \times 10^{-4}$$

- **C)** Strong coupling



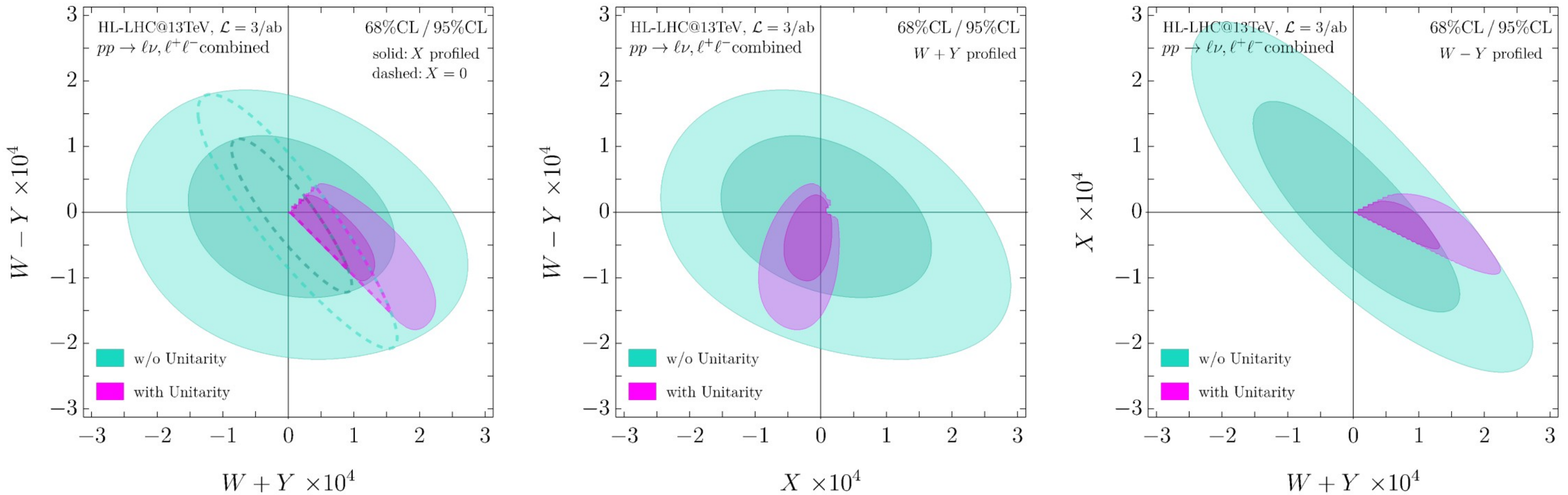
EW coupling runs strong at some scale not much above

$$\Lambda \sim m_\star \frac{4\pi}{g} \sim 100 \text{ TeV}$$

Future projections

Including or not unitarity constraints has a dramatic impact on future Drell-Yan projections

$$W + Y > 0 \text{ determines the overall scale} \rightarrow \ \text{unitarity: } X^2 + (W-Y)^2 < (W+Y)^2$$



If the central value stays the same, at HL-LHC the current tension will grow to ~ 8 sigma level

Conclusions

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Basic principles set nontrivial constraints on the space of consistent IR dynamics

Universal theories are subject to those constraints,

implying nontrivial consequences for the interpretation of collider data

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Thank you.