New (amplitude) relations between gluons and scalars

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Based on 2310.13041 [hep-th]
w/ G. Durieux and J. Roosmale Nepveu

Motivations

Why scattering amplitudes ?

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Old topic (60s)

(Close to) **Physical objects**

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- Challenging objects from a **technical perspective**

  Ex : precision for collider processes or gravitational waves — recursion relations, generalized unitarity, …

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(Close to) **Physical objects**

- Challenging objects from a **technical perspective**

  Ex : precision for collider processes or gravitational waves — recursion relations, generalized unitarity, …

- Interesting objects from a **conceptual perspective**

  Ex : bootstrap, soft theorems, geometry, … and **some universal behaviors**

Universal algebras and building blocks

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Algebraic relations…

… realized on building blocks

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Algebraic relations… **… realized on building blocks**

At four points,

$$c_s = f^{abe} f^{ecd} \; , \; c_t = \ldots$$

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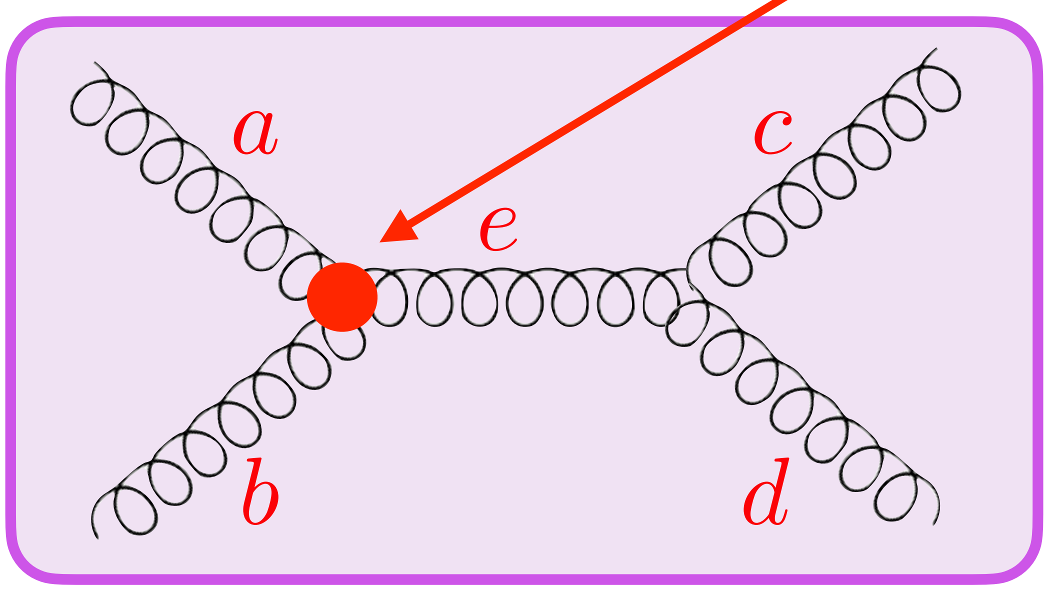

Universal algebras and building blocks

Algebraic relations…

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$$c_s + c_t + c_u = 0$$

… realized on building blocks

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Color-kinematics duality
and **double copy**
[Bern/Carrasco/Johansson '09,'10]

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(beyond 4pts, loops also,…)

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[Broedel/Schlotterer/Stieberger '13]

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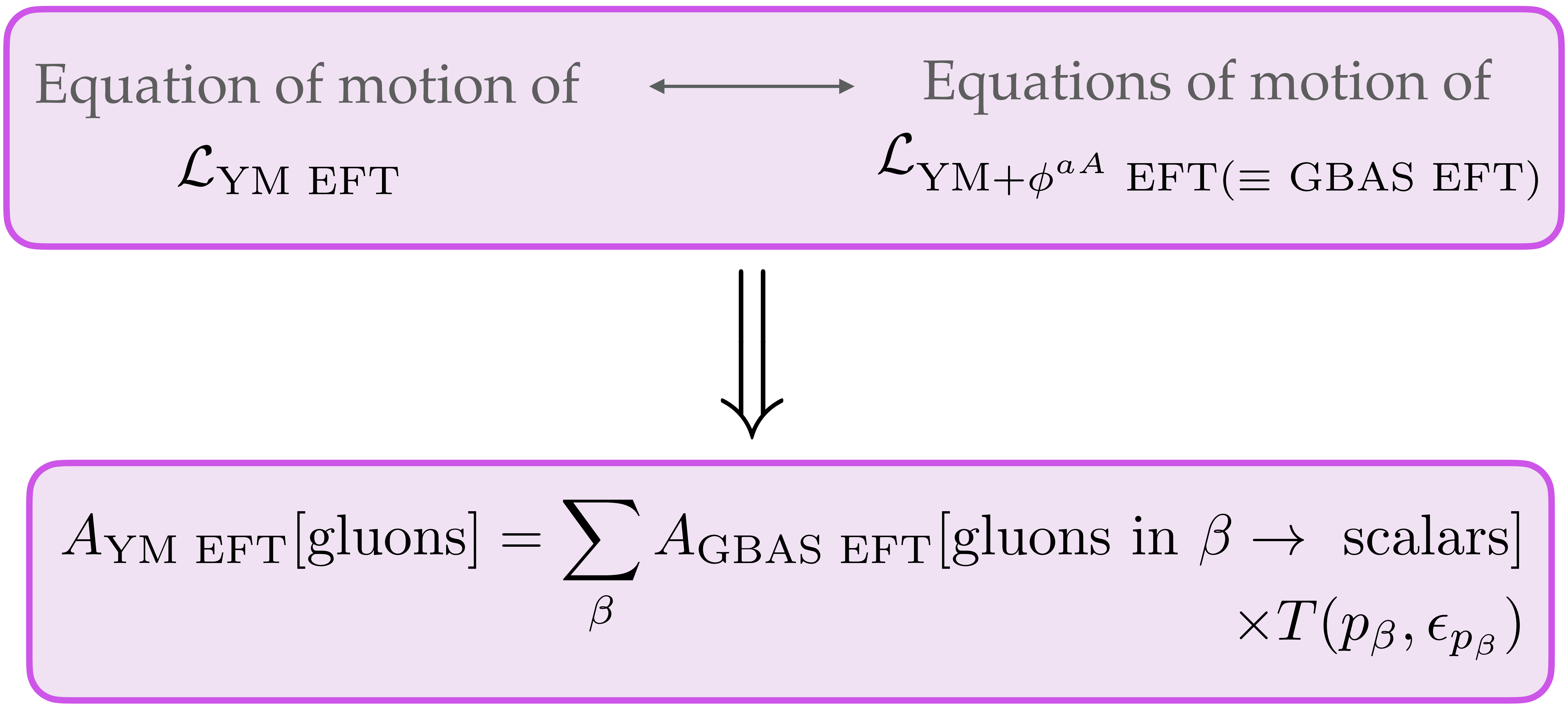
[Broedel/Schlotterer/Stieberger '13]

What about **effective field theories** in general?

[Broedel/Dixon '12, …, QB/Durieux/Grojean/Machado/RoosmaleNepveu '21, …]

Result

Equation of motion of $\mathcal{L}_{\text{YM EFT}}$ $\longleftrightarrow$ Equations of motion of $\mathcal{L}_{\text{YM}+\phi^{aA}\ \text{EFT}(\equiv\ \text{GBAS EFT})}$


Equation of motion of
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$A_{\text{YM EFT}}[\text{gluons}] = \sum_{\beta} A_{\text{GBAS EFT}}[\text{gluons in } \beta \to \text{ scalars}] \times T(p_\beta, \epsilon_{p_\beta})$

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Our work

Minimal YM :
covariant CK duality

[Cheung/Mangan '21]

Gluon field strengths propagate as scalars

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$$\left.\begin{array}{c} D^\mu F^a_{\mu\nu} = -J^a_\nu \\ \text{D + Bianchi} \end{array}\right\} \implies D^2 F^a_{\mu\nu} + g\, f^{abc} F^b_{\rho[\mu} F^{c\rho}_{\nu]} = -D_{[\mu} J^a_{\nu]}$$

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D + Bianchi

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$$F^a_{\mu\nu} \to \Phi^a_A \qquad F^b_{\rho[\mu} F^{c\rho}_{\nu]} \to f^{ABC}\Phi^b_B \Phi^c_C \qquad D_{[\mu} J^a_{\nu]} \to J^a_A$$

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GBAS theory ! Same algebra for color and spacetime

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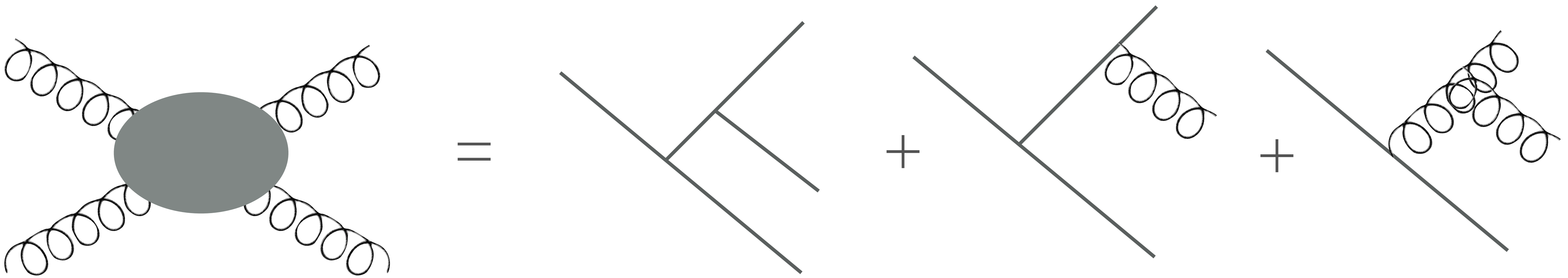

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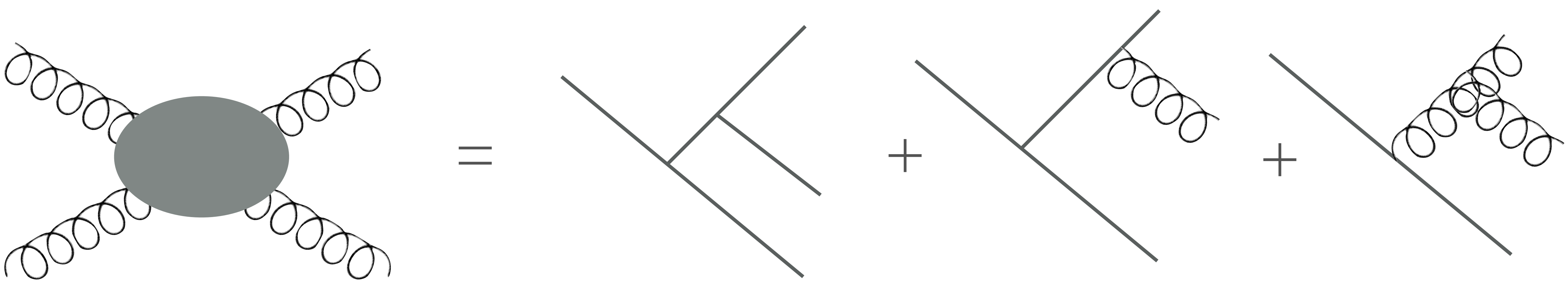

(very schematic)

$$A_{\mathrm{YM}}(gggg) = \frac{\epsilon}{p}\left[A_{\mathrm{GBAS}}(\Phi\Phi\Phi\Phi)(\epsilon p)^3 + A_{\mathrm{GBAS}}(\Phi\Phi\Phi g)(\epsilon p)^2 + A_{\mathrm{GBAS}}(\Phi g g g)(\epsilon p)\right]$$

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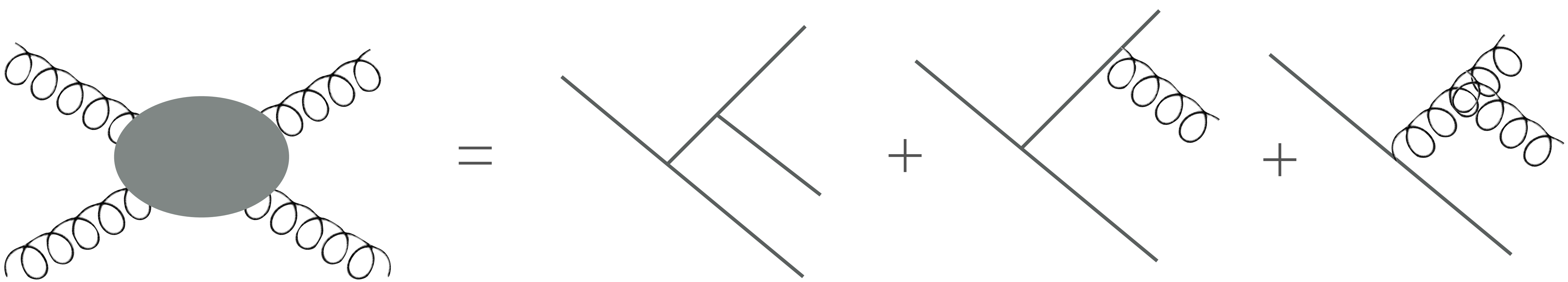

(very schematic)

$$A_{\text{YM}} = \frac{\epsilon}{p} \sum A_{\text{GBAS}}(\text{at least one scalar})(\epsilon p)^{\#}$$

Works at all multiplicities

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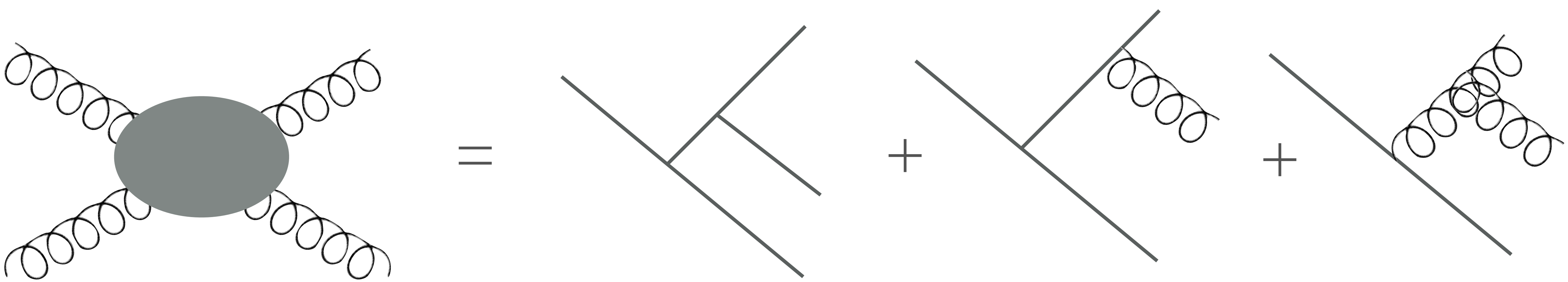

(very schematic)

$$A_{\rm YM} = \frac{\epsilon}{p} \sum A_{\rm GBAS}(\text{at least one scalar})(\epsilon p)^{\#}$$

Works at all multiplicities : **new closed-form YM amplitudes**

YM EFTs :
covariant CK duality ?

Gluon field strengths still propagate as scalars

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$$\mathcal{L}_{\text{YM EFT}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{g}{3\,\Lambda^2} f^{abc} F^{a\ \nu}_{\mu} F^{b\ \rho}_{\nu} F^{c\ \mu}_{\rho} + \mathcal{O}\left(1/\Lambda^4\right) + A^a_\mu J^{a\,\mu}_A$$

Equation of motion :

$$D^\mu F^a_{\mu\nu} + \frac{g}{\Lambda^2} f^{abc} F^b_{\mu\rho} D_\nu F^{c\,\mu\rho} = -J^a_\nu$$

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$\mathcal{O}(\Lambda^0)$ → scalar !

↕

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↕

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Renormalizable YM :

$$A_{\text{YM}} = \frac{\epsilon}{p} \sum A_{\text{GBAS}}(\text{at least one scalar})(\epsilon p)^{\#}$$

Dim-6 YM :

$$A_{\text{dim-6 YM}} = \sum A_{\text{GBAS}}(\text{at least one gluon and one scalar})(\epsilon p)^{\#}$$

Next orders ?

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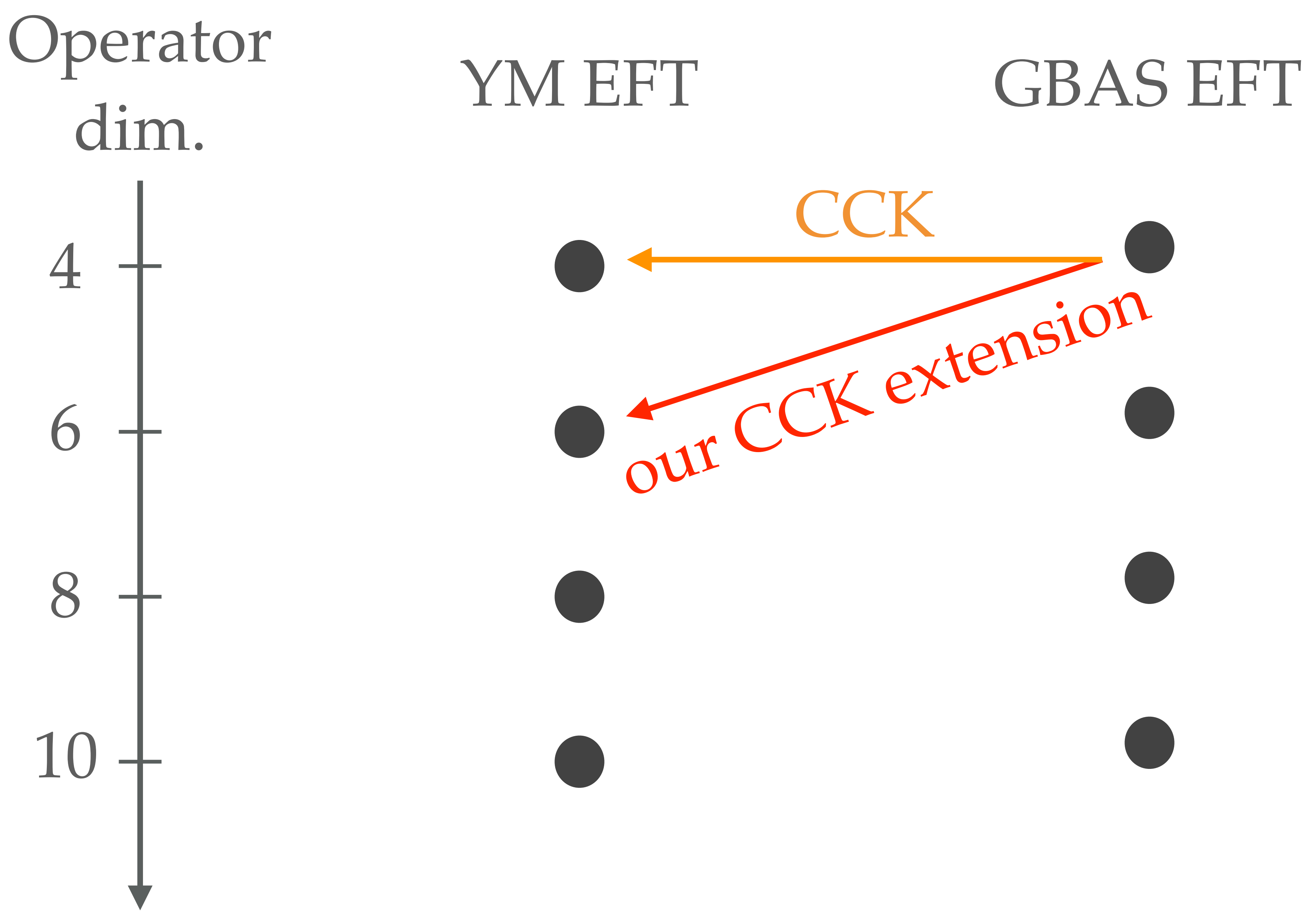

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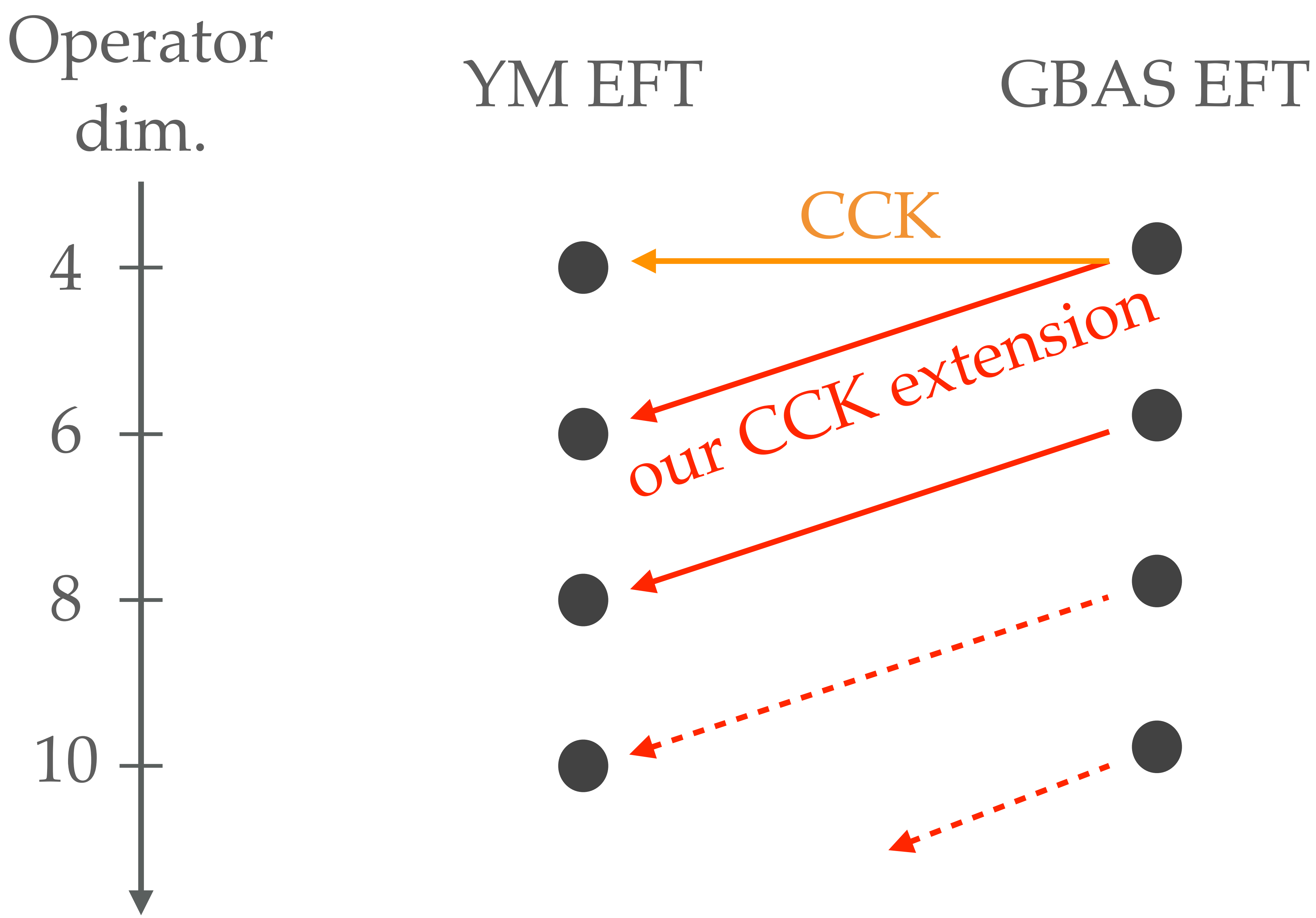

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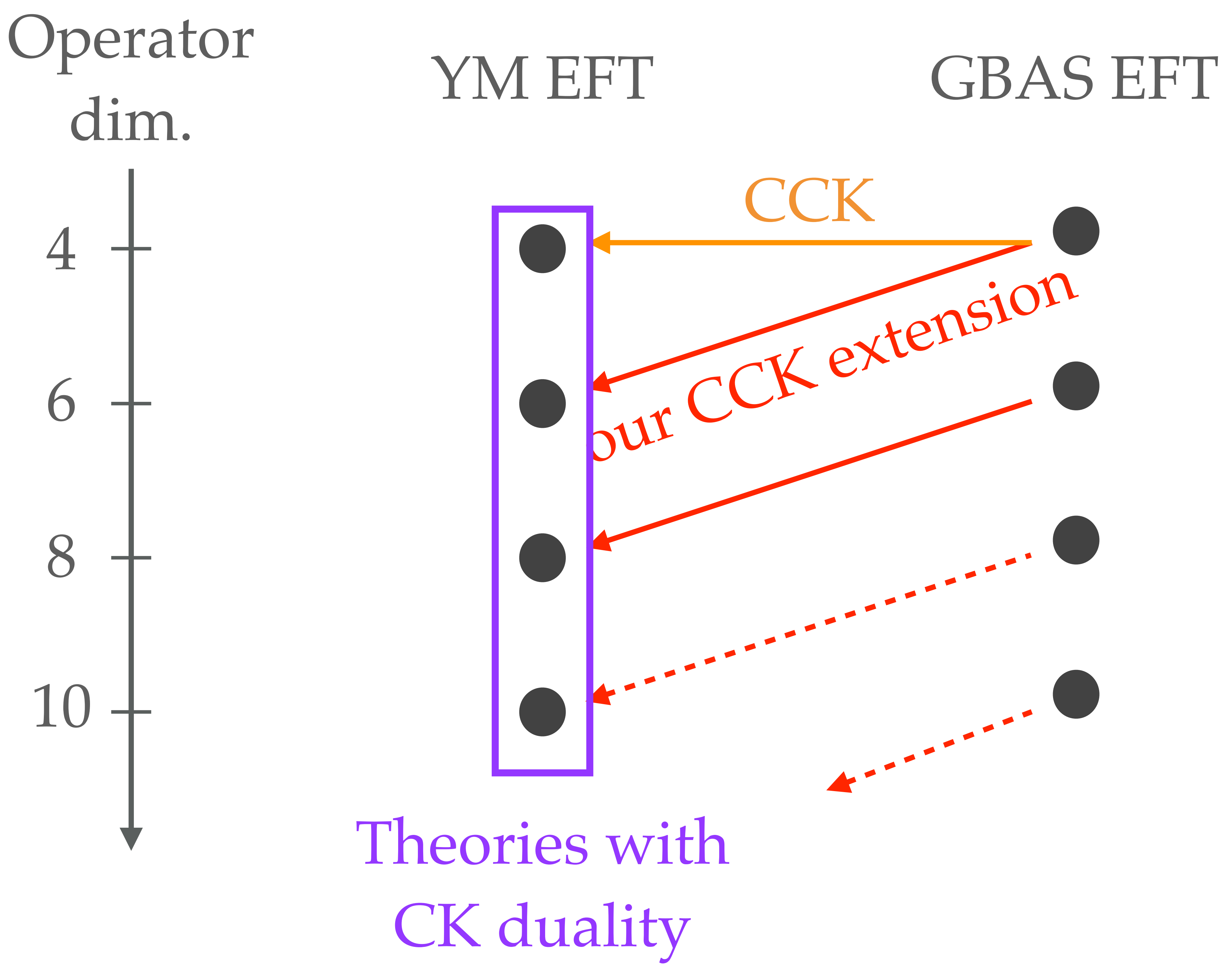
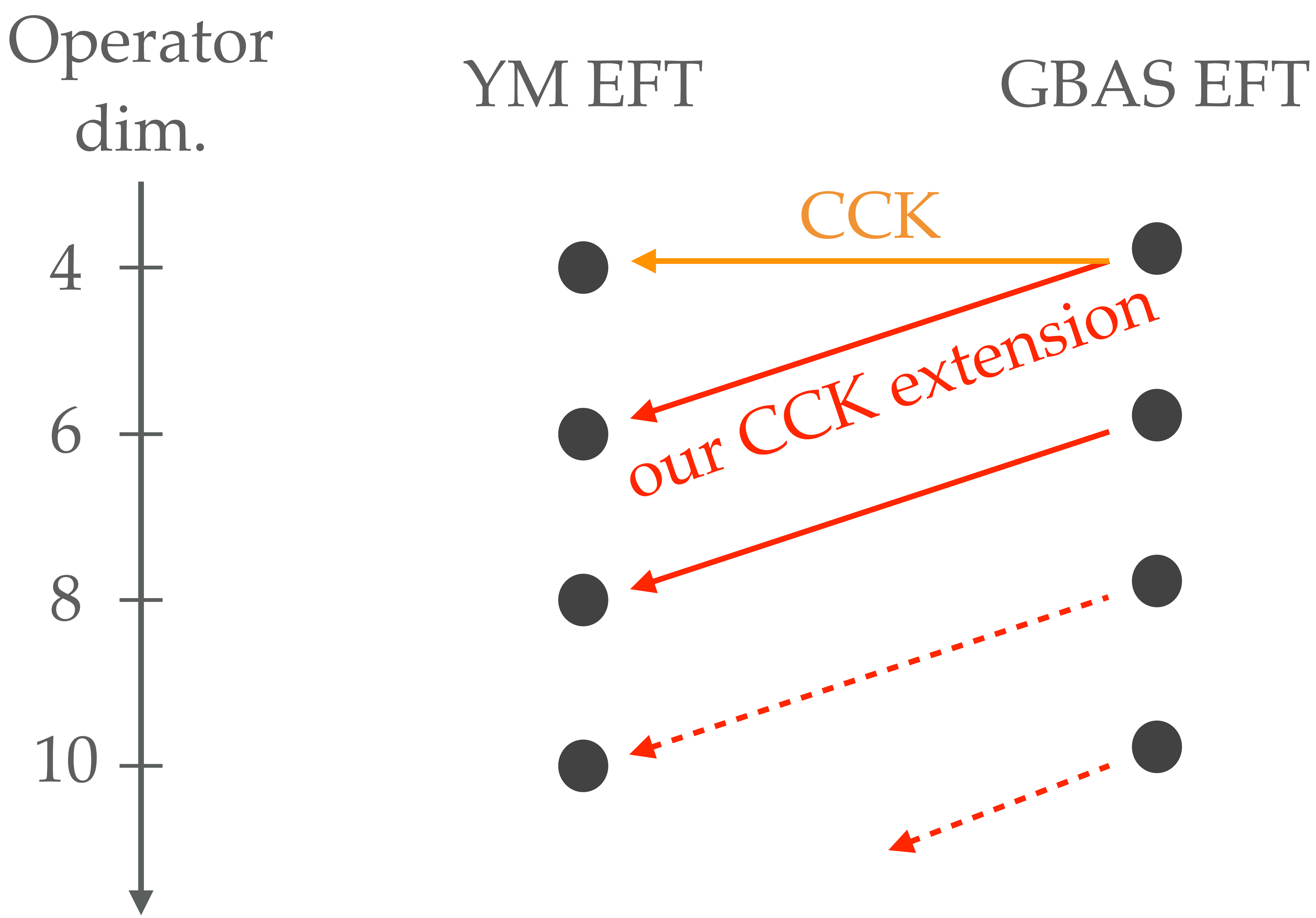

Operator dim.
YM EFT
GBAS EFT
CCK
4
6
8
10
our CCK extension
Theories with CK duality

Next orders ?



Also for dim-8 YM :

$$A_{\text{dim-8 YM}} = \sum_{\text{all}} A_{\text{GBAS}} (\epsilon p)^{\#}$$

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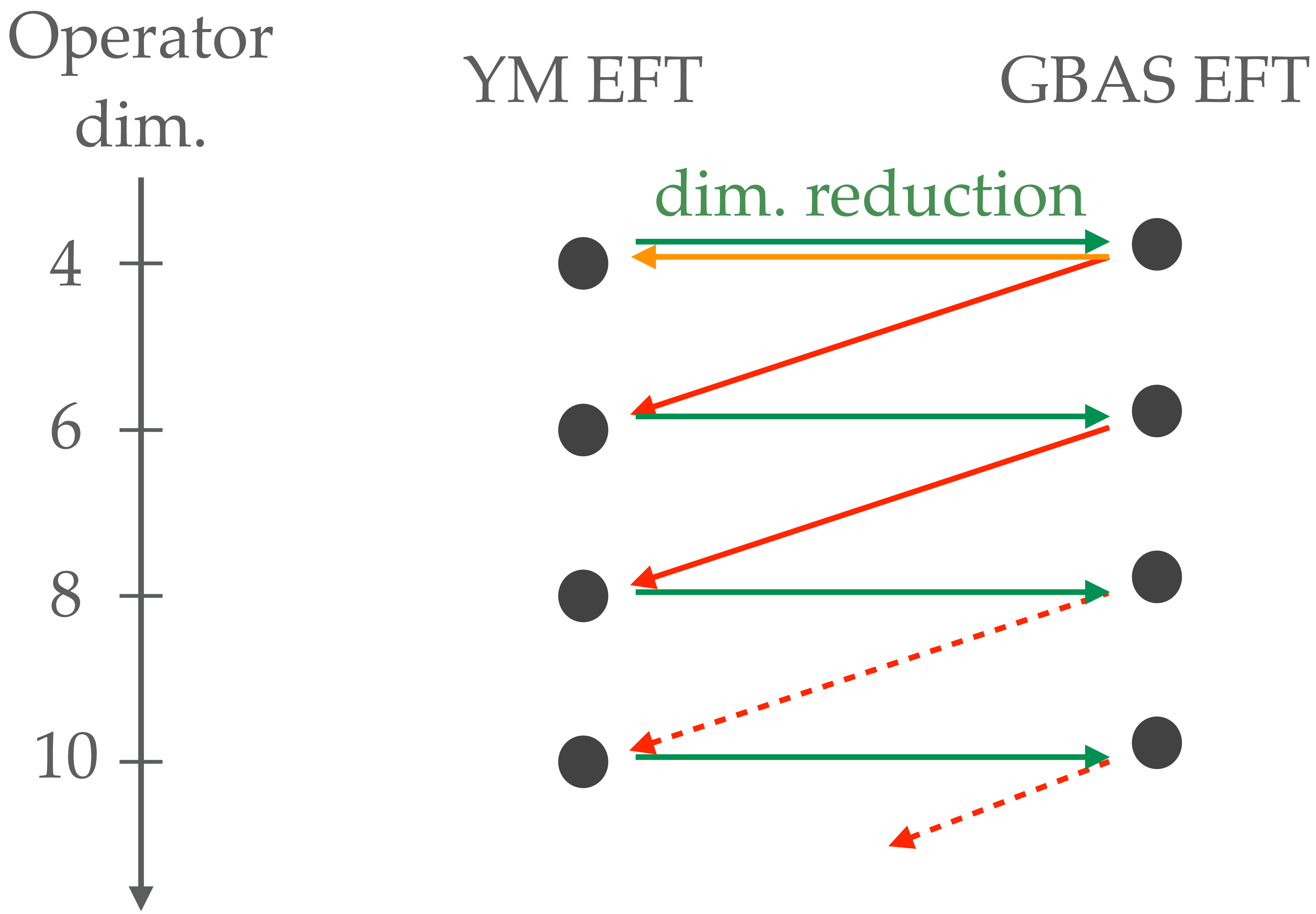

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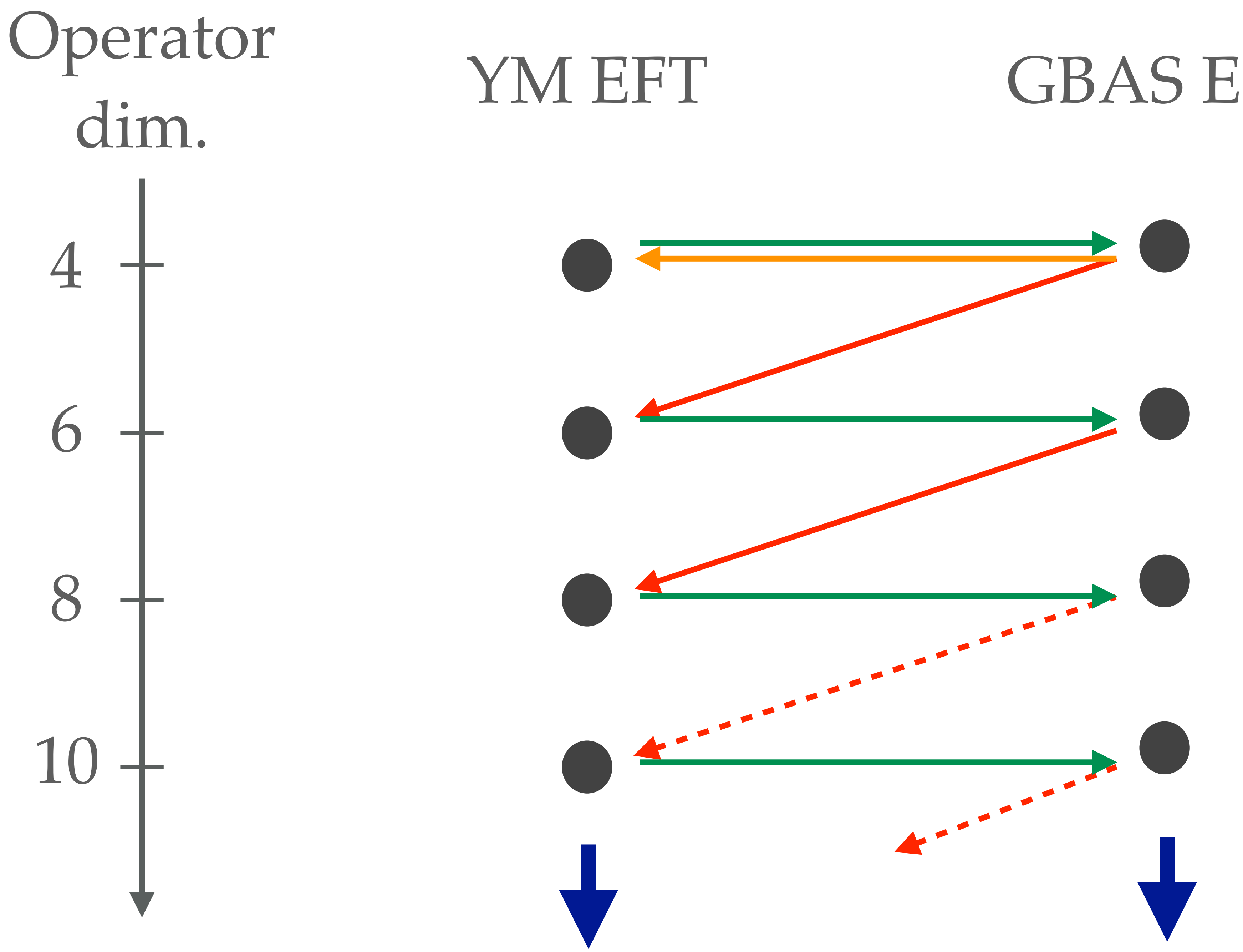

Resums to $\mathrm{YM} + (DF)^2$ $\overset{?}{\longleftrightarrow}$ Resums to $\mathrm{GBAS} + (DF)^2$

Involved in the double copy of the heterotic and bosonic strings

[Johansson/Nohle '17]

Summary and outlook

Surprises in the tree level scattering of gluons !

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Dynamics related to that of colored bi-adjoint scalars, all-multiplicity relations between amplitudes, all-multiplicity results for YM EFT amplitudes

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Many leftover questions :

- Why are the CK-dual theories relevant here ? Are they the only ones?
- What do we learn about the kinematic algebra of YM EFTs ?
- Why is dimension reduction relevant here ? How does it act on the YM+(DF)^2 theories ?
- Gravity, pions, … ?
- …