High-energy, high-density and hot QCD

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Outline

- High-energy QCD

  - study of hadronic and nuclear wave functions at small values of x, using perturbation theory in the presence of strong color fields

  - establish a long-distance/short-distance factorization framework in the strong field regime

- Hot QCD

  - study of QCD at finite temperature, using perturbative methods and lattice QCD

  - characterization of the quark-gluon plasma created in relativistic heavy-ion collisions

- Dense QCD

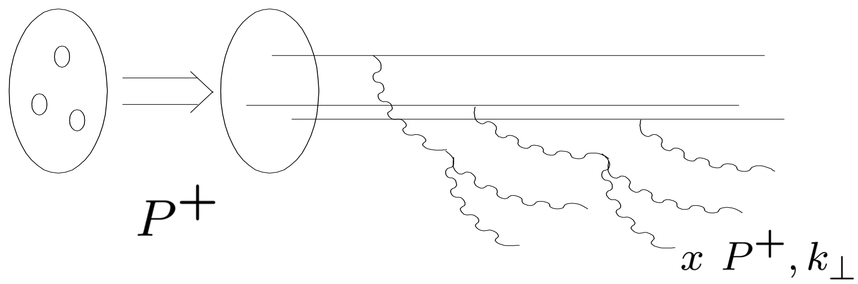
  - study of QCD at finite baryon density, using non-perturbative methods and effective models

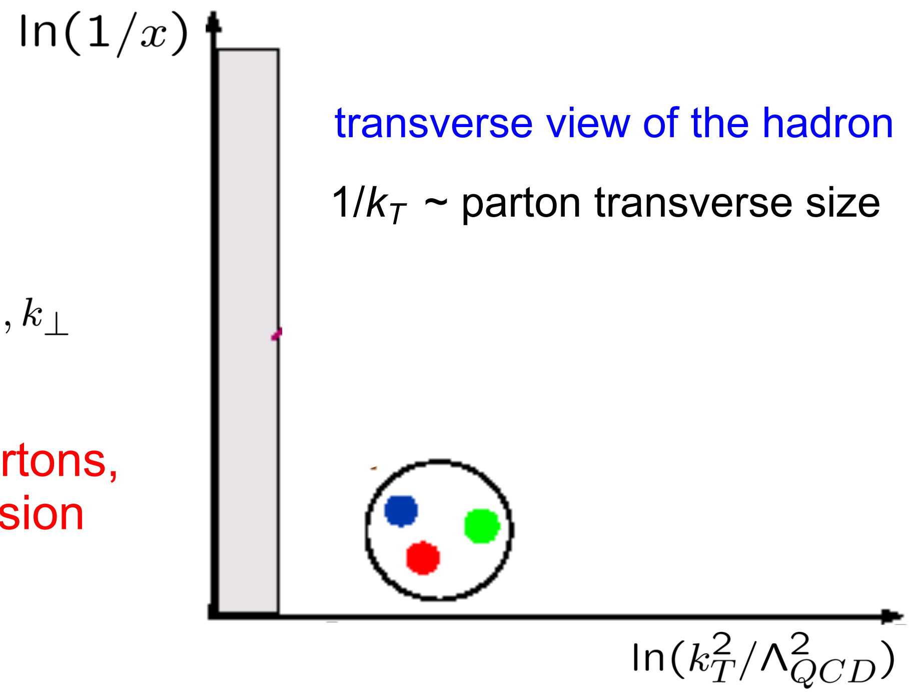
  - explore the QCD phase diagram, investigate confinement and chiral symmetry breaking

high-energy QCD

From independent partons...

the parton content of high-energy hadrons:





when a hadron is a dilute system of partons, they interact incoherently during a collision

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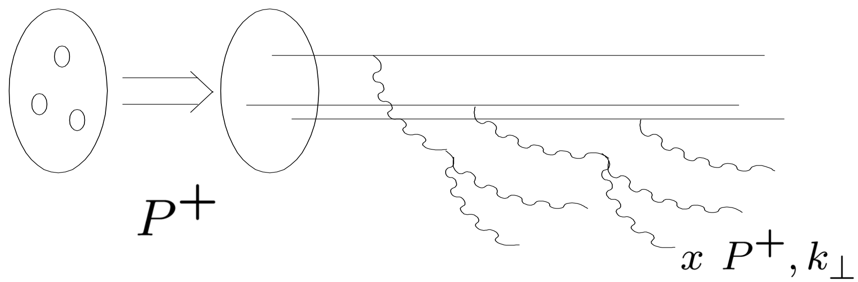
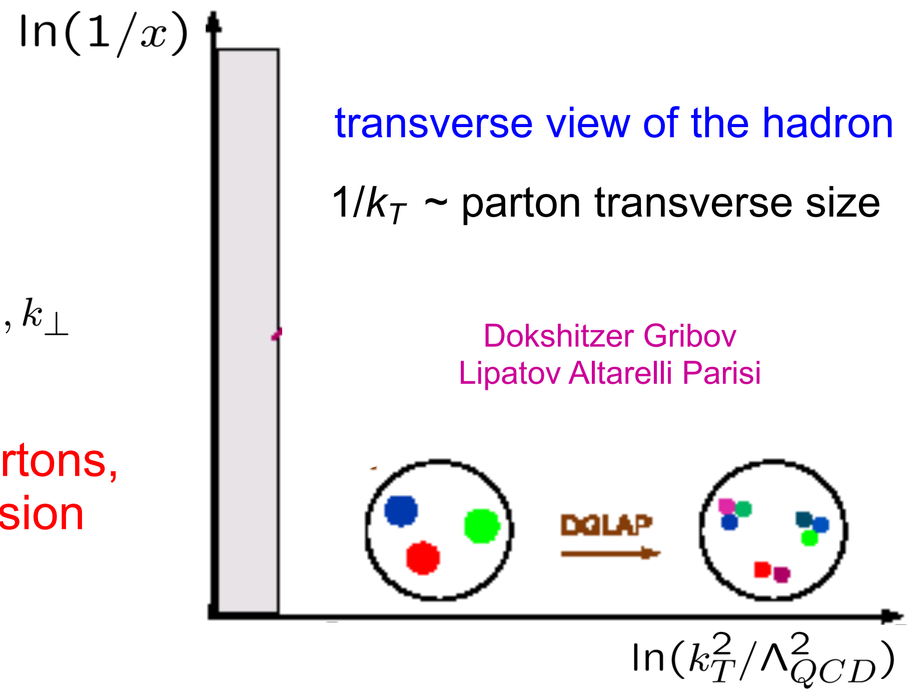




when a hadron is a dilute system of partons, they interact incoherently during a collision

standard QCD evolution: as k_T increases, the hadron gets more dilute

standard QCD factorization: probabilistic sum of partonic cross-sections

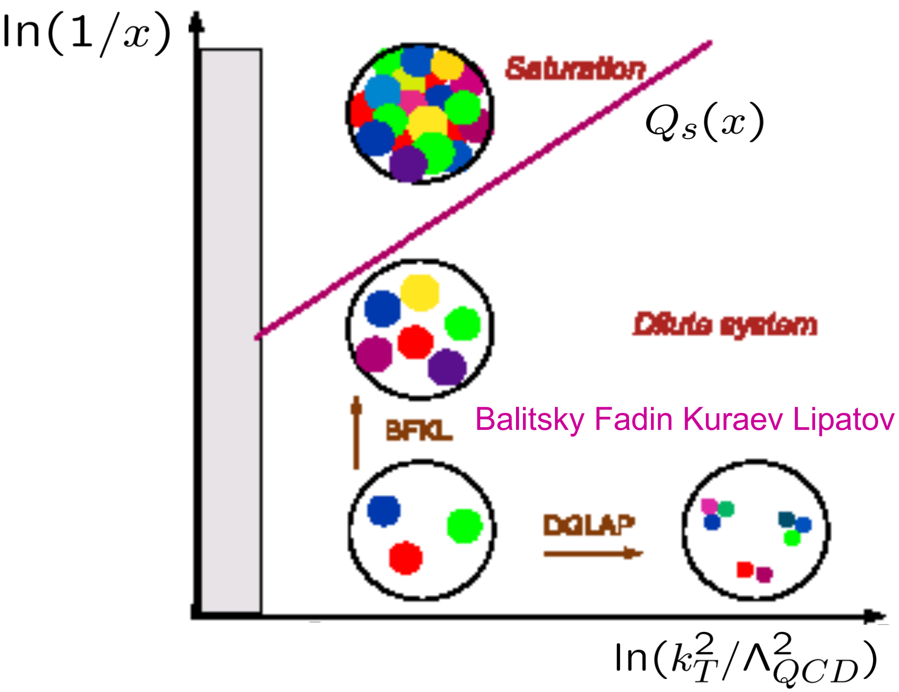
$$d\sigma_{AB\to X} = \sum_{ij} \int dx_1 dx_2\ f_{i/A}(x_1,\mu^2) f_{j/B}(x_2,\mu'^2)\ d\hat{\sigma}_{ij\to X} + \mathcal{O}\left(\Lambda_{QCD}^2/M^2\right)$$

...to collective behavior

when x gets smaller and smaller, the hadron is no longer dilute, the partons start interacting coherently

the Λ_{QCD}^2/M^2 power corrections get enhanced by $x^{-\lambda}$

for heavy-nuclei, those density effect are further amplified by $A^{1/3}$



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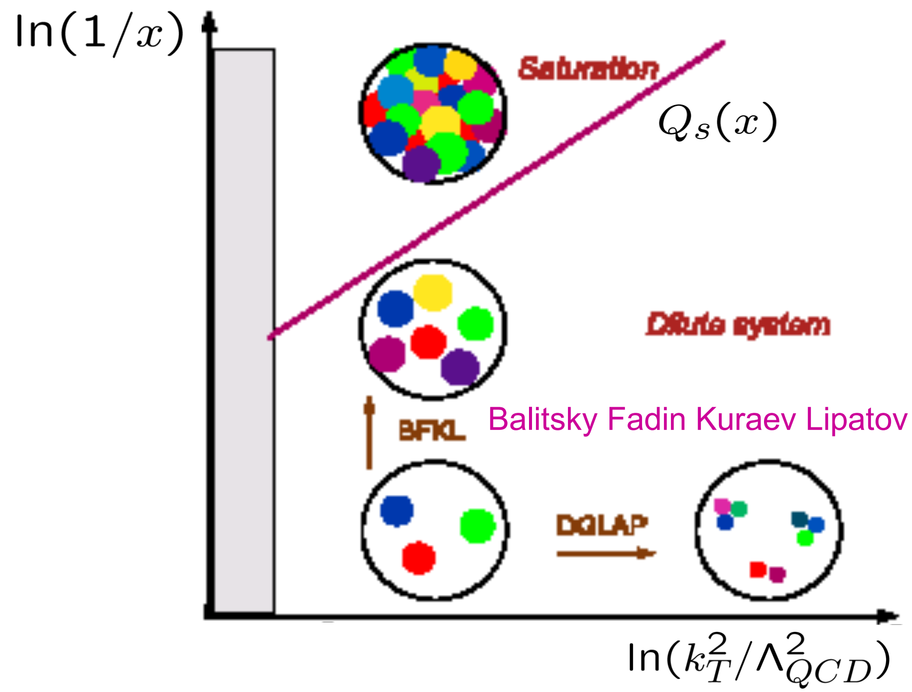


an alternate long-distance/short-distance factorization scheme is needed

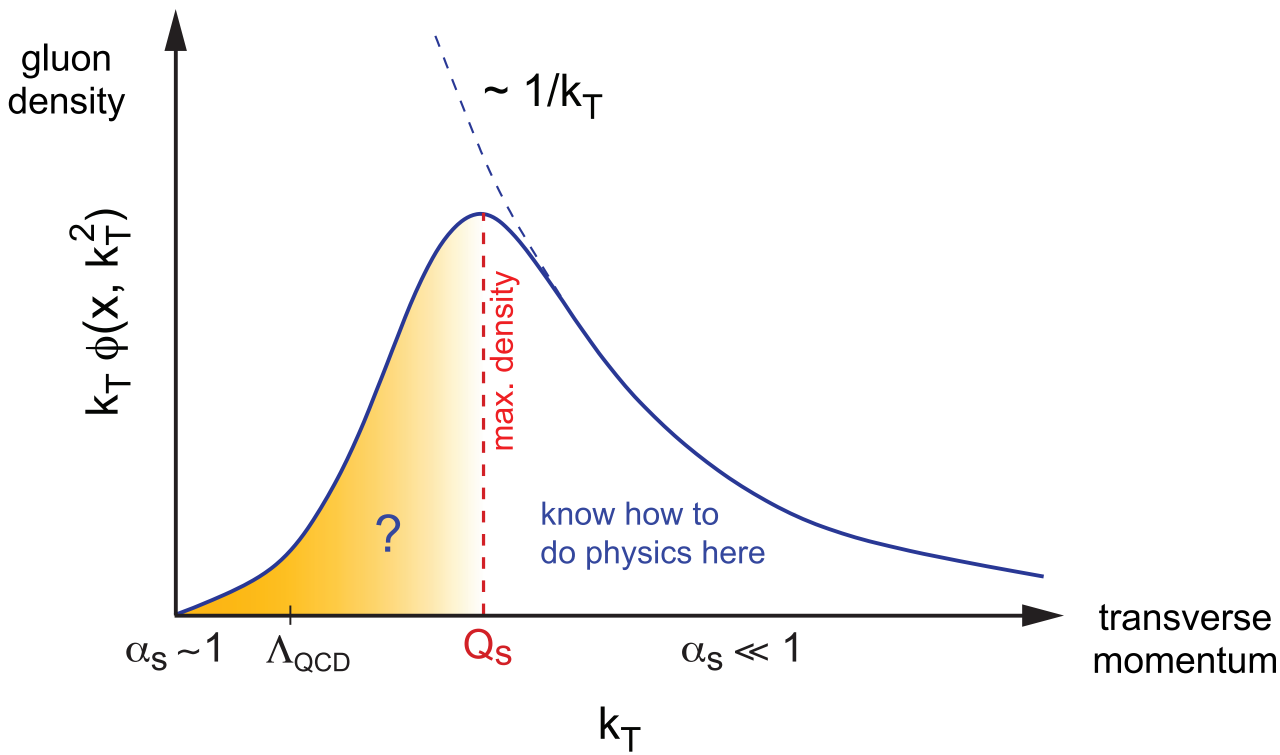
it involves effective degrees of freedom (Wilson lines, Reggeized gluons, …), new operators governed by an effective action (Color Glass Condensate, Lipatov's action, …)

→ an approximation of QCD suited to describe physics at large parton densities

The saturation scale

The saturation scale $Q_S(x)$ is the momentum scale which characterizes the transition between the dilute and dense regimes

at small-x, the typical gluon transverse momentum is no more Λ_{QCD} , it is instead $Q_S(x)$



the dynamics is non-linear, but the theory stays weakly coupled $\alpha_s(Q_s) \ll 1$

Future Prospects

the field of high-energy QCD has recently entered the NLO era: higher-order corrections of several kinds to be computed

- next to leading order in α_S: essential to prove factorization and assess robustness of predictions

  in most cases, perturbation theory must be done in conjunction with all-order resummations of various large logarithms

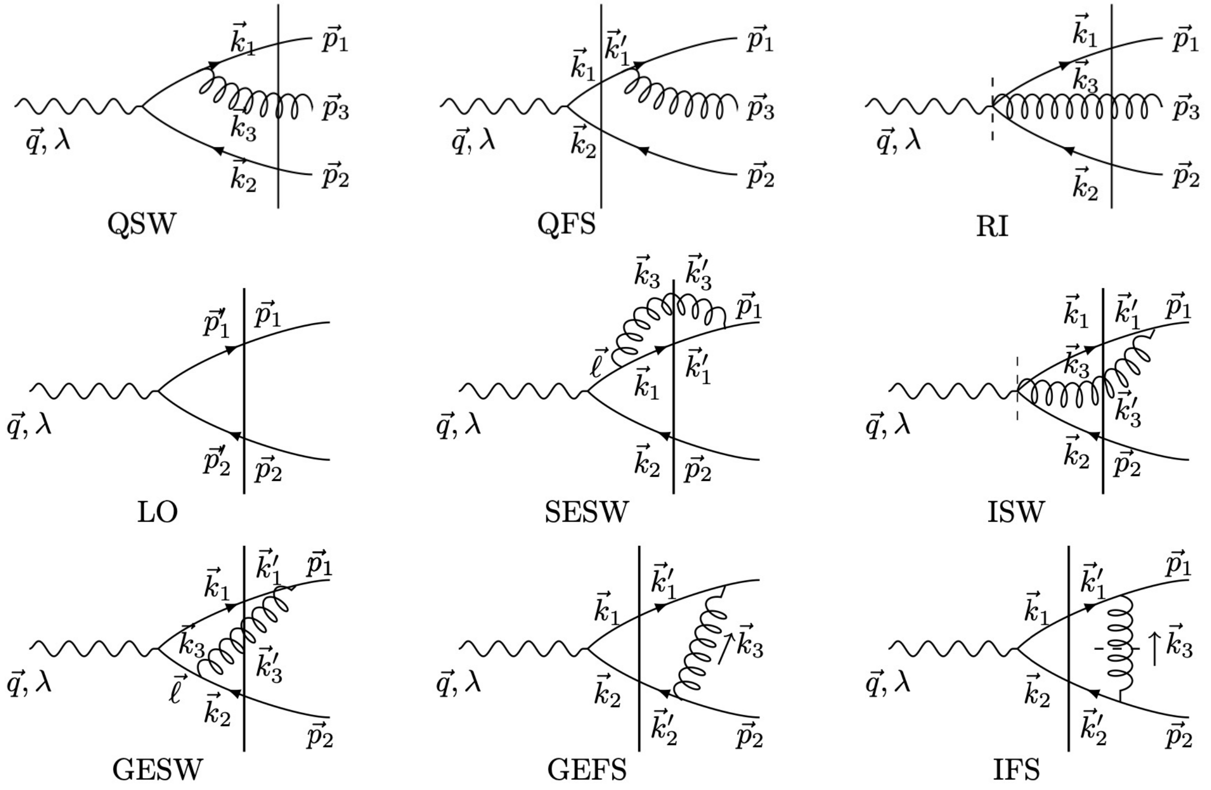
- next-to-eikonal corrections: energy-suppressed but give access to spin-dependent observables

- next-to-planar corrections: going beyond the large-Nc limit

these must be addressed for less and less inclusive observables measured in experiments: exclusive and diffractive cross sections, correlation measurements, global event properties …

A recent example

NLO calculation of di-jet production

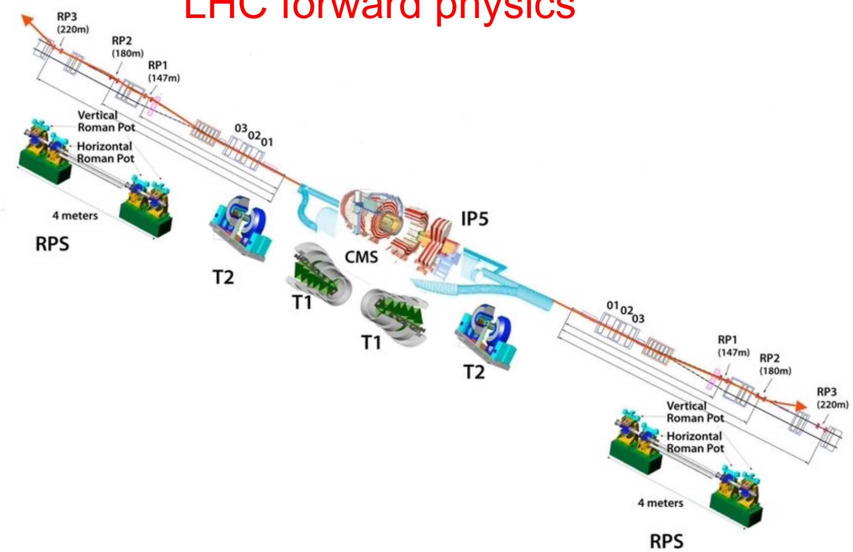


the French community is strongly involved in those NLO calculations

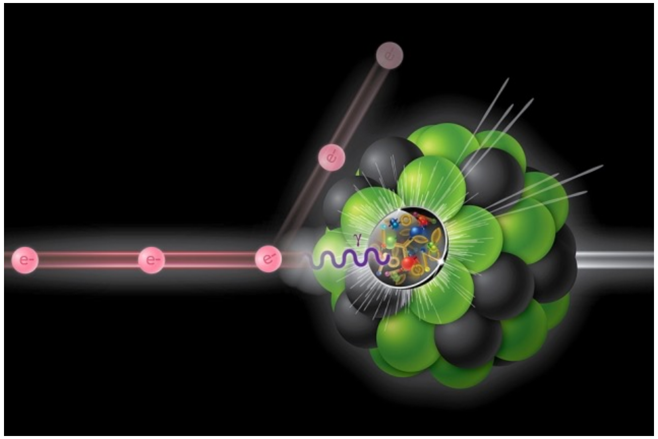
Altinoluk, Boussarie, CM and Taels (2020); Caucal, Salazar and Venugopalan (2021)
Taels, Altinoluk, Beuf and CM (2022); Fucilla, Grabovsky, Li, Szymanowski and Wallon (2023)
Iancu and Mulian (2023)

When is this important ?

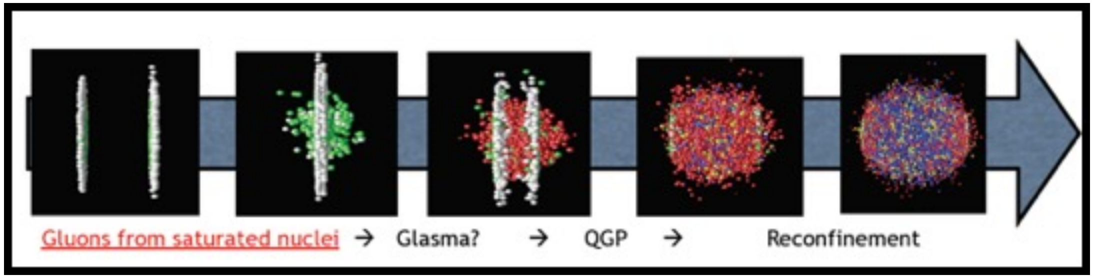
LHC forward physics

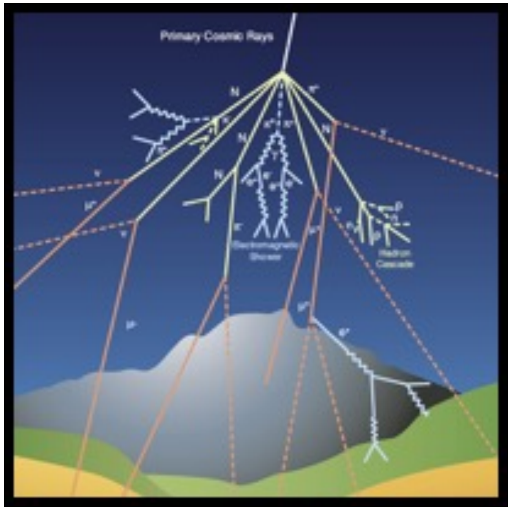


Electron Ion Collider (EIC)



initial stages of heavy-ion collisions

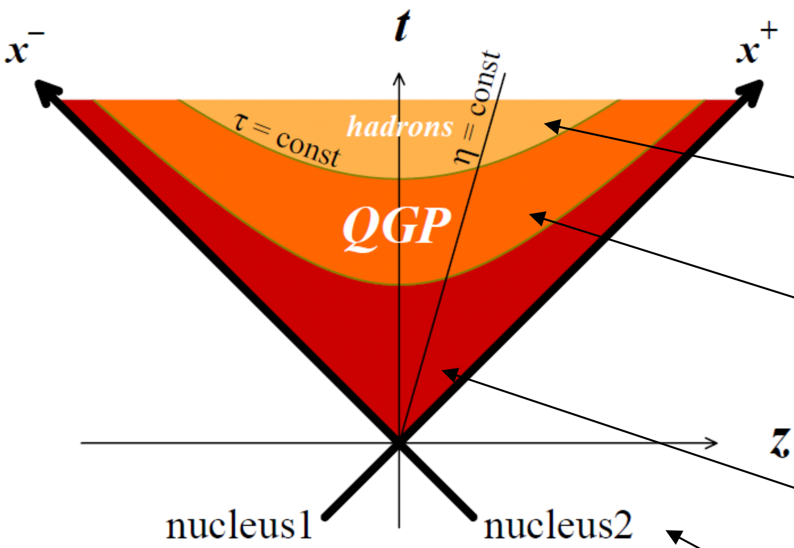




high-energy cosmic rays

Relativistic Heavy-Ion Collisions

main goal: produce and study the quark-gluon-plasma



space-time picture of heavy-ion collisions

5. Individual hadrons

freeze out

4. Hadron gas

cooling with expansion

3. Quark Gluon Plasma

thermalization, expansion

2. Pre-equilibrium state

collision

1. Nuclei (initial condition)

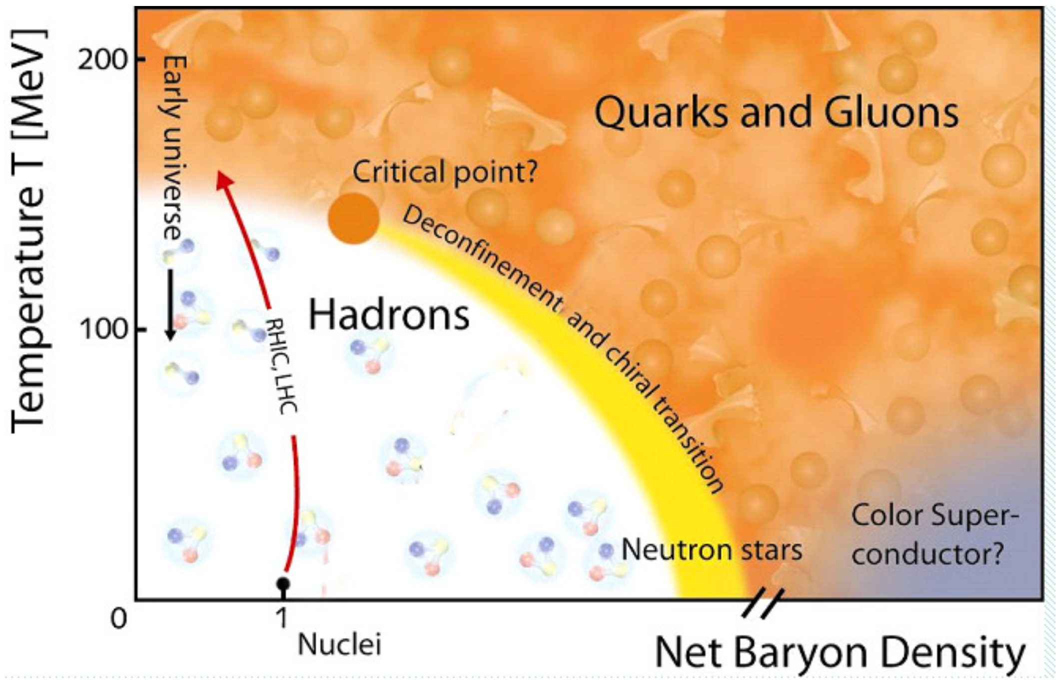
one observes the system after it has gone through
a complicated evolution involving different aspect of QCD

to understand each stage and the transition between them has been challenging

hot and/or dense QCD:
the phase diagram

The QCD phase diagram

rich structure: early universe, critical point, deconfinement phase transition, chiral symmetry restoration, neutron stars, color superconductivity, …

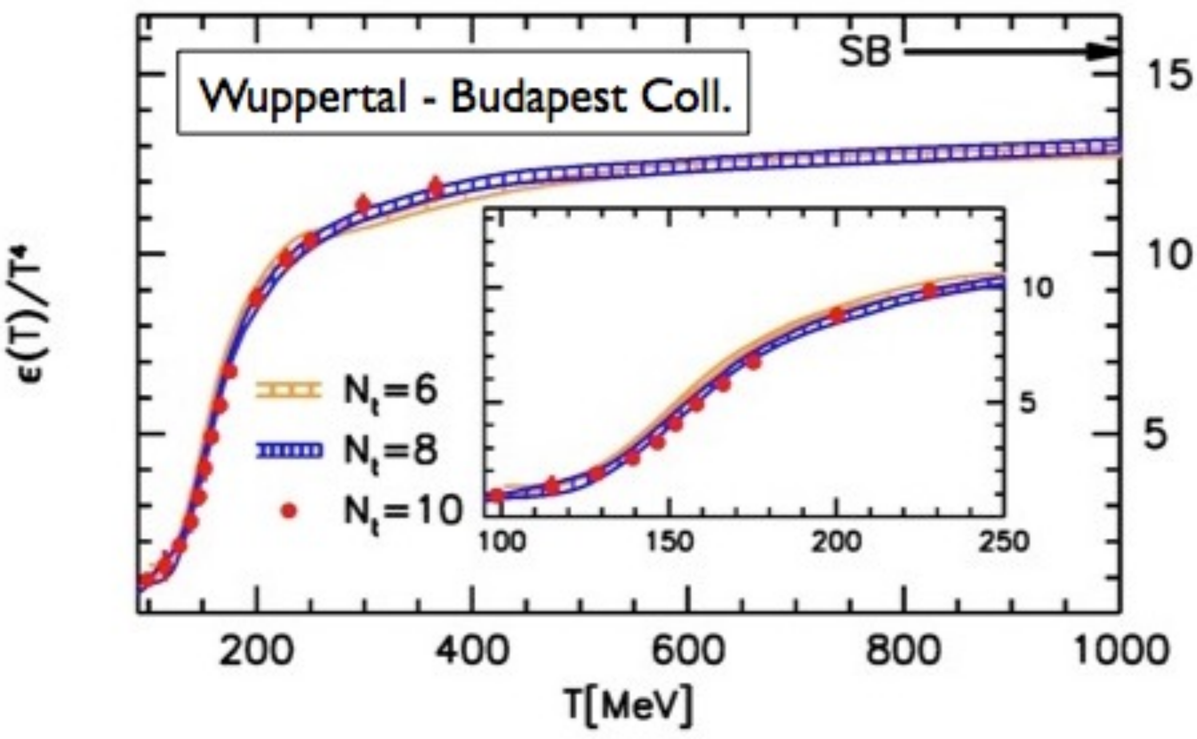


high temperature or baryon density : perturbative computations

zero net baryon density: lattice QCD simulations

Finite temperature lattice QCD

- it is now possible to use a realistic pion mass



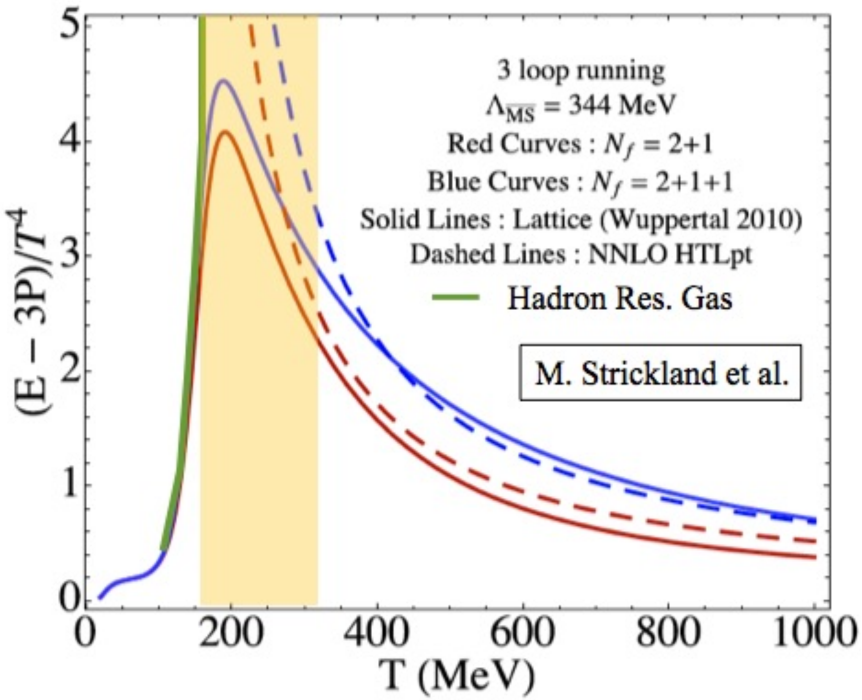
lattice QCD deals with a simpler QGP compared to heavy-ion collisions: it is static, fully-thermalized and baryon-less

Reproducing lattice results

- except around *Tc*, we know how to approximate QCD well enough

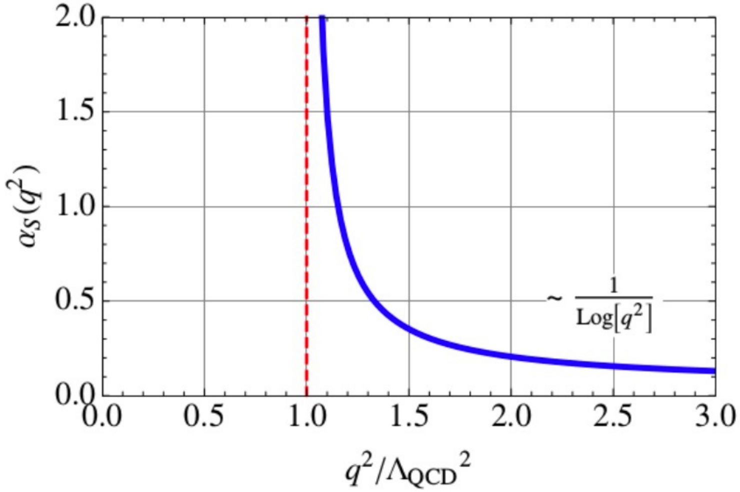
Hadron Resonance Gas model works until 0.9 *Tc*

Hard-Thermal-Loop QGP works above 2 *Tc*

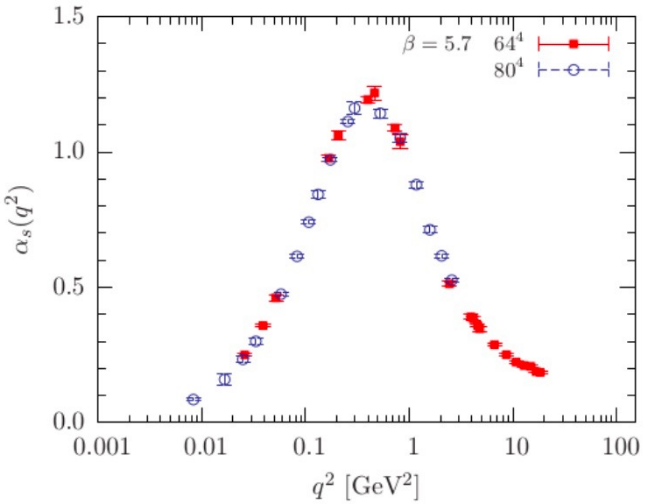


Perturbative approaches near T_c

- perturbation theory works well at high scales



(Faddeev-Popov)



(Lattice)

gauge-fixing plagued by ambiguities which prevent access to the infrared

but the pure gauge coupling does not necessarily get large

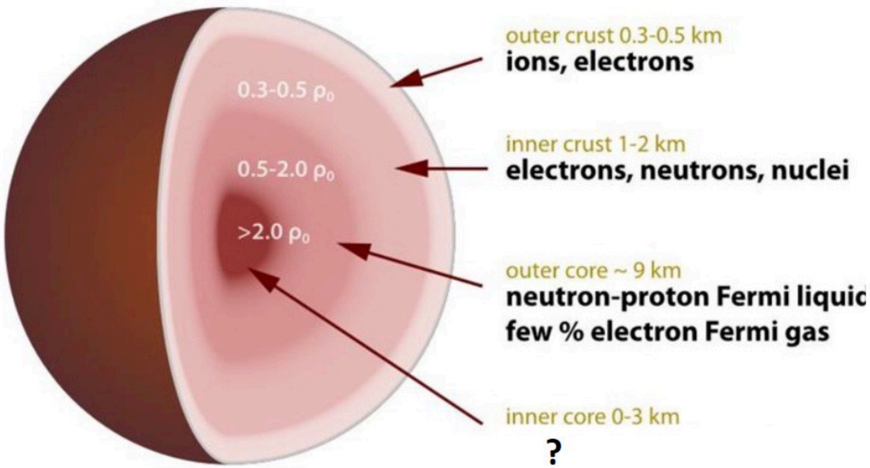
- one can approach the problem using effective models solved by semi-perturbative methods, e.g. the Curci-Ferrari model

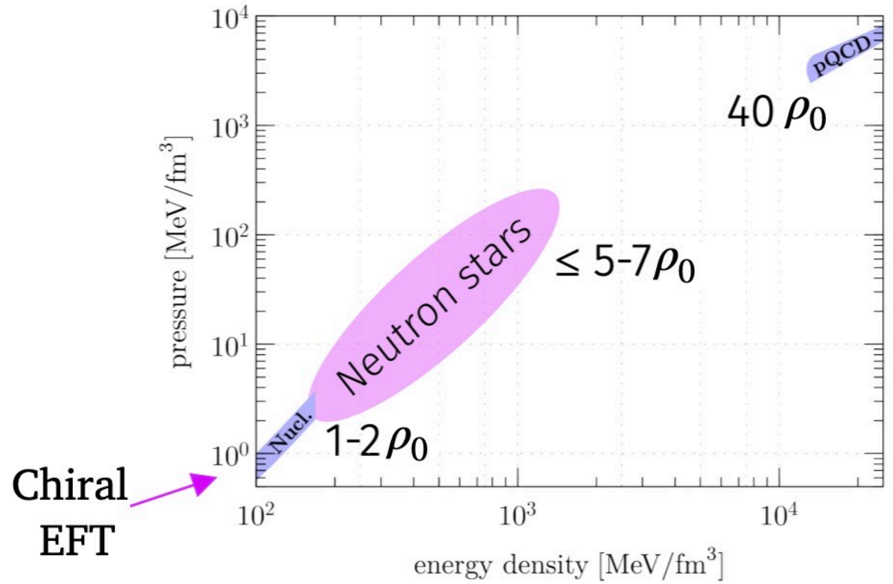
T_c (MeV)	lattice	fRG	1-loop	2-loop
SU(2)	295	230	238	284
SU(3)	270	275	185	254

Reinosa et al. (2015-16)

Neutron stars

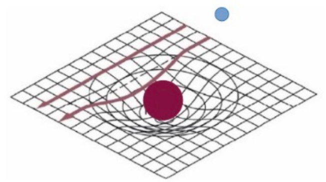
- At T=0: No Lattice. But we have astrophysics and both particle and nuclear physics



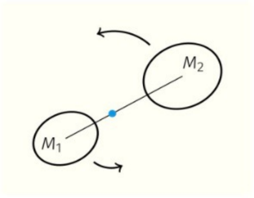


- EoS of the inner core? Upper and lower bound from pQCD and Chiral EFT + **astrophysical measurements** (including **GW data** from binary neutron star mergers)

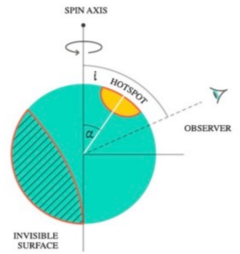
Equation of state in inner core



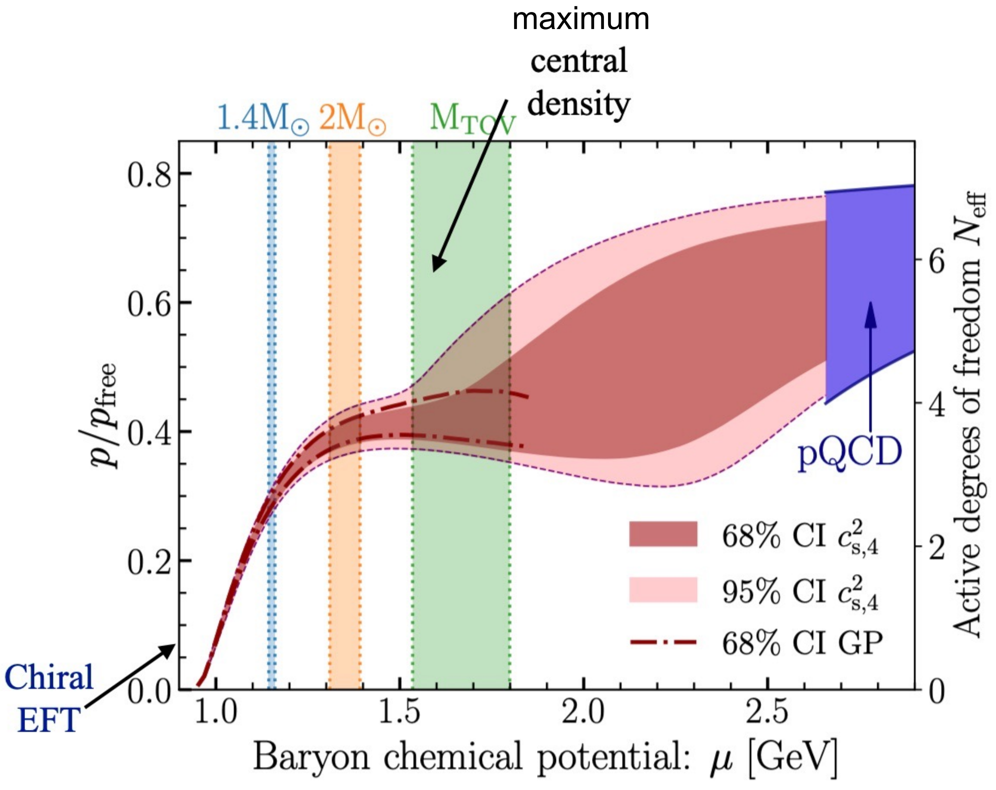
Masses



Deformabilities



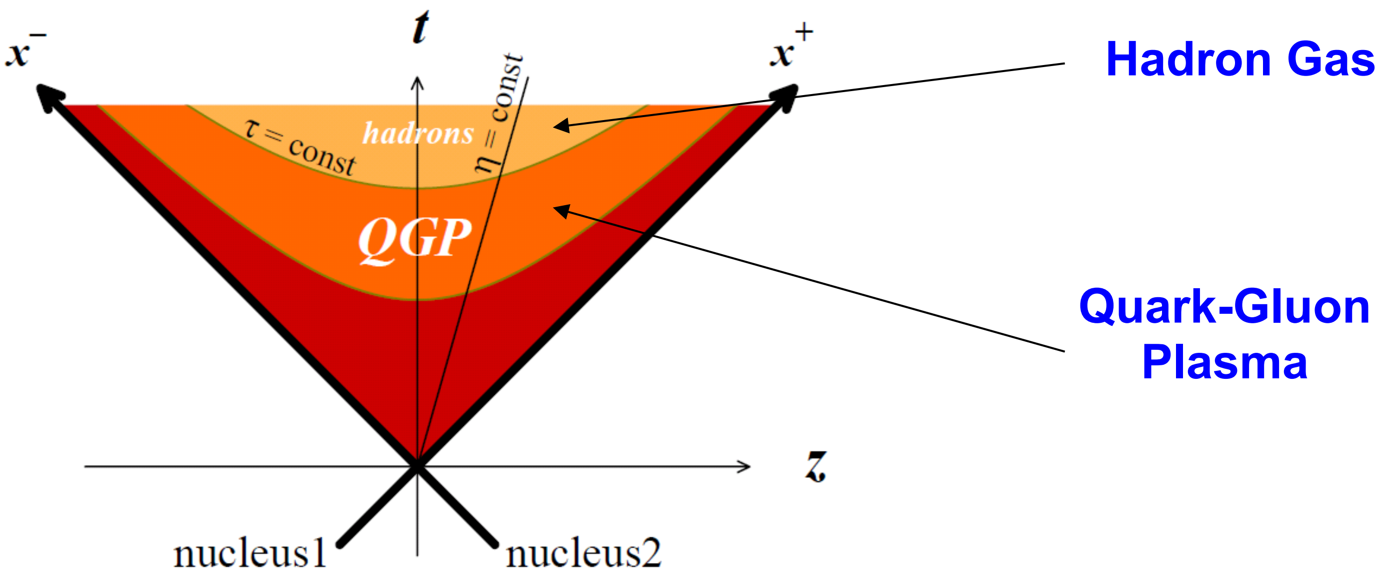
Radii, compactness

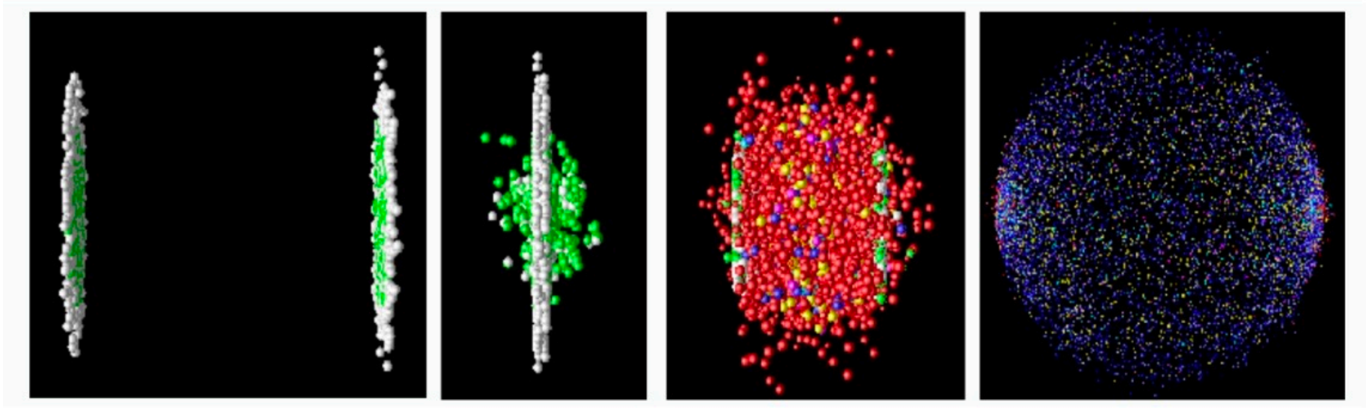


Annala, Gorda, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen, 2303.11356

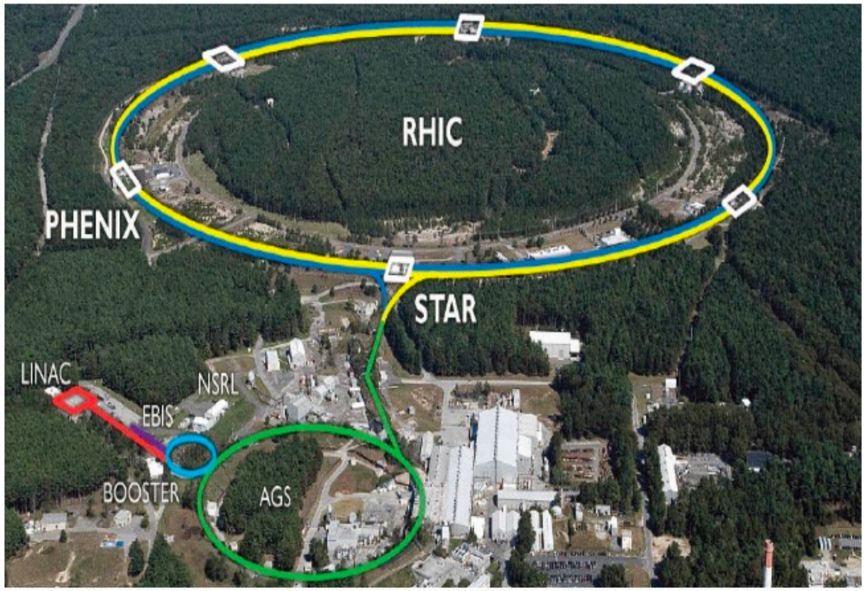
Number of degrees of freedom consistent with deconfined quark matter!

hot QCD at colliders





Heavy-ion Programs



Relativistic Heavy Ion Collider (RHIC)

Au-Au collisions

$$\sqrt{s_{\mathrm{NN}}} = 7.7 - 200\,\mathrm{GeV}$$

(Also d-Au, He-Au, Cu-Cu, O-O…)

Large Hadron Collider (LHC)

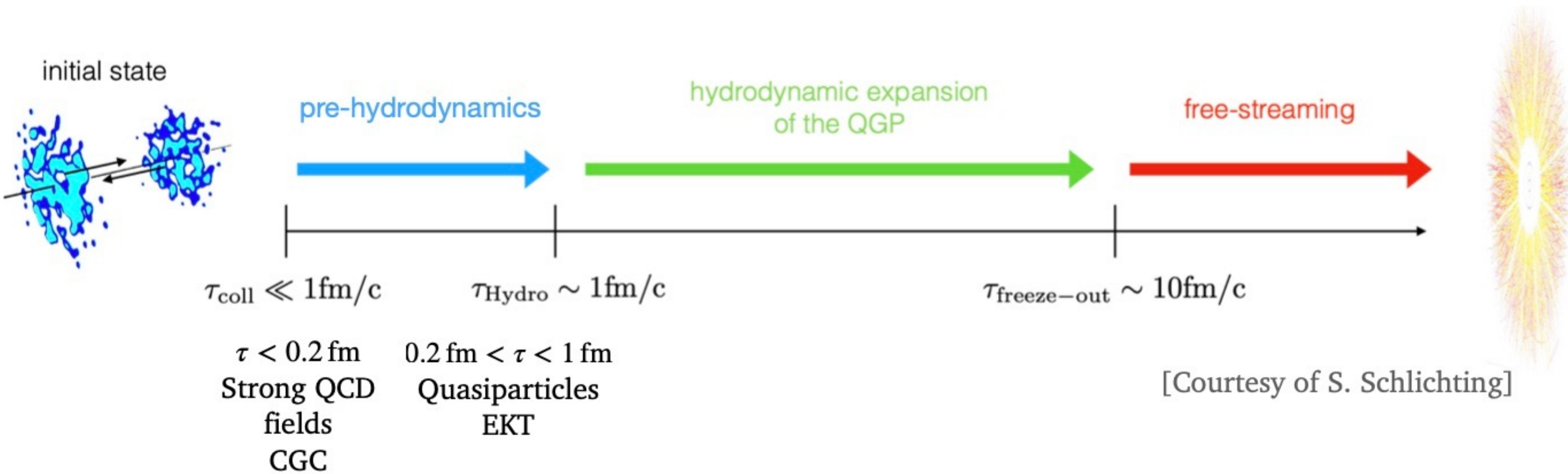
Pb-Pb collisions

$$2010 - 2011 : \sqrt{s_{\mathrm{NN}}} = 2.76\,\mathrm{TeV}$$
$$2011 - 2015 : \sqrt{s_{\mathrm{NN}}} = 5.02\,\mathrm{TeV}$$
$$2023 - 2025 : \sqrt{s_{\mathrm{NN}}} = 5.36\,\mathrm{TeV}$$

(Also p-Pb, Xe-Xe)

Heavy-ion Collisions

- Dynamical description of heavy-ion collisions from underlying theory of QCD remains a challenge
- Standard picture based on **effective descriptions of QCD** exploiting the clear separation of time scales



[Courtesy of S. Schlichting]

Relativistic hydrodynamics

Solve numerically: $\delta_\mu T^{\mu\nu} = 0$

Input: EoS from Lattice

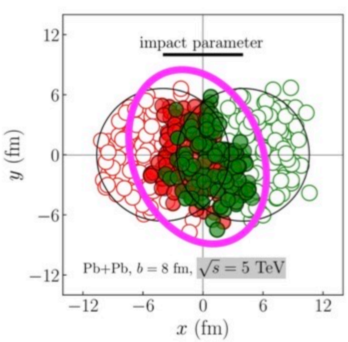
Output: extracted from data

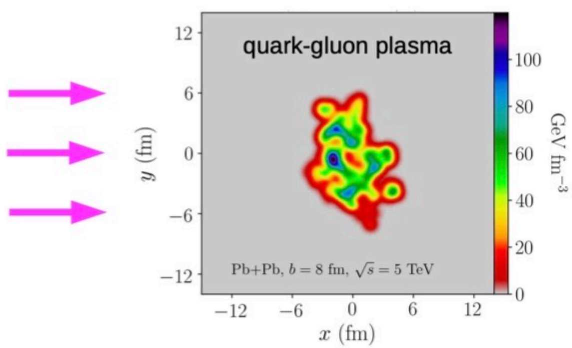
$$T_{\mu\nu} = \varepsilon\, u_\mu u_\nu + p[\varepsilon]\Delta_{\mu\nu} - \eta[\varepsilon]\,\sigma_{\mu\nu} - \zeta[\varepsilon]\Delta_{\mu\nu}\nabla_\mu u^\mu + \mathcal{O}$$

$$\sigma_{\mu\nu} = \Delta_{\mu\alpha}\Delta_{\nu\beta}(\nabla^\alpha u^\beta + \nabla^\beta u^\alpha) - \frac{2}{3}\Delta_{\mu\nu}\Delta_{\alpha\beta}\nabla^\alpha u^\beta,$$

$$\Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu,$$

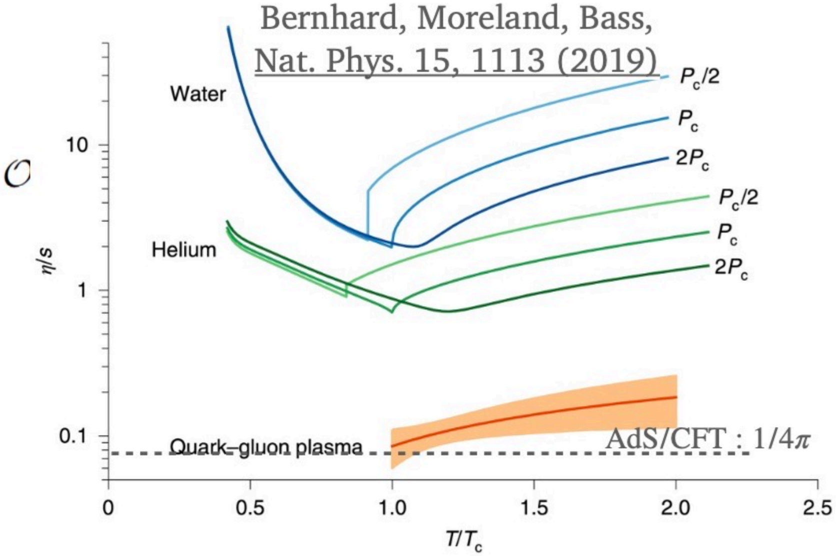
+ initial condition





Global fits

Bernhard, Moreland, Bass, Nat. Phys. 15, 1113 (2019)

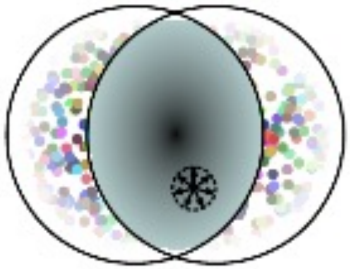


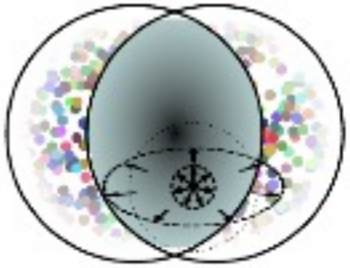
See also:
Schenke, Shen, Tribedy, Phys. Rev. C 102 (2020) 044905
JETSCAPE,s Phys. Rev. C 103 (2021) 054904
Nijs, van der Schee, Gürsoy, Snellings, Phys. Rev. C 103 (2021) 054909

Very small η/s: **most perfect** fluid in Nature

The QGP flows like a fluid

the initial momentum distribution is isotropic

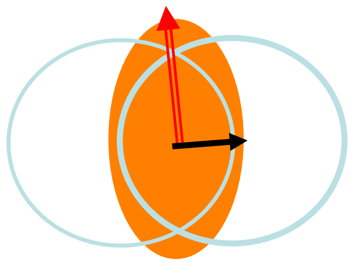




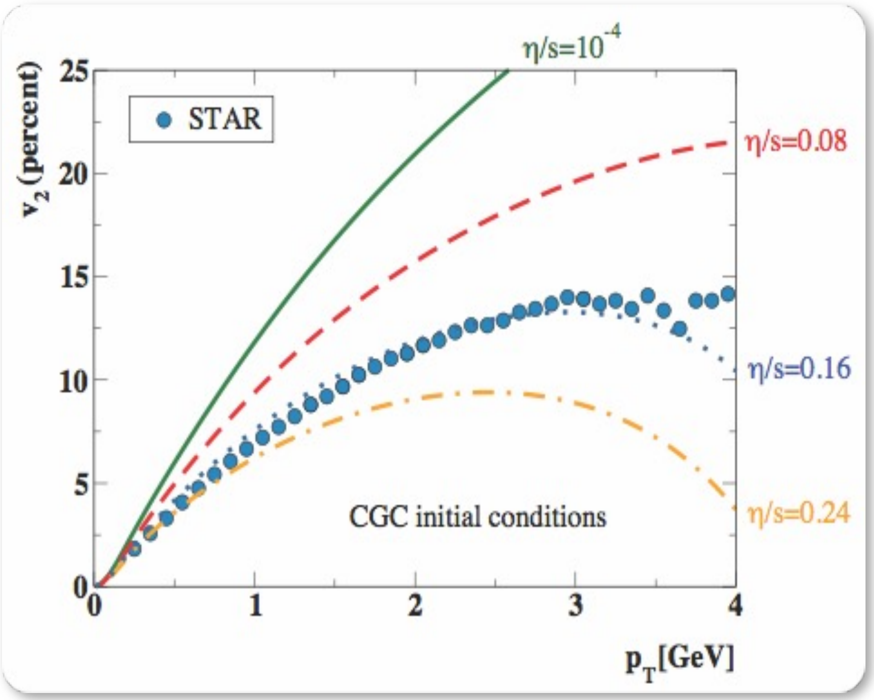
strong interactions induce pressure gradients
the expansion turns the space anisotropy
into a momentum anisotropy

a complete causal formulation of relativistic viscous hydro was developed

Elliptic flow



$$v_2(p_T, b) = \frac{\int d\phi \cos(2\phi)\ d\sigma_{AA}/d^2p_T d^2b}{\int d\phi\ d\sigma_{AA}/d^2p_T d^2b}$$



a measure of the viscosity

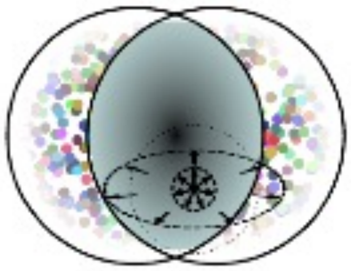
AdS/CFT bound: $\frac{\eta}{s} > \frac{1}{4\pi}$

Two ingredients needed for flow

flow is an **initial spatial anisotropy** turned into a momentum anisotropy by the **hydrodynamic expansion** of the medium

final-state harmonic ⟵ $$v_n = \kappa_n \varepsilon_n$$ ⟶ initial-state harmonic

v_2 has two components: a geometric one and one due to fluctuations (the geometric component vanishes in central collisions)



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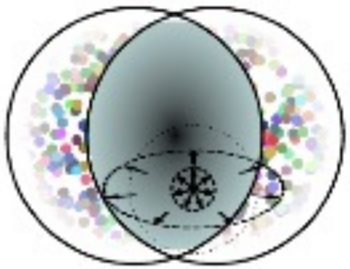
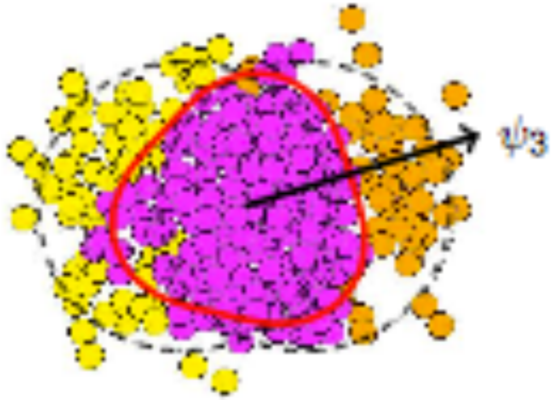




v_3 is only due to fluctuations

The eccentricity harmonics

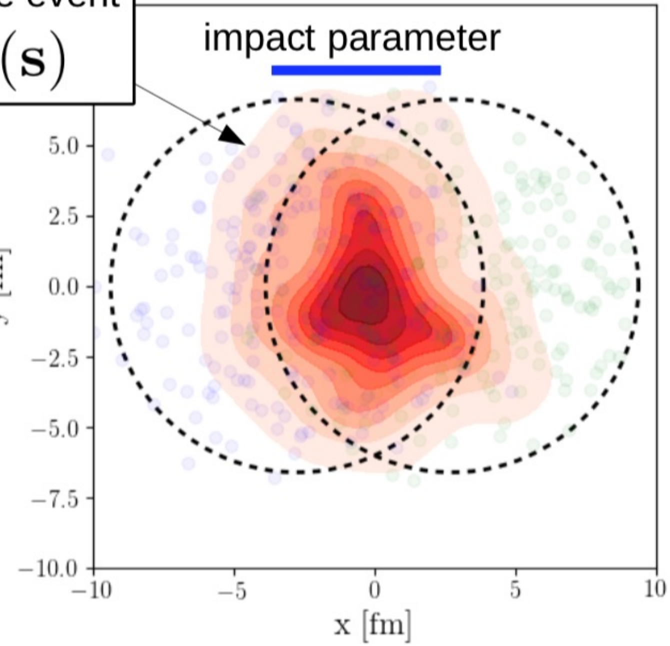
How do we calculate the initial anisotropy?

[Teaney, Yan **1010.1876**]

$$\varepsilon_n \equiv \frac{\int_{\mathbf{s}} \mathbf{s}^n \rho(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \rho(\mathbf{s})}$$

$$\mathbf{s} = x + iy$$

density field in one event $\rho(\mathbf{s})$



Origin of anisotropy:

n=2
elliptic flow ⟶ **geometry** + **fluctuations**
[PHOBOS Collaboration **nucl-ex/0610037**]

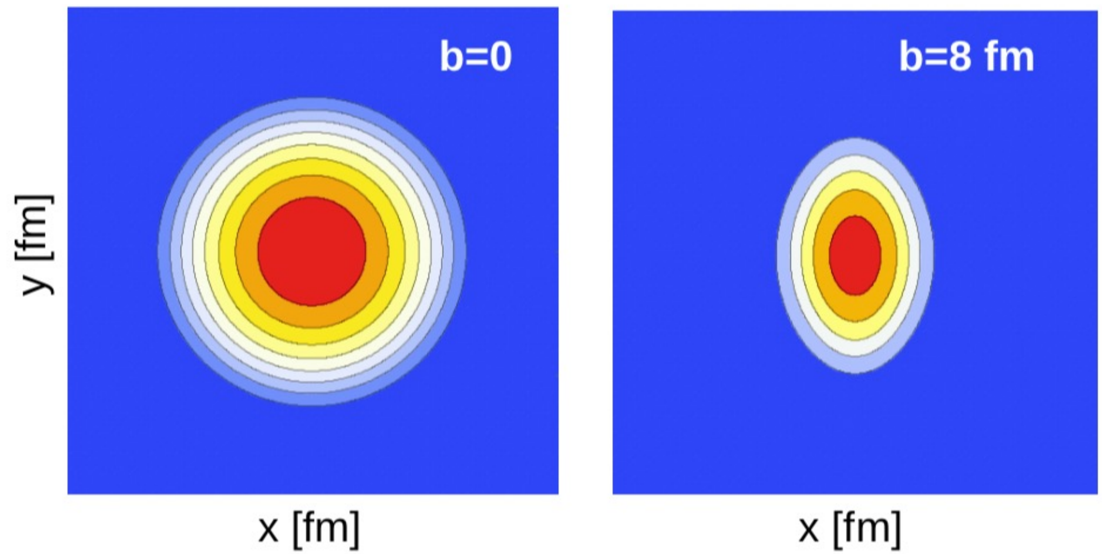
n=3
triangular flow ⟶ **fluctuations only**
[Alver, Roland **1003.0194**]

The theoretical input is a model for $\rho(\mathbf{s})$ and its fluctuations.

What is needed ?

- $\langle \rho(\mathbf{s}) \rangle$

The average density.



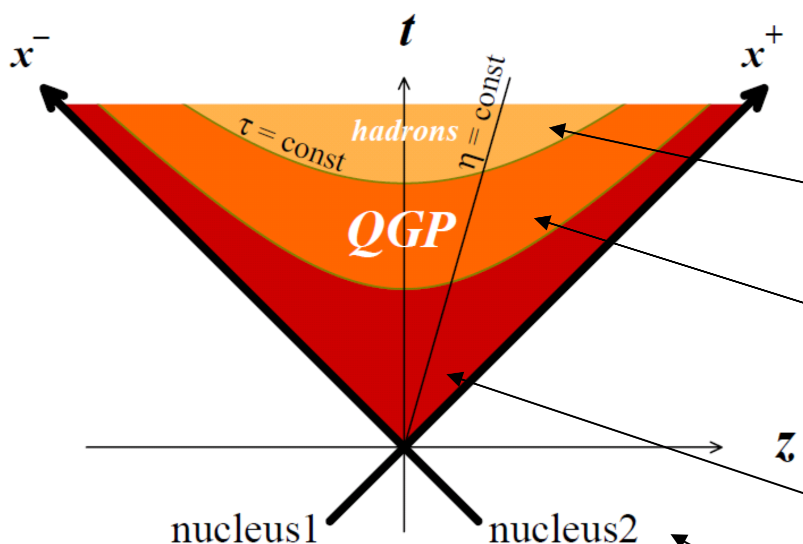
- **The fluctuations** around the average.

$$S(\mathbf{s}_1, \mathbf{s}_2) = \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2) \rangle - \langle \rho(\mathbf{s}_1) \rangle \langle \rho(\mathbf{s}_2) \rangle$$

in particular the integral $\xi(\mathbf{s}) = \int_{\mathbf{r}} S\left(\mathbf{s} + \frac{\mathbf{r}}{2}, \mathbf{s} - \frac{\mathbf{r}}{2}\right)$

→ compute the 1-point and 2-point energy correlators

Initial eccentricity fluctuations from first principles ?



5. Individual hadrons

freeze out

4. Hadron gas

cooling with expansion

3. Quark Gluon Plasma

thermalization, expansion

2. Pre-equilibrium state

collision

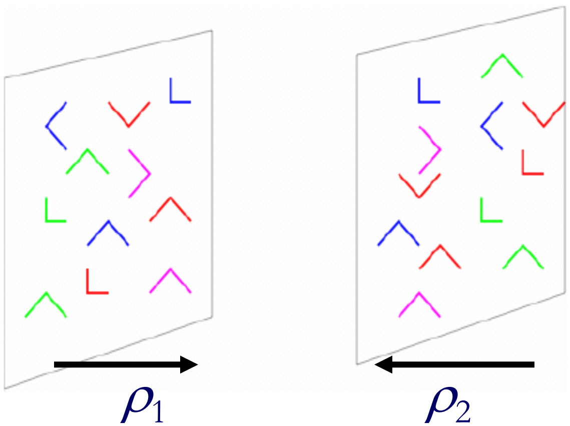
1. Nuclei (initial condition) ← CGC

$$v_n = \kappa_n \varepsilon_n$$

CGC = Color Glass Condensate

The collision of two CGCs

- the initial condition for the time evolution in heavy-ion collisions



before the collision:

$$J^\mu = \delta^{\mu+}\delta(x^-)\rho_1(x_\perp) + \delta^{\mu-}\delta(x^+)\rho_2(x_\perp)$$

$$\rho_1 \sim 1/g \qquad \rho_2 \sim 1/g$$

the distributions of ρ contain the small-x evolution of the nuclear wave functions

$$|\Phi_{x_1}[\rho_1]|^2 \qquad |\Phi_{x_2}[\rho_2]|^2$$

$\rho(x_\perp) = -\nabla^2\alpha(x_\perp)$ denotes the color charge which generates the field

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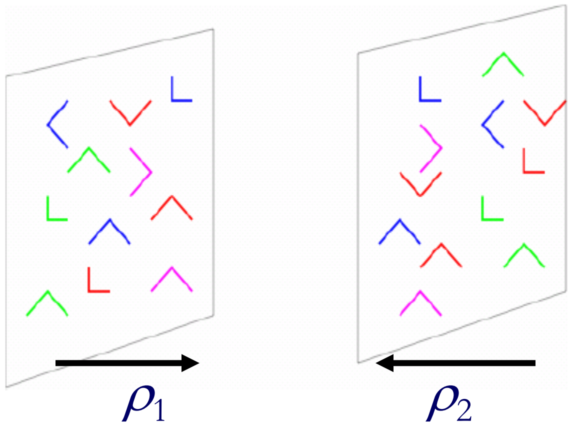

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- after the collision

the gluon field is a complicated function of the two classical color sources

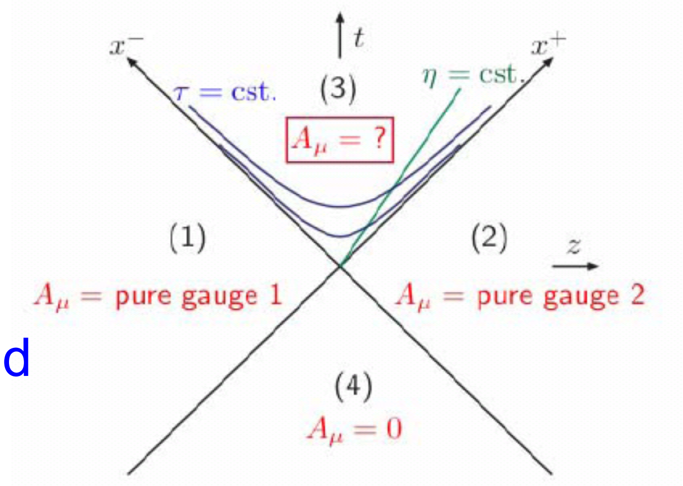
the field decays, once it is no longer strong (classical)
a particle description is again appropriate

“strong-field” QCD factorization

- solve Yang-Mills equations

$$[D_\mu, F^{\mu\nu}] = J^\nu \longrightarrow \mathcal{A}_\mu[\rho_1, \rho_2]$$

this is done numerically (it can be done analytically in the p+A case)



- express observables in terms of the field

determine $O[\mathcal{A}_\mu]$, in general a non-linear function of the sources

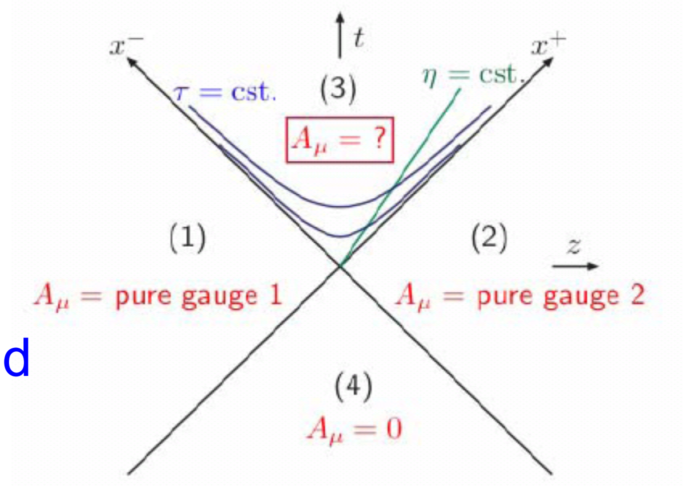
e.g. $$T^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^{\lambda\sigma} F_{\lambda\sigma} - F^{\mu\lambda} F^{\nu}_{\lambda}$$

“strong-field” QCD factorization

- solve Yang-Mills equations

$$[D_\mu, F^{\mu\nu}] = J^\nu \longrightarrow \mathcal{A}_\mu[\rho_1, \rho_2]$$

this is done numerically (it can be done analytically in the p+A case)



- express observables in terms of the field

determine $O[\mathcal{A}_\mu]$, in general a non-linear function of the sources

e.g. $$T^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^{\lambda\sigma} F_{\lambda\sigma} - F^{\mu\lambda} F^\nu_\lambda$$

- perform the averages over the color charge densities

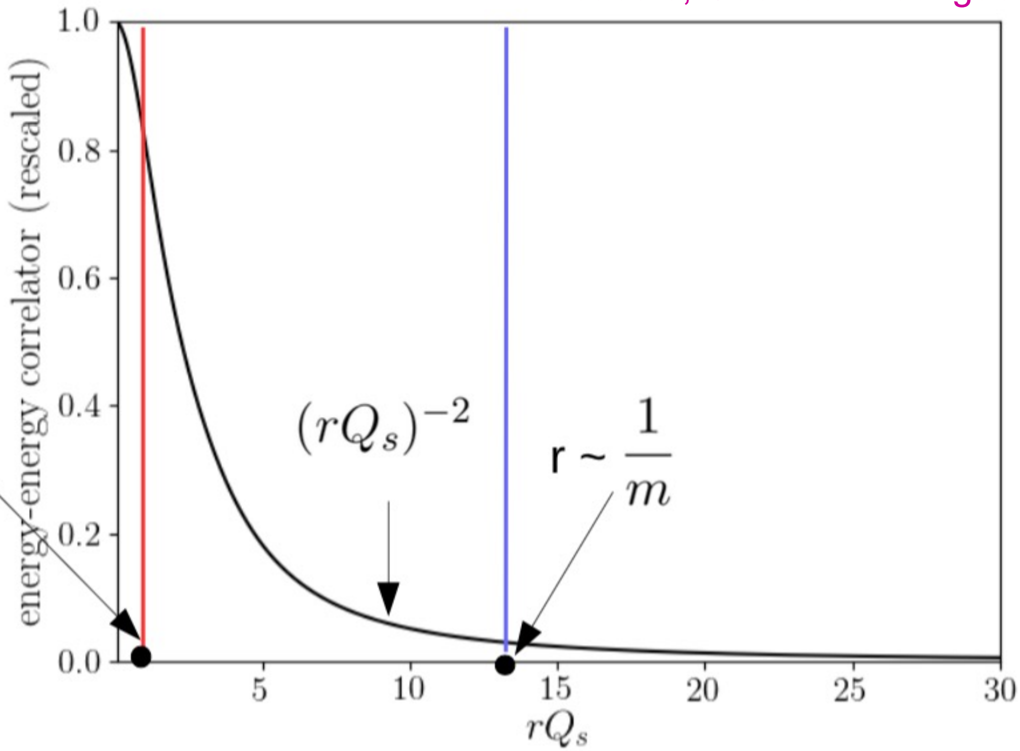
$$\langle O \rangle = \int D\rho_1 D\rho_2 |\Phi_{x_1}[\rho_1]|^2 |\Phi_{x_2}[\rho_2]|^2 O[\mathcal{A}_\mu]$$

⟶ each nucleus is characterized by its saturation scale $Q_s^2(\mathbf{s}) \propto T(\mathbf{s})$

nuclear thickness

Relevant features of $S(s_1,s_2)$

Albacete, Guerrero-Rodriguez and CM (2019)



$r \sim \dfrac{1}{Q_s}$ → ~ 0.1 fm

$r \sim \dfrac{1}{m}$

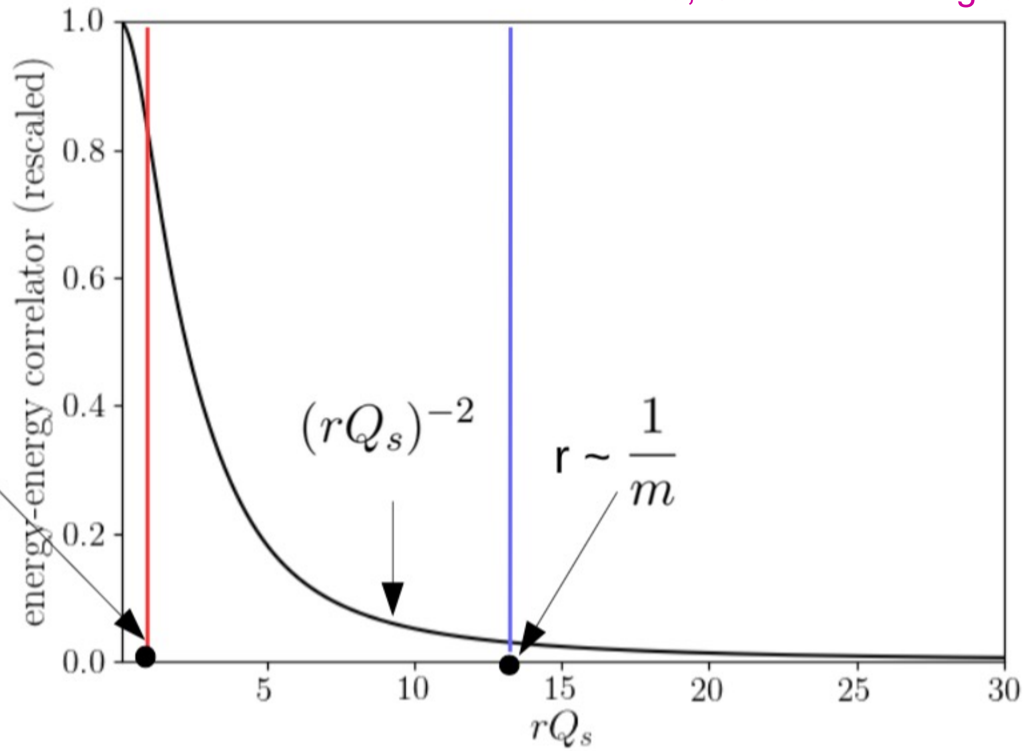
$$\frac{1}{Q_s} \ll \frac{1}{m} \ll R$$

system size ~ 10 fm

scale of color neutrality ~ 1 fm

Relevant features of $S(s_1,s_2)$

Albacete, Guerrero-Rodriguez and CM (2019)



r ~ $\frac{1}{Q_s}$

~ 0.1 fm

system size
~ 10 fm

$$\frac{1}{Q_s} \ll \frac{1}{m} \ll R$$

scale of color neutrality
~ 1 fm

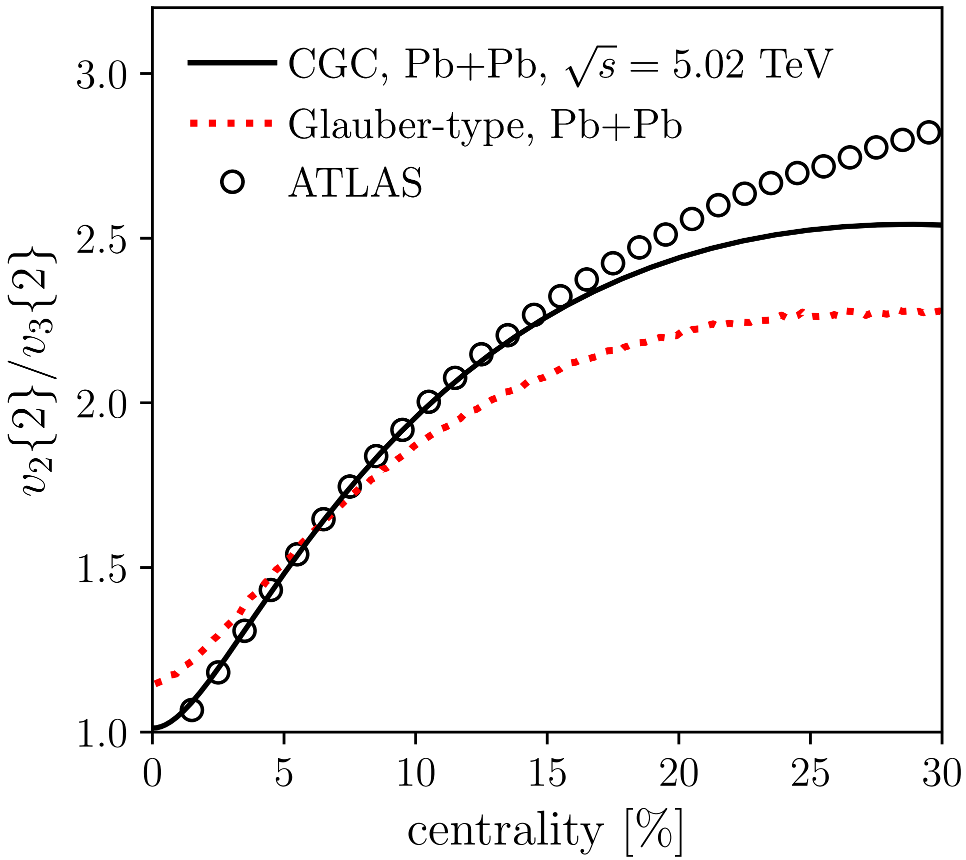
$$\langle \rho(\mathbf{s}) \rangle = \frac{4}{3g^2} Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s})$$

$$\xi(\mathbf{s}) = \frac{16\pi}{9g^4} Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s}) \left(Q_A^2(\mathbf{s}) \ln \left(1 + \frac{Q_B^2(\mathbf{s})}{m^2}\right) + Q_B^2(\mathbf{s}) \ln \left(1 + \frac{Q_A^2(\mathbf{s})}{m^2}\right)\right)$$

Giacalone, Guerrero-Rodriguez, Luzum, CM and Ollitrault (2019)

Comparison to data

only applicable for central collisions



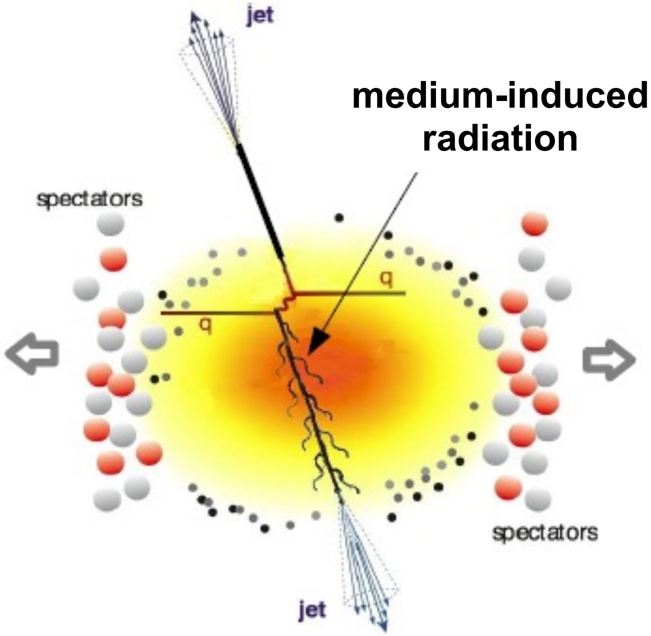
$$\frac{\epsilon_2}{\epsilon_3}(\mathbf{b}=\mathbf{0}) = \frac{8}{\sqrt{75}} \simeq 0.92$$ compensates $\kappa_2/\kappa_3 \simeq 1.1$

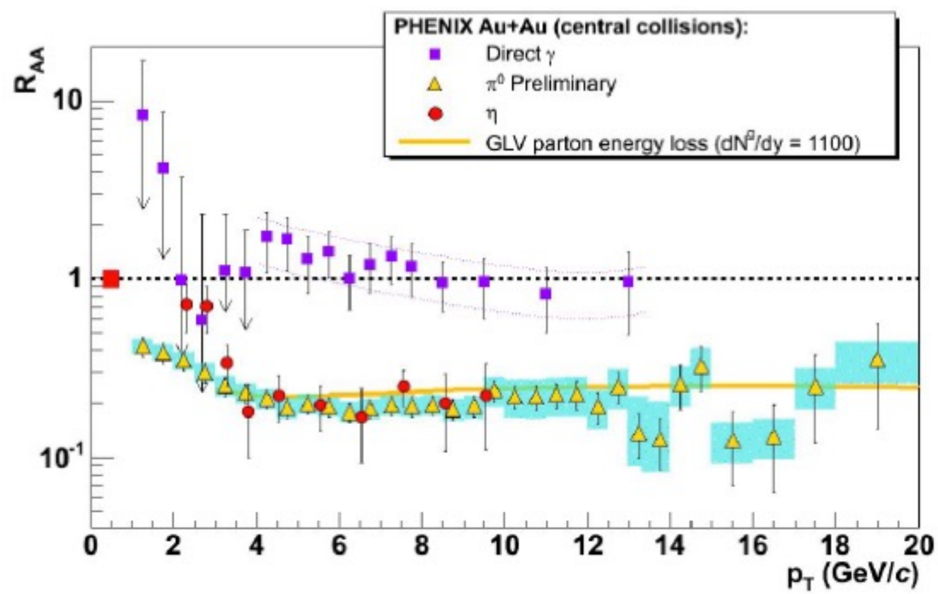
Hard probes

rare high-p_T particles created at early times that have propagated through the evolving quark-gluon plasma

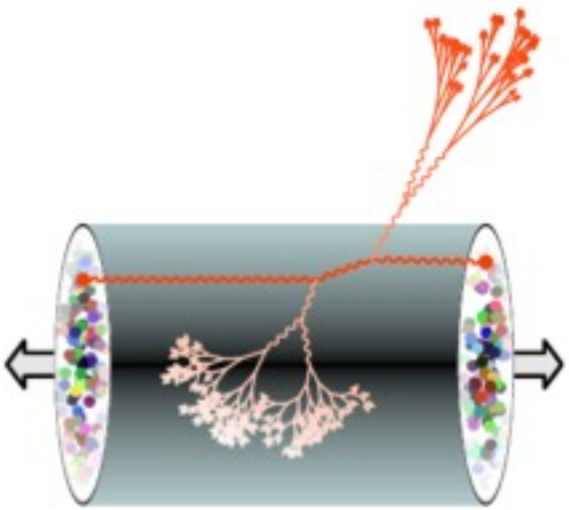
Nuclear modification factor

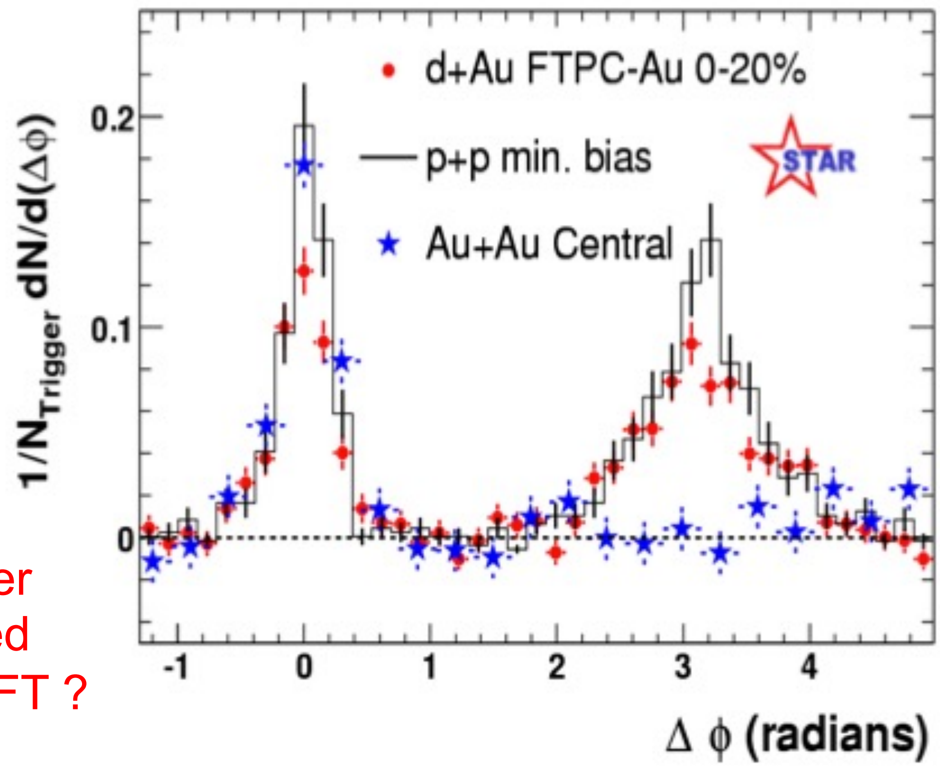
$$R_{AA}(p_T, b) = \frac{d\sigma_{AA}/dp_T d^2 b}{T_{AA}(b) d\sigma_{pp}/dp_T}$$





Jet quenching





can such a strong stopping power happen in pQCD ? or do we need strong-coupling dynamics ? AdS/CFT ?

this will be discussed by Carlota Andres in the next talk, along with novel techniques to address the problem

Conclusions

- QCD under extreme conditions (strong fields, high temperature, large baryon densities) is a vey rich field, there are many aspect I didn’t mention

- several ongoing experimental programs:

  FAIR, NICA (low energies)
  RHIC and the planned EIC (medium-energies)
  LHC (high energies)

- heavy-ions collisions (e+A, p+A, A+A) are considered in future collider discussions

Thank you for your attention!