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Review of Matching Effective Field Theories

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LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

SORBONNE UNIVERSITÉ

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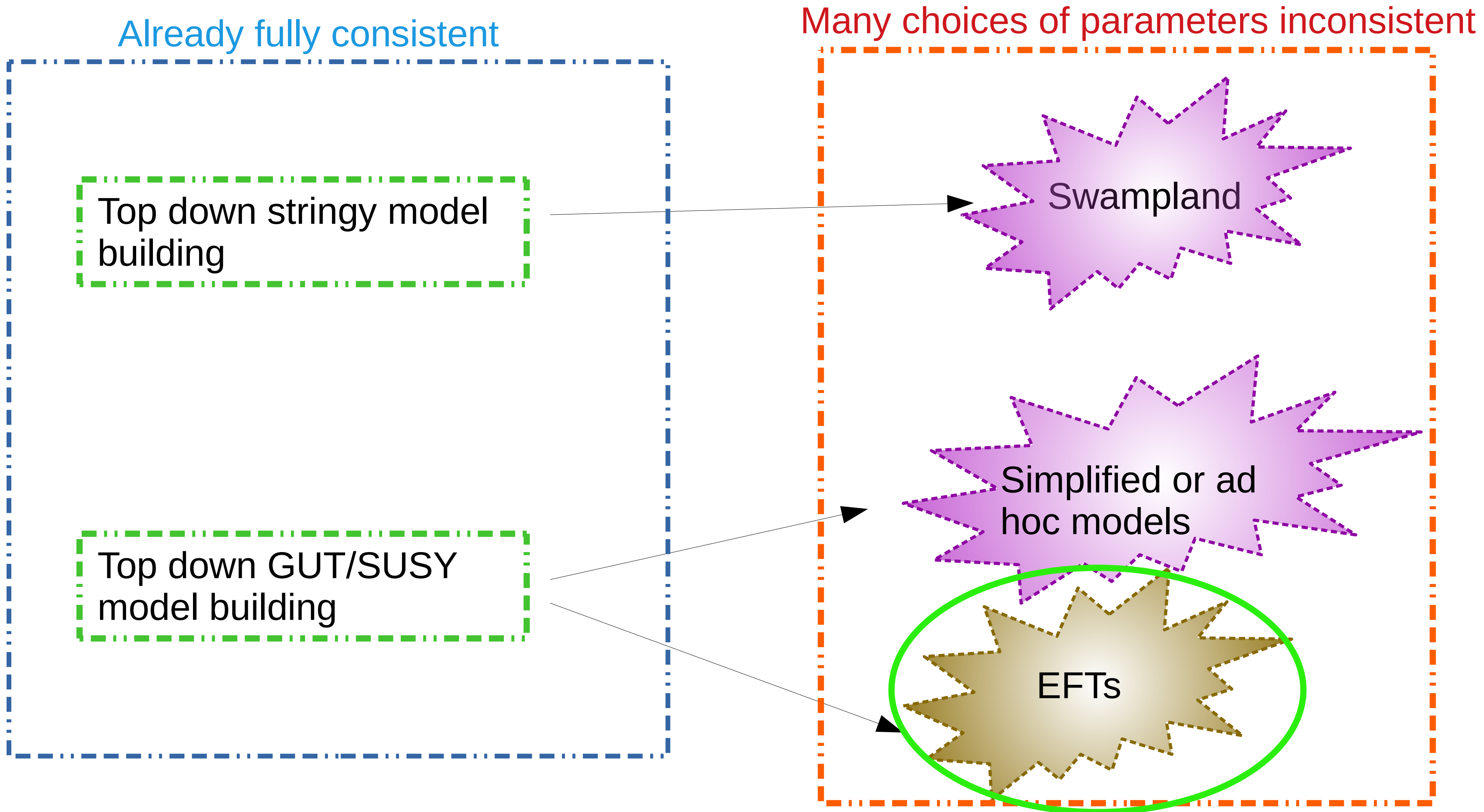
In recent years, there has been a distinct trend in the community:



Jargon warning

SM + higher-dimension operators made from SM fields in complete gauge multiplets (respects gauge symmetries). Usually up to dimension 6, contains 2499 operators in Warsaw basis

SMEFT

Generalisation of the SMEFT such that the Higgs is written as a real scalar and the underlying electroweak symmetry is explicitly broken.

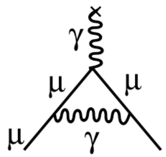
HEFT

Theory below the W/Z/higgs/top mass consisting of four-fermion operators and dipole operators. Generalisation of Fermi theory.

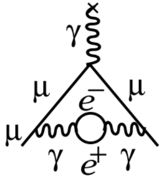
WET/ LEFT

Example: **Dipole operator**

QED diagrams

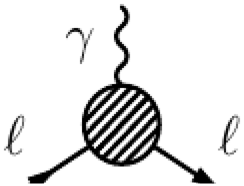


(a)



(b)

The lepton vertex function is responsible for a lot of physics:



$$= \quad (-ie\mathsf{q}_e)\,\bar{u}(p')\Gamma^\mu(p,p')u(p)$$

$$\Gamma^\mu(p,p') = \gamma^\mu F_E(k^2) + i\frac{\sigma^{\mu\nu}k_\nu}{2m_\ell}F_M(k^2) + \frac{\sigma^{\mu\nu}k_\nu}{2m_\ell}\gamma_5 F_D(k^2) + \frac{k^2\gamma^\mu - k^\mu \not{k}}{m_\ell^2}\gamma_5 F_A(k^2)$$

Gauge coupling renormalisation

Anomalous magnetic moment

Electric Dipole moment

Anapole

$$i\varepsilon_\mu(k)\sigma^{\mu\nu}k_\nu \sim \partial_0 A_i \sigma^{0i} + \partial_i A_j \sigma^{ij} \sim \mathbf{E}\cdot\sigma\gamma_5 + i\mathbf{B}\cdot\sigma$$

SMEFT → $(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$, $(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$

WET/ LEFT → $\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr}F_{\mu\nu}$

HEFT → $\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr}hF_{\mu\nu}$

Then within the study of EFTs we have a huge number of 'bottom-up' activities:

- Construction of bases: SMEFT up to high dimensions of operators, WET/LEFT with N flavours, HEFT vs SMEFT, …
- EFT RGEs
- Flavour constraints
- EWPT
- Global fits
- Building ALP EFTs (e.g. arXiv:2112.00553 and many many more)
- Building DM EFTs
- Positivity constraints
- Reinterpretation of LHC limits
- ...

In absence of a choice of model, can in principle be model agnostic and interpret all data in terms of EFTs

These are not the subject of this talk

BUT if you do have a favourite model, it can be useful to match onto an EFT

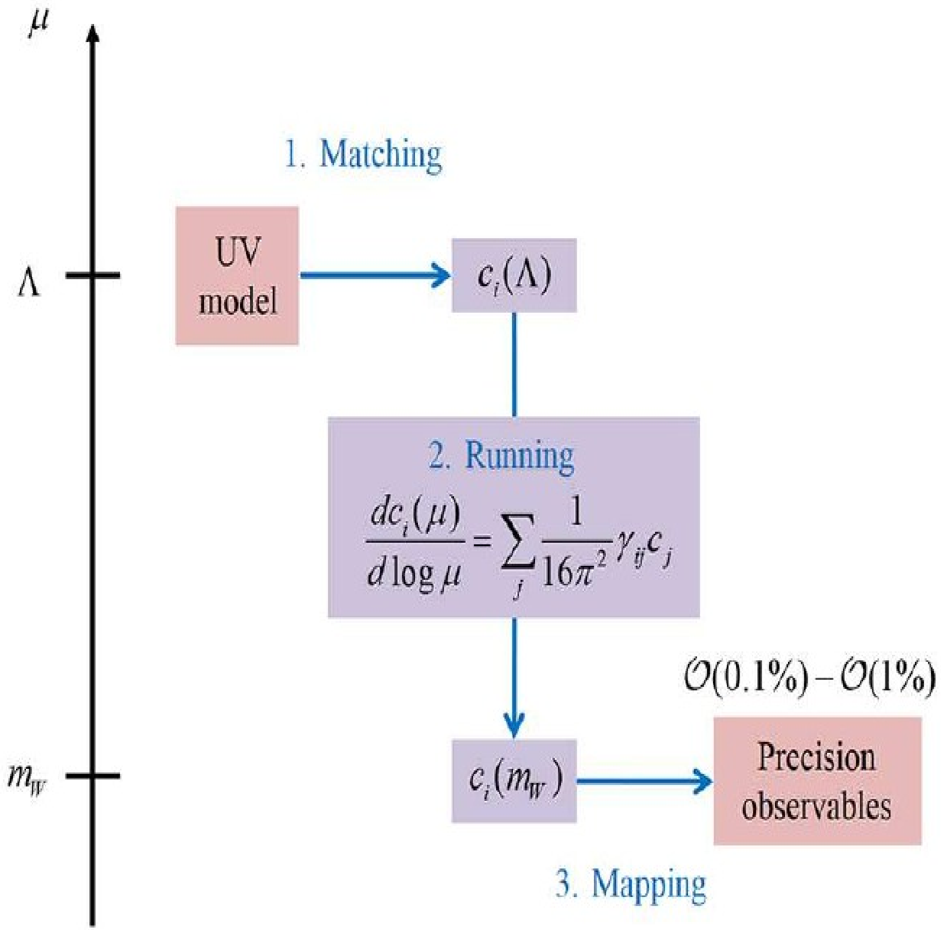
This has also attracted a lot of interest recently, in particular:

- Functional matching at one loop: arXiv:1412.1837 + many afterwards
- … e.g. 1706.07765, 1810.06994 (gauge bosons), 2006.16260 (fermions) ...
- `CoDEx`: arXiv:1808.04403
- Implementation in Mathematica: `SuperTracer`, arXiv:2012.07851, `STrEAM`, arXiv:2012.08506
- `MatchMaker`, arXiv:2112.10787
- `Matchete`, arXiv:2212.04510
- Fierz identities at one loop: arXiv:2208.10513, arXiv:2306.16449
- Using on-shell methods: arXiv:2308.00035, arXiv:2309.10851
- Functional matching at two loops: arXiv:2311.13630

This review is to decrypt what is going on and why

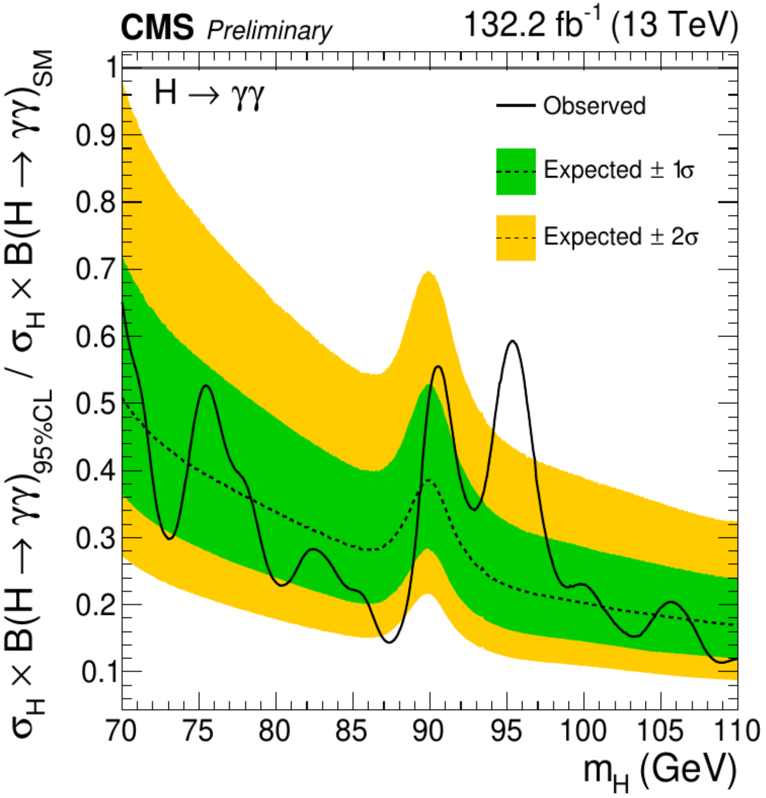
Why should you match?

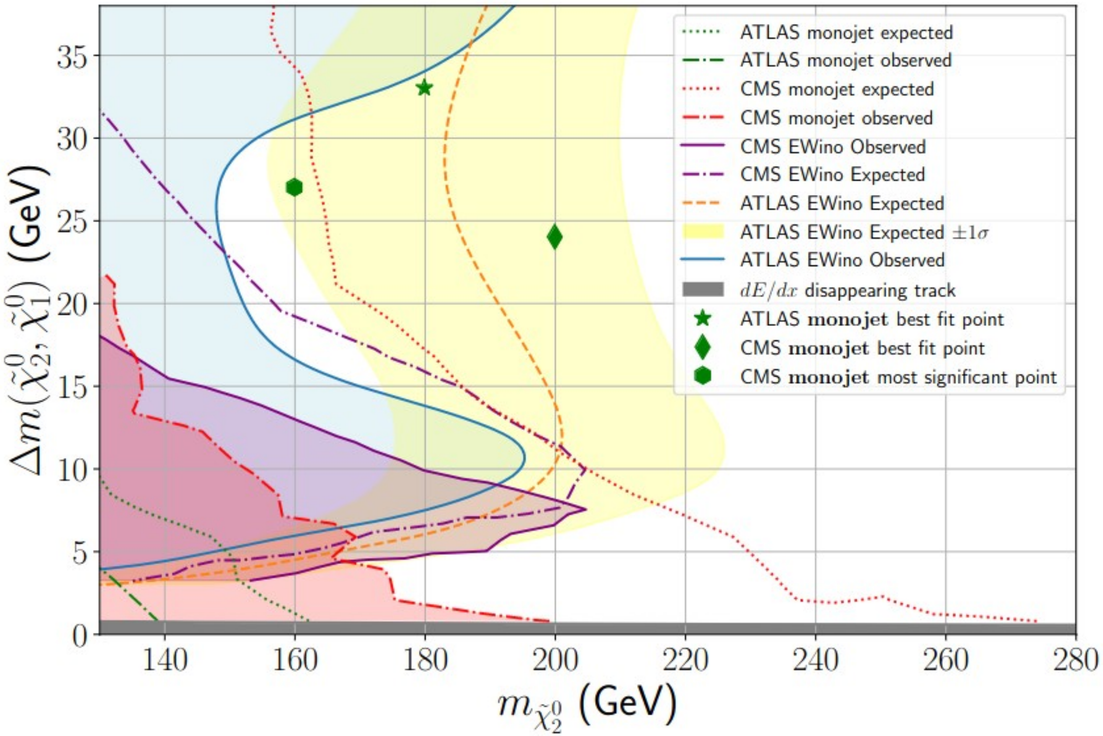
- Simplest reason: to take advantage of the technology developed for EFTs! E.g. flavour constraints such as $\mu \to e\gamma, \quad \mu \to 3e$
- Most important: **precision!** In the EFT we resum large logarithms. BSM models could consist of a tower of widely separated scales. No other way to treat them (modulo some ideas about multiscale renormalisation ...)
- Comparison with LHC information … especially once bounds on operators become more important than direct searches.
- Comparison of explicit model with positivity bounds etc.



From 1412.1837

Yes, there may (still) be motivation for light (electroweak-charged) particles!





So ideally need to be flexible about the EFT above the weak scale

Typically BSM models are either:

From some UV framework (SUSY, composite Higgs, Randall-Sundrum, …)

- May be heavy and leave only SM at low energies
- SUSY → interested in matching to the SMEFT, or SMEFT+DM
- Composite Higgs → HEFT, or vector-like quark models

or

Simplified models (add one/few particle, minimal DM, vector-like quarks, …)

- May already include effective operators as a portal
- **Should be built in to an EFT if particles are light**

Flexible about choice of EFT → genericity

Muon g-2

Only a handful of SMEFT operators are important for lepton g-2:

$$\begin{aligned}\Delta a_\ell^{250\ \mathrm{GeV}} = \frac{m_\ell}{m_\mu}\mathrm{Re}\Bigg[& \begin{matrix}2.9_\mu\\ 2.8_e\end{matrix} \times 10^{-3}\widetilde{C}_{\substack{eB\\ \ell\ell}} - \begin{matrix}1.6_\mu\\ 1.5_e\end{matrix} \times 10^{-3}\widetilde{C}_{\substack{eW\\ \ell\ell}} \\
& - \begin{matrix}4.3_\mu\\ 4.1_e\end{matrix} \times 10^{-5}\widetilde{C}^{(3)}_{\substack{\ell equ\\ \ell\ell 33}} - \left(2.6 + 0.37 c_T^{(c)}\right) \times 10^{-6}\widetilde{C}^{(3)}_{\substack{\ell equ\\ \ell\ell 22}} \\
& - 7.9 \times 10^{-8}\widetilde{C}_{\substack{\ell e\\ \ell 33\ell}} + \left(5.7 c_T - \begin{matrix}0.49_\mu\\ 0.48_e\end{matrix}\right) \times 10^{-8}\widetilde{C}^{(3)}_{\substack{\ell equ\\ \ell\ell 11}} + 1.4 \times 10^{-8}\widetilde{C}^{(1)}_{\substack{\ell equ\\ \ell\ell 33}} \\
& + \left(\begin{matrix}10_\mu\\ 9.8_e\end{matrix} + 2.5 c_T^{(c)}\right) \times 10^{-9}\widetilde{C}^{(1)}_{\substack{\ell equ\\ \ell\ell 22}} - \begin{matrix}4.6_\mu\\ 4.7_e\end{matrix} \times 10^{-9}\widetilde{C}_{\substack{\ell e\\ \ell 22\ell}} \\
& + \frac{m_\ell}{m_\mu}\Bigg\{ \begin{matrix}2.5_\mu\\ 2.4_e\end{matrix} \times 10^{-8}\left(\widetilde{C}_{HWB} + i\widetilde{C}_{H\widetilde{W}B}\right) - \begin{matrix}1.8_\mu\\ 1.7_e\end{matrix} \times 10^{-8}\left(\widetilde{C}_{HB} + i\widetilde{C}_{H\widetilde{B}}\right) \\
& - \begin{matrix}6.0_\mu\\ 5.7_e\end{matrix} \times 10^{-9}\left(\widetilde{C}_{HW} + i\widetilde{C}_{H\widetilde{W}}\right) + 3.8 \times 10^{-9}\widetilde{C}_{\substack{He\\ \ell\ell}} - \begin{matrix}3.7_\mu\\ 3.6_e\end{matrix} \times 10^{-9}\widetilde{C}^{(1)}_{\substack{Hl\\ \ell\ell}} \\
& + \begin{matrix}3.6_\mu\\ 3.3_e\end{matrix} \times 10^{-9}\widetilde{C}^{(3)}_{\substack{Hl\\ \ell\ell}} + \begin{matrix}1.8_\mu\\ 1.7_e\end{matrix} \times 10^{-9}\widetilde{C}_{HD} + \begin{matrix}2.1_\mu\\ 2.0_e\end{matrix} \times 10^{-9}\widetilde{C}_{W} \\
& + 1.1 \times 10^{-9} i\widetilde{C}_{\widetilde{W}}\Bigg\}\Bigg]\end{aligned}$$

Dipoles, contribute at leading order (one loop)

$$\begin{aligned}\Delta a_{\mathrm{exp}} &= 41 \times 10^{-11} \\ \left|a_{\mathrm{exp}} - a_{\mathrm{R-ratio}}\right| &= 251 \times 10^{-11}\end{aligned}$$

Nowadays if we can compute the SMEFT coefficients, can include running effects equivalent to the leading logs of 2-loop fixed-order ...

From Aebischer *et al*, 2102.08954

EDMs

$$d_\ell = -\frac{e_{\rm QED}\,\mathsf{q}_e}{2m_\ell}F_D(0)$$

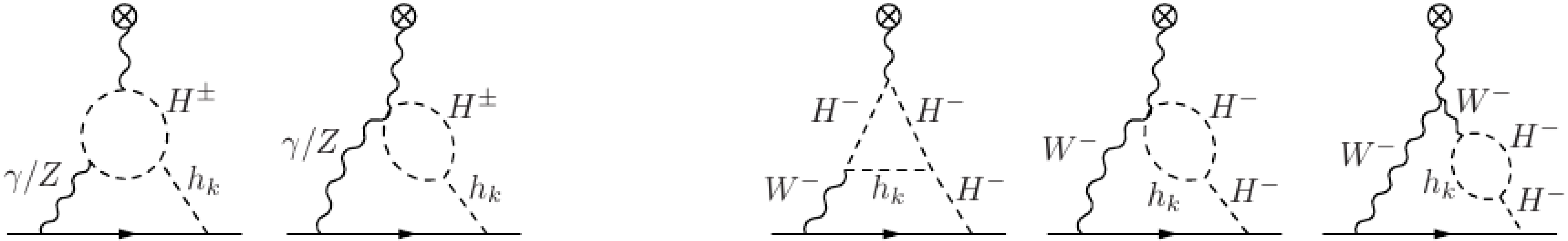
Lepton EDMs have very tight constraints from experiments:

$$d_\ell^{SM} \lesssim 10^{-38} e\,\text{cm}$$

$$|d_e| < 1.1 \times 10^{-29}\ e\text{–cm} \qquad @\,90\%\,\text{CL}$$

$$|d_\mu| < 1.5 \times 10^{-19}\ e\text{–cm} \qquad @\,90\%\,\text{CL}$$

Yet key contributions to EDMs only appear at two loops

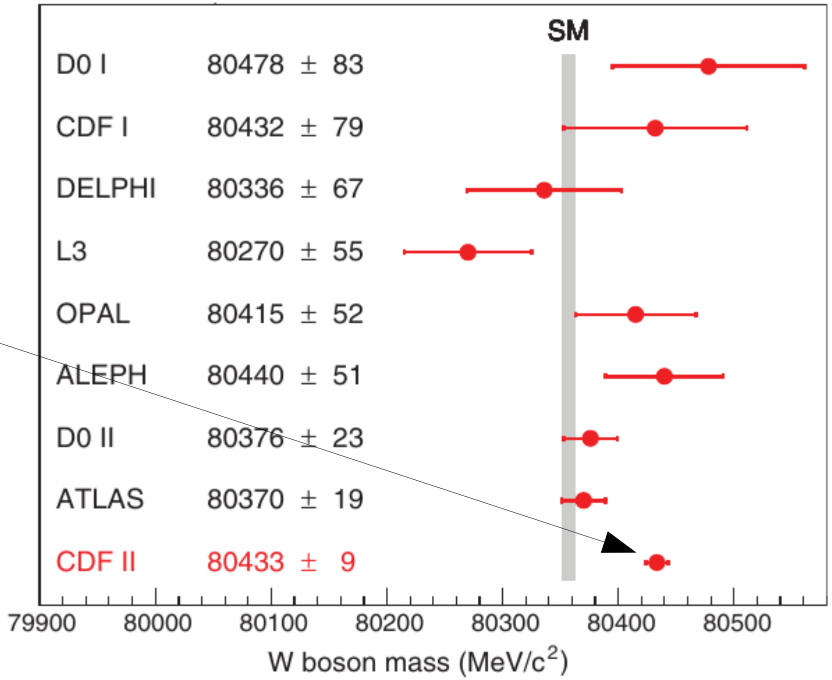


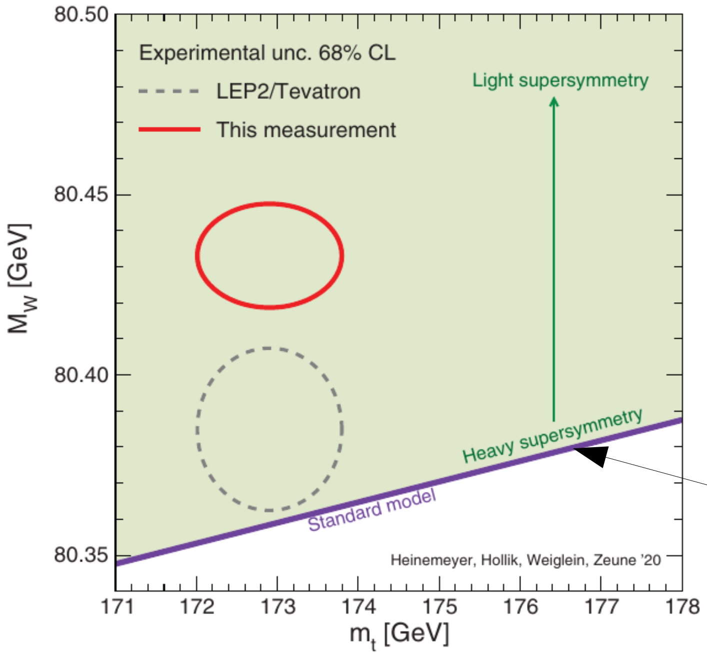
Explicit calculations only for a couple of models (here THDM 2009.01258)

(see also gluino phase in MSSM)

W boson mass

The biggest reaction from the theory community in 2022 was the CDF II W boson mass measurement





Could be explained by heavy particles, either at EW scale or much higher (multi-TeV) via tree-level effect

Models to enhance the W mass

Currently > 500 citations to the CDF paper!

Explanations generally one or more of:

- EFT fits
- Models with extra SU(2) reps (e.g. triplet) with a vev
- Models with light EW states (loops)
- Models with a Z' that mixes with the Z (we fix the Z mass and the weak mixing angles from observations)

$$\rho \equiv 1 + \Delta\rho_{\rm tree} + \Delta\rho = 1 + 4\frac{v_T^2}{v^2} + \Delta\rho.$$

$$\Delta_{\rm tree} M_W^2 = \frac{c_W^2}{c_W^2 - s_W^2} (M_W^2)_{\rm SM} \Delta\rho_{\rm tree},$$

$$\Delta_{\rm tree} s_W^2 = -\frac{s_W^2 c_W^2}{c_W^2 - s_W^2} \Delta\rho_{\rm tree},$$

$$\Delta\rho \equiv \frac{M_W^2}{c_W^2 M_Z^2} - 1$$

Traditionally in a fixed order computation (expanding to one loop) we use:

$$s_W^2 c_W^2 = \frac{\pi\alpha(0)}{\sqrt{2}G_F M_Z^2}\left(1 + \Delta\alpha - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW}(0)}{M_W^2} + \delta_{VB}\right)$$

To extract the weak angle

$$\alpha(0) = \frac{g_2^2(M_Z)g_Y^2(M_Z)}{4\pi[g_2^2(M_Z)+g_Y^2(M_Z)]}\left[1 - \Delta\alpha_{\rm had}^{(5)}(M_Z) - \Delta\alpha_{\rm pert}\right] \equiv \alpha(M_Z)[1-\Delta\alpha]$$

For the EM gauge coupling

$$M_Z^2 = \frac{\pi\alpha(M_Z)v^2}{s_W^2 c_W^2} - \Pi_{ZZ}(M_Z^2)$$

To extract the electroweak vev

Typically this is done iteratively (which breaks perturbation theory …)

Then we compute the W mass from $M_W^2 = \hat{M}_W^2(Q) - \mathcal{R}e\,\Pi_{WW}^T(M_W^2)$

In the SM the **two loop computation** is required to reach the experimental precision!

In BSM models typically only one loop computation is available

W mass using the SMEFT

Corrections to the SM value can be conveniently parametrised in the SMEFT:

$$\delta m_W^2 = \hat{m}_W^2 (\sqrt{2}\delta G_F + 2\frac{\delta g_2}{\hat{g}_2})$$

$$= -\frac{s_{2\hat{\theta}}\bar{v}_T^2}{4c_{2\hat{\theta}}}\left(\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}C_{HD} + \frac{s_{\hat{\theta}}}{c_{\hat{\theta}}}(4C_{H\ell}^{(3)} - 2C_{\ell\ell}) + 4C_{HWB}\right)$$

$$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$$

$$\varphi^\dagger \tau^I \varphi\, W_{\mu\nu}^I B^{\mu\nu}$$

$$\left(\varphi^\dagger D^\mu \varphi\right)^\star \left(\varphi^\dagger D_\mu \varphi\right)$$

$$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$

Ideally, would extract all the operators at the matching scale and run, this would resum large logs, even if the result is of form

$$\Delta\mathcal{O}_6 \sim \frac{1}{M^2}\log M^2/Q^2$$

This should be the approach when the matching is actually available!

c.f.

Athron *et al*, 2204.05285

$$M_W^2 = (M_W^2)_{SM}\left(1 + \frac{s_W^2}{c_W^2 - s_W^2}\left[\frac{c_W^2}{s_W^2}(\Delta\rho_{\text{tree}} + \Delta\rho) - \Delta r_W - \Delta\alpha\right]\right)$$

Can make a 1-1 mapping between this and combinations of SMEFT coefficients at the matching scale!

Tree-level part

e.g.

$$C_{HD} = \frac{2}{v^2}\left[-\Delta\rho_0\left(1 + \frac{\Pi_{WW}(0)}{M_W^2}\right) + \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right]$$

Just need to separate out

$$\delta_{VB} = \frac{1}{16\pi^2}\left[\frac{V}{g_2} - \frac{2c_W^2 M_Z^2}{g_2^2}\mathrm{Re}(B)\right]$$

$C_{H\ell}^{(3)}$ $C_{\ell\ell}$

For now, we just match the total combination and neglect running in the SMEFT

Example procedure:

Run up SM parameters to matching scale

Guess parameters in high-energy theory based on tree-level matching plus gauge thresholds on first run. On subsequent runs use stored Yukawas

Compute loop masses in HET and “effective SM” subset of theory. Compute

$$c_W^2 s_W^2 = (c_W^2 s_W^2)_{\rm SM} \times \frac{(1+\Delta\alpha+\Delta\hat{r})}{1+\Delta\rho_{\rm tree}}$$

Match quantities in the SM from the high-energy ones via pole matching. Compute

$$\left(1+\frac{s_W^2}{c_W^2-s_W^2}\left[\frac{c_W^2}{s_W^2}(\Delta\rho_{\rm tree}+\Delta\rho)-\Delta r_W-\Delta\alpha\right]\right)$$

Once converged: run down to top mass and compute Higgs and W masses in the SM wih the above correction arXiv:2208.05867 with Benakli, Ke and Slavich

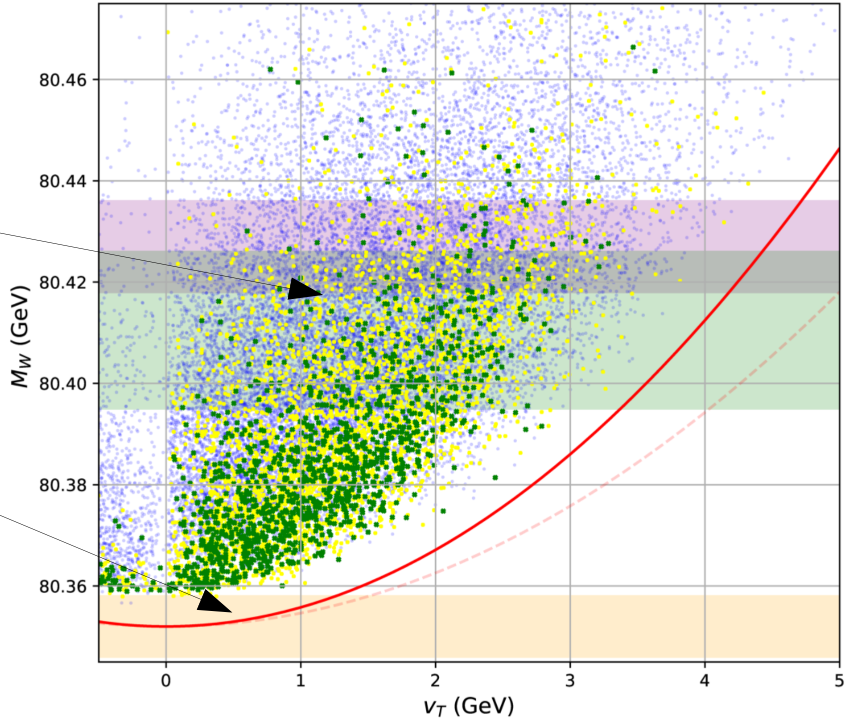
We use

$$\Delta\rho_0 = \frac{M_W^2(\text{tree})}{M_Z^2(\text{tree})}\frac{g_1^2+g_2^2}{g_2^2}$$

The tree-level relations are determined in the high-energy theory

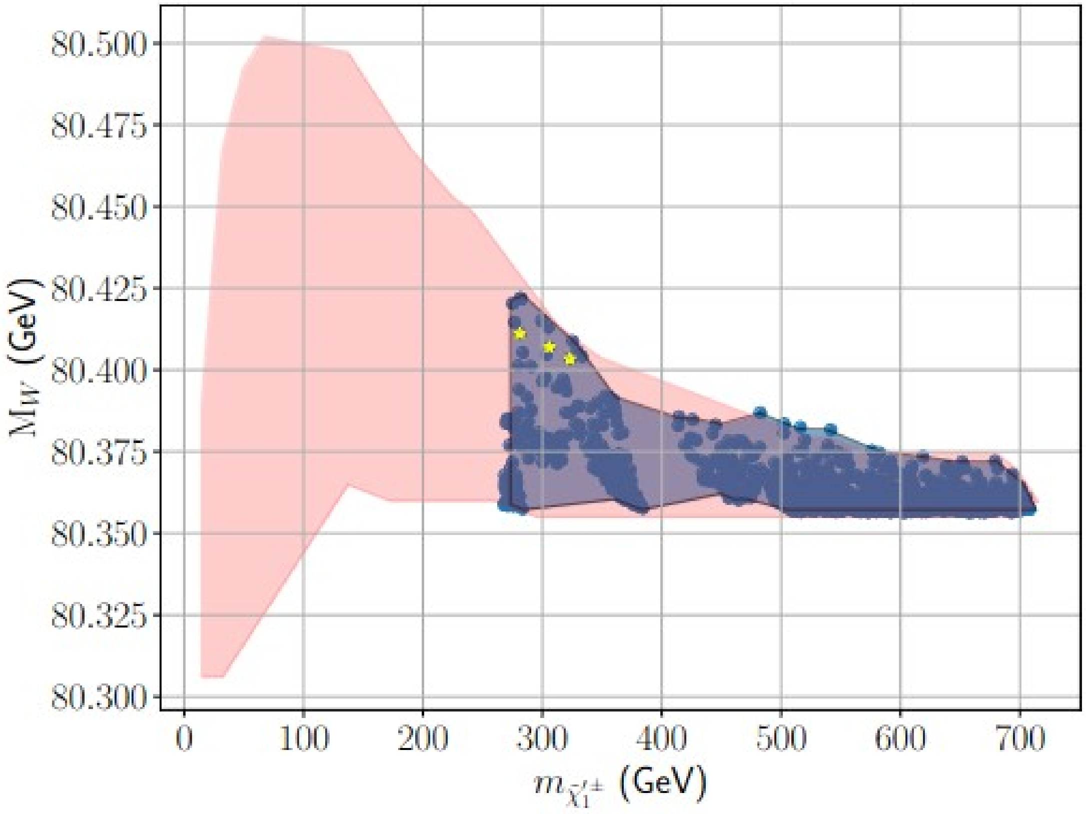
Can incorporate tree-level shifts, which occur in models with extra vevs, e.g. triplets in Dirac Gaugino models

$$\rho = 1 + 4\frac{v_T^2}{v^2} \longrightarrow M_W^2 = \rho c_W^2 M_Z^2$$



Or shifts from Z' mixing with the Z boson … such as from heterotic-inspired Z' models

e.g. A. Faraggi & MDG, 2312.13411



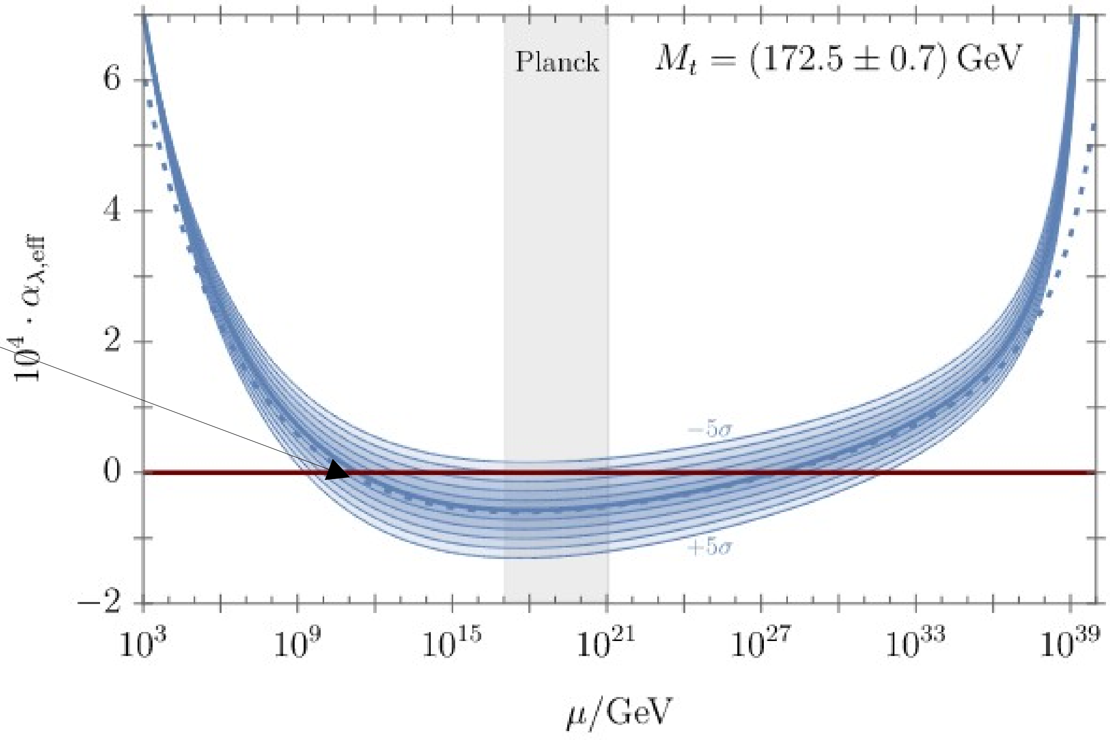
Although in this model the Z' has to be too heavy to affect the W mass in any substantial way, enhancements come through quantum corrections

Matching of renormalisable operators

Integrating out heavy fields changes the SM couplings – especially the Higgs quartic!

This is important because in pure SM the SM quartic looks like it runs negative at an intermediate scale

What if this signals that it should be replaced by a high-energy theory below this scale?



From 1807.07546

The classic example is SUSY, which predicts the Higgs quartic coupling at tree-level

$$\lambda_{\rm SM}^{\rm tree}(M_S) \;=\; \frac{1}{4}\left(g^2+g'^2\right)\,\cos^2 2\beta$$

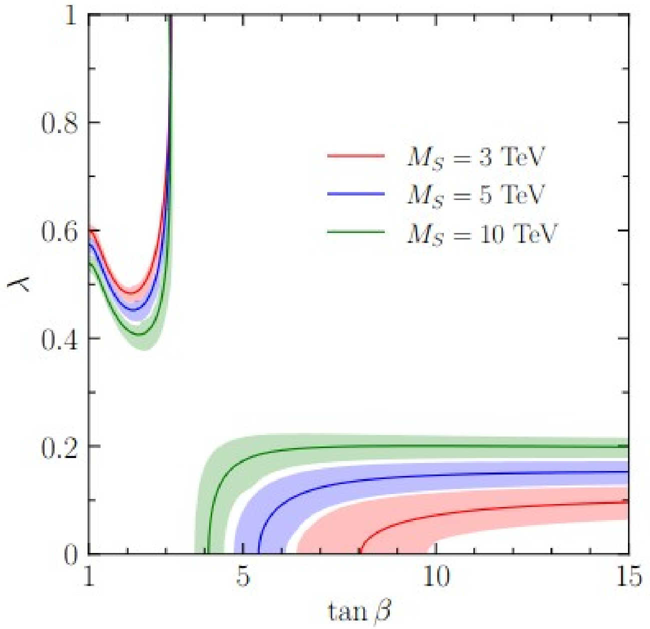
Scalar trilinears (e.g. in the NMSSM) change this at tree-level:

$$\lambda_{\rm SM}^{\rm tree}(M_S) \;=\; \frac{1}{4}\left(g^2+g'^2\right)\,\cos^2 2\beta \;+\; \frac{\lambda^2}{2}\,\sin^2 2\beta \;-\; \frac{\left[2\,\lambda^2\,v_s-\lambda\,(A_\lambda+2\,\kappa\,v_s)\,\sin 2\beta\right]^2}{2\,\kappa\,v_s\,(A_\kappa+4\,\kappa\,v_s)}$$

Loop corrections can be very important, and corrections up to two-loop order are known

See e.g. 2012.15629 for a review of the literature

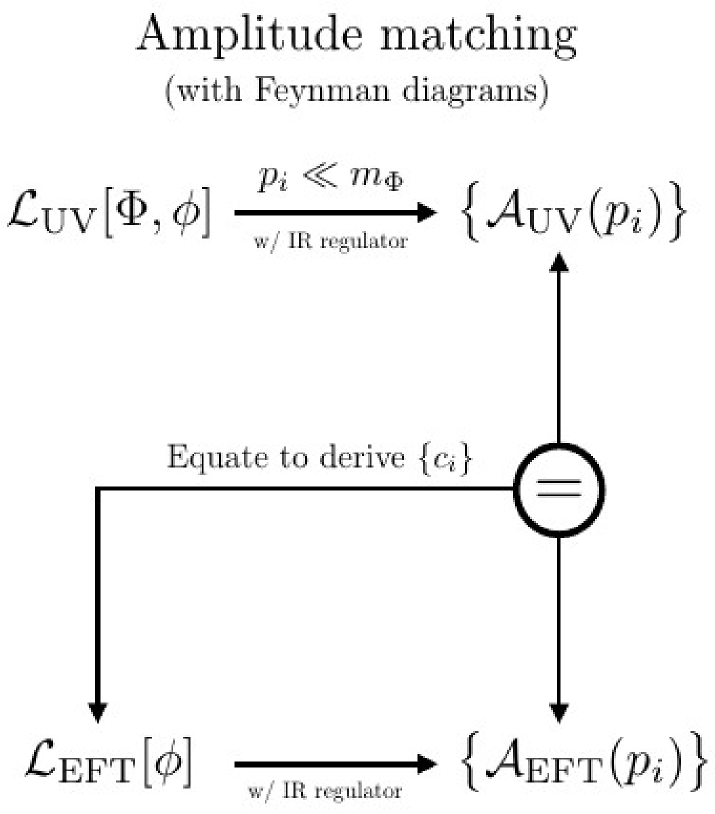
2206.04618

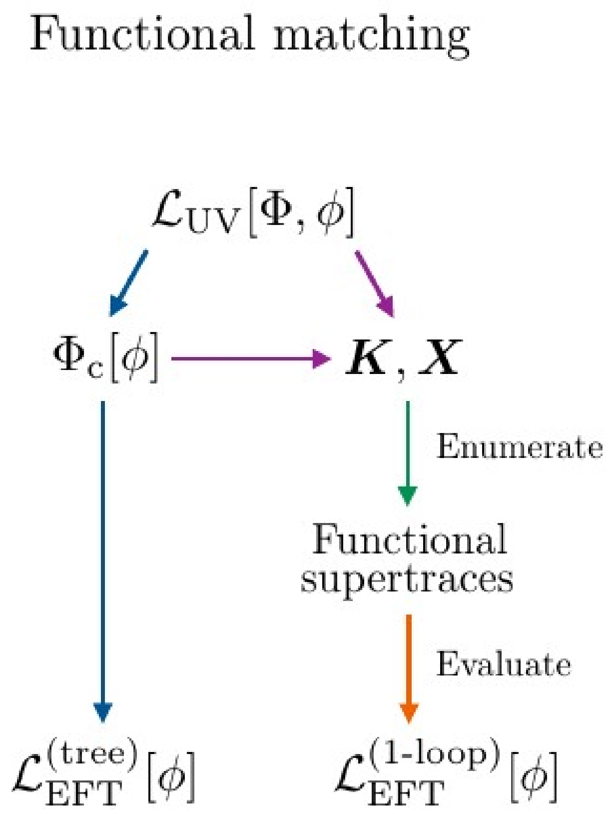


How do you match?

Bing, Microsoft. "An image to demonstrate how to match." Generated by Bing chat mode using graphic_art tool, 2023.

Two main approaches





Adapted from Cohen et al, 2011.02484

Functional matching @ 1 loop

Recall Coleman-Weinberg effective potential:

$$V_{\text{eff}} = V_0 - \frac{i}{2}\text{tr}\int \frac{d^dk}{(2\pi)^d}\log[k^2 - m^2_{eff}(\phi_{cl})]$$

$$\begin{aligned} V^{(1)}_{\text{eff}} &= \sum_i (-1)^{2s_i}(2s_i+1)\left[\frac{m_i^4}{64\pi^2}\left(\log\frac{m_i^2}{\mu^2} - c_i\right)\right] \\ &\equiv \text{STr}\left[\frac{m^4_{eff}}{64\pi^2}\left(\log\frac{m^2_{eff}}{\mu^2} - c\right)\right], \end{aligned}$$

The mass depends on the classical fields, so we can expand the above to obtain the effective potential as a power series in them.

To find the effective operators in an EFT, we integrate over the 'heavy' fields

$$\exp\left\{i\mathcal{L}_W[\phi_L]\right\} = \int \mathcal{D}\phi_H \exp\left\{i\mathcal{L}[\phi_L, \phi_H]\right\}$$

… which is equivalent to solving this equation for the heavy fields:

$$\frac{\delta\Gamma[\phi_{cl,L}, \phi_{cl,H}]}{\delta\phi_H} = 0$$

Functional matching @ 1 loop

Generalise the one-loop effective potential to include derivative operators, gauge boson and fermion background fields: compute effective operators by expanding:

$$\int \mathrm{d}^d x\, \mathcal{L}_{\mathrm{EFT}}^{(1\text{-loop})}[\phi] = \frac{i}{2}\,\mathrm{STr}\log \boldsymbol{K}\Big|_{\mathrm{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\Big[\big(\boldsymbol{K}^{-1}\boldsymbol{X}\big)^n\Big]\Big|_{\mathrm{hard}}$$

Integration over 'hard' momenta handles IR divergences from massless (SM …) fields in loop

Kinetic term

$$\mathcal{L}_{\mathrm{UV}} \supset \frac{1}{2}\,\bar{\varphi}\,\boldsymbol{K}\,\varphi = \frac{1}{2}\sum_i \bar{\varphi}_i\, K_i\, \varphi_i\,,$$

$$\delta^2\mathcal{L}_{\mathrm{UV}} = 2\,\mathcal{L}_{\mathrm{UV}}[\varphi+\delta\varphi]\Big|_{\mathcal{O}(\delta\varphi^2)} \equiv \delta\bar{\varphi}\,(\boldsymbol{K}-\boldsymbol{X}_{\mathrm{UV}})\,\delta\varphi = \delta\varphi^T\boldsymbol{R}\,(\boldsymbol{K}-\boldsymbol{X}_{\mathrm{UV}})\,\delta\varphi$$

$$K_i = \begin{cases} P^2 - m_i^2 & (\text{spin-}0) \\ \not{P} - m_i & (\text{spin-}\tfrac{1}{2}) \\ -\eta^{\mu\nu}(P^2 - m_i^2) + \left(1-\tfrac{1}{\xi}\right)P^\mu P^\nu & (\text{spin-}1) \end{cases}$$

interactions

$$\boldsymbol{X} \equiv \boldsymbol{X}_{\mathrm{UV}}\Big|_{\Phi=\Phi_{\mathrm{c}}[\phi]}$$

$$P_\mu \equiv i D_\mu$$

First subtlety: **redundancies**

Once we have generated all possible operators consisting of ‘light’ fields, either through functional matching or computing diagrams, find that they are not independent:

Integration by parts → conservation of momentum

$$\phi_L^2 \Box \phi_L^2 = 2\phi_L^2 (\partial_\mu \phi_L)^2 + 2\phi_L^3 \Box \phi_L$$

Field redefinitions

$$\phi \to N\phi + F(\phi)$$

Equations of motion

$$(D^\mu D_\mu \varphi)^j = m^2 \varphi^j - \lambda \left(\varphi^\dagger \varphi\right) \varphi^j - \bar{e}\, \Gamma_e^\dagger l^j + \varepsilon_{jk} \bar{q}^k\, \Gamma_u u - \bar{d}\, \Gamma_d^\dagger q^j$$

Fierz identies

$$(P_{L/R})[P_{R/L}] = \frac{1}{2}(\gamma^\rho P_{R/L}][\gamma_\rho P_{L/R})$$

(more on these in a minute)

Second subtlety: **infra-red divergences**

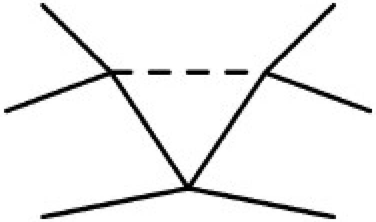
Integration of loops involving light fields leads to divergences when the light-field masses $\rightarrow 0$

$$\int d^d q \frac{1}{(q^2 - M^2)(q^2 - m^2)(q^2 - m^2)} \sim -\frac{1}{\epsilon_{IR}}\frac{1}{M^2} + \text{finite}$$

These should cancel out in the matching!

$$\sim \frac{1}{m_H^2} \log \frac{m^2}{M^2} + \text{finite}$$

- Same divergence should be present in loops of just the EFT – subtract them off!
- Can use the method of regions to handle this directly:



$$= \frac{i}{16\pi^2} 45 \frac{\kappa \lambda^2}{M^2}\Bigg|_{\text{hard}} + \cancel{\frac{i}{16\pi^2} 45 \frac{\kappa \lambda^2}{M^2} \ln\left(\frac{m^2}{M^2}\right)\Bigg|_{\text{soft}}} + \mathcal{O}(M^{-4})$$

See e.g. 1607.02142

Third subtlety: **evanescent operators**

Not all four-fermion operators are independent:

$$\mathcal{O}^{(3)}_{lequ} = (\bar{L}^a_L \sigma_{\mu\nu} e_R)\varepsilon_{ab}(\bar{Q}^b_L \sigma^{\mu\nu} u_R) \;\to\; \mathcal{O}_{luqe} = (\bar{L}^a_L u_R)\varepsilon_{ab}(\bar{Q}^b_L e_R)\,,$$

$$\mathcal{O}_{le} = (\bar{L}_L \gamma^\mu L'_L)(\bar{e}'_R \gamma_\mu e_R) \;\to\; \mathcal{O}_{le\bar{e}'\bar{l}'} = (\bar{L}_L e_R)(\bar{e}'_R L'_L)\,,$$

Fierz identities arise from completeness relation on Dirac operators

$$1 = \sum |e_i\rangle\langle e_i| = \sum |N_A|^2 (\Gamma^A)^*_{ij} (\Gamma^A)_{kl}, \qquad \Gamma_A = \{\delta_{ij}, \gamma^\mu, \sigma^{\mu\nu}, \gamma^\mu\gamma^5, \gamma^5\}$$

But this is only true in integral dimensions: in dimensional regularisation have

$$X_1 \otimes X_2 = \sum_i b_i(X_1,\, X_2)\,\Gamma_i \otimes \tilde{\Gamma}_i + E(X_1,\, X_2)$$

Any Lorentz product

Coefficient of Fierz ID

Evanescent operator, vanishes when d=4.

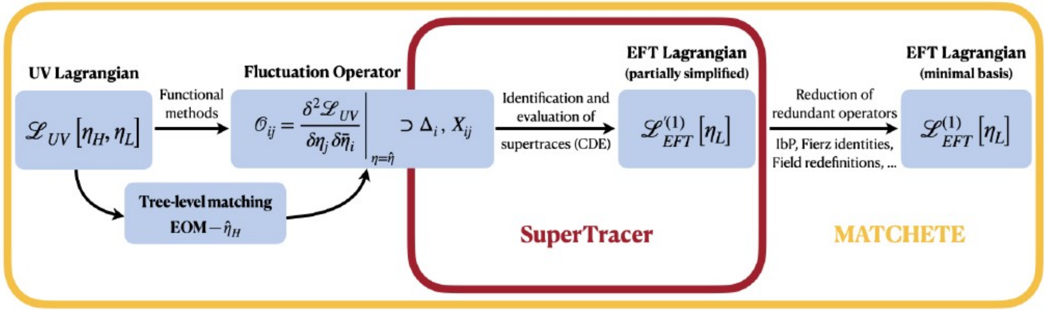
Solution:

1) Construct a ***complete*** basis of operators in the EFT that are ***not independent*** under normalisation and equations of motion, only integration by parts/commutation of derivatives/Fierz identities. This is called a ***Green basis.***
2) Match Green functions of the high energy theory (with IR singularities subtracted) onto this basis in the EFT.
3) Canonically normalise fields, remove redundancies by applying equations of motion and field redefinitions.
4) Evanescent operators can be dealt with either by:
   a) Including them in the Green basis,
   b) Choosing a renormalisation scheme that removes them,
   c) Redefining Fierz identities at the appropriate loop (e.g. arXiv:2306.16449)

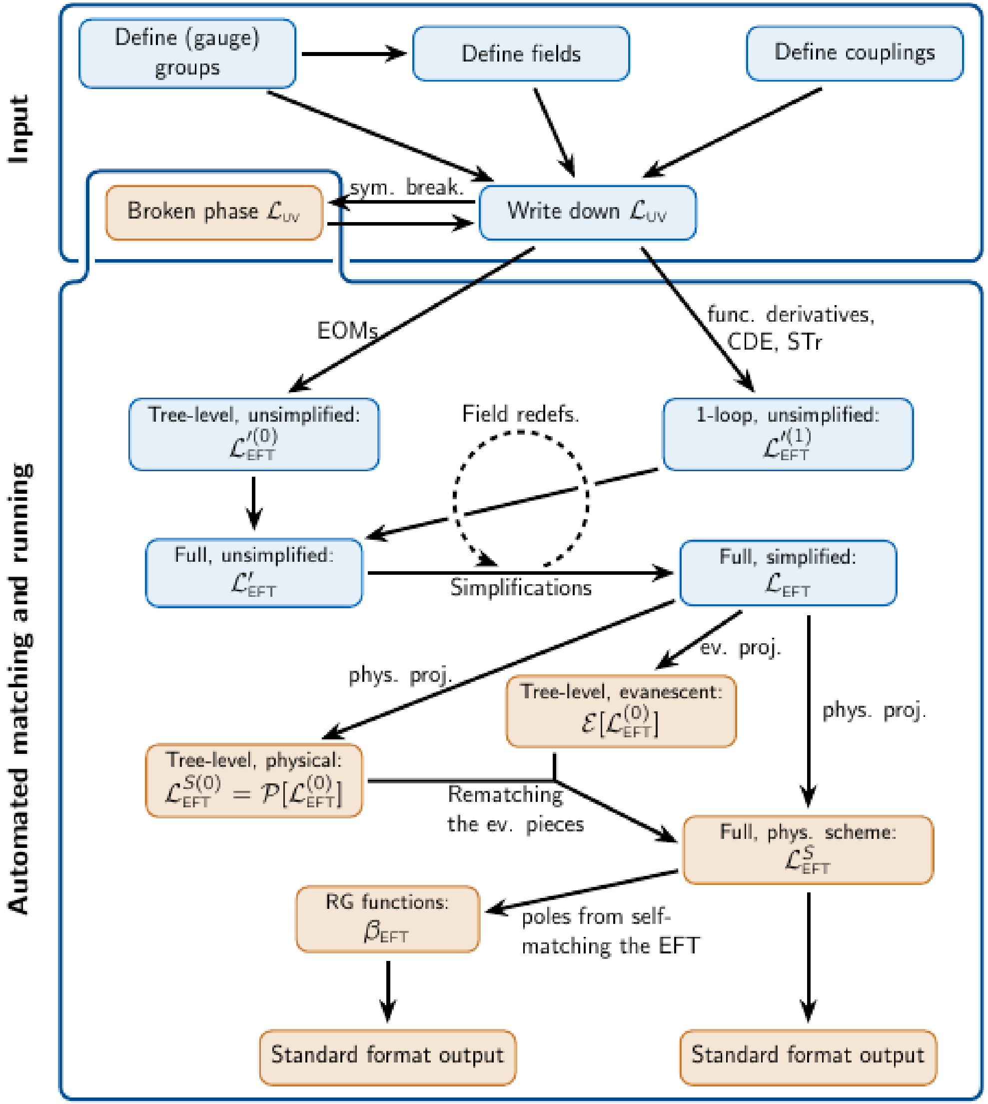
In practice: codes

Computing matching by hand for thousands of operators would be tedious or impossible → need automation by codes

- Mathematica codes for computing supertraces: `SuperTracer`, arXiv:2012.07851; `STrEAM`, arXiv:2012.08506
- `CoDEx`: arXiv:1808.04403 computes the same and matches to example bases.
- `MatchMaker`, arXiv:2112.10787 takes models defined in FeynRules files in background gauge and unbroken phase, and generates automatically using diagrammatic method via QGRAF and FORM.
- `Matchete`, arXiv:2212.04510 uses `SuperTracer` to compute effective operators, partly automates the solution of redundancies.



E.g. `Matchete` roadmap:



Still incomplete – claim is that functional approach should ultimately be more efficient than diagrammatic.

Outlook

- There is lots of activity on complementarity between top-down and bottom-up approaches even in the context of EFTs.
- Full automation of the one-loop matching process to the SMEFT is not yet available via functional methods: still work in progress.
- From my point of view, full integration of top-down model construction to matching is a key goal.
- Automation of basis generation to matching would be ideal.
- Matching computations can also be used to compute RGEs.
- Groups have started looking at two-loop matching (relevant for EW, rare processes, **EDMs**, …)
- ....