MicrOMEGAs 6: new developments and physics applications

Rencontres de Physique des Particules

January 2024

Andreas Goudelis

LPC – Clermont Ferrand

A bit of history: $\chi_1{}^0$ freeze-out relic density

Already since the early 80's, more and more groups wanted to compute the lightest neutralino relic abundance in the Minimal Supersymmetric Standard Model:

· 28 new particles (+ 17 from the SM).

· All of them can, in principle, coannihilate (either with $\chi_1{}^0$ or among them).

· In full generality, this amounts to:

- Calculating matrix elements for ~3000 processes.
- Writing down numerical code to evaluate them.
- Writing down the relevant Boltzmann equations.
- Writing down numerical code to solve them.

People *did* do all that and, by the early 2000's, started developing public codes.

→ Common feature: all relevant expressions were *hard-coded*.

↓

This procedure had to be repeated for every new model

Development of micrOMEGAs

MicrOMEGAs is a numerical code for the calculation of dark matter properties

micrOMEGAs: A program for calculating the relic density in the MSSM G. Bélanger[1], F. Boudjema[1], A. Pukhov[2], A. Semenov[1]	Neutralino DM relic density in the MSSM. Based on CompHEP for ME calculation. arXiv:hep-ph/0112278
micrOMEGAs 2.0: a program to calculate the relic density of dark matter in a generic model . G. Bélanger[1], F. Boudjema[1], A. Pukhov[2], A. Semenov[3]	Freeze-out calculation of DM relic density in generic extensions of the SM. CalcHEP. arXiv:hep-ph/0607059
micrOMEGAs_3 : a program for calculating dark matter observables G. Bélanger[1], F. Boudjema[1], A. Pukhov[2], A. Semenov[3]	Asymmetric DM, semi-annihilations, generalized thermodynamics, DD/ID/LHC. arXiv:1305.0237
micrOMEGAs4.1: two dark matter candidates G. Bélanger[1], F. Boudjema[1], A. Pukhov[2], A. Semenov[3]	Two generic frozen-out dark matter components. arXiv:1407.6129
micrOMEGAs5.0 : freeze-in G. Bélanger[1†], F. Boudjema[1‡], A. Goudelis[2§], A. Pukhov[3¶], B. Zaldivar[1††]	Incorporation of freeze-in dark matter production mechanism (one-component). arXiv:1801.03509
micrOMEGAs 6.0: N-component dark matter G. Alguero[1], G. Bélanger[2], F. Boudjema[2], S. Chakraborti[3], A. Goudelis[4], S. Kraml[1], A. Mjallal[2], A. Pukhov[5]	Arbitrary number of (frozen in/out) dark matter components. arXiv:2312.14894

+ intermediate versions. Until 2013, the *only* DM code to handle generic SM extensions.

So, what is micrOMEGAs ?

https://lapth.cnrs.fr/micromegas/

A C/Fortran code to compute dark matter observables for generic dark matter candidates (current version: v6). For any BSM model, the code can:

· Figure out which processes are relevant for the evolution of the freeze-out/freeze-in dark matter cosmic abundance.

· Compute the relevant matrix elements.

Based on CalcHEP. By default tree-level $1/2 \leftrightarrow 2$, possibility for some $2 \to 3/4$. Possibility to replace $\langle\sigma v\rangle$ with own expression.

· Solve the necessary Boltzmann equations.

· Compute additional observables, compare to EXP limits, link to other packages.

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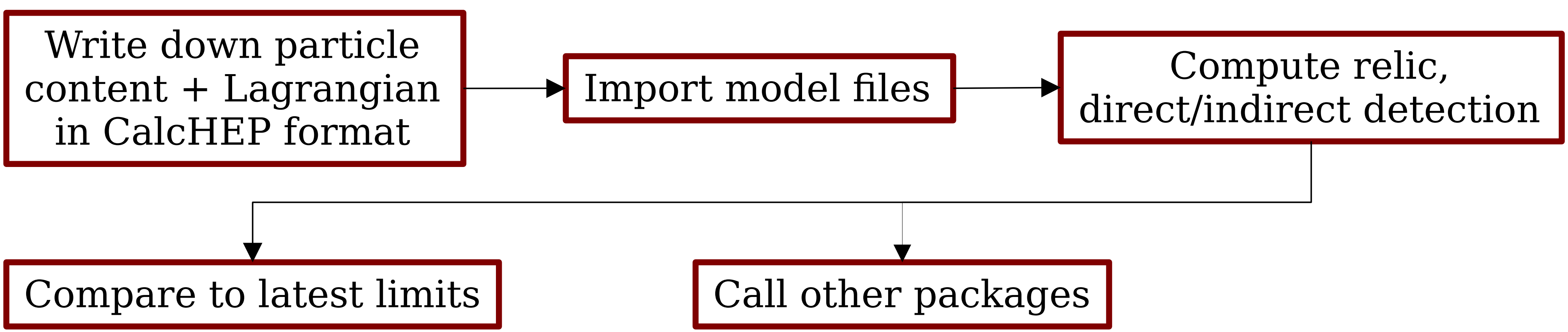


What is new in MO6 ?

Numerous new features have been implemented in the latest version :

· Major upgrade : possibility to compute the DM cosmic abundance in models with multiple WIMP+FIMP dark matter candidates + consistent computation of relevant experimental constraints.

· Major upgrade : inclusion of conversion-driven freeze-out (“co-scattering”) and decay terms.

· Possibility to define (and, partly, check) which sets of particles are in thermal equilibrium.

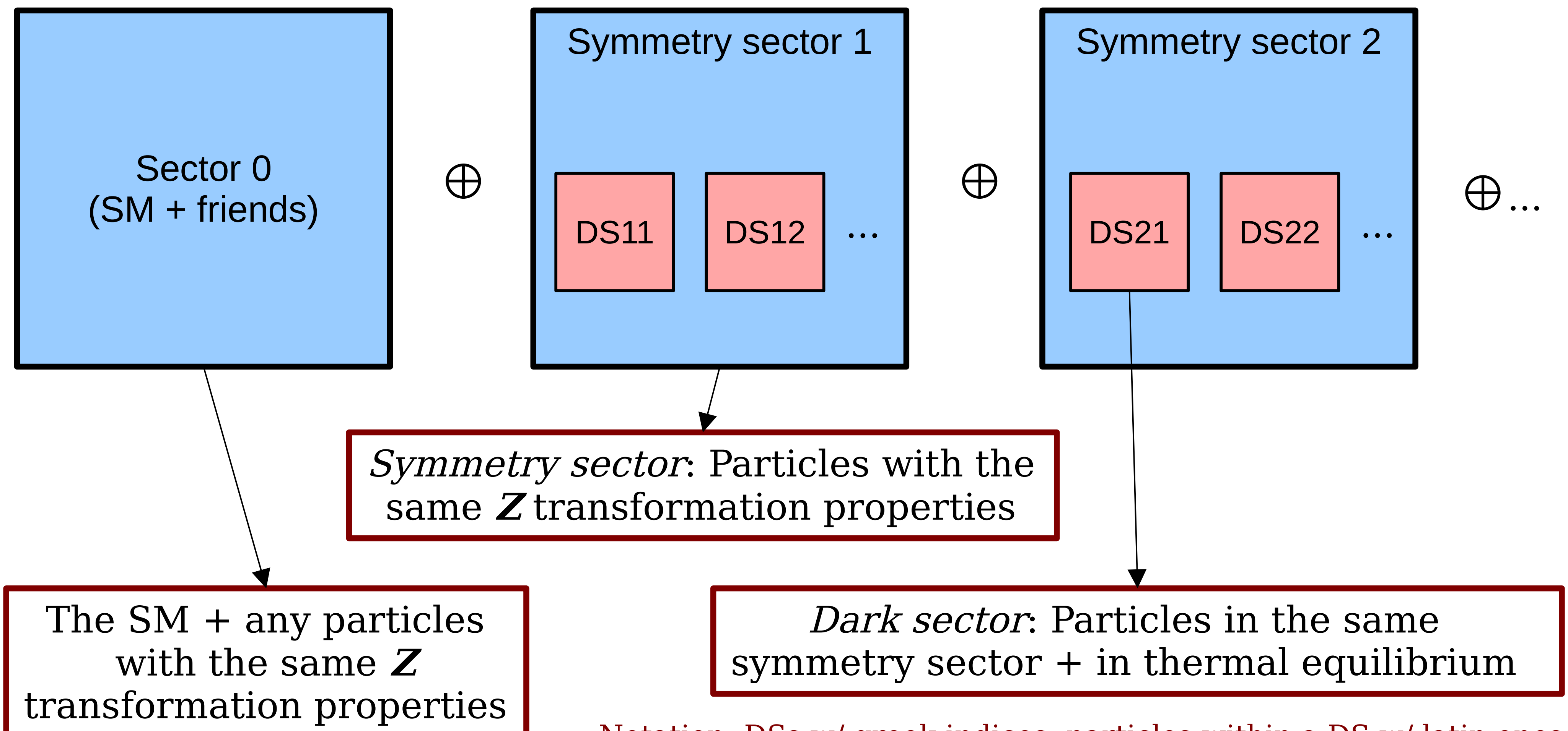
· Possibility to include $2 \rightarrow 3$ and $2 \rightarrow 4$ processes in single-component DM models.

· Improvements in freeze-in computations.

· Additional functionalities for direct/indirect detection.

Multi-component dark matter : strategy

Types of models handled in MO : one (or more) discrete symmetries $\boldsymbol{Z}_i$ are imposed at the Lagrangian. Different (sets of) particles may transform differently under the direct product $\boldsymbol{Z} = \boldsymbol{Z}_1 \otimes \boldsymbol{Z}_2 \otimes \ldots \otimes \boldsymbol{Z}_N$ of these symmetries. We divide the model content in *sectors*.



Notation: DSs w/ greek indices, particles within a DS w/ latin ones

Multi-WIMP case

Any DS may (or may not) contain a dark matter candidate. The evolution of the μ-th candidate's abundance as a function of the entropy density follows :

$$3H\frac{dY_\mu}{d\mathfrak{s}} = \sum_{\alpha\leq\beta;\ \gamma\leq\delta} Y_\alpha Y_\beta C_{\alpha\beta}\langle v\sigma_{\alpha\beta\gamma\delta}\rangle(\delta_{\mu\alpha}+\delta_{\mu\beta}-\delta_{\mu\gamma}-\delta_{\mu\delta})$$

where:

$$\langle v\sigma_{\alpha\beta\gamma\delta}\rangle = \frac{1}{C_{\alpha\beta}\bar{n}_\alpha(T)\bar{n}_\beta(T)} \sum_{\substack{a\in\alpha, b\in\beta, c\in\gamma, d\in\delta \\ \text{if}(\alpha=\beta)a\leq b;\ \text{if}(\gamma=\delta)c\leq d}} \bar{N}_{a,b\to c,d}$$

$$\bar{N}_{a,b\to c,d} = \frac{T g_a g_b}{8\pi^4}\int\sqrt{s}p_{ab}^2(s)K_1(\frac{\sqrt{s}}{T})C_{ab}\sigma_{a,b\to c,d}(s)ds$$

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$$\bar{N}_{a,b\to c,d} = \frac{Tg_a g_b}{8\pi^4}\int\sqrt{s}p_{ab}^2(s)K_1(\frac{\sqrt{s}}{T})C_{ab}\sigma_{a,b\to c,d}(s)ds$$

If some particle species in a DS decay slowly, we get additional terms of the type :

$$\frac{1}{\mathfrak{s}^2(T)}\sum_{\alpha;\ \gamma\leq\delta}\left(\frac{Y_\alpha}{\bar{Y}_\alpha}-\frac{Y_\beta}{\bar{Y}_\beta}\frac{Y_\gamma}{\bar{Y}_\gamma}\right)(\delta_{\mu\alpha}-\delta_{\mu\beta}-\delta_{\mu\gamma})\sum_{a\in\alpha,c\in\beta,d\in\gamma}\bar{N}_{a\to c,d}$$

where: $\bar{N}_{a\to c,d} = \frac{Tg_a}{2\pi^2}m_a^2\Gamma^0(a\to c,d)K_1\left(\frac{m_a}{T}\right)$

Including co-scattering, freeze-in

Co-scattering corresponds to processes of the type $\mu + 0 \to \nu + 0$. It turns out that these contributions enter the Boltzman eqs. similarly to decay terms $\mu \to \nu + 0$

$$3H\frac{dY_\mu}{d\mathfrak{s}} \approx (Y_\mu - Y_\nu \frac{\bar{Y}_\mu}{\bar{Y}_\nu})\,\Gamma_{\mu\to\nu}$$

where:

$$\Gamma_{\mu\to\nu} = Y_0 \langle \sigma_{\mu 0 \nu 0} v\rangle(T) + \frac{\sum_{a\in\mu, c\in\nu} g_a m_a^2 \Gamma^0(a\to c,0) K_1\left(\frac{m_a}{T}\right) + \sum_{a\in\nu, c\in\mu} g_a m_a^2 \Gamma^0(a\to c,0) K_1\left(\frac{m_a}{T}\right)}{\sum_{a\in\mu} g_a m_a^2 K_2\left(\frac{m_a}{T}\right)}$$

can be seen as an effective width between sectors μ and ν.

Freeze-in can also be implemented through the same set of equations, but setting the initial DM abundance to zero as usual.

· Important difference wrt single-component case: DM annihilations *are* taken into account.

NB: Kinetic equilibrium is assumed even for FIMPs, otherwise need to solve un-integrated Boltzmann eqs!

Validation and example results

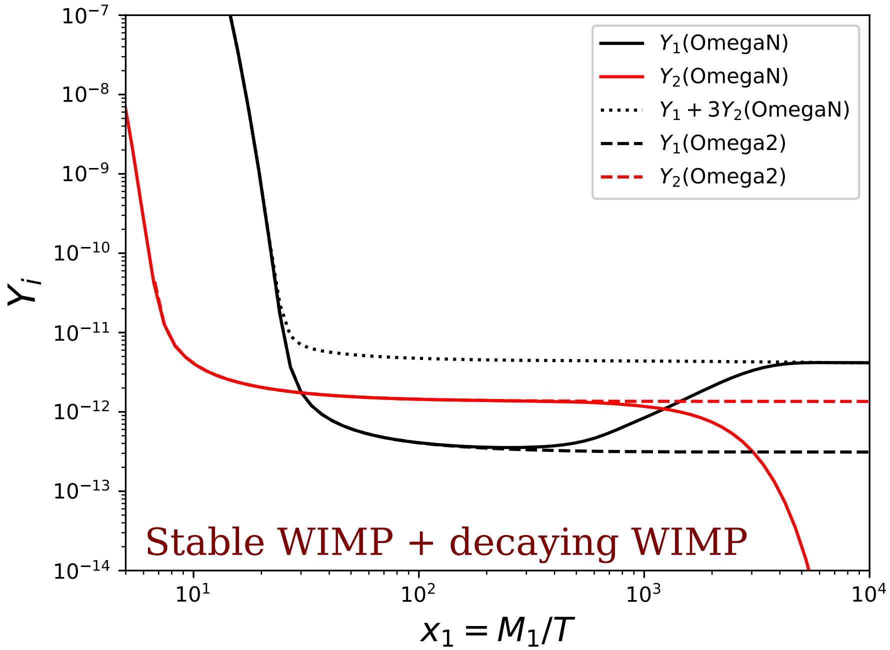
The code was validated using different models as examples :

· Singlet scalar (sanity checks for single-component DM, 1 WIMP or 1 FIMP).

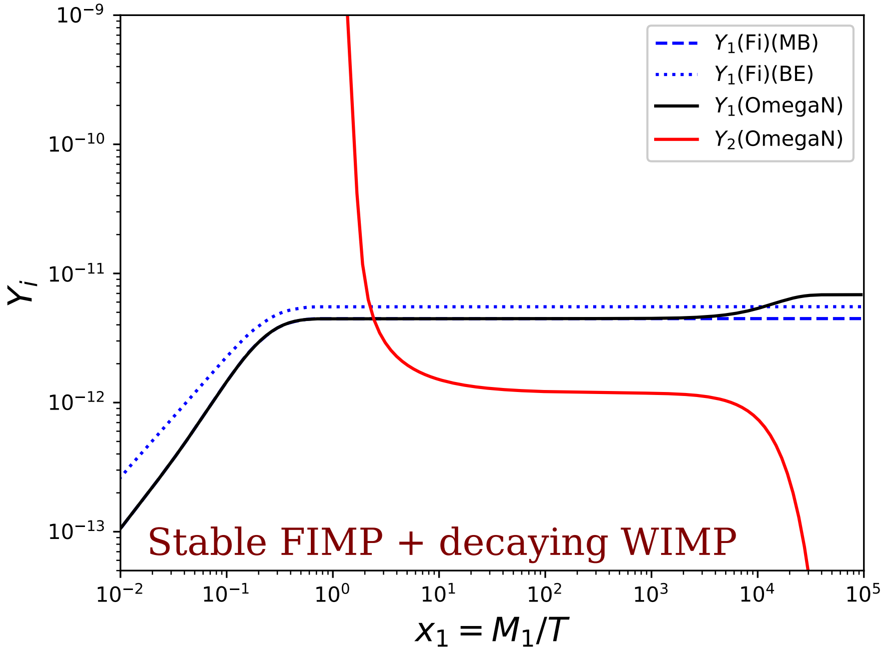
· Z5M (two singlets w/ $\boldsymbol{Z}_5$ symmetry, 2 WIMPs or 1 WIMP + 1 FIMP).

· Z4IDSM (Inert Doublet plus Singlet w/ $\boldsymbol{Z}_4$ symmetry, 1 WIMP + 1 FIMP).

Two examples from the Z5M :



Decay occurs through $\lambda\ \varphi_2\ \varphi_1^3$ coupling



Decay occurs through $\mu\ \varphi_2\ \varphi_1^2$ coupling

Excellent agreement w/ previous versions until decays become relevant.

Issues with t-channels : the problems

Although in principle quite straightforward, processes involving particle exchange in the t-channel may present some peculiarities :

Spin-1 particle exchange leads to constant σ at high temperatures → $Y_{\rm DM} \sim T_{\rm R}$ even for renormalizable models.

Issue only appears in FI

If a stable particle is exchanged in the t-channel, σ diverges as the particle becomes on-shell.

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In particular, *in a medium*, at finite temperature :

· The vector mass receives a T-dependent contribution that scales as $M^2 \sim T^2$.

· Every particle (even a stable one) has a finite absorption probability (“width”).

Issues with *t*-channels : solutions

Computing full-blown thermal corrections to masses/widths is beyond the scope of micrOMEGAs.

Matrix elements calculated at *tree-level*

NB: We *cannot* simply replace T-dependent masses (gauge invariance).

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Observation : consider $e^+ e^- \to \nu_e \overline{\nu}_e$ and compute the integrated cross-section with a cut c on the scattering angle

$$\sigma(\sqrt{s}, M_W, c) = 4\hat{\sigma}_{e\nu_e} \left(\frac{1}{2\mu^2 + c} + \log\left(\frac{2\mu^2 + c}{2}\right) + \mu^2 \log\left(\frac{2\mu^2 + c}{2}\right) + 1 - c \right.$$
$$\left. + \frac{c^2}{4(-2 + c - 2\mu^2)} + \frac{c(c-4)}{4(c + 2\mu^2)} - (1 + \mu^2)\log\left(1 + \mu^2 - c/2\right)\right)$$

Where $\mu^2 = M_W^2/s, \quad \hat{\sigma}_{e\nu_e} = \pi\alpha^2/(8 s_W^4 s)$ and $\{\mu, c\}$ enter both singularities through the same combination $2\mu^2 + c$

> The effect of a T-dependent mass can be captured by a zero-temperature calculation with a T-dependent cut on the scattering angle (or the p_T).

In practice: user-defined p_T cuts for all relevant particles

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For stable particles: introduce a small width ~M/100 for t-channel particles.

Improvements for other observables

- **Direct detection:**

In general multi-component models, one cannot naïvely impose DD limits: simple rescaling by the fraction of each component is not enough.

In MO6 a function is provided in order to compute whether a model is excluded or not by the leading DD experiments.

- **Indirect detection:**

In previous versions the DM annihilation – induced photon spectra (Pythia 6) were tabulated down to DM masses of ~2 GeV. For lighter DM, need to consider different final states.

In MO6 the gamma-ray tables have been updated/improved to include annihilations into light leptons, pions, Kaons.

- **Structure formation:**

Free-streaming length of DM particles through:

$$\lambda_{FS} = \int_{T_2}^{T_1} \left(1 + \left(\frac{a(T)m}{a(T_1)p}\right)^2\right)^{-\frac{1}{2}} \frac{dT}{a(T)\overline{H}(T)T}$$

Summary and outlook

· MicrOMEGAs can now handle scenarios with multiple dark matter components.

· MicrOMEGAs can now handle conversion-driven freeze-out (“co-scattering”) and decaying long-lived particles.

· Numerous improvements have been introduced both for the calculation of the relic density (taming singular behaviours, multi-body final states) and of different observables (DD/ID, free-streaming length).

· Next version? We’ll see… :)