On the evaluation of the HVP contribution to a_μ and α_{QED}

(and implications for the EW fit and α_S)

Bogdan MALAESCU

LPNHE, CNRS

Based on: 1908.00921(DHMZ), 2006.04822(WP Theory Initiative), 2008.08107(BM,MS), 2302.01359(DDMPR), 2308.04221(BMW & DMZ), 2308.05233(BaBar), 2312.02053(DHLMZ)

In collaboration & useful discussions with:
Michel DAVIER, Andreas HOECKER, Zhiqing ZHANG, Anne-Marie LUTZ (DHLMZ)
Laurent LELLOUCH, the BMW collaboration; Matthias SCHOTT, the Gfitter group
David DIAZ-CALDERON, Toni PICH, Antonio RODRIGUEZ-SANCHEZ

Rencontres de Physique des Particules 2024 (RPP)

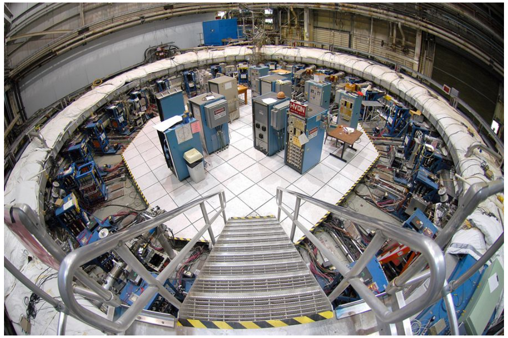
24/01/2024

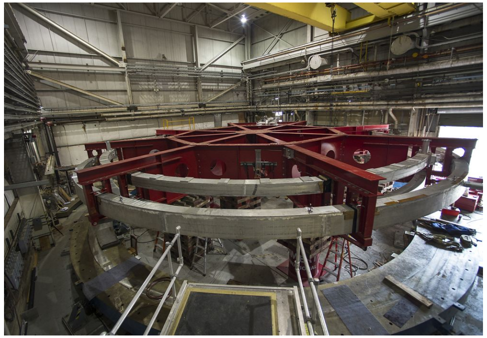
Content of the talk

- Introduction: the $(g-2)_\mu$ experiment & theoretical prediction
- Data on $e^+e^- \to$ hadrons and their combination
- Relevance of uncertainties on uncertainties and on correlations
- Results on a_μ
- Relevance of a_μ and α_{QED} results for the EW fit and for $\alpha_S(M_Z)$
- Comparison between dispersive and lattice HVP results
- A *new possible perspective on a_μ* and *Conclusions*

From BNL to Fermilab

BNL → 1 month long trip for the g-2 storage ring





→ Fermilab
July 26, 2013

This is NOT an UFO !!! ;-)

The $(g\text{-}2)_\mu$: definition & experimental measurement

- Magnetic dipole moment of a charged lepton: $\vec{\mu} = g\frac{e}{2m}\vec{s}$
- "anomaly" = deviation w.r.t. Dirac's prediction: $a = \frac{g-2}{2}$

- Experimental "ingredients" to measure a_μ:

→ Polarised muons from pion decays (parity violation)

spin orientation

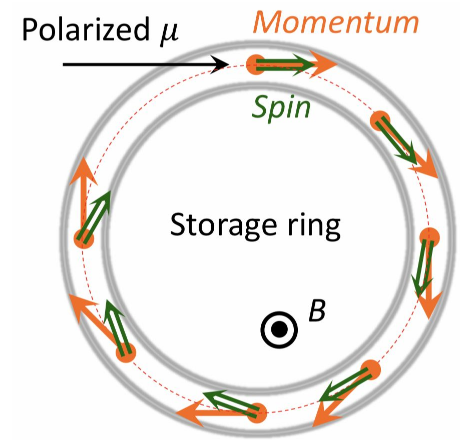
$\bar{\nu}_\mu \longleftarrow \pi^- \longrightarrow \mu^-_{\text{polarised}}$

→ "Anomalous frequency"

(difference between spin precession and cyclotron frequency)

proportional to a_μ for the "magic γ"

$$\vec{\omega}_a = \frac{e}{m_\mu c}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right] \approx \frac{e}{m_\mu c} a_\mu \vec{B}$$



→ Parity violation in muon decays

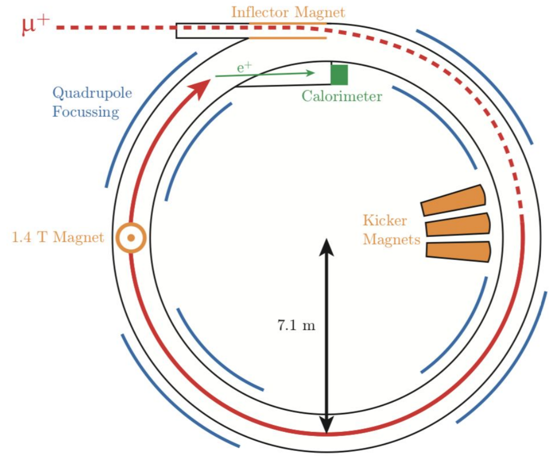
(electron emitted in the direction opposite to the muon spin)

$\mu^-_{\text{polarised}} \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$

a_μ^{Exp}(BNL): $(11\ 659\ 208.9 \pm 6.3)\ 10^{-10}$

→ Expected uncertainty reduction by a factor 4 with the experiment at Fermilab

- improved apparatus and enhanced statistics: more intense (x20) and pure muon beam; B-field mapped every 3 days with special trolley with probes pulled through beampipe (homogeneity ~ ppm); tracking system for electron detectors etc.

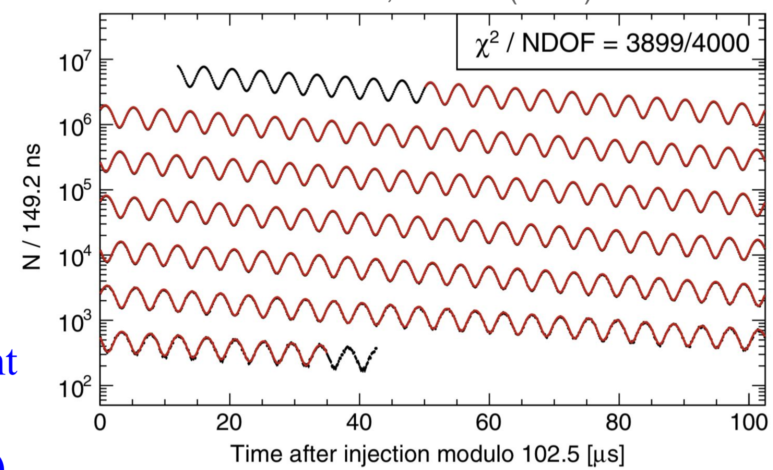


- 1st publication: similar precision & good agreement with BNL (7th of April 2021) PRL 126, 141801 (2021)

a_μ^{Exp}(Fermilab): $(11\ 659\ 204.0 \pm 5.1 \pm 1.8)\ 10^{-10}$ → 6% of total data

a_μ^{Exp}(Fermilab + BNL): $(11\ 659\ 206.1 \pm 4.1)\ 10^{-10}$ (0.35 ppm)

PRD 103, 072002 (2021)



- 2nd publication: uncertainty reduction by a factor ~2 (10th of August 2023) PRL 131, 161802 (2023) (+Run 2 & 3 data)

a_μ^{Exp}(Fermilab): $(11\ 659\ 205.5 \pm 2.4)\ 10^{-10}$ (0.20 ppm)

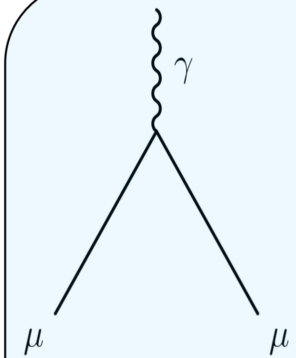
a_μ^{Exp}(Fermilab + BNL): $(11\ 659\ 205.9 \pm 2.2)\ 10^{-10}$ (0.19 ppm)

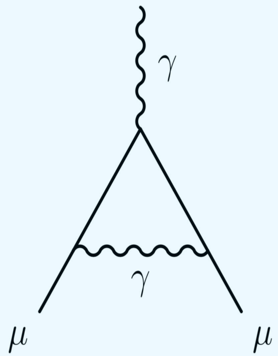
→ One of the most precise quantities ever measured

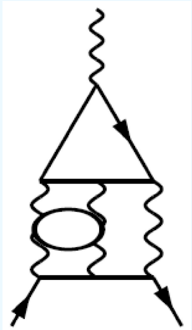
- Expectation for final publication: another factor 2 improvement for the statistical precision

→ Initiative for a measurement using slow muons (KEK, Japan)

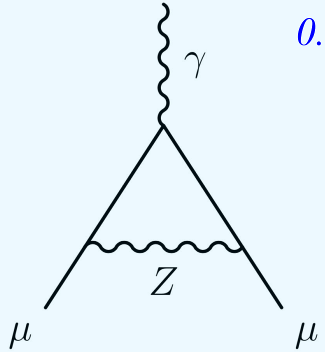
Why is it (so) complicated to compute one number ? (*very precisely*)



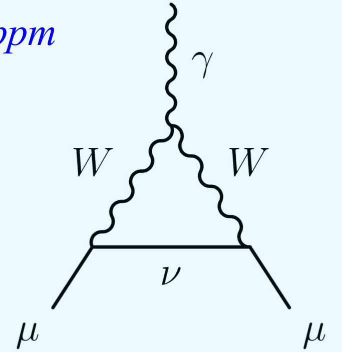




0.001 ppm



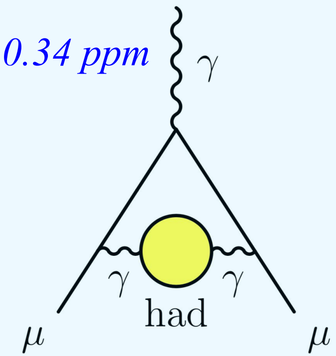
0.01 ppm



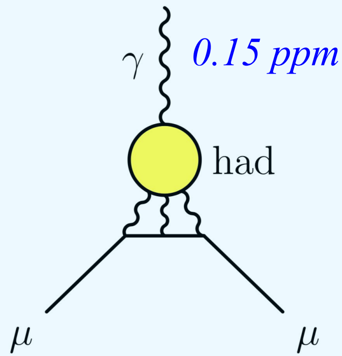
\+ Many other diagrams at higher orders…

QED up to O(α^5) (Kinoshita et al.)

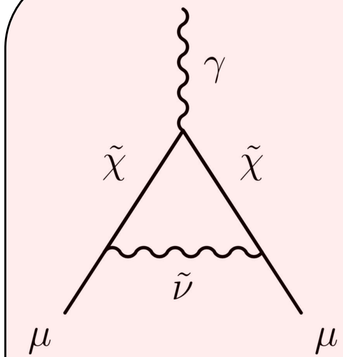
0.34 ppm

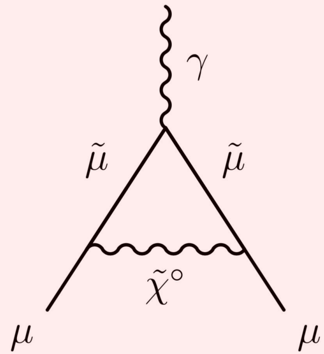


0.15 ppm



Dominant uncertainties: non-perturbative...





? ? ?

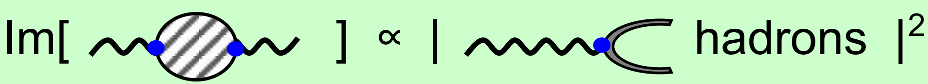
Hadronic Vacuum Polarization and Muon $(g\text{-}2)_\mu$

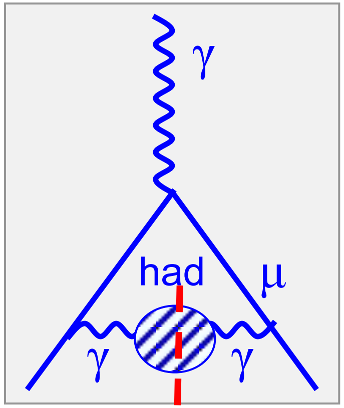
Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece

Cannot be calculated from QCD (low mass scale), but one can use experimental data on $e^+e^- \to$ hadrons cross section

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

$$12\pi \ \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\left[e^+e^- \to \text{hadrons}\,(\gamma_{FSR})\right]}{\sigma_{pt}} \equiv R(s)$$

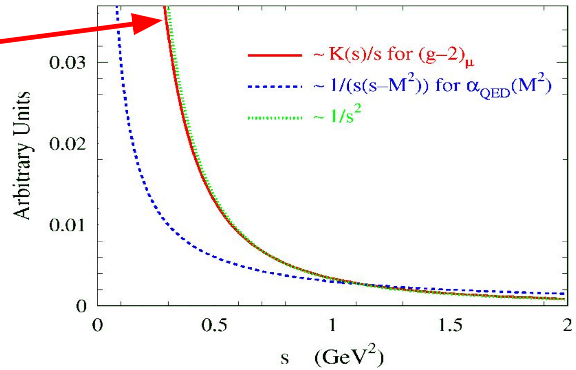


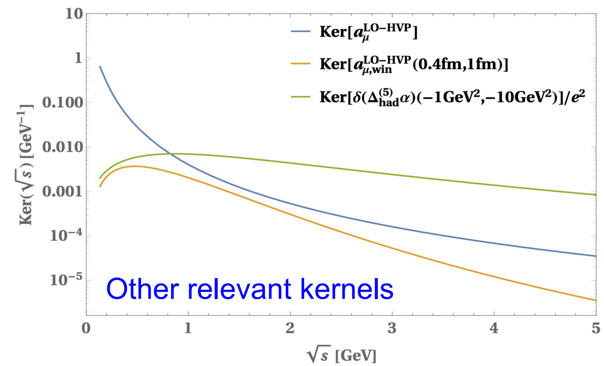


$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \ \frac{K(s)}{s} \ R(s)$$

Dispersion relation

Bouchiat and Michel, 1961

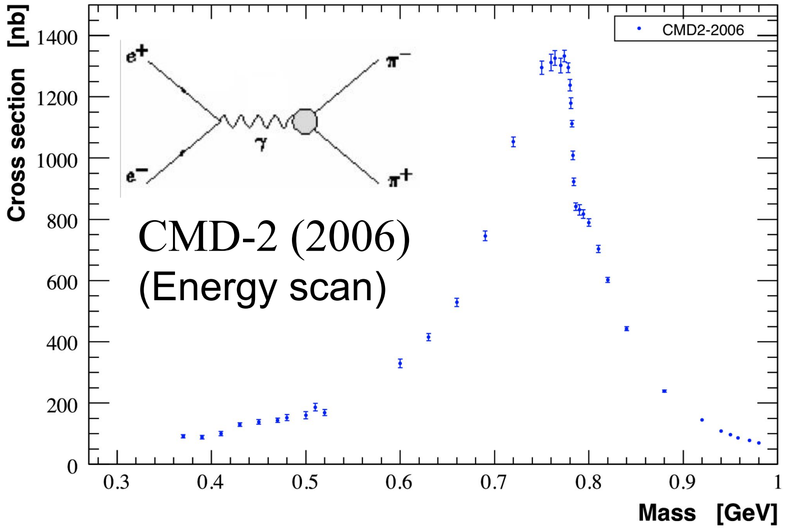


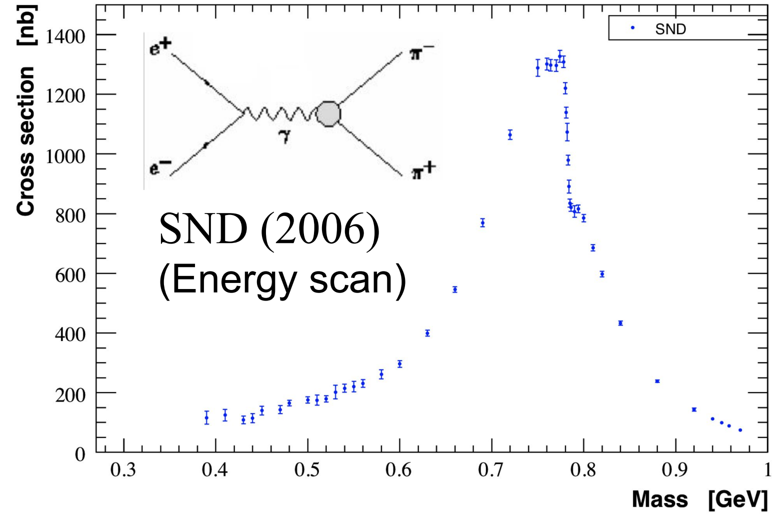


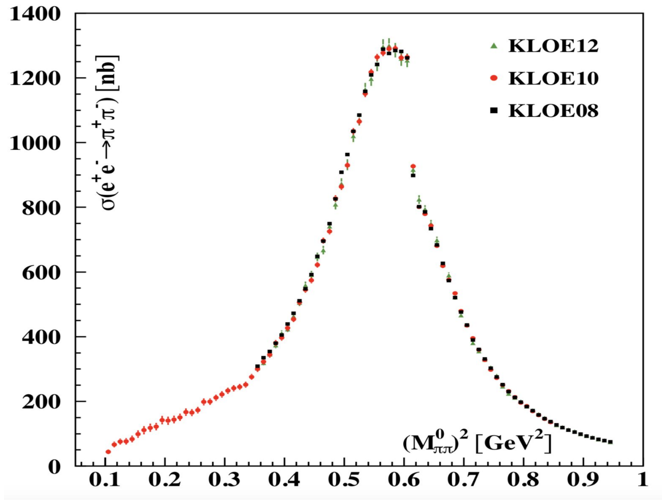
→ Precise $\sigma(e^+e^- \to \text{hadrons})$ measurements at low energy are very important

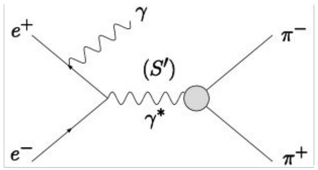
→ Alternatively, one can use hadronic τ decays data + IB corrections

HVP: Data on $e^+e^- \rightarrow$ hadrons



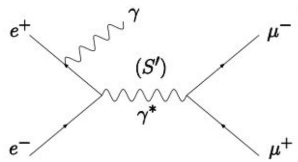


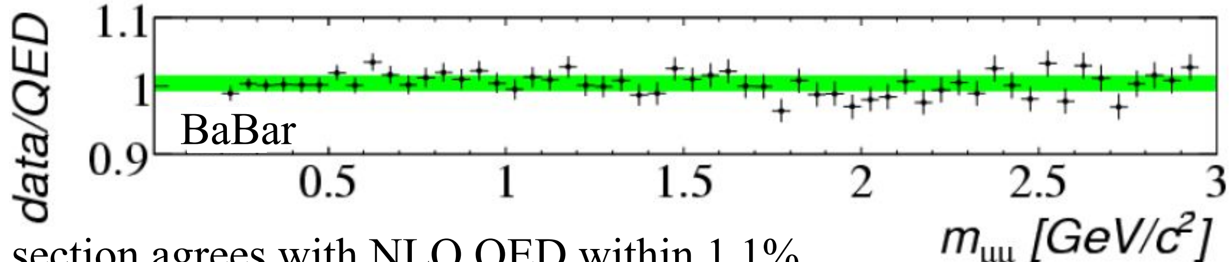




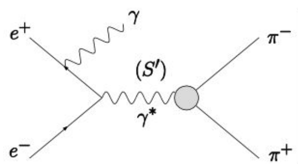
KLOE (08&10) + μμ (12) (ISR)

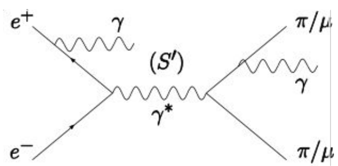
BaBar results (arXiv:0908.3589, PRL 103, 231801 (2009); arXiv:1205.2228(PRD)

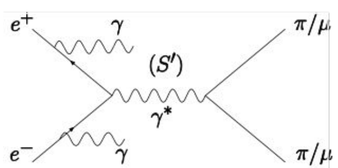


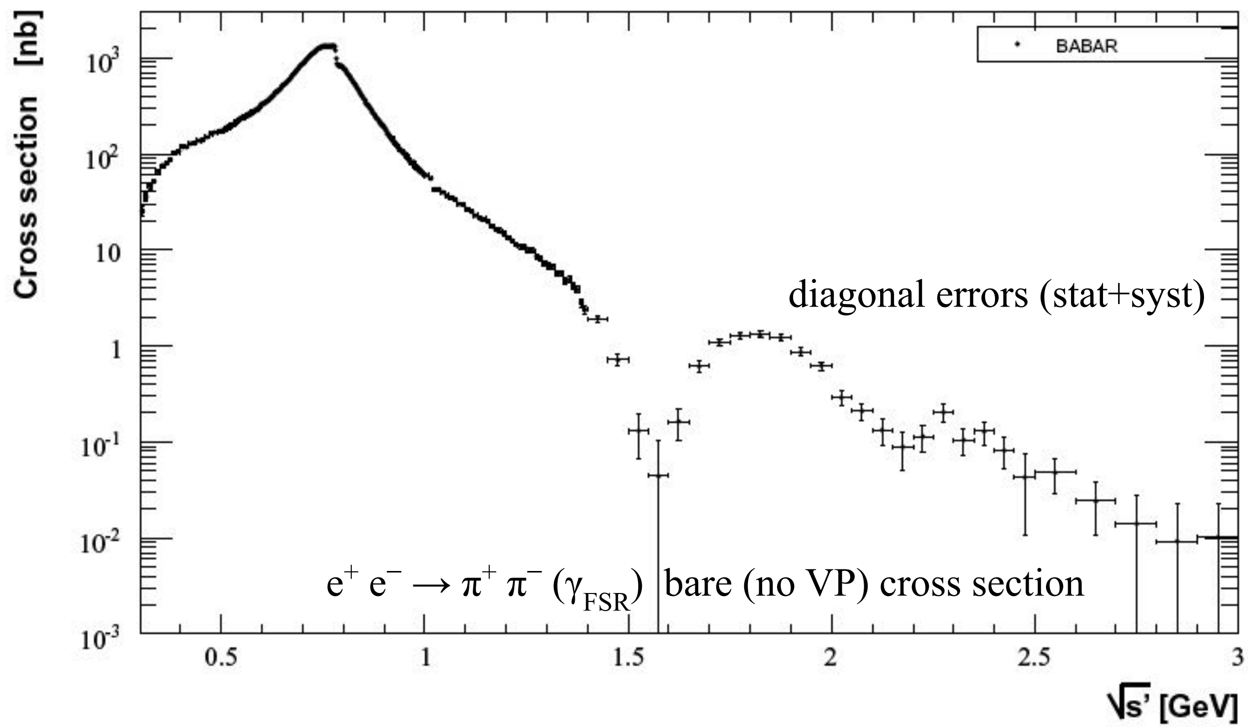


Absolute $\mu^+\mu^-$ cross section agrees with NLO QED within 1.1%

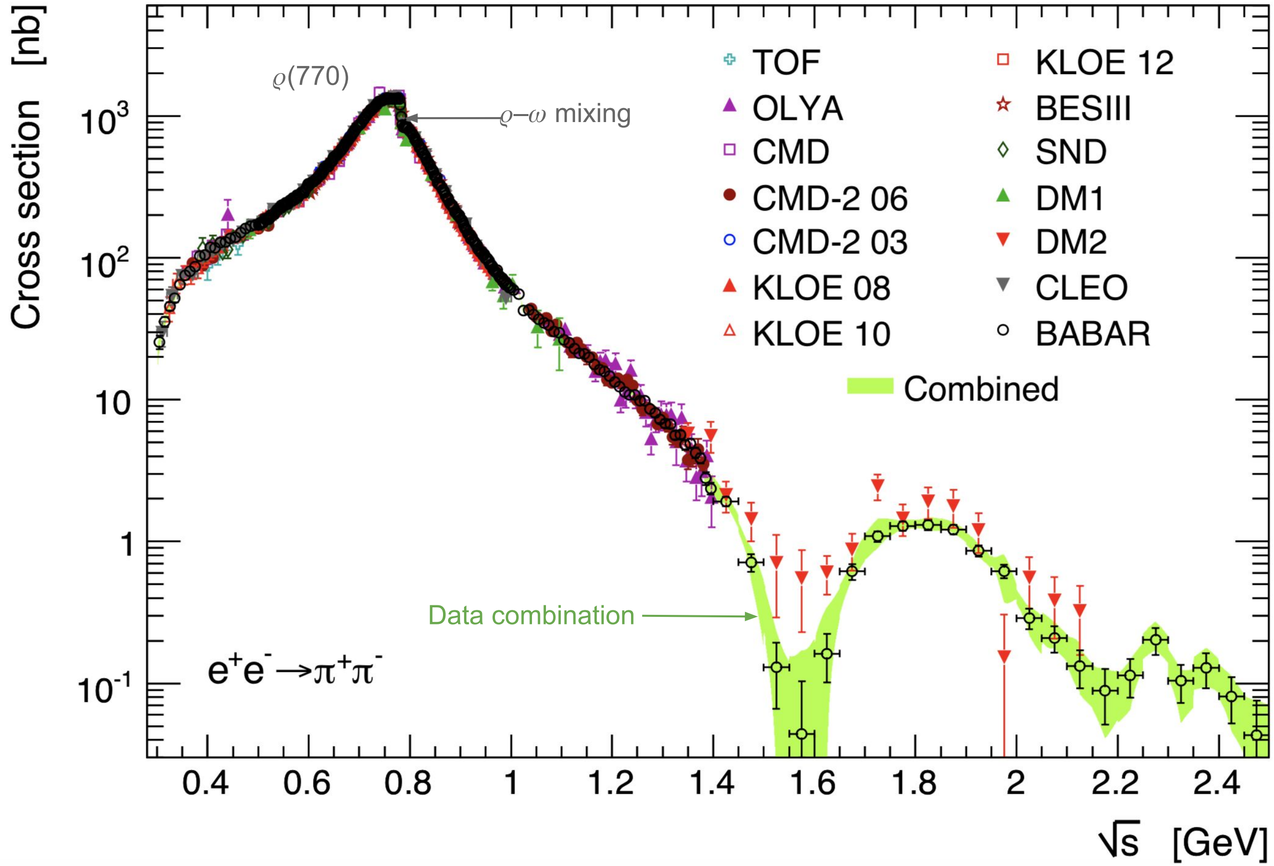








Combination for the $e^+e^- \to \pi^+\pi^-$ channel (DHMZ '19)



→ Procedure and software (*HVPTools - Since 2009*) for combining differential cross section data

→ New since TI White Paper, in next slides: SND20, CMD3, Updated BESIII cov matrix

Combine cross section data: goal and requirements

→ Goal: combine experimental spectra with arbitrary point spacing / binning

→ Requirements:

- Properly propagate uncertainties and correlations

- *Between measurements (data points/bins) of a given experiment*

  (covariance matrices and/or detailed split of uncertainties in sub-components)

- *Between experiments* (common systematic uncertainties, e.g. VP)

  based on detailed information provided in publications

- *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones

  BABAR luminosity (ISR or BhaBha), efficiencies (photon, Ks, Kl, modeling);
  BABAR radiative corrections; $4\pi2\pi^0-\eta\omega$
  CMD2 $\eta\gamma - \pi^0\gamma$; CMD2/3 luminosity; SND luminosity;
  FSR; hadronic VP (old experiments)

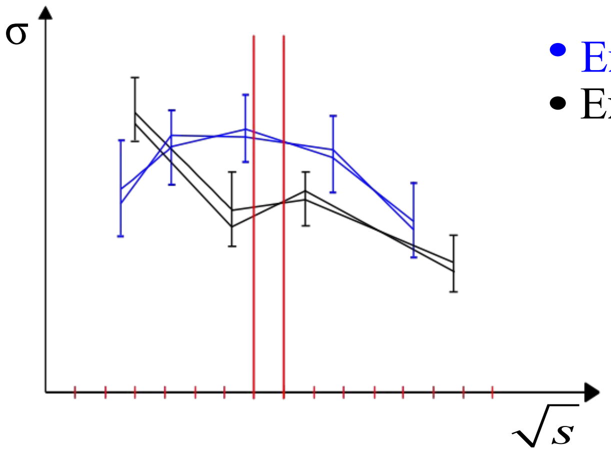
  (1st motivation for using DHMZ uncertainties as "baseline" in the g-2 TI White Paper)

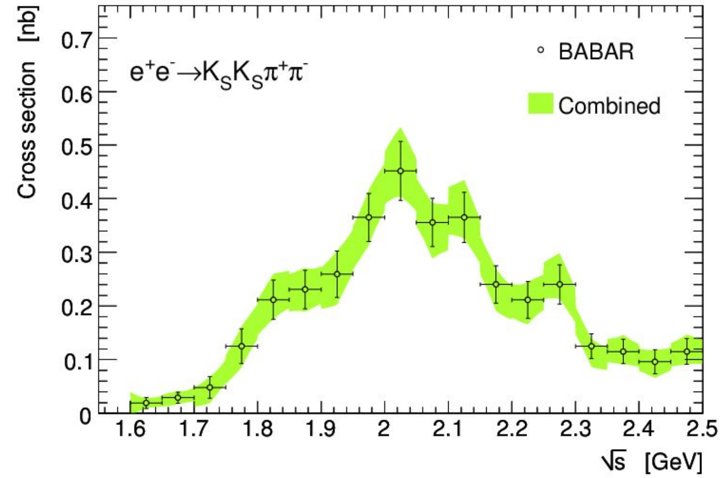
- Minimize biases

- Optimize g-2 integral uncertainty

  (*without overestimating the precision with which the uncertainties of the measurements are known*)

Combination procedure implemented in HVPTools software





→ Define a (fine) final binning (to be filled and used for integrals etc.)

→ Linear/quadratic splines to interpolate between the points/bins of each experiment
- for binned measurements: preserve integral inside each bin
- closure test: replace nominal values of data points by Gounaris-Sakurai model and re-do the combination
  → *(non-)negligible bias for (linear)quadratic interpolation*

→ Fluctuate data points taking into account correlations & re-do the splines for each (pseudo-)experiment
- each uncertainty fluctuated coherently for all the points/bins that it impacts
- eigenvector decomposition for (statistical) covariance matrices

Combination procedure implemented in HVPTools software

For each final bin:

→ Compute an average value for each measurement and its uncertainty

→ Compute correlation matrix between experiments

→ Minimize χ^2 and get average coefficients (weights)

→ Compute average between experiments and its uncertainty

Evaluation of integrals and propagation of uncertainties:

→ Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys

→ The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation

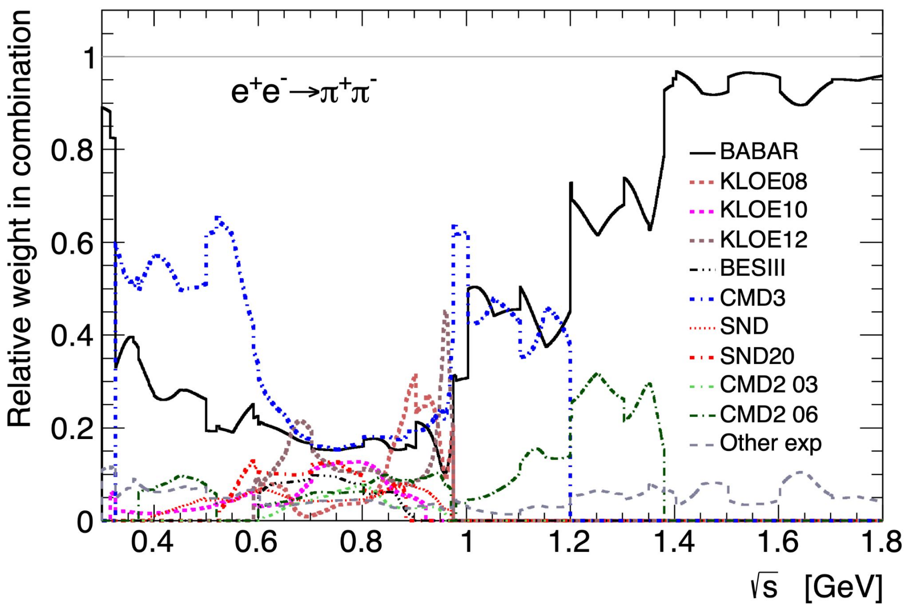
→ Uncertainties also propagated through $\pm1\sigma$ shifts of each uncertainty:
- allows to account for correlations between different channels (for integrals and spectra)

→ *Checked consistency between the different approaches*

Combination procedure: weights of various measurements

For each final bin:
→ Minimize χ^2 and get average coefficients

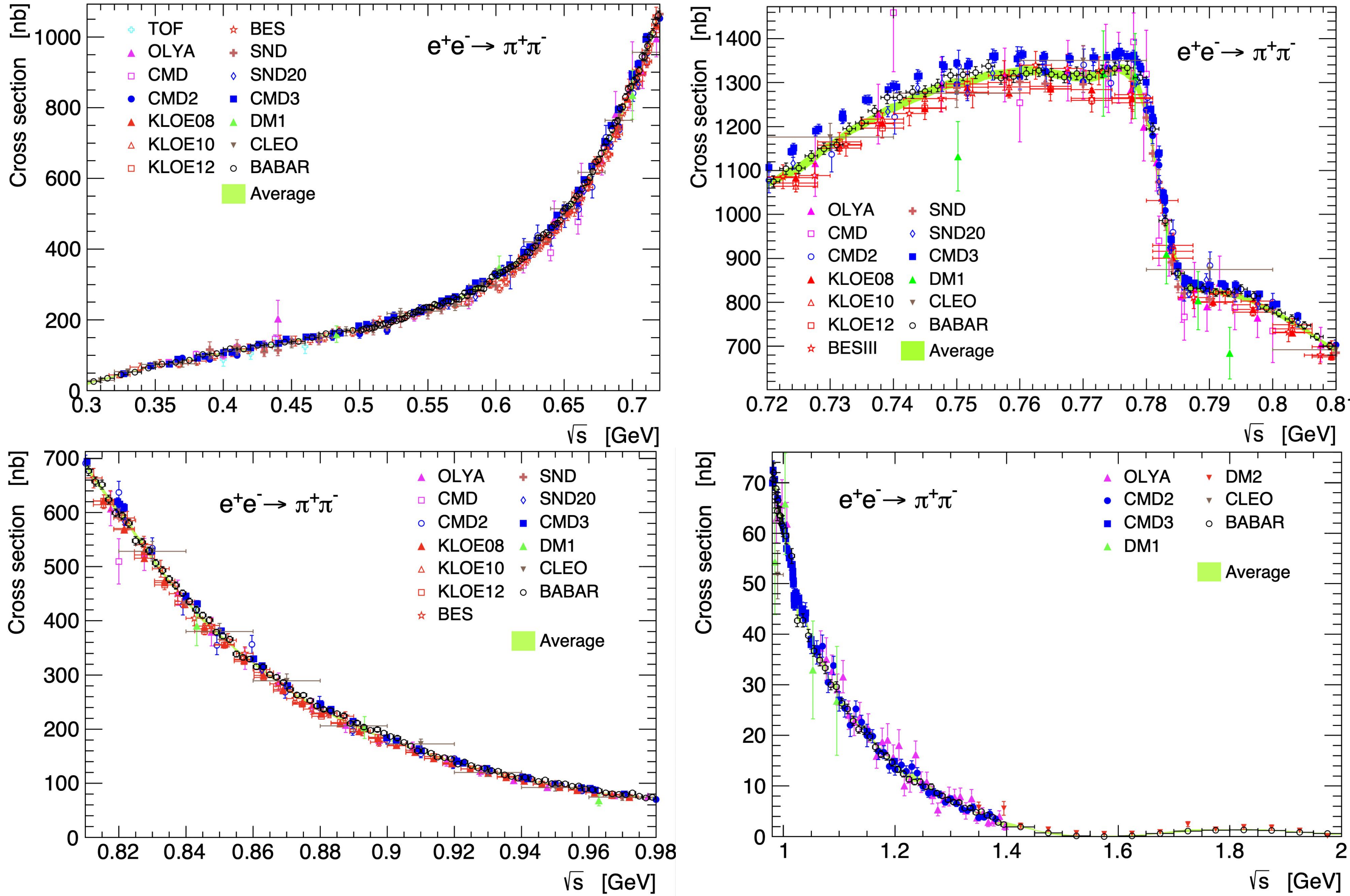
Note: average weights must account for bin sizes / point spacing of measurements
(do not over-estimate the weight of experiments with large bins)
→ Weights in fine bins evaluated using a common (large) binning for measurements + interpolation
→ Compare the precisions on the same footing



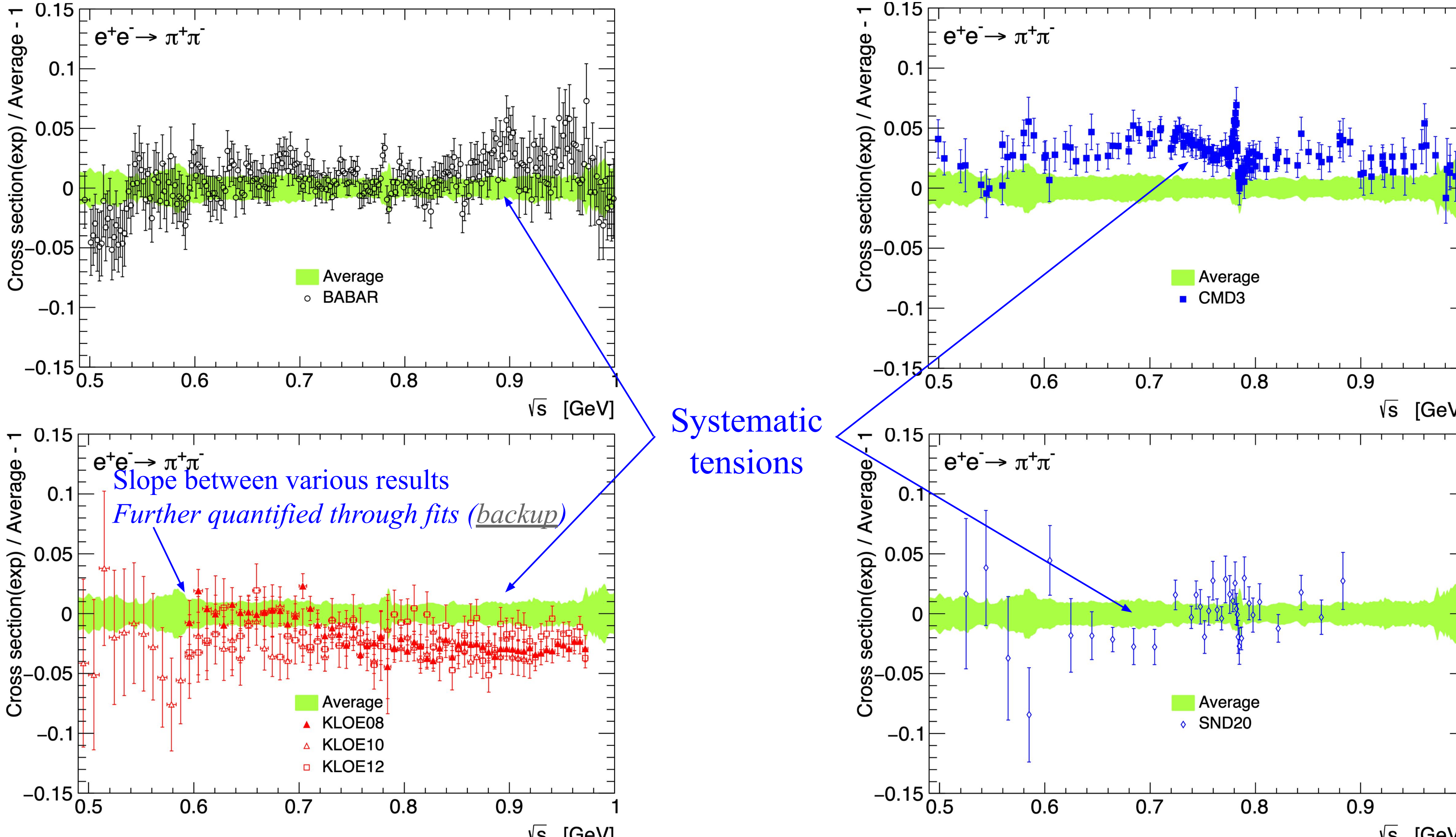
→ Bins used by KLOE larger than the ones by BABAR in ρ-ω interference region (factor ~3)

→ Average dominated by BaBar, CMD3, KLOE, SND20
BaBar covering full range

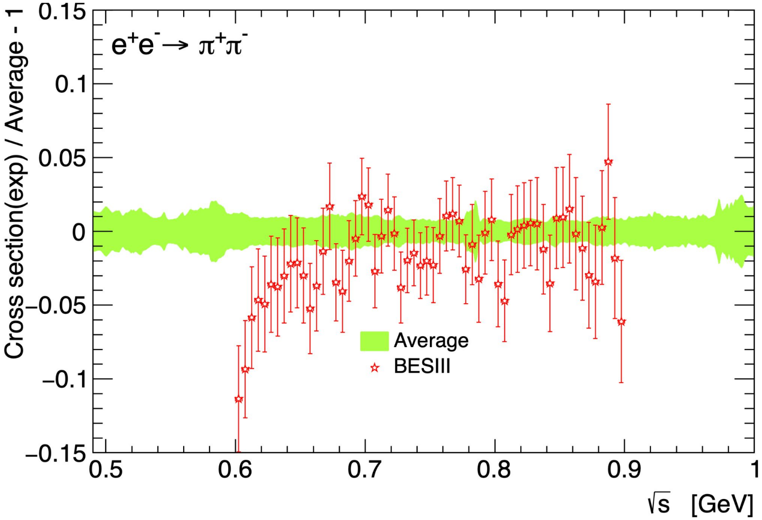
Combining the $e^+e^- \to \pi^+\pi^-$ data

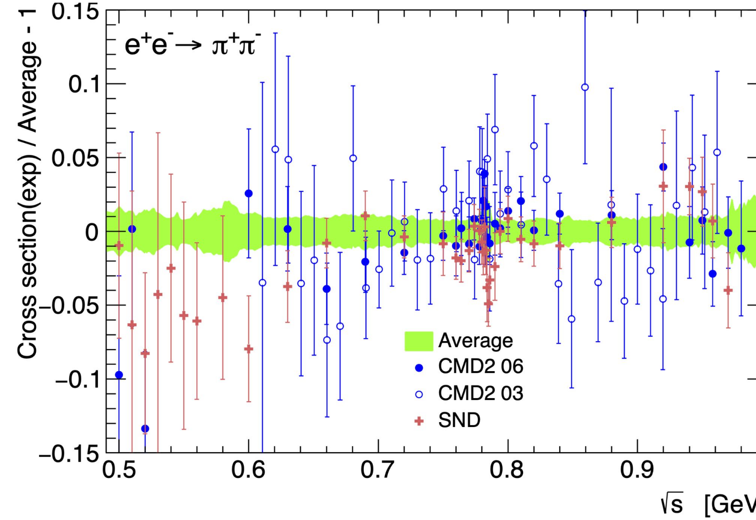


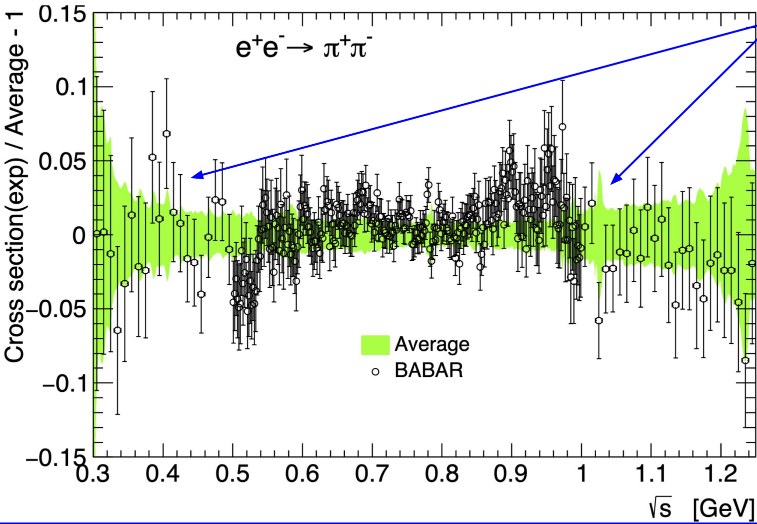
Combining the $e^+e^- \to \pi^+\pi^-$ data: relative differences



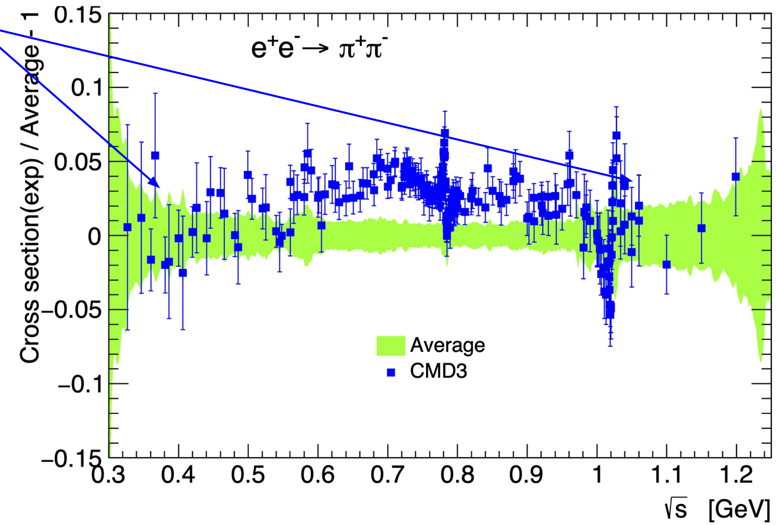
Combining the $e^+e^- \to \pi^+\pi^-$ data: relative differences





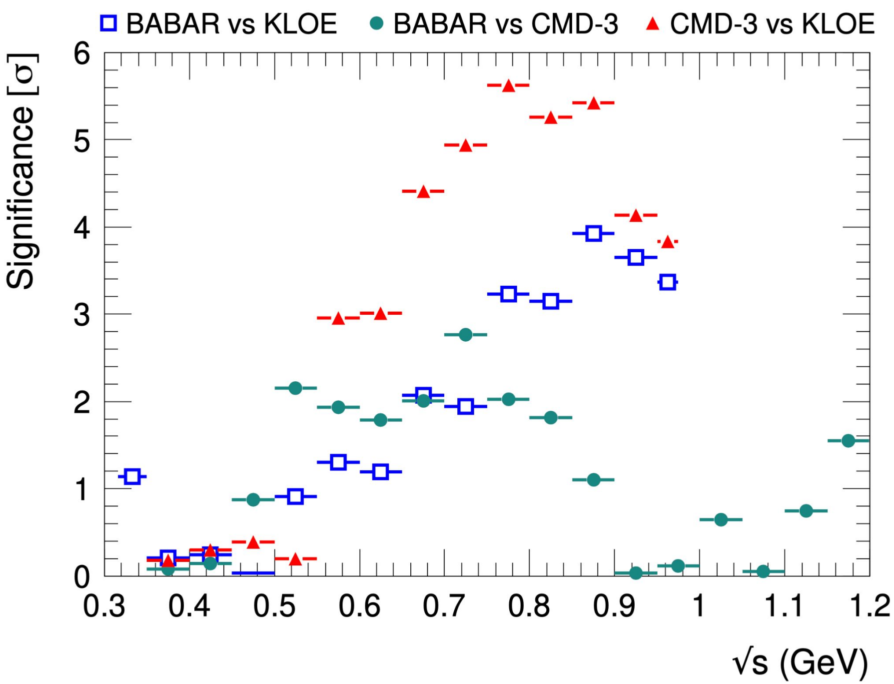


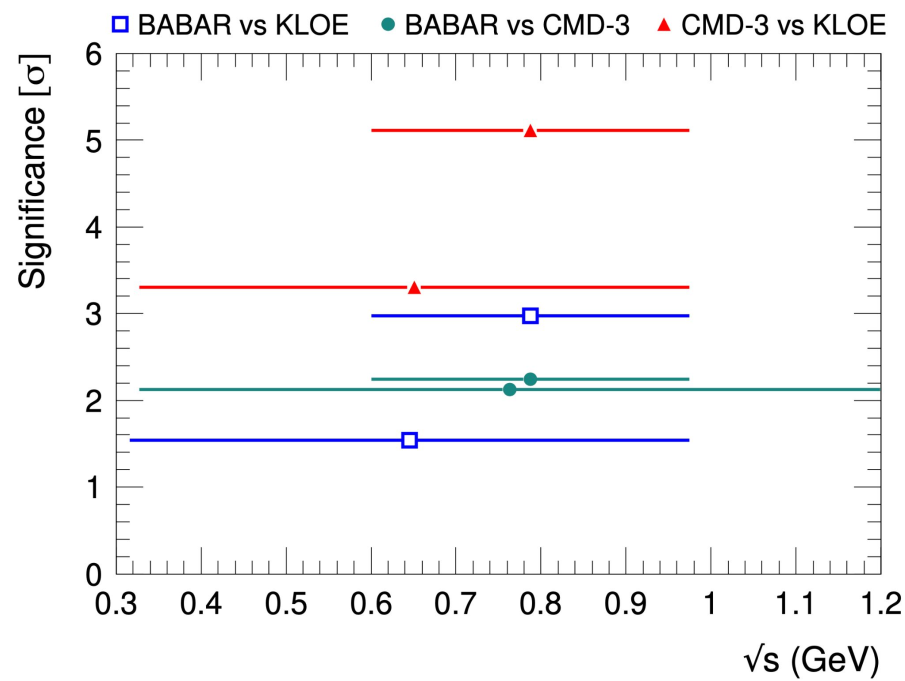
Reasonable BABAR/CMD3 agreement at low & high E



Quantitative comparisons for a_μ^{HVP}

→ Comparison of integrals computed on restricted energy ranges, for individual experiments:
significance of the difference between different experiments, taking into account correlations





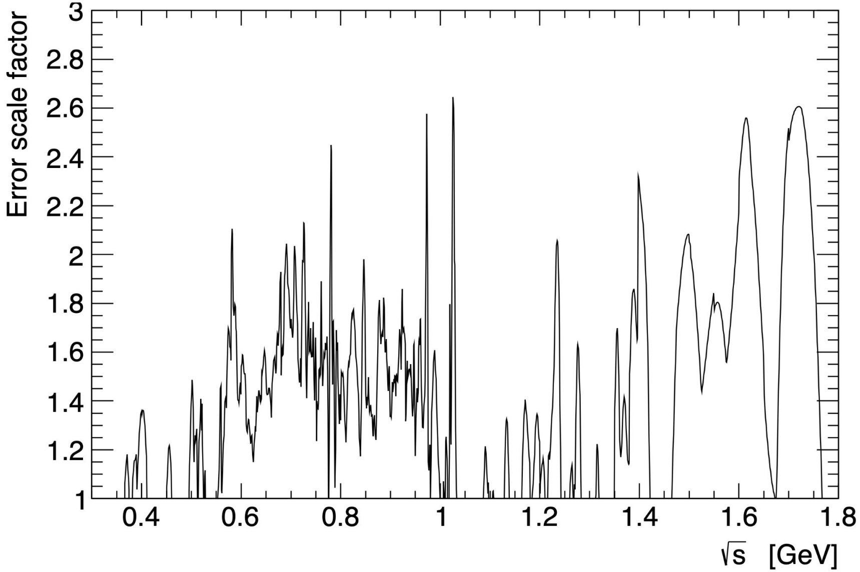
→ Largest tensions between CMD3 and KLOE

Combination procedure: compatibility between measurements

For each final bin:

→ χ^2 /ndof: test locally the level of agreement between input measurements, *taking into account correlations*

→ Scale uncertainties in bins with χ^2 /ndof > 1 (PDG): *locally* conservative; Adopted by KNT since ’17



→ Tension between measurements, especially between KLOE & CMD3, which provide the smallest / largest cross-sections in the ρ region:

Indication of underestimated uncertainties

Motivates conservative uncertainty treatment

in combination fit (evaluation of weights / fits based on analyticity & unitarity to constrain uncertainties at low √s - *backup*)

→ Observed (systematic) tension between measurements, beyond the local χ^2 /ndof rescaling

→ (*Since 2019*) Included extra (dominant) uncertainty: 1/2 difference between integrals w/o either BABAR or KLOE *(2nd motivation for using DHMZ uncertainties as “baseline” in the TI WP)*

Extra uncertainty started to be adopted in other studies (2205.12963)

However, *tensions are larger now and we need to understand their source!*

Two panel TI discussions with 49 questions addressed to CMD3 did not allow to identify any major problem. CMD2 / CMD3 tension still open question!

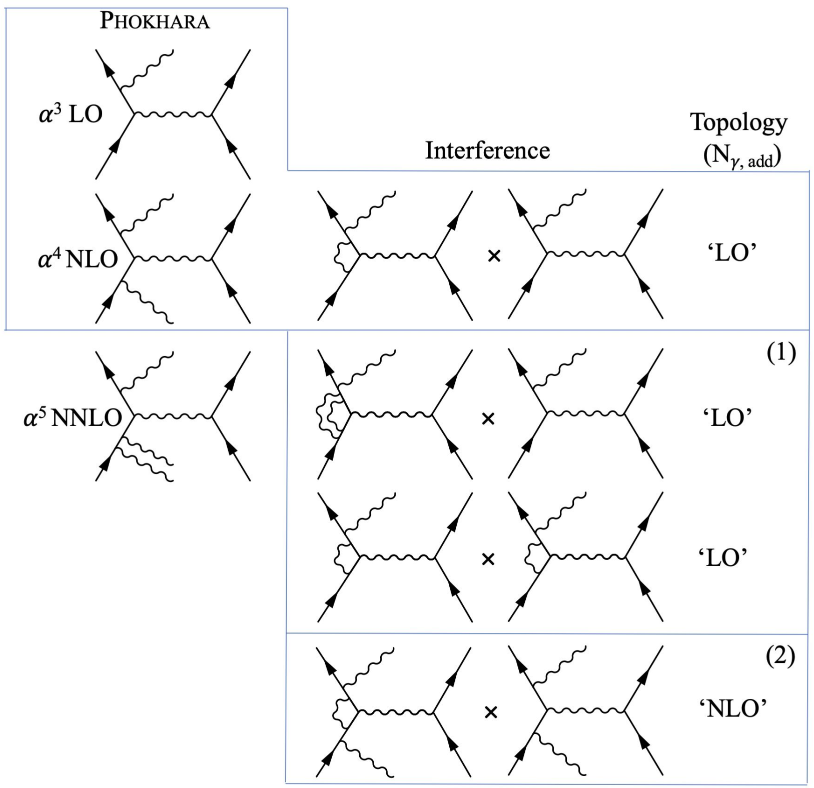
Impact of higher order photon emissions: *Unique '(N)NLO' BaBar study*

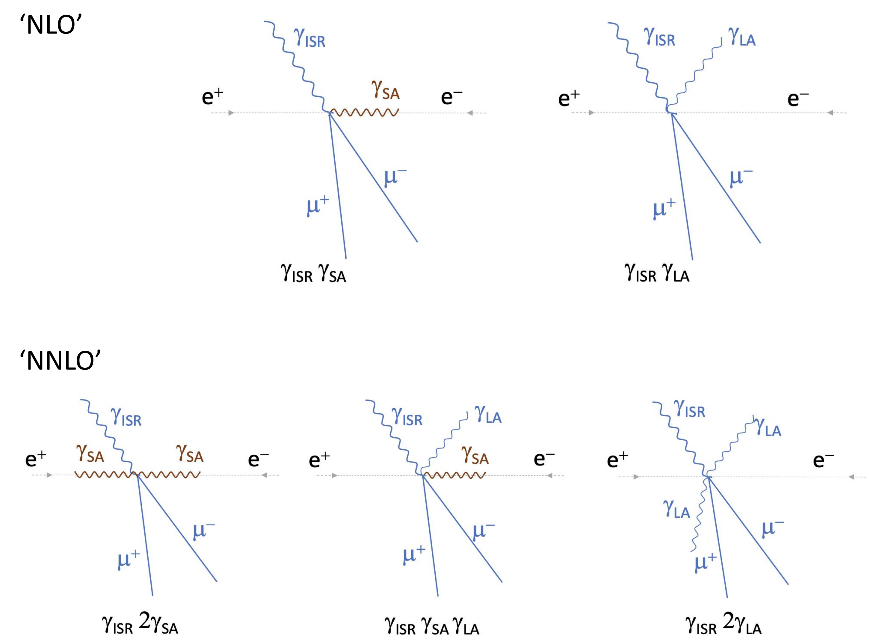
→ Studied in-situ in BaBar data, using kinematic fits: test the most frequently used Monte Carlo generators

- PHOKHARA: full NLO matrix element for ISR and FSR
- AFKQED: NLO and NNLO, with collinear approximation for additional ISR

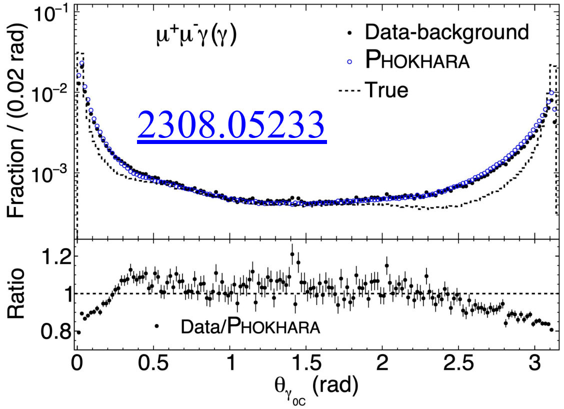
Generic ISR diagrams

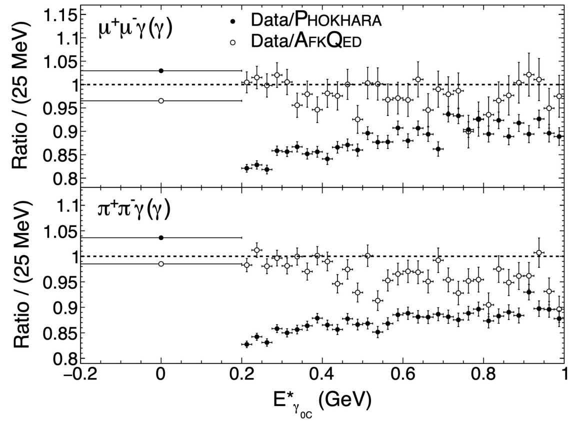
2308.05233

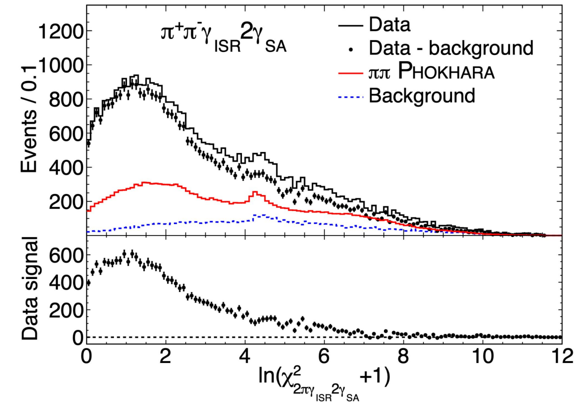


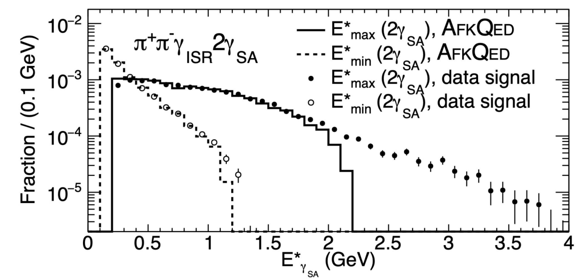


BaBar results on higher order photon emissions









→ NLO small-angle ISR in PHOKHARA higher than in data; large-angle ratios consistent with unity

→ AFKQED: reasonable description of rate and energy distributions for '(N)NLO' data

→ NNLO contributions clearly observed in data

→ BaBar measurements with loose selection incorporate NLO and HO radiation minimising MC-dependence

→ Other ISR measurements select 'LO' topology and rely on PHOKHARA for hard NLO (but with no NNLO)

→ Aspects further emphasized through studies based on fast simulation (2312.02053)

Uncertainties on uncertainties and correlations

Numerous indications of uncertainties on uncertainties and on correlations, with a direct impact on combination fits

→ Shapes of systematic uncertainties *evaluated* in ~wide mass ranges with sharp transitions

→ One standard deviation is statistically not well defined for systematic uncertainties

→ Systematic uncertainties like acceptance, tracking efficiency, background etc. not necessarily fully correlated between low and high mass

→ Are all systematic uncertainty components fully independent between each-other? (e.g. tracking and trigger)

→ *Yield uncertainties on uncertainties and on correlations*

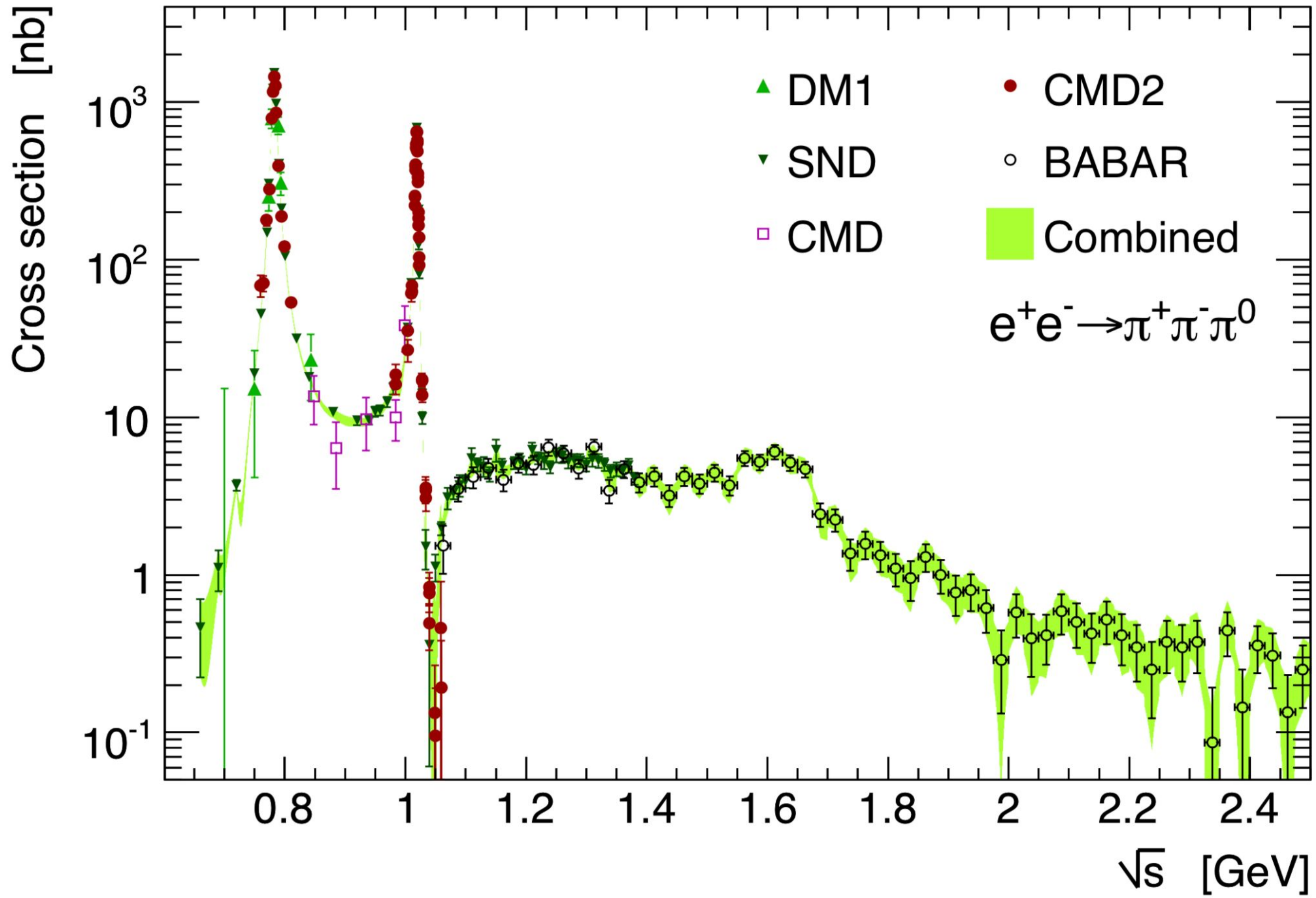
→ Tensions between measurements (BABAR/KLOE/CMD3; 3 KLOE results etc.): *experimental indications of underestimated uncertainties*

→ *Statistical methods* (χ^2 with correlations, likelihood fits, ratios of measured quantities etc.) *should not over-exploit the information on the amplitude and correlations of uncertainties*

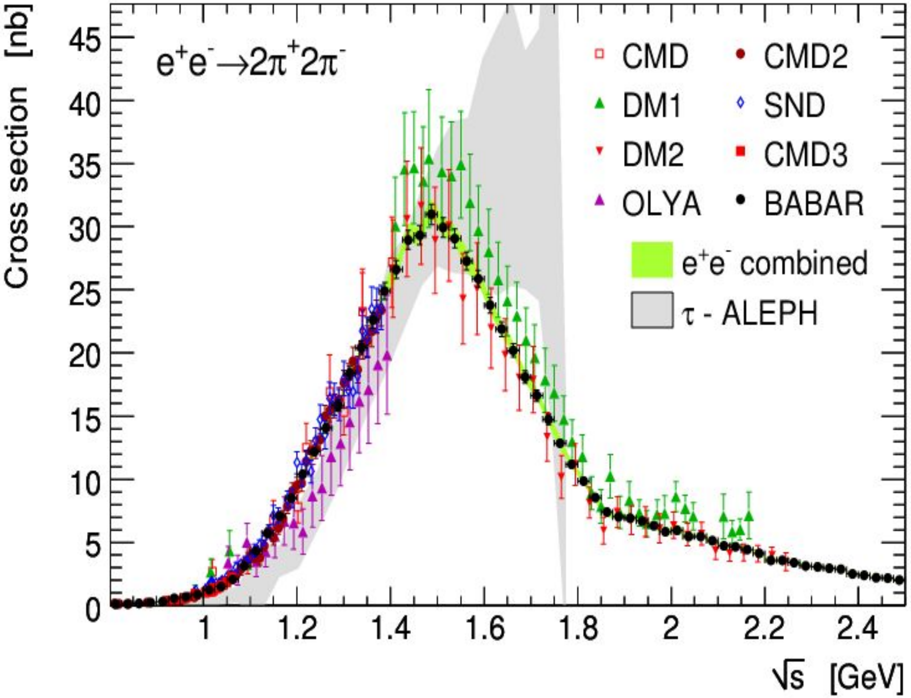
Topic of general interest, in other fields too (see backup)

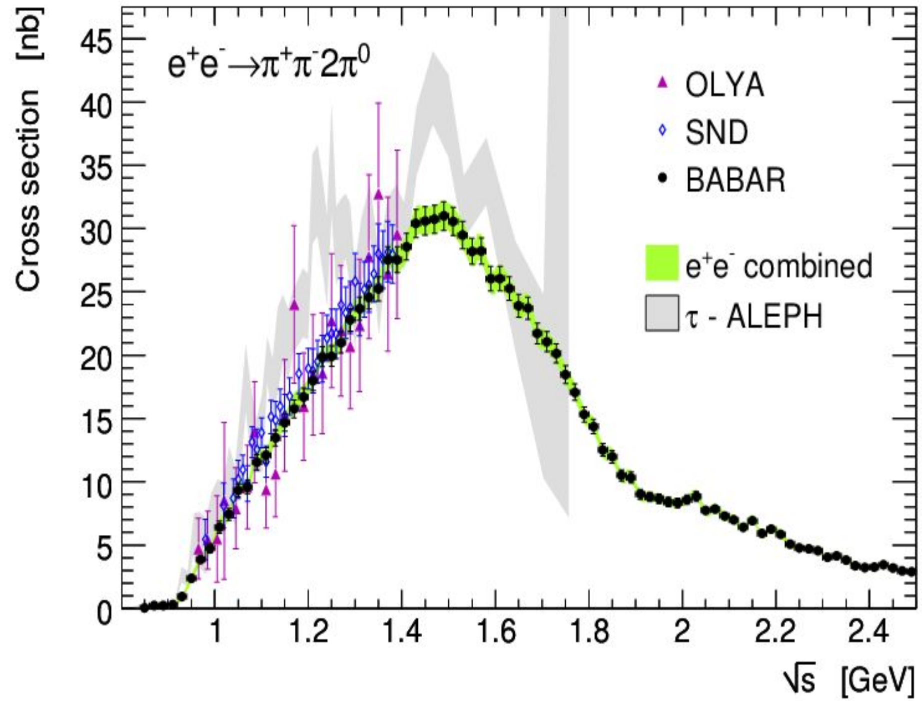
Combination of measurements for various channels and total HVP contribution

Combination for the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ channel



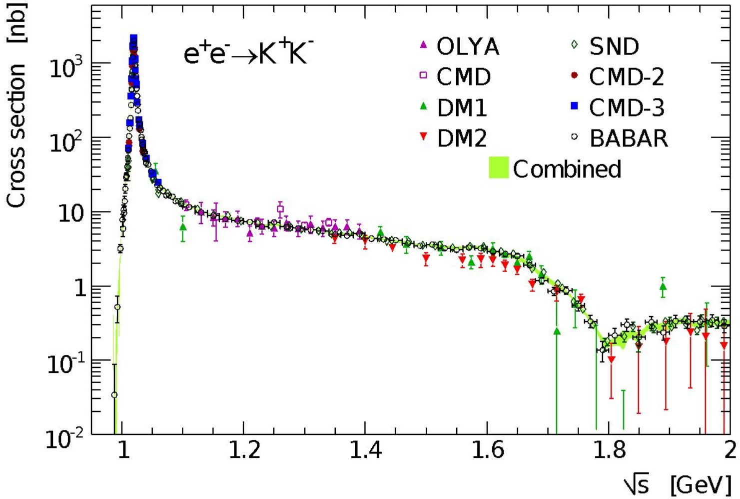
$e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$

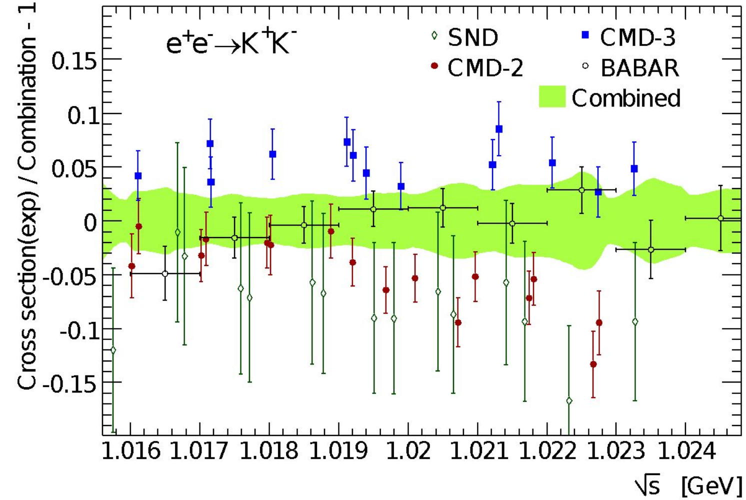


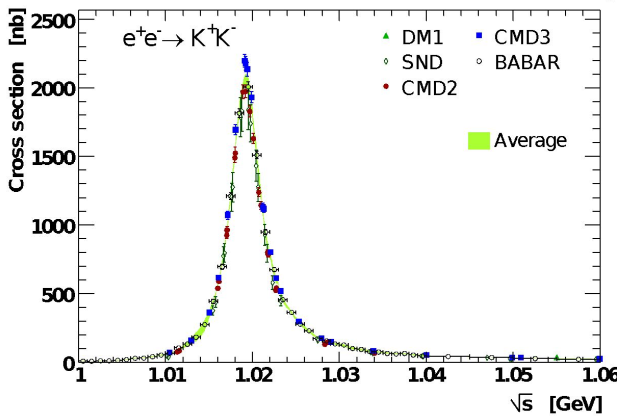


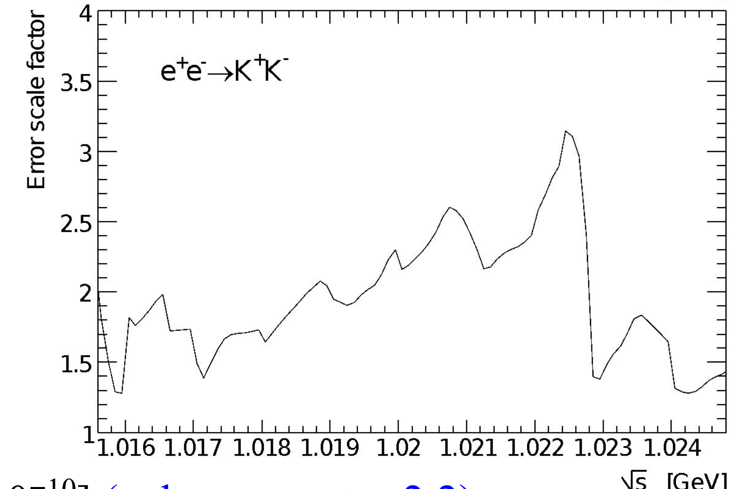
→ Essentially normalization differences w.r.t. τ data: *cross-checks very desirable*

Combination for the $e^+e^- \to K^+K^-$ channel





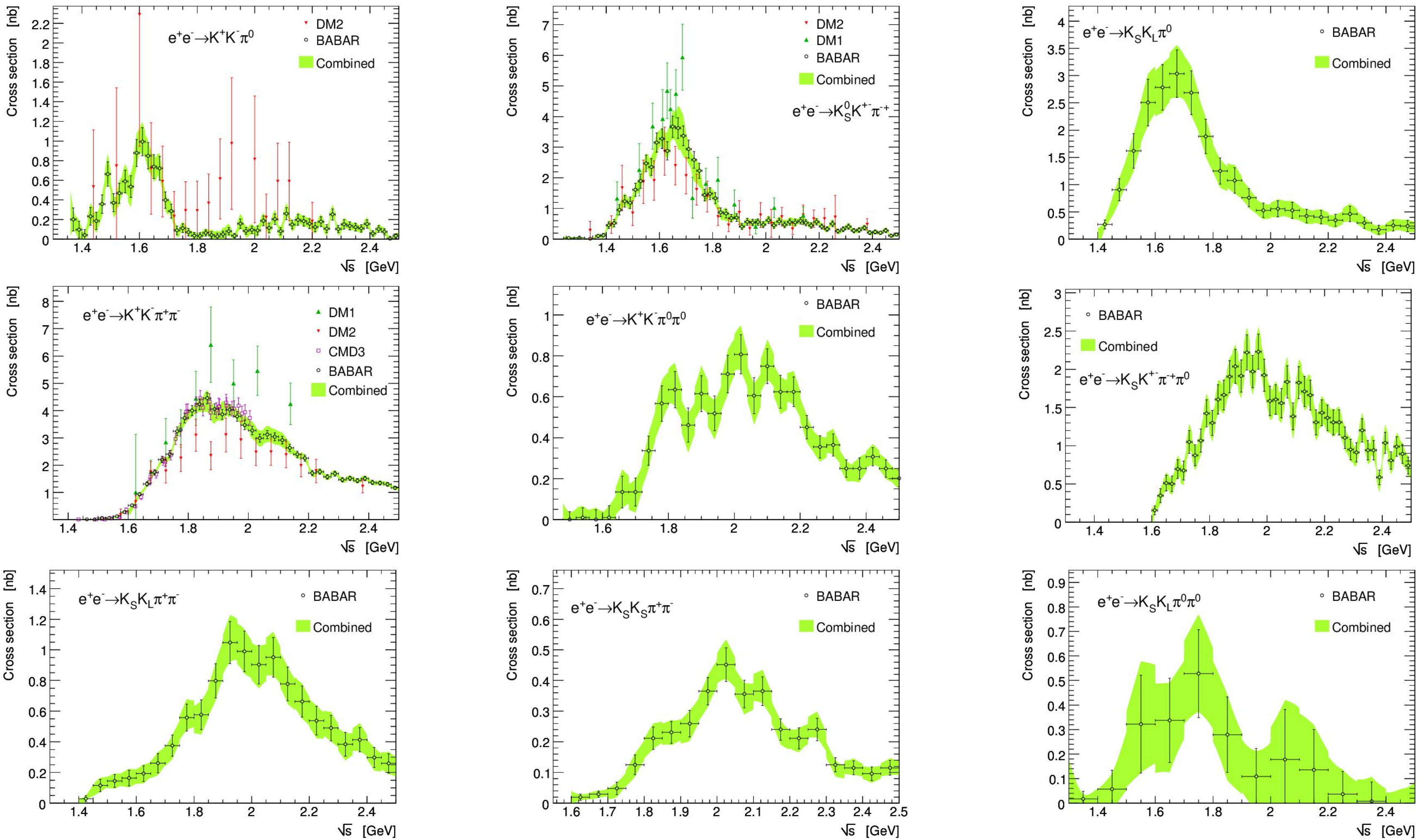




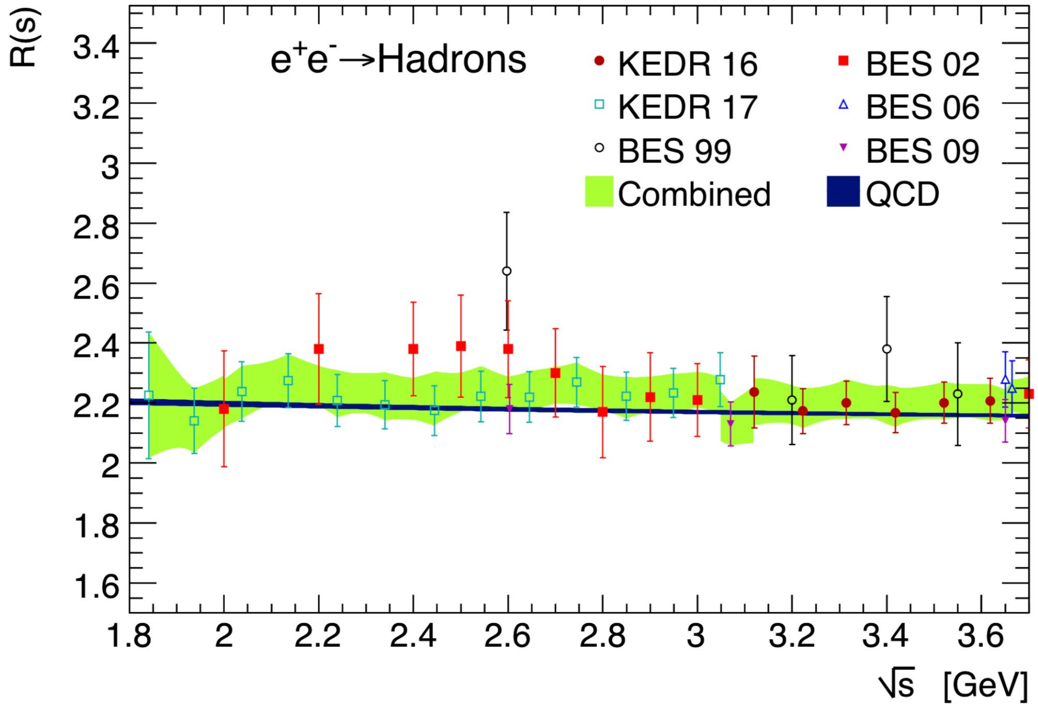
→ Tension between measurements

→ a_μ[→1.8GeV]: 23.08 ± 0.20 (stat.) ± 0.40 (syst.) [10^{-10}] (enhancement x 2.2)

Combination for the $e^+e^- \to KK\pi$ and $KK2\pi$ channels



Contributions from the 1.8 – 3.7 GeV region



→ Contribution evaluated from pQCD (4 loops) + $O(\alpha_s^{\ 2})$ quark mass corrections

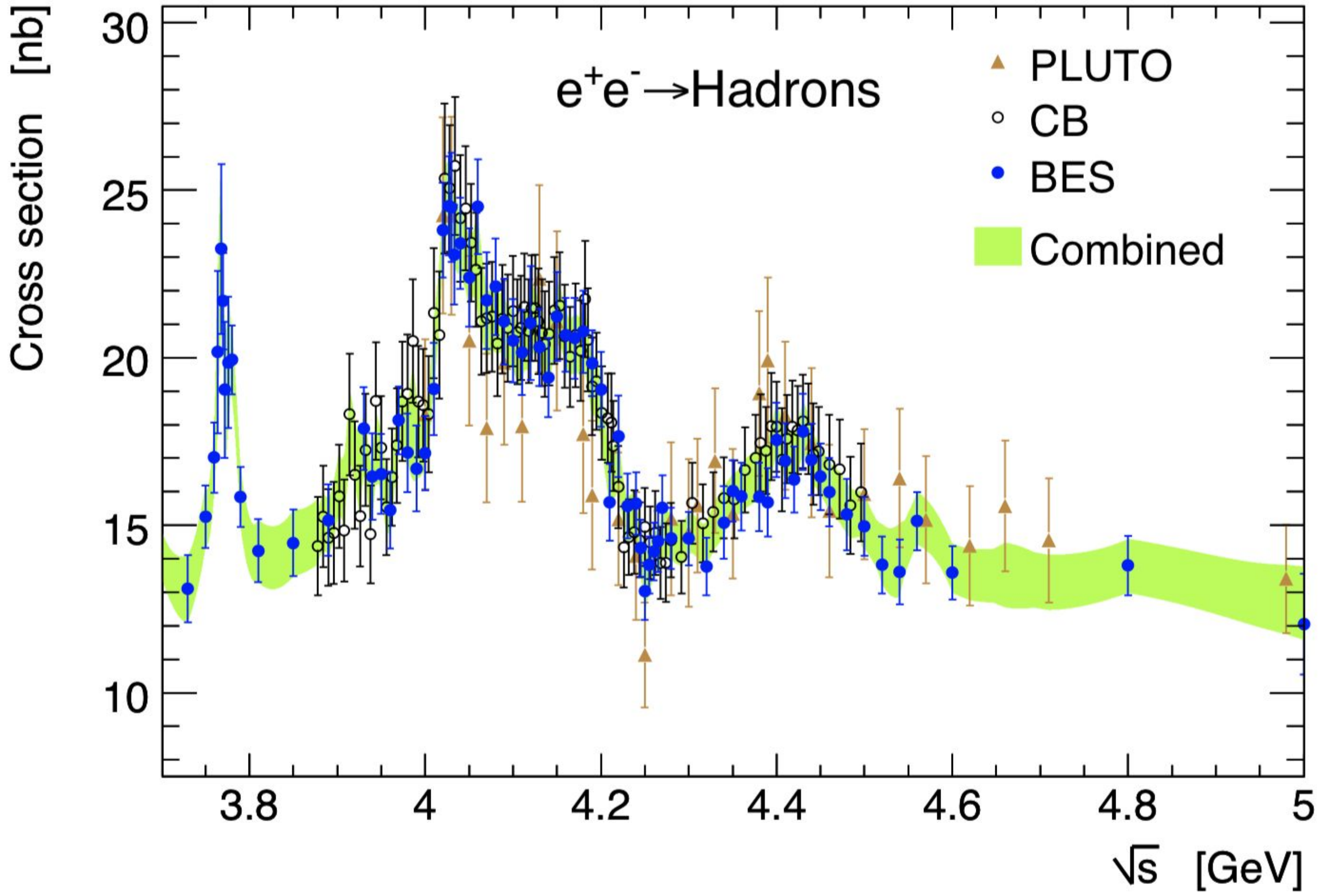
→ Uncertainties: α_s, truncation of perturbative series, CIPT/FOPT, m_q

→ 1.8-2.0 GeV: 7.65±0.31(data excl.); 8.30±0.09(QCD); added syst. 0.65 [10^{-10}]

→ 2.0-3.7 GeV: 25.82±0.61(data); 25.15 ± 0.19(QCD); agreement within 1σ

→ BES III results to be included: ~tension with pQCD and with KEDR 16 (*backup*)

Contributions from the charm resonance region



Situation in arXiv:1908.00921 (EPJC)

Channel	$a_\mu^{\rm had,LO}$ $[10^{-10}]$	$\Delta\alpha_{\rm had}(m_Z^2)$ $[10^{-4}]$
$\pi^0\gamma$	$4.41 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm 3.23 \pm 0.55$	$34.50 \pm 0.06 \pm 0.20 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.21 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega 2\pi$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi,\pi\gamma,\eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non-$K\overline{K},3\pi,\pi\gamma,\eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\overline{K}\omega$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	6.20 ± 0.11	7.00 ± 0.13
$\psi(2S)$ (BW integral)	1.56 ± 0.05	2.48 ± 0.08
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
$R_{\rm QCD}$ $[1.8 - 3.7$ GeV$]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\rm dual}$	$24.27 \pm 0.18 \pm 0.28_{\rm dual}$
$R_{\rm QCD}$ $[5.0 - 9.3$ GeV$]_{udsc}$	6.86 ± 0.04	34.89 ± 0.18
$R_{\rm QCD}$ $[9.3 - 12.0$ GeV$]_{udscb}$	1.20 ± 0.01	15.53 ± 0.04
$R_{\rm QCD}$ $[12.0 - 40.0$ GeV$]_{udscb}$	1.64 ± 0.00	77.94 ± 0.13
$R_{\rm QCD}$ $[> 40.0$ GeV$]_{udscb}$	0.16 ± 0.00	42.70 ± 0.05
$R_{\rm QCD}$ $[> 40.0$ GeV$]_t$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$694.0 \pm 1.0 \pm 3.5 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\rm QCD}$	$275.29 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.15_\psi \pm 0.55_{\rm QCD}$

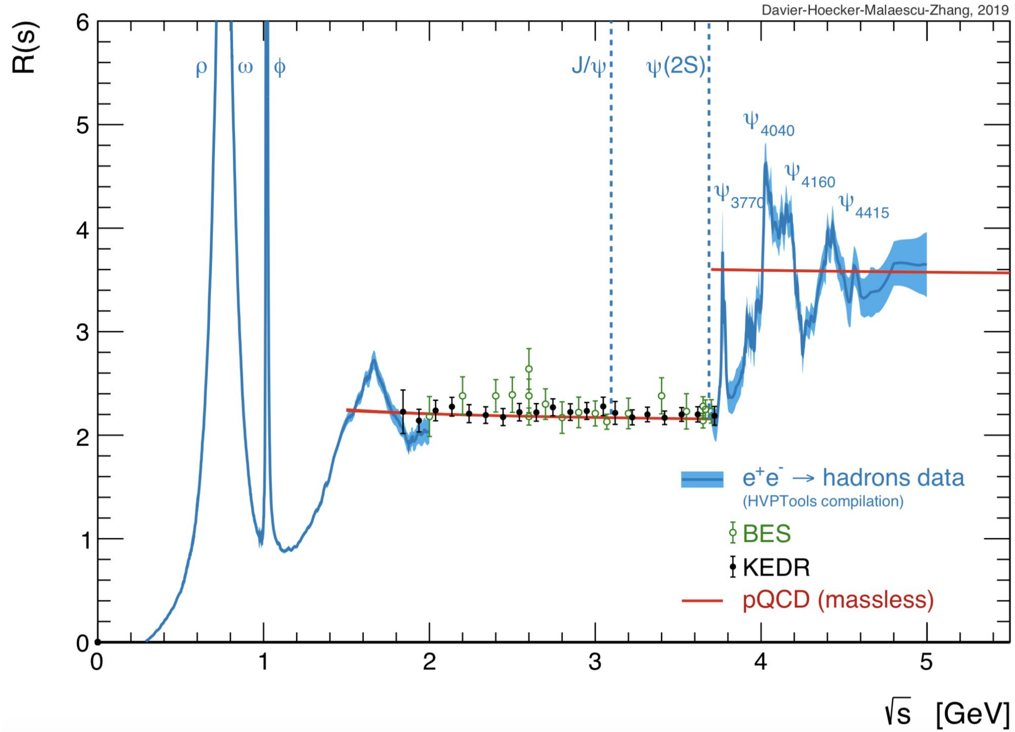
→ 32 exclusive channels are integrated up to 1.8 GeV

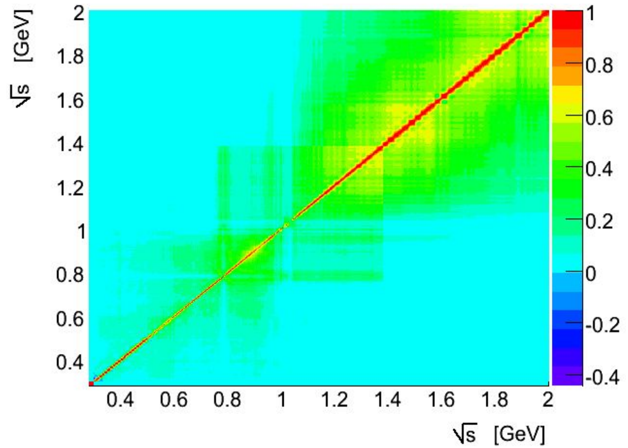
Relative contributions to a_μ from missing channels (estimated based on isospin symmetry)

→ 0.87 ± 0.15 % (DEHZ 2003)
→ 0.69 ± 0.07 % (DHMZ 2010)
→ 0.09 ± 0.02 % (DHMZ 2017)
→ 0.016 ± 0.016 % (DHMZ 2019)
(*Nearly complete set of exclusive measurements from BABAR*)

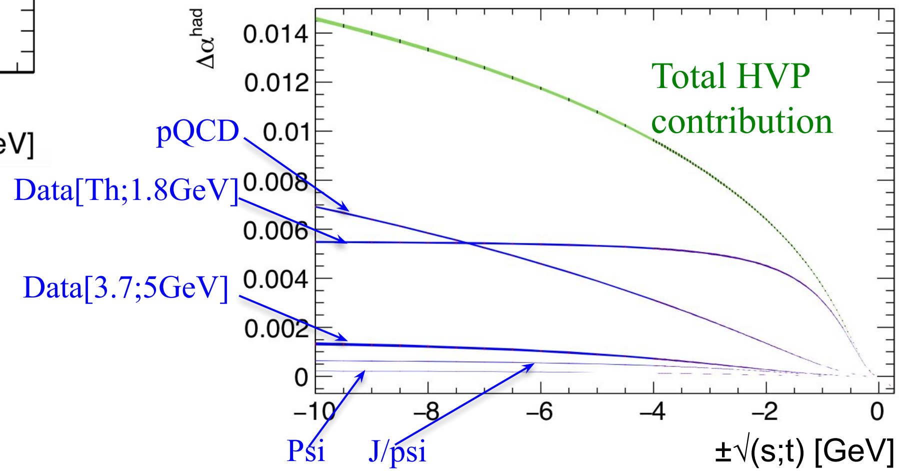
Estimation procedures also adopted by KNT

$R_{e+e-} \rightarrow$ Hadrons





Sum of *32 exclusive channels* with *full propagation of correlations*



→ Performed non-trivial check: a_μ and $\Delta\alpha_{had}$ from sum of individual channels and from Ree integral < 1.8 GeV

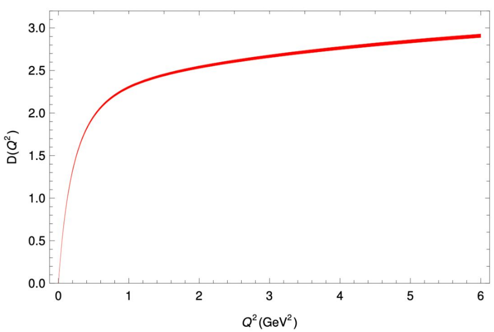
→ Enables the determination of the various HVP contributions to the "running" of α_{QED}

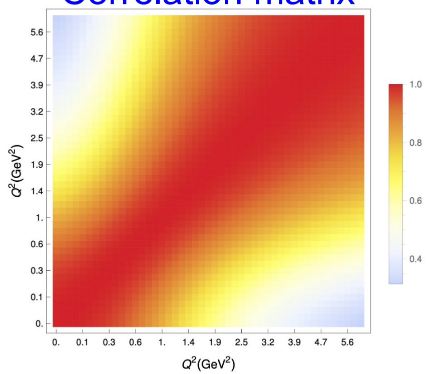
α_S extraction from the Adler function and test of the RGE

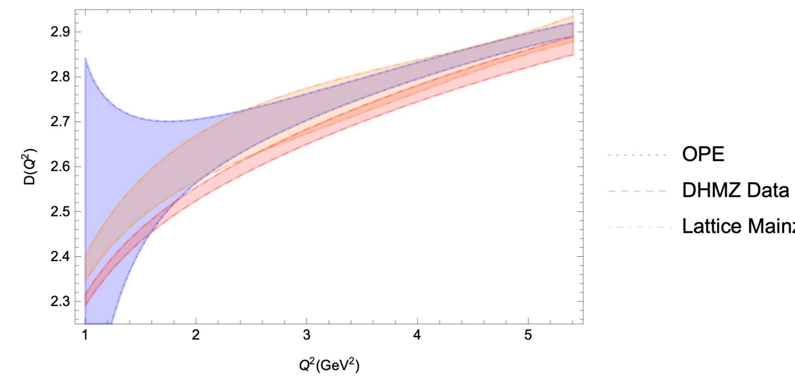
$$D(Q^2) \equiv \sum_{i,j} Q_i Q_j \, D_{ij}(Q^2) = 3\pi Q^2 \, \frac{d\Delta\alpha_{\rm had}(Q^2)}{\alpha \, dQ^2}$$

2302.01359

→ Experimental values with correlations; Theoretical predictions: perturbative + non-perturbative corrections (OPE)



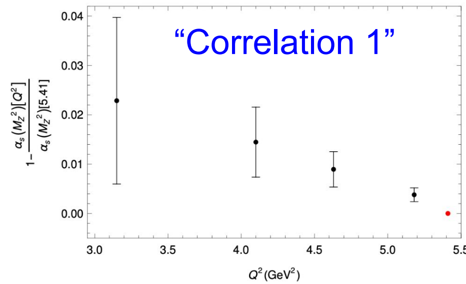


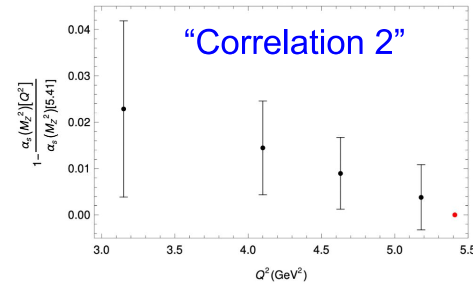


→ Using W.Av. $\alpha_S(M_Z)$ value ~0.118: OPE prediction in good agreement with Lattice QCD, above dispersive

→ Fit DHMZ data: $\alpha_s^{(n_f=5)}(M_Z^2) = 0.1136 \pm 0.0025$

→ Performing RGE test and evaluating its precision, with different correlation scenarios for theory uncertainties





Comparing lattice QCD and data-driven results in systematically improvable ways

2308.04221 (BMW & DMZ)

Primary observables: lattice calculations; comparisons w.r.t. dispersive

→ Lattice: employ simulations to compute electromagnetic-current two-point function

$$C(t) = \frac{1}{3e^2}\sum_{i=1}^{3}\int d^3x\,\langle J_i(\vec{x},t)J_i(0)\rangle$$

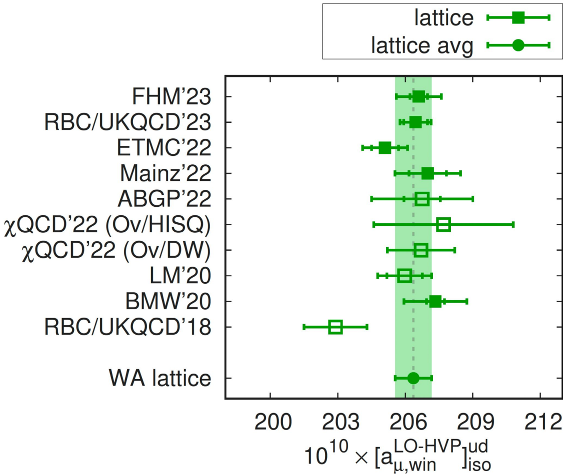
$$\frac{J_\mu}{e} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c - \frac{1}{3}\bar{b}\gamma_\mu b + \frac{2}{3}\bar{t}\gamma_\mu t$$

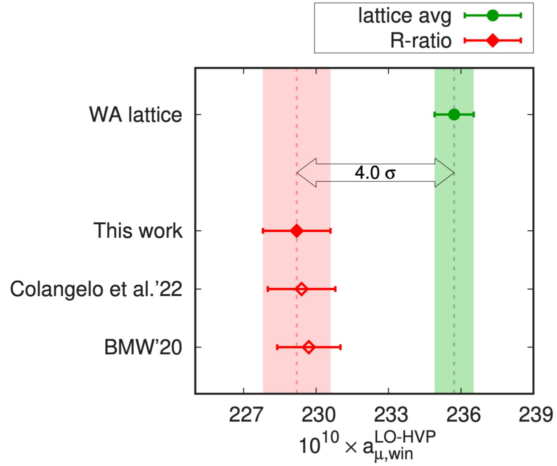
$a_\mu^{\text{LO-HVP}}$, $a_{\mu,\text{win}}^{\text{LO-HVP}}$, $\delta\left(\Delta\alpha_{\text{had}}^{(5)}\right) \equiv \Delta\alpha_{\text{had}}^{(5)}(-1 \to -10\text{GeV}^2), \ldots$ are weighted sums of $C(t)$ over t

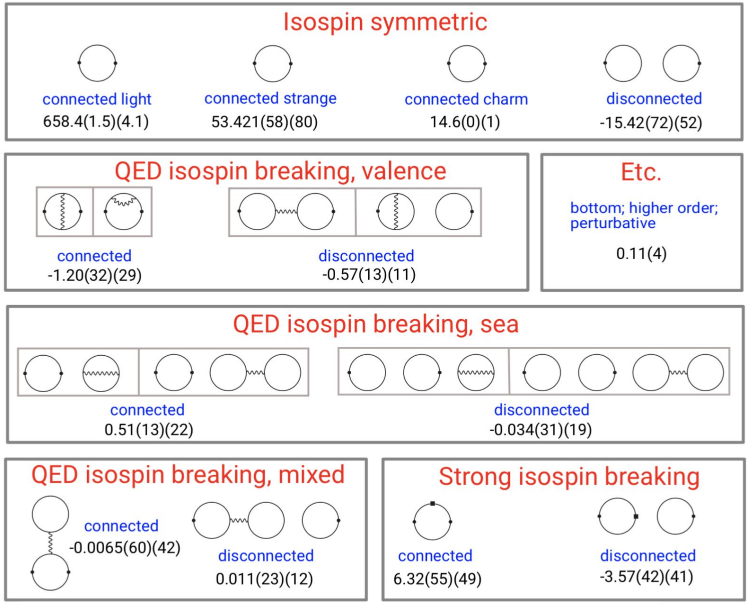
Based on BMW'20 (+) with preliminary $\delta\left(\Delta\alpha_{\text{had}}^{(5)}\right)$

→ Tensions between lattice and data-driven (DD) HVP results

$\left[\Delta a_\mu^{\text{LO-HVP}}\right]_{\text{lat-DD}} \sim 2.1\sigma$ $\quad \left[\Delta a_{\mu,\text{win}}^{\text{LO-HVP}}\right]_{\text{lat-DD}} \sim 4.0\sigma$







All contributions to C(t), with all limits taken:

$a \to 0$, $L \to \infty$, $M_\pi \to M_\pi^\varphi$, ...

$$C(t) = \frac{1}{24\pi^2} \int_0^\infty ds\, \sqrt{s} R(s)\, e^{-|t|\sqrt{s}}$$

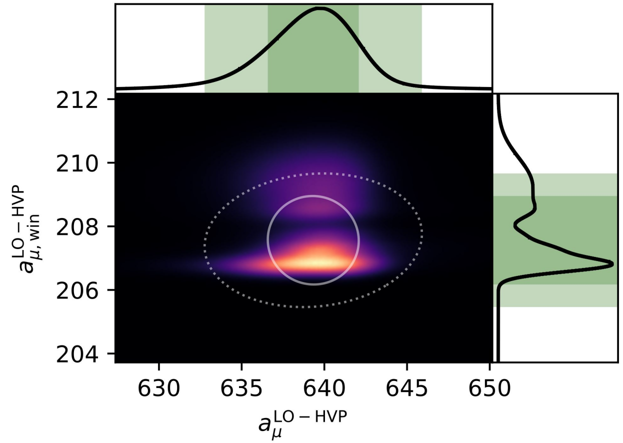
→ R-ratio → lattice: "straightforward" (integrate R-ratio)

→ Lattice → R-ratio: inverse Laplace transform (ill-posed problem)

New in this study:

→ Correlations among lattice HVP observables (stat. resampling and syst. histogram, with flat and Akaike Information Criterion (AIC) weights)

→ Uncertainties on these correlations (important for checking stability of inverse problem)



(connected *ud*)

→ Developed statistical approach that:

- Provides useful information with limited lattice input
- Can be systematically improved with more lattice input
- Can (eventually) incorporate physical constraints
- Includes measure of agreement of lattice & R-ratio results with comparison hypothesis
- Accounts for all correlations in lattice and R-ratio observables ...
- ... including uncertainties on these

Testing lattice

→ 1-by-1 comparison of moment integrals

Observable	lattice BMW'20	data-driven	diff.	% diff.	σ	p-value [%]
$a_\mu^{\text{LO-HVP}} \times 10^{10}$	707.5(5.5)	694.0(4.0)	13.5(6.8)	1.9(1.0)	2.0	4.7
$a_{\mu,\text{win}}^{\text{LO-HVP}} \times 10^{10}$	236.7(1.4)	229.2(1.4)	7.5(2.0)	3.2(0.8)	3.8	0.01
$\left[\Delta_{\text{had}}^{(5)}\alpha(-10\,\text{GeV}^2) - \Delta_{\text{had}}^{(5)}\alpha(-1\,\text{GeV}^2)\right] \times 10^4$	48.67(0.32)	48.02(0.32)	0.65(0.45)	1.3(0.9)	1.4	15.

→ Simultaneous comparisons with correlations

$$\chi^2(a_j) = \sum_{j,k}\left[a_j^{\text{lat}} - a_j\right]\left[C_{\text{lat}}^{-1}\right]_{jk}\left[a_k^{\text{lat}} - a_k\right] + \sum_{j,k}\left[a_j^{\text{R}} - a_j\right]\left[C_{\text{R}}^{-1}\right]_{jk}\left[a_k^{\text{R}} - a_k\right]$$

$$\chi^2_{\min} = \sum_{j,k}\left[a_j^{\text{lat}} - a_j^{\text{R}}\right]\left[\left(C_{\text{lat}} + C_{\text{R}}\right)^{-1}\right]_{jk}\left[a_k^{\text{lat}} - a_k^{\text{R}}\right]$$

# observ.	χ^2/dof	p-value [%]
2	14.4/2 – 18.8/2	0.002 – 0.017
3	14.4/3 – 18.8/3	0.009 – 0.63

→ Some dilution compared to $a_{\mu,\text{win}}^{\text{LO-HVP}}$ alone, but still significant tension

→ (Taking into account the shapes of integral kernels - *backup*) Differences could be explained by: a *C(t)* that is enhanced in $t \sim [0.4, 1.5]$ fm, also probably for $t \gtrsim 1.5$ fm, with possible suppression for $t \lesssim 0.4$ fm

Testing R-ratio: methodology

→ Chop a_j^{R} into contributions $a_{j,b}^{\mathrm{R}}$ from same $\sqrt{s}$-intervals I_b for all j : $\quad a_j^{\mathrm{R}} = \sum_b a_{j,b}^{\mathrm{R}}$

→ To accommodate lattice results a_j^{lat}, allow common rescaling of $a_{j,b}^{\mathrm{R}}$, for all j, in certain I_b : $\quad a_j^{\mathrm{lat}} = \sum_b \gamma_b \cdot a_{j,b}^{\mathrm{R}}$

- Simplest interpretation: R-ratio rescaled in I_b
- However, constrains shape of R-ratio modification in limited way: *physical deformation may be allowed*

→ If $N_j \geq N_b$, system (over-)constrained: solved here for one γ via weighted average and/or χ^2 minimization, while avoiding too strong assumptions about the knowledge of uncertainties and correlations

$$a_j^{\mathrm{lat}} = \sum_{b\,\in A} \gamma \cdot a_{j,b}^{\mathrm{R}} + \sum_{b\,\in B} a_{j,b}^{\mathrm{R}} \qquad \rightarrow \gamma = \frac{a_j^{\mathrm{lat}} - \sum_{b\,\in B} a_{j,b}^{\mathrm{R}}}{\sum_{b\,\in A} a_{j,b}^{\mathrm{R}}} \equiv \tilde{\gamma}_j\,;$$

$$\chi^2(\gamma) = \sum_{j,k} [\gamma - \tilde{\gamma}_j] \left[\left(C_{\mathrm{lat}}^{\tilde{\gamma}} + C_{\mathrm{R}}^{\tilde{\gamma}} \right)^{-1} \right]_{jk} [\gamma - \tilde{\gamma}_k]$$

$$\chi^2(\gamma) = \sum_{j,k} \left[a_j^{\mathrm{lat}} - \sum_{b\,\in A} \gamma \cdot a_{jb} - \sum_{b\,\in B} a_{jb} \right] \left[C_{\mathrm{lat}}^{-1} \right]_{jk} \left[a_k^{\mathrm{lat}} - \sum_{c\,\in A} \gamma \cdot a_{kc} - \sum_{c\,\in B} a_{kc} \right]$$
$$+ \sum_{(jb),(kc)} \left[a_{jb}^{\mathrm{R}} - a_{jb} \right] \left[C_{\mathrm{R}}^{-1} \right]_{(jb)(kc)} \left[a_{kc}^{\mathrm{R}} - a_{kc} \right]$$

→ Somewhat different interpretation, still compatible results

Sensitivity to the lattice statistical uncertainties on covariance matrix

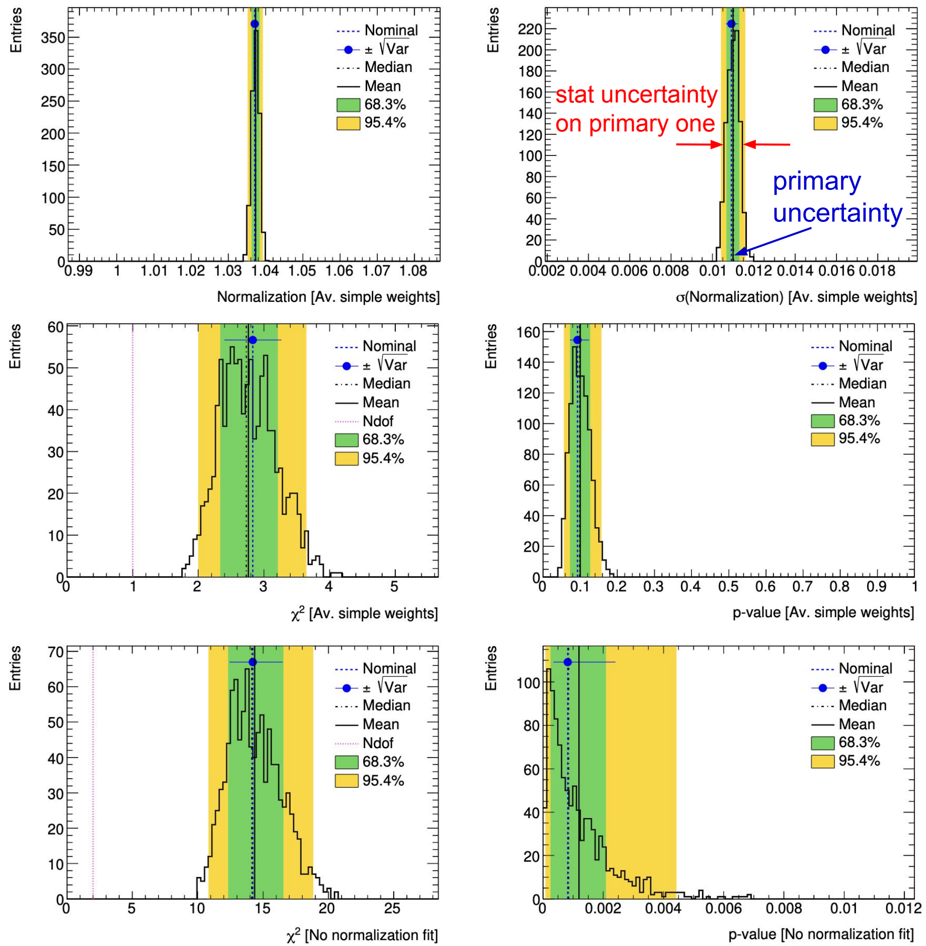
→ Employ 2nd order sampling (bootstraps on jackknife samples) to build distributions for the quantities of interest: re-run procedure with fluctuated lattice covariance matrix

→ Quantiles of these distributions to quantitatively evaluate the impact

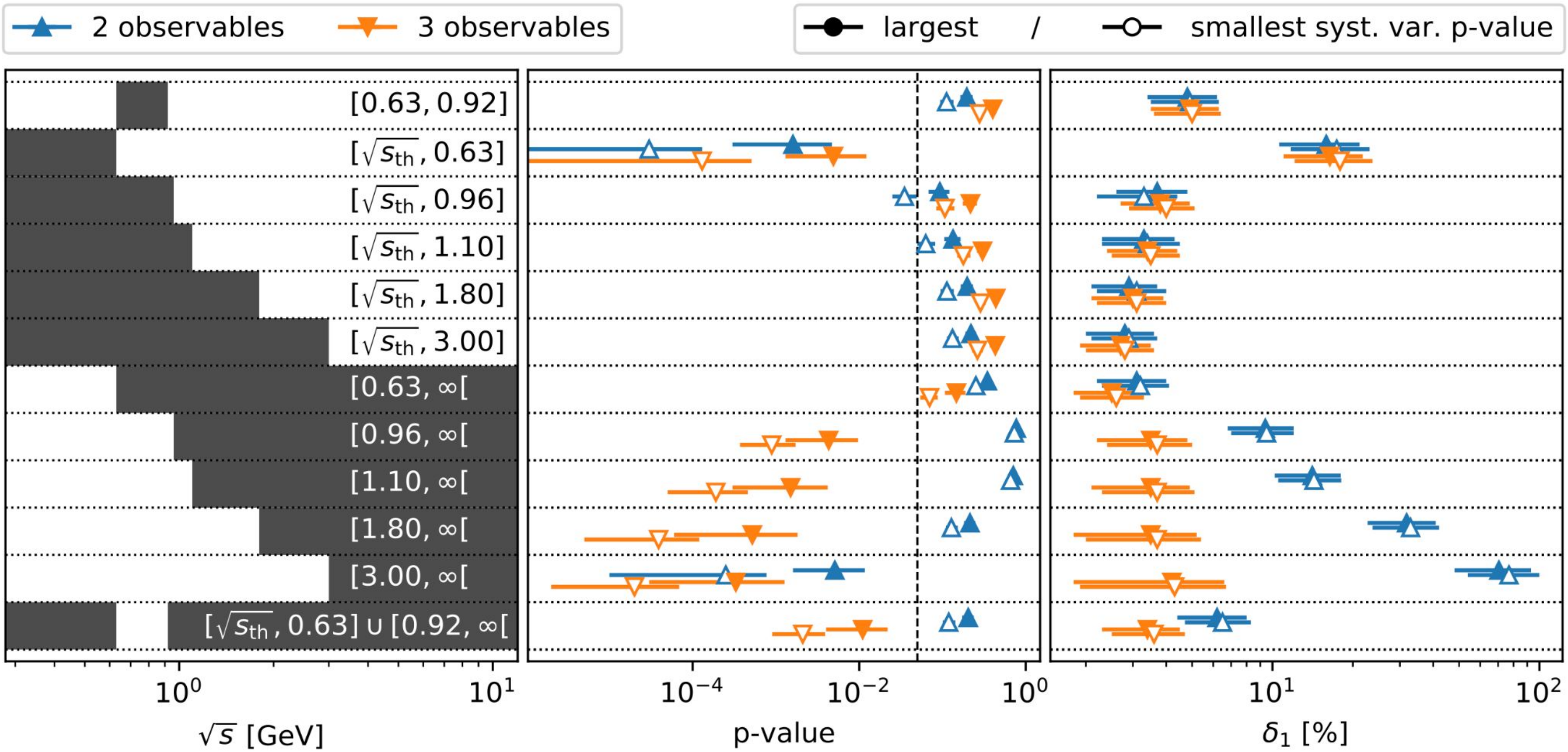
→ Normalisation factor and its uncertainty from fit precisely determined

→ Conclusions about χ^2 and p-values stable within lattice statistical uncertainties on covariance matrix

2 input moment integrals; Normalisation fit < 0.96 GeV;
Lattice covariance matrix "0"



Consider $a_j = a_\mu^{\text{LO-HVP}}, a_{\mu,\text{win}}^{\text{LO-HVP}}$ (2 observables) with $a_j = \delta\left(\Delta\alpha_{\text{had}}^{(5)}\right)$ (3 observables)

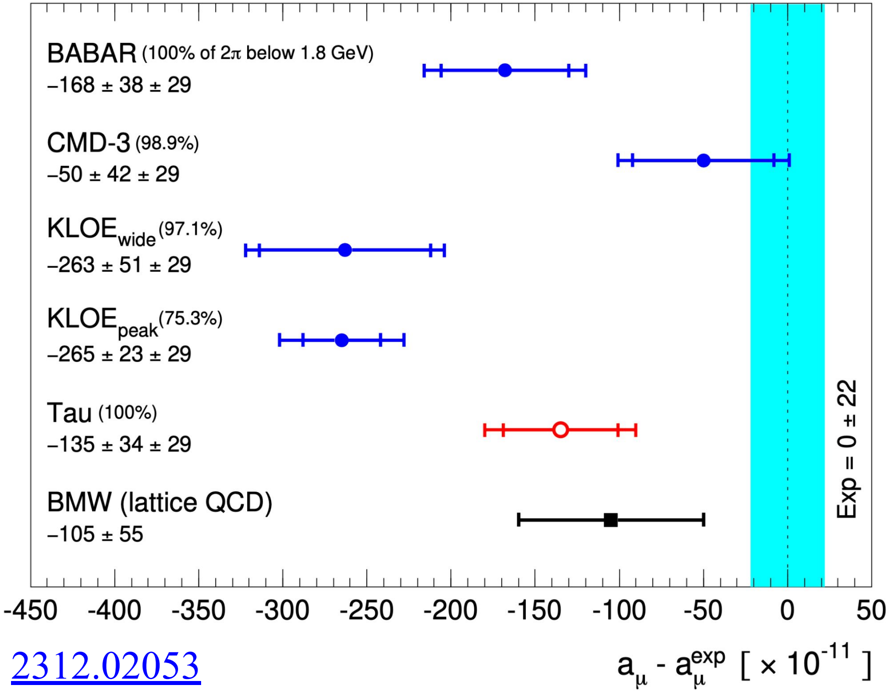


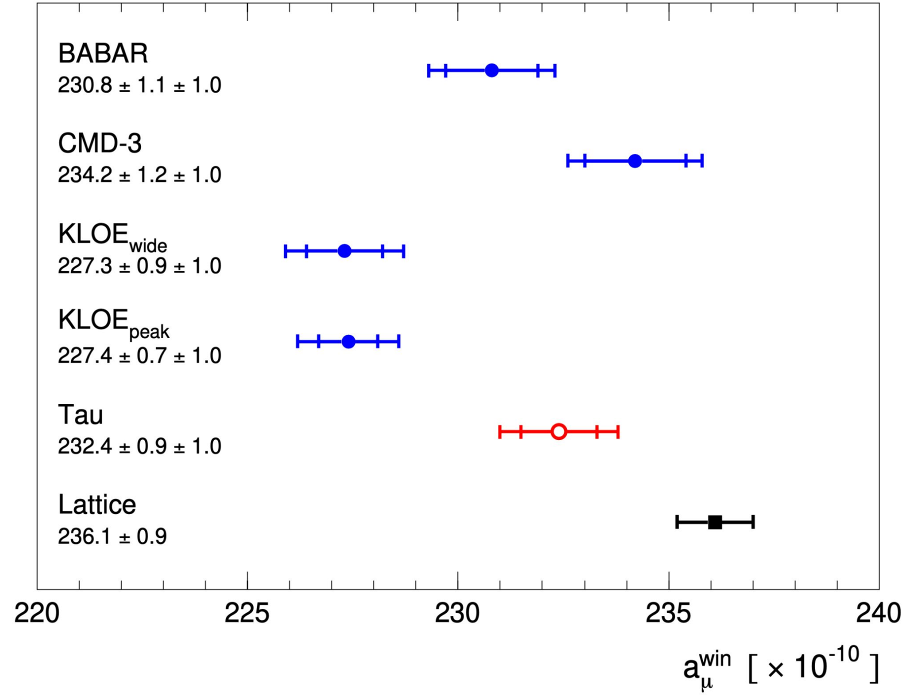
→ Differences could be explained by enhancing measured R-ratio around (/any larger interval including) ρ-peak

→ Outcome of the studies stable within stat. and syst. uncertainties on lattice covariance matrices

→ Rescalings beyond the uncertainties of $R^{e^+e^-}$ → No problems for EW fits in case of 3-observable

A new possible perspective on a_μ (HVP)





2312.02053

→ The τ-based HVP contribution close to the values provided by BABAR and CMD-3

→ Their combination (3.8σ > KLOEpeak) is compatible with BMW for a_μ, but a 2.9σ tension persists for a_μ^{win}

→ The BMW-based prediction is 1.8σ below the experimental value; *not* incompatible with the EW fit (backup)

Combining BABAR, CMD-3, τ (+BMW): 2.5σ (2.8σ) difference with experiment

When including KLOE in the dispersive calculation: > 5σ w.r.t. experiment

→ *Tests of MC generators using KLOE data & in-situ studies of impact on the analysis are very much desirable*

Remarks and Conclusions

We have an interesting, long standing, multifaceted problem…
… And very important elements to solve the puzzle started to become available !

→ Future e+e- measurements very important: independent 2π measurement from BaBar w/o PID ~this year

Guiding ideas:

→ Need *rigorous* and *realistic* treatment of uncertainties and correlations at all levels
(Underestimated uncertainties do not bring scientific progress & can put studies on wrong path)

→ Caution about significance:
statistics-dominated measurement; prediction uncertainty limited by non-Gaussian systematic effects

→ Studies for understanding differences between data-driven and Lattice QCD approaches need to follow similar standards as the g-2 experiment: *double-blinding*

Backup

Lepton Magnetic Anomaly: from Dirac to QED

- Magnetic dipole moment of a charged lepton: $\vec{\mu} = g \frac{e}{2m} \vec{s}$

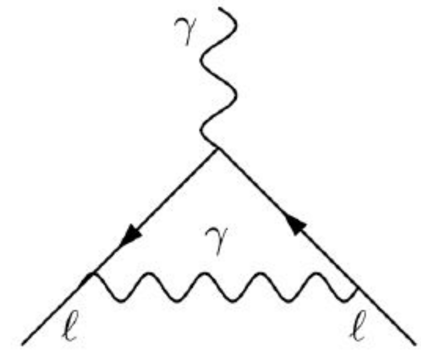
  Dirac (1928) $g_e=2$ $a_e=0$

- "anomaly" = deviation w.r.t. Dirac's prediction: $a = \frac{g-2}{2}$

anomaly discovered:
Kusch-Foley (1948) $a_e = (1.19 \pm 0.05)\ 10^{-3}$

and explained by $O(\alpha)$ QED contribution:
Schwinger (1948) $a_e = \alpha/2\pi = 1.16\ 10^{-3}$

first triumph of QED

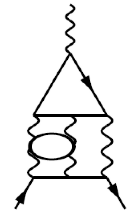


$\Rightarrow$ a_e sensitive to quantum fluctuations of fields

More Quantum Fluctuations

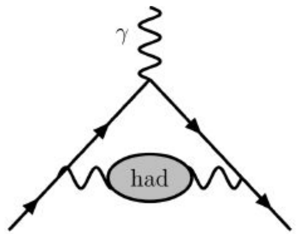
$$a = a^{\rm QED} + a^{\rm had} + a^{\rm weak} \quad + \, ? \; a^{\text{new physics}} \; ?$$

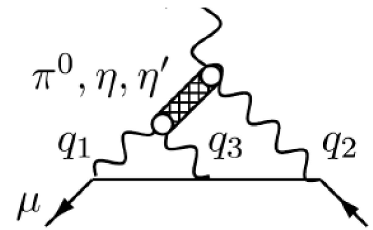
typical contributions:



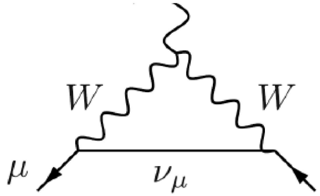
QED up to $O(\alpha^5)$ (Kinoshita et al.)

Hadrons vacuum polarization light-by-light (dispersive & lattice QCD)

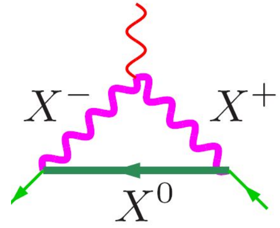




Electroweak



new physics at high mass scale



$\delta a_l \propto \frac{m_l^2}{M^2}$ $\Rightarrow$ a_μ much more sensitive to high scales

Theory initiative: prepare the Standard Model prediction for $(g\text{-}2)_\mu$

PAST WORKSHOPS AND EVENTS

Sixth Plenary Workshop of the Muon g-2 Theory Initiative
http://muong-2.itp.unibe.ch/
to be held at the University of Bern (Bern, Switzerland), 4-8 September 2023.

2nd CMD3 discussion meeting
https://indico.ijclab.in2p3.fr/event/9697/
A virtual event hosted by IJCLab on 20 July 2023.

Scientific seminar and discussion on new CMD-3 result
https://indico.fnal.gov/event/59052/
A virtual event hosted by Fermilab on 37 Mar 2023.

Fifth Plenary Workshop of the Muon g-2 Theory Initiative
https://higgs.ph.ed.ac.uk/
held at the Higgs Centre, University of Edinburgh (Edinburgh, United Kingdom), 5-9 September 2022.

Fourth Plenary Workshop of the Muon g-2 Theory Initiative
https://agenda.hepl.phys.nagoya-u.ac.jp/indico/conferenceDisplay.py?confId=1691
A virtual workshop hosted by KEK (Tsukuba, Japan), held on 28 June - 02 July 2021.

The hadronic vacuum polarization from lattice QCD at high precision
https://indico.cern.ch/event/956699/
A virtual topical workshop of the Muon g-2 Theory Initiative, 16-20 Nov 2020.

Hadronic contributions to (g-2) µ
https://indico.fnal.gov/event/21626/
held at the Institute for Nuclear Theory, University of Washington, Seattle, WA, 9-13 September 2019

Second workshop of the Muon g-2 Theory Initiative
https://wwwth.kph.uni-mainz.de/g-2/
held at the Helmholtz Institute Mainz, University of Mainz, Mainz, Germany, 18-22 June 2018

Muon g-2 Theory Initiative Hadronic Light-by-Light working group workshop
https://indico.phys.uconn.edu/event/1/
held at the University of Connecticut, Storrs, CT, 12-14 March 2018

Workshop on Hadronic Vacuum Polarization Contributions to Muon g-2
https://www-conf.kek.jp/muonHVPws/
held at KEK, Tsukuba, Japan, 12-14 Feb 2018

First workshop of the Muon g-2 Theory Initiative
https://indico.fnal.gov/event/13795/
held in St. Charles, IL, USA, 3-6 June 2017

Put together in a *coherent & conservative* way the results of various groups, *before the Fermilab result*

https://muon-gm2-theory.illinois.edu

White Paper: arXiv:2006.04822 (Phys. Rept.)

Theory initiative white paper executive summary & new results

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

→ Dominant uncertainty: HVP LO → *Merging* of model independent results: DHMZ and KNT (and CHHKS for $\pi^+\pi^-$ & $\pi^+\pi^-\pi^0$) Central value from simple average; BABAR-KLOE tension & correlations between channels from DHMZ; Max(DHMZ & KNT uncertainties) in each channel

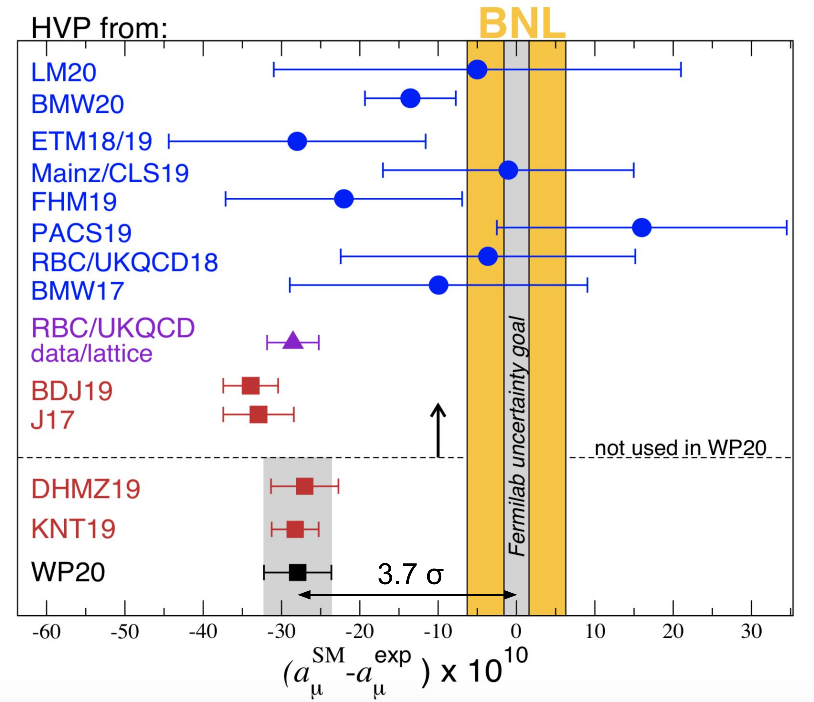
→ HLbL also has an important uncertainty

→ Lattice QCD (+QED) results become more and more interesting; *Precision of BMW20* (*to be cross-checked* by other lattice groups) became *similar to the one of dispersive approaches*; *Good agreement* using Euclidean time windows (related to HVP with suppression of very low and high energies) for which various groups achieved similar precision; *If BMW20 result is fully confirmed, the difference w.r.t. dispersive results* to be understood.

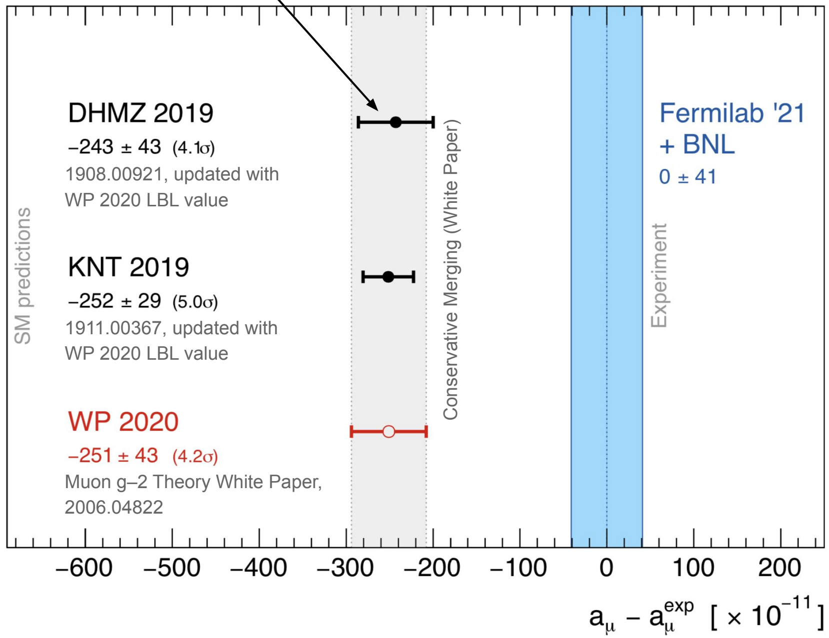
→ A tension between the BNL measurement and the reference SM prediction: ~ 3.7 σ (~ 4.2 σ including FNAL)

→ Tension significantly smaller when using BMW20 for the LO HVP

Status of a_μ before/with 1st Fermilab result



Important to account for BABAR-KLOE diff. & inter-channel correlations

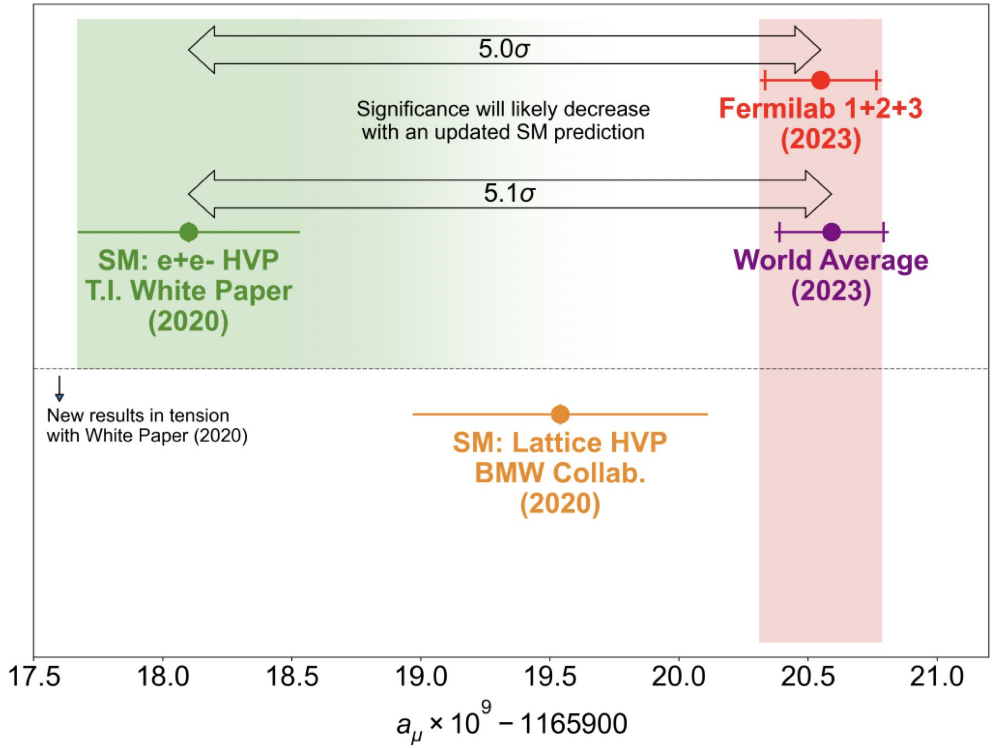


→ Caution about significance:
statistics-dominated measurement; prediction uncertainty limited by non-Gaussian systematic effects

→ Nevertheless, large discrepancy between measurement and reference SM prediction
(to be significantly improved in view of the forthcoming updates of the Fermilab measurement)

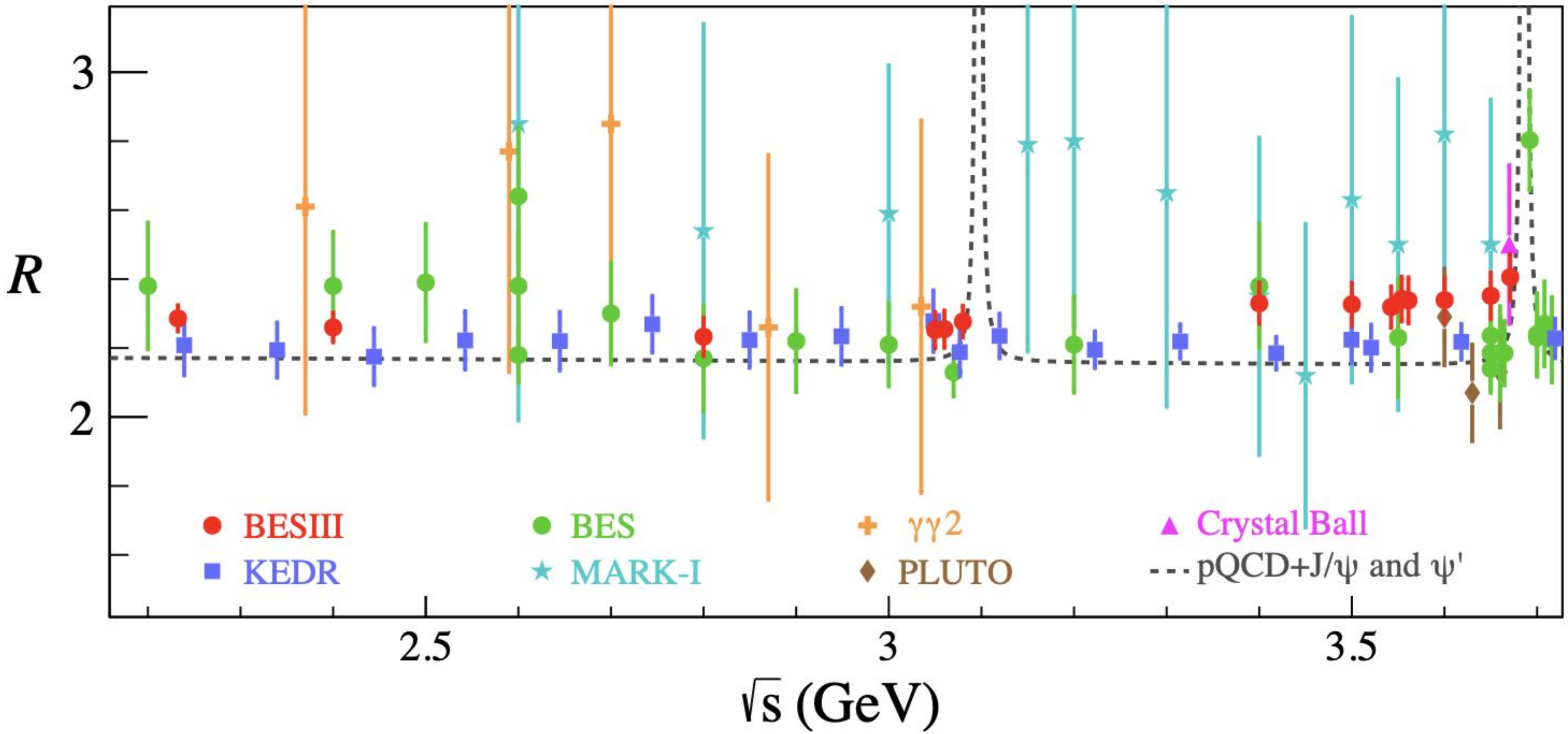
→ Tension significantly smaller when using BMW20 for the LO HVP (TBC by other lattice groups), *not* incompatible with the EW fit (*see below*)

Status of a_μ @ publication of 2nd Fermilab result



Comparison of inclusive measurements with pQCD

arXiv:2112.11728



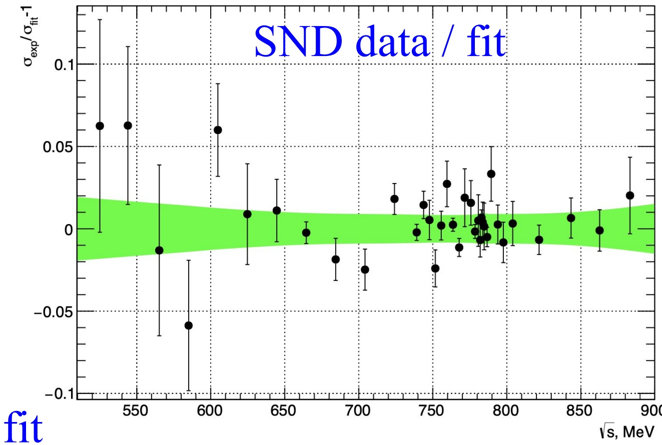
→ BES III results to be included: ~tension with pQCD and with KEDR 16

→ Another example of "*uncertainties on the uncertainties*" / *systematic effects to be understood* at the level of precision that is claimed

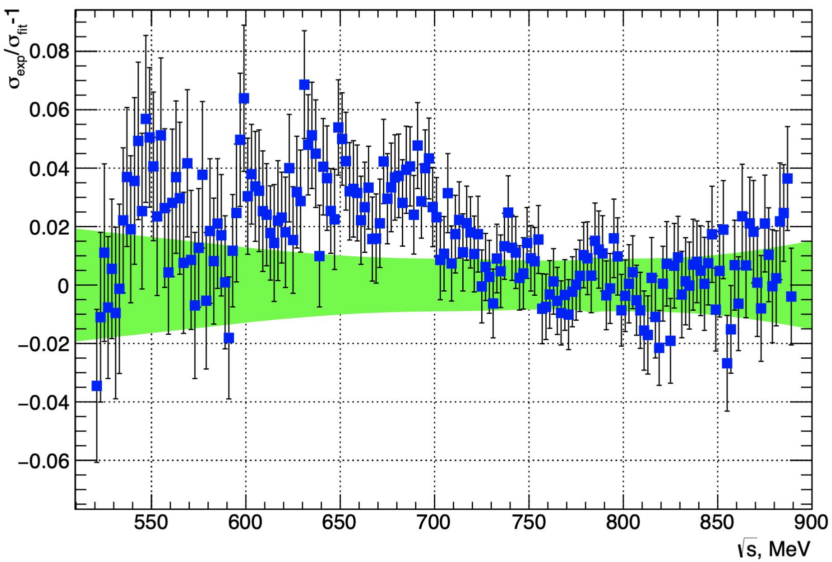
Back

Comparison of SND measurement with BABAR and KLOE

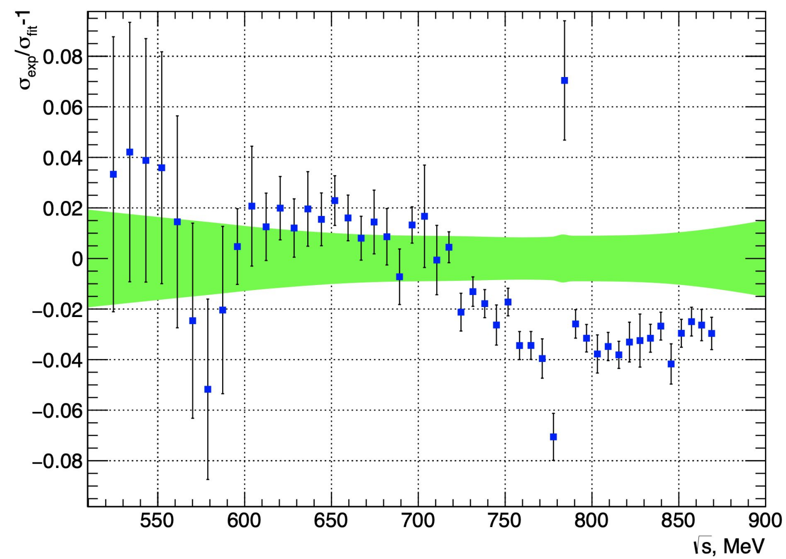
2004.00263



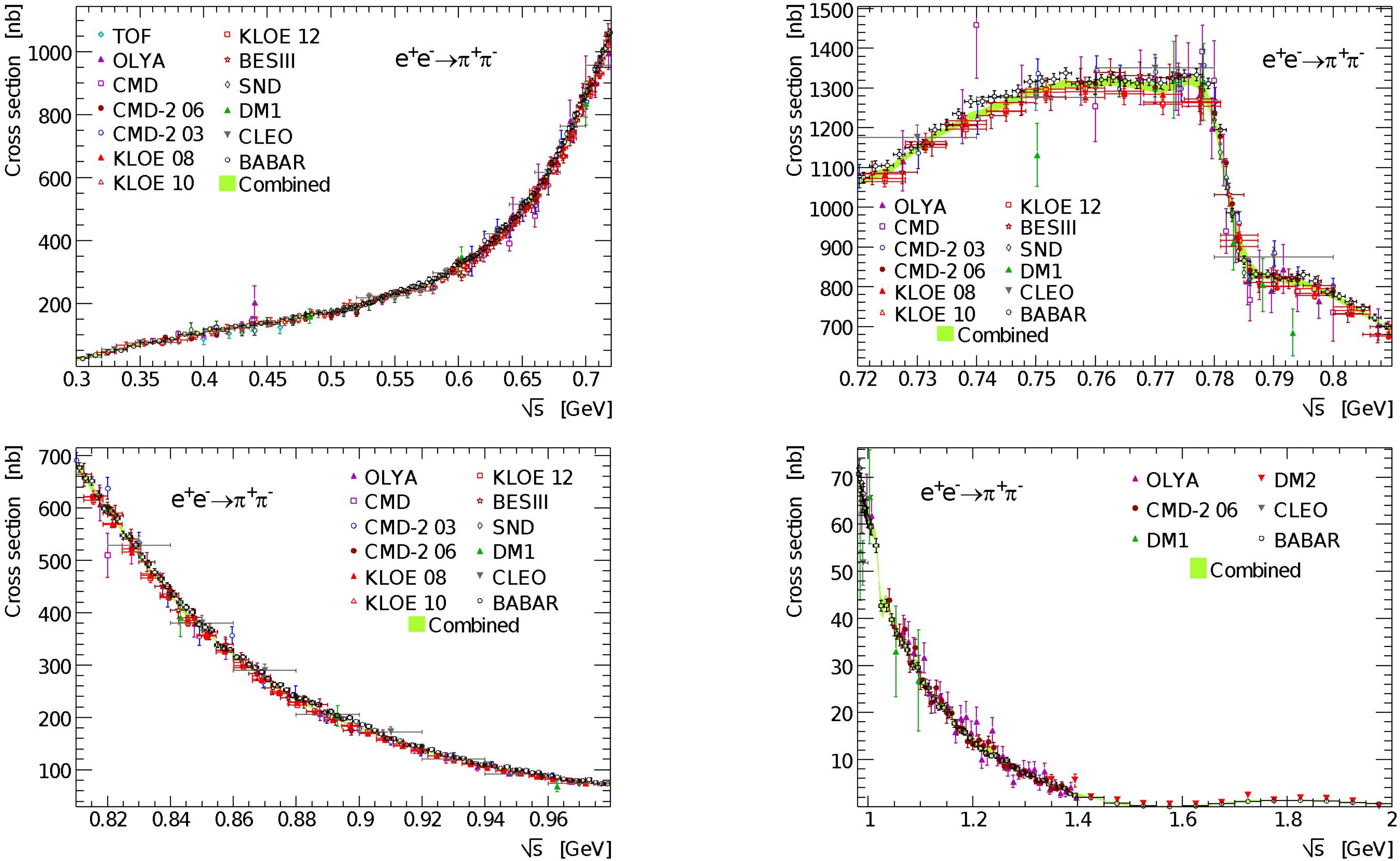
BABAR / SND fit



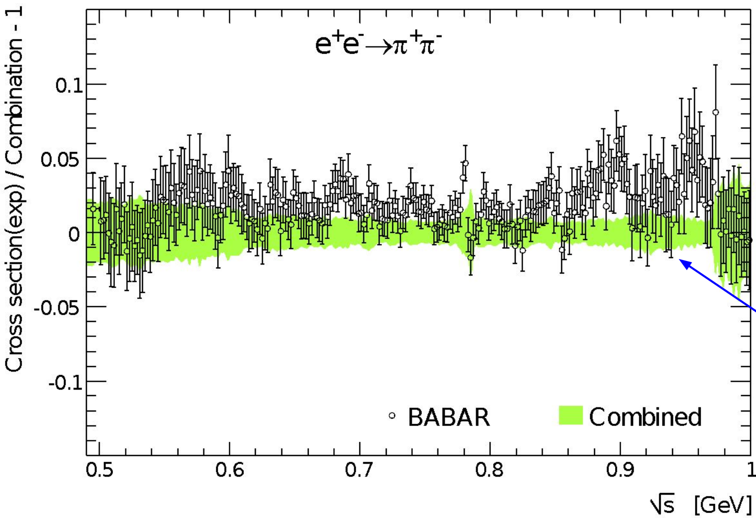
KLOE / SND fit

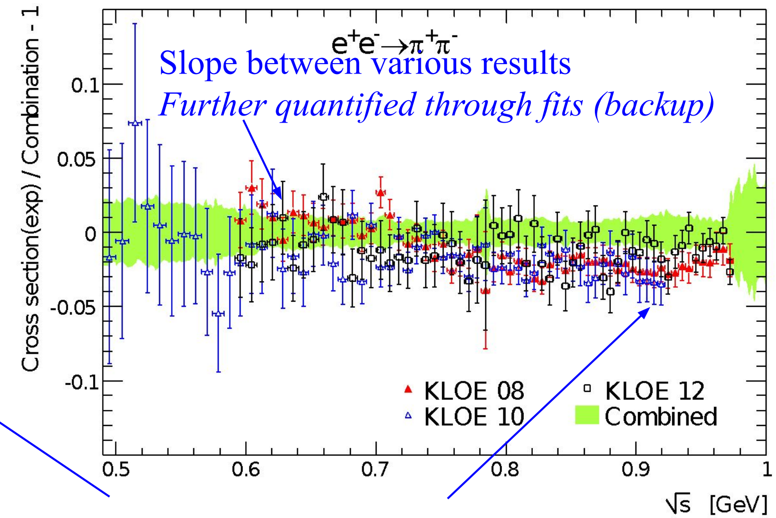


Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel (DHMZ '19)



More on the combination for the $e^+e^- \to \pi^+\pi^-$ channel (DHMZ '19)



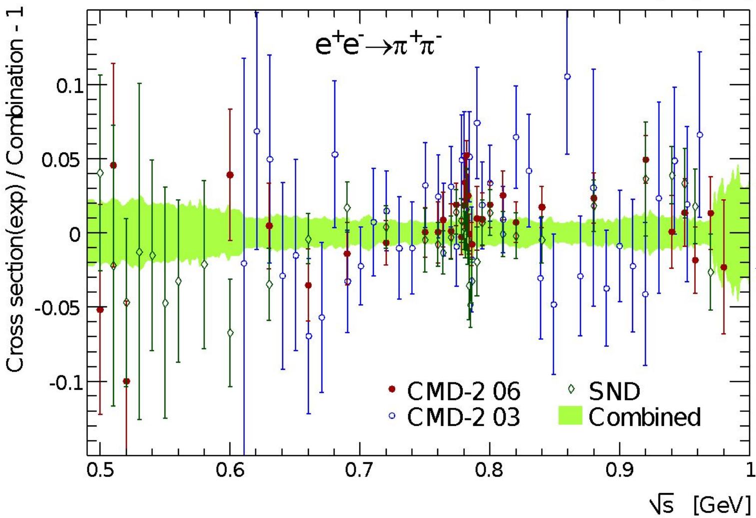


Slope between various results

Further quantified through fits (backup)

Local tension & systematic trends

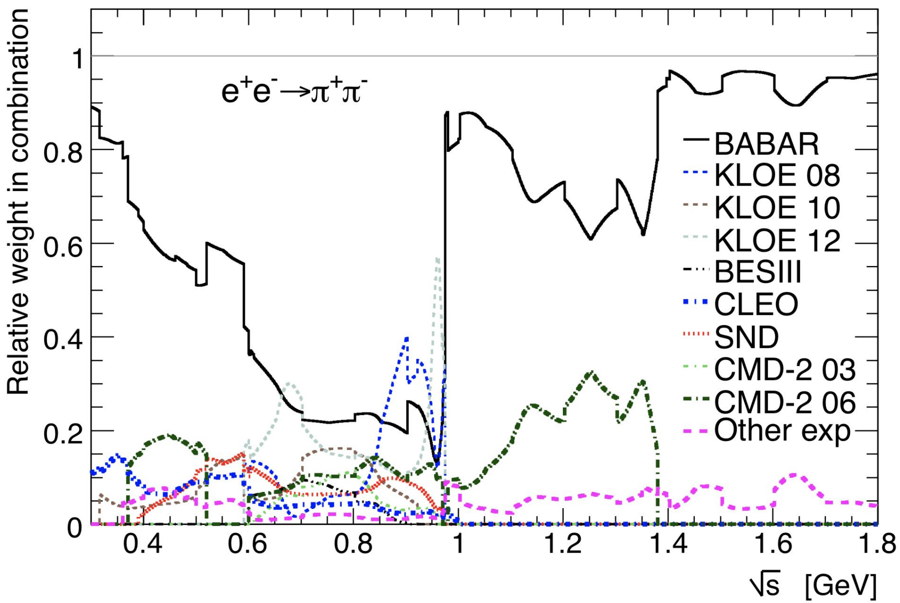
Indication of "uncertainties on uncertainties" (i.e. unaccounted biases)

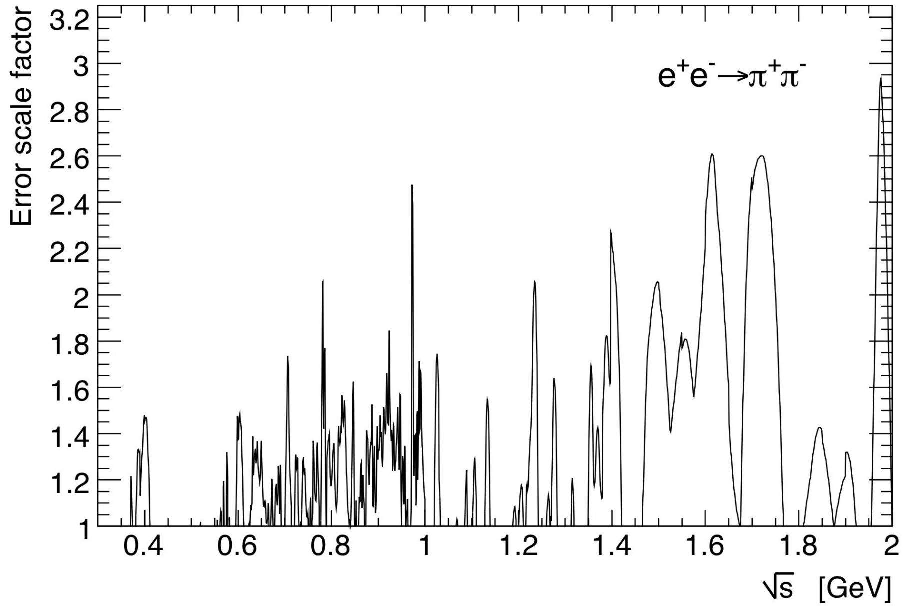


Other experiments not yet precise enough to discriminate

(see however update from SND: ~significant tension with KLOE above 720 MeV)

Combining the $e^+e^- \to \pi^+\pi^-$ data: weights and tension (DHMZ '19)





Improving a_μ through fits for the $e^+e^- \to \pi^+\pi^-$ channel (*Since 2019*)

→ Fit bare form-factor using 6 param. model based on *analyticity* and *unitarity*

$$|F_\pi^0|^2 = |R(s) \times J(s)|^2$$

$$R(s) = 1 + \alpha_V s + \frac{\kappa s}{m_\omega^2 - s - i m_\omega \Gamma_\omega}$$

(1611.09359, C. Hanhart et al.)

$$J(s) = e^{1-\frac{\delta_1(s_0)}{\pi}} \left(1 - \frac{s}{s_0}\right)^{\left[1-\frac{\delta_1(s_0)}{\pi}\right]\frac{s_0}{s}} \left(1 - \frac{s}{s_0}\right)^{-1} e^{\frac{s}{\pi}\int_{4m_\pi^2}^{s_0} dt \frac{\delta_1(t)}{t(t-s)}}$$

Omnès integral

(hep-ph/0402285, F.J. Yndurain et al.)

$$\cot \delta_1(s) = \frac{\sqrt{s}}{2k^3}\left(m_\rho^2 - s\right)\left[\frac{2m_\pi^3}{m_\rho^2\sqrt{s}} + B_0 + B_1\omega(s)\right]$$

$$k = \frac{\sqrt{s - 4m_\pi^2}}{2}$$

(1102.2183, F.J. Yndurain et al.)

$$\omega(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}} \qquad \sqrt{s_0} = 1.05 \text{ GeV}$$

→ Conservative χ^2 (diagonal matrix) & local rescaling of input uncertainties

→ Full propagation of uncertainties & correlations using pseudo-experiments

DHMZ - 1908.00921

Back

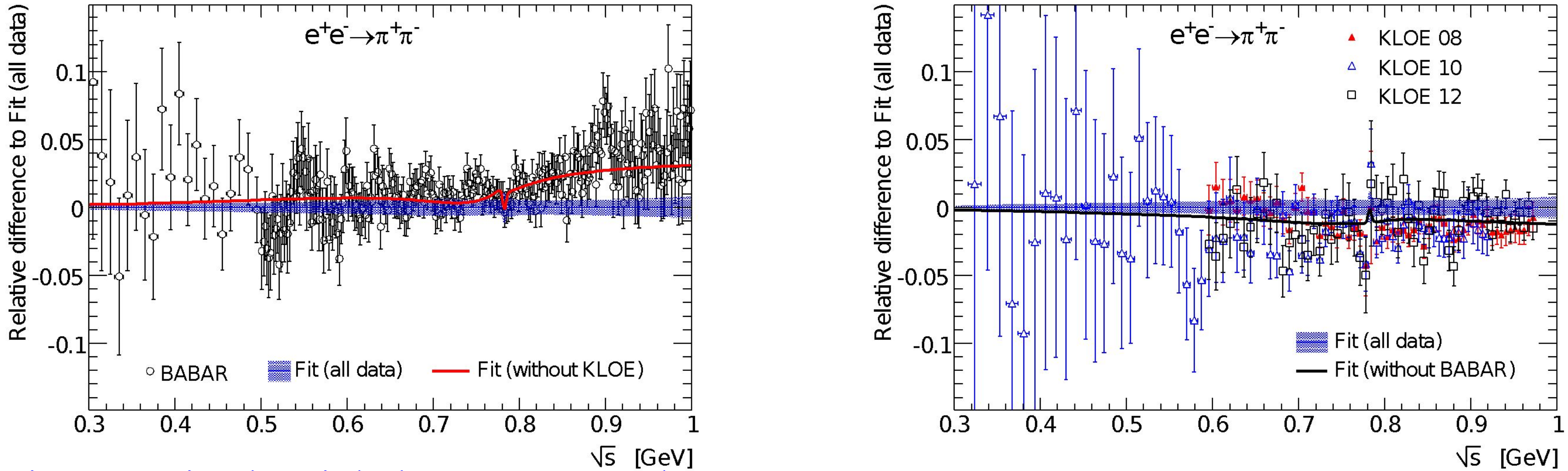
Fit parameters, uncertainties and correlations $e^+e^- \to \pi^+\pi^-$

	α_V	$\kappa[10^{-4}]$	B_0	B_1	m_ρ [MeV]	m_ω [MeV]
α_V	0.133 ± 0.020	0.52	−0.45	−0.97	0.90	−0.25
$\kappa[10^{-4}]$		21.6 ± 0.5	−0.33	−0.57	0.64	−0.08
B_0			1.040 ± 0.003	0.40	−0.40	0.29
B_1				-0.13 ± 0.11	−0.96	0.20
m_ρ [MeV]					774.5 ± 0.8	−0.17
m_ω [MeV]						782.0 ± 0.1

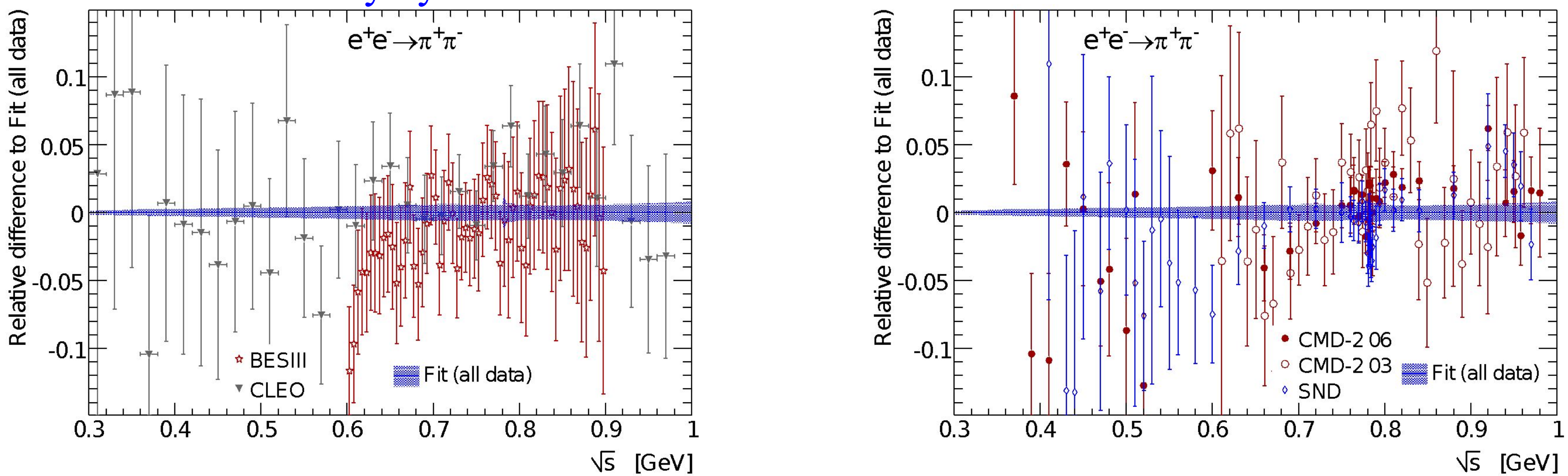
→ κ corresponds to a Br ($\omega \to \pi^+\pi^-$) of $(2.09 \pm 0.09) \cdot 10^{-2}$, in agreement with the result extracted from the fit of arXiv:1810.00007, $(1.95 \pm 0.08) \cdot 10^{-2}$. Both values disagree with the PDG average $(1.51 \pm 0.12) \cdot 10^{-2}$, dominated by the result of arXiv:1611.09359 which uses fits to essentially the same data.

→ The fitted ω mass is found to be lower than the PDG average obtained from 3π decays by $(0.65 \pm 0.12 \pm 0.12_{PDG})$ MeV, in agreement with previous fits of the $\rho - \omega$ interference in the 2π spectrum (see e.g. arXiv:1205.2228 and arXiv:1810.00007).

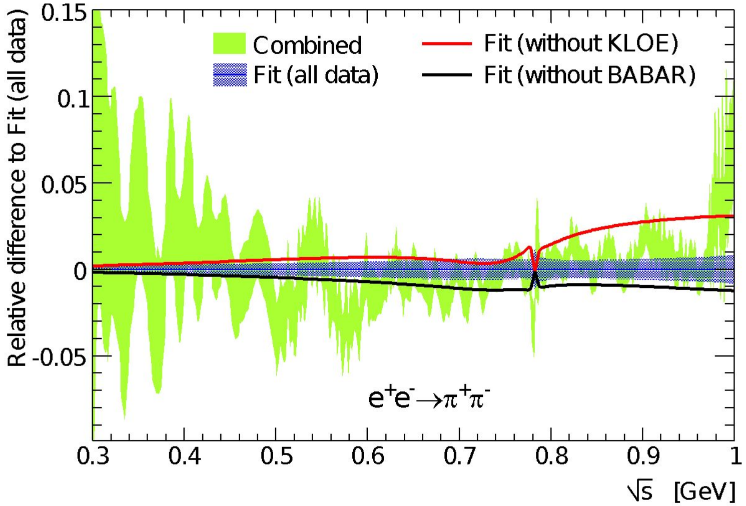
Fit performed up to 1 GeV: comparison with data



→ Fit constrained mainly by BABAR and KLOE measurements



Fit performed up to 1 GeV, Result used up to 0.6 GeV



→ Use fit only below 0.6 GeV for a_μ integral:

- where data is less precise and scarce
- less impacted by potential uncertainties of inelastic effects

√s range [GeV]	a_μ^{had} [10^{-10}] Fit	a_μ^{had} [10^{-10}] Data Integration
0.3 - 0.6	$109.80 \pm 0.37_{exp} \pm 0.36_{para*}$	$109.6 \pm 1.0_{exp}$

→ The difference 0.2 ± 0.9 (72% correlation accounted for)

→ The fit improves the precision by a factor ~2

(*) Parameter uncertainty corresponds to variations with/without the B_1 term in the phase shift formula and $\sqrt{s_0}$ varied from 1.05 GeV to 1.3 GeV (absolute values summed linearly), *checked to be statistically significant*

Combined results: Fit [<0.6GeV] + Data[0.6-1.8GeV]

→ Full uncertainty propagation using the same pseudo-experiments as for the spline-based combination: 62% correlation among the two contributions

$\sqrt{s}$ range [GeV]	a_μ^{had} $[10^{-10}]$ All data	a_μ^{had} $[10^{-10}]$ All but BABAR	a_μ^{had} $[10^{-10}]$ All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{total}$	$505.0 \pm 2.1_{total}$	$510.6 \pm 2.2_{total}$

→ The difference "All but BABAR" and "All but KLOE" = 5.6, to be compared with 1.9 uncertainty with "All data"

- The local error inflation is not sufficient to amplify the uncertainty
- Global tension (normalisation/shape) not previously accounted for
- Potential underestimated uncertainty in at least one of the measurements?
- Other measurements not precise enough to discriminate BABAR / KLOE

→ Given the fact we do not know which dataset is problematic, we decide to:

- Add half of the discrepancy (2.8×10^{-10}) as an uncertainty (corrected local PDG inflation to avoid double counting)
- Take ("All but BABAR" + "All but KLOE") / 2 as central value

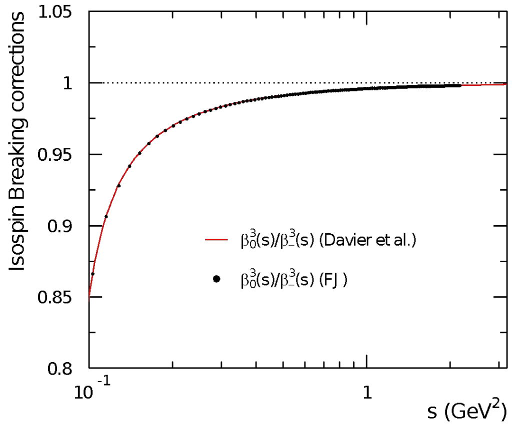
Channel	$a_\mu^{\mathrm{had,LO}}$ $[10^{-10}]$	$\Delta\alpha_{\mathrm{had}}(m_Z^2)$ $[10^{-4}]$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm \boxed{3.23} \pm 0.55$	$34.50 \pm 0.06 \pm \boxed{0.20} \pm 0.04$

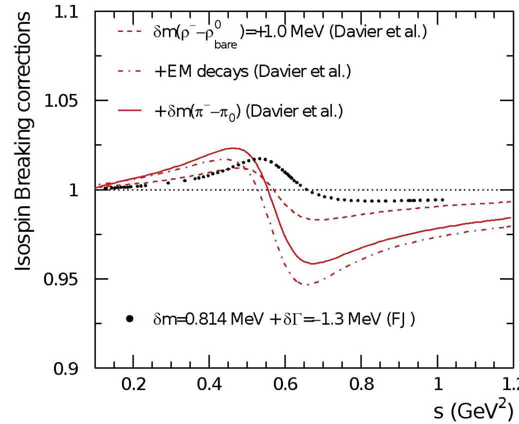
→ *Potential precision improvement* for a_μ ; less important for $\Delta\alpha_{had}(m_Z^2)$, BABAR-KLOE syst. ~16% of total uncertainty

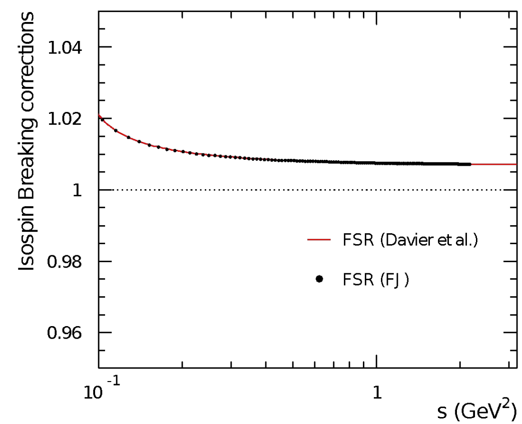
Comparison with IB-corrected τ data

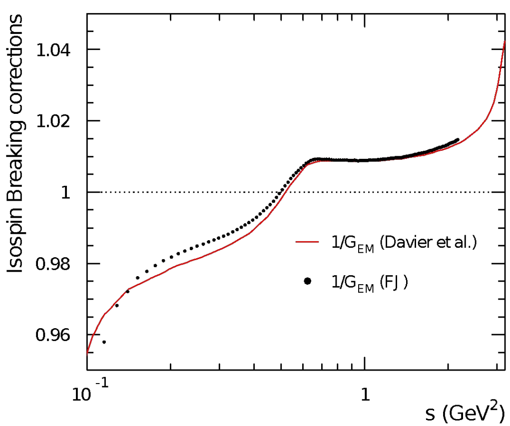
$$v_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{X^-}}{\mathcal{B}_e}\frac{1}{N_X}\frac{dN_X}{ds}\times\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{R_{\rm IB}(s)}{S_{\rm EW}} \qquad R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)}\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

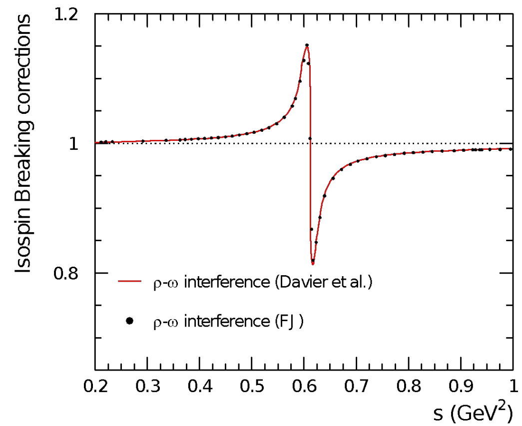
→ Comparing corrections used by Davier et al. with the ones by F. Jegerlehner







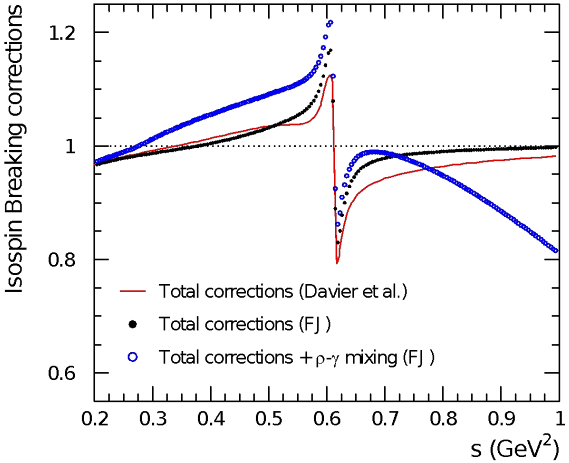


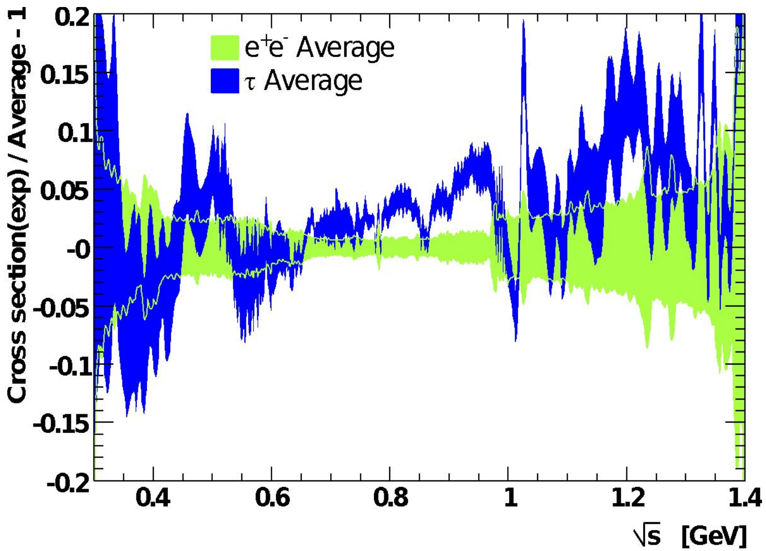


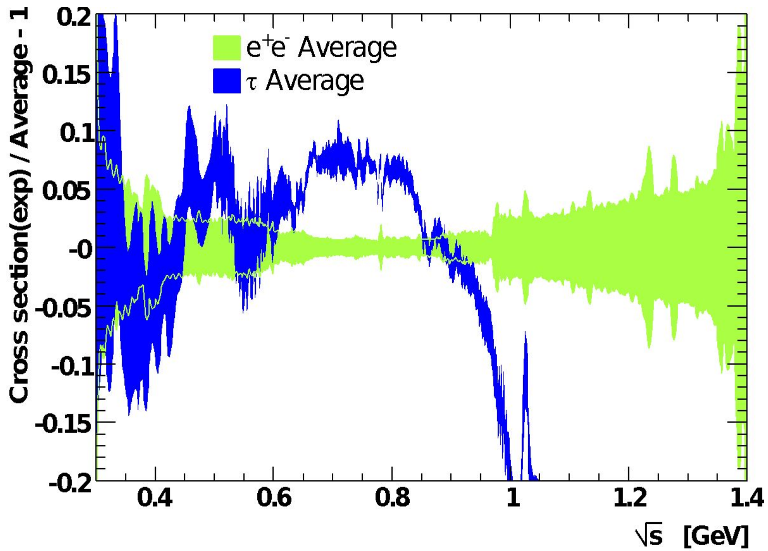
Plots by Z. Zhang, based on private communication with F. Jegerlehner

→ for a_μ, $e^+e^- - \tau$ difference of 2.2 σ (Davier et al.)

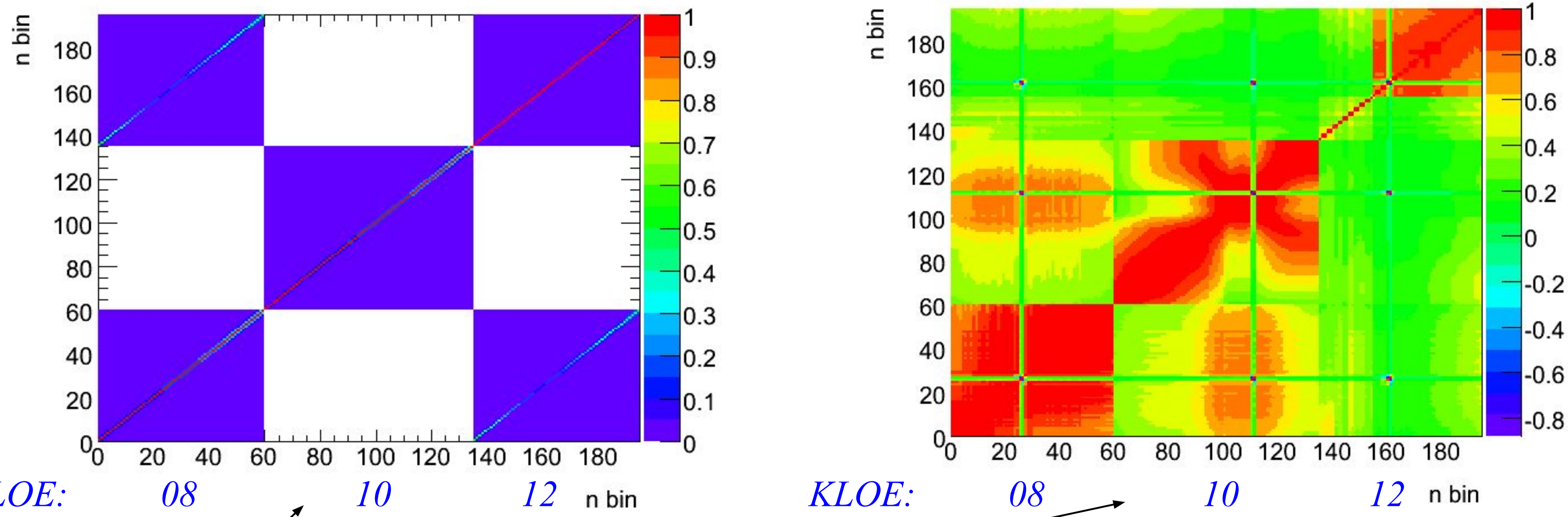
→ the ρ–γ mixing correction proposed in arXiv:1101.2872 (FJ) seems to over-estimate the $e^+e^- - \tau$ difference





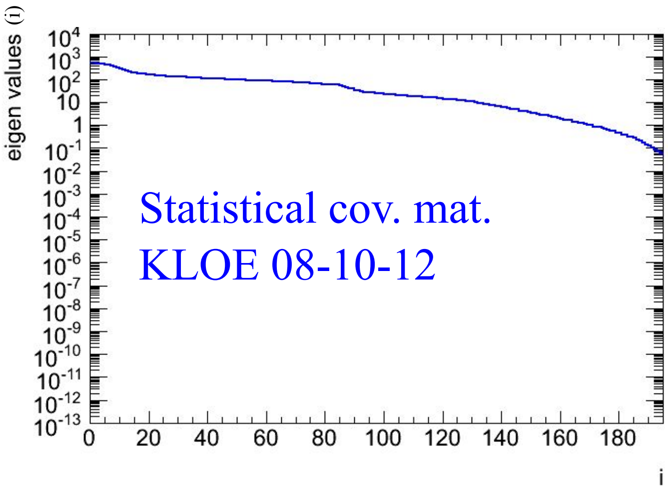


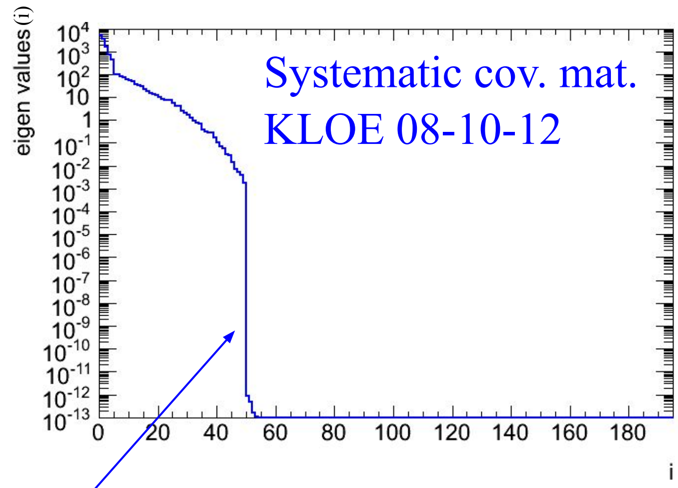
Treatment of the KLOE correlation matrices



→ Statistical and systematic correlation matrices among the 3 measurements

Treatment of the KLOE data – eigenvector decomposition





$$C = S \cdot D \cdot S^T$$

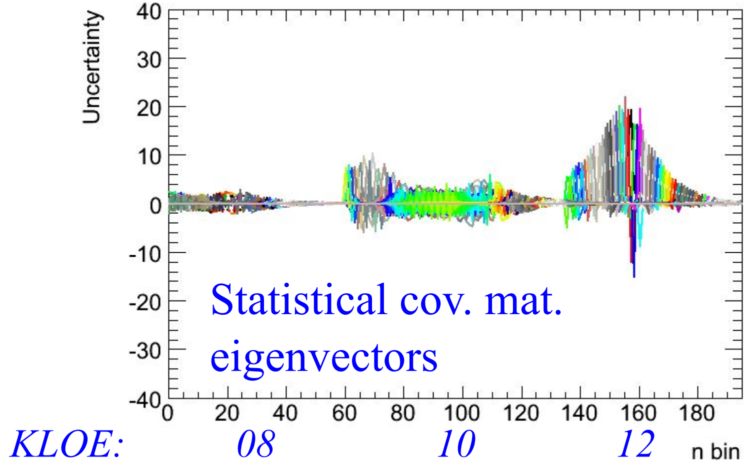
$$D = \begin{pmatrix} \ddots & 0 & 0 \\ 0 & \sigma_i^2 & 0 \\ 0 & 0 & \ddots \end{pmatrix}$$

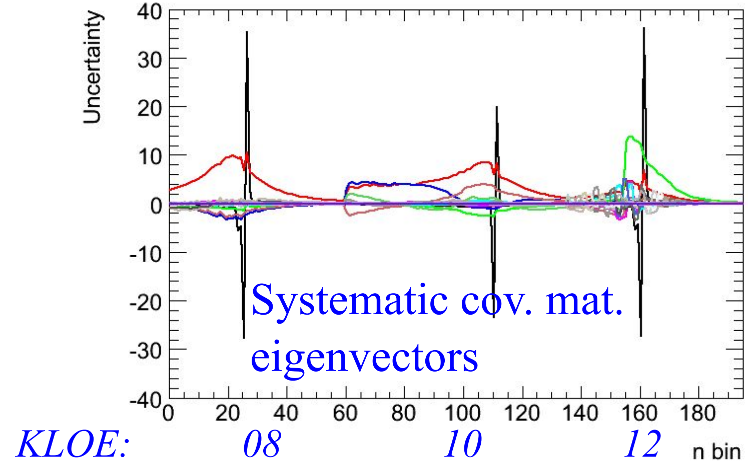
$$S = \begin{pmatrix} V_1 & \dots & V_n \end{pmatrix}$$

→ "counting" the number of independent components (50) used to build the covariance matrix

→ Problem of negative eigenvalues for previous systematic covariance matrix solved
(informed KLOE collaboration about the problem in summer 2016)

Treatment of the KLOE data – eigenvector decomposition





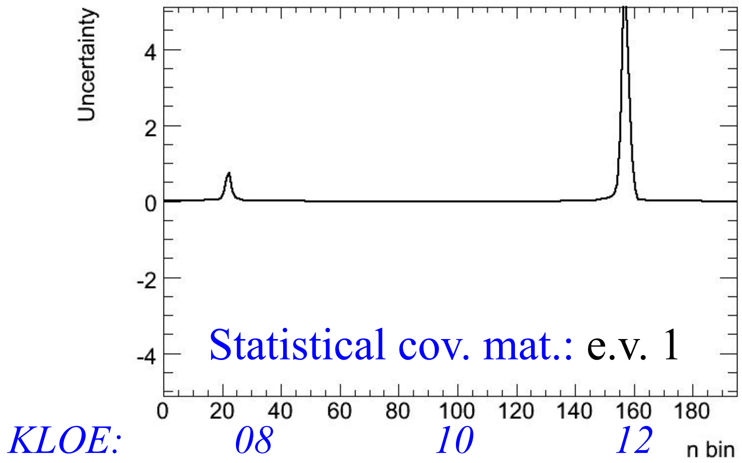
→ Each normalized eigenvector (σ_i*V_i) treated as an uncertainty fully correlated between the bins

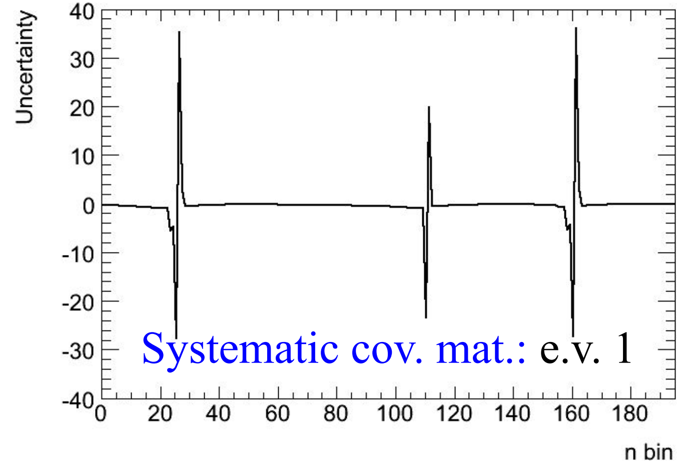
→ All these uncertainties are independent between each-other

$$C = \sum_{i=1}^{N_{bins}} \sigma_i^2 \cdot C(V_i)$$

→ Checked exact matching with the original matrices + with all a_μ integrals and uncertainties published by KLOE

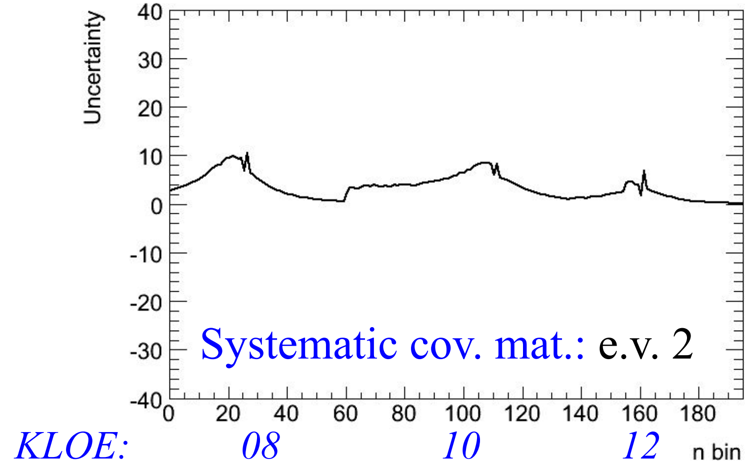
Treatment of the KLOE data – eigenvector decomposition



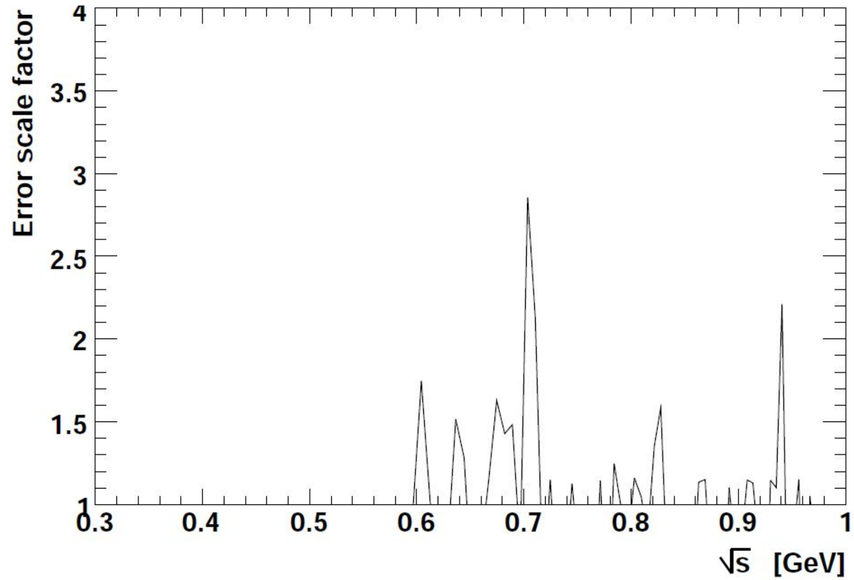


→ Eigenvectors carry the general features of the correlations:

- long-range for systematics
- ~short-range for statistical uncertainties + correlations between KLOE 08 & 12



Local comparison of the 3 KLOE measurements



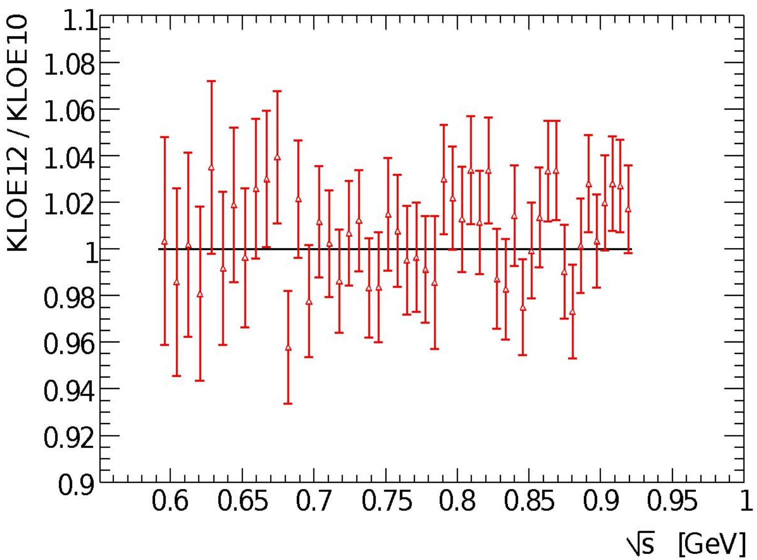
→ Local χ^2 /ndof test of the local compatibility between KLOE 08 & 10 & 12, taking into account the correlations: some tensions observed

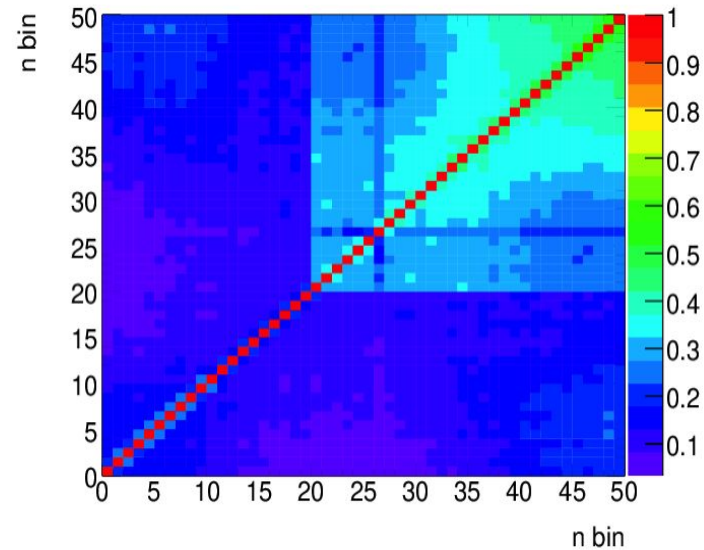
→ Does not probe general trends of the difference between the measurements (e.g. slopes in the ratio)

Ratios between measurements

→ Compute ratio between pairs of KLOE measurements

→ Full propagation of uncertainties and correlations using pseudo-experiments (agreement with analytical linear uncertainty propagation)





→ Good agreement between KLOE 10 and KLOE 12

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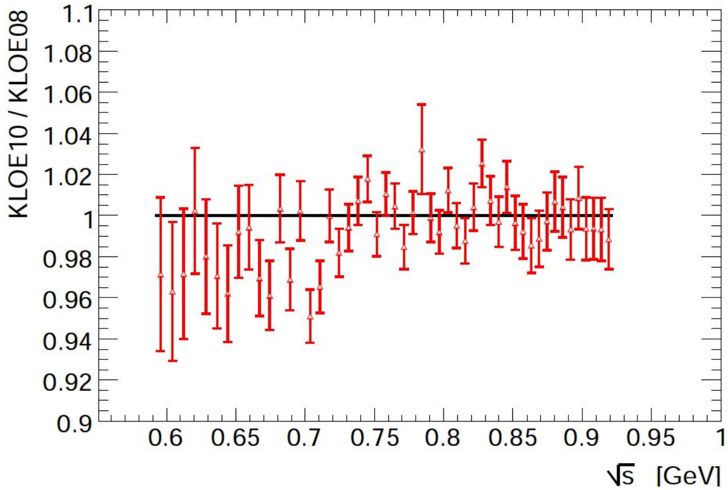
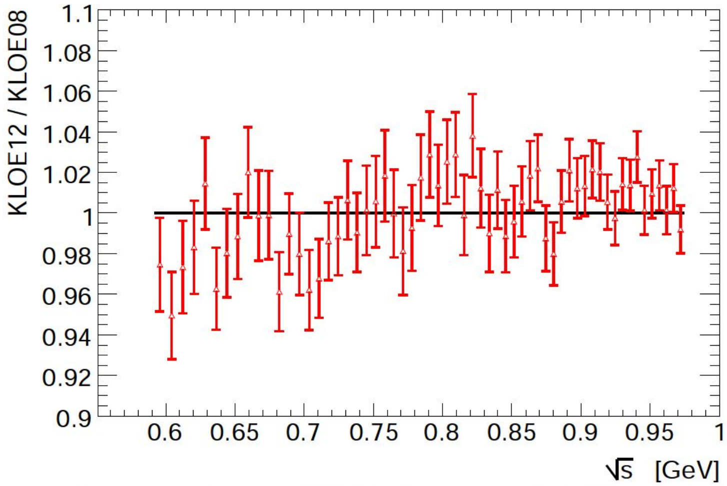
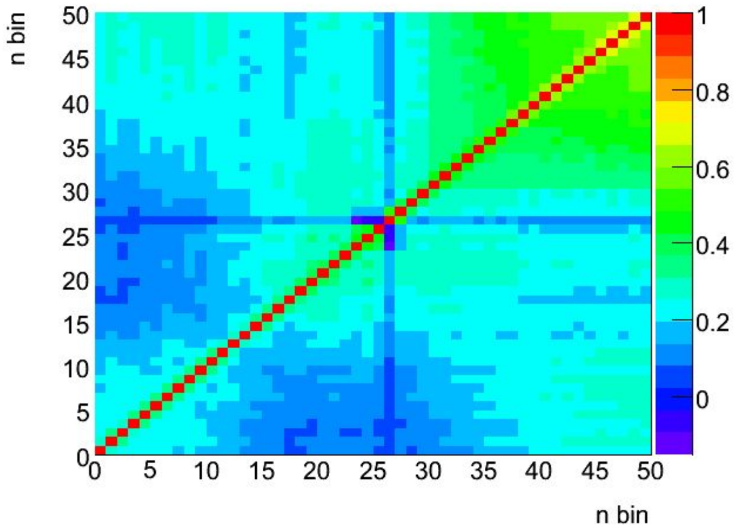
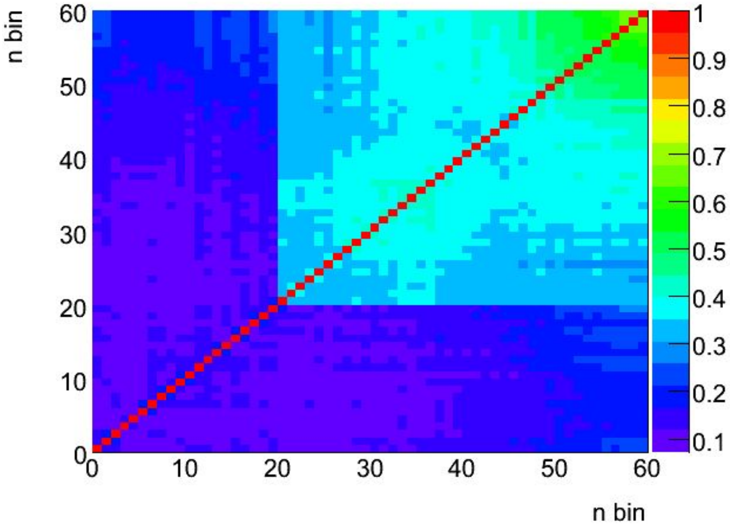
Ratios between measurements









Direct comparison of the 3 KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

KLOE 10 / KLOE 08

χ^2 [0.35;0.85] GeV^2 : 79.0 / 50(DOF)
p-value= 0.0056

χ^2 [0.35;0.58] GeV^2 : 46.2 / 23(DOF)
p-value= 0.0028

χ^2 [0.58;0.85] GeV^2 : 29.7 / 27(DOF)
p-value= 0.33

χ^2 [0.64;0.85] GeV^2 : 20.7 / 21(DOF)
p-value= 0.47

KLOE 12 / KLOE 08

χ^2 [0.35;0.95] GeV^2 : 73.7 / 60(DOF)
p-value= 0.11

χ^2 [0.35;0.58] GeV^2 : 21.8 / 23(DOF)
p-value= 0.53

χ^2 [0.35;0.64] GeV^2 : 27.5 / 29(DOF)
p-value= 0.55

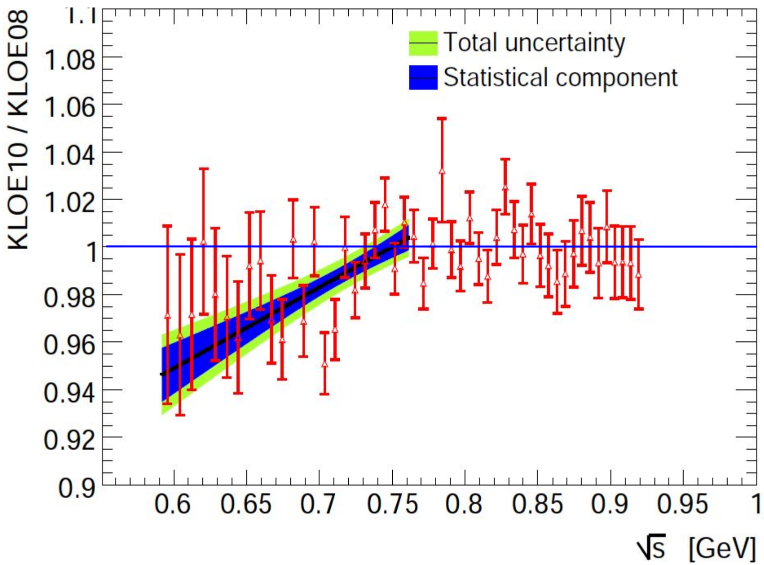
χ^2 [0.64;0.95] GeV^2 : 39.4 / 31(DOF)
p-value= 0.14

Quantitative comparisons of the KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: $\pm 1\sigma$ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit



Comparison with Unity:

χ^2 [0.35;0.85] GeV^2 : 79.0 / 50(DOF)

p-value= 0.0056

χ^2 [0.35;0.58] GeV^2 : 46.2 / 23(DOF)

p-value= 0.0028

χ^2 [$p0 + p1\sqrt{s}$]: 36.1 / 21(DOF)

p-value= 0.02

p0 : 0.745 ± 0.085

p1 : 0.341 ± 0.117

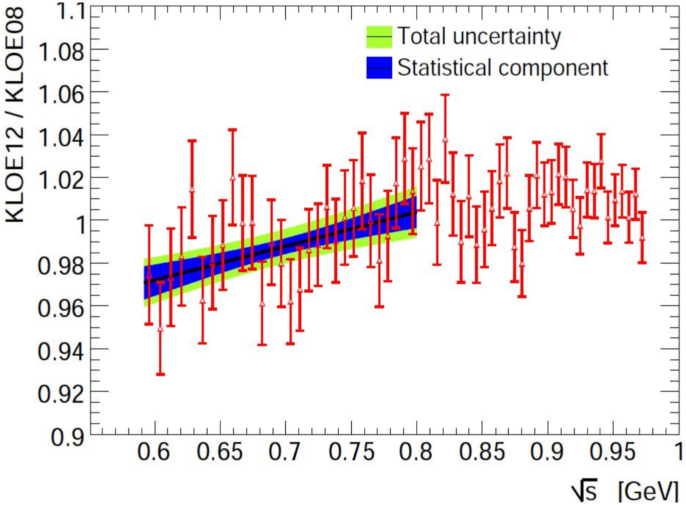
→ Significant shift & slope (~2.5-3σ) at low $\sqrt{s}$, no significant shift at high $\sqrt{s}$
Similar shift & slope for KLOE 12 / KLOE 08 (*see below*)

→ Should motivate conservative treatment of uncertainties and correlations in combination

Direct comparison of the 3 KLOE measurements

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: ±1σ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit

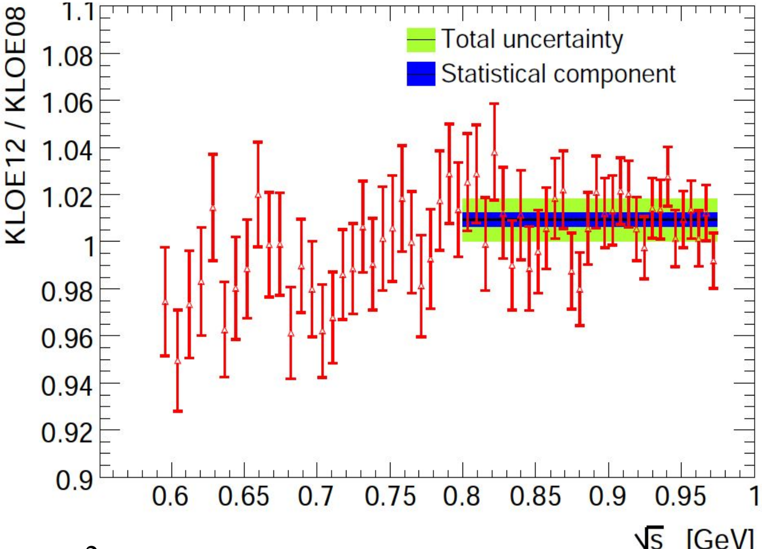


χ^2 [p0 + p1√s]: 20.7 / 27(DOF)

p-value= 0.80

p0 : 0.876 ± 0.056

p1 : 0.159 ± 0.081



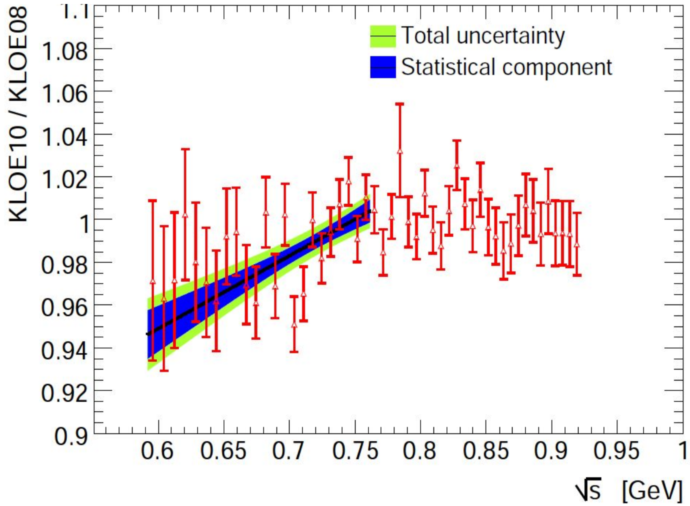
χ^2 [p0]: 38.4 / 30(DOF)

p-value= 0.14

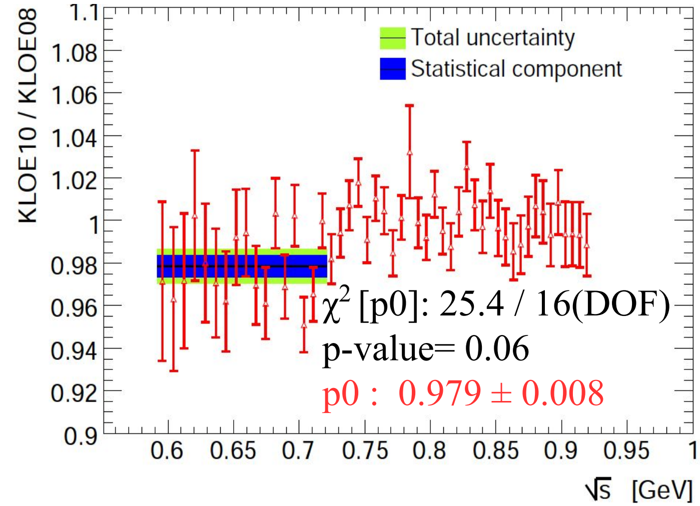
p0 : 1.009 ± 0.009

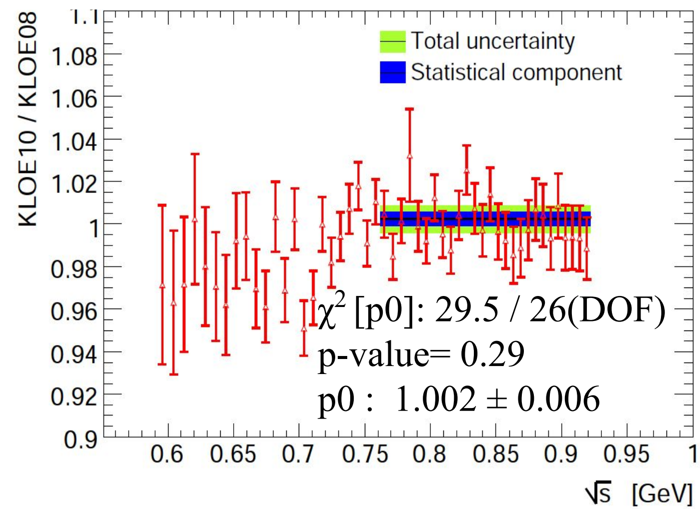
→ Significant shift and slope (~2σ) at low √s, no significant shift at high √s

Direct comparison of the 3 KLOE measurements



χ^2 [p0 + p1√s]: 36.1 / 21(DOF)
p-value= 0.02
p0 : 0.745 ± 0.085
p1 : 0.341 ± 0.117

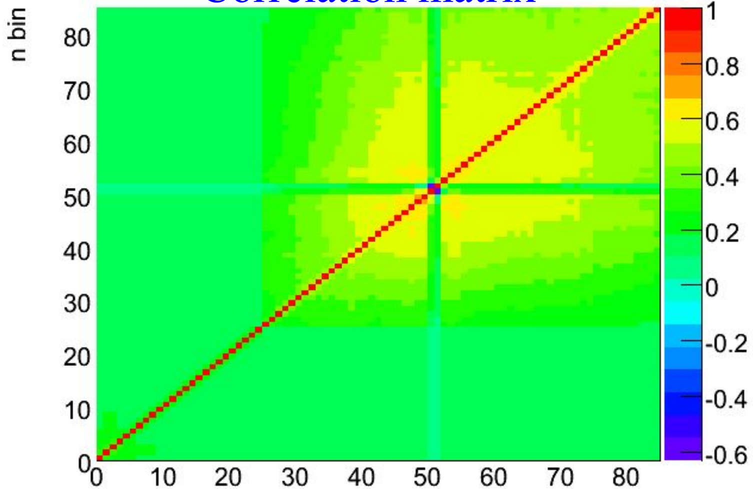


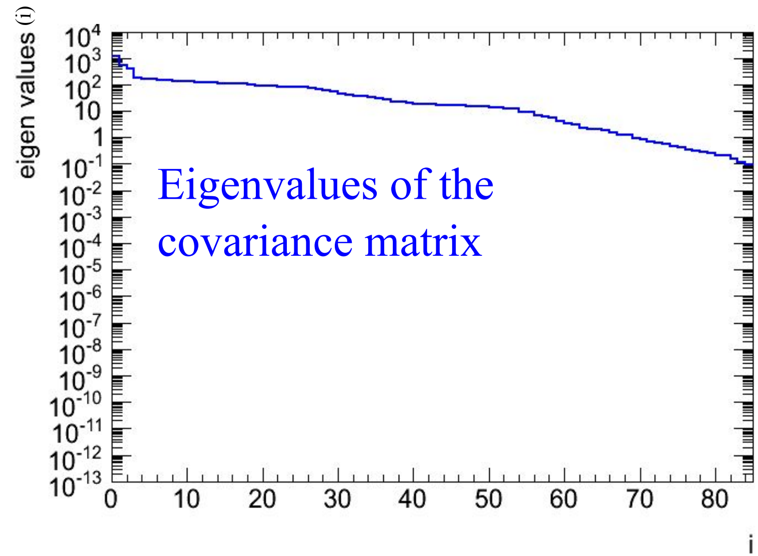


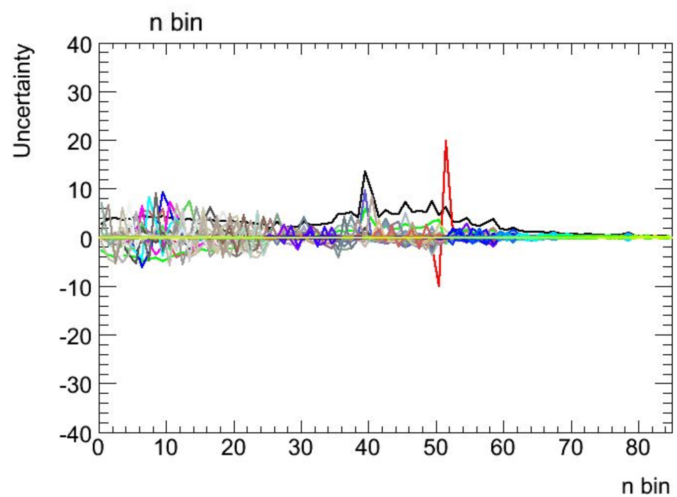
→ Significant shift and slope (~2.5-3σ) at low √s,
no significant shift at high √s

Treatment of the combined KLOE data

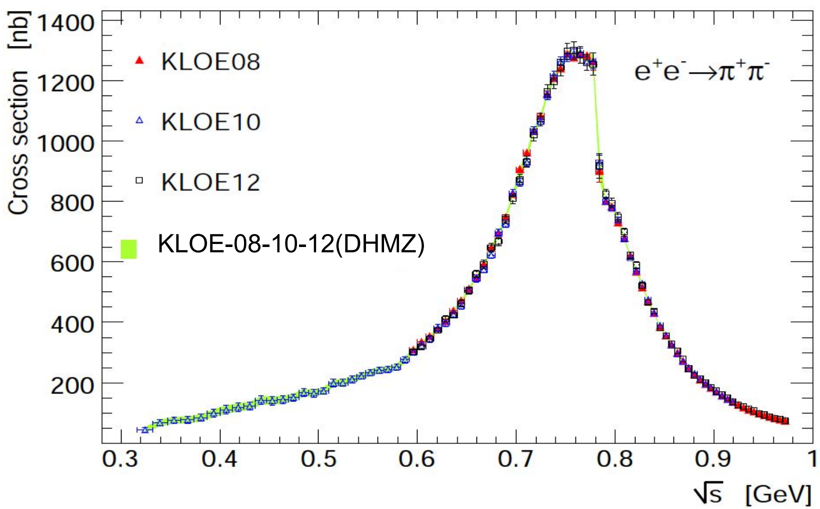
Correlation matrix

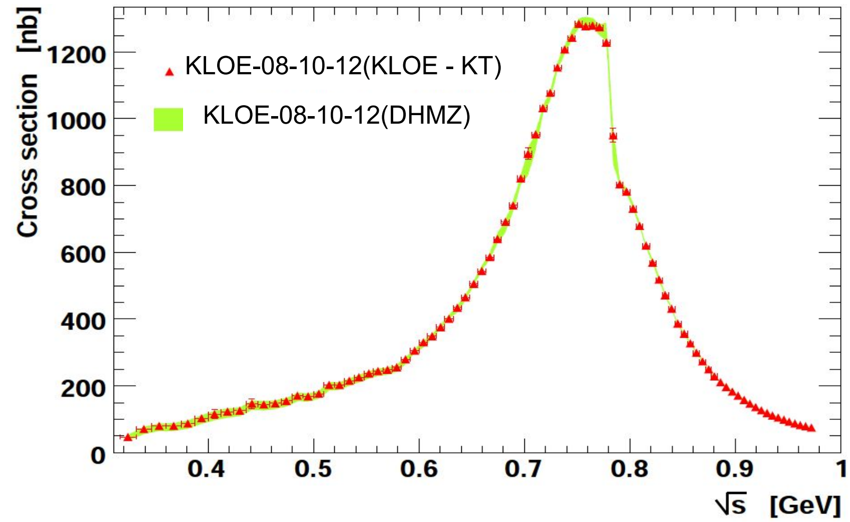


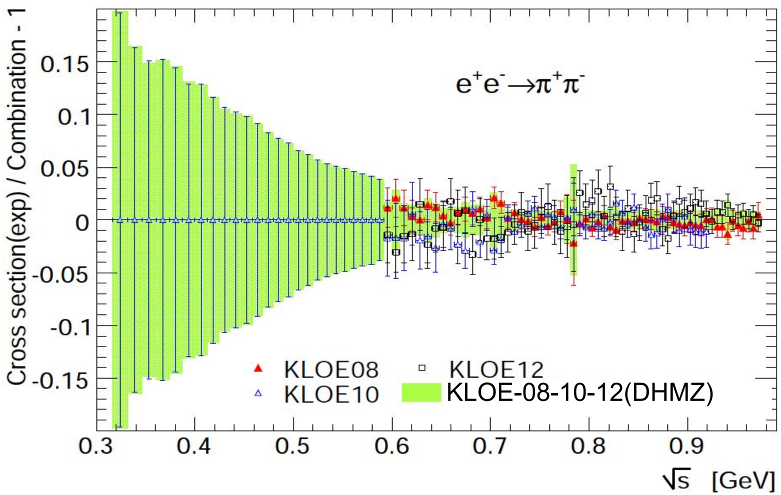


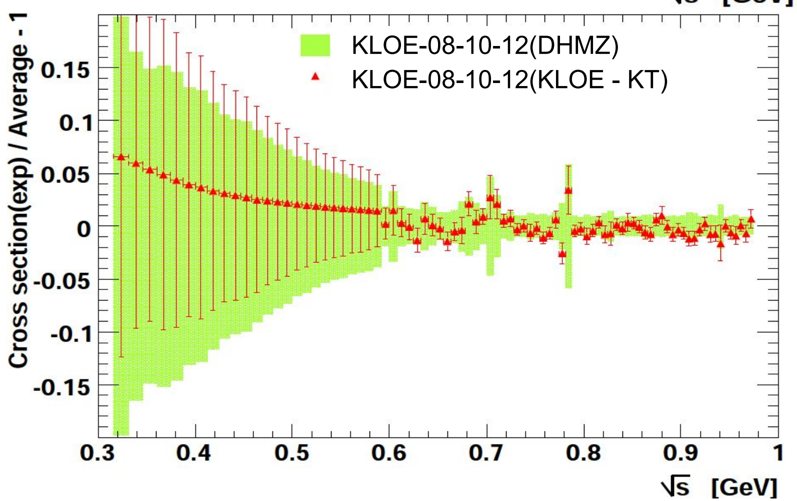


Combining the 3 KLOE measurements









Comparison of / consequences for combination methods

Analysis aspect	DHMZ	KNT
Blinding	Not necessary (No ad-hoc choices to make)	Included for upcoming update
Binning	Fine ($\leq$ 1 MeV) final binning for average and integrals. Large (O(100 MeV) or less) common binning @ intermediate step: compare statistics of experiments coherently for deriving weights in fine bins.	Re-bin data into "clusters". Scans over cluster configurations for optimisation.
Closure test	Using model for spectrum: negligible bias. (since 2009)	Not performed
Additional constraints	Analyticity constraints for 2π channel.	None
Fitting	χ^2 minimisation with correlated uncertainties incorporated locally (in fine & large bins), for deriving weights. Full propagation of uncertainties & correlations.	χ^2 minimisation with correlated uncertainties incorporated globally.
Integration / interpolation	Av. of quadratic splines (3rd order polynomial), integral preservation in bins of measurements. Analyticity-based function for 2π ($<$ 0.6 GeV).	Trapezoidal for continuum, quintic for resonances.
Uncertainty inflation	Local χ^2 uncertainty inflation. (since 2009) Extra BABAR-KLOE systematic. (since 2019)	Local χ^2 uncertainty inflation. (adopted since 2017)
Inter-channel correlations	Taken into account. (since 2010)	Not included.
Missing channels	Estimated based on isospin symmetry. (since 1997 - ADH)	Adopted in subsequent updates

→ Large DHMZ/KNT differences for the resulting uncertainties, as well as for the shapes of the combined spectra

WP TI	DHMZ19	KNT19
$a_\mu^{\text{HVP, LO}} \times 10^{10}$	694.0(4.0)	692.8(2.4)

→ CHS approach for 2π and 3π: Analyticity and global χ^2 fit (See talk by Peter Stoffer)

$a_\mu^{\pi\pi}$ contribution [0.28; 1.8] GeV – spline-based (2018)

→ Updated result:

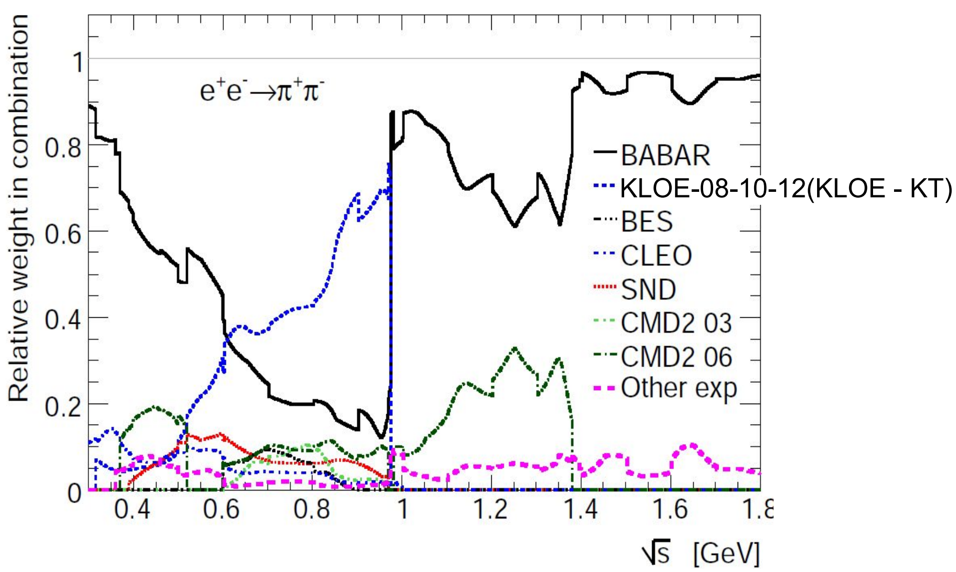
506.70 ± 2.32 ($\pm$ 1.01 (stat.) $\pm$ 2.08 (syst.)) [10^{-10}]

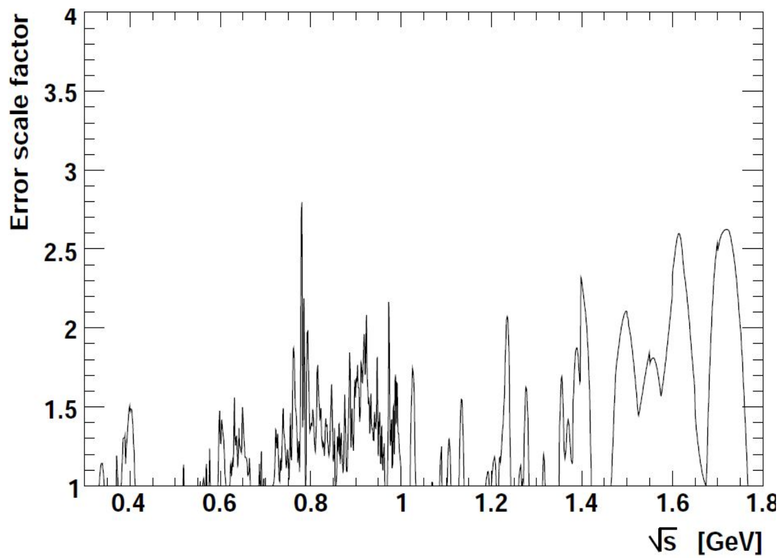
(after uncertainty enhancement by ~14% caused by the tension between inputs, taken into account through a local rescaling)

Total uncertainty: 5.9 (2003) → 2.8 (2011) → 2.6 (2017) → 2.3 (2018)

$a_\mu^{\pi\pi}$ contribution [0.28; 1.8] GeV – spline-based (2018)

→ with KLOE-08-10-12 (KLOE-KT) used as input: 506.55 ± 2.38 [10^{-10}]
(after uncertainty enhancement by 18% caused by the tension between inputs, taken into account through a local rescaling)
→ Compensation between uncertainty reduction for KLOE-08-10-12 (KLOE-KT), inducing a change of weights in DHMZ combination, and tension enhancement





Uncertainties on uncertainties and on correlations

Topic of general interest, in other fields too

1908.00921(DHMZ), 2006.04822(WP Theory Initiative)

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Two different approaches for combining (e^+e^-) data

DHMZ:

→ χ^2 computed locally (in each fine bin), taking into account correlations between measurements (see previous slides)

→ Used to determine the weights on the measurements in the combination and their level of agreement

→ Uncertainties and correlations propagated using pseudo-experiments or ±1σ shifts of each uncertainty component

KNT:

→ χ^2 computed globally (for full mass range)

$$\chi_I^2 = \sum_{i=1}^{N_{\mathrm{tot}}} \sum_{j=1}^{N_{\mathrm{tot}}} \left(R_i^{(m)} - \mathcal{R}_m^{i,I}\right) \mathbf{C}_I^{-1}\left(i^{(m)}, j^{(n)}\right)\left(R_j^{(n)} - \mathcal{R}_n^{j,I}\right)$$

KNT (1802.02995)

$$\chi^2 = \sum_{i=1}^{195} \sum_{j=1}^{195} \left(\sigma^0_{\pi\pi(\gamma)}(i) - \bar{\sigma}^0_{\pi\pi(\gamma)}(m)\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right)\left(\sigma^0_{\pi\pi(\gamma)}(j) - \bar{\sigma}^0_{\pi\pi(\gamma)}(n)\right)$$

KLOE-KMT (1711.03085)

→ relies on description of correlations on long ranges

→ *One of the main sources of differences for the uncertainty on* a_μ

Evaluation of uncertainties and correlations (e^+e^-)

	$\sigma_{\pi\pi\gamma}$	$\sigma^0_{\pi\pi}$	F_π	$\Delta^{\pi\pi}a_\mu$
Reconstruction Filter	negligible			
Background subtraction	Tab. 1			0.3%
Trackmass	0.2%			
Pion cluster ID	negligible			
Tracking efficiency	0.3%			
Trigger efficiency	0.1%			
Acceptance	Tab. 2			0.2%
Unfolding	Tab. 3			negligible
L3 filter	0.1%			
$\sqrt{s}$ dependence of H	-	Tab. 4		0.2%
Luminosity	0.3%			
Experimental systematics				0.6%
FSR resummation	-	0.3%		
Radiator function H	-	0.5%		
Vacuum Polarization	-	0.1%	-	0.1%
Theory systematics				0.6%

$M^2_{\pi\pi}$ range (GeV^2)	Systematic error (%)
$0.35 \leq M^2_{\pi\pi} < 0.39$	0.6
$0.39 \leq M^2_{\pi\pi} < 0.43$	0.5
$0.43 \leq M^2_{\pi\pi} < 0.45$	0.4
$0.45 \leq M^2_{\pi\pi} < 0.49$	0.3
$0.49 \leq M^2_{\pi\pi} < 0.51$	0.2
$0.51 \leq M^2_{\pi\pi} < 0.64$	0.1
$0.64 \leq M^2_{\pi\pi} < 0.95$	-

KLOE 08 (0809.3950)

KLOE 10 (1006.5313)

	$\sigma_{\pi\pi\gamma}$	$\sigma^{bare}_{\pi\pi}$	$\lvert F_\pi\rvert^2$	$\Delta a^{\pi\pi}_\mu$
	threshold ; ρ-peak			(0.1 - 0.85 GeV^2)
Background Filter	0.5% ; 0.1%			negligible
Background subtraction	3.4% ; 0.1%			0.5%
$f_0 + \rho\pi$ bkg.	6.5% ; negl.			0.4%
Ω cut	1.4% ; negl.			0.2%
Trackmass cut	3.0% ; 0.2%			0.5%
π-e PID	0.3% ; negl.			negligible
Trigger	0.3% ; 0.2%			0.2%
Acceptance	1.9% ; 0.3%			0.5%
Unfolding	negl. ; 2.0%			negligible
Tracking	0.3%			
Software Trigger (L3)	0.1%			
Luminosity	0.3%			
Experimental syst.				1.0%
FSR treatment	-	7% ; negl.		0.8%
Radiator function H	-	0.5%		
Vacuum Polarization	-	Ref. [34]	-	0.1%
Theory syst.				0.9%

→ Systematics *evaluated* in ~wide mass ranges with sharp transitions

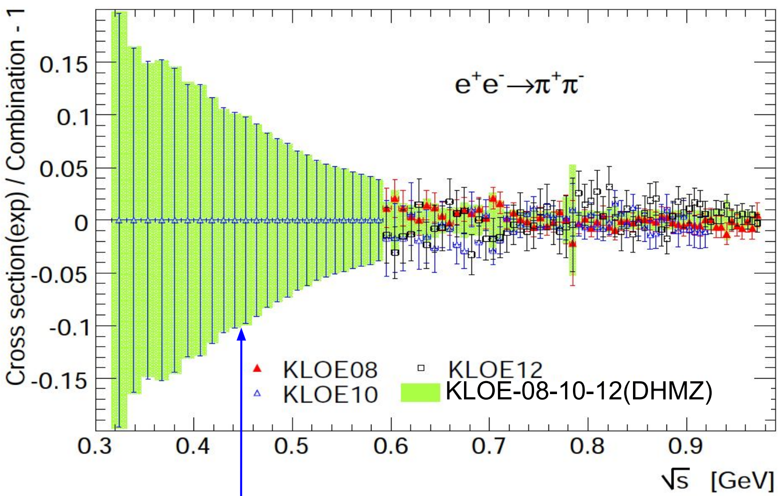
Evaluation of uncertainties and correlations (e^+e^-)

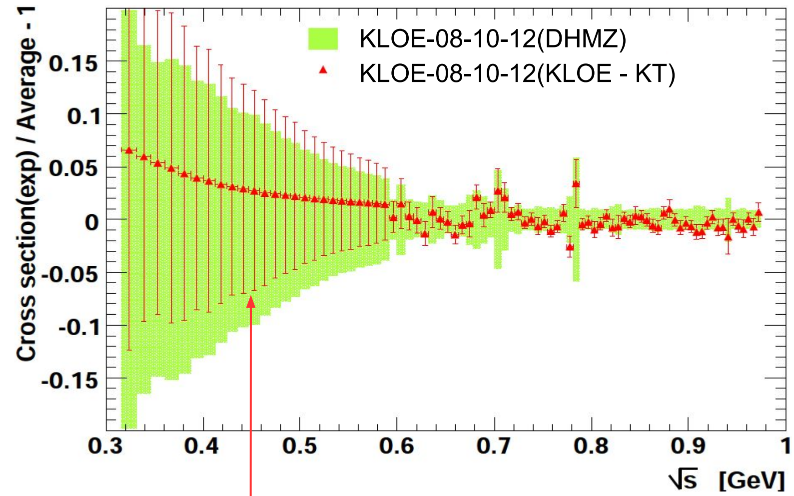
BABAR (1205.2228)

Sources	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.9	0.9-1.2	1.2-1.4	1.4-2.0	2.0-3.0
trigger/ filter	5.3	2.7	1.9	1.0	0.7	0.6	0.4	0.4
tracking	3.8	2.1	2.1	1.1	1.7	3.1	3.1	3.1
π-ID	10.1	2.5	6.2	2.4	4.2	10.1	10.1	10.1
background	3.5	4.3	5.2	1.0	3.0	7.0	12.0	50.0
acceptance	1.6	1.6	1.0	1.0	1.6	1.6	1.6	1.6
kinematic fit (χ^2)	0.9	0.9	0.3	0.3	0.9	0.9	0.9	0.9
correl $\mu\mu$ ID loss	3.0	2.0	3.0	1.3	2.0	3.0	10.0	10.0
$\pi\pi/\mu\mu$ non-cancel.	2.7	1.4	1.6	1.1	1.3	2.7	5.1	5.1
unfolding	1.0	2.7	2.7	1.0	1.3	1.0	1.0	1.0
ISR luminosity	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
sum (cross section)	13.8	8.1	10.2	5.0	6.5	13.9	19.8	52.4

→ Systematics *evaluated* in ~wide mass ranges with sharp transitions
(statistics limitations when going to narrow ranges)

Combining the 3 KLOE measurements





Local combination (DHMZ)

Information propagated between mass regions, through shifts of systematics - relying on correlations, amplitudes and shapes of systematics (KLOE-KT)

Combining the 3 KLOE measurements - $a_\mu^{\pi\pi}$ contribution

KLOE08 a_μ[0.6 ; 0.9] : 368.3 ± 3.2 $[10^{-10}]$
KLOE10 a_μ[0.6 ; 0.9] : 365.6 ± 3.3
KLOE12 a_μ[0.6 ; 0.9] : 366.8 ± 2.5

→ Correlation matrix:

	08	10	12
08	1	0.70	0.35
10	0.70	1	0.19
12	0.35	0.19	1

→ Amount of independent information provided by each measurement

→ KLOE-08-10-12(DHMZ) - a_μ[0.6 ; 0.9] : 366.5 ± 2.8 (Without χ^2 rescaling: ± 2.2)
→ Conservative treatment of uncertainties and correlations (*not perfectly known*) in weight determination

→ KLOE-08-10-12(KLOE-KT) - a_μ[0.6 ; 0.9]GeV : 366.9 ± 2.2 (Includes χ^2 rescaling)

→ Assuming perfect knowledge of the correlations to minimize average uncertainty

χ^2 definitions and properties

$$\chi^2(\mathbf{d};\mathbf{t}) = \sum_{i,j}(d_i - t_i)\cdot\left[C^{-1}(\mathbf{t})\right]_{ij}\cdot(d_j - t_j) \qquad C_{ij} = C_{ij}^{stat} + \sum_k s_i^k \cdot s_j^k$$

$$\chi^2(\mathbf{d};\mathbf{t}) = \min_{\beta_a}\left\{\sum_{i,j}\left[d_i - \left(1 + \sum_a \beta_a \cdot \left(\boldsymbol{\epsilon}_a^{\pm}(\beta_a)\right)_i\right)t_i\right]\cdot\left[C_{\rm su}^{-1}(\mathbf{t})\right]_{ij}\right.$$
$$\left.\cdot\left[d_j - \left(1 + \sum_a \beta_a \cdot \left(\boldsymbol{\epsilon}_a^{\pm}(\beta_a)\right)_j\right)t_j\right] + \sum_a \beta_a^2\right\},$$

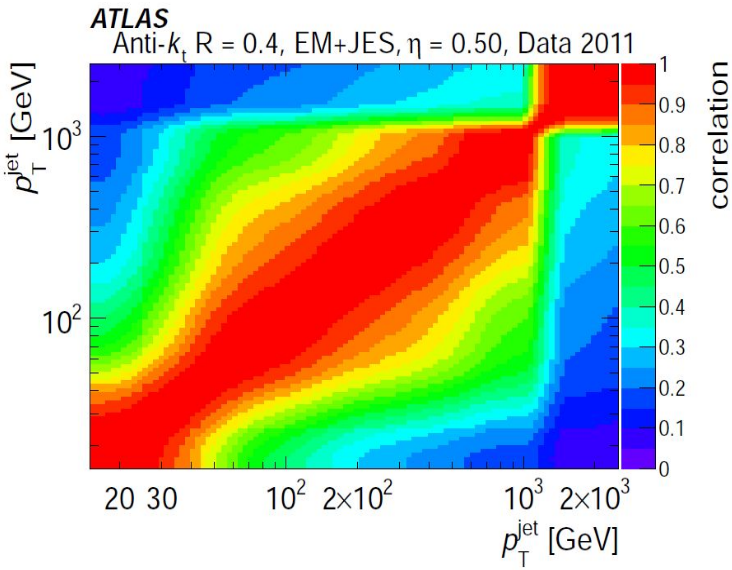
→ Two χ^2 definitions, with systematic uncertainties included in covariance matrix or treated as fitted "nuisance parameters"

→ Equivalent for symmetric Gaussian uncertainties
(1312.3524 - ATLAS)

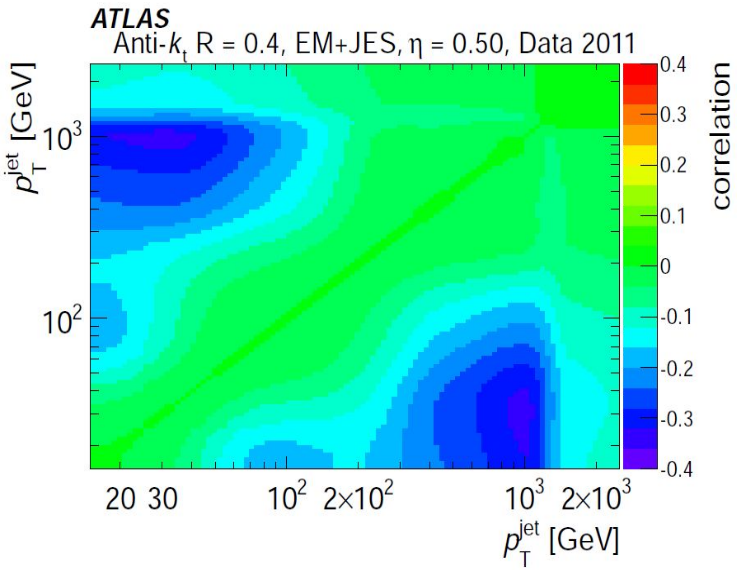
→ *Both approaches assume the knowledge of the amplitude, shape (phase-space dependence) and correlations of systematic uncertainties*

Example: published uncertainties on correlations

1406.0076 – ATLAS jet energy scale uncertainties



Nominal correlation scenario



Weaker - stronger correlation scenarios

Comparing lattice QCD and data-driven results in systematically improvable ways

2308.04221 (BMW & DMZ)

Lattice ↔ R-ratio comparison: *requirements*

$$C(t) = \frac{1}{24\pi^2} \int_0^\infty ds\, \sqrt{s} R(s)\, e^{-|t|\sqrt{s}}$$

→ R-ratio → lattice: "straightforward" (integrate R-ratio)

→ Lattice → R-ratio: inverse Laplace transform (ill-posed problem)

(Former) Status for lattice calculations:

→ Very few HVP quantities computed on lattice with:

- All contributions to C(t): flavors, various contractions, QED and SIB corrections
- All limits taken: $a \to 0$, $L \to \infty$, $M_\pi \to M_\pi^\varphi$, ...

→ None with correlations among lattice HVP observables

→ None with uncertainties on these correlations (important for checking stability of inverse problem)

→ Want approach that:

- Provides useful information with limited lattice input
- Can be systematically improved with more lattice input
- Can (eventually) incorporate physical constraints
- Includes measure of agreement of lattice & R-ratio results with comparison hypothesis
- Accounts for all correlations in lattice and R-ratio observables ...
- ... including uncertainties on these

Lattice covariances: method

→ Uncertainties and correlations critical for comparisons

→ Use extension of BMW uncertainty method with stat. resampling and syst. histogram, with flat and Akaike Information Criterion (AIC) weights

→ Applicable for observables: $\{a_j\} = \left\{a_\mu^{\text{LO-HVP}}, a_{\mu,\text{win}}^{\text{LO-HVP}}, \delta\left(\Delta\alpha_{\text{had}}^{(5)}\right), \dots\right\}$

$$H(A,B) = \sum_{\substack{\phi,\psi_A^{\text{flat}},\psi_A^{\text{AIC}} \\ \psi_B^{\text{flat}},\psi_B^{\text{AIC}}}} w_A\left(\phi,\psi_A^{\text{flat}},\psi_A^{\text{AIC}}\right) \times w_B\left(\phi,\psi_B^{\text{flat}},\psi_B^{\text{AIC}}\right) \times \mathcal{N}_2\left(A,B,\overline{A}\left(\phi,\psi_A^{\text{flat}},\psi_A^{\text{AIC}}\right),\overline{B}\left(\phi,\psi_B^{\text{flat}},\psi_B^{\text{AIC}}\right),C_{\text{stat}}^{(1)}\left(\phi,\psi_A^{\text{flat}},\psi_A^{\text{AIC}},\psi_B^{\text{flat}},\psi_B^{\text{AIC}}\right)\right)$$

with $w\left(\phi,\psi^{\text{flat}},\psi^{\text{AIC}}\right) = \frac{\text{AIC}(\phi,\psi^{\text{flat}},\psi^{\text{AIC}})}{\sum_{\psi^{\text{AIC}}}\text{AIC}(\phi,\psi^{\text{flat}},\psi^{\text{AIC}})}$

correlated / independent choices

→ Build covariance matrix from quantiles of three 1D distributions $\left(a_\mu^{\text{LO-HVP}}, a_{\mu,\text{win}}^{\text{LO-HVP}}, a_\mu^{\text{LO-HVP}} + a_{\mu,\text{win}}^{\text{LO-HVP}}\right)$

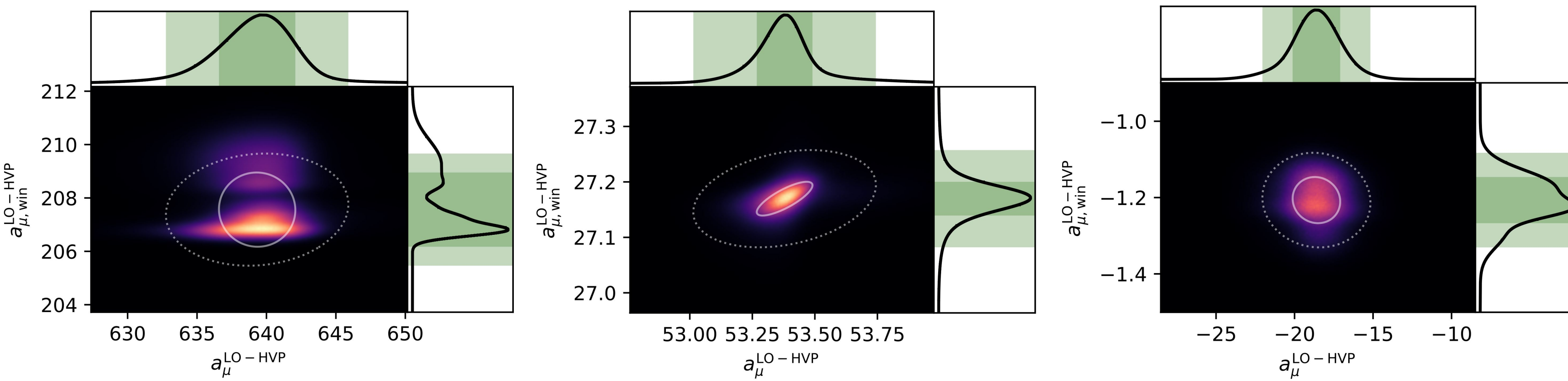
→ Separate stat. & syst. by solving (for $\lambda = 2$)

$C_{\text{stat}} + C_{\text{syst}} = C$

$\lambda C_{\text{stat}} + C_{\text{syst}} = C_\lambda$

→ $\delta\left(\Delta\alpha^{(5)}_{\rm had}\right)$ largely uncorrelated with other two observables

→ Uncertainties and correlations of $a_\mu^{\text{LO-HVP}}$ & $a_{\mu,\text{win}}^{\text{LO-HVP}}$ contributions (units of 10^{-10})



(connected *ud*) (connected *s*) (disconnected)

→ Double peak structure due to the variation $\alpha_S^{(n\,=\,0,\,3)}$ in continuum extrapolation

→ Taken into account by considering 1σ & 2σ quantiles

Uncertainties on lattice covariances

→ *Uncertainties on covariance matrix* could potentially compromise the inverse problem

→ *Stat. error on error* estimated from bootstrap on only 48 jackknife samples (sufficient for this study)

→ *Syst. error on error* from:

- For: ud, s, QED, SIB connected, and disconnected
  → Get uncertainties from 1 or 2σ quantiles
  → 0 or 100% correlations in a → 0 uncertainties of $T = a_\mu^{\text{LO-HVP}}$ and $W = a_{\mu,\text{win}}^{\text{LO-HVP}}$, with $C = T - W$

$$C_{TW} = C_{TW}^{\text{other}} + \begin{bmatrix} (dW)^2 + (dC)^2 & \{0,1\} \times (dW)^2 \\ \{0,1\} \times (dW)^2 & (dW)^2 \end{bmatrix}_{\text{cont}}$$

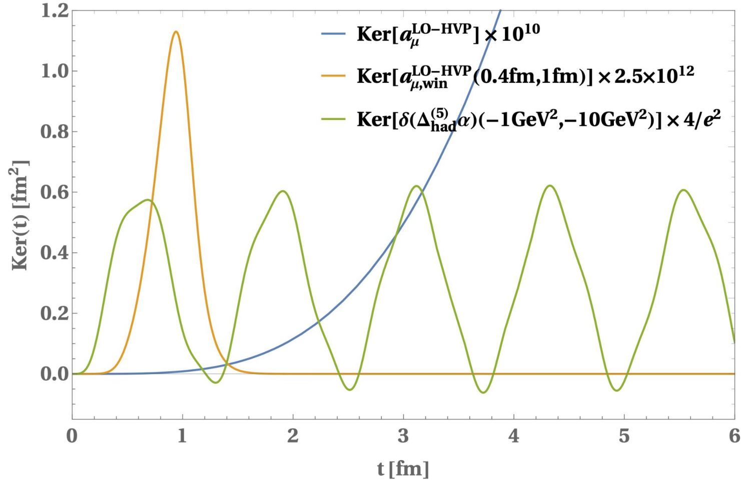
- Similarly for c

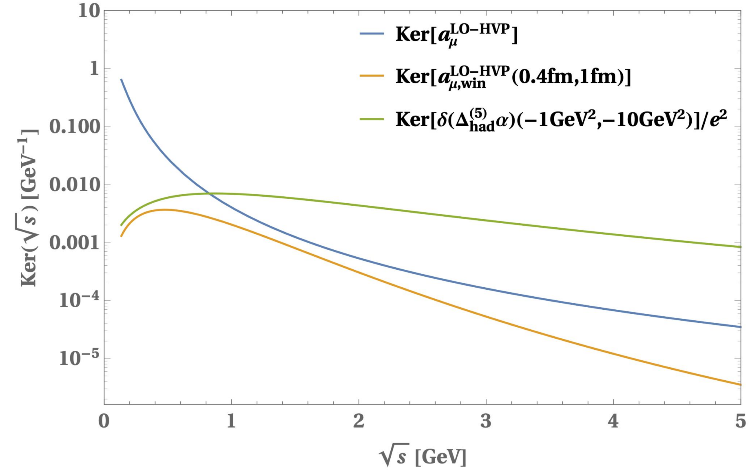
→ Result (in units of 10^{-20}):

$$C_{\text{lat}}^{1\sigma,0\%} = \begin{bmatrix} 30.13(4.88) & -0.05(0.03) \\ -0.05(0.03) & 1.95(0.47) \end{bmatrix} \qquad C_{\text{lat}}^{2\sigma,0\%} = \begin{bmatrix} 34.04(16.80) & 0.32(0.05) \\ 0.32(0.05) & 1.12(0.07) \end{bmatrix}$$

$$C_{\text{lat}}^{1\sigma,100\%} = \begin{bmatrix} 30.13(4.88) & 1.56(0.03) \\ 1.56(0.03) & 1.95(0.47) \end{bmatrix} \qquad C_{\text{lat}}^{2\sigma,100\%} = \begin{bmatrix} 34.04(16.80) & 1.94(0.05) \\ 1.94(0.05) & 1.12(0.07) \end{bmatrix}$$

Consequences of direct lattice / dispersive moments comparison for *C(t)*





→ SD:ID:LD windows

- 10%:33%:57% for $a_\mu^{\text{LO-HVP}}$
- 70%:29%:1% for $\delta\left(\Delta\alpha_{\text{had}}^{(5)}\right)$

+ Taking into account the tensions and agreements above:

→ Excess in *C(t)* for $t \sim [0.4, 1.5]$ fm

→ Probably for $t \gtrsim 1.5$ fm

→ Possible suppression for $t \lesssim 0.4$ fm (mainly based on preliminary $\delta\left(\Delta\alpha_{\text{had}}^{(5)}\right)$)

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Testing R-ratio: summary of results

Modifications to measured R-ratio that could explain lattice results are:

→ Possible in ρ-peak interval [0.63, 0.92] GeV for 2 & 3 observables

- Requires rescaling of observables in that interval by ~ (5.0 ± 1.5)%

→ Disfavored in interval below ρ-peak, [$\sqrt{s_{th}}$, 0.63 GeV]

→ Possible in [$\sqrt{s_{th}}$, $\sqrt{s_{max}}$] with $\sqrt{s_{max}}$: 0.96 → 3.0GeV that include ρ-peak, for 2 & 3 observables

- Rescalings ~(4±1)% → (3±1)% for $\sqrt{s_{max}}$ ↗

→ Possible in [$\sqrt{s_{min}}$, ∞[with $\sqrt{s_{min}}$: 0.63 → 1.8 GeV, for 2 observables

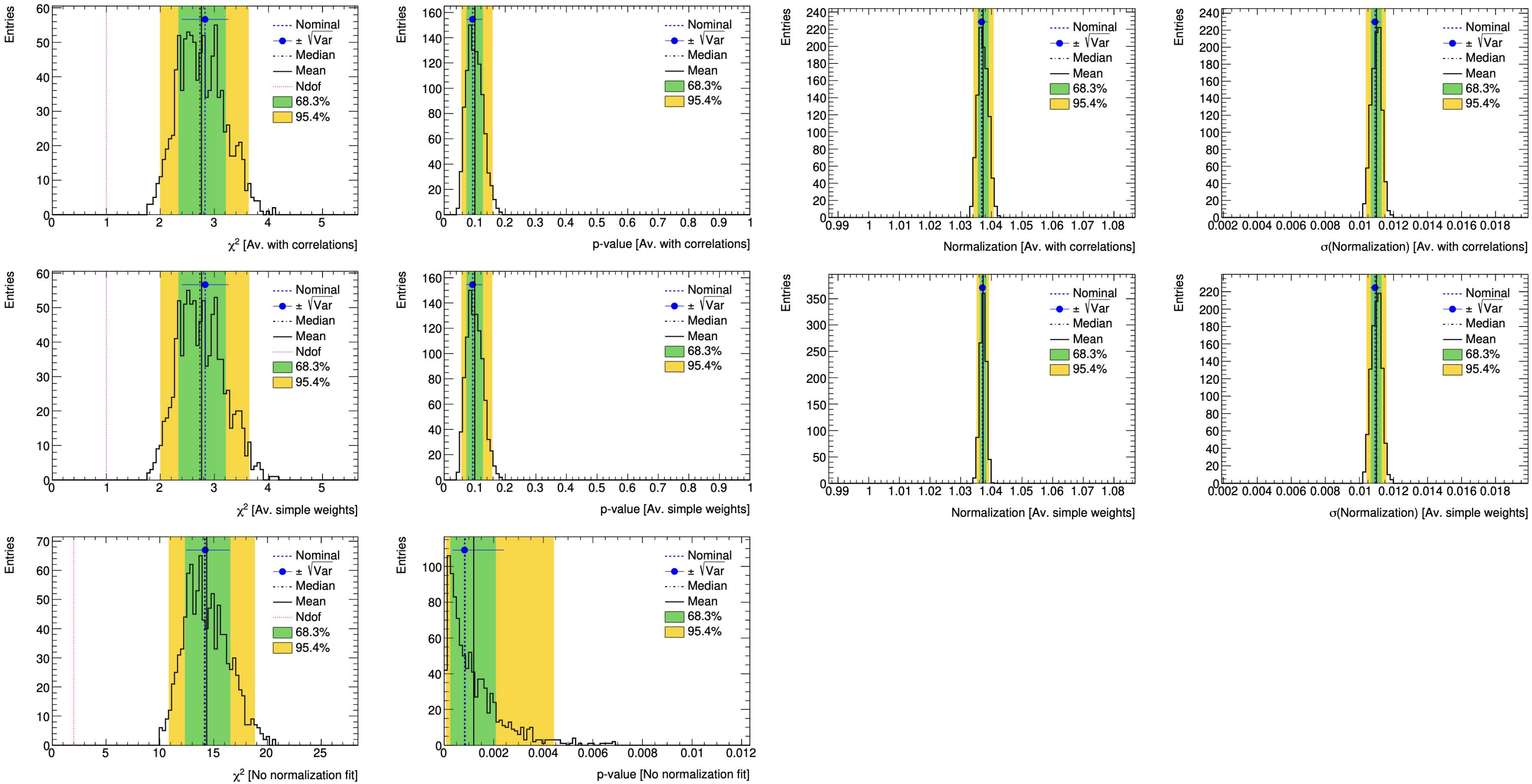
- Rescalings ~(3±1)% → (32±9)% for $\sqrt{s_{min}}$ ↗

→ Disfavored in [3.0 GeV, ∞[, for 2 & 3 observables

→ Adding $\delta\left(\Delta\alpha_{\mathrm{had}}^{(5)}\right)$ constraint eliminates the possibility of rescalings in [$\sqrt{s_{min}}$, ∞[with $\sqrt{s_{min}}$: 0.96 → 3.0 GeV that do not include ρ-peak

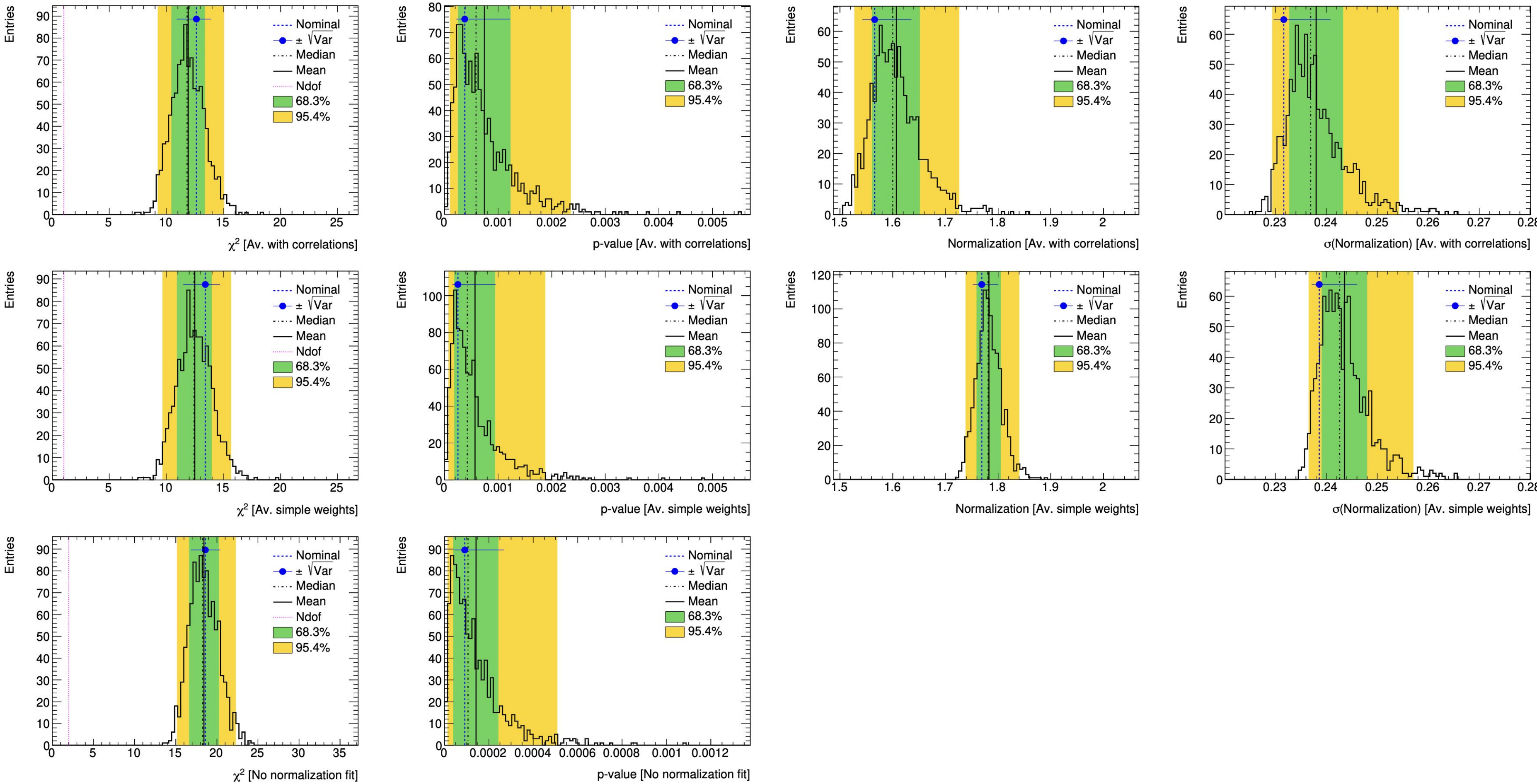
Results - Normalisation < 0.96 GeV; lattice covariance matrix “0”

2 input moment integrals



Results - Normalisation > 3 GeV; lattice covariance matrix “3”

2 input moment integrals



Testing R-ratio: results

Number of observables	I_1 [GeV]	Lat.cov.	$\delta_1 \equiv (\gamma_1 - 1)$	$\chi^2/ndof$	p-value	$\delta_1 \times \Delta\alpha^{(5)}_{\rm had}(M_Z^2)[I_1] \times 10^4$
2	$[\sqrt{s_{\rm th}}, 0.63]$	0	$15.9(5.3)[^{+0.9}_{-0.8}]\%$	$10.0[^{+2.4}_{-1.9}]/1$	$0.16[^{+0.31}_{-0.13}]\%$	0.80
2	$[\sqrt{s_{\rm th}}, 0.63]$	3	$17.4(5.7)[^{+0.6}_{-0.5}]\%$	$17.4[^{+2.2}_{-1.9}]/1$	$0.003[^{+0.010}_{-0.004}]\%$	0.88
2	$[0.63, \infty[$	0	$3.1(0.9)[^{+0.05}_{-0.05}]\%$	$0.9[^{+0.1}_{-0.1}]/1$	$34.6[^{+3.2}_{-3.2}]\%$	8.49
2	$[0.63, \infty[$	3	$3.2(0.9)[^{+0.02}_{-0.02}]\%$	$1.3[^{+0.1}_{-0.1}]/1$	$25.2[^{+2.8}_{-2.2}]\%$	8.71
3	$[\sqrt{s_{\rm th}}, 0.63]$	0	$16.4(5.4)[^{+0.9}_{-0.7}]\%$	$10.6[^{+2.2}_{-1.7}]/2$	$0.49[^{+0.73}_{-0.36}]\%$	0.83
3	$[\sqrt{s_{\rm th}}, 0.63]$	3	$17.9(5.8)[^{+0.6}_{-0.5}]\%$	$17.8[^{+2.1}_{-1.9}]/2$	$0.013[^{+0.038}_{-0.016}]\%$	0.91
3	$[0.63, \infty[$	0	$2.5(0.7)[^{+0.08}_{-0.07}]\%$	$3.8[^{+0.6}_{-0.5}]/2$	$14.7[^{+4.2}_{-4.0}]\%$	6.68
3	$[0.63, \infty[$	3	$2.6(0.7)[^{+0.04}_{-0.04}]\%$	$5.3[^{+0.5}_{-0.4}]/2$	$7.0[^{+1.8}_{-1.6}]\%$	6.96
2	$[\sqrt{s_{\rm th}}, 0.96]$	0	$3.7(1.1)[^{+0.1}_{-0.1}]\%$	$2.8[^{+0.5}_{-0.4}]/1$	$9.3[^{+2.8}_{-2.5}]\%$	1.32
2	$[\sqrt{s_{\rm th}}, 0.96]$	3	$3.9(1.1)[^{+0.06}_{-0.06}]\%$	$4.4[^{+0.4}_{-0.5}]/1$	$3.5[^{+1.4}_{-1.0}]\%$	1.39
2	$[0.96, \infty[$	0	$9.4(2.6)[^{+0.04}_{-0.04}]\%$	$0.09[^{+0.01}_{-0.009}]/1$	$77.0[^{+1.2}_{-1.3}]\%$	22.59
2	$[0.96, \infty[$	3	$9.5(2.5)[^{+0.02}_{-0.02}]\%$	$0.12[^{+0.01}_{-0.01}]/1$	$72.9[^{+1.5}_{-1.1}]\%$	22.75
3	$[\sqrt{s_{\rm th}}, 0.96]$	0	$3.8(1.1)[^{+0.09}_{-0.09}]\%$	$3.1[^{+0.4}_{-0.3}]/2$	$21.7[^{+4.3}_{-4.2}]\%$	1.36
3	$[\sqrt{s_{\rm th}}, 0.96]$	3	$4.0(1.1)[^{+0.06}_{-0.05}]\%$	$4.5[^{+0.4}_{-0.4}]/2$	$10.7[^{+3.2}_{-2.3}]\%$	1.42
3	$[0.96, \infty[$	2	$3.5(1.3)[^{+0.2}_{-0.2}]\%$	$10.9[^{+2.2}_{-1.6}]/2$	$0.43[^{+0.54}_{-0.30}]\%$	8.35
3	$[0.96, \infty[$	3	$3.7(1.3)[^{+0.1}_{-0.1}]\%$	$14.1[^{+1.5}_{-1.2}]/2$	$0.089[^{+0.083}_{-0.052}]\%$	8.91
2	$[\sqrt{s_{\rm th}}, 1.1]$	0	$3.3(1.0)[^{+0.1}_{-0.1}]\%$	$2.2[^{+0.4}_{-0.3}]/1$	$13.4[^{+3.2}_{-2.9}]\%$	1.40
2	$[\sqrt{s_{\rm th}}, 1.1]$	3	$3.4(1.0)[^{+0.05}_{-0.04}]\%$	$3.5[^{+0.3}_{-0.4}]/1$	$6.3[^{+1.9}_{-1.3}]\%$	1.46
2	$[1.1, \infty[$	0	$14.1(3.9)[^{+0.07}_{-0.08}]\%$	$0.1[^{+0.02}_{-0.02}]/1$	$70.9[^{+1.6}_{-1.6}]\%$	33.01
2	$[1.1, \infty[$	3	$14.3(3.8)[^{+0.04}_{-0.04}]\%$	$0.2[^{+0.02}_{-0.02}]/1$	$65.8[^{+1.8}_{-1.4}]\%$	33.31
3	$[\sqrt{s_{\rm th}}, 1.1]$	0	$3.4(1.0)[^{+0.07}_{-0.07}]\%$	$2.4[^{+0.3}_{-0.3}]/2$	$30.3[^{+4.5}_{-4.4}]\%$	1.44
3	$[\sqrt{s_{\rm th}}, 1.1]$	3	$3.5(1.0)[^{+0.04}_{-0.04}]\%$	$3.5[^{+0.3}_{-0.3}]/2$	$17.8[^{+3.8}_{-2.8}]\%$	1.49
3	$[1.1, \infty[$	2	$3.5(1.4)[^{+0.2}_{-0.2}]\%$	$13.0[^{+2.9}_{-2.0}]/2$	$0.15[^{+0.27}_{-0.12}]\%$	8.14
3	$[1.1, \infty[$	3	$3.7(1.4)[^{+0.1}_{-0.1}]\%$	$17.1[^{+1.9}_{-1.6}]/2$	$0.019[^{+0.027}_{-0.014}]\%$	8.70
2	$[\sqrt{s_{\rm th}}, 1.8]$	0	$2.9(0.8)[^{+0.1}_{-0.1}]\%$	$1.7[^{+0.3}_{-0.2}]/1$	$19.8[^{+3.4}_{-3.2}]\%$	1.63
2	$[\sqrt{s_{\rm th}}, 1.8]$	3	$3.1(0.9)[^{+0.03}_{-0.03}]\%$	$2.5[^{+0.2}_{-0.3}]/1$	$11.3[^{+2.4}_{-1.8}]\%$	1.69
2	$[1.8, \infty[$	0	$31.8(9.1)[^{+0.6}_{-0.6}]\%$	$1.5[^{+0.2}_{-0.2}]/1$	$21.4[^{+3.4}_{-3.2}]\%$	70.17
2	$[1.8, \infty[$	3	$32.9(9.2)[^{+0.4}_{-0.3}]\%$	$2.3[^{+0.2}_{-0.2}]/1$	$12.8[^{+2.5}_{-1.8}]\%$	72.62
3	$[\sqrt{s_{\rm th}}, 1.8]$	0	$3.0(0.9)[^{+0.05}_{-0.05}]\%$	$1.7[^{+0.2}_{-0.2}]/2$	$43.7[^{+4.9}_{-5.2}]\%$	1.65
3	$[\sqrt{s_{\rm th}}, 1.8]$	3	$3.1(0.9)[^{+0.03}_{-0.03}]\%$	$2.5[^{+0.2}_{-0.3}]/2$	$28.6[^{+4.5}_{-3.5}]\%$	1.70
3	$[1.8, \infty[$	2	$3.5(1.7)[^{+0.2}_{-0.1}]\%$	$15.1[^{+3.6}_{-2.4}]/2$	$0.052[^{+0.130}_{-0.046}]\%$	7.79
3	$[1.8, \infty[$	3	$3.7(1.7)[^{+0.08}_{-0.08}]\%$	$20.3[^{+2.4}_{-2.0}]/2$	$0.0039[^{+0.0081}_{-0.0034}]\%$	8.18
2	$[\sqrt{s_{\rm th}}, 3.0]$	0	$2.8(0.8)[^{+0.06}_{-0.06}]\%$	$1.5[^{+0.2}_{-0.2}]/1$	$22.0[^{+3.4}_{-3.2}]\%$	2.03
2	$[\sqrt{s_{\rm th}}, 3.0]$	3	$2.9(0.8)[^{+0.03}_{-0.03}]\%$	$2.3[^{+0.2}_{-0.2}]/1$	$13.2[^{+2.5}_{-1.9}]\%$	2.10
2	$[3.0, \infty[$	0	$70.5(22.4)[^{+3.6}_{-3.2}]\%$	$7.8[^{+1.7}_{-1.4}]/1$	$0.51[^{+0.66}_{-0.35}]\%$	143.42
2	$[3.0, \infty[$	3	$76.9(23.9)[^{+2.4}_{-2.2}]\%$	$13.4[^{+1.6}_{-1.5}]/1$	$0.025[^{+0.052}_{-0.024}]\%$	156.38
3	$[\sqrt{s_{\rm th}}, 3.0]$	0	$2.7(0.8)[^{+0.05}_{-0.05}]\%$	$1.7[^{+0.3}_{-0.2}]/2$	$43.1[^{+5.8}_{-6.1}]\%$	1.97
3	$[\sqrt{s_{\rm th}}, 3.0]$	3	$2.8(0.8)[^{+0.03}_{-0.03}]\%$	$2.7[^{+0.3}_{-0.3}]/2$	$26.3[^{+4.3}_{-3.7}]\%$	2.04
3	$[3.0, \infty[$	2	$4.2(2.4)[^{+0.09}_{-0.08}]\%$	$16.0[^{+3.9}_{-2.6}]/2$	$0.033[^{+0.094}_{-0.030}]\%$	8.59
3	$[3.0, \infty[$	3	$4.3(2.4)[^{+0.06}_{-0.05}]\%$	$21.7[^{+2.7}_{-2.2}]/2$	$0.0020[^{+0.0049}_{-0.0018}]\%$	8.80
2	$[0.63, 0.92]$	0	$4.8(1.4)[^{+0.1}_{-0.1}]\%$	$1.7[^{+0.3}_{-0.2}]/1$	$19.6[^{+3.4}_{-3.2}]\%$	1.42
2	$[0.63, 0.92]$	3	$4.9(1.4)[^{+0.06}_{-0.05}]\%$	$2.5[^{+0.2}_{-0.3}]/1$	$11.2[^{+2.4}_{-1.8}]\%$	1.47
2	$[\sqrt{s_{\rm th}}, 0.63] \cup [0.92, \infty[$	0	$6.2(1.8)[^{+0.1}_{-0.1}]\%$	$1.6[^{+0.2}_{-0.2}]/1$	$20.4[^{+3.4}_{-3.2}]\%$	15.33
2	$[\sqrt{s_{\rm th}}, 0.63] \cup [0.92, \infty[$	3	$6.5(1.8)[^{+0.07}_{-0.07}]\%$	$2.4[^{+0.2}_{-0.2}]/1$	$11.9[^{+2.4}_{-1.8}]\%$	15.88
3	$[0.63, 0.92]$	0	$4.9(1.4)[^{+0.08}_{-0.08}]\%$	$1.8[^{+0.2}_{-0.2}]/2$	$40.2[^{+4.0}_{-4.1}]\%$	1.45
3	$[0.63, 0.92]$	3	$5.0(1.4)[^{+0.05}_{-0.05}]\%$	$2.6[^{+0.2}_{-0.2}]/2$	$27.9[^{+4.0}_{-3.1}]\%$	1.50
3	$[\sqrt{s_{\rm th}}, 0.63] \cup [0.92, \infty[$	0	$3.4(1.1)[^{+0.2}_{-0.1}]\%$	$9.0[^{+1.8}_{-1.3}]/2$	$1.1[^{+1.1}_{-0.7}]\%$	8.30
3	$[\sqrt{s_{\rm th}}, 0.63] \cup [0.92, \infty[$	3	$3.6(1.1)[^{+0.08}_{-0.08}]\%$	$12.4[^{+1.3}_{-1.1}]/2$	$0.21[^{+0.18}_{-0.12}]\%$	8.80

Considering more observables in the data-driven approach

→ Enhancement of available information limited by the (anti-)correlations among the moment integrals

Moment integral	Correlation coefficients																		
$\Delta\alpha_{\mathrm{had}}^{(5)}(-10\ \mathrm{GeV}^2)$	1																		
$\Delta\alpha_{\mathrm{had}}^{(5)}(-9\ \mathrm{GeV}^2)$	0.999	1																	
$\Delta\alpha_{\mathrm{had}}^{(5)}(-8\ \mathrm{GeV}^2)$	0.999	0.999	1																
$\Delta\alpha_{\mathrm{had}}^{(5)}(-7\ \mathrm{GeV}^2)$	0.996	0.998	0.999	1															
$\Delta\alpha_{\mathrm{had}}^{(5)}(-6\ \mathrm{GeV}^2)$	0.993	0.995	0.998	0.999	1														
$\Delta\alpha_{\mathrm{had}}^{(5)}(-5\ \mathrm{GeV}^2)$	0.986	0.990	0.994	0.997	0.999	1													
$\Delta\alpha_{\mathrm{had}}^{(5)}(-4\ \mathrm{GeV}^2)$	0.976	0.981	0.986	0.991	0.995	0.999	1												
$\Delta\alpha_{\mathrm{had}}^{(5)}(-3\ \mathrm{GeV}^2)$	0.960	0.966	0.973	0.980	0.986	0.993	0.998	1											
$\Delta\alpha_{\mathrm{had}}^{(5)}(-2\ \mathrm{GeV}^2)$	0.931	0.939	0.948	0.957	0.967	0.977	0.987	0.996	1										
$\Delta\alpha_{\mathrm{had}}^{(5)}(-1\ \mathrm{GeV}^2)$	0.874	0.885	0.896	0.909	0.923	0.938	0.955	0.973	0.990	1									
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[0, 0.1]$ fm	0.806	0.791	0.774	0.753	0.728	0.698	0.660	0.611	0.543	0.442	1								
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[0.1, 0.4]$ fm	0.959	0.955	0.949	0.942	0.931	0.916	0.895	0.864	0.813	0.723	0.864	1							
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[0.4, 0.7]$ fm	0.876	0.887	0.899	0.912	0.926	0.940	0.954	0.966	0.972	0.958	0.428	0.786	1						
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[0.7, 1]$ fm	0.711	0.726	0.743	0.762	0.784	0.809	0.838	0.873	0.91	0.961	0.206	0.509	0.893	1					
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[1, 1.3]$ fm	0.604	0.619	0.636	0.656	0.678	0.705	0.738	0.778	0.831	0.901	0.123	0.365	0.775	0.973	1				
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[1.3, 1.6]$ fm	0.553	0.568	0.584	0.604	0.626	0.653	0.686	0.728	0.783	0.861	0.093	0.305	0.710	0.941	0.993	1			
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[1.6, 2.6]$ fm	0.508	0.522	0.537	0.556	0.577	0.604	0.636	0.677	0.733	0.814	0.074	0.260	0.647	0.891	0.963	0.987	1		
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[2.6, 4]$ fm	0.419	0.431	0.445	0.461	0.479	0.502	0.530	0.567	0.617	0.694	0.052	0.197	0.523	0.753	0.840	0.885	0.944	1	
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[4, \infty[$ fm	0.312	0.321	0.332	0.344	0.358	0.375	0.397	0.426	0.466	0.528	0.034	0.137	0.381	0.565	0.646	0.698	0.787	0.942	1

→ Employing blinding approach in BMW - DHMZ collaboration: here sharing only uncertainties and correlations for dispersive result while pending lattice-based calculations of new moments

Considering more observables in the data-driven approach

→ Quantify available information through the distribution of eigenvalues for covariance, correlation and normalized covariance matrices (complementary information): strong correlations yield small eigenvalues

Moment integral		Eigenvalues		
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[t_{\min},\, t_{\max}]$	Total uncertainty	Covariance	Correlation	Normalized covariance
$[0, 0.1]$ fm	$8.18 \cdot 10^{-12}$	$3.12 \cdot 10^{-20}$	4.85	$2.04 \cdot 10^{-4}$
$[0.1, 0.4]$ fm	$3.86 \cdot 10^{-11}$	$3.52 \cdot 10^{-21}$	1.76	$8.39 \cdot 10^{-5}$
$[0.4, 0.7]$ fm	$6.43 \cdot 10^{-11}$	$6.38 \cdot 10^{-22}$	$3.32 \cdot 10^{-1}$	$1.45 \cdot 10^{-5}$
$[0.7, 1]$ fm	$8.00 \cdot 10^{-11}$	$1.53 \cdot 10^{-22}$	$4.88 \cdot 10^{-2}$	$2.06 \cdot 10^{-6}$
$[1, 1.3]$ fm	$7.90 \cdot 10^{-11}$	$8.77 \cdot 10^{-24}$	$4.27 \cdot 10^{-3}$	$1.87 \cdot 10^{-7}$
$[1.3, 1.6]$ fm	$6.32 \cdot 10^{-11}$	$4.64 \cdot 10^{-25}$	$4.47 \cdot 10^{-4}$	$1.88 \cdot 10^{-8}$
$[1.6, \infty[$ fm	$1.15 \cdot 10^{-10}$	$7.89 \cdot 10^{-26}$	$1.51 \cdot 10^{-5}$	$6.34 \cdot 10^{-10}$

$[C_{\mathrm{R}}]_{ij} / \left(a_i^{\mathrm{R}} \cdot a_j^{\mathrm{R}}\right)$

Moment integral		Eigenvalues		
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}}$ $[t_{\min},\, t_{\max}]$	Total uncertainty	Covariance	Correlation	Normalized covariance
$[0, 0.1]$ fm	$8.18 \cdot 10^{-12}$	$2.76 \cdot 10^{-20}$	6.07	$2.49 \cdot 10^{-4}$
$[0.1, 0.4]$ fm	$3.86 \cdot 10^{-11}$	$3.17 \cdot 10^{-21}$	1.99	$9.28 \cdot 10^{-5}$
$[0.4, 0.7]$ fm	$6.43 \cdot 10^{-11}$	$5.11 \cdot 10^{-22}$	$6.86 \cdot 10^{-1}$	$2.72 \cdot 10^{-5}$
$[0.7, 1]$ fm	$8.00 \cdot 10^{-11}$	$1.33 \cdot 10^{-22}$	$2.29 \cdot 10^{-1}$	$9.71 \cdot 10^{-6}$
$[1, 1.3]$ fm	$7.90 \cdot 10^{-11}$	$1.52 \cdot 10^{-23}$	$2.12 \cdot 10^{-2}$	$8.76 \cdot 10^{-7}$
$[1.3, 1.6]$ fm	$6.32 \cdot 10^{-11}$	$1.20 \cdot 10^{-24}$	$2.36 \cdot 10^{-3}$	$9.89 \cdot 10^{-8}$
$[1.6, 2.6]$ fm	$9.32 \cdot 10^{-11}$	$2.91 \cdot 10^{-25}$	$3.90 \cdot 10^{-4}$	$1.59 \cdot 10^{-8}$
$[2.6, 4]$ fm	$2.01 \cdot 10^{-11}$	$2.15 \cdot 10^{-26}$	$3.43 \cdot 10^{-5}$	$1.41 \cdot 10^{-9}$
$[4, \infty[$ fm	$2.64 \cdot 10^{-12}$	$1.78 \cdot 10^{-27}$	$5.83 \cdot 10^{-7}$	$2.50 \cdot 10^{-11}$

→ 2 extra moment integrals add ~1 d.o.f.

Considering more observables in the data-driven approach

Moment integral	Total uncertainty	Eigenvalues Covariance	Correlation	Normalized covariance
$\Delta\alpha^{(5)}_{\rm had}(-10\ {\rm GeV}^2)$	$4.80 \cdot 10^{-5}$	$1.48 \cdot 10^{-8}$	14.74	$5.21 \cdot 10^{-4}$
$\Delta\alpha^{(5)}_{\rm had}(-9\ {\rm GeV}^2)$	$4.64 \cdot 10^{-5}$	$2.17 \cdot 10^{-10}$	3.26	$1.34 \cdot 10^{-4}$
$\Delta\alpha^{(5)}_{\rm had}(-8\ {\rm GeV}^2)$	$4.48 \cdot 10^{-5}$	$4.09 \cdot 10^{-12}$	$7.36 \cdot 10^{-1}$	$2.92 \cdot 10^{-5}$
$\Delta\alpha^{(5)}_{\rm had}(-7\ {\rm GeV}^2)$	$4.29 \cdot 10^{-5}$	$1.94 \cdot 10^{-14}$	$2.40 \cdot 10^{-1}$	$1.02 \cdot 10^{-5}$
$\Delta\alpha^{(5)}_{\rm had}(-6\ {\rm GeV}^2)$	$4.09 \cdot 10^{-5}$	$2.22 \cdot 10^{-16}$	$2.16 \cdot 10^{-2}$	$8.90 \cdot 10^{-7}$
$\Delta\alpha^{(5)}_{\rm had}(-5\ {\rm GeV}^2)$	$3.85 \cdot 10^{-5}$	$3.40 \cdot 10^{-18}$	$2.45 \cdot 10^{-3}$	$1.02 \cdot 10^{-7}$
$\Delta\alpha^{(5)}_{\rm had}(-4\ {\rm GeV}^2)$	$3.57 \cdot 10^{-5}$	$1.99 \cdot 10^{-20}$	$4.17 \cdot 10^{-4}$	$1.68 \cdot 10^{-8}$
$\Delta\alpha^{(5)}_{\rm had}(-3\ {\rm GeV}^2)$	$3.23 \cdot 10^{-5}$	$9.27 \cdot 10^{-23}$	$3.92 \cdot 10^{-5}$	$1.59 \cdot 10^{-9}$
$\Delta\alpha^{(5)}_{\rm had}(-2\ {\rm GeV}^2)$	$2.78 \cdot 10^{-5}$	$1.40 \cdot 10^{-23}$	$2.33 \cdot 10^{-6}$	$8.94 \cdot 10^{-11}$
$\Delta\alpha^{(5)}_{\rm had}(-1\ {\rm GeV}^2)$	$2.07 \cdot 10^{-5}$	$7.03 \cdot 10^{-25}$	$1.15 \cdot 10^{-7}$	$4.78 \cdot 10^{-12}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[0, 0.1]$ fm	$8.18 \cdot 10^{-12}$	$1.47 \cdot 10^{-25}$	$3.46 \cdot 10^{-10}$	$1.22 \cdot 10^{-14}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[0.1, 0.4]$ fm	$3.86 \cdot 10^{-11}$	$7.96 \cdot 10^{-28}$	$3.76 \cdot 10^{-13}$	$1.19 \cdot 10^{-17}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[0.4, 0.7]$ fm	$6.43 \cdot 10^{-11}$	$1.66 \cdot 10^{-28}$	$2.28 \cdot 10^{-13}$	$7.40 \cdot 10^{-18}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[0.7, 1]$ fm	$8.00 \cdot 10^{-11}$	$4.50 \cdot 10^{-30}$	$9.74 \cdot 10^{-14}$	$3.10 \cdot 10^{-18}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[1, 1.3]$ fm	$7.90 \cdot 10^{-11}$	$5.25 \cdot 10^{-31}$	$3.07 \cdot 10^{-14}$	$9.72 \cdot 10^{-19}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[1.3, 1.6]$ fm	$6.32 \cdot 10^{-11}$	$8.57 \cdot 10^{-32}$	$1.40 \cdot 10^{-14}$	$4.38 \cdot 10^{-19}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[1.6, 2.6]$ fm	$9.32 \cdot 10^{-11}$	$-4.56 \cdot 10^{-27}$	$-2.24 \cdot 10^{-14}$	$-7.06 \cdot 10^{-19}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[2.6, 4]$ fm	$2.01 \cdot 10^{-11}$	$-2.01 \cdot 10^{-23}$	$-3.79 \cdot 10^{-14}$	$-1.19 \cdot 10^{-18}$
$a^{\rm LO\text{-}HVP}_{\mu,\rm win}$ $[4, \infty[$ fm	$2.64 \cdot 10^{-12}$	$-1.12 \cdot 10^{-22}$	$-1.22 \cdot 10^{-13}$	$-3.83 \cdot 10^{-18}$

$\leftarrow [C_{\rm R}]_{ij} / \left(a_i^{\rm R} \cdot a_j^{\rm R}\right)$

→ 10 extra moment integrals, but no additional independent d.o.f.

Detailed conclusions for lattice / dispersive comparisons

→ Presented flexible method for comparing lattice QCD and data-driven HVP results

→ Find that discrepancies/agreements between lattice and data-driven results for
$a_\mu^{\text{LO-HVP}}$, $a_{\mu,\text{win}}^{\text{LO-HVP}}$, $\delta\left(\Delta\alpha_{\text{had}}^{(5)}\right)$

On lattice side, result from:

- a $C(t)$ that is enhanced in $t \sim [0.4, 1.5]$ fm
- also probably for $t \gtrsim 1.5$ fm
- with possible suppression for $t \lesssim 0.4$ fm (mainly based on preliminary $\delta\left(\Delta\alpha_{\text{had}}^{(5)}\right)$)

On data-driven side, could be explained by:

- enhancing measured R-ratio around ρ-peak
- or in any larger interval including ρ-peak

→ Lattice and measured R-ratio correlations of uncertainties critical for drawing such conclusions

Detailed conclusions for lattice / dispersive comparisons

→ Important to check that uncertainties on uncertainties and correlations do not spoil picture, especially for inverse problem

- checked here for lattice stat and syst uncertainties
- must do so for measured R-ratio uncertainties

→ Also important not to share results between 2 approaches before they are final (mutual blinding)

→ With more HVP observables, many generalizations possible, also including physics-driven constraints

→ However, limit on independent HVP observables in data-driven and lattice approaches

→ Same methods can be used to combine determinations of lattice and data-driven results for HVP observables, once differences are understood

→ No problems with EW fits in case of 3-observable comparisons (not shown)

Some references to related work on HVP

→ Windows proposed in RBC/UKQCD arXiv:1801.07224

→ Discussed in context of detailed comparison in Colangelo et al arXiv:2205.12963

→ Consequences of rescaling of measured R-ratio studied in Crivellin et al arXiv:2003.04886, Keshavarzi et al arXiv:2006.12666, de Rafael arXiv:2006.13880, Malaescu et al arXiv:2008.08107

→ Consequences of lattice $\Delta a_\mu^{\text{LO-HVP}}$ on $\pi^+\pi^-$ contributions to R-ratio with physical constraints in Colangelo et al arXiv:2010.07943

→ Use of Backus-Gilbert method for reconstruction of smeared R-ratio from lattice C(t) in Hansen et al arXiv:1903.06476, Alexandrou et al arXiv:2212.08467

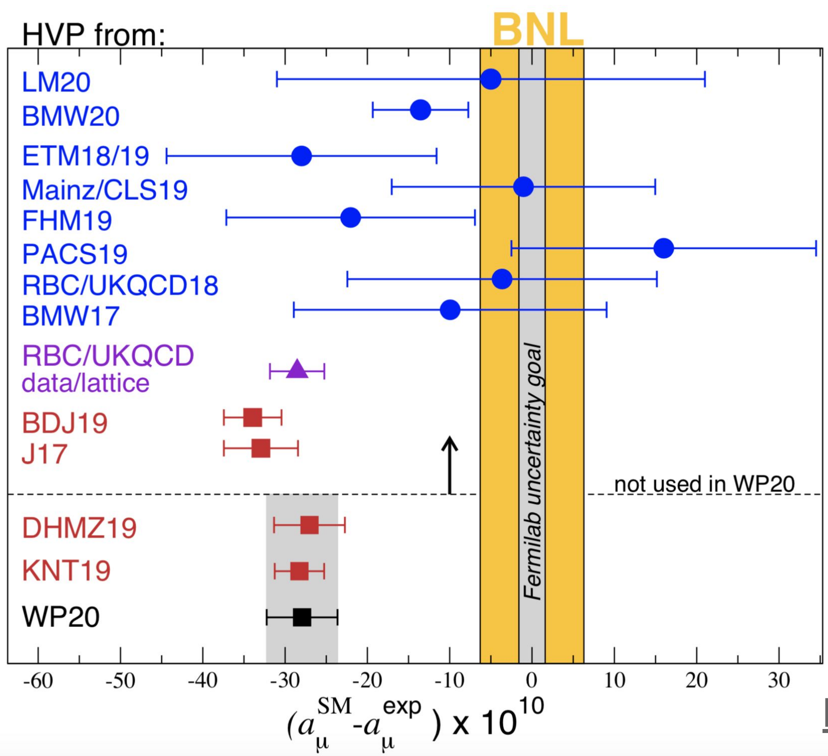
→ Proposal for comparing measured R-ratio and lattice C(t) via spectral-width sumrules in Boito et al arXiv:2210.13677

… (many other references for reconstructing spectral functions from lattice correlators)

Impact of correlations between a_μ and α_{QED} on the EW fit

2008.08107(BM, Matthias Schott)

See also: Crivellin et al, 2003.04886; Keshavarzi et al., 2006.12666 ;de Rafael, 2006.13880; Colangelo et al, 2010.07943



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Approaches considered for treating the a_μ - α_{QED} correlations

Studied approaches probing different hypotheses concerning the possible source(s) of the a_μ tension(s) :

(0) Scaling factor applied to the HVP contribution from some energy range of the hadronic spectrum

→ Approaches taking into account (*for the first time*) the full correlations between the uncertainties of the HVP contributions to a_μ and α_{QED} , based on input from DHMZ 19 (arXiv:1908.00921): correlations between points/bins of a measurement in a given channel, between different measurements in the same channel, between different channels; full treatment of the BABAR-KLOE tension in the $\pi^+\pi^-$ channel

Computation (Energy range)	$a_\mu^{\text{HVP, LO}}$ $[10^{-10}]$	$\Delta\alpha_{\text{had}}(M_Z^2)$ $[10^{-4}]$	ρ
Phenomenology (Full HVP)	694.0 ± 4.0	275.3 ± 1.0	44%
Phenomenology ([Th.; 1.8 GeV])	635.5 ± 3.9	55.4 ± 0.4	86%
Phenomenology ([Th.; 1 GeV])	539.8 ± 3.8	36.3 ± 0.3	99.5%
Lattice (Full HVP)BMW 20 (v1)	712.4 ± 4.5	-	-

(1) Cov. matrix of a_μ and α_{QED} (Pheno) described by a nuisance parameter (NP_1) impacting both quantities (used to shift a_μ to some "target" value - coherent shift applied to α_{QED}) and another one (NP_2) impacting only α_{QED} (used in the EW fit)
Note: "target" values chosen in order to reach agreement with the BMW 20 prediction / Experimental a_μ ($\pm 1\sigma$)

Uncertainty components	$a_\mu^{\text{HVP, LO}}$	$\Delta\alpha_{\text{had}}(M_Z^2)$
NP_1	$\sigma(a_\mu^{\text{HVP, LO}})$	$\sigma(\Delta\alpha_{\text{had}}(M_Z^2)) \cdot \rho$
NP_2	0	$\sigma(\Delta\alpha_{\text{had}}(M_Z^2)) \cdot \sqrt{1-\rho^2}$

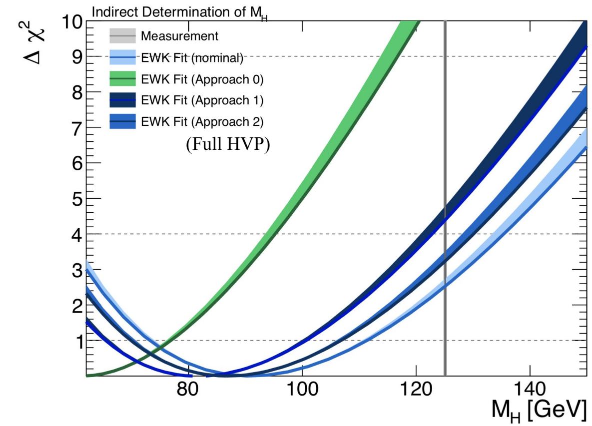
(2) Include the HVP contribution to a_μ as extra parameter in the EW fit, constrained by the Pheno & BMW 20 values
Note: Also accounted for the coherent impact of α_S on the HVP contribution and on the EW fit

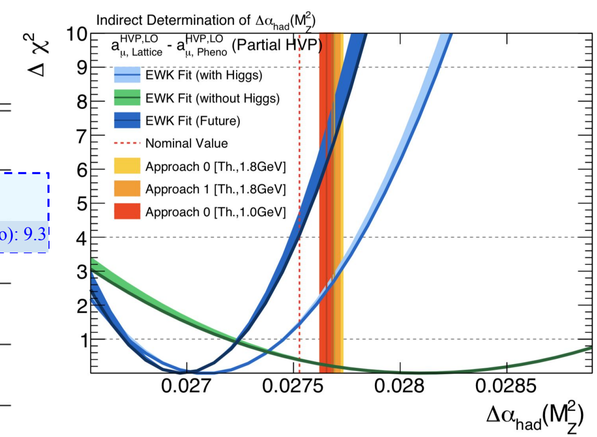
Results: comparing the Phenomenology & BMW 20 values

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	*Approach 0* Scaling factor	*Approach 0* $\Delta'\alpha_{\text{had}}(M_Z^2)$	*Approach 1* Shift NP$_1$	*Approach 1* $\sigma'\left(\Delta\alpha_{\text{had}}(M_Z^2)\right)$	*Approach 1* $\Delta'\alpha_{\text{had}}(M_Z^2)$
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	1.027	0.02826	4.6	$9.0\cdot10^{-5}$	0.02774
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	1.020	0.02808	3.5	$9.0\cdot10^{-5}$	0.02769
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	1.029	0.02769	4.7	$9.5\cdot10^{-5}$	0.02768
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	1.022	0.02765	3.5	$9.5\cdot10^{-5}$	0.02764
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1 GeV])	1.034	0.02765	-	-	-
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1 GeV])	1.026	0.02762	-	-	-

→ Large scaling factors (w.r.t. exp. uncertainties) & significant shifts of NP$_1$

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	Nominal $\Delta'\alpha_{\text{had}}(M_Z^2)$	Nominal χ^2/ndf	*Approach 0* $\Delta'\alpha_{\text{had}}(M_Z^2)$	*Approach 0* χ^2/ndf	*Approach 1* $\Delta'\alpha_{\text{had}}(M_Z^2)$	*Approach 1* χ^2/ndf	*Approach 2* $\Delta'\alpha_{\text{had}}(M_Z^2)$	*Approach 2* χ^2/ndf
	0.02753	18.6/16 (p=0.29)	-	-	-	-	0.02753	28.1/17 (p=0.04) χ^2(BMW20-Pheno): 9.3
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	-	-	0.02826	27.6/16 (p=0.04)	0.02774	20.3/16 (p=0.21)	-	
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	-	-	0.02769	19.9/16 (p=0.22)	0.02768	19.8/16 (p=0.23)	-	-
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.0 GeV])	-	-	0.02765	19.6/16 (p=0.24)	-	-	-	-





→ Addressing the BMW 20 - Pheno difference for a_μ has little impact on the EW fit, except for the unrealistic scenario rescaling the full HVP contribution

Note: Similar conclusions for the comparison with the Experimental a_μ value (see next slides)

Scaling factors and NP shifts

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	*Approach 0*		*Approach 1*		
	Scaling factor	$\Delta'\alpha_{\text{had}}(M_Z^2)$	Shift NP_1	$\sigma'\left(\Delta\alpha_{\text{had}}(M_Z^2)\right)$	$\Delta'\alpha_{\text{had}}(M_Z^2)$
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	1.027	0.02826	4.6	$9.0 \cdot 10^{-5}$	0.02774
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ (Full HVP)	1.020	0.02808	3.5	$9.0 \cdot 10^{-5}$	0.02769
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	1.029	0.02769	4.7	$9.5 \cdot 10^{-5}$	0.02768
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1.8 GeV])	1.022	0.02765	3.5	$9.5 \cdot 10^{-5}$	0.02764
$a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1 GeV])	1.034	0.02765	-	-	-
$(a_{\mu\,(\text{Lattice})}^{\text{HVP, LO}} - 1\sigma) - a_{\mu\,(\text{Pheno})}^{\text{HVP, LO}}$ ([Th.; 1 GeV])	1.026	0.02762	-	-	-
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM (Pheno)}}$ (Full HVP)	1.037	0.02856	6.6	$9.0 \cdot 10^{-5}$	0.02782
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM (Pheno)}}$ (Full HVP)	1.028	0.02831	5.0	$9.0 \cdot 10^{-5}$	0.02775
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM (Pheno)}}$ ([Th.; 1.8 GeV])	1.041	0.02776	6.6	$9.5 \cdot 10^{-5}$	0.02774
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM (Pheno)}}$ ([Th.; 1.8 GeV])	1.031	0.02770	5.0	$9.5 \cdot 10^{-5}$	0.02769
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM (Pheno)}}$ ([Th.; 1 GeV])	1.048	0.02771	-	-	-
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM (Pheno)}}$ ([Th.; 1 GeV])	1.036	0.02766	-	-	-

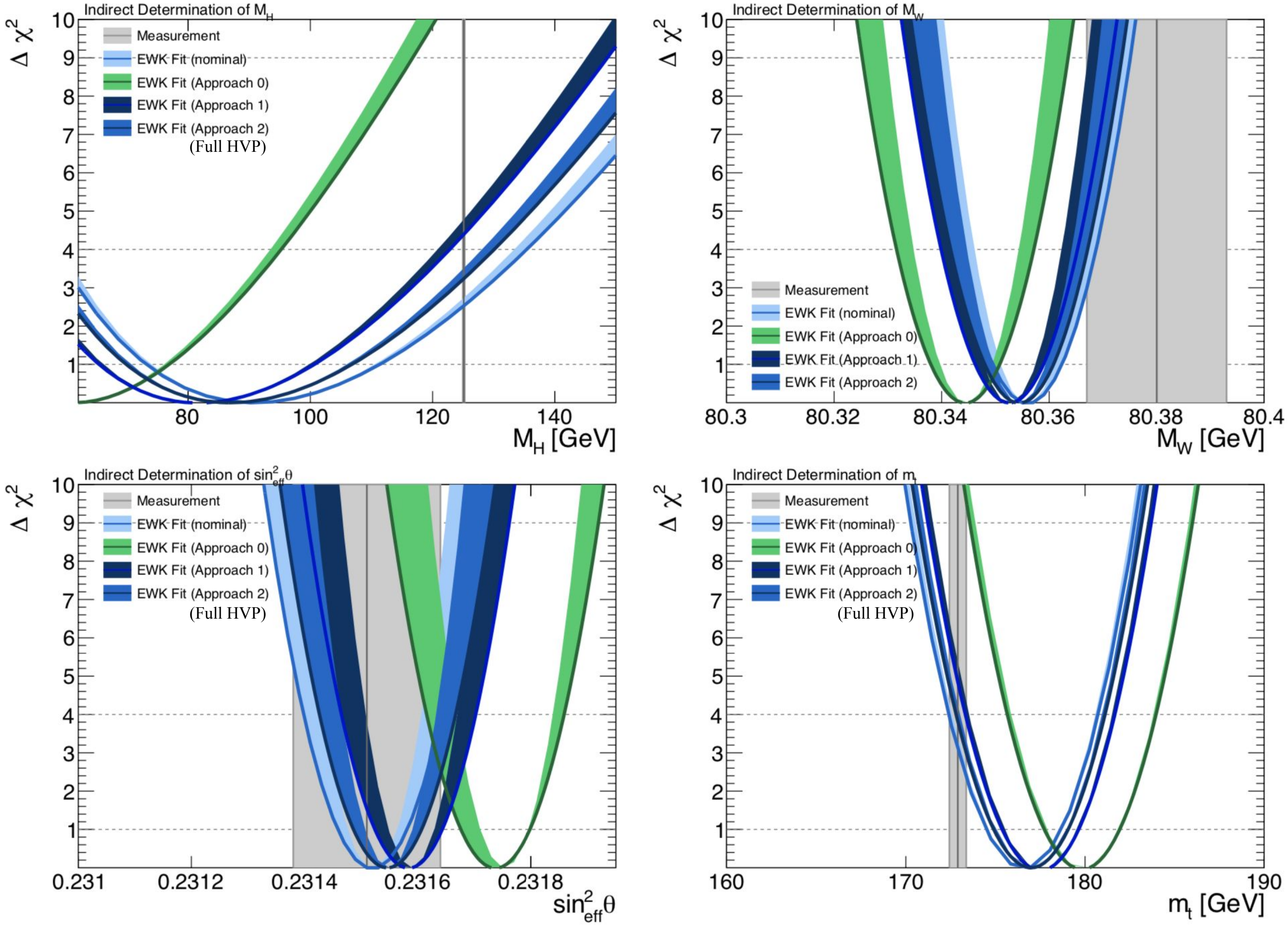
→ Large scaling factors (w.r.t. uncertainties) & significant shifts of NP_1

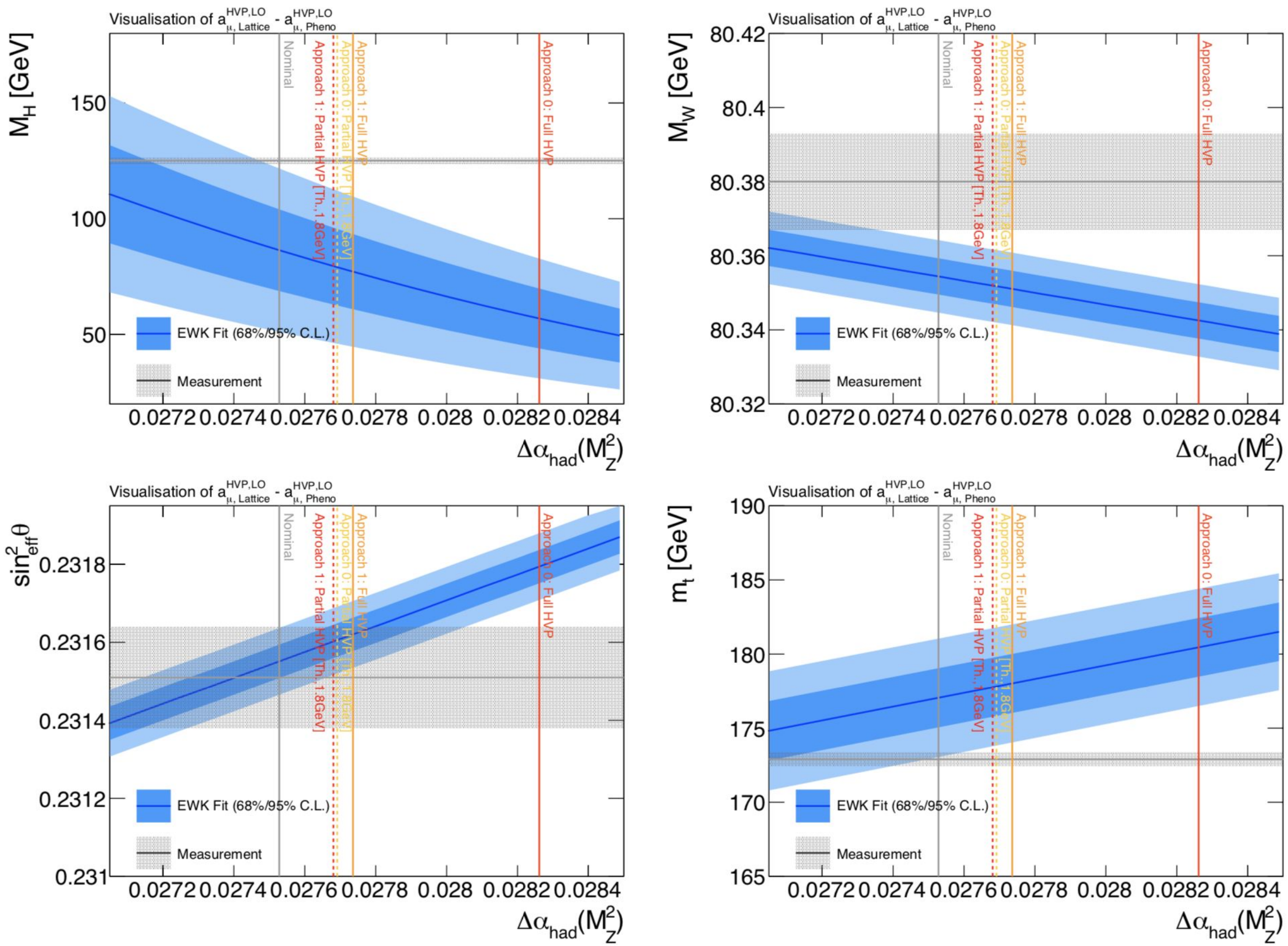
EW fit inputs and χ^2 results

LEP/LHC/Tevatron					
M_Z [GeV]	91.188 ± 0.002	R_c^0	0.1721 ± 0.003	M_H [GeV]	125.09 ± 0.15
$\sigma^0_{\rm had}$ [nb]	41.54 ± 0.037	R_b^0	0.21629 ± 0.00066	M_W [GeV]	80.380 ± 0.013
Γ_Z [GeV]	2.495 ± 0.002	A_c	0.67 ± 0.027	m_t [GeV]	172.9 ± 0.5
A_l (SLD)	0.1513 ± 0.00207	A_l (LEP)	0.1465 ± 0.0033	$\sin^2\theta^l_{\rm eff}$	0.2314 ± 0.00023
$A^l_{\rm FB}$	0.0171 ± 0.001	m_c [GeV]	$1.27^{+0.07}_{-0.11}$ GeV	**After HL-LHC**	
$A^c_{\rm FB}$	0.0707 ± 0.0035	m_b [GeV]	$4.20^{+0.17}_{-0.07}$ GeV	M_W [GeV]	80.380 ± 0.008
$A^b_{\rm FB}$	0.0992 ± 0.0016	$\alpha_s(M_Z)$	0.1198 ± 0.003	$\sin^2\theta^l_{\rm eff}$	0.2314 ± 0.00012
R_l^0	20.767 ± 0.025	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ $[10^{-5}]$	2760 ± 9	m_t [GeV]	172.9 ± 0.3

$a_\mu^{\rm HVP,\,LO}$ shift (Energy range)	Nominal		*Approach* 0		*Approach* 1		*Approach* 2	
	$\Delta'\alpha_{\rm had}(M_Z^2)$	χ^2/ndf	$\Delta'\alpha_{\rm had}(M_Z^2)$	χ^2/ndf	$\Delta'\alpha_{\rm had}(M_Z^2)$	χ^2/ndf	$\Delta'\alpha_{\rm had}(M_Z^2)$	χ^2/ndf
	0.02753	18.6/16 (p=0.29)	-	-	-	-	0.02753	28.1/17 (p=0.04) χ^2(BMW20-Pheno): 9.3
$a_\mu^{\rm HVP,\,LO}$(Lattice) $- a_\mu^{\rm HVP,\,LO}$(Pheno) (Full HVP)	-	-	0.02826	27.6/16 (p=0.04)	0.02774	20.3/16 (p=0.21)	-	
$a_\mu^{\rm HVP,\,LO}$(Lattice) $- a_\mu^{\rm HVP,\,LO}$(Pheno) ([Th.; 1.8 GeV])	-	-	0.02769	19.9/16 (p=0.22)	0.02768	19.8/16 (p=0.23)	-	-
$a_\mu^{\rm HVP,\,LO}$(Lattice) $- a_\mu^{\rm HVP,\,LO}$(Pheno) ([Th.; 1.0 GeV])	-	-	0.02765	19.6/16 (p=0.24)	-	-	-	-
$a_\mu^{\rm Exp} - a_\mu^{\rm SM}$ (Pheno) (Full HVP)	-	-	0.02856	33.6/16 (p=0.01)	0.02782	21.2/16 (p=0.17)	-	-
$a_\mu^{\rm Exp} - a_\mu^{\rm SM}$ (Pheno) ([Th.; 1.8 GeV])	-	-	0.02776	20.6/16 (p=0.19)	0.02774	20.4/16 (p=0.20)	-	-
$a_\mu^{\rm Exp} - a_\mu^{\rm SM}$ (Pheno) ([Th.; 1.0 GeV])	-	-	0.02771	20.1/16 (p=0.22)	-	-	-	-

EW fit results: χ^2 scans




Visualisation of $a_{\mu, Lattice}^{HVP,LO} - a_{\mu, Pheno}^{HVP,LO}$
M_H [GeV]
150
100
50
Nominal
Approach 1: Partial HVP [Th., 1.8GeV]
Approach 0: Partial HVP [Th., 1.8GeV]
Approach 1: Full HVP
Approach 0: Full HVP
EWK Fit (68%/95% C.L.)
Measurement
0.0272 0.0274 0.0276 0.0278 0.028 0.0282 0.0284
$\Delta\alpha_{had}(M_Z^2)$
Visualisation of $a_{\mu, Lattice}^{HVP,LO} - a_{\mu, Pheno}^{HVP,LO}$
M_W [GeV]
80.42
80.4
80.38
80.36
80.34
80.32
EWK Fit (68%/95% C.L.)
Measurement
$\Delta\alpha_{had}(M_Z^2)$
Visualisation of $a_{\mu, Lattice}^{HVP,LO} - a_{\mu, Pheno}^{HVP,LO}$
$\sin^2_{eff}\theta$
0.2318
0.2316
0.2314
0.2312
0.231
EWK Fit (68%/95% C.L.)
Measurement
$\Delta\alpha_{had}(M_Z^2)$
Visualisation of $a_{\mu, Lattice}^{HVP,LO} - a_{\mu, Pheno}^{HVP,LO}$
m_t [GeV]
190
185
180
175
170
165
EWK Fit (68%/95% C.L.)
Measurement
$\Delta\alpha_{had}(M_Z^2)$

EW fit results: indirect determination of $\Delta\alpha_{had}(M_Z^2)$