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Exorcizing a Model of Fuzzy Electroweak Interactions

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Based on arXiv:2307.11741 & arXiv:2311.08311

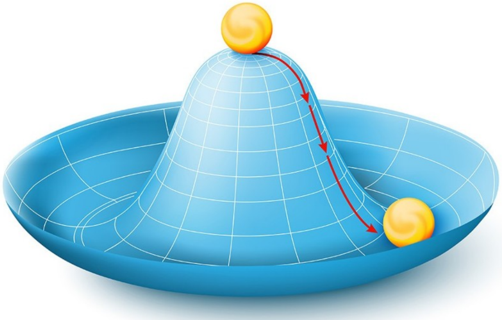
Summary:

1. Why Fuzziness?
2. Fuzzy Tachyon Condensation
3. Fuzzy Electroweak Model
4. Conclusion & Outlook

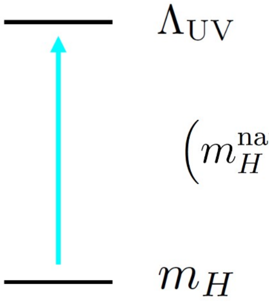
1. Why Fuzziness? → Beating Electroweak Naturalness

Higgs Mechanism

EWSB → Light Scalar:



Wilsonian EFT → EW Naturalness:



higher-spin symmetry

$$\left(m_H^{\rm nat}\right)^2 = \mathcal{O}(1) \times \frac{y_t^2}{16\pi^2}\Lambda_{UV}^2$$

dilatation symmetry

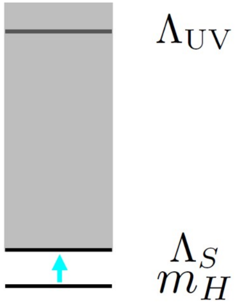
Gauge Hierarchy: $\Lambda_{EW} \sim 100$ GeV << $\Lambda_P \sim 10^{18}$ GeV

$$m_H^2 = c\Lambda_{\rm UV}^2$$

WE HAVE A CONTRADICTION

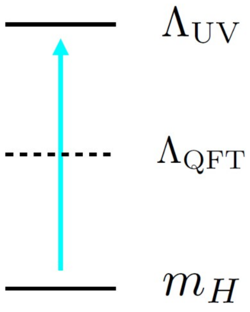
$$\sqrt{c}\Lambda_{\rm UV} \gg m_H^{\rm exp}$$

$\xrightarrow[\text{Symmetry?}]{\text{New}}$



→ Bunch of new particles @ $\Lambda_S \sim$ TeV Scale (LHC)

→ Little Hierarchy Problem!



Radical: Breakdown of (local) QFT
(e.g. UV/IR mixing)

1. Why Fuzziness? → Exorcizing Higher-Derivative QFT's

∂^4-Scalar QFT → Polynomial in ∂ → Local

$$S=\int d^4x\left[\frac{1}{2}\phi(\Box-\alpha\Box^2)\phi-\frac{1}{2}m^2\phi^2\right]$$

Propagator: UV ~ $1/k^4$ → Better UV behavior!

$$m=0 \rightarrow G(k^2)=-\frac{1}{k^2(1+\alpha k^2)}=-\frac{1}{k^2}+\frac{\alpha}{1+\alpha k^2}$$

Poles: $k^2=0$ & $k^2=-1/\alpha<0$

Particle ✓

Ghost ✗

~~Stability~~ ($-i\varepsilon$)

or

~~Unitarity~~ ($+i\varepsilon$)

Woodard, arXiv:1506.02210

Platania, arXiv:2206.04072

Kubo, Kugo, arXiv:2308.09006

Prototype → ∂^∞-Scalar QFT → Weakly Nonlocal!

$$S=\int d^4x\left[\frac{1}{2}\phi(x)\gamma(\Box)\phi(x)-V(\phi)\right], \quad \gamma(\Box)=\sum_{n=0}^{\infty}c_n\Box^n$$

Ghost-free form factor → UV finiteness!

$$\gamma(\Box)=(\Box-m^2)e^{-l_*^2\Box} \quad \rightarrow \quad G(k^2)=-\frac{e^{-l_*^2k^2}}{k^2+m^2}$$

Scalar Naturalness: $\lambda\,\phi^4$

$$\delta m^2=\frac{\lambda}{32\pi^2}\Lambda^2, \quad \Lambda=1/l_*$$

↓

Stabilization of EWSB?

↓

TeV Fuzziness! → LHC?

Biswas, Okada, arXiv:1407.3331

Buoninfante & al, arXiv:1805.03559

Pole: $k^2=-m^2$

Particle ✓

Summary:

1. Why Fuzziness?

→ 2. Fuzzy Tachyon Condensation

3. Fuzzy Electroweak Model

4. Conclusion & Outlook

2. Fuzzy Tachyon Condensation → The Ghosts Strikes Back!

Tachyon Condensation ϕ^4 → $\mu^2 > 0$

→ Z_2 spontaneous symmetry breaking

$$\mathcal{L} = \underbrace{\frac{1}{2}\phi\left(\Box+\mu^2\right)e^{-l_*^2\Box}\phi}_{\text{nonlocal mass}} - \underbrace{\frac{\lambda}{4!}\phi^4}_{\text{local mass}}$$

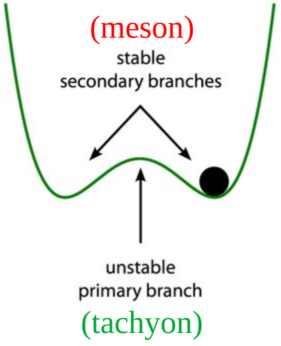
$\phi(x) = v + \sigma(x)$

nonlocal mass local mass

no ghost-free factorization!

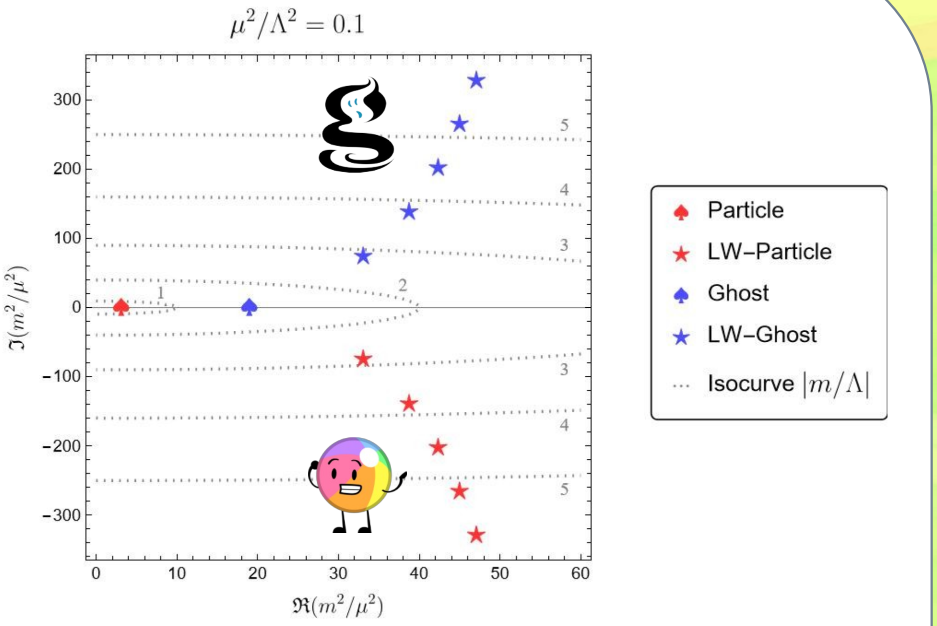
$$G(k^2) = -\frac{e^{-l_*^2 k^2}}{k^2+m^2}$$ ✖

∞ tower
of ghost-like excitations!



(meson)
stable
secondary branches

unstable
primary branch
(tachyon)



Galli, Koshelev, arXiv:1011.5672

Biswas, Kapusta, Reddy, arXiv:1201.1580

Hashi, Isono, Noumi, Shiu, Soler, arXiv:1805.02676

Koshelev, Tokareva, arXiv:2006.06641

Nortier, arXiv:2307.11741

2. Fuzzy Tachyon Condensation → Revenge of the Ghosts!

Tachyon Condensation ϕ^4 → $\mu^2 > 0$

→ Z_2 spontaneous symmetry breaking

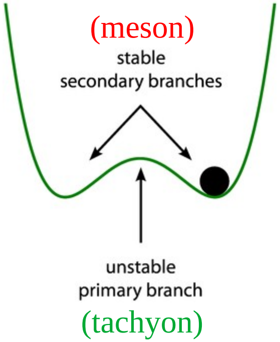
$$\mathcal{L} = \underbrace{\frac{1}{2}\phi\left(\Box+\mu^2\right)e^{-l_*^2\Box}\phi}_{\text{nonlocal mass}} - \underbrace{\frac{\lambda}{4!}\phi^4}_{\text{local mass}}$$

$\phi(x) = v + \sigma(x)$

no ghost-free factorization!

$$G(k^2) = -\frac{e^{-l_*^2k^2}}{k^2+m^2}$$ ✖

∞ tower
of ghost-like excitations!



Possible solution → Local mass counter-term:

$$-\frac{\delta\mu^2}{2}\phi^2$$

Adjust δ_μ → Ghost-free σ-vacuum (asymptotic states)

BUT Higgs mechanism:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}\,e^{-l_*^2\Box}\,F^{\mu\nu} - \left(\mathcal{D}_\mu\Phi\right)^\dagger\,e^{-l_*^2\mathcal{D}^2}\,\left(\mathcal{D}^\mu\Phi\right) + \mu^2\,\Phi^\dagger\,e^{-l_*^2\mathcal{D}^2}\,\Phi - \lambda\,|\Phi|^4$$

→ Ghosts arise also in the gauge sector!

But no local gauge invariant mass counter-term…

→ Go back home… or find a new solution!

Galli, Koshelev, arXiv:1011.5672

Biswas, Kapusta, Reddy, arXiv:1201.1580

Hashi, Isono, Noumi, Shiu, Soler, arXiv:1805.02676

Koshelev, Tokareva, arXiv:2006.06641

Nortier, arXiv:2307.11741

Summary:

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→ 3. Fuzzy Electroweak Model

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3. Fuzzy Electroweak Model → Star-Product: A New Hope!

Φ(x) = field in a representation *r* of the gauge group

ϑ(z) = real entire function on complex plane

$\vartheta(-z) = \vartheta(z), \quad z = p^2/\Lambda^2$ → spacelike ~ timelike *p*

Non-covariant star-product (asterisk) of fields:

$$\overline{\Phi}(x) *_r \Phi(x) = \overline{\Phi}(x)\, e^{\vartheta_r\left(\overleftarrow{\partial_\mu}\,\eta_r^{\mu\nu}\,\overrightarrow{\partial_\nu}\right)}\,\Phi(x)\,, \qquad \eta_r^{\mu\nu} = \frac{\eta^{\mu\nu}}{\Lambda_r^2}\,,$$

$$= e^{\vartheta_r\left(\partial_\mu^{(1)}\,\eta_r^{\mu\nu}\,\partial_\nu^{(2)}\right)}\,\overline{\Phi}(x_1)\cdot\Phi(x_2)\Big|_{x_1=x_2=x}\,,$$

$$= \int \frac{d^4p_1\, d^4p_2}{(2\pi)^8}\; e^{-i(p_1+p_2)\cdot x + \vartheta\left(p_1\cdot p_2/\Lambda_r^2\right)}\,\tilde{\overline{\Phi}}(p_1)\cdot\tilde{\Phi}(p_2)$$

→ Linear & commutative but non-associative in general!

Important property: $v * \Phi(x) = v \cdot \Phi(x)$

Nortier, arXiv:2307.11741

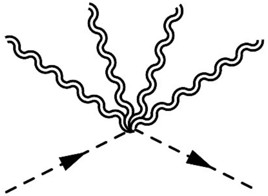
Chattopadhyay, Nortier, arXiv:2311.08311

Covariant star-product of fields:

$$\overline{\Phi}(x) \star_r \Phi(x) = \overline{\Phi}(x)\, e^{\vartheta_r\left(\overleftarrow{\mathcal{D}_\mu}\,\eta_r^{\mu\nu}\,\overrightarrow{\mathcal{D}_\nu}\right)}\,\Phi(x)\,,$$

$$= e^{\vartheta_r\left(\mathcal{D}_\mu^{(1)}\,\eta_r^{\mu\nu}\,\mathcal{D}_\nu^{(2)}\right)}\,\overline{\Phi}(x_1)\cdot\Phi(x_2)\Big|_{x_1=x_2=x}\,, \qquad \left[\mathcal{D}_\mu^{(1)}, \mathcal{D}_\nu^{(2)}\right] = 0\,,$$

$$= \underbrace{\overline{\Phi}(x) *_r \Phi(x)}_{\text{non-covariant star-product}} + \underbrace{\mathcal{O}\left(\frac{g}{\Lambda^4}\,|\Phi|^2\,A\right)}_{\text{covariant dressing = gauge cloud}}$$



Important property: $v \star v = e^{\vartheta\left(\frac{g^2}{\Lambda^2}A^2\right)}\,v^2 = v^2 + \mathcal{O}\left(A^4\right)$

3. Fuzzy Electroweak Model → Toy Abelian Higgs Model

Goal → Propagators ~

→ Ghost-free!

$$G_h(k) = -\frac{e^{-\vartheta_h\left(\frac{k^2}{\Lambda_h^2}\right)}}{k^2 + m_h^2}$$

$$G_A(k) = -\frac{e^{-\vartheta_0\left(\frac{k^2}{\Lambda_0^2}\right)}}{k^2 + m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2}\right)$$

$$\mathcal{L} = -\frac{1}{4}\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - (\mathcal{D}_\mu H^*) \star_h (\mathcal{D}^\mu H) + \mu^2 H^* \star_h H - \lambda (H^* \star_h H) *_0 (H^* \star_h H)$$

drop gauge cloud:

$\star \mapsto *$

unitary gauge:

$$H(x) = \frac{1}{\sqrt{2}}[v + h(x)]$$

$$\begin{aligned}\mathcal{L}_U \supset & -\frac{1}{4}\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - \partial_\mu H *_h \partial^\mu H \\ & - g^2 (H \cdot A_\mu) *_h (H \cdot A^\mu) \\ & + \mu^2 H^* *_h H - \lambda (H *_h H) *_0 (H *_h H)\end{aligned}$$

Ghost-free factorization

→ $*_0 \equiv *_h \quad \Rightarrow \quad \Lambda_0 \equiv \Lambda_h \;\; \& \;\; \vartheta_0(z) \equiv \vartheta_h(z)$

$$\begin{aligned}\mathcal{L}_U \supset & -\frac{1}{4}\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - \frac{g^2 v^2}{2} A_\mu A^\mu \\ & - \frac{1}{2}\partial_\mu h \partial^\mu h - \lambda v^2 h *_0 h \\ & - \lambda v\, h *_0 (h *_0 h) - \frac{\lambda}{4}(h *_0 h) *_0 (h *_0 h)\end{aligned}$$

Alternative possibility for Higgs quartic term:

$$|H|^4, \quad |H|^2 *_0 |H|^2, \quad \left(H^\dagger \star_h H\right) \cdot \left(H^\dagger \star_h H\right)$$

→ Local mass counter-terms to be ghost-free!

3. Fuzzy Electroweak Model → A Fuzzy Standard Model (FSM)

Pure gauge sector:

$$-\frac{1}{2}\,\mathrm{tr}\left[\mathcal{G}_{\mu\nu} \star_c \mathcal{G}^{\mu\nu}\right] - \frac{1}{2}\,\mathrm{tr}\left[\mathcal{W}_{\mu\nu} \star_w \mathcal{W}^{\mu\nu}\right] - \frac{1}{4}\,\mathcal{B}_{\mu\nu} \ast_w \mathcal{B}^{\mu\nu}$$

Pure Higgs sector:

$$\begin{aligned} &- \mathcal{D}_\mu H^\dagger \star_w \mathcal{D}^\mu H + \mu^2\, H^\dagger \star_w H \\ &- \lambda \left(H^\dagger \star_w H\right) \ast_w \left(H^\dagger \star_w H\right) \end{aligned}$$

$$\Big\downarrow \ \text{EWSB}$$

$$\begin{aligned} &- \frac{1}{4}\,\mathcal{A}_{\mu\nu} \ast_w \mathcal{A}^{\mu\nu} - \frac{1}{2}\,\mathcal{W}^{+}_{\mu\nu} \ast_w \mathcal{W}^{-\mu\nu} - \frac{1}{4}\,\mathcal{Z}_{\mu\nu} \ast_w \mathcal{Z}^{\mu\nu} \\ &- \frac{m_A^2}{2}\, A_\mu \ast_w A^\mu - m_W^2\, W^+_\mu \ast_w W^{-\mu} - \frac{m_Z^2}{2}\, Z_\mu \ast_w Z^\mu\,, \end{aligned}$$

with $m_W = g_w\,\frac{v}{2}\,,\ m_Z = \frac{m_W}{\cos\theta_w}$ and $m_A = 0\,.$

→ Same masses as SM!

Fermionic sector:

$$\begin{aligned} &- \overline{Q}^i_L \star_q \left(\frac{1}{2}\,\delta_{ij}\gamma^\mu \mathcal{D}_\mu Q^j_L\right) - \overline{d}^i_R \star_q \left(\frac{1}{2}\,\delta_{ij}\gamma^\mu \mathcal{D}_\mu d^j_R\right) - \overline{u}^i_R \star_q \left(\frac{1}{2}\,\delta_{ij}\gamma^\mu \mathcal{D}_\mu u^j_R\right) \\ &- \overline{L}^i_L \star_\ell \left(\frac{1}{2}\,\delta_{ij}\gamma^\mu \mathcal{D}_\mu L^j_L\right) - \overline{e}^i_R \star_\ell \left(\frac{1}{2}\,\delta_{ij}\gamma^\mu \mathcal{D}_\mu e^j_R\right) + \mathrm{H.c.} \end{aligned}$$

Yukawa couplings (1 generation):

$$\begin{aligned} &- \lambda_d \left(\overline{Q}_L \cdot H\right) \star_q d_R - \lambda_u \left(\overline{Q}_L \cdot \overline{H}\right) \star_q u_R - \lambda_e \left(\overline{L}_L \cdot H\right) \star_\ell e_R + \mathrm{H.c.} \\ &\supseteq -m_d\, \overline{d}_L \ast_q d_R - m_u\, \overline{u}_L \ast_q u_R - m_e\, \overline{e}_L \ast_\ell e_R + \mathrm{H.c.}\,, \end{aligned}$$

with $\quad m_d = \frac{\lambda_d v}{\sqrt{2}}\,, \quad m_u = \frac{\lambda_u v}{\sqrt{2}}\,, \quad m_e = \frac{\lambda_e v}{\sqrt{2}}$

Yukawa couplings (*N* generations):

$$-\lambda_d^{ij} \left(\overline{Q}^i_L \cdot H\right) \star_q d^j_R - \lambda_u^{ij} \left(\overline{Q}^i_L \cdot \overline{H}\right) \star_q u^j_R - \lambda_e^{ij} \left(\overline{L}^i_L \cdot H\right) \star_\ell e^j_R + \mathrm{H.c.}$$

Linearity of star-product → Same flavor structure as SM!

→ 1 *CP* violating phase, GIM mechanism

→ Approximate flavor symmetries:

$$U(3)_Q \times U(3)_u \times U(3)_d \times U(3)_L \times U(3)_e$$

(broken only by Yukawa couplings)

3. Fuzzy Electroweak Model → Renormalizable Gauge/Gravity Theories?

Kuz'min, Sov.J.Nucl.Phys. 50 (1989) 1011
Tomboulis, arXiv:hep-th/9702146
Modesto, Rachwal, arXiv:1503.00261

Fuzziness + Gauge invariance (local) → Competition: propagators vs vertices

$$\mathcal{L}_{YM} = -\frac{1}{2}\,\mathrm{tr}\left[F_{\mu\nu}\, e^{H(-\ell_*^2\mathcal{D}^2)}\, F^{\mu\nu}\right] \qquad \text{(Yang-Mills)}$$

$$\mathcal{L}_{GR} = -\frac{2}{\kappa_D^2}\sqrt{-g}\left[R - G_{\mu\nu}\,\frac{e^{H(-\ell_*^2\Box)}-1}{\Box}\,R^{\mu\nu}\right] \qquad \text{(Gravity)}$$

Power Counting Theorem → Asymptotically Polynomial: *p(z)*

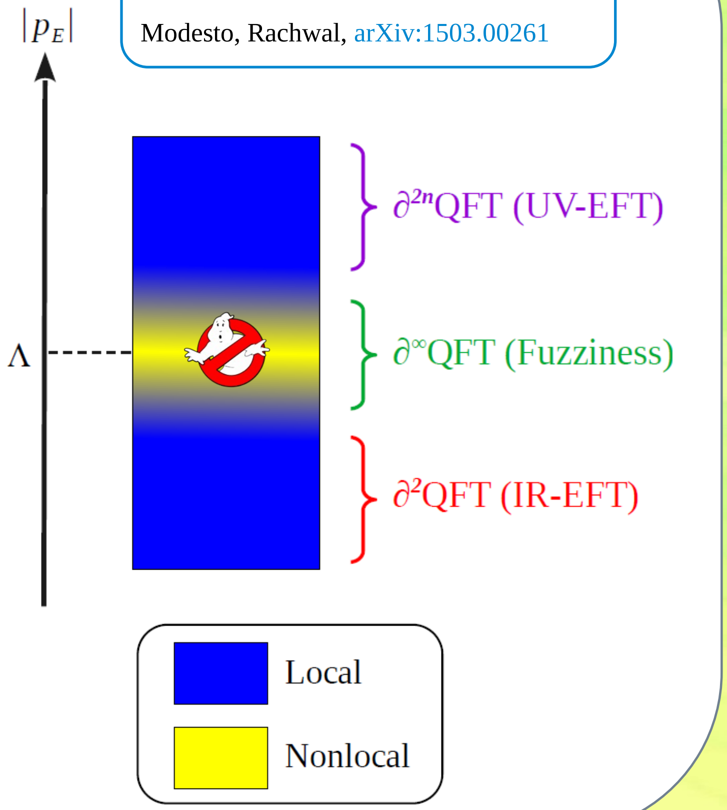
$$e^{H(z)} = e^{\frac{1}{2}\left[\Gamma\left(0,p(z)^2\right)+\gamma_E+\log\left(p(z)^2\right)\right]}$$

$$= \underbrace{e^{\frac{\gamma_E}{2}}\sqrt{p(z)^2}\,\bigg\{1+}_{\text{UV Locality}}\underbrace{\left[\frac{e^{-p(z)^2}}{2p(z)^2}\left(1+O\left(\frac{1}{p(z)^2}\right)\right)+O\left(e^{-2p(z)^2}\right)\right]}_{\text{IR Nonlocal Dressing}}\bigg\}$$

→ Interpolates btw "Normal QFT's" & "Higher-Derivative QFT's"

D = *4* → Super-renormalizability! (at least 1 loop UV-divergences)

→ What about the FSM?



4. Conclusion & Outlook

Recap:

Fuzzy Interactions → Naturalness & Quantum Gravity

Ghost-Free Condition → Issues with Tachyon Condensation! → New Star-Product & FSM

Other attempts:

Hashi, Isono, Noumi, Shiu, Soler, arXiv:1805.02676

Modesto, arXiv:2103.05536

Outlook:

→ Better understanding of loop corrections: fuzzy window ~ Λ for a gauge theory?

→ Study of naturalness in the FSM (i.e. loop corrections to m_H)

→ TeV scale phenomenology? → Little Hierarchy Problem?

→ Include gravity with the star-product

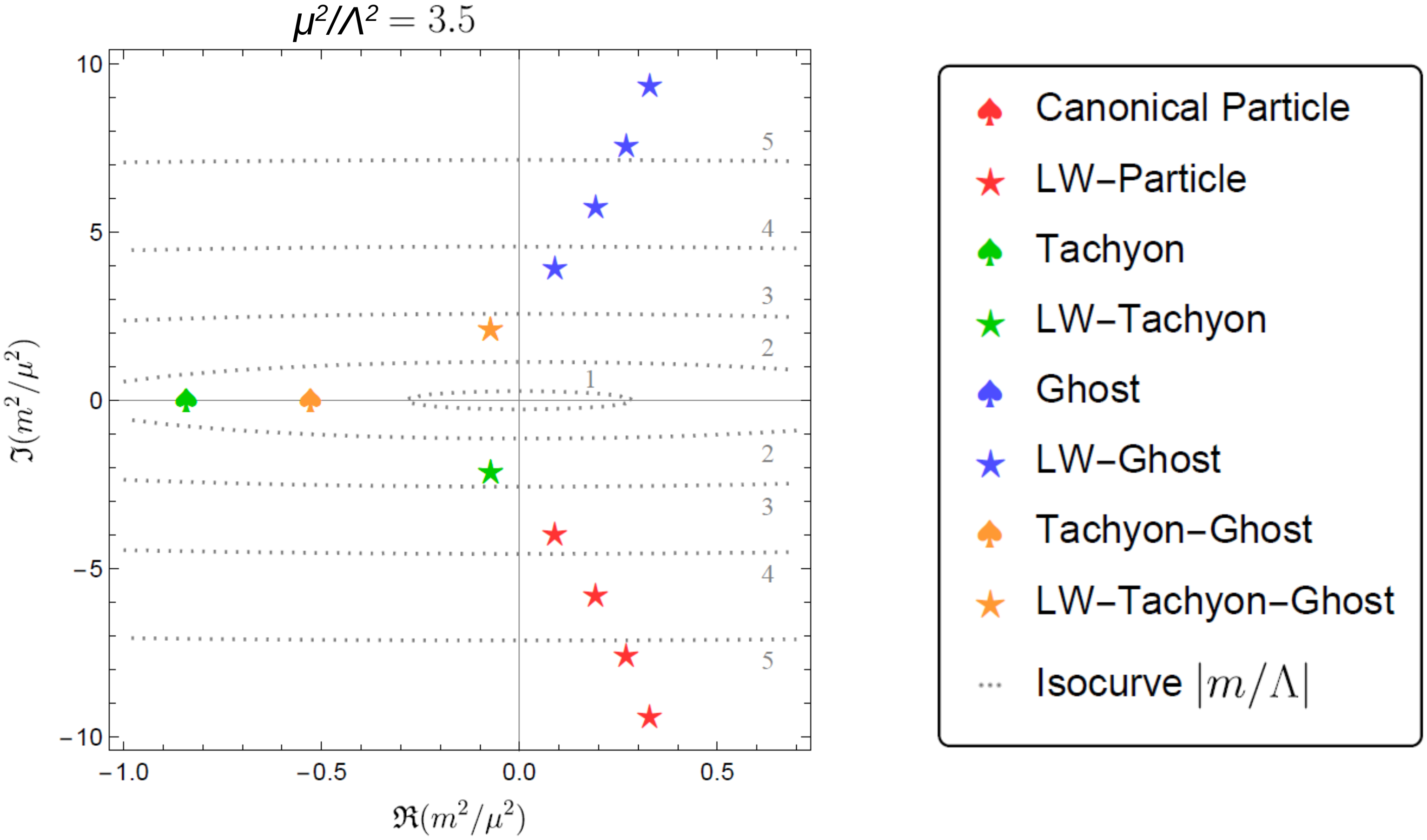
→ ...

Fuzziness → Another “Flashy” BSM Scenario?

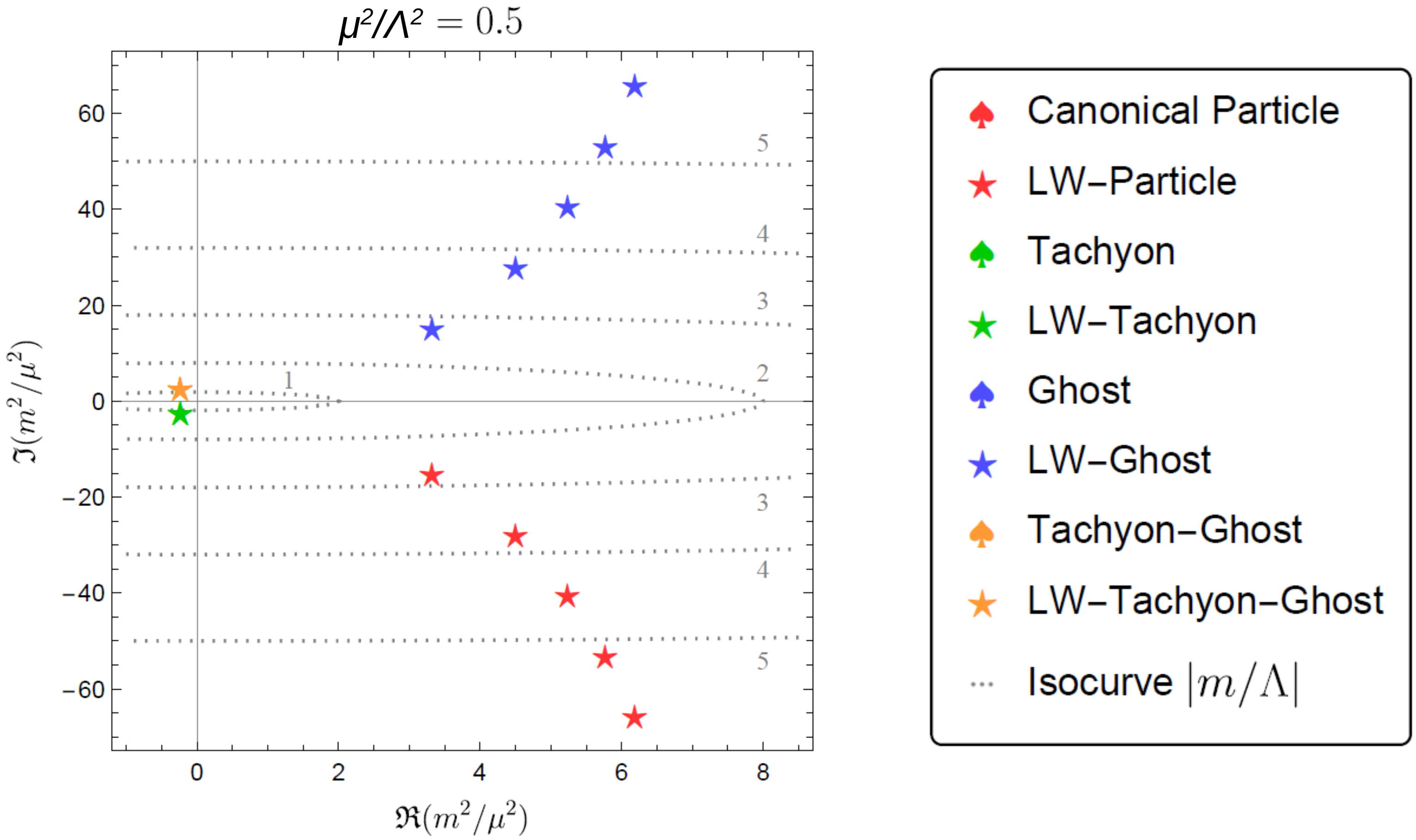
“Barbenheimer” Meme

Thank You for your Attention!

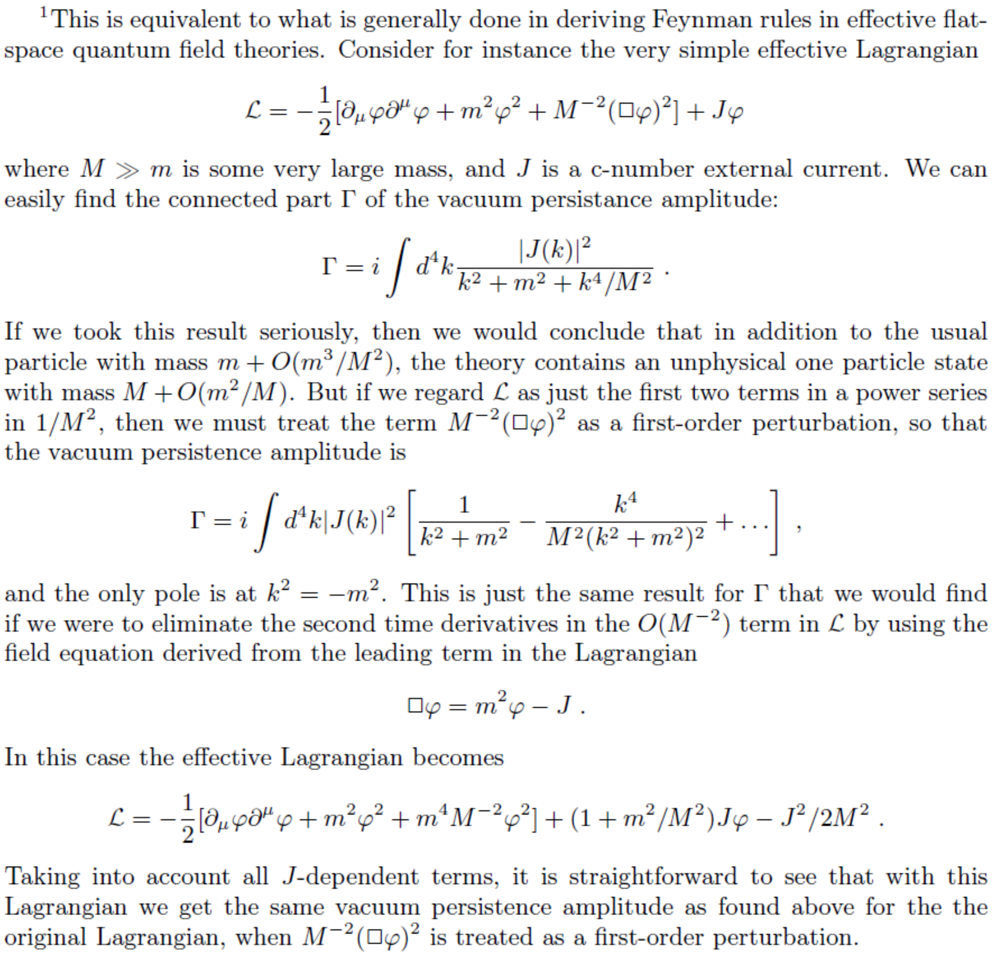
Appendix → Inverted Hierarchy



Appendix → Ahierarchy



Appendix → No Ghosts in an EFT (Weinberg's Footnote, arXiv:0804.4291)



[1] This is equivalent to what is generally done in deriving Feynman rules in effective flat-space quantum field theories. Consider for instance the very simple effective Lagrangian

$$\mathcal{L} = -\frac{1}{2}[\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2 + M^{-2}(\Box\varphi)^2] + J\varphi$$

where $M \gg m$ is some very large mass, and J is a c-number external current. We can easily find the connected part Γ of the vacuum persistance amplitude:

$$\Gamma = i\int d^4k \frac{|J(k)|^2}{k^2 + m^2 + k^4/M^2} \ .$$

If we took this result seriously, then we would conclude that in addition to the usual particle with mass $m + O(m^3/M^2)$, the theory contains an unphysical one particle state with mass $M + O(m^2/M)$. But if we regard $\mathcal{L}$ as just the first two terms in a power series in $1/M^2$, then we must treat the term $M^{-2}(\Box\varphi)^2$ as a first-order perturbation, so that the vacuum persistence amplitude is

$$\Gamma = i\int d^4k |J(k)|^2 \left[\frac{1}{k^2 + m^2} - \frac{k^4}{M^2(k^2+m^2)^2} + \dots\right] \ ,$$

and the only pole is at $k^2 = -m^2$. This is just the same result for Γ that we would find if we were to eliminate the second time derivatives in the $O(M^{-2})$ term in $\mathcal{L}$ by using the field equation derived from the leading term in the Lagrangian

$$\Box\varphi = m^2\varphi - J \ .$$

In this case the effective Lagrangian becomes

$$\mathcal{L} = -\frac{1}{2}[\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2 + m^4M^{-2}\varphi^2] + (1 + m^2/M^2)J\varphi - J^2/2M^2 \ .$$

Taking into account all J-dependent terms, it is straightforward to see that with this Lagrangian we get the same vacuum persistence amplitude as found above for the the original Lagrangian, when $M^{-2}(\Box\varphi)^2$ is treated as a first-order perturbation.

Appendix → String Field Theory

String Field $\Psi \equiv$ Infinite Tower of Spins [Witten, NPB 268 (1986) 253-294]:

$$|\Psi\rangle = \left[\phi(x) + A_\mu(x)\alpha^\mu_{-1} + B_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1} + \cdots\right] c_1|0\rangle\,.$$

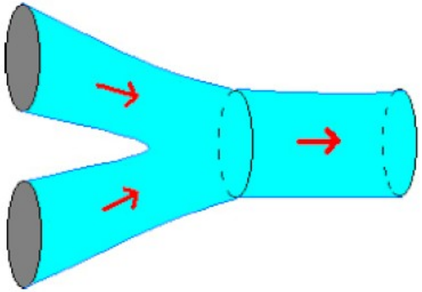
Open String Field Theory ($M_s = 1$), 0-level Truncated Action ($- + + \cdots +$):

$$S = \frac{1}{g_s^2}\int d^d x\left[\frac{1}{2}\phi\Box\phi - \frac{e^{3r_*}}{3}\widetilde{\phi}^3\right]\,, \quad \widetilde{\phi}(x) = e^{r_*\Box}\phi(x)\,, \quad r_* = \log\left(\frac{3^{3/2}}{4}\right) \simeq 0.2616\,.$$

$\widetilde{\phi}(x) \equiv$ Smeared Field via Nonlocality from ∞-Derivative Operator (Nonlocal Length Scale η):

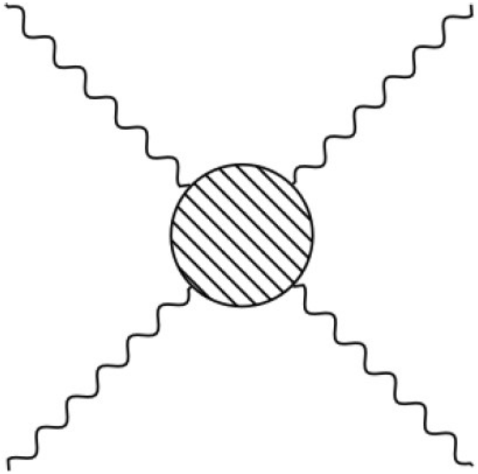
$$e^{\eta^2\partial_x^2}\,\delta(x) = \sqrt{\frac{1}{4\pi\eta^2}}\,e^{-\frac{x^2}{4\eta^2}}\,,$$

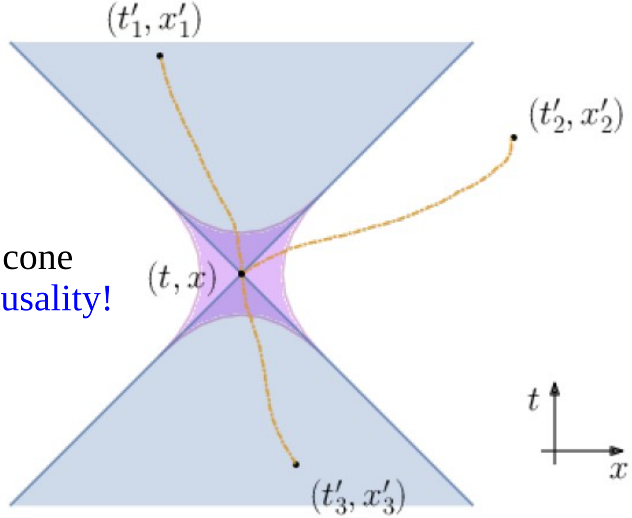
$$= \delta(x) + \sum_{n=1}^{N-1}\frac{\eta^{2n}}{n!}\,\delta^{(n)}(x) + \mathcal{O}(\eta^{2N})\,.$$



Appendix → Macrocausality

- Initial Value Problem: ∞-Derivatives $\not\Rightarrow$ ∞ Number of Initial Data N_0.
  $\Rightarrow N_0 = 2 \times$ Number of Poles [Barnaby, Kamran, JHEP 02 (2008) 008].
- No Superluminal Propagation [Erbin, Firat, Zwiebach, JHEP 01 (2022) 167].
- Weak Nonlocality ⇒ Smeared Vertices ⇒ Minimal Uncertainty in Time Resolution!
  [Carone, PRD 95 (2017) 4, 045009] & [Giaccari, Modesto (2018), arXiv:1803.08748]:

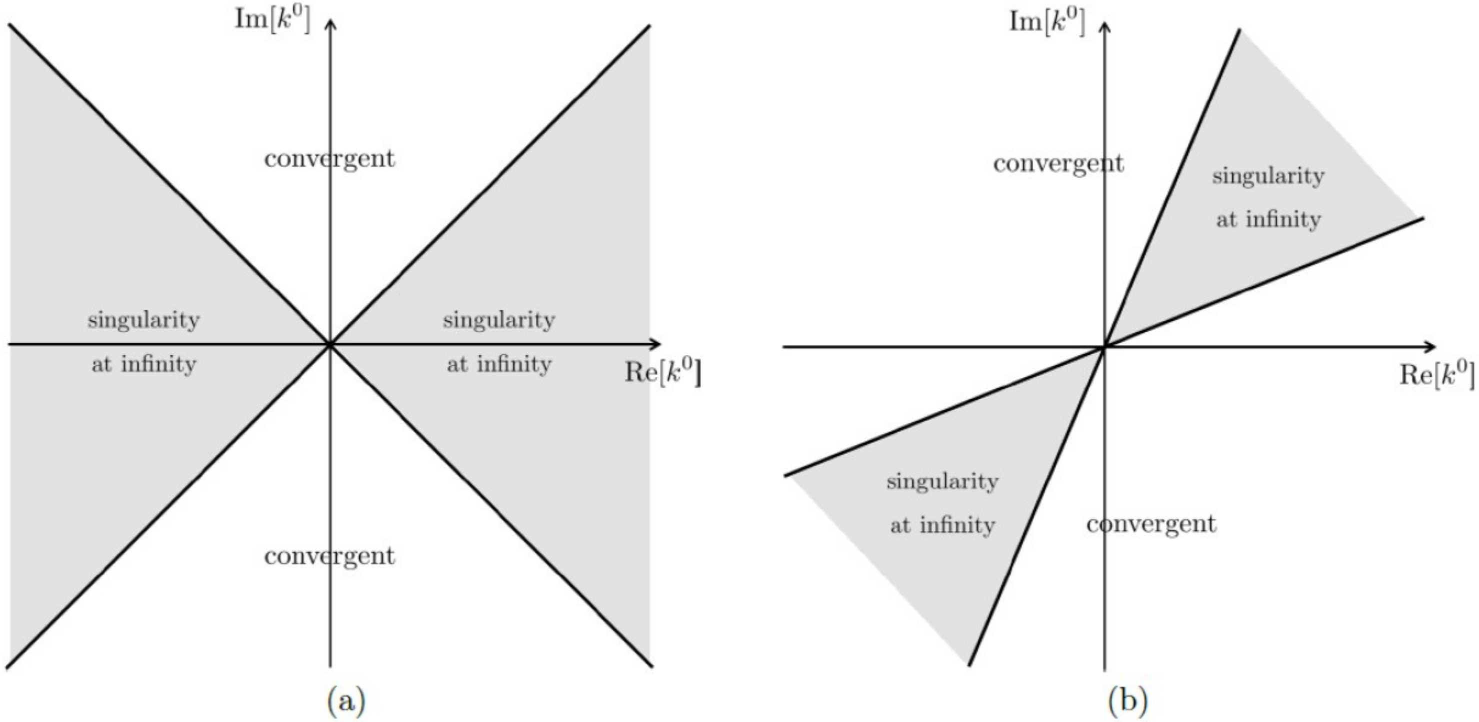




Fuzzy light cone
→ Macrocausality!

Appendix → Analyticity & Unitarity (1/2)

Some Form Factors Blow Up for $E \gg \Lambda_\phi \Rightarrow$ Perturbative Unitarity Lost!
[Koshelev, Tokareva, PRD 104 (2021) 2, 025016]

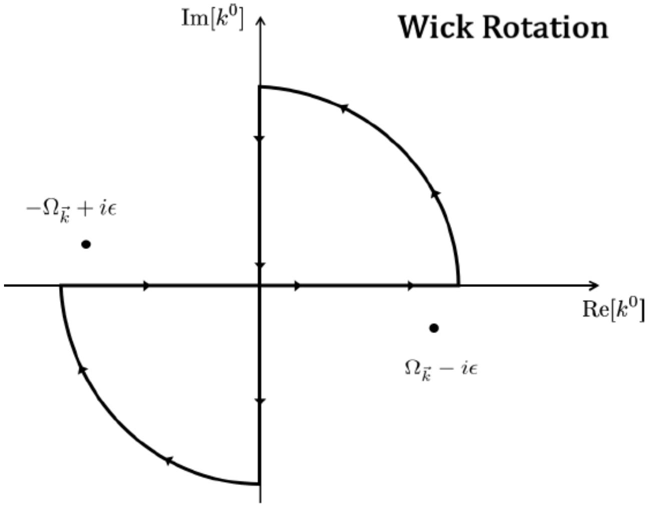


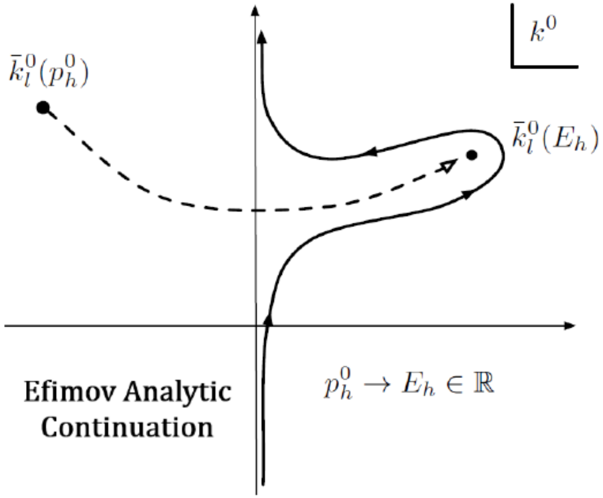
(a) Stringy Form Factor $e^{-\Box/\Lambda_\phi^2}$.

(b) Form Factor $e^{\Box^2/\Lambda_\phi^4}$.

Appendix → Analyticity & Unitarity (2/2)

- Essential Singularity at Complex Infinity $\Rightarrow$ Wick Rotation is Forbidden!
- Kallen–Lehmann Spectral Representation $\not\Rightarrow$ Unitarity Violation! [Calcagni, Rachwal (2022), arXiv:2210.04914]
- Euclidean $\rightarrow$ Minkowskian Signature by Efimov Analytic Continuation $\Rightarrow$ Cutkosky Rules:
  - Contour Prescription: [Efimov, Sov.J.Nucl.Phys. 4 (1967) 2, 309-315].
  - SFT: [Pius, Sen, JHEP 10 (2016) 024] & [de Lacroix, Erbin, Sen, JHEP 05 (2019) 139].
  - Review Nonlocal Scalars: [Buoninfante, PRD 106 (2022) 12, 126028].



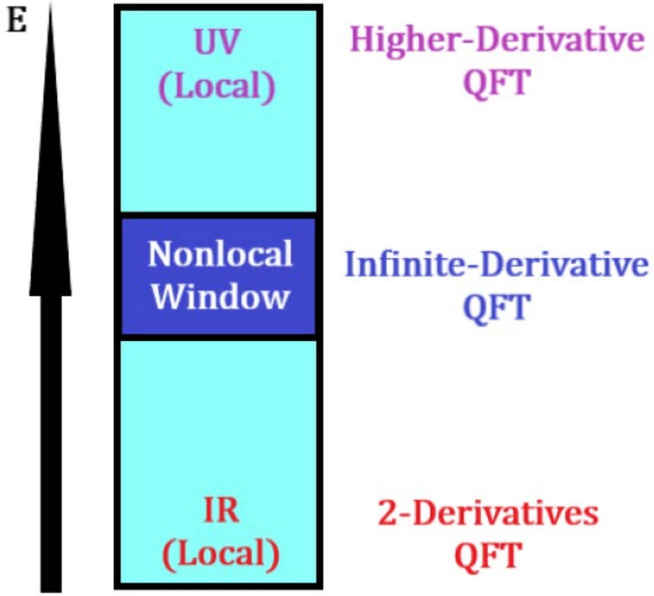


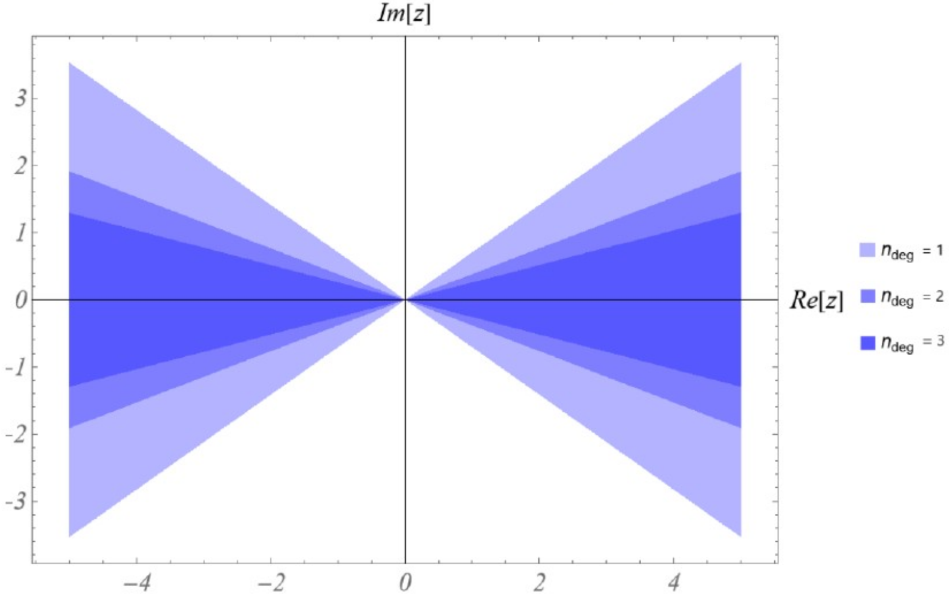
Appendix → Form Factor (1/2)

Gost-Free & Asymptotically Polynomial Form Factor $e^{\vartheta(z)} \to p_n(z)$, $|z| \to \infty_c$ in Conical Region $\mathcal{C}$:

$$\vartheta(z) = \int_0^{p_n(z)} d\omega \, \frac{1 - e^{-\omega^2}}{\omega} \; ; \text{ Polynomial } p_n(z) \text{ of Degree } n_{deg} \in \mathbb{N}^* ;$$

$$\mathcal{C} = \{ z | \quad -\theta < \arg(z) < \theta \quad \cup \quad \pi - \theta < \arg(z) < \pi + \theta \}, \qquad \theta = \frac{\pi}{4 n_{\rm deg}}$$



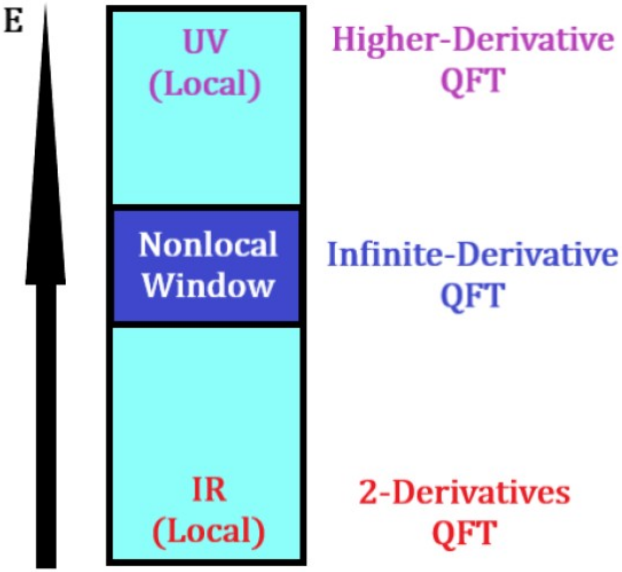


Appendix → Form Factor (2/2)

My Recipe for Renormalization Group Improved Perturbation Theory:

- $Q_0 \ll 1/\eta \Rightarrow$ IR Expansion of $e^{\vartheta(-\eta^2 p^2)} \Rightarrow$ Local EFT: 2-Derivative Kinetic Term.
- $Q_0 \gg 1/\eta \Rightarrow$ UV Expansion of $e^{\vartheta(-\eta^2 p^2)} \Rightarrow$ Local EFT: Higher-Derivative Kinetic Term.
- $Q_0 \sim 1/\eta \Rightarrow$ Ghost-Free Renorm. Scheme $\Rightarrow$ Nonlocal QFT: ∞-Derivative Kinetic Term.

$\Rightarrow$ Must Be Checked in a Concrete Model.



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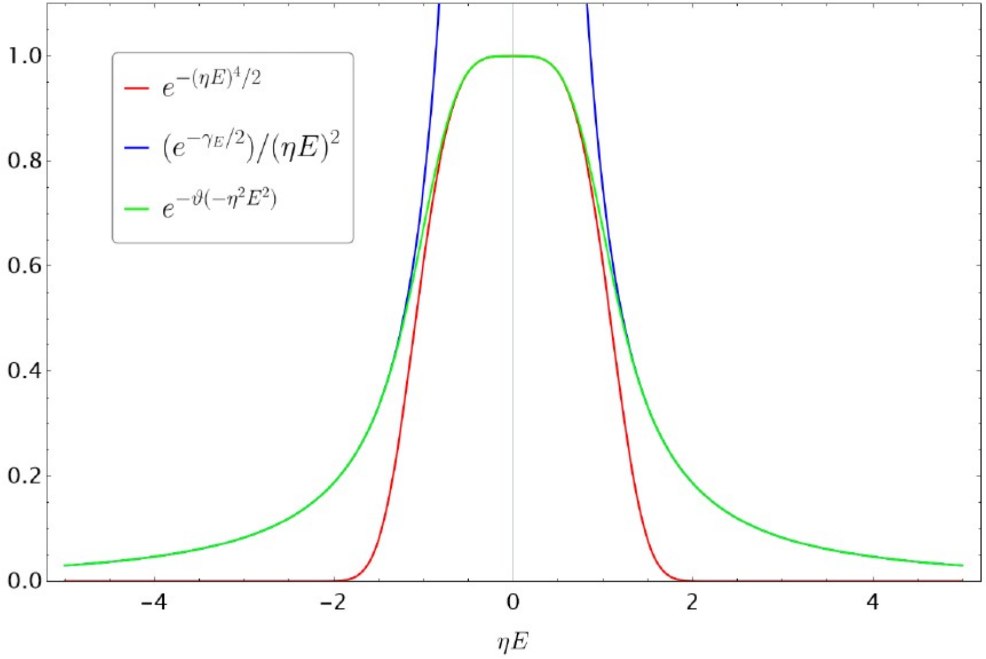
UV (Local) — Higher-Derivative QFT

Nonlocal Window — Infinite-Derivative QFT

IR (Local) — 2-Derivatives QFT



Appendix → Renormalization of Fuzzy Yang-Mills

- Renormalized ∞-Derivative Yang-Mills Lagrangian [Tomboulis (1997), arXiv:hep-th/9702146]:

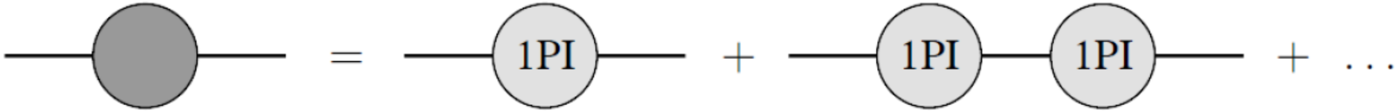
$$\mathcal{L}_{YM} = -\underset{\text{Local YM (Renormalized)}}{\frac{1}{2g^2_{YM}(\eta)}\,\mathrm{Tr}\left[\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right]} - \underset{\text{Higher-Deriv. (Not Renormalized)}}{\frac{\alpha}{2}\,\mathrm{Tr}\left[\mathcal{F}_{\mu\nu}\left(e^{\vartheta\left(\eta^2 D^2_\mu\right)}-1\right)\mathcal{F}^{\mu\nu}\right]} + \text{G.F. terms} + \text{counterterms}$$

Ghost-Free Renormalization Scheme: $\alpha\, g^2_{YM}(\eta) = 1$ at Scale $Q_0 \sim 1/\eta$.

- No Ghosts at Perturbative Level as in SFT!
[Pius, Sen, JHEP 10 (2016) 024] & [de Lacroix, Erbin, Sen, JHEP 05 (2019) 139].

- Dressed Propagator ⇒ Nonperturbative Resummation of 1PI Diagrams:
  - Shapiro: Infinite Tower of Complex Conjugate Poles [Shapiro, PLB 744 (2015) 67-73].
  - Modesto: Shapiro Ghosts Outside Radius of Convergence of Dressed Propagator ⇒ Not an Issue!

⇒ No Proof of Ghost-Freedom at Nonperturbative Level (Open Issue).

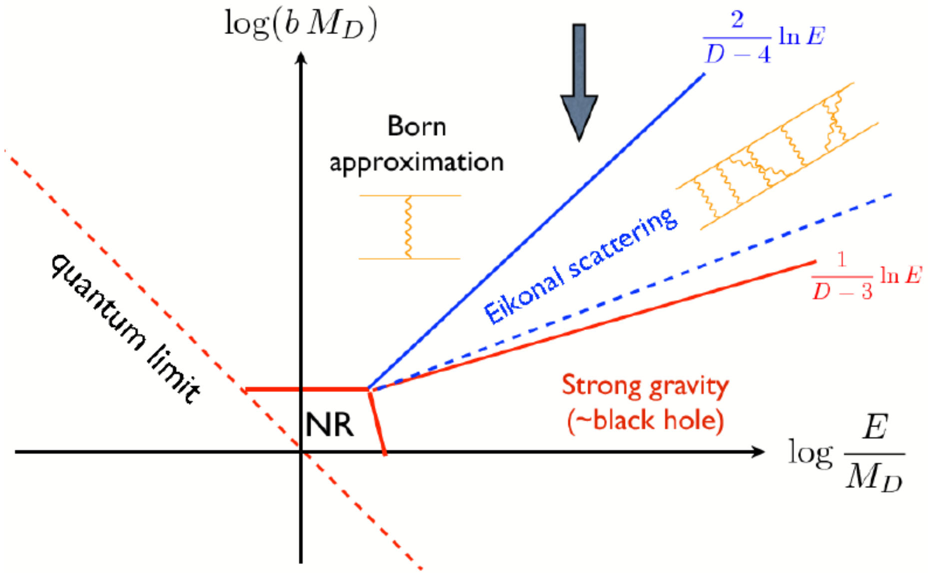


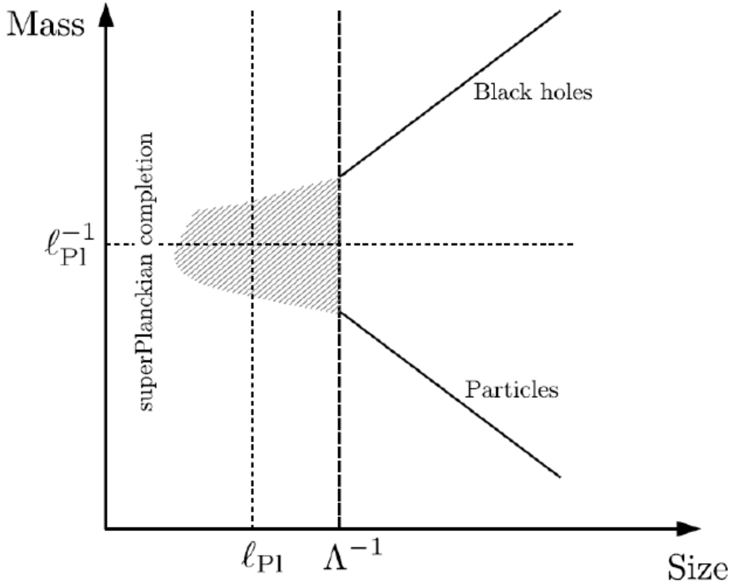
Appendix → Black Holes vs Fuzzstar (1/2)

- $\sqrt{s} \gg \Lambda_P$ & $b \lesssim r_{sch} = 2G\sqrt{s} \Rightarrow$ Black Hole Production (Mass $m = \sqrt{s}$):
[Dvali, Gomez, Isermann, Lust, Stieberger, NPB 893 (2015)]

$$G + G \longrightarrow BH \longrightarrow N \gg 1\,.$$

- Multiparticle Scattering $\Rightarrow$ Collective Transmutation of $\Lambda_{nloc} \Rightarrow \eta_{eff} = N^{\beta}\,\eta$ with $\beta > 0$.
[Buoninfante, Ghoshal, Lambiase, Mazumdar, PRD 99 (2019) 4, 044032].





Appendix → Black Holes vs Fuzzstar (2/2)

- String-Inspired Nonlocal Gravity [Biswas, Gerwick, Koivisto, Mazumdar, PRL 108 (2012) 031101]:

$$S_{GR} = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{g}\left[\mathcal{R} + \mathcal{G}_{\mu\nu}\,\frac{e^{-\Box/M_s^2}-1}{\Box/M_s^2}\,\mathcal{R}^{\mu\nu}\right]\,,\quad M_s \mapsto \frac{M_s}{\sqrt{N}}\,.$$

- BH ↦ Fuzzstar (No Singularity & Horizon) ∼ Fuzzball in String Theory:
  - [Koshelev, Mazumdar, PRD 96 (2017) 8, 084069];
  - [Buoninfante, Koshelev, Lambiase, Mazumdar, JCAP 09 (2018) 034];
  - [Buoninfante, Koshelev, Lambiase, Marto, Mazumdar, JCAP 06 (2018) 014];
  - Echoes in Gravitational Waves? [Buoninfante, Mazumdar, Peng, PRD 100 (2019) 10, 104059].
- Regular Gravitational Potential:

$$\Phi(r) = -\frac{Gm}{r}\,\mathrm{erf}\left(\frac{M_s r}{2}\right)\,,$$

$$\xrightarrow[r\gg 2/M_s]{} -\frac{Gm}{r}\,,$$

$$\xrightarrow[r\ll 2/M_s]{} \frac{GmM_s}{\sqrt{\pi}}\,.$$