Inflation along an accidentally flat direction

Giacomo Ferrante

Work in progress
with
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Outline

1. The Inflationary Paradigm
2. (Hybrid) Natural Inflation
3. Accidental Inflation
4. Cosmic Strings
5. Conclusions

The Inflationary Paradigm

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Slow-roll: $V(\varphi) \gg \frac{1}{2}\dot{\varphi}^2$

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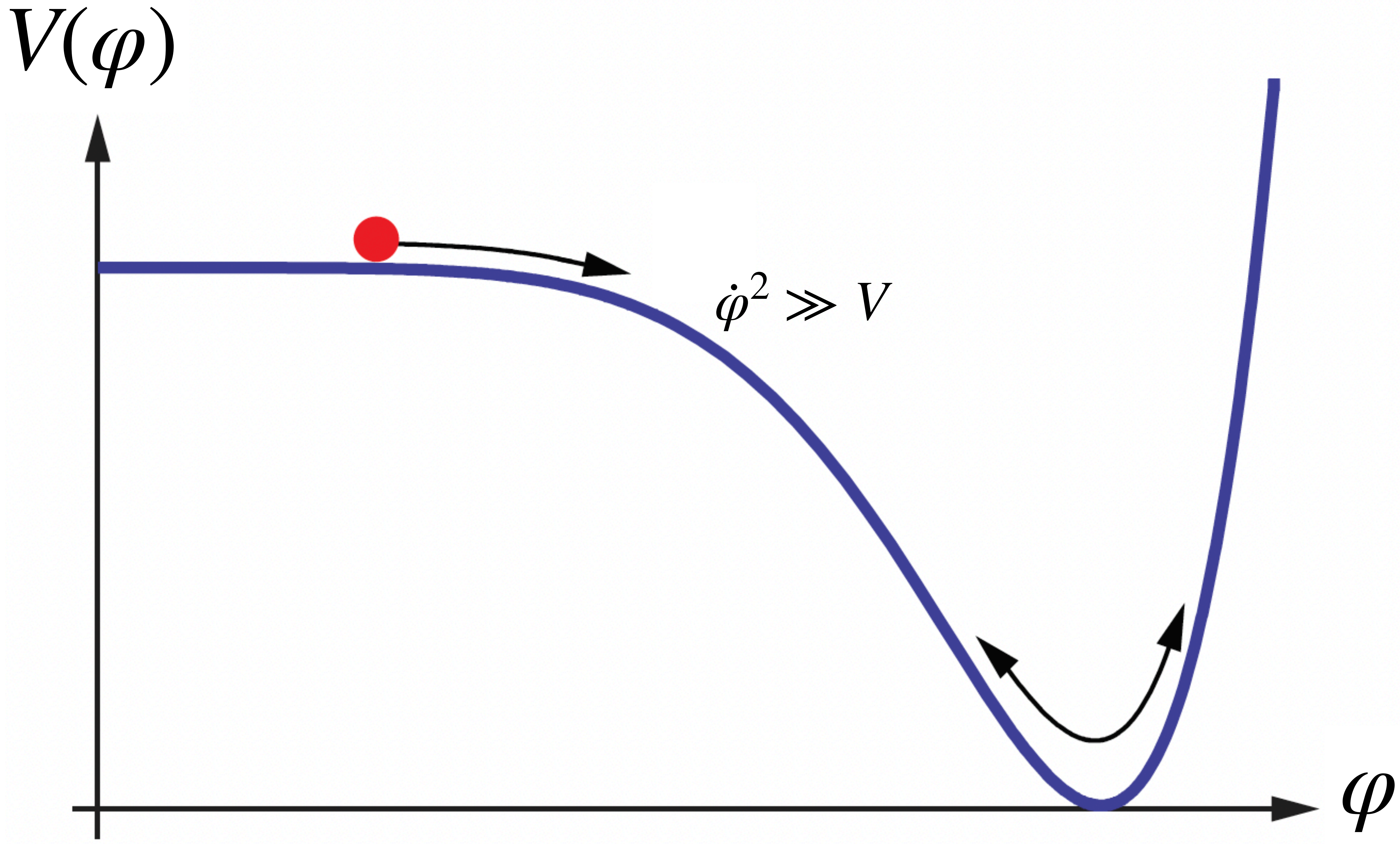

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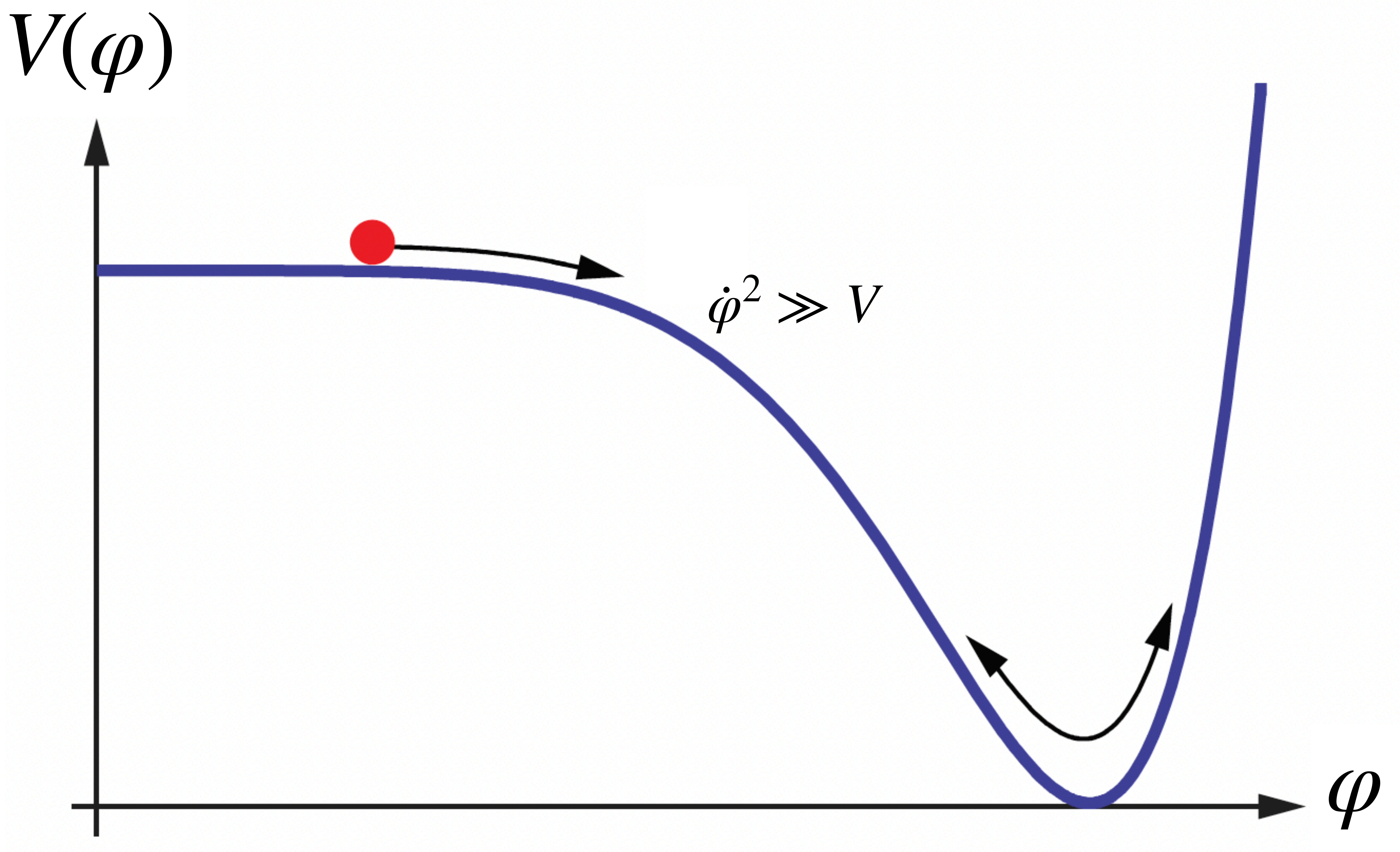

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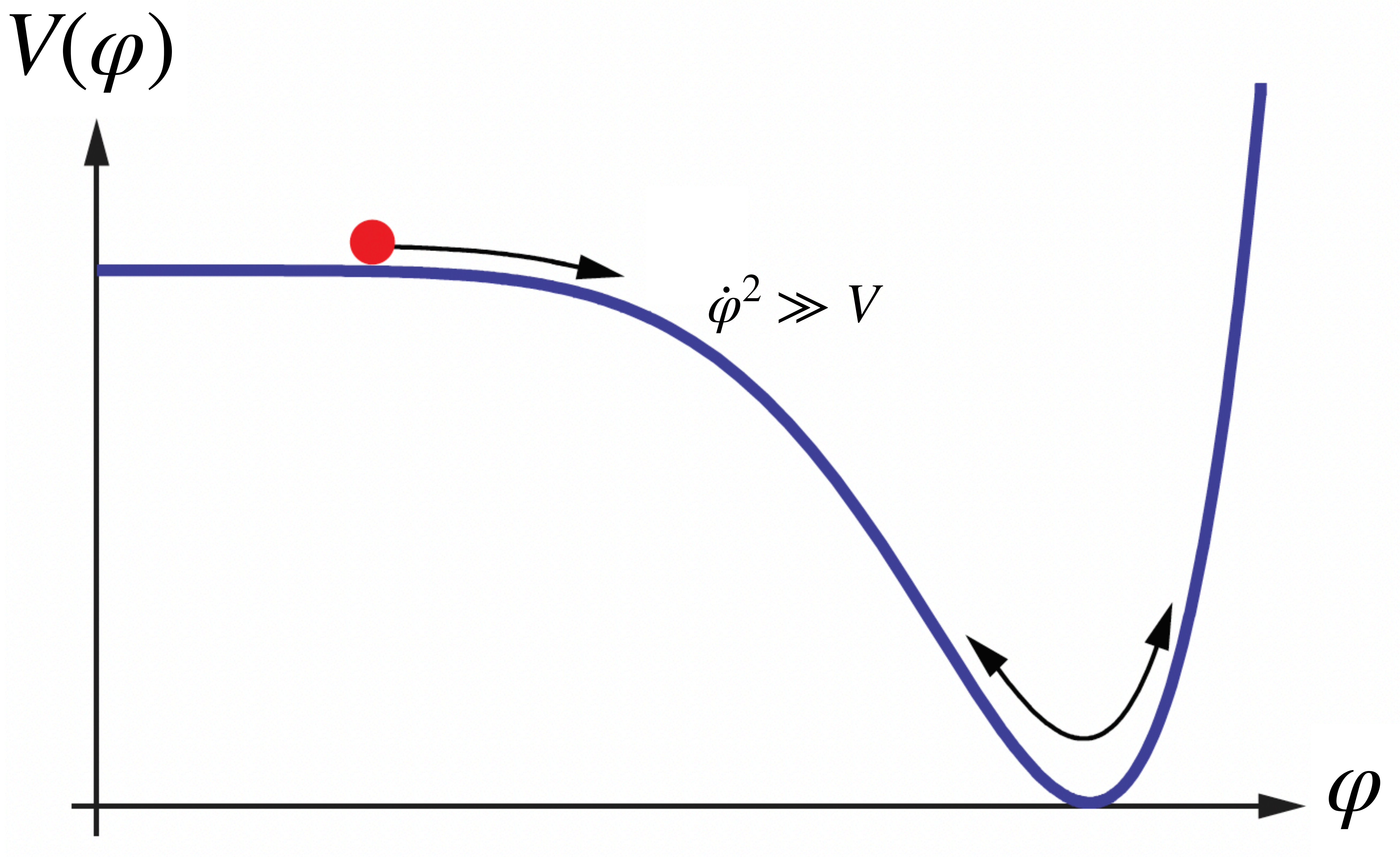

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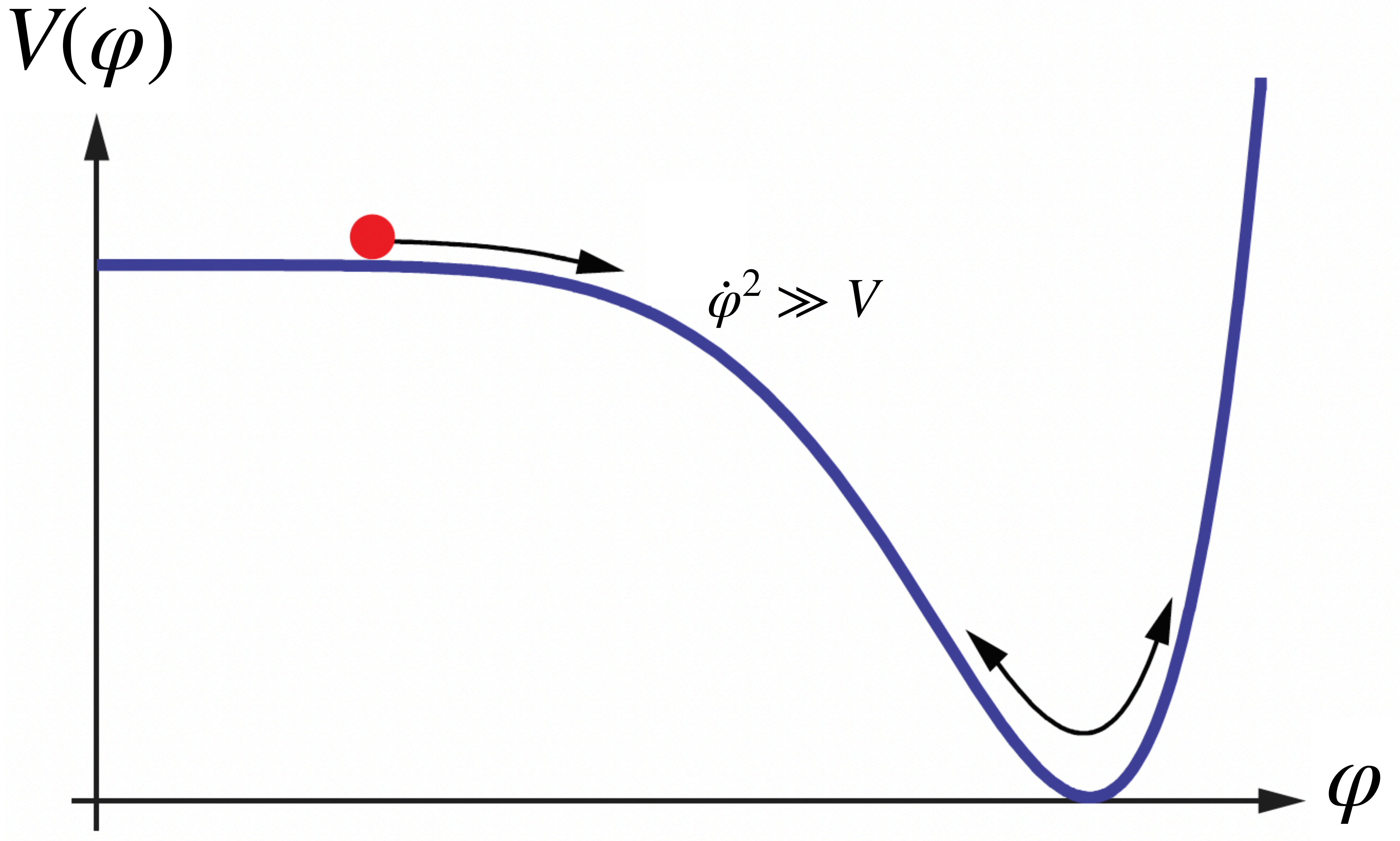

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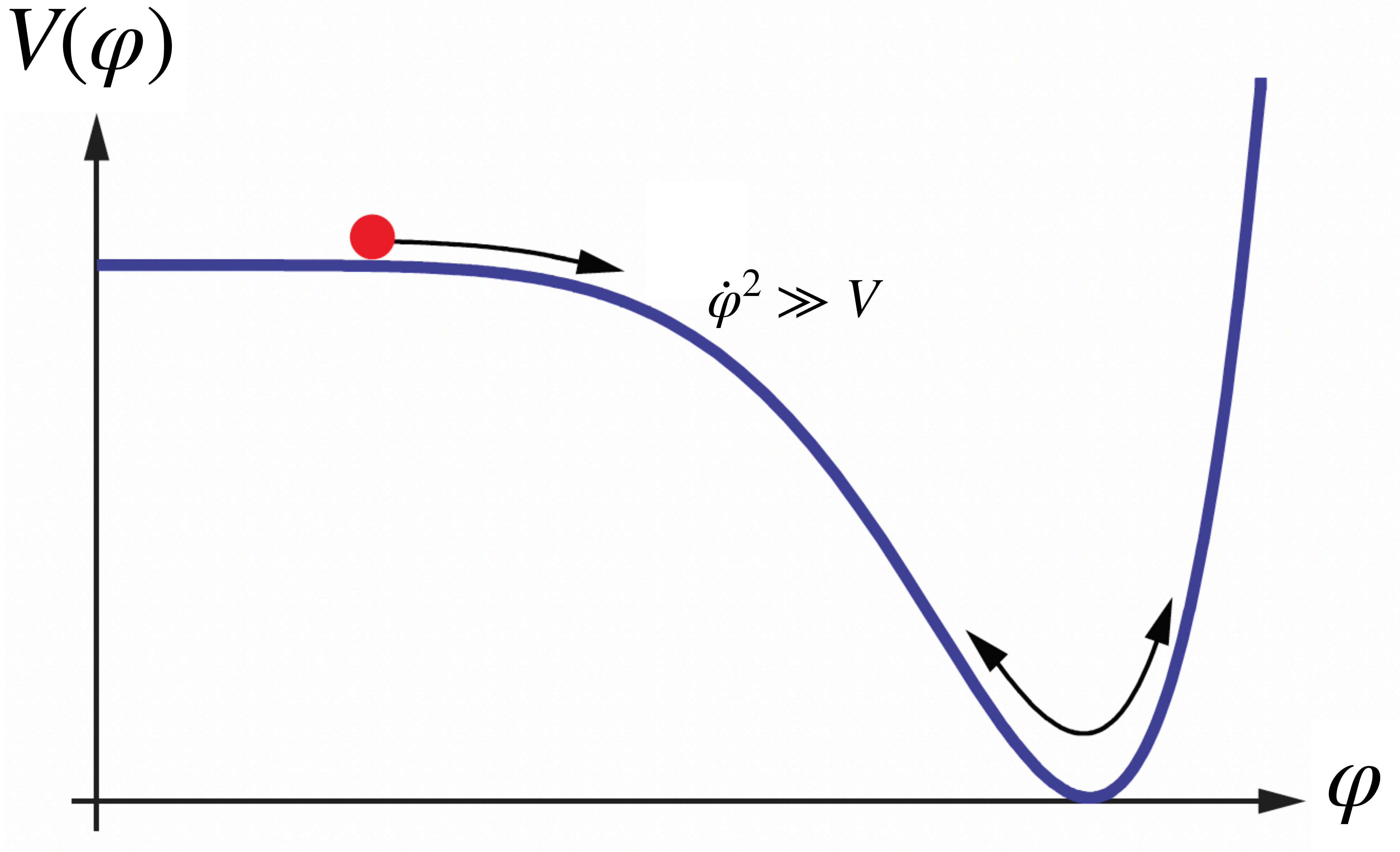

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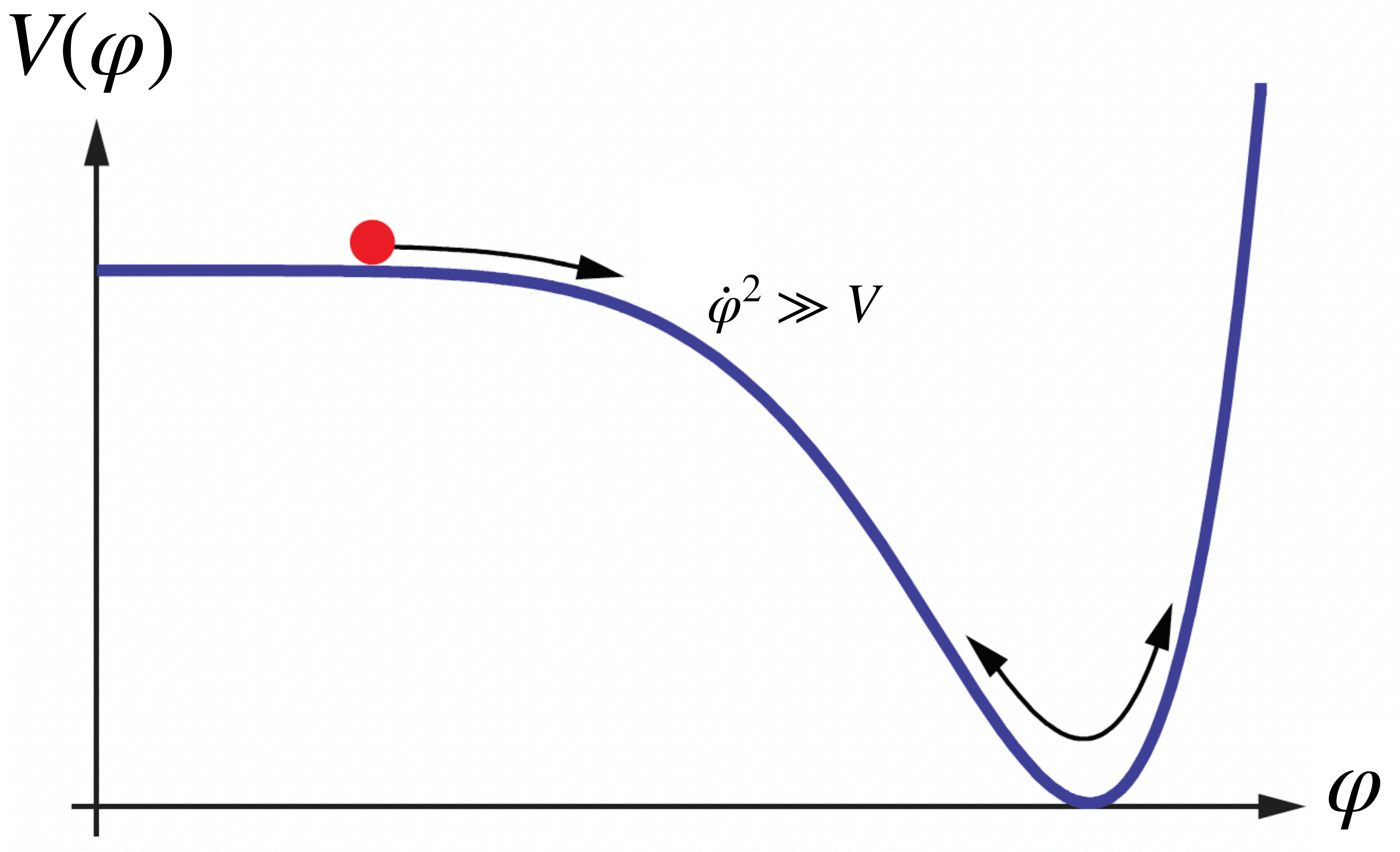

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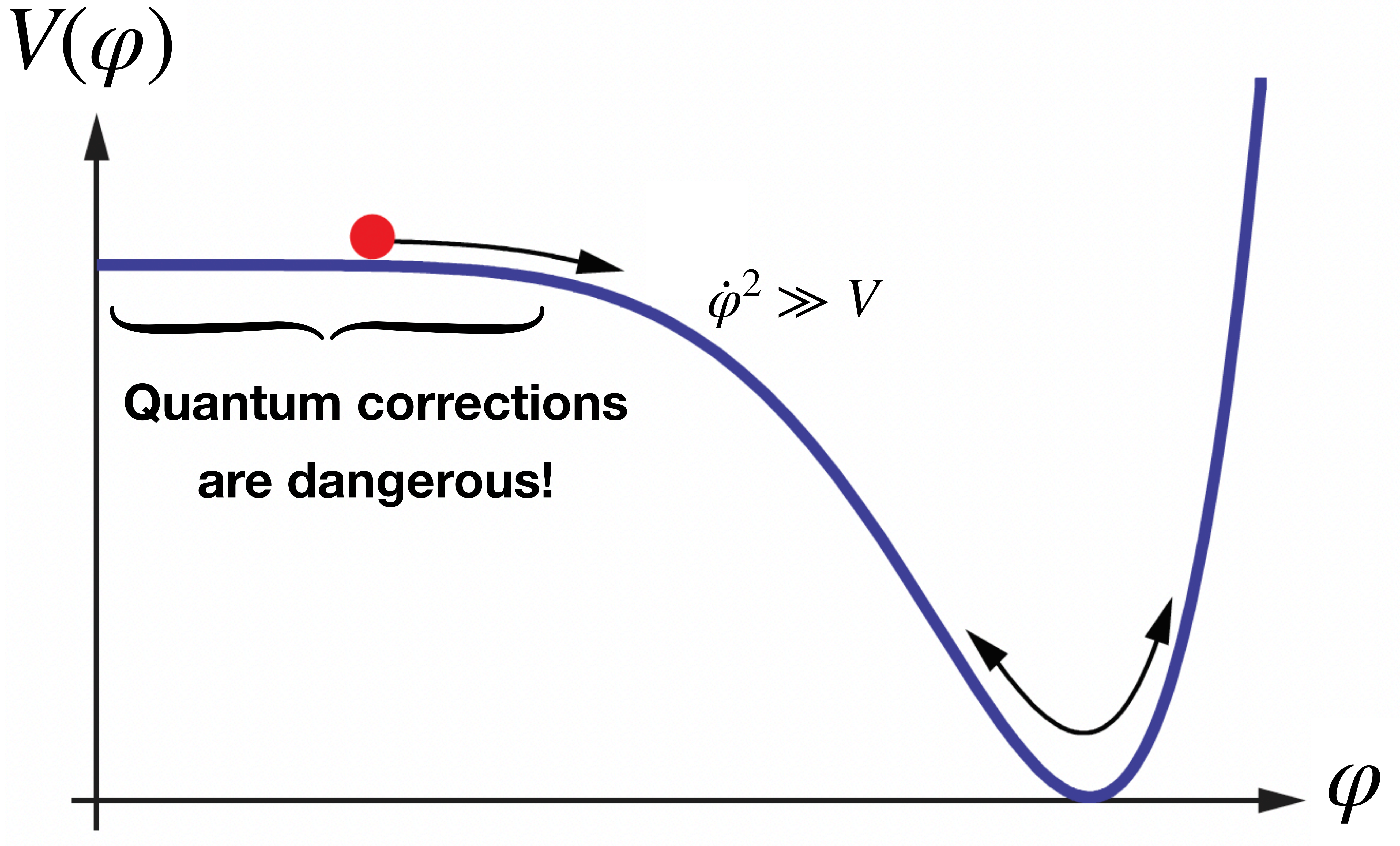

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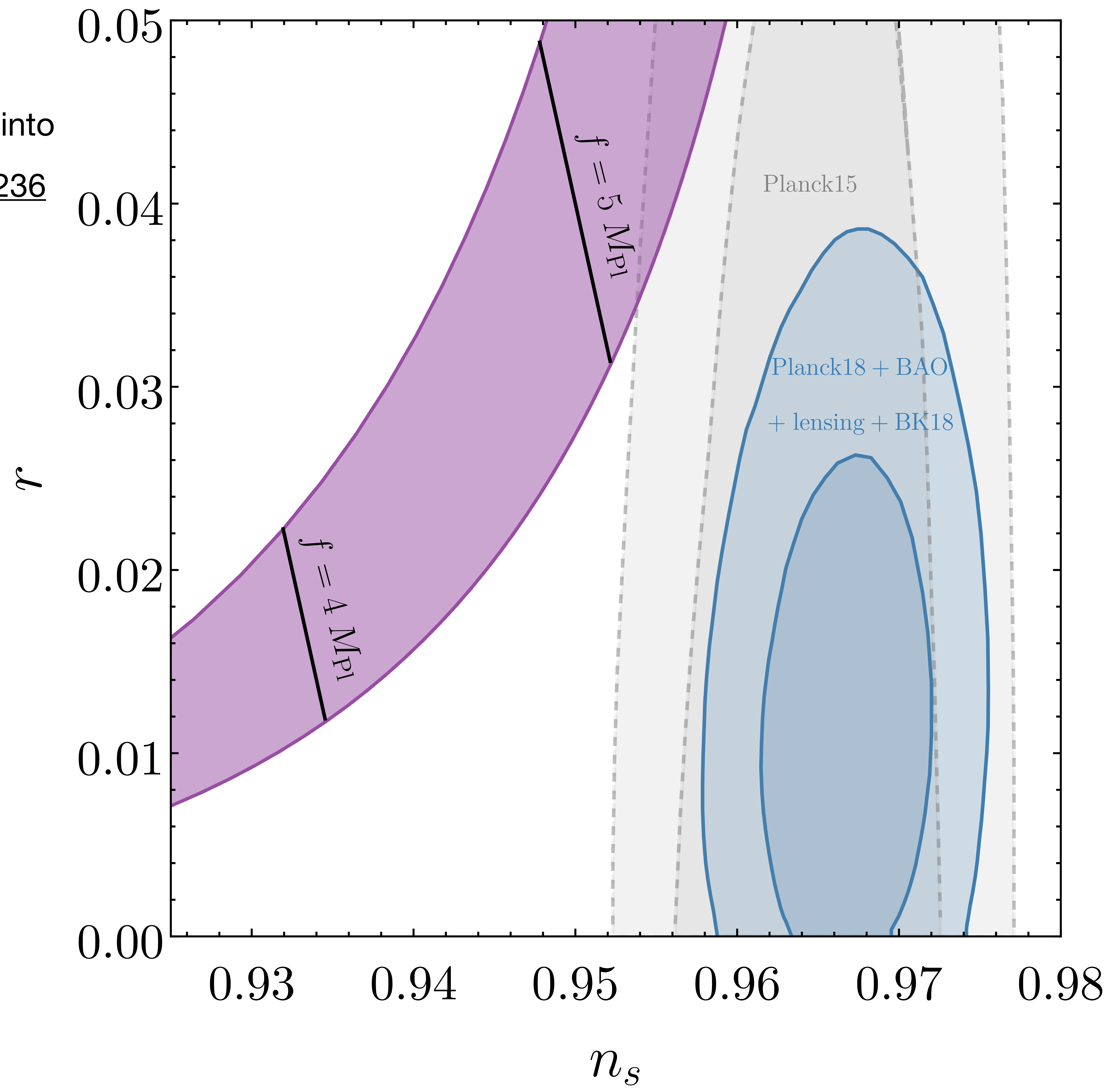

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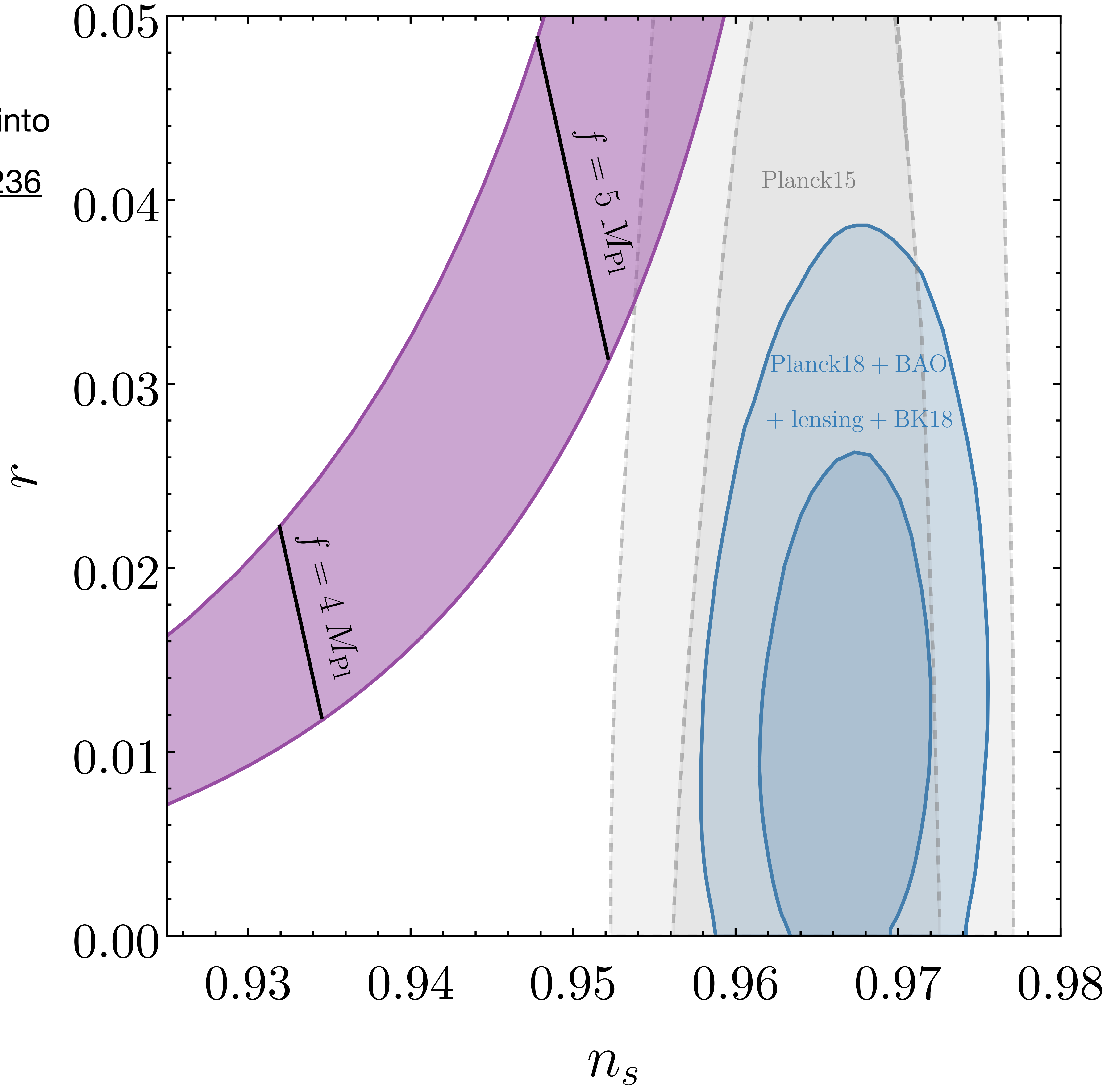

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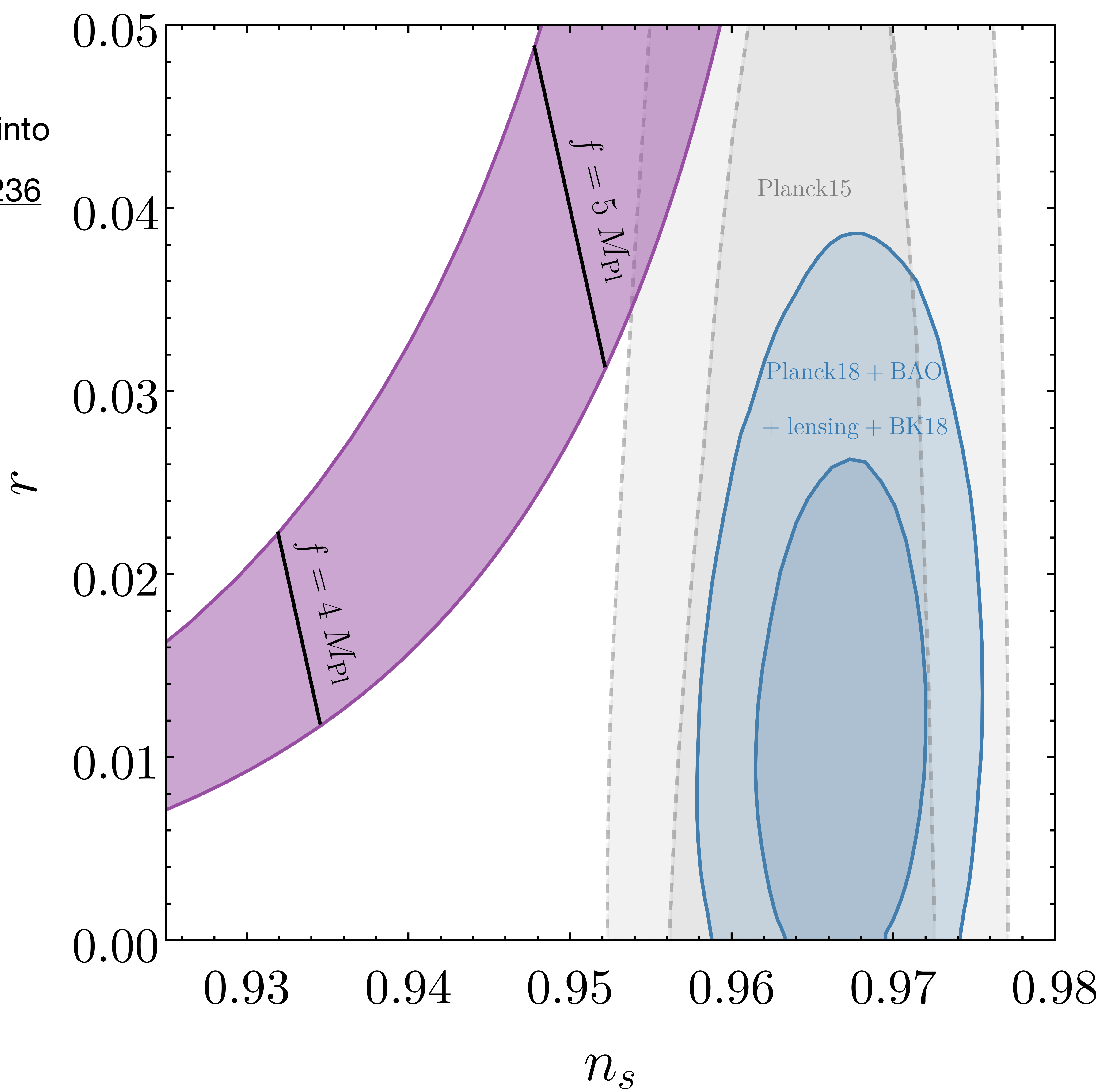

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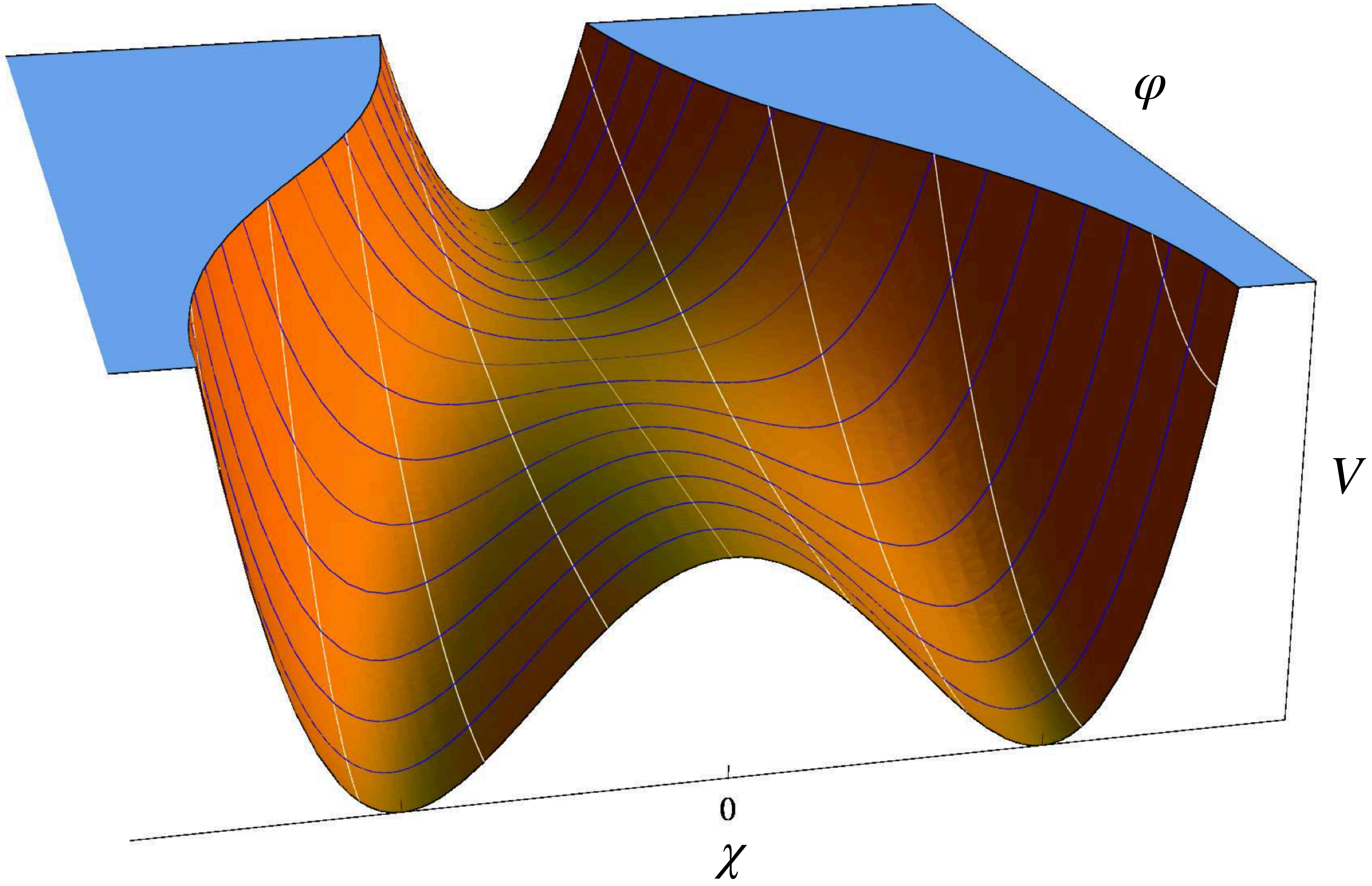
Adapted from: M. Civiletti et al, 1303.3602



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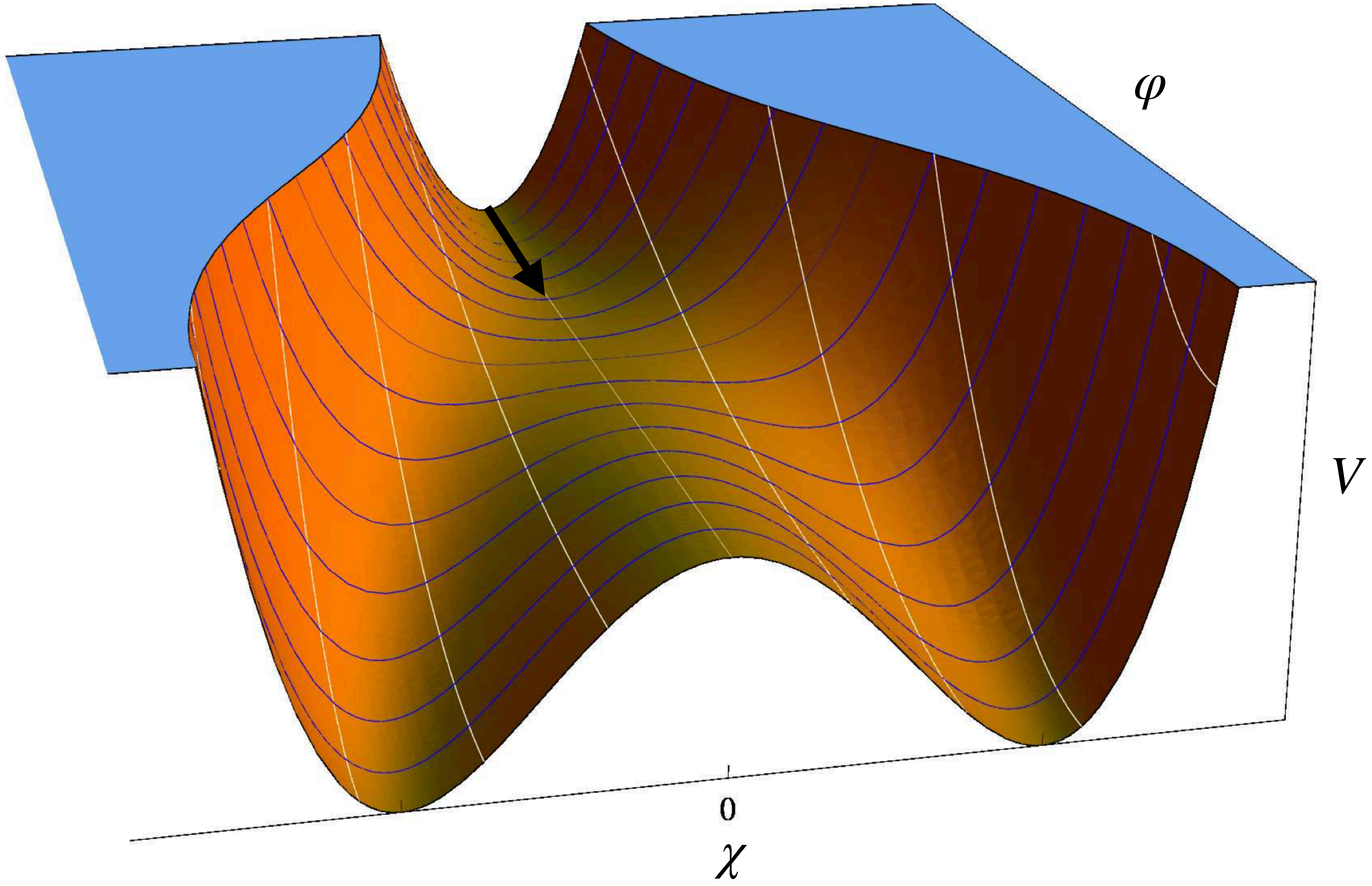

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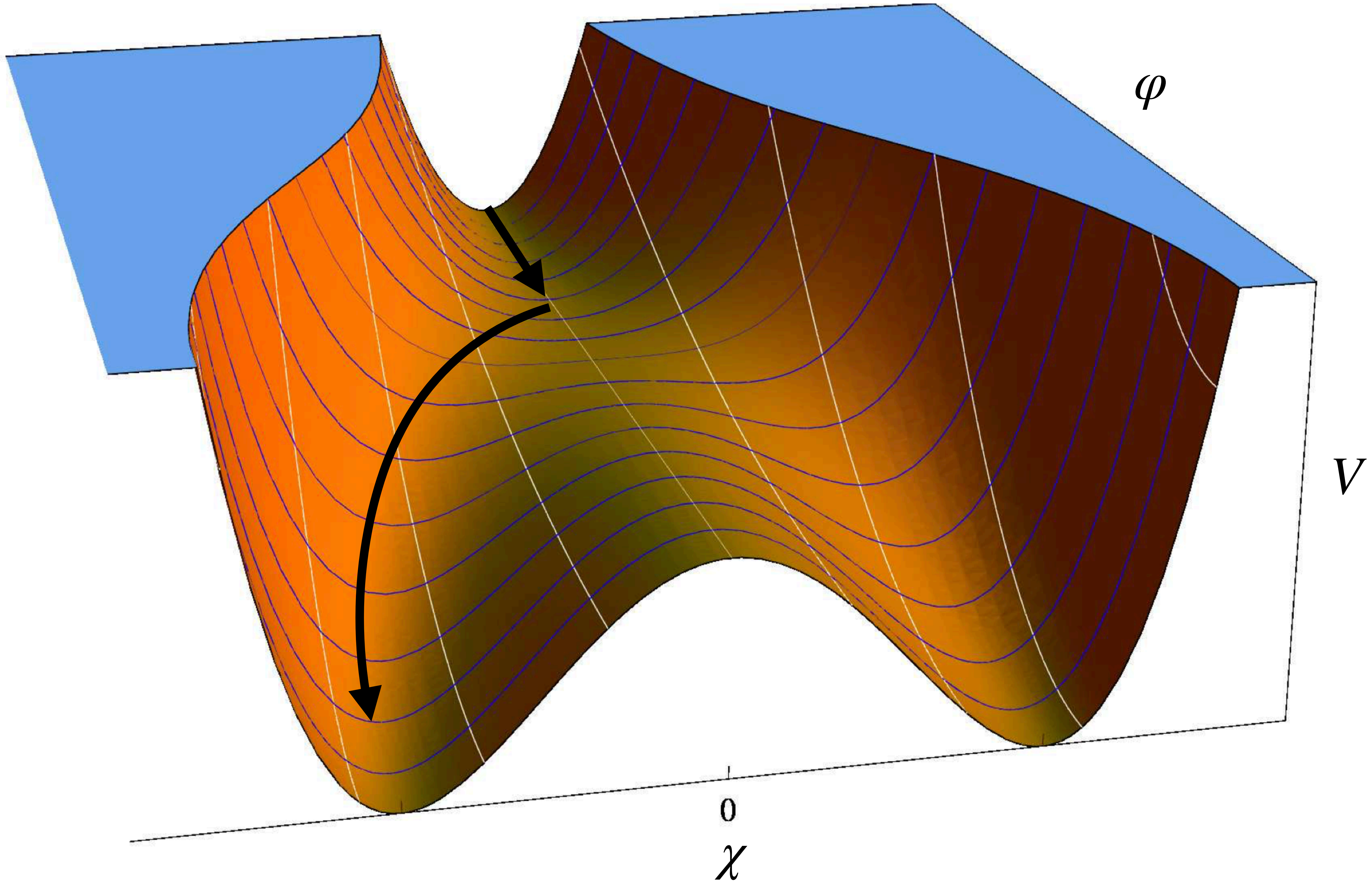

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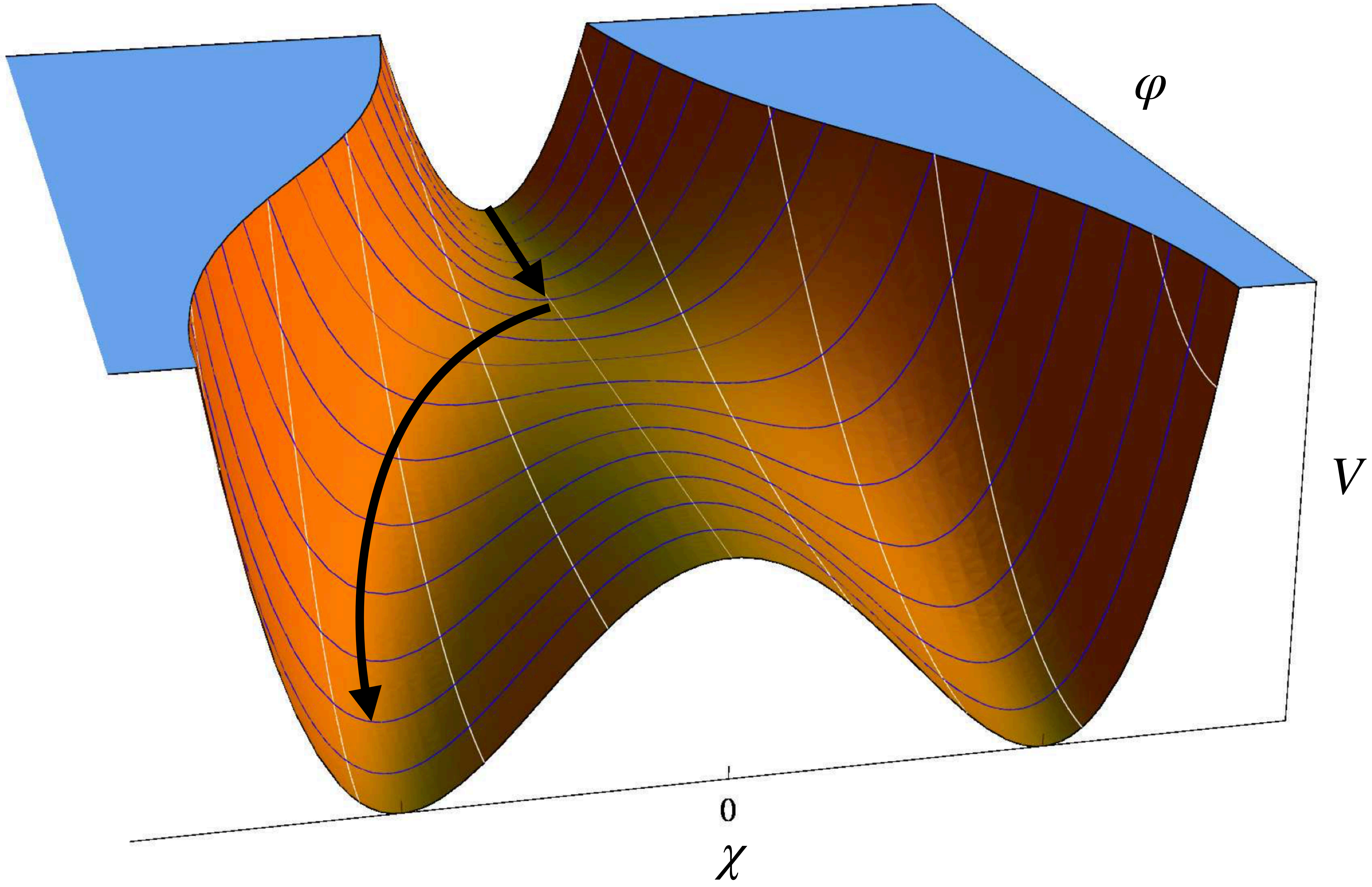

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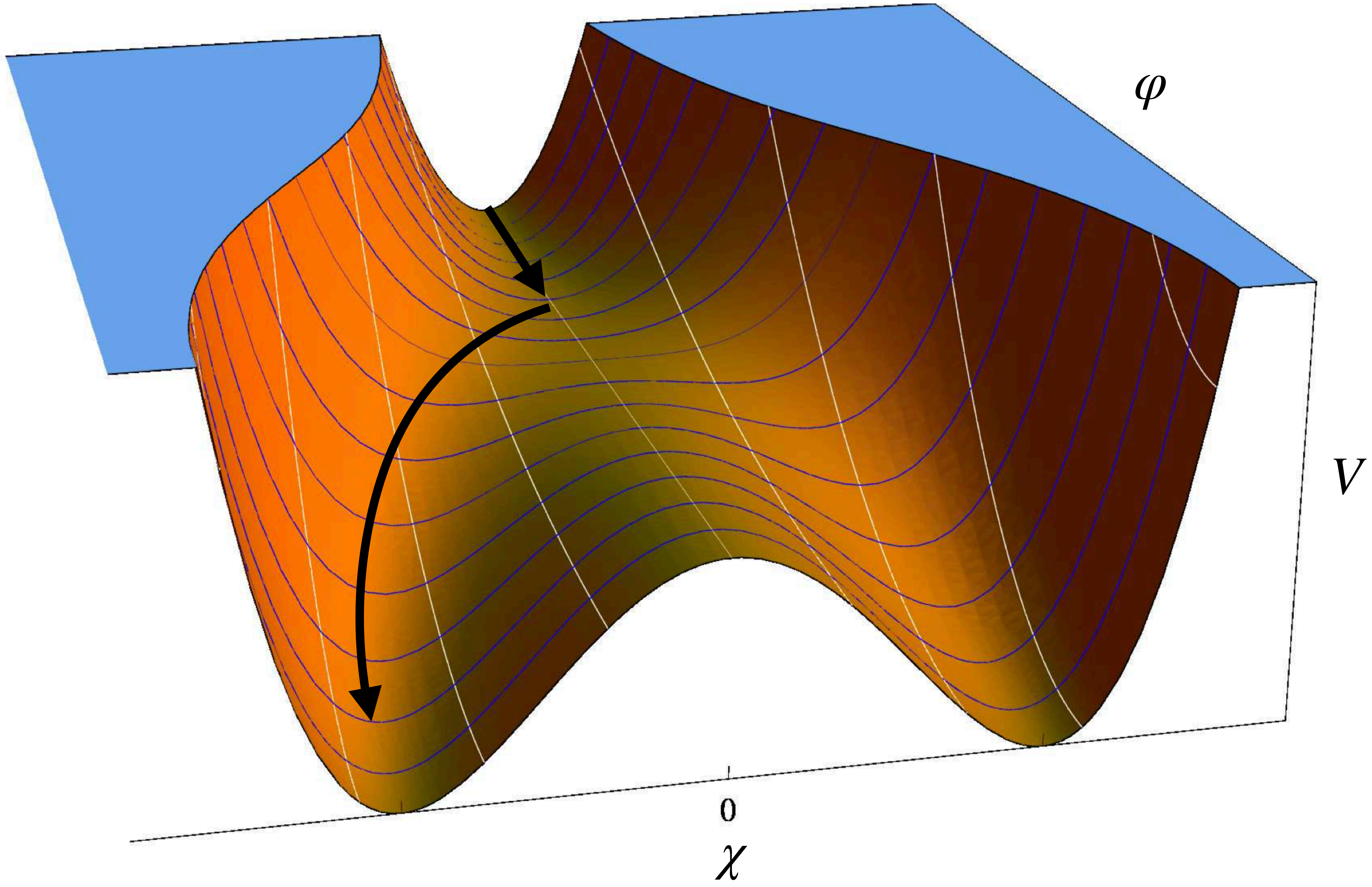

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→ BUT: **Fine-tuning** problems require **ad-hoc discrete symmetries**

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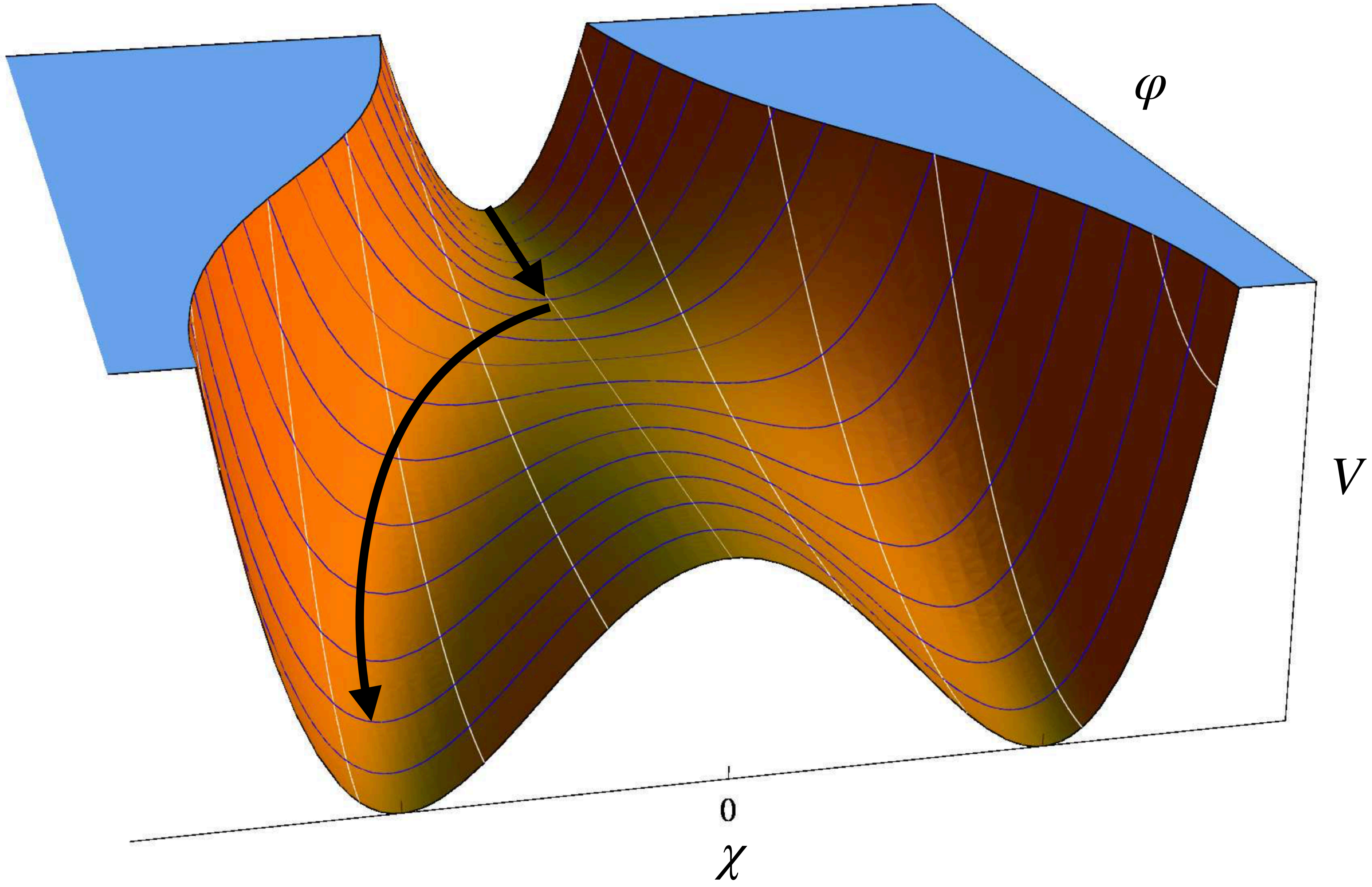

Accidental Inflation

Minimal model

F. Brümmer, GF, M. Frigerio, T. Hambye

2307.10092

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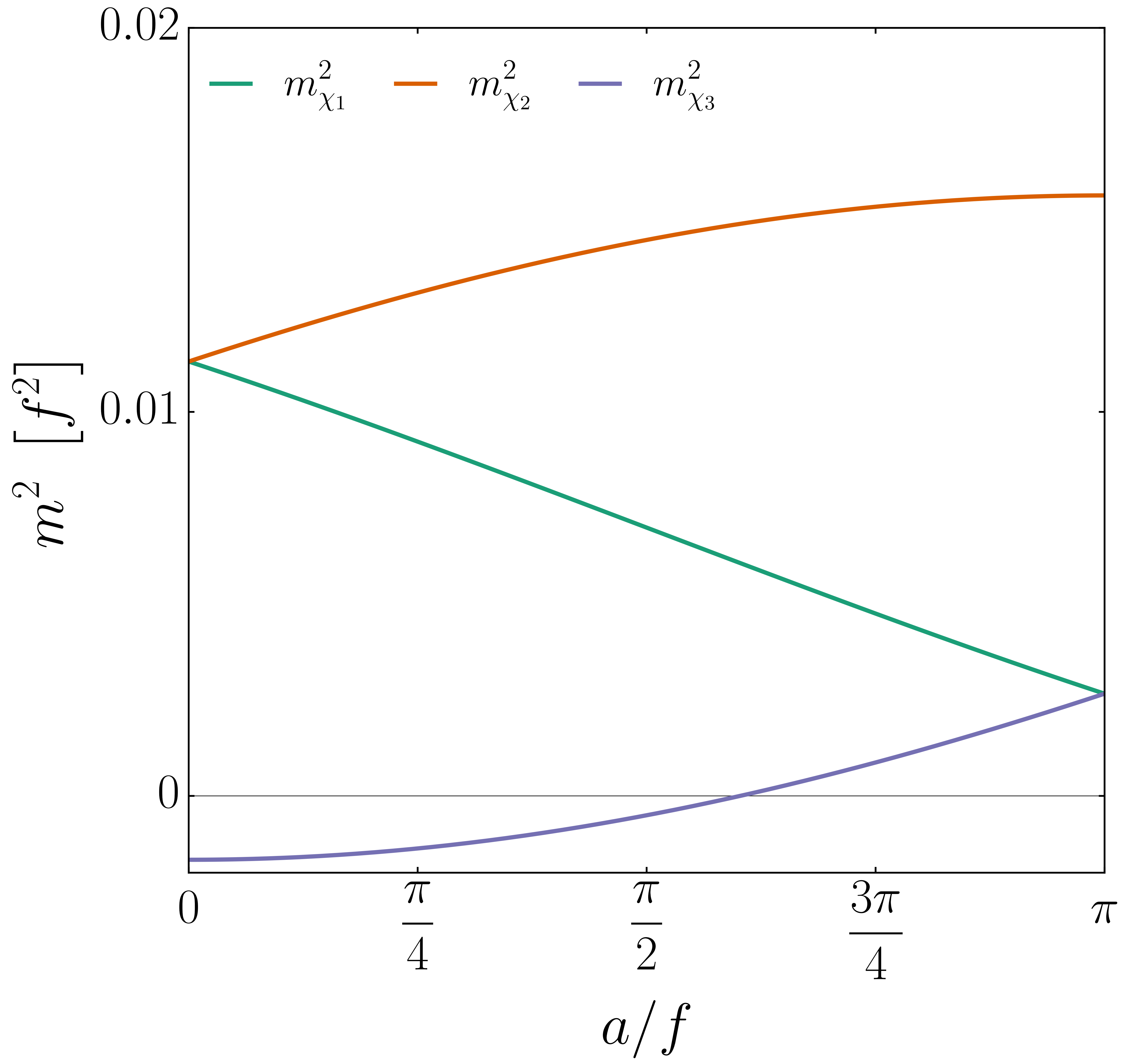

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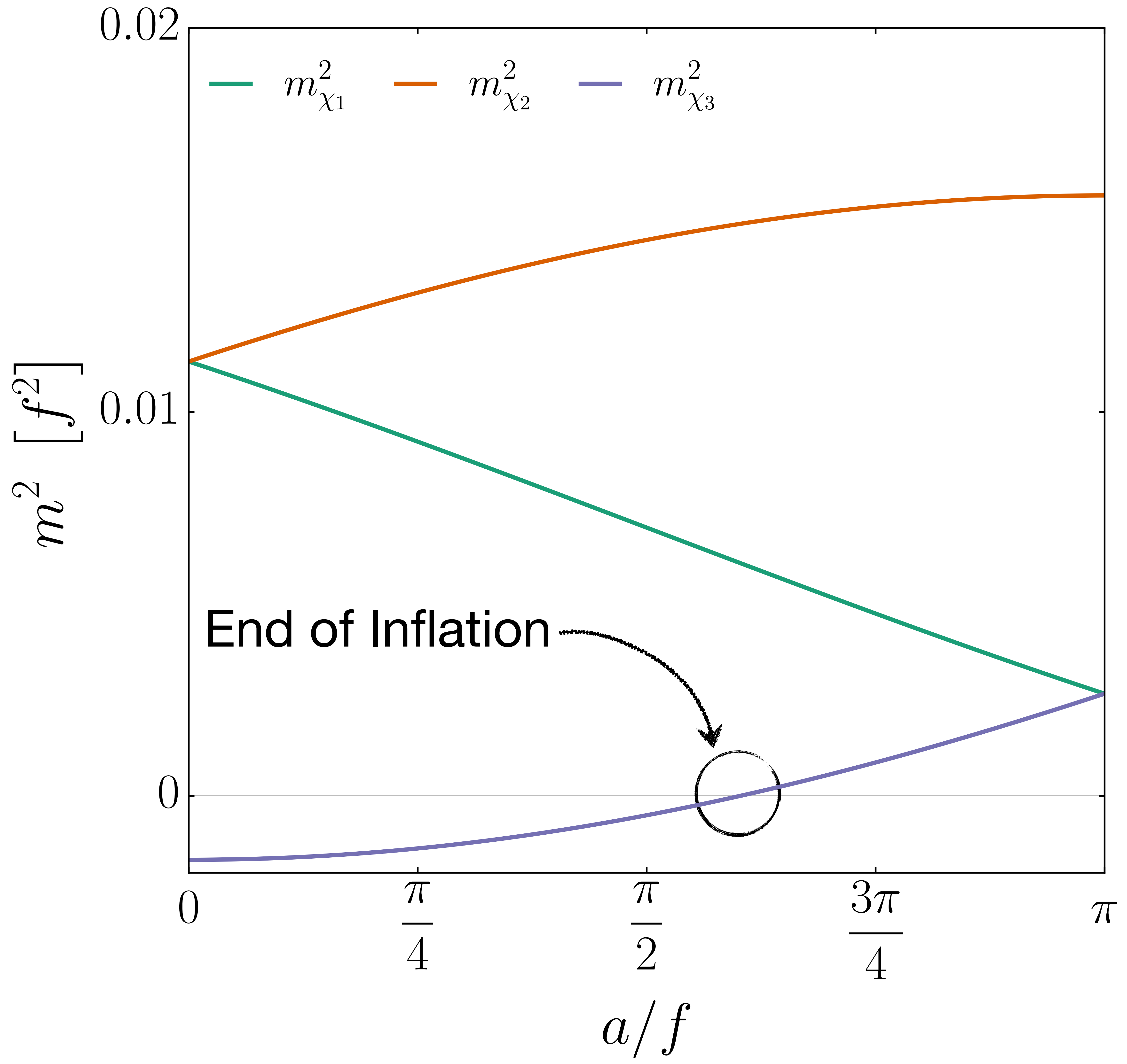

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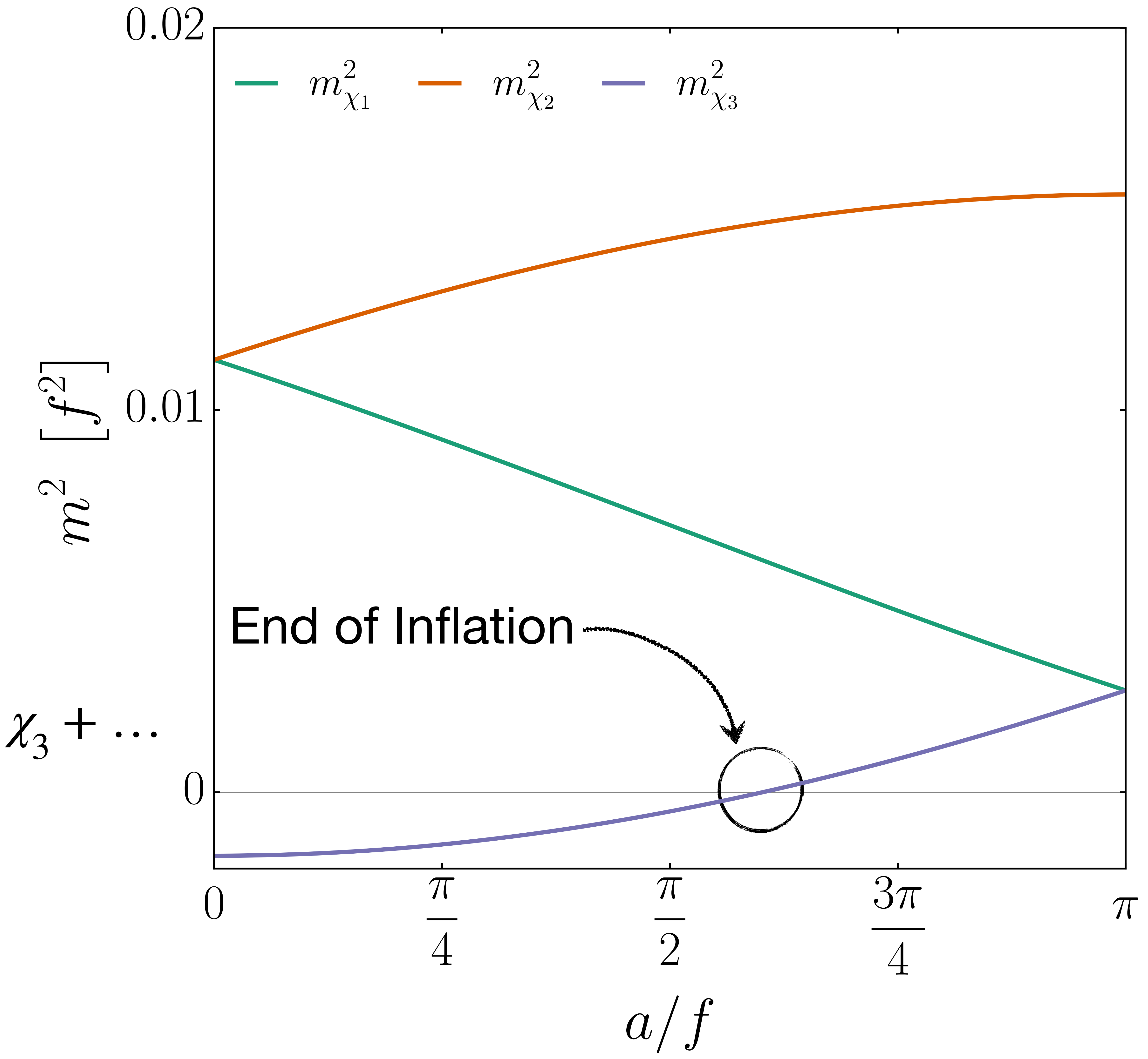

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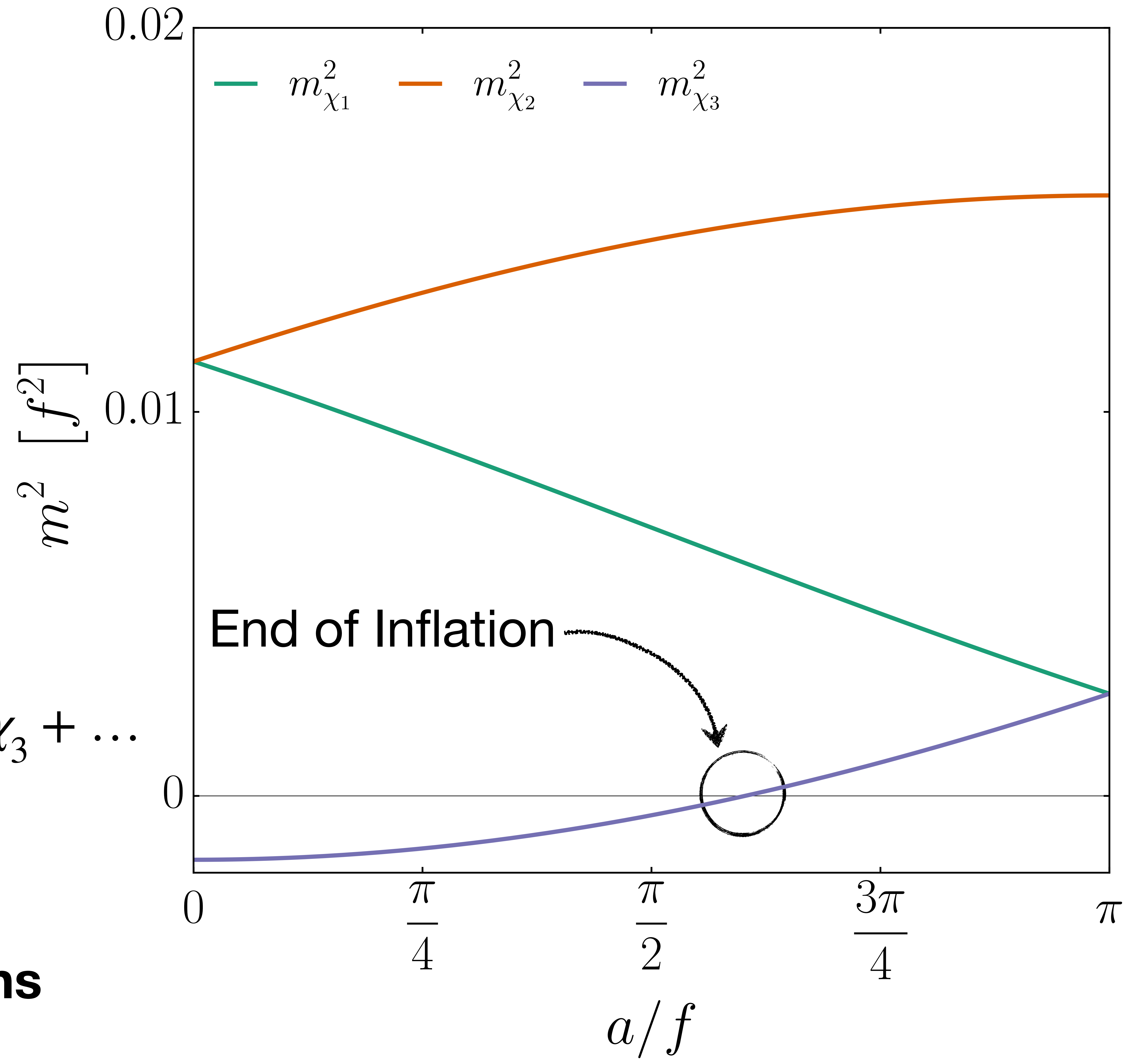

Inflaton = Accident $\Longrightarrow$ **Protection from ALL higher-order corrections**

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CMB

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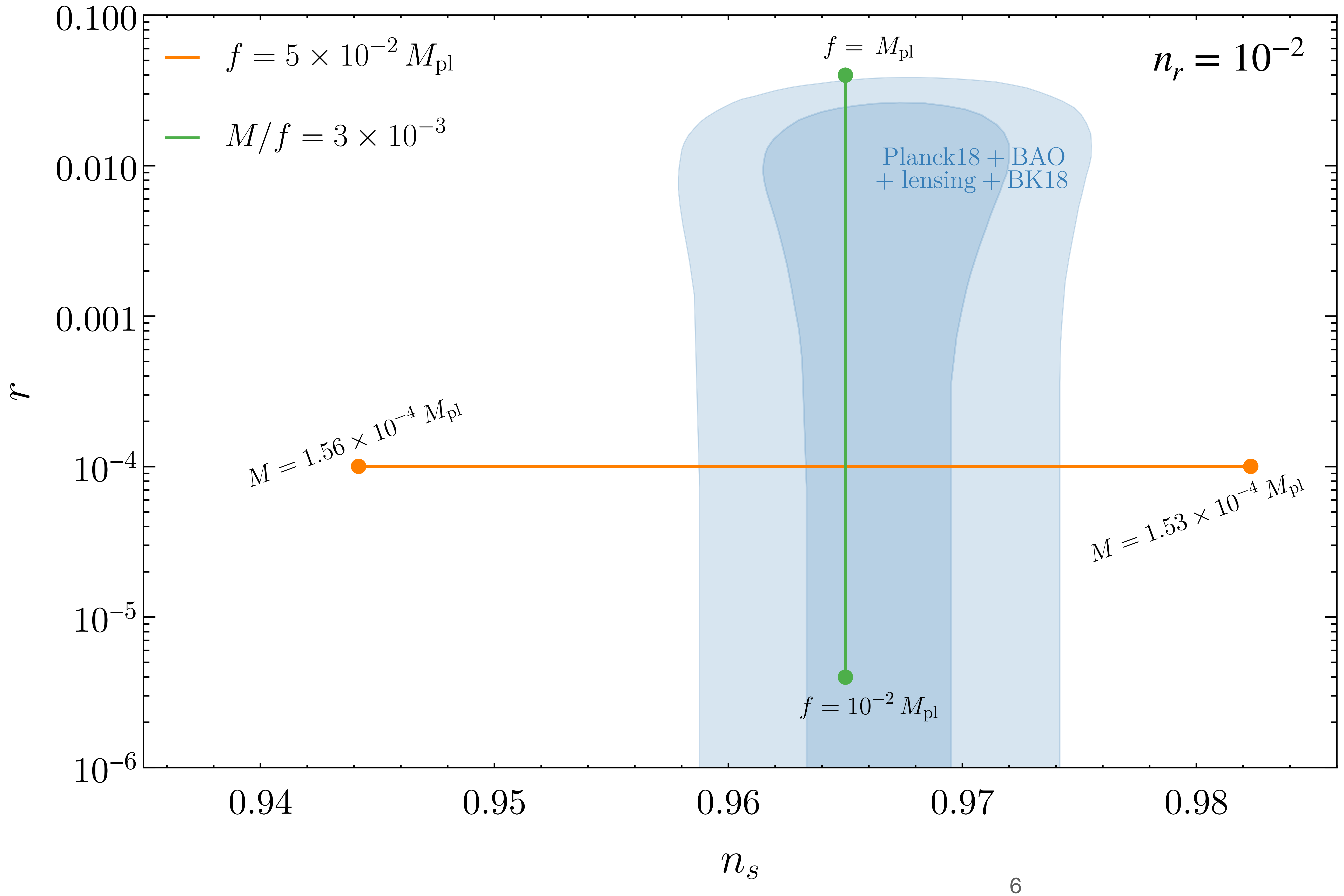

Planck, 1807.06211

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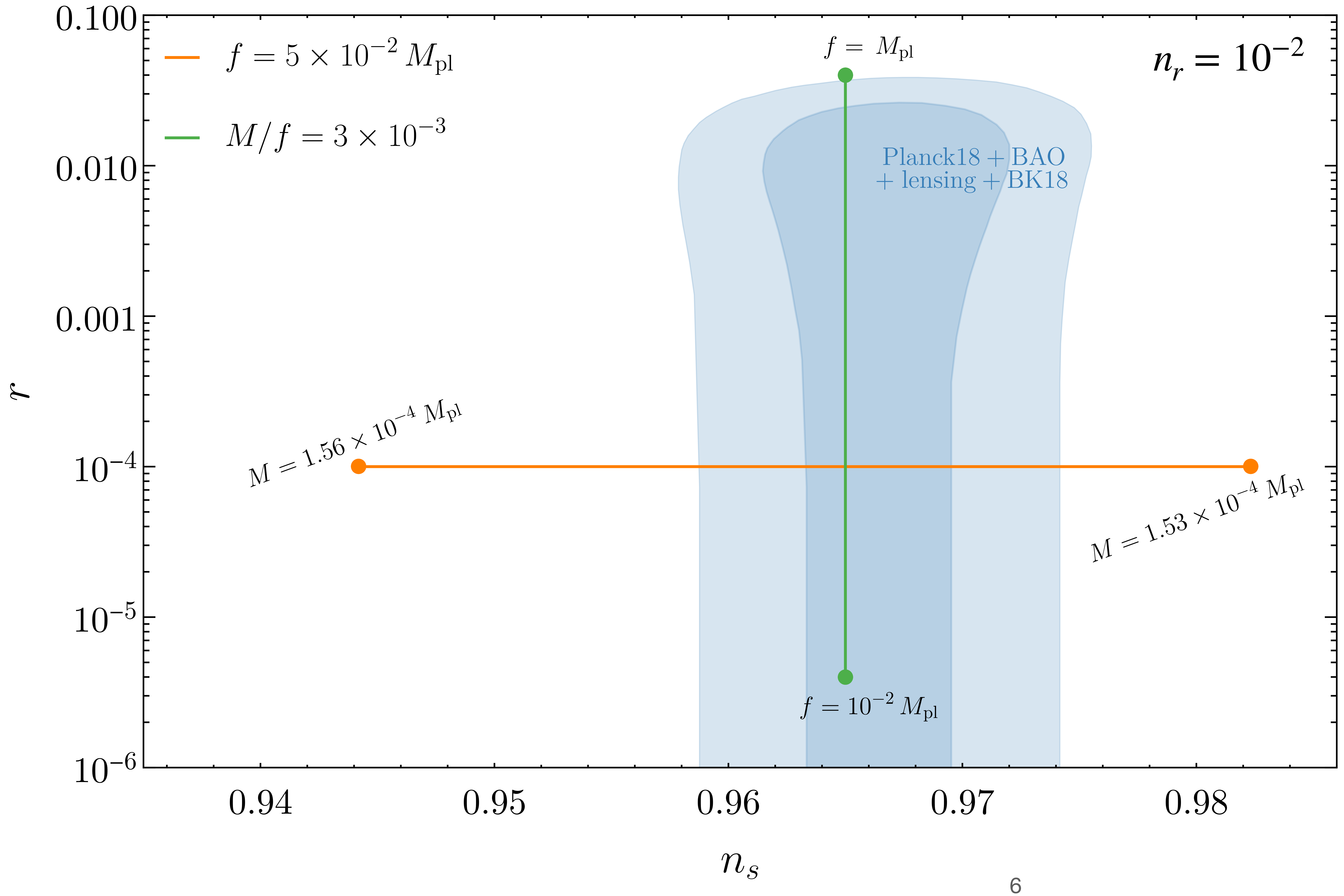

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Successful inflation for f natural

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NANOGrav, 2306.16219

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and ongoing work

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    - Naturalness of the parameters

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and ongoing work

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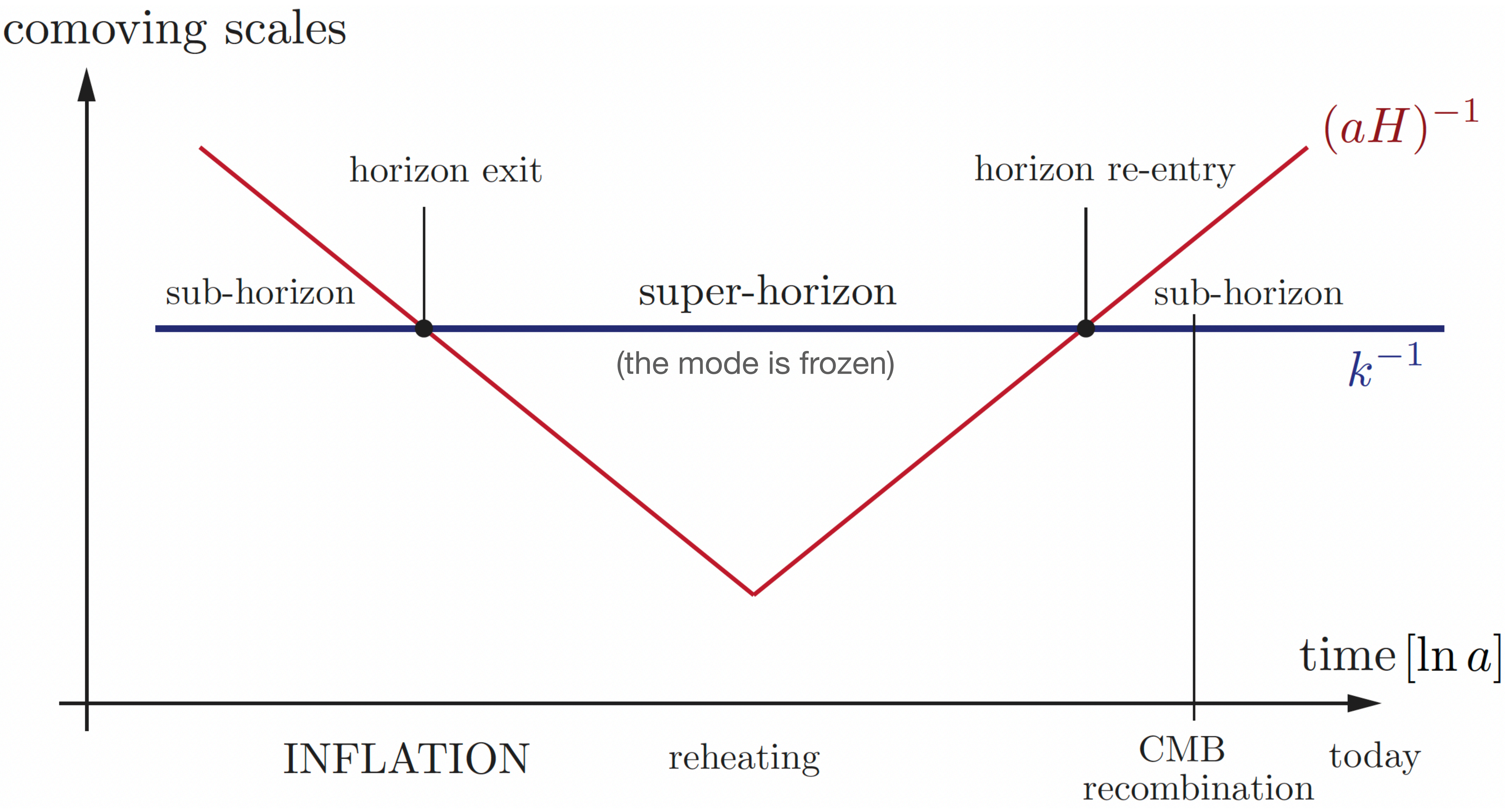
    - Naturalness of the parameters
    - Model with unstable Domain Walls
    - Preheating

Thank you for your attention!

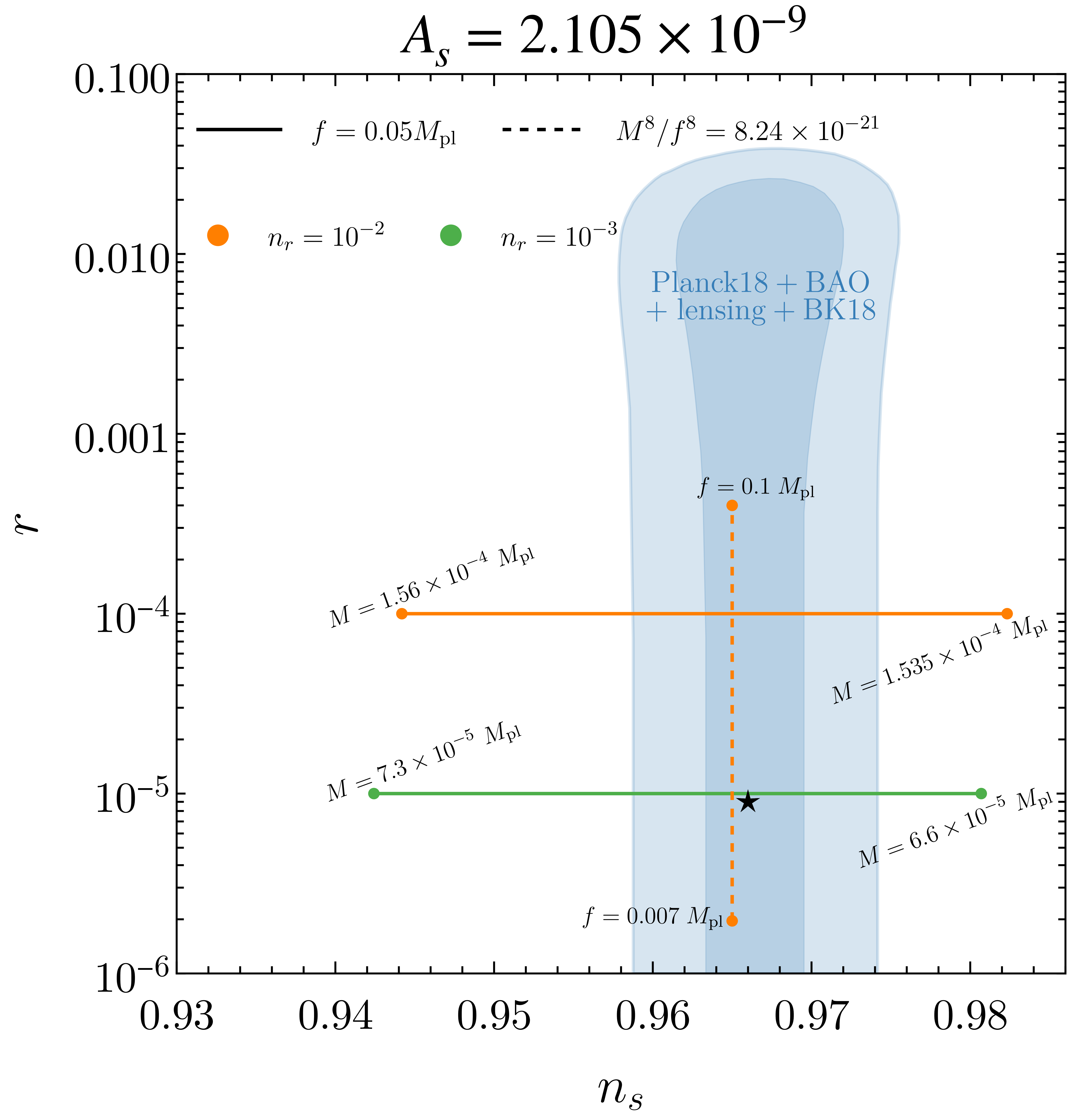
Backup Slides



“The Physics of Inflation”, D. Baumann

$$V \supset \left(\mu_\chi^2 + \epsilon\, \phi^\dagger \phi\right) \chi^\dagger \chi + \frac{1}{2}\left[\zeta\, T^a_{AB} T^b_{BC} \phi^{*A} \phi^C \chi^{*a} \chi^b + \vartheta\, T^a_{AB} (i \varepsilon^a_{bc}) \phi^{*A} \phi^B \chi^{*b} \chi^c\right]$$
$$+ \frac{\lambda_\chi}{4} (\chi^\dagger \chi)^2 + \frac{\lambda'_\chi}{4} |\chi^T \chi|^2$$

$$M^4 = \frac{v^4}{640\, \pi^2} \left(9\, g_2^4 + \frac{\kappa^5}{\delta^3}\, T_6\left(\frac{\delta}{\kappa}\right) + 128\, \zeta^2 \left(\frac{\tilde{\mu}_\chi^2}{\zeta v^2}\right)^5 \widetilde{T}_6\left(\frac{\zeta v^2}{\tilde{\mu}_\chi^2}\right)\right)$$



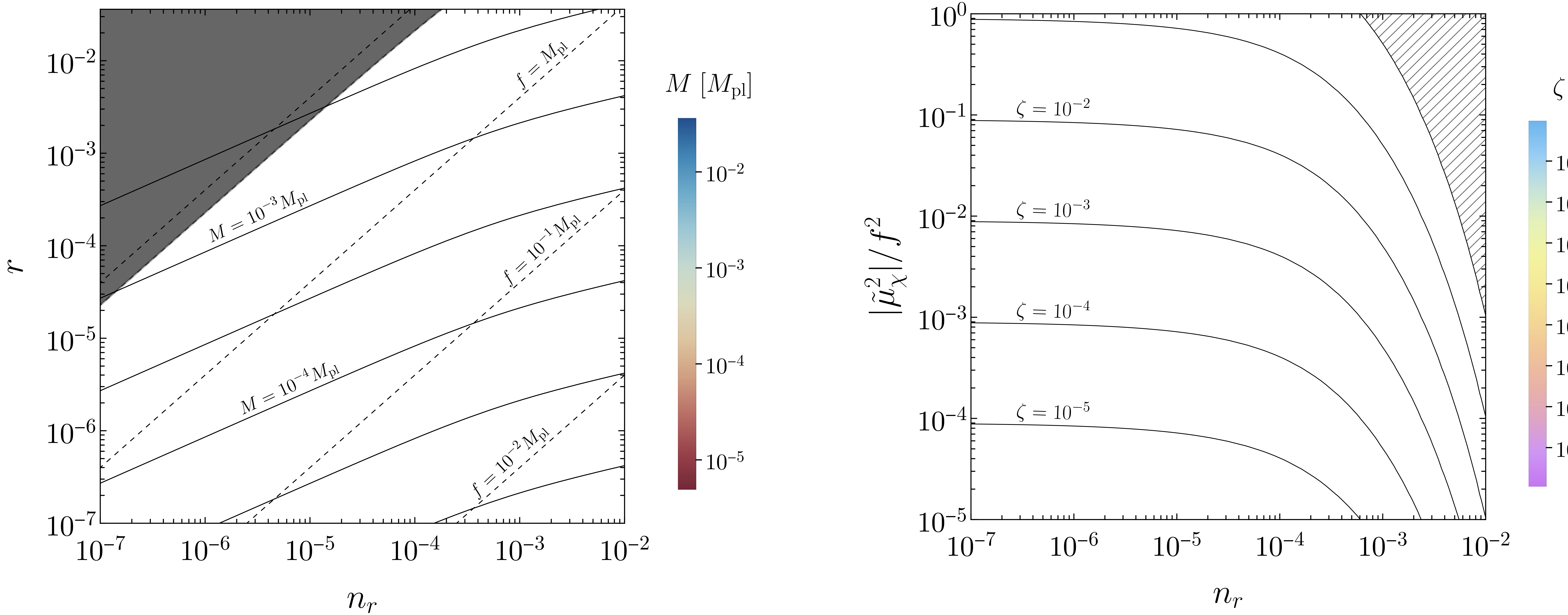
$$\bigstar : \begin{cases} V_0 = 3.1 \times 10^{-13}\, M_{\rm Pl}^4, \quad f = 2.89 \times 10^{-2}\, M_{\rm Pl}, \\ M^4 = 1.1 \times 10^{-17}\, M_{\rm Pl}^4, \quad \tilde{\mu}_\chi^2 = -\left(9.1 \times 10^{-4}\, M_{\rm Pl}\right)^2, \\ \zeta = 1.73 \times 10^{-2} \end{cases}$$

$$A_s \simeq \frac{2}{3\pi^2 r}\frac{V_0}{M_{\rm P}^4}, \qquad r \simeq 8\frac{M_{\rm P}^2}{f^2}\frac{M^8}{V_0^2}\sin^2\frac{a_*}{f},$$

$$n_s \simeq 1 + 2\frac{M_{\rm P}^2}{f^2}\frac{M^4}{V_0}\cos\frac{a_*}{f}, \qquad n_r \simeq \frac{1}{4}\frac{M_{\rm P}^2}{f^2}r.$$

$$V_0 = \frac{3\pi^2}{2} A_s r M_{\mathrm{P}}^4, \qquad M^4 = \frac{3\pi^2}{16} \frac{A_s}{n_r} \sqrt{2n_r + (n_s - 1)^2} r^2 M_{\mathrm{P}}^4,$$

$$f = \frac{1}{2} \sqrt{\frac{r}{n_r}} M_{\mathrm{P}}, \qquad \frac{a_*}{f} = \arccos\left[\frac{n_s - 1}{\sqrt{2n_r + (n_s - 1)^2}}\right].$$



$M\ [M_{\rm pl}]$
r
n_r
$M = 10^{-3} M_{\rm pl}$
$M = 10^{-4} M_{\rm pl}$
$f = M_{\rm pl}$
$f = 10^{-1} M_{\rm pl}$
$f = 10^{-2} M_{\rm pl}$
$|\tilde{\mu}_\chi^2|/f^2$
ζ
$\zeta = 10^{-2}$
$\zeta = 10^{-3}$
$\zeta = 10^{-4}$
$\zeta = 10^{-5}$

r
$f \geq M_{\rm P}$
$\lambda_\chi \geq 1$
$f \leq M_{\rm P}/24\pi$
n_r
$M\ [M_{\rm pl}]$
$|\tilde{\mu}_\chi^2|/f^2$
f too large or too small
ϵ un-natural
n_r
ζ

NANOGrav, 2306.16219

$$(G\mu)^{CMB} \lesssim 10^{-7} \Longrightarrow v_\chi \lesssim 10^{-4}\, M_{\rm Pl}$$

$$f_* \;\sim\; \frac{k_*}{\lambda^{1/4} v}\, 6 \times 10^{10} Hz \;,$$

$$h^2 \Omega^*_{gw} \;\sim\; 2 \times 10^{-6}\, \frac{\lambda v^4}{k_*^2\, M_{\rm Pl}^2} \;,$$