CHIRAL FERMIONS IN GRAVITY

ANOMALIES & PATH INTEGRAL METHODS

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Based on 2312.13222 [hep-th], J. Quevillon, R. Zwicky, R.L.
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QUANTUM ANOMALY

* Definition

$S[\phi, B]$ such that $\delta_\theta S = 0 \Leftrightarrow \partial j = 0$ Noether th

Quantum effective action: $\boxed{W[B] = -i \log \int \mathcal{D}\phi \, e^{iS[\phi, B]}}$

$$\delta_\theta W = \int \theta \, \mathcal{A} \Leftrightarrow \langle \partial j \rangle \neq 0$$

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* Example

Chiral anomaly

$$S = \int \bar{\Psi}(i\partial\!\!\!/ - V\!\!\!/ - A\!\!\!/\gamma_5)\Psi$$

Classical	Quantum
$\delta_\theta^V S = 0$	$\delta_\theta^V W[V,A] = 0$
$\delta_\theta^A S = 0$	$\Rightarrow \boxed{\delta_\theta^A W[V,A] \neq 0}$

WEYL SYMMETRY

"Scale symmetry in gravity"

* Example: massless fermion

$$S = \int \sqrt{g}\, \bar{\Psi} \left(i \not{\partial} + i \cancel{\not{\omega}} \right) \Psi$$

spin-connection

$$\delta_\sigma^W \Psi = -\frac{3}{2}\sigma \Psi$$

$$\delta_\sigma^W g_{\mu\nu} = 2\sigma g_{\mu\nu}$$

$$\delta_\sigma^W \omega_\mu = \dots$$

$$\delta_\sigma^W S = \int \delta g_{\mu\nu} \frac{\delta S}{\delta g_{\mu\nu}} = \int \sqrt{g}\, \sigma g_{\mu\nu} T^{\mu\nu} = 0, \quad T^{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S}{\delta g_{\mu\nu}}$$

on-shell

$$\boxed{\text{Class. Weyl inv.} \Leftrightarrow \delta_\sigma^W S = 0 \Leftrightarrow T^\mu{}_\mu = 0}$$

WEYL ANOMALY (Capper & Duff '74)

"Breaking of scale invariance by quantum scale"

Quantum fluctuations of the vacuum ⇒ Casimir effect

Hawking radiation

⋮

$$\delta_\sigma^W W[g_{\mu\nu}] = \int \sqrt{g}\, \sigma\, g_{\mu\nu} \langle T^{\mu\nu} \rangle$$

Anomaly:

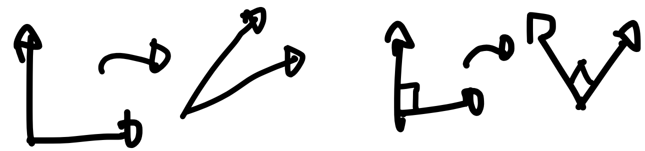
$$\boxed{\mathcal{A}_{Weyl} = g_{\mu\nu} \langle T^{\mu\nu} \rangle = \frac{1}{16\pi^2}\left(-\frac{1}{20} E + \frac{11}{360} W^2 - \frac{1}{30} \Box R \right)}$$

Euler density (topological): $E = R_{\mu\nu\rho\sigma}^2 - 4 R_{\mu\nu}^2 + R^2$

Weyl tensor squared: $W^2 = R_{\mu\nu\rho\sigma}^2 - 2 R_{\mu\nu}^2 + \frac{1}{3} R^2$

SPACETIME SYMMETRIES

* **Lorentz symmetry:** boosts & rotations



$\frac{\delta \mathcal{L}}{\delta \alpha}$, $\alpha_{\mu\nu} = -\alpha_{\nu\mu}$

$$\boxed{\text{class. Lorentz inv.} \Leftrightarrow T^{\mu\nu} - T^{\nu\mu} = 0 \ \text{on-shell}}$$

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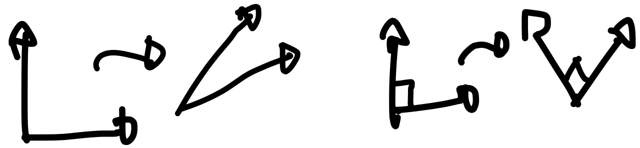
* **Lorentz symmetry**: boosts & rotations 

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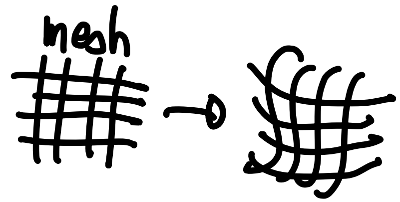
* **Diffeomorphism invariance**: change of coordinates 

$\delta_\xi \mathcal{L}$, ξ^μ

$$\boxed{\text{class. diffeo. inv.} \Leftrightarrow D^\nu T_{\mu\nu} - \underbrace{\omega_{\mu ab} T^{ab}}_{\text{antisym}} = 0 \text{ on-shell}}$$

$\omega_{\mu ab} = -\omega_{\mu ba}$
(& pure gravity)

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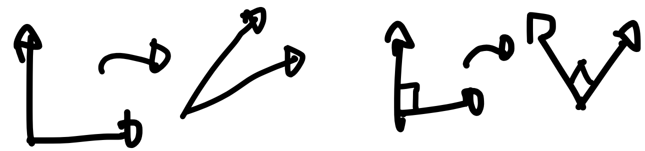
* **Lorentz symmetry:** boosts & rotations 

δ^L_α, $\alpha_{\mu\nu} = -\alpha_{\nu\mu}$

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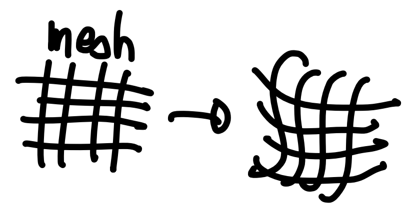
* **Diffeomorphism invariance:** change of coordinates 

δ^d_ξ, ξ^μ

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(& pure gravity)

* **Anomalies:** in $d=4$ (mod 4) $\delta^L_\alpha W = \delta^d_\xi W = 0$ for chiral fermions
(& pure gravity)

(Alvarez-Gaume & Witten 83')

in $d=2$ (mod 4) $\delta^L_\alpha W, \delta^d_\xi W \neq 0$

GENERAL CONSIDERATIONS

What is the most generic form of $\mathcal{A}_{Weyl}$?

* Wess-Zumino consistency conditions (WZcc)

WZcc $\Longleftrightarrow$ $\exists F$ such that $\mathcal{A} = \delta F$ integrability condition

$$\Rightarrow \boxed{\mathcal{A}_{Weyl} = aE + cW^2 + d\,\Box R \text{ for class. Weyl inv. theory}}$$ (Duff 77')

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$\Rightarrow$ $\boxed{\mathcal{A}_{\text{Weyl}} = aE + cW^2 + d\,\Box R \text{ for class. Weyl inv. theory}}$ (Duff 74')

* Pontryagin density: $R\tilde{R} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta}$

Respects WZcc $\Rightarrow$ $\mathcal{A}_{\text{Weyl}} = aE + cW^2 + d\,\Box R + e R\tilde{R}$

$R\tilde{R}$ CP-odd $\Rightarrow$ SM source for baryogenesis, ...

Was omitted.

CHECK POINT

1) Anomaly

* $\delta_\theta S[\phi, B] = 0$

but $\delta_\theta W = \int \theta(x)\, \mathcal{A} \neq 0$

* $W[B] = -i \log \int \mathcal{D}\phi \, e^{iS}$

2) Gravity symmetries

* Weyl
* Diffeo
* Lorentz

$\delta_\sigma^W W = \int \sigma \, \mathcal{A}_{Weyl}$

$\delta^d W = \delta^{\mathcal{L}} W = 0$

in $d = 4$

3) General form of the Weyl anomaly

$$\mathcal{A}_{Weyl} = g_{\mu\nu} \langle T^{\mu\nu} \rangle = aE + cW^2 + d\,\Box R + \underbrace{e\, R\tilde{R}}_{?}$$

THE CONTROVERSY

Weyl fermion : $S = \int d^4x \sqrt{g}\, \bar{\Psi} (i\partial\!\!\!/ + i\omega\!\!\!/) P_L \Psi$ neutrino ν_L

Bonora et al 2014 : $\mathcal{A}_{Weyl} \supset e R\tilde{R}$, $e = \frac{i}{1536\pi^2}$

From 2014 to 2023 : - many indep. computations
- various approaches
- some find $e=0$ others $e = \frac{i}{1536\pi^2}$

↳ e purely imaginary ⇒ unitarity violation in SM
⇒ need ν_R ?

↳ $CPT \circ iR\tilde{R} = -iR\tilde{R}$, surprising but no so crazy
(Lorentz inv. broken)

Why is it a difficult calculation ?

DIFFICULTIES

* Regularisation $\Rightarrow$ spurious anomalies in other spacetime symmetries (diffeomorphism, Lorentz)

* Ill-defined propagator $\int \bar{\Psi}\, i\not{\partial} P_L \Psi$ not invertible $\Rightarrow$ no propagator

Right-handed spectator breaks Lorentz invariance.

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+ covariantisation $(h \to R)$: dubious if $\exists$ Lorentz/diffeo anomalies

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$\hookrightarrow$ Path integral : needs to be properly defined

PATH INTEGRAL

Goal: Compute $\int dx\, \sigma\, \partial_\mu J^\mu_{Weyl} = \delta_\sigma^W W$

Make sense of $W = -i \log \int d\mu\, e^{iS}$, $S = \int dx \sqrt{g}\, \bar{\Psi}\, i \cancel{D} P_L \Psi$

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1) Propagator: $(\cancel{i\not{D} P_L})^{-1}$ $\quad S = \int dx \sqrt{g}\, \bar{\Psi}_L\, i\sigma\cdot D\, \underbrace{\Psi_L}_{\text{2-components}}$ $\quad (i\sigma\cdot D)^{-1}$ ✓

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2) Measure: $d\mu = \cancel{\mathcal{D} \bar{\Psi}_L \mathcal{D} \Psi_L}$

Invariant measure: $d\mu = \mathcal{D}(g^{1/4} \bar{\Psi}_L)\, \mathcal{D}(g^{1/4} \Psi_L)$

(Fujikawa 81', Toms 87')

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3) Path integral definition:

$$W = -i\log\int d\mu\, e^{iS} = -i\log\det i\sigma\cdot D \equiv -i\log\prod_n \lambda_n \quad \text{where} \quad i\sigma\cdot D\,\varphi_n = \lambda_n \varphi_n$$

Gaussian int.

Ok if $\lambda_n \neq 0 \Leftrightarrow$ no instanton $\Leftrightarrow$ perturbative set-up

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⚠ $\underbrace{i\sigma\cdot D\,\varphi_n}_{R} = \underbrace{\lambda_n\varphi_n}_{L}$ is meaningless since $i\sigma\cdot D : L \to R$

$R = L$?

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Way out: $\int \sigma\cdot\mathcal{J}_{\mathrm{Weyl}} = \delta W = \mathrm{Tr}\underbrace{(\delta\, i\sigma\cdot D)(i\sigma\cdot D)^{-1}}_{\substack{L\to L\\ R\to R}}$ ✓

(Leutwyler & Mallik 86')

PATH INTEGRAL

4) Careful regularisation

$$\mathcal{O}^{-1} = \lim_{\Lambda\to\infty} \mathcal{O}^{\dagger} \int_{1/\Lambda^2}^{\infty} dt \; e^{-t\,\mathcal{O}^{\dagger}\mathcal{O}}$$

$\mathcal{O}^{\dagger}\mathcal{O} > 0$

$\mathcal{O}^{\dagger}$: hermitian conjugate

$$\Rightarrow \boxed{\int d^4x \, \sigma \, \mathcal{A}^{\text{Weyl}} \equiv \lim_{\Lambda\to\infty} \mathrm{Tr}\left(\delta^{W}_{\sigma}\, i\sigma\cdot D\right)\left(i\sigma\cdot D\right)^{-1} e^{-\frac{(i\sigma\cdot D)^{\dagger}\, i\sigma\cdot D}{\Lambda^2}}}$$

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5) Computation of anomalies

⚠ May break diffeo & Lorentz invariance $\Rightarrow$ no "manifestly covariant" computations

$\Rightarrow$ We follow

JHEP 11 (2023) 045, J. Quevillon, R.L

Result:

$$\boxed{\begin{gathered}\mathcal{A}^\mu_{\text{diffeo}} = \mathcal{A}^{\mu\nu}_{\text{Lorentz}} = 0 \\ \mathcal{A}_{\text{Weyl}} \not\ni R\tilde{R}\end{gathered}}$$

JHEP 12 (2023) 064 J. Quevillon, R. Zwicky, R.L.

Gravity-gauge Anomaly Constraints on $T^{\mu\nu}$

2312.13222 [hep-th], J. Quevillon, R. Zwicky, R.L.

* Weyl fermion: $g_{\mu\nu}\langle T^{\mu\nu}\rangle \not\ni R\tilde{R}, F\tilde{F}$

  gauge sector $F\tilde{F} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$

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  → Most generic ansatz for $\frac{\delta W_{ct}}{\delta g_{\mu\nu}}$ in dim. reg & use $\mathcal{A}^{\text{Weyl}} \sim g^{(d)}_{\mu\nu}\frac{\delta W_{ct}}{\delta g_{\mu\nu}}$

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→ Includes explicit Weyl sym breaking

& formal proof of $\mathcal{A}_{\mathrm{Weyl}} = g_{\mu\nu}\langle T^{\mu\nu}\rangle - \langle T^{\mu}{}_{\mu}\rangle$

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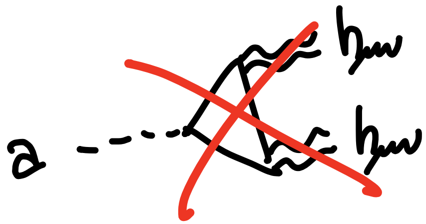
→ Mixed gravity-gauge anomalies ⇒

new pheno. constraints on global symmetries

CDE IN GRAVITY

Want to do QFT in gravity?



Feynman diagrams $\Rightarrow g = \eta + h$

$\Rightarrow$ infested by nasty $h_{\mu\nu}, \partial_\mu h_{\rho\sigma}, h^\mu{}_\nu, \partial^2 h, h^2, \partial^2 h^2 \ldots$

Adopt the CDE and say GOODBYE to $h_{\mu\nu}$!

Get $S_{eff}^{1\text{-loop}}$ once and for all $\Rightarrow$ get any one-loop amplitude

J. Quevillon & R.L 23' $\Rightarrow$ Now available for chiral fermions
$\Rightarrow$ New unexplored effective operators

Already adopted by: - Rémy Larue "It changed my life!"
- Jérémie Quevillon "I love my Ph.D student!"

SUMMARY

* **Weyl fermion:** - path integral approach powerful
  - some work to have a well-defined quantity
  - $\mathcal{A}_{\mathrm{Weyl}} = g_{\mu\nu}\langle T^{\mu\nu}\rangle \not\ni R\tilde{R}$

* **Model indep. approach:**
  - Finiteness of $\mathcal{A}_{\mathrm{diffeo}}$, $\mathcal{A}_{\mathrm{Lorentz}}$, $\mathcal{A}_{\mathrm{gauge}} \Rightarrow \mathcal{A}_{\mathrm{Weyl}} \not\ni R\tilde{R}, F\tilde{F}$
  - New pheno constraints on global symmetries via mixed anomalies

THANK YOU! :)

Rémy Larue

APPENDIX: WEYL, SCALE, CONFORMAL

* Scale, conformal, Weyl transformations

Flat spacetime

Scale : $x \to x' = e^{\sigma} x$

$\phi(x) \to \phi'(x') = e^{\Delta\sigma} \phi(x)$ Δ canonical mass dimension

Conformal $\supset$ Scale (dilatation) (+ rotat°, translat°, special conformal)

Curved spacetime

Conformal : only if $\exists$ Conformal Killing Vector Field (ie cte curvature spacetime)

Weyl : Change of spacetime $g'_{\mu\nu}(x) = e^{2\sigma} g_{\mu\nu}(x)$ $\forall g$ + $\phi \to e^{\Delta\sigma}\phi$

Generalisation of scale transfo. in curved spacetime

Trade $x \to x'$ for $g_{\mu\nu} \to g'_{\mu\nu}$.

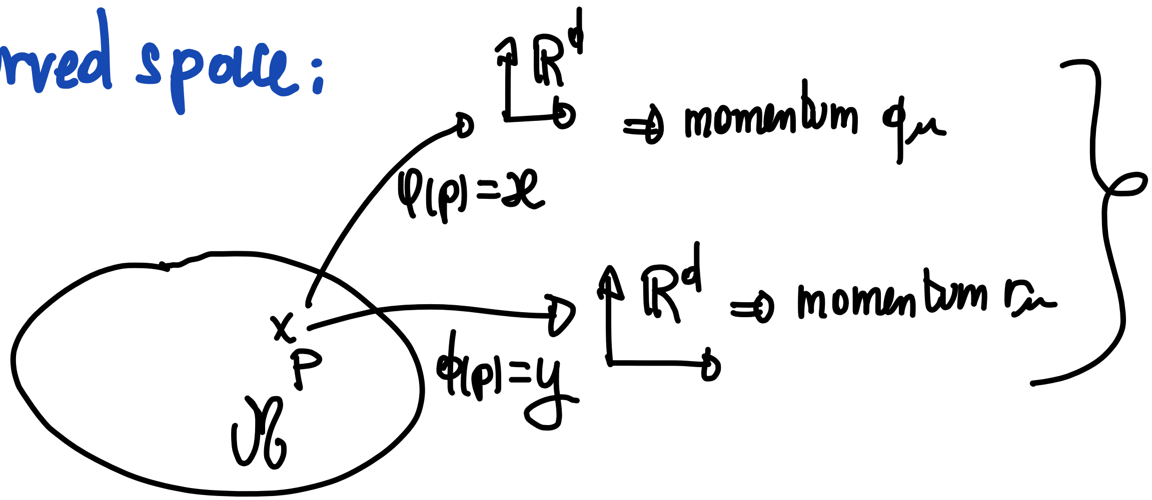
APPENDIX: curved space momentum representation

* Flat space:

$$F(x,y) = f(x,\partial_x)\,\delta(x-y) = f(x,\partial_x)\int \frac{d^4q}{(2\pi)^4}\, e^{iq\cdot(x-y)}$$
$$= \int \frac{d^4q}{(2\pi)^4}\, e^{iq\cdot(x-y)} f(x,\partial_x + iq)$$

$$\mathrm{Tr}F = \int d^4x\, F(x,x) = \int d^4x\, \frac{d^4q}{(2\pi)^4} f(x,\partial_x+iq) \quad \text{(divergent)}$$

* Curved space:



Non-unicity of momentum representation

Problem: $q\cdot x$ is coordinate dependent, ie $q\cdot x \neq r\cdot y$

APPENDIX: curved space momentum representation

Indeed : $X = x^\mu \partial_\mu$ does not define a vector :

Diffeomorphism $x' = f(x)$ f & f^{-1} are C^∞

Vector $V = v^\mu \frac{\partial}{\partial x^\mu} \Rightarrow v'^\mu = v^\nu \frac{\partial x'^\mu}{\partial x^\nu}$

$x'^\mu = f^\mu(x) \neq x^\nu \frac{\partial x'^\mu}{\partial x^\nu}$ in general

Possible way out: Make a specific choice of coordinate

Riemann Normal Coordinates

(Bunch & Parker 79'
Binetruy & Gaillard 89'
Parker & Toms 09')

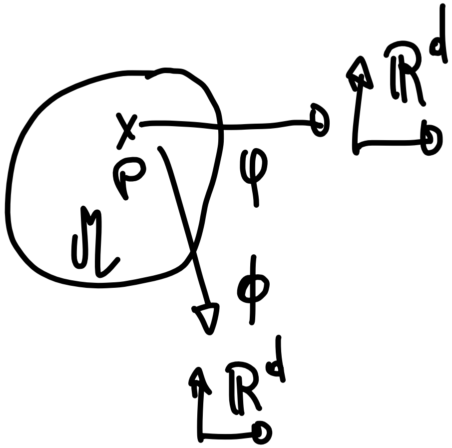
Only possible for covariant result

$\Rightarrow \nexists$ diffeo & Lorentz anomalies

APPENDIX: curved space momentum representation

(J. Quevillon, R.L 23')

Define momentum representation independently of a choice of coordinates



$\varphi : \mathcal{M} \to \mathbb{R}^d$
$p \to x(p)$

In (flat) $\mathbb{R}^d$ can define conjugate variable to x^μ:

q_μ such that $\dfrac{\partial q_\mu}{\partial x^\nu} = \partial_\nu q_\mu = 0$

$$\Rightarrow e^{-iq\cdot x} \frac{\partial}{\partial x^\mu} e^{iq\cdot x} = \partial_\mu + i q_\mu \qquad (\partial_\nu q^\mu \neq 0)$$

$\phi(p) \Rightarrow y(p) \Rightarrow r_\mu$ such that $\dfrac{\partial r_\mu}{\partial y^\nu} = 0$

Define $Q = q_\mu dx^\mu = r_\mu dy^\mu \Rightarrow q_\mu = r_\nu \dfrac{\partial y^\nu}{\partial x^\mu}$ transforms covariantly.

We can show that: $\to \forall \mathcal{O} = \mathcal{O}^{\mu_1 \dots \mu_n} \partial_{\mu_1} \cdots \partial_{\mu_n}$ covariant, $e^{-iq\cdot x} \mathcal{O} e^{iq\cdot x} = e^{-ir\cdot y} \mathcal{O} e^{ir\cdot y}$

despite $q\cdot x \neq r\cdot y$

$\to$ Invariant measure

$$d^d x \, d^d q = d^d y \, d^d r$$

APPENDIX: index theorems

* Topological anomaly: ABJ anomaly

$$S=\int \bar{\Psi} i\not{D}\Psi \qquad U(1)_A:\ \Psi' = e^{i\theta\gamma_5}\Psi\,,\quad \bar{\Psi}' = \bar{\Psi}e^{i\theta\gamma_5}$$

$$\Rightarrow \int \theta\, \mathcal{A}_{ABJ} = \mathrm{Tr}\,\theta\gamma_5 \xrightarrow{\text{regularise}} \lim_{\Lambda\to\infty} \mathrm{Tr}\,\theta\gamma_5\, e^{-\not{D}^2/\Lambda^2}$$

$$= \theta \sum_n \int \varphi_n^\dagger \gamma_5 \varphi_n \qquad = \lim_{\Lambda\to\infty} \theta \sum_n e^{-\frac{\lambda_n^2}{\Lambda^2}} \int \varphi_n^\dagger \gamma_5 \varphi_n$$

$$\| \qquad\qquad\qquad \|$$

$$\theta(n_+ - n_-) \qquad\qquad \text{finite}$$

since $\int \varphi_n^\dagger \gamma_5 \varphi_n = \begin{cases} 0 & \text{if } \lambda_n \neq 0 \\ \pm 1 & \text{if } \lambda_n = 0 \end{cases}$ $\qquad i\not{D}\varphi_n = \lambda_n \varphi_n\,,\ (i\not{D})^\dagger = i\not{D}$

Remark: $\mathrm{Tr}\,\theta\gamma_5 \sim \mathrm{tr}\,\gamma_5 \times \delta(0) \sim +1-1+1-1+1\ldots$

APPENDIX: index theorems

gauge, diffeo, Lorentz : topological & 1-loop exact too

* **Weyl anomaly** : not topological, not 1-loop exact

$$\int \sigma \, \frac{\delta}{\delta} \mathcal{L} \propto \operatorname{Tr} \sigma \times \mathbb{1} \xrightarrow{\text{regularise}} \lim_{\Lambda\to\infty} \operatorname{Tr} \sigma \, e^{-\not{D}^2/\Lambda^2}$$

$$= \sigma \sum_n \underbrace{\int \varphi_n^+ \varphi_n}_{=1} \qquad = \lim_{\Lambda\to\infty} \sigma \sum_n e^{-\frac{\lambda_n^2}{\Lambda^2}} \underbrace{\int \varphi_n^+ \varphi_n}_{=1}$$

$$= \sigma \sum_\lambda \dim E(\lambda) \qquad = \lim_{\Lambda\to\infty} \sigma \sum_\lambda e^{-\frac{\lambda^2}{\Lambda^2}} \dim E(\lambda)$$

divergent $\neq$ finite

$E(\lambda) = \{ \varphi \mid i\not{D}\varphi = \lambda \varphi \}$

APPENDIX: index theorems

* ABJ, diffeo, gauge, Lorentz anomalies:

$$\int \mathcal{A} \sim \mathrm{Tr}\,\gamma_5 \ldots \propto n_+ - n_- \qquad \text{P-odd} \Rightarrow F\tilde{F},\ R\tilde{R}$$

$$n_\pm = \dim\{\psi \mid i\not{D}\psi = 0\ \&\ \gamma_5\psi = \pm\psi\}$$

* Weyl anomaly:

selects $\lambda = 0$

$$\int \sigma \mathcal{A}_{\text{Weyl}} = \mathrm{Tr}\,\sigma \times \mathbb{1} \xrightarrow{\text{reg}} \lim_{\Lambda\to\infty} \sigma \sum_\lambda e^{-\frac{\lambda^2}{\Lambda^2}} \dim E(\lambda)$$

$$\mathrm{Tr}\,\sigma \times \mathbb{1} = \sigma \sum_\lambda \dim E(\lambda)$$

$$= \sigma \dim E(0)$$

$$= \sigma (n_+ + n_-) \qquad \text{P even} \Rightarrow F^2,\ R^2,\ \ldots$$

$$E(\lambda) = \{\psi \mid i\not{D}\psi = \lambda\psi\}$$