Probing Reheating with Graviton Bremsstrahlung

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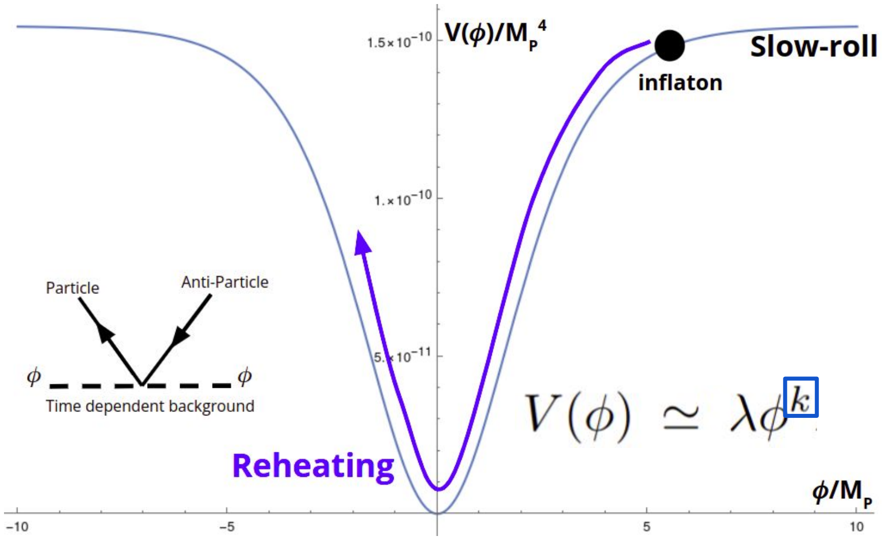
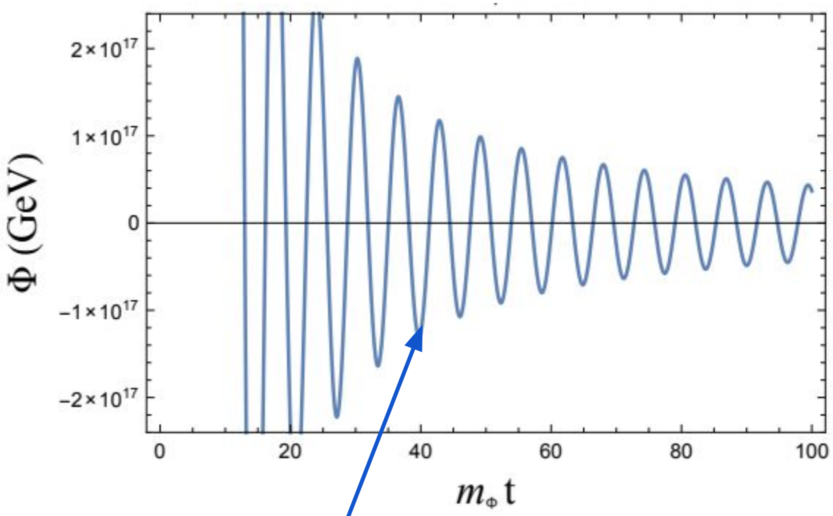
Les Rencontres de Physique des Particules **- 24th January 2024**

Based on :

Probing Reheating with Graviton Bremsstrahlung, Nicolás Bernal, **SC**, Yann Mambrini, Yong Xu, **2311.12694**

1- Reheating after inflation





Redshifts of envelop and frequency of the oscillations depend on the shape of the potential near the minimum

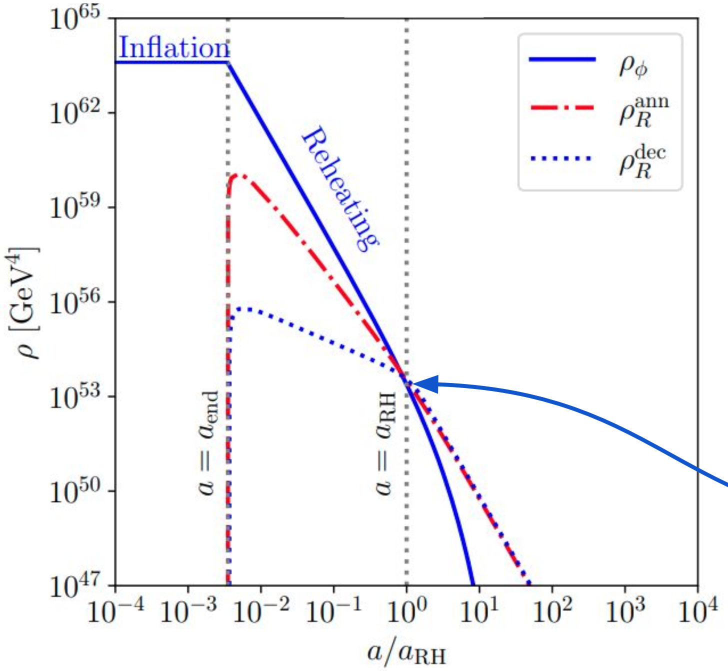
EOM : $\ddot{\phi}(t) + 3H\dot{\phi}(t) + \boxed{\Gamma\dot{\phi}(t)} + V'(\phi(t)) = 0$

After (slow-roll) inflation, the inflaton falls down into the minimum of its potential and starts to oscillate coherently

$$w = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\langle\dot{\phi}^2\rangle - \langle V(\phi)\rangle}{\frac{1}{2}\langle\dot{\phi}^2\rangle + \langle V(\phi)\rangle} = \boxed{\frac{k-2}{k+2}}$$

During reheating : oscillating background field with small couplings to the other quantum fields

➔ Perturbative particle production, then particles thermalize and constitute the thermal bath



➔ Compute particle production rate

➔ Solve Boltzmann equations for energy densities

$$\frac{d\rho_\phi}{dt} + 3H(1+w_\phi)\rho_\phi \simeq -(1+w_\phi)\Gamma_\phi \rho_\phi$$
$$\frac{d\rho_R}{dt} + 4H\rho_R \simeq (1+w_\phi)\Gamma_\phi \rho_\phi\,.$$

$$\rho_R(T_{\rm RH}) = \rho_\phi(T_{\rm RH}) = 3\, M_P^2\, H_{\rm RH}^2$$

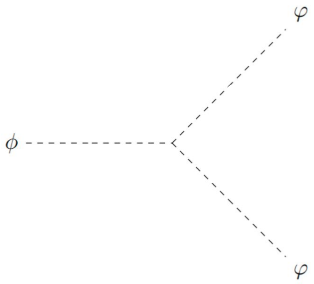
➔ Define the end of the reheating at equality between energy densities, at a temperature $T = T_{RH}$

Probing Reheating with Graviton Bremsstrahlung, Bernal, SC, Mambrini and Xu, **2311.12694**

2- Different perturbative processes during reheating

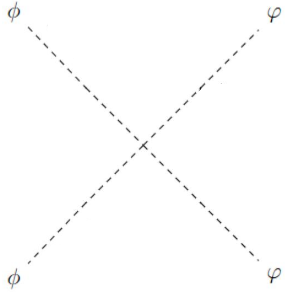
We consider different couplings between the inflation and matter fields (scalars and/or fermions)

$$\mathcal{L} \supset -\mu\,\phi\,\varphi^2 - \sigma\,\phi^2\,\varphi^2 - y_\psi\,\overline{\Psi}\,\Psi\,\phi$$



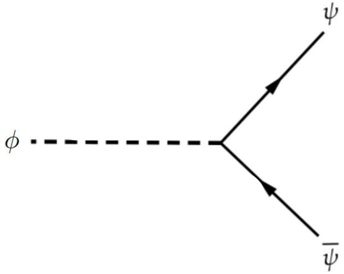
$$\Gamma^{1\to 2} \simeq \frac{m_\phi}{8\pi}\left(\frac{\mu_{\rm eff}}{m_\phi}\right)^2$$

Decay to bosons



$$\Gamma^{2\to 2} \simeq \frac{\sigma_{\rm eff}^2\,\rho_\phi}{8\pi\,m_\phi^3}$$

Scattering to bosons

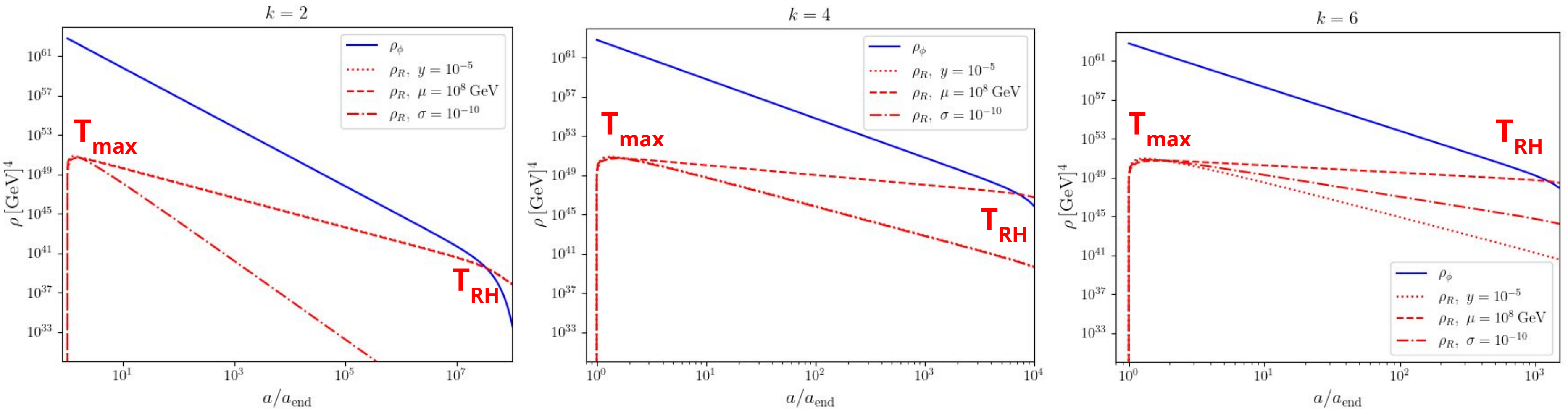


$$\Gamma_{\phi\to\overline{\Psi}\Psi} = \frac{y_{\rm eff}^2}{8\pi} m_\phi$$

Decay to fermions

Inflaton Oscillations and Post-Inflationary Reheating, Garcia, Kaneta, Mambrini, Olive, **2012.10756**

⚠ Effective inflation mass, m_ϕ ,can be time dependent for non quadratic shape of the inflaton potential near the minimum (k>2)



➔ Different "redshifts" of produced energy density from T_{max} to T_{RH}

➔ T_{RH} depends both on couplings and shape of inflaton potential

Evolution of produced energy density

channel	$k=2$	$k=4$	$k=6$
$\phi\to\bar{f}f$	$T\propto a^{-3/8}$	$T\propto a^{-3/4}$	$T\propto a^{-15/16}$
$\phi\to bb$	$T\propto a^{-3/8}$	$T\propto a^{-1/4}$	$T\propto a^{-3/16}$
$\phi\phi\to bb$	$T\propto a^{-1}$	$T\propto a^{-3/4}$	$T\propto a^{-9/16}$

Evolution of inflaton energy density and effective mass

$$\rho_\phi(a)\simeq\rho_\phi(a_{\rm end})\left(\frac{a_{\rm end}}{a}\right)^{6k/k+2}$$

$$m_\phi(a)\simeq m_\phi(a_{\rm end})\left(\frac{a_{\rm end}}{a}\right)^{3(k-2)/k+2}$$

Inflaton Oscillations and Post-Inflationary Reheating, Garcia, Kaneta, Mambrini, Olive, **2012.10756**

Reheating description and constraints

$$\mathrm{Couplings} + \mathrm{m}_\phi(\mathrm{a}_{\mathrm{end}}) + \rho_\phi(\mathrm{a}_{\mathrm{end}}) \rightarrow \mathrm{T}_{\mathrm{max}}$$

And redshifts of energy densities from potential shape (parameter k) ➔ T_{RH} is determined

Can trade $\mathrm{Couplings} + \rho_\phi(\mathrm{a}_{\mathrm{end}})$ for T_{RH} and T_{max} as free parameters

For our analysis, we let $\mathrm{m}_\phi(\mathrm{a}_{\mathrm{end}})$ free, but for a specific inflationary model it is determined

Constraint on the tensor-to-scalar ratio $r < 0.035$ *(BICEP/Keck 2018)*
➔ upper bound on T_{RH} for each scenario as a function of T_{max}

$$T_{\mathrm{RH}} \lesssim 10^{15}\,\mathrm{GeV}\left(\frac{\mathrm{T}_{\mathrm{max}}}{\mathrm{T}_{\mathrm{RH}}}\right)^{-\mathrm{k}/(\mathrm{k}-1)} \qquad \phi \rightarrow \bar{f} f$$

$$T_{\mathrm{RH}} \lesssim 10^{15}\,\mathrm{GeV}\left(\frac{\mathrm{T}_{\mathrm{max}}}{\mathrm{T}_{\mathrm{RH}}}\right)^{-\mathrm{k}} \qquad \phi \rightarrow bb$$

$$T_{\mathrm{RH}} \lesssim 10^{15}\,\mathrm{GeV}\left(\frac{\mathrm{T}_{\mathrm{max}}}{\mathrm{T}_{\mathrm{RH}}}\right)^{-\mathrm{k}/3} \qquad \phi\phi \rightarrow bb$$

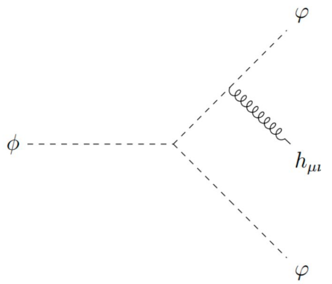
3- Graviton Bremsstrahlung during reheating

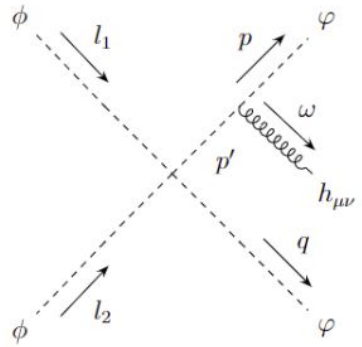
➔ Look at particle origin for stochastic GWs background via graviton bremsstrahlung

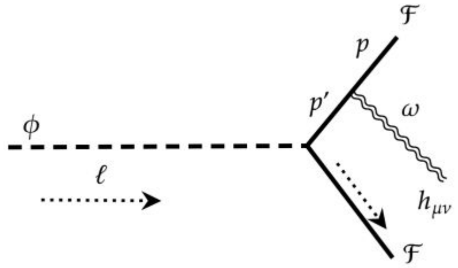
➔ Gravitons as effective metric perturbations over the classical background

perturbative reheating + $\sqrt{-g}\,\mathcal{L} \supset -\frac{1}{M_P} h_{\mu\nu} T^{\mu\nu}$

graviton

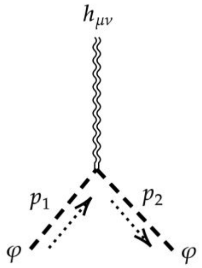




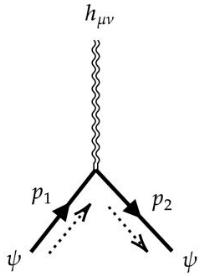


➔ Generate a spectrum of stochastic GWs at high frequencies, that depends on the details of reheating (redshifts of energy densities, couplings to the inflaton)

➔ Compute Bremsstrahlung differential rate of gravitons production from the different vertices



$$\frac{i}{M_P}\left[p_{1\mu}p_{2\nu}+p_{1\nu}p_{2\mu}-\eta_{\mu\nu}\left(p_1 . p_2-m^2\right)\right]$$



$$-\frac{i}{4M_P}\left[(p_1+p_2)_\mu\gamma_\nu+(p_1+p_2)_\nu\gamma_\mu-2\eta_{\mu\nu}\left(\gamma^\alpha p_{1\alpha}+\gamma^\beta p_{2\beta}-2m\right)\right]$$

➔ Impose transverse traceless polarizations for on-shell outgoing gravitons

$$\frac{d\Gamma^{(1)}_{\phi\to \mathrm{GW}}}{dE_{\mathrm{GW}}}\equiv\frac{E_{\mathrm{GW}}}{\rho_\phi}\sum_{n=1}^{\infty}\int|\mathcal{M}_n^{(1)}|^2\frac{d\mathrm{LIPS}_3}{dE_{\mathrm{GW}}}$$

where E_{GW} is the outgoing graviton energy, and integrate over the 3-body PS

$$\dot{\rho}_{\mathrm{GW}}+4H\rho_{\mathrm{GW}}=\Gamma^{(1)}_{\phi\to\mathrm{GW}}\rho_\phi$$

➔ Solve Boltzmann equation for gravitational energy density sourced during reheating

➔ Compute the relic density of stochastic GWs as it can be observed today

$$\Omega^0_{\rm GW} = \frac{1}{\rho^0_c}\,\frac{d\rho_{\rm GW}}{d\ln E_\omega}\bigg|_{a_0} = \Omega^{(0)}_\gamma\,\frac{g_\star(T_{\rm RH})}{g_\star(T_0)}\left[\frac{g_{\star s}(T_0)}{g_{\star s}(T_{\rm RH})}\right]^{4/3}\frac{d(\rho_{\rm GW}(T_{\rm RH})/\rho_R(T_{\rm RH}))}{d\ln E_\omega}$$

$$h_c(f) \equiv \frac{H_0}{f}\sqrt{\frac{3}{2\pi^2}\,\Omega_{\rm GW}(f)}$$

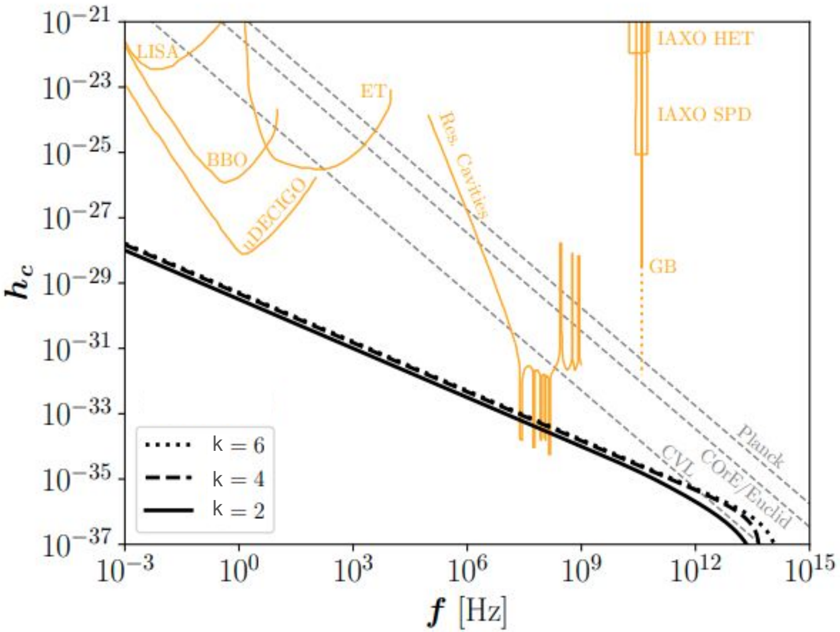
➔ Redshifts of gravitational vs. radiation energy density during reheating leads to suppression/enhancement of the spectrum of GWs

➔ Present GW frequency associated with the graviton energy $E_\omega(a_{RH})$ at the end of reheating

$$f = \frac{E_\omega}{2\pi}\,\frac{a_{\rm RH}}{a_0} = \frac{E_\omega}{2\pi}\,\frac{T_0}{T_{\rm RH}}\left[\frac{g_{\star s}(T_0)}{g_{\star s}(T_{\rm RH})}\right]^{1/3}$$

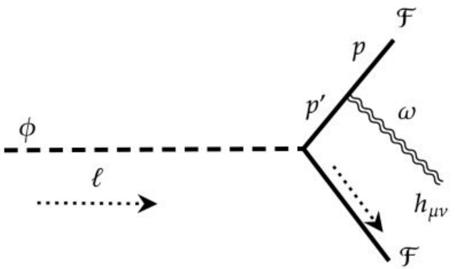
➔ Gravitons energy $E_\omega(a)$ ~ $m_\phi(a)$ at production so we expect a spectrum at very high frequency GWs

4 - Results : GWs spectra



$m_\phi(a_{end}) = 10^{17}$ GeV, $T_{RH} = 10^{13}$ GeV, and $T_{max}/T_{RH} = 10$

Previous analysis : Yukawa decay



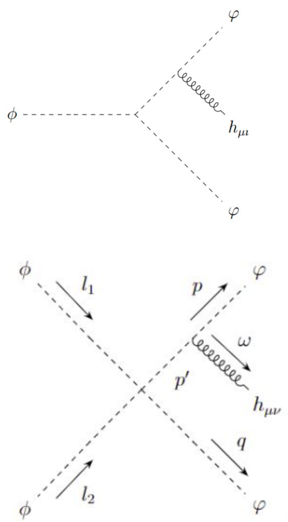
➔ Coupling allowing to reheat is the same as the one for the graviton Bremsstrahlung

Nakayama and Tang, **1810.04975**
Huang, Yin, **1905.08510**
Barman, Bernal, Xu, Zapata, **2301.11345** and **2305.16388**

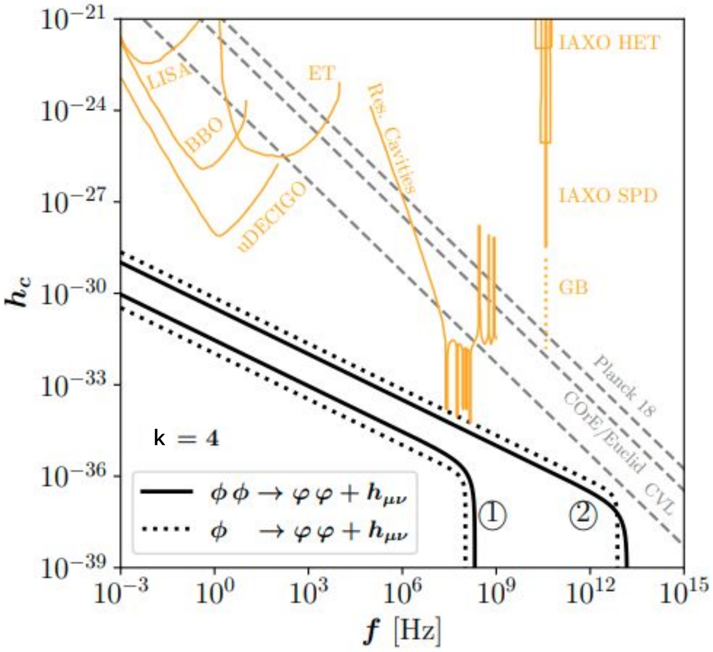
➔ High frequency GWs can be probed by resonant cavities for a very high inflaton mass and high T_{RH}

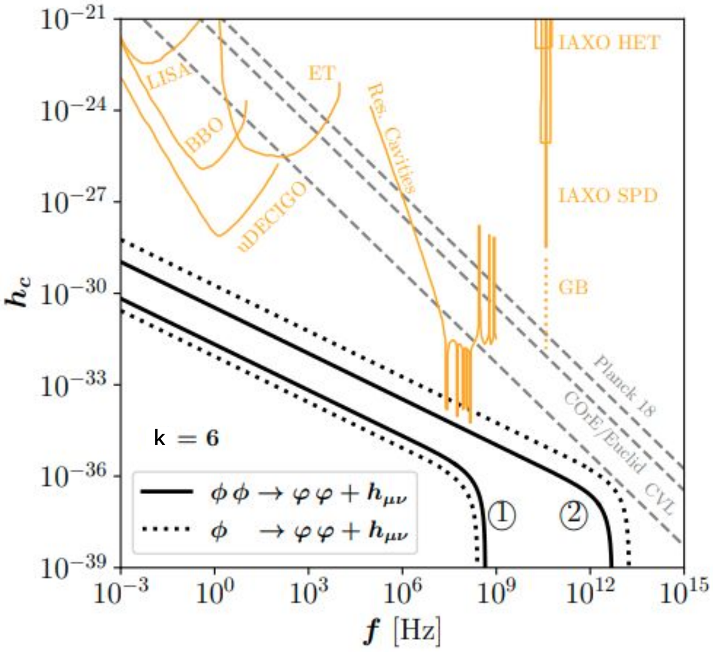
➔ Strong dependance on the inflation mass but weak dependance on the potential (parameter k)

In our last work : Bosonic reheating



Probing Reheating with Graviton Bremsstrahlung, Bernal, SC, Mambrini and Xu, **2311.12694**





① **$m_\phi(a_{end}) = 10^{13}$ GeV, $T_{RH} = 10^{13}$ GeV, $\rho_{end} = 10^{62}$ GeV^4 (T-alpha attractor model)**

② **$m_\phi(a_{end}) = 10^{15}$ GeV, $T_{RH} = 10^{13}$ GeV, $T_{max}/T_{RH} = 5$**

➔ Distinctive spectra due to the different dilution of energy densities for scatterings and decays

➔ Strong dependency on m_ϕ , and k dependance can distinguish between the scenarios

An interesting possibility : T_{RH} generated independently from the emission of gravitons

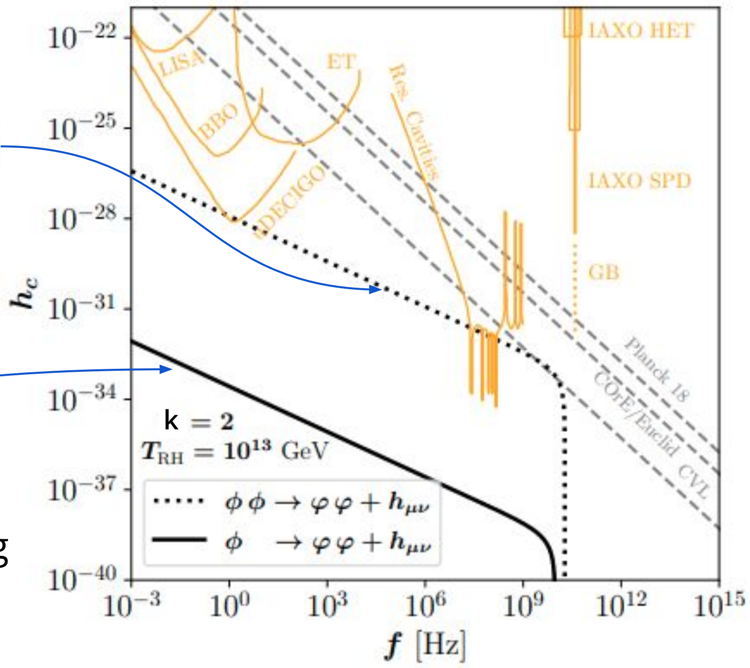
Example : Scatterings to emit gravitons and decays to reheat efficiently

For k = 2

$$\Omega_{GW}^{2\to3}h^2 \simeq 10^{-19}\,\sigma^2 \left(\frac{T_{RH}}{10^{10}\ \mathrm{GeV}}\right)^{\frac{7}{3}} \left(\frac{\rho_{end}}{6.25\times10^{62}\ \mathrm{GeV}^4}\right)^{\frac{1}{6}} \left(\frac{10^{13}\ \mathrm{GeV}}{m_\phi}\right)^2 \left(\frac{f}{10^8\ \mathrm{Hz}}\right)$$

Now, T_{RH} independent of σ

$$\Omega_{GW}^{1\to3}h^2 \simeq 10^{-23}\left(\frac{T_{RH}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{m_\phi}{10^{13}\,\mathrm{GeV}}\right)\left(\frac{f}{10^8\ \mathrm{Hz}}\right)$$



➔ Different redshifts lead to enhancement of relic GWs originating from inflaton scattering

➔ Can be probed by resonant cavities and future GWs experiments, even for standard large scale inflation models

Probing Reheating with Graviton Bremsstrahlung, Bernal, SC, Mambrini and Xu, **2311.12694**

Summary

- Unavoidable source of stochastic GWs background from particle origin during reheating
- Couplings and dynamics of the inflaton during reheating influence the GWs relic
- Probes of reheating in resonant cavities and future GWs detectors for large inflaton mass and large TRH
- Distinctive signal for bosonic reheating due to dilution effects for different inflaton potential near the minimum

Thank you for your attention !

Backup Slides

ΔN_{eff} Contributions and Constraints

Energy density in GW before BBN acts as (dark) radiation, thus its impact on BBN captured in terms of N_{eff} (number of effective neutrinos)

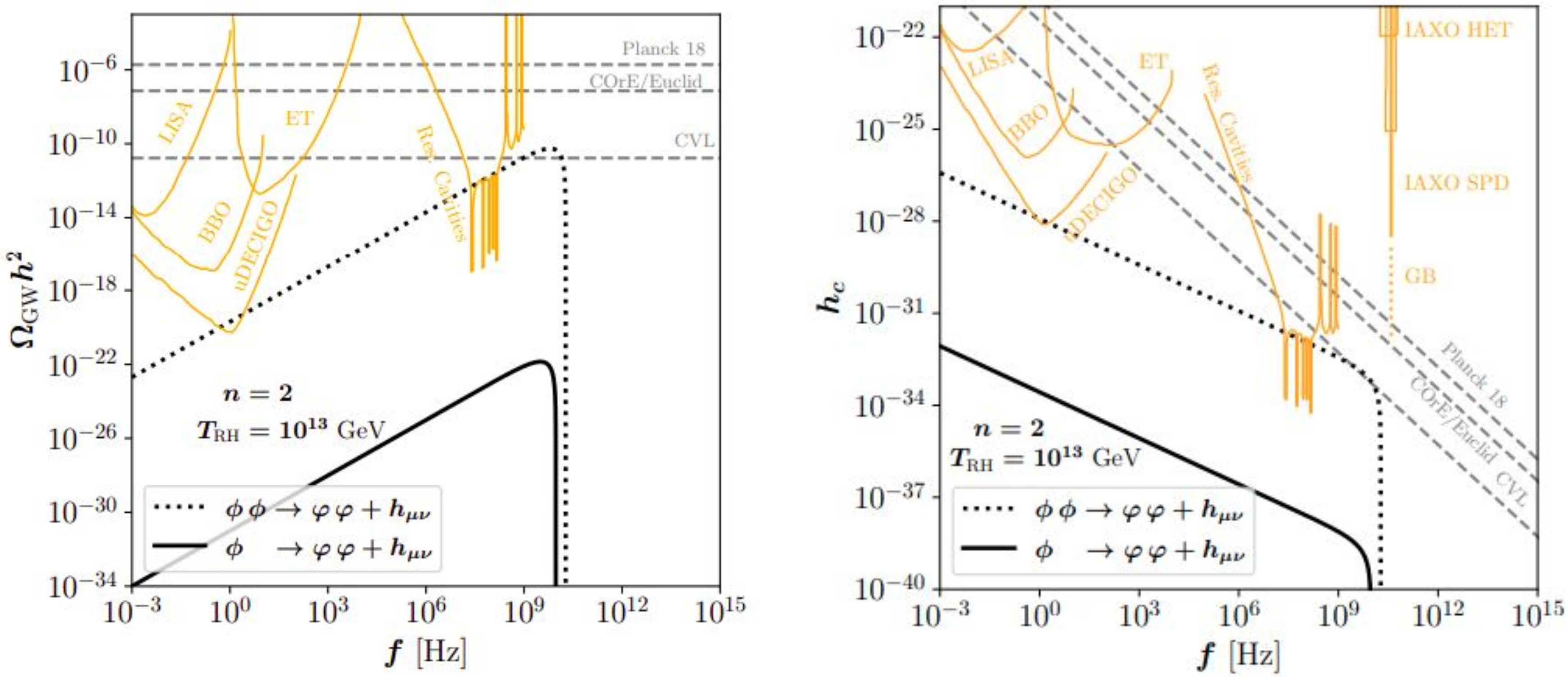
$$\rho_{\rm rad}(T) = \rho_\gamma + \rho_\nu + \boxed{\rho_{\rm GW}} = \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 \boxed{N_{\rm eff}}\right] \rho_\gamma(T)$$

$$\Delta N_{\rm eff} \equiv N_{\rm eff} - N_{\rm eff}^{\rm SM} = \frac{8}{7}\left[\frac{11}{4}\frac{g_{\star s}(T \lesssim m_e)}{g_{\star s}(T_{\rm rh})}\right]^{\frac{4}{3}} \frac{g_\star(T_{\rm rh})}{2} \frac{\rho_{\rm GW}}{\rho_R}\bigg|_{T_{\rm rh}}$$

➔ Excess of GW energy density around BBN can be constrained by present and future bounds on ΔNeff from CMB, BBN, and combined

$\Delta N_{\rm eff}$	Experiments
0.34	Planck legacy data [62]
0.14	BBN+CMB combined [63]
0.06	CMB-S4 [64]
0.027	CMB-HD [65]
0.013	COrE [66], Euclid [67]
0.06	PICO [68]

Bremsstrahlung-induced gravitational waves in monomial potentials during reheating, Barman, Bernal, Xu, Zapata, **2305.16388**



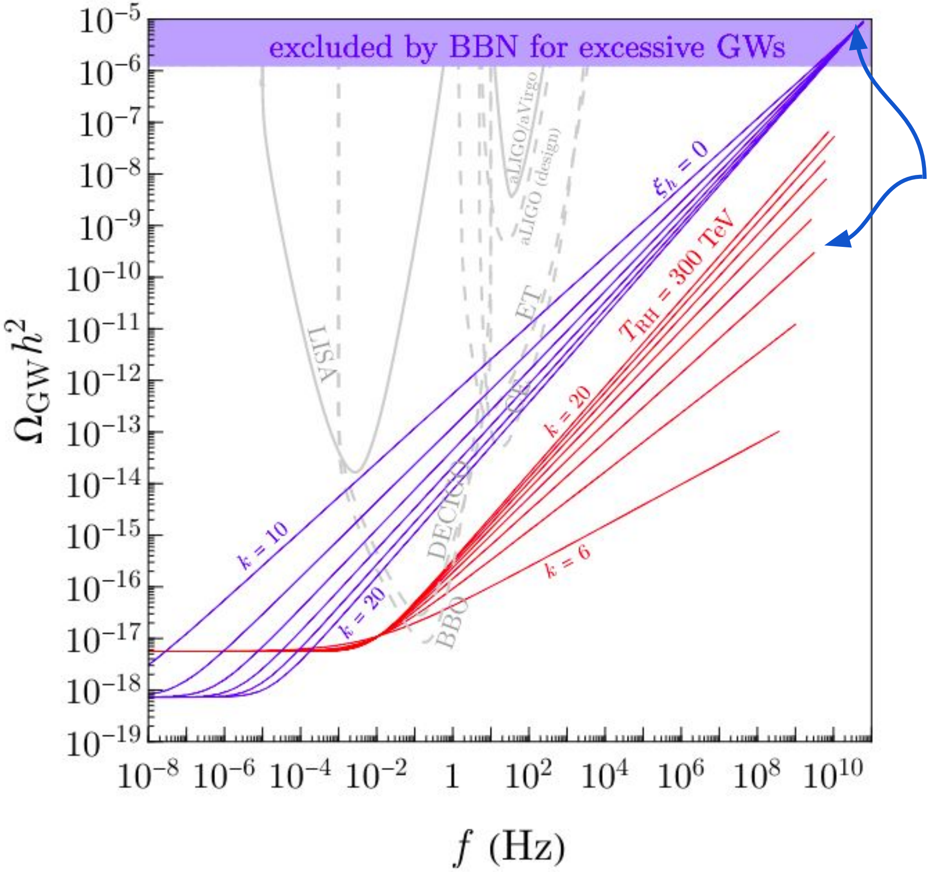
$$\Omega_{\rm GW}^{2\to3} h^2 \simeq 10^{-19}\,\sigma^2 \left(\frac{T_{\rm RH}}{10^{10}\ \text{GeV}}\right)^{\frac{7}{3}} \left(\frac{\rho_{\rm end}}{6.25\times10^{62}\ \text{GeV}^4}\right)^{\frac{1}{6}} \left(\frac{10^{13}\ \text{GeV}}{m_\phi}\right)^{2} \left(\frac{f}{10^{8}\ \text{Hz}}\right)$$

$$\Omega_{\rm GW}^{1\to3} h^2 \simeq 10^{-23} \left(\frac{T_{\rm RH}}{10^{10}\,\text{GeV}}\right) \left(\frac{m_\phi}{10^{13}\,\text{GeV}}\right) \left(\frac{f}{10^{8}\,\text{Hz}}\right)$$

➜ Primordial GWs re-entering the horizon during reheating, if inflaton redshifts faster than radiation, are enhanced.

➜ GWs spectrum scales with the frequency as $\Omega^0_{GW}h^2 \propto f^{\,k-4/k-1}$

➜ The slope of this spectrum can probe the shape of the inflaton potential near the minimum



The largest enhancement is for the mode that re-enters the horizon right after inflation

Figure 4 : Primordial GWs strength as function of its frequency f. Blue curves fix $\xi_h = 0$ and Red curves fix T_{RH} = 300 TeV, for k in [6,20]. The sensitivity of several future GWs experiments are shown.

Inflation described as an exponential expansion of the Universe driven by an homogeneous scalar field ϕ in the potential :

Flat direction in the potential → Slow roll

$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^k$$

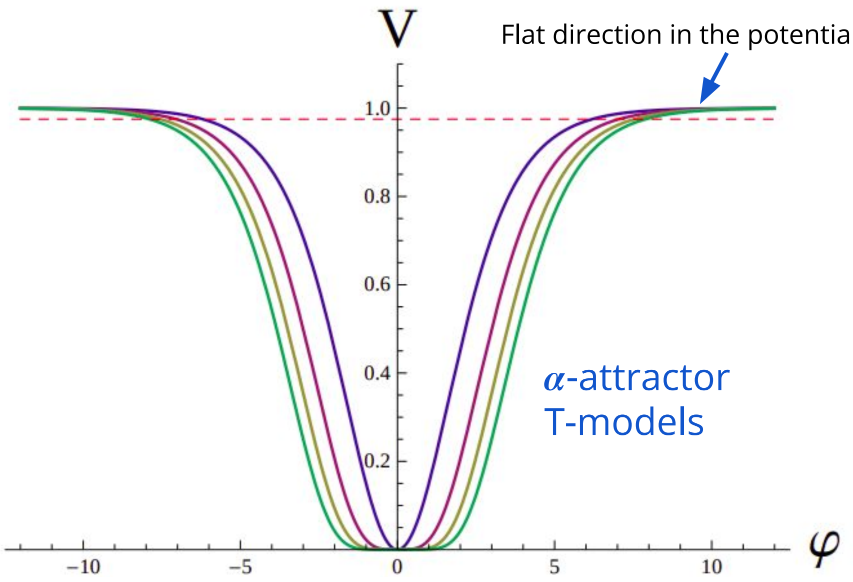


Figure 1: Potentials for the T-Model inflation $\tanh^{2n}(\varphi/\sqrt{6})$ for $n = 1, 2, 3, 4$

From *Universality Class in Conformal Inflation*, Kallosh and Linde, JCAP (2013)

$$A_{S*} \simeq \frac{V_*}{24\pi^2\epsilon_* M_P^4} \simeq \frac{6^{\frac{k}{2}}}{8k^2\pi^2}\lambda \sinh^2\left(\sqrt{\frac{2}{3}}\frac{\phi_*}{M_P}\right)\tanh^k\left(\frac{\phi_*}{\sqrt{6}M_P}\right)$$

λ determined by the scalar power spectrum amplitude of the CMB “*As*”

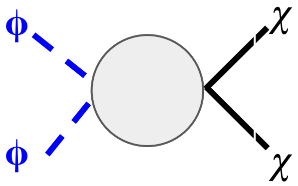
→ Planck measurements give for k=2 : *λ ~ 10⁻¹¹* for N ~ 50 efolds

$$\lambda \simeq \frac{18\pi^2 A_{S*}}{6^{k/2} N_*^2}$$

Reheating and Post-inflationary Production of Dark Matter, Marcos A.G. Garcia, Kunio Kaneta, Yann Mambrini, Keith A. Olive, Phys.Rev.D (2020)

Boltzmann approach

Assuming that the local background geometry is Minkowskian, we compute transition probability



Initial state inflaton ϕ as a coherently oscillating homogeneous condensate with no momentum

From this, production rate can be computed which is the right hand side of the Boltzmann equations

$$\dot{n}_\chi + 3Hn_\chi = R^{(\mathrm{N})}_{\phi\phi\to\chi\chi}$$

$$\frac{d\rho_\phi}{dt} + 3H(1+w_\phi)\rho_\phi \simeq -(1+w_\phi)\Gamma_\phi\rho_\phi$$

$$\frac{d\rho_R}{dt} + 4H\rho_R \simeq (1+w_\phi)\Gamma_\phi\rho_\phi\,.$$

See Boltzmann or Bogoliubov? Approaches Compared in Gravitational Particle Production, Kunio Kaneta, Sung Mook Lee, Kin-ya Oda, **2206.10929**

Inflaton scattering

Potential near the minimum is a power k-dependent monomial

$$V(\phi) = \lambda \frac{\phi^k}{M_P^{k-4}}, \quad \phi \ll M_P$$

Treat the time dependent condensate as a time dependent coupling with an amplitude and quasi-periodic function which is k-dependent

$$\phi(t) \;=\; \phi_0(t) \cdot \mathcal{P}(t)$$

➔ An homogeneous field experiencing coherent oscillations

$$V(\phi) = V(\phi_0) \sum_{n=-\infty}^{\infty} \mathcal{P}_n^k e^{-in\omega t} = \rho_\phi \sum_{n=-\infty}^{\infty} \mathcal{P}_n^k e^{-in\omega t}$$

Expand the quasi-periodic function in Fourier modes

with $\omega = m_\phi \sqrt{\frac{\pi k}{2(k-1)}} \frac{\Gamma(\frac{1}{2}+\frac{1}{k})}{\Gamma(\frac{1}{k})}$

Each Fourier mode adds its contribution to the scattering amplitude with its energy *En = n.ω*

Gravitational portals in the early Universe, SC, Y. Mambrini, K. A. Olive, S Verner **2112.15214**

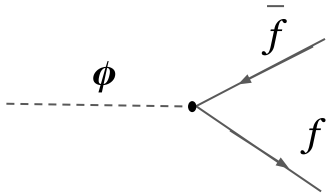
Particle production

Perturbative reheating : considering an oscillating background field with small couplings to the other quantum fields
➔ Particle production

↓

<u>Example :</u> Yukawa like interaction

$$\mathcal{L}_{\phi,bath} = y_\phi \phi \bar{f} f \quad \Rightarrow \quad \Gamma_\phi = \frac{y_\phi^2}{8\pi} m_\phi$$



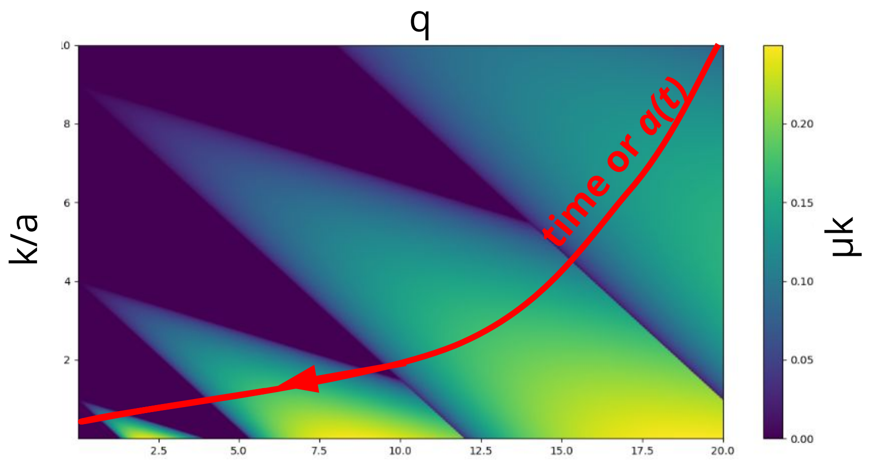
Constitute the primordial bath that will thermalize

See Freeze-in from preheating, Garcia , Kaneta, Mambrini, Olive, Verner, **2109.13280**

Classical non-perturbative approach : **p**reheating
Time dependent background coupled to fields leads to parametric resonance, tachyonic instabilities etc…

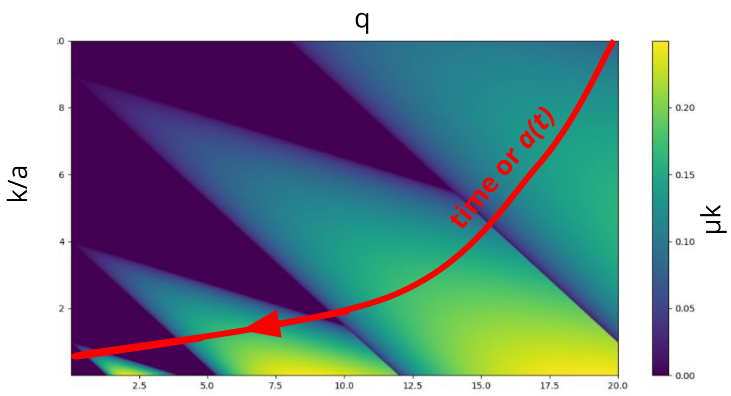
$$\chi_k'' + \left(\frac{k^2}{m_\phi^2 a^2} + 2q - 2q\cos(2z) \right) \chi_k = 0$$

EOM for Fourier modes in the oscillating background



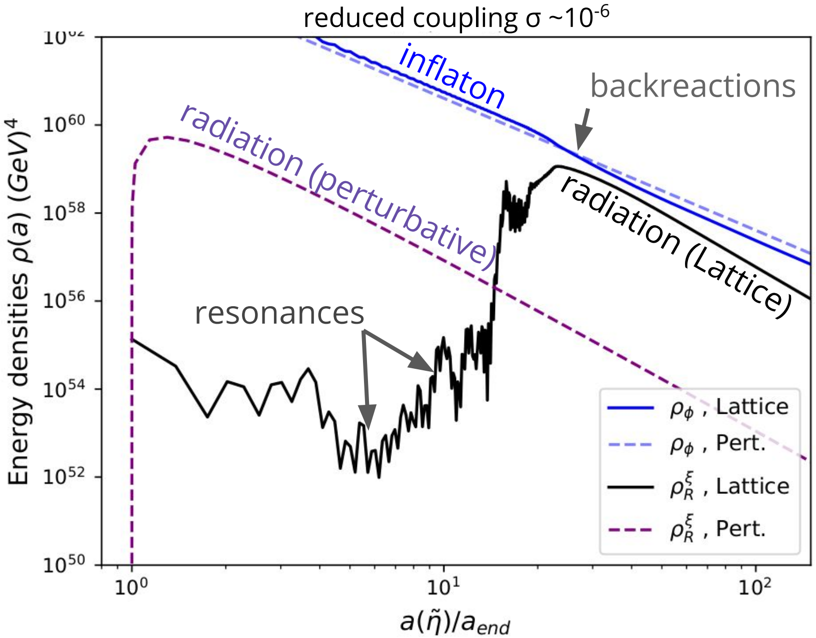
Instabilities in the colored regions
=> increasing occupation number of the modes

Preheating : non-perturbative processes



Instabilities in the colored regions
=> number of occupation increasing $\chi_k \propto exp[\mu_{k,q} z]$

with q ~ σ.(ϕ / M_P)



Preheating corresponds to the first oscillations of the background => resonances and exponential production

For large couplings, reach a regime of large backreactions of the fields on the background

See *Freeze-in from preheating*, Garcia, Kaneta, Mambrini, Olive, Verner, JCAP (2022)