Hidden Supersymmetric Dark Sectors

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Introduction : Dark Matter in SUSY formalism

- DM candidate in the MSSM : Lightest SUSY particle (LSP), neutralino or gravitino :

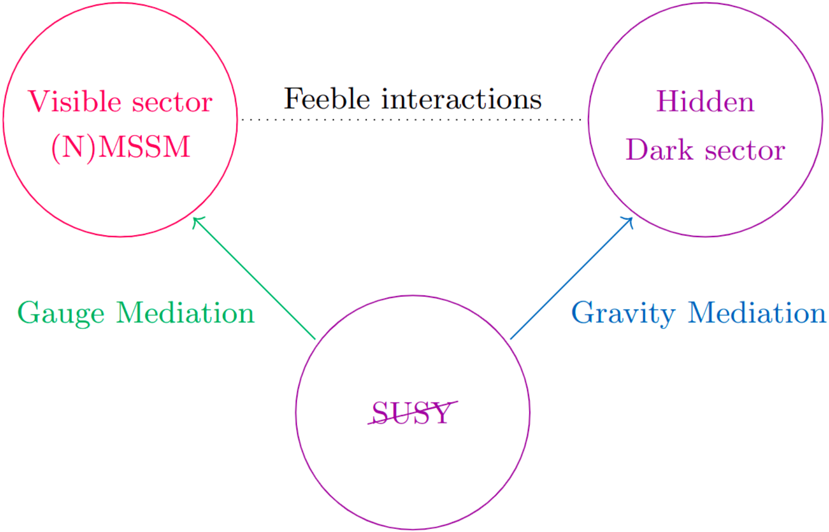
  $\rightarrow$ No signals detected, strong constraints on MSSM parameter space

- Alternative to LSP : hidden sector, exact SUSY at the DM mass scale

  $\rightarrow$ Hierarchy between the SUSY breaking scales in the visible and hidden sectors

Content

I. The model


Visible sector
(N)MSSM
Feeble interactions
Hidden
Dark sector
Gauge Mediation
Gravity Mediation
SUSY

I. SUSY breaking mediation : Gravity mediation

- Messenger : chiral superfield X such as $\langle F_X \rangle \neq 0$
- Interaction with the dark sector :

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \left(\frac{c}{M_P} X W^{\alpha}_{\mathrm{HS}} W_{\mathrm{HS}\ \alpha} + \text{h.c.} \right) . \tag{1}$$

- SUSY breaking scale in the hidden sector :

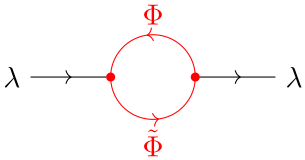
$$m_{\mathrm{HS}} \sim \frac{\langle F_X \rangle}{M_P} . \tag{2}$$

I. SUSY breaking mediation : Gauge mediation

- SUSY breaking parameter : spurion X with $\langle X \rangle = M_G + \theta^2 \langle F_X \rangle$
- Messengers : chiral superfields Φ, $\tilde{\Phi}$
- Interaction :

$$\int \mathrm{d}^2\theta X \Phi \tilde{\Phi}. \tag{3}$$

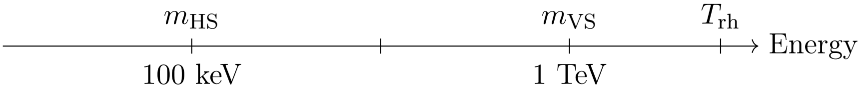
- SUSY Breaking scale in the visible sector :



$$m_{\mathrm{VS}} \sim \frac{g_G^2}{16\pi^2} \frac{\langle F_X \rangle}{M_G}. \tag{4}$$

I. SUSY breaking mediation : order of magnitudes

- Assumptions : $m_{\text{DM}} \geq 100$ keV, $\quad g_G \sim 1$,



$$\longrightarrow \quad \langle F_X \rangle \leq 10^{15} \text{ GeV}^2, \qquad M_G \leq 10^{10} \text{ GeV}.$$

II. Example of Dark sector : Super Yang Mills (SYM)

- Super Yang Mills : gluons (v^μ) and gluinos (λ) dynamics, SU(N_c) gauge group

- Lagrangian :

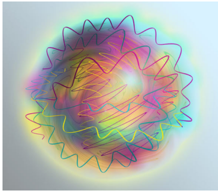
$$\begin{aligned}\mathcal{L}_{\text{SYM}} &= \frac{1}{32\pi}\text{Im}\left(\tau\int \mathrm{d}^2\theta \text{Tr} W^\alpha W_\alpha\right) \\ &= \text{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu D_\mu\bar{\lambda} + \frac{1}{2}D^2\right] + \frac{\theta_{\text{SYM}}}{32\pi^2}g^2 \text{Tr} F_{\mu\nu}\tilde{F}^{\mu\nu}\end{aligned} \tag{5}$$

- ✳ $\tau = \frac{\theta_{\text{SYM}}}{2\pi} + \frac{4\pi i}{g^2}$
- ✳ $F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - ig[v_\mu, v_\nu]$
- ✳ D : Auxiliary field
- ✳ $D_\mu\bar{\lambda} = \partial_\mu\lambda - ig[v_\mu, \lambda]$
- ✳ $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

II. Example of Dark sector : Super Yang Mills (SYM)

- SYM is expected to confine at a scale Λ : uncoloured bound states made of gluons and gluinos $\rightarrow$ glueballs $(v^{\mu}v_{\mu})$ and gluinoballs $(\lambda\lambda)$
- Suitable dark matter candidates :

✓Electrically neutral ✓Uncoloured
✓Weakly interacting ✓Stable



[Image credit : TU Wien]

II. Dark Matter production : defining the feeble interactions

- Simplest mediator fields : heavy chiral superfields $\mathcal{F}_i$ and $\tilde{\mathcal{F}}_i$ in conjugate representation of $\mathrm{SU}(N_c)$

1. If the visible sector is the MSSM, we can add 2 sets of $\mathcal{F}$ fields :
   $i = Q$, $\mathcal{F}_Q$ is a $\mathrm{SU}(2)_\mathrm{w}$ doublet, $\quad$ $i = U$, $\mathcal{F}_U$ is a up-type singlet

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \; \left[M_{\mathcal{F}} \left(\mathcal{F}_Q \tilde{\mathcal{F}}_Q + \mathcal{F}_U \tilde{\mathcal{F}}_U \right) + \lambda_M H_u \mathcal{F}_Q \tilde{\mathcal{F}}_U \right] + \text{h.c.} \tag{6}$$

Integrating out the heavy fields, we get a dim 6 operator :

$$\mathcal{L}_{\mathrm{dim6}} \supset \frac{1}{32\pi} \int \mathrm{d}^2\theta \left[\mathrm{Im}\left(\tau \mathrm{Tr} W^\alpha W_\alpha \right) \left(1 + \frac{1}{\Lambda_M^2} H_u H_u^\dagger + \ldots \right) \right] \tag{7}$$

II.Dark Matter production : defining the feeble interactions

2. If the visible sector is the NMSSM :

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \; \left[M_{\mathcal{F}} \mathcal{F}\tilde{\mathcal{F}} + \lambda_M \hat{N} \mathcal{F}\tilde{\mathcal{F}} \right] + \mathrm{h.c} \tag{8}$$

Leading dim 5 operator :

$$\mathcal{L}_{\mathrm{dim5}} \supset \frac{1}{32\pi} \int \mathrm{d}^2\theta \left[\mathrm{Im} \left(\tau \mathrm{Tr} W^{\alpha} W_{\alpha} \right) \left(1 + \frac{1}{\Lambda_M} (\hat{N} + \hat{N}^{\dagger}) + ... \right) \right] \tag{9}$$

- Both cases : Λ_M determines the production rate in the dark sector

$$\frac{1}{\Lambda_M} \sim \frac{\lambda_M}{4\pi M_{\mathcal{F}}} \tag{10}$$

II. Dark Matter production

- Thermally decoupled sectors and non-renormalizable operator $\rightarrow$ UV freeze-in mechanism

- Dark Matter production before confinement in the hidden sector

- Hypotheses for a UV freeze-in mechanism : [Elahi, Kolda, Unwin 2015]

  - Gluon n_v and gluinos n_λ number densities are initially negligible
  - Dark and visible sectors are never at thermal equilibrium
  - Confinement energy of gluons/gluinos in glueballs/gluinoballs is negligible

II. Dark Matter production

✳ K_1 : Bessel function of 2nd kind

- Boltzmann equations (simplified) :

$$\frac{\mathrm{d}n_{\mathrm{HS}}}{\mathrm{d}t} + 3Hn_{\mathrm{HS}} \simeq \frac{T}{512\pi^5}\int_0^\infty \mathrm{d}s|\mathcal{M}|^2\sqrt{s}K_1(\sqrt{s}/T). \tag{11}$$

- Amplitudes for both cases :

$$|\mathcal{M}_{\mathrm{dim5}}|^2 \sim \frac{N_c}{\Lambda_M^2}s \qquad \text{and} \qquad |\mathcal{M}_{\mathrm{dim6}}|^2 \sim \frac{N_c}{\Lambda_M^4}s^2. \tag{12}$$

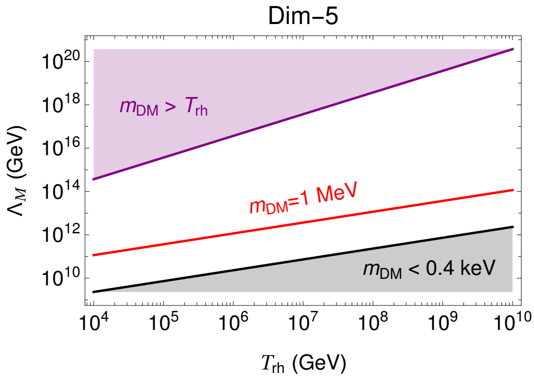
II. Dark Matter production

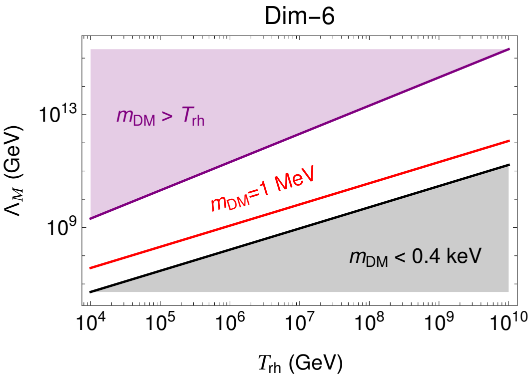
- DM relic density $\Omega_{\rm DM} = \frac{m_{\rm DM} Y_{\rm DM} s_0}{\rho_c}$:

$$\Omega_{\rm dim5} h^2 \simeq 0.134 \times 10^{21} N_c \frac{T_{\rm rh} m_{\rm DM}}{\Lambda_M^2}$$

and

$$\Omega_{\rm dim6} h^2 \simeq 0.185 \times 10^{21} N_c \frac{T_{\rm rh}^3 m_{\rm DM}}{\Lambda_M^4}$$





III. Low energy Dynamics : Veneziano-Yankeliowicz effective theory

- Difficulty to describe the confined theory with SYM [Veneziano, Yankeliowicz 1982]
- Veneziano and Yankeliowicz idea : introduction of a chiral superfield S such as

$$S = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y), \qquad y_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta}. \tag{13}$$

$$\phi(y) \equiv \frac{\beta(g)}{2g}\lambda^\alpha\lambda_\alpha, \qquad \sqrt{2}\psi_\alpha(y) \equiv -\frac{\beta(g)}{2g}\left(-i\lambda_\alpha D + (\sigma^{\mu\nu}\lambda)_\alpha F_{\mu\nu}\right), \tag{14}$$

$$F(y) \equiv -\frac{\beta(g)}{g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{i}{2}\bar{\lambda}\bar{\sigma}\bar{\nabla}\lambda + \frac{1}{2}D^2 - \frac{i}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{i}{2}\partial_\mu J^{\mu 5}\right). \tag{15}$$

III. Low energy Dynamics : Veneziano-Yankeliowicz effective theory

- Veneziano-Yankeliowicz Lagrangian :

$$\mathcal{L}_{\mathrm{VY}}^{N_c} = \frac{9N_c^2}{\alpha}(S^\dagger S)^{\frac{1}{3}}\Big|_D + \left[\frac{2N_c}{3}\,S\left(\log\left(\frac{S}{\Lambda^3}\right)^{N_c} - N_c\right)\Big|_F + \text{h.c.}\right] \tag{16}$$

✳ Λ : dynamical energy scale ✳ α : order 1 parameter

Issue : glueballs appear in the auxiliary field

III. Low energy Dynamics : Veneziano-Yankeliowicz effective theory

- Idea : add a glueball chiral superfield χ :

$$\chi = \phi_\chi + \sqrt{2}\theta\psi_\chi + \theta^2 F_\chi \qquad [\chi] = 0 \tag{17}$$

[Merlatti, Sannino 2004]

- Generalization of $\mathcal{L}_{\text{VY}}^{N_c}$:

✳ γ : fixed parameter

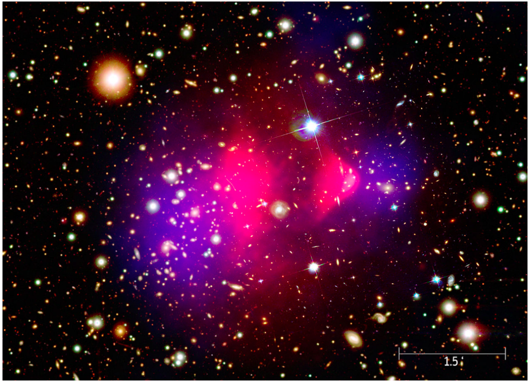
$$\begin{aligned}\mathcal{L}_{\text{gVY}}^{N_c} = & \frac{9N_c^2}{\alpha}(S^\dagger S)^{\frac{1}{3}}\left(1+\gamma\chi\chi^\dagger\right)\Big|_D \\ & + \frac{2N_c}{3}S\left(\log\left(\frac{S}{\Lambda^3}\right)^{N_c} - N_c - N_c\ln\left(-e\frac{\chi}{N_c}\ln\chi^N\right)\right)\Big|_F + \text{h.c.}\end{aligned} \tag{18}$$

III. Bound on Dark Matter mass

- Use bounds on self-scattering interactions from Bullet clusters :

$$\frac{\sigma_{\mathrm{DM}}}{m_{\mathrm{DM}}} \leq 2 \text{ cm.g}^{-1} \qquad (19)$$

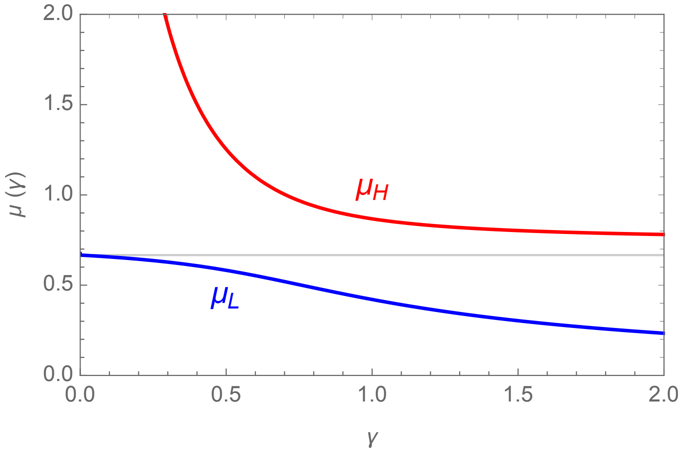
[Robertson, Massey, Eke 2016]



[Image credit : Chandra 2004]

III. Bound on Dark Matter mass

- Diagonalize scalar potential to get mass eigenstates ϕ_L (light) and ϕ_H (heavy) :



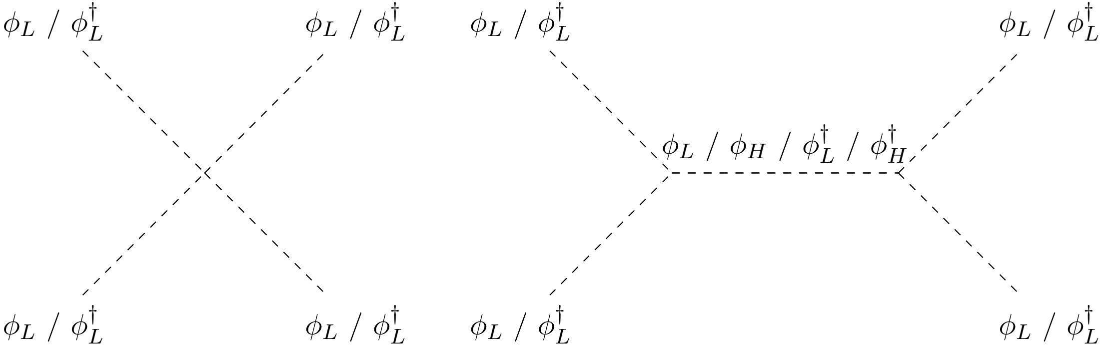
$$m_{L,H} = \alpha\Lambda\ \mu_{L,H}(\gamma). \tag{20}$$

$$m_S = \frac{2}{3}\alpha\Lambda. \tag{21}$$

III. Bound on Dark Matter mass

- Terms in the Lagrangian allowing scattering of the light eigenstate :

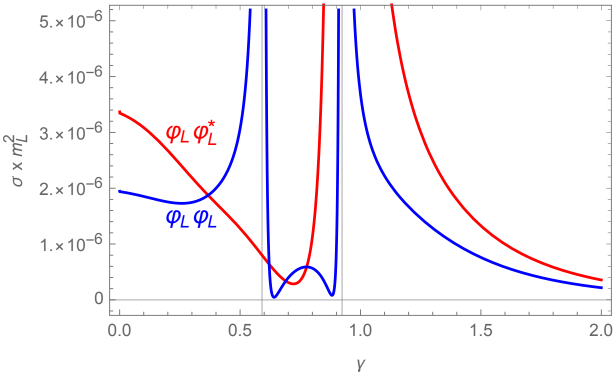
$$\mathcal{L} \supset C_{31}\ (\varphi_L^3 \varphi_L^\dagger + \text{h.c.}) + C_{22}\ \varphi_L^2 (\varphi_L^\dagger)^2 + m_L \left(c_{21}\ \varphi_L^2 \varphi_L^\dagger + \text{h.c.} \right) \\ + m_H \left(c_{20}^H\ \varphi_L^2 \varphi_H^\dagger + c_{11}^H\ \varphi_L \varphi_L^\dagger \varphi_H^\dagger + \text{h.c.} \right) . \tag{22}$$



III. Bound on Dark Matter mass

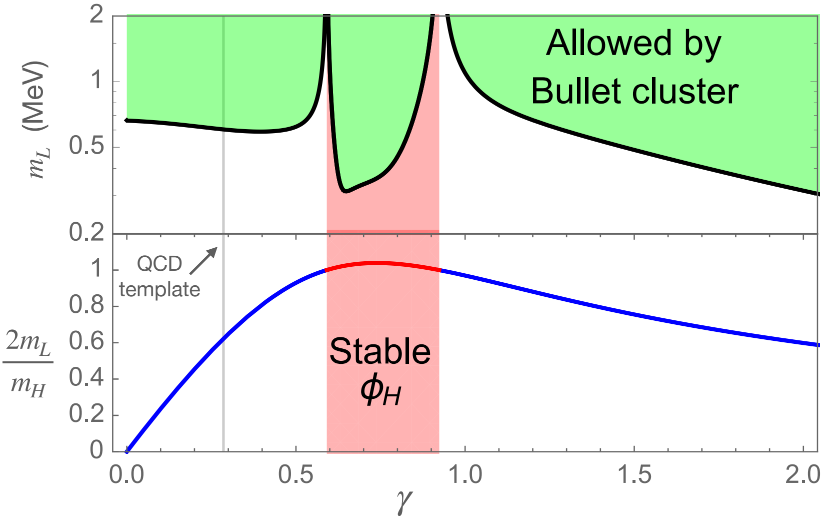
- DM cross section :

$$\sigma(\varphi_L\varphi_L^\dagger) = \sum_f \frac{|\mathcal{A}(\varphi_L\varphi_L^\dagger \to f)|^2}{128\pi\ m_L^2}\,, \quad \sigma(\varphi_L\varphi_L) = \sum_f \frac{|\mathcal{A}(\varphi_L\varphi_L \to f)|^2}{128\pi\ m_L^2}\,, \tag{23}$$



$$\sigma_{\text{DM}} = \frac{\sigma(\varphi_L\varphi_L^\dagger) + \sigma(\varphi_L\varphi_L)}{2} \sim \frac{\alpha^6}{N_c^4}\frac{|\tilde{A}(\gamma)|^2}{128\pi m_L^2} \tag{24}$$

III. Bound on Dark Matter mass



Conclusion

- SUSY hidden sectors offer new possibilities for Dark Matter
- Example with a SYM hidden sectors where predictions can be made :
  - Dark matter are gluons and gluinos bound states called glueballs and gluinoballs
  - DM production through UV Freeze-in
  - Constraints on the DM mass using Bullet Cluster data
- Outlooks :
  - Constraints from Domain Walls in SYM theory ?
  - Possibility to construct the same kind of model using different SUSY theories

Why Supersymmetry (SUSY) ?

- SUSY is the natural extension of Poincaré algebra, attractive formalism

- Superfields in superspace $(y_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$ with $y_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta}$:

  - Chiral : $\Phi(y,\theta) = \phi(y) + \sqrt{2}\psi(y)\theta + F(y)\theta^2, \qquad \bar{D}_{\dot{\alpha}}\Phi = 0.$

  - Vector : $V(y,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}v_\mu(y) + i\theta^2\bar{\theta}\bar{\lambda}(y) - i\theta\bar{\theta}^2\lambda(y) + \frac{1}{2}\theta^2\bar{\theta}^2(D(y) - i\partial_\mu v^\mu(y)).$

✳ ϕ : scalar field

✳ ψ_α, λ_α : spinors

✳ v^μ : vector field

✳ F, D : auxiliary fields

Dark Matter in SUSY formalism

- Minimal Supersymmetric Standard Model (MSSM) :

	Superfield	$SU(3)$	$SU(2_L)$	$U(1)_Y$	Particles
Quarks/Squarks	$\hat{Q}$	3	2	1/6	(u_L, d_L), $(\tilde{u}_L, \tilde{d}_L)$
	$\hat{U}^c$	$\bar{3}$	1	$-2/3$	$\bar{u}_R$, $\tilde{u}_R^*$
	$\hat{D}^c$	$\bar{3}$	1	1/3	$\bar{d}_R$, $\tilde{d}_R^*$
Leptons/Sleptons	$\hat{L}$	1	2	$-1/2$	(ν_L, e_L), $(\tilde{\nu}_L, \tilde{e}_L)$
	$\hat{E}^c$	1	1	1	$\bar{e}_R$, $\tilde{e}_R^*$
Higgs/Higgsinos	$\hat{H}_u$	1	2	1/2	$(H_u, \tilde{h}_u)$
	$\hat{H}_d$	1	2	$-1/2$	$(H_d, \tilde{h}_d)$
Gauge/Gauginos	$\hat{G}^a$	8	1	0	G^μ, $\tilde{g}$
	$\hat{W}^i$	1	3	0	W_i^μ, $\tilde{w}_i$
	$\hat{B}$	1	1	0	B^μ, $\tilde{b}$

- Next to Minimal Supersymmetric Standard Model (NMSSM) : MSSM + $\hat{N}$ superfield

Dark Matter production

- Comoving number density (yield) $Y_{\text{HS}} = \frac{n_{\text{HS}}}{s_e}$:

$$Y_{\text{dim5}} \simeq \frac{45 M_P N_c}{128\pi^7 1.66 g_*^s \sqrt{g_*^\rho}} \frac{T_{\text{rh}}}{\Lambda_M^2} \quad \text{and} \quad Y_{\text{dim6}} \simeq \frac{1485 M_P N_c}{1024\pi^7 1.66 g_*^s \sqrt{g_*^\rho}} \frac{T_{\text{rh}}^3}{\Lambda_M^4}. \tag{25}$$

g_*^s / g_*^ρ : number of effective degrees of freedom

- DM relic density $\Omega_{\text{DM}} = \frac{m_{\text{DM}} Y_{\text{DM}} s_0}{\rho_c}$:

$$\Omega_{\text{dim5}} h^2 \simeq 0.134 \times 10^{21} N_c \frac{T_{\text{rh}} m_{\text{DM}}}{\Lambda_M^2} \quad \text{and} \quad \Omega_{\text{dim6}} h^2 \simeq 0.185 \times 10^{21} N_c \frac{T_{\text{rh}}^3 m_{\text{DM}}}{\Lambda_M^4}. \tag{26}$$

Scalar potential

- Developing the gVY Lagrangian gives the interactions between the scalar parts of the glueballs (ϕ_χ) and the gluinoballs (ϕ) :

$$V(\phi,\bar{\phi},\phi_\chi,\bar{\phi}_\chi) = (\phi\bar{\phi})^{\frac{2}{3}}\frac{4N^2\alpha}{9}\left[\left|\log\left(\frac{\phi}{-e\Lambda^3\phi_\chi\log\phi_\chi}\right)\right|^2 + \frac{1+\gamma\phi_\chi\bar{\phi}_\chi}{9\gamma}\left|\frac{\log\phi_\chi+1}{\phi_\chi\log\phi_\chi}\right|^2 \right.$$
$$\left. + \frac{\log\phi_\chi+1}{3\log\phi_\chi}\log\left(\frac{\bar{\phi}}{-e\Lambda^3\bar{\phi}_\chi\log\bar{\phi}_\chi}\right) + \frac{\log\bar{\phi}_\chi+1}{3\log\bar{\phi}_\chi}\log\left(\frac{\phi}{-e\Lambda^3\phi_\chi\log\phi_\chi}\right)\right].$$

Bound on Dark Matter mass

- We note :

$$C_x = \frac{\alpha^3}{N_c^2}\, F_x(\gamma)\,, \quad c_x = \sqrt{\frac{\alpha^3}{N_c^2}}\, f_x(\gamma)\,, \quad c_x^H = \sqrt{\frac{\alpha^3}{N_c^2}}\, f_x^H(\gamma) \tag{27}$$

- Amplitudes for the different scattering processes at 0 velocity, $\zeta = m_L^2/m_H^2$:

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L^\dagger) = \frac{\alpha^3}{N_c^2}\left[4F_{22} + \frac{20}{3}f_{21}^2 + 4(f_{20}^H)^2 + 4(f_{11}^H)^2\left(1 - \frac{1}{4\zeta - 1}\right)\right], \tag{28}$$

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L) = \frac{\alpha^3}{N_c^2}\left[6F_{31} + \frac{20}{3}f_{21}^2 + 2f_{20}^H f_{11}^H\left(2 - \frac{1}{4\zeta - 1}\right)\right], \tag{29}$$

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L^\dagger\varphi_L^\dagger) = i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L)\,, \tag{30}$$

$$i\mathcal{A}(\varphi_L\varphi_L \to \varphi_L\varphi_L) = \frac{\alpha^3}{N_c^2}\left[4F_{22} + \frac{20}{3}f_{21}^2 - 4(f_{20}^H)^2\left(\frac{1}{4\zeta - 1}\right) + 8(f_{11}^H)^2\right], \tag{31}$$

$$i\mathcal{A}(\varphi_L\varphi_L \to \varphi_L\varphi_L^\dagger) = i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L)\,. \tag{32}$$