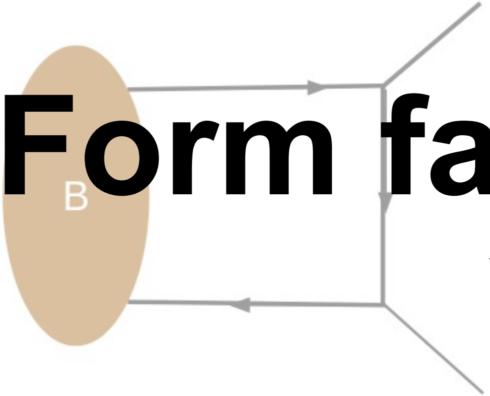
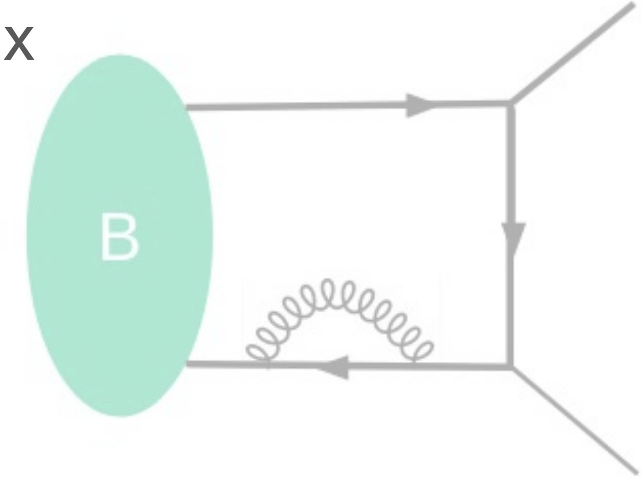


Form factors for *b*→*sll*

Yann Monceaux

10/10/2023



Theoretical framework:

$b \longrightarrow sll$ in the weak effective theory

At the scale m_b $$H_{eff} = H_{eff,sl} + H_{eff,had}$$

- $$H_{eff,sl} = -\underbrace{\frac{4G_F\alpha_{em}^2}{\sqrt{2}}V_{tb}V_{ts}^*}_{\mathcal{N}} \sum_{i=7,9,10,S,P} (C_i^l O_i^l + C_i'^l O_i'^l)$$

$$O_7^{(\prime)} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_{R(L)}b)F^{\mu\nu}$$
$$O_9^{(\prime)} = (\bar{s}\gamma_\mu P_{R(L)}b)(\bar{l}\gamma^\mu l)$$
$$O_{10}^{(\prime)} = (\bar{s}\gamma_\mu P_{R(L)}b)(\bar{l}\gamma^\mu\gamma_5 l)$$

- $$H_{eff,had} = -\mathcal{N}\frac{1}{\alpha_{em}^2}\left(C_8O_8 + C_8' + O_8' + \sum_{i=1,\dots,6} C_iO_i\right) + \text{h.c}$$

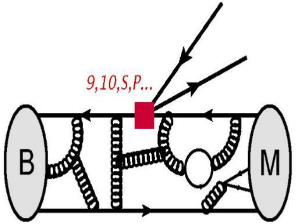
$$O_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b)$$
$$\dots$$

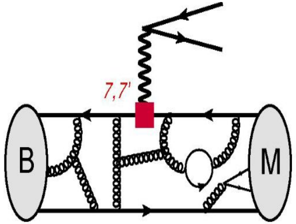
Amplitude of $B \to K^{(*)}ll$ decays

$$\mathcal{A}(B \to K^{(*)}l^+l^-) = \mathcal{N}\left\{(C_9 L_V^\mu + C_{10} L_A^\mu)\mathcal{F}_\mu(q^2) - \frac{L_V^\mu}{q^2}\left[C_7 \mathcal{F}_\mu{}^T(q^2) + \mathcal{H}_\mu(q^2)\right]\right\}$$

- **Local** $\mathcal{F}_\mu(q^2) = \langle \bar{K}^{(*)}(k)|O^{had}_{7,9,10}|\bar{B}(k+q)\rangle$

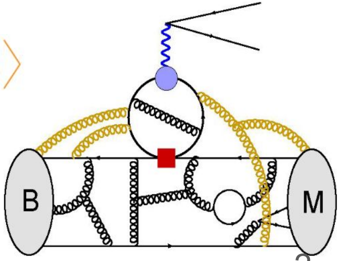
  Parametrized with local Form Factors





Diagrams by Javier Virto

- **Non-Local** $\mathcal{H}_\mu(q^2) = i\int d^4x e^{iq.x}\langle K^{(*)}(k)|T\{j^{em}_\mu(x), C_i O_i(0)\}|\bar{B}(k+q)\rangle$

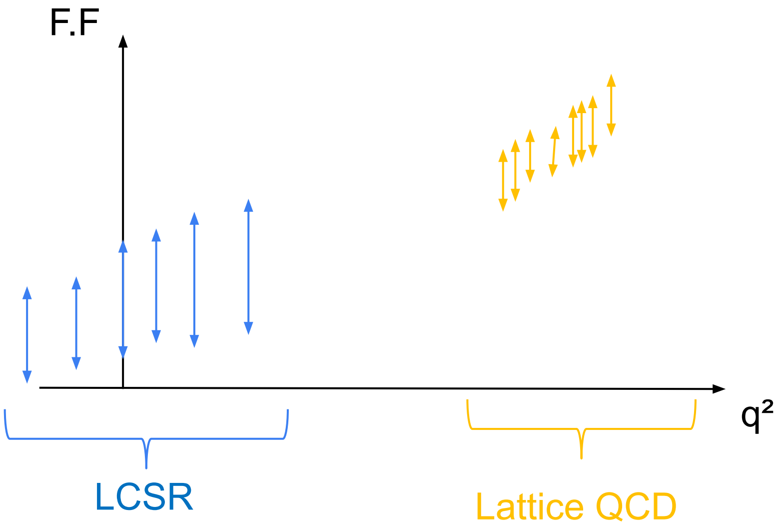


Local Form Factors computation

- At high-q^2 : computed on the lattice

- At low-q^2 : (mostly) Light-Cone Sum Rule (LCSR)

Challenging systematic uncertainties



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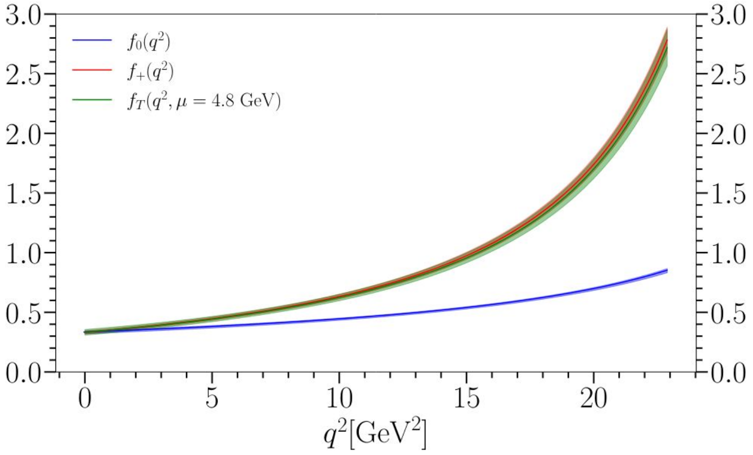
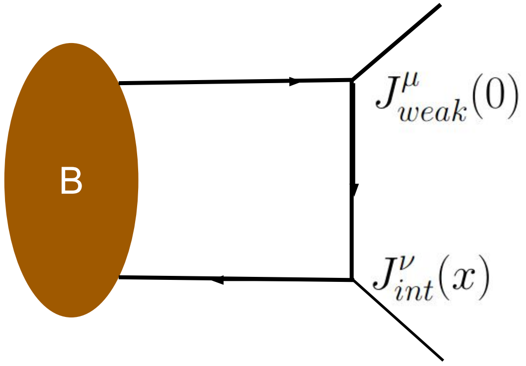
HPQCD (Lattice QCD)



Procedure for Light-Cone Sum Rules

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$

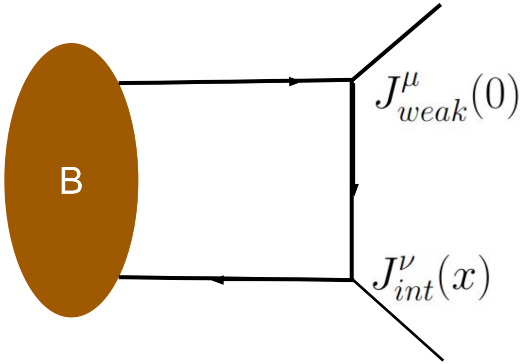
B to vacuum correlation function



Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$ (expansion in growing twists)

Procedure for Light-Cone Sum Rules



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B to vacuum correlation function

Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$ (expansion in growing twists)

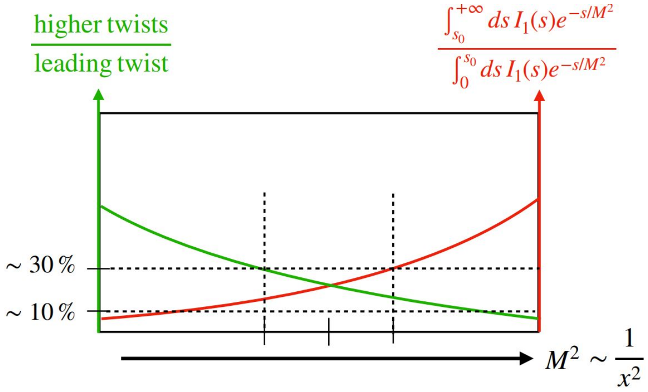
Match both expression

Borel parameter and scale

$$F(q^2) = \frac{f_B m_B}{K^{(F)}} \int_0^{s_0} ds I_1(s) e^{-(s - m^2)/M^2}$$

- Borel parameter M² : compromise between supression of higher twists, and continuum and excited states contribution

We reproduced Gubernari et al *1811.00983* by taking the same window for M².



Range of the Borel parameter

E.g. for $B \to K$: $M^2 \in [0.5, 1.5]\,\mathrm{GeV}^2$

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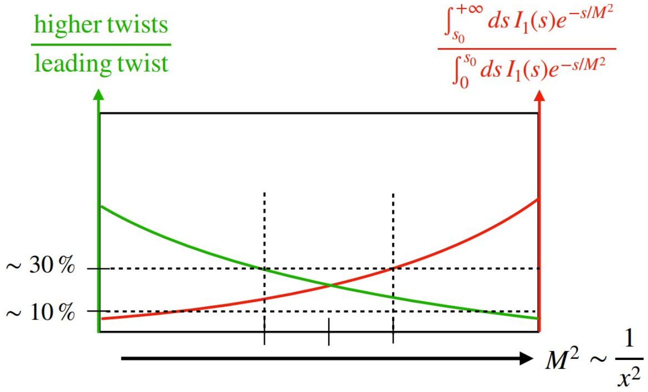
- Borel parameter M² : compromise between supression of higher twists, and continuum and excited states contribution

We reproduced Gubernari et al *1811.00983* by taking the same window for M²

However we found a better compromise at lower M².

<u>Problem</u> : Khodjamirian et al argue that the renormalisation scale is M. At low M² need the radiative corrections !



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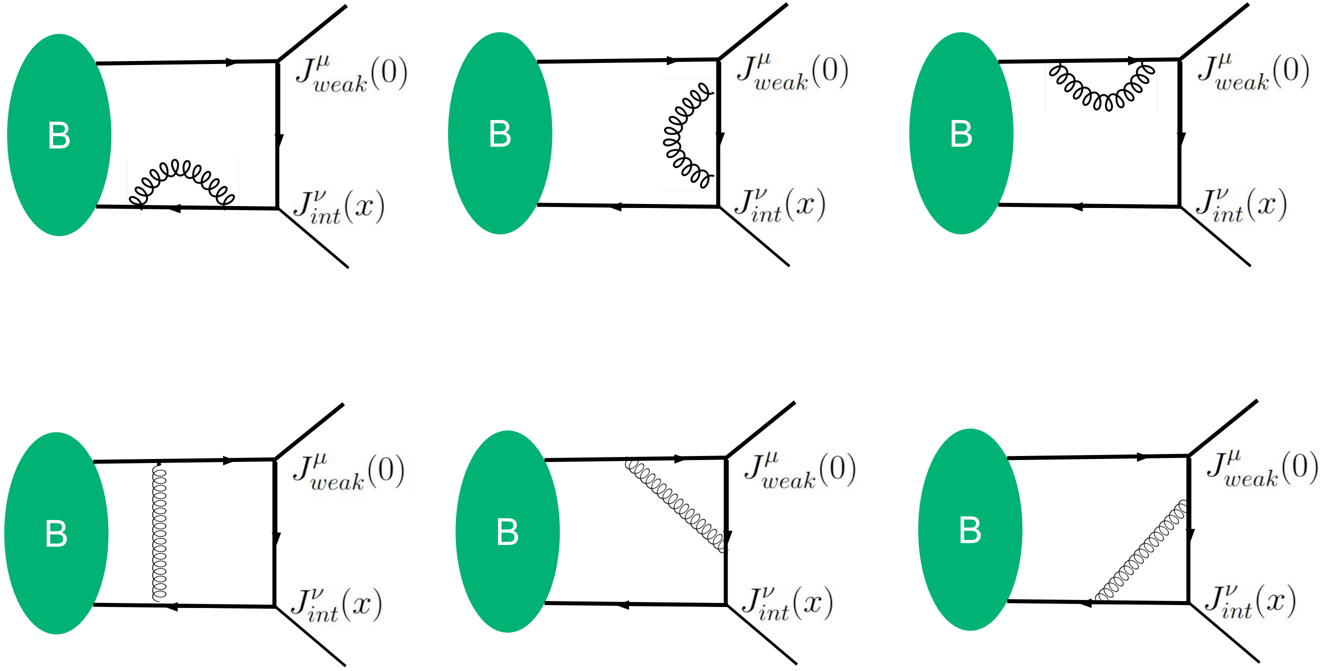
The radiative corrections



2212.11624:

"The higher-order QCD corrections [...] can bring about consistently O(30 %) reductions of the counterpart leading-order LCSR predictions"

Factorisation

$$\Pi^{\mu\nu} = \int \frac{d^4\tilde{t}}{(2\pi)^4} \Phi^B(\tilde{t})_{\alpha\beta} T^{\mu\nu}(\tilde{t})_{\beta\alpha}$$

B-meson DAs :
non perturbative

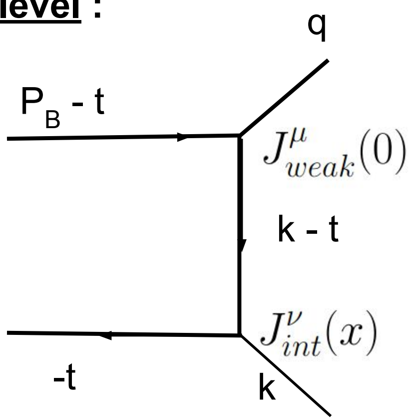
Hard-Scattering Kernel :
perturbative

The hard-scattering kernel does not depend on the external state : can take a partonic external state

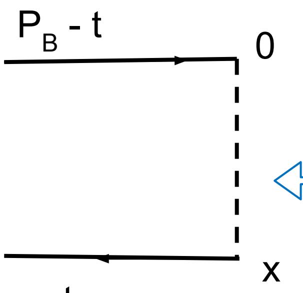
Factorisation

$$\Pi^{\mu\nu,\bar{b}q} = \int \frac{d^4\tilde{t}}{(2\pi)^4} \Phi^{\bar{b}q}(\tilde{t})_{\alpha\beta} T^{\mu\nu}(\tilde{t})_{\beta\alpha}$$

<u>**At tree level**</u> **:**



$\Pi^{\mu\nu,\bar{b}q,(0)}$



A Wilson line : connects the two quarks

$\Phi^{\bar{b}q,(0)}$

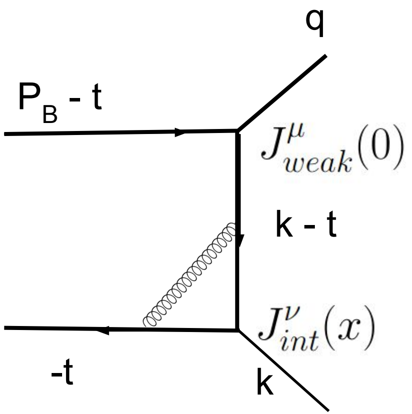
Factorisation : At order α_s

$$\Pi^{\mu\nu,(1)} = \Phi^{(1)} \otimes T^{\mu\nu,(0)} + \Phi^{(0)} \otimes T^{\mu\nu,(1)}$$

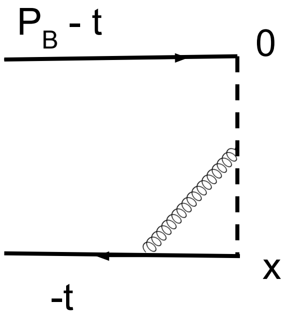
$$\Rightarrow \Phi^{(0)} \otimes T^{\mu\nu,(1)} = \Pi^{\mu\nu,(1)} - \Phi^{(1)} \otimes T^{\mu\nu,(0)}$$

$T^{\mu\nu,(1)}$ should be such that there are only hard contributions, soft and collinear divergences should cancel in the subtraction

Factorisation : an example



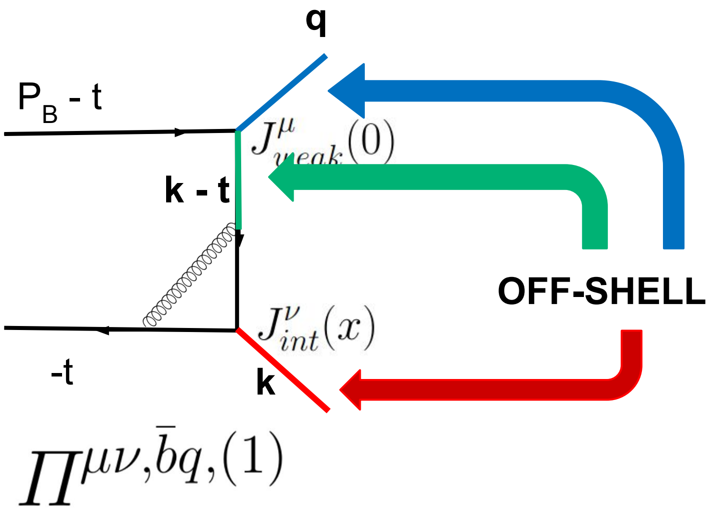
$\Pi^{\mu\nu,\bar{b}q,(1)}$

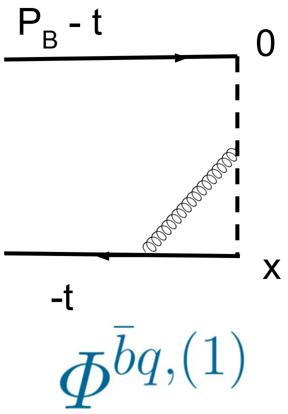


$\Phi^{\bar{b}q,(1)}$

➡ Check soft and collinear divergences should cancel in the subtraction diagram by diagram

Factorisation : an example





Check soft and collinear divergences should cancel in the subtraction diagram by diagram

Passarino-Veltman reduction

Scalar integrals are calculable.
For non-scalar integrals : Passarino-Veltman reduction

$$\int \frac{d^D l}{(2\pi)^D} \frac{l^\mu}{l^2((l+p_1)^2-m_1^2)((l+p_2)^2-m_2^2)} = C_1 p_1^\mu + C_2 p_2^\mu$$

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$$\Rightarrow \quad [l \cdot p_1] = \int \frac{d^D l}{(2\pi)^D} \frac{l \cdot p_1}{l^2((l+p_1)^2-m_1^2)((l+p_2)^2-m_2^2)} = C_1 p_1^2 + C_2 p_1 \cdot p_2$$

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$$\Rightarrow \quad \begin{pmatrix} [l \cdot p_1] & [l \cdot p_2] \end{pmatrix} = \begin{pmatrix} C_1 & C_2 \end{pmatrix} \begin{pmatrix} p_1^2 & p_1 \cdot p_2 \\ p_1 \cdot p_2 & p_2^2 \end{pmatrix}$$

Invert this matrix

Passarino-Veltman reduction

$$\int \frac{d^D l}{(2\pi)^D} \frac{l \cdot p_1}{l^2((l+p_1)^2 - m_1^2)((l+p_2)^2 - m_2^2)} = \int \frac{d^D l}{(2\pi)^D} \frac{\frac{1}{2}((l+p_1)^2 - m_1^2 + m1^2 - l^2 - p_1^2)}{l^2((l+p_1)^2 - m_1^2)((l+p_2)^2 - m_2^2)}$$

Passarino-Veltman reduction

$$\int \frac{d^D l}{(2\pi)^D} \frac{l \cdot p_1}{l^2((l+p_1)^2-m_1^2)((l+p_2)^2-m_2^2)} = \int \frac{d^D l}{(2\pi)^D} \frac{\frac{1}{2}((l+p_1)^2-m_1^2+m1^2-l^2-p_1^2)}{l^2((l+p_1)^2-m_1^2)((l+p_2)^2-m_2^2)}$$

$$= \int \frac{d^D l}{(2\pi)^D} \frac{\frac{1}{2}}{l^2((l+p_2)^2-m_2^2)} + \int \frac{d^D l}{(2\pi)^D} \frac{\frac{1}{2}(m1^2-p_1^2)}{l^2((l+p_1)^2-m_1^2)((l+p_2)^2-m_2^2)} - \int \frac{d^D l}{(2\pi)^D} \frac{\frac{1}{2}}{((l+p_1)^2-m_1^2)((l+p_2)^2-m_2^2)}$$

Scalar integrals : known analytically

For tensor integrals, similar procedure !

Plan

1. Calculate the hard-scattering kernel for each diagram
2. Resum large logarithms
3. Calculate form factors at NLO (and evaluate if lower M^2 is feasible)
4. Publish results!

5. Non-local contributions ?

Amplitude of B → K(*)ll decays

$$\mathcal{A}(B \to K^{(*)} l^+ l^-) = \mathcal{N}\left\{(C_9 L_V^\mu + C_{10} L_A^\mu)\mathcal{F}_\mu(q^2) - \frac{L_V^\mu}{q^2}\left[C_7 \mathcal{F}_\mu{}^T(q^2) + \mathcal{H}_\mu(q^2)\right]\right\}$$

- **Non-Local** $\mathcal{H}_\mu(q^2) = i\int d^4x e^{iq.x}\langle K^{(*)}(k)|T\{j_\mu^{em}(x), C_i O_i(0)\}|\bar{B}(k+q)\rangle$

→ **Main source of uncertainty**

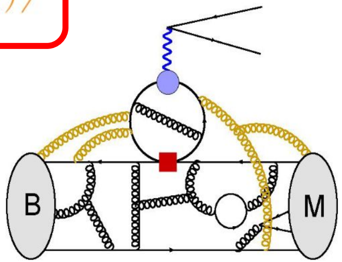


Diagram by Javier Virto

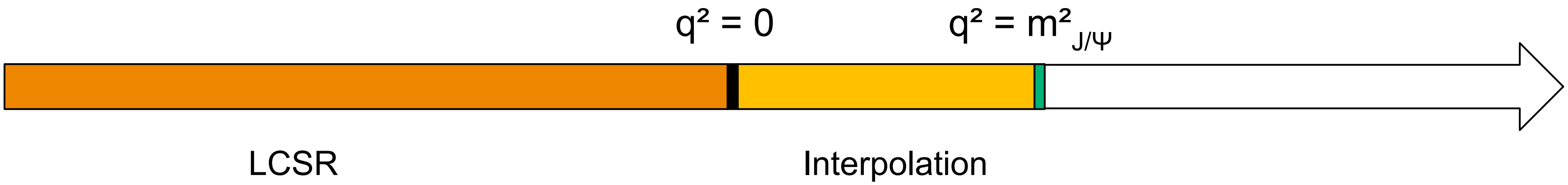
Non-local contributions

$$\mathcal{H}_\mu(q^2) = i\int d^4x\, e^{iq.x} \langle K^{(*)}(k)|T\{j_\mu^{em}(x), C_iO_i(0)\}|\bar{B}(k+q)\rangle$$

At leading power in α_s: Proportional to local Form Factors (1)
+ non-perturbative soft-gluon corrections (2)

(2) can be computed using LCSR a negative q^2.

Gubernari et al use experimental data at $q^2 = m^2_{J/\Psi}$



Thank you for you attention !