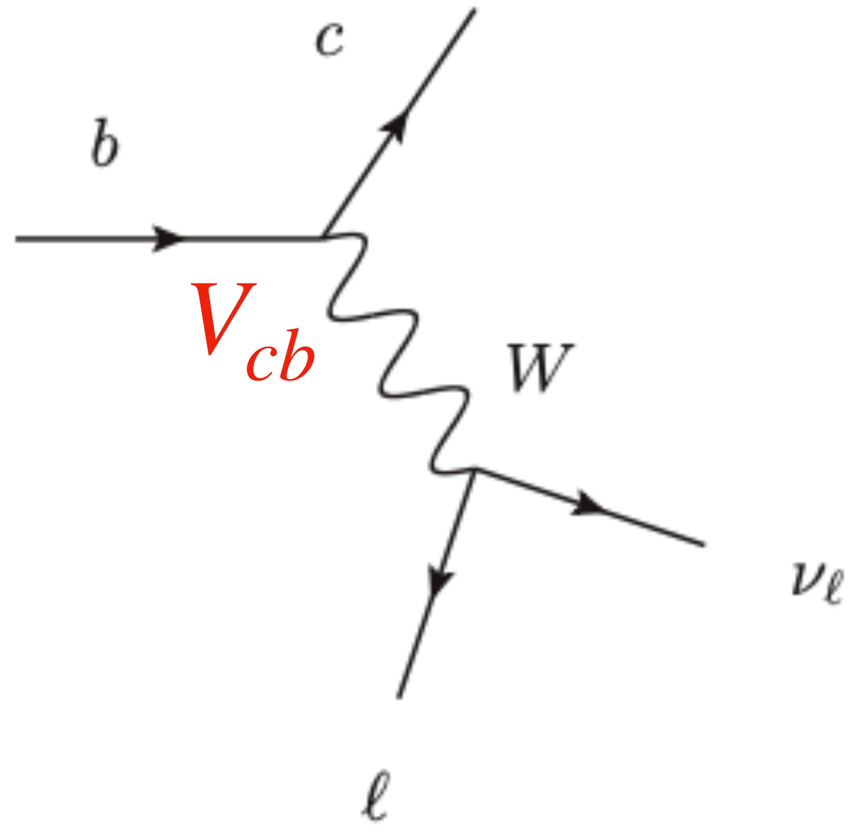
The interplay of $\mathbf{V_{cb}}$ evaluation and $\mathbf{b \to c \ell \nu}$ phenomenology

Nazila's Mini-Workshop Oct 2023 - IP2I

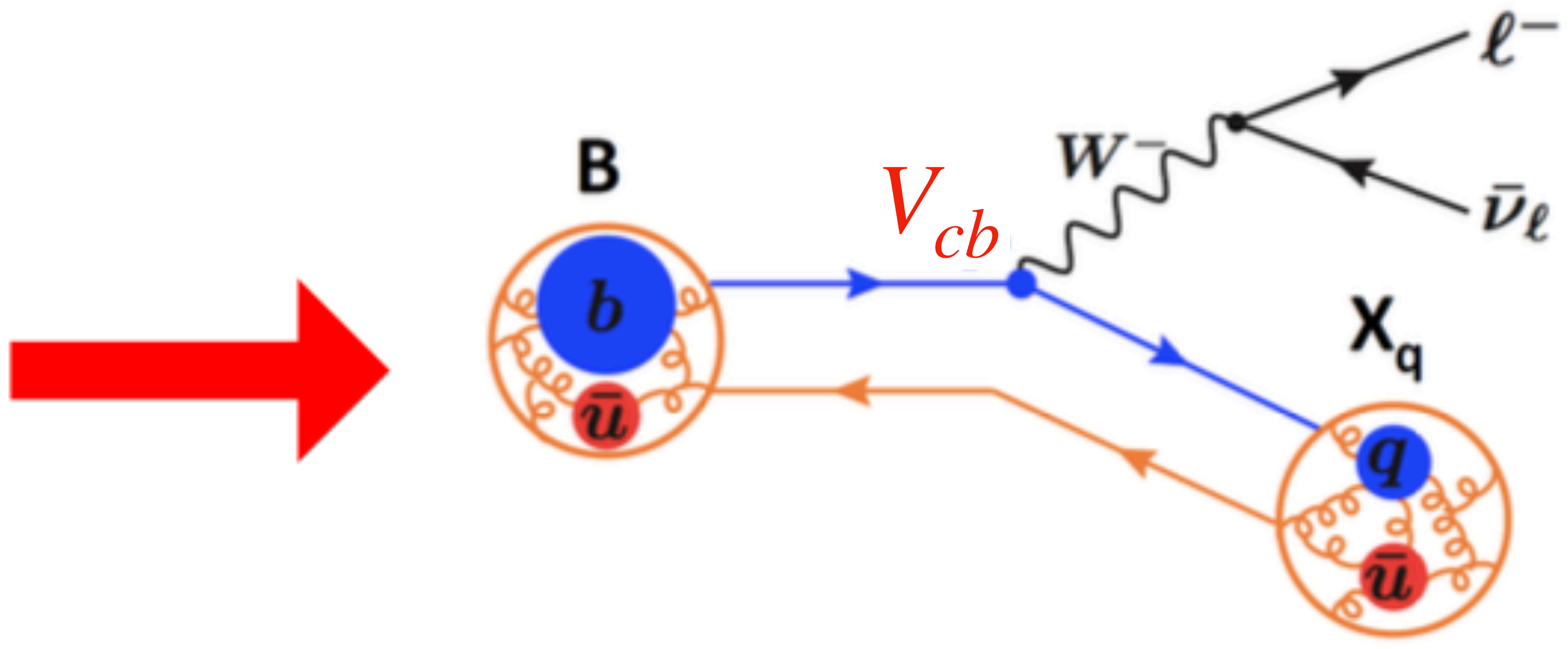
Alexandre Carvunis - 10/10/2023

The Vcb Puzzle

Exclusive vs. Inclusive determination of Vcb



$\mu_{\text{partonic}} = m_b$



$\mu_{\text{hadronic}} = \Lambda_{\text{QCD}}$

$$\bar{B} \to X_c \ell \bar{\nu}$$

Inclusive decay rate

Measurement from B factories (Belle and Babar)

Prospects for $\Lambda_b \to X_c \ell \bar{\nu}$ at LHCb

$$\bar{B} \to D^{(*)} \ell \bar{\nu}$$

Exclusive decay rate

$\bar{B} \to D^{(*)} \ell \bar{\nu}$ data from Belle

$\bar{B}_s \to D_s^{(*)}$ data from LHCb

The Vcb Puzzle

Exclusive vs. Inclusive determination of Vcb

Inclusive

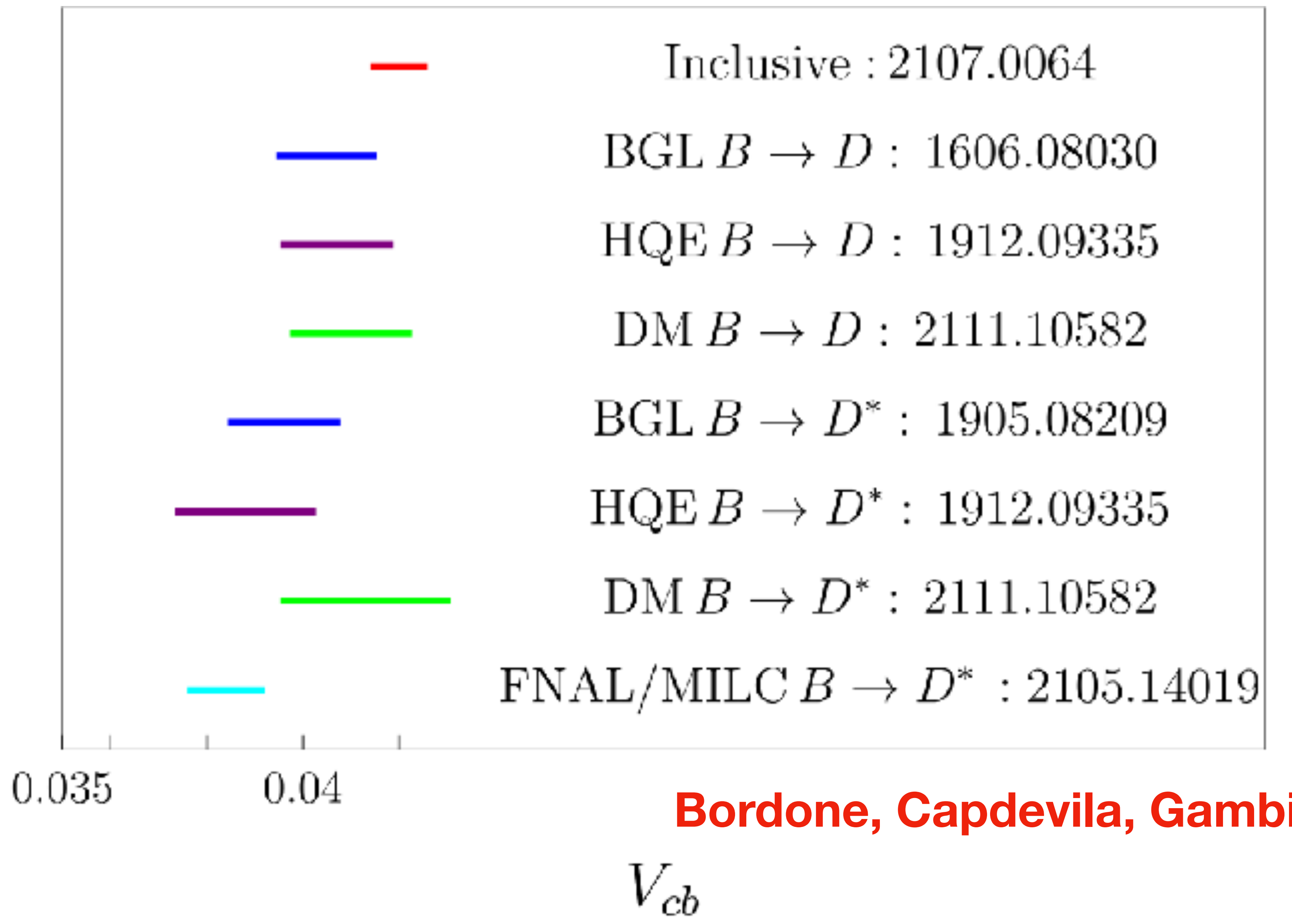
$$V_{cb}^{\text{incl}} = (42.2 \pm 0.8) \times 10^{-3}$$

Exclusive

$$V_{cb}^{\text{excl}} = (39.5 \pm 0.9) \times 10^{-3}$$

Average from PDG 2020

Large variability between determinations using different processes, FF parameterization, lattice input, etc… See next slides



Bordone, Capdevila, Gambino ‘21

The Vcb Puzzle

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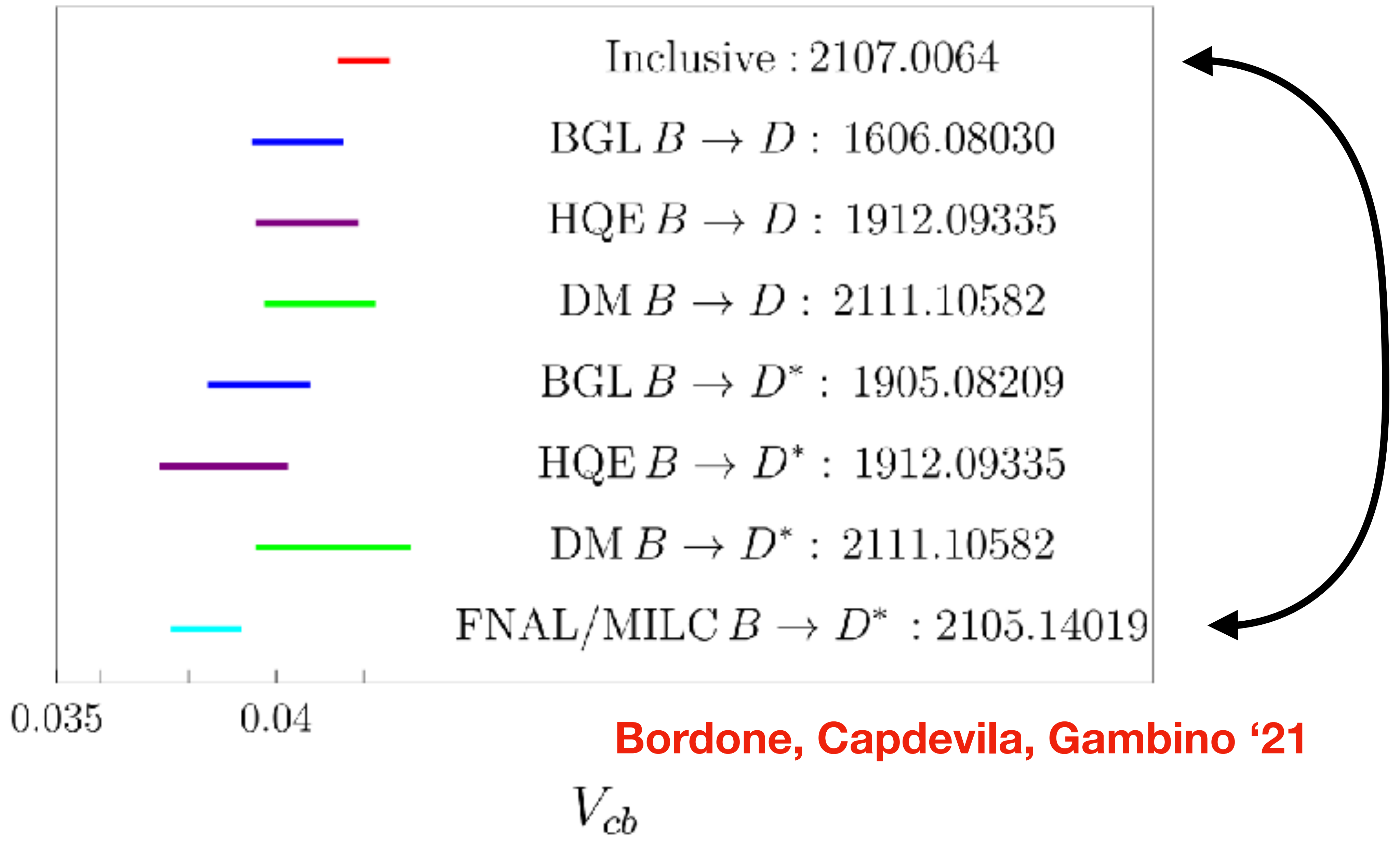

Bordone, Capdevila, Gambino '21

Exclusive Vcb: Theory prediction of $d\Gamma/dw(\bar{B} \to D^* \ell \bar{\nu})$

$$\frac{d\Gamma}{dw}\left(\bar{B} \to D^* \ell \bar{\nu}_\ell\right) = \frac{G_F^2 m_B^5}{48\pi^3} \left|V_{cb}\right|^2 \left(w^2 - 1\right)^{1/2} P(w) \left(\eta_{\rm ew} \mathscr{F}(w)\right)^2$$

Phase space factor → $P(w)$; EW corrections → $\eta_{\rm ew}$; Form Factor → $\mathscr{F}(w)$

$w = v \cdot v'$ product of initial and final velocities

$r = m_{D*}/m_B$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = h_V(w)\,\varepsilon^{\mu\nu\rho\sigma} v_{B,\nu} v_{D^*,\rho} \epsilon^*_\sigma,$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu \gamma^5 b|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = i h_{A_1}(w)\,(1+w)\epsilon^{*\mu} - i\left[h_{A_2}(w) v_B^\mu + h_{A_3}(w) v_{D^*}^\mu\right] \epsilon^* \cdot v_B$$

$$P(w)|\mathcal{F}(w)|^2 = |h_{A_1}(w)|^2 \left\{ 2\frac{r^2 - 2rw + 1}{(1-r)^2}\left[1 + \frac{w-1}{w+1} R_1^2(w)\right] + \left[1 + \frac{w-1}{1-r}(1 - R_2(w))\right]^2 \right\},$$

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)}, \quad R_2(w) = \frac{h_{A_3}(w) + r\,h_{A_2}(w)}{h_{A_1}(w)}.$$

Theory prediction of $d\Gamma/dw(\bar{B} \to D^*\ell\bar{\nu})$

$$\frac{d\Gamma}{dw}\left(\bar{B} \to D^*\ell\bar{\nu}_\ell\right) = \frac{G_F^2 m_B^5}{48\pi^3}\left|V_{cb}\right|^2\left(w^2-1\right)^{1/2}P(w)\left(\eta_{\mathrm{ew}}\mathcal{F}(w)\right)^2$$

- $\mathcal{F}(1) = 1$ from Heavy Quark Symmetry in the infinite quark mass limit (w=1 -> 0 recoil)
- $\mathcal{F}(w)$ extrapolated from $w = 1$ using parameterizations:
  - From analycity and unitarity constraints: Boyd, Grinstein, and Lebed param. (BGL)

$$F(z) = \frac{1}{P_F(z)\phi_F(z)}\sum_{n=0}^{\infty} a_n z^n \,, \; z = (\sqrt{w+1}-\sqrt{2})/(\sqrt{w+1}+\sqrt{2})$$

  - From BGL and using Heavy Quark Symmetry: Caprini, Lellouch, and Neubert param. (CLN)

$$h_{A_1}(w) = h_{A_1}(1)\left[1 - 8\rho^2 z + \left(53\rho^2 - 15\right) z^2 + \left(231\rho^2 - 91\right) z^3\right]$$ (one parameter)

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Typically used up to n=3

  - From BGL and using Heavy Quark Symmetry: Caprini, Lellouch, and Neubert param. (CLN)

$$h_{A_1}(w) = h_{A_1}(1)\left[1 - 8\rho^2 z + \left(53\rho^2 - 15\right) z^2 + \left(231\rho^2 - 91\right) z^3\right]$$ (one parameter)

Inconsistent with subleading terms in the $1/m_{c/b}$ expansion

Theory prediction of $d\Gamma/dw(\bar{B} \to D^*\ell\bar{\nu})$

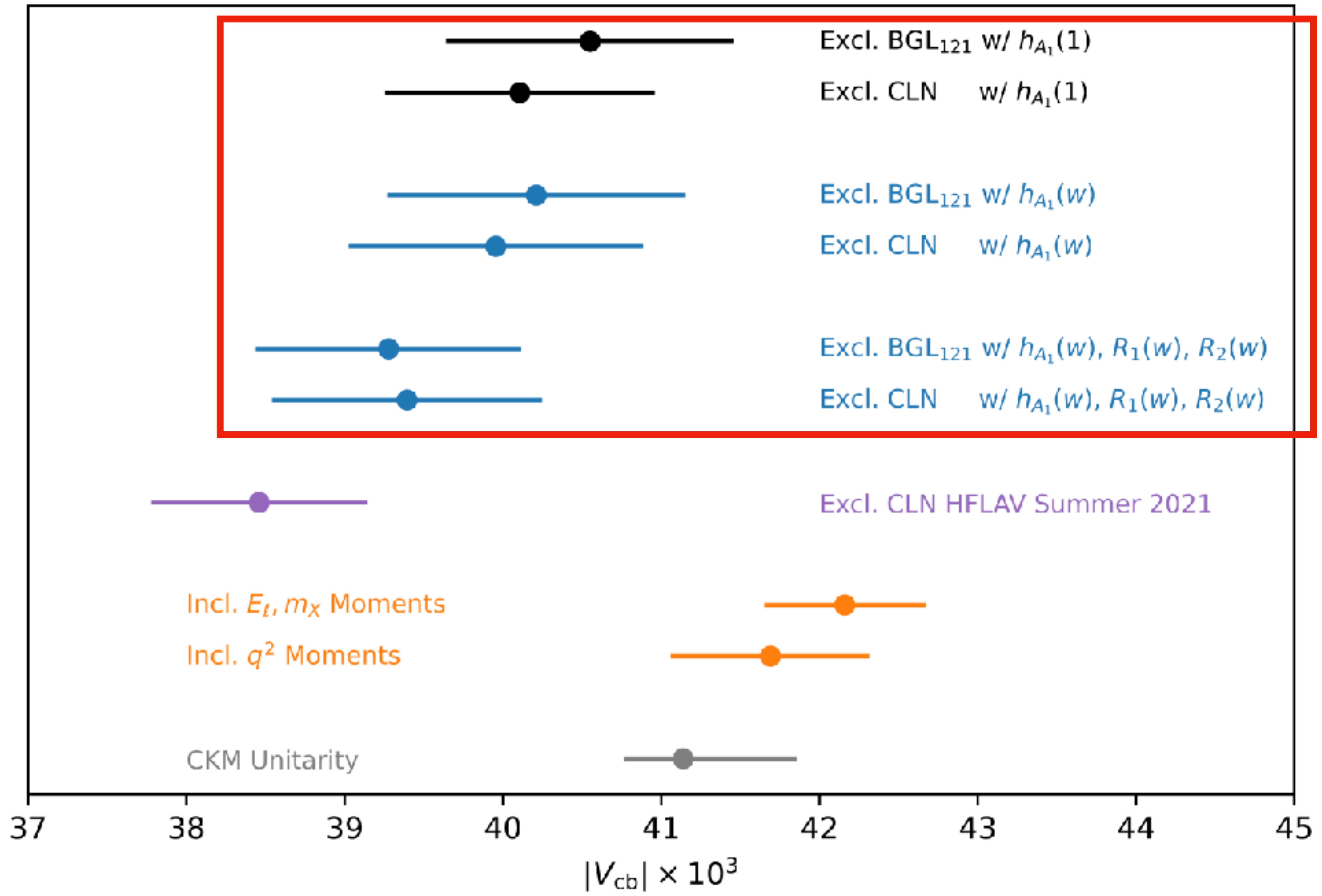
$$\frac{d\Gamma}{dw}\left(\bar{B} \to D^*\ell\bar{\nu}_\ell\right) = \frac{G_F^2 m_B^5}{48\pi^3}\left|V_{cb}\right|^2 \left(w^2 - 1\right)^{1/2} P(w)\left(\eta_{\rm ew}\mathcal{F}(w)\right)^2$$

- From lattice, $\mathcal{F}(1) = 0.904 \pm 0.012$ (Fermilab/MILC + HPQCD)
- From sum rules, $\mathcal{F}(1) = 0.86 \pm 0.01 \pm 0.02$ (Gambino, Manuel, Uraltsev 2012) -> yields results closer to exclusive measurements
- Typically fit to data provides :

$$\eta_{\rm ew}\mathcal{F}(1)\,|V_{cb}| = (35.27 \pm 0.52) \times 10^{-3} \ \ (\mathrm{CLN}).$$

Exclusive measurement by Belle Collab. - 2301.07529

$B \to D^* \ell \bar{\nu}_\ell$

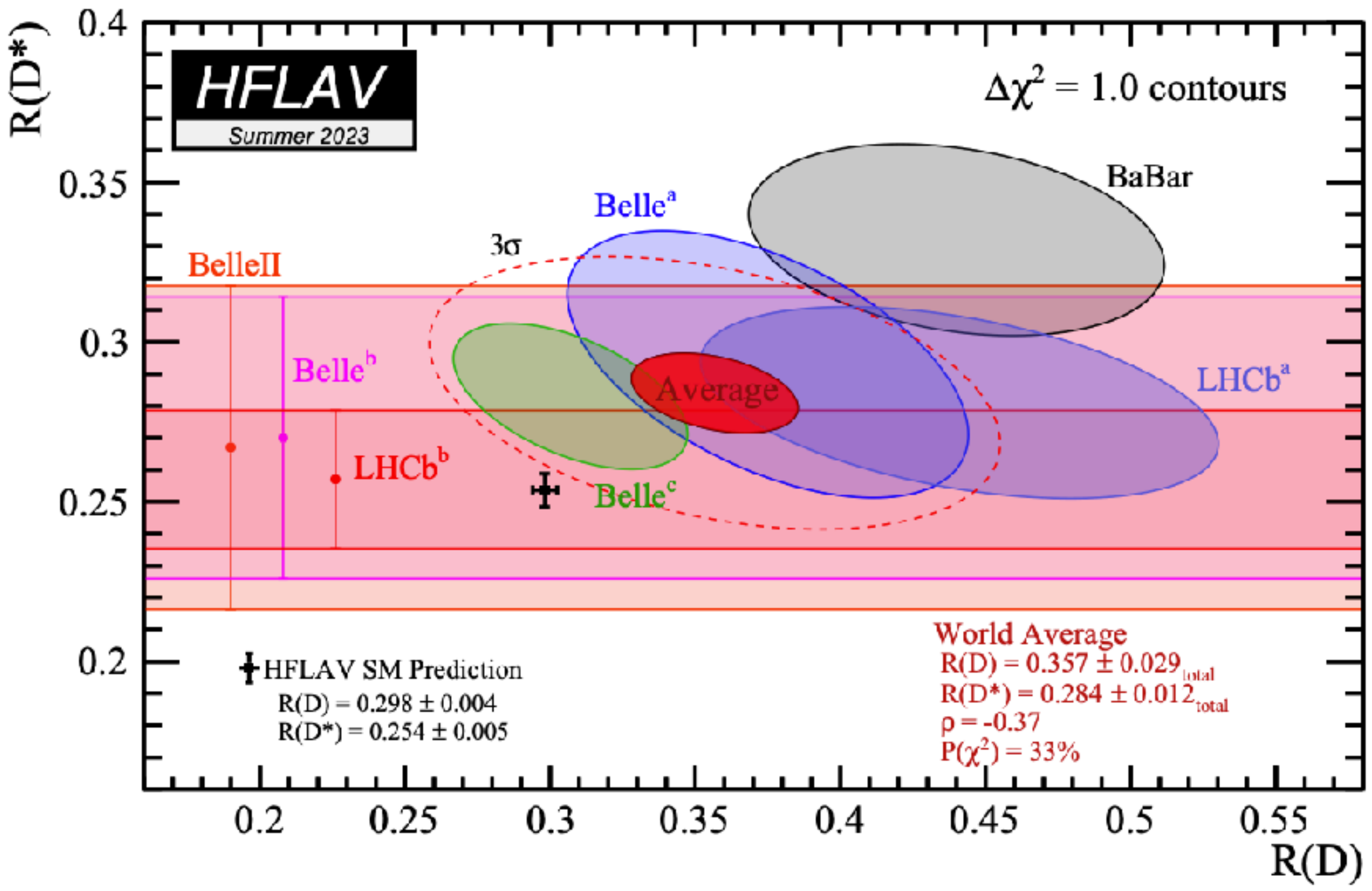


Belle 2023
With various FF parameterizations
and LQCD inputs

Take home message: **Vcb is theory dependent**

Status of the $b \to c\ell\nu$ anomalies and recent data
LFUV tests

- $$R_{D^{(*)}} \equiv \frac{\mathscr{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathscr{B}(B \to D^{(*)}\ell\bar{\nu})}, \quad \ell = \mu, e$$
- Longstanding tension $\sim 3\sigma$, hinting at LFUV
- ~~Particularly interesting in the light of hints of LFUV in neutral current B decays~~ No LFUV in neutral current B decays anymore, but some $b \to s\ell\ell$ anomalies are still standing.
- Ratios of BR($b \to c\ell\nu$) are independent of Vcb but depend on the form factor parametrization
- Hence in a global fit including RD-RD* and BRs must be consistent in the f.f. parametrization



Status of the $b \to c\ell\nu$ anomalies and recent data
LFUV tests

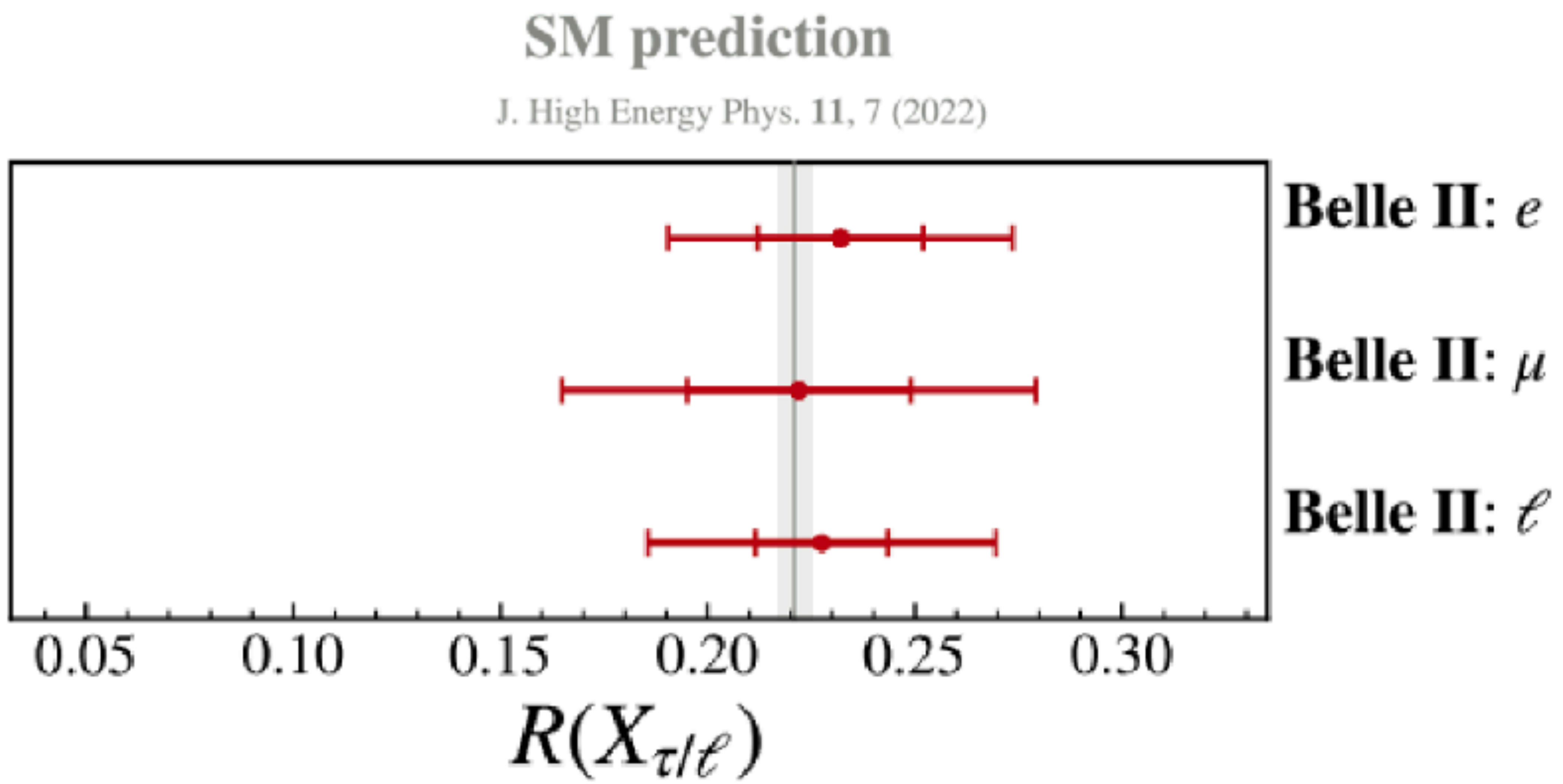
- Other ratios are now available:

LHCb 2022: $\mathcal{R}(\Lambda_c^+) = 0.242 \pm 0.026 \pm 0.040 \pm 0.059,$ $R(\Lambda_c^+)_{\rm SM} = 0.332 \pm 0.008$

LHCb 2017: $\mathcal{R}(J/\psi) = \dfrac{\mathcal{B}(B_c^+ \to J/\psi\,\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\,\mu^+\nu_\mu)} = 0.71 \pm 0.17\,(\text{stat}) \pm 0.18\,(\text{syst}).$ $R(J/\psi)_{\rm SM} = 0.259 \pm 0.004$

CMS 2023: $\mathrm{R}(\mathrm{J}/\psi) = \dfrac{\mathcal{B}(\mathrm{B_c^+} \to \mathrm{J}/\psi\,\tau^+\nu_\tau)}{\mathcal{B}(\mathrm{B_c^+} \to \mathrm{J}/\psi\,\mu^+\nu_\mu)} = 0.17^{+0.18}_{-0.17}\,(\text{stat.})^{+0.21}_{-0.22}\,(\text{syst.})^{+0.19}_{-0.18}\,(\text{theo.}) = 0.17 \pm 0.33,$

Belle II 2023: $R(X_c) \equiv \dfrac{B(B_c \to X_c\tau\nu)}{B(B_c \to X_c\ell\nu)}$



Global fits for b->clv

EFT assuming NP in the tau sector only

$$\mathscr{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \big[(1 + C_{V_L})\, O^{\tau}_{V_L} + C_{V_R} O^{\tau}_{V_R} + C_{S_R} O^{\tau}_{S_R} + C_{S_L} O^{\tau}_{S_L} + C_T O^{\tau}_T \big],$$

$$O^{\tau}_{V_{L,R}} = (\bar{c}\gamma^{\mu} P_{L,R} b)(\bar{\tau}\gamma_{\mu} P_L \nu_{\tau})$$

$$O^{\tau}_{S_{L,R}} = (\bar{c} P_{L,R} b)(\bar{\tau} P_L \nu_{\tau}),$$

$$O^{\tau}_{T} = (\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu_{\tau})\,.$$

Observables can conveniently be expressed in polynomials of WCs

Global fits for b->clv

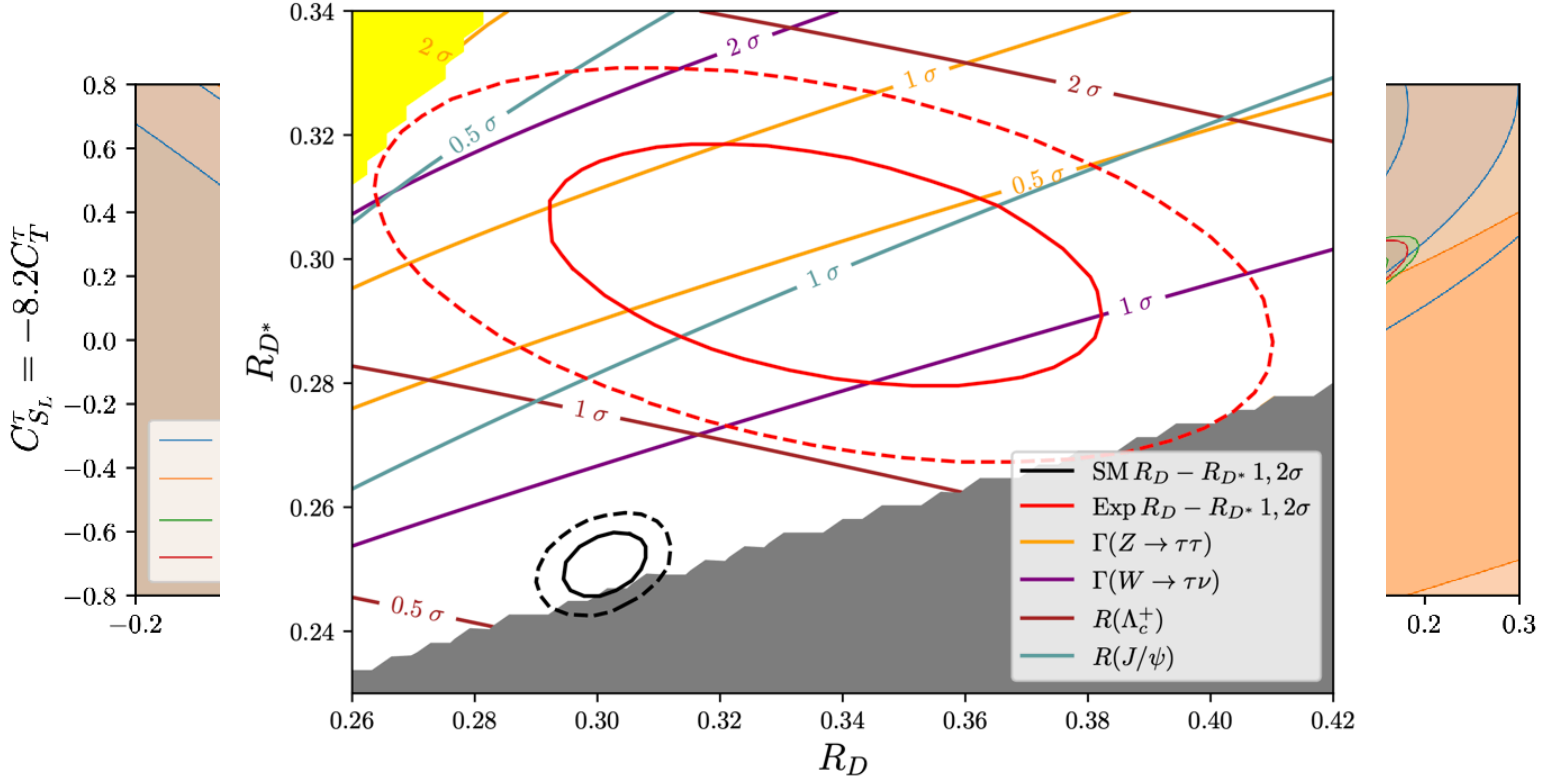
Available data

Observable	Measurement
R_D	BaBar [403], Belle [183, 404]
R_{D^*}	BaBar [403], Belle [183, 404, 405], LHCb [406, 407]
$F_L(B_0 \to D^* \tau \bar{\nu})$	Belle [408]
$\mathrm{BR}(B_c \to \tau \nu)$	LEP [409]
$\frac{1}{\Gamma}\frac{d\Gamma}{dq^2}(B \to D^{(*)} \tau \bar{\nu})$	BaBar [403], Belle [183]
$R_{J/\psi}$	LHCb [195]
R_{Λ_c}	LHCb [197]

+ 2023 data: CMS R(J/Psi), Belle II R(Xc), R(D*)

Global fits for b->clv

Preliminary plots



To do

- State-of-the-art global fit of WET WCs of $b \to c\ell\nu$
- Study the impact of f.f. parametrization on the fit result, account for it as a systematic uncertainty

In collaboration with Peter Stangl and Marzia Bordone

Inclusive prediction for $B \to X_c \ell \bar{\nu}$

- From the optical theorem: $$\Gamma_{tot} = \frac{1}{m_B} \mathrm{Im} \int d^4x \left\langle B(p) \left| T\left\{ \mathscr{H}^{\dagger}_{\text{eff}}(x) \mathscr{H}_{\text{eff}}(0) \right\} \right| B(p) \right\rangle$$

- HQE : $\sum_{n,i} \frac{1}{m_b^n} \mathscr{C}_{n,i} \mathscr{O}_{n+3,i}$

- $\mathscr{C}_{n,i} \propto y_i, z_i$ calculated perturbatively

- $\left\langle B(p) \left| \mathscr{O}_{n+3,i} \right| B(p) \right\rangle$ are non perturbative
  - Need to be determined with e.g. LQCD
  - Can be extracted from data
  - Large n -> loss predictive power

$$\begin{aligned}
\Gamma = & |V_{cb}|^2 \frac{G_F^2 m_b^5(\mu)}{192\pi^3} \eta_{\text{ew}} \times \\
& \Bigg[z_0^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_0^{(1)}(r) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 z_0^{(2)}(r) + \cdots \\
& + \frac{\mu_\pi^2}{m_b^2} \left(z_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_2^{(1)}(r) + \cdots \right) \\
& + \frac{\mu_G^2}{m_b^2} \left(y_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_2^{(1)}(r) + \cdots \right) \\
& + \frac{\rho_D^3}{m_b^3} \left(z_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_3^{(1)}(r) + \cdots \right) \\
& + \frac{\rho_{LS}^3}{m_b^3} \left(y_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_3^{(1)}(r) + \cdots \right) + \ldots \Bigg]
\end{aligned}$$

$r \equiv m_c / m_b$

Inclusive prediction for $B \to X_c \ell \bar{\nu}$

$$
\begin{aligned}
\Gamma =& |V_{cb}|^2 \frac{G_F^2 m_b^5(\mu)}{192\pi^3} \eta_{\rm ew} \times \\
& \left[z_0^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_0^{(1)}(r) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 z_0^{(2)}(r) + \cdots \right. \\
& + \frac{\mu_\pi^2}{m_b^2} \left(z_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_2^{(1)}(r) + \cdots \right) \\
& + \frac{\mu_G^2}{m_b^2} \left(y_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_2^{(1)}(r) + \cdots \right) \\
& + \frac{\rho_{\rm D}^3}{m_b^3} \left(z_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_3^{(1)}(r) + \cdots \right) \\
& \left. + \frac{\rho_{\rm LS}^3}{m_b^3} \left(y_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_3^{(1)}(r) + \cdots \right) + \ldots \right]
\end{aligned}
$$

Parameters $\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ are the non-perturbative input.
Can be fitted on the moments of the distributions of:
charged-lepton energy, hadronic invariant mass, …

$$
\langle E_e^n \rangle_{E_e > E_{\rm cut}} = \int_{E_{\rm cut}}^{E_{\rm max}} \frac{d\Gamma}{dE_e} E_e^n dE_e \Bigg/ \int_{E_{\rm cut}}^{E_{\rm max}} \frac{d\Gamma}{dE_e} dE_e \ .
$$

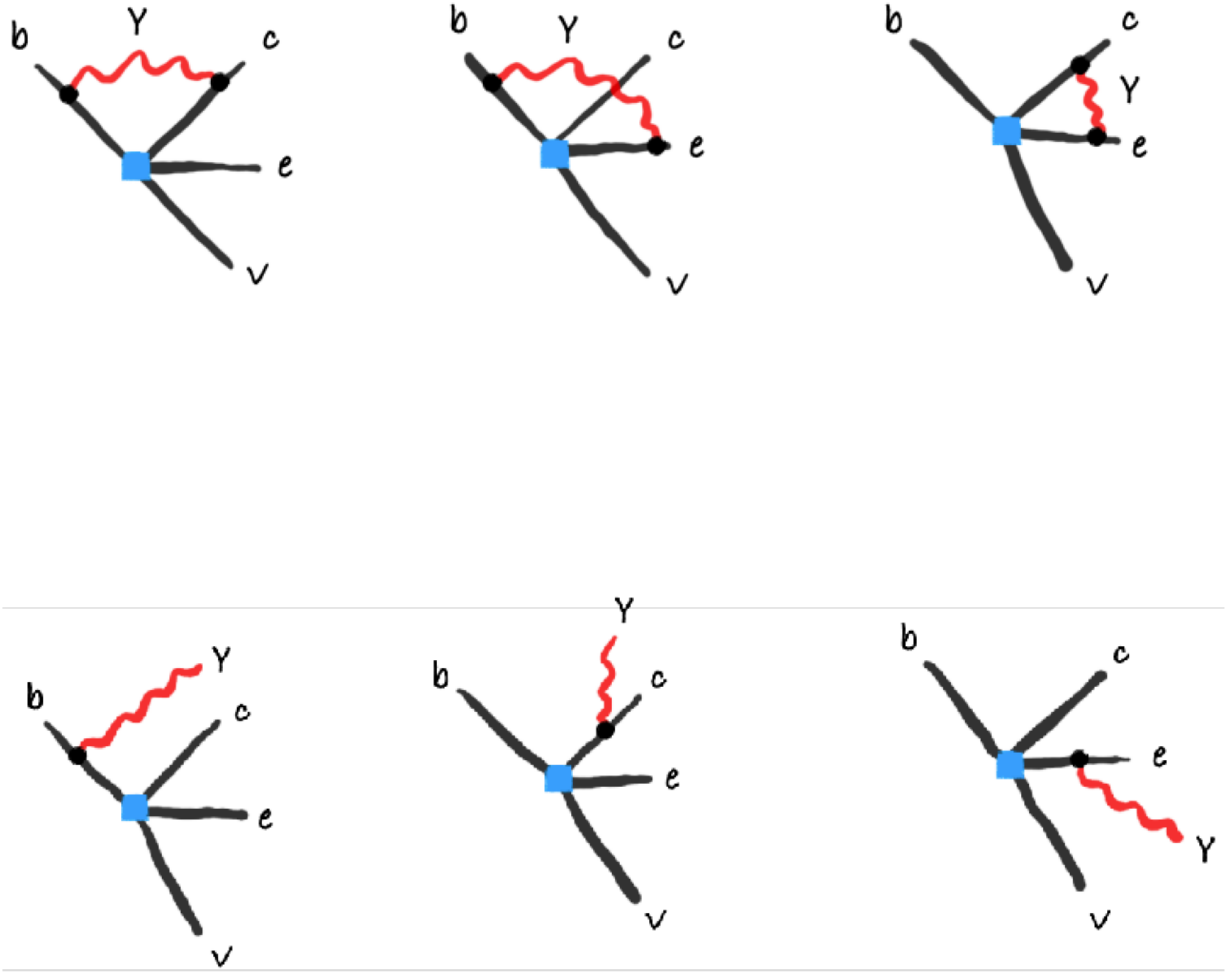
QED effects in inclusive semi-leptonic B decays

Bigi, Bordone, Gambino et al. - *2309.02849*

- Compute soft QED $O(\alpha)$ correction to $B \to X_c e \nu$ including real and virtual corrections

- Improvement over PHOTOS which is currently used by BaBar and Belle to subtract photon radiation

- Roughly decrease the value of V_{cb} by $0.4\,\%$



To do

- **Inclusive Vcb**: Generalize the correction to the moments of the hadronic and leptonic invariant mass distribution, these moments play an important role in the HQE global fits to experimental data

- **Exclusive Vcb**: Similar computation needs to be done for EW and QED corrections to $B \to D^* \ell \nu$

In collaboration with Paolo Gambino and Martin Jung

Thanks!

The Cabibbo-Kobayashi-Maskawa matrix

$$-\mathscr{L}_{\text{Yukawa}} = Y_d^{ij}\bar{Q}_L^i H d_R^j + Y_u^{ij}\bar{Q}_L^i \tilde{H} u_R^j + \text{ h.c.}$$

$$M^f_{\text{diag}} = V_L^f Y^f V_R^{f\dagger}(v/\sqrt{2}) \qquad V_{CKM} \equiv V_L^u V_L^{d\dagger}$$

$$\mathcal{L}_{\text{SM}} \supset \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \Gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_\mu$$

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Wolfenstein parametrization

$$\bar{\rho} = \rho(1-\lambda^2/2+\dots)$$
$$\bar{\eta} = \eta(1-\lambda^2/2+\dots)$$