

Three body forces revealed in strongly deformed nuclei

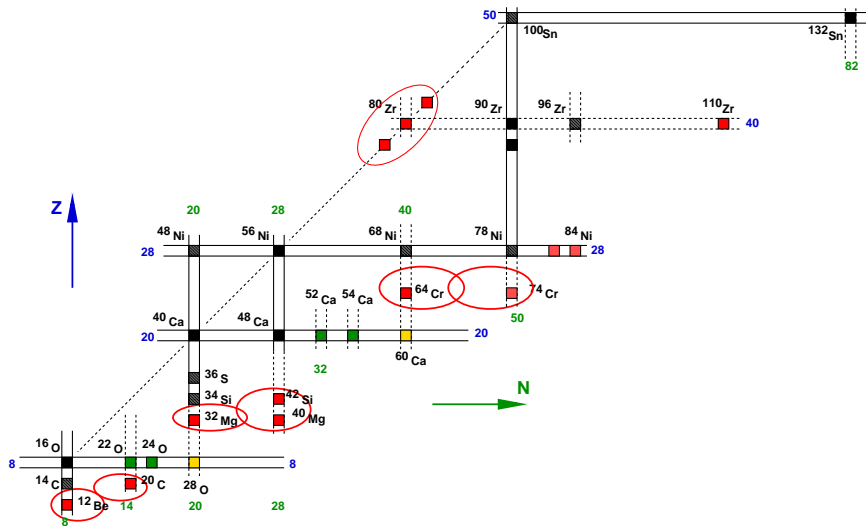
Frédéric Nowacki



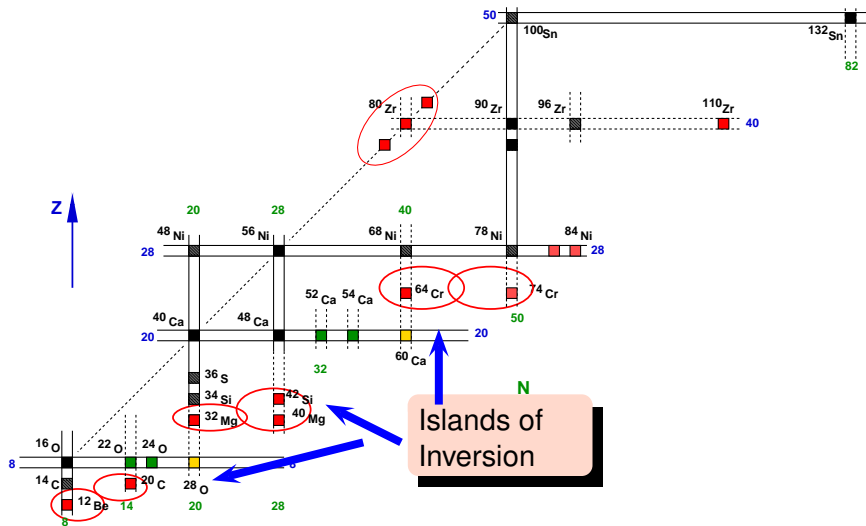
Kickoff meeting of the CNRS-MSU International Research Laboratory on Nuclear Physics and Nuclear Astrophysics



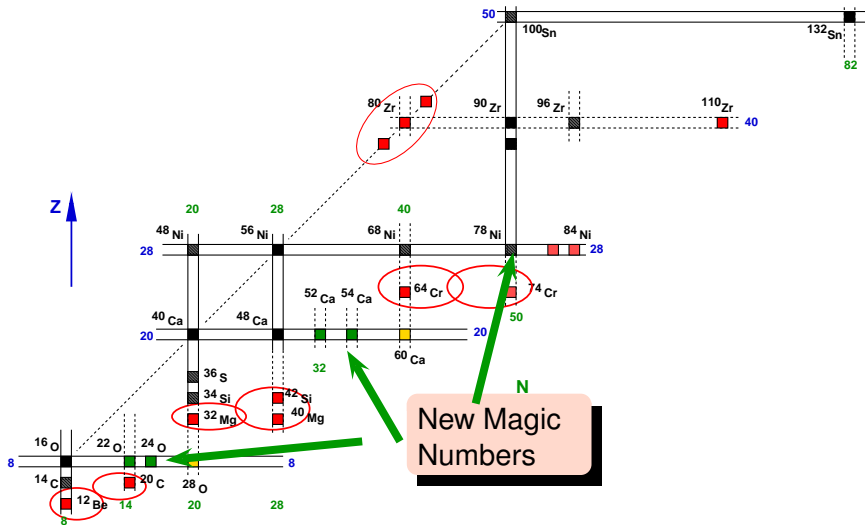
Landscape of medium mass nuclei



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Three-Body Forces and the Limit of Oxygen Isotopes

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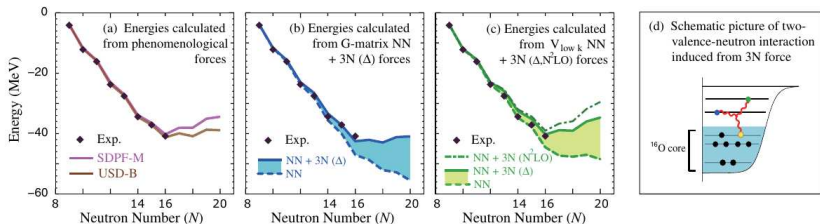
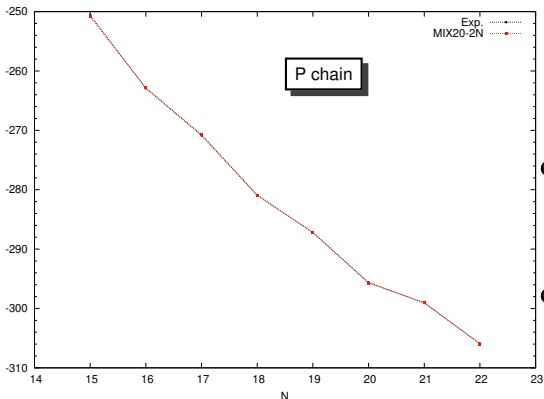


FIG. 4 (color online). Ground-state energies of oxygen isotopes measured from ^{16}O , including experimental values of the bound 16–24 O. Energies obtained from (a) phenomenological forces SDPF-M [13] and USD-B [14], (b) a G matrix and including FM $3N$ forces due to Δ excitations, and (c) from low-momentum interactions $V_{\text{low } k}$ and including chiral EFT $3N$ interactions at $N^2\text{LO}$ as well as only due to Δ excitations [25]. The changes due to $3N$ forces based on Δ excitations are highlighted by the shaded areas. (d) Schematic illustration of a two-valence-neutron interaction generated by $3N$ forces with a nucleon in the ^{16}O core.

At the drip line



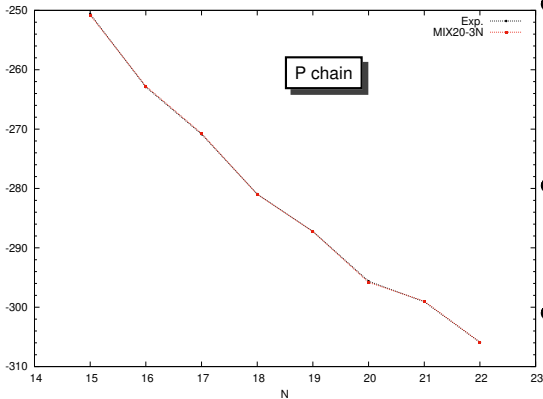
Duflo-Zuker master term:

$$BE \sim N\epsilon + \frac{N(N-1)}{2} V_{NN}$$

Nowacki/Poves 2023

- At the neutron drip line, the ESPE's of ^{28}O are completely at variance with those of ^{40}Ca at the stability valley. The change from the standard ESPE's of ^{16}O to the anomalous ones in ^{28}O is totally due to the interactions of *sd* shell neutrons among themselves
- Notice that the *sd* shell orbits remain always below the *pf* shell with the $\nu 0f_{7/2}$ and $\nu 0p_{3/2} - 0p_{1/2}$ orbitals DO get inverted
- The monopole part of the neutron-proton interaction restores the N=20 shell gap when the valley of stability is approached
- New ^{28}O data from NeuLAND-SAMOURAI collaboration (Kondo et al., NATURE 620, 965 (2023))
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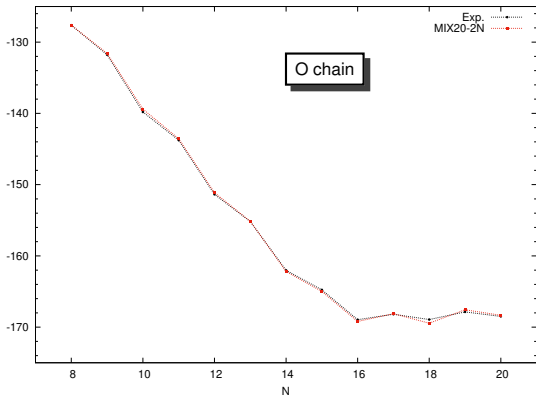
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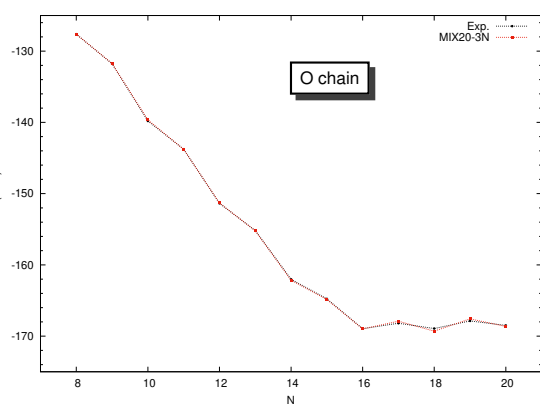
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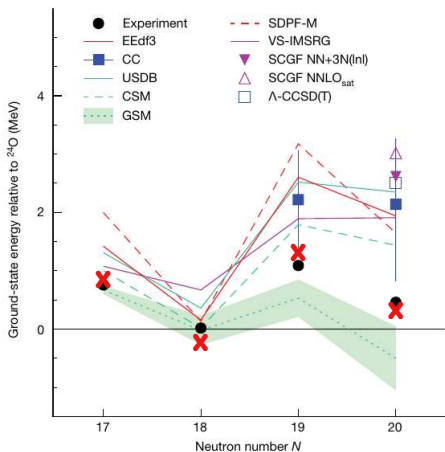
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Nowacki/Poves 2023

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Evolution of Shell Structure in Neutron-Rich Calcium Isotopes

G. Hagen,^{1,2} M. Hjorth-Jensen,^{3,4} G. R. Jansen,³ R. Machleidt,⁵ and T. Papenbrock^{1,2}

¹Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

²Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA

³Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway

⁴National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy,
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⁵Department of Physics, University of Idaho, Moscow, Idaho 83844, USA

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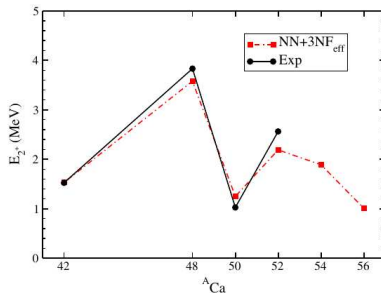


FIG. 2 (color online). (Excitation energies of $J^\pi = 2^+$ states in the isotopes ^{42,48,50,52,54,56}Ca (experiment: black circles, theory: red squares)

Testing *ab initio* nuclear structure in neutron-rich nuclei: Lifetime measurements of second 2^+ state in ^{16}C and ^{20}O

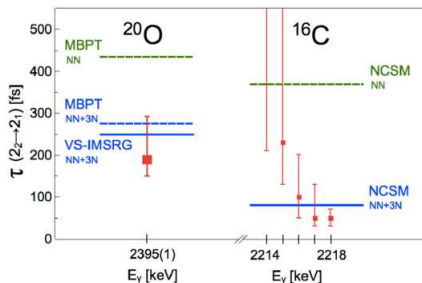
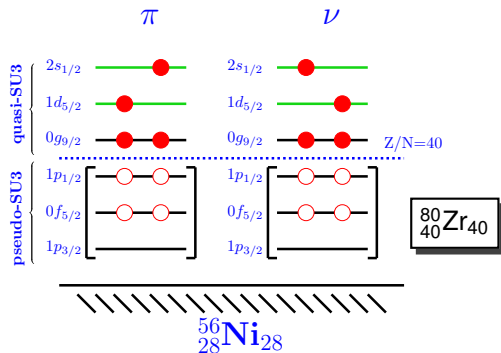


FIG. 4. Partial lifetime for $2_2^+ \rightarrow 2_1^+$ decays. Left: ^{20}O , experiment (symbol) compared to MBPT (dashed lines, with and without 3N interactions), and *ab initio* VS-IMSRG (solid line) results. Right: ^{16}C , experiment (symbols) for assumed E_γ energies [see Fig. 3(j) and 3(k)], including the uncertainty from a $\leq 13\%$ branching ratio. The solid (dashed) lines show *ab initio* NCSM predictions with (without) 3N interactions. The MBPT results

H.O. vs Spin-Orbit shell closure at N=Z



- p shell: ${}^{16}\text{O}$
spherical/doubly magic
- sd shell: ${}^{40}\text{Ca}$
spherical/doubly magic
- pf shell: ${}^{80}\text{Zr}$
deformed nucleus

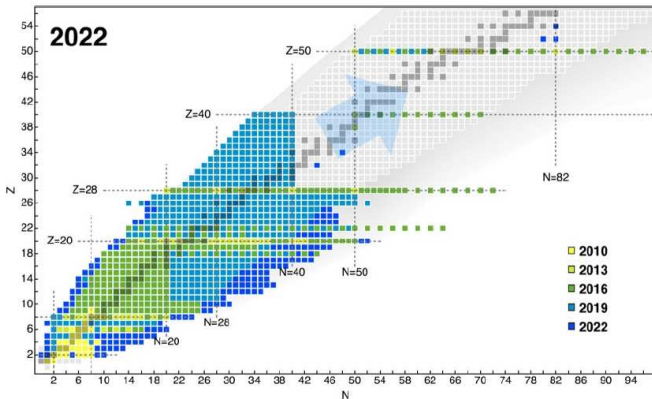
- Low-lying states in H.O. N=Z=8: **CS** , 4p4h, 8p8h
- Low-lying states in H.O. N=Z=20: **CS** , 4p4h,8p8h
- Low-lying states in H.O. N=Z=40: ???

Ab-initio predictions ?



Ab Initio Progress: How Heavy Can We Go?

Tremendous progress in ab initio reach, largely due to polynomially scaling methods!



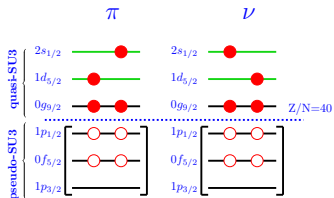
Jason Holt 2023

Island of Inversion at the $N=Z$ line

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NSCL/GRETINA Experiment



R.D.O. Llewellyn *et al.*, Phys. Rev. Lett. **124**, 152501 (2020)

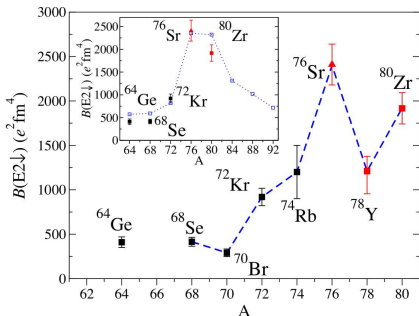


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei

- ZBM3 valence space: extension of JUN45 to pseudo-SU3 + Quasi-SU3
- New effective interaction (Realistic TBME + Monopole “3N” constraints)
- SM + DNO-SM for most **deformed cases**

Discrete Non-Orthogonal Shell Model

Generator Coordinate Method: $|\Psi_{\text{eff}}\rangle = \sum_i f_i |\Phi_i\rangle$

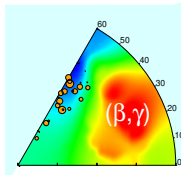
- 1) Deformed Hartree-Fock (HF) Slater determinants
- 2) Restoration of rotational symmetry
- 3) Mixing of shapes:

$$|\Psi_{\text{eff}}\rangle = \text{[deformed sphere]} + \text{[deformed sphere]} + \text{[deformed sphere]} + \dots$$

Intrinsic/Laboratory Description

- **Deformation structure of nuclear states:** $\{J_{\alpha}^{\pi}\}$, $q = (\beta, \gamma)$

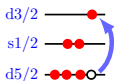
$$M_{\alpha}^{(J)}(q, K) = \sum_{q', K'} [\hat{N}^{1/2}]_{K'K}^{(J)}(q', q) f_{\alpha}^{(J)}(q', K')$$



- ◇ Probability of a configuration (β, γ) :

$$P_{\alpha}^{(J)}(q) = \sum_K |M_{\alpha}^{(J)}(q, K)|^2$$

- **particle-hole interpretation:**



M-scheme

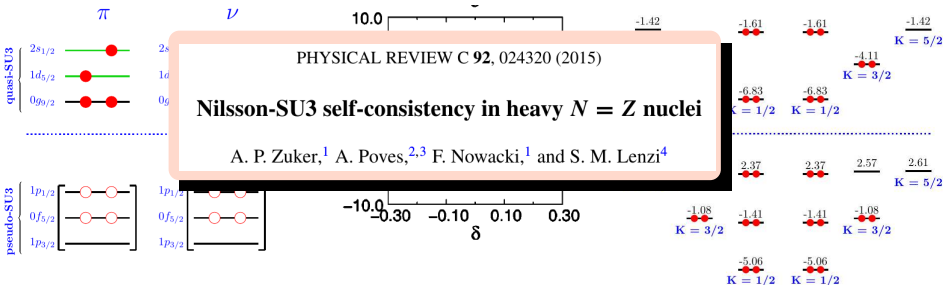
- ***K*-quantum numbers:**

$$P_{\alpha}^{(J)}(K) = \sum_q |M_{\alpha}^{(J)}(q, K)|^2$$

Nilsson-SU3 estimates

single particle energy levels

single particle quadrupole moments

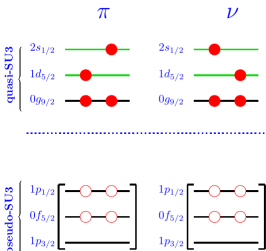


⁵⁶₂₈Ni₂₈

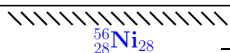
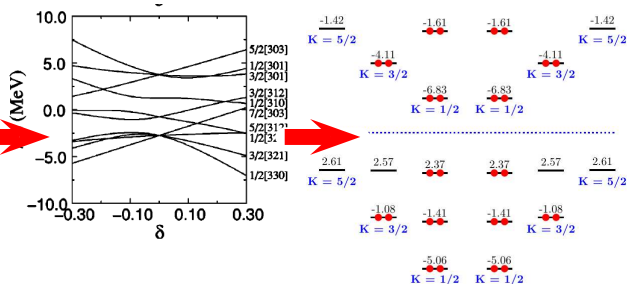
nucleus	NpNh*	B(E2)(e ² .fm ⁴)		
		ZRP	PHF	Exp. DNO-SM
⁷⁶ Sr	4p-4h	924	806	
	8p-8h	2189	2101	2390(240)
	12p-12h	2316	2300	
⁸⁰ Zr	4p-4h	587	637	
	8p-8h	1713	1509	1910(180)
	12p-12h	2663	2396	2325

Nilsson-SU3 estimates

single particle energy levels



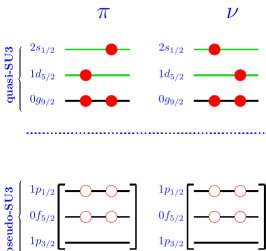
single particle quadrupole moments



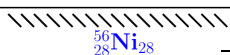
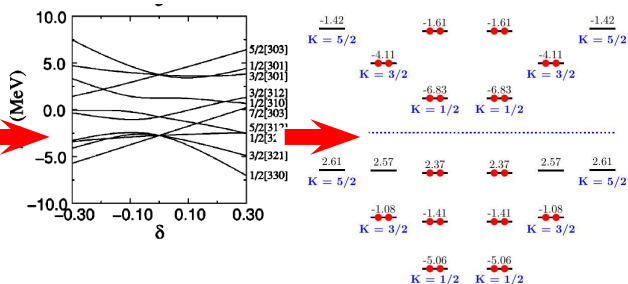
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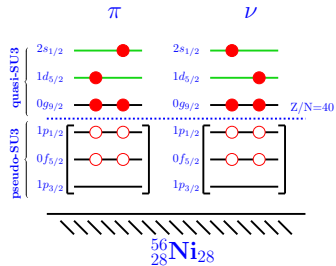
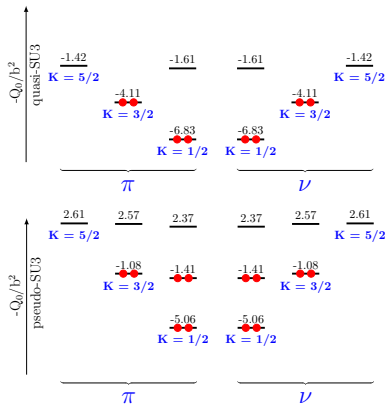
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Island of Inversion at the N=Z line

Strongly deformed states at $N = Z$

- Configuration mixing in ^{72}Kr
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NSCL/GRETINA Experiment



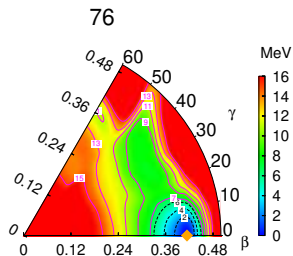
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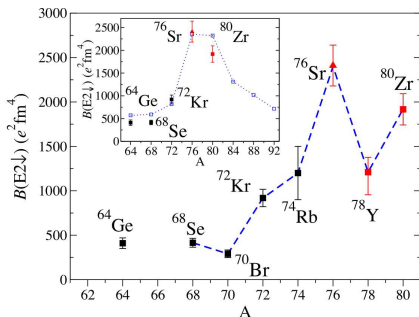
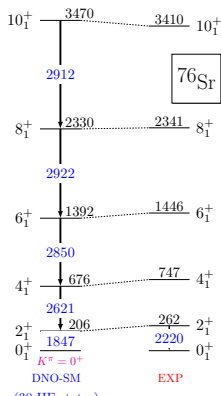


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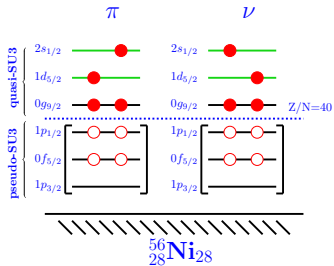
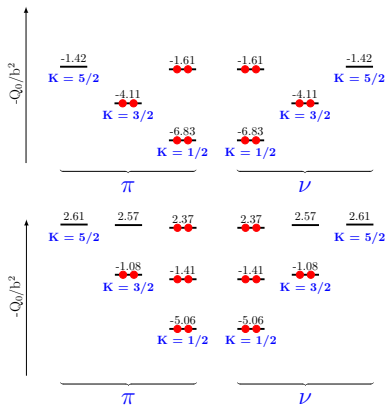


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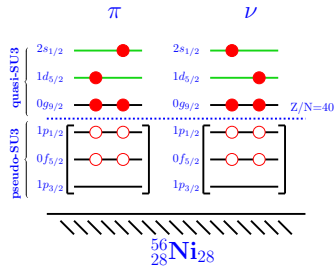
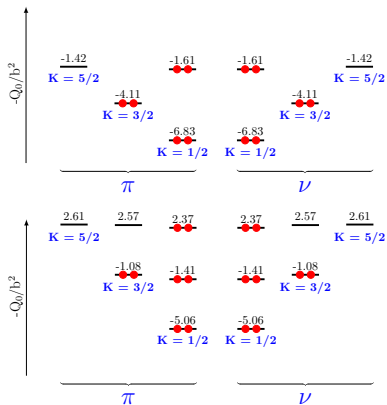
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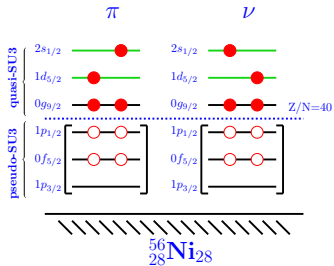
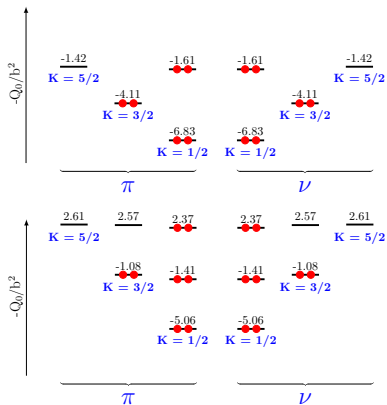
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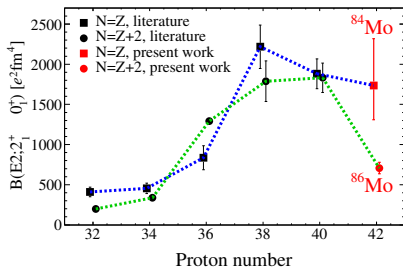
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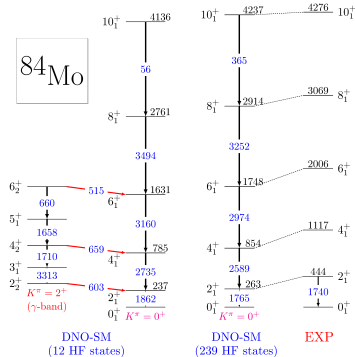
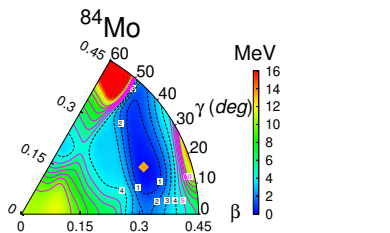
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J. Ha, F. Recchia *et al.*, to be submitted

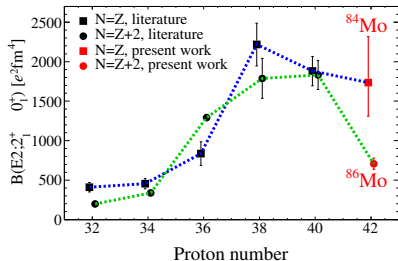
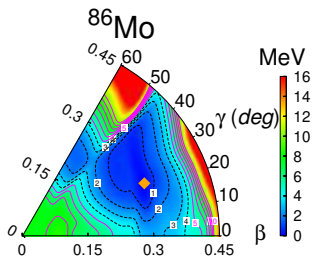


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NSCL/GRETINA Experiment



J. Ha, F. Recchia *et al.*, to be submitted

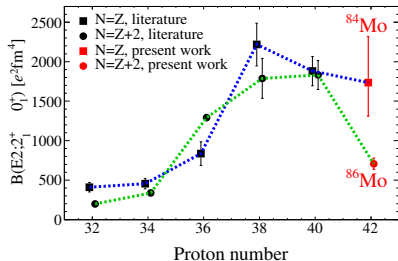
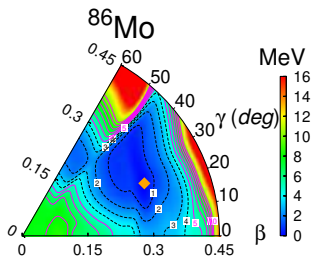
nucleus	Np-Nh*	B(E2)(e ² .fm ⁴)				
		ZRP	PHF	Exp.	DNO-SM* SM	
^{84}Mo	4p-4h	1104	1193	1740⁺⁵⁸⁰₋₇₃₀	1765	-
	8p-8h	1891	1732			
^{86}Mo	0p-0h	542	196	707(71)	1184	731
	2p-2h	1030	871			
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Shell closures and 2N forces only

PHYSICAL REVIEW C **74**, 061302(R) (2006)

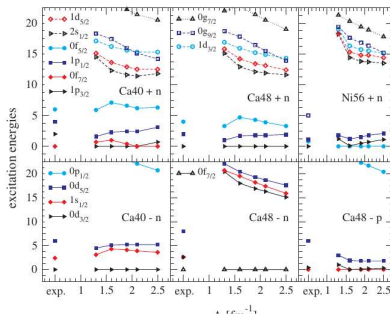
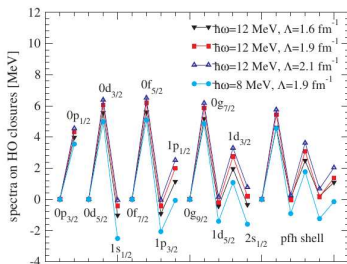
Shell-model phenomenology of low-momentum interactions

Achim Schwenk^{1,*} and Andrés P. Zuker^{2,†}

¹Nuclear Theory Center, Indiana University, 2401 Milo B. Sampson Lane, Bloomington, Indiana 47408, USA

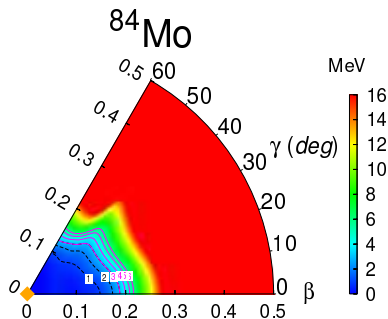
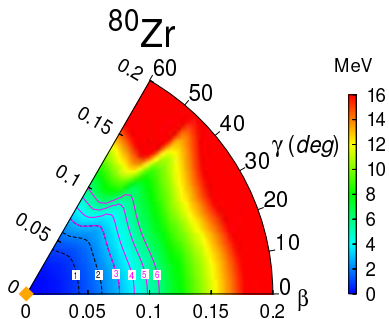
²Institut de Recherches Subatomiques, IN2P3-CNRS, Université Louis Pasteur, F-67037 Strasbourg, France

(Received 14 January 2005; revised manuscript received 20 September 2006; published 12 December 2006)



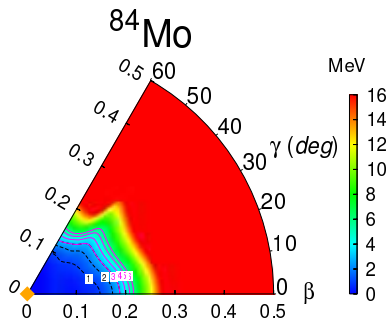
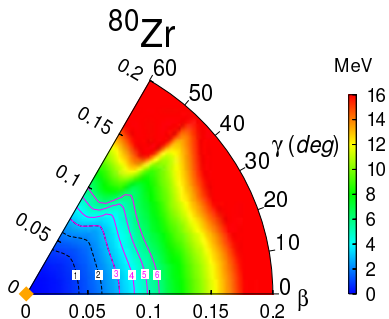
- no Spin-orbit shell closures in ^{12}C , ^{22}O , ^{48}Ca , ^{56}Ni
- too strong H. O. shell closures ^{16}O , ^{40}Ca , ... and ^{80}Zr !!!

N3LO NN calculations



nucleus	NpNh*	B(E2)(e ² .fm ⁴)				
		ZRP	PHF	Exp.	DNO-SM	N3LO
^{80}Zr	4p-4h	587	637			
	8p-8h	1713	1509	1910(180)	2325	0.03
	12p-12h	2663	2396			
^{84}Mo	4p-4h	1104	1193	1740⁺⁵⁸⁰₋₇₃₀	1740	174
	8p-8h	1891	1732			

N3LO NN calculations



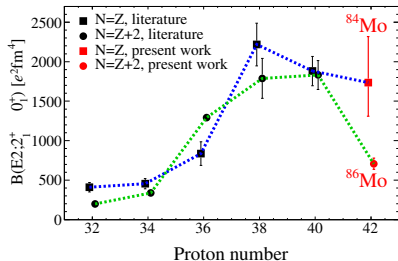
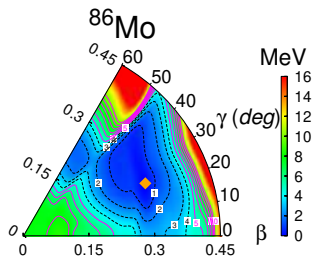
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NSCL/GRETINA Experiment



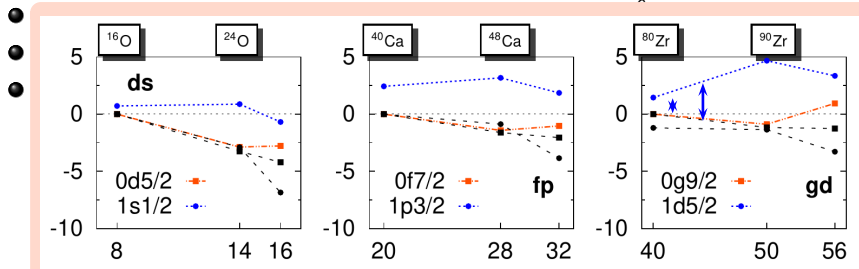
J. Ha, F. Recchia *et al.*, to be submitted

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^{86}Mo

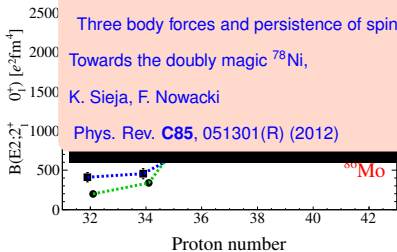


Three body forces and persistence of spin-orbit shell gaps in medium-mass nuclei:

Towards the doubly magic ^{78}Ni ,

K. Sieja, F. Nowacki

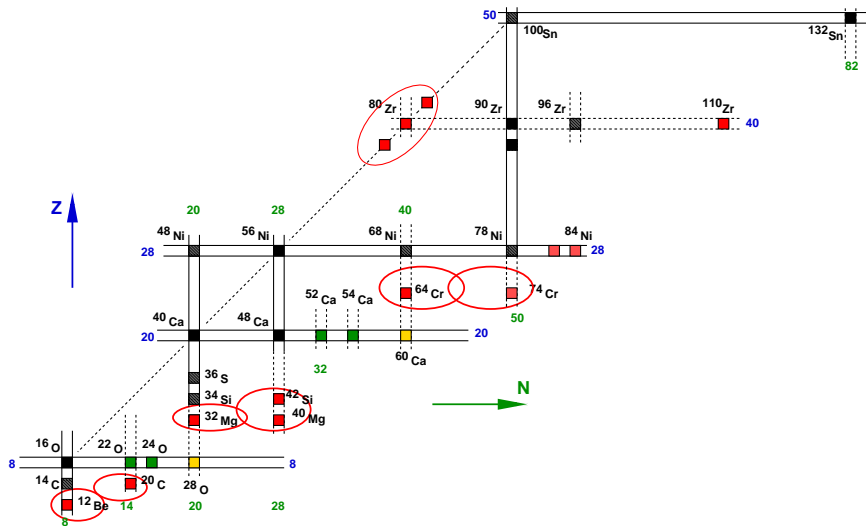
Phys. Rev. **C85**, 051301(R) (2012)



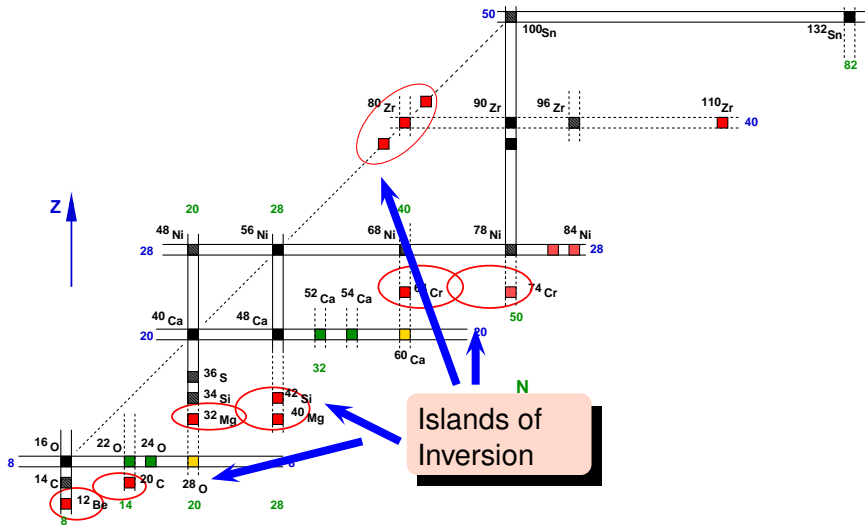
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J. Ha, F. Recchia *et al.*, to be submitted

Landscape of medium mass nuclei



Landscape of medium mass nuclei



Summary

- Monopole drift develops in all regions but the Interplay between correlations (pairing + quadrupole) and spherical mean-field (monopole field) determines the physics.
- New "island of inversion" or "island of deformation" present for neutron-rich systems show up also at N=Z line with very deformed rotors dominated by Many-particles-Many-holes configurations.
- Shape transition between ^{84}Mo and ^{86}Mo and first fingerprint of 3N forces in deformed systems
- Around $A \sim 80$, an "island of enhanced collectivity" show very deformed rotors dominated by Many-particles-Many-holes configurations.
- First FRIB in-beam spectroscopy of ^{62}Cr , A. Gade et al. to be submitted to NATURE
- Island of Inversion in $^{84,86}\text{Mo}$ @GRETINA/MSU, F. Recchia et al. to be submitted to NATURE
- FRIB-TA topical program: Theoretical justifications for FRIB (May 2023)

Open questions for the future

- Are Ab-initio approaches accurate enough to describe the whole chart of isotopes within the same framework (2N , 3N hamiltonian etc ...)? So far 34 “non-implausible”
- How to simplify such calculations ? (see N. Smirnova's talk)
3N effects mainly monopole 3N effects ?
How to implement simple global monopole effects ?
- What are the data needed to better constrain theoretical models ?
- Better interplay between EDF and SM/CI expertises ?
- How to disentangle continuum effects from bound systems description ?

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- D. D. Dao, K. Sieja, S. Courtin
- G. Martinez-Pinedo, A. Poves, S. Lenzi
- A. Gade, O. Sorlin, A. Obertelli