Calculation of Form Factors in Neutral Current B Decays

Nazila’s mini workshop @ IP2I

Alexandre Carvunis - IP2I (Lyon) - 04/05/2023

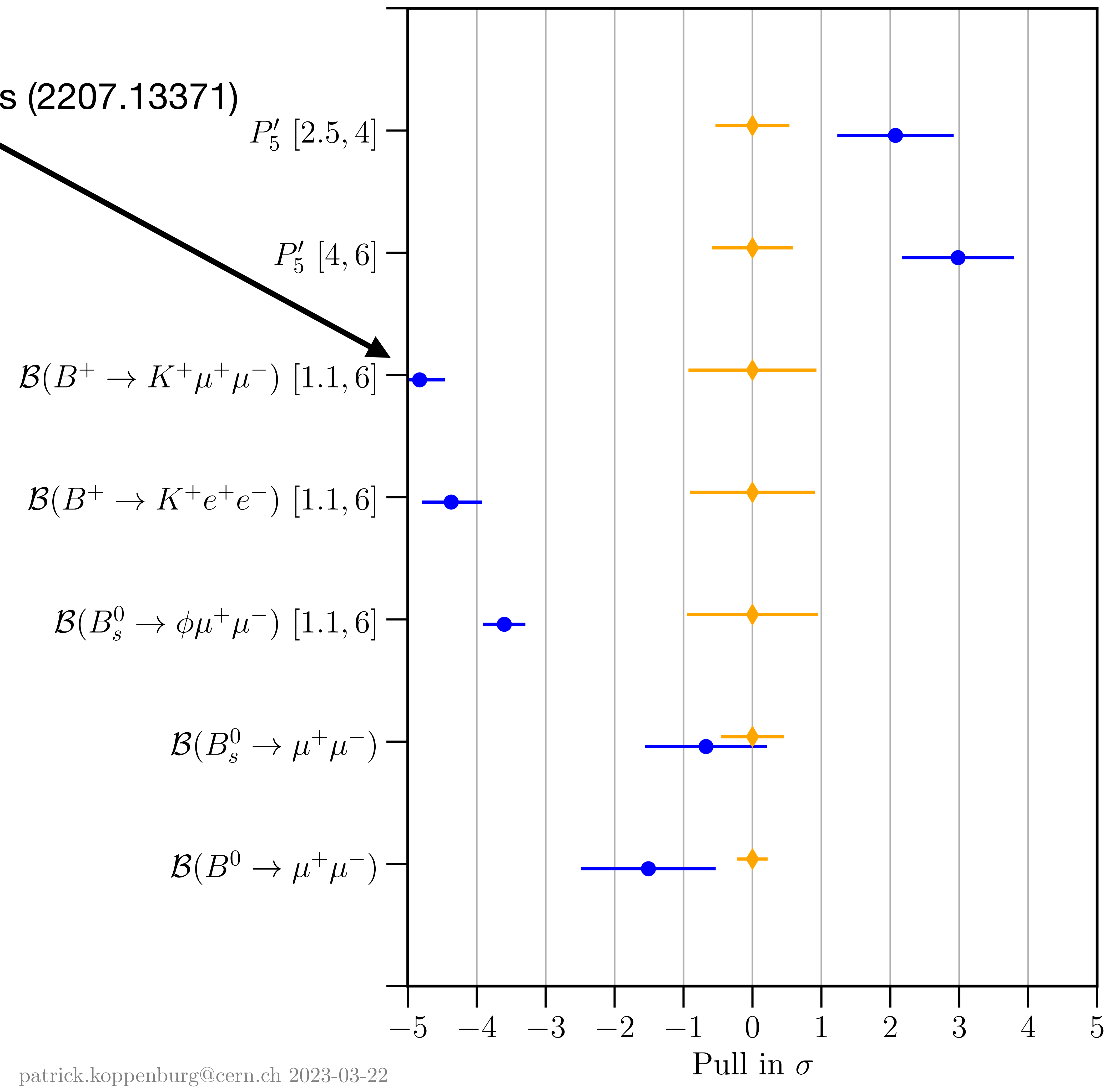
In collaboration with Nazila Mahmoudi and Yann Monceaux

Motivation: B-anomalies status (see Yann's talk)

$$q^2 = (p_\ell + p_{\ell'})^2$$

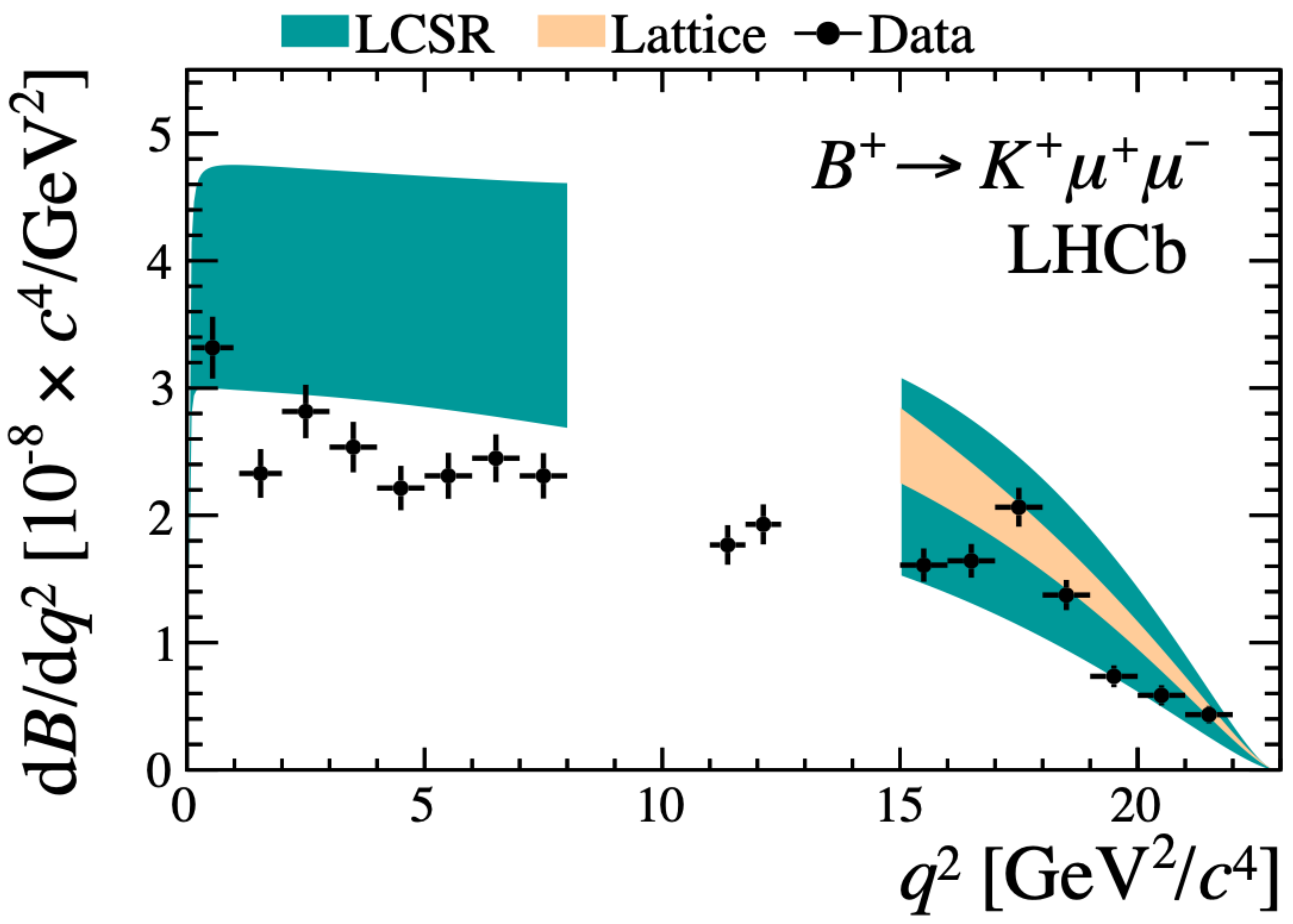
- $b \to s\ell\ell$
  - Anomalies in 'clean' observables gone:
    - $R_K - R_{K*}$ (LHCb 2022)
    - $BR(B_s \to \mu\mu)$ (LHCb 2021)
  - $P_2, P_5'(B \to K^*\mu\mu)$ (LHCb 2021) and Q_5 (Belle 2017) still standing
  - Largest deviations in $BR(b \to s\ell\ell)$ at low-q2, but theoretically challenging
- $b \to c\ell\bar{\nu}$
  - $R(D^*)$ alone compatible with the SM
  - $R(D) - R(D^*)$ still around 3 sigma (theoretically clean)

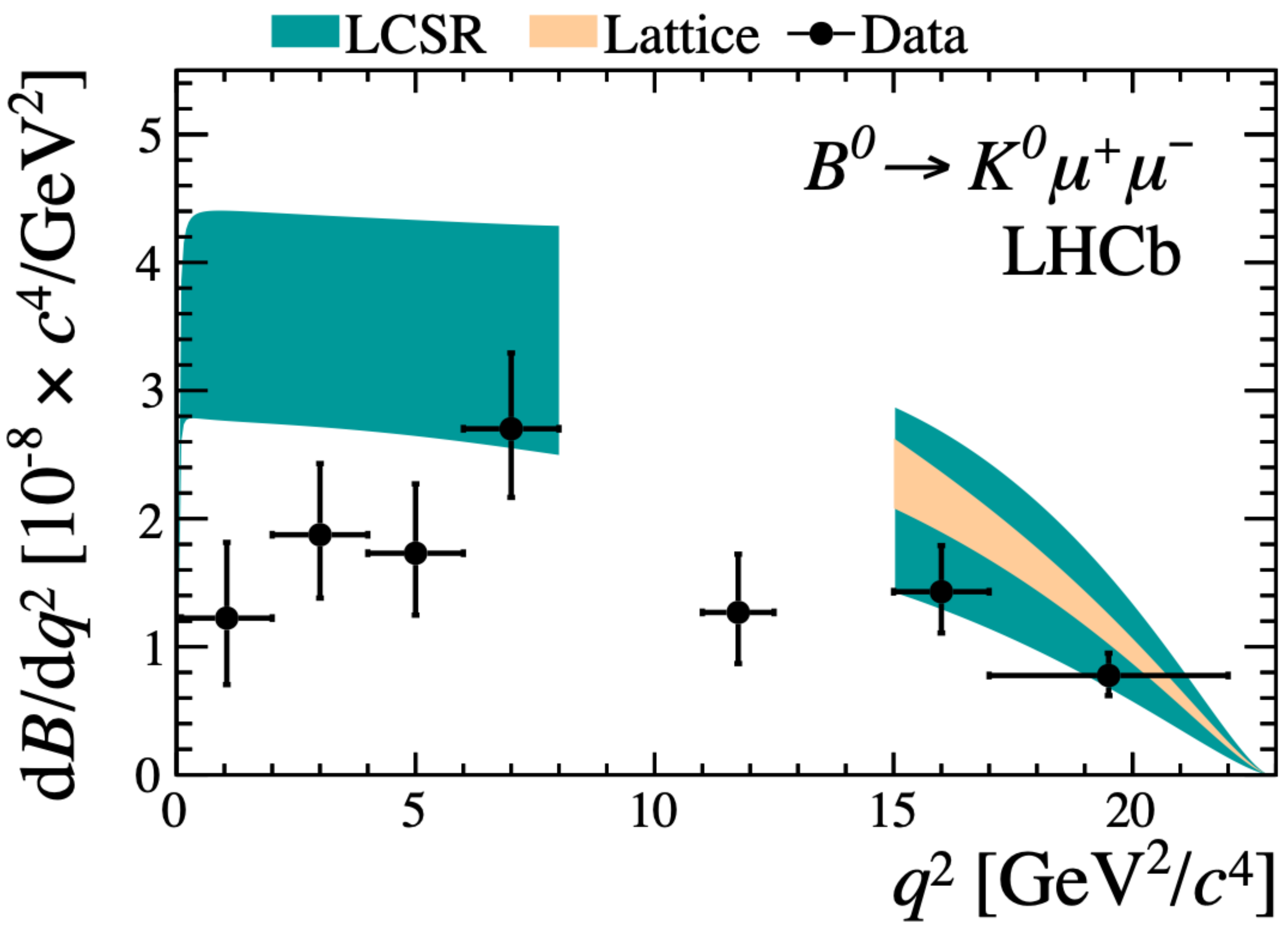
HPQCD, Parrot, Bouchard, Davies (2207.13371)



The $\mathrm{dBR/dq^2}(\mathrm{B} \to \mathrm{K}\mu\mu)$ discrepancy

LHCb 2014 - 1403.8044





Discrepancy significance: $\sim 2\sigma$ per bin

TH uncertainty dominates the assessment of the significance of the anomaly

Theoretical Framework

$b \to s\ell\ell$ in the weak effective theory

- At the scale m_b $\quad H_{eff} = H_{eff,sl} + H_{eff,had}$

Semileptonic Operators:

$$H_{\text{eff,sl}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10,S,P} \left(C_i^\ell O_i^\ell + C_i'^\ell O_i'^\ell \right)$$

$$O_7^{(\prime)} = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}, \quad O_9^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell), \quad O_{10}^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$C_7^{\text{SM}} \simeq -0.3 \qquad C_9^{\text{SM}} \simeq -4 \qquad C_{10}^{\text{SM}} \simeq 4$$

Hadronic operators

$$H_{\text{eff,had}} = -\mathcal{N} \frac{16\pi^2}{e^2} \left(C_8 O_8 + C_8' O_8' + \sum_{i=1,\ldots,6} C_i O_i \right) + \text{ h.c.}$$

$$O_1 = \left(\bar{s}\gamma_\mu P_L T^a c\right) \left(\bar{c}\gamma^\mu P_L T^a b\right), \quad O_2 = \left(\bar{s}\gamma_\mu P_L c\right) \left(\bar{c}\gamma^\mu P_L b\right), \quad \ldots$$

Amplitude of $B \to M\ell\ell$ decays

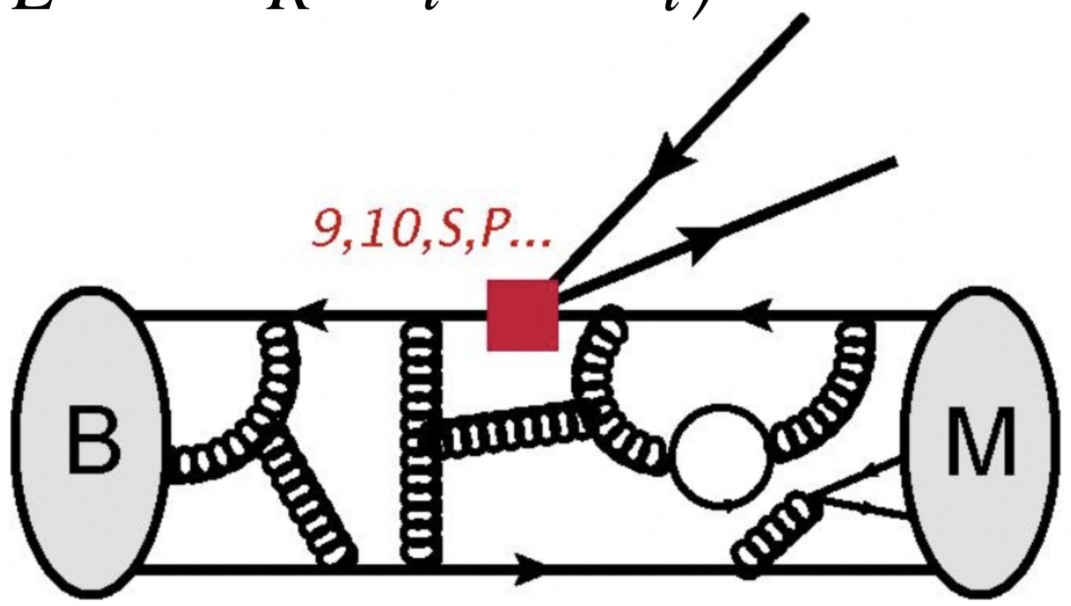
$$\mathcal{M}(B \to M\ell\ell) = \left\langle M\ell\ell \left| H_{\text{eff}} \right| B \right\rangle = \mathcal{N}\left[\left(A_V^\mu + T^\mu\right)\bar{u}_\ell\gamma_\mu v_\ell + A_A^\mu\,\bar{u}_\ell\gamma_\mu\gamma_5 v_\ell + A_S\,\bar{u}_\ell v_\ell + A_P\,\bar{u}_\ell\gamma_5 v_\ell\right]$$

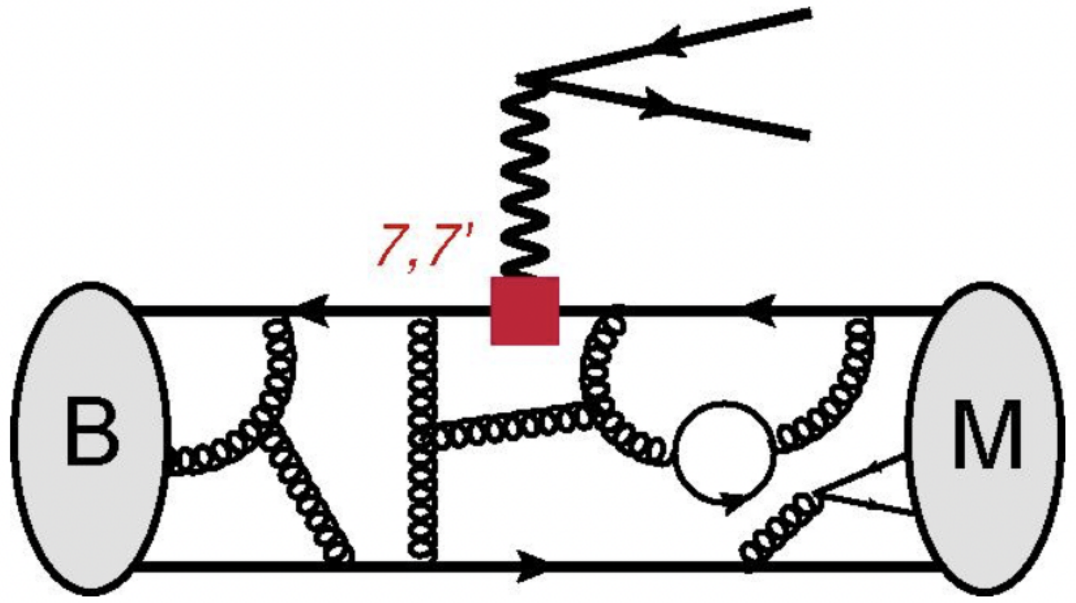
Local contributions

$$A_V^\mu = -\frac{2im_b}{q^2}C_7\left\langle M \left| \bar{s}\sigma^{\mu\nu}q_\nu P_R b \right| B\right\rangle + C_9\left\langle M \left| \bar{s}\gamma^\mu P_L b \right| B\right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$

$$A_A^\mu = C_{10}\left\langle M \left| \bar{s}\gamma^\mu P_L b \right| B\right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$

$$A_{S,P} = C_{S,P}\left\langle M \left| \bar{s}P_R b \right| B\right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$





Amplitude of $\mathrm{B} \to \mathrm{M}\ell\ell$ decays

Local contributions - definition of the form factors

- 3 independent f.f. for B to pseudoscalar meson:

$$\left\langle P(k) \middle| \bar{q}_1 \gamma^\mu b \middle| B(p) \right\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right] f_+^{B\to P} + \frac{m_B^2 - m_P^2}{q^2} q^\mu f_0^{B\to P}$$

$$\left\langle P(k) \middle| \bar{q}_1 \sigma^{\mu\nu} q_\nu b \middle| B(p) \right\rangle = \frac{i f_T^{B\to P}}{m_B + m_P} \left[q^2 (p+k)^\mu - \left(m_B^2 - m_P^2\right) q^\mu \right]$$

- 7 independent f.f. for B to vector meson:

$$\left\langle V(k,\eta) \middle| \bar{q}_1 \gamma^\mu b \middle| B(p) \right\rangle = \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_\rho k_\sigma \frac{2 V^{B\to V}}{m_B + m_V}$$

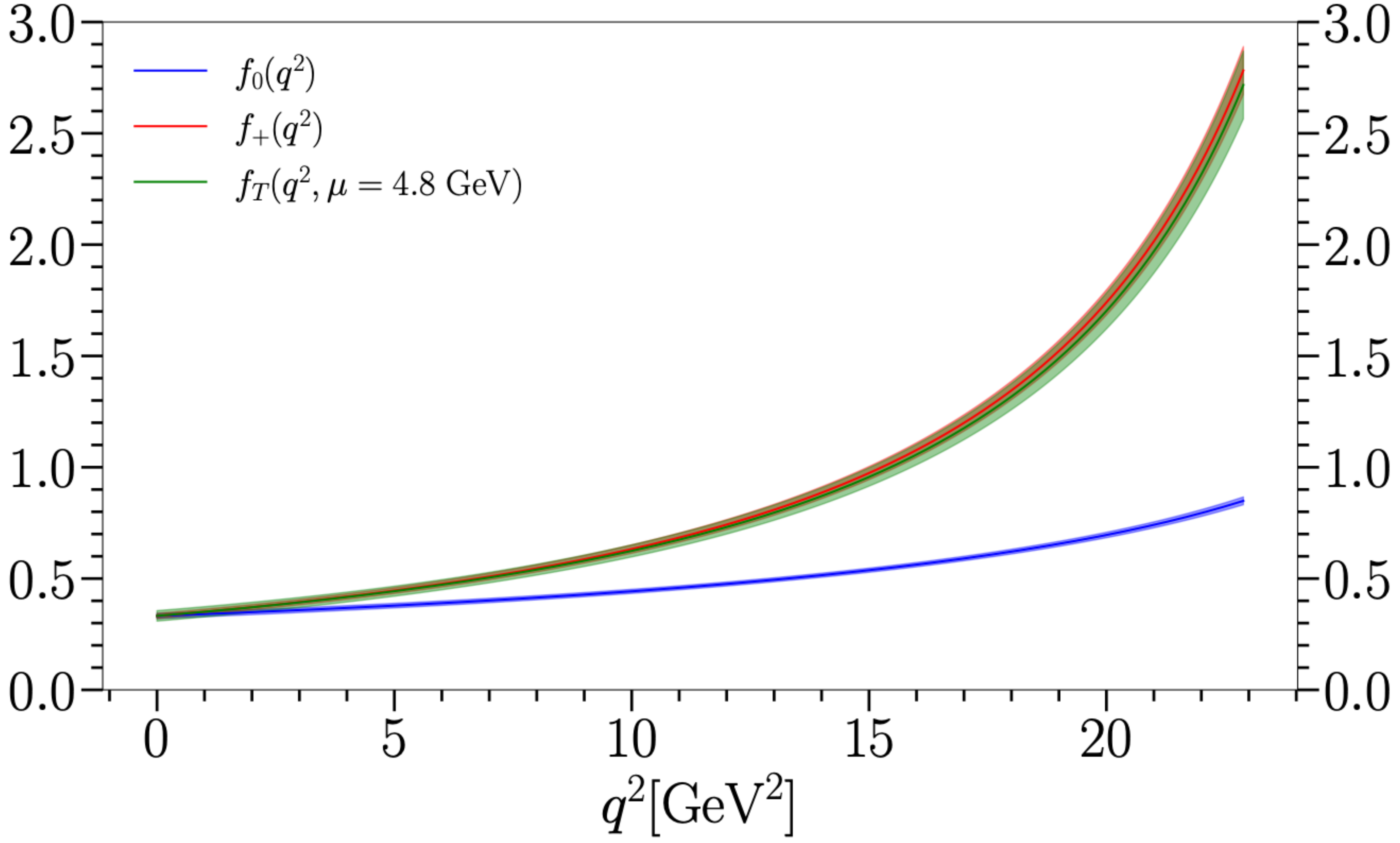
$$\left\langle V(k,\eta) \middle| \bar{q}_1 \gamma^\mu \gamma_5 b \middle| B(p) \right\rangle = i\eta_\nu^* [g^{\mu\nu} \left(m_B + m_V\right) A_1^{B\to V} - \frac{(p+k)^\mu q^\nu}{m_B + m_V} A_2^{B\to V} - q^\mu q^\nu \frac{2m_V}{q^2} \left(A_3 - A_0\right)]$$

$$\left\langle V(k,\eta) \middle| \bar{q}_1 i\sigma^{\mu\nu} q_\nu b \middle| B(p) \right\rangle = \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_\rho k_\sigma 2 T_1^{B\to V}$$

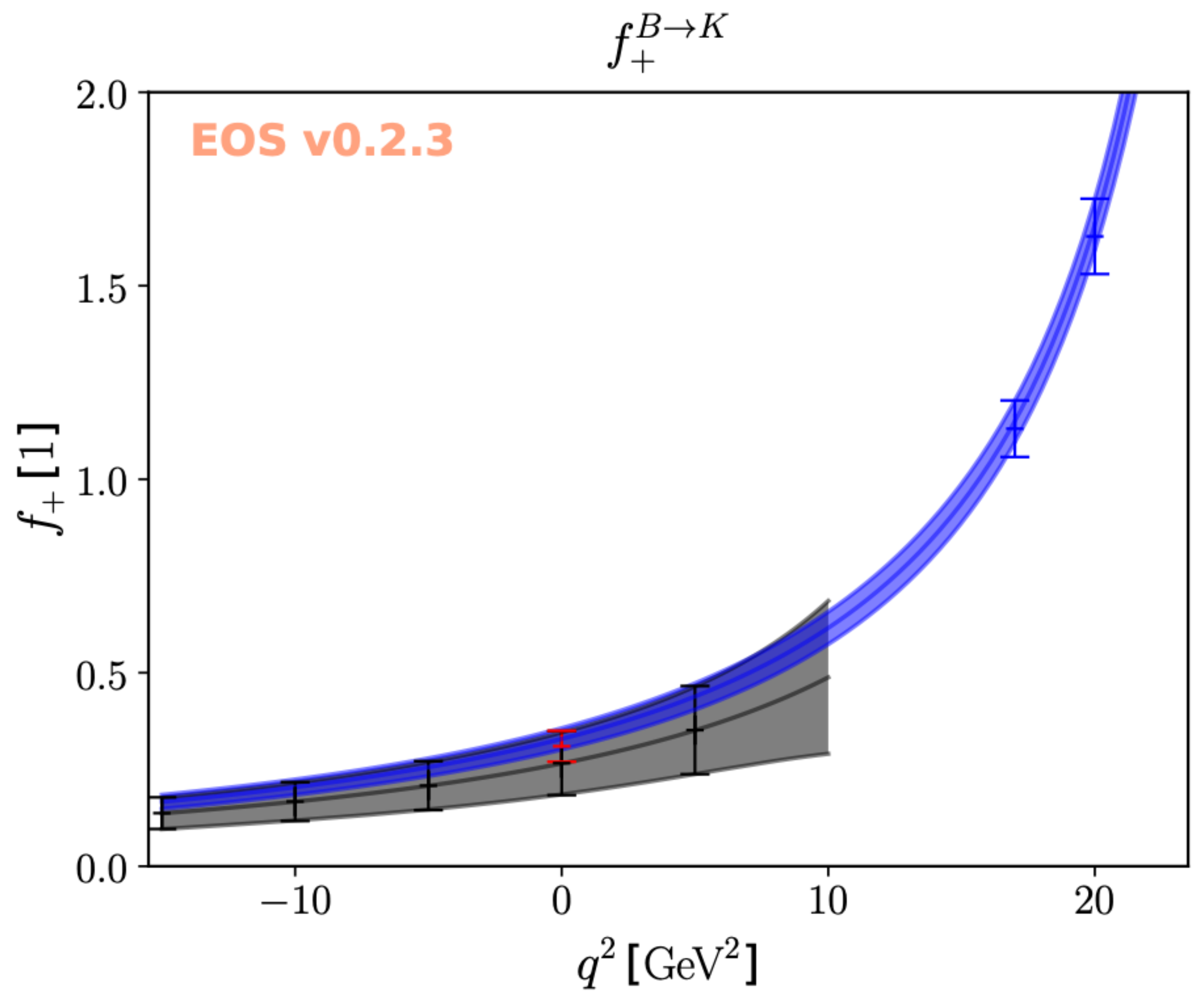
$$A_3^{B\to V} \equiv \frac{m_B + m_V}{2m_V} A_1^{B\to V} - \frac{m_B - m_V}{2m_V} A_2^{B\to V}.$$

$$\left\langle V(k,\eta) \middle| \bar{q}_1 i\sigma^{\mu\nu} q_\nu \gamma_5 b \middle| B(p) \right\rangle = i\eta_\nu^* [\left(g^{\mu\nu} \left(m_B^2 - m_V^2\right) - (p+k)^\mu q^\nu\right) T_2^{B\to V} + q^\nu \left(q^\mu - \frac{q^2}{m_B^2 - m_V^2} (p+k)^\mu \right) T_3^{B\to V}]$$

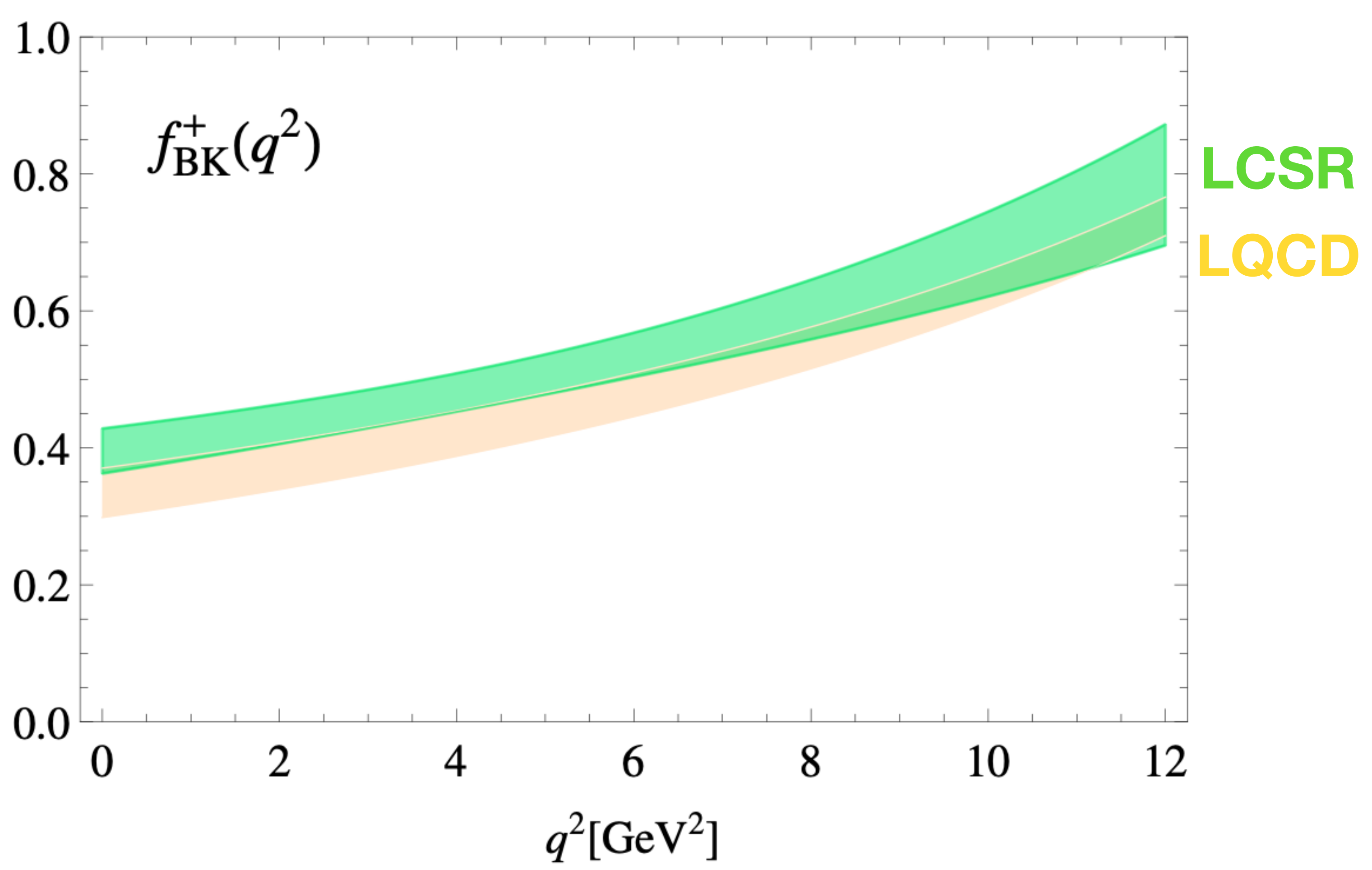
HPQCD 2022 vs LCSR



Parrot, Bouchard, Davies 2207.12468v2

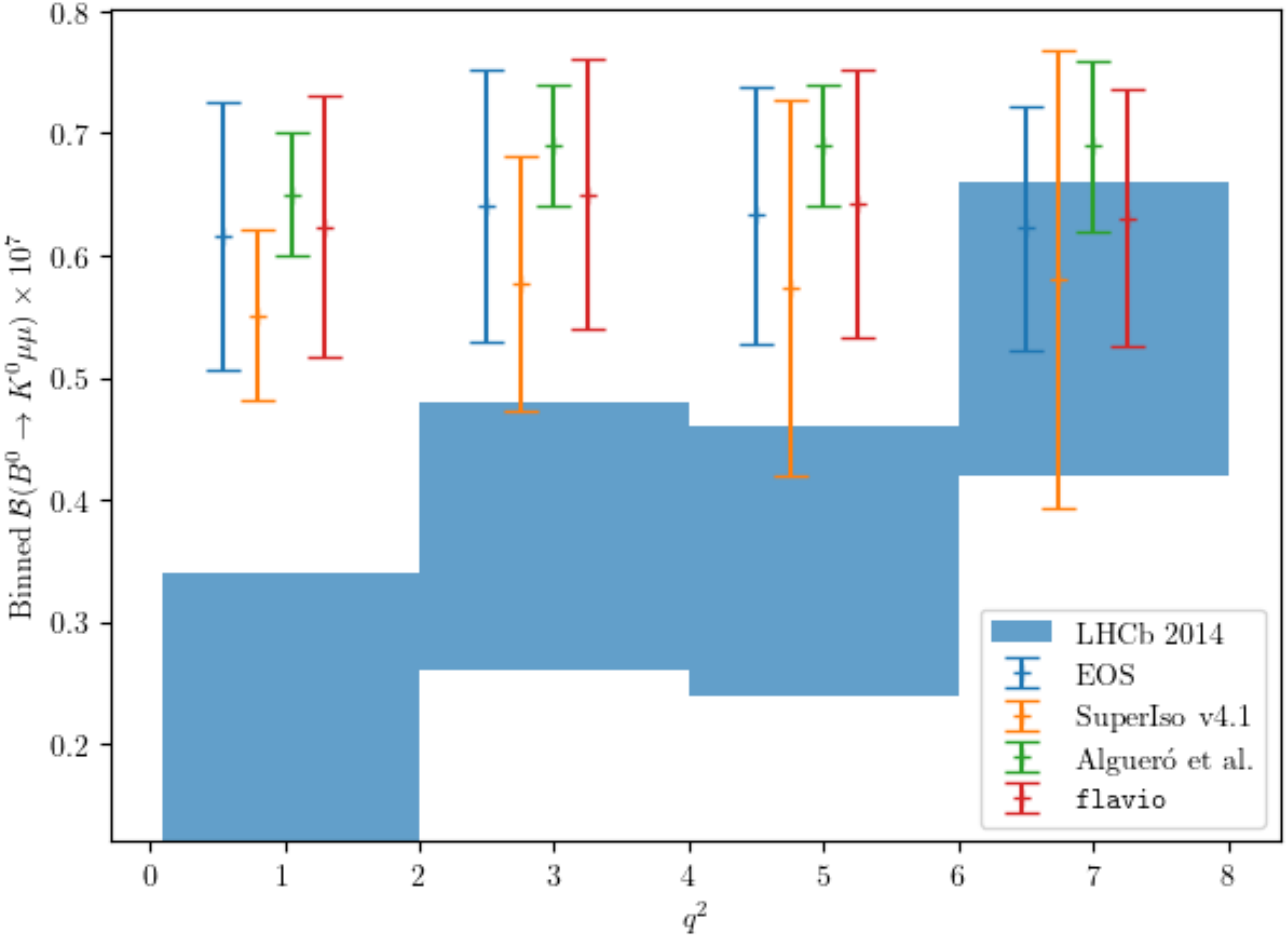


Gubernari, Kokulu, van Dyk 1811.00983



Khodjamirian, Rusov 1703.04765v2

The $\mathrm{dBR}/\mathrm{dq}^2(\mathrm{B} \to \mathrm{K}\mu\mu)$ discrepancy



Local FFs	Non-local FFs (not exhaustive)
Gubernari et al. 2018	Soft FF approach Bobeth et al. 2007
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HPQCD 2022	
Gubernari et al. 2018	Straub et al. 1411.3161

Amplitude of $\mathrm{B} \to \mathrm{M}\ell\ell$ decays

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Local contributions

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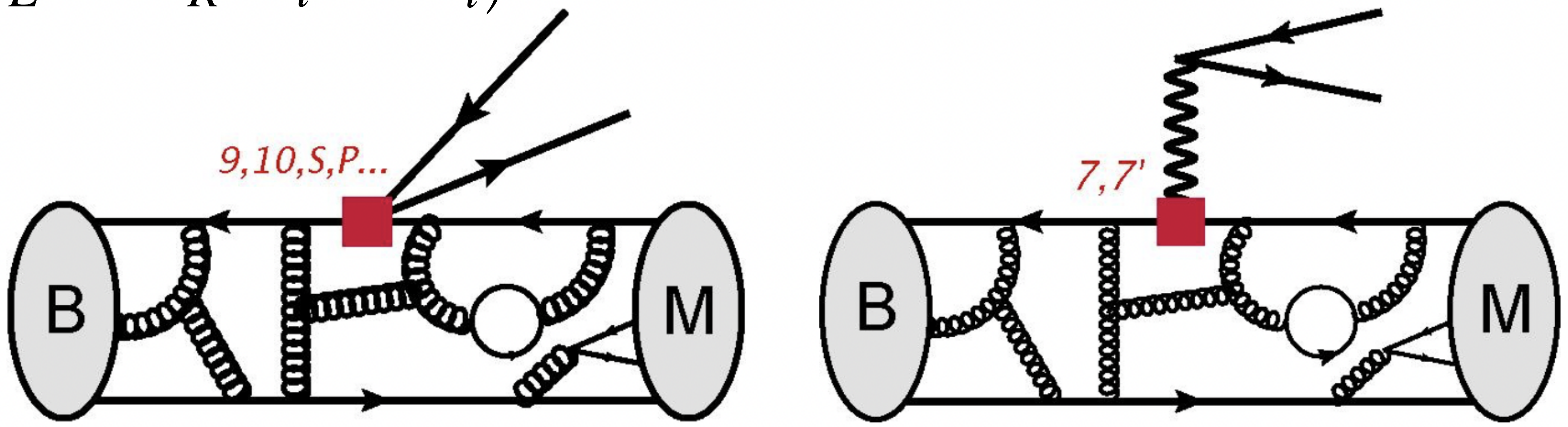
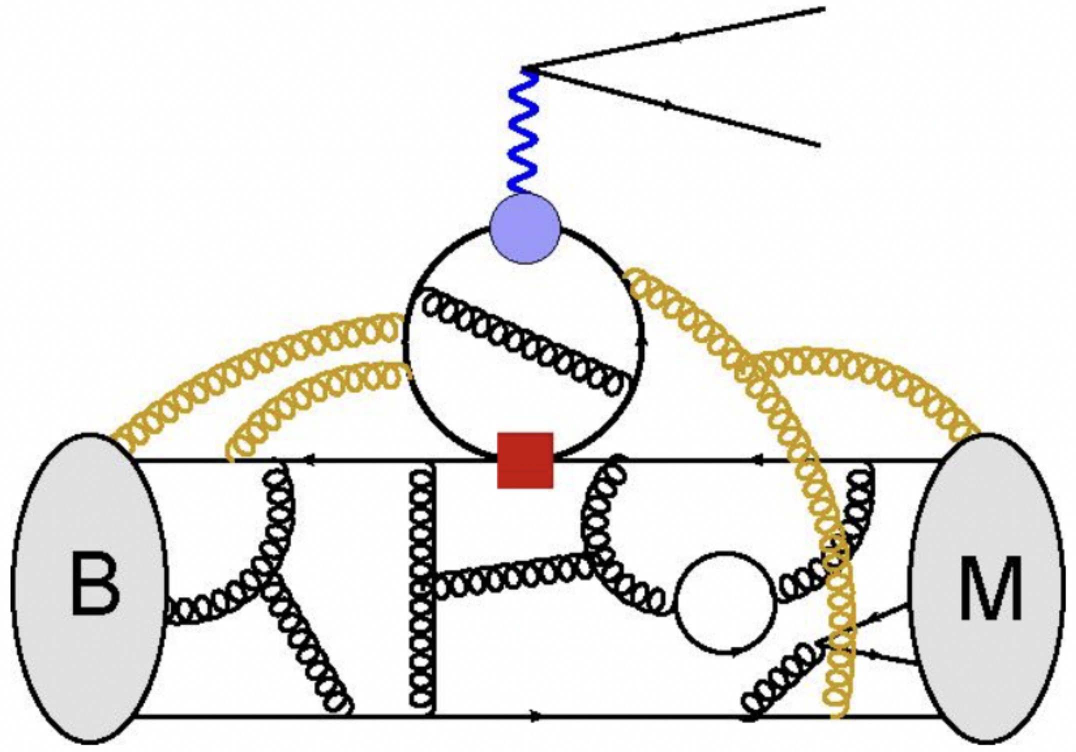


Non-Local contributions

$$T^\mu = \frac{-16i\pi^2}{q^2}\sum_{i=1,\ldots,6,8} C_i \int dx^4 e^{iq\cdot x}\left\langle M\left|T\left\{j^\mu_{\text{em}}(x), O_i(0)\right\}\right|B\right\rangle,$$

$$j^\mu_{\text{em}} = \sum_q Q_q\bar{q}\gamma^\mu q$$

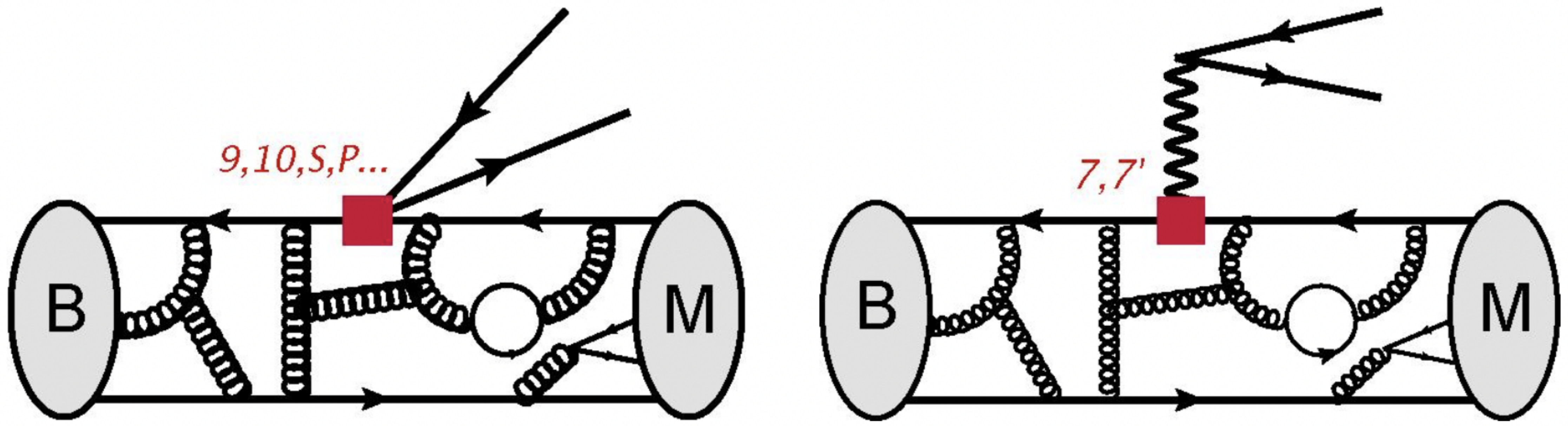


Calculation of the matrix elements

$$\mathcal{M}(B \to M\ell\ell) = \left\langle M\ell\ell \left| H_{\text{eff}} \right| B \right\rangle = \mathcal{N}\left[\left(A_V^\mu + T^\mu\right) \bar{u}_\ell \gamma_\mu v_\ell + A_A^\mu \bar{u}_\ell \gamma_\mu \gamma_5 v_\ell + A_S \bar{u}_\ell v_\ell + A_P \bar{u}_\ell \gamma_5 v_\ell\right]$$

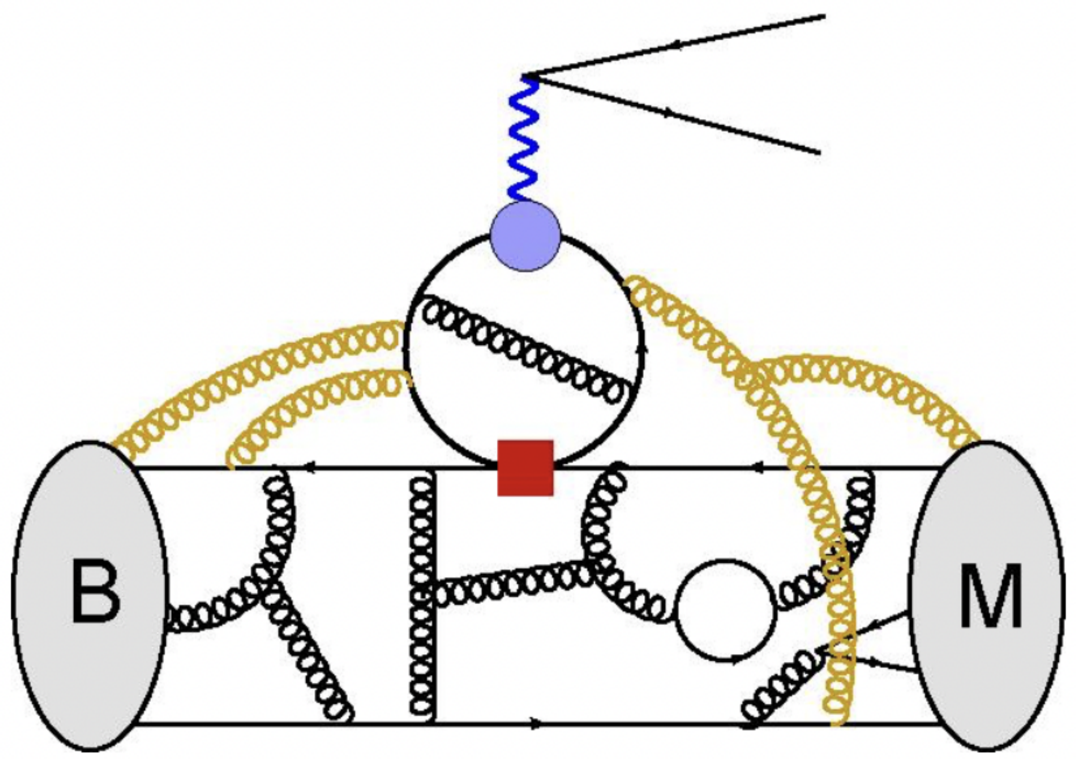
Local contributions

- At high-q2, computed on the lattice
- At low-q2, analytic approach: e.g. Light-Cone Sum Rule (LCSR)



Non-Local contributions

- At low-q2 from QCD factorization (QCDF)
- Beyond QCDF contributions are not well understood, main source of uncertainty



Calculation of the matrix elements

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Local contributions

- At high-q2, computed on the lattice
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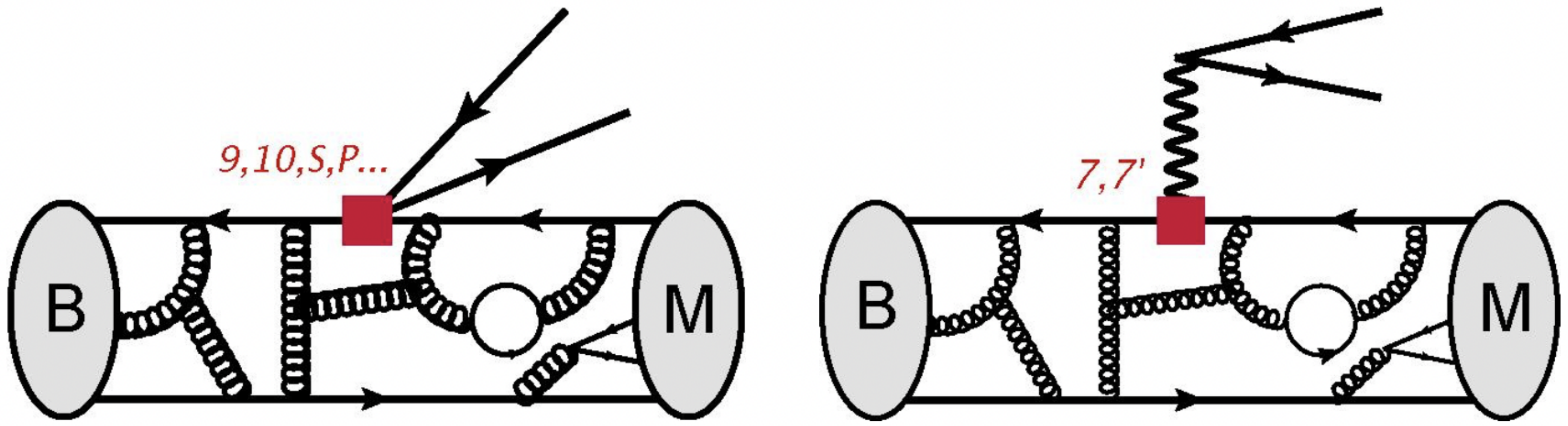
The estimation of a systematic error associated with the method is *challenging*



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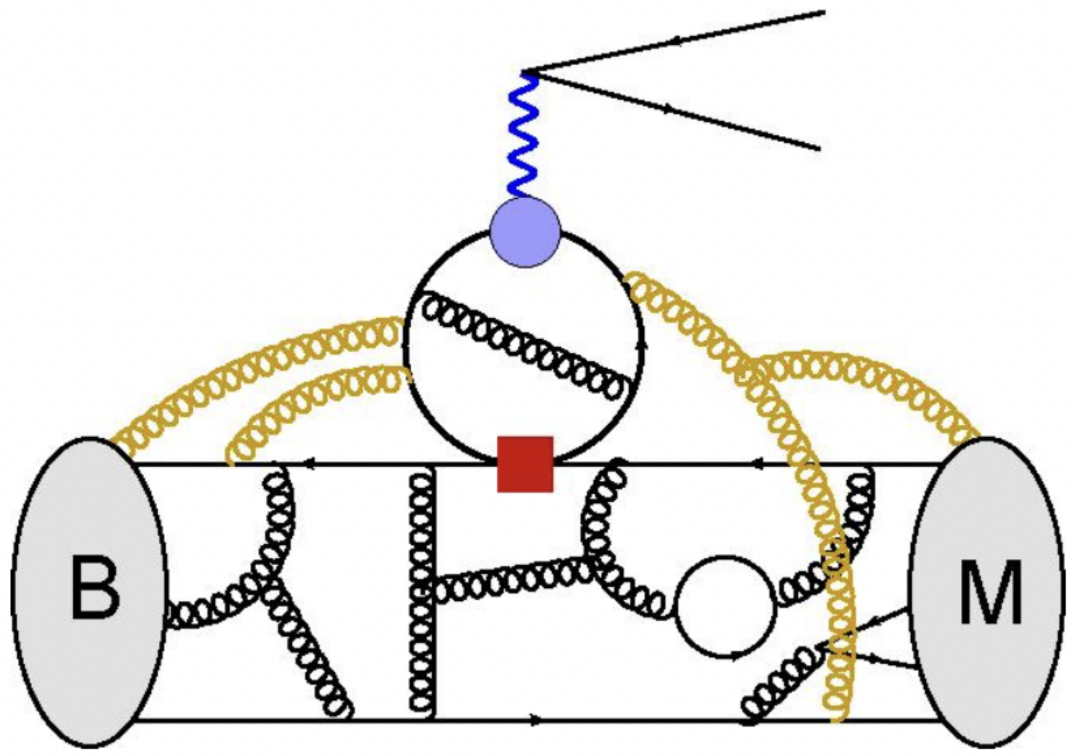
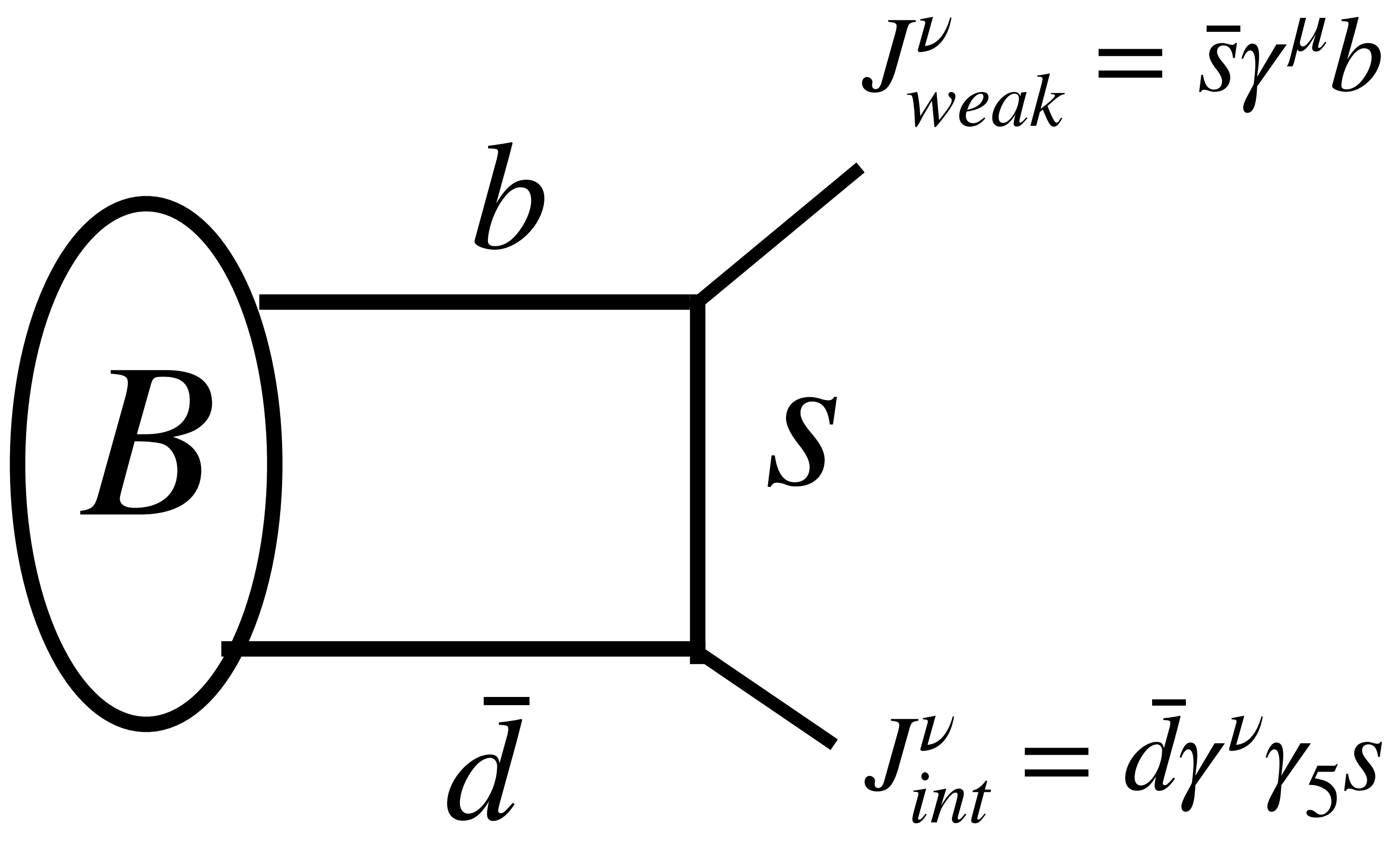


Procedure for Light Cone Sum Rules

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0|TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)|\bar{B}(P_B = q+k) >$$

Correlation function of B to vacuum
(also possible with final meson to vacuum)



1) Express Π in function of the non-perturbative quantities that we want to calculate

2) Compute Π perturbatively

3) 1) = 2) + use of quark-hadron duality

Procedure for Light Cone Sum Rules

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0\,|\,TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)\,|\,\bar{B}(P_B = q+k) >$$

Hadronic unitarity relation
&
Dispersion relation

Density of excited states
of the final meson

∝ Light hadron decay constant   What we want to compute

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{\langle 0\,|\,j_\nu\,|\,M(k)\rangle\langle M(k)\,|\,j_\mu\,|\,B\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{\infty} ds\frac{\rho^{\mu\nu}}{s-k^2}$$

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$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0\,|\,TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)\,|\,\bar{B}(P_B = q+k) >$$

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$$K^{(F)}\frac{F\left(q^2\right)}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho(s)}{s-k^2}$$

Procedure for Light Cone Sum Rules

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0\,|\,TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)\,|\,\bar{B}(P_B = q+k) >$$

Hadronic unitarity relation
&
Dispersion relation

HQET - heavy m_b limit

Density of excited states of the final meson

∝ Light hadron decay constant   What we want to compute

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{\langle 0\,|\,j_\nu\,|\,M(k)\rangle\langle M(k)\,|\,j_\mu\,|\,B\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{\infty} ds \frac{\rho^{\mu\nu}}{s-k^2}$$

$$\Pi^{\mu\nu} = \int d^4x \int \frac{d^4p'}{(2\pi)^4} e^{i(k-p').x}\left[\Gamma_2^{\nu}\frac{\not{p}' + m_1}{m_1^2 - p'^2}\Gamma_1^{\mu}\right]_{\alpha\beta} < 0\,|\,\bar{q}_2^{\alpha}(x)h_v^{\beta}(0)\,|\,\bar{B}(v) >$$

$$K^{(F)}\frac{F\left(q^2\right)}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho(s)}{s-k^2}$$

Procedure for Light Cone Sum Rules

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0|TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)|\bar{B}(P_B = q+k) >$$

Hadronic unitarity relation
&
Dispersion relation

HQET - heavy m_b limit

Density of excited states
of the final meson

∝ Light hadron decay constant

What we want to compute

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{\langle 0|j_\nu|M(k)\rangle\langle M(k)|j_\mu|B\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{\infty} ds \frac{\rho^{\mu\nu}}{s-k^2}$$

$$\Pi^{\mu\nu} = \int d^4x \int \frac{d^4p'}{(2\pi)^4} e^{i(k-p').x} \left[\Gamma_2^{\nu}\frac{\not{p}'+m_1}{m_1^2-p'^2}\Gamma_1^{\mu}\right]_{\alpha\beta} < 0|\bar{q}_2{}^{\alpha}(x)h_v^{\beta}(0)|\bar{B}(v) >$$

Integral dominated by terms
on the light cone $x^2 \ll 1/\Lambda_{QCD}^2$

Near the LC: Expansion in twists
(Twist = dimension - spin)
In terms of **LC B-meson distribution amplitudes**

$$K^{(F)}\frac{F(q^2)}{m_M^2-k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho(s)}{s-k^2} = f_B m_B \int_0^{+\infty} ds \sum_{n=1}^{+\infty} \frac{I_n(s)}{(s-k^2)^n}$$

$k^2 \ll \Lambda_{\text{had}}^2$

$\tilde{q} \leq m_b^2 + m_b k^2/\Lambda_{\text{had}}$

Quark-Hadron Duality at leading order in twist

$$K^{(F)}\frac{F\left(q^2\right)}{m^2-k^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\frac{\rho(s)}{s-k^2} = \Pi = f_B m_B\int_0^{+\infty} ds\,\frac{I_1(s)}{s-k^2}$$

$$\mathcal{B}_{M^2}f(k^2) = \lim_{\substack{-k^2,n\to\infty \\ \frac{-k^2}{n}=M^2}} \frac{(-q^2)^{n+1}}{n!}\left(\frac{d}{dk^2}\right)^n f(k^2)$$

Borel transform

$$K^{(F)}F\left(q^2\right)e^{-m^2/M^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\rho(s)\,e^{-s/M^2} = \Pi = f_B m_B\int_0^{+\infty} ds\,I_1(s)e^{-s/M^2}$$

M^2 : Borel parameter

s_0 : Duality threshold

Semi-global quark hadron duality: there is a s_0 such that

$$\frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\,\rho(s)\,e^{-s/M^2} \simeq \int_{s_0}^{+\infty} ds\,\mathrm{Im}\,\Pi^{\mathrm{pert}}(\mathrm{q}^2,\mathrm{s})\,e^{-s/M^2} \simeq \mathrm{f_B m_B}\int_{s_0}^{+\infty} ds\,I_1(s)e^{-s/M^2}$$

unknown systematic error

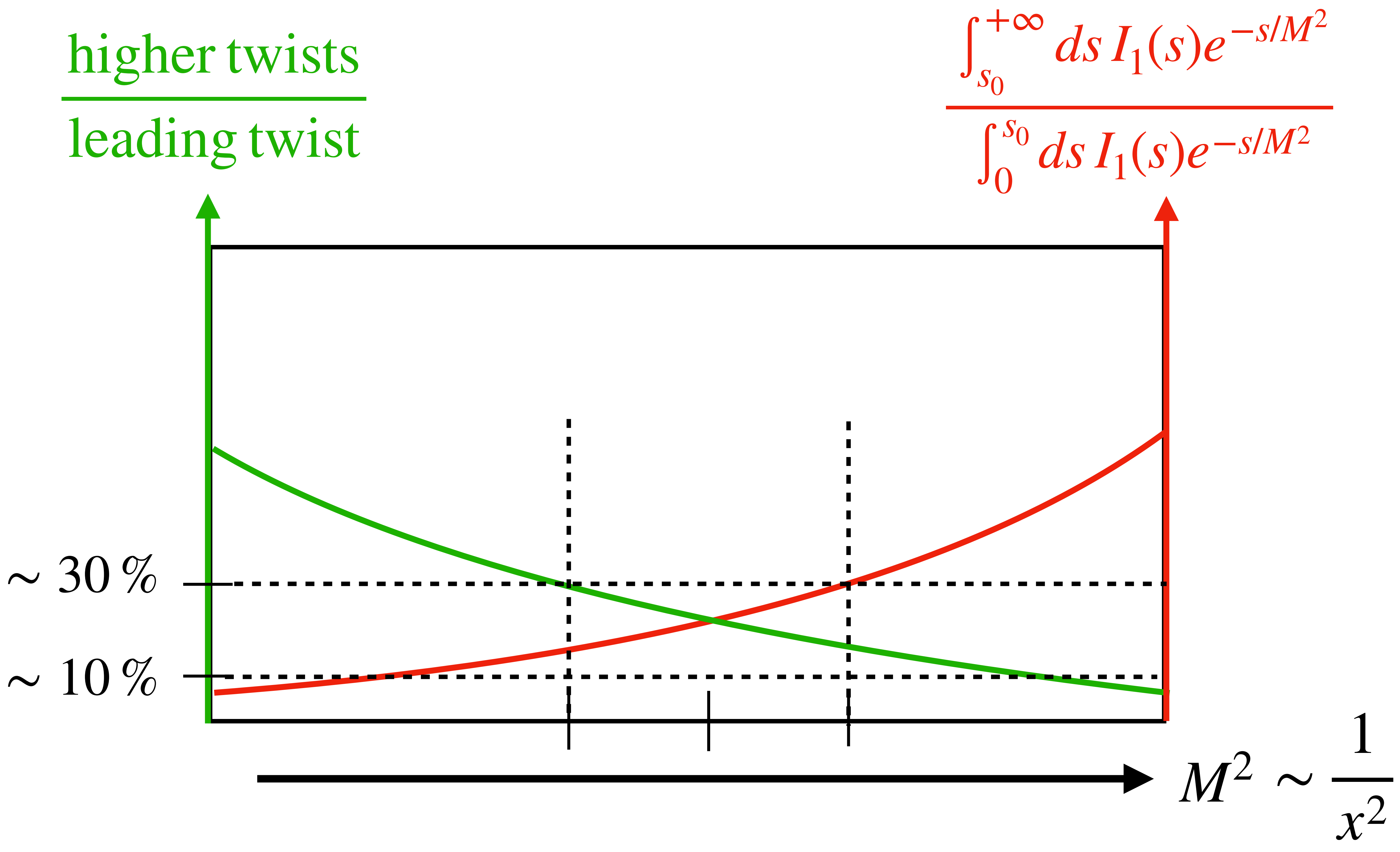
$$F\left(q^2\right) = \frac{f_B m_B}{K^{(F)}}\int_0^{s_0} ds\,I_1(s)\,e^{\frac{-s+m^2}{M^2}}$$

How to determine the threshold parameter s_0

$$F\left(q^2\right) = \frac{f_B m_B}{K^{(F)}} \int_0^{s_0} ds\, I_1(s)\, e^{\frac{-s+m^2}{M^2}}$$

Threshold s_0 can be determined by looking for independence wrt M^2

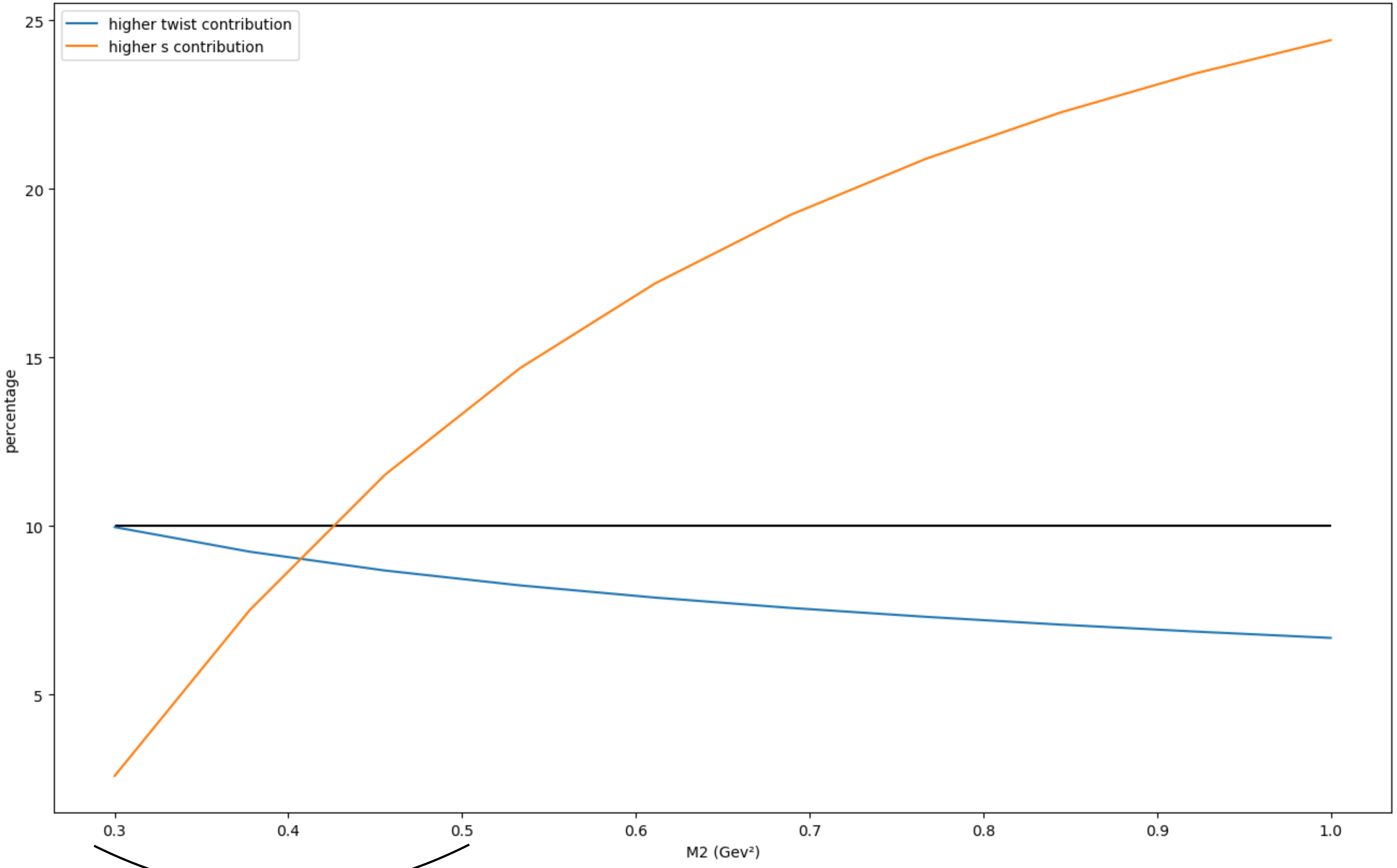
Daughter sum rule: $\frac{d}{dM^2} F(q^2) = 0$



Range of the Borel parameter

E.g. for $B \to K$: $M^2 \in [0.5, 1.5]\,\mathrm{GeV}^2$

H… r s_0



$\frac{e^{-s/M^2}}{e^{-s/M^2}}$

$M^2 \sim \frac{1}{x^2}$

Updated range for M^2

Quark-Hadron Duality with higher-twists

B-meson DA's

$$\Pi = K^{(F)}F\left(q^2\right) e^{-m^2/M^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\rho(s)\, e^{-s/M^2} = f_B m_B \int_0^{+\infty} ds\, e^{-s/M^2}\left(I_1(s) + \sum_{n=2}^{+\infty}\frac{I_n(s)}{(n-1)!M^{2(n-1)}}\right)$$

- Rewrite the correlation function as

$$\Pi = \text{surface terms}\,|_{s=0} + \int_0^{+\infty} e^{-s/M^2}\sum_n\sum_j (d/ds)^j I_n(s)$$

- Apply QHD on the integral term only

$$\int_{s_0^h}^{+\infty}\rho(s)e^{-s/M^2} \simeq \int_{s_0}^{+\infty} e^{-s/M^2}\sum_n\sum_j (d/ds)^j I_n(s)$$

Because of the surface terms:

- No relative suppression of some higher twist contributions at high M^2 ✗
- Daughter sum rules for pseudo Goldstone boson breaks, must use s_0 from other methods ✗

Quark-Hadron Duality with higher-twists

Our prescription

$$\Pi = K^{(F)}F\left(q^2\right)e^{-m^2/M^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\rho(s)\,e^{-s/M^2} = f_B m_B \int_0^{+\infty} ds\, e^{-s/M^2}\left(I_1(s) + \sum_{n=2}^{+\infty}\frac{I_n(s)}{(n-1)!M^{2(n-1)}}\right)$$

$$\Pi = \text{surface terms}\,|_{s=0} + \int_0^{+\infty} e^{-s/M^2}\sum_n\sum_j (d/ds)^j I_n(s)$$

$$\Pi = \int_{-\epsilon}^{+\infty} e^{-s/M^2}\left(\sum_n\sum_j (d/ds)^j I_n(s) + \sum_{n=2}^{+\infty} G(I_n, \delta(s))\right)$$

QHD*: $$\frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\rho(s)e^{-s/M^2} \simeq f_B m_B \int_{\sigma_0}^{+\infty} d\sigma \sum_{n=1}^{+\infty} I_n(\sigma)\frac{e^{-s/M^2}}{(n-1)!M^{2(n-1)}}$$

- Relative suppression of some higher twist contributions at high M^2 ✓
- Daughter sum rules for pseudo Goldstone boson works ✓
- Associated error? ✗

Results

Following closely the procedure of Gubernari et al. 2018

s_0 from SVZ sum rules Khodjamirian-Mannel hep-ph/0308297

Form Factor $q^2 = 0$	Our Result	Gubernari et al. 2018	Other results
$f_+^{B\to\pi}$	0.249 ± 0.064 PRELIMINARY	0.21 ± 0.07	0.258 ± 0.031 [1] 0.25 ± 0.05 [2] 0.301 ± 0.023 [3] 0.280 ± 0.037 [4]
$f_T^{B\to\pi}$	0.259 ± 0.065 PRELIMINARY	0.19 ± 0.06	0.253 ± 0.028 [1] 0.21 ± 0.04 [2] 0.273 ± 0.021 [3] 0.26 ± 0.06 [4]
$f_+^{B\to K}$	0.376 ± 0.068 PRELIMINARY	0.27 ± 0.08	0.331 ± 0.041 [1] 0.31 ± 0.04 [2] 0.395 ± 0.033 [3] 0.364 ± 0.05 [4]
$f_T^{B\to K}$	0.367 ± 0.053 PRELIMINARY	0.25 ± 0.07	0.358 ± 0.037 [1] 0.27 ± 0.04 [2] 0.381 ± 0.027 [3] 0.363 ± 0.08 [4]

- Same input on B-meson DA's as *Gubernari, Kokulu, van Dyk 2018* (twist 4, exponential model for DA's, Braun et al. 2017)
- In agreement with previous calculations
- Non-negligible dependence on the surface terms in the quark-hadron duality
- $BR(B \to K\mu\mu) \propto \sim |f^{B->K}|^2$ up to interference terms

[1] Ball and Zwicky 2005, light meson DA's
[2] Khodjamirian, Mannel, Offen 2007, B meson DA's
[3] Khodjamirian, Rusov, LCSR + CKM
[4] Lu, Shen, Wang, Wei, LCSR + QCD SR up to twist 6

Conclusion

- Deviations in neutral current B decays subsist in BR's and angular observables
- In BR, TH uncertainty as large or larger than EXP and is dominated by local form factor uncertainties
- A more accurate assessment of local form factors is needed
- In LCSR :
    - Quark Hadron Duality introduces an unknown systematic error
    - In the presence of higher twists in LCSR with B-meson DA's, QHD is not consistent with the suppression of some higher twist contributions near the light cone and requires external input for s_0
    - Including the surface term in the QHD fixes the issue and yields non-negligible corrections to the form factors
    - New results are compatible with the literature
- *Coming soon*:
    - Full form factors for $B \to K^{(*)}, D^{(*)}, \pi, \rho$ + fit with LQCD
    - SM predictions of relevant observables with this method

Backup

LCSR: The correlation function

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0\,|\,TJ^{\nu}_{int}(x)J^{\mu}_{\rm weak}(0)\,|\,\bar{B}(P_B = q+k) >$$

Unitarity relation

$$2\mathrm{Im}(\Pi^{\mu\nu}) = \sum_{\rm X}\int \mathrm{d}\tau_{\rm X} < 0\,|\,\mathrm{J}^{\nu}_{\rm int}\,|\,\mathrm{X} > < \mathrm{X}\,|\,\mathrm{J}^{\mu}_{\rm weak}\,|\,\overline{\mathrm{B}} > (2\pi)^4\delta^{(4)}(\mathrm{k}-\mathrm{P_X})$$

Dispersion relation

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{1}{\pi}\int_{t_{min}}^{+\infty} ds\frac{\mathrm{Im}\,\Pi^{\mu\nu}(\mathrm{q}^2,\mathrm{s})}{s-k^2}$$

∝ decay constant of the light meson

What we want to compute

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{< 0\,|\,j_\nu\,|\,M(k) > < M(k)\,|\,j_\mu\,|\,B >}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{\infty} ds\frac{\rho^{\mu\nu}}{s-k^2}$$

Continuum, a priori unknown

$$\left\langle 0\,\middle|\,\bar{q}_2\gamma^\nu\gamma_5 q_1\,\middle|\,P(k)\right\rangle = ik^\nu f_P$$

$$\left\langle 0\,\middle|\,\bar{q}_2\gamma^\nu q_1\,\middle|\,V(k,\eta)\right\rangle = i\eta^\nu m_V f_V$$

Light-Cone Sum Rules

B-meson distribution amplitude

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0|TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)|\bar{B}(P_B = q+k) >$$

Heavy Quark Effective Theory

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0|TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)|\bar{B}_v(h_v = \tilde{q}+k) > + \mathcal{O}(1/m_b)$$

$$\Pi^{\mu\nu} = \int d^4x \int \frac{d^4p'}{(2\pi)^4} e^{i(k-p').x} \left[\Gamma_2^{\nu}\frac{\not{p}'+m_1}{m_1^2-p'^2}\Gamma_1^{\mu}\right]_{\alpha\beta} < 0|\bar{q}_2{}^{\alpha}(x)h_v^{\beta}(0)|\bar{B}(v) >$$

Perturbative piece (Fully calculable)

Can be expressed as a function of B-meson distribution amplitudes

$$< 0|\bar{q}_2{}^{\alpha}(x)h_v^{\beta}(0)|\bar{B}(v) > = -\frac{if_Bm_B}{4}\int_0^{+\infty} dw e^{-iwv.x}\Phi_{2p}(w)^{\beta\alpha}$$
$$= \sum_t -\frac{if_Bm_B}{4}\int_0^{+\infty} dw e^{-iwv.x}\Phi^t_{2p}(w)^{\beta\alpha}$$

Condition for Perturbativity and Light-Cone dominance:

$$\tilde{q} \leq m_b^2 + m_b k^2/\Lambda_{\text{had}}$$
$$k^2 \ll \Lambda_{\text{had}}^2$$

Near the light-cone ($x^2 \ll 1/\Lambda^2_{\text{QCD}}$) the DA's are expanded in a series of operators with increasing (twist = dimension - spin)
At $x^2 = 0$, the only non-zero contribution is twist 2

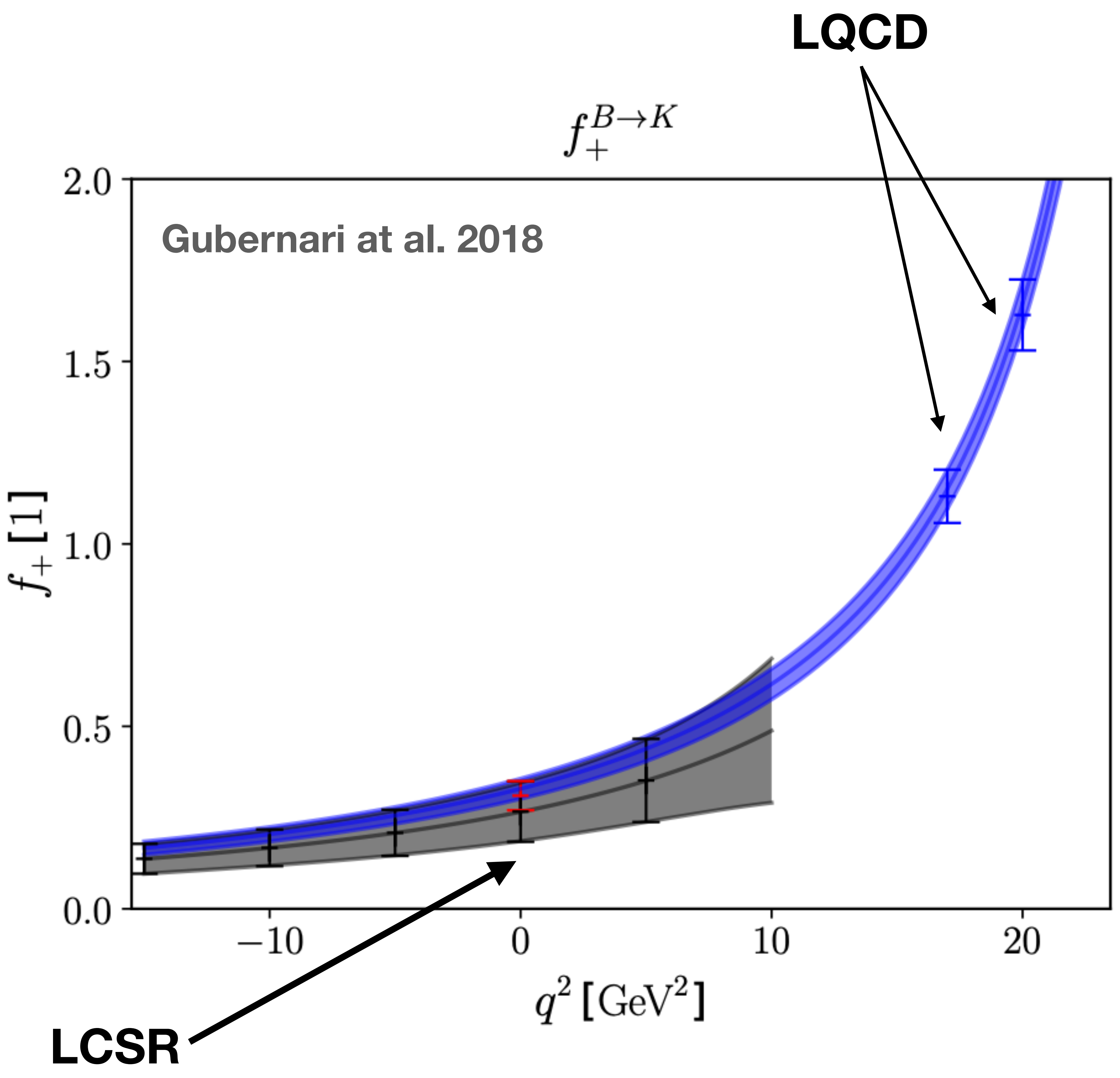
Hadronic Form Factors on the full q^2 range

- LCSR valid for $q^2 \ll m_b^2$
- LQCD works at low-recoil $q^2 \approx (m_B - m_M)^2$
- It is customary to interpolate them in the BSZ expansion

$$F\left(q^2\right) \equiv \frac{1}{1 - q^2/m_{R,F}^2} \sum_{k=0}^{n} \alpha_k^{(F)} \left[z\left(q^2\right) - z(0)\right]^k$$

with typically $n = 2$ or 3

$$z(t) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \qquad t_\pm = (m_B \pm m_{P,V})^2$$



$$K^{(F)}\frac{f^{(F)}\left(q^2\right)}{m^2-k^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\frac{\rho(s)}{s-k^2}=f_Bm_B\int_0^{+\infty}d\sigma\sum_{n=1}^{+\infty}\frac{1}{(n-1)!}\frac{1}{s-k^2}\left(\frac{d}{d\sigma}\frac{1}{s'}\right)^{n-1}I_n^{(F)}(s)$$

$$+f_Bm_B\sum_{n=2}^{+\infty}\frac{1}{(n-1)!}\sum_{j=1}^{n-1}\frac{(n-j-1)!}{\left(s-k^2\right)^{n-j}}\frac{1}{s'}\left(\frac{d}{d\sigma}\frac{1}{s'}\right)^{j-1}\left(I_n^{(F)}(s)\right)\Bigg|_{\sigma=0}$$

$$F=\frac{f_BM_B}{K^{(F)}}\sum_{n=1}^{\infty}\Bigg\{(-1)^n\int_0^{\sigma_0}d\sigma e^{\left(-s\left(\sigma,q^2\right)+m_{P,V}^2\right)/M^2}\frac{1}{(n-1)!\left(M^2\right)^{n-1}}I_n^{(F)}$$

$$-\left[\frac{(-1)^{n-1}}{(n-1)!}e^{\left(-s\left(\sigma,q^2\right)+m_{P,V}^2\right)/M^2}\sum_{j=1}^{n-1}\frac{1}{\left(M^2\right)^{n-j-1}}\frac{1}{s'}\left(\frac{\mathrm{d}}{\mathrm{d}\sigma}\frac{1}{s'}\right)^{j-1}I_n^{(F)}\right]_{\sigma=\sigma_0}\Bigg\},$$