

Some of my memories with Anamaría and Classical dS from Supergravity and Strings

**Fernando Quevedo
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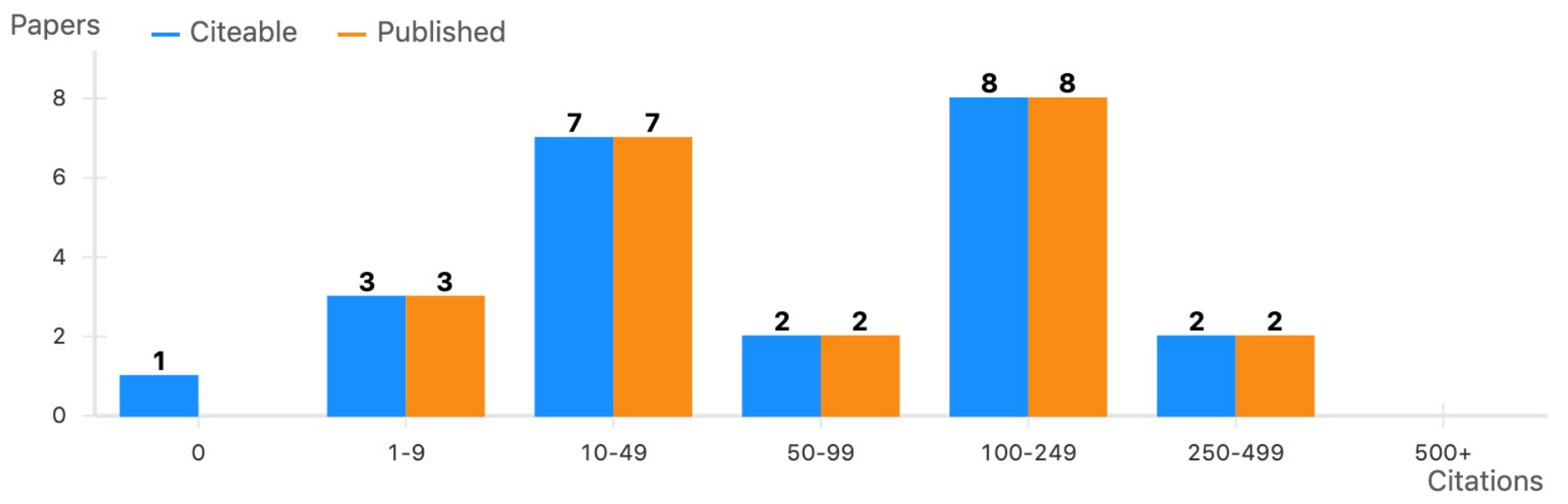
**AnLy LAPP Annecy
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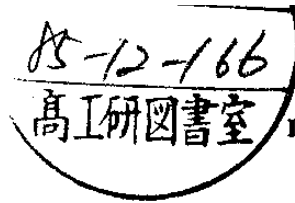
Joint Publications Highlights

Citation Summary

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	Citeable ?	Published ?
Papers	23	22
Citations	2,295	2,295
h-index ?	17	17
Citations/paper (avg)	99.8	104.3





Low Energy Effective Action for the Superstring

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ABSTRACT

We construct the low energy $D = 4$, $N = 1$ supergravity that arises in superstring theories for an arbitrary number of generations. The couplings of all massless modes that carry low-energy gauge quantum numbers are calculated by truncating the heavy Kaluza-Klein modes of the ten-dimensional effective field theory. The resulting action is compared to the most general effective action compatible with the symmetries of the underlying ten-dimensional field (and string) theories. This comparison indicates which features of the truncation correctly approximate the correct low-energy action.

Degenerate orbifolds

A. Font, L.E. Ibáñez¹, H.-P. Nilles², F. Quevedo

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[https://doi.org/10.1016/0550-3213\(88\)90524-X](https://doi.org/10.1016/0550-3213(88)90524-X) 

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Abstract

We present evidence for the existence of new four-dimensional string theories, obtained from a smooth variation of background fields in the twisted sectors of symmetric and asymmetric orbifolds. Flat directions only in the untwisted sector are shown to reproduce previously constructed models in terms of Wilson lines, exhibiting a Three-Higgs-Rule (THR). The new models provide a mechanism to lower the rank of the gauge group, lead to more flexible Yukawa couplings and give a strict separation of hidden and observable sectors, which are usually mixed in $(2, 0)$ -models. Even though Fayet-Iliopoulos terms are induced in some of the models due to the presence of anomalous $U(1)$'s supersymmetry remains, in general, unbroken. Particular examples of the new models correspond to “blown up” versions of $(2, 0)$ -orbifolds.

Yukawa couplings in degenerate orbifolds: towards a realistic $SU(3)\times SU(2)\times U(1)$ superstring

A. Font, L.E. Ibáñez¹, H.-P. Nilles^{2,3}, F. Quevedo

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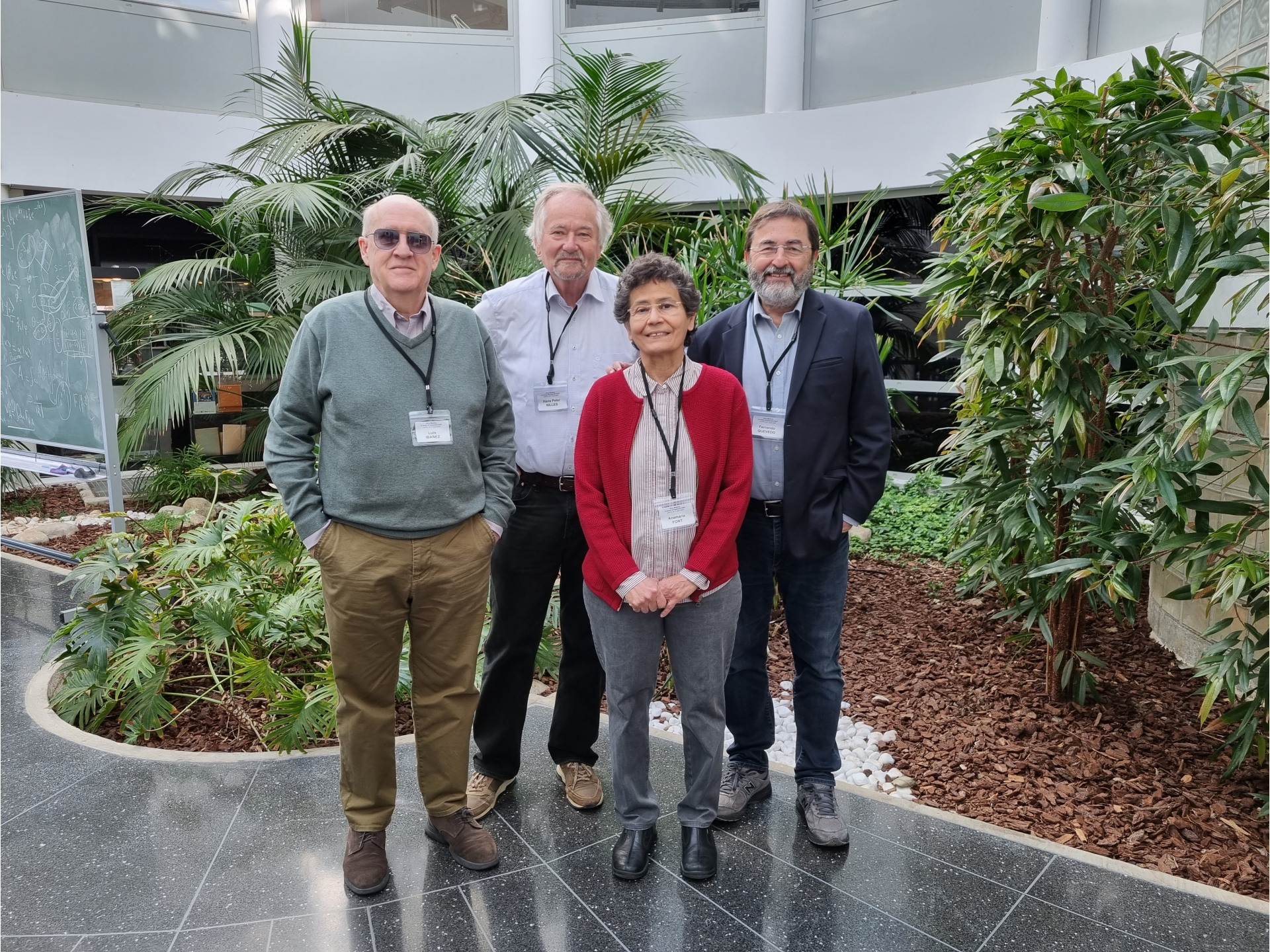
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[https://doi.org/10.1016/0370-2693\(88\)90357-7](https://doi.org/10.1016/0370-2693(88)90357-7) 

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Abstract

We discuss the construction of $SU(3)\times SU(2)\times U(1)$ three-generation superstrings through the “degenerate orbifolds” recently described by the authors. These are lower rank models continuously connected to rank-sixteen or twenty-two models through flat directions in the potential of the scalar fields. The structure of Yukawa couplings is carefully investigated and special attention is paid to nonrenormalizable interactions in determining the flat directions and the induced cubic couplings that are forbidden in the original model. The importance of twisted oscillator modes and moduli to this effect is explained. One specific example is presented in detail and its phenomenological consequences such as quark and lepton masses, proton stability and neutrino masses are discussed. In this example there are built-in “stringy” symmetries that protect Higgs



LUIS IBARRA

JOSE PABLO WILLES

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ANTONIO GONZALEZ

$Z_N \times Z_M$ orbifolds and discrete torsion

A. Font, [L.E. Ibáñez](#), [F. Quevedo](#)

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Abstract

We extend previous work on Z_N -orbifolds to the general $Z_N \times Z_M$ abelian case for the (2, 2) and (0, 2) models. We classify the corresponding (2, 2) compactifications and show that a number of models obtained by tensoring minimal $N = 2$ superconformal theories can be constructed as $Z_N \times Z_M$ -orbifolds. Furthermore, $Z_N \times Z_M$ -orbifolds allow the addition of discrete torsion which leads to new (2, 2) compactifications not considered previously. Some of the latter have negative Euler characteristic and Betti numbers equal to those of some complete intersection Calabi-Yau (CICY) manifolds. This suggests the existence of a previously overlooked connection between CICY models and orbifolds.

Supersymmetry breaking from duality invariant gaugino condensation

[A. Font](#)^a, [L.E. Ibáñez](#)^b, [D. Lüst](#)^b, [F. Quevedo](#)^c

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Abstract

It is known that the formation of gaugino condensates can be a source of supersymmetry breaking in string theory. We study the constraints imposed by target space modular invariance on the formation of such condensates. We find that the dependence of the vacuum energy on the moduli of the internal variety is such that the theory is forced to be compactified. The radius of compactification is of the order of the string scale and in the process target space duality is spontaneously broken.

Strong–weak coupling duality and non-perturbative effects in string theory

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Received 13 July 1990

We conjecture the existence of a new discrete symmetry of the modular type relating weak and strong coupling in string theory. The existence of this symmetry would strongly constrain the non-perturbative behaviour in string partition functions and introduces the notion of a maximal (minimal) coupling constant. An effective lagrangian analysis suggests that the dilaton vacuum expectation value is dynamically fixed to be of order one. In supersymmetric heterotic strings, supersymmetry (as well as this modular symmetry itself) is generically spontaneously broken.

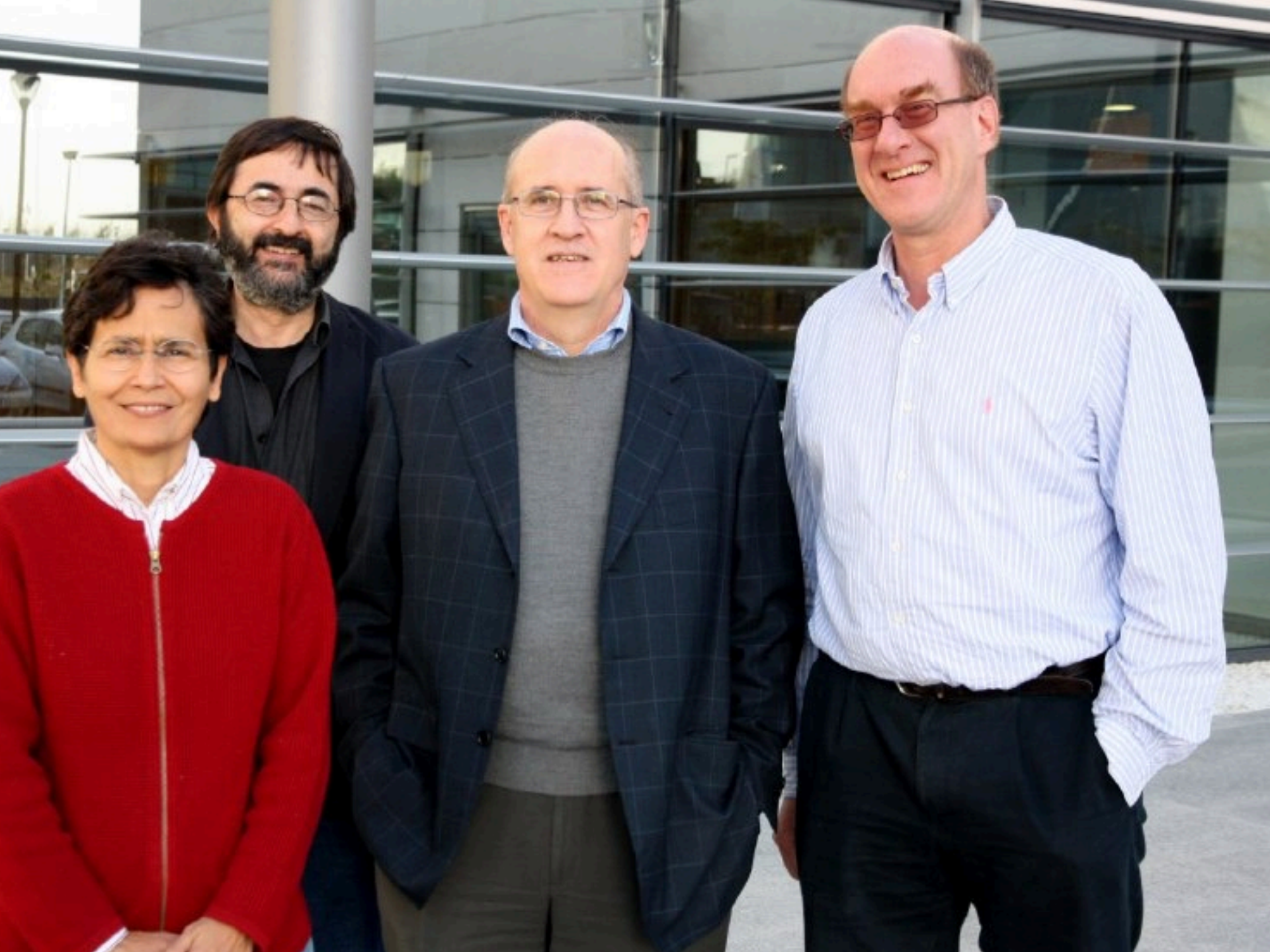
The membranes include those of the string theory with a duality transformation for the dilaton field (Type II A strings, however, are not selfdual but “dual” to type II B strings [22,23]). Of course, there is at the moment no idea about how a ten-dimensional heterotic string could be obtained from an eleven-dimensional extended structure, but that is certainly an open possibility. If this was the case duality in both T and S would be expected.

The S -duality we are discussing includes an invariance under the transformation of the string coupling constant $g \rightarrow 1/g$. Montonen and Olive [24] conjectured some time ago that this type of duality invariance does in fact occur in field theory models of the Georgi–Glashow type (and for any other gauge group with adjoint scalars). They argued that both the spec-

. Then it is natural to conjecture theory, the one which describes elementary and solitonic states, should symmetry of the type described in the conjecture that the Montonen–Olive symmetry of the heterotic strings will lead to gauge symmetry as in eq. (2).

lagrangian should be explicitly dual-appens with the $R \rightarrow 1/R$ duality. te complete. For small g (big R) strings dominate and the “dual” modes in T -duality are very massive. The opposite occurs. Thus a duality

$g \rightarrow 1/g$ in the effective four-dimensional field theory should exist if the above arguments are correct. It follows that inequivalent theories are characterized by coupling constants g smaller (larger) than some critical value. [The notion of a maximal (minimal) coupling could possibly be understood in the sense that the coupling constant determines the “size” of the internal gauge group manifold which should not be “smaller” than the typical scale in string theory.] In analogy with T -duality, one also expects the continuous Peccei–Quinn symmetry $S \rightarrow S + 1a$, $a \in \mathbb{R}$ not to be completely broken but a discrete subgroup ($a \in \mathbb{R}$) to survive. In this way the effective field theory should be invariant under a full modular group $SL(2, \mathbb{R})$.



PERIODS FOR CALABI–YAU AND LANDAU–GINZBURG VACUA

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ABSTRACT

The complete structure of the moduli space of Calabi–Yau manifolds and the associated Landau–Ginzburg theories, and hence also of the corresponding low-energy effective theory that results from (2,2) superstring compactification, may be determined in terms of certain holomorphic functions called periods. These periods are shown to be readily calculable for a great many such models. We illustrate this by computing the periods explicitly for a number of classes of Calabi–Yau manifolds. We also point out that it is possible to read off from the periods certain important information relating to the mirror manifolds.

Chains of $N=2$, $D=4$ heterotic/type II duals

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Abstract

We report on a search for $N = 2$ heterotic strings that are dual candidates of type II compactifications on Calabi-Yau threefolds described as $K3$ fibrations. We find many new heterotic duals by using standard orbifold techniques. The associated type II compactifications fall into chains in which the proposed duals are heterotic compactifications related one another by a sequential Higgs mechanism. This breaking in the heterotic side typically involves the sequence $SU(4) \rightarrow SU(3) \rightarrow SU(2) \rightarrow 0$, while in the type II side the weights of the complex hypersurfaces and the structure of the $K3$ quotient singularities also follow specific patterns. Some qualitative features of the relationship between each model and its dual can be understood by fiber-wise application of string-string duality.

A Comment on Continuous Spin Representations of the Poincaré Group and Perturbative String Theory

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ABSTRACT: We make a simple observation that the massless continuous spin representations of the Poincaré group are not present in perturbative string theory constructions. This represents one of the very few model-independent low-energy consequences of these models.

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4D dS from 6D Supergravity and Strings

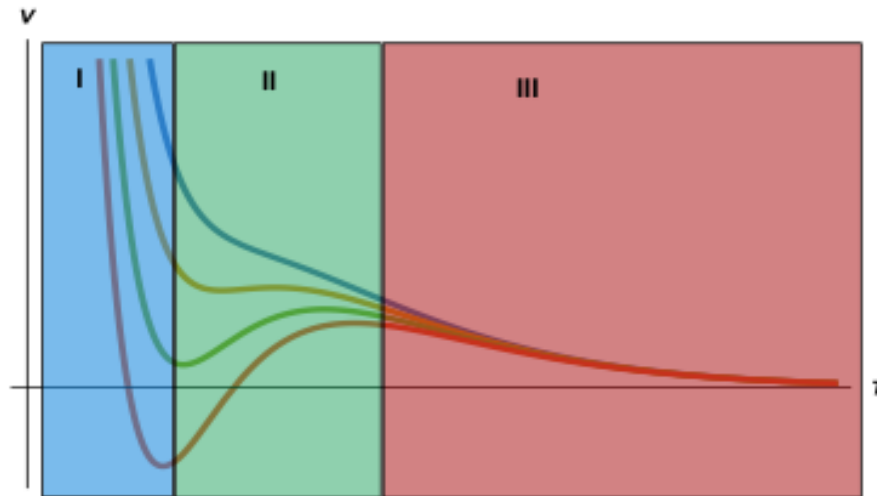
Two Related Questions

- Classical de Sitter from supergravity and string theory.
- Extensions to the landscape (Λ CDM) with light scalars.

Both: Multifield set-ups and accelerated expansion

Obstacles for dS from UV Theory

- Classical No-Go Theorems
- Dine Seiberg problem



Different approaches

- **String flux compactification EFTs** (e.g. KKLT, LVS)

Review: L. McCallister and FQ, [2310.20559](#)

- **Classical solutions?** (evading no-go theorems)

Classical de Sitter solutions

Classical no-go theorem

Gibbons, De Wit, Maldacena-Nunez...

- The gravity action does not contain higher curvature corrections.
- The potential is non-positive, $V \leq 0$.
- The theory contains massless fields with positive kinetic terms.
- The d dimensional effective Newton's constant is finite.

$$ds_D^2 = \Omega^2(y) \left(dx_d^2 + \hat{g}_{mn} dy^n dy^m \right)$$

$$\frac{1}{(D-2)\Omega^{D-2}} \nabla^2 \Omega^{D-2} = R + \Omega^2 \left(-T^\mu{}_\mu + \frac{d}{D-2} T^L{}_L \right) \geq 0 \quad \text{For de Sitter} \quad R \geq 0$$

But integrating: $\int d^{(D-d)}y \sqrt{\hat{g}} \left(\hat{\nabla} \Omega^{(D-2)} \right)^2 \leq 0$ **So no de Sitter**

Ways out

- Quantum effects,...
- Relax assumptions (e.g. $V \leq 0$)

De Sitter from 6D (1,0) Gauged Supergravity

Matter content

- *Gravity multiplet.* Metric g_{MN} a self-dual antisymmetric tensor B_{MN}^+ , one left-handed gravitino Ψ_M^α .
- *Tensor multiplet.* One anti self-dual antisymmetric tensor B_{MN}^- , one scalar ϕ , one right-handed fermion ψ (tensorino).
- *Vector multiplet.* One vector A_M and one fermion λ (gaugino).
- *Hypermultiplet:* Two complex scalars q^1, q^2 and one right-handed Weyl fermion ξ (hyperino) .

In general n_T tensor, n_V vector and n_H hyper multiplets

Scalar fields

- From tensor multiplets n_T real scalars

$$SO(1, n_T)/SO(n_T)$$

$$j^\alpha \quad \alpha = 1, \dots, n_T + 1 \quad \Omega_{\alpha\beta} j^\alpha j^\beta = 1 \quad g_{\alpha\beta} = 2j_\alpha j_\beta - \Omega_{\alpha\beta}$$

$$\mathbf{n_T=1} \quad j^0 = \sinh \varphi, \quad j^1 = \cosh \varphi$$

- From hypermultiplets

$$q^U \quad (U = 1, \dots, 4n_H)$$

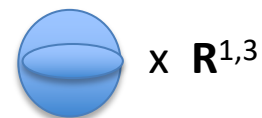
Quaternionic manifold

6D Supergravity (Salam-Sezgin)

$$S = - \int d^D x \sqrt{-g} \left[\frac{1}{2\kappa^2} g^{MN} (R_{MN} + \partial_M \varphi \partial_N \varphi) + \frac{1}{2} \sum_r \frac{1}{(p_r + 1)!} e^{-p_r \varphi} F_r^2 + \mathcal{A} e^\varphi \right],$$

D=6, r=2, A>0

- **Positive potential (evades Maldacena-Nunez theorem)**
- **Chiral**
- **No maximally symmetric solution in 6D (Dine-Seiberg problem in 6D?)**
- **Maximally symmetric in 4D**
- **Maximally symmetric smooth solution: $S^2 \times$ Minkowski, N=1 SUSY.**



General 4D Solutions

Gibbons et al 2004
Burgess et al 2005

$$\mathcal{L}_6 = R * \mathbf{1} - *d\phi \wedge d\phi - \frac{1}{2}e^{-\varphi} * F_{(2)} \wedge F_{(2)} - \frac{1}{2}e^{-2\varphi} * H_{(3)} \wedge H_{(3)} - 8g^2 e^\varphi * \mathbf{1}$$

Runaway potential! 6D Dine-Seiberg problem?

$$ds^2 = \hat{g}_{MN} dx^M dx^N = W^2(y) g_{\mu\nu}(x) dx^\mu dx^\nu + \tilde{g}_{ij}(y) dy^i dy^j$$

$$\hat{g}_{\mu\nu} = W^2 g_{\mu\nu}, \quad \hat{R}_{\mu\nu} = R_{\mu\nu} + \frac{1}{n}(W^{2-n} \tilde{\nabla}^2 W^n) g_{\mu\nu} \quad \text{and} \quad \hat{\square}\varphi = W^{-n} \tilde{\nabla}_i (W^n \tilde{g}^{ij} \partial_j \varphi),$$

$$\frac{1}{n} \int_M d^d y \sqrt{\tilde{g}} W^{n-2} R = - \sum_\alpha \int_{\Sigma_\alpha} d^{d-1} y \sqrt{\tilde{g}} N_i \left[W^n \tilde{g}^{ij} \partial_j \left(\ln W + \frac{2\varphi}{D-2} \right) \right]$$

No singularities/boundaries imply $R=H^2=0$ e.g. $S^2 \times \mathbb{R}^{1,3}$

(uniqueness of Salam-Sezgin solution)

General Solutions

Burgess et al 2005



★ 3-Branes

Asymptotic near brane solutions (n=4, d=2):

$$\varphi \approx q \ln r \quad \text{and} \quad ds^2 \approx r^{2w} g_{\mu\nu}(x) dx^\mu dx^\nu + dr^2 + r^{2\alpha} f_{ab}(z) dz^a dz^b,$$

$$nw + \alpha(d - 1) = 1. \quad nw^2 + \alpha^2(d - 1) + q^2 = 1.$$

**Kasner constraints
(BKL: Belinsky et al)**

$$-\frac{1}{\sqrt{n}} \leq w \leq \frac{1}{\sqrt{n}}, \quad -\frac{1}{\sqrt{d-1}} \leq \alpha \leq \frac{1}{\sqrt{d-1}} \quad \text{and} \quad -1 \leq q \leq 1.$$

Flat Solutions

Gibbons et al.

$$ds^2 = \hat{g}_{MN} dx^M dx^N = W^2 q_{\mu\nu} dx^\mu dx^\nu + a^2 d\theta^2 + a^2 W^8 d\eta^2,$$

$$e^\varphi = W^{-2} e^{-\lambda_3 \eta}$$

$$W^4 = \left(\frac{Q\lambda_2}{4g\lambda_1} \right) \frac{\cosh[\lambda_1(\eta - \eta_1)]}{\cosh[\lambda_2(\eta - \eta_2)]}$$

$$a^{-4} = \left(\frac{gQ^3}{\lambda_1^3 \lambda_2} \right) e^{-2\lambda_3 \eta} \cosh^3[\lambda_1(\eta - \eta_1)] \cosh[\lambda_2(\eta - \eta_2)]$$

$$F = \left(\frac{Qa^2}{W^2} \right) e^{-\lambda_3 \eta} d\eta \wedge d\theta.$$

Numerical de Sitter solution

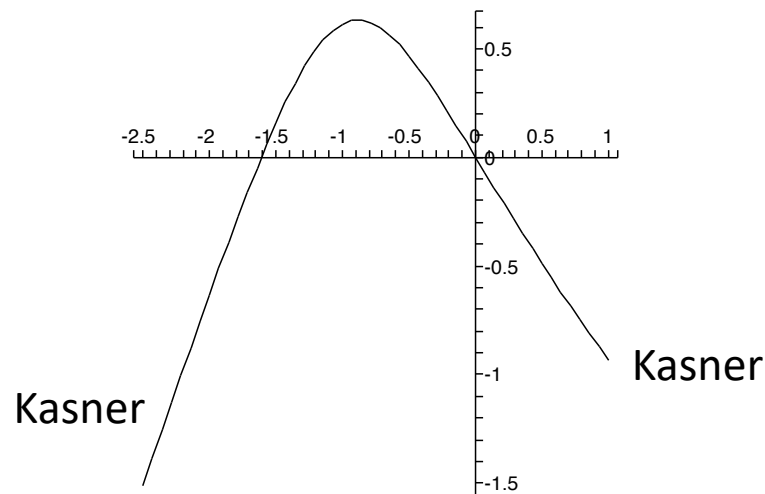
$$X'' + e^{2X} = 0$$

$$Y'' + e^{2Y} - \epsilon e^{2Y+Z} = 0$$

$$Z'' + \frac{\epsilon}{2} e^{2Y+Z} = 0,$$

$$e^{-X} = \lambda_1^{-1} \cosh[\lambda_1(\eta - \eta_1)].$$

**X,Y,Z linear combinations
of log W, log a, φ
 $\epsilon = H^2$**



Burgess et al 2005

Solutions stable under small perturbations!

6D (1,0) Supergravity From String Theory?

- **M-theory/IIA on hyperbolic manifold $H^{(2,2)}$**

$$x_1^2+x_2^2-x_3^2-x_4^2=\rho^2$$

Consistent truncations give Salam-Sezgin theory

Cvetic, Gibbons, Pope hep-th/0308026

- **F-theory on elliptic Calabi-Yau**

Grimm, Pugh 1302.3223

10D String on $H^{(2,2)} \times S^1$

(ρ, α, β) (z)

Any solution to the 6D equations from:

$$\mathcal{L}_6 = R * \mathbf{1} - *d\phi \wedge d\phi - \frac{1}{2}e^{-\varphi} * F_{(2)} \wedge F_{(2)} - \frac{1}{2}e^{-2\varphi} * H_{(3)} \wedge H_{(3)} - 8g^2 e^\varphi * \mathbf{1}$$

Can be uplifted to solutions of 10D (string) equations:

$$\mathcal{L}_{10} = \hat{R} * \hat{\mathbf{1}} - \frac{1}{2} * \hat{d}\hat{\phi} \wedge d\hat{\phi} - \frac{1}{2}e^{-\hat{\phi}} * \hat{F}_{(3)} \wedge \hat{F}_{(3)}$$

From:

$$d\hat{s}_{10}^2 = (\cosh 2\rho)^{1/4} \left[e^{-\phi/4} ds_6^2 + e^{\phi/4} dz^2 + \frac{e^{\phi/4}}{2\bar{g}^2} \left(d\rho^2 + \frac{\cosh^2 \rho}{\cosh 2\rho} (D\alpha)^2 + \frac{\sinh^2 \rho}{\cosh 2\rho} (D\beta)^2 \right) \right]$$

$$\hat{F}_{(3)} = H_{(3)} + \frac{\sinh 2\rho}{2\bar{g}(\cosh 2\rho)^2} d\rho \wedge D\alpha \wedge D\beta + \frac{1}{2\bar{g} \cosh 2\rho} F_{(2)} \wedge (\cosh^2 \rho D\alpha - \sinh^2 \rho D\beta)$$

$$e^{\hat{\phi}} = (\cosh 2\rho)^{-1/2} e^\varphi$$

Cvetic et al 2003

Then the 6D de Sitter solutions can be uplifted to 10D !!!

6D Supergravity from F-theory

Grimm et al 2013

11D M-theory to 5D on elliptically fibred CY_3 and uplift to D=6

$h_{12} + 1$ hypermultiplets, $h_{11} - 1$ tensor multiplets

$$S^{(6)} = \int_{\mathcal{M}_6} \left[\frac{1}{2} \hat{R} \hat{*} 1 - \frac{1}{4} \hat{g}_{\alpha\beta} \hat{G}^\alpha \wedge \hat{*} \hat{G}^\beta - \frac{1}{2} \hat{g}_{\alpha\beta} d\hat{j}^\alpha \wedge \hat{*} d\hat{j}^\beta - \frac{1}{2} \hat{h}_{UV} \hat{D}\hat{q}^U \wedge \hat{*} \hat{D}\hat{q}^V \right. \\ \left. - 2\Omega_{\alpha\beta} \hat{j}^{\alpha\beta} C_{IJ} \hat{F}^I \wedge \hat{*} \hat{F}^J - \Omega_{\alpha\beta} b^\alpha C_{IJ} \hat{B}^\beta \wedge \hat{F}^I \wedge \hat{F}^J - \hat{V}^{(6)} \hat{*} \hat{1} \right],$$

6D potential from D7 fluxes

$$\hat{V}_{\text{flux}}^{(6)} = \frac{1}{32\Omega_{\alpha\beta} \hat{j}^{\alpha\beta} \hat{\mathcal{V}}^2} C^{-1ij} \theta_i \theta_j.$$

From 6D to 4D

$$ds^2 = W(r)^2 q_{\mu\nu} dx^\mu dx^\nu + a(r)^2 d\theta^2 + dr^2 = e^{2\Gamma(r)} q_{\mu\nu} dx^\mu dx^\nu + e^{2\Omega(r)} d\theta^2 + dr^2$$

Field equations

$$\ddot{\varphi} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\varphi} = \tilde{V} e^{\varphi-2\chi} - 2C \dot{\Delta}^2 e^{-\varphi+2\Delta-2\Omega}$$

$$\ddot{\chi} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\chi} = -\frac{k^2}{4} e^{-2\chi+2\Delta-2\Omega} - 4\tilde{V} e^{\varphi-2\chi}$$

$$\ddot{\Gamma} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\Gamma} = 3H^2 e^{-2\Gamma} - \frac{1}{2} \left(\ddot{\varphi} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\varphi} \right)$$

$$\ddot{\Omega} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\Omega} = -4C \dot{\Delta}^2 e^{-\varphi+2\Delta-2\Omega} - \frac{k^2}{8} e^{-2\chi+2\Delta-2\Omega} - \frac{1}{2} \left(\ddot{\varphi} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\varphi} \right)$$

$$\ddot{\Delta} + (\dot{\Omega} + 4\dot{\Gamma}) \dot{\Delta} = \dot{\Delta} \dot{\varphi} + 2\dot{\Omega} \dot{\Delta} - \dot{\Delta}^2 + \frac{k^2}{32C} e^{\varphi-2\chi}$$

Constraint

$$6H^2 e^{-2\Gamma} - 4\dot{\Omega} \dot{\Gamma} - 6\dot{\Gamma}^2 + \frac{1}{2} \dot{\varphi}^2 + \frac{1}{4} \dot{\chi}^2 + 2C e^{-\varphi-2\Omega+2\Delta} \dot{\Delta}^2 - \tilde{V} e^{\varphi-2\chi} - \frac{k^2}{16} e^{-2\chi-2\Omega+2\Delta} = 0$$

$\chi = \log \text{volume}$, $\Gamma = \log W$, $\Omega = \log a$, $\Delta = \log A$

$H^2 > 0$ de Sitter

Asymptotic solutions

Near brane solutions:

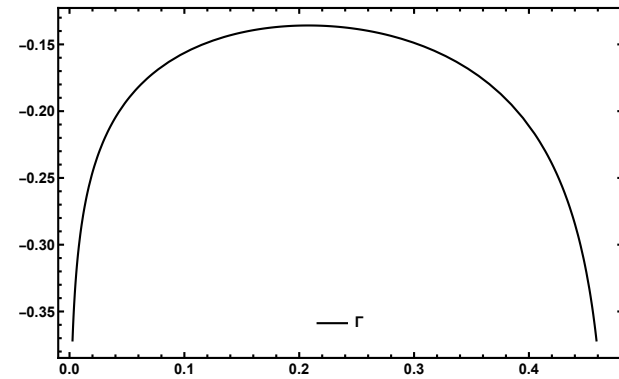
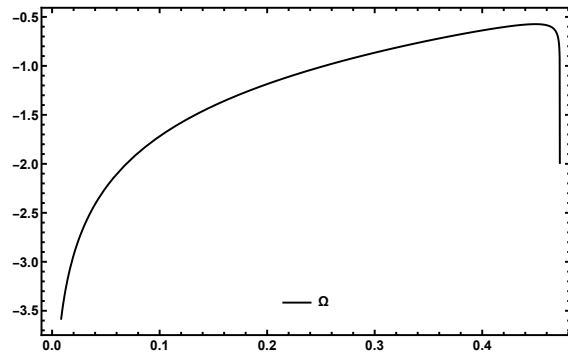
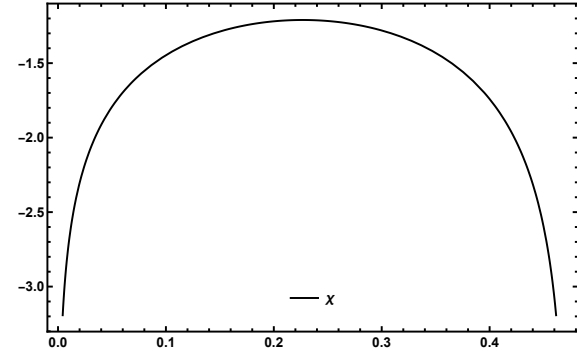
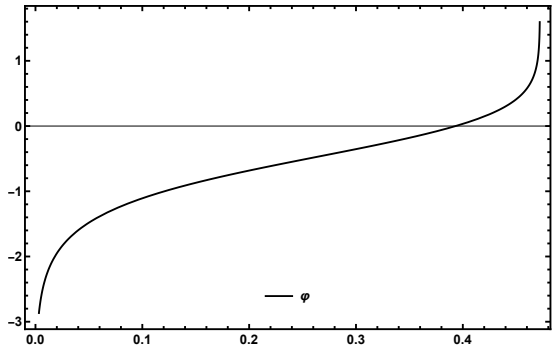
$$\begin{aligned}
 \varphi &= q \ln r + \ln u & \frac{1}{2}q^2 + \frac{1}{4}s^2 - 6w^2 - 4\alpha w + \frac{6}{x^2}H^2 + \frac{2Cz^2}{uy^2}\delta^2 - \frac{u}{v^2}\tilde{V} &= 0 \\
 \chi &= s \ln r + \ln v & (\alpha + 4w - 1)q - \frac{u}{v^2}\tilde{V} + \frac{2Cz^2}{uy^2}\delta^2 &= 0 \\
 \Gamma &= w \ln r + \ln x & (\alpha + 4w - 1)s + \frac{4u}{v^2}\tilde{V} &= 0 \\
 \Omega &= \alpha \ln r + \ln y & (\alpha + 4w - 1)w - \frac{3}{x^2}H^2 + (\alpha + 4w - 1)\frac{q}{2} &= 0 \\
 \Delta &= \delta \ln r + \ln z & (\alpha + 4w - 1)\delta - (q + 2\alpha - \delta)\delta &= 0 \\
 & & 2\delta - q - 2\alpha &= 0 \\
 & & q - 2s + 2 &= 0 \\
 & & w - 1 &= 0
 \end{aligned}$$

Kasner constraints
(BKL: Belinsky et al)

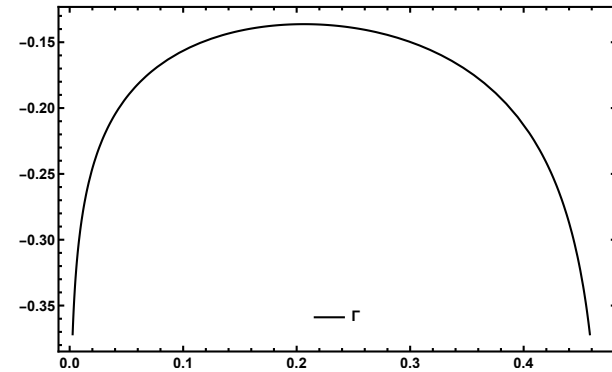
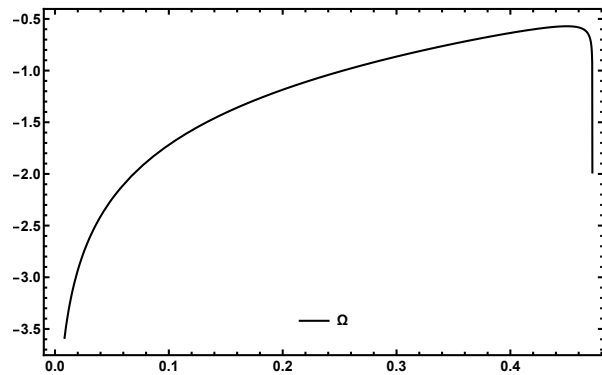
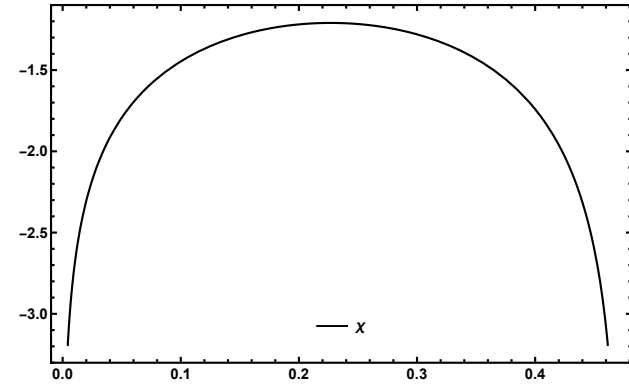
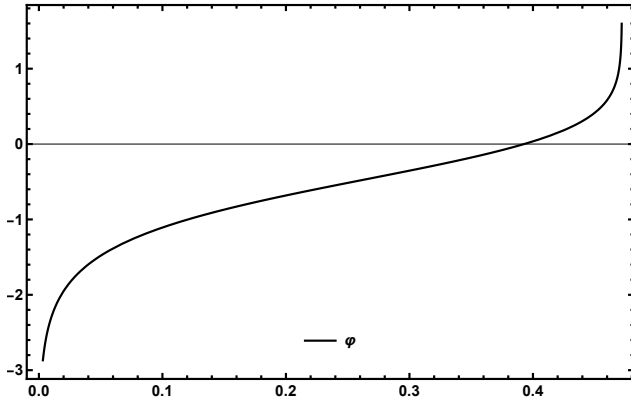
or

$$\begin{aligned}
 q = -\frac{2}{9}, \quad s = \frac{8}{9}, \quad \alpha = \frac{1}{9}, \quad w = 1, \quad \delta = 0, \quad \frac{u}{v^2}\tilde{V} = -\frac{56}{81}, \quad \frac{3H^2}{x^2} = \frac{224}{81} \\
 q = -\frac{2}{9}, \quad s = \frac{8}{9}, \quad \alpha = \frac{1}{9}, \quad w = \frac{1}{9}, \quad \delta = 0, \quad \frac{u}{v^2}\tilde{V} = \frac{8}{81}
 \end{aligned}$$

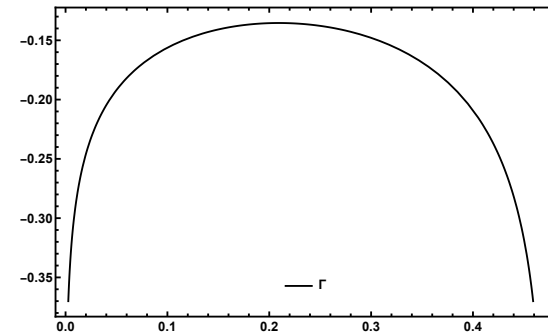
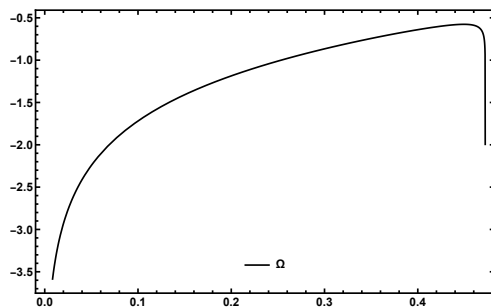
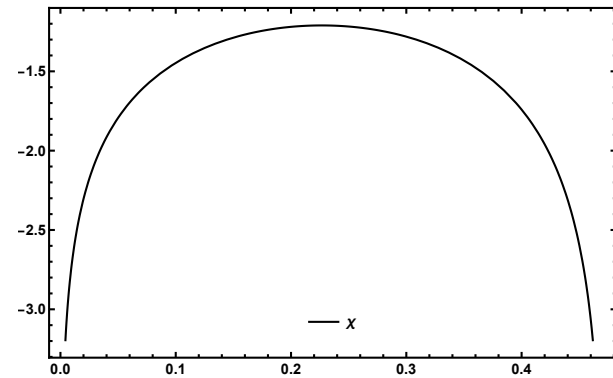
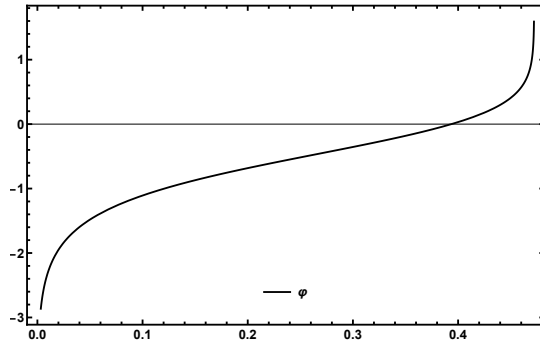
Numerical Solutions $H^2=0$



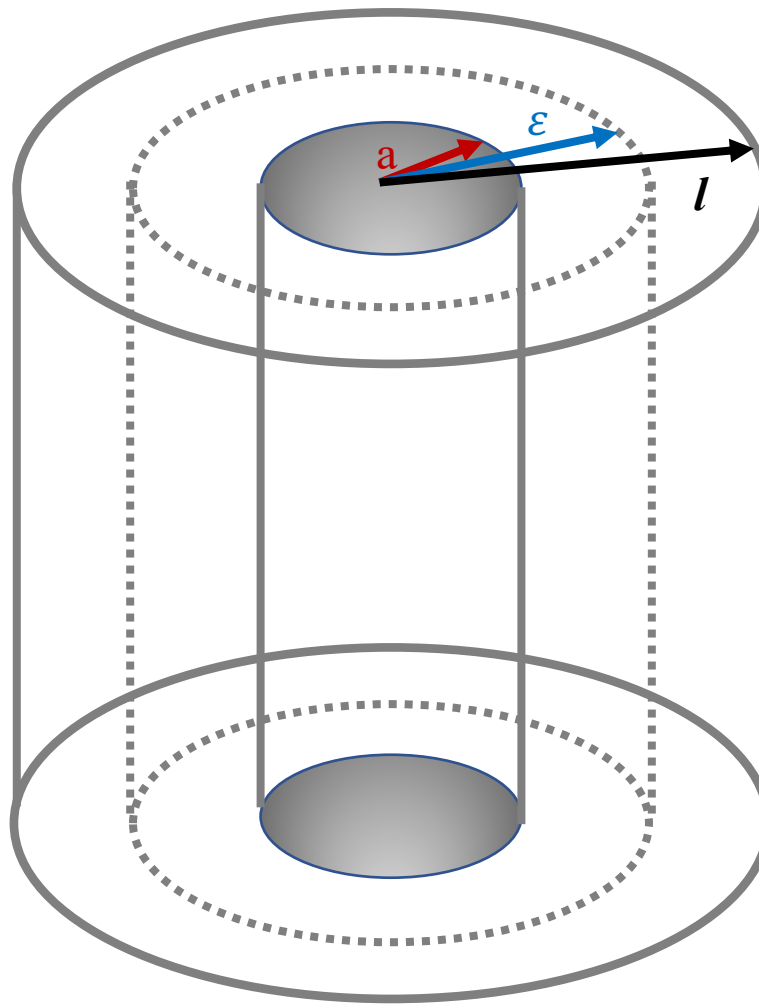
Numerical AdS Solutions $H^2 \leq 0$



Numerical dS Solutions $H^2 \geq 0$



Singularities?



PPEFT,...Stay tuned...

Personal Memories

In Austin 1984?



Geneva 1990



ICTP Prize 1998



Trieste 2019





Paris 2023





P
A
R
I
S

2
0
2
3





Thank you Anamaría and Congratulations!!!