The Power of Modular Symmetry

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Outline

- Joint work with Anamaria
- 1988 paper on "Concept of Naturalness in string theory"
- Traditional (discrete) flavor symmetries
- Modular (discrete) flavor symmetries
- Eclectic Flavor Group
- Local Flavor Unification
- Specific properties of Modular Symmetry
- UV-IR relation:
  the hidden power of modular flavor symmetry

Influence of winding modes on low-energy effective theory

(Work with Baur, Knapp-Perez, Liu, Ratz, Ramos-Sanchez, Trautner, Vaudrevange, 2019-23)

Work with Anamaria

We met at CERN after she had come to Annecy. Joint work:

- Degenerate orbifolds,
  A. Font, L.E. Ibanez, H.P. Nilles and F. Quevedo,
  Nucl.Phys.B 307 (1988) 109-129,
- Yukawa couplings in degenerate orbifolds: towards a realistic $SU(3) \times SU(2) \times U(1)$ superstring
  A. Font, L.E. Ibanez, H.P. Nilles and F. Quevedo,
  Phys.Lett.B 210 (1988) 101,
- On the Concept of Naturalness in String Theory,
  A. Font, L.E. Ibanez, H.P. Nilles and F. Quevedo,
  Physics. Lett. B213(1988)274

Earliest work on Yukawa couplings, scalar potential, flat directions, blow-up modes in orbifold compactifications

Stringy Miracles

Today I shall make a connection to the third paper

- On the Concept of Naturalness in String Theory, A. Font, L.E. Ibanez, H.P. Nilles and F. Quevedo, Physics. Lett. B210(1988)101

It elaborated on a conformal field theory selection rule in string theory (called "Rule 4")

- that could not be understood through the symmetries of the low-energy effective action of massless modes
- in particular the vanishing of couplings of fields localized at the same fixed point

This remained a puzzle till recently and found its explanation in modular symmetry and a UV-IR relation

Various Types of Symmetries

It is all a question of properties of (flavor) symmetries.

Typically we are dealing with traditional flavor symmetries:

- they are linearly realised
- need flavon fields for symmetry breakdown

Another type of symmetries are modular symmetries

- motivated by string theory dualities (Lauer, Mas, Nilles, 1989)
- applied recently to the question of flavor (Feruglio, 2017)
- modular symmetries are nonlinearly realised!
- Yukawa couplings are modular forms

Combine with traditional flavor symmetries to the so-called "eclectic flavor group" (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

String Geometry of extra dimensions

Strings are extended objects and this reflects itself in special aspects of geometry (including winding modes). We have:

- normal symmetries of extra dimensions as observed in quantum field theory – traditional flavor symmetries.
- String duality transformations lead to modular or symplectic flavor symmetries that cannot be realised linearly in low-energy effective theory.
- They still give restrictions on the low-energy action
- provide constraints from the UV-sector of the theory

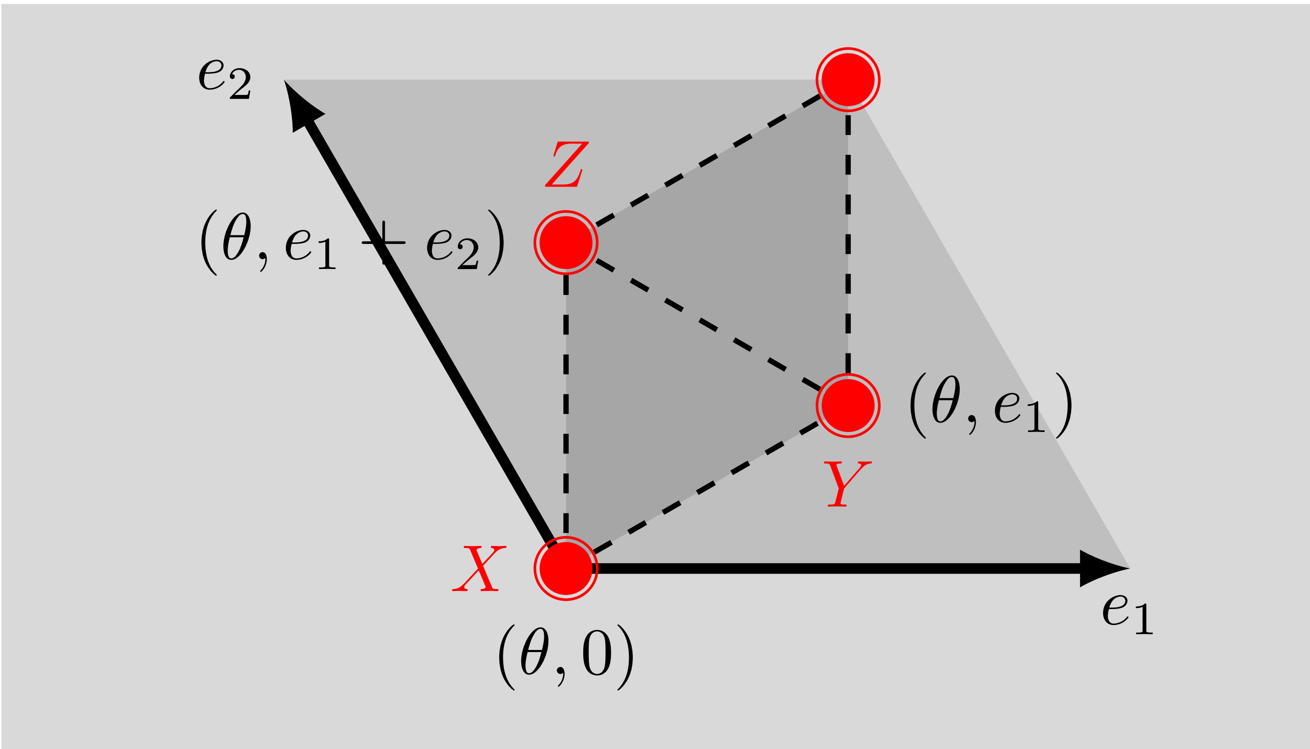
In the following we illustrate with a simple example

- twisted 2D-torus with localized matter fields

Traditional Flavor Symmetries

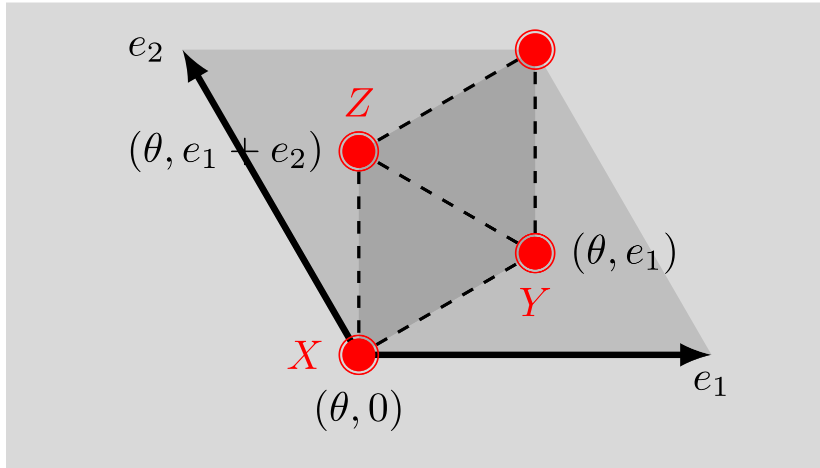
In string theory discrete symmetries can arise form geometry and string selection rules.

As an example we consider the orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from string theory selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – various singlet, doublet and triplet reps.
- e.g. flavor symmetry for three families of quarks (as triplets of $\Delta(54)$)

String dualities

Consider a particle on a circle with radius R

- discrete spectrum of momentum modes (KK-modes)
- density of spectrum is governed by m/R (m integer)
- heavy modes decouple for $R \to 0$

Now consider a string

- KK modes as before m/R
- Strings can wind around circle
- spectrum of winding modes governed by nR
- massless modes for $R \to 0$

T-duality

This interplay of momentum and winding modes is the origin of T-duality where one simultaneously interchanges

- momentum $\rightarrow$ winding
- $$R \rightarrow 1/R$$

This transformation maps a theory to its T-dual theory: it is a map not a symmetry

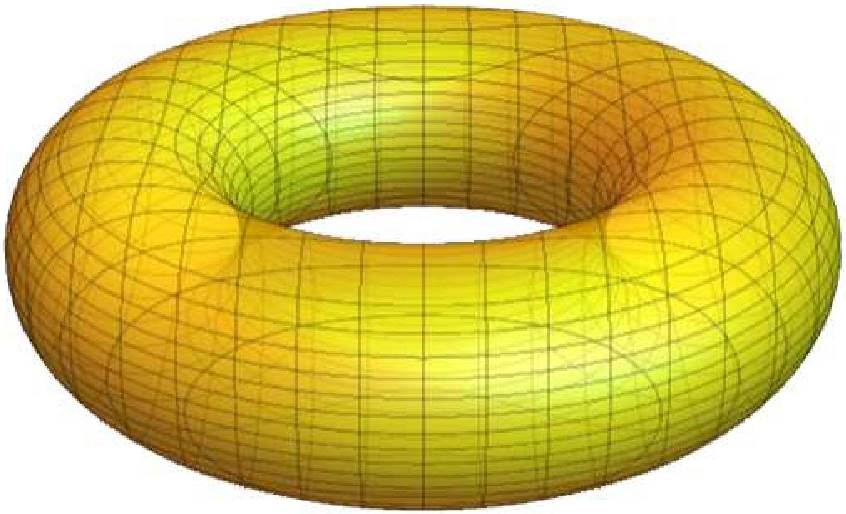
- self-dual point is $R^2 = 1 = \alpha' = 1/M_{\mathrm{string}}^2$

If the string scale M_{string} is large, the low energy effective theory describes the momentum states and the winding states are heavy.

Does T-duality restrict the low-energy effective theory?

Torus compactification

Strings can wind around several cycles



Complex modulus M (in complex upper half plane)

Modular Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D=2$ these transformations are connected to the group $SL(2,Z)$ acting on Kähler and complex structure moduli.

The group $SL(2,Z)$ is generated by two elements

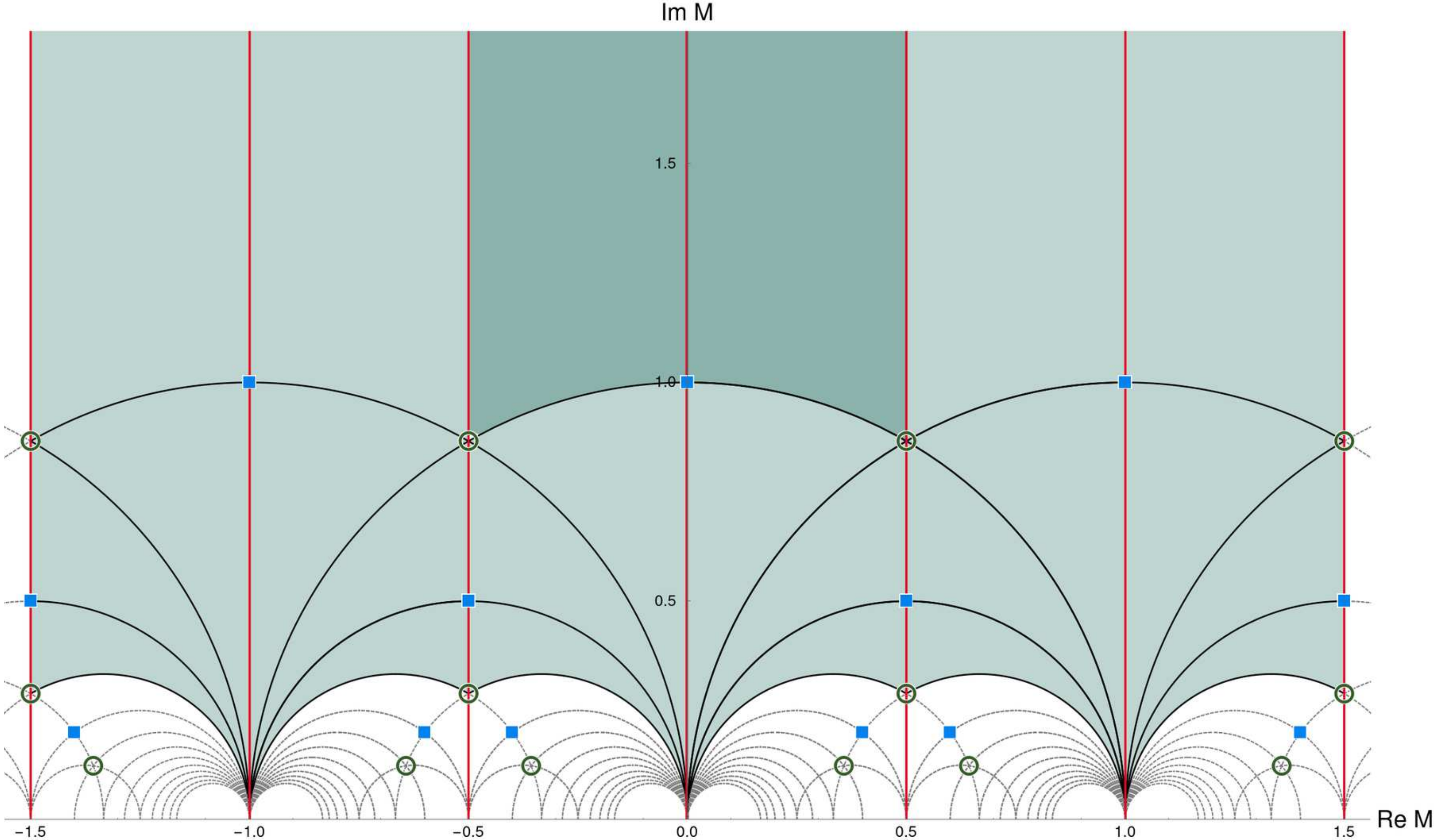
$$\mathrm{S}\ ,\ \mathrm{T}:\quad \text{with}\ \ \mathrm{S}^4=(\mathrm{ST})^3=1\ \ \text{and}\ \ \mathrm{S}^2\mathrm{T}=\mathrm{TS}^2.$$

A modulus M transforms as

$$\mathrm{S}:\quad M\to-\frac{1}{M}\quad \text{and}\quad \mathrm{T}:M\to M+1$$

Further transformations might include $M\to-\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

Fundamental Domain



Three fixed points at M$= i$, $\omega = \exp(2\pi i/3)$ and $i\infty$

Modular Forms

String dualities give important constraints on the action of the theory via the modular group $SL(2,Z)$:

$$\gamma: \quad M \to \frac{aM+b}{cM+d}$$

with $ad-bc=1$ and integer a,b,c,d.

Matter fields transform as (i) representations $\rho(\gamma)$ and (ii) modular functions of weight k

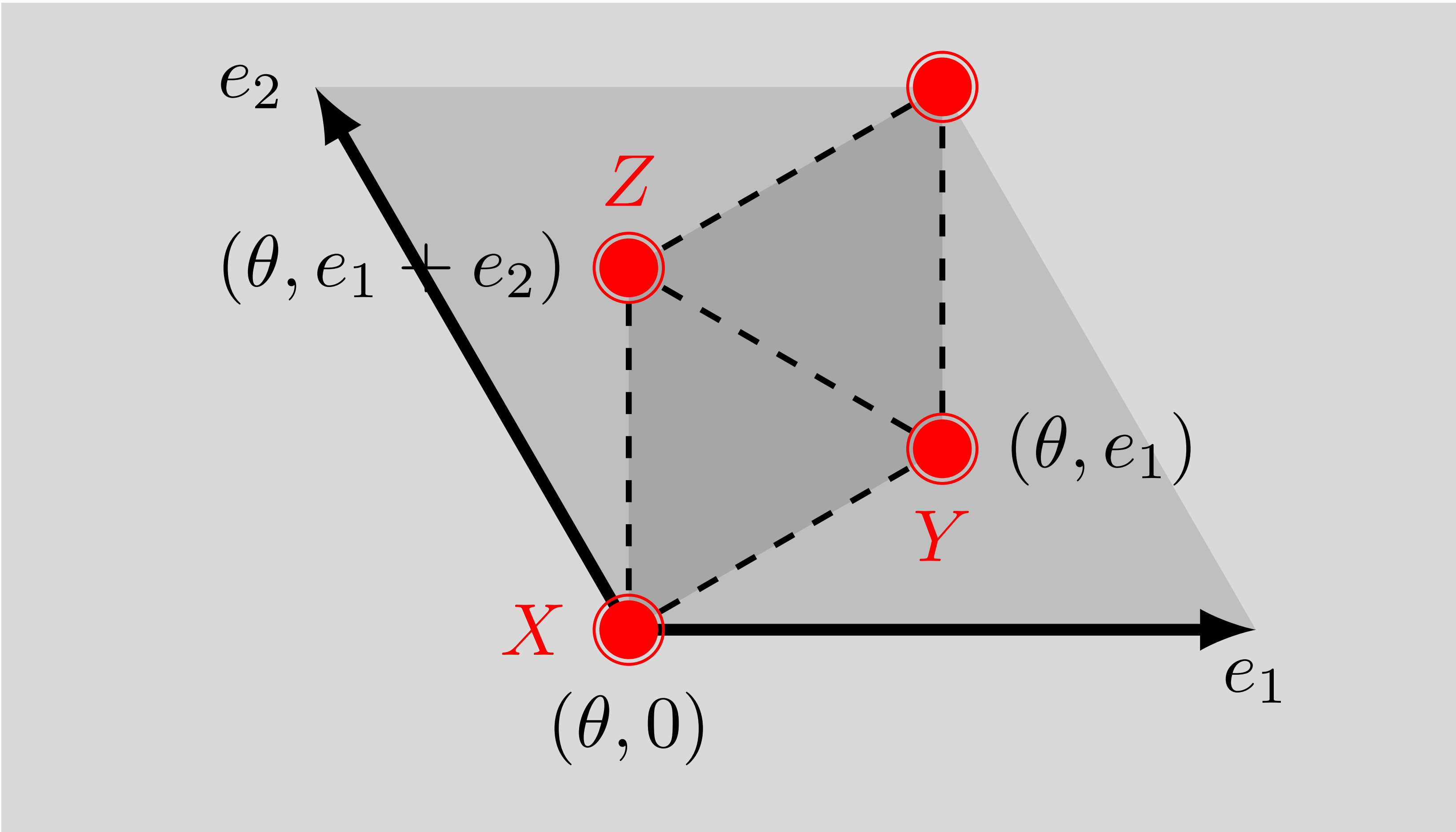
$$\gamma: \quad \phi \to (cM+d)^k \rho(\gamma)\phi \ .$$

Yukawa-couplings transform as modular forms.

$G = K + \log|W|^2$ must be invariant under T-duality

(Ferrara, Lüst, Theisen, 1989)

Towards Modular Flavor Symmetry



Modular flavor symmetry

On the T_2/Z_3 orbifold some of the moduli are frozen,

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2,Z)$

- $\Gamma(3) = SL(2,3Z)$ as a mod(3) subgroup of $SL(2,Z)$
- discrete modular flavor group $\Gamma_3' = SL(2,Z)/\Gamma(3)$
- the discrete modular group is $\Gamma_3' = T' \sim SL(2,3)$ (which acts nontrivially on twisted fields); the double cover of $\Gamma_3 \sim A_4$ (which acts only on the modulus).
- the CP transformation $M \to -\overline{M}$ completes the picture.

Full discrete modular group is $GL(2,3)$.

Eclectic Flavor Groups

We have thus two types of flavor groups

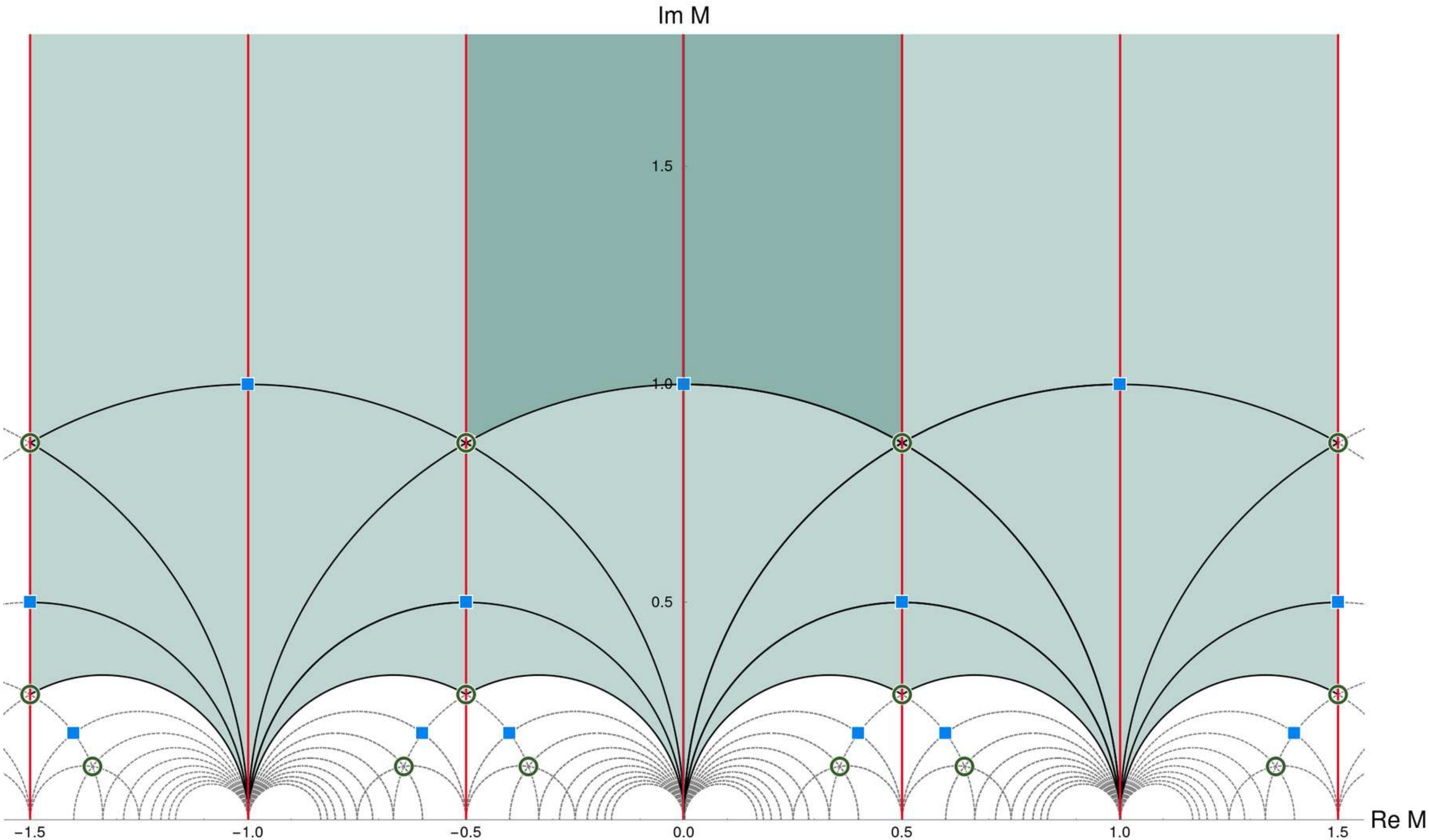
- the traditional flavor group that is universal in moduli space (here $\Delta(54)$)
- the modular flavor group that transforms the moduli nontrivially (here T')

The eclectic flavor group is defined as the multiplicative closure of these groups. Here we obtain for T_2/Z_3

- $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2,3)$
- $SG[1296, 2891]$ from $\Delta(54)$ and $GL(2,3)$ including CP

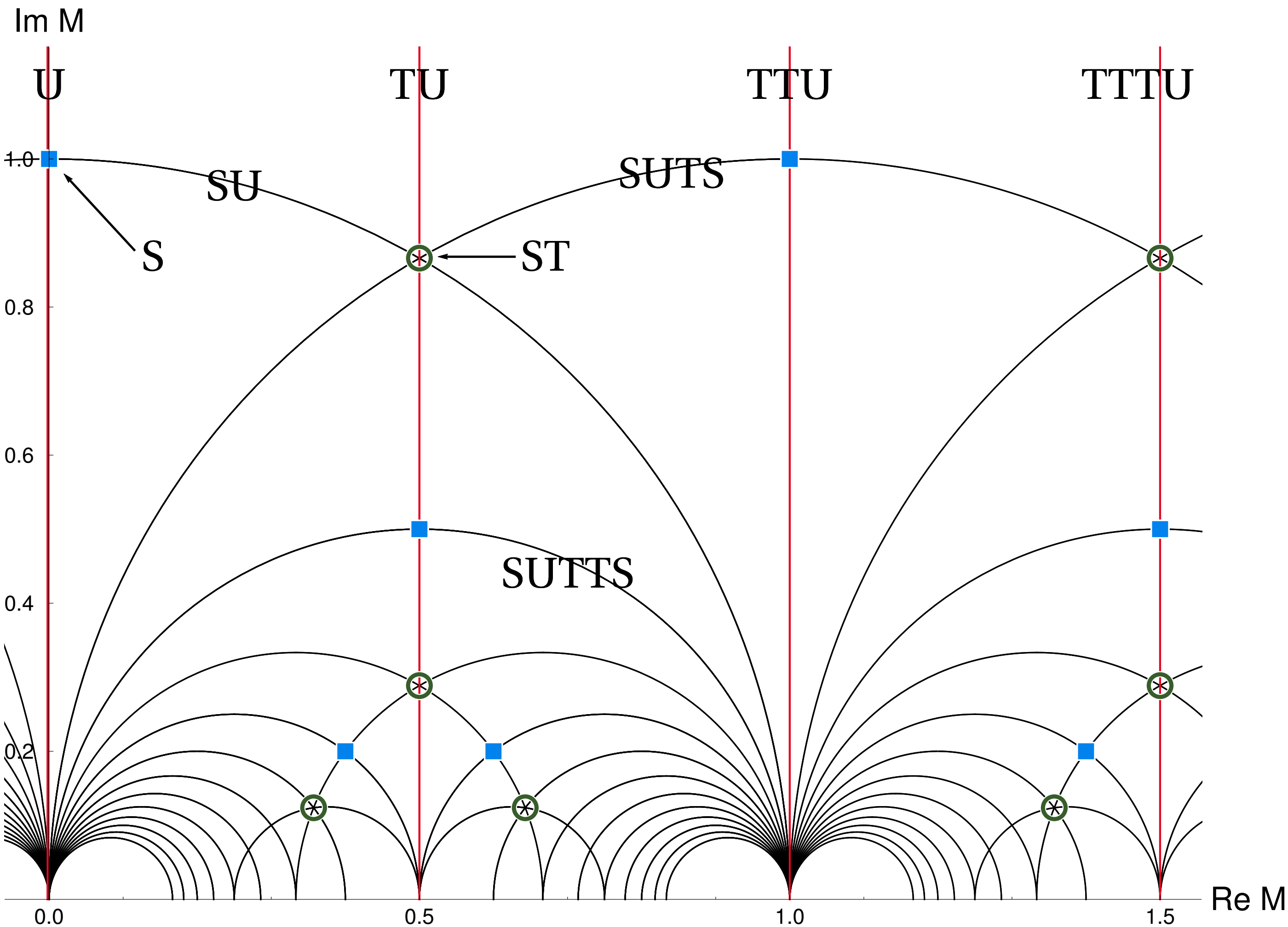
The eclectic group is the largest possible flavor group for the given system, but it is not necessarily linearly realized.

Local Flavor Unification



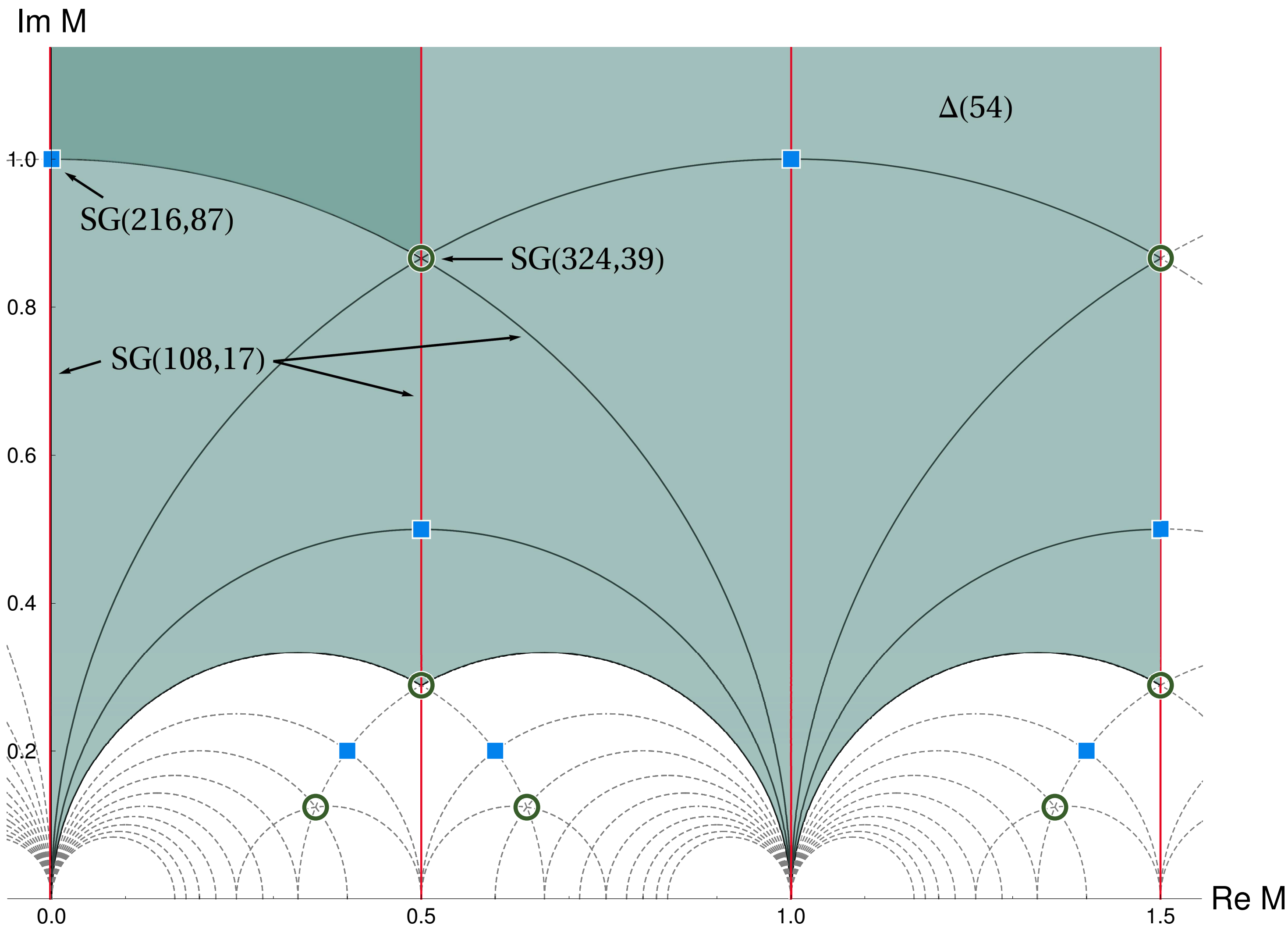
Moduli space of $\Gamma(3)$

Fixed lines and points



$$\mathrm{S} : M \to -\frac{1}{M}, \quad \mathrm{T} : M \to M + 1 \quad \text{and} \quad \mathrm{U} : M \to -\overline{M}$$

Moduli space of flavour groups



"Local Flavor Unification"

Comparison

Traditional and modular flavor symmetries are fundamentally different

- linear versus non-linear realization
- traditional is subgroup of $SU(3)_{\text{flavor}}$
- modular symmetry is not a subgroup of $SU(3)_{\text{flavor}}$
- Yukawa couplings are modular forms (that depend nontrivially on the modulus)
- local enhancement at specific locations

This peculiar behaviour of modular flavor symmetry allows a description of the influence of winding modes on the low energy effective theory (thus gives a UV-IR connection)

Back to our 1988 Paper

String dualities connect winding to momentum modes. Winding modes are heavy. Could there be nonetheless an effect at low energies?

- "Stringy Miracles" and naturalness in string theory – need introduction of "Rule 4" (Font, Ibanez, Nilles, Quevedo, 1988)
- selection rules of CFT lead to vanishing of certain couplings that could not be understood through the symmetries of the low energy effective theory
- extended later including "Rule 5" and "Rule 6"

  (Kobayashi, Parameswaran, Ramos-Sanchez, Zavala, 2011)
- these "Stringy Miracles" remained a puzzle till recently

Calculations with eclectic flavor symmetries explain "Rule 4"

(Nilles, Ramos-Sanchez, Vaudrevange, 2020)

"Stringy Miracles"

Yukawa couplings of twisted fields are modular forms that depend nontrivially on the modulus M.

Consider, for example, the twisted fields of the T_2/Z_3 orbifold, located at the fixed points X, Y and Z.

Ususally the allowed couplings are:

$$f(M)(X^3 + Y^3 + Z^3) + g(M)XYZ$$

with both non-vanishing modular forms $f(M)$ and $g(M)$.

Stringy miracles are cases where $f(M)$ is absent.

Can we identify the reason for this peculiar situation?

Modular Flavor

What is the reason for this?

- It is the presence of the discrete modular flavor symmetry and the modular weights.
- modular group $SL(2, Z)$ with $S^4 = 1$ and $S^2 \neq 1$
- $PSL(2, Z)$ with $S^2 = 1$ acts on moduli
- additional Z_2 corresponds to the double cover of finite modular group (originates from CFT selection rules)
- it is also part of the traditional flavor group. It looks "traditional" but it is intrinsically "modular"
- Modular weights of matter fields and Yukawa couplings play a crucial role

(Work in progress)

Example T_2/Z_3

Superpotential is restricted by the eclectic flavor group

- $SG[648, 533] = \Omega(1)$ from $\Delta(54)$ and T'
- a Z_2 symmetry is common to $\Delta(54)$ and T'
- responsible for double cover T' of A_4
- extends $\Delta(27)$ to $\Delta(54)$ (connected to $S^2 = -1$)
- $\Delta(54)$ contains nontrivial singlet $1'$ as well as two 3-dimensional representations 3_1 and 3_2
- vev of $1'$ breaks $\Delta(54)$ to $\Delta(27)$ with one triplet rep.
- twisted oscillator modes transform as $1'$ rep. of $\Delta(54)$

This Z_2 as part of $\Delta(54)$ together with the action of T' completes the explanation of the "Stringy Miracles".

Summary

String theory provides the necessary ingredients for flavor:

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- the concept of local flavour unification

The eclectic flavor group provides the basis:

- it includes a non-universality of flavor symmetry in moduli space
- leads to a non-trivial UV-IR relation:
  the "hidden power" of modular flavor symmetry