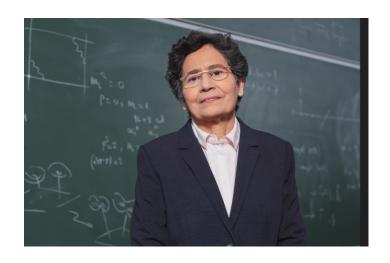


# Dualities, Correspondences and Emergent String Transitions

Dieter Lüst, LMU and MPI München



AnLy Meeting in honour of Anamaria Font, Annecy, 10th. April 2024

### L'Oréal-UNESCO International Award (2023)



## Laureate for Latin America and the Caribbean:

PROFESSOR ANAMARÍA FONT:

Professor of Physics, Central University of Venezuela

Prof. Anamaría Font is recognized for her work in theoretical particle physics, with a particular focus on developing the theory of superstrings. This describes, in a unified and consistent way, the elementary particles of nature. Her research has enabled further understanding of the theory's consequences for the structure of matter and quantum gravity, which are also relevant to the description of black holes and the first moments after the big bang.

Starting about 35 years ago we enjoyed a very pleasant and fruitful collaboration with Anamaria, in particular on various aspect of duality symmetries in string theory.

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The term duality refers to a situation where two seemingly different physical systems turn out to be equivalent in a nontrivial way. If two theories are related by a duality symmetry, it means that one theory can be transformed in some way so that it ends up looking just like the other theory.

### Correspondence principle in quantum mechanics: Wave - particle duality

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#### Maxwell electromagnetism:

Field equation - Bianchi identity duality

$$F_{\mu\nu} = 0 \longleftrightarrow {}^*F_{\mu\nu} = 0$$

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[P. Dirac (1931)]

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N=4, N=2 supersymmetric field theories:

#### String theory: T-duality symmetry

T - duality related different space-time geometries.

Circle compactification: 
$$R \longleftrightarrow \frac{\alpha'}{R} = 0$$

[K. Kikkawa, M. Yamasaki (1984); B. Stathiapalan (1987)

Momentum - winding exchange:  $p = \frac{n}{R} \longleftrightarrow \tilde{p} = \frac{mR}{\alpha'}$ 

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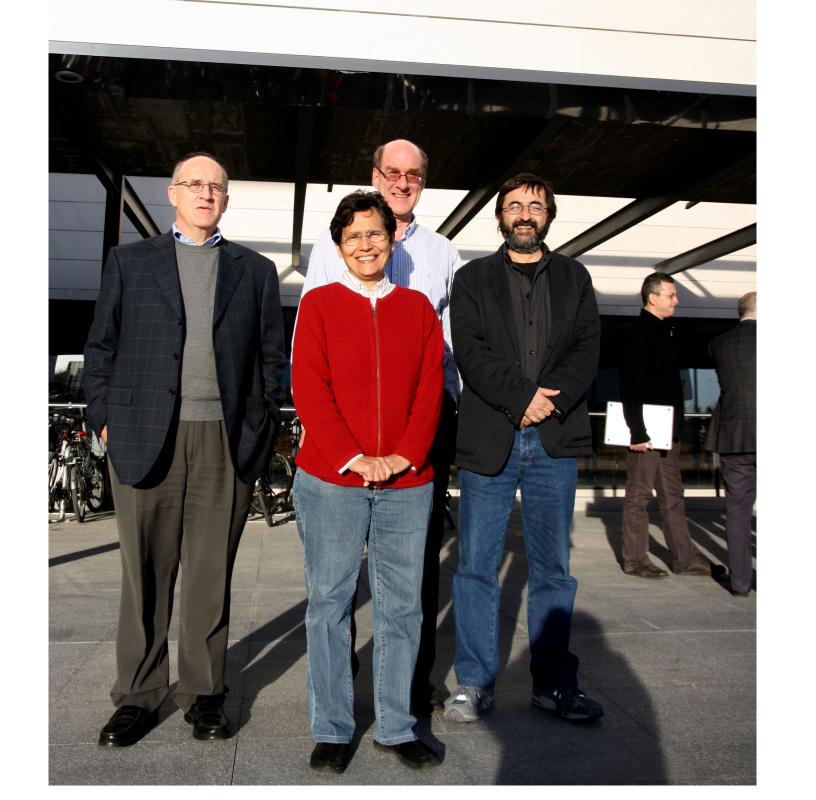
Momentum - winding exchange:  $p = \frac{n}{R} \longleftrightarrow \tilde{p} = \frac{mR}{\alpha'}$ 

T - duality in effective SUGRA field theory:

Invariance of action under discrete symmetry

[S. Ferrara, D.L., A. Shapere, S. Theisen (1989)]

$$T 
ightarrow rac{aT - ib}{icT + d} \quad (T = R^2 + iB) \qquad \Rightarrow \qquad W(T) \sim rac{1}{\eta(T)^6}$$



Volume 245, number 3, 4 PHYSICS LETTERS B 16 August 1990

#### Supersymmetry breaking from duality invariant gaugino condensation

A. Font a, L.E. Ibáñez b, D. Lüst b and F. Quevedo c

- <sup>a</sup> Departamento de Fisica, Universidad Central de Venezuela, Aptdo. 20513, Caracas 1020-A, Venezuela
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- <sup>c</sup> Theoretical Division LANL, Los Alamos, NM 87545, USA

Received 3 May 1990

It is known that the formation of gaugino condensates can be a source of supersymmetry breaking in string theory. We study the constraints imposed by target space modular invariance on the formation of such condensates. We find that the dependence of the vacuum energy on the moduli of the internal variety is such that the theory is forced to be compactified. The radius of compactification is of the order of the string scale and in the process target space duality is spontaneously broken.

One of the major open problems in fourdimensional string theories is the breaking of spacetime (N = 1) supersymmetry. Because of phenomenological reasons it is desirable that space-time supersymmetry is broken spontaneously below the Planck scale, and the most promising scenario realizing this requirement is the mechanism of gaugino condensation in the so-called hidden sector [1-7]. Gaugino condensation cannot, up to now, be directly analyzed in string theory; however, there are strong arguments that it actually occurs at the level of the low-energy effective field theory. More recently it was shown [8-10] that the effective supergravity action following from string compactification on orbifolds or even Calabi-Yau manifolds is severely constrained by an underlying string symmetry, the so-called target space modular invariance. The target space modular group  $PSL(2, \mathbb{Z})$  acts on the complex scalar T as

$$T \rightarrow \frac{aT - ib}{icT + d}$$
,  $a, b, c, d \in \mathbb{Z}$ ,  $ad - bc = 1$ , (1)

where  $\langle T \rangle$  is the background modulus associated to the overall scale of the internal six-dimensional space on which the string is compactified. Specifically,  $T = R^2 + iB$  with R being the "radius" of the internal space and B an internal axion. The target space modular transformations contain the well-known duality transformation  $R \rightarrow 1/R$  [11] as well as dis-

crete shifts of the axionic background B, and the T moduli space has to be restricted to the fundamental region  $SU(1, 1)/[U(1) \times PSL(2, \mathbb{Z})]$ .

Although duality respectively target space modular invariance is only shown [12] to be an unbroken symmetry at any order of string perturbation theory, one also expects that non-perturbative string effects respect these discrete symmetries. Adopting this point of view, the effective action describing the spontaneous supersymmetry breaking via non-perturbative effects, like gaugino condensates must be also invariant under the modular transformation on T. In fact it was shown in ref. [8] that the non-perturbative, purely T-dependent effective superpotential must be a modular form like

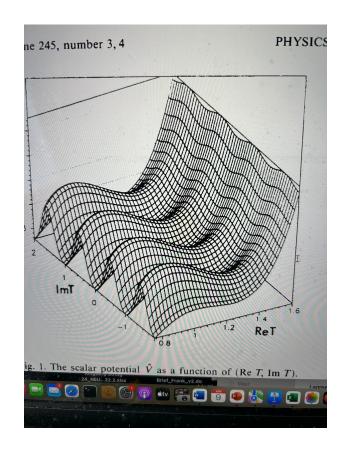
$$W(T) \sim \eta(T)^{-6},\tag{2}$$

where  $\eta(T) = q^{1/24} \prod_n (1-q^n)$  is the well-known Dedekind function,  $q = \exp(-2\pi T)$ , and the resulting gravitino mass is of the form  $m_{3/2}^2 \sim 1/(T+T^*)^3 |\eta(T)|^{12}$ . We will show that gaugino condensation provides a natural dynamical realization for exactly this kind of effective actions. Using the relation  $W \sim \langle \lambda \lambda \rangle \sim \exp[(3/2b_0)f(T)]$ , this could be seen as if the gauge kinetic function of the N=1 supergravity action is of the form  $f(T)=-b_0\log[\eta(T)^4]$ . Interestingly enough, this kind of expression for the gauge kinetic function was recently derived from a direct one-loop string calculation [13].

Modular superpotential generated by gaugino condensation:

$$W(T) \sim \exp[\frac{3}{2b}f(T)] = \frac{1}{\eta(T)^6}$$

Potential with Kähler moduli stabilisation:



#### S-duality proposal (1990)

Volume 249, number 1 PHYSICS LETTERS B 11 October 1990

#### Strong-weak coupling duality and non-perturbative effects in string theory

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- a Departamento de Fisica, Universita Central de Venezuela, Aptdo 20513, Caracas 1020-A, Venezuela
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Received 13 July 1990

We conjecture the existence of a new discrete symmetry of the modular type relating weak and strong coupling in string theory. The existence of this symmetry would strongly constrain the non-perturbative behaviour in string partition functions and introduces the notion of a maximal (minimal) coupling constant. An effective lagrangian analysis suggests that the dilaton vacuum expectation value is dynamically fixed to be of order one. In supersymmetric heterotic strings, supersymmetry (as well as this modular symmetry itself) is generically spontaneously broken

Modular invariance appears in a variety of physical problems [1]. These symmetries involve an invariance under the inversion of coupling constants along with the discrete translations of a "theta term". The first example of this type of symmetry in field theory was discovered by Cardy [2] who showed that the phase structure of the abelian Higgs model on the lattice exhibits such a type of invariance under inversion of couplings and shift of the  $\theta$ -parameter. These transformations generate an infinite discrete group  $SL(2, \mathbb{Z})$ . In the context of string theory such a sort of symmetry seems also ubiquous. The one loop partition functions in terms of the world sheet modular parameter  $\tau$  must be explicitly modular invariant.

to the existence of the  $B_{mn}$  antisymmetric tensor which acts as a  $\theta$ -parameter. In more realistic six-dimensional compactifications (like e.g. orbifolds) the same structure (conveniently generalized) is also found. This target-space modular invariance strongly constrains the form of the low-energy effective action as a function of the compactification moduli [5]. It can also give interesting information about the possible form of non-perturbative string corrections (like e.g. supersymmetry breaking [6,7]) if duality were an exact symmetry of string theory (possibly broken spontaneously but not explicitly) [6].

In the present letter we conjecture the existence of a further modular invariance symmetry in string theWe conjectured the existence of a further modular invariance symmetry under which the string dilaton S (whose vev yields the string coupling constant) gets inverted:

$$S o rac{aS - ib}{icS + d}$$

We conjectured that the Montonen-Olive duality of the heterotic string will lead to this modular symmetry.

We conjectured that S-duality in the 4D heterotic string is based on the string-five brane duality, with the 5-branes wrapped around 4-cycles and look like strings.

[M. Duff (1988); A. Strominger (1990)]

We investigated the consequences of S-field modular invariance for the effective string action in terms of modular superpotentials:

$$W(S) \sim \frac{1}{\eta(S)^2}$$



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#### Heterotic T-fects, 6D SCFTs, and F-theory

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 $\label{eq:energy} E\text{-}\textit{mail:} \ \ \text{afont@fisica.ciens.ucv.ve, inaki@mpp.mpg.de,} \\ \ \ \text{dieter.luest@lmu.de, stefano.massai@lmu.de,} \\ \ \ \text{christoph.mayrhofer@lmu.de}$ 

ABSTRACT: We study the (1,0) six-dimensional SCFTs living on defects of non-geometric heterotic backgrounds (T-fects) preserving a  $E_7 \times E_8$  subgroup of  $E_8 \times E_8$ . These configurations can be dualized explicitly to F-theory on elliptic K3-fibered non-compact Calabi-Yau threefolds. We find that the majority of the resulting dual threefolds contain non-resolvable singularities. In those cases in which we can resolve the singularities we explicitly determine the SCFTs living on the defect. We find a form of duality in which distinct defects are described by the same IR fixed point. For instance, we find that a subclass of non-geometric defects are described by the SCFT arising from small heterotic instantons on ADE singularities.

Keywords: F-Theory, Superstrings and Heterotic Strings, Field Theories in Higher Dimensions, Supersymmetric gauge theory

ARXIV EPRINT: 1603.09361

#### Correspondences and phase transitions

In physical systems we often encounter correspondences and transitions between different phases of matter.

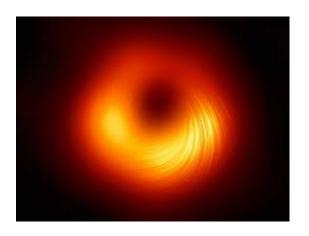
Example: Water - ice - steam phase transition

Phase transitions can be described by considering the entropy or free energy of the system as function of temperature

We will now consider different phases of stringy matter.

### Correspondences and phase transitions between particles and black holes in quantum gravity

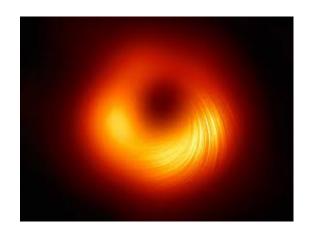
#### BH formed by collapse of particles:



[Picture from Event Horizon Telescope]

### Correspondences and phase transitions between particles and black holes in quantum gravity

#### BH formed by collapse of particles:



[Picture from Event Horizon Telescope]

Here we want to consider the correspondence between certain particles and certain black holes.

Phase transition between species and black holes in the moduli space of quantum gravity.

Horowitz/Polchinski: string - BH correspondence

[G. Horowitz, J. Polchinski (1996)]

#### Particle Species:

(Light) Particles in EFT of quantum gravity: interact gravitationally, otherwise weakly interacting

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Important for the fundamental understanding of quantum gravity

#### Species scale:

EFT cut-off of quantum gravity

#### Swampland distance conjecture:

Light species appear at large distances in moduli space

#### Relation to black holes:

Thermodynamic nature of species

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Relation to black holes:

Thermodynamic nature of species

Phenomenological importance

EW Hierarchy problem

 $10^{32}$  copies of SM

[Arkani-Hamed, Dimopoulos, Dvali (1998); Dvali, Redi (2009); Ettengruber, Zander, Eller (2024)]

Cosmological hierarchy problem - dark dimension

 $\Lambda_{cc} \leftrightarrow$  light species

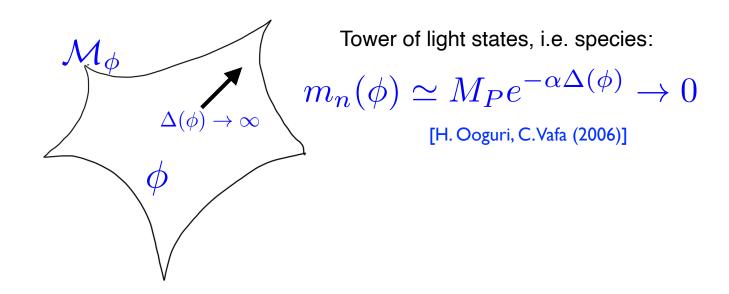
[Palti, Vafa, D.L. (2019); Montero, Vafa, Valenzuela (2022)]

Species as dark matter candidates

[Gonzalo, Montero, Obied, Vafa 2022)]

#### Infinte distance / emergent string conjecture:

Quantum Gravity moduli space:



The light tower of states, i.e. species, at large distances are given by either light string excitations or light KK modes.

[S. Lee, W. Lerche, T. Weigand (2019)]

#### Black Hole - Species Correspondence

Particles

- Geometry

Species

- Minimal BH as species bound state

There is a transition between the species and the BH description:

Large coupling

Small coupling





Collective BH description

Individual description as qm particles

#### Species Scale:

$$\Lambda_{sp} \simeq rac{M_P}{(N_{sp})^{rac{1}{d-2}}}$$
 [G. Dvali (2007)]

 $N_{sp}$  : Number of particles below  $\ \Lambda_{sp}$  .

 $\left\{\begin{array}{c} M_P \\ \Lambda_{sp} \\ \vdots \\ N_{sp} \end{array}\right\}$ 

.... regardless of their masses

.... it depends on moduli fields: 
$$N_{sp}=N_{sp}(\phi)$$

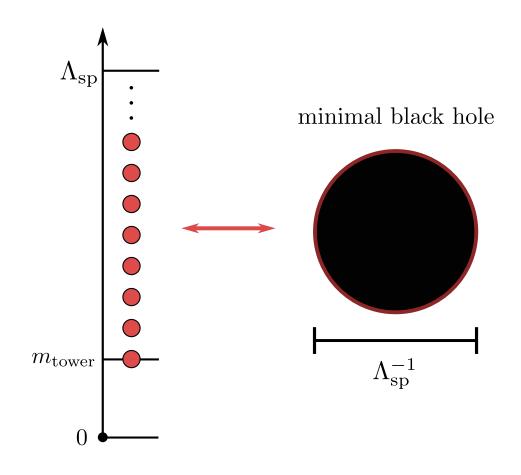
#### Alternative definition of the species scale:

#### Schwarzschild radius of a minimal black hole.

$$L_{\rm sp} = \Lambda_{\rm sp}^{-1} \simeq R_{BH,min}$$

[G. Dvali, M. Redi (2009)]

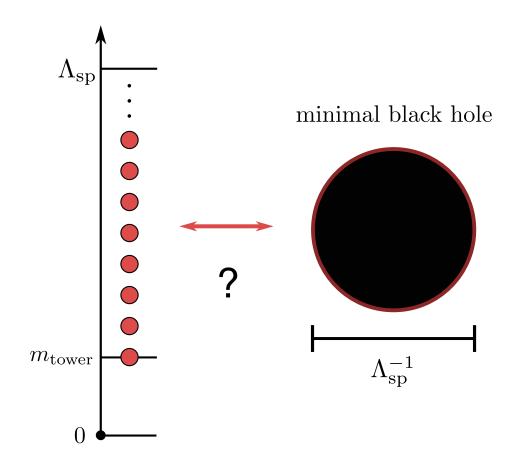
[N. Cribiori, D.L., G. Staudt (2022)]



#### Alternative definition of the species scale:

Schwarzschild radius of a minimal black hole.

$$L_{
m Sp} = \Lambda_{
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 [G. Dvali, M. Redi (2009)] [N. Cribiori, D.L., G. Staudt (2022)]



But what towers can build a minimal black hole?

#### Species Thermodynamics:

[N. Cribiori, D.L., C. Montella (2013); I. Basile, D.L. C. Montella (2013); I. Basile, N. Cribiori, D.L. C. Montella (2014); A. Herraez, D.L., J. Masias, M. Scalisi, to appear]

#### Definition of species entropy:

The species entropy is defined as the entropy of the associated minimal BH:

$$\mathcal{S}_{sp} := \mathcal{S}_{BH,min} \simeq (L_{sp})^{d-2} M_P^{d-2}$$

#### Species temperature

The species temperature is defined as the temperature of the associated minimal BH:

$$T_{sp} := T_{BH,min} \simeq (L_{sp})^{-1} = \Lambda_{sp}$$

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We will require that species entropy and species temperature follow the known laws of black hole thermodynamics.

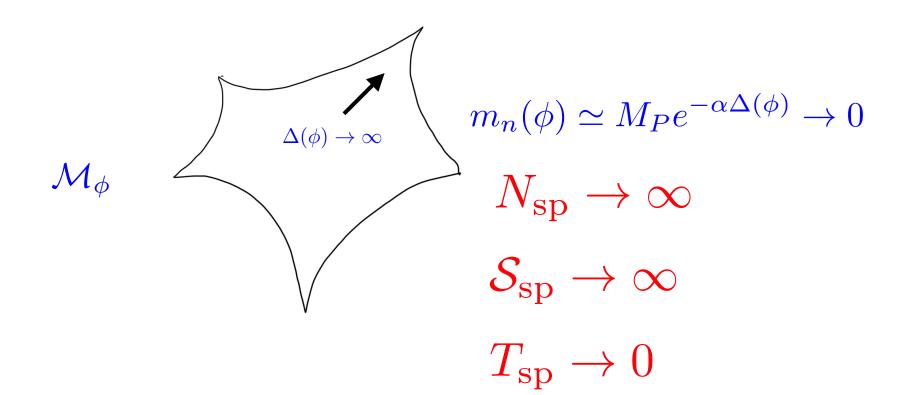
#### BH-species entropy conjecture:

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019); N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

Large species entropy limit:

$$S_{sp} \to \infty \quad \Rightarrow \quad m_{sp} = \left(\frac{1}{S_{sp}}\right)^{\gamma} \to 0$$

Quantum Gravity moduli space:



Mass, temperature and entropy of Schwarzschild-like BHs satisfy the following general thermodynamic laws:

$$S_{BH} \simeq (R_{BH})^{d-2} M_P^{d-2} \simeq (M_{BH})^{\frac{d-2}{d-3}} M_P^{\frac{2-d}{d-3}}$$

$$T_{BH} = (R_{BH})^{-1} \implies S_{BH} T_{BH}^{d-2} = M_P^{d-2}$$

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These relations are consistent with the first law of thermodynamics:

$$dM_{BH} = T_{BH}dS_{BH}$$

$$\frac{1}{T_{BH}} = \frac{\partial S_{BH}}{\partial M_{BH}}$$

#### Black Hole - Species Correspondence

BH thermodynamic laws must be fulfilled in order for the BH - Species Correspondence to work!

$$S_{\rm sp} = \Lambda_{\rm sp}^{2-d}, \quad E_{\rm sp} = \Lambda_{\rm sp}^{3-d}$$

Species thermodynamics will help to a provide bottom-up evidences of the emergent string conjecture.

[I. Basile, D.L. C. Montella (2013)]

#### General species tower and emergent string conjecture:

General ansatz for species tower:

Mass spectrum:  $m_n = m \, f(n) \,, \quad n = 1, \dots, N$ 

Degeneracy at each level:  $d_n$ 

Moduli dependent tower mass scale:  $m=m(\phi)$ 

Species scale:  $\Lambda_{
m sp} = m_N = m \, f(n)$ 

Species number:  $N_{sp} = \sum_{n=1}^{N} d_n$ 

Species energy:  $E_{sp} = \sum_{n=1}^{\infty} d_n \, m_n$ 

(No binding energy and no thermal fluctuations)

#### Bottum-up solution:

$$f(n) = n^{1/p}, \quad p \ge 1, \quad d_n = 1$$

$$N_{sp} = m^{\frac{p(2-d)}{p+d-2}}, \quad \Lambda_{sp} = m^{\frac{p}{p+d-2}}$$

$$S_{sp} = N_{sp} = \Lambda_{sp}^{2-d}, \quad E_{sp} = \sum_{n=0}^{N} n^{\frac{1}{p}} m = \frac{p}{p+1} \Lambda_{sp}^{3-d}$$

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All thermodynamic BH relations are satisfied



### KK tower and string tower



## Two possible interpretations of the tower:

[A. Castellano, A. Herraez, L. Ibanez, (2021)]

(i) p is finite  $\implies$  KK tower

$$m \equiv m_{kk} = 1/R$$
,  $p: \# \text{ of compact dimensions}$  
$$\Lambda_{sp} \equiv \Lambda_{UV} = M_P^{(d+p)}$$

### KK tower and string tower



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(ii)  $p=\infty \implies$  string tower  $m\equiv M_s$ 

$$N_{sp} = g_s^{-2}, \quad \Lambda_{sp} = M_s = g_s^{\frac{2}{d-2}} M_P^{(d)}$$

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==> Emergent string conjecture.

Energy of the particles = Mass of the black hole

$$\Lambda_{BH} = \Lambda_{sp}^{3-d} >> \Lambda_{sp} \quad (d > 3)$$

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String states: 
$$\Lambda_{BH} = \frac{M_s}{g_s^2}$$

(Entropy of weakly coupled strings agrees with BH entropy)

[G. Horowitz, J. Polchinski (1996)]

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$$\Lambda_{BH}=(m_{KK})^{rac{p(3-d)}{p+d-2}}$$

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Hierarchy of scales:

$$m_{KK}$$
  $\Lambda_{sp}$   $M_P$   $\Lambda_{BH}$ 

#### Summary:

 There is a correspondence picture: species as particles - species as minimal black hole.

Strong constraints on the form of the species tower.

==> Emergent string conjecture.

The thermodynamic conditions on the species are systematic and rigorous.

One should complement these results from string S-matrix approach.

This correspondence might be also related to the emergence proposal: species build space-time geometry

#### Anamaria as a person:



Enthusiastic physicist

Faithful and reliable colleague and collaborator

Warm and very helpful person

Very good friend of mine

