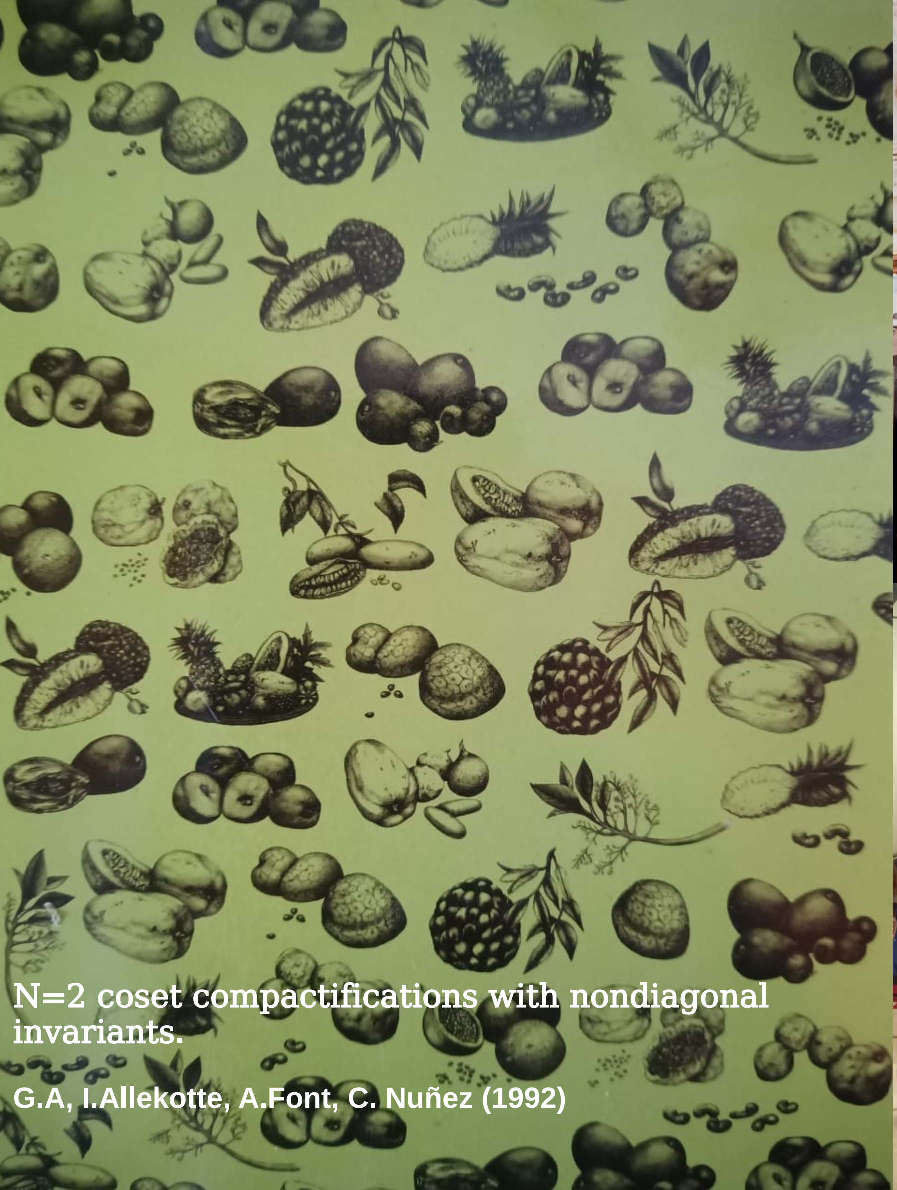
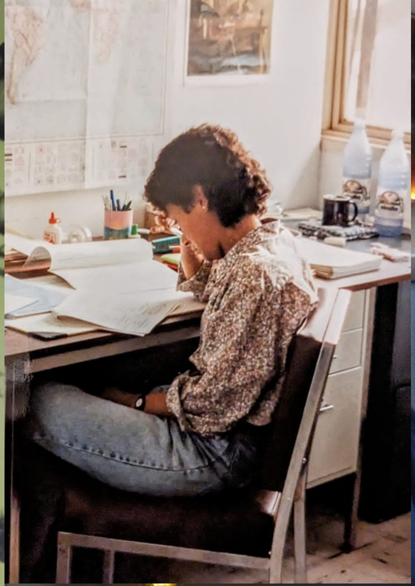


N=2 coset compactifications with nondiagonal invariants.

G.A, I.Allekotte, A.Font, C. Nuñez (1992)



Anly workshop
(2024)

Back to Heterotic:

On asymmetric orbifold models and rank reduction

Anly workshop

April 10-11, Annecy, France

G. Aldazabal, CAB-IB, Bariloche, Argentina

Based on collaborations with: E. Andrés, A. Font, K. Narain, I. Zadeh and B. Acharya

Motivation and aims:

- Asymmetric orbifolds, providing a world-sheet description of non-geometric compactifications, not fully explored. New models.

- Asymmetric orbifolds, appear to be required to study certain duality conjectures. i.e. M-theory with K3 involutions and heterotic string compactifications in D=7.

- Asymmetric orbifolds, provide an adequate framework to built up heterotic string models with reduced rank gauge group

Recent work on asymmetric orbifolfds:

Groot-Nibbelink, Vaudrevange ’17, Harvey, Moore ’18, Kaan Baykara, Harvey ’21, Faraggi, Groot-Nibbelink ’21, Acharya, G. A, Font, Narain, Zadeh ‘22, Groot-Nibbelink, Percival ’23, Kaan Baykara, Hamada, Tarazi, Vafa ’23, Gkountoumis, Hull, Stemerdink,Vandoren Fraiman, Freitas(talk) ‘24..

Outline:

- Basics of heterotic asymmetric orbifolds
- Non-supersymmteric D=7, $T^3/\mathbb{Z}_2$ example and M-theory on K3 involutions
- Supersymmetric D=6 $T^4/\mathbb{Z}_N$ models.
- Summary and outlook

Heterotic string in D=10

Gross, Harvey, Martinec, Rhom '85

Basics and notation:

Left sector — Right sector

X_L^M, Y^I — $X_R^M, \bar{\psi}_R^M$ — $M = 0,\ldots,9;\ I = 1,\ldots 16$

$\downarrow$

$P \in \Gamma(16,0)$

Modular invariance $\Rightarrow$ even, self-dual lattice: $\Gamma(16,0) = \Gamma_8 \oplus \Gamma_8,\ \Gamma_{16}$ (two possible lattices)

$$\Gamma_8 q = \{P \equiv (m_1,\ldots,m_{8q}), (\tfrac{1}{2},\ldots,\tfrac{1}{2}) + (m_1,\ldots,m_{8q}) | m_i \in \mathbb{Z}, \textstyle\sum_i^{8q} m_i = even\}$$

massless states P=0 (Cartan's) , $P^2 = 2$(roots $\in$ charged)

$\Gamma_8 \oplus \Gamma_8 \rightarrow E_8 \times E_8$ heterotic

$\Gamma_{16} \rightarrow SO(32)$ heterotic

Heterotic string in D=9-d

Narain '86

Left sector	Right sector	
$X_L^\mu,$	$X_R^\mu, \bar\psi_R^\mu$	$\mu = 0, \ldots, 9-d;$
$X_L^i,\ Y_L^I$	$X_R^i, \bar\psi_R^i$	$i = 1, \ldots d;\ I = 1, \ldots 16$

Modular invariance $\Rightarrow$ even, self-dual lattice: $\Gamma(16+d, d)$

signature $(16+d, d)$ $\quad (P_L, P_R) : P_L^2 - P_R^2 = even$

Infinite lattices parameterized by background values of :

$g_{ij}\,(metric), b_{ij}\ (Kalb - Ramond), A_i^I\ (WL)$

Torus partition function

$$Z(1,1) = Tr_{\mathcal{H}}(q^{L_0}\bar{q}^{L_0})$$

Ibáñez, Uranga '12
Polchinski '98
Blumenhagen, Lüst, Theisen '13

$$\mathrm{Z}_{(1,1)} = \frac{1}{(\sqrt{\tau_2}\bar{\eta}\eta)^{8-d}}\,\frac{1}{\bar{\eta}^4}\left(\sum_{p\in \mathrm{V}} - \sum_{p\in \mathrm{Sp}}\right)\bar{q}^{\frac{1}{2}p^2}\,\frac{1}{\bar{\eta}^d\eta^{16+d}}\sum_{P\in\Gamma}\bar{q}^{\frac{1}{2}P_R^2}q^{\frac{1}{2}P_L^2}$$

$$m_L^2 = \frac{1}{2}P_L^2 + N_L - 1, \qquad m_R^2 = \frac{1}{2}P_R^2 + N_R + \frac{1}{2}p^2 - \frac{1}{2},$$

$$\text{massless states}: \; P_L = 0, N_L = 1; P_L^2 = 2,\; N_L = 0 \,\&\, p^2 = 1, P_R = 0, N_R = 0$$

$$\Downarrow$$

$(10-d)$ -dimensional theory with 16 supercharges:

sugra multiplet (with d graviphotons)

gauge multiplet $U(1)^{d+16}$ (at generic moduli), enhancement (at specific) moduli i.e. $SO(2(d+16))$

Keep states invariant under automorpisms, $\Rightarrow$ (asymmetric) orbifolds

Abelian Z_M orbifold partition function:

- orbifold generator g with $g^M = 1$

$$Z = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau,\bar{\tau}) \qquad \text{where} \qquad Z(\tau,\bar{\tau}) = \sum_{m=0}^{M-1}\left[\frac{1}{M}\sum_{l=0}^{M-1} Z(g^m,g^l)\right]$$

$$Z(g^m,g^l)] = Tr_{\mathcal{H}_m}(g^l q^{L_0}\bar{q}^{L_0})$$

$\mathcal{H}_m$: g^m- twisted Hilbert space

- orbifold projection on sector m : $\frac{1}{M}\sum_{l=0}^{M-1}$

- $\sum_{m=0}^{M-1}\frac{1}{M}\sum_{l=0}^{M-1}$ → Modular invariant

Asymmetric orbifolds

Narain, Sarmadi, Vafa `87

generator $g \to \Theta$

$\Theta = (\theta_L, \theta_R)$ acts differently on *L* and *R* of $\Gamma = \Gamma(16+d, d)$

Invariant lattice: $I = \{P \in \Gamma | \Theta P = P\}$

Normal lattice: $N = I^{\perp} = \{P \in \Gamma | P.Q = 0, \forall\, Q \in I\}$

Full lattice: $\Gamma \supset \{P : (P_N, P_I) \,\text{with}\, P_N \in N^*,\ P_I \in I^*\}$

$$I^*/I = N^*/N$$

$$\Gamma = (N, I) \bigoplus_{w \in N^*/N} (w, \zeta(w)) \qquad \zeta : N^*/N \to I^*/I$$

$\Uparrow$

Glue vectors (correlated classes): even norm, integer scalar product

Example:

$$\Gamma_8 \equiv E_8 = \{P = (m_1, \ldots, m_8), (\tfrac{1}{2}, \ldots, \tfrac{1}{2}) + (m_1, \ldots, m_8), |m_i \in \mathbb{Z}, \sum_i m_i = even\}$$

$$\Theta P = (P_1, P_2, P_3, P_4; -P_5, -P_6, -P_7, -P_8)$$ reflection

$$I = \{(m_1, \ldots, m_4; \mathbf{0}), \sum_i m_i = even\} = D_4, \qquad N = \{\mathbf{0}; (m_5, \ldots, m_8), \sum_i m_i = even\} = D_4$$

$$I^*/I = N^*/N = Z_2 \times Z_2$$

$$\mathrm{Sc}_4 = D_4 = (m_1, m_2, m_3, m_4)$$

$$E_8 = (D_4, D_4) \oplus (V_4, V_4) \oplus (S_4, S_4) \oplus (C_4, C_4)$$

$$\left.\begin{aligned} \mathrm{V}_4 &= (1,0,0,0) + (m_1, m_2, m_3, m_4) \\ S_4 &= (\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}) + (m_1, m_2, m_3, m_4) \end{aligned}\right\} \in N^*$$

$$\mathrm{C}_4 = (-\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}) + (m_1, m_2, m_3, m_4)$$

$$\Gamma = (N, I) \bigoplus_{w \in N*/N} (w, \zeta(w))$$

$$P_L^2 = 2 \longrightarrow \mathbf{248} = (\mathbf{28}, \mathbf{1}) + (\mathbf{1}, \mathbf{28}) + (\mathbf{8}_v, \mathbf{8}_v) \oplus (\mathbf{8}_s, \mathbf{8}_s) \oplus (\mathbf{8}_c, \mathbf{8}_c)$$

Asymmetric orbifolds:

generator g :

- g rotates $s(\bar{s})$ $L(R)$ (internal) oscillators
- g on fermions $\rightarrow v_f$ can break all, half, or none supersymmteries
- $g|P_N, P_I >= e^{2i\pi P.v}|\Theta P_N, P_I >,\qquad \Theta = (\Theta_L, \Theta_R)$

Allows to build the untwisted sector partition function: $Z = Z_{st}\, Z_f\, \mathcal{O}\, \mathcal{L}$

$$Z_{(1,g)} = Tr_{\mathcal{H}_0}(g q^{L_0} \bar{q}^{L_0}) \longrightarrow \mathcal{L}(1,g) = \sum_{P\in I} q^{\frac{1}{2}P_L^2} \bar{q}^{\frac{1}{2}P_R^2} e^{2i\pi P.v}$$

Modular transformations $\longrightarrow$ twisted sectors

$$Z_{(1,g)} \xrightarrow{\tau\rightarrow -\frac{1}{\tau}} Z_{(g,1)} \qquad \mathcal{L}(g,1) = \sum_{P\in I^*} q^{\frac{1}{2}(P+v)_L^2} \bar{q}^{\frac{1}{2}(P+v)_R^2}$$

$$Z_{(g,1)} \xrightarrow{\tau\rightarrow\tau+l} Z_{(g,g^l)} \qquad Z_{(g,g^M)} \overset{g^M=1}{=} Z_{(g,1)} \longrightarrow \frac{1}{2}Mv^2 + \cdots \in \mathbb{Z}$$

Examples of heterotic asymmetric orbifolds:

- D=7, non-supersymmetric, $\mathrm{T}^3/\mathbb{Z}_2$
- D=6, 16 superchages, $\mathrm{T}^4/\mathbb{Z}_N$

Heterotic on $\mathrm{T}^3/\mathbb{Z}_2$

Motivation: $\mathrm{M}/\mathrm{K}_3 \approx \mathrm{Het}/T^3$ — Hull,Townsend; Witten '95

↓

$$\text{M-theory on } (\mathrm{K}_3/\theta) \overset{?}{\approx} \mathrm{Het}\, T^3/\mathbb{Z}_2$$

θ a Nikulin $\mathbb{Z}_2$ involution — Nikulin 1983

M-theory on K3 :16 supercharges, rank 22 gauge group, membranes wrapped on 2-cycles of K3 charged with charge lattice $\Gamma(19,3)$

Heterotic on T^3 :16 supercharges, rank 22 gauge group, even self-dual $\Gamma(19,3)$ momentum lattice

M-theory on (K_3/θ)

- θ involution reflects holomorphic 2-form (non-symplectic), leaves volume form invariant.

- 75 such involutions characterized $I \in \Gamma(19,3)$, $\theta I = I$.

- I rank $r = 20 - s$, signature $(r-1,1) = (19-s,1)$. $I^*/I = Z_2^a$

- I uniquely specified by (r, a, δ) triplets with

$$\delta_I = \delta = \begin{cases} 0 & \text{if } P_I^2 \in \mathbb{Z} \ \ \forall P_I^2 \in I^* \ , \\ 1 & \text{otherwise .} \end{cases}$$

Nikulin points:

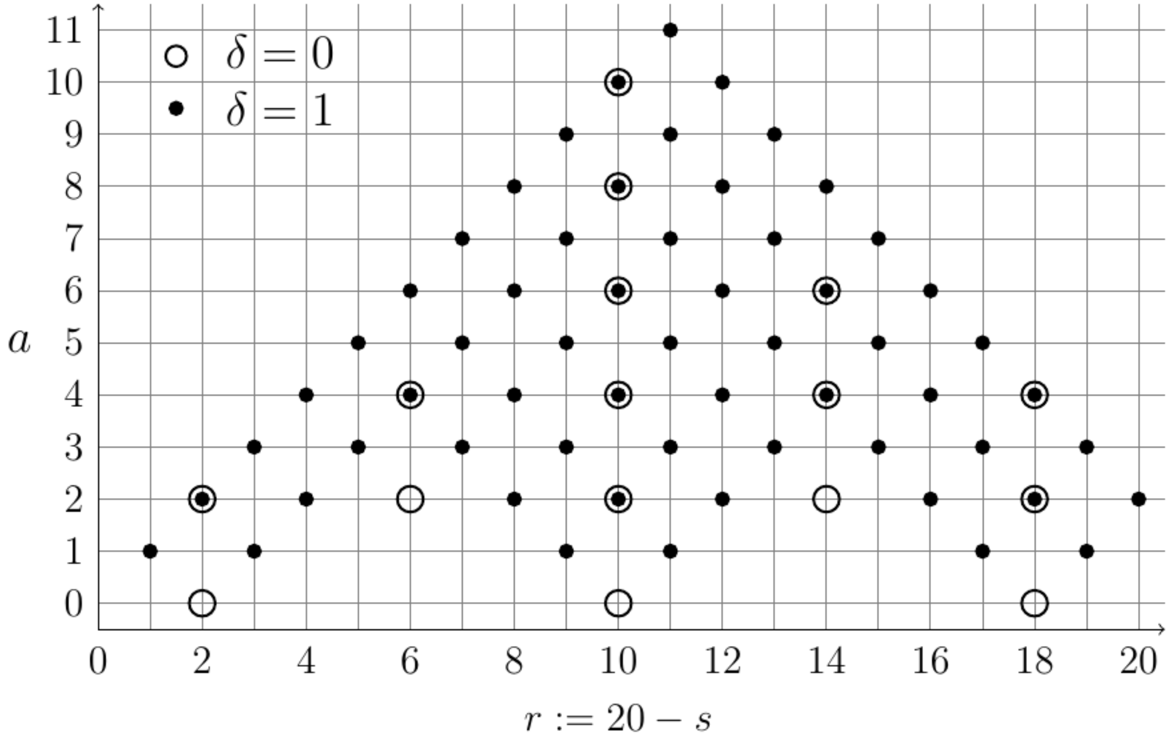


Figure 1: Points (r, a, δ) determining all 75 invariant lattices of signature $(r-1, 1)$ which are embedded primitively in the K3 lattice $\Gamma_{(19,3)}$.

Example: $(\text{r,a,}\delta) = (14, 4, 0) \Rightarrow I = U(2) + D_4 + E_8, \qquad N = U(2) + D_4$

Heterotic on $\mathrm{T}^3/\mathbb{Z}_2$

B. Acharya, G. A, A. Font, K. Narain, I. Zadeh '22

- Non-supersymmetric $(\mathrm{non-symplectic})$
- asymmetric orbiblod $\mathrm{I} \in \Gamma(19,3)$, I rank $r = 20 - s$, signature $(19 - s, 1)$

→ $\Theta = (\theta_L, \theta_R)$ Reflects $(s, 2)$, (P_L, P_R) momenta

- g reflects $s(2)$ $L(R)$ (internal) oscillators
- g^2 on fermions -1 breaks all supersymmetries, $g^4 = 1$, $\Rightarrow \mathrm{Z}_4$
- $g|P_N, P_I >= f(P_N) e^{2i\pi P.v} | - P_N, P_I >$, shift vector v, $4v \in I$, M.I. → $2\mathrm{v}^2 + \frac{s}{4} \in \mathbb{Z}$

$\mathrm{Z}_4, f(P_N)$ phase, $f(P_N) f(-P_N) = e^{2i\pi P_N^2} = e^{2i\pi P_I^2} = e^{2i\pi P_I.w}$, $\mathrm{w} \in I^*, w^2 + \frac{1}{2}(s-2) \in 2\mathbb{Z}$

Z_2

- Gravity multiplet → graviton, dilaton, Kalb-Ramond, 1 graviphoton

  $\mathrm{U}(1)^{19}$ → $\mathrm{U}(1)^{r-1}$

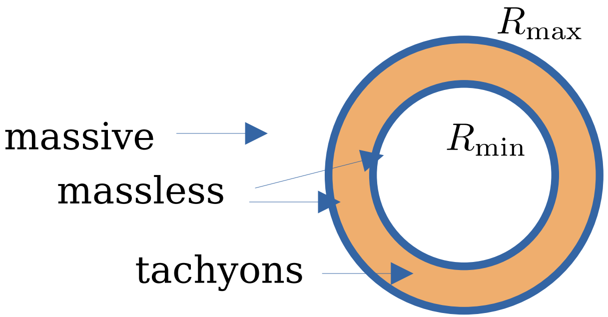
  Further enhancements at special moduli points and from $(\mathrm{P}_I, P_N), P_N \neq 0$

- All $(\mathrm{r,a},\delta)$ triplets reproduced except $(1,1,1)$, $(2,2,1)$

  I has no place for shift v satisfying modular invariance

- Tachyons do not appear in the untwisted sector.

twisted sector:



Heterotic asymmetric orbifolds in D=6:

$T^4/\mathbb{Z}_N$ with 16 superchages and rank reduction

Set up:

Z_N

- acts on s=2m of 20 L-movers, eigenvalues $e^{\pm 2i\pi t_i}$, $i = 1, \ldots, m$, $t_i = \frac{n_i}{N}$

  (20 − s) L-movers and all 4 R invariant

- no action on world-sheet fermions, all 16 supersymmetries unbroken

- Θ on $\Gamma(20, 4)$

  I Invariant lattice with signature (20 − s, 4)

  N normal lattice, signature (s, 0)

Rank reduction $\Rightarrow$ N without roots $(P^2 > 2)$ → Leech lattice Λ

Niemeier '78

Lerche,Schellekens, Warner ' 89

24 even self-dual lattices (24,0)

23 Niemeier lattices:

Root lattices with correlated classes →

Table 11.3 The 23 Euclidean self-dual semi-simple Lie algebra lattices in 24 dimensions (conjugacy classes in square brackets should be cyclically permuted)

Lie algebra	Glue vector
D_{24}	(S)
$D_{16}E_8$	(S,0)
E_8^3	(0,0,0)
A_{24}	(5)
D_{12}^2	(S,V), (V,S)
$A_{17}E_7$	(3, 1)
$D_{10}E_7^2$	(S,1,0), (C,0,1)
$A_{15}D_9$	(2, S)
D_8^3	(S,V,V), (V,S,V), (V,V,S)
A_{12}^2	(1, 5)
$A_{11}D_7E_6$	(1,S,1)
E_6^4	$(1, 0, 1, \bar{1})$, $(1, \bar{1}, 0, 1)$, $(1, 1, \bar{1}, 0)$
$A_9^2D_6$	(2, 4, 0), (5,0,S), (0,5,C)
D_6^4	Even permutations of (0,S,V,C)
A_8^3	(1, 1, 4), (4, 1, 1), (1, 4, 1)
$A_7^2D_5^2$	(1,1,S,V), (1,7,V,S)
A_6^4	(1, 2, 1, 6), (1, 6, 2, 1), (1, 1, 6, 2)
$A_5^4D_4$	(2, [0, 2, 4], 0), (3,3,0,0,S), (3,0,3,0,V), (3,0,0,3,C)
D_4^6	(S,S,S,S,S,S), (0,[0,V,C,C,V])
A_4^6	(1, [0, 1, 4, 4, 1])
A_3^8	(2, [2, 0, 0, 1, 0, 1, 1])
A_2^{12}	(2, [1, 1, 2, 1, 1, 1, 2, 2, 2, 1, 2])
A_1^{24}	(1, [0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 1])

Leech lattice Λ $(P^2 \geq 4)$

$T^4/\mathbb{Z}_N$ Orbifold on the Leech lattice Λ

- Rank reduction $\Rightarrow$ N without roots, look at automorphisms of Λ

Conway Group= Co_0

- Λ sublattices fixed by Co_0 elements have been classified — Höhn, Mason '16

290 distinct invariant lattices I (r , 0), with normal N (s, 0), r + s = 24

- Look for I, with signature (r-4,4) such that $I^*/I = N^*/N$

- Θ Automorphism of N preserves correlated classes in N^*/N

$$\Gamma(24,0) \Rightarrow \Gamma(20,4)$$

$\mathrm{T}^4/\mathbb{Z}_{10}$ asymmetric orbifold with rank 4 Island? (HM100)

s=20

N$\in (20,0)$

$$G_N = \begin{pmatrix}
4 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 2 & 1 & 2 & 2 & 0 & -2 & 1 & -2 & -2 & -1 & -2 & -1 \\
1 & 4 & -2 & -1 & 1 & -2 & 1 & -2 & 2 & -1 & 2 & 0 & 1 & -1 & 2 & -1 & -2 & -2 & 0 & -2 \\
-1 & -2 & 4 & -1 & -1 & 2 & 1 & 0 & -2 & -1 & -1 & 0 & 1 & 0 & -1 & 2 & 1 & 0 & -1 & 2 \\
1 & -1 & -1 & 4 & 0 & 0 & 0 & 2 & 1 & 2 & -1 & 1 & -2 & 0 & -1 & 0 & -1 & 0 & -1 & -1 \\
-1 & 1 & -1 & 0 & 4 & -2 & 0 & 1 & 1 & 1 & -1 & -2 & 0 & 2 & 0 & 0 & 1 & 1 & 2 & -1 \\
-1 & -2 & 2 & 0 & -2 & 4 & 0 & 1 & -1 & 0 & -1 & 1 & -1 & 0 & 0 & 2 & 1 & 1 & 0 & 2 \\
-1 & 1 & 1 & 0 & 0 & 0 & 4 & 0 & -1 & -2 & -1 & -1 & 0 & 0 & -1 & 2 & 0 & -2 & 0 & 0 \\
-1 & -2 & 0 & 2 & 1 & 1 & 0 & 4 & 0 & 2 & -2 & -1 & -2 & 1 & -2 & 1 & 1 & 2 & 1 & 0 \\
2 & 2 & -2 & 1 & 1 & -1 & -1 & 0 & 4 & 1 & 2 & 1 & 0 & -1 & 2 & -1 & -2 & 0 & 0 & -2 \\
1 & -1 & -1 & 2 & 1 & 0 & -2 & 2 & 1 & 4 & -1 & 1 & -2 & 1 & 0 & -1 & 0 & 2 & 0 & -1 \\
2 & 2 & -1 & -1 & -1 & -1 & -1 & -2 & 2 & -1 & 4 & 1 & 2 & -2 & 2 & -2 & -2 & -1 & -1 & -1 \\
2 & 0 & 0 & 1 & -2 & 1 & -1 & -1 & 1 & 1 & 1 & 4 & 0 & -1 & 1 & -1 & -2 & -1 & -2 & 0 \\
0 & 1 & 1 & -2 & 0 & -1 & 0 & -2 & 0 & -2 & 2 & 0 & 4 & -1 & 1 & 0 & -1 & -1 & 0 & 0 \\
-2 & -1 & 0 & 0 & 2 & 0 & 0 & 1 & -1 & 1 & -2 & -1 & -1 & 4 & 0 & 0 & 2 & 2 & 2 & 0 \\
1 & 2 & -1 & -1 & 0 & 0 & -1 & -2 & 2 & 0 & 2 & 1 & 1 & 0 & 4 & -1 & -1 & 0 & 0 & -1 \\
-2 & -1 & 2 & 0 & 0 & 2 & 2 & 1 & -1 & -1 & -2 & -1 & 0 & 0 & -1 & 4 & 1 & 0 & 1 & 1 \\
-2 & -2 & 1 & -1 & 1 & 1 & 0 & 1 & -2 & 0 & -2 & -2 & -1 & 2 & -1 & 1 & 4 & 2 & 2 & 2 \\
-1 & -2 & 0 & 0 & 1 & 1 & -2 & 2 & 0 & 2 & -1 & -1 & -1 & 2 & 0 & 0 & 2 & 4 & 2 & 0 \\
-2 & 0 & -1 & -1 & 2 & 0 & 0 & 1 & 0 & 0 & -1 & -2 & 0 & 2 & 0 & 1 & 2 & 2 & 4 & 0 \\
-1 & -2 & 2 & -1 & -1 & 2 & 0 & 0 & -2 & -1 & -1 & 0 & 0 & 0 & -1 & 1 & 2 & 0 & 0 & 4
\end{pmatrix}$$

$$\mathrm{N}^*/N = Z_2^3 \times Z_{10}$$

$$\Theta = \begin{pmatrix}
-1 & 0 & 0 & 0 & -2 & 0 & -2 & 0 & 0 & -1 & -1 & 0 & -1 & 0 & 1 & -1 & -1 & -1 & 0 & 1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 2 & -2 & 3 & 0 & -2 \\
-1 & 1 & -1 & -1 & 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & -1 & 1 & -1 & -2 & 1 & 0 \\
1 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 2 & 0 & 2 & 0 & 1 & 0 & -2 & 3 & -1 & 1 & 0 & -2 \\
-1 & 1 & -1 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 3 & 1 & 1 & 1 & -3 & 2 & -1 & 0 & -1 & -2 \\
1 & 0 & 2 & -1 & -1 & 0 & -1 & 0 & -2 & -1 & -2 & -1 & 0 & -1 & 2 & -2 & 2 & -1 & 1 & 2 \\
1 & 1 & 1 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 2 & 0 \\
0 & -2 & 0 & 0 & -1 & -1 & 2 & 0 & -2 & 0 & -2 & 0 & -1 & -1 & 2 & -1 & 0 & 1 & -1 & 1 \\
-1 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & -1 & 1 & -1 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 2 \\
0 & -1 & 0 & 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 2 & -1 & -1 \\
1 & 1 & 1 & 0 & 1 & 1 & -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 & 1 & 0 & 1 & 1 & -1 \\
-1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & -1 & 1 & -1 & 1 & -1 & -1 \\
1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -1 & 0 & -1 & 1 & -1 & 1 & 0 & 1 & 1 \\
0 & -1 & 0 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & -1 & 1 & -1 & 1 & 0 & -1 & 1 \\
0 & -1 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 \\
1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{pmatrix}.$$

$\mathrm{e}^{\pm 2i\pi t_i}$ eigenvalues

$$t = \frac{1}{10}(1,1,1,2,3,3,3,4,5,5)$$

$$G_I = \begin{pmatrix} 4 & -2 & -2 & 2 \\ -2 & 4 & 0 & 0 \\ -2 & 0 & 4 & -2 \\ 2 & 0 & -2 & 4 \end{pmatrix} \longrightarrow P.P' \in 2\mathbb{Z}$$

I$\in (0,4)$ $I = A_4(-2)$ (no roots)

$$g|P_L, P_R >= e^{2i\pi P.v}|\theta P_L, P_R > \qquad 10v \in I$$

$$Z_{(1,g)} \xrightarrow{\text{S,T}} \qquad Z(\tau,\bar{\tau}) = \sum_{m=0}^{9}\left[\frac{1}{10}\sum_{l=0}^{9} Z(g^m, g^l)\right]$$

Full partition function can be computed

modular invariance: $\quad 10v^2 \in 2\mathbb{Z}$

$$\mathrm{e}^{2i\pi P_I.5v} = e^{i\pi P_I.10v} = 1 \Rightarrow 5v \in I^* \longrightarrow \qquad \mathrm{g}^5 : P + 5v = P' \in I^*$$

massless states

Partition function → spectrum

Untwisted sector

No-massless states $(P_L^2 \geq 4)$

Twisted sector

v=0: 20 massless states → $\mathrm{U}(1)^{20}$

v$\neq 0$: Can be chosen such that there are no massless states in twisted sectors, except

4 in g^5 → $\mathrm{U}(1)^4$

Further check on massive states sector

SO(5,1)

e.g. $Z_{(g,1)} = 24 + 1280q^{\frac{1}{5}} + \ldots$

24=3×8 3 massless vector multiplets

1280=16×80 16 massive states

Further compactification to D=5 with spectator circle with a shift

$$(\mathrm{T}^4 \times S^1)/\mathbb{Z}_{10}, \qquad \Gamma[21,5] = \Gamma[20,4] + U[1,1]$$

$$\text{g:}\ (\Theta, v_U) \qquad v_U = \frac{1}{10}\left(\frac{n}{2R} + mR, \frac{n}{2R} - mR\right)$$

1-vector multiplet in untwisted sector → U(1)   almost an island

Other candidates, work in progress

Summary and outlook:

- Further develop asymmetric orbifold constructions to deal with non-geometric compactifications
- $T^3/\mathbb{Z}_2,\ D = 7$

  Non-supersymmetric. Nikulin involutions of $\text{M-theory on }(K_3/\theta)$ reproduced (except 2).

- $T^4/\mathbb{Z}_M,\ D = 6,$ 16 (and 8, not discussed here) supercharges and rank reduction.

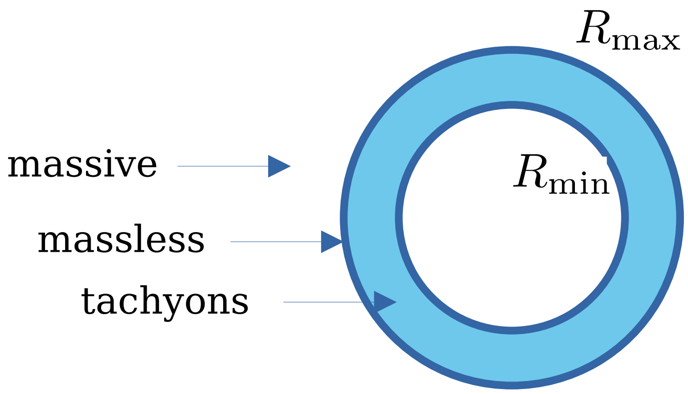
  Normal lattice $\mathcal{N}$ with no-roots required. Contructions using Leech lattice Λ

  $T^4/\mathbb{Z}_{10}$ Candidate for D=6 island (no gauge group) example.

  Obstruction due to Modular Invariance $U(1)^4$ but $U(1)$ in D=5

- Explore D=4 dimensions

¡Gracias!


R_{max}
massive
R_{min}
massless
tachyons