Extensions of scale-separated AdS_4 flux vacua

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April 11, 2024

AnLy meeting in honor of Anamaria Font

Based on

D. Andriot, G. T, "Extensions of a scale-separated AdS_4 solution and their mass spectrum," [2310.06115].

G. T, "Anisotropic a scale-separated AdS_4 flux vacua," [2309.16542].

Outline

- Introduction : Motivation, Scale-separation
- DGKT - CFI model : features and characteristics
- Op-planes : Smearing and beyond smearing approximation
- Scalings: Smeared and next-to-smeared order generalizations
- The role of flux quantization
- Next-to-smeared order VS α'-corrections to the potential ?
- Investigating NNLO EoMs

Introduction

Motivation - Scope

- DGKT-CFI (4d N=1 SUGRA EFT with AdS) possess rare properties like *scale separation* and *moduli stabilization*, with smeared O6-planes

  Critisism: McOrist, Sethi [1208.0261], Saracco, Tomasiello [1201.5378]

  Relevant conjectures: D. Lüst, Palti, Vafa [1906.05225]

- Dimensions of conformal operators of relevant CFT have special features

$$\Delta = \frac{1}{2}\left(3 \pm \sqrt{9 + 12m^2 \frac{M_p^2}{|V|_0|}}\right)$$

  see Apers, Conlon, Ning, Revello [2202.09330],Quirant [2204.00014], Apers, Montero Van Riet, Wrase [2202.00682]

- Will going beyond the smeared approximation change the nice properties of DGKT-CFI? Junghans [2003.06274]

- Does backreaction modify the scalar potential and masses $\rightarrow \Delta$?

Scale separation

Condition to estimate whether there is a large energy gap between extra dimensional states of the fields and the vacuum energy of the EFT

$$\frac{\langle V \rangle}{m_{\mathsf{KK}}^2} \equiv \frac{L_{\mathsf{KK}}^2}{L_{\Lambda}^2} \ll 0$$

4d: DeWolfe, Giryavets, Kachru, Taylor [0505160]
Camara, Font, Ibanez [0506066]
Tsimpis [1206.5900]
Marchesano, Quirant [1908.11386]
Cribiori, Junghans ,Hemelryck , Van Riet, Wrase [2107.00019]
Carrasco, Coudarchet, Marchesano, Prieto [2309.00043]
G.T. [2309.16542]

3d: Farakos, G.T, Van Riet [2005.05246]
Van Hemelryck [2128116]
Farakos, Morittu, G.T. [2304.14372]

2d: D. Lüst, Tsimpis, [2004.07582]

Scale separation

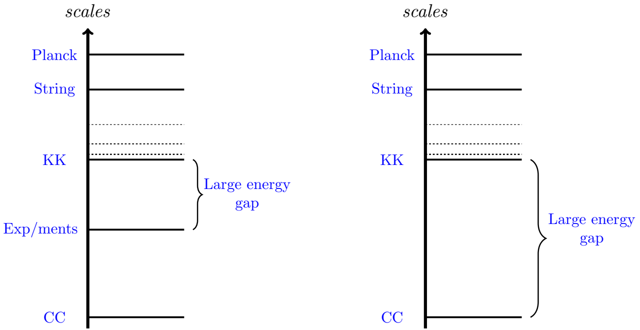


Figure: On the left, we have experimental locations where we can reach the highest energies (e.g. CMS and ATLAS), with the KK scale depicted above them, as we have not yet observed any effects that can be interpreted as signatures of extra dimensions. On the right, we present the universal scales typically considered in compactifications.

AdS vacua and scale separation

We will focus on: DeWolfe, Giryavets, Kachru, Taylor [0505160]
Camara, Font, Ibanez [0506066]

Massive type IIA supergravity on $X^6 = \frac{T^6}{\mathbb{Z}_3 \times \mathbb{Z}_3}$

$$\downarrow$$

AdS$_4$ $\mathcal{N} = 1$ supergravity

- Characteristics: Smeared O6-planes, Romans mass, unbounded flux
- Tadpole cancellation with fluxes and flux quantization
- Moduli stabilization (dilaton + 3 T^2 torii sizes)

Classical regime

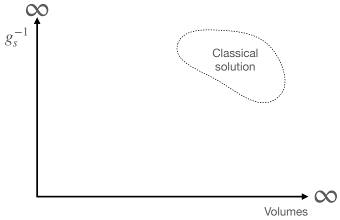
String theory solution at large volume and weak coupling regime can be described by 10d supergravities – Perturbative solution

- Weak coupling : string loops neglected

$$g_s = e^{\phi} \ll 1$$

- Large radii : sub-leading α' corrections

$$r_i \gg l_s$$



AdS flux vacua from type IIA

Massive type IIA supergravity

The bosonic type IIA action ($p = 0, 2, 4$) in string frame

$$S_{\mathsf{IIA}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}X\sqrt{-G}\left[e^{-2\phi}\left(R_{10} + 4\partial_M\phi\partial^M\phi - \frac{1}{2}|H_3|^2\right) - \frac{1}{2}\sum_p |F_p|^2\right]$$

Op-planes : Span $p+1$ dimensions of the 10d space and wrap internal cycles

$$S_{\mathsf{O}p} = -T_{\mathsf{O}p}\int \mathrm{d}^{10}X\,\sqrt{-G}e^{-\phi}\sum_i \delta(\alpha_i) \;+\; Q_{\mathsf{O}p}\int C_{p+1}\wedge\delta_{9-p}\,,$$

	AdS$_4$	y^1	y^2	y^3	y^4	y^5	y^6
O6$_1$	$\otimes$	$\otimes$	–	$\otimes$	–	$\otimes$	–
O6$_2$	$\otimes$	$\otimes$	–	–	$\otimes$	–	$\otimes$
O6$_3$	$\otimes$	–	$\otimes$	$\otimes$	–	–	$\otimes$
O6$_4$	$\otimes$	–	$\otimes$	–	$\otimes$	$\otimes$	–

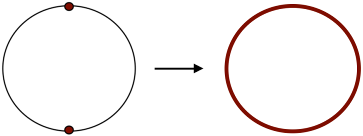
Table: The O6-planes fill the AdS$_4$ and wrap 3-cycles α_i.

Smearing approximation

Replace ***singular*** delta form-function with ***regular*** normalized volume form

$$\delta_{9-p} \;\rightarrow\; j_{9-p}$$

- Local sources are distributed globally all over the cycles



[image from Tomasiello's talk]

- Fields ignore local backreaction : Not exact field profile

$$\phi(y) \approx \phi\,, \qquad w(y) \approx w\,, \qquad \mathrm{d}F_p = \mathrm{d}\star_6 F_p = 0\,, \qquad R_{mn} = 0$$

- Equations of motion (and 4d potential) are simplified

Fluxes and tadpole cancellation

Fluxes ansatz :

$$F_4 = \sqrt{2}\, e_i \tilde{w}^i\,, \qquad H_3 = -p\beta_0\,, \qquad F_0 = -\sqrt{2}\, m_0\,, \qquad F_2 = 0$$

$$\uparrow$$

$$e_i \sim \; f_4\, {\color{red} n}$$

Relevant Bianchi identity and equations of motions :

$$0 = H_3 \wedge F_0 - Q_{\mathsf{O6}} \sum j_{\beta_i} \quad \xrightarrow{\int} \quad p\, m_0 = \pm\{1,2\}$$

$$0 = H_3 \wedge \star_6 F_4$$

The flux ${\color{red} n}$ of F_4 is unconstrained !

Scalings

Smeared equations of motion

Smeared 10d equations of motion:

$$\text{dilaton e.o.m.} : \quad 2\mathcal{R}_4^S + e^{\phi}\frac{T_{10}}{7} - |H_3|^2 = 0$$

$$\text{4d Einstein} : \quad 4\mathcal{R}_4^S = e^{\phi}\frac{T_{10}}{7} - 2|H_3|^2 + e^{2\phi}(|F_0|^2 - 3|F_4|^2)$$

$$+ \quad \text{6d Einstein} : \quad \dots$$

Detailed balance analysis (see Petrini, Solard, Van Riet [1308.1265]):

Equations of motion have the following form:

$$(\text{fluxes})^a \times (\text{radii})^b \times (\text{dilaton})^c + \dots = number$$

General scalings

Quantities expressed in terms of the F_4 scalings:

$$F_4^{(1)} \sim n^{f_1}\,, \qquad F_4^{(2)} \sim n^{f_2}\,, \qquad F_4^{(3)} \sim n^{f_3}\,.$$

- String coupling

$$e^{\phi} \sim n^{-\frac{1}{4}(f_1+f_2+f_3)}\,.$$

- Subvolumes

$$\upsilon_1 \sim n^{\frac{1}{2}(-f_1+f_2+f_3)}\,, \qquad \upsilon_2 \sim n^{\frac{1}{2}(f_1-f_2+f_3)}\,, \qquad \upsilon_3 \sim n^{\frac{1}{2}(f_1+f_2-f_3)}\,.$$

- Separation of scales?

$$\frac{L^2_{\mathsf{KK}_i}}{L^2_{\mathsf{AdS}}} \sim n^{-f_i}\,, \qquad i = 1, 2, 3.$$

Scaling investigation

Large values of n :

1. Scale separation, weak coupling & large volume

$$v_i \gg 1\,, \qquad e^{\phi} < 1\,, \qquad \frac{L^2_{\mathsf{KK}_i}}{L^2_{\mathsf{AdS}}} \ll 1\,,$$

   as long as

$$f_1 > 0\,, \qquad 0 < f_2 \leq f_1\,, \qquad f_1 - f_2 < f_3 < f_1 + f_2\,.$$

   e.g. anisotropic case for $f_1 = f_2 = 2$ and $f_3 = 3$

$$v_1 = v_2 \sim N^{3/2}\,, \qquad v_3 \sim N^{1/2}$$

   DGKT-CFI: Isotropic scale separation $f_i = 1$, weak coupling $\&$ large volume

$$v_1 = v_2 = v_3 \sim n^{1/2}\,, \qquad g_s \sim n^{-3/4}\,, \qquad \frac{L^2_{\mathsf{KK}}}{L^2_{\mathsf{AdS}}} \sim n^{-1}$$

2. Scale separation, weak coupling & one small (shrinking) subvolume
3. Scale separation, weak coupling & small constant subvolumes
4. Broken scale separation, weak coupling & one small subvolume

Beyond smeared solution

Methodology

Junghans [2003.06274]: *expansion method* to go beyond the smeared solutions

- DGKT admits an unbounded discretized parameter n (F_4 flux)
- All quantities can be expressed as n to some power

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Field is expanded in a perturbative way:

$$\phi = \phi^{(0)} n^a + \phi^{(1)}(y) n^b + \dots \,, \qquad a > b$$
$$F_p = F_p^{(0)} n^c + F_p^{(1)} n^d + \dots \,, \qquad c > d$$

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$$F_p = \underbrace{F_p^{(0)}}_{\text{DGKT}} n^c + \underbrace{F_p^{(1)}}_{\text{corrections}} n^d + \dots , \qquad c > d$$

see also: Marchesano, Palti, Quirant, Tomasiello [2003.13578]
Emelin, Farakos, G. T [2202.13431]

Beyond the smeared solution

Expansion ansatz

$$
\begin{aligned}
g_{mn} &= g_{mn}^{(0)} n^{1/2} + g_{mn}^{(1)} n^{-1/2} \\
e^{A} \equiv w &= w^{(0)} n^{3/4} + w^{(1)} n^{-1/4} \\
e^{-\phi} \equiv \tau &= \tau^{(0)} n^{3/4} + \tau^{(1)} n^{-1/4}
\end{aligned}
$$

$$
\begin{aligned}
F_0 &= F_0^{(0)} n^0 \\
F_2 &= F_2^{(0)} n^{1/2} + F_2^{(1)} n^0 \\
H_3 &= H_3^{(0)} n^0 + H_3^{(1)} n^{-1} \\
F_4 &= F_4^{(0)} n + F_4^{(1)} n^{1/2} \\
F_6 &= 0 + F_6^{(1)} n^{1}
\end{aligned}
$$

Corrected field strengths remain harmonic as long as the Bianchi and flux equations are satisfied trivially

$$
\begin{aligned}
\mathrm{d}F_4^{(1)} &= 0 \\
\mathrm{d}(*_6 F_p^{(1)}) &= 0 \\
\mathrm{d}(*_6 H_3^{(1)}) &= 0
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Expansion ansatz

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$$F_0 = F_0^{(0)} n^0$$
$$F_2 = F_2^{(0)} n^{1/2} + F_2^{(1)} n^0$$
$$H_3 = H_3^{(0)} n^0 + H_3^{(1)} n^{-s_3}$$
$$F_4 = F_4^{(0)} n + F_4^{(1)} n^{1-s_4}$$
$$F_6 = 0 + F_6^{(1)} n^{s_6}$$

Corrected field strengths remain harmonic as long as the Bianchi and flux equations are satisfied trivially

$$\mathrm{d}F_4 - H_3 \wedge F_2 = 0$$
$$e^{-4A}\mathrm{d}(e^{4A} *_6 F_p) + H_3 \wedge *_6 F_{p+2} = 0$$
$$e^{-4A}\mathrm{d}(e^{4A-2\phi} *_6 H_3) - \sum_{p=0}^{4} F_p \wedge *_6 F_{p+2} = 0$$

Expansion ansatz

$$g_{mn} = g_{mn}^{(0)} n^{1/2} + g_{mn}^{(1)} n^{-1/2}$$
$$e^{A} \equiv w = w^{(0)} n^{3/4} + w^{(1)} n^{-1/4}$$
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$$F_0 = F_0^{(0)} n^0$$
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Corrected field strengths remain harmonic as long as the Bianchi and flux equations are satisfied trivially

$$\mathrm{d}F_4^{(1)} = 0$$
$$\mathrm{d}(*_6 F_p^{(1)}) = 0$$
$$\mathrm{d}(*_6 H_3^{(1)}) = 0$$

As long as: $s_4 < 2 - s_6$ and $s_3 < 2 - s_6$

NLO corrections

- New set of equations of motion
- Those NLO equations will include 1^{th} order fields, e.g.

$$0 = \nabla^2 \tau^{(1)} - \frac{1}{2}\tau^{(0)}|H_3^{(0)}|^2 + \cdots + \frac{1}{2}\delta$$

Exact 1st order corrections:

$$g^{(1)} \sim \tau^{(1)} \sim w^{(1)} \sim \frac{1}{4\pi r}$$

$$F_2^{(1)} \sim -\frac{1}{\pi} \star_6^{(0)} (\mathrm{d}\frac{1}{r} \wedge \mathrm{d}y^2 \wedge \mathrm{d}y^4 \wedge \mathrm{d}y^6)$$

NLO contribution to the scalar potential

Introduce general warped metric $M_{10} = M_4 \times_w X^6$:

$$V \sim \int \mathrm{d}^6 y \sqrt{|g_6|}\, e^{4A-2\phi} \Bigg(-\mathcal{R}_6 + 12 e^{-2A} (\partial e^A)^2 + 8 e^{-A} \Delta_6 e^A - 4(\partial\phi)^2 + \frac{1}{2}|H_3|^2 + \frac{e^{2\phi}}{2}\Big[F_0^2 + |F_2|^2 + |F_4|^2\Big] - e^{\phi} \sum_I \frac{T_{10}^I}{7} \Bigg)$$

- Smeared potential $V^{(0)} \sim n^{-9/2}$
- (NLO) terms – corrections to the potential:

$$V^{(1)} \sim \left(\mathcal{R}_6^{(1)} + e^{-A}\Delta_6 e^{A^{(1)}} + \mathrm{vol}_{||_I}^{(0)} \wedge \mathrm{d}F_2^{(1)}\right) n^{-9/2}$$

$V^{(1)}$ terms make up total derivatives and vanish.

Corrections from fluxes

The NLO correction comes from flux corrections:

$$|F_p^{(1)}|^2 \sim F_p^{(0)} F_p^{(1)} (g^{-1\,(0)})^p \, n^b$$

Considering the bulk scalar potential, we get:

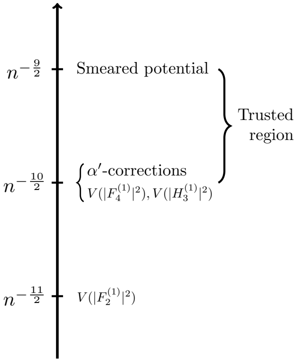
$$V^{(1)} \sim 0^{th}\text{order} \times \left(\frac{1}{2}|H_3^{(1)}|^2 + \frac{e^{2\phi}}{2} \left| F_4^{(1)} \right|^2 - e^{\phi} \sum_I \frac{(T_{10}^I)^{(1)}}{7} \right) n^{-10/2}$$

Thus:

$$V = V^{(0)} n^{-9/2} + V^{(1)} n^{-10/2}$$

Where is the correction $F_2^{(1)} F_2^{(1)}$?

Scaling hierarchy

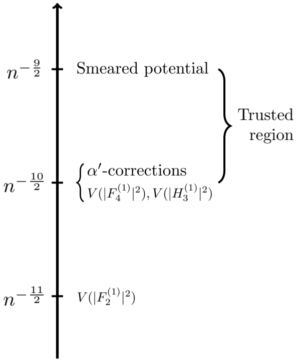


- First α'-corrections: arise at $\alpha'^3 \longrightarrow$ 8-derivative terms

$$V(\alpha'^3) \sim (\dots) \times (g^{(0)^{-4}})(g^{(0)})^{-1}(\partial)^8 g^{(0)} \sim n^{-\frac{10}{2}}$$

- Set fluxes above α'-corrections: $\quad 0 < s_4 < \frac{1}{2} \quad \& \quad 0 < s_3 < 1$

Scaling hierarchy



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$$V(\alpha'^3) \sim (\dots) \times (g^{(0)^{-4}})(g^{(1)})^{-1}(\partial)^8 g^{(1)} \sim n^{-\frac{14}{2}}$$

- Set fluxes above α'-corrections: $\quad 0 < s_4 < \frac{1}{2} \quad \& \quad 0 < s_3 < 1$

Flux quantization

Flux quantization

Flux correction quantized over the same 4- and 3- cycles as smeared ones:

$$\int_i F_4 = \big(\underset{\substack{\uparrow \\ e_i}}{f_{4i}}\, n + \underset{\substack{\uparrow \\ e_i^{(1)}}}{f_{4i}^{(1)}} n^{1-s_4} \big)\,, \qquad \int H_3 = \big(\underset{\substack{\uparrow \\ |h_3| = 1 \text{ or } 2}}{h_3} n^0 + h_3^{(1)} n^{-s_h} \big)$$

with $\{h_3\,, h_3^{(1)} n^{-s_h}\}\,, \{f_{4i}\, n\,,\ f_{4i}^{(1)} n^{1-s_4}\} \ \in \mathbb{Z}$

- $h_3^{(1)} n^{-s_h}$ is not valid with large n expansion

Thus : $\quad H_3^{(1)} = 0$

Next-to-next to leading order

NNLO?

Look for subdominant EoM:

- From the 6d Einstein EoM (one flux corrected):

$$\frac{H^{(1)}_{mpq} H^{(0)\,pq}_{n}}{e^{2\phi^{(0)}}} - \frac{g^{(0)}_{mn}}{2} \frac{|H^{(1)}|^2}{e^{2\phi^{(0)}}} + \frac{1}{3} F^{(1)}_{4\,mpqr} F^{(0)\,pqr}_{4\,n} - 3 \frac{g^{(0)}_{mn}}{4} |F^{(1)}_4|^2 = 0$$

NNLO?

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- From the 6d Einstein EoM (one flux corrected):

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- Solution for $\forall\, i \neq j \in \{1,2,3\}$:

$$e_i\, e^{(1)}_i\, v_i^2 + e_j\, e^{(1)}_j\, v_j^2 = 0 \;\Rightarrow\; e^{(1)}_i = 0 \;\Rightarrow\; F^{(1)}_4 = 0$$

Holds for Junghans flux scaling, no need to generalize:

The NLO corrections to the fluxes $F_4^{(1)}$ vanish.

Axions

Scaling hierarchy including axions

$$(overall)\times \sim \left|F_2 + F_0 B_2\right|^2 + \left|F_4 + C_1 \wedge H_3 + F_2 \wedge B_2 + \frac{1}{2} F_0 \; B_2 \wedge B_2\right|^2$$

$n^{-\frac{9}{2}}$ — DGKT potential

$n^{-\frac{10}{2}}$ —
- α'-corrections
- $F_2^{(1)} F_0 B_2^{(0)}, (F_4 + F_0 B_2^2)^{(0)} F_2^{(1)} B_2^{(0)}, \ldots$

$n^{-\frac{11}{2}}$ — Most NLO corrections

Recap

- Study whether corrections due to backreaction affect the scalar potential
- Imposed scaling conditions to have harmonic $H_3^{(1)}$ and $F_4^{(1)}$ from quantization
- Quantization of H_3, show that relevant flux has to vanish
- NNLO equations of motion show that $F_4^{(1)} = 0$
- $F_2^{(1)}$ as well as $w(y)\partial_m w(y)$ terms are very subdominant to α'-corrections.
- Unsmearing DGKT-CFI and considering harmonic corrections to fluxes seem not to affect the potential above α'-corrections.

Thank you!

Metric and warped 4d action

10d string frame metric

$$\mathrm{d}s^2 = \tau(x)^{-2}\, e^{2A(y)}\, g_{\mu\nu}(x)\, \mathrm{d}x^\mu \mathrm{d}x^\nu + \rho(x)\, g_{mn}(x,y)\, \mathrm{d}y^m \mathrm{d}y^n$$

Bianchi identities:

$$\int \mathrm{d}^4x \sqrt{|g_4|}\Bigg(\frac{M_p^2}{2}\mathcal{R}_4 - V_{\text{part}} \\ - \frac{M_p^2}{4}\left(\mathcal{V}_{6\phi}^{-2}\,(\partial \mathcal{V}_{6\phi})^2 - \frac{1}{2}\partial_\mu g_{mn}\,\partial^\mu g^{mn} + |\partial B_2|^2 + |\partial C_{1,3}|^2 \right)\Bigg) + \mathcal{S}_{F_6}\ ,$$

with

$$V_{\text{part}} = \frac{M_p^2}{2}(2\pi\sqrt{\alpha'})^{-6}\,\mathcal{V}_{6\phi}^{-2} \int \mathrm{d}^6y \sqrt{|g_6|}\, e^{4A-2\phi}\, \Bigg(-\mathcal{R}_6 + 12 e^{-2A}(\partial e^A)^2 \\ + 8 e^{-A}\Delta_6 e^A - 4(\partial\phi)^2 - e^\phi \sum_I \frac{T_{10}^I}{7} + \frac{1}{2}|H_3|^2 \\ + \frac{e^{2\phi}}{2}\left[F_0^2 + \Big|F_2 + F_0 B_2\Big|^2 + \Big|F_4 + C_1\wedge H_3 + F_2\wedge B_2 + \frac{1}{2}F_0\ B_2\wedge B_2\Big|^2 \right]\Bigg)$$

NLO equations of motion

$$\mathsf{d}F_2^{(1)} = 2\sum_i (j_{i3} - \delta_{i3})$$

$$\nabla^2 \tau^{(1)} = -\frac{3}{2}\sum_i (j_{\pi_i} - \delta(\pi_i))$$

$$\nabla^2 w^{(1)} = \frac{1}{2}\frac{w^{(0)}}{\tau^{(0)}}\sum_i (j_{\pi_i} - \delta(\pi_i))$$

$$\tau^{(0)} R^{(1)}_{mn} - 4\frac{\tau^{(0)}}{w^{(0)}}\nabla_m \partial_n w^{(1)} - 2\nabla_m \partial_n \tau^{(1)} = \sum_i \left(\frac{1}{2} g^{(0)}_{mn} - \Pi^{(0)}_{i,mn}\right)(j_{\pi_i} - \delta(\pi_i))$$

Critical points and masses

Stabilization points:

- Subvolumes and dilaton (saxions)

$$v_i = \frac{1}{|e_i|}\sqrt{\frac{5}{3}\left|\frac{e_1 e_2 e_3}{\kappa m_0}\right|}, \quad e^{\phi} = \frac{5}{4\sqrt{2}}\frac{|p|}{\sqrt{\kappa\, v_1 v_2 v_3 |m_0|}},$$

- Axions

$$b_i = \xi = 0$$

The conformal dimensions are related to the masses via

$$\Delta = \frac{1}{2}\left(3 \pm \sqrt{9 + 12 m^2 \frac{M_p^2}{|V|_0|}}\right)$$

and are integers:

$$\Delta_{sax} = \Big(10, 6, 6, 6\Big), \qquad \Delta_{ax} = \begin{cases} \Big(11, 5, 5, 5\Big) & \text{for } s_1 s_2 s_3 = -1 \\ \Big(2, 8, 8, 8\Big) & \text{for } s_1 s_2 s_3 = 1 \end{cases}$$

General equations of motions

Bianchi identities:

$$\mathsf{d}H_3 = 0$$
$$\mathsf{d}F_0 = 0$$
$$\mathsf{d}F_2 - H_3 \wedge F_0 = \sum_I \frac{T_{10}^I}{7} \mathsf{vol}_{\perp_I}$$
$$\mathsf{d}F_4 - H_3 \wedge F_2 = 0$$

More specificaly:

$$\mathsf{d}(*_6^{(0)} F_2^{(1)}) = 0$$
$$\mathsf{d}(*_6^{(0)} F_4^{(1)}) n^{-s_4} + H_3^{(0)} \wedge *_6^{(0)} F_6^{(1)} n^{s_6 - 2} = 0$$
$$\mathsf{d}(*_6^{(0)} F_6^{(1)}) = 0$$
$$\mathsf{d}({\tau^{(0)}}^2 *_6^{(0)} H_3^{(1)}) n^{-s_3} - F_4^{(0)} \wedge *_6^{(0)} F_6^{(1)} n^{s_6 - 2} = 0$$

General equations of motions

The dilaton equation

$$
\begin{aligned}
0 = & 2e^{-\phi-2A}\mathcal{R}_4^S + 2e^{-\phi}\mathcal{R}_6 - e^{-\phi}|H_3|^2 + \frac{T_{10}}{7} \\
& - 24e^{-\phi-2A}(\partial e^A)^2 - 16e^{-\phi-A}\Delta_6 e^A - 8\Delta_6 e^{-\phi} - 32e^{-A}\partial_m e^A \partial^m e^{-\phi}
\end{aligned}
$$

The 4d Einstein

$$
\begin{aligned}
0 = & - e^{-2\phi-2A}\mathcal{R}_4^S - \frac{1}{2}e^{-2\phi}|H|^2 - \frac{1}{4}\sum_{q=0}^{6}(q-1)|F_q|^2 + \frac{e^{-\phi}}{4}\frac{T_{10}}{7} + 4e^{-2\phi-A}\Delta_6\, e^A \\
& + 12e^{-2\phi-2A}(\partial e^A)^2 + e^{-\phi}\Delta_6\, e^{-\phi} + (\partial e^{-\phi})^2 + 12e^{-\phi-A}\partial_m e^A \partial^m e^{-\phi}
\end{aligned}
$$

The trace-reversed 6d Einstein

$$
\begin{aligned}
0 = & - e^{-2\phi}\mathcal{R}_{mn} + \frac{e^{-2\phi}}{4}H_{mpq}H_n{}^{pq} + \frac{1}{2}\left(F_{2\ mp}F_{2\ n}{}^{p} + \frac{1}{3!}F_{4\ mpqr}F_{4\ n}{}^{pqr}\right) \\
& + \frac{e^{-\phi}}{2}T_{mn} - \frac{g_{mn}}{8}e^{-2\phi}|H|^2 + \frac{g_{mn}}{16}\left(\sum_{q=0}^{6}(1-q)|F_q|^2 + 8|F_6|^2\right) - g_{mn}\frac{7}{16}e^{-\phi}\frac{T_{10}}{7} \\
& + \frac{g_{mn}}{4}\left(e^{-\phi}\Delta_6\, e^{-\phi} + (\partial e^{-\phi})^2 + 4e^{-\phi-A}\partial_p e^A \partial^p e^{-\phi}\right) \\
& + 4e^{-2\phi-A}\nabla_{nm} e^A + 2e^{-\phi}\nabla_m \partial_n e^{-\phi} - 2\partial_m e^{-\phi}\partial_n e^{-\phi}
\end{aligned}
$$

4d theory at NLO - Axions

$$
\begin{aligned}
&|F_2 + F_0 B_2|^2 \\
&\sim \left((F_0 B_2^{(0)})^2 \, n^{-9/2} + 2F_2^{(1)} F_0 B_2^{(0)} \, n^{-10/2} + 2F_0^2 B_2^{(0)} B_2^{(1)} \, n^{-10/2+b} \right)
\end{aligned}
$$

- 1st term: standard LO term
- 2nd term: same level as α'-corrections
- 3rd term: below α'-corrections for $b < 0$

$$
\begin{aligned}
&\left| F_4 + C_1 \wedge H_3 + F_2 \wedge B_2 + \frac{1}{2} F_0 \; B_2 \wedge B_2 \right|^2 \\
&\sim \left(F_4^{(0)} + \frac{1}{2} (F_0 \, B_2^2)^{(0)} \right) \left(F_4^{(1)} n^{1/2 - s_4} + F_2^{(1)} \, B_2^{(0)} n^0 + F_0^{(0)} \, B_2^{(0)} B_2^{(1)} n^b \right) n^{-10/2}
\end{aligned}
$$

- 1st term: above α'-corrections for $0 < s_4 < 1/2$
- 2nd term: same level as α'-corrections
- 3rd term: below α'-corrections for $b < 0$

4d warped action

$$\mathcal{S}_{F_6} = -\frac{M_p^2}{4}\int \mathsf{d}^4x\sqrt{|g_4|}\ \frac{(2\pi\sqrt{\alpha'})^6}{\mathcal{V}_{6\phi}^2\int \mathsf{d}^6y\sqrt{|g_6|}e^{-4A}}\left((2\pi\sqrt{\alpha'})^{-6}\ \int_6 (F_6+\dots)\right)^2$$

$$F_6+\dots = F_6 + C_3\wedge H + F_4\wedge B_2 + C_1\wedge H_3\wedge B_2 + \frac{1}{2}F_2\wedge B_2\wedge B_2 + \frac{1}{6}F_0 B_2^3$$

4d theory at NLO - Axions

F_6-type contributions at NLO, we get schematically

$$\mathcal{S}_{F_6} \sim n^{-10/2-1} \times \left(C_3 \wedge H + F_4 \wedge B_2 + \frac{1}{6} F_0 \; B_2 \wedge B_2 \wedge B_2 \right)^{(0)} \times \left(F_6^{(1)} \, n^{s_6} + sub. \right)$$

- $F_6^{(1)}$ gives linear B_0 and C_3 terms above α'-corrections level
- This changes the DGKT vacua

J Scaling ($s_4 = 1/2$ and $s_6 = 1$) :

- Liner B_0 and C_3 at α'-corrections level.
- non-trivial $F_6^{(1)}$ from the flux equation.

General scaling

- Liner 0^{th} order axions B_0 and C_3 above α'-corrections.
- $F_6^{(1)}$ vanish for $s_6 > 1$