Exceptional Field Theory to study Supergravity compactifications

Bastien Duboeuf

Laboratoire de Physique (ENS de Lyon)

April 9th 2024

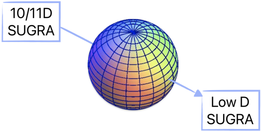
Based on [2212.01135][2306.11789][2311.00742][24 ▪▪ , ▪▪▪▪]
with Michele Galli, Emanuel Malek, Henning Samtleben.

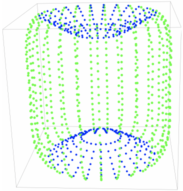
ENS DE LYON

Lab PHYS ENS de LYON

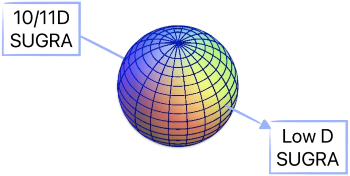
Plan

- Motivation
- Tools
  - Consistent Truncations
  - Exceptional Field Theory
- Examples
  - New $\mathrm{SO}(7)$ solution of 11d Supergravity
  - Spectrum of $\mathrm{AdS}_4\times\mathrm{S}^7$ squashed sphere
  - n-point couplings fo 10d/11d Sugra
  - Cubic couplings on $\mathrm{AdS}_5\times\mathrm{S}^5$





- Compactifications of Supergravities : $\mathcal{M}_{10/11} = \mathcal{M}_{int} \times \mathcal{M}_{ext}$.



- New 10/11 d solutions describing compactification scenarios.
- Expand fluctuations in harmonics

$$\phi(x, y) = \sum_{\Sigma} \phi^{\Sigma}(x) \mathcal{Y}^{\Sigma}$$

- Kaluza-Klein fluctuations dynamics $\rightarrow$ lower dimensional theory
- Infinitely many fields (KK towers) $\rightarrow$ spectrum, *n*-point functions

- Spectrum : diagonalize Laplacians on internal manifold, disentangle mass eigenstates from different higher-dimensional origin, flux compactifications : higher-dimensional p-forms, ... $\rightarrow$ (super)-symmetries help [Salam, Strathdee]
- Cubic couplings : lengthy brut force calculations, non-linear field redefinitions ... [Arutyonov, Frolov, Lee, Minwalla, Rangamani]
- Important problem : study stability, for holography, ...
- Question : what about less symmetrical backgrounds ?

> Exceptional Field Theories (ExFT) offer new tools to access vacua with lower or no (super)-symmetries.

- Compactification of Maximal Supergravity on n-Tori : $\mathrm{SL}(n) \to \mathrm{E}_{n(n)}$. [Cremmer,Julia]

Exceptional Group depending on the dimension

Dimension	Global Symmetry	Maximal Compact Subgroup
5	$E_{6(6)}$	Usp(8)
4	$E_{7(7)}$	SU(8)
3	$E_{8(8)}$	SO(16)

- Exceptional Field Theories
  - reformulation of 10/11-d Supergravities "exceptionally" covariant.
- Example : $E_{7(7)}$-ExFT : reformulation of 11-d Supergravity.

Lab PHYS ENS de LYON

- E_7 Lagrangian [Hohm, Samtleben]

$$\mathcal{L}_{E_7} = \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}_{\mu\nu}{}^{N} + e^{-1} \mathcal{L}_{top} - \mathcal{V}(\mathcal{M}, g)$$

  - M,N,. . .= 1,. . .,56 fundamental indices of E_7
  - 56 vectors $\rightarrow \mathcal{A}_\mu{}^M$
  - Scalar target space $E_7/Usp(8) \rightarrow \mathcal{M}_{MN}$ generalized metric : encodes internal geometry and fluxes
- $\mathcal{L}_{E_7} \equiv \mathcal{L}_{\mathrm{SUGRA}11D}$
- Split 4 external (x) + 7 internal (y) coordinates $\rightarrow \{x^\mu, y^m\}$
- $y^m \hookrightarrow Y^M$ coordinates subject to a section constraint
- $\mathcal{V}$ generalized scalar curvature for $\mathcal{M}$ [Coimbra, Strickland-Constable, Waldram]

Consistent Truncation

- Consistent Truncation : truncate fields to lower dimensional theory.
- Can harmlessly put fields to zero.

$$\mathcal{L} = g_{LLH}\phi_L^2\phi_H \longrightarrow \begin{cases} \Box\phi_L = g_{LLH}\phi_L\phi_H \\ \Box\phi_H = g_{LLH}\phi_L^2 \end{cases}$$

light modes better not source the heavy ones.

- Low dimensional solution $\rightarrow$ higher dimensional solution
- Remaining scalars $\rightarrow$ background geometry, fluxes, ...

- ✗ The dangerous couplings are those which are linear in the "heavy" modes.
- ✗ Consistent Truncations imply the vanishing of infinitely many couplings.
- ✗ and there is much more . . .

Warning

Consistent truncations are not effective theories. You throw away fields which masses are comparable with the ones you keep.

- Parametrize

$$\mathcal{M}_{MN}(x,Y) = U_M{}^A(Y)U_N{}^B(Y)M_{AB}(x)$$
$$\mathcal{A}(x,Y)_\mu{}^M = \rho(Y)^{-1}(U(Y)^{-1})_A{}^M A(x)_\mu{}^A$$

$M_{AB}(x)$ and $A(x)_\mu{}^A$ fields of 4 dimensional $\mathcal{N} = 8$ Supergravity

with $U \in E_{7(7)}$ globally well defined vielbein (Generalised parallelizable) and $\rho(Y)$ a scale factor.

- $M_{AB}(x)$ can be re-written in term of an internal vielbein

$$M_{AB}(x) = \mathcal{V}_A{}^{\underline{A}}(x)\mathcal{V}_B{}^{\underline{B}}(x)\delta_{\underline{AB}}\,, \text{with} \quad \mathcal{V}_A{}^{\underline{A}}(x) = exp(\phi_s(x)^\alpha \mathbb{T}_\alpha)_A{}^{\underline{A}}$$

ϕ_s 70 scalars of 11d SUGRA

- Consistent Truncation $\longrightarrow$ Y dependence in the EOM factors out

$$EOM_{11D}(x,Y) = U(Y)(EOM_{4D}(x))U(Y)^t$$

- ExFT techniques : find solutions, spectroscopy ... theories $\subset \mathcal{N} = 8$ SUGRA

- Look for background solution with G symmetry.
- Within 11d theory $\rightarrow$ keep G-singlets : consistent truncation.
- Concrete realization : $\mathcal{M}_{11d} = \mathrm{AdS}_{11-k} \times \mathrm{S}^k$ with G = isometries of S^k.
- S^k can be written as G/H
  - ✗ L a coset representative of $\mathrm{S}^k \rightarrow \delta_\Lambda L(y) = \Lambda L(y) - L(y) h_\Lambda(y)$, $\Lambda \in G$ and $h_\Lambda \in H$
  - ✗ $W(x) = \mathcal{C}_H(E_{7(7)}) \Rightarrow W(x)h(y) = h(y)W(x)$

$$\Rightarrow L(y)W(x)L^{-1}(y) \text{ G singlets}$$

- Embedding into ExFT

$$\mathcal{V}(x, Y) = U_0(Y) exp(\phi_s(x)^{\alpha\Sigma}\mathbb{T}_\alpha \mathcal{Y}^\Sigma) = U_0(Y) L(y)W(x)L^{-1}(y)$$

with ϕ_s G-singlets, $\mathbb{T}_\alpha$ $\mathrm{E}_{7(7)}$ generators and $\mathcal{Y}^\Sigma$ harmonics.

- From this ansatz [Cassani,Petrini,Josse,Waldram]

$$\mathcal{M}_{scalar} = \mathcal{C}_H\left(\frac{E_{7(7)}}{SU(8)}\right)$$

$$\mathcal{M}_{scalar} = \mathcal{C}_H\left(\frac{E_{7(7)}}{SU(8)}\right)$$

- G-singlets in full theory $\equiv$ H-singlets in $\mathrm{E}_{7(7)}$
- Extends previous ExFT techniques to new consistent truncations of 11d SUGRA
- Find new solutions, compute spectra and couplings ...

Sphere	Coset	SUSY	Target Space
S^7	$\frac{Usp(4)}{SU(2)}$	$\mathcal{N}=4$	$\frac{SO(6,3)}{SO(6)\times SO(3)} \times \frac{SL(2)}{SO(2)}$
S^7	$\frac{SU(4)}{SU(3)}$	$\mathcal{N}=2$	$\frac{SL(2)}{SO(2)} \times \frac{SU(2,1)}{U(2)}$
S^7	$\frac{Usp(4)\times SU(2)}{SU(2)\times SU(2)}$	$\mathcal{N}=1$	$\frac{SL(2)}{SO(2)} \times \frac{SL(2)}{SO(2)}$
S^7	$\frac{SO(7)}{G_2}$	$\mathcal{N}=1$	$\frac{SL(2)}{SO(2)}$
S^6	$\frac{G_2}{SU(3)}$	$\mathcal{N}=2$	$\frac{SL(2)}{SO(2)} \times \frac{SU(2,1)}{U(2)}$
S^5	$\frac{SU(3)}{SU(2)}$	$\mathcal{N}=4$	$\frac{SO(5,2)}{SO(5)\times SO(2)} \times \mathbb{R}^+$
S^3	$SU(2)$	$\mathcal{N}=2$	$\frac{SL(5)}{SO(5)}$
$S^3\times S^3$	$SU(2)\times SU(2)$	$\mathcal{N}=8$	$\frac{E_{7(7)}}{SU(8)}$

Finding new solutions

[Malek, Galli, Samtleben, Duboeuf]

- Look for $4d$ SO(7) solutions in 11-dimensional Supergravity
- Write $S^7 \simeq S^6 \times_\omega I$ with $S^6 = \frac{SO(7)}{SO(6)}$
- $11d$ uplift

$$\mathcal{V}(x, Y) = U_0(Y) L(y) W(x, \omega)$$

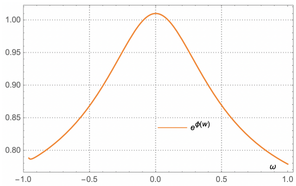
with $L(y) \in \frac{SO(7)}{SO(6)}$, $W(x, \omega) \in \frac{SL(2)}{SO(2)} \times \mathbb{R}^+$ 3 scalars depending on x and ω.

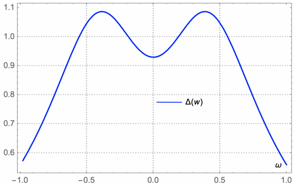
- $11d$ metric is of the form

$$ds^2 = \Delta^{-1} ds^2_{(4)} + e^{3\phi(\omega)}(1-\omega^2)^{-1}\Delta^{-1} d\omega^2 + e^{-\phi(\omega)/2}(1-\omega^2)\Delta^{1/2} ds^2_{S^6}$$

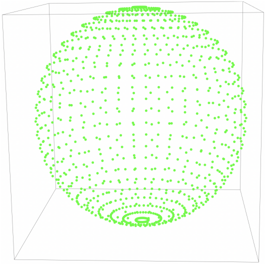
- $\Delta(\omega)$ combination of all the scalars

- Extremize scalar potential $\Rightarrow$ differential equations in ω.
- Recovers known (constant) SO(7) solutions [de Wit, Nicolai, Warner]
- Solve numerically : new solution

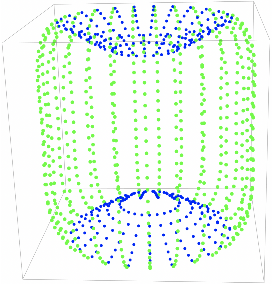




- Corresponds to squashing the round sphere



$\longrightarrow$



Kaluza-Klein Spectroscopy

[Malek, Galli, Samtleben, Duboeuf]

- Fluctuations around backgrounds given by a consistent truncation
- Need harmonics $\mathcal{Y}^\Sigma \to$ expand fluctuations. Harmonics $\in \mathrm{Rep}(SO(8))$, the maximally symmetric point of the internal manifold (topology), here S^7

$$\phi(x, Y) = \sum_\Sigma \phi^\Sigma \mathcal{Y}^\Sigma$$

- 4-dimensional $\mathcal{N} = 8$ SUGRA $SO(8)$ theory : $\mathcal{Y}^\Sigma \in \mathrm{Rep}(SO(8))$ ($\Sigma = ((i_1 \dots i_n))$, completely symmetric and traceless representation, ie $[n, 0, 0, 0]$).
- Use harmonics of round S^7 for all others deformed spheres

- ExFT : nice description of the linearized fluctuations around $11d$ backgrounds.
- Excite fluctuations $\rightarrow$ masses of Kaluza-Kleins modes.

$$\mathcal{M}_{MN}(x,Y) = U_M{}^A(Y) U_N{}^B(Y) \left(M_{AB}(x) + \sum_\Sigma \mathcal{Y}^\Sigma \phi(x)^{\alpha\Sigma} \mathbb{T}_{\alpha A}{}^C \delta_{CB} \right)$$

$$\mathcal{A}(x,Y)_\mu{}^M = \rho(Y)^{-1} (U(Y)^{-1})_A{}^M \sum_\Sigma \mathcal{Y}^\Sigma A(x)_\mu{}^{A\Sigma}$$

$$g(x,Y)_{\mu\nu} = \rho(Y)^{-2} \left(\eta_{\mu\nu} + \sum_\Sigma \mathcal{Y}^\Sigma h(x)_{\mu\nu}{}^\Sigma \right)$$

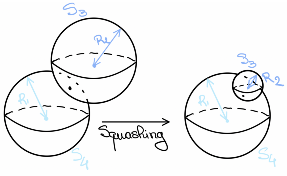
α adjoint index of E_7 ; A,B, ... fundamental indices of E_7 ; $\mathbb{T}$ generators of E_7

- New structure $\rightarrow$ lowest KK modes $\otimes$ scalar harmonics
- Use only of scalar harmonics

Lab PHYS ENS de LYON

- 11d SUGRA on $AdS_4 \times S^7$: masses on round S^7 known since the 80's [Englert, Nicolai, Sezgin, Casher, Rooman]

- S^7 is an S^4 fibered with S^3



- Other solution : the squashed S^7_s [Karlsson, Duff, Nilsson, Pope]

$$S^7 \cong \frac{SO(8)}{SO(7)} \cong \frac{Sp(2) \times Sp(1)_0}{Sp(1)_L \times Sp(1)_D}$$

- From $\mathcal{N} = 8$ to $\mathcal{N} = 1$ vacuum.
- beyond standard consistent truncations : deformation triggered by scalars at level 2 and 4 of the KK towers.
- Use ExFT techniques and the ansatz

$$\mathcal{V}(x, Y) = U_0(Y)L(y)W(x, \omega)$$

Lab PHYS ENS de LYON

- The mass matrix for scalars reads

$$\left(\mathbb{M}_{\text{spin}-0}\right)^{\alpha}{}_{\beta} = \left(\mathbb{M}^{(0)}_{\text{spin}-0}\right)^{\alpha}{}_{\beta} + \left(\mathbb{N}^{\alpha}{}_{\beta}{}^{C} - \mathbb{N}_{\beta}{}^{\alpha C}\right)\partial_C + \partial_C \mathbb{N}^{\alpha}{}_{\beta}{}^{C} + \delta^{\alpha}_{\beta}\,\mathbb{M}_{\text{spin}-2}$$

$$-\frac{1}{24}\Pi^{\alpha}{}_{A}\,\Pi^{A}{}_{\beta}\,,$$

$$\left(\mathbb{M}^{(0)}_{\text{spin}-0}\right)^{\alpha}{}_{\beta} = X_{AE}{}^{F}X_{BF}{}^{E}\,(\mathbb{T}^{\alpha}\mathbb{T}_{\beta})_{A}{}^{B} + \frac{1}{7}\left(X_{AE}{}^{F}X_{BE}{}^{F} + X_{EA}{}^{F}X_{EB}{}^{F} + X_{EF}{}^{A}X_{EF}{}^{B}\right)(\mathbb{T}^{\alpha}\mathbb{T}_{\beta})_{A}{}^{B}$$

$$+\frac{2}{7}\left(X_{AC}{}^{E}X_{BD}{}^{E} - X_{AE}{}^{C}X_{BE}{}^{D} - X_{EA}{}^{C}X_{EB}{}^{D}\right)(\mathbb{T}^{\alpha})_{A}{}^{B}\,(\mathbb{T}_{\beta})_{C}{}^{D} + \frac{1}{6}\,X_{A}{}^{\alpha}\,X_{A,\beta}\,,$$

$$\mathbb{N}^{\alpha}{}_{\beta}{}^{C} = -2X_{A}{}^{\alpha}\mathbb{T}_{\beta,C}{}^{A} - 2X_{A\beta}\mathbb{T}^{\alpha}{}_{C}{}^{A} - [\mathbb{T}^{\alpha},\,\mathbb{T}_{\beta}]_{A}{}^{B}\left(X_{CB}{}^{A} + \frac{7}{2}X_{AB}{}^{C}\right)\,.$$

- $\left(\mathbb{M}_{\text{spin}-0}\right)(y)^{\alpha}{}_{\beta}$: y-dependent mass matrix $\rightarrow$ level mixing

- Resolved spectrum organized in long multiplets ($\mathcal{N} = 1$)

$$\bigoplus L_\Delta[J, s] \otimes [p, q, r]$$

with the spin J, the $[p, q, r]$ $Sp(2) \times Sp(1)_0$ representation, $s \in \frac{1}{2}\mathbb{Z}$, and Δ the conformal dimension associated to the corresponding state in the dual CFT.

- Universal formula

$$\Delta(J, s, p, q, r) = 1 + \frac{5}{3}s + \frac{1}{3}\sqrt{(3J + 2s^2)^2 + 5\mathcal{C}_3(p, q, r)}$$

with $\mathcal{C}_3$ a combinations of Casimirs operators.

- With $L_\Delta[J, s] \equiv L_\Delta[J] \otimes \{-s, -s+1, \ldots, s\}$ we have for example

$$[k, q, k]_{k\geq 2, q\geq 2} : \quad L_\Delta[\frac{3}{2}, 0] \oplus L_\Delta[\frac{3}{2}, \frac{1}{2}] \oplus L_\Delta[\frac{1}{2}, \frac{1}{2} \otimes \frac{1}{2}] \oplus L_\Delta[0, \frac{1}{2} \otimes 1]$$

- Partial results for spectrum on S^7_s [Karlsson, Duff, Nilsson, Pope] $\rightarrow$ now complete answer.

n-point couplings

[Malek, Samtleben, Duboeuf]

- Extend previous techniques to n-point couplings. E.g. : cubic couplings.

$$\mathcal{L} \sim g_{I_1 I_2 I_3} \phi^{I_1} \phi^{I_2} \phi^{I_3}$$

- Focus on $AdS_5 \times S^5 \longrightarrow E_{6(6)}$-ExFT.
- Holography : three point functions $\langle \mathcal{O}_{I_1} \mathcal{O}_{I_2} \mathcal{O}_{I_3} \rangle$, with $\mathcal{O}_I = \mathcal{O}^{i_1 \cdots i_3} = \mathrm{Tr}[X^{i_1} \ldots X^{i_n}]$ [Lee, Minwalla, Rangamani, Seiberg, Arutyunov, Frolov]
- Some couplings on $AdS_5 \times S^5$ known and matched with $\mathcal{N} = 4$ SYM $\rightarrow$ lengthy

$$G(s^{I_1}, s^{I_2}, s^{I_3}) = \frac{\pi^3}{(1/2\Sigma + 2)! 2^{1/2(\Sigma - 2)}} \frac{k_1! k_2! k_3!}{\alpha_1! \alpha_2! \alpha_3!} \frac{128\Sigma((1/2\Sigma)^2 - 1)((1/2\Sigma)^2 - 4)\alpha_1 \alpha_2 \alpha_3}{(k_1 + 1)(k_2 + 1)(k_3 + 1)} c_{I_1 I_2 I_3} s^{I_1} s^{I_2} s^{I_3}$$

with $I_i = (\!(i_1 \ldots i_{k_i})\!)$, $\Sigma = k_1 + k_2 + k_3$, $\alpha_1 = 1/2(k_2 + k_3 - k_1)$ and so on, $c_{I_1 I_2 I_3}$ the unique $SO(6)$ invariant.

Use ExFT techniques to extract universal formulas for cubic couplings

$$\mathcal{G}(\phi^{\alpha\Sigma}\phi^{\beta\Omega}\phi^{\gamma\Gamma}) = \phi^{\alpha\Sigma}\phi^{\beta\Delta}\phi^{\gamma\Gamma}\left(XX_{\alpha\beta\gamma,\Sigma\Delta\Gamma} + X\mathcal{T}_{\alpha\beta\gamma,\Sigma\Delta\Gamma} + \mathcal{T}\mathcal{T}_{\alpha\beta\gamma,\Sigma\Delta\Gamma}\right)$$

with $X_{AB}{}^{C}$ the embedding tensor and $\mathcal{T}_{A\Sigma}{}^{\Omega}$ SO(6) generators.

- Schematically, cubic couplings look like

$$S_{cubic} \sim \int dy \phi^{\alpha\Sigma}\phi^{\beta\Omega}\phi^{\gamma\Gamma} g_{\alpha\Sigma,\beta\Omega,\gamma\Gamma}{}^{\Sigma'\Omega'\Gamma'}\mathcal{Y}_{\Sigma}\mathcal{Y}_{\Omega}\mathcal{Y}_{\Delta}$$

  ϕ any scalar field
- introducing

$$\int dy \mathcal{Y}_{\Sigma}\mathcal{Y}_{\Omega}\mathcal{Y}_{\Delta} = c_{\Sigma\Omega\Delta}$$

  we can rewrite

$$S_{cubic} \sim \phi^{\alpha\Sigma}\phi^{\beta\Omega}\phi^{\gamma\Gamma} g_{\alpha\Sigma,\beta\Omega,\gamma\Gamma}{}^{\Sigma'\Omega'\Gamma'} c_{\Sigma'\Omega'\Gamma'}$$

$$S_{cubic} \sim \phi^{\alpha\Sigma}\phi^{\beta\Omega}\phi^{\gamma\Gamma} g_{\alpha\Sigma,\beta\Omega,\gamma\Gamma}{}^{\Sigma'\Omega'\Gamma'} c_{\Sigma'\Omega'\Gamma'}$$

- Crucial point : no level mixing induced by $g_{\alpha\Sigma,\beta\Omega,\gamma\Gamma}{}^{\Sigma'\Omega'\Gamma'}$
- Up to permutation $\Sigma'\Omega'\Gamma' \equiv \Sigma\Omega\Gamma$

Non vanishing couplings between $\phi^{\alpha\Sigma}\phi^{\beta\Omega}\phi^{\gamma\Gamma}$

$\Leftrightarrow$

$c_{\Sigma\Omega\Delta}$ is non-vanishing

- Let $\Sigma = ((i_1 \dots i_{n_1}))$, $\Omega = ((j_1 \dots j_{n_2}))$ and $\Gamma = ((k_1 \dots k_{n_3}))$, and $\alpha_1 = \frac{1}{2}(n_3 + n_2 - n_1)$ (similar for α_2 and α_3)
- Condition of non vanishing c-symbol

$$\alpha_i \in \mathbb{N}$$

- Generalize previous arguments to n-point couplings for $s^{I_1} \in [n+2,0,0]$: $\mathcal{G}(s^{I_1},\dots,s^{I_n})$
- We have : $s^I \in \phi^{ab,\Sigma} \Rightarrow |I| = |\Sigma| + 2$
- From group theory

$$|I_j| \leq \sum_{i\neq j} |I_i|, \quad \forall j$$

- Therefore from ExFT

$$|\Sigma_j| \leq \sum_{i\neq j} |\Sigma_i|, \quad \forall j \quad \Leftrightarrow \quad |I_j| + 2(n-2) \leq \sum_{i\neq j} |I_i|, \quad \forall j$$

- Put differently

$$\sum_{i\neq j}^{n} |I_i| - |I_j| \leq 2(n-3) \quad \Rightarrow \quad \mathcal{G}(s^{I_1},\dots,s^{I_n}) = 0$$

Lab PHYS ENS de LYON

$$\sum_{i\neq j}^{n} |I_i| - |I_j| \leq 2(n-3) \quad \Rightarrow \quad \mathcal{G}(s^{I_1}, \ldots, s^{I_n}) = 0$$

- The truncation of KK spectrum to $\mathcal{N} = 8$ SUGRA is consistent

$$I_1 > 2 \text{ and } |I_{i\geq 2}| = 2 \quad \Rightarrow \quad \mathcal{G}(s^{I_1}, s^{I_2}, \ldots, s^{I_n}) = 0$$

- Can be generalized to scalars living in [n-2,0,0] or $[n,0,0]_{\pm}$, and fields with spin-1 and spin-2. Eg : $t^I \in [n-2,0,0]$

$$\left(\sum_j |I_j| + \sum_{l\neq i} |J_l|\right) - |J_i| \leq 2(m-n+1) \quad \Rightarrow \quad \mathcal{G}(s^{I_1}, \ldots, s^{I_m}, t^{J_1}, \ldots, t^{J_n}) = 0$$

- Can be generalized to $AdS_4 \times S^7$ and $AdS_7 \times S^4$ with R-symmetry SO(8) and SO(5).
- Proof of the old conjectures of [D'Hoker,Pioline] for the vanishing of near-extremal correlators between chiral primaries.

Cubic Couplings in Exceptional Field Theory

$$\mathcal{G}(\phi^{\alpha\Sigma}\phi^{\beta\Omega}\phi^{\gamma\Gamma}) = \phi^{\alpha\Sigma}\phi^{\beta\Delta}\phi^{\gamma\Gamma}\Big(XX_{\alpha\beta\gamma,\Sigma\Delta\Gamma} + X\mathcal{T}_{\alpha\beta\gamma,\Sigma\Delta\Gamma} + \mathcal{T}\mathcal{T}_{\alpha\beta\gamma,\Sigma\Delta\Gamma}\Big)$$

- For $AdS_5 \times S^5$ reduces to

$$\begin{aligned}\mathcal{G}(\phi,\phi,\phi) = \phi^{\alpha\Sigma}\phi^{\beta\Delta}\phi^{\gamma\Gamma}\Bigg[& -\frac{1}{6}c^{\Sigma\Delta\Gamma}X_{\underline{AB}}{}^{\underline{C}}X_{\underline{DC}}{}^{\underline{B}}\mathbb{T}_{\beta\gamma\alpha\underline{A}}{}^{\underline{D}} \\ & +\mathcal{T}_{\underline{B}}{}^{\Sigma\Lambda}c^{\Lambda\Delta\Gamma}X_{\underline{AC}}{}^{\underline{D}}\mathbb{T}_{[\alpha\gamma]\underline{D}}{}^{\underline{C}}\mathbb{T}_{\beta\underline{B}}{}^{\underline{A}} \\ & +\mathcal{T}_{\underline{B}}{}^{\Sigma\Lambda}\mathcal{T}_{\underline{A}}{}^{\Lambda\Omega}c^{\Omega\Delta\Gamma}\left[6\mathbb{T}_{\gamma\alpha\beta\underline{B}}{}^{\underline{A}} - \frac{1}{2}\mathbb{T}_{\gamma\underline{B}}{}^{\underline{A}}\kappa_{\alpha\beta}\right]\Bigg]\end{aligned}$$

with $\mathcal{T}$ generators of $SO(6)$ in the representation $[n,0,0]$, $\mathbb{T}$ generators of $SO(6)$ in the fundamental representation.

- For eg :

$$\mathcal{G}(s^{I_1},s^{I_2},s^{I_3}) = a(n_1,n_2,n_3)\left(\frac{\sigma}{2}+2\right)\left(\frac{\sigma}{2}+1\right)\mathcal{C}^{I_1I_2I_3}\,s^{I_1}s^{I_2}s^{I_3}$$

$$= a(k_1,k_2,k_3)\frac{128\Sigma((1/2\Sigma)^2-1)((1/2\Sigma)^2-4)\alpha_1\alpha_2\alpha_3}{(k_1+1)(k_2+1)(k_3+1)}\mathcal{C}_{I_1I_2I_3}s^{I_1}s^{I_2}s^{I_3}$$

Lab PHYS ENS de LYON

- New approach to find new solutions of higher-dimensional SUGRA : inclusion of consistent truncations of the full 11*d* theory.
  - ✗ New SO(7) solution of 11*d* SUGRA with infinitely many scalar fields.
- New tools for the analysis of Kaluza-Klein spectra from ExFT.
  - ✗ Resolved spectrum for the background $AdS_4 \times S^7_s$.
- Extension of ExFT techniques to cubic and higher order couplings.
  - ✗ New hidden structures $\to$ infinitely many vanishing couplings
  - ✗ New cubic couplings for $AdS_5 \times S^5$.
- Access to vacua
  - ✗ with few or no (super-)symmetries.
  - ✗ consistent truncations of the full 11*d* SUGRA.
- Universal patterns in mass spectra & cubic couplings : holography !

Thank you !