
A Unified Framework for Mitigating Foregrounds and Systematic Effects for Tensor-to-Scalar Ratio and Birefringence Angle Measurements

CMB France #5, December 4th 2023

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THE UNIVERSITY OF TOKYO

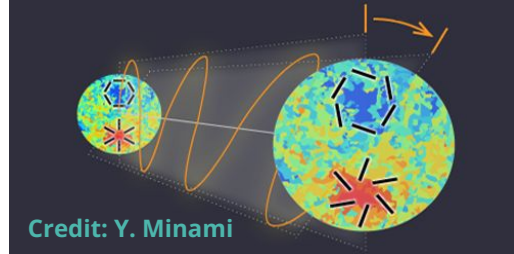


CD3



Cosmic Birefringence

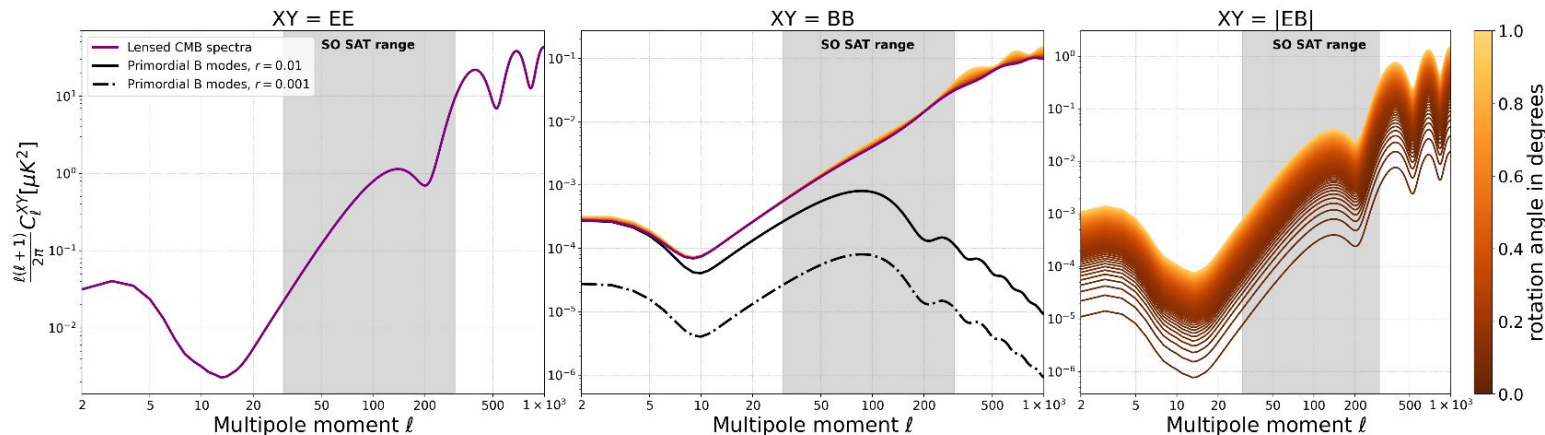
Cosmic Birefringence: rotation of linear polarisation plane of CMB photons



Correlation between E- and B-modes \rightarrow parity violation mechanism (Chern-Simons coupling from axion-like particles)

$$C_{\ell}^{EB,o} = \frac{1}{2} \sin(4\beta) (C_{\ell}^{EE} - C_{\ell}^{BB})$$

Hints of $\beta_b = 0.34^{\circ} \pm 0.09^{\circ}$ (3.6σ) Planck+WMAP data ([Eskilt & Komatsu 2022](#)) based on the Minami & Komatsu method using assumptions about foreground EB correlations for calibration.



The Tensor to Scalar Ratio

Primordial B-modes generated by **tensor perturbations** from **inflation**.

Smoking gun of inflation.

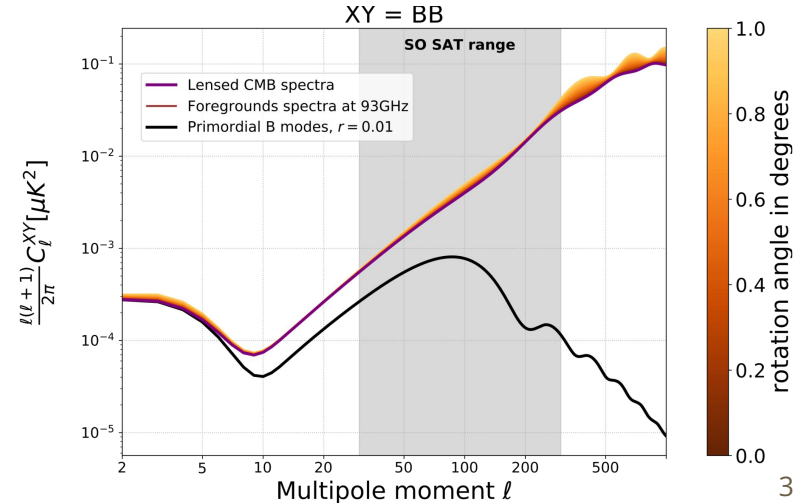
Primordial B-modes amplitude parametrised by: **r**

r constraints $r < 0.032$ (95% C.L.) (**Tristram et al. 2021**)

B-modes are also affected by polarization

angle rotation:

$$C_{\ell}^{BB,o} = \sin^2(2\beta)C_{\ell}^{EE} + \cos^2(2\beta)C_{\ell}^{BB}$$

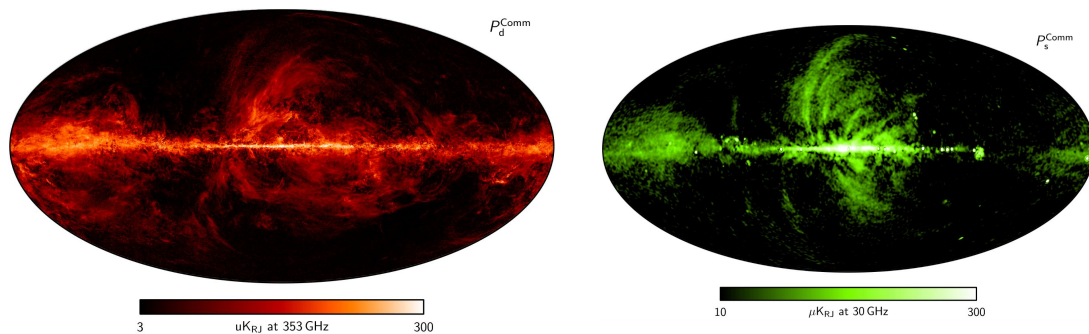


Observation challenges: Galactic Foregrounds

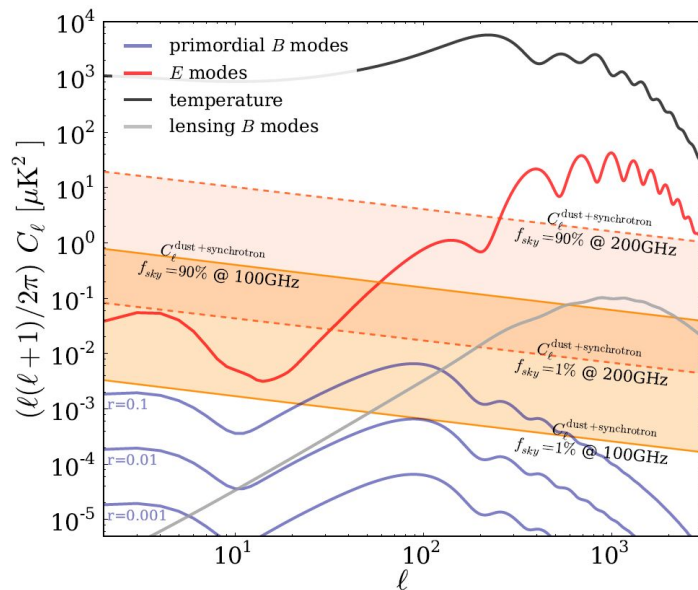
Dust emission: Asymmetric dust grains in the galaxy aligned with magnetic fields.

Synchrotron emissions: charged particles accelerated along Galactic magnetic fields.

No EB correlation measured yet. But physical motivation for it ([Clark et al. 2021](#)).



Credit: Planck Collaboration



Credit: Errard et al. 2016

Observation challenges: Polarisation Angle Miscalibration

Miscalibration of telescope polarisation angle \Rightarrow similar to isotropic birefringence!

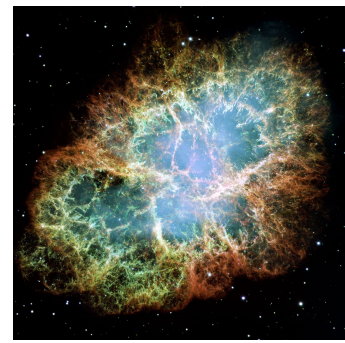
Need a way to lift the degeneracy between birefringence angle and polarization angles

Creates **$E \rightarrow B$ leakage** that **pollute primordial B-modes** $\rightarrow r$

$Q \leftrightarrow U$ mixing at different frequencies will **bias foreground cleaning** \rightarrow more residuals



Polarisation Angle Calibrations



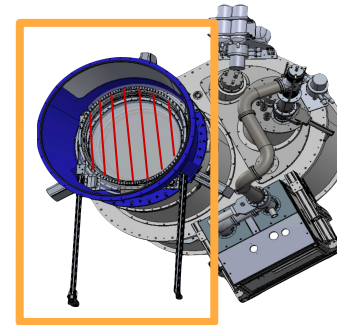
Credit: Nasa/Hubble

Ground based telescope → many polarization angle calibration methods. From observation and hardware:

- Measurements of the crab nebula (tau A) $\sigma(\alpha) \approx 0.27^\circ$ **Aumont et al. 2020**
- Wire-grid $\sigma(\alpha) \lesssim 1^\circ$ **Bryan et al. 2018**
- Drone with polarised source $\sigma(\alpha) \lesssim 0.1^\circ$ **Nati et al. 2017**

Analysis based:

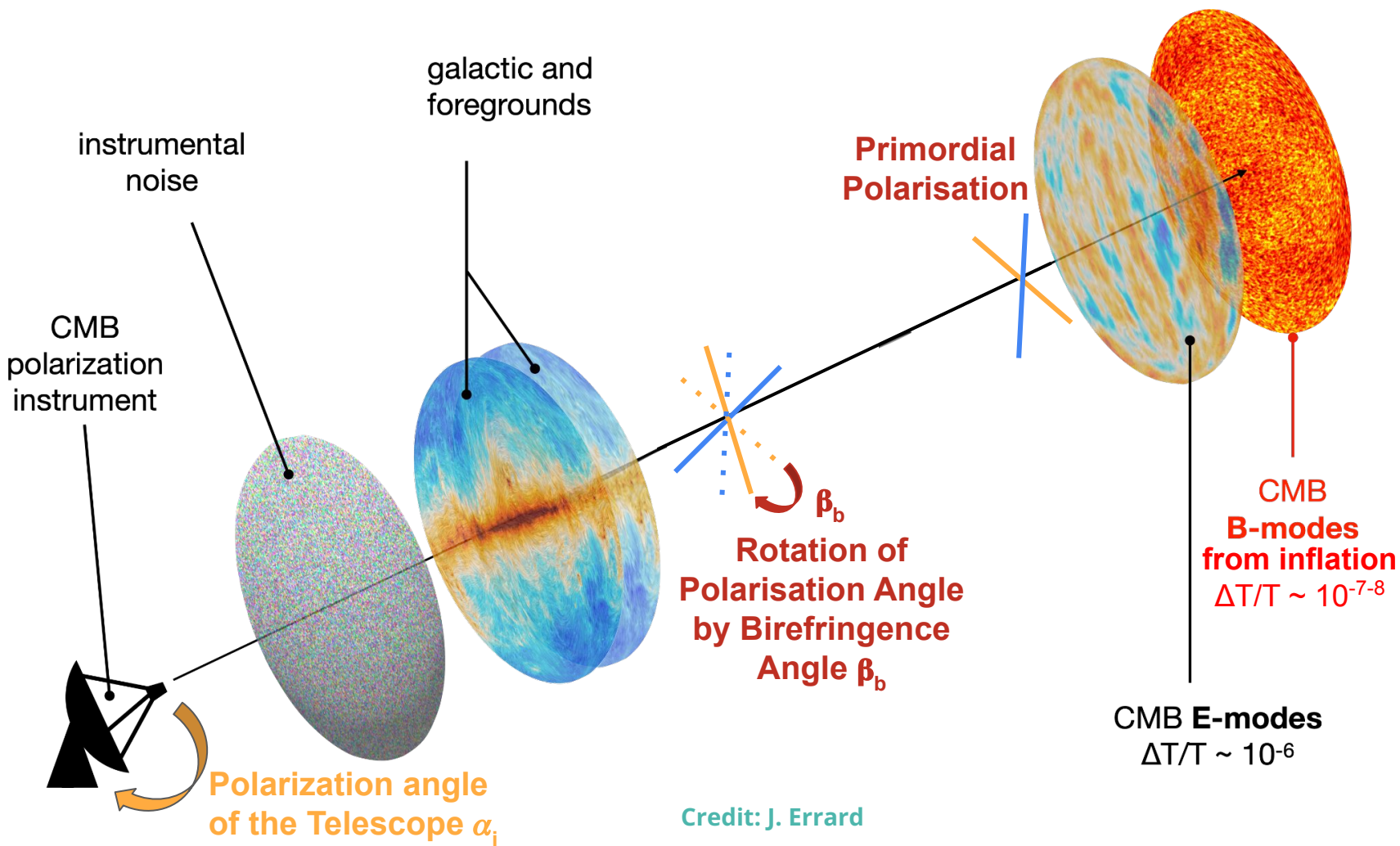
- Self-calibration **Keating et al. 2012**
- Foreground calibration **Minami et al. 2020**

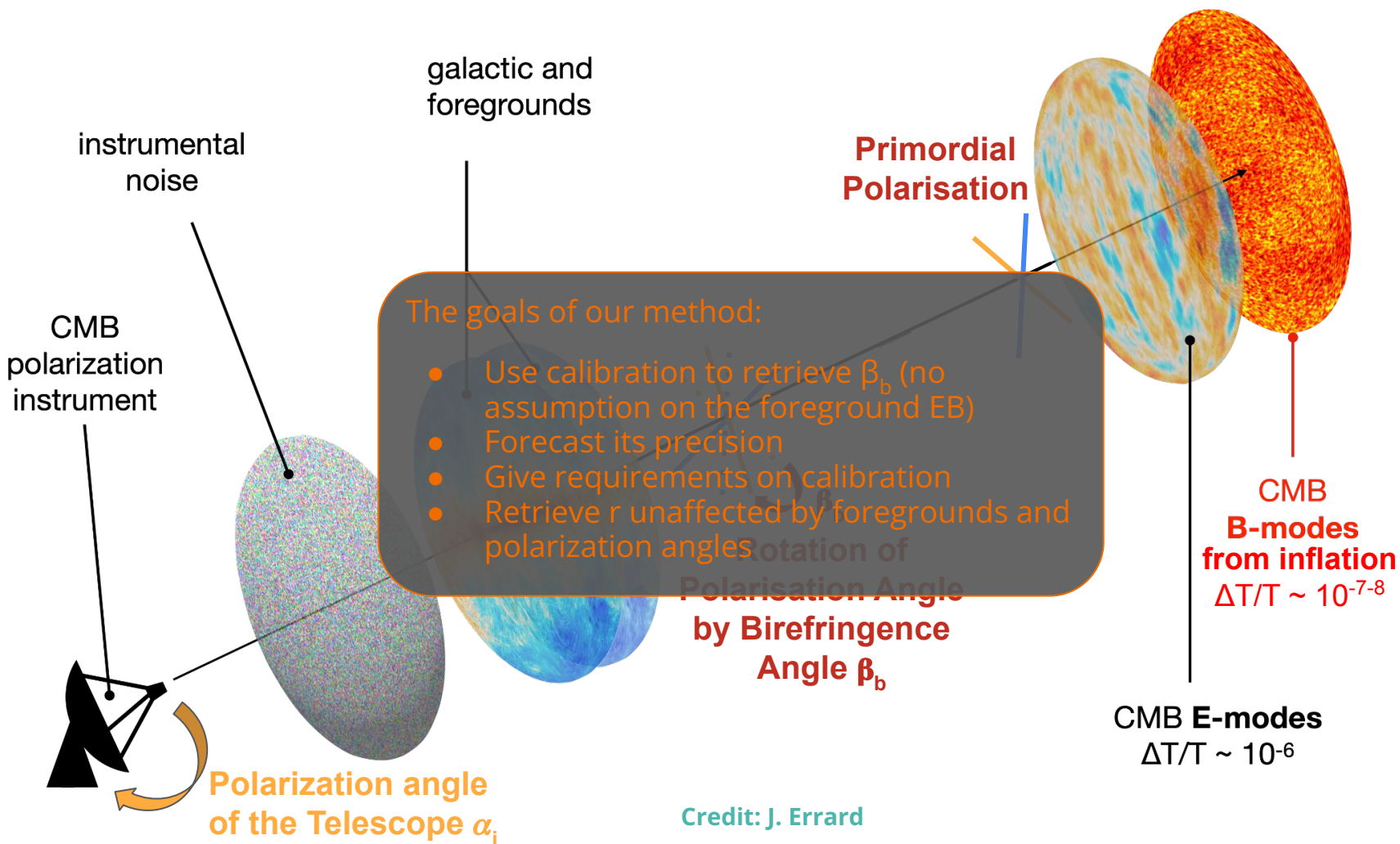


Wire grid



Credit: F. Nati





Credit: J. Errard

Map-Based Parametric Component Separation

$$d = \boxed{?} A \mathcal{B} c + n$$

$$\mathcal{B}(\{\beta_b\}) = \begin{pmatrix} \cos(2\beta_b) & \sin(2\beta_b) & 0 & 0 & 0 & 0 \\ -\sin(2\beta_b) & \cos(2\beta_b) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The birefringence matrix

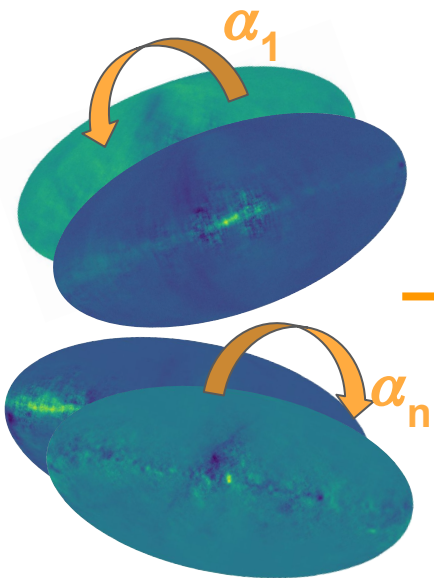
$$A(\{\beta_{fg}\}) = \begin{pmatrix} 1 & 0 & A_1^d & 0 & A_1^s & 0 \\ 0 & 1 & 0 & A_1^d & 0 & A_1^s \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & A_n^d & 0 & A_n^s & 0 \\ 0 & 1 & 0 & A_n^d & 0 & A_n^s \end{pmatrix}$$

CMB
Dust
Synchrotron

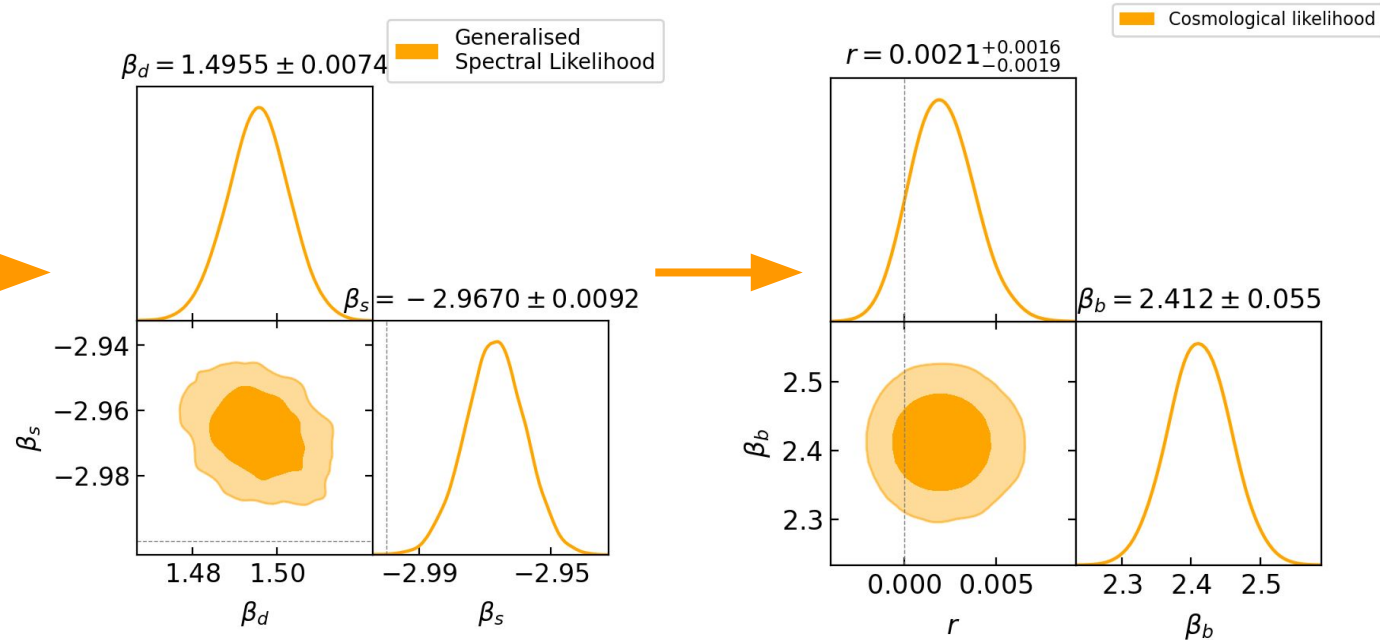
$T_d = 20K, \beta_d$
 β_s

The mixing matrix

Map-Based Parametric Component Separation in the Presence of Uncontrolled Systematics



Non-zero relative polarisation angles



Map-Based Parametric Component Separation

$$\mathbf{X}(\{\alpha_1, \dots, \alpha_{n_f}\}) = \begin{pmatrix} \cos(2\alpha_1) & \sin(2\alpha_1) & & & & 0 \\ -\sin(2\alpha_1) & \cos(2\alpha_1) & & & & \\ & & \ddots & & & \\ & & & \cos(2\alpha_{n_f}) & \sin(2\alpha_{n_f}) & \\ 0 & & & -\sin(2\alpha_{n_f}) & \cos(2\alpha_{n_f}) & \end{pmatrix}$$

$$\mathbf{B}(\{\beta_b\}) = \begin{pmatrix} \cos(2\beta_b) & \sin(2\beta_b) & 0 & 0 & 0 & 0 \\ -\sin(2\beta_b) & \cos(2\beta_b) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The miscalibration matrix

The birefringence matrix

$$\mathbf{d} = \mathbf{X} \mathbf{A} \mathbf{B} \mathbf{c} + \mathbf{n}$$

$$\mathbf{A}(\{\beta_{fg}\}) = \begin{pmatrix} 1 & 0 & A_1^d & 0 & A_1^s & 0 \\ 0 & 1 & 0 & A_1^d & 0 & A_1^s \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & A_n^d & 0 & A_n^s & 0 \\ 0 & 1 & 0 & A_n^d & 0 & A_n^s \end{pmatrix}$$

CMB
Dust
Synchrotron

$T_d = 20K, \beta_d$
 β_s

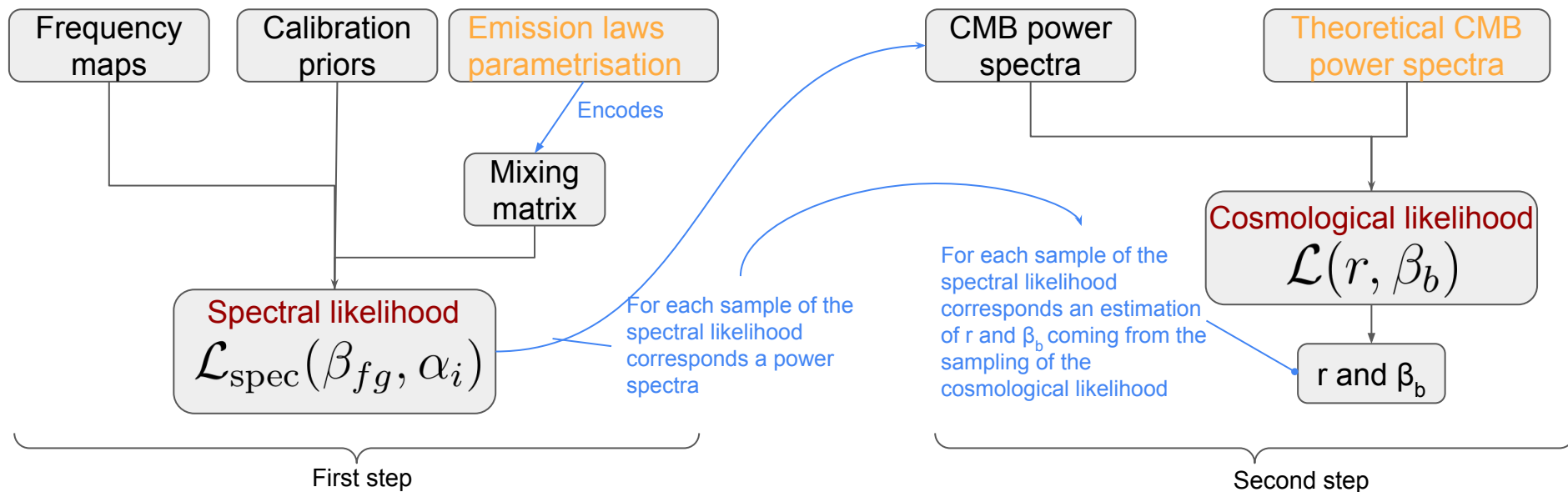
The mixing matrix

We define the generalised mixing matrix:

$$\mathbf{\Lambda} = \mathbf{X} \mathbf{A}$$

Pipeline Summary: A Two Step Analysis

Jost et al. PRD 2023



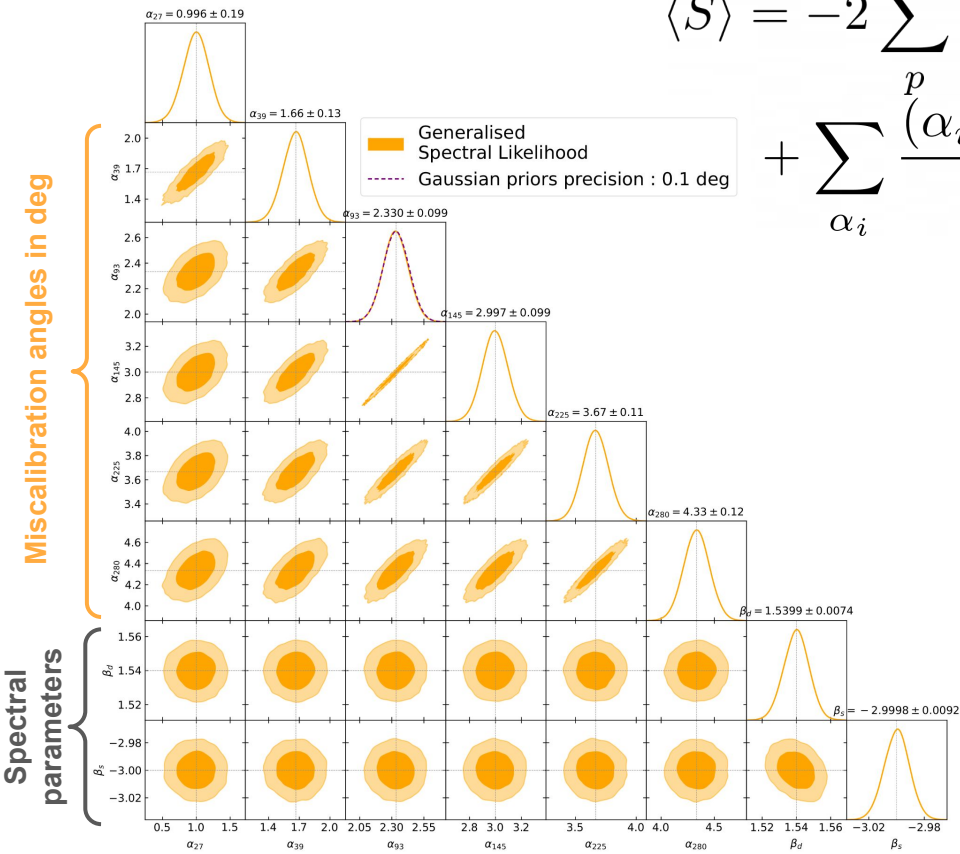
Stompór et al. MNRAS 2009
Vergès et al. PRD 2020

Relative Angles Retrieved by the Spectral Likelihood

$$\langle S \rangle = -2 \sum \text{tr} \left(\mathbf{N}_p^{-1} \mathbf{\Lambda}_p (\mathbf{\Lambda}_p^t \mathbf{N}_p^{-1} \mathbf{\Lambda}_p)^{-1} \mathbf{\Lambda}_p^t \mathbf{N}_p^{-1} \langle \mathbf{d}_p \mathbf{d}_p^t \rangle \right)$$

$$+ \sum_{\alpha_i} \frac{(\alpha_i - \hat{\alpha}_i)^2}{2\sigma_{\alpha_i}^2}$$

Calibration prior to lift degeneracy between global polarization angle and β_b

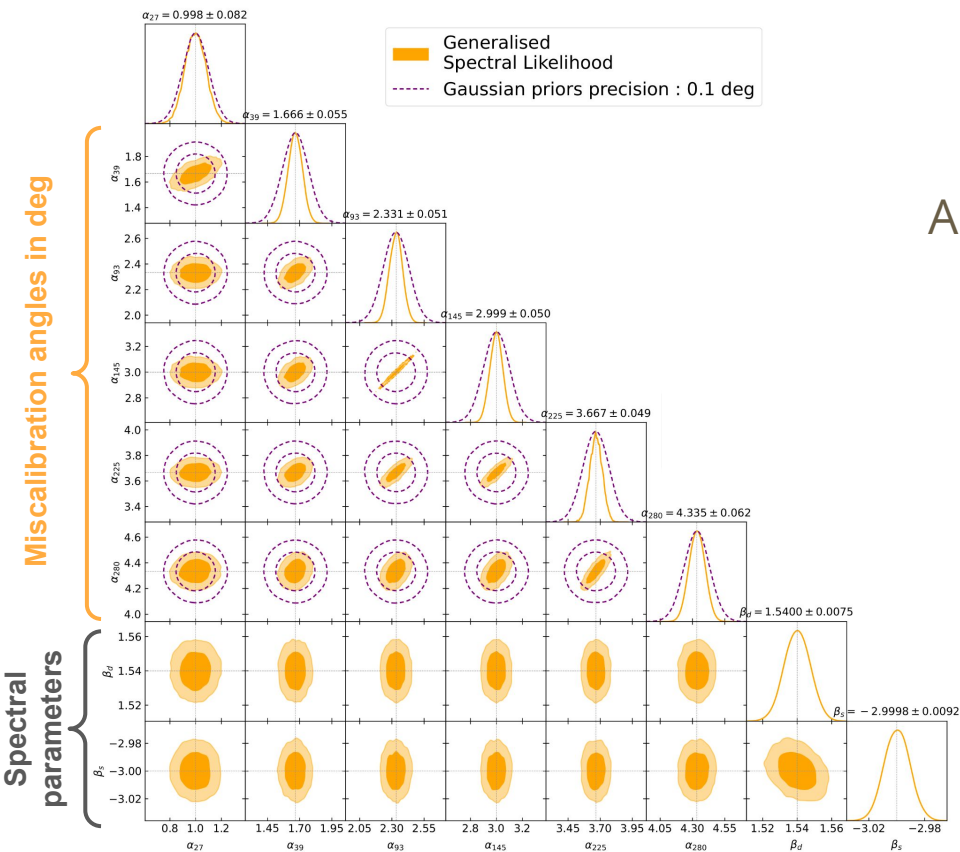


Test case in **Jost et al. PRD 2023**: SO SAT-like survey

Spectral parameters are correctly estimated (d0s0 input)

With only one prior of $\sigma(\alpha_{\text{prior}}) = 0.1^\circ$ all polarization angle are retrieved with $\sigma(\alpha_i) \geq 0.1^\circ$

Spectral Likelihood With Multiple Priors



Adding priors improves the precision:

- 6 priors $\sigma(\alpha_{\text{prior}}) = 0.1^\circ$
- $\sigma(\alpha_i) \geq 0.05^\circ$

Cosmological Likelihood With Multiple Priors

Estimate both r and β_b

Priors \Rightarrow **no bias on β_b**

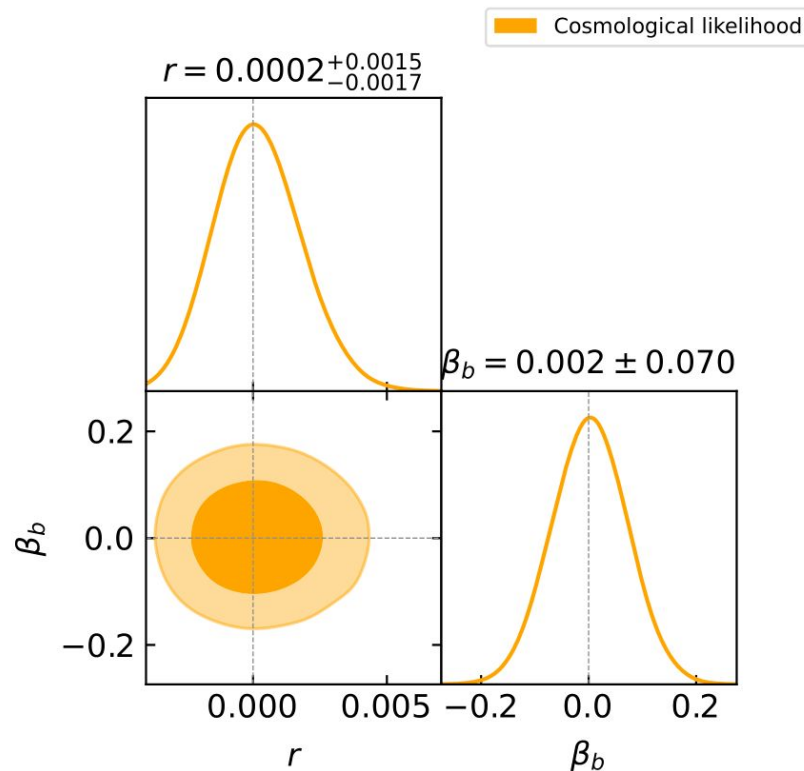
With $6 \sigma(\alpha_{\text{prior}}) = 0.1^\circ \Rightarrow \sigma(\beta_b) = 0.07^\circ$

Enough for 5σ detection with current hints

No significant impact on $\sigma(r)$

With biased priors:

- $\Delta(\beta_b) \approx \frac{1}{n_{\text{prior}}} \sum \Delta\alpha_{\text{prior}}$
- **r retrieved **without bias** (global angle marginalization)**



Setting and Relaxing Requirements on Polarization Angle Calibration

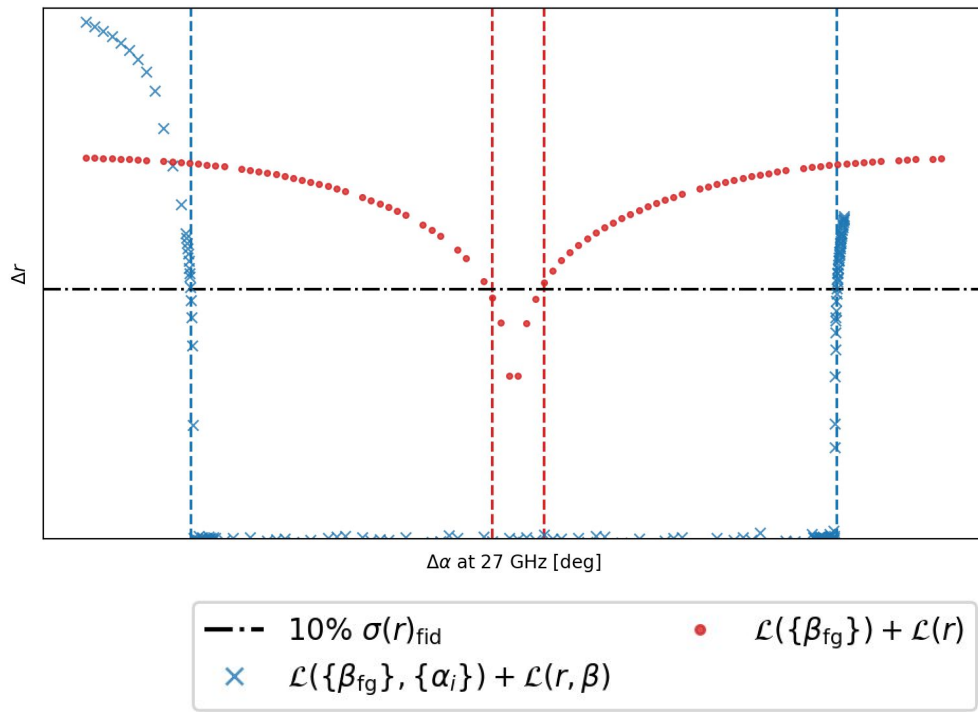
Set all $\alpha_i = 0$ except on the considered channel $\alpha_{27} = \Delta\alpha$

Use one prior, $\sigma(\alpha_{\text{prior},27}) = 0.1^\circ$, centered at 0° .

Run the pipeline and get Δr with respect to $\Delta\alpha \Rightarrow$ get requirement on polarization angle systematic error.

Relaxed requirements compared to case where angles are ignored.

This is with d0s0: how complex foreground residuals with non-zero EB would impact this type of results?



Measuring Isotropic Birefringence with LiteBIRD



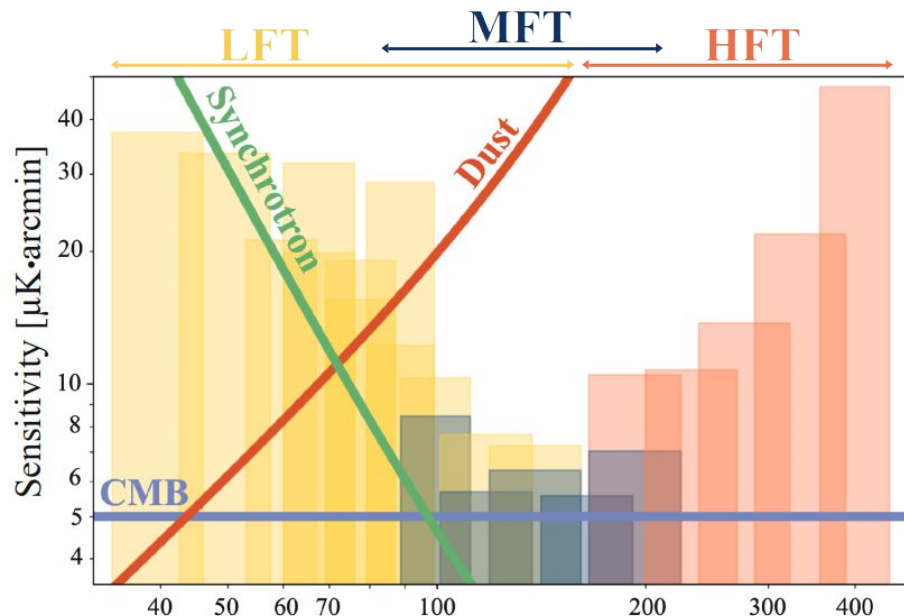
On behalf of **LiteBIRD's Cosmic Birefringence Project Study Group**,
P. Diego-Palazuelos, M. Bortolami, E. de la Hoz, J. Errard, A. Gruppuso, R. Sullivan et al.

LB's wide frequency range:

- Efficient component separation
- Better foreground models
- Cross-correlation of low and high frequencies reduces the impact of EB mismodeling

LB's Full sky survey:

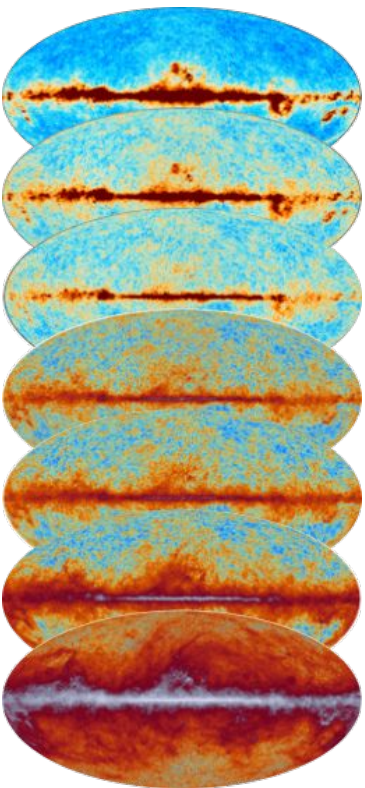
- Access to more modes
- Low ℓ EB modes can probe the axion-like particle mass and distinguish different ALP and early dark energy models



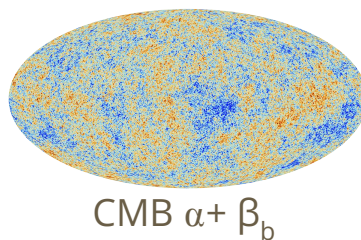
Multiple Pipelines are Developed



Frequency maps



Standard
comp-sep →



CMB $\alpha + \beta_b$

Harmonic
space →

Pixel
space →

D-estimator
Gruppuso et al. JCAP 2016

Peak Stacking
Planck XLIX A&A 2016

Foreground calibration
+ dust + synchrotron models →

Minami & Komatsu
Minami et al 2020

Modified comp-sep $\mathcal{L}(\{\mathcal{A}_{\text{comp}}\}, \{\beta_{fg}\}, \{\alpha_i\})$
+ $\mathcal{L}_{\text{cosmo}}(\beta_b)$ →

“Modified B-SeCRET”
De la Hoz et al JCAP 2022

Modified comp-sep $\mathcal{L}_{\text{spec}}(\{\beta_{fg}\}, \{\alpha_i\})$
+ $\mathcal{L}_{\text{cosmo}}(r, \beta_b)$ →

“Modified FGBuster”
Jost et al PRD 2023

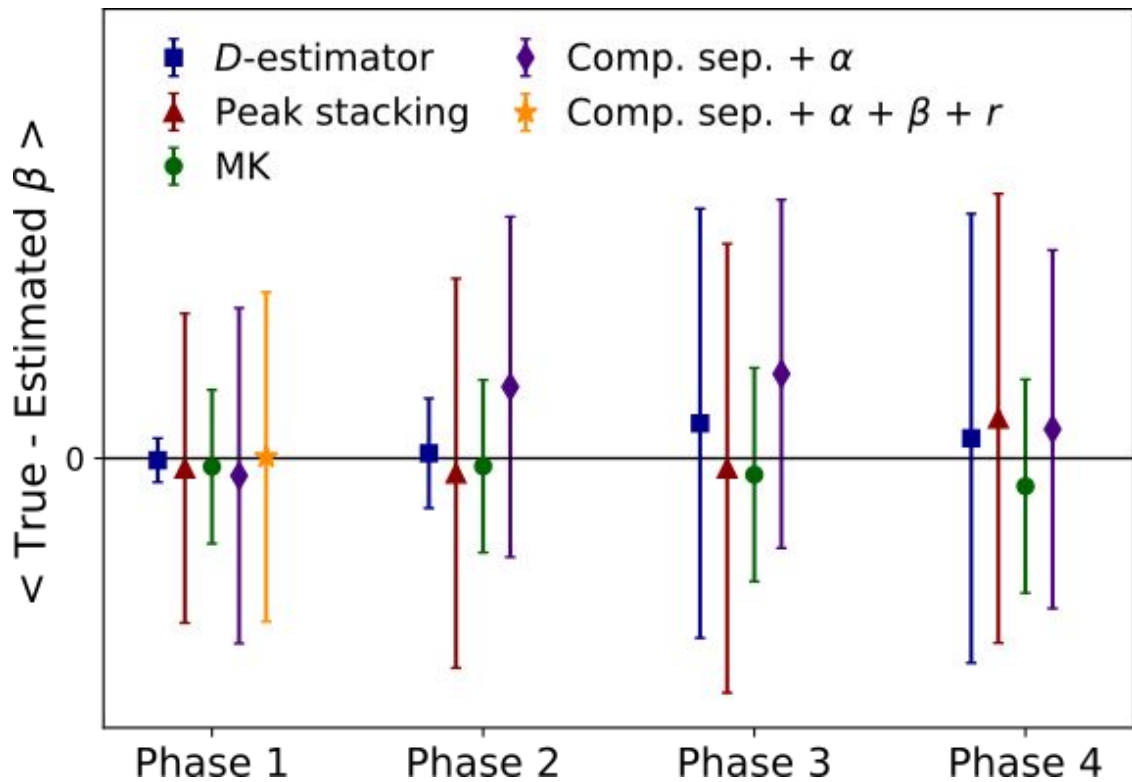
A Forecast with Different Degrees of Complexity

Phase1: CMB ($\beta=0$) + noise
+ simple foregrounds (s0d0)

Phase2: CMB ($\beta=0$) + noise
+ **complex foregrounds (s1d1)**

Phase3: CMB ($\beta=0$) + noise
+ complex foregrounds (s1d1)
+ **systematics ($\alpha_i \neq 0$)**

Phase4: **CMB ($\beta \neq 0$)** + noise
+ complex foregrounds (s1d1) +
systematics ($\alpha_i \neq 0$)



Take Home Messages

A method to retrieve r and β_b in presence of foreground and systematic effects, using calibration priors.

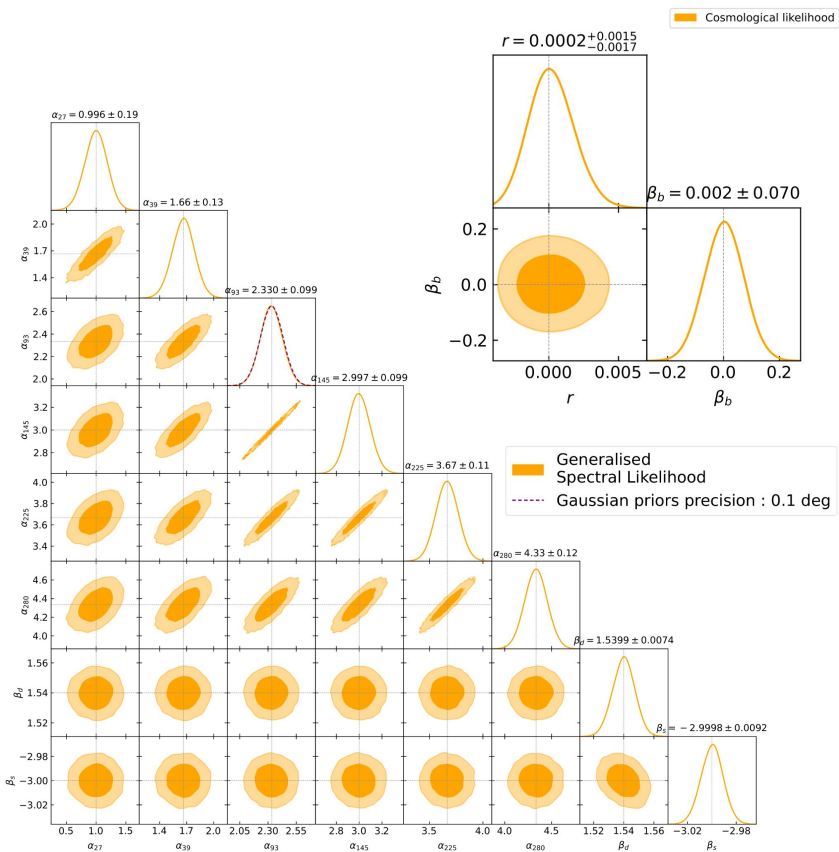
Generalised parametric component separation method that includes polarization angles:

- Relative angle are constrained by the system

Cosmological parameters estimation:

- β_b retrieved thanks to calibration prior. Its precision is improved with multiple priors
- r estimated without bias coming from E→B leakage

Keep an eye on arxiv for the application of this method and others in the LiteBIRD birefringence forecast!



THANK YOU !



Source : Deborah Kellner



Backup Slides

Source : Deborah Kellner

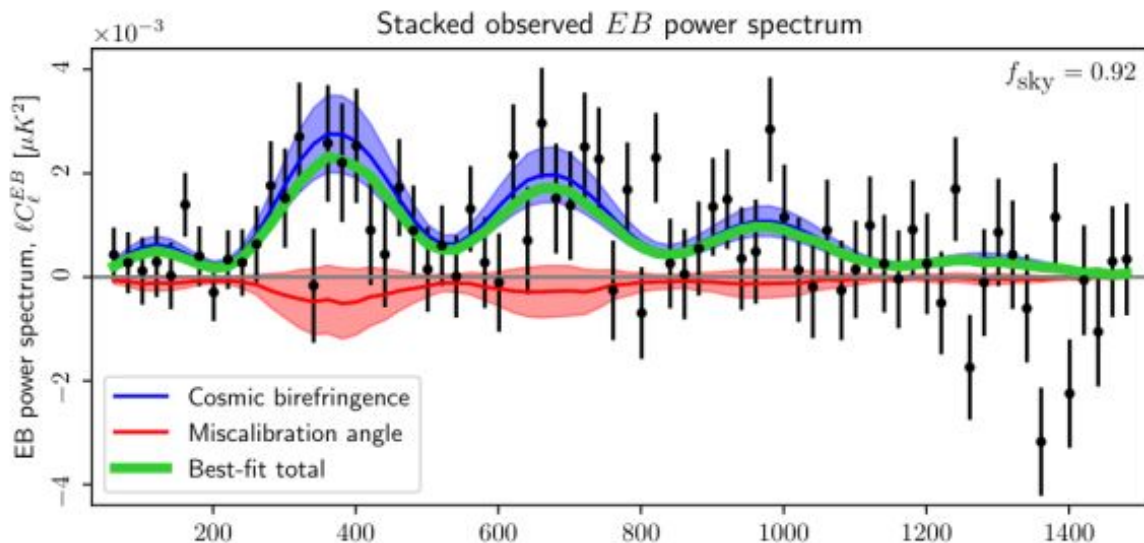
Calibrating against Galactic foregrounds



$$\beta = -\frac{1}{2}g_{\phi\gamma} \int \frac{\partial\phi}{\partial t} dt \quad \blacktriangleright \quad \begin{array}{l} \text{Galactic emission not significantly rotated by } \beta \\ \text{Use foregrounds as our calibrator} \end{array}$$

Minami+ PTEP 2019

$$C_{\ell}^{EB,o} = \frac{\tan(4\alpha)}{2} \left(C_{\ell}^{EE,o} - C_{\ell}^{BB,o} \right) + \frac{1}{\cos(4\alpha)} C_{\ell}^{EB,fg} + \frac{\sin(4\alpha)}{2 \cos(4\alpha)} \left(C_{\ell}^{EE,cmb} - C_{\ell}^{BB,cmb} \right)$$



Model the EB correlation of Galactic synchrotron and dust emissions

Clark+ ApJ 2021

Tightest constraint to date (3.6σ)

$$\beta = 0.342^{\circ} \begin{array}{l} +0.094^{\circ} \\ -0.091^{\circ} \end{array}$$

from the joint analysis of *Planck* and WMAP data

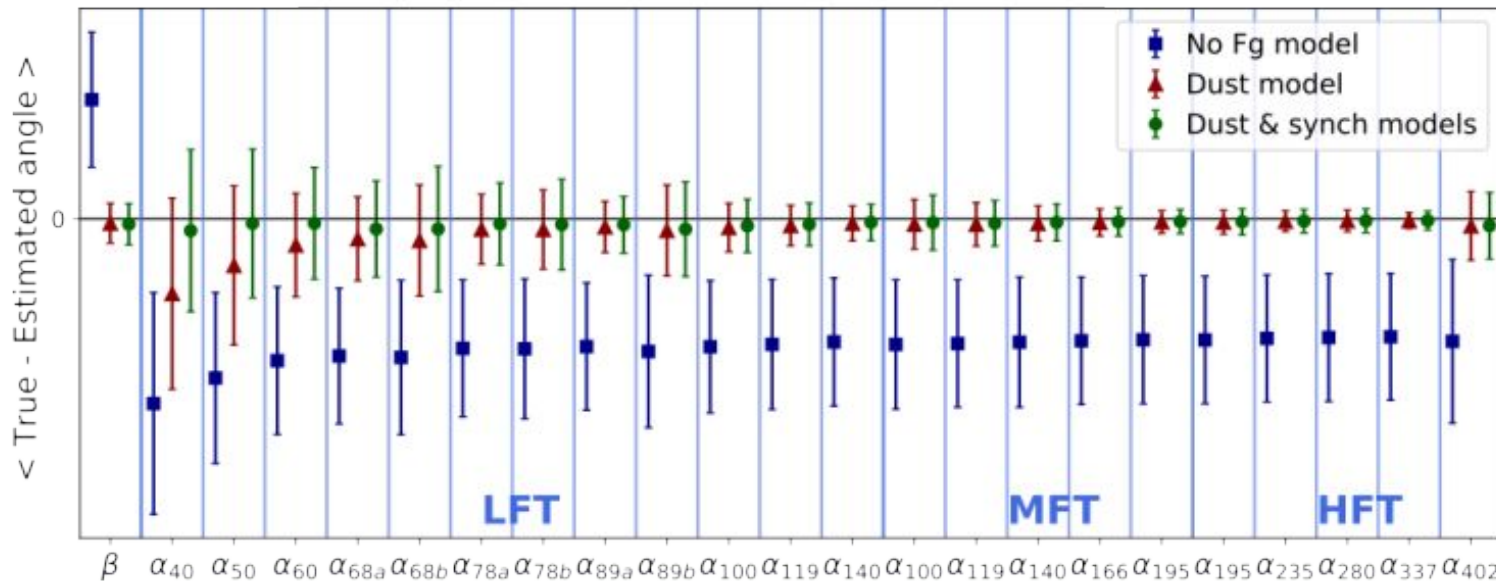
Eskilt & Komatsu PRD 2022

Minami-Komatsu (MK) technique

Considering dust and synchrotron contributions separately

$$C_{\ell}^{EB,o} = \frac{1}{2} \tan(4\alpha) \left(C_{\ell}^{EE,o} - C_{\ell}^{BB,o} \right) + \frac{\sin(4\beta)}{2 \cos(4\alpha)} \left(C_{\ell}^{EE,cmb} - C_{\ell}^{BB,cmb} \right) + \frac{1}{\cos(4\alpha)} \left(C_{\ell}^{EB,s} + C_{\ell}^{EB,d} + C_{\ell}^{E_s B_d} + C_{\ell}^{E_d B_s} \right)$$

Diego-Palazuelos+ JCAP 2023



D-estimator

$$D_\ell(\hat{\beta}) = C_\ell^{EB,o} - \frac{1}{2} \tan(4\hat{\beta}) \left(C_\ell^{EE,o} - C_\ell^{BB,o} \right)$$

$$\langle D_\ell(\hat{\beta} = \beta) \rangle = 0$$

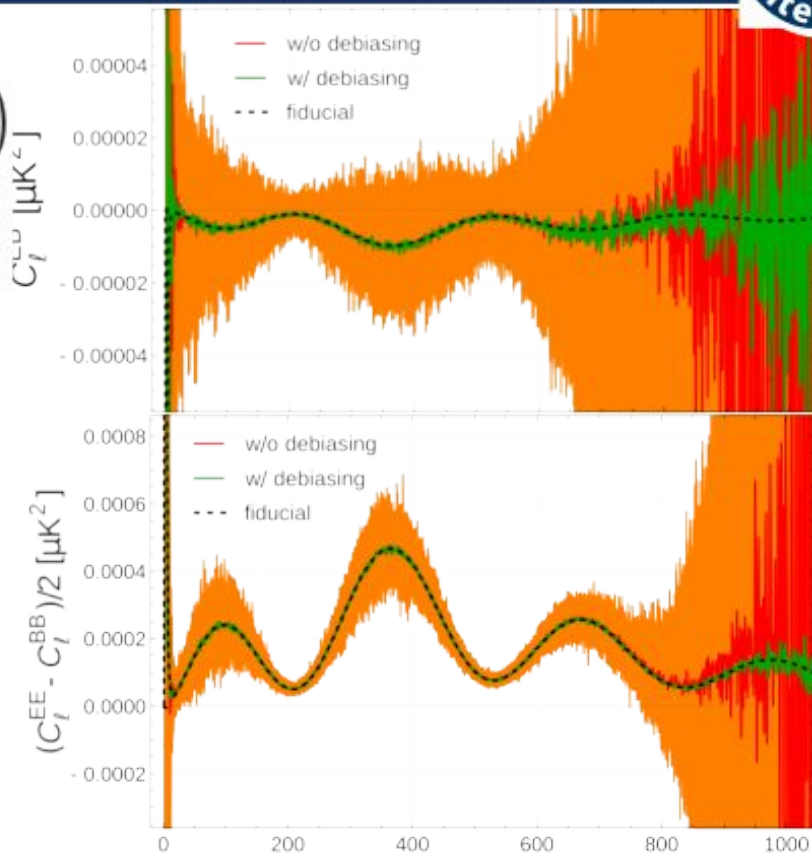
Find the zeros minimizing

$$\chi^2(\hat{\beta}) = \sum_{\ell\ell'} D_\ell(\hat{\beta}) M_{\ell\ell'}^{-1} D_{\ell'}(\hat{\beta})$$

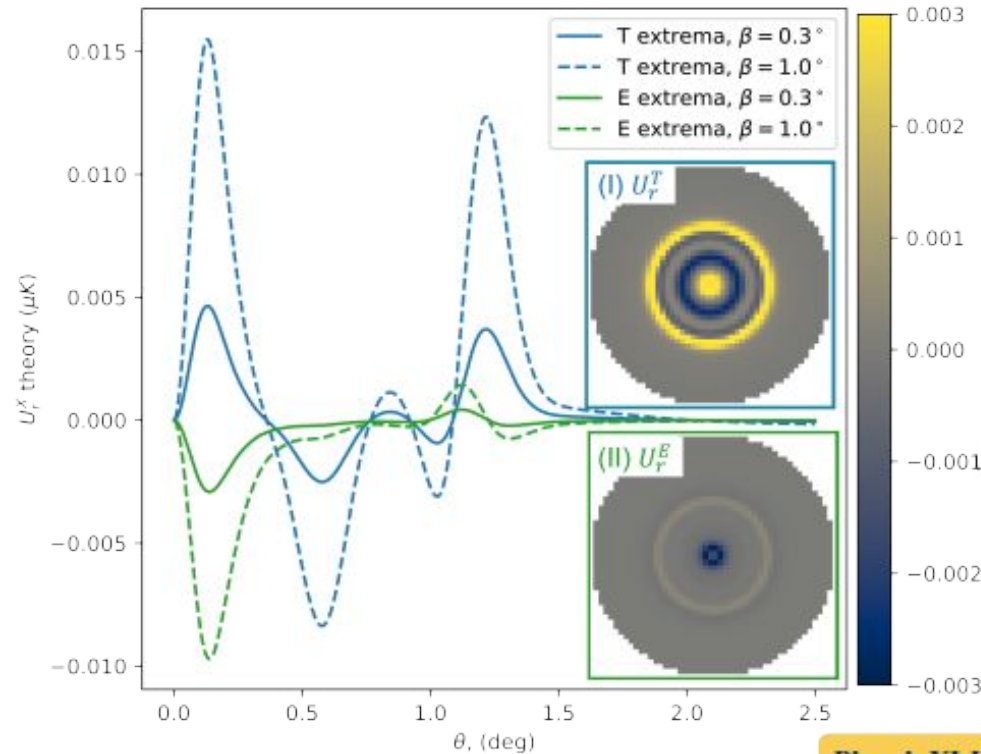
Build the covariance matrix from simulations to account for foreground debiasing and the extra dispersion caused by α miscalibrations

$$M_{\ell\ell'} = \langle D_\ell D_{\ell'} \rangle$$

Gruppuso+ JCAP 2016



Stacking of peaks



Find local extrema in T and E anisotropies

Transform the Stokes parameters and stack peaks

$$Q_r(\theta) = -Q(\theta) \cos(2\phi) - U(\theta) \sin(2\phi)$$

$$U_r(\theta) = Q(\theta) \sin(2\phi) - U(\theta) \cos(2\phi)$$

Radial profile around peaks is sensitive to β

$$\langle U_r^T \rangle(\theta) = -\sin(2\beta) \int \frac{\ell d\ell}{2\pi} W_\ell^T W_\ell^P J_2(\ell\theta) \times (\bar{b}_\nu + \bar{b}_\zeta \ell^2) C_\ell^{TE}$$

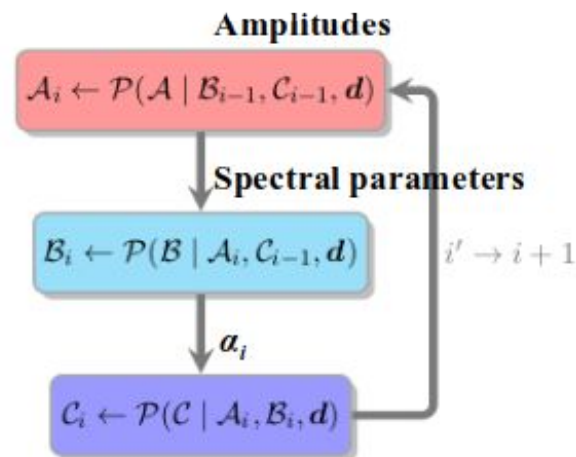
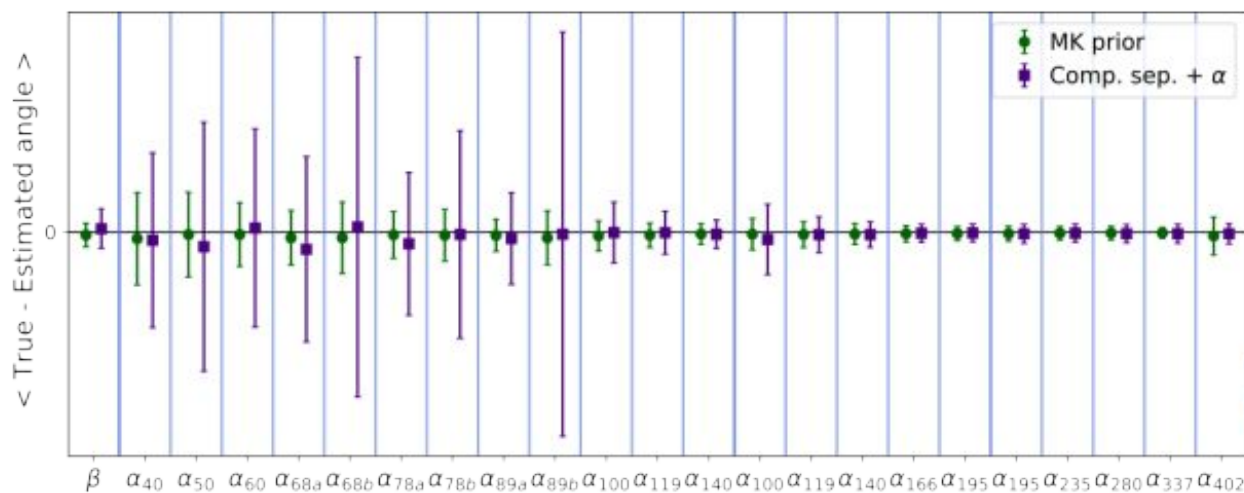
$$\langle U_r^E \rangle(\theta) = -\frac{1}{2} \sin(4\beta) \int \frac{\ell d\ell}{2\pi} W_\ell^E W_\ell^P J_2(\ell\theta) \times (\bar{b}_\nu + \bar{b}_\zeta \ell^2) (C_\ell^{EE} - C_\ell^{BB})$$

Component separation + α_i



$$\begin{pmatrix} Q(\nu, \theta) \\ U(\nu, \theta) \end{pmatrix}_p = \begin{pmatrix} c^Q \\ c^U \end{pmatrix}_p + \begin{pmatrix} a_s^Q \\ a_s^U \end{pmatrix}_p \frac{1}{u(\nu)} \left(\frac{\nu}{\nu_s}\right)^{\beta_s} + \begin{pmatrix} a_d^Q \\ a_d^U \end{pmatrix}_p \frac{1}{u(\nu)} \left(\frac{\nu}{\nu_d}\right)^{\beta_d-2} \frac{B(\nu, T_d)}{B(\nu_d, T_d)}$$

$$\begin{pmatrix} Q^o(\nu, \alpha, \theta) \\ U^o(\nu, \alpha, \theta) \end{pmatrix}_p = \begin{pmatrix} \cos(2\alpha) & -\sin(2\alpha) \\ \sin(2\alpha) & \cos(2\alpha) \end{pmatrix} \begin{pmatrix} Q(\nu, \theta) \\ U(\nu, \theta) \end{pmatrix}_p$$

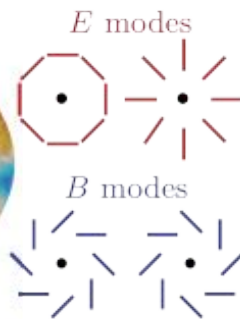
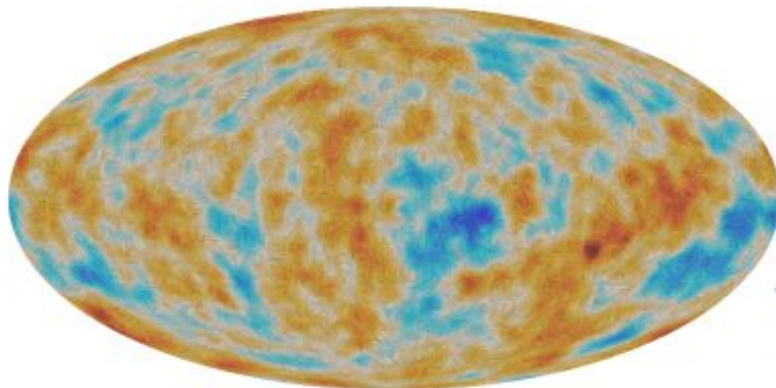
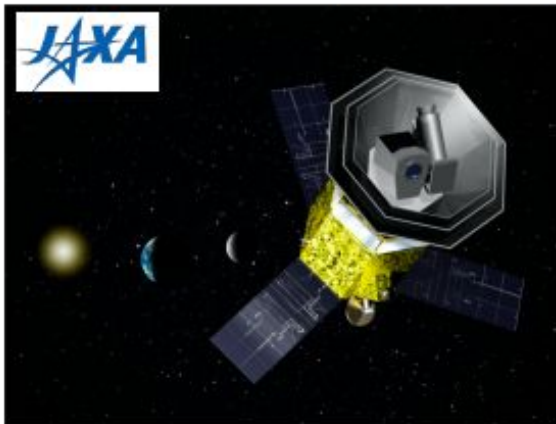


Gaussian priors on spectral parameters M_i result as prior on α_i

LiteBIRD overview

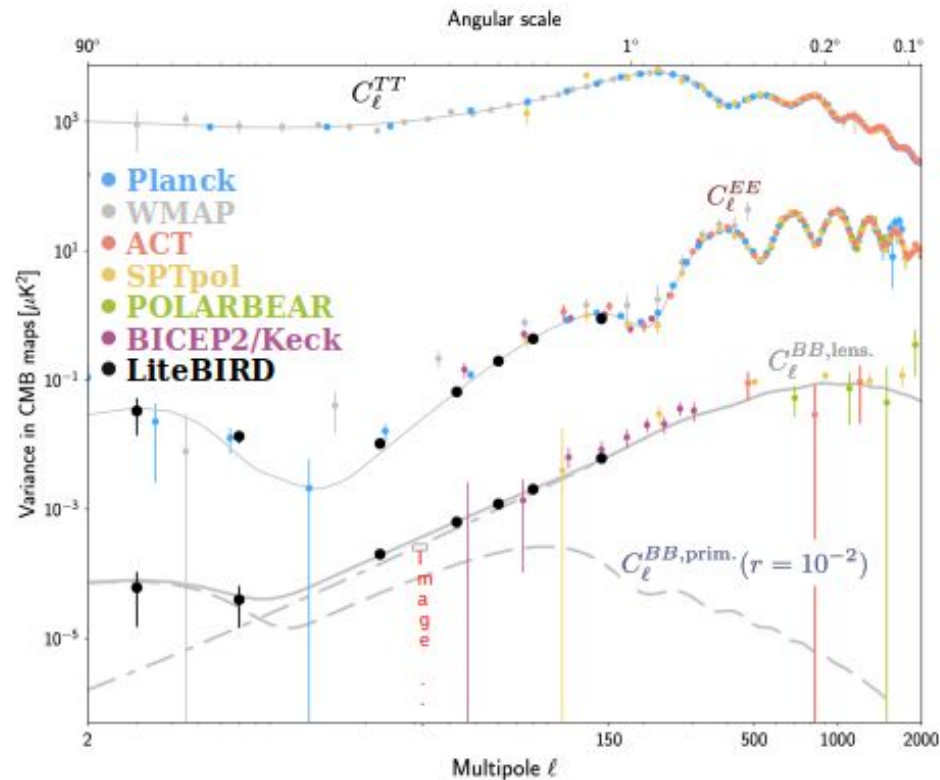
- Lite (Light) satellite for the study of *B*-mode polarization and Inflation from cosmic background Radiation Detection
- JAXA's L-class mission selected in May 2019
- Expected launch in late 2032 (JFY) with JAXA's H3 rocket
- **All-sky 3-year survey**, from Sun-Earth Lagrangian point L2
- Large frequency coverage (**40–402 GHz**, 15 bands) at **70–18 arcmin** angular resolution for precision measurements of the **CMB *B*-modes**
- Final combined sensitivity: **2.2 $\mu\text{K}\cdot\text{arcmin}$**

LiteBIRD PTEP 2023



LiteBIRD main scientific objectives

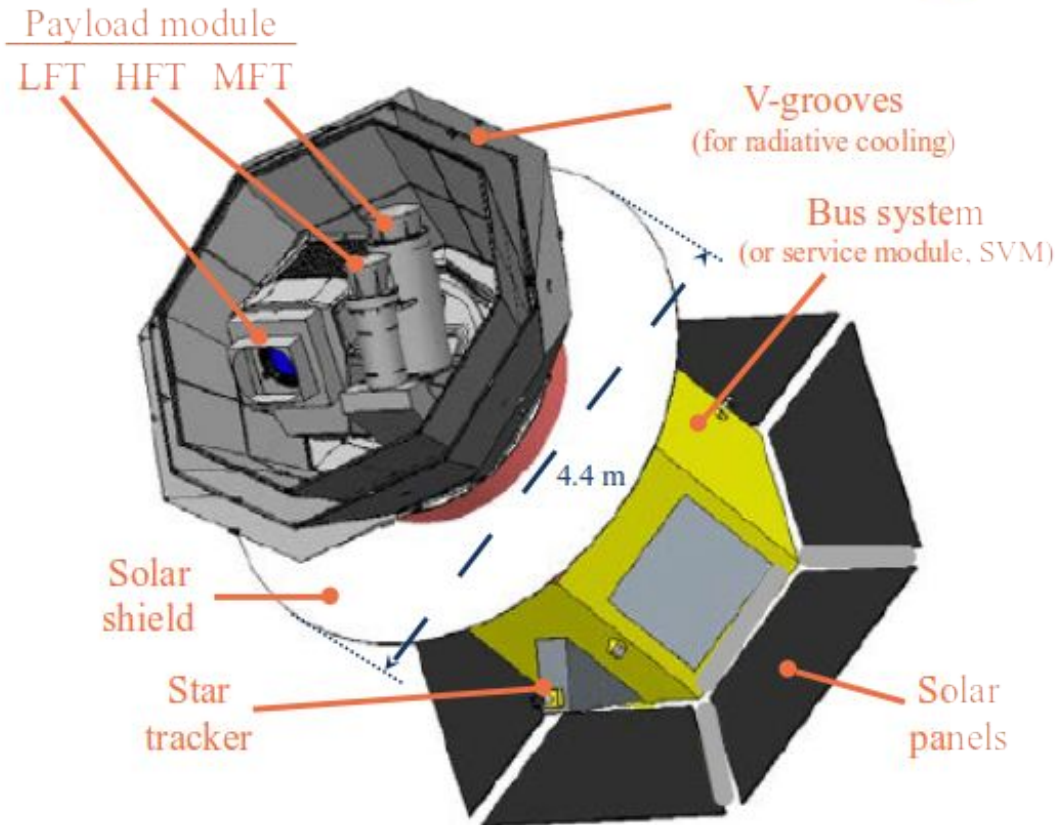
- Definitive search for the ***B*-mode signal** from **cosmic inflation** in the CMB polarization
 - Making a discovery or ruling out well-motivated inflationary models
 - Insight into the quantum nature of gravity
- The inflationary (i.e. primordial) *B*-mode power is proportional to the **tensor-to-scalar ratio, r**
- Current best constraint: $r < 0.032$ (95% C.L.) (Tristram+ 2021, combining BK18 and Planck PR4)
- LiteBIRD will improve current sensitivity on r by a factor ~ 50
- L1-requirements (no external data):
 - For $r = 0$, **total uncertainty of $\delta r < 0.001$**
 - For $r = 0.01$, 5σ detection of the reionization ($2 < \ell < 10$) and recombination ($11 < \ell < 200$) peaks independently
- Huge discovery impact (evidence for inflation, knowledge of its energy scale, ...)



LiteBIRD spacecraft overview

- **3 telescopes** are used to provide the **40-402 GHz** frequency coverage
 1. **LFT** (low frequency telescope)
 2. **MFT** (middle frequency telescope)
 3. **HFT** (high frequency telescope)
- Multi-chroic transition-edge sensor (TES) **bolometer arrays** cooled to **100 mK**
- Polarization modulation unit (PMU) in each telescope with **rotating half-wave plate** (HWP), for $1/f$ noise and systematics reduction
- Optics cooled to **5 K**

- Mass: 2.6 t
- Power: 3.0 kW
- Data: 17.9 Gb/day



The Generalised Spectral Likelihood

I generalise the spectral log likelihood from [Stompor et al. 2009](#), similarly as in [Vergès et al. 2020](#):

$$\langle S \rangle = -2 \sum_p \text{tr} \left(\mathbf{N}_p^{-1} \mathbf{\Lambda}_p (\mathbf{\Lambda}_p^t \mathbf{N}_p^{-1} \mathbf{\Lambda}_p)^{-1} \mathbf{\Lambda}_p^t \mathbf{N}_p^{-1} \langle \mathbf{d}_p \mathbf{d}_p^t \rangle \right)$$

For forecasting purposes we **average over CMB and noise** realisations.

To lift the degeneracy we add priors to the likelihood:

$$S' \equiv \langle S \rangle + \sum_{\alpha_i} \frac{(\alpha_i - \hat{\alpha}_i)^2}{2\sigma_{\alpha_i}^2}$$




Credit: F. Nati

The Cosmological Likelihood

With $\{\beta_{fg}\}$ and $\{\alpha_i\}$ we estimate a CMB map. Imperfect component separation will lead to residuals.

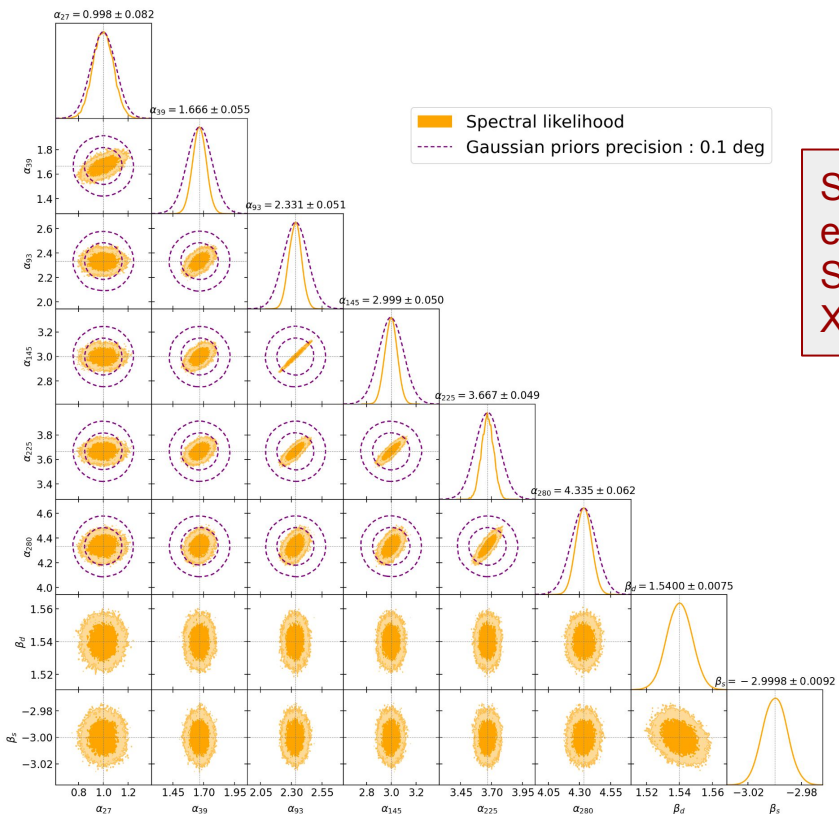
Its power spectra is used to estimate cosmological parameters:

$$\langle S^{cos} \rangle = f_{sky} \sum_{\ell=l_{min}}^{\ell_{max}} \frac{(2\ell+1)}{2} (Tr(\mathbf{C}_\ell^{-1} \mathbf{E}_\ell) + \ln(\det(\mathbf{C}_\ell)))$$

 Data after generalised component separation

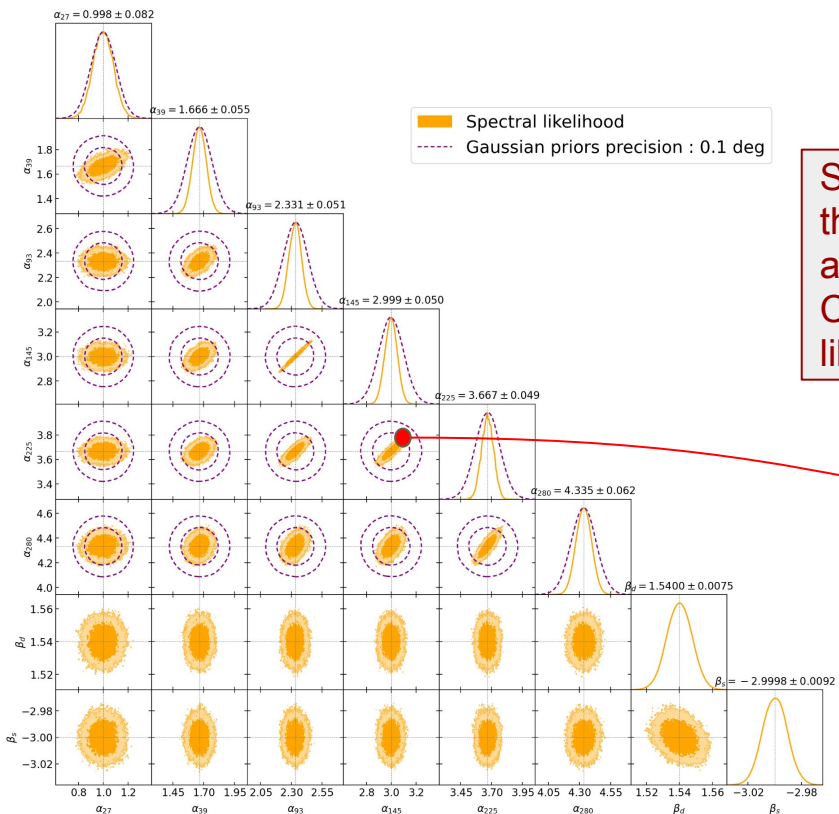
$$\mathbf{C}_\ell(r, \beta_b) \equiv \mathcal{R}(\beta_b) \begin{pmatrix} C_\ell^{EE,p} & 0 \\ 0 & rC_\ell^{BB,p} + A_L C_\ell^{BB,lens} \end{pmatrix} \mathcal{R}^{-1}(\beta_b) + C_\ell^{\text{noise}}$$

How to Have a Statistically Robust Method ?

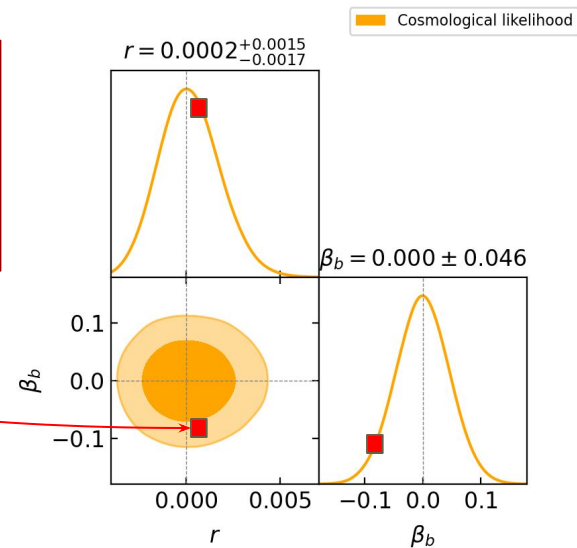


Step 1 : Sampling the ensemble averaged Spectral likelihood $X\{\alpha\}.A\{\beta_{fg}\}$

How to Have a Statistically Robust Method ?



Step 2 : Draw from the ensemble averaged Cosmological likelihood



Method Validation

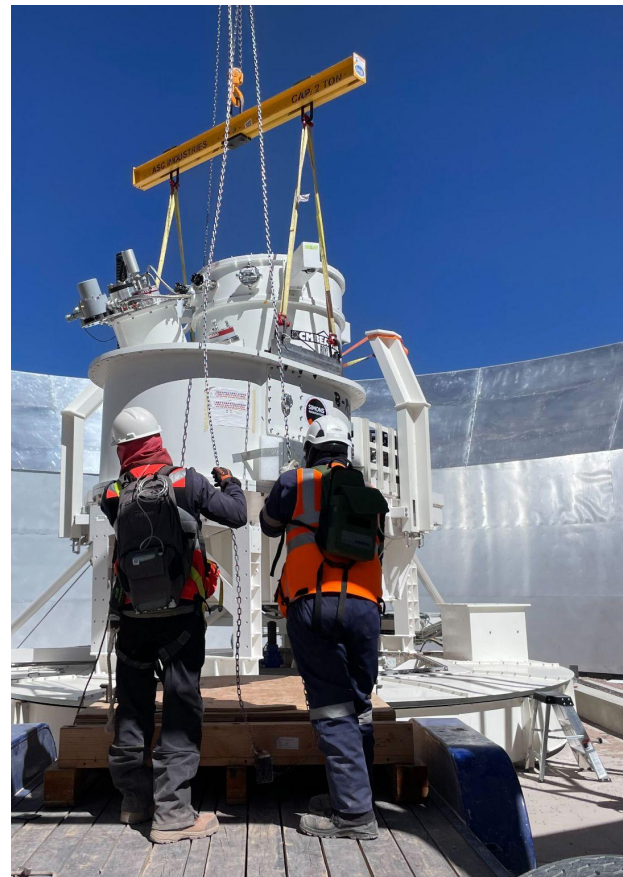
SO SAT like survey: 6 frequency bands, characteristics noise, 10% sky coverage.

Priors:

- $\sigma(\alpha_i) = 0.1^\circ$
- Priors are centred at the true value of polarisation angles.
- One vs multiple priors.

Forecast input sky:

- CMB maps from Planck power spectra, $r = 0.0$, $\beta_b = 0.0^\circ$
- PySM foreground maps with different degrees of complexity (d0s0, d1s1, d7s3 in order of complexity...)
([Thorne et al 2016](#), [Zonca et al. 2021](#))



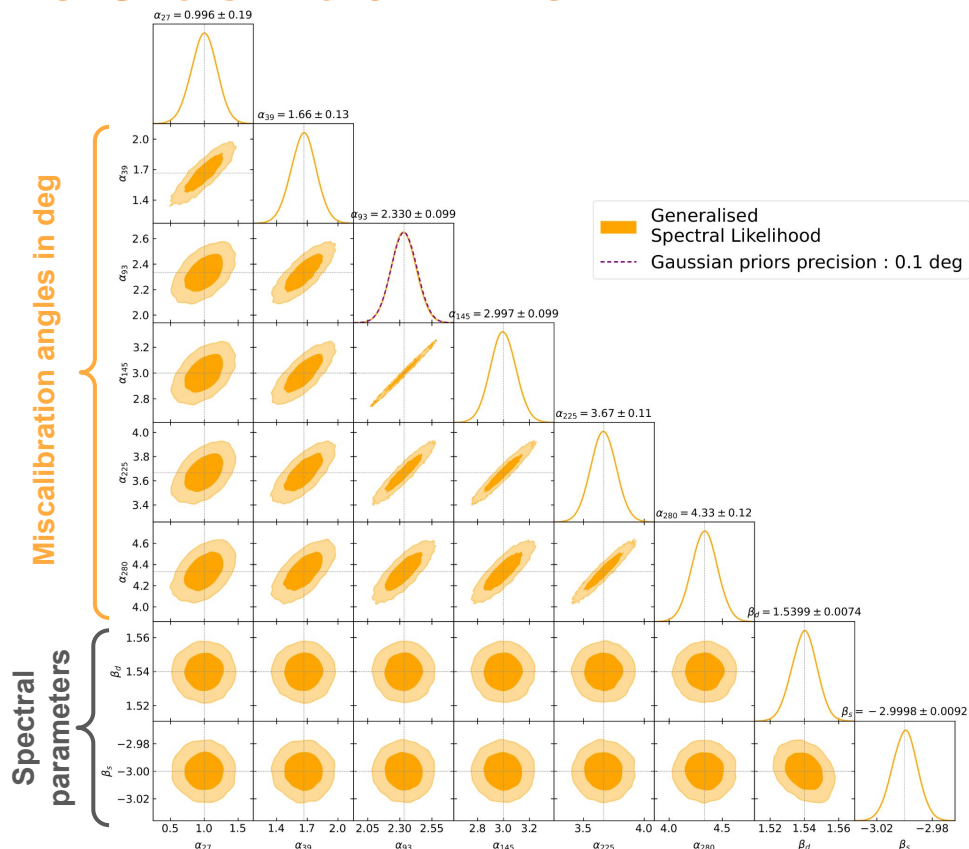
Credit: Remington Gerras

Simple Foregrounds and One Calibration Prior

- Input CMB: $\mathbf{r} = \mathbf{0.0}$; $\beta_b = 0.0^\circ$
- Input fg: **PySM** models (Thorne et al 2016, Zonca et al. 2021) d0s0:
 - dust: MBB, spatially constant spectral indices
 - synchrotron: power law, spatially constant spectral indices
- 1 prior on 93 GHz: $\sigma(\alpha_i) = 0.1^\circ$

Foreground cleaning is ok

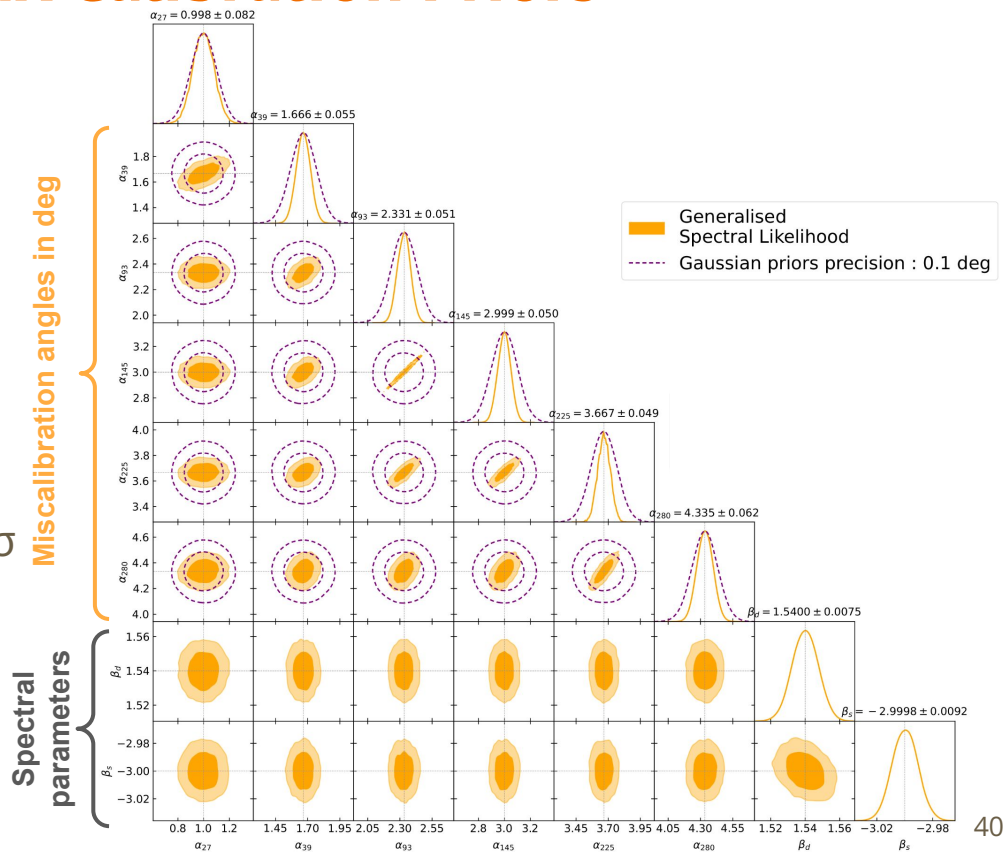
Miscalibration: one prior enough



Simple Foregrounds and Six Calibration Priors

- Input CMB: $\mathbf{r} = \mathbf{0.0}$; $\beta_b = 0.0^\circ$
- Input fg: **PySM** models (**Thorne et al 2016, Zonca et al. 2021**) d0s0:
 - dust: MBB, spatially constant spectral indices
 - synchrotron: power law, spatially constant spectral indices
- **Prior on all frequency channels:** $\sigma(\alpha_i) = 0.1^\circ$

Overall $\sigma(\alpha)$ improved wrt priors precisions!



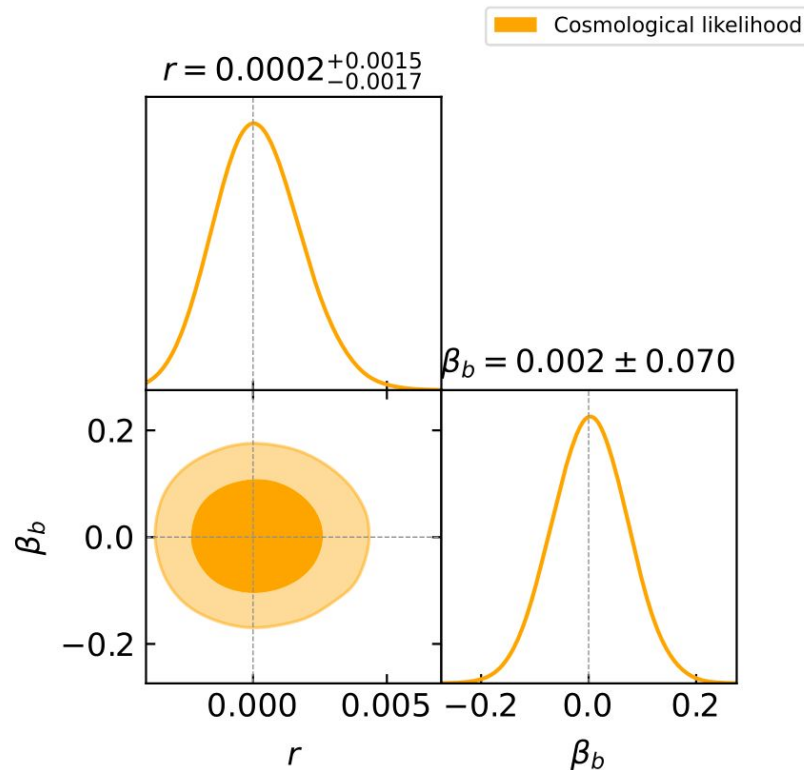
Simple Foregrounds and Six Calibration Priors

- Simple foregrounds: **d0s0**
- **Prior on all frequency channels**

r and β_b correctly estimated

$\sigma(r)$: same order as SO SAT forecast with $\sigma(r) = 2.1 \cdot 10^{-3}$ ([Ade et al. 2018](#))

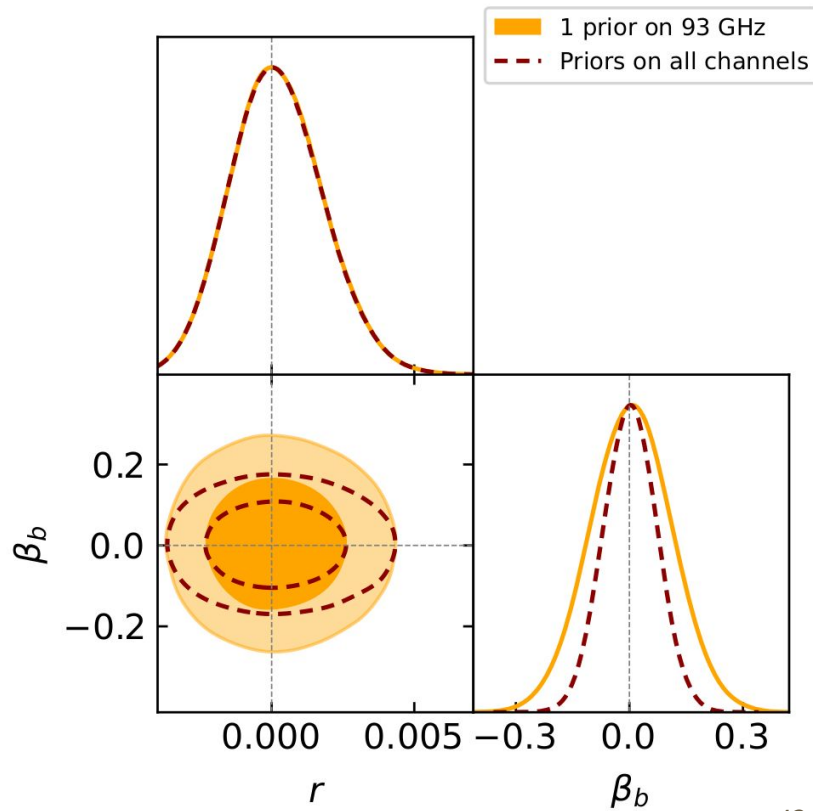
$\sigma(\beta_b)$: improved wrt prior precision!



Results: Simple Foregrounds and Six Calibration Priors

- d0s0
- Prior on all frequency channels

$\sigma(\beta_b)$: improved wrt prior precision!



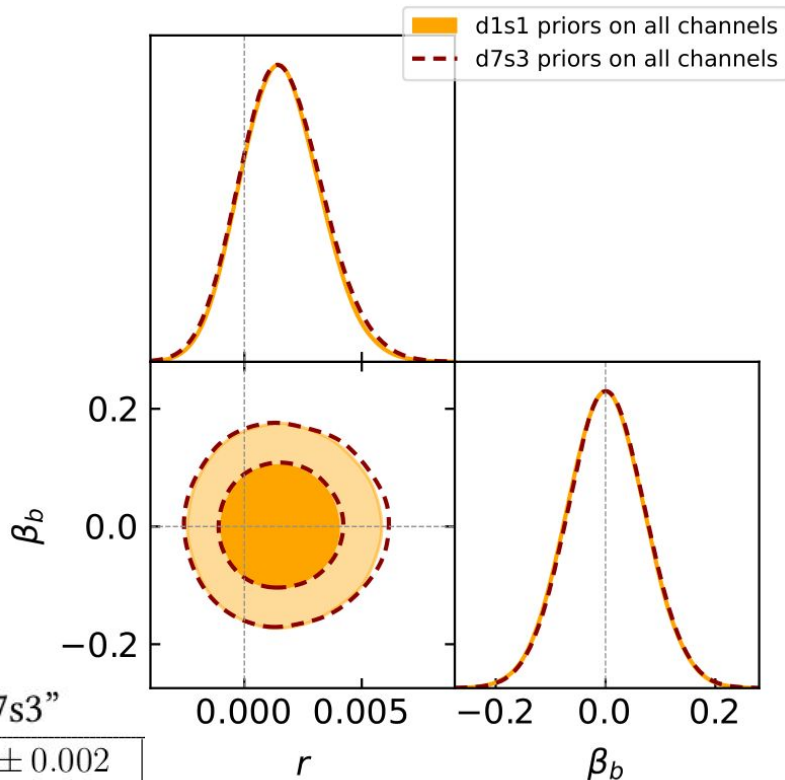
Complex Foregrounds and Six Calibration Priors

Foreground emissions **don't follow** the assumption used in the mixing matrix:

- **d1s1**: spatially varying foreground spectral indices
- **d7s3**: dust emission is non parametric and synchrotron has a curvature term
- **Prior on all frequency channels**: $\sigma(\alpha_i) = 0.1^\circ$

r : biased due to foreground residuals

β_b : no noticeable effect

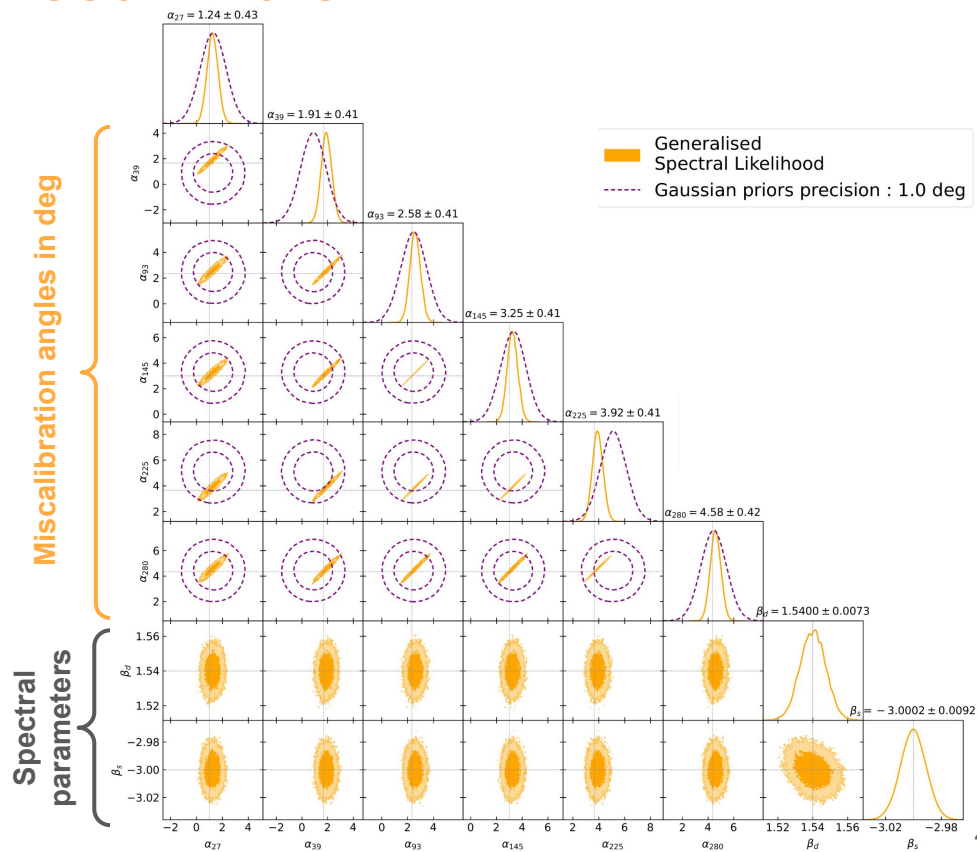


| | “d1s1” | “d7s3” |
|--------------------|-------------------|-------------------|
| r | 0.002 ± 0.002 | 0.002 ± 0.002 |
| $\beta_b [^\circ]$ | 0.00 ± 0.07 | 0.00 ± 0.07 |

Simple Foregrounds and Biased Priors

- d0s0:
 - dust: MBB, spatially constant spectral indices
 - synchrotron: power law, spatially constant spectral indices
- **Prior on all channels, $\sigma(\alpha_i) = 1^\circ$**
- **Priors randomly biased by $N(0, 1^\circ)$**

α_i biased by the same value: the mean of the biases



Simple Foregrounds and Biased Priors

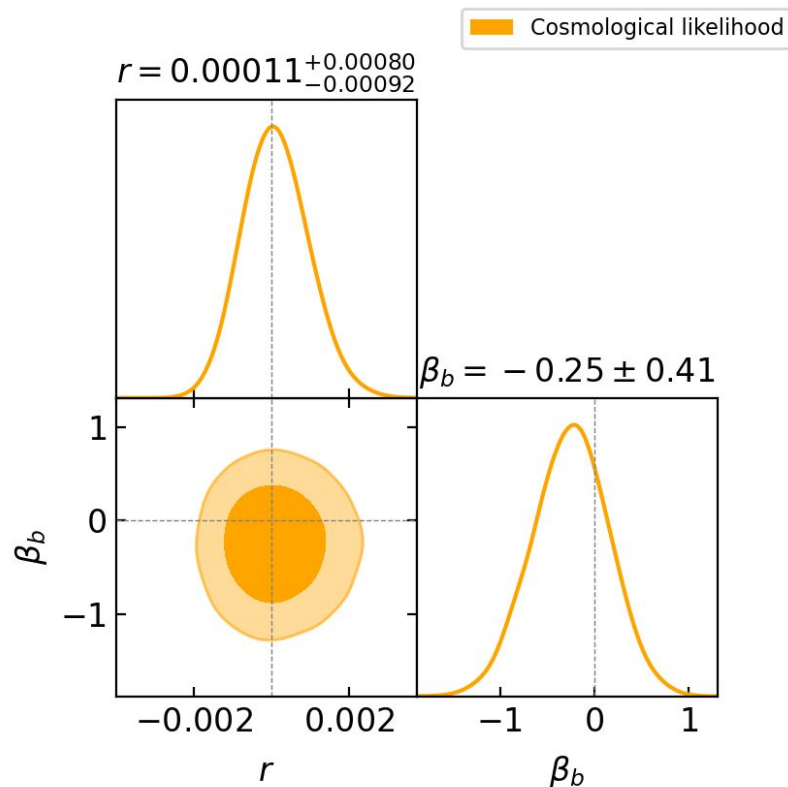
- Simple foregrounds **d0s0**:
 - dust: MBB, isotropic spectral indices
 - synchrotron: power law, isotropic spectral indices
- **Prior on all channels**, $\sigma(\alpha_i) = 1^\circ$
- **Priors randomly biased by $N(0,1^\circ)$**

β_b biased by the same value as α_i

For β_b trade-off between statistical uncertainty and possible bias.

r is unbiased: we marginalise over a global angle, removing any E→B leakage either from α_i or β_b

We can always be confident that r is not affected by α_i and β_b .



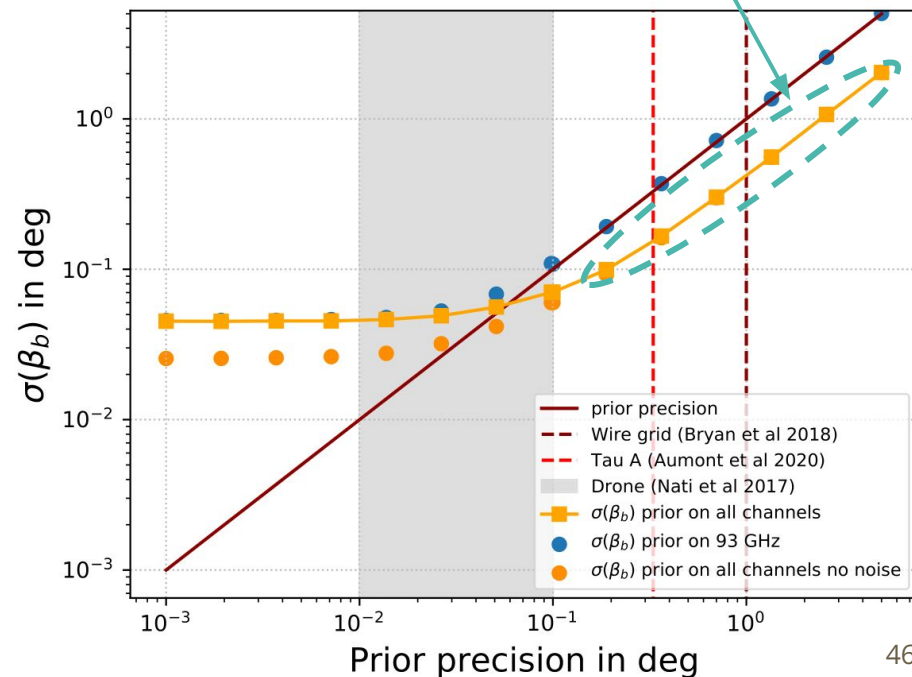
Evolution of Uncertainty wrt Prior Precision

$$\sigma(\beta_b) \approx \left(\sum_1^6 \frac{1}{\sigma_{\alpha_i}^2} \right)^{-\frac{1}{2}}$$
$$= \frac{\sigma_{\alpha_i}}{\sqrt{6}}$$

We can set calibration requirement.

- Simple Foregrounds: d0s0
- 3 cases:
 - 1 prior
 - 6 priors ■
 - 6 priors and no noise ●

Noise represents ~42% of $\sigma(\beta_b)$ in the SO SATs like survey used here



Cosmic Birefringence

I focus in particular on **spatially constant** and **time independent** cosmic birefringence:

$$\tilde{C}_\ell^{EE} = C_\ell^{EE} \cos^2(2\beta_b) + C_\ell^{BB} \sin^2(2\beta_b)$$

$$\tilde{C}_\ell^{BB} = C_\ell^{EE} \sin^2(2\beta_b) + C_\ell^{BB} \cos^2(2\beta_b)$$

$$\tilde{C}_\ell^{EB} = (C_\ell^{EE} - C_\ell^{BB}) \frac{\sin(4\beta_b)}{2},$$

Method Validation

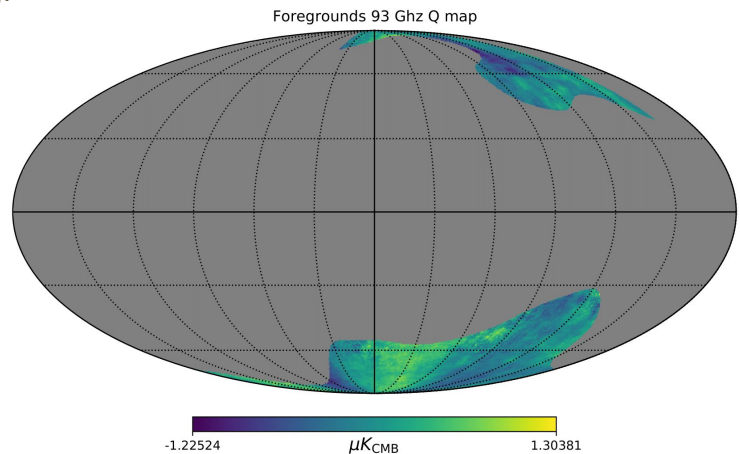
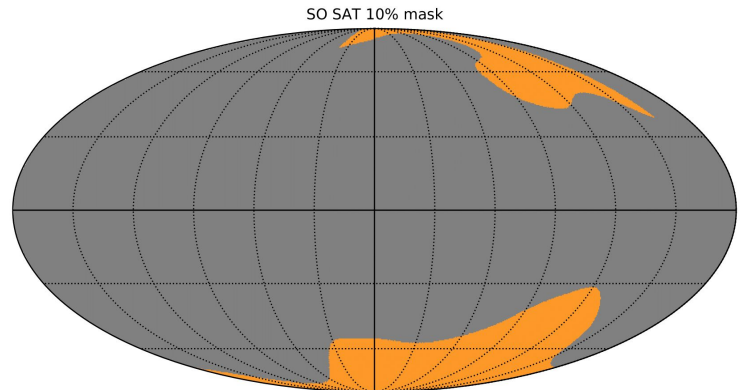
SO SAT characteristics noise, 10% sky coverage, $I_{\min} = 30$, $I_{\max} = 300$, 30 000 detectors, first light by the end of the year

Priors:

- as a benchmark we use $\sigma(\alpha_i) = 0.1^\circ$
- Unless precised otherwise, priors are centred at the true value of polarisation angles.
- Different calibration methodology explored e.g. one vs multiple priors.

Forecast input sky:

- average over CMB maps generated from Planck power spectra with $r = 0.0$, $\beta_b = 0.0^\circ$
- PySM foreground maps with different degrees of complexity (d0s0, d1s1, d7s3 in order of complexity...) (Thorne et al 2016, Zonca et al. 2021)



Foreground Models

Dust template: maps at 545 GHz in intensity and 353 GHz in polarisation from the 2015 Commander Planck+WMAP+Haslam 408 MHz (Plank 2016)

d1, spectral index map from commander (assumes same spectral index for temperature and polarisation)

d7 Hensley and Draine 2012 + Hensley 2015: Emission modeled after dust size, shape temperature and ferromagnetic iron inclusion

$$\log_{10} \mathcal{U}_p = (4 + \beta_{d,p}) \log_{10} \left(\frac{T_{d,p}}{\langle T_d \rangle} \right),$$

$$Q_{\nu,p}^{d7} = A_{d,p}^Q \frac{f_{\nu}(\mathcal{U}_p)}{f_{\nu_0}(\mathcal{U}_p)}$$

$$U_{\nu,p}^{d7} = A_{d,p}^U \frac{f_{\nu}(\mathcal{U}_p)}{f_{\nu_0}(\mathcal{U}_p)}.$$

Synchrotron template: 23 GHz map from WMAP 9 yr (Bennett et al. 2013)

s1, Miville-Deschênes et al. (2008): combination of WMAP (Hinshaw et al. 2007) and Haslam 408 MHz data (Haslam et al. 1982)

s3, global curvature index $C = -0.052$ (Kogut et al 2012)

$$Q_{\nu,p}^{s3} = A_{s,p}^Q \left(\frac{\nu}{\nu_0} \right)^{\beta_{s,p} + 2 + C \ln(\nu/\nu_0)}$$

$$U_{\nu,p}^{s3} = A_{s,p}^U \left(\frac{\nu}{\nu_0} \right)^{\beta_{s,p} + 2 + C \ln(\nu/\nu_0)},$$