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# Updated Constraints on Hubble Tension solutions

With recent SPT-3G and SH0ES data

Ali Rida Khalife

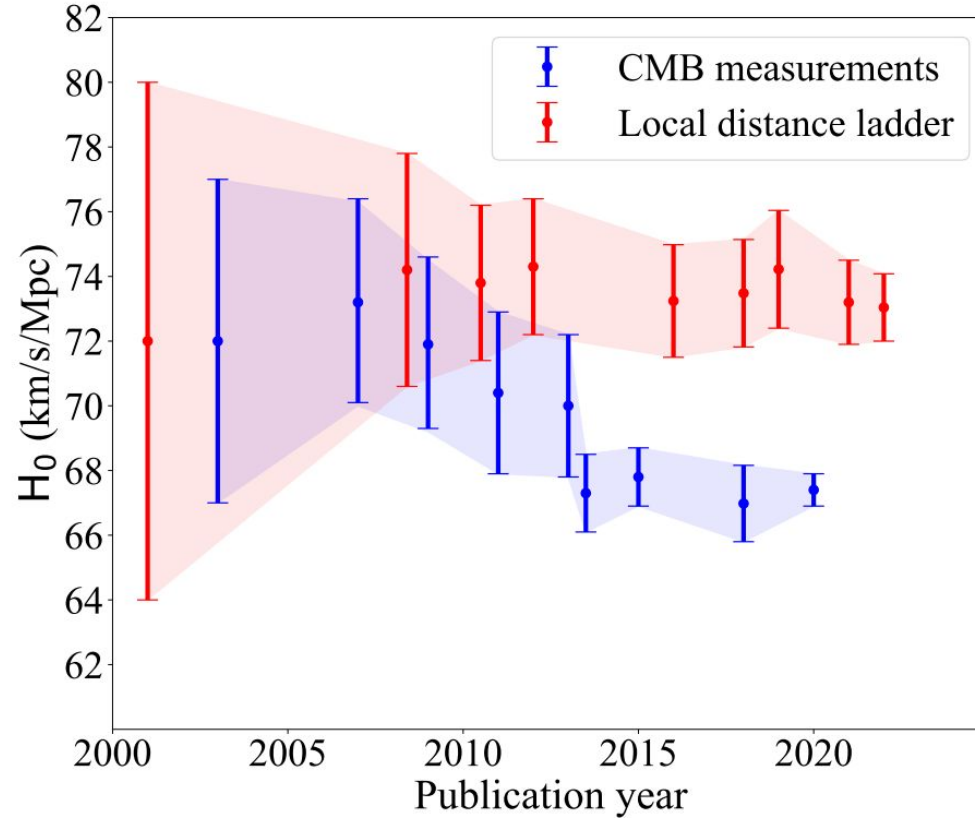
Collaborators: Mariam Bahrami, Sven Günther and Julien Lesgourgues from RWTH Aachen  
Silvia Galli and Karim Benabed from IAP

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Thanks to the great support from the IAP CMB team:  
Federica Guidi, Aristide Doussot, Eric Hivon, Etienne Camphuis, Lennart Balkenhol and Aline Vitrier

04/12/2023, CMB-France

# The Trouble with Hubble



Source: [Hubble Tension: The Evidence of New Physics](#)

# Goal of the Project

Use the full [SPT3G 2018](#) data, in combination with others, to evaluate the potential of Cosmological models to solve the Hubble Tension.

Comparing with recent [SH0ES analysis](#):  $H_0 = 73.29 \pm 0.90 \text{ km/s/Mpc}$  ([2306.00070](#)).

Study 5 classical  $\Lambda$ CDM extensions + 3 Elaborate Models (+extensions).

Assess these models with Tension metrics.

Update  $H_0$  Olympics paper ([2107.1029](#)) with new metrics and with massive neutrinos.

# How to Solve it

- Solutions to the Hubble Tension include changing the Physics pre-recombination or in the late universe
- Note:  $100 \times \theta = 1.04075 \pm 0.00028$  ([Balkenhol et al.](#))

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \text{sin}_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

Sound Speed  $\rightarrow$  (points to the upper integral)

$H(z)$   $\rightarrow$  (points to the upper integral)

Flat, closed or open  $\rightarrow$  (points to  $\text{sin}_K$ )

$H(z)/H_0$   $\rightarrow$  (points to the lower integral)

# $\Lambda$ CDM Extensions

Extending  $\Lambda$ CDM with 3 degenerate massive neutrinos( $\Sigma m_\nu$ ) and:

- Chevallier-Polarski-Linder (CPL) Dark Energy ( $\omega(a) = \omega_0 + \omega_a(1-a)$ );  $a \equiv$  scale factor
- Free streaming Dark Radiation ( $N_{\text{eff}}$ )
- Spatial Curvature( $\Omega_K$ )
- Self Interacting Dark Radiation ( $N_{\text{SIDR}}$ )

# Elaborate Models

- **Varying electron mass ( $m_e$ ):**

Compactification in higher dimensional theories results in scalar fields that alter the effective mass of elementary particles, specifically electrons.

Recombination rate is affected  Recombination time changes

More details: [Hart & Chulba, 2018](#)(1705.03925); [Planck 2015](#)(1406.7482)

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

# Elaborate Models

- Varying electron mass ( $m_e$ )
  - $+\Sigma m_\nu$ : Study interplay between masses of the two species



# Elaborate Models

- Varying electron mass ( $m_e$ )
  - $+\Sigma m_\nu$
  - $+\Omega_K$ : Changing the time of recombination changes the distance

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

More details: [Sekigushi & Takahashi \(2020\)](#) (2007.03381)

# Elaborate Models

- Varying electron mass ( $m_e$ )
  - $+\Sigma m_\nu$
  - $+\Omega_K$
  - $+\Sigma m_\nu + \Omega_K$

# Elaborate Models

- **Varying electron mass ( $m_e$ )**
  - $+\Sigma m_\nu$
  - $+\Omega_K$
  - $+\Sigma m_\nu + \Omega_K$
- **Early Dark Energy:**

Also motivated by higher dimensional theories. A brief period of accelerated expansion around matter-radiation equality.

Free parameters:  $\theta_i$ ,  $z_c$  and  $f_{\text{EDE}}$

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

EDE

More details: [Smith & Poulin, 2023](#) (2309.03265); [Poulin et al, 2023](#) (2302.09032)

# Elaborate Models

- Varying electron mass ( $m_e$ )
  - $+\Sigma m_\nu$
  - $+\Omega_K$
  - $+\Sigma m_\nu + \Omega_K$
- Early Dark Energy ( $\theta_i, z_c, f_{\text{EDE}}$ )
- The Majoron:

Breaking lepton number symmetry produces a pseudo-scalar ( $\varphi$ ) that gives neutrinos their mass (like the Higgs). A particle Physics motivated SIDR.

Free parameters:  $m_\varphi, \Gamma_{\text{eff}}$  and  $N_{\text{DR}}$

More details: [Escudero & Witte, 2020](#) (1909.04044); [Escudero & Witte, 2021](#) (2103.03249)

# Tension Metrics

- *Marginalised Posterior Compatibility Level (MPCL):*  
What's the probability of getting 0 in the distribution of the difference between SH0ES and a model's  $H_0$  posteriors?

$$\mathcal{P}(\delta) = \mathcal{N} \int dH_0 \mathcal{P}_{\text{model}}(H_0) \mathcal{P}_{\text{SH0ES}}(H_0 - \delta) \simeq \mathcal{N}' \sum_i w_i \mathcal{P}_{\text{SH0ES}}(H_{0,i} - \delta)$$

Normalisation

Normalisation

Weights from chains

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$$q = \int_0^{\delta'} d\delta \mathcal{P}(\delta) .$$

Probability of finding  $\delta$  in  $[0, \delta']$ , such that  $\mathcal{P}(\delta') = \mathcal{P}(0)$



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$$n = \sqrt{2} \operatorname{erf}^{-1}(q)$$

Tension in units of  $\sigma$ , denoted by:  $Q_{\text{MPCL}}$

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$$n = \sqrt{2} \operatorname{erf}^{-1}(q)$$

Assuming Gaussian posteriors



$$n = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

# Tension Metrics

- *Marginalised Posterior Compatibility Level (MPCL):*

$$n = \sqrt{2} \operatorname{erf}^{-1}(q) \quad \text{Tension in units of } \sigma, \text{ denoted by } Q_{\text{MPCL}}$$

- *Difference of the Maximum A Posteriori (DMAP):*

$$Q_{\text{DMAP, model}} \equiv \sqrt{\chi_{\text{min, model, } \mathcal{D}+\text{SH0ES}}^2 - \chi_{\text{min, model, } \mathcal{D}}^2} ; \chi^2 = -2 \ln \mathcal{L} ; \mathcal{D} \equiv \text{data set}$$

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- *Akaike Information Criterion (AIC):*

$$\Delta \text{AIC}_{\text{model}} = \chi_{\min, \text{model}, \mathcal{D}+SH0ES}^2 - \chi_{\min, \Lambda\text{CDM}, \mathcal{D}+SH0ES}^2 \quad ; \quad N \equiv \# \text{ of parameters} \\ + 2(N_{\text{model}} - N_{\Lambda\text{CDM}}) .$$

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- *AIC without SH0ES*

# Data Sets and Numerical Tools

- Data sets:
  - SPT-3G 2018: TT,TE,EE
  - Planck 2018: TT,TE,EE+Lensing
  - BAO: 6dFGS+SDSS MGS, DR12-16
  - ACT: DR4
  - Pantheon SN Ia
- Theory Codes: [CLASS](#) and [CAMB](#)
- Monte Carlo Sampler: [COBAYA](#)
- Minimizing  $\chi^2$ : [Py-BOBYQA](#)
- New cosmological emulator ([arXiv:2307.01138](#))
- Our reference data set: SPT+Planck+BAO+Pantheon

# Main Results

Models	$H_0(\text{km/s/Mpc})$	$Q_{\text{MPCL}}(\sigma)$	$Q_{\text{DMAP}}(\sigma)$	w/o SH0ES		w/ SH0ES	
				$\Delta\chi^2$	$\Delta\text{AIC}$	$\Delta\chi^2$	$\Delta\text{AIC}$
$\Lambda\text{CDM}$	$67.56(67.58)^{+0.38}_{-0.38}$	6.0	5.8	0	0	0	0
$+\Sigma m_\nu$	$67.60(67.01)^{+0.49}_{-0.43}$	5.9	—	—	—	—	—
$+\Sigma m_\nu + \text{CPL}$	$67.94(67.89)^{+0.78}_{-0.79}$	4.5	—	—	—	—	—
$+\Sigma m_\nu + N_{\text{eff}}$	$68.25(67.45)^{+0.62}_{-0.76}$	4.2	—	—	—	—	—
$+\Sigma m_\nu + \Omega_K$	$67.67(66.88)^{+0.62}_{-0.62}$	5.1	—	—	—	—	—
$+\Sigma m_\nu + N_{\text{SIDR}}$	$68.53(69.06)^{+0.69}_{-0.92}$	3.8	4.0	-0.1	3.9	-17.1	-13.1
$m_e$	$68.00(68.03)^{+1.06}_{-1.07}$	3.8	3.9	0.0	2.0	-18.0	-16.0
$m_e + \Sigma m_\nu$	$68.22(67.70)^{+1.09}_{-1.23}$	3.5	3.6	-0.9	3.1	-21.6	-17.6
$m_e + \Omega_K$	$68.20(67.42)^{+1.63}_{-1.60}$	<b>2.9</b>	3.1	-1.0	3.0	-24.7	-20.7
$m_e + \Omega_K + \Sigma m_\nu$	$69.75(67.75)^{+1.85}_{-2.93}$	<b>1.5</b>	<b>3.0</b>	-0.9	5.1	-25.8	-19.8
EDE	$68.18(68.55)^{+0.42}_{-0.79}$	3.8	<b>2.7</b>	-4.6	1.4	-31.1	-25.1
Majoron	$68.55(68.08)^{+0.48}_{-0.70}$	4.3	—	—	—	—	—

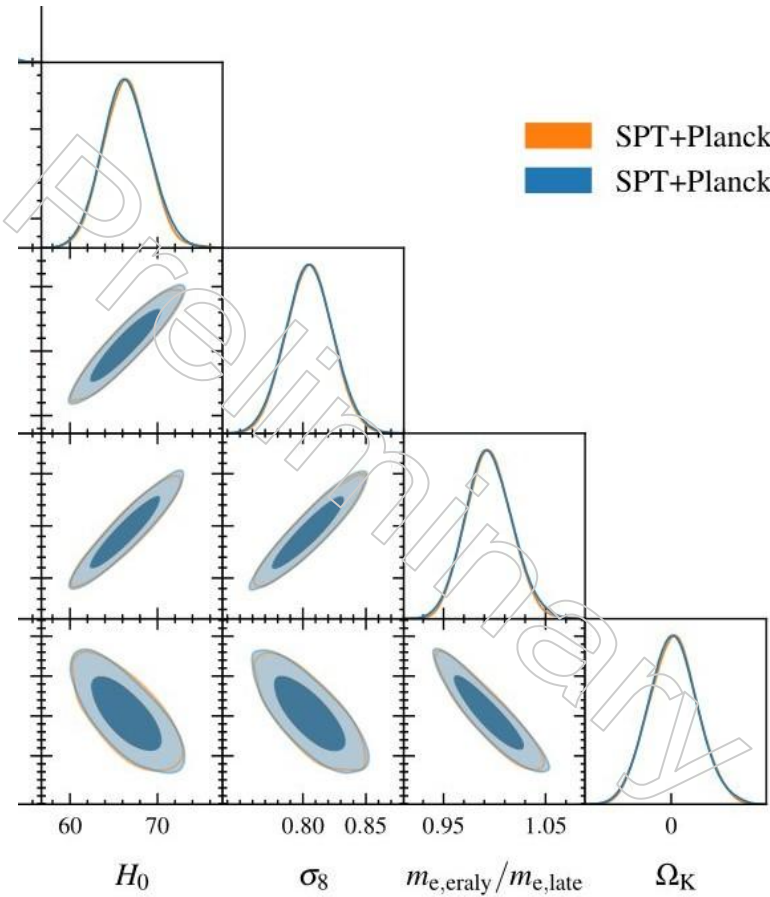
# Compare with Olympics Paper

Model	$\Delta N_{\text{param}}$	$M_B$	Gaussian Tension	$Q_{\text{DMAP}}$ Tension		$\Delta\chi^2$	$\Delta\text{AIC}$		Finalist
$\Lambda\text{CDM}$	0	$-19.416 \pm 0.012$	$4.4\sigma$	$4.5\sigma$	X	0.00	0.00	X	X
$\Delta N_{\text{nr}}$	1	$-19.395 \pm 0.019$	$3.6\sigma$	$3.8\sigma$	X	-6.10	-4.10	X	X
SIDR	1	$-19.385 \pm 0.024$	$3.2\sigma$	$3.3\sigma$	X	-9.57	-7.57	✓	✓ 🟡
mixed DR	2	$-19.413 \pm 0.036$	$3.3\sigma$	$3.4\sigma$	X	-8.83	-4.83	X	X
DR-DM	2	$-19.388 \pm 0.026$	$3.2\sigma$	$3.1\sigma$	X	-8.92	-4.92	X	X
SI $\nu$ +DR	3	$-19.440^{+0.037}_{-0.039}$	$3.8\sigma$	$3.9\sigma$	X	-4.98	1.02	X	X
Majoron	3	$-19.380^{+0.027}_{-0.021}$	$3.0\sigma$	$2.9\sigma$	✓	-15.49	-9.49	✓	✓ 🟡
primordial B	1	$-19.390^{+0.018}_{-0.024}$	$3.5\sigma$	$3.5\sigma$	X	-11.42	-9.42	✓	✓ 🟡
varying $m_e$	1	$-19.391 \pm 0.034$	$2.9\sigma$	$2.9\sigma$	✓	-12.27	-10.27	✓	✓ 🟡
varying $m_e + \Omega_k$	2	$-19.368 \pm 0.048$	$2.0\sigma$	$1.9\sigma$	✓	-17.26	-13.26	✓	✓ 🟡
EDE	3	$-19.390^{+0.016}_{-0.035}$	$3.6\sigma$	$1.6\sigma$	✓	-21.98	-15.98	✓	✓ 🟡
NEDE	3	$-19.380^{+0.023}_{-0.040}$	$3.1\sigma$	$1.9\sigma$	✓	-18.93	-12.93	✓	✓ 🟡
EMG	3	$-19.397^{+0.017}_{-0.023}$	$3.7\sigma$	$2.3\sigma$	✓	-18.56	-12.56	✓	✓ 🟡
CPL	2	$-19.400 \pm 0.020$	$3.7\sigma$	$4.1\sigma$	X	-4.94	-0.94	X	X
PEDE	0	$-19.349 \pm 0.013$	$2.7\sigma$	$2.8\sigma$	✓	2.24	2.24	X	X
GPEDE	1	$-19.400 \pm 0.022$	$3.6\sigma$	$4.6\sigma$	X	-0.45	1.55	X	X
DM $\rightarrow$ DR+WDM	2	$-19.420 \pm 0.012$	$4.5\sigma$	$4.5\sigma$	X	-0.19	3.81	X	X
DM $\rightarrow$ DR	2	$-19.410 \pm 0.011$	$4.3\sigma$	$4.5\sigma$	X	-0.53	3.47	X	X

Table I of  
[2107.10291](https://arxiv.org/abs/2107.10291)



# The Power of an Emulator



— SPT+Planck+BAO → 3 days of running time  
— SPT+Planck+BAO using emulator → 10 hrs of running time

[arXiv:2307.01138](https://arxiv.org/abs/2307.01138)

<https://github.com/svenguenter/cobaya>

# Conclusions and Future Plans

- Classical extensions of  $\Lambda$ CDM are interesting, but cannot solve the HT.
- Only  $m_e + \Omega_k (+\Sigma m_\nu)$  and EDE remain in the competition.
- Further investigation of these models, theoretically, is needed.
- Revisit these models, along with others, with upcoming SPT-3G 2019/2020 and ACT DR6 data.
- Stay on the lookout for the paper next week!

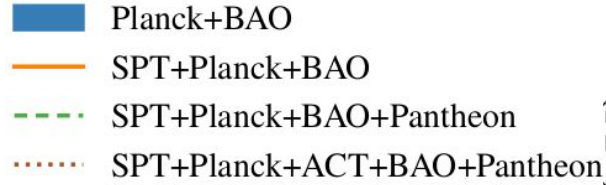
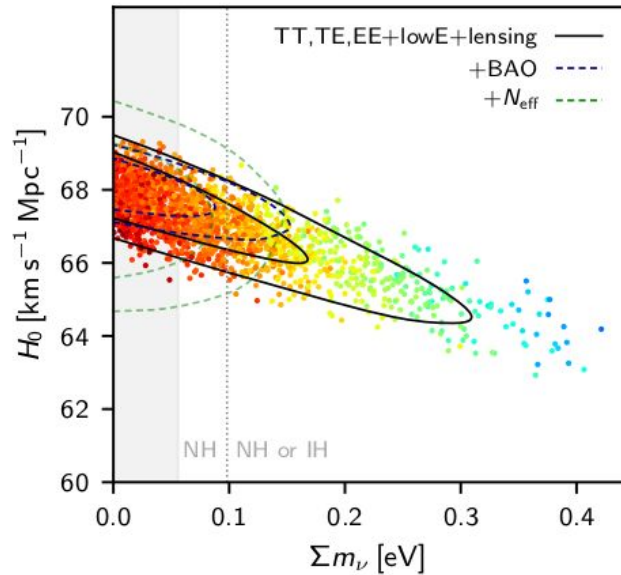
# Back Up

# Further Results

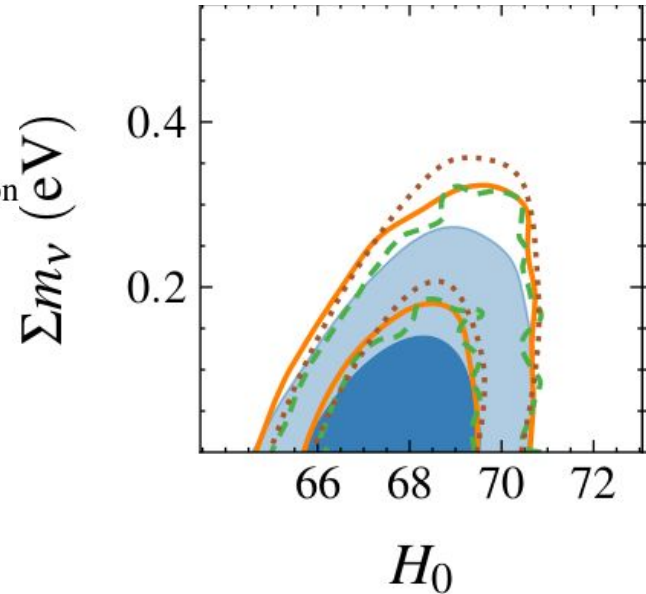
- SDSS-DR16 made the difference for the  $m_e$  model

# Further Results

- SDSS-DR16 made the difference for the  $m_e$  model
- Flip in degeneracy direction of  $H_0$ - $\Sigma m_\nu$  when varying  $m_e$



Planck 2018 ([Aghanim et al.](#))



# Further Results

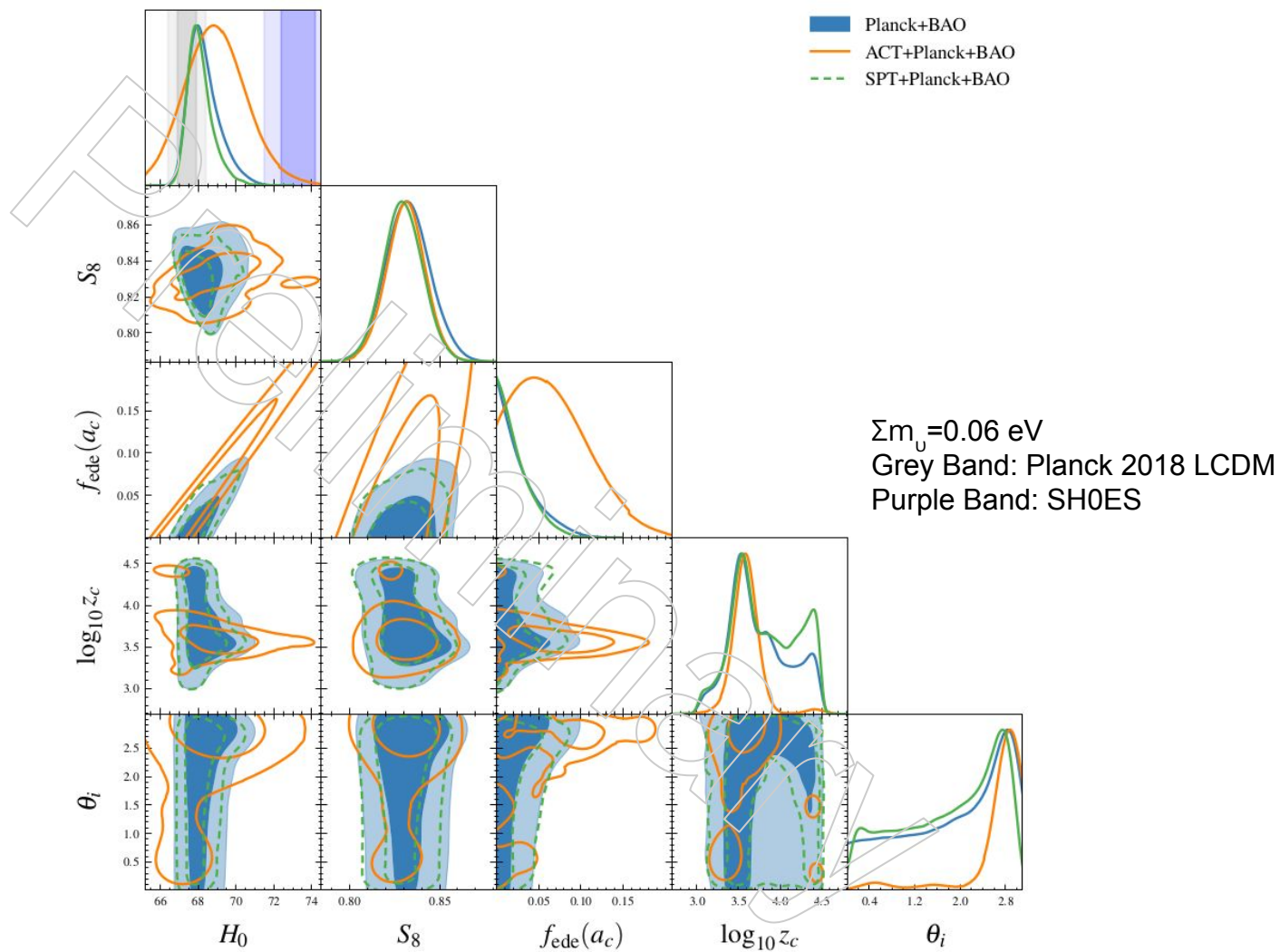
- SDSS-DR16 made the difference for the  $m_e$  model
- Flip in degeneracy direction of  $H_0$ - $\Sigma m_U$  when varying  $m_e$
- SPT-3G improved polarization data made a difference for  $m_e + \Omega_K$

$$H_0(\text{Planck+BAO}): \quad 69.1^{+2.1}_{-2.1}$$

$$H_0(\text{SPT+Planck+BAO}): \quad 67.7^{+1.9}_{-1.8}$$

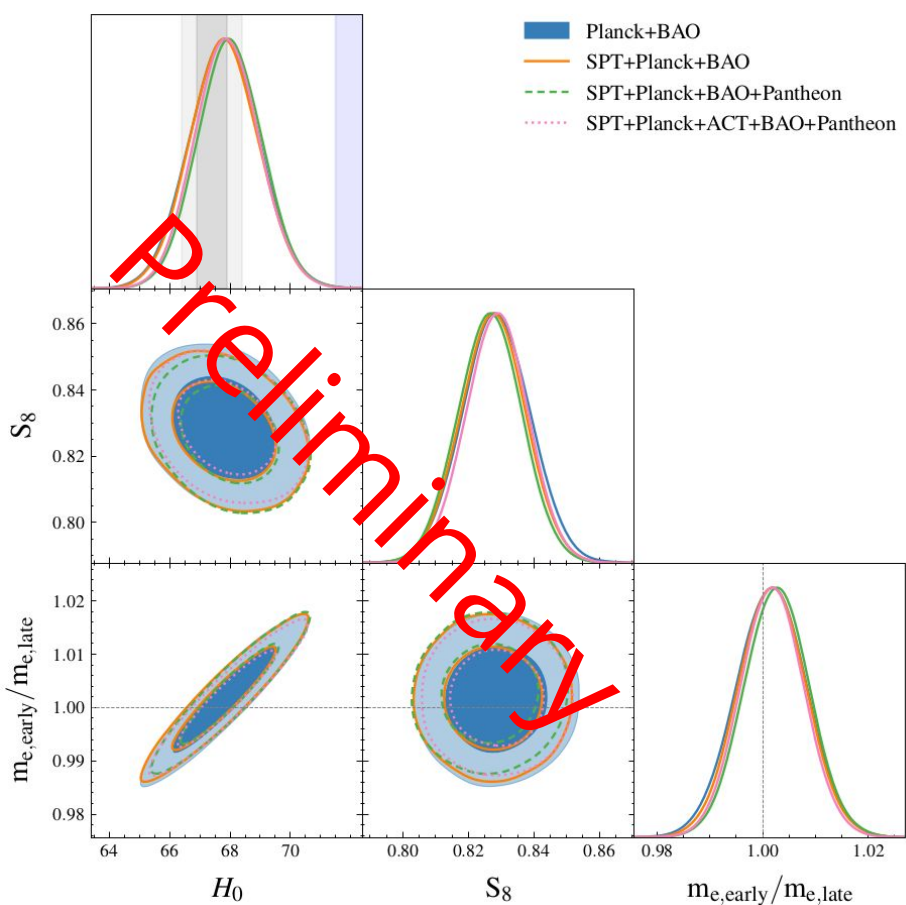
# Further Results

- SDSS-DR16 made the difference for the  $m_e$  model
- Flip in degeneracy direction of  $H_0 - \Sigma m_U$  when varying  $m_e$
- SPT-3G improved polarization data made a difference for  $m_e + \Omega_K$
- Preference for EDE from ACT-DR4 is still present





# Varying Electron Mass: Results

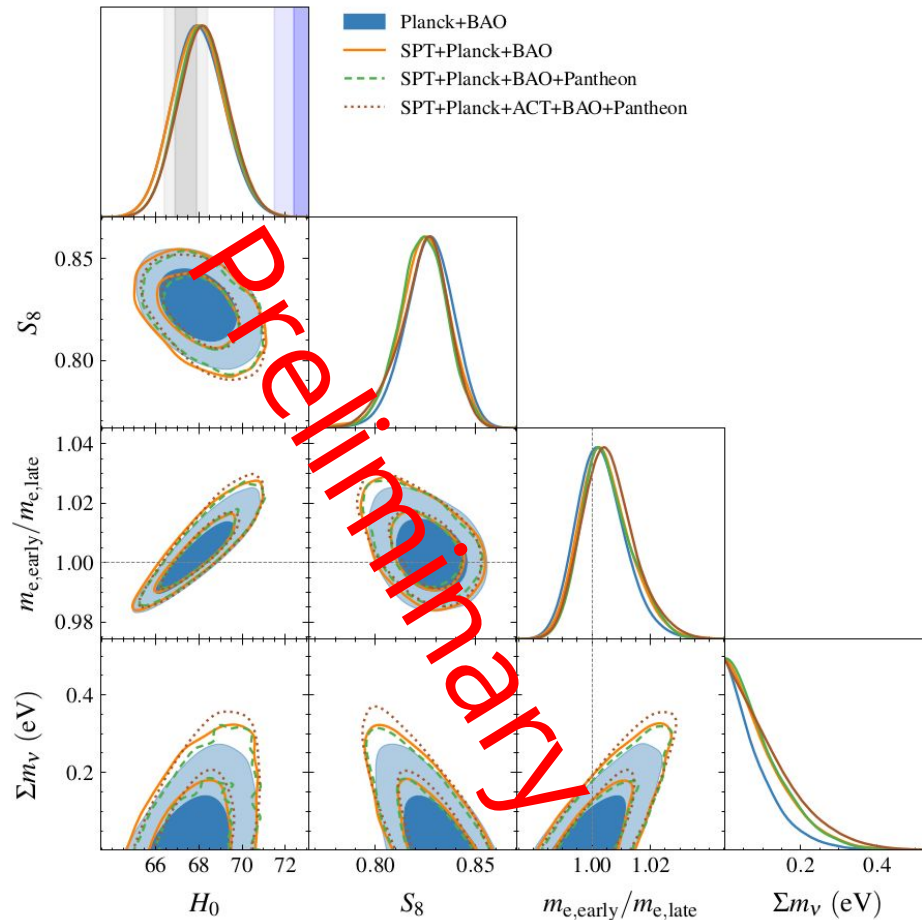


$\Sigma m_\nu = 0.06$  eV

Grey Band: Planck 2018 LCDM

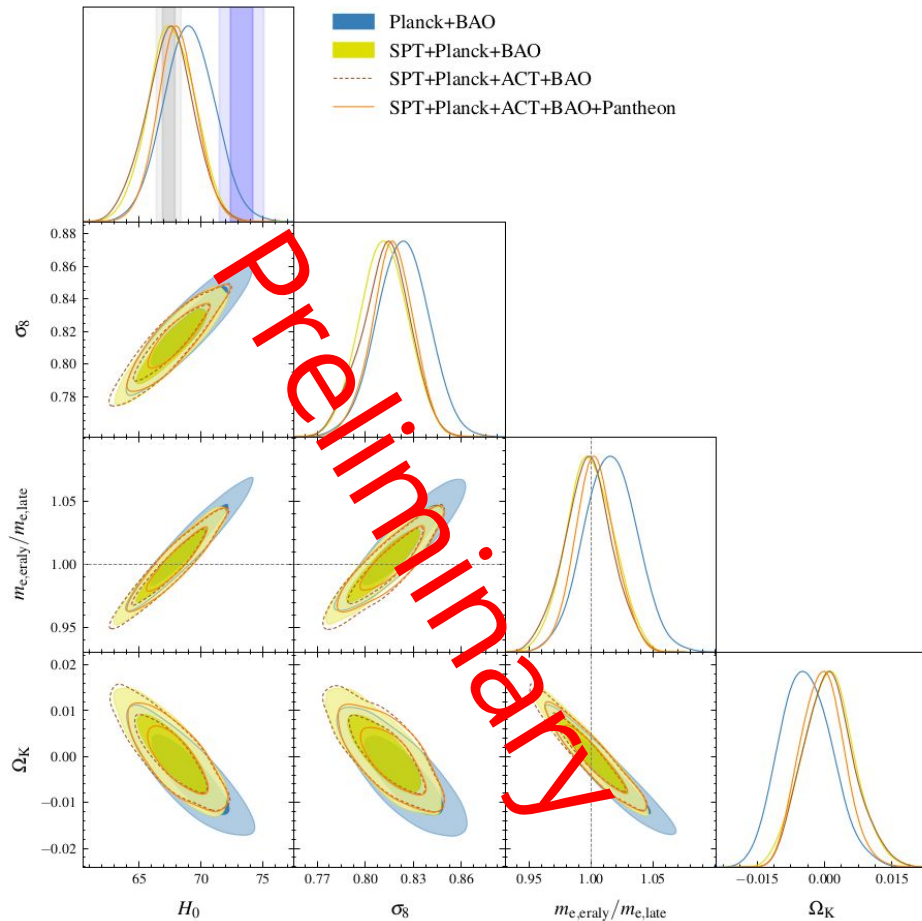
Purple Band: SH0ES

# Me+Mnu: Results

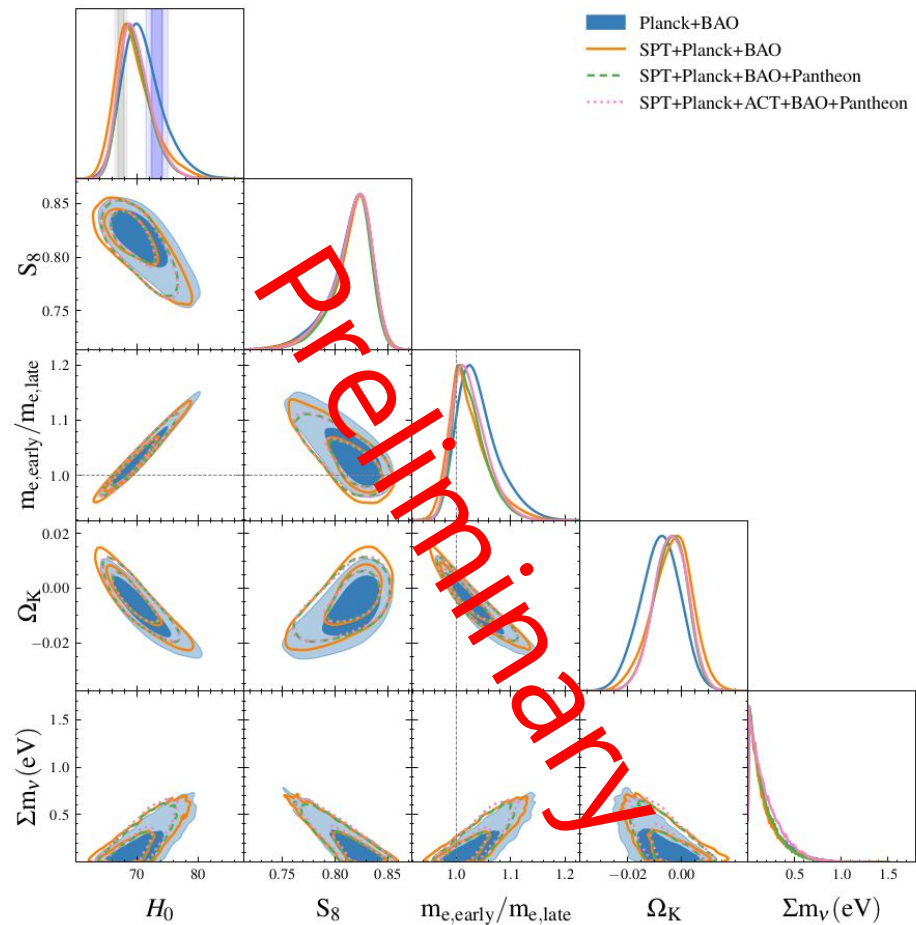


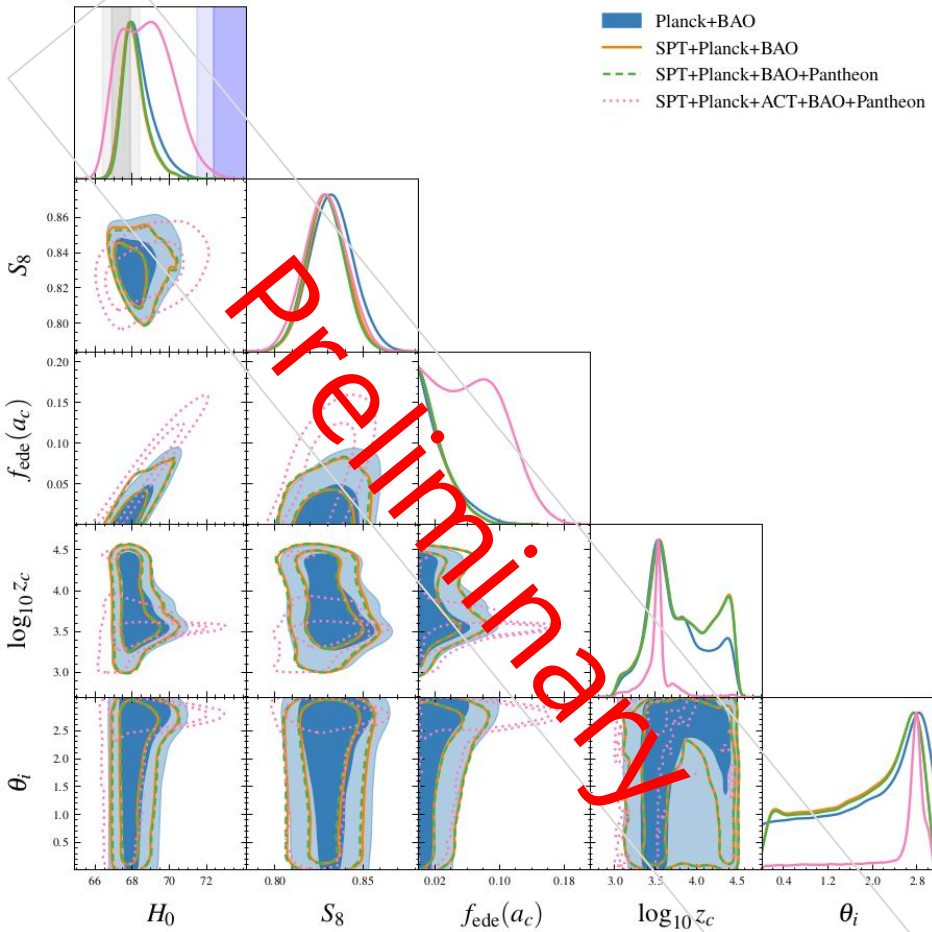
Grey Band: Planck 2018 LCDM  
Purple Band: SH0ES

# Me+0mk

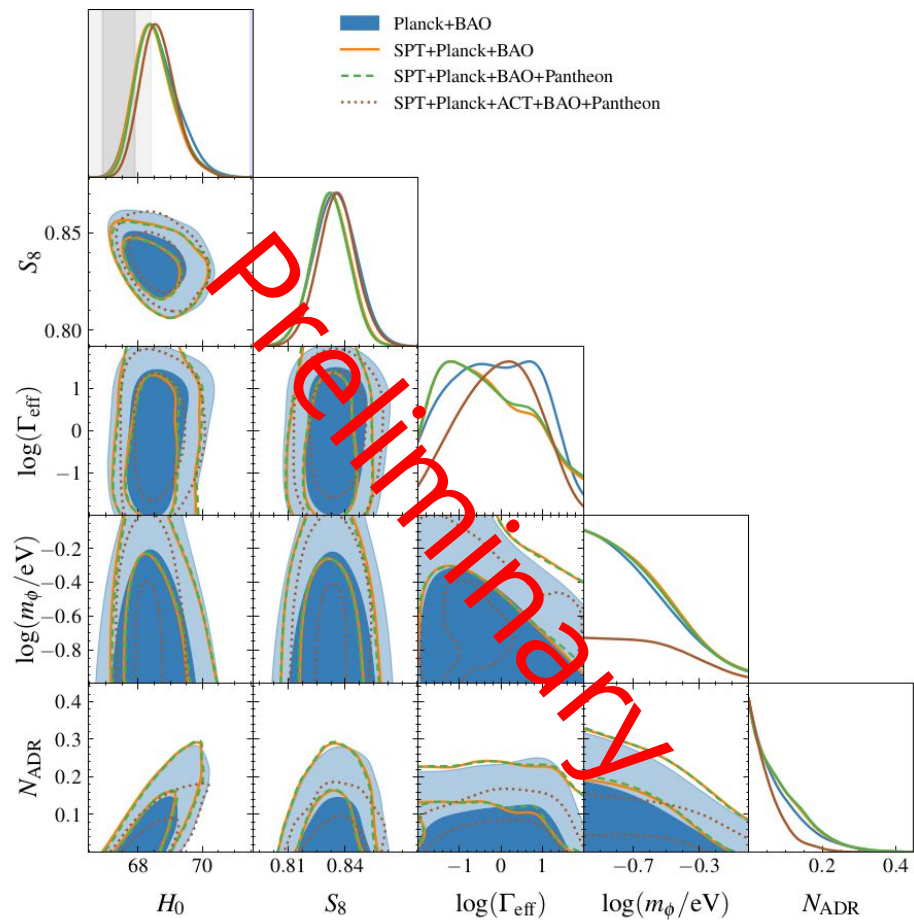


# Me+Mnu+Omk





# Majoron



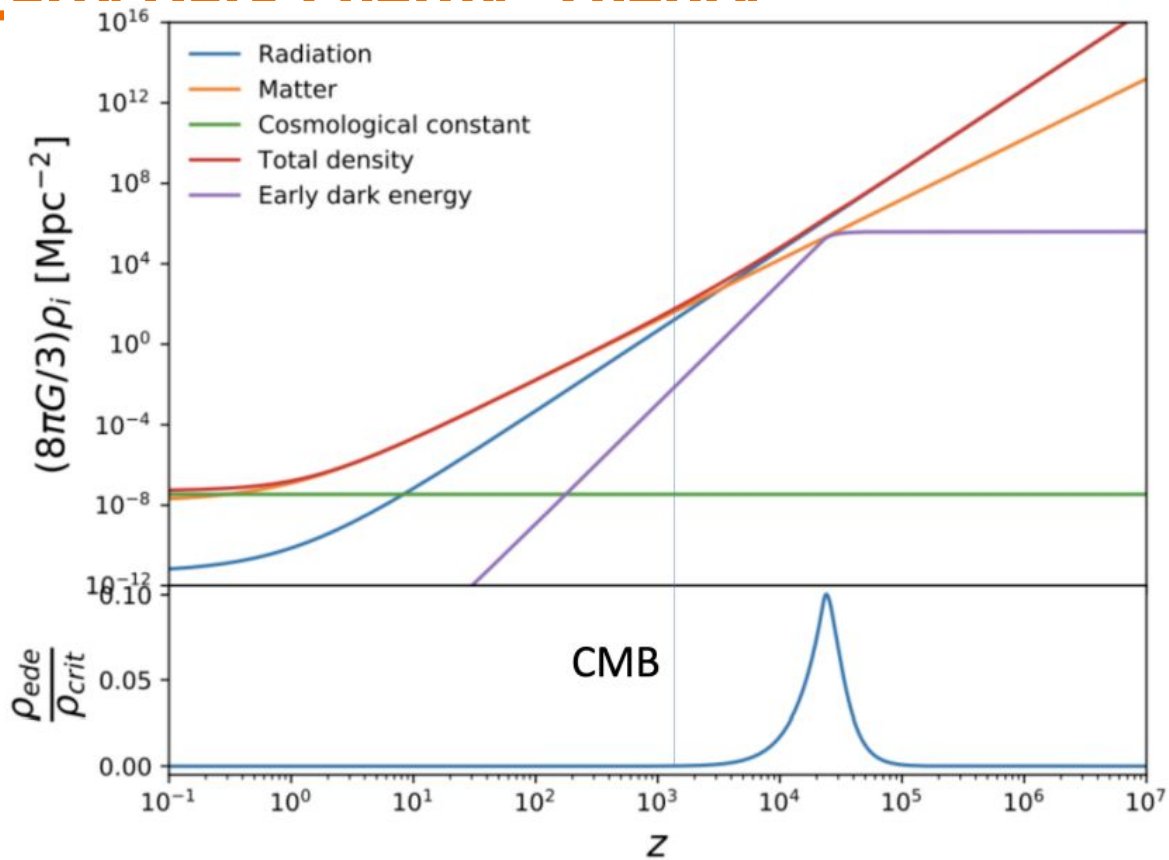
Sub eV mass

# Early Dark Energy

- A short phase of accelerated expansion around matter-radiation equality.
- A scalar field around that time oscillating or slowly rolling along its potential.
- Same mechanism as Inflation, but different scalar field and time.
- Early decrease in the sound horizon is compensated by an increase in  $H_0$ .
- References: [H<sub>0</sub> Olympics](#); [0205340](#); [2007.03381](#); [1912.03986](#); [1811.04083](#)

Most recent results: [arXiv:2309.03265](#)

# Early Dark Energy Theory

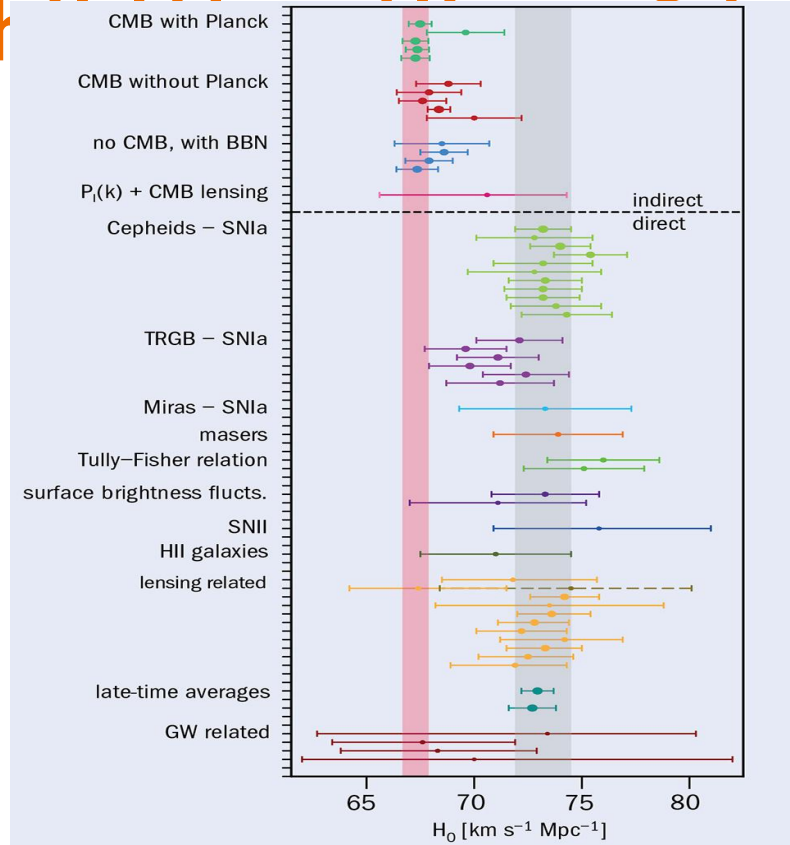


[Kamionkowski & Riess\(2022\)](#)



# The Trouble with

it



Source: [In the Realm of the Hubble Tension](#)

## Varying Electron Mass: Theory

- In Gravity theories with higher dimensions, compactification of the latter results in scalar fields.
- These scalars are gravitationally coupled to Standard Model fields, particularly electrons
- The result is an effective electron mass that could differ from the one we measure in the lab.

# Varying Electron Mass: Theory

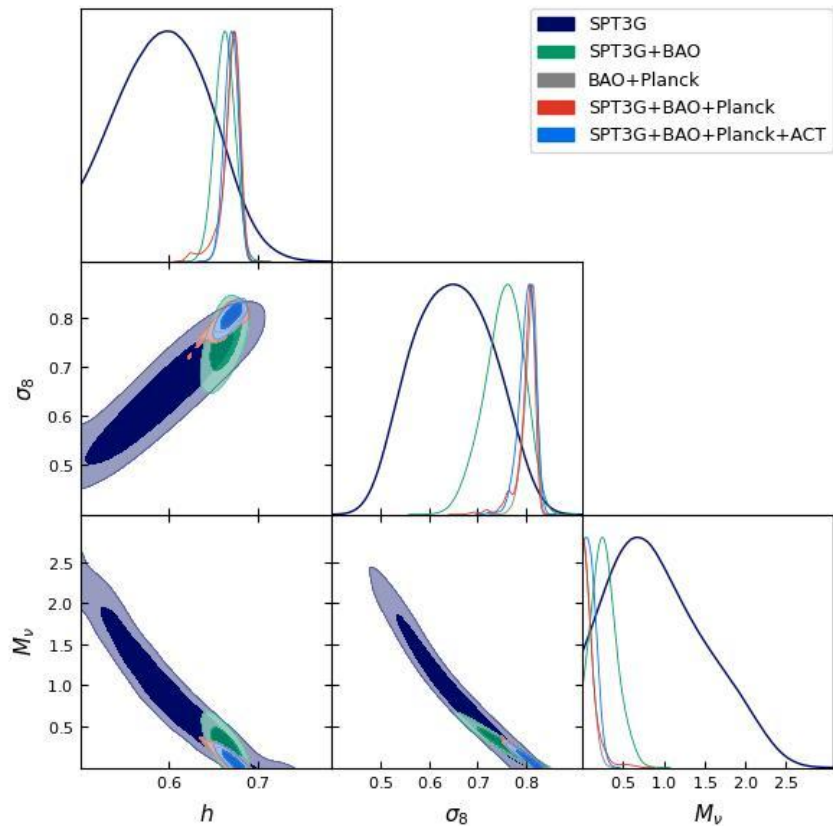
- Varying the electron mass affects Hydrogen/Helium recombination.
- Varying properties of Hydrogen/Helium formation efficiently impacts the time when recombination happens

$$\theta = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

# Varying Electron Mass: Theory

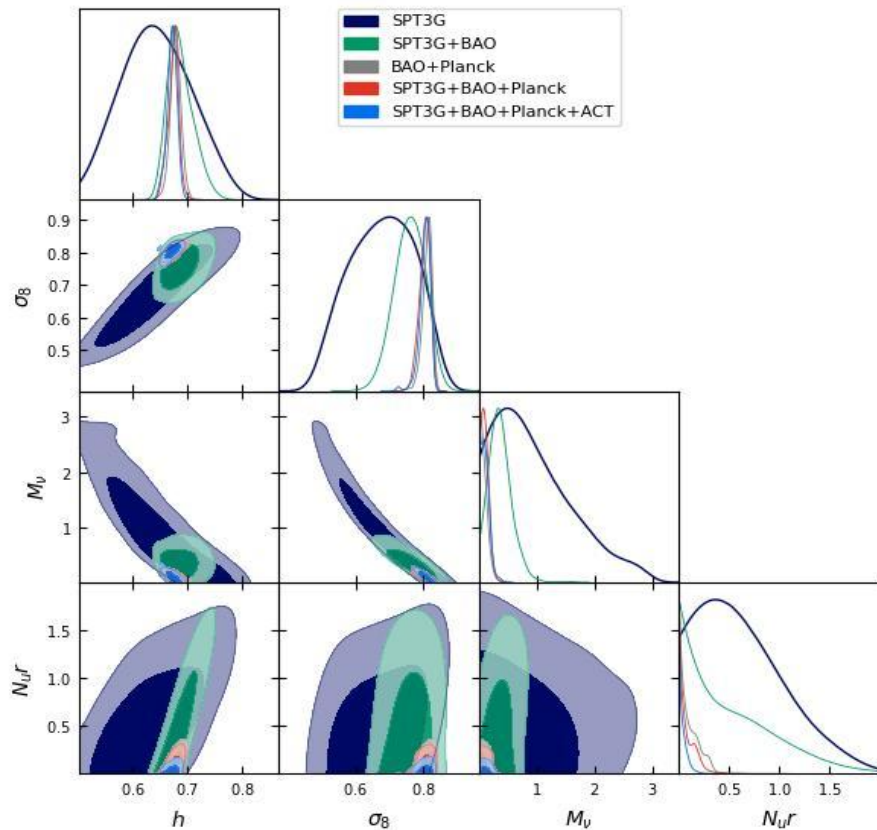
- Varying the electron mass affects Hydrogen/Helium formation rates.
- Varying properties of Hydrogen/Helium formation efficiently impacts the time when recombination happens.
- Allowing a non-zero curvature on top of that provides a better fit by adjusting the  $D_A$  to BAO and other late time probes.
- In practice, however, the model is described with a step function.

# $\Lambda$ CDM + $\Sigma m_\nu$

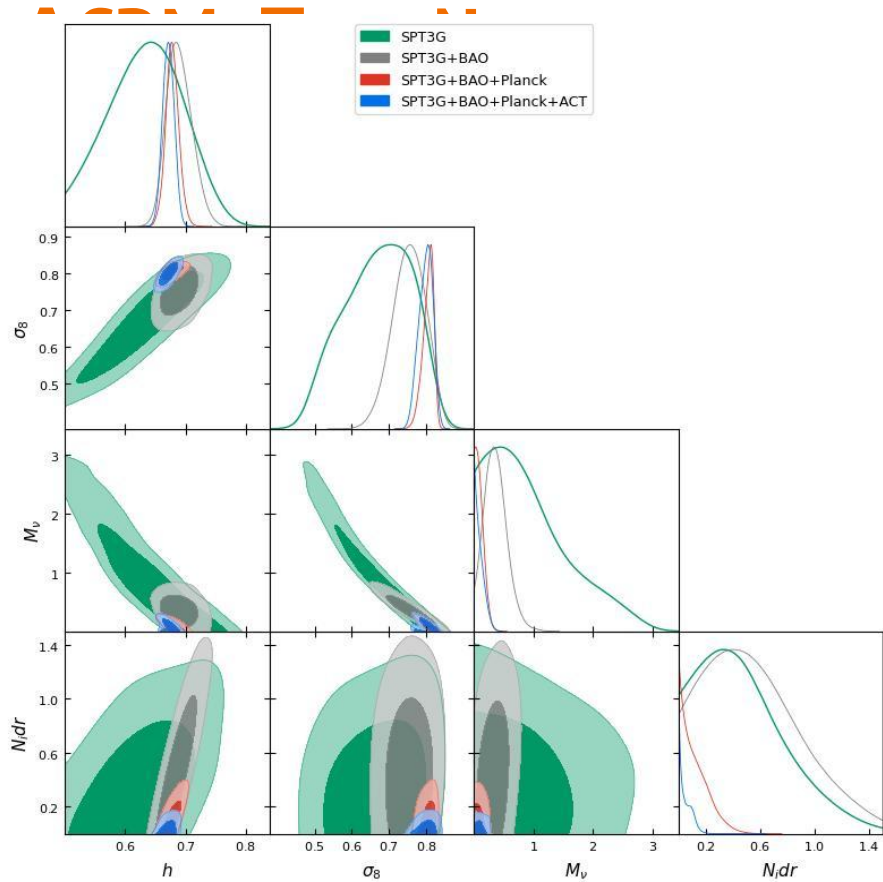


Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$67.00 \pm 0.82$
$\sigma_8$	$0.803 \pm 0.019$

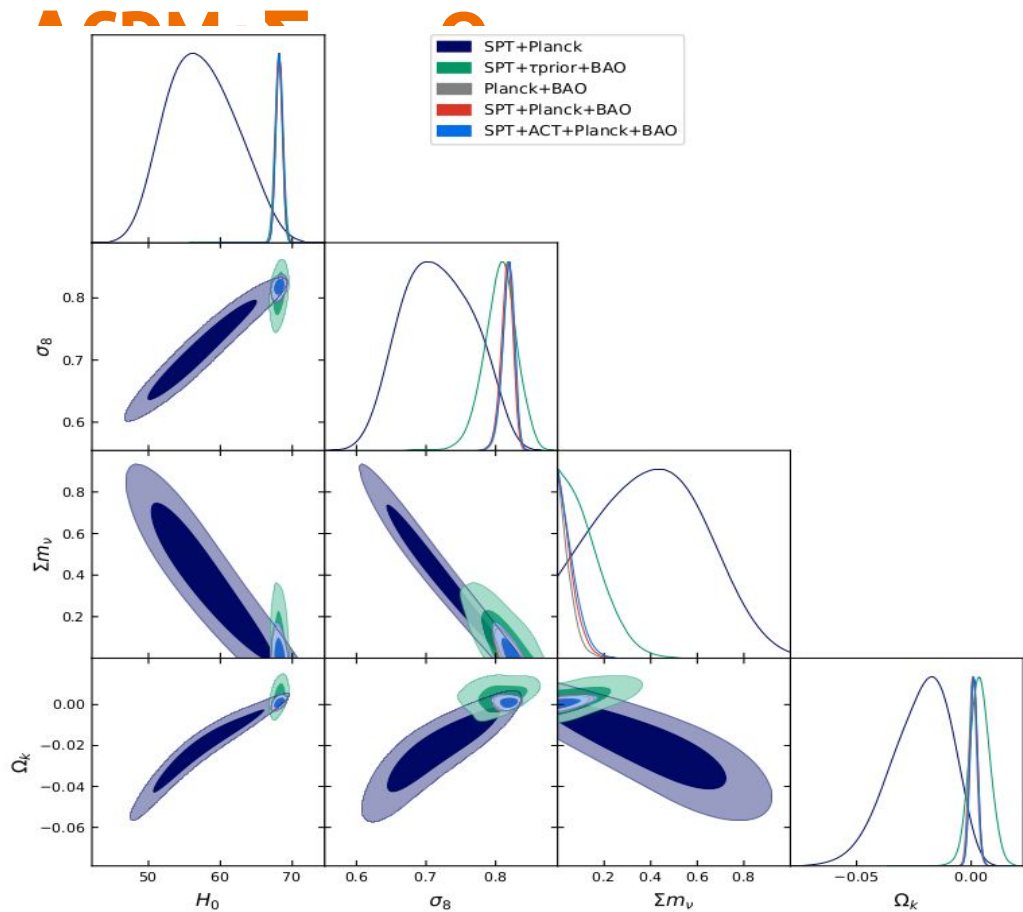
# $\Lambda$ CDM + $\Sigma_m$ + $N_{eff}$



Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$67.10 \pm 0.85$
$\sigma_8$	$0.812 \pm 0.009$

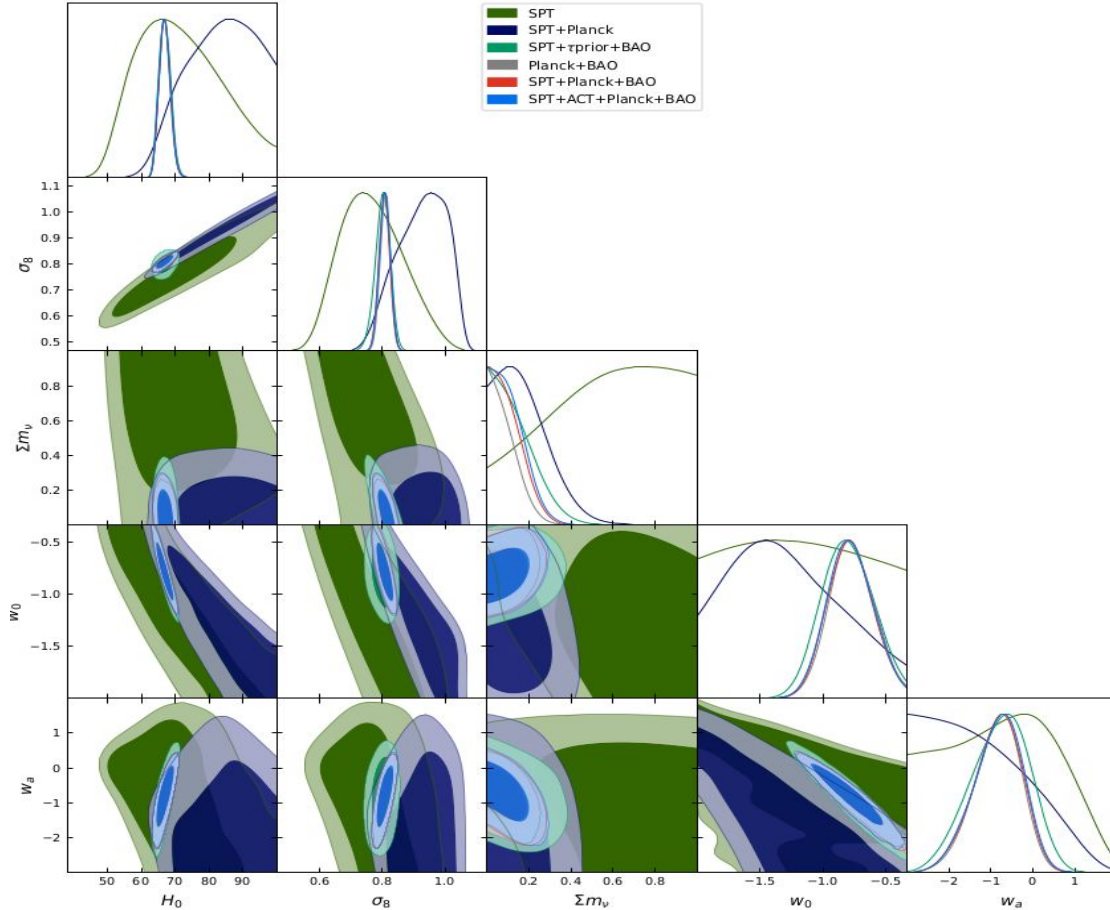


Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$67.22 \pm 0.91$
$\sigma_8$	$0.801 \pm 0.022$



Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$68.16 \pm 0.46$
$\sigma_8$	$0.818 \pm 0.009$

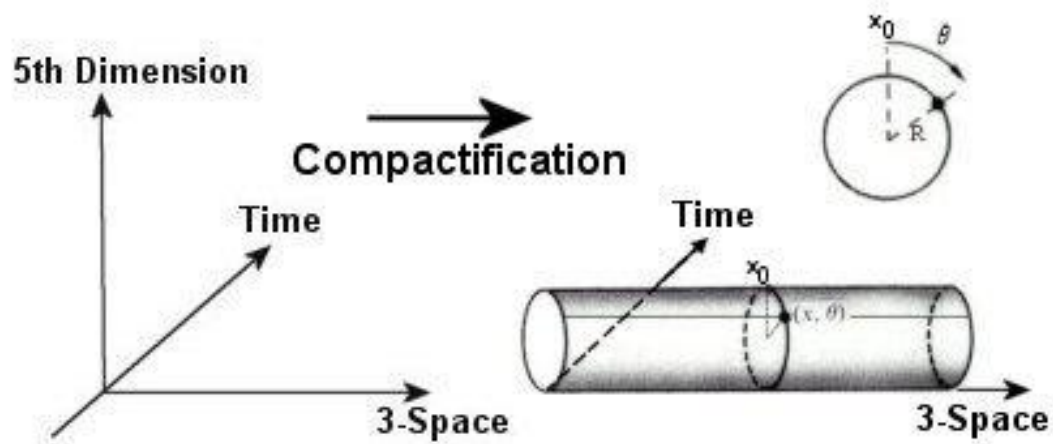




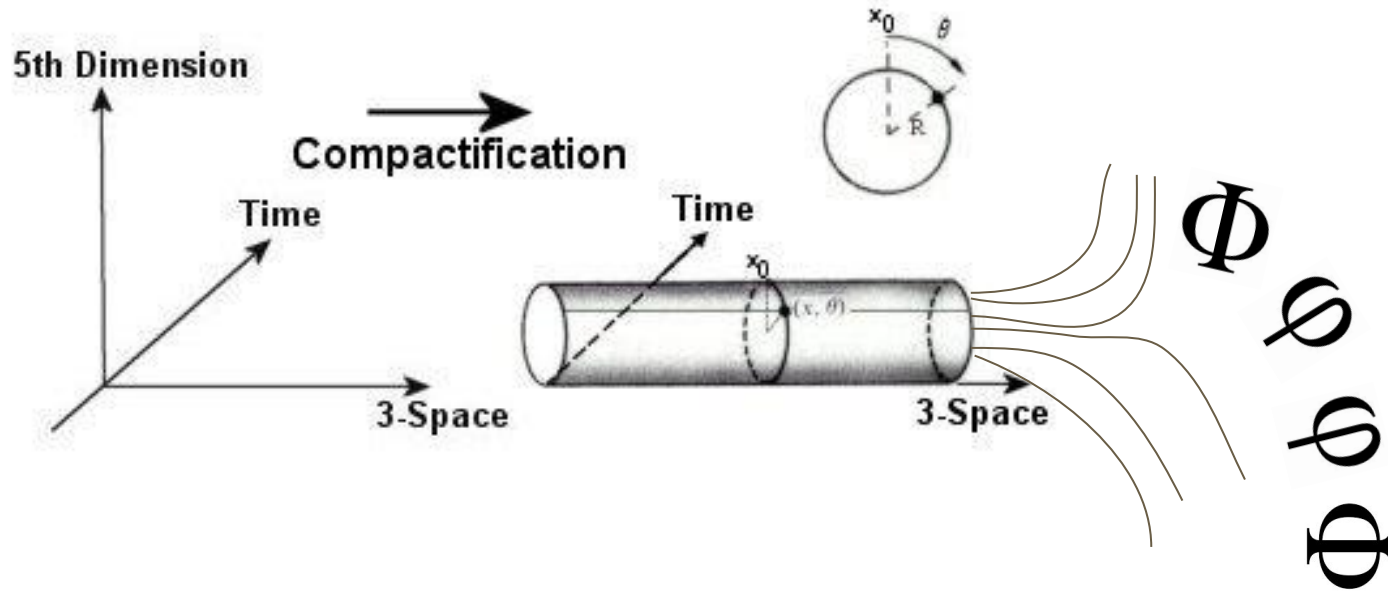
Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$66.89 \pm 1.62$
$\sigma_8$	$0.808 \pm 0.017$

# SLIDES FOR A GENERAL AUDIENCE TALK

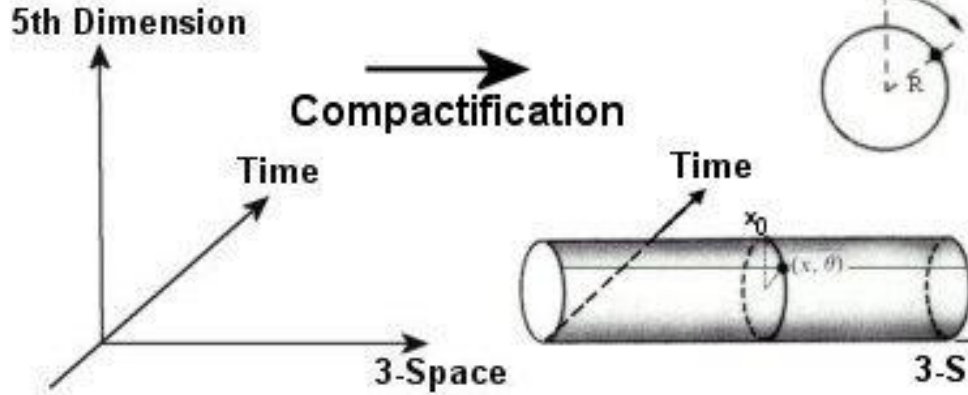
# Varying

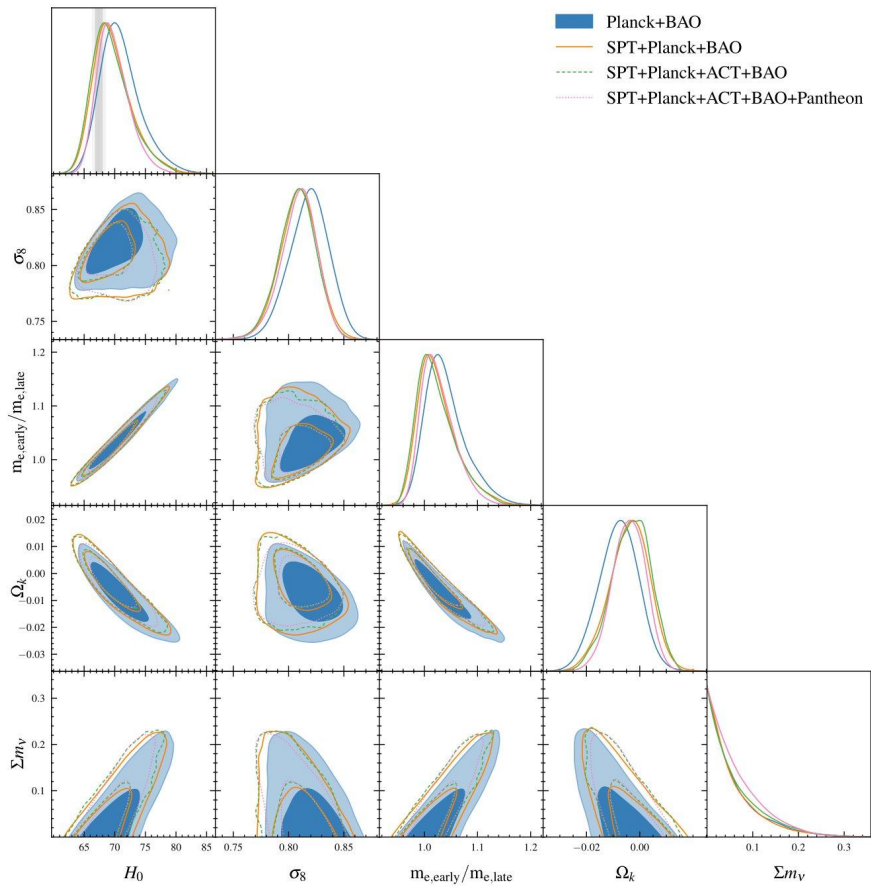


# Varying Electron Mass Theory

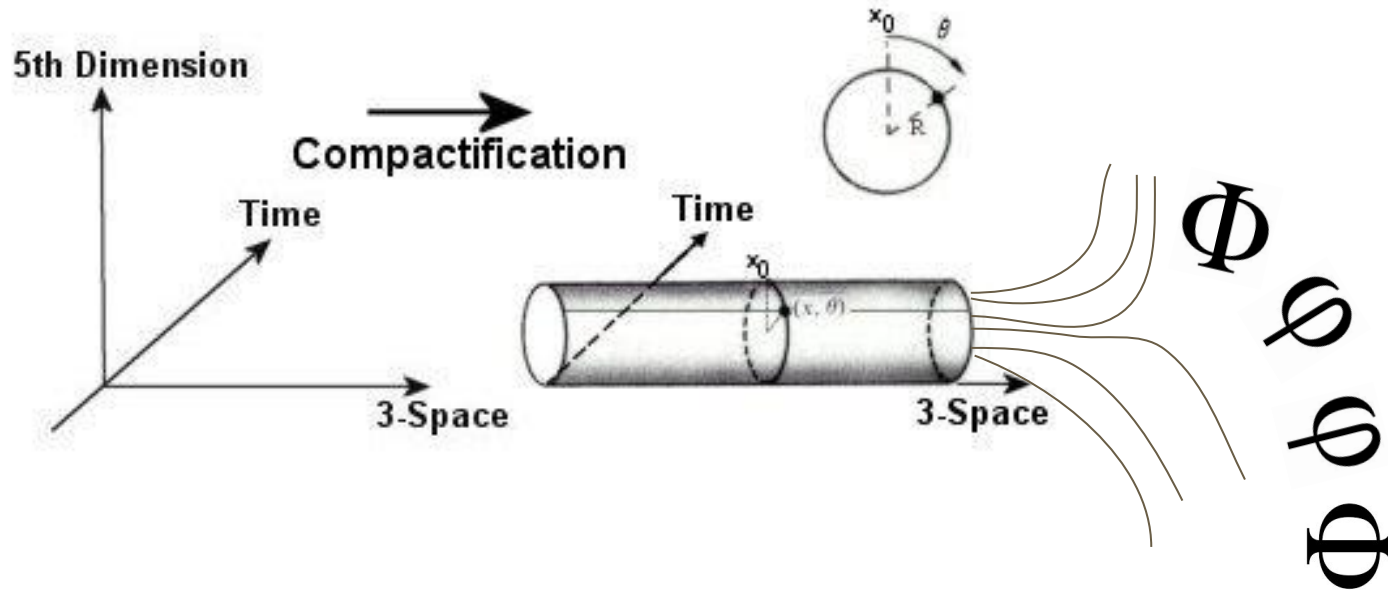


# Varying Electron Mass: Theory

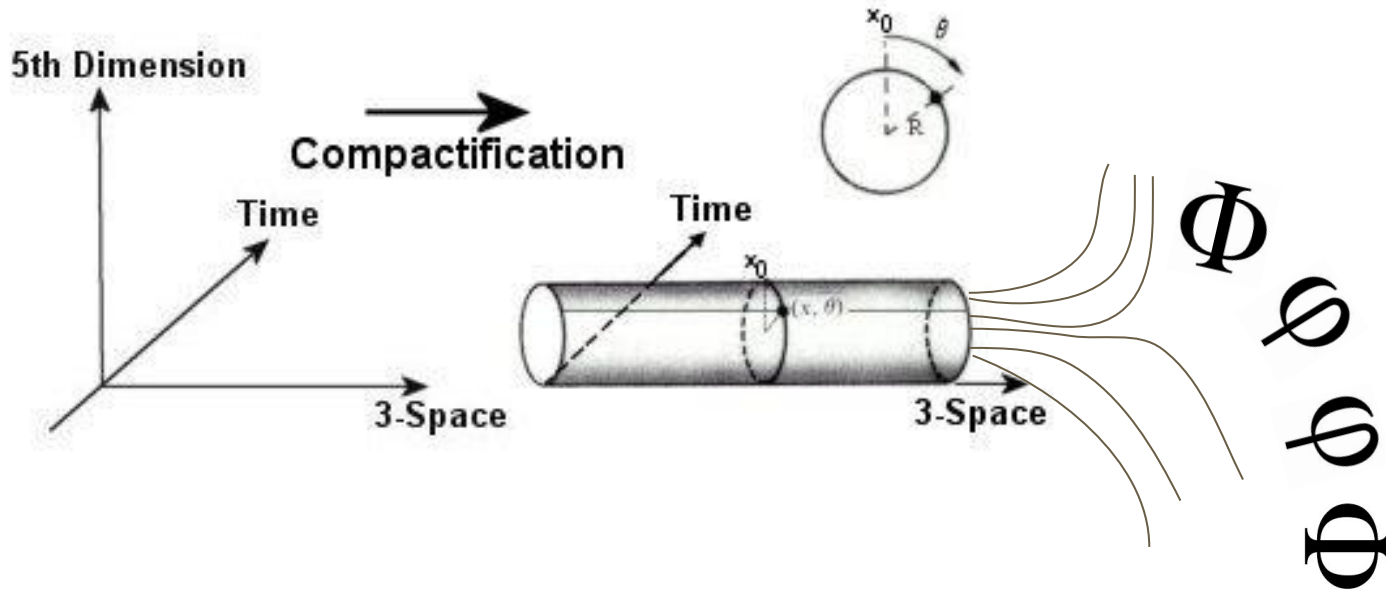




# Early Dark Energy Theory



# Early Dark Energy Theory



$$V(\phi) = \Lambda_{\text{ede}}^4 [1 - \cos(\phi/f_{\text{ede}})]^n$$

[Kamionkowski & Riess\(2022\)](#)