Detectability of higher harmonics with LISA

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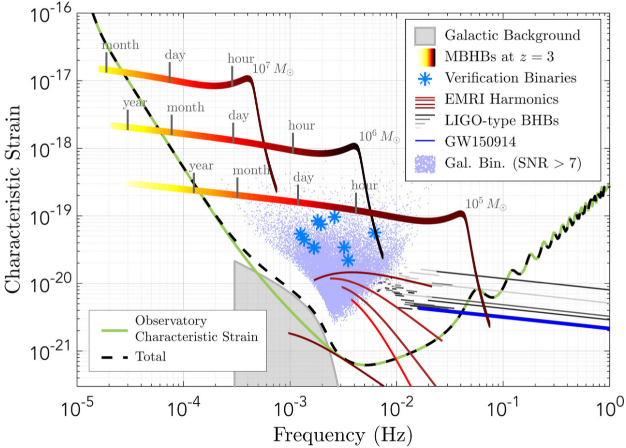
Motivation

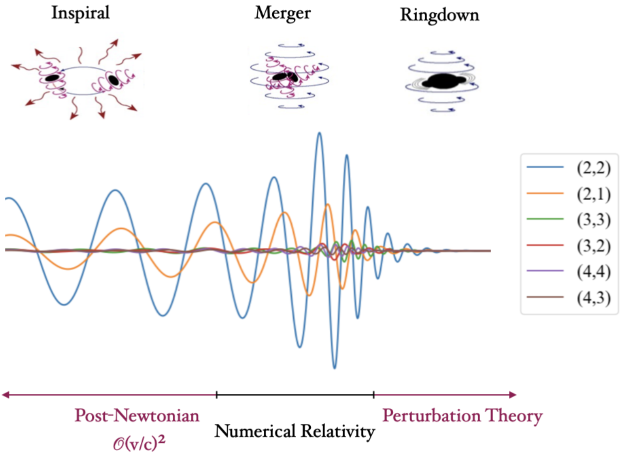
- SMBHB sources with high SNR ⟶ Test General Relativity



credit: LISA consortium proposal 2017[1]

[1] P. Amaro-Seoane *et al.*, *arXiv e-prints*, arXiv:1702.00786 (fév. 2017).

$$h^{IMR}(t,\Xi,\theta,\varphi) = \sum_{lm} A_{lm}(t,\Xi)e^{i\phi_{lm}(t,\Xi)} \; {}_{-2}Y^{lm}(\theta,\varphi)$$



Are we able to disentangle modes?

`Lisabeta` software[2] $\longrightarrow$ Introduce LISA response to the waveform

- IMR (Inspiral-Merger-Ringdown) Phenomenological waveforms in frequency domain
- Pass the waveform through LISA $\longrightarrow$ LISA response can be integrated as a transfer function in Fourier's domain for each harmonic

$$\mathcal{H}^I_{lm}(f,\Xi,\Theta) = \mathcal{T}^I_{lm}(f,\Theta)h_{lm}(f,\Xi)$$

where $\mathcal{T}^I_{lm}(f,\Theta)$ includes:

  - Orbits of the constellation
  - Delays with respect to SSB's frame and to LISA's CoM frame
  - TDI (Time delay interferometry)

Signal:

- Inject a signal with `PhenomHM`[3] (IMR waveform) with 6 harmonics: (l,m) = (2, 2), (2, 1), (3, 3), (3, 2), (4, 4), (4, 3)
- Random source from LDC-Sangria catalogue with a SNR $\sim$ 750

M ($M_\odot$)	2.28×10^6	χ_1	-0.55
q	2.76	χ_2	0.23

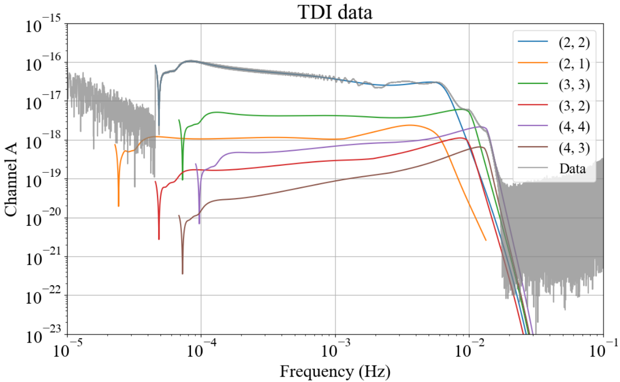
[2] S. Marsat *et al.*, *Physical Review D* **103** (avr. 2021).

[3] L. London *et al.*, *Physical Review Letters* **120** (avr. 2018).

Compare different models $M_i(l,m)$ with different number of modes with a Bayesian analysis.

- Dynesty[4] : nested sampler → estimate model evidence

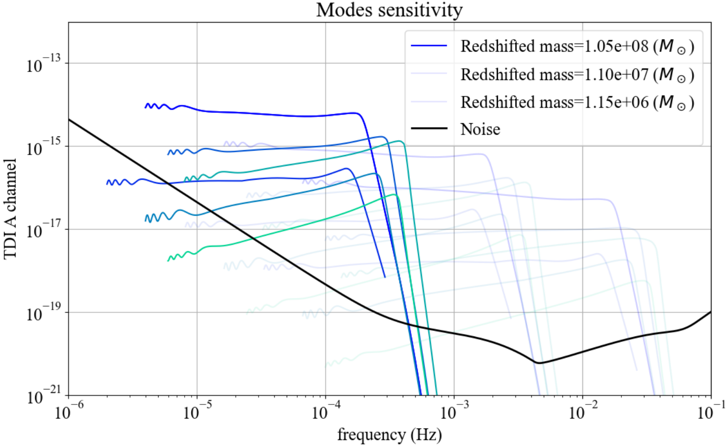
$$\mathcal{Z} = \int_\Theta \mathcal{L}(\theta)\pi(\theta)d\theta \rightarrow \text{ Bayes factor: } \quad \mathcal{B} = \frac{\mathcal{Z}_i}{\mathcal{Z}_j}$$



[4] J. S. Speagle, *Monthly Notices of the Royal Astronomical Society* **493**, 3132-3158 (avr. 2020).

Modes' SNR contribution

The detectability of the harmonics depends on the SNR of each mode. Which depend on the mass



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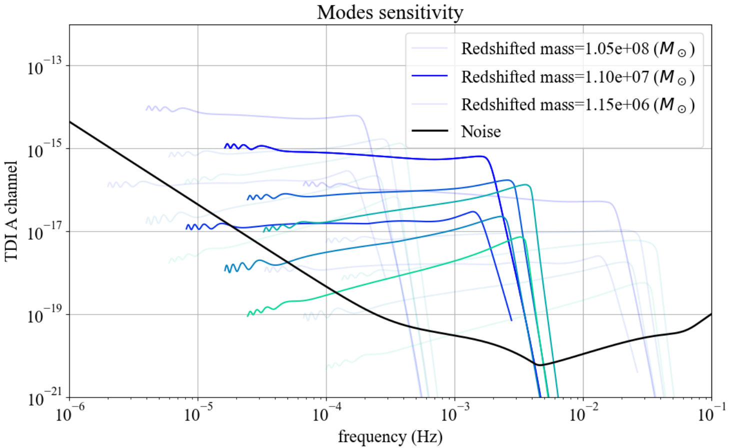

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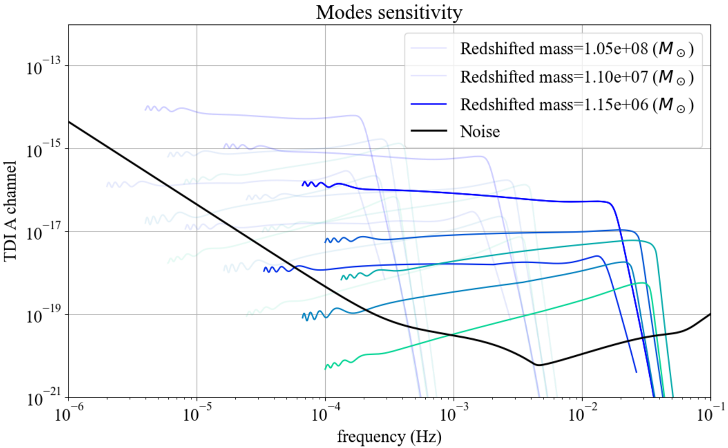
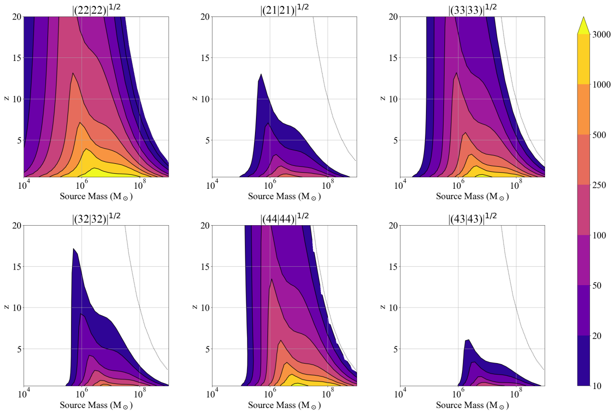


The SNR builds up in time and frequency:

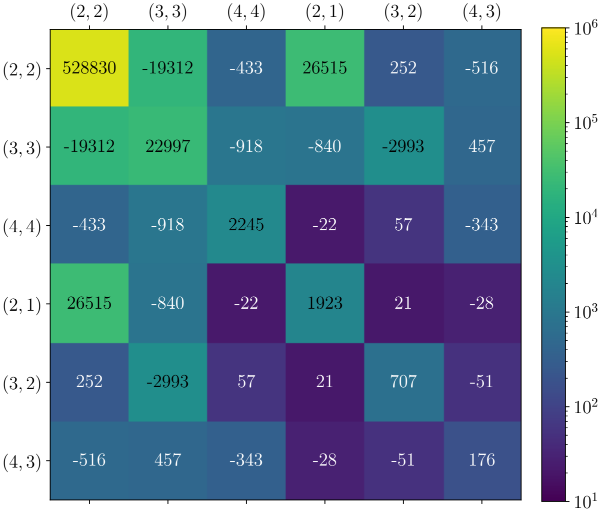
$$\rho^2 = \sum_{lm} \sum_{l'm'} \sum_{I} 4\,\mathrm{Re} \int \frac{\mathcal{H}^I_{lm}(f)\mathcal{H}^{I\,*}_{l'm'}(f)}{S_n(f)} df$$

$$\rho^2 = \sum_{lm} \sum_{l'm'} (lm|l'm')$$



Example source

$$(lm|l'm') = \sum_I 4\,\mathrm{Re} \int \frac{\mathcal{H}^I_{lm}(f)\mathcal{H}^I_{l'm'}{}^*(f)}{S_n(f)}\,df$$



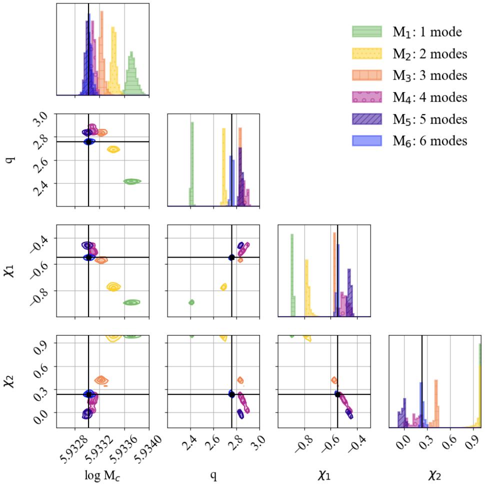
Detectability of higher harmonics

(2, 2)	M_1
(2, 2), (3, 3)	M_2
(2, 2), (3, 3), (4, 4)	M_3
(2, 2), (3, 3), (4, 4), (2, 1)	M_4
(2, 2), (3, 3), (4, 4), (2, 1), (3, 2)	M_5
(2, 2), (3, 3), (4, 4), (2, 1), (3, 2), (4, 3)	M_6

Parameter	True value	Estimated value M_6 in noisy data
$\log M_c$ $(M_\odot)$	5.93302	$5.93304^{+0.00009}_{-0.00010}$
q	2.759	$2.759^{+0.013}_{-0.023}$
χ_1	-0.549	$-0.548^{+0.011}_{-0.021}$
χ_2	0.232	$0.231^{+0.057}_{-0.030}$

Bayes factor $\longrightarrow$ $\log \mathcal{B}$

$\log(\mathcal{Z}_1/\mathcal{Z}_6)$	-6873
$\log(\mathcal{Z}_2/\mathcal{Z}_6)$	-1015
$\log(\mathcal{Z}_3/\mathcal{Z}_6)$	-259
$\log(\mathcal{Z}_4/\mathcal{Z}_6)$	-134
$\log(\mathcal{Z}_5/\mathcal{Z}_6)$	-100



Using an incorrect template results in a modelling error[5] $\Delta\theta_i$

If the statistical error ($\sigma_{\theta_i} \propto \mathrm{SNR}^{-1}$) is smaller than the modelling error, the bias in the parameters becomes relevant.

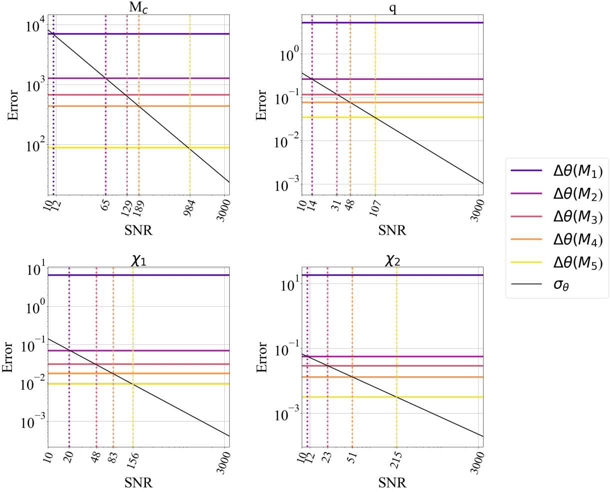
$$\begin{cases} \sigma_{\theta_i} = \sqrt{\Gamma^{-1}_{ii}} \\ \Delta\theta_i = \sum_j \Gamma^{-1}_{ij} \left(\frac{\partial \mathcal{H}_k}{\partial \theta_j} \middle| \delta \mathcal{H}_k \right) \end{cases}$$

Fisher matrix:

$$\Gamma_{ij} = \left(\frac{\partial \mathcal{H}}{\partial \theta_i} \middle| \frac{\partial \mathcal{H}}{\partial \theta_j} \right)$$

where

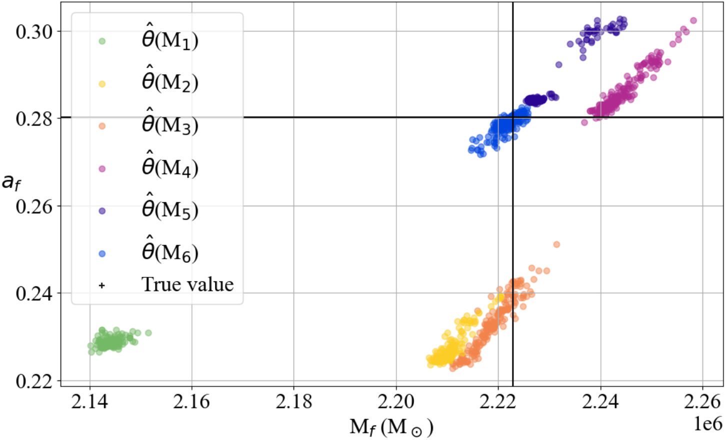
$$(a|b) = 4\mathcal{R} \int_0^\infty \frac{a(f) b^*(f)}{S_n(f)} df$$



[5] C. Cutler, M. Vallisneri, *Physical Review D* **76** (nov. 2007).

Final mass and spin

We can obtain the final mass and the spin of the remnant BH from the progenitors[6] :



[6] S. Husa *et al.*, *Phys. Rev. D* **93**, 044006 (4 fév. 2016).

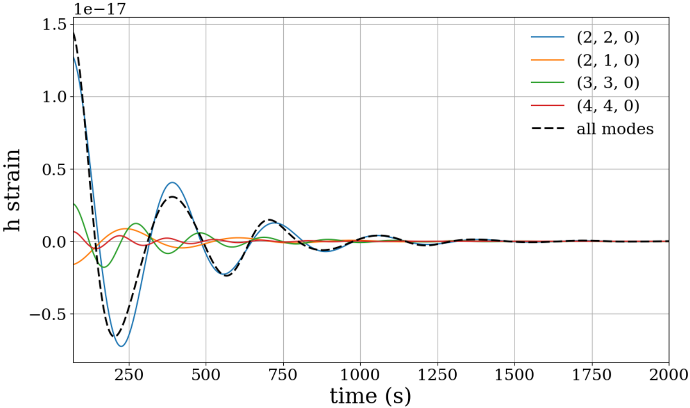
BH's spectroscopy to test no-hair theorem

Sum of sinusoidal decaying waves → quasi-normal modes (QNMs)

$$h_{lmn}(t,\Xi) = A_{lmn}(\Xi)e^{-t/\tau_{lmn}} \cos(\omega_{lmn}t + \phi_{lmn}(\Xi))$$

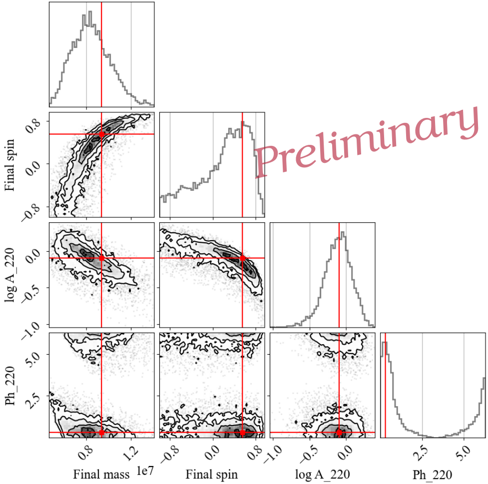
$$h(t,\Xi,\theta,\varphi) = \sum_{lmn} h_{lmn}(t,\Xi)\ {}_{-2}S^{lmn}(j\tilde{\omega}_{lmn},\theta,\varphi)$$

$$\tilde{\omega}_{lmn} = \underbrace{\omega_{lmn}}_{\text{freq}}(M_f,j_f) + \underbrace{i/\tau_{lmn}}_{\text{damping time}}(M_f,j_f)$$



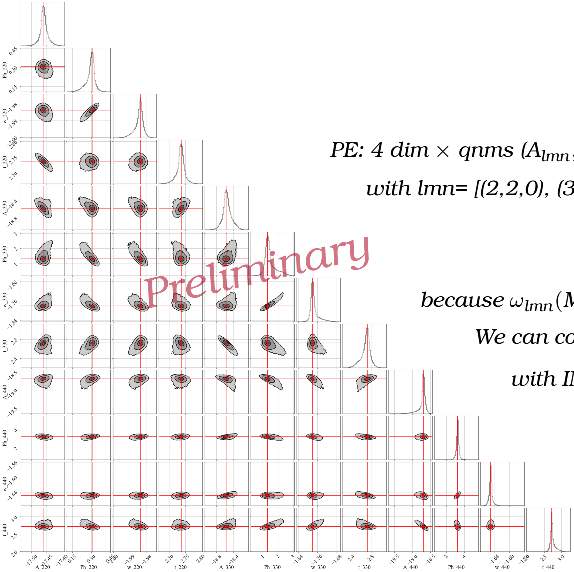
$$h(t,\Xi) = \sum_{lmn} A_{lmn}(\Xi) e^{-t/\tau_{lmn}} \cos(\omega_{lmn} t + \phi_{lmn}(\Xi))_{-2}S^{lmn}(j\tilde{\omega}_{lmn},\theta,\varphi)$$

$$h(t,M_f,j_f) = A_{220} e^{-t/\tau_{220}(M_f,j_f)} \cos\left(\omega_{220}(M_f,j_f)t + \phi_{220}\right)$$



QNMs (Agnostic case)

Agnostic case $\rightarrow h(t) = \sum_{lmn} A_{lmn} e^{i\phi_{lmn}} e^{-it(\omega_{lmn} - i/\tau_{lmn})}$



PE: 4 dim $\times$ *qnms* $(A_{lmn}, \phi_{lmn}, \omega_{lmn}, \tau_{lmn})$

with lmn= [(2,2,0), (3,3,0), (4,4,0)]

because $\omega_{lmn}(M_f, j_f), \tau_{lmn}(M_f, j_f)$

We can compare M_f, j_f

with IMR values

Conclusions

- We are able to discriminate waveform models and therefore their harmonics with a Bayesian analysis.
- We see how the use of an incorrect template of modes causes bias in the source parameter estimation.
- Given a certain SNR we can constrain the number of modes needed to estimate the parameters without significant bias, in the case of a waveform with 6 modes.
- Biased parameters can lead to misinterpretation in GR tests.
- To test GR with SMBHB events with a SNR of a few hundreds we will need higher harmonics with the inclusion of precession, eccentricity, mixing of the modes, ...

Next steps:

- Bayesian analysis using only the ringdown to study the sensitivity to the remnant's quasi-normal modes.
- Issues:
  - start of the ringdown
  - mixing of the modes
  - non-linear effects

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Thanks for your attention!

Back up slides

Modes and quasi-normal modes

We can write the GW emission from a rotating BH's ringdown as:

$$h_{lmn}(t) = A_{lmn} e^{-t/\tau_{lmn}} \cos(\omega_{lmn} t + \phi_{lmn})$$

$$h^{PT}(t,\theta,\varphi) = \sum_{l,m,n} h_{lmn}(t) {}_{-2}S_{lmn}(j_f\, \tilde{\omega}_{lmn},\theta,\varphi)$$

In NR the modes decomposition is in terms of spherical harmonics:

$$h^{NR}(t,\theta,\varphi) = \sum_{lm} A_{lm} e^{i\,\phi_{lm} t} {}_{-2}Y_{lm}(\theta,\varphi)$$

Press and Teukolsky noted:

$$\boxed{{}_{-2}S_{lmn} = {}_{-2}Y_{lm}} + j_f\, \tilde{\omega}_{lmn} \sum_{l \neq l'} {}_{-2}Y_{l'm} c_{l'lm} + \mathcal{O}(j_f\, \tilde{\omega}_{lmn})^2$$

where $c_{l'lm}$ are related to Clebsch-Gordan coefficients.

$$h^{NR}_{l'm}(t) \simeq \sum_{l,n} h^{PT}_{lmn}(t) = \sum_{l,n} A_{lmn} \sigma_{l'lmn} e^{i\,\tilde{\omega}_{lmn} t}$$

where[7]

$$\sigma_{l'lmn} = \int_{\Omega} {}_{-2}S_{lm}(j_f\, \tilde{\omega}_{lmn},\theta,\varphi) {}_{-2}\overline{Y}_{l'm}(\theta,\varphi) d\Omega$$

[7] L. London *et al.*, *Physical Review D* **90** (déc. 2014).

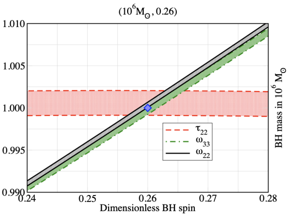
Testing the no-hair theorem with QNM

Knowing values of $\tilde{\omega}_{lm}$ one can find the mass and spin through a parametrization[8,9] :

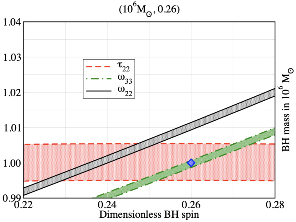
$$M\omega = f_1 + f_2(1-j)^{f_3}$$
$$Q = q_1 + q_2(1-j)^{q_3}$$
$$Q_{lmn} = \omega_{lmn}\tau_{lmn}/2$$

$$\omega_{lmn}^{nonGR} = \omega_{lmn}(1+\delta\omega_{lmn})$$
$$\tau_{lmn}^{nonGR} = \tau_{lmn}(1+\delta\tau_{lmn})$$

$$\begin{cases} IMR(M_f, a_f) \\ \tilde{\omega}_{lmn}(M_f, a_f) \end{cases}$$



Mass = 10^6 $M_\odot$, j = 0.26



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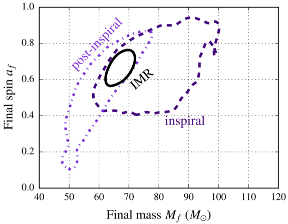

Mass = 62 $M_\odot$, j = 0.67

[8] E. Berti *et al.*, *Physical Review D* **73** (mars 2006).

[9] S. Gossan *et al.*, *Physical Review D* **85** (juin 2012).