Spacetime-Symmetry Breaking and the Generation of Gravitational Waves

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Based on 2307.13302 and 2307.13374

Symmetries?

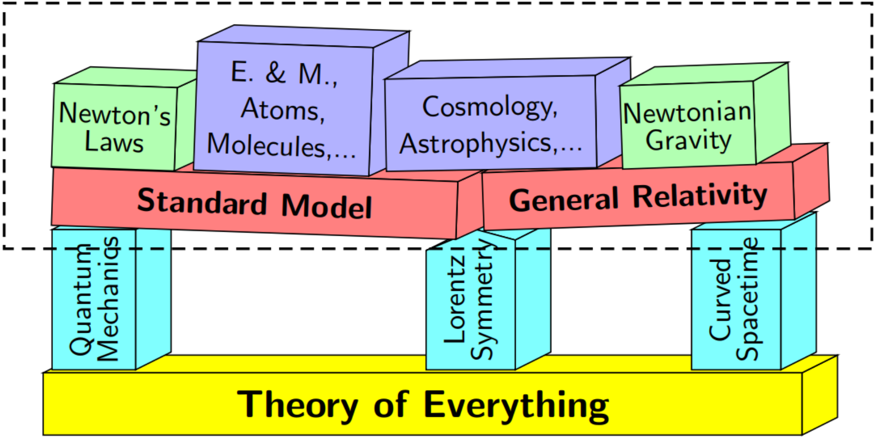


Image credit: Matt Mewes, Cal Poly

Spacetime symmetries

- General relativity defined in geometrical terms: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$, no torsion

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G\, T^{\mu\nu}$$

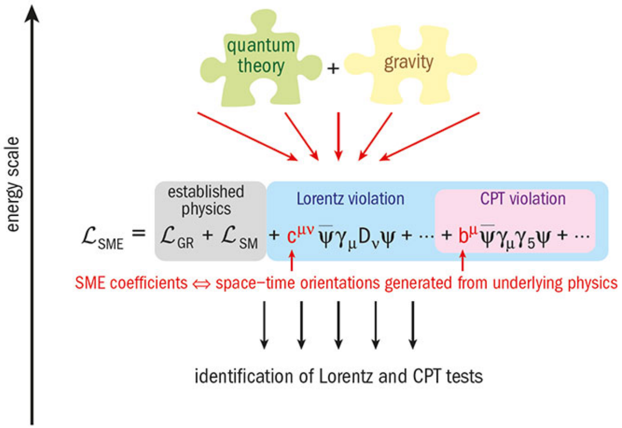
- Geometry
- Matter

The related conservation laws lead to the Bianchi identities ($+\,T_{[\mu\nu]} = 0$):

$$\nabla_\mu G^{\mu\nu} = 0$$

- Related symmetries:
  - Local Lorentz symmetry (**6 rotations**): $B'^\mu = \Lambda^\mu_\nu B^\nu$
  - Diffeomorphism symmetry (**4 translations**): $B^\mu \to B^\mu + (\partial_\nu \xi^\mu)\, B^\nu - \xi^\nu \partial_\nu B^\mu$

Standard-Model Extension



- Data Tables `0801.0287`
- Indiana Center for Spacetime Symmetries (conferences, workshops, summer schools)

The pure-gravity sector

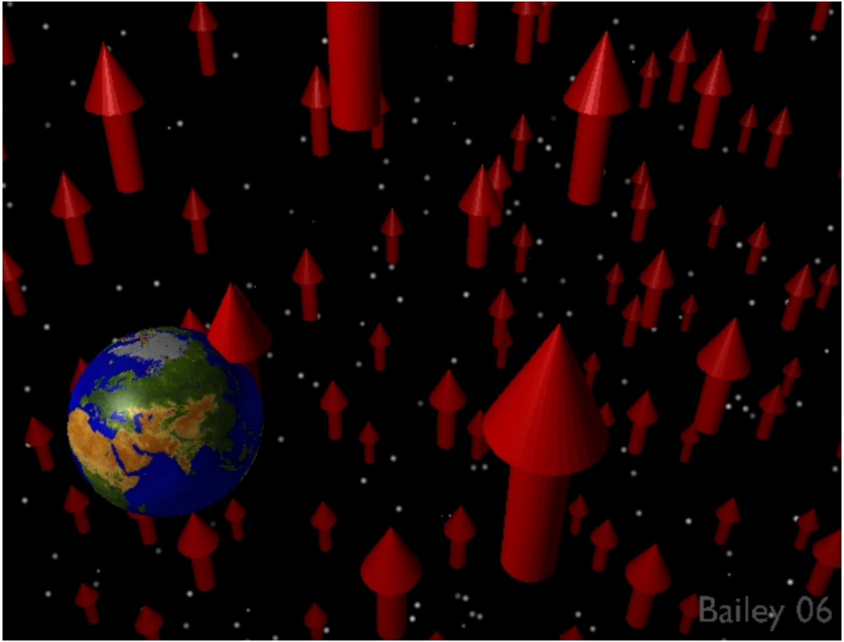
Pure gravity: generic Lagrangian up to dimension-6 operators

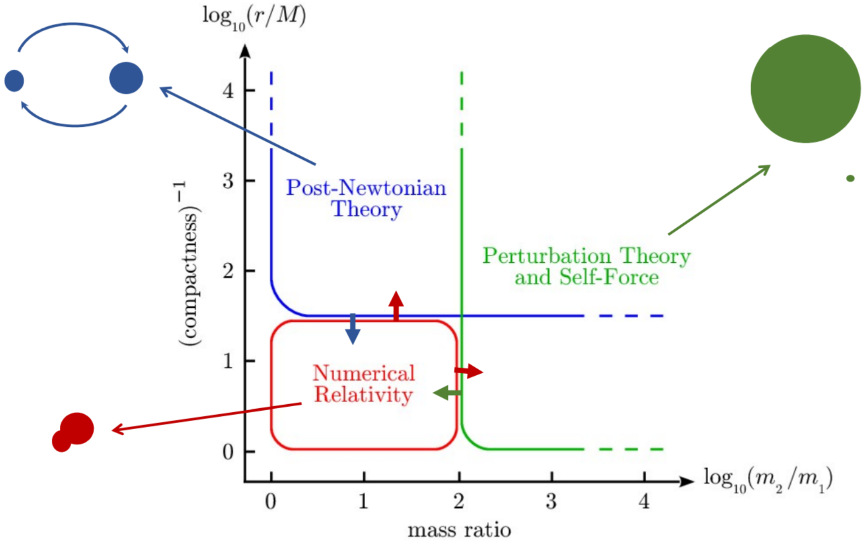
$$\mathcal{L} = \frac{\sqrt{-g}}{2\kappa}\Big[R + \underbrace{\left(k^{(4)}\right)_{\alpha\beta\gamma\delta}}_{20} R^{\alpha\beta\gamma\delta} + \underbrace{\left(k^{(5)}\right)_{\alpha\beta\gamma\delta\kappa}}_{60} \nabla^{\kappa} R^{\alpha\beta\gamma\delta}$$

$$+\tfrac{1}{2}\underbrace{\left(k^{(6)}\right)_{\alpha\beta\gamma\delta\kappa\lambda}}_{126} \left\{\nabla^{\kappa}, \nabla^{\lambda}\right\} R^{\alpha\beta\gamma\delta} + \underbrace{\left(k^{(6)}\right)_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}}_{210} R^{\alpha\beta\gamma\delta} R^{\kappa\lambda\mu\nu}\Big] + \mathcal{L}'$$

- $\nabla_{\mu} k \neq 0$, thanks to property of Riemannian geometry
- Dynamical coefficients iff spontaneous breaking, contained in $\mathcal{L}'$
- Any spontaneous breaking will involve a choice

Altschul+Bailey PRD 2010 consider generic potentials for antisymmetric tensors. For the pure-gravity sector, see e.g. Kostelecky PRD 2004, Bailey+Kostelecky PRD 2006, O'Neal-Ault+Bailey+NAN PRD 2021


Bailey 06



- Galactic binaries, science case for LISA! (Image credit: Le Tuc 2014, Bernard)

“But what’s the Lagrangian?”

$$h_{\alpha\beta} \equiv \eta_{\alpha\beta} - g_{\alpha\beta}$$

$$\mathcal{L} = \frac{1}{8\kappa}\epsilon^{\mu\rho\alpha\kappa}\epsilon^{\nu\sigma\beta\lambda}\eta_{\kappa\lambda}h_{\mu\nu}\partial_\alpha\partial_\beta h_{\rho\sigma} + \frac{1}{8\kappa}h_{\mu\nu}\left(\hat{s}^{\mu\rho\nu\sigma} + \hat{q}^{\mu\rho\nu\sigma} + \hat{k}^{\mu\rho\nu\sigma}\right)h_{\rho\sigma}$$

- General Relativity
- Symmetry-breaking contribution

$$\begin{aligned}
\hat{s}^{\mu\rho\nu\sigma} &= s^{(d)\ \mu\rho\epsilon_1\nu\sigma\epsilon_2\ldots\epsilon_{d-2}}\partial_{\epsilon_1}\ldots\partial_{\epsilon_{d-2}}, \\
\hat{q}^{\mu\rho\nu\sigma} &= q^{(d)\ \mu\rho\epsilon_1\nu\epsilon_2\sigma\epsilon_3\ldots\epsilon_{d-2}}\partial_{\epsilon_1}\ldots\partial_{\epsilon_{d-2}}, \\
\hat{k}^{\mu\rho\nu\sigma} &= k^{(d)\ \mu\epsilon_1\nu\epsilon_2\rho\epsilon_3\sigma\epsilon_4\ldots\epsilon_{d-2}}\partial_{\epsilon_1}\ldots\partial_{\epsilon_{d-2}},
\end{aligned}$$

$$G_L^{\mu\nu} + M^{\mu\nu\rho\sigma}h_{\rho\sigma} - \frac{\kappa}{c^4}\tau^{\mu\nu} = 0$$

- We have already discarded the Landau-Lifshitz pseudotensor

Order-by-order solutions

- Adopt an order-by-order solution scheme, where GR is the zeroth order

$$\bar{h}^{\mu\nu} = \bar{h}^{(0)\mu\nu} + \bar{h}^{(1)\mu\nu}$$

$$\Box\bar{h}^{(0)\mu\nu} = -\frac{2\kappa}{c^4}\tau^{\mu\nu} \quad \Box\bar{h}^{(1)\mu\nu} = 2\bar{M}^{\mu\nu\rho\sigma}\bar{h}^{(0)}_{\rho\sigma}$$

- GR solution is well known

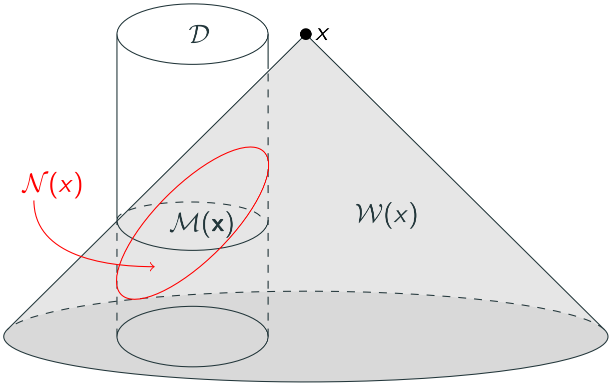
$$\bar{h}^{(0)\mu\nu}(x) = \frac{\kappa}{4\pi c^4}\int d^4y\, G(x-y)\tau^{\mu\nu}(y)$$

- First-order solution is non trivial

$$\boxed{\bar{h}^{(1)\mu\nu} = -\frac{\kappa}{8\pi^2 c^4}\int d^4y d^4z G(x-y)G(y-z)\bar{M}^{\mu\nu\alpha\beta}\tau_{\alpha\beta}(z)}$$

Post-Newtonian solution for an isolated source

- Integrate over the **near zone** and the **wave zone** separately



Formal solutions

- At the zeroth order, we find

$$\bar{h}^{(0)\mu\nu}(x) = \frac{\kappa}{4\pi c^4}\int_{\mathcal{N}(x)} d^3x' \frac{\tau^{\mu\nu}(\tau, \mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \frac{\kappa}{4\pi c^4}\int_{\mathcal{W}(x)} d^3x' \frac{\tau^{\mu\nu}(\tau, \mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

- which becomes the sources of first order

$$\bar{h}^{(1)\mu\nu}(x) = -\frac{1}{2\pi}\int_{\mathcal{N}_{\mathcal{W}}(x)} d^3x' \frac{\bar{M}^{\mu\nu\rho\sigma}\bar{h}^{(0)}_{\rho\sigma}(\tau, \mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} - \frac{1}{2\pi}\int_{\mathcal{W}_{\mathcal{W}}(x)} d^3x' \frac{\bar{M}^{\mu\nu\rho\sigma}\bar{h}^{(0)}_{\rho\sigma}(\tau, \mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

- Contains $\hat{s}^{\mu\nu\alpha\beta}$, $\hat{q}^{\mu\nu\alpha\beta}$, $\hat{k}^{\mu\nu\alpha\beta}$ and ∂^n_μ

Post-Newtonian expansion in the near zone

- Our approach is valid to 1PN → linear in $\bar{h}$. Remember that nPN: $\frac{1}{c^n}$

$$\text{First-order source term: } \bar{M}^{\mu\nu\rho\sigma}\bar{h}^{(0)}_{\rho\sigma} = \partial\,\partial\,\bar{h}^{(0)} + \partial\,\partial\,\partial\,\bar{h}^{(0)} + \dots$$

- Number of derivatives allowed is limited! (Also need to be linear in $\bar{h}$)

$$\leq 2 \text{ time derivatives } \rightarrow \bar{h}^{(0)00} = \frac{4}{c^2}U + \frac{1}{c^4}\left(7U^2 + 4\psi - 4V + 2\frac{\partial^2 X}{\partial t^2}\right) + \mathcal{O}(c^{-5})$$

$$\leq 1 \text{ time derivative } \rightarrow \bar{h}^{(0)0j} = \frac{4}{c^3}U^j + \mathcal{O}(c^{-5})$$

$$\text{etc } \bar{h}^{(0)ij} = \frac{1}{c^4}\left(4W^{jk} + U^2\delta^{jk} + 4\chi^{jk}\right) + \mathcal{O}(c^{-5}),$$

All details in 2307.13302. First PN work in the SME: Bailey+Kostelecky PRD 2006

Toy solution

$$\text{Choose the source as: } -\frac{1}{2c}\hat{\hat{s}}^{jkmi}\partial_t\partial_i\bar{h}^{(0)}_{m0}$$

- PN potentials involved become $U_j(x) = U_j^A(x) + U_j^{\neg A}(x)$

$$U_j^A(x) = \sum_A \frac{Gm_A v_j^A}{|\mathbf{x} - \mathbf{r_A}(t)|}, U_j^{\neg A}(x) = \sum_{B\neq A} \frac{Gm_B v_j^B}{|\mathbf{x} - \mathbf{r_B}(t)|}$$

Complications:

- Singular in the integration domain

Solutions:

- Schwartz distributional derivative + the generalised Gel'fand-Shilov formula
- Hadamard Finite Part procedure

New effects

$$\bar{h}^{(1)\mu\nu}(x) = -\frac{1}{2\pi r}\sum_{\ell=0}^{\infty}\frac{n_L}{\ell! c^\ell}\left(\frac{d}{d\tau}\right)^\ell \int_{\mathcal{M}} d^3x' \tau^{(0)\mu\nu}(\tau, \mathbf{x}') x'^L$$

- New effects: Monopole and dipole contributions!
- We can work out the quadrupole solution as

$$\boxed{\bar{h}^{(1)jk} \supseteq \bar{h}^{\mathrm{GR}jk}_{\mathcal{N}\mathcal{W}} - \frac{4G}{3c^4 r}\tilde{\bar{s}}^{jkmi}\ddot{I}^{\mathrm{GR}}_{im} + \mathcal{O}(c^{-5})}$$

Polarisations

- In general, a metric gravity theory has ≤ 6 polarisations

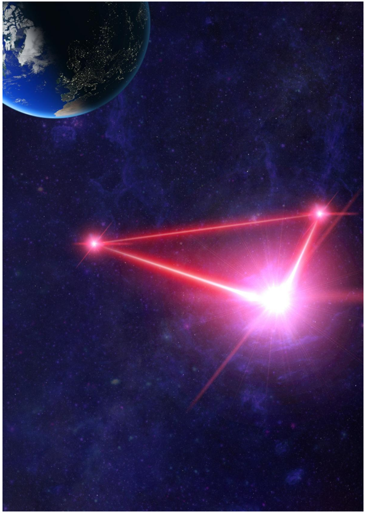
$$\frac{d^2\xi_j}{dt^2} \sim R_{0i0j}\xi^k$$

- So projecting $R0i0j$ onto a spatial basis tells us about the polarisations
- Using a different solution technique (see 2307.13374) we find 2 d.o.f. at first order in coefficients.
- At second order, there is an additional longitudinal polarisation

$$\begin{aligned} R_{0202} - R_{0101} = \frac{G}{\bar{r}}\Big[&(e_{1i}e_{1j} - e_{2i}e_{2j})(1 - \tfrac{2}{3}\bar{s}_{00}) - 2((\bar{s}_{tr})_{1i}e_{1j} - (\bar{s}_{tr})_{2i}e_{2j}) \\ &- 2(\bar{s}_{01}e_{1i}n_j - \bar{s}_{02}e_{2i}n_j) + \tfrac{1}{2}((\bar{s}_{tr})_{11} - (\bar{s}_{tr})_{22})(\delta_{ij} - n_i n_j)\Big]\overset{(4)}{I}{}^{ij} \\ R_{0102} = \frac{G}{\bar{r}}\Big[&-(e_1)_i(e_2)_j(1 - \tfrac{2}{3}\bar{s}_{00}) + (\bar{s}_{tr})_{1i}(e_2)_j + (\bar{s}_{tr})_{2i}(e_1)_j \\ &- \tfrac{1}{2}(\bar{s}_{tr})_{12}(\delta_{ij} - n_i n_j) + \bar{s}_{01}(e_2)_i n_j + \bar{s}_{02}(e_1)_i n_j\Big]\overset{(4)}{I}{}^{ij} \end{aligned}$$

Summary

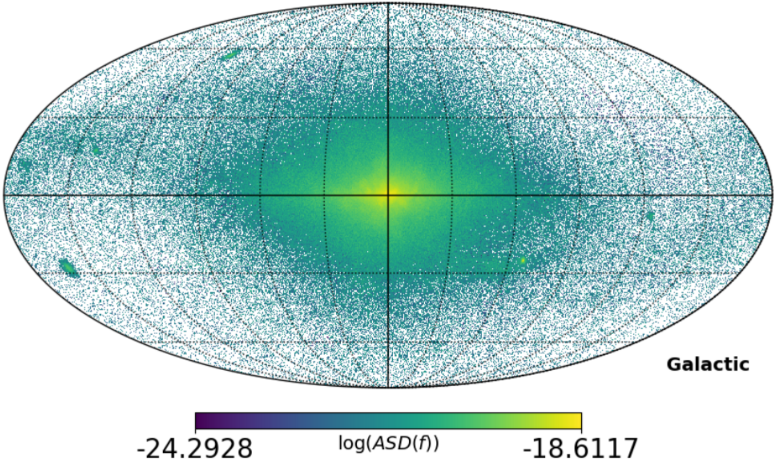
- Systematic gravitational-waves searches for spacetime-symmetry breaking largely focus on propagation effects
- First attempt at modified solutions within the Standard-Model Extension EFT (`2307.13302` and `2307.13374`)
- Monopole and dipole contributions appear
- Corrections proportional to known quantities
- Stay tuned for waveforms and LISA data



Backup slides

"Any signal would be much too weak."

Answer: Yes and no



“Are there other choices for the hatted potentials?”

Operator $\widehat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$	Tableau	Gauge invariant	CPT	d	Number
$s^{(d)\mu\rho\circ\nu\sigma\circ\circ^{d-4}}$	$[\mu\,\nu\,\cdots]$ / $[\rho\,\sigma]$ / $[\circ\,\circ]$	yes	even	even, ≥ 4	$(d-3)(d-2)(d+1)$
$s^{(d,1)\mu\rho\nu\sigma\circ^{d-2}}$	$[\mu\,\nu\,\cdots]$ / $[\rho\,\sigma]$	no	even	even, ≥ 2	$(d-1)(d+2)(d+3)$
$s^{(d,2)\mu\rho\circ\nu\sigma\circ\circ^{d-4}}$	$[\mu\,\nu\,\circ\,\cdots]$ / $[\rho\,\sigma]$ / $[\circ]$	no	even	even, ≥ 4	$\frac{4}{3}(d-2)d(d+2)$
$q^{(d)\mu\rho\circ\nu\circ\sigma\circ\circ^{d-5}}$	$[\mu\,\nu\,\sigma\,\cdots]$ / $[\rho\,\circ\,\circ]$ / $[\circ]$	yes	odd	odd, ≥ 5	$\frac{5}{2}(d-4)(d-1)(d+1)$
$q^{(d,1)\mu\rho\nu\sigma\circ\circ^{d-3}}$	$[\mu\,\nu\,\sigma\,\circ\,\cdots]$ / $[\rho]$	no	odd	odd, ≥ 3	$\frac{1}{2}(d+1)(d+3)(d+4)$
$q^{(d,2)\mu\rho\nu\circ\sigma\circ^{d-3}}$	$[\mu\,\nu\,\sigma\,\cdots]$ / $[\rho\,\circ]$	no	odd	odd, ≥ 3	$(d-1)(d+2)(d+3)$
$q^{(d,3)\mu\rho\circ\nu\sigma\circ^{d-3}}$	$[\mu\,\nu\,\sigma\,\cdots]$ / $[\rho]$ / $[\circ]$	no	odd	odd, ≥ 3	$\frac{1}{2}d(d+1)(d+3)$
$q^{(d,4)\mu\rho\nu\circ\sigma\circ\circ\circ^{d-5}}$	$[\mu\,\nu\,\sigma\,\circ\,\cdots]$ / $[\rho\,\circ\,\circ]$	no	odd	odd, ≥ 5	$\frac{5}{3}(d-3)(d+1)(d+2)$
$q^{(d,5)\mu\rho\circ\nu\circ\sigma\circ\circ^{d-5}}$	$[\mu\,\nu\,\sigma\,\circ\,\cdots]$ / $[\rho\,\circ]$ / $[\circ]$	no	odd	odd, ≥ 5	$\frac{4}{3}(d-2)d(d+2)$
$k^{(d)\mu\circ\nu\circ\rho\circ\sigma\circ\circ^{d-6}}$	$[\mu\,\nu\,\rho\,\sigma\,\cdots]$ / $[\circ\,\circ\,\circ\,\circ]$	yes	even	even, ≥ 6	$\frac{5}{2}(d-5)d(d+1)$
$k^{(d,1)\mu\nu\rho\sigma\circ^{d-2}}$	$[\mu\,\nu\,\rho\,\sigma\,\cdots]$	no	even	even, ≥ 2	$\frac{1}{6}(d+3)(d+4)(d+5)$
$k^{(d,2)\mu\circ\nu\rho\sigma\circ\circ^{d-4}}$	$[\mu\,\nu\,\rho\,\sigma\,\circ\,\cdots]$ / $[\circ]$	no	even	even, ≥ 4	$\frac{1}{2}(d+1)(d+3)(d+4)$
$k^{(d,3)\mu\circ\nu\circ\rho\sigma\circ^{d-4}}$	$[\mu\,\nu\,\rho\,\sigma\,\cdots]$ / $[\circ\,\circ]$	no	even	even, ≥ 4	$(d-1)(d+2)(d+3)$
$k^{(d,4)\mu\circ\nu\circ\rho\circ\sigma\circ\circ^{d-6}}$	$[\mu\,\nu\,\rho\,\sigma\,\circ\,\cdots]$ / $[\circ\,\circ\,\circ]$	no	even	even, ≥ 6	$\frac{5}{3}(d-3)(d+1)(d+2)$

TABLE I: Operators in the quadratic action for linearized gravity.

Credit: Kostelecky+Mewes PLB 2017

"What's the symmetry structure of the hatted potentials?"

μ	ν	$\dots$
ρ	σ	
$\circ$	$\circ$	

μ	ν	σ	$\dots$
ρ	$\circ$	$\circ$	
$\circ$			

μ	ν	ρ	σ	$\dots$
$\circ$	$\circ$	$\circ$	$\circ$	

We write the fluctuations around the Minkowski metric and its inverse as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + \mathcal{O}(h^2), \tag{1}$$

whereas the gothic metric and associated potentials are defined as

$$\mathfrak{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}, \quad \mathfrak{h}^{\mu\nu} = \eta^{\mu\nu} - \mathfrak{g}^{\mu\nu}, \tag{2}$$

and we temporarily introduce the gothic metric potential $\mathfrak{h}^{\mu\nu}$ to distinguish it from $h^{\mu\nu}$. The gothic metric can be expanded in $h^{\mu\nu}$ as

$$\begin{aligned} \mathfrak{g}^{\mu\nu} = \left(1 + \tfrac{1}{2}h + \ldots\right)\left(\eta^{\mu\nu} - h^{\mu\nu}\right) = \eta^{\mu\nu} - h^{\mu\nu} \\ + \tfrac{1}{2}\eta^{\mu\nu} h + \mathcal{O}(h^2), \end{aligned} \tag{3}$$

where we can plug in the definition of the trace-reversed potential $\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$ to see that

$$\mathfrak{g}^{\mu\nu} = \eta^{\mu\nu} - \bar{h}^{\mu\nu}, \tag{4}$$

and the gothic potential therefore reads

$$\mathfrak{h}^{\mu\nu} = \bar{h}^{\mu\nu} + \mathcal{O}(h^2). \quad \square \tag{5}$$

- Explicit or Spontaneous breaking
- Observer or Particle transformations
- Local, Global, Global-Local, Manifold transformations