Spectral separation of the cosmological SGWB for LISA in context of galactic and astrophysical background

Journée LISA France

by Guillaume Boileau, Laboratoire Lagrange, Observatoire de la Côte d'Azur

on November 20, 2023

» Overview

Introduction

Spectral separation methods

Results 6 parameter A-MCMC

Spectral separation with a modulated galactic foreground

Spectral separation with cosmic strings

Prospects for LISA to detect a gravitational-wave background from first order phase transitions

Conclusion

Current Position: Starting a postdoc in October at Lagrange (OCA), with Astrid Lamberts (CNES Founding)

* Developing / Consolidating / Preparing the pipeline of Catalogs (L2/L3)
* Figure of Merit calculation on catalogs and Global Fit output
* Developing techniques to compare Global Fits
* Generating new populations of Double White Dwarfs (DWD)

Introduction

* Stochastic Gravitational Wave Background (SGWB) in LISA band

Introduction

* Stochastic Gravitational Wave Background (SGWB) in LISA band

» Stochastic background in LISA band

Stochastic Background : Superposition of a large number of independent sources (unresolved sources):

* White dwarf binaries in our galaxy Lamberts *et al.* simulation of the waveform with galactic population of Double White Dwarf (DWD)
  $s(t) = \sum_{i=1}^{N} \sum_{P=+,\times} h_{P,i}(f_{orb,i}, M1_i, M2_i, X_i, Y_i, Z_i, t) \times F_P(\theta, \phi, t)\mathbf{D}(\theta, \phi, f)_P : \mathbf{e}_P$
  with a modulated waveform (Adams & Cornish)

* Binary Black Holes and Binary Neutron Stars from LIGO/Virgo Band
  * $\Omega_{GW} \simeq 1.8 \times 10^{-9} - 2.5 \times 10^{-9}$ at 25 Hz Chen *et al.* (2019)
  * $\Omega_{GW} \simeq 4.97 \times 10^{-9} - 2.58 \times 10^{-8}$ at 25 Hz Périgois *et al.* (2020)

* Cosmological sources: Phase transition, Preheating, Cosmic strings ... (early universe)

» **Stochastic background in LISA band**

Energy density spectrum:

Model of Energy density spectrum SGWB sources

$$\Omega_{GW}(f) = \frac{A_1 \left(\frac{f}{f_*}\right)^{\alpha_1}}{1 + A_2 \left(\frac{f}{f_*}\right)^{\alpha_2}} + \Omega_{Astro} \left(\frac{f}{f_*}\right)^{\alpha_{Astro}} + \Omega_{GW,Cosmo}(f)$$

with $\alpha = 2/3$ for the astrophysical component and low frequency
DWD ($\Omega_{DWD,LF}(f) = \frac{A_1}{A_2} \left(\frac{f}{f_*}\right)^{\alpha_1 - \alpha_2}$) and different Cosmological
models

<u>Goal:</u> Detecting a cosmological SGWB with LISA in the presence
of an astrophysical background and Galactic foreground

Spectral separation methods

* LISA noise model
* MCMC: Markov Chain Monte Carlo
* Fisher Information Matrix

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» LISA noise model

Time-series XYZ to AET

$A = \frac{1}{\sqrt{2}}(Z - X)$
$E = \frac{1}{\sqrt{6}}(X - 2Y + Z)$
$T = \frac{1}{\sqrt{3}}(X + Y + Z)$

PSD AET

$N_A = N_E = N_X(f) - N_{XY}(f)$
$N_T = N_X(f) + 2N_{XY}(f)$

$$\begin{cases} N_X(f) = \left(4S_s(f) + 8\left(1 + \cos^2\left(\frac{f}{f_*}\right)\right)S_a(f)\right)|1 - e^{-\frac{2if}{f_*}}|^2 \\ N_{XY}(f) = -\left(2S_s(f) + 8S_a(f)\right)\cos\left(\frac{f}{f_*}\right)|1 - e^{-\frac{2if}{f_*}}|^2 \end{cases}$$

$$\begin{cases} S_s(f) = N_{Pos} \\ S_a(f) = \frac{N_{acc}}{(2\pi f)^4}\left(1 + \left(\frac{f_1}{f}\right)^2\right) \end{cases}$$

with $f_* = \frac{c}{2\pi L}$, $L = 2.5 \times 10^9$ m, $f_1 = 0.4$ mHz
Acceleration noise: $N_{Acc} = 1.44 \times 10^{-48}$ s^{-4} Hz^{-1}
Optical Metrology System noise: $N_{Pos} = 3.6 \times 10^{-41}$ Hz^{-1}

Spectral separation methods

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» MCMC (Markov Chain Monte Carlo)

<u>Likelihood function</u>, (d = data, θ = parameter)

> Likelihood
>
> $$\mathcal{L}(\mathbf{d}|\theta) = -\frac{1}{2} \sum_{k=0}^{N} \left[\frac{d_A^2}{S_A + N_A} + \frac{d_E^2}{S_E + N_E} + \frac{d_T^2}{N_T} \right.$$
> $$\left. + \log \left(8\pi^3 (S_A + N_A)(S_E + N_E)N_T \right) \right]$$

with N_I as the LISA noise of channel $I = [A, E, T]$ and $S_I(f) = \frac{3H_0^2}{4\pi^2} \frac{\sum_i \Omega_{GW,i}}{\mathcal{R}_I(f)f^3}$
the SGWB.

  * Posterior distribution $p(\theta|d) \propto p(\theta)\mathcal{L}(\mathbf{d}|\theta)$
  * Using log-uniform and uniform priors $p(\theta) = \prod_i U(\theta_i, a_i, b_i)$
  * Estimation parameters: $\theta_{\text{LISA}} + \theta_{\text{Astro}} + \theta_{\text{Galac}} + \theta_{\text{Cosmo}}$.
⟹ Using a Metropolis-Hastings sampler

Spectral separation methods

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» Fisher Information Matrix

Fisher Information Matrix

$$F_{ab} = \frac{1}{2}\text{Tr}\left(\mathcal{C}^{-1}\frac{\partial \mathcal{C}}{\partial \theta_a}\mathcal{C}^{-1}\frac{\partial \mathcal{C}}{\partial \theta_b}\right)$$

$$= \frac{1}{2}\sum_{I=A,E,T}\sum_{k=0}^{N}\frac{\frac{\partial S_I(f)+N_I(f)}{\partial \theta_a}\frac{\partial S_I(f)+N_I(f)}{\partial \theta_b}}{(S_I(f)+N_I(f))^2}$$

Co-variance Matrix

$$\mathcal{C}(\theta, f) = \begin{pmatrix} S_A + N_A & 0 & 0 \\ 0 & S_E + N_E & 0 \\ 0 & 0 & N_T \end{pmatrix}$$

with N_I is the LISA noise of the channel $I = [A, E, T]$ and
$S_I \propto \sum_\alpha \Omega_\alpha \left(\frac{f}{f_{ref}}\right)^{\alpha-3}$ the SGWB.

$\implies \sqrt{F_{aa}^{-1}} = \sigma_a$

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Results 6 parameter A-MCMC

» **Context: Stochastic background Astrophysical and flat cosmological components**
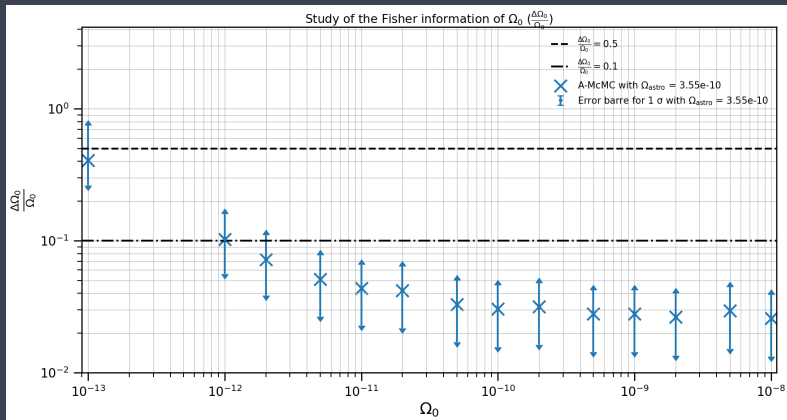
Energy density spectrum:

Model of Energy density spectrum SGWB sources

$$\Omega_{GW}(f) = \Omega_{Astro} \left(\frac{f}{f_*} \right)^{\alpha_{Astro}} + \Omega_{Cosmo} \left(\frac{f}{f_*} \right)^{\alpha_{Cosmo}}$$

with $\alpha_{Astro} = 2/3$ for the astrophysical component and $\alpha_{Cosmo} = 0$ for the cosmological component
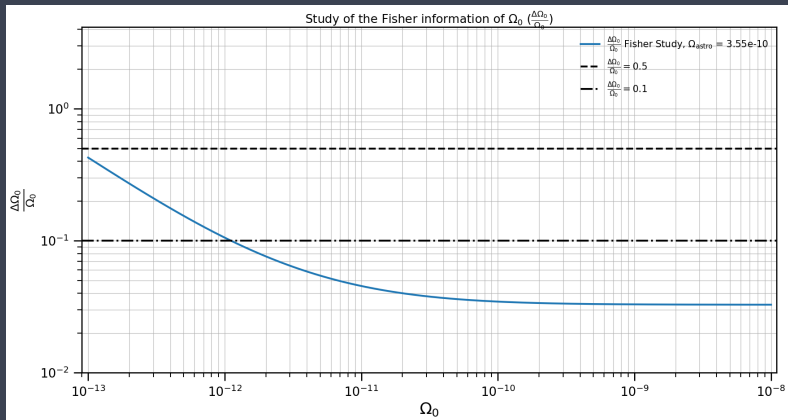
<u>Goal:</u> Detecting a cosmological SGWB with LISA in the presence of an astrophysical background
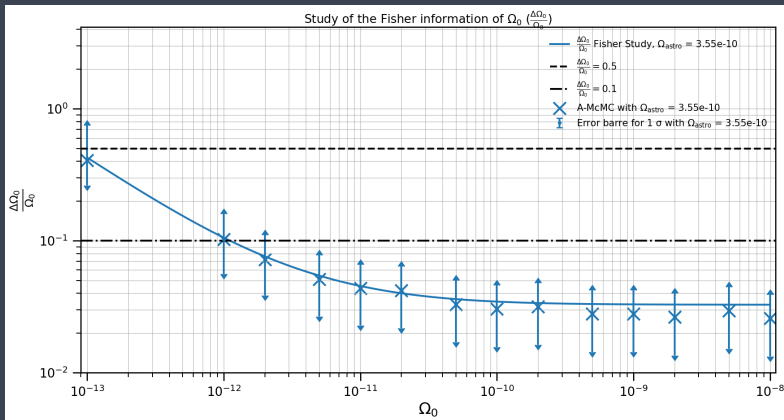
» Results 6 parameter A-MCMC

Uncertainty of the estimation of the Cosmological Amplitude from the Fisher Information study in line (with the Cramer-Rao calculation) and the parametric estimation from the A-MCMC in scatters for the channel A with the noise channel T. The upper horizontal dash line represents the error level 50%. In fact, above the line, the error is greater than 50%.
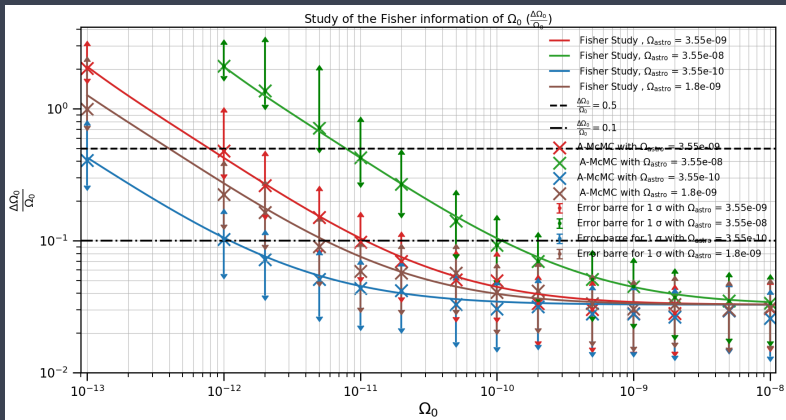
» Results 6 parameter A-MCMC

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» Results 6 parameter A-MCMC

Prediction of the measurement limit of Cosmological Amplitude in 4 cases of isotropic astrophysical background with 2 noise parameters (acceleration noise : $N_{Acc} = 1.44 \times 10^{-48}$ s^{-4}Hz^{-1} and Optical Metrology System noise $N_{Pos} = 3.6 \times 10^{-41}$ Hz^{-1}) of 4 years mission data measurement:

Limit for BBH/BNS + Cosmo + LISA noise

* $\Omega_{astro} = 3.55 \times 10^{-8}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.8 \times 10^{-12}$
* $\Omega_{astro} = 3.55 \times 10^{-9}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.8 \times 10^{-13}$
* $\Omega_{astro} = 1.8 \times 10^{-9}$ (25 Hz): $\Omega_{Cosmo,lim} = 3.6 \times 10^{-13}$
* $\Omega_{astro} = 3.55 \times 10^{-10}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.6 \times 10^{-14}$

$\Longrightarrow$ paper G. Boileau *et. al.* (PhysRevD.103.103529).

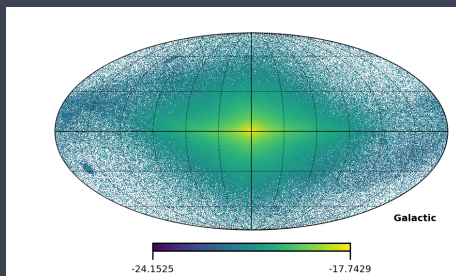*@ Guillaume Boileau, Nelson Christensen, Renate Meyer, Neil J. Cornish*

Spectral separation with a modulated galactic foreground

* Orbital Modulation of the white dwarf binaries in our Galaxy
* 10 parameter runs of A-MCMC (LISA noise + BBH/BNS + DWD + Cosmo)

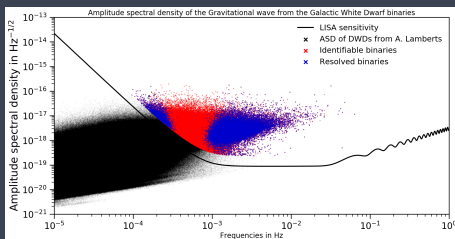## Spectral separation with a modulated galactic foreground

* Orbital Modulation of the white dwarf binaries in our Galaxy
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» **Population of the white dwarf binaries in our Galaxy**

Map Galactic DWD for $f_{GW} \geq 1 \times 10^{-5}$ Hz. DWD distribution from Lamberts *et. al.* Galactic coordinates *GLON*, *GLAT* with $N_{side} = 256$.

ASD galactic DWD from A. Lamberts (2019), **black**: $\simeq 35\,000\,000$ binaries, **red**: binaries $SNR > 7$ and **blue**: $SNR > 7$ + LISA bin ($\simeq 32\,000$ binaries)

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Spectral separation with a modulated galactic foreground

* Orbital Modulation of the white dwarf binaries in our Galaxy
* 10 parameter runs of A-MCMC (LISA noise + BBH/BNS + DWD + Cosmo)

» **Context: Stochastic background In LISA paper of BBH/BNS prediction**

Energy density spectrum:

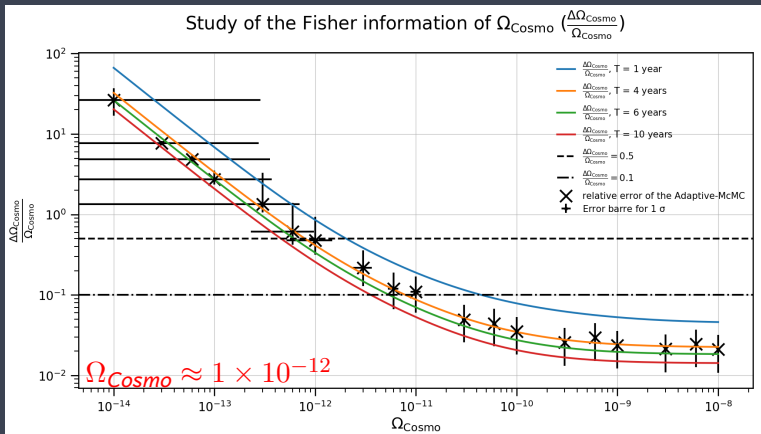Model of Energy density spectrum SGWB sources

$$\Omega_{GW}(f) = \frac{A_1 \left(\frac{f}{f_*}\right)^{\alpha_1}}{1 + A_2 \left(\frac{f}{f_*}\right)^{\alpha_2}} + \Omega_{Astro} \left(\frac{f}{f_*}\right)^{\alpha_{Astro}} + \Omega_{Cosmo} \left(\frac{f}{f_*}\right)^{\alpha_{Cosmo}}$$

with $\alpha_{Astro} = 2/3$ for the astrophysical component and low frequency DWD : $\Omega_{DWD,LF}(f) = \frac{A_1}{A_2} \left(\frac{f}{f_*}\right)^{\alpha_1 - \alpha_2}$.
Cosmological model : $\alpha_{Cosmo} = 0$

<u>Goal:</u> Detecting a cosmological SGWB with LISA in the presence of an astrophysical background and Galactic foreground

» 10 parameter runs of A-MCMC (LISA noise + BBH/BNS + DWD + Cosmo)

Study of the Fisher information of Ω_{Cosmo} ($\frac{\Delta\Omega_{Cosmo}}{\Omega_{Cosmo}}$)

$\Omega_{Cosmo} \approx 1 \times 10^{-12}$

$\Longrightarrow$ paper for MNRAS G. Boileau *et al.* (10.1093/mnras/stab2575)

@ Guillaume Boileau, Astrid Lamberts, Nelson Christensen, Neil J. Cornish, Renate Meyer

Spectral separation with cosmic strings

* Models
* Uncertainty and Deviance Information Criterion (DIC)
* Result

Spectral separation with cosmic strings

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* Uncertainty and Deviance Information Criterion (DIC)
* Result

» Models : cosmic strings

Cosmic strings:

* predictions from field theory, stable topological defects
* formed during symmetry breaking phase transitions in the early Universe

Energy Spectral Density

$$\Omega_{\text{GW}, G\mu}(G\mu, \mathcal{M}_i, f), i \in [1, 2, 3]$$

* $\mathcal{M}_1$: Auclair *et. al* analytical model of loops produced by the network of long chains described by a single free parameter (size of loops at the time of their formation) Kibble *et. al*
* $\mathcal{M}_2$: simulations of the Blanco-Pillado, Olum and Shlar (BOS) model. Networks of cosmic strings present between the eras of radiation and matter.
* $\mathcal{M}_3$: simulation of Lorenz, Ringeval and Sakellariadou (LRS), sister simulation to model 2, calculates and considers different quantities. (the power in the age of matter differs from that of model 2)
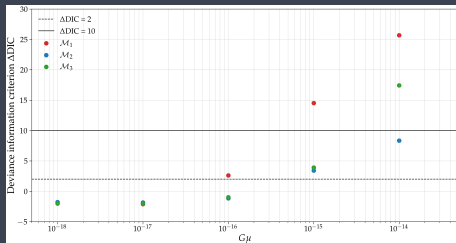
Spectral separation with cosmic strings

» Uncertainty: Cosmic Strings
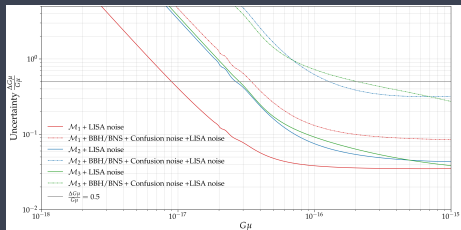
$G\mu$ uncertainty from Fisher information for 3 models $\mathcal{M}_i$. Solid lines : CS + LISA noise. Dot-dashed lines: CS + LISA noise + galactic foreground astrophysical SGWB.

DIC values for different scenarios. $\Delta DIC < 2$: Not worth more than a bare mention, ΔDIC 2 to 10: positive, $\Delta DIC >$ 10: very strong.

Spectral separation with cosmic strings

* Models
* Uncertainty and Deviance Information Criterion (DIC)
* Result

» Result

* Development of a discrete MCMC with a library (limit $\frac{\Delta G\mu}{G\mu}$)
* Good overlap between Fisher study and MCMC
* Development of DIC technic
* Future study: add MBHB and EMRIs GWB

	LISA noise + Cosmic strings		
	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
$G\mu_{lim}$	1×10^{-17}	3×10^{-17}	3×10^{-17}
	LISA noise + DWD + BBH/BNS + Cosmic strings		
	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
$G\mu_{lim}$	3×10^{-17}	1×10^{-16}	2×10^{-16}

Measurement limit

$G\mu \approx 10^{-16} \longrightarrow 10^{-15}$

$\Longrightarrow$ paper G. Boileau *et. al.* (PhysRevD.105.023510)
@ *Guillaume Boileau, Alexander C. Jenkins, Mairi Sakellariadou, Renate Meyer, Nelson Christensen*

Prospects for LISA to detect a gravitational-wave background from first order phase transitions

* Models
* Result

Prospects for LISA to detect a gravitational-wave background from first order phase transitions

* Models
* Result

» First order phase transitions

* Quantum and thermal fluctuation from bubbles in the early Universe
* Production of GWs: Collision of Bubble / subsequent sound waves / Magneto hydrodynamic turbulence
* Simulation: Sound shell model (SSM) for GWs production Hindmarsh *et. al*
* SSM approximated by a double broken power law (Caprini *et. al*, Guo *et. al*, Gowling & Hindmarsh)

Broken Power law first order phase transitions

$$\Omega_{\text{GW},PT}(f) = \Omega_P \left(\frac{f}{f_{\text{p}}} \right)^9 \left(\frac{1 + r_{\text{b}}^4}{r_{\text{b}}^4 + \left(\frac{f}{f_{\text{p}}} \right)^4} \right)^{(9-b)/4} \left(\frac{b+4}{b+4 - m + m \left(\frac{f}{f_{\text{p}}} \right)^2} \right)^{(b+4)/2}$$

with Ω_P amplitude at the frequency peak f_{p}, r_{b} the ratio between the two breaks, b the spectral slope between the two breaks, and $m = \frac{9r_{\text{b}}^4 + b}{r_{\text{b}}^4 + 1}$.

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Prospects for LISA to detect a gravitational-wave background from first order phase transitions

* Models
* Result

» Result
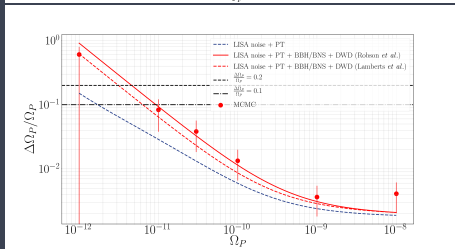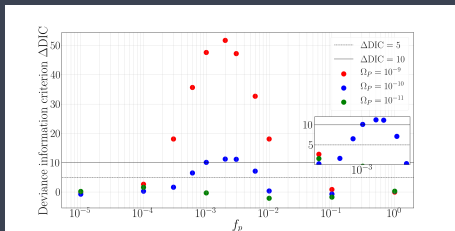
<u>Context:</u> Phase transition model
($r_b = 0.4, = 1$) + Astrophysical
background + DWD + LISA noise,
Limit for $\Delta DIC > 5$:

 * $\Omega_P = 1 \times 10^{-9} \rightarrow$
   $f_p \in [2 \times 10^{-3}, 4 \times 10^{-2}]$ Hz

 * $\Omega_P = 1 \times 10^{-10} \rightarrow$
   $f_p \in [5 \times 10^{-3}, 7 \times 10^{-3}]$ Hz

Limit from Fisher ($r_b = 0.4$, $b = 1$,
$f_p = 1$mHz):

 * $\frac{\Delta \Omega_P}{\Omega_P} = 0.01 \rightarrow \Omega_P \sim 1 \times 10^{-10}$

 * $\frac{\Delta \Omega_P}{\Omega_P} = 0.1 \rightarrow \Omega_P \sim 1 \times 10^{-11}$

 * Similar result for Fisher (lines)
   and MCMC (scatter)

$\Longrightarrow$ Paper G. Boileau *et. al* (JCAP 02 (2023) 056)

Guillaume Boileau, Nelson Christensen, Chloe Gowling, Mark Hindmarsh, Renate Meyer.

Conclusion

» Conclusion

– We provide evidence that it is possible for LISA to measure the cosmological SGWB :

Measurement limit

$\Omega_{Cosmo,lim} = 8 \times 10^{-14} - 8 \times 10^{-12}$

Limitation

BBH and BNS principal limitation for the Cosmological background

– We use Lamberts *et al.* (10 October 2019), possibility to generate white dwarf waveform with other catalogs
– able to estimate more complex backgrounds, like broken power laws, or spectrum with peaks. Our method can be easily expanded with more complex cosmological backgrounds
$\implies$ papers G. Boileau *et al.* (PhysRevD.103.103529) and MNRAS G. Boileau *et al.* (10.1093/mnras/stab2575)

» Conclusion Cosmological Sources

Cosmic Strings

$G\mu \approx 10^{-16} \longrightarrow 10^{-15}$

$\Longrightarrow$ Paper G. Boileau *et. al.* (PhysRevD.105.023510)

First Order Phase Transition

* DIC: $\Omega_P = 1 \times 10^{-9} \to f_p \in [2 \times 10^{-3}, 4 \times 10^{-2}]$ Hz
  $\Omega_P = 1 \times 10^{-10} \to f_p \in [5 \times 10^{-3}, 7 \times 10^{-3}]$ Hz
* Fisher/MCMC: $\frac{\Delta\theta}{\theta} \sim 1\%$: $\Omega_P = 1 \times 10^{-10}$ and $f_p = 3$ mHz
  $\frac{\Delta\theta}{\theta} \sim 10\%$: $r_b = 0.2$ and $b = 1$

Limitations of the Study

* LISA noise assumed stationary, Gaussian, and *AET* are uncorrelated
* Future study: add MBHB and EMRIs GWB or other cosmological sources

The End : Thank You !