Leptogenesis in the Universe

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Abstract

Leptogenesis is a class of scenarios in which the cosmic baryon asymmetry originates from an initial lepton asymmetry generated in the decays of heavy sterile neutrinos in the early Universe. We explain why leptogenesis is an appealing mechanism for baryogenesis. We review its motivations, the basic ingredients, and describe subclasses of effects, like those of lepton flavours, spectator processes, scatterings, finite temperature corrections, the role of the heavier sterile neutrinos and quantum corrections. We then address leptogenesis in supersymmetric scenarios, as well as some other popular variations of the basic leptogenesis framework.

Contents

1.1 The Baryon Asymmetry of the Universe

1.1.1 Observations

Up to date no traces of cosmological antimatter have been observed. The presence of a small amount of antiprotons and positrons in cosmic rays can be consistently explained by their secondary origin in cosmic particles collisions or in highly energetic astrophysical processes, but no antinuclei, even as light as anti-deuterium or as tightly bounded as anti-α particles, has ever been detected.

The absence of annihilation radiation $p\bar{p} \to \ldots \pi^0 \to \ldots 2\gamma$ excludes significant matter-antimatter admixtures in objects up to the size of galactic clusters $\sim 20\,\mathrm{Mpc}$ [1]. Observational limits on anomalous contributions to the cosmic diffuse γ-ray background and the absence of distortions in the cosmic microwave background (CMB) implies that little antimatter is to be found within $\sim 1\,\mathrm{Gpc}$ and that within our horizon an equal amount of matter and antimatter can be excluded [2]. Of course, at larger super-horizon scales the vanishing of the average asymmetry cannot be ruled out, and this would indeed be the case if the fundamental Lagrangian is C and CP symmetric and charge invariance is broken spontaneously [3].

Quantitatively, the value of baryon asymmetry of the Universe is inferred from observations in two independent ways. The first way is by confronting the abundances of the light elements, D, ^{3}He, ^{4}He, and ^{7}Li, with the predictions of Big Bang nucleosynthesis (BBN) [4, 5, 6, 7, 8, 9]. The crucial time for primordial nucleosynthesis is when the thermal bath temperature falls below $T \lesssim 1\,\mathrm{MeV}$. With the assumption of only three light neutrinos, these predictions depend on a single parameter, that is the difference between the number of baryons and anti-baryons normalized to the number of photons:

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma}\Big|_0, \tag{1.1}$$

where the subscript 0 means "at present time". By using only the abundance of deuterium, that is particularly sensitive to η, Ref. [4] quotes:

$$10^{10}\,\eta = 5.7 \pm 0.6 \qquad\qquad (95\%\ \text{c.l.})\,. \tag{1.2}$$

In this same range there is also an acceptable agreement among the various abundances, once theoretical uncertainties as well as statistical and systematic errors are accounted for [6].

The second way is from measurements of the CMB anisotropies (for pedagogical reviews, see Refs. [10, 11]). The crucial time for CMB is that of recombination, when the temperature dropped down to $T \lesssim 1\,\mathrm{eV}$ and neutral hydrogen can be formed. CMB observations measure the relative baryon contribution to the energy density of the Universe multiplied by the square of the (reduced) Hubble constant $h \equiv H_0/(100\,\mathrm{km\,sec^{-1}\,Mpc^{-1}})$:

$$\Omega_B h^2 \equiv h^2 \frac{\rho_B}{\rho_{\mathrm{crit}}}\,, \tag{1.3}$$

that is related to η through $10^{10}\eta = 274\ \Omega_B\ h^2$. The physical effect of the baryons at the onset of matter domination, which occurs quite close to the recombination epoch, is to provide extra gravity which enhances the compression into potential wells. The consequence is enhancement of the compressional phases which translates into enhancement of the odd peaks in the spectrum. Thus, a measurement of the odd/even peak disparity constrains the baryon energy density. A fit to the most recent observations

(WMAP7 data only, assuming a ΛCDM model with a scale-free power spectrum for the primordial density fluctuations) gives at 68% c.l. [12]

$$10^2\,\Omega_B h^2 = 2.258^{+0.057}_{-0.056}\,. \tag{1.4}$$

There is a third way to express the baryon asymmetry of the Universe, that is by normalizing the baryon asymmetry to the entropy density $s = g_*(2\pi^2/45)T^3$, where g_* is the number of degrees of freedom in the plasma, and T is the temperature:

$$Y_{\Delta B} \equiv \left.\frac{n_B - n_{\bar{B}}}{s}\right|_0 . \tag{1.5}$$

The relation with the previous definitions is given by the conversion factor $s_0/n_{\gamma 0} = 7.04$. $Y_{\Delta B}$ is a convenient quantity in theoretical studies of the generation of the baryon asymmetry from very early times, because it is conserved throughout the thermal evolution of the Universe. In terms of $Y_{\Delta B}$ the BBN results eq. (1.2) and the CMB measurement eq. (1.4) (at $95\% c.l.$) read:

$$Y_{\Delta B}^{BBN} = (8.10 \pm 0.85) \times 10^{-11}, \qquad Y_{\Delta B}^{CMB} = (8.79 \pm 0.44) \times 10^{-11}. \tag{1.6}$$

The impressive consistency between the determinations of the baryon density of the Universe from BBN and CMB that, besides being completely independent, also refer to epochs with a six orders of magnitude difference in temperature, provides a striking confirmation of the hot Big Bang cosmology.

1.1.2 Theory

From the theoretical point of view, the question is where the Universe baryon asymmetry comes from. The inflationary cosmological model excludes the possibility of a fine tuned initial condition, and since we do not know any other way to construct a consistent cosmology without inflation, this is a strong veto.

The alternative possibility is that the Universe baryon asymmetry is generated dynamically, a scenario that is known as *baryogenesis*. This requires that baryon number (B) is not conserved. More precisely, as Sakharov pointed out [13], the ingredients required for baryogenesis are three:

1. B violation is required to evolve from an initial state with $Y_{\Delta B} = 0$ to a state with $Y_{\Delta B} \neq 0$.
2. C and CP violation: If either C or CP were conserved, then processes involving baryons would proceed at the same rate as the C- or CP-conjugate processes involving antibaryons, with the overall effect that no baryon asymmetry is generated.
3. Out of equilibrium dynamics: Equilibrium distribution functions $n_{\rm eq}$ are determined solely by the particle energy E, chemical potential μ, and by its mass which, because of the CPT theorem, is the same for particles and antiparticles. When charges (such as B) are not conserved, the corresponding chemical potentials vanish, and thus $n_B = \int \frac{d^3p}{(2\pi^3)} n_{\rm eq} = n_{\bar{B}}$.

Although these ingredients are all present in the Standard Model (SM), so far all attempts to reproduce quantitatively the observed baryon asymmetry have failed.

1. In the SM B is violated by the triangle anomaly. Although at zero temperature B violating processes are too suppressed to have any observable effect [14], at high temperatures they occur with unsuppressed rates [15]. The first condition is then quantitatively realized in the early Universe.

2. SM weak interactions violate C maximally. However, the amount of CP violation from the Kobayashi-Maskawa complex phase [16], as quantified by means of the Jarlskog invariant[17], is only of order 10^{-20}, and this renders impossible generating $Y_{\Delta B} \sim 10^{-10}$ [18, 19, 20].

3. Departures from thermal equilibrium occur in the SM at the electroweak phase transition (EWPT) [21, 22]. However, the experimental lower bound on the Higgs mass implies that this transition is not sufficiently first order as required for successful baryogenesis [23].

This shows that baryogenesis requires new physics that extends the SM in at least two ways: It must introduce new sources of CP violation and it must either provide a departure from thermal equilibrium in addition to the EWPT or modify the EWPT itself. In the past thirty years or so, several new physics mechanisms for baryogenesis have been put forth. Some among the most studied are *GUT baryogenesis* [24, 25, 26, 27, 28, 29, 30, 31, 32, 33], *Electroweak baryogenesis* [21, 34, 35], the *Affleck-Dine mechanism* [36, 37], *Spontaneous Baryogenesis* [38, 39]. However, soon after the discovery of neutrino masses, because of its connections with the seesaw model [40, 41, 42, 43, 44] and its deep interrelations with neutrino physics in general, the mechanism of baryogenesis via *Leptogenesis* acquired a continuously increasing popularity. Leptogenesis was first proposed by Fukugita and Yanagida in Ref. [45]. Its simplest and theoretically best motivated realization is precisely within the seesaw mechanism. To implement the seesaw, new Majorana $SU(2)_L$ singlet neutrinos with a large mass scale M are added to the SM particle spectrum. The complex Yukawa couplings of these new particles provide new sources of CP violation, departure from thermal equilibrium can occur if their lifetime is not much shorter than the age of the Universe when $T \sim M$, and their Majorana masses imply that lepton number is not conserved. A lepton asymmetry can then be generated dynamically, and SM sphalerons will partially convert it into a baryon asymmetry [46]. A particularly interesting possibility is "thermal leptogenesis" where the heavy Majorana neutrinos are produced by scatterings in the thermal bath starting from a vanishing initial abundance, so that their number density can be calculated solely in terms of the seesaw parameters and of the reheat temperature of the Universe.

This review is organized as follows: in Section 1.2 the basis of leptogenesis are reviewed in the simple scenario of the one flavour regime, while the role of flavour effects is described in Section 1.3. Theoretical improvements of the basic pictures, like spectator effects, scatterings and CP violation in scatterings, thermal corrections, the possible role of the heavier singlet neutrinos, and quantum effects are reviewed in Section 1.4. Leptogenesis in the supersymmetric seesaw is reviewed in Section 1.5, while in Section 1.6 we mention possible leptogenesis realizations that go beyond the type-I seesaw. Finally, in Section 1.7 we draw the conclusions.

1.2 N_1 Leptogenesis in the Single Flavour Regime

The aim of this section is to give a pedagogical introduction to leptogenesis [45] and establish the notations. We will consider the classic example of leptogenesis from the lightest right-handed (RH) neutrino N_1 (the so-called N_1 leptogenesis) in the type-I seesaw model [40, 43, 41, 44] in the single flavour regime. First in Section 1.2.1 we introduce the type-I seesaw Lagrangian and the relevant parameters. In Section 1.2.2, we will review the CP violation in RH neutrino decays induced at 1-loop level. Then in Section 1.2.3, we will write down the classical Boltzmann equations taking into account of only decays and inverse decays of N_1 and give a simple but rather accurate analytical estimate of the solution. In Section 1.2.4 we will relate the lepton asymmetry generated to the baryon asymmetry of the Universe. Finally in Section 1.2.5, we will discuss the lower bound on N_1 mass and the upper bound on light neutrino mass scale from successful leptogenesis.

1.2.1 Type-I seesaw, neutrino masses and leptogenesis

With $m\,(m \geq 2)$[1] singlet RH neutrinos N_{R_i} $(i = 1, m)$, we can add the following Standard Model (SM) gauge invariant terms to the SM Lagrangian

$$\mathcal{L}_I = \mathcal{L}_{SM} + i\overline{N_{R_i}}\partial\!\!\!/ N_{R_i} - \left(\frac{1}{2}M_i\overline{N^c_{R_i}}N_{R_i} + \epsilon_{ab}Y_{\alpha i}\overline{N_{R_i}}\ell^a_\alpha H^b + h.c.\right), \tag{1.7}$$

where M_i are the Majorana masses of the RH neutrinos, $\ell_\alpha = (\nu_{\alpha L}, \alpha^-_L)$ with $\alpha = e, \mu, \tau$ and $H = (H^+, H^0)$ are respectively the left-handed (LH) lepton and Higgs $SU(2)_L$ doublets and $\epsilon_{ab} = -\epsilon_{ba}$ with $\epsilon_{12} = 1$. Without loss of generality, we have chosen the basis where the Majorana mass term is diagonal. The physical mass eigenstates of the RH neutrinos are the Majorana neutrinos $N_i = N_{R_i} + N^c_{R_i}$. Since N_i are gauge singlets, the scale of M_i is naturally much larger than the electroweak (EW) scale $M_i \gg \langle\Phi\rangle \equiv v = 174$ GeV. Hence after EW symmetry breaking, the light neutrino mass matrix is given by the famous seesaw relation [40, 43, 41, 44]

$$m_\nu \simeq -v^2 Y \frac{1}{M} Y^T. \tag{1.8}$$

Assuming $Y \sim \mathcal{O}(1)$ and $m_\nu \simeq \sqrt{\Delta m^2_{atm}} \simeq 0.05$ eV, we have $M \sim 10^{15}$ GeV not far below the GUT scale.

Besides giving a natural explanation of the light neutrino masses, there is another bonus: the *three* Sakharov's conditions[13] for leptogenesis are implicit in eq. (1.7) with the *lepton number violation* provided by M_i, the *CP-violation* from the complexity of $Y_{i\alpha}$ and the *departure from thermal equilibrium condition* given by an additional requirement that N_i decay rate Γ_{N_i} is not very fast compared to the Hubble expansion rate of the Universe $H(T)$ at temperature $T = M_i$ with

$$\Gamma_{N_i} = \frac{(Y^\dagger Y)_{ii} M_i}{8\pi}, \qquad H(T) = \frac{2}{3}\sqrt{\frac{g_* \pi^3}{5}}\frac{T^2}{M_{pl}}, \tag{1.9}$$

where $M_{pl} = 1.22 \times 10^{19}$ GeV is the Planck mass, g_* (=106.75 for the SM excluding RH neutrinos) is the total number of relativistic degrees of freedom contributing to the energy density of the Universe.

To quantify the departure from thermal equilibrium, we define the *decay parameter* as follows

$$K_i \equiv \frac{\Gamma_{N_i}}{H(M_i)} = \frac{\widetilde{m}_i}{m_*}, \tag{1.10}$$

where $\widetilde{m}_i$ is the *effective neutrino mass* defined as[47]

$$\widetilde{m}_i \equiv \frac{(Y^\dagger Y)_{ii} v^2}{M_i}, \tag{1.11}$$

with $m_* \equiv \frac{16\pi^2 v^2}{3M_{pl}}\sqrt{\frac{g_*\pi}{5}} \simeq 1 \times 10^{-3}$ eV. The regimes where $K_i \ll 1$, $K_i \approx 1$ and $K_i \gg 1$ are respectively known as weak, intermediate, and strong washout regimes.

[1] Neutrino oscillation data and leptogenesis both require $m \geq 2$.

1.2.2 CP asymmetry

The CP asymmetry in the decays of RH neutrinos N_i can be defined as

$$\epsilon_{i\alpha} = \frac{\gamma\left(N_i \to \ell_\alpha H\right) - \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}{\sum_\alpha \gamma\left(N_i \to \ell_\alpha H\right) + \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)} \equiv \frac{\Delta\gamma^\alpha_{N_i}}{\gamma_{N_i}}, \tag{1.12}$$

where $\gamma(i \to f)$ is the thermally averaged decay rate defined as[2]

$$\gamma(i \to f) \equiv \int \frac{d^3 p_i}{(2\pi)^3 2E_i} \frac{d^3 p_f}{(2\pi)^3 2E_f} (2\pi)^4 \delta^{(4)}(p_i - p_f) |\mathcal{A}(i \to f)|^2 e^{-E_i/T}, \tag{1.13}$$

where $\mathcal{A}(i \to f)$ is the decay amplitude. Ignoring all thermal effects [48, 49], eq. (1.12) simplifies to

$$\epsilon_{i\alpha} = \frac{|\mathcal{A}_0\left(N_i \to \ell_\alpha H\right)|^2 - |\mathcal{A}_0\left(N_i \to \overline{\ell_\alpha} H^*\right)|^2}{\sum_\alpha |\mathcal{A}_0\left(N_i \to \ell_\alpha H\right)|^2 + |\mathcal{A}_0\left(N_i \to \overline{\ell_\alpha} H^*\right)|^2}, \tag{1.14}$$

where $\mathcal{A}_0(i \to f)$ denotes the decay amplitude at zero temperature. Eq. (1.14) vanishes at tree level but is induced at 1-loop level through the interference between tree and 1-loop diagrams shown in Figure 1.1. There are two types of contributions from the 1-loop diagrams: the self-energy or wave diagram (middle) [50] and the vertex diagram (right) [45]. At leading order, we obtain the CP asymmetry [51]:

$$\begin{aligned} \epsilon_{i\alpha} &= \frac{1}{8\pi} \frac{1}{(Y^\dagger Y)_{ii}} \sum_{j \neq i} \mathrm{Im}\left[(Y^\dagger Y)_{ji} Y_{\alpha i} Y^*_{\alpha j}\right] g\left(\frac{M_j^2}{M_i^2}\right) \\ &+ \frac{1}{8\pi} \frac{1}{(Y^\dagger Y)_{ii}} \sum_{j \neq i} \mathrm{Im}\left[(Y^\dagger Y)_{ij} Y_{\alpha i} Y^*_{\alpha j}\right] \frac{M_i^2}{M_i^2 - M_j^2}, \end{aligned} \tag{1.15}$$

where the loop function is

$$g(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right]. \tag{1.16}$$

The first term in eq. (1.15) comes from L-violating wave and vertex diagrams, while the second term is from the L-conserving wave diagram. The terms of the form $(M_i^2 - M_j^2)^{-1}$ in eq. (1.15) are from the wave diagram contributions which can resonantly enhance the CP asymmetry if $M_i \approx M_j$ (resonant leptogenesis scenario, see Section 1.6.1)[3]. Let us also note that at least two RH neutrinos are needed, otherwise the CP asymmetry vanishes because the Yukawa couplings combination becomes real.

In the one flavour regime, we sum over the flavour index α in eq. (1.15) and obtain

$$\epsilon_i \equiv \sum_\alpha \epsilon_{i\alpha} = \frac{1}{8\pi} \frac{1}{(Y^\dagger Y)_{ii}} \sum_{j \neq i} \mathrm{Im}\left[(Y^\dagger Y)^2_{ji}\right] g\left(\frac{M_j^2}{M_i^2}\right), \tag{1.17}$$

where the second term in eq. (1.15) vanishes because the combination of the Yukawa couplings is real.

[2]Here the Pauli-blocking and Bose-enhancement statistical factors have been ignored and we also assume Maxwell-Boltzmann distribution for the particle i i.e. $f_i = e^{-E_i/T}$. See Refs. [48, 49] for detailed studies of their effects.

[3]Notice that the resonant term becomes singular in the degenerate limit $M_i = M_j$. This singularity can be regulated by using for example an effective field-theoretical approach based on resummation [52].

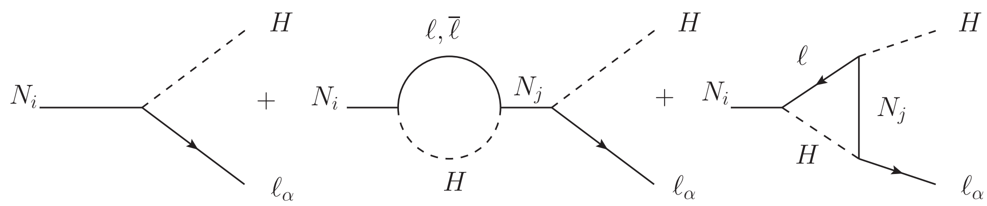

Figure 1.1: The CP asymmetry in type-I seesaw leptogenesis results from the interference between tree and 1-loop wave and vertex diagrams. For the 1-loop wave diagram, there is an additional contribution from L-conserving diagram to the CP asymmetry which vanishes when summing over lepton flavours.

1.2.3 Classical Boltzmann equations

We work in the one flavour regime and consider only the decays and inverse decays of N_1. If leptogenesis occurs at $T \gtrsim 10^{12}$ GeV, then the charged lepton Yukawa interactions are out of equilibrium, and this defines the one flavour regime. The assumption that only the dynamics of N_1 is relevant can be realized if for example the reheating temperature after inflation is $T_{RH} \ll M_{2,3}$ such that $N_{2,3}$ are not produced. In order to scale out the effect of the expansion of the Universe, we will introduce the *abundances*, i.e. the ratios of the particle densities $n_i = \int d^3p f_i$ to the entropy density $s = \frac{2\pi^2}{45} g_* T^3$:

$$Y_i \equiv \frac{n_i}{s}. \tag{1.18}$$

The evolution of the N_1 density and the lepton asymmetry $Y_{\Delta L} = 2Y_{\Delta \ell} \equiv 2(Y_\ell - Y_{\bar{\ell}})$ [4] can be described by the following classical Boltzmann equations (BE)[53]

$$\frac{dY_{N_1}}{dz} = -D_1(Y_{N_1} - Y_{N_1}^{eq}), \tag{1.19}$$

$$\frac{dY_{\Delta L}}{dz} = \epsilon_1 D_1(Y_{N_1} - Y_{N_1}^{eq}) - W_1 Y_{\Delta L}, \tag{1.20}$$

where $z \equiv M_1/T$ and the decay and washout terms are respectively given by

$$D_1(z) = \frac{\gamma_{N_1} z}{sH(M_1)} = K_1 z \frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)}, \qquad W_1(z) = \frac{1}{2} D_1(z) \frac{Y_{N_1}^{eq}(z)}{Y_\ell^{eq}}, \tag{1.21}$$

with $\mathcal{K}_n$ the n-th order modified Bessel function of second kind. $Y_{N_1}^{eq}$ and Y_ℓ^{eq} read:[5]

$$Y_N^{eq}(z) = \frac{45}{2\pi^4 g_*} z^2 \mathcal{K}_2(z), \qquad Y_\ell^{eq} = \frac{15}{4\pi^2 g_*}. \tag{1.22}$$

From eq. (1.19) and eq. (1.20), the solution for $Y_{\Delta L}$ can be written down as follows

$$Y_{\Delta L}(z) = Y_{\Delta L}(z_i) e^{-\int_{z_i}^{z} dz' W_1(z')} - \int_{z_i}^{z} dz' \epsilon_1(z') \frac{dY_{N_1}}{dz'} e^{-\int_{z'}^{z} dz'' W_1(z'')} \tag{1.23}$$

where z_i is some initial temperature when N_1 leptogenesis begins, and we have assumed that any preexisting lepton asymmetry vanishes $Y_{\Delta L}^0(z_i) = 0$. Notice that ignoring thermal effects, the CP asymmetry is independent of the temperature $\epsilon_1(z) = \epsilon_1$ (c.f. eq. (1.17)).

[4] The factor of 2 comes from the $SU(2)_L$ degrees of freedoms.

[5] To write down a simple analytic expression for the equilibrium density of N_1, we assume Maxwell-Boltzmann distribution. However, follwing [54], the normalization factor Y_ℓ^{eq} is obtained from a Fermi-Dirac distribution.

Weak washout regime

In the weak washout regime ($K_1 \ll 1$), the initial condition on the N_1 density $Y_{N_1}(z_i)$ is important. If we assume thermal initial abundance of N_1 i.e. $Y_{N_1}(z_i) = Y^{eq}_{N_1}(0)$, we can ignore the washout when N_1 starts decaying at $z \gg 1$ and we have

$$Y^t_{\Delta L}(\infty) \simeq -\epsilon_1 \int_0^\infty dz' \frac{dY^{eq}_{N_1}}{dz'} = \epsilon_1 Y^{eq}_{N_1}(0). \tag{1.24}$$

On the other hand, if we have zero initial N_1 abundance i.e. $Y_{N_1}(z_i) = 0$, we have to consider the opposite sign contributions to lepton asymmetry from the inverse decays when N_1 is being populated ($Y_{N_1} < Y^{eq}_{N_1}$) and from the period when N_1 starts decaying ($Y_{N_1} > Y^{eq}_{N_1}$). Taking this into account the term which survives the partial cancellations are given by [55] [6]

$$Y^0_{\Delta L}(\infty) \simeq \frac{27}{16}\epsilon_1 K_1^2 Y^{eq}_{N_1}(0). \tag{1.25}$$

Strong washout regime

In the strong washout regime ($K_1 \gg 1$) any lepton asymmetry generated during the N_1 creation phase is efficiently washed out. Here we adopt the *strong washout balance approximation*[56] which states that in the strong washout regime, the lepton asymmetry at each instant takes the value that enforces a balance between the production and the destruction rates of the asymmetry. Equating the decay and washout terms in eq. (1.20), we have

$$Y_{\Delta L}(z) \approx -\frac{1}{W(z)}\epsilon_1 \frac{dY_{N_1}}{dz} \simeq -\frac{1}{W(z)}\epsilon_1 \frac{dY^{eq}_{N_1}}{dz} = \frac{2}{zK_1}\epsilon_1 Y^{eq}_\ell, \tag{1.26}$$

where in the second approximation, we assume $Y_{N_1} \simeq Y^{eq}_{N_1}$. The approximation no longer holds when $Y_{\Delta L}$ freezes and this happens when the washout decouples at z_f i.e. $W(z_f) < 1$. Hence, the final lepton asymmetry is given by[7]

$$Y_{\Delta L}(\infty) = \frac{2}{z_f K_1}\epsilon_1 Y^{eq}_\ell = \frac{\pi^2}{6 z_f K_1}\epsilon_1 Y^{eq}_{N_1}(0). \tag{1.27}$$

The freeze out temperature z_f depends mildly on K_1. For $K_1 = 10$ - 100 we have for example $z_f = 7$ - 10. We also see that independently of initial conditions, in the strong regime $Y_{\Delta L}(\infty)$ goes as K_1^{-1}.

1.2.4 Baryon asymmetry from EW sphaleron

The final lepton asymmetry $Y_{\Delta L}(\infty)$ can be conveniently parametrized as follows

$$Y_{\Delta L}(\infty) = \epsilon_1 \eta_1 Y^{eq}_{N_1}(0), \tag{1.28}$$

where η_1 is known as the *efficiency factor*. In the weak washout regime ($K_1 \ll 1$) from eq. (1.24) we have $\eta_1 = 1$ ($= \frac{27}{16}K_1^2 < 1$) for thermal (zero) initial N_1 abundance. In the strong washout regime ($K_1 \gg 1$), from eq. (1.27), we have $\eta_1 = \frac{\pi^2}{6 z_f K_1} < 1$.

[6] This differs from the efficiency in Ref. [55] by the factor $\frac{12}{\pi^2}$, which is due to the different normalization Y^{eq}_ℓ eq. (1.22).
[7] Compare this to a more precise analytical approximation in Ref. [55].

If leptogenesis ends before EW sphaleron processes become active ($T \gtrsim 10^{12}$ GeV), the $B-L$ asymmetry $Y_{\Delta_{B-L}}$ is simply given by

$$Y_{\Delta_{B-L}} = -Y_{\Delta L}\,. \tag{1.29}$$

At the later stage, the $B-L$ asymmetry is partially transfered to a B asymmetry by the EW sphaleron processes through the relation [57]

$$Y_{\Delta B}(\infty) = \frac{28}{79} Y_{\Delta_{B-L}}(\infty)\,, \tag{1.30}$$

that holds if sphalerons decouple before EWPT. This relation will change if the EW sphaleron processes decouple after the EWPT [57, 58] or if threshold effects for heavy particles like the top quark and Higgs are taken into account [58, 59].

1.2.5 Davidson-Ibarra bound

Assuming a hierarchical spectrum of the RH neutrinos ($M_1 \ll M_2$, M_3) and that the dominant lepton asymmetry is from the N_1 decays, from eq. (1.17) the CP asymmetry from N_1 decays can be written as

$$\epsilon_1 = -\frac{3}{16\pi}\frac{1}{(Y^\dagger Y)_{11}}\sum_{j\neq 1}\mathrm{Im}\left[(Y^\dagger Y)^2_{j1}\right]\frac{M_1}{M_j}. \tag{1.31}$$

Assuming three generations of RH neutrinos ($n = 3$) and using the Casas-Ibarra parametrization [60] for the Yukawa couplings

$$Y_{\alpha i} = \frac{1}{v}\left(\sqrt{D_{m_N}}R\sqrt{D_{m_\nu}}U_\nu^\dagger\right)_{\alpha i}, \tag{1.32}$$

where $D_{m_N} = \mathrm{diag}(M_1, M_2, M_3)$, $D_{m_\nu} = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$ and R any complex orthogonal matrix satisfying $R^T R = RR^T = 1$, eq. (1.31) becomes

$$\epsilon_1 = -\frac{3}{16\pi}\frac{M_1}{v^2}\frac{\sum_i m_{\nu_i}\mathrm{Im}(R^2_{1i})}{\sum_i m_{\nu_i}|R_{1i}|^2}. \tag{1.33}$$

Using the orthogonality condition $\sum_i R^2_{1i} = 1$, we then obtain the Davidson-Ibarra (DI) bound [61]

$$|\epsilon_1| \leq \epsilon^{DI} = \frac{3}{16\pi}\frac{M_1}{v^2}(m_{\nu_3} - m_{\nu_1}) = \frac{3}{16\pi}\frac{M_1}{v^2}\frac{\Delta m^2_{atm}}{m_{\nu_1} + m_{\nu_3}}, \tag{1.34}$$

where m_{ν_3} (m_{ν_1}) is the heaviest (lightest) light neutrino mass. Applying the DI bound on eqs. (1.28)–(1.30), and requiring that $Y_{\Delta B}(\infty) \geq Y_B^{CMB} \simeq 10^{-10}$, we obtain

$$M_1\left(\frac{0.1\,\mathrm{eV}}{m_{\nu_1} + m_{\nu_3}}\right)\eta_1^{max}(M_1) \gtrsim 10^9\,\mathrm{GeV}, \tag{1.35}$$

where the $\eta_1^{max}(M_1)$ is the efficiency factor maximized with respect to K_1 eq. (1.10) for a particular value of M_1. This allows us to make a plot of region which satisfies eq. (1.35) on the (M_1, m_{ν_1}) plane and hence obtain bounds on M_1 and m_{ν_1}. Many careful numerical studies have been carried out and it was found that successful leptogenesis with a hierarchical spectrum of the RH neutrinos

requires $M_1 \gtrsim 10^9$ GeV [61, 62, 63] and $m_{\nu_1} \lesssim 0.1$ eV [64, 65, 55, 66]. This bound implies that the RH neutrinos must be produced at temperatures $T \gtrsim 10^9$ GeV which in turn implies the reheating temperature after inflation has to be $T_{RH} \gtrsim 10^9$ GeV in order to have sufficient RH neutrinos in the thermal bath. To conclude this section, let us note that the DI bound eq. (1.34) holds if and only if all the following conditions apply:
(1) N_1 dominates the contribution to leptogenesis.
(2) The mass spectrum of RH neutrinos are hierarchical $M_1 \ll M_2, M_3$.
(3) Leptogenesis occurs in the unflavoured regime $T \gtrsim 10^{12}$ GeV.
As we will see in the following sections, violation of one or more of the above conditions allows us to lower somewhat the scale of leptogenesis.

1.3 Lepton Flavour Effects

1.3.1 When are lepton flavour effects relevant?

The first leptogenesis calculations were performed in the single lepton flavour regime. In short, this amounts to assuming that the leptons and antileptons which couple to the lightest RH neutrino N_1 maintain their coherence as flavour superpositions throughout the leptogenesis era, that is $\ell_1 = \sum_\alpha c_{\alpha 1}\ell_\alpha$ and $\overline{\ell}'_1 = \sum_\alpha c'^*_{\alpha 1}\overline{\ell}_\alpha$. Note that at the tree-level the coefficients c and c'^* are simply the Yukawa couplings: $c_{\alpha 1} = Y_{\alpha 1}$ and $c'^*_{\alpha 1} = Y^*_{\alpha 1}$. However it should be kept in mind that since CP is violated by loops, beyond the tree level approximation the antilepton state $\overline{\ell'_1}$ is not the CP conjugate of the ℓ_1, that is $c'_{\alpha 1} \neq c_{\alpha 1}$.

The single flavour regime is realized only at very high temperatures ($T \gtrsim 10^{12}$ GeV) when both ℓ_1 and ℓ'_1 remain coherent flavour superpositions, and thus are the correct states to describe the dynamics of leptogenesis. However, at lower temperatures scatterings induced by the charged lepton Yukawa couplings occur at a sufficiently fast pace to distinguish the different lepton flavours, ℓ_1 and ℓ'_1 decohere in their flavour components, and the dynamics of leptogenesis must then be described in terms of the flavour eigenstates ℓ_α. Of course, there is great interest to extend the validity of quantitative leptogenesis studies also at lower scale $T \lesssim 10^{12}$ GeV, and this requires accounting for flavour effects. The role of lepton flavour in leptogenesis was first discussed in Ref. [67], however the authors did not highlight in what the results were significantly different from the single flavour approximation. Therefore, until the importance of flavour effects was fully clarified in Refs. [68, 69, 70] they had been included in leptogenesis studies only in a few cases [71, 72, 73, 74, 75]. Nowadays lepton flavour effects have been investigated in full detail [76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89] and are a mandatory ingredient of any reliable analysis of leptogenesis.

The specific temperature when leptogenesis becomes sensitive to lepton flavour dynamics can be estimated by requiring that the rates of processes Γ_α ($\alpha = e, \mu, \tau$) that are induced by the charged lepton Yukawa couplings h_α become faster than the Universe expansion rate $H(T)$. An approximate relation gives [90, 91]

$$\Gamma_\alpha(T) \simeq 10^{-2} h_\alpha^2 T \,, \tag{1.36}$$

which implies that[8]

$$\Gamma_\alpha(T) > H(T) \qquad \text{when} \qquad T \lesssim T_\alpha \,, \tag{1.37}$$

[8] In supersymmetric case, since $h_\alpha = m_\alpha/(v_u \cos\beta)$, we have $T \lesssim T_\alpha\,(1 + \tan^2\beta)$.

where $T_e \simeq 4 \times 10^4$ GeV, $T_\mu \simeq 2 \times 10^9$ GeV, and $T_\tau \simeq 5 \times 10^{11}$ GeV. Notice that to fully distinguish the three flavours it is sufficient that the τ and μ Yukawa reactions attain thermal equilibrium. It has been pointed out that besides being faster than the expansion of the Universe, the charged lepton Yukawa interactions should also be faster than the N_1 interactions [69, 83, 84]. In general whenever $\Gamma_\tau(M_1) > H(M_1)$ we also have $\Gamma_\tau(M_1) > \Gamma_{N_1}(M_1)$. However, there exists parameter space where $\Gamma_\tau(M_1) > H(M_1)$ but $\Gamma_\tau(M_1) < \Gamma_{N_1}(M_1)$. This scenario was studied in Ref. [83].

1.3.2 The effects on CP asymmetry and washout

The CP violation in N_i decays can manifest itself in two ways [69]:

(i) The leptons and antileptons are produced at different rates,

$$\gamma_i \neq \bar{\gamma}_i, \tag{1.38}$$

where $\gamma_i \equiv \gamma(N_i \to \ell_i H)$ and $\bar{\gamma}_i \equiv \gamma(N_i \to \overline{\ell_i'} H^*)$.

(ii) The leptons and antileptons produced are not CP conjugate states,

$$CP(\overline{\ell_i'}) = \ell_i' \neq \ell_i, \tag{1.39}$$

that is, due to loops effects they are slightly misaligned in flavour space.

We can rewrite the CP asymmetry for N_i decays from eq. (1.12) as follows

$$\epsilon_{i\alpha} = \frac{P_{i\alpha}\gamma_i - \bar{P}_{i\alpha}\bar{\gamma}_i}{\gamma_i + \bar{\gamma}_i} = \frac{P_{i\alpha} + \bar{P}_{i\alpha}}{2}\,\epsilon_i + \frac{P_{i\alpha} - \bar{P}_{i\alpha}}{2} \simeq P^0_{i\alpha}\epsilon_i + \frac{\Delta P_{i\alpha}}{2}, \tag{1.40}$$

where terms of order $\mathcal{O}(\epsilon_i\,\Delta P_{i\alpha})$ and higher have been neglected. $P_{i\alpha}$ is the projector from state ℓ_i into flavour state ℓ_α and $\Delta P_{i\alpha} = P_{i\alpha} - \bar{P}_{i\alpha}$. At tree level, clearly, $P_{i\alpha} = \bar{P}_{i\alpha} \equiv P^0_{i\alpha}$ where the tree level flavour projector is given by

$$P^0_{i\alpha} = \frac{Y_{\alpha i} Y^*_{\alpha i}}{(Y^\dagger Y)_{ii}}. \tag{1.41}$$

From eq. (1.40), we can identify the two types of CP violation, the first term being of type (i) eq. (1.38) while the second being of type (ii) eq. (1.39). Since $\sum_\alpha P_{i\alpha} = \sum_\alpha \bar{P}_{i\alpha} = 1$, when summing over flavour indices α, the second term vanishes $\sum_\alpha \Delta P_{i\alpha} = 0$. Note that the lepton-flavour-violating but L-conserving terms in the second line of eq. (1.15) is part of type (ii). In fact, they come from $d = 6$ L-conserving operators which have nothing to do with the unique $d = 5$ L-violating operator (the Weinberg operator [92]) responsible for neutrino masses. However, in some cases they can still dominate the CP asymmetries but, as we will see in Section 1.3.4, lepton flavour equilibration effects [93] then impose important constraints on their overall effects. Note also that due to flavour misalignment, the CP asymmetry in a particular flavour direction $\epsilon_{i\alpha}$ can be much larger and even of opposite sign from the total CP asymmetry ϵ_i. In fact the relevance of CP violation of type (ii) in the flavour regimes is what allows to evade the DI bound eq. (1.34). As regards the washout of the lepton asymmetry of flavour α, it is proportional to

$$W_{i\alpha} \propto P_{i\alpha}\gamma_i + \bar{P}_{i\alpha}\bar{\gamma}_i \simeq P^0_{i\alpha} W_i, \tag{1.42}$$

which results in a reduction of washout by a factor of $P^0_{i\alpha} \leq 1$ compared to unflavoured case. As we will see next, the new CP-violating sources from flavour effects and the reduction in the washout could result in great enhancement of the final lepton asymmetry and, as was first pointed out in Ref. [69], leptogenesis with a vanishing total CP asymmetry $\epsilon_i = 0$ also becomes possible.

1.3.3 Classical flavoured Boltzmann equations

Here again we only consider leptogenesis from the decays and inverse decays of N_1. In this approximation, the BE for Y_{N_1} is still given by eq. (1.19) while the BE for $Y_{\Delta L_\alpha}$ the lepton asymmetry in the flavour α is given by[9]

$$\frac{dY_{\Delta L_\alpha}}{dz} = \epsilon_{1\alpha} D_1 (Y_{N_1} - Y_{N_1}^{eq}) - P_{1\alpha}^0 W_1 Y_{\Delta L_\alpha}. \tag{1.43}$$

Notice that as long as L violation from sphalerons is neglected (see section 1.4) the BE for $Y_{\Delta L_\alpha}$ are independent of each other, and hence the solutions for the weak and strong washout regimes are given respectively by eq. (1.25)) and eq. (1.27), after replacing $\epsilon_1 \to \epsilon_{1\alpha}$ and $K_1 \to K_{1\alpha} \equiv P_{1\alpha}^0 K_1$.

As an example let us assume that leptogenesis occurs around $T \sim 10^{10}$ GeV, that is in the two-flavour regime. Due to the fast τ Yukawa interactions $\ell_1(\ell_1')$ gets projected onto $\ell_\tau(\ell_\tau')$ and a coherent mixture of $e+\mu$ eigenstate $\ell_{e+\mu}(\ell_{e+\mu}')$. For illustrative purpose, here we consider a scenario in which lepton flavour effects are most prominent. We take both $K_{1\tau}, K_{1e+\mu} \gg 1$, so that both $Y_{\Delta L_\tau}$ and $Y_{\Delta L_{e+\mu}}$ are in the strong regime. From eq. (1.27) we can write down the solution:

$$\begin{aligned} Y_{\Delta L}(\infty) &= Y_{\Delta L_\tau}(\infty) + Y_{\Delta L_{e+\mu}}(\infty) \\ &= \frac{\pi^2}{6 z_f K_1} Y_{N_1}^{eq}(0) \left(\frac{\epsilon_{1\tau}}{P_{1\tau}^0} + \frac{\epsilon_{1e+\mu}}{P_{1e+\mu}^0} \right) \\ &\simeq \frac{\pi^2}{3 z_f K_1} \epsilon_1 Y_{N_1}^{eq}(0) + \frac{\pi^2}{12 z_f K_1} Y_{N_1}^{eq}(0) \left(\frac{\Delta P_{1\tau}}{P_{1\tau}^0} + \frac{\Delta P_{1e+\mu}}{P_{1e+\mu}^0} \right), \end{aligned} \tag{1.44}$$

where in the last line we have used eq. (1.40). If $P_{1\tau}^0 \simeq P_{1e+\mu}^0$, then since $\Delta P_{1\tau} + \Delta P_{1e+\mu} = 0$ the second term approximately cancels, and eq. (1.44) reduces to

$$Y_{\Delta L}(\infty) \simeq \frac{\pi^2}{3 z_f K_1} \epsilon_1 Y_{N_1}^{eq}(0) \,. \tag{1.45}$$

We see that the final asymmetry is enhanced by a factor of 2 compared to the unflavoured case. If there exists some hierarchy between the flavour projectors, then the second term in eq. (1.44) plays an important role and can further enhance the asymmetry. For example we can have $P_{1\tau}^0 > P_{1e+\mu}^0$ while $\Delta P_{1\tau} \ll \Delta P_{1e+\mu}$. In this case, the second term can dominate over the first term. Finally from eq. (1.44) we also notice that leptogenesis with $\epsilon_1 = 0$, the so-called *purely flavoured leptogenesis* (PFL)[10], can indeed proceed [69, 95, 96, 97, 98]. In this scenario some symmetry has to be imposed to realize the condition $\epsilon_1 = 0$, as for example an approximate global lepton number $U(1)_L$. In the limit of exact $U(1)_L$ the active neutrinos will be exactly massless. Instead of the seesaw mechanism, the small neutrino masses is explained by $U(1)_L$ which is slightly broken by a small parameter μ (the "inverse seesaw") [99] which is technically natural since the Lagrangian exhibits an enhanced symmetry when $\mu \to 0$ [100]. In the next section, we will discuss another aspect of flavour effects which are in particular crucial for PFL.

[9] To study the transition between different flavour regimes (from one to two or from two to three flavours), a density matrix formalism has to be used [68, 84, 94].

[10] This can also refer to the case where the total CP asymmetry is negligible $\epsilon_1 \approx 0$.

1.3.4 Lepton flavour equilibration

Another important effect is lepton flavour equilibration (LFE) [93]. LFE processes violate lepton flavour but conserve total lepton number e.g. $\ell_\alpha H \to \ell_\beta H$, and can proceed e.g. via off-shell exchange of $N_{2,3}$. In thermal equilibrium, LFE processes can quickly equilibrate the asymmetries generated in different flavours. In practice this would be equivalent to a situation where all the flavour projectors eq. (1.41) are equal, in which case the flavoured BE eq. (1.43) can be summed up into a single BE:

$$\frac{dY_{\Delta L}}{dz} = \epsilon_1 D_1 (Y_{N_1} - Y_{N_1}^{eq}) - P_{1\alpha}^0 W_1 Y_{\Delta L}, \quad (1.46)$$

where $P_{1\alpha}^0 = 1/2\,(1/3)$ in the two (three) flavours regime. In this case the BE is just like the unflavoured case but with a reduced washout which, in the strong washout regime, would result in enhancement of a factor of 2 (3) in the two (three) flavours regime (c.f. eq. (1.45)). Clearly, LFE can make PFL with $\epsilon_1 = 0$ impotent [93, 56]. Since LFE $N_{2,3}$ processes scale as T^3 while the Universe expansion scales as T^2, in spite of the fact that PFL evades the DI bound, they eventually prevent the possibility of lowering too much the leptogenesis scale. A generic study in PFL scenario taking into account LFE effects concluded that successful leptogenesis still requires $M_1 \gtrsim 10^8$ GeV [97]. A more accurate study in the same direction recently carried out in Ref. [98], showed that in fact the leptogenesis scale can be lowered down to $M_1 \sim 10^6$ GeV.

1.4 Beyond the Basic Boltzmann Equations

Within factors of a few, the amount of baryon asymmetry that is generated via leptogenesis in N_1 decays is determined essentially by the size of the (flavoured) CP asymmetries and by the rates of the (flavoured) washout reactions. However, to obtain more precise results (say, within an $\mathcal{O}(1)$ uncertainty) several additional effects must be taken into account, and the formalism must be extended well beyond the basic BE discussed in the previous Sections. In the following we review some of the most important sources of corrections, namely spectator processes (Section 1.4.1), scatterings with top quarks and gauge bosons (Section 1.4.2), thermal effects (Section 1.4.3), contributions from heavier RH neutrinos (Section 1.4.4), and we also discuss the role of quantum corrections evaluated in the quantum BE approach (Section 1.4.5). Throughout this review we use integrated BE, i.e. we assume kinetic equilibrium for all particle species, and thus we use particles densities instead than particles distribution functions. Corrections arising from using non-integrated BE have been studied for example in Refs. [101, 102, 103, 104], and are generally subleading.

1.4.1 Spectator processes

Reactions that without involving violation of $B-L$ can still affect the final amount of baryon asymmetry are classified as *"spectator processes"* [105, 106]. The basic way through which they act is that of redistributing the asymmetry generated in the lepton doublets among the other particle species. Since the density asymmetries of the lepton doublets are what weights the rates of the washout processes, it can be expected that spectator processes would render the washouts less effective and increase the efficiency of leptogenesis. However, in most cases this is not true: proper inclusion of spectator processes implies accounting for all the particle asymmetries, and in particular also for the density asymmetry of the Higgs $Y_{\Delta H}$ [106]. This was omitted in Section 1.2 but in fact has to be added to the density asymmetry of the leptons $Y_{\Delta\ell}$ in weighting for example washouts from inverse decays. Eq. (1.20) would then become:

$$\frac{dY_{\Delta L}}{dz} = \epsilon_1 D_1 (Y_{N_1} - Y_{N_1}^{eq}) - 2\,(Y_{\Delta\ell} + Y_{\Delta H})\, W_1 \qquad (1.47)$$

where the factor of two in front of the washout term counts the leptons and Higgs gauge multiplicity. Clearly, in some regimes in which $Y_{\Delta\ell}$ and $Y_{\Delta H}$ are not sufficiently diluted by interacting with other particles, this can have the effect of enhancing the washout rates and suppressing the efficiency.

In the study of spectator processes it is fundamental to specify the range of temperature in which leptogenesis occurs. This is because at each specific temperature T, particle reactions must be treated in a different way depending if their characteristic time scale τ (given by inverse of their thermally averaged rates) is [89, 107]

(1) much shorter than the age of the Universe: $\tau \ll t_U(T)$;
(2) much larger than the age of the Universe: $\tau \gg t_U(T)$;
(3) comparable with the Universe age: $\tau \sim t_U(T)$.

Spectator processes belong to the first type of reactions which occur very frequently during one expansion time. Their effects can be accounted for by imposing on the thermodynamic system the chemical equilibrium condition appropriate for each specific reaction, that is $\sum_I \mu_I = \sum_F \mu_F$, where μ_I denotes the chemical potential of an initial state particle, and μ_F that of a final state particle[11]. The numerical values of the parameters that are responsible for these reactions only determine the precise temperature T when chemical equilibrium is attained but, apart from this, have no other relevance, and do not appear explicitly in the formulation of the problem. Reactions of type (2) cannot have any effect on the system, since they basically do not occur. All physical processes are blind to the corresponding parameters, that can be set to zero in the effective Lagrangian. In most cases this results in exact global symmetries corresponding to conserved charges, and these conservation laws impose constraints on the particle chemical potentials. Reactions of type (3) in general violate some symmetry, and thus spoil the corresponding conservation conditions, but are not fast enough to enforce chemical equilibrium. These are the only reactions that need to be studied by means of BE, and for which the precise value of the parameters that control their rates is of utmost importance.

A simple case to illustrate how to include spectator processes is the one flavour regime at particularly high temperatures (say $T \gtrsim 10^{13}\,\mathrm{GeV}$). The Universe expansion is fast implying that except for processes induced by the large Yukawa coupling of the top and for gauge interactions, all other $B-L$-conserving reactions fall in class (ii). Then there are several conserved quantities as for example the total number density asymmetries of the RH leptons as well as those of all the quarks except the top. Since electroweak sphalerons are also out of equilibrium, B is conserved too (and vanishing, if we assume that there is no preexisting asymmetry). $B = 0$ then translates in the condition:

$$2Y_{\Delta Q_3} + Y_{\Delta t} = 0\,, \qquad (1.48)$$

where $Y_{\Delta Q_3}$ is the density asymmetries of one degree of freedom of the top $SU(2)_L$ doublet and color triplet which, being gauge interactions in equilibrium, is the same for all the six gauge components, and $Y_{\Delta t}$ is the density asymmetry of the $SU(2)_L$ singlet top. Hypercharge is always conserved, yielding

$$Y_{\Delta Q_3} + 2Y_{\Delta t} - Y_{\Delta\ell} + Y_{\Delta H} = 0\,. \qquad (1.49)$$

[11]The relation between chemical potentials and particle density asymmetries is given in eq. (1.85).

Finally, in terms of density asymmetries chemical equilibrium for the top-Yukawa related reactions $\mu_{Q_3} + \mu_H = \mu_t$ translates into

$$Y_{\Delta Q_3} + \frac{1}{2} Y_{\Delta H} = Y_{\Delta t} \,. \tag{1.50}$$

We have three conditions for four density asymmetries, which allows to express the Higgs density asymmetry in terms of the density asymmetry of the leptons as $Y_{\Delta H} = \frac{2}{3} Y_{\Delta \ell}$. Moreover, given that only the LH lepton degrees of freedom are populated, we have $Y_{\Delta L} = 2Y_{\Delta \ell}$ so that the coefficient weighting W_1 in eq. (1.47) becomes $2\ (Y_{\Delta \ell} + Y_{\Delta H}) = \frac{5}{3} Y_{\Delta L}$ and the washout is accordingly stronger.

With decreasing temperatures, more reactions attain chemical equilibrium, and accounting for spectator processes becomes accordingly more complicated. When the temperature drops below $T \sim 10^{12}$ GeV, EW sphalerons are in equilibrium, and baryon number is no more conserved. Then the condition eq. (1.48) is no more satisfied and, more importantly, the BE eq. (1.47) is no more valid since sphalerons violate also lepton number with in-equilibrium rates. However, sphalerons conserve $B - L$, which is then violated only by slow reactions of type (3), and we should then write down a BE for this quantity. Better said, since at $T \lesssim 10^{12}$ GeV all the third generation Yukawa reactions, including the ones of the τ-lepton, are in equilibrium, the dynamical regime is that of two flavours in which the relevant quasi-conserved charges are $\Delta_\tau = B/3 - L_\tau$ and $\Delta_{e\mu} = B/3 - L_{e\mu}$. The fact that only two charges are relevant is because there is always a direction in e-μ space which remains decoupled from N_1. The corresponding third charge $\Delta'_{e\mu}$ is then exactly conserved, its value can be set to zero and the corresponding BE dropped. In this regime, the BE corresponding to eq. (1.47) becomes:

$$-\frac{dY_{\Delta_\alpha}}{dz} = \epsilon_\alpha D_1 (Y_{N_1} - Y_{N_1}^{eq}) - 2\ (Y_{\Delta \ell_\alpha} + Y_{\Delta H})\, W_1 \qquad (\alpha = \tau, e\mu)\,. \tag{1.51}$$

To rewrite these equations in a solvable closed form, $Y_{\Delta \ell_\tau}$, $Y_{\Delta \ell_{e\mu}}$ and $Y_{\Delta H}$ must be expressed in terms of the two charge densities Y_{Δ_τ} and $Y_{\Delta_{e\mu}}$. This can be done by imposing the hypercharge conservation condition eq. (1.49) and the chemical equilibrium conditions that, in addition to eq. (1.50), are appropriate for the temperature regime we are considering. They are [106]: 1. QCD sphaleron equilibrium; 2. EW sphaleron equilibrium; 3. b-quark and τ-lepton Yukawa equilibrium. The 'rotation' from the particle density asymmetries $Y_{\Delta \ell_\alpha}, Y_{\Delta H}$ to the charge densities Y_{Δ_α} can be expressed in terms of the A matrix introduced in [67] $Y_{\Delta \ell_\alpha} = A^\ell_{\alpha\beta} Y_{\Delta_\beta}$ $(\alpha, \beta = \tau, e\mu)$ and C-vector $Y_{\Delta H} = C^H_\alpha Y_{\Delta_\alpha}$ introduced in [69]. For the present case, with the ordering $(e\mu, \tau)$ they are [69]:

$$A^\ell = \frac{1}{460} \begin{pmatrix} 196 & -24 \\ -9 & 156 \end{pmatrix} \qquad \text{and} \qquad C^H = \frac{1}{230} (41,\ 56)\,. \tag{1.52}$$

It is important to stress that in each temperature regime there are always enough constraints (conservation laws and chemical equilibrium conditions) to allow to express all the relevant particle density asymmetries in terms of the quasi-conserved charges Y_{Δ_α}. This is because each time a conservation law has to be dropped (like B conservation above) it gets replaced by a chemical equilibrium condition (like EW sphalerons equilibrium), and each time the chemical potential of a new particle species becomes relevant, it is precisely because a new reaction involving that particle attains chemical equilibrium, enforcing the corresponding condition. As regards the quantitative corrections ascribable to spectator processes, several numerical studies have confirmed that they generally remain below order one. Thus, differently from flavour effects, for order of magnitude estimates they can be neglected.

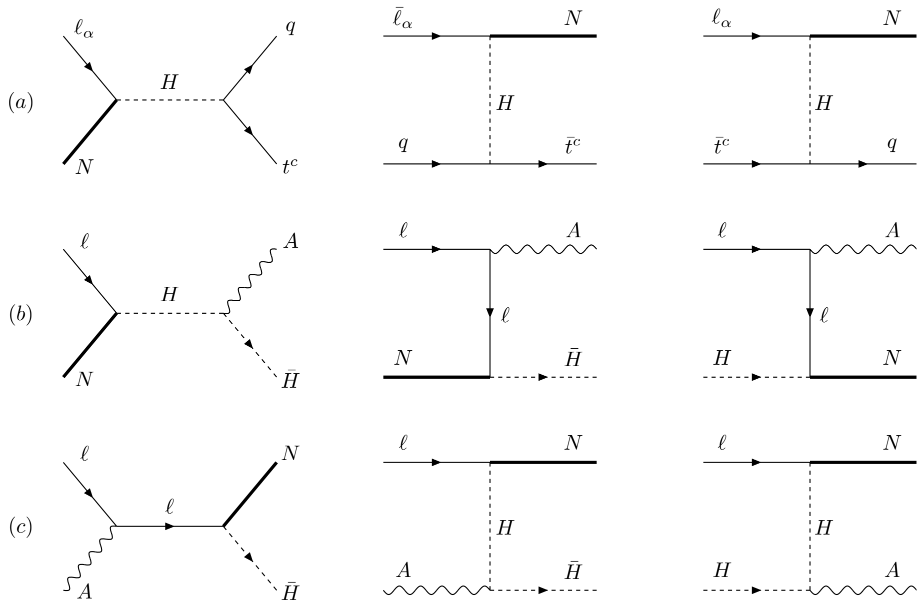

Figure 1.2: Diagrams for various $2 \leftrightarrow 2$ scattering processes: (a) scatterings with the top quarks, (b), (c) scatterings with the gauge bosons ($A = B, W_i$ with $i = 1, 2, 3$).

1.4.2 Scatterings and CP violation in scatterings

Scattering processes are relevant for the production of the N_1 population, because decay and inverse decay rates are suppressed by a time dilation factor $\propto M_1/T$. The $N_1 = \bar{N}_1$ particles can be produced by scatterings with the top quark in s-channel H-exchange $qt^c \to N\ell_\alpha$ and $\bar{q}\bar{t}^c \to N\bar{\ell}_\alpha$, by t-channel H-exchange in $q\bar{\ell}_\alpha \to N\bar{t}^c$, $\bar{q}\ell_\alpha \to Nt^c$ and by u-channel H-exchange in $\ell_\alpha\bar{t}^c \to Nq$, $t^c\bar{\ell}_\alpha \to N\bar{q}$, see the diagrams labeled (a) in Figure 1.2. Several scattering channels with gauge bosons also contribute to the production of N_1. The corresponding diagrams are labeled (b) and (c) in the same figure.

When the effects of scatterings in populating the N_1 degree of freedom are included, for consistency CP violation in scatterings must also be included. In doing so some care has to be put in treating properly also all the processes of higher order in the couplings ($h_t^2\lambda^4$, $g^2\lambda^4$, where g is a gauge coupling) with an on-shell intermediate state N_1 subtracted out. This can be done by following the procedure adopted in Ref. [108], and we refer to that paper for details.

In first approximation, the CP asymmetry in scattering processes is the same as in decays and inverse decays [70, 109]. This result was first found in Refs. [110, 75, 111] for the case of resonant leptogenesis, and was later derived in Ref. [70] for the case of hierarchical N_j. A full calculation of the CP asymmetry in scatterings involving the top quark was carried out in Ref. [108], and the validity of approximating it with the CP asymmetry in decays was analyzed, finding that the approximation is generally good for sufficiently strong RH neutrino hierarchies, e.g. $M_2/M_1 \gg 10$. Corrections up to several tens of percent can appear around temperatures of order $T \sim M_2/10$, and can be numerically relevant in case of milder hierarchies.

Regarding the scattering processes with gauge bosons such as $N\ell_\alpha \to A\bar{H}$ $NH \to A\bar{\ell}_\alpha$ or $NA \to \ell_\alpha H$, their effects in leptogenesis were estimated in Ref. [108] under the assumption that it can also be factorized in terms of the decay CP asymmetry. However, with respect to scatterings involving the top quark, there is a significant difference that now box diagrams in which the gauge boson is attached to a lepton or Higgs in the loop of the vertex-type diagrams are also present, leading to more complicated expressions that were explicitly calculated in Ref. [112]. There it was shown that the presence of box diagrams implies that for scatterings with gauge bosons the CP asymmetry is different from the decay CP asymmetry even for hierarchical RH neutrinos. Still, this difference remains within a factor of two [112] so that related effects are in any case not very large. In general, it turns out that CP asymmetry in scatterings is more relevant at high temperatures ($T > M_1$) when the scattering rates are larger than the decay rate. Hence, it can be of some relevance to the final value of the baryon asymmetry when some of the lepton flavours are weakly washed out, and some memory of the asymmetries generated at high temperature is preserved in the final result.

1.4.3 Thermal corrections

At the high temperatures at which leptogenesis occurs, the light particles involved in the leptogenesis processes are in equilibrium with the hot plasma. Thermal effects give corrections to several ingredients in the analysis: (i) coupling constants, (ii) particle propagators (leptons, quarks, gauge bosons and the Higgs) and (iii) CP-violating asymmetries, which we briefly discuss below. A detailed study of thermal corrections can be found in Ref. [49].

Coupling constants

Renormalization of gauge and Yukawa couplings in a thermal plasma is studied in Ref. [113]. In practice, it is a good approximation to use the zero-temperature renormalization group equations for the couplings, with a renormalization scale $\Lambda \sim 2\pi T$ [49]. The value $\Lambda > T$ is related to the fact that the average energy of the colliding particles in the plasma is larger than the temperature.

The renormalization effects for the neutrino couplings are also well known [114, 115]. In the non-supersymmetric case, to a good approximation these effects can be described by a simple rescaling of the low energy neutrino mass matrix $m(\mu) = r \cdot m$, where $1.2 \lesssim r \lesssim 1.3$ for $10^8\,\mathrm{GeV} \lesssim \mu \lesssim 10^{16}\,\mathrm{GeV}$ [49], and therefore can be accounted for by increasing the values of the neutrino mass parameters (for example, $\tilde{m}$) as measured at low energy by $\approx 20\%-30\%$ (depending on the leptogenesis scale). In the supersymmetric case one expects a milder enhancement, but uncertainties related with the precise value of the top-Yukawa coupling can be rather large (see Figure 3 in Ref. [49]).

Decays and scatterings

In the thermal plasma, any particle with sizable couplings to the background acquires a thermal mass which is proportional to the plasma temperature. Consequently, decay and scattering rates get modified. Particle thermal masses have been thoroughly studied in both the SM and the supersymmetric SM [116, 117, 91, 118, 119, 120]. The singlet neutrinos have no gauge interactions, their Yukawa couplings are generally small and, during the relevant era, their bare masses are of the order of the temperature or larger. Consequently, to a good approximation, corrections to their masses can be neglected. We thus need to account for the thermal masses of the leptons and Higgs doublets and, when scatterings are included, also of the third generation quarks and of the gauge bosons (and of their superpartners in the supersymmetric case). For a qualitative discussion, it is enough to keep in mind that, within the leptogenesis temperature range, we have $m_H(T) \gtrsim m_{q_3,t}(T) \gg m_\ell(T)$. The most important effects are related to four classes of leptogenesis processes:

(i) *Decays and inverse decays.* Since thermal corrections to the Higgs mass are particularly large ($m_H(T) \approx 0.4\,T$), decays and inverse decays become kinematically forbidden in the temperature range in which $m_H(T) - m_\ell(T) < M_{N_1} < m_H(T) + m_\ell(T)$. For lower temperatures, the usual processes $N_1 \leftrightarrow \ell H$ can occur. For higher temperatures, the Higgs is heavy enough that it can decay: $H \leftrightarrow \ell N_1$. A rough estimate of the kinematically forbidden region yields $2 \lesssim T/M_1 \lesssim 5$. The important point is that these corrections are effective only at $T > M_1$. In the parameter region $\tilde{m} > 10^{-3}\,\mathrm{eV}$, that is favoured by the measurements of the neutrino mass-squared differences, the N_1 number density and its L-violating reactions attain thermal equilibrium at $T \approx M_1$ and erase quite efficiently any memory of the specific conditions at higher temperatures. Consequently, in the strong washout regime, these corrections have practically no effect on the final value of the baryon asymmetry.

(ii) $\Delta L = 1$ *scatterings with top quark.* A comparison between the corrected and uncorrected rates of the top-quark scattering $\gamma^{\rm top}_{H_s} \equiv \gamma(q_3\,\bar{t} \leftrightarrow \ell\,N_1)$ with the Higgs exchanged in the s-channel, and of the sum of the t- and u-channel scatterings $\gamma^{\rm top}_{H_{t+u}} \equiv \gamma(q_3\,N_1 \leftrightarrow \ell\,t) + \gamma(\bar{t}\,N_1 \leftrightarrow \ell\,\bar{q}_3)$ shows that the only corrections appearing at low temperatures, and thus more relevant, are for $\gamma^{\rm top}_{H_{t+u}}$ (see Figure 7.1 in Ref. [109]). They reduce the scattering rates and suppress the related washouts. This peculiar situation arises from the fact that in the zero temperature limit there is a large enhancement $\sim \ln(M_{N_1}/m_H)$ from the quasi-massless Higgs exchanged in the t- and u-channels, which disappears when the Higgs thermal mass $m_H(T) \sim T \sim M_{N_1}$ is included.

(iii) $\Delta L = 1$ *scatterings with the gauge bosons.* Here the inclusion of thermal masses is required to avoid IR divergences that would arise when massless ℓ (and H) states are exchanged in the t- and u-channels. A naive use of some cutoff for the phase space integrals to control the IR divergences can yield incorrect estimates of the gauge bosons scattering rates [49]) and would be particularly problematic at low temperatures, where gauge bosons scatterings dominate over top-quark scatterings.

CP asymmetries

CP asymmetries arise from the interference of tree level and one-loop amplitudes when the couplings involved have complex phases and the loop diagrams have an absorptive part. This last requirement is satisfied whenever the loop diagram can be cut in such a way that the particles in the cut lines can be produced on shell. For the CP asymmetry in decay (at zero temperature) this is guaranteed by the fact that the Higgs and the lepton final states coincide with the states circulating in the loops. However, in the hot plasma in which N_1 decays occur, the Higgs and the lepton doublets are in thermal equilibrium and their interactions with the background introduce in the CP asymmetries a dependence on the temperature $\epsilon \to \epsilon(T)$ that arises from various effects:

- *i)* Absorption and re-emission of the loop particles by the medium require the use of finite temperature propagators.
- *ii)* Stimulation of decays into bosons and blocking of decays into fermions in the dense background require proper modification of the final states density distributions.
- *iii)* Thermal motion of the decaying N's with respect to the background breaks the Lorentz symmetry and affects the evaluation of the CP asymmetries.
- *iv)* Thermal masses should be included in the finite temperature resummed propagators, and they also modify the fermion and boson dispersion relations. Their inclusion yields the most significant modifications to the zero temperature results for the CP asymmetries.

The first three effects were investigated in Ref. [48] while the effects of thermal masses was included in Ref. [49]. In principle, at finite temperature, there are additional effects related to new cuts that involve the heavy $N_{2,3}$ neutrino lines. These new cuts appear because the heavy particles in the loops may absorb energy from the plasma and go on-shell. However, for hierarchical spectrum, $M_{2,3} \gg M_1$, the related effects are Boltzmann suppressed by $\exp(-M_{2,3}/T)$ that at $T \sim M_1$ is a tiny factor. For a non-hierarchical spectrum, the effect of these new cuts can however be sizable. A detail study can be found in Ref. [121].

Propagators and statistical distributions

Particle propagators at finite temperature are computed in the real time formalism of thermal field theory [122, 123]. In this formalism, ghost fields dual to each of the physical fields have to be introduced, and consequently the thermal propagators have 2×2 matrix structures. For the one-loop computations of the absorptive parts of the Feynman diagrams, the relevant propagator components are just those of the physical lepton and Higgs fields. The usual zero temperature propagators $-iS^0_\ell(p, m_\ell) = (\not p - m_\ell + i0^+)^{-1}$ and $-iD^0_H(p, m_H) = (p^2 - m_H^2 + i0^+)^{-1}$ acquire an additive term that is proportional to the particle density distribution $n_{\ell,H} = [\exp(E_{\ell,H}/T) \pm 1]^{-1}$:

$$\delta S_\ell(T) = -2\pi\, n_\ell\, (\not p - m_\ell)\, \delta(p^2 - m_\ell^2)\,, \tag{1.53}$$

$$\delta D_H(T) = +2\pi\, n_H\, \delta(p^2 - m_H^2)\,. \tag{1.54}$$

For the fermionic thermal propagators, there are other higher order corrections (see Ref. [49]). Unlike the case of bosons, the interactions of the fermions with the thermal bath lead to two different types of excitations with different dispersion relations, that are generally referred to as 'particles' and 'holes' [49]. The contributions of these two fermionic modes were studied in Refs. [124, 125, 126] where it was argued that in the strong washout regime they could give non-negligible effects [126]. The leading effects in *i)* are proportional to the factor $-n_\ell + n_H - 2n_\ell\, n_H$ that vanishes when the final states thermal masses are neglected, because the Bose-Einstein and Fermi-Dirac statistical distributions depend on the same argument, $E_\ell = E_H = M_1/2$. As a consequence, the thermal corrections to the fermion and boson propagators (n_ℓ and n_H) and the product of the two thermal corrections ($n_\ell\, n_H$) cancel each other. This was interpreted as a complete compensation between stimulated emission and Pauli blocking. As regards the effects in *ii)*, they lead to overall factors that cancel between numerator and denominator in the expression for the CP asymmetry[12]. More recently, on the basis of a first principle derivation of the CP asymmetry within a quantum BE approach (see Section 1.4.5) it has been claimed that the statistical factor induced by thermal loops is instead $-n_\ell + n_H$, which does not vanish even in the massless approximation. This would result in a further enhancement in the CP asymmetry from the thermal effects [127].

Particle motion

Given that the decaying particle N_1 is moving with respect to the background (with velocity $\vec\beta$) the fermionic decay products are preferentially emitted in the direction anti-parallel to the plasma velocity (for which the thermal distribution is less occupied), while the bosonic ones are emitted preferentially in

[12] A similar cancellation holds also in the supersymmetric case. However, because of the presence of the superpartners $\tilde\ell, \tilde H$ both as final states and in the loops, the cancellation is more subtle and it involves a compensation between the two types of corrections *i)* and *ii)*. We refer to Ref. [48] for details.

the forward direction (for which stimulated emission is more effective). Particle motion then induces an angular dependence in the decay distribution at order $\mathcal{O}(\beta)$. In the total decay rate the $\mathcal{O}(\beta)$ anisotropy effect is integrated out, and only $\mathcal{O}(\beta^2)$ effects remain [48]. Therefore, while accounting for thermal motion does modify the zero temperature results, these corrections are numerically small [48, 49] and generally negligible.

Thermal masses

When the finite values of the light particle thermal masses are taken into account, the arguments of the Bose-Einstein and Fermi-Dirac statistical distributions are different. It is a good approximation [49] to use for the particle energies $E_{\ell,H} = M_1/2 \mp (m_H^2 - m_\ell^2)/2M_1$. Since now $E_\ell \neq E_H$, the prefactor $-n_\ell + n_H - 2n_\ell n_H$ that multiplies the thermal corrections does not vanish anymore, and sizable corrections become possible. The most relevant effect is that the CP asymmetry vanishes when, as the temperature increases, the sum of the light particles thermal masses approaches M_1 [49]. This is not surprising, since the particles in the final state coincide with the particles in the loop, and therefore when the decay becomes kinematically forbidden, also the particles in the loop cannot go on the mass shell. When the temperature is large enough that $m_H(T) > m_\ell(T) + M_1$ the Higgs can decay, and then there is a new source of lepton number asymmetry associated with $H \to \ell N_1$. The CP asymmetry in Higgs decays ϵ_H can be up to one order of magnitude larger than the CP asymmetry in N_1 decays [49]. While this could represent a dramatic enhancement of the CP asymmetry, ϵ_H is non-vanishing only at temperatures $T \gtrsim T_H \sim 5M_1$, when the kinematic condition for its decays is satisfied. Therefore, in the strong washout regime, no trace of this effect survives. On the other hand, rather large couplings are required in order that Higgs decays can occur before the phase space closes: the decay rate can attain thermal equilibrium only when $\tilde{m} \gtrsim (T_H/M_1)^2 m_* \gg m_*$, and therefore, in the weak washout regime ($\tilde{m} \lesssim m_*$), these decays always remain strongly out of equilibrium. This means that only a small fraction of the Higgs particles have actually time to decay, and the lepton-asymmetry generated in this way is accordingly suppressed.

In summary, while the corrections to the CP asymmetries can be significant at $T \gtrsim M_1$ (and quite large at $T \gg M_1$ for Higgs decays), in the low temperature regime, where the precise value of ϵ plays a fundamental role in determining the final value of the baryon asymmetry, there are almost no effects, and the zero temperature results still give a reliable approximation.

1.4.4 Decays of the heavier right-handed neutrinos

In leptogenesis studies, the effects of $N_{2,3}$ are often neglected, which in many cases is not a good approximation. This is obvious for example when N_1 dynamics is irrelevant for leptogenesis: $\epsilon_1 \ll 10^{-6}$ cannot provide enough CP asymmetry to account for baryogenesis, and $\tilde{m}_1 \ll m_*$ implies that N_1 washout effects are negligible. It is then clear that any asymmetry generated in $N_{2,3}$ decays can survive, and becomes crucial for the success of leptogenesis. Another case in which it is intuitively clear that $N_{2,3}$ effects can be important, is when the RH neutrino spectrum is compact, which means that $M_{2,3}$ have values within a factor of a few from M_1. Then N_1 and $N_{2,3}$ contributions to leptogenesis can be equally important and must be summed up. A model with compact RH neutrino spectrum in which $N_{2,3}$ dynamics is of crucial importance was recently discussed in Ref. [128].

It is less obvious that $N_{2,3}$ effects can also be important for a hierarchical RH spectrum and when N_1 is strongly coupled. This can happen because decoherence effects related to N_1-interactions can project the asymmetry generated in $N_{2,3}$ decays onto a flavour direction that remains protected against

N_1 washouts [67, 129, 130, 131]. Let us illustrate this with an example. Let us assume that a sizable asymmetry is generated in N_2 decays, while N_1 leptogenesis is inefficient and fails, that is:

$$\tilde{m}_2 \not\gg m_*, \qquad \tilde{m}_1 \gg m_*. \tag{1.55}$$

Assuming also a strong hierarchy and that leptogenesis occurs thermally guarantees that [131]:

$$n_{N_1}(T \sim M_2) \approx 0, \quad n_{N_2}(T \sim M_1) \approx 0. \tag{1.56}$$

Thus, the dynamics of N_2 and N_1 are decoupled: there are neither N_1-related washout effects during N_2 leptogenesis, nor N_2-related washout effects during N_1 leptogenesis. The N_2 decays into a combination of lepton doublets that we denote by ℓ_2:

$$|\ell_2\rangle = (Y^\dagger Y)_{22}^{-1/2} \sum_\alpha Y_{\alpha 2} |\ell_\alpha\rangle. \tag{1.57}$$

The second condition in eq. (1.55) implies that already at $T \gtrsim M_1$ the N_1-Yukawa interactions are sufficiently fast to quickly destroy the coherence of ℓ_2. Then a statistical mixture of ℓ_1 and of the state orthogonal to ℓ_1 builds up, and it can be described by a suitable diagonal density matrix. Let us consider the simple case where both N_2 and N_1 decay at $T \gtrsim 10^{12}$ GeV, so that flavour effects are irrelevant. A convenient choice for an orthogonal basis for the lepton doublets is $(\ell_1, \ell_0, \ell_0')$ where, without loss of generality, ℓ_0' satisfies $\langle \ell_0' | \ell_2 \rangle = 0$. Then the asymmetry ΔY_{ℓ_2} produced in N_2 decays decomposes into two components:

$$\Delta Y_{\ell_0} = c^2\, \Delta Y_{\ell_2}, \qquad \Delta Y_{\ell_1} = s^2\, \Delta Y_{\ell_2}, \tag{1.58}$$

where $c^2 \equiv |\langle \ell_0 | \ell_2 \rangle|^2$ and $s^2 = 1 - c^2$. The crucial point here is that we expect, in general, $c^2 \neq 0$ and, since $\langle \ell_0 | \ell_1 \rangle = 0$, ΔY_{ℓ_0} is protected against N_1 washout. Consequently, a finite part of the asymmetry ΔY_{ℓ_2} from N_2 decays survives through N_1 leptogenesis. A more detailed analysis [131] finds that ΔY_{ℓ_1} is not entirely washed out, resulting in the final lepton asymmetry $Y_{\Delta L} = (3/2)\Delta Y_{\ell_0} = (3/2) c^2\, \Delta Y_{\ell_2}$.

For 10^9 GeV$\lesssim M_1 \lesssim 10^{12}$ GeV, flavour issues modify the quantitative details, but the qualitative picture, and in particular the survival of a finite part of ΔY_{ℓ_2}, still holds. In contrast, for $M_1 \lesssim 10^9$ GeV, the full flavour basis $(\ell_e, \ell_\mu, \ell_\tau)$ is resolved and thus there are no directions in flavour space where an asymmetry is protected, so that Y_{ℓ_2} can be erased entirely. A dedicated study in which the various flavour regimes for $N_{1,2,3}$ decays are considered can be found in Ref. [132].

In conclusion $N_{2,3}$ leptogenesis cannot be ignored, unless one of the following conditions holds:

1. The reheat temperature is below M_2.
2. The asymmetries and/or the washout factors vanish, $\epsilon_{N_2}\eta_2 \approx 0$ and $\epsilon_{N_3}\eta_3 \approx 0$.
3. N_1-related washout is still significant at $T \lesssim 10^9$ GeV.

1.4.5 Quantum Boltzmann equations

So far we have analyzed the leptogenesis dynamics by adopting the classical BE of motion. An interesting question which has attracted some attention recently [133, 85, 134, 135, 136, 127, 137, 94, 138, 121, 139] is under which circumstances the classical BE can be safely applied to get reliable results and, conversely, when a more rigorous quantum approach is needed. Quantum BE are obtained starting from the non-equilibrium quantum field theory based on the Closed Time-Path (CTP) formulation

[140]. Both, CP violation from wave-function and vertex corrections are incorporated. Unitarity issues are resolved and an accurate account of all quantum-statistical effects on the asymmetry is made. Moreover, the formulation in terms of Green functions bears the potential of incorporating corrections from Thermal Field Theory within the CTP formalism.

In the CTP formalism, particle number densities are replaced by Green's functions obeying a set of equations which, under some assumptions, can be reduced to a set of kinetic equations describing the evolution of the lepton asymmetry and the RH neutrinos. These kinetic equations are non-Markovian and present memory effects. In other words, differently from the classical approach where every scattering in the plasma is independent from the previous one, the particle abundances at a given time depend upon the history of the system. The more familiar energy-conserving delta functions are replaced by retarded time integrals of time-dependent kernels and cosine functions whose arguments are the energy involved in the various processes. Therefore, the non-Markovian kinetic equations include the contribution of coherent processes throughout the history of the kernels and the relaxation times are expected to be typically longer than the one dictated by the classical approach.

If the time range of the kernels are shorter than the relaxation time of the particles abundances, the solutions to the quantum and the classical BE differ only by terms of the order of the ratio of the timescale of the kernel to the relaxation timescale of the distribution. In thermal leptogenesis this is typically the case. However, there are situations where this does not happen. For instance, in the case of resonant leptogenesis, two RH (s)neutrinos N_1 and N_2 are almost degenerate in mass and the CP asymmetry from the decay of the first RH neutrino N_1 is resonantly enhanced if the mass difference $\Delta M = (M_2 - M_1)$ is of the order of the decay rate of the second RH neutrino Γ_{N_2} . The typical timescale to build up coherently the CP asymmetry is of the order of $1/\Delta M$, which can be larger than the timescale $\sim 1/\Gamma_{N_1}$ for the change of the abundance of the N_1's.

Since we need the time evolution of the particle asymmetries with definite initial conditions and not simply the transition amplitude of particle reactions, the ordinary equilibrium quantum field theory at finite temperature is not the appropriate tool. The most appropriate extension of the field theory to deal with non-equilibrium phenomena amounts to generalizing the time contour of integration to a closed time-path. More precisely, the time integration contour is deformed to run from t_0 to $+\infty$ and back to t_0. The CTP formalism is a powerful Green's function formulation for describing non-equilibrium phenomena in field theory. It allows to describe phase-transition phenomena and to obtain a self-consistent set of quantum BE. The formalism yields various quantum averages of operators evaluated in the in-state without specifying the out-state. On the contrary, the ordinary quantum field theory yields quantum averages of the operators evaluated with an in-state at one end and an out-state at the other.

For example, because of the time-contour deformation, the partition function in the in-in formalism for a complex scalar field is defined to be

$$\begin{aligned} Z\left[J, J^\dagger\right] &= \mathrm{Tr}\left[\mathcal{T}\left(\exp\left[i\int_C \left(J(x)\phi(x) + J^\dagger(x)\phi^\dagger(x)\right) d^4x\right]\right)\rho\right] \\ &= \mathrm{Tr}\left[\mathcal{T}_+\left(\exp\left[i\int \left(J_+(x)\phi_+(x) + J_+^\dagger(x)\phi_+^\dagger(x)\right) d^4x\right]\right)\right. \\ &\times \left.\mathcal{T}_-\left(\exp\left[-i\int \left(J_-(x)\phi_-(x) + J_-^\dagger(x)\phi_-^\dagger(x)\right) d^4x\right]\right)\rho\right], \end{aligned} \quad (1.59)$$

where C in the integral denotes that the time integration contour runs from t_0 to plus infinity and then back to t_0 again. The symbol ρ represents the initial density matrix and the fields are in the

Heisenberg picture and defined on this closed time-contour (plus and minus subscripts refer to the positive and negative directional branches of the time path, respectively). The time-ordering operator along the path is the standard one ($\mathcal{T}_+$) on the positive branch, and the anti-time-ordering ($\mathcal{T}_-$) on the negative branch. As with the Euclidean-time formulation, scalar (fermionic) fields ϕ are still periodic (anti-periodic) in time, but with $\phi(t,\vec{x}) = \phi(t-i\beta,\vec{x})$, $\beta = 1/T$. The temperature T appears due to boundary condition, and time is now explicitly present in the integration contour.

We must now identify field variables with arguments on the positive or negative directional branches of the time path. This doubling of field variables leads to six different real-time propagators on the contour. These six propagators are not independent, but using all of them simplifies the notation. For a generic charged scalar field ϕ they are defined as

$$\begin{aligned}
G^{>}_{\phi}(x,y) &= -G^{-+}_{\phi}(x,y) = -i\langle\phi(x)\phi^{\dagger}(y)\rangle, \\
G^{<}_{\phi}(x,y) &= -G^{+-}_{\phi}(x,y) = -i\langle\phi^{\dagger}(y)\phi(x)\rangle, \\
G^{t}_{\phi}(x,y) &= G^{++}_{\phi}(x,y) = \theta(x,y)G^{>}_{\phi}(x,y) + \theta(y,x)G^{<}_{\phi}(x,y), \\
G^{\bar{t}}_{\phi}(x,y) &= G^{--}_{\phi}(x,y) = \theta(y,x)G^{>}_{\phi}(x,y) + \theta(x,y)G^{<}_{\phi}(x,y), \\
G^{r}_{\phi}(x,y) &= G^{t}_{\phi} - G^{<}_{\phi} = G^{>}_{\phi} - G^{\bar{t}}_{\phi}, \quad G^{a}_{\phi}(x,y) = G^{t}_{\phi} - G^{>}_{\phi} = G^{<}_{\phi} - G^{\bar{t}}_{\phi},
\end{aligned} \tag{1.60}$$

where the last two Green's functions are the retarded and advanced Green's functions respectively and $\theta(x,y) \equiv \theta(t_x - t_y)$ is the step function.

For a generic fermion field ψ the six different propagators are analogously defined as

$$\begin{aligned}
G^{>}_{\psi}(x,y) &= -G^{-+}_{\psi}(x,y) = -i\langle\psi(x)\overline{\psi}(y)\rangle, \\
G^{<}_{\psi}(x,y) &= -G^{+-}_{\psi}(x,y) = +i\langle\overline{\psi}(y)\psi(x)\rangle, \\
G^{t}_{\psi}(x,y) &= G^{++}_{\psi}(x,y) = \theta(x,y)G^{>}_{\psi}(x,y) + \theta(y,x)G^{<}_{\psi}(x,y), \\
G^{\bar{t}}_{\psi}(x,y) &= G^{--}_{\psi}(x,y) = \theta(y,x)G^{>}_{\psi}(x,y) + \theta(x,y)G^{<}_{\psi}(x,y), \\
G^{r}_{\psi}(x,y) &= G^{t}_{\psi} - G^{<}_{\psi} = G^{>}_{\psi} - G^{\bar{t}}_{\psi}, \quad G^{a}_{\psi}(x,y) = G^{t}_{\psi} - G^{>}_{\psi} = G^{<}_{\psi} - G^{\bar{t}}_{\psi}.
\end{aligned} \tag{1.61}$$

From the definitions of the Green's functions, one can see that the hermiticity properties

$$\left(i\gamma^0 G_{\psi}(x,y)\right)^{\dagger} = i\gamma^0 G_{\psi}(y,x), \;\; (iG_{\phi}(x,y))^{\dagger} = iG_{\phi}(y,x), \tag{1.62}$$

are satisfied. For interacting systems, whether in equilibrium or not, one must define and calculate self-energy functions. Again, there are six of them: Σ^t, $\Sigma^{\bar{t}}$, $\Sigma^<$, $\Sigma^>$, Σ^r and Σ^a. The same relationships exist among them as for the Green's functions in (1.60) and (1.61), such as

$$\Sigma^r = \Sigma^t - \Sigma^< = \Sigma^> - \Sigma^{\bar{t}}, \quad \Sigma^a = \Sigma^t - \Sigma^> = \Sigma^< - \Sigma^{\bar{t}}. \tag{1.63}$$

The self-energies are incorporated into the Green's functions through the use of Dyson's equations. A useful notation may be introduced which expresses four of the six Green's functions as the elements of two-by-two matrices

$$\widetilde{G} = \begin{pmatrix} G^t & \pm G^< \\ G^> & -G^{\bar{t}} \end{pmatrix}, \quad \widetilde{\Sigma} = \begin{pmatrix} \Sigma^t & \pm\Sigma^< \\ \Sigma^> & -\Sigma^{\bar{t}} \end{pmatrix}, \tag{1.64}$$

where the upper signs refer to the bosonic case and the lower signs to the fermionic case. For systems either in equilibrium or in non-equilibrium, Dyson's equation is most easily expressed by using the matrix notation

$$\widetilde{G}(x,y) = \widetilde{G}^0(x,y) + \int d^4z_1 \int d^4z_2\, \widetilde{G}^0(x,z_1)\widetilde{\Sigma}(z_1,z_2)\widetilde{G}(z_2,y), \tag{1.65}$$

where the superscript "0" on the Green's functions means to use those for noninteracting system. It is useful to notice that Dyson's equation can be written in an alternative form, instead of (1.65), with $\widetilde{G}^0$ on the right in the interaction terms,

$$\widetilde{G}(x,y) = \widetilde{G}^0(x,y) + \int d^4z_3 \int d^4z_4\, \widetilde{G}(x,z_3)\widetilde{\Sigma}(z_3,z_4)\widetilde{G}^0(z_4,y). \tag{1.66}$$

Eqs. (1.65) and (1.66) are the starting points to derive the quantum BE describing the time evolution of the CP-violating particle density asymmetries.

To proceed, one has to choose a form for the propagators. For a generic fermion ψ (and similarly for scalars) one may adopt the real-time propagator in the form $G^t_\psi(\mathbf{k}, t_x - t_y)$ in terms of the spectral function $\rho_\psi(\mathbf{k}, k_0)$

$$\begin{aligned} G^t_\psi(\mathbf{k}, t_x - t_y) &= \int_{-\infty}^{+\infty} \frac{dk^0}{2\pi}\, e^{-ik^0(t_x-t_y)}\, \rho_\psi(\mathbf{k},k^0) \\ &\times \left\{\left[1 - f_\psi(k^0)\right]\theta(t_x-t_y) - f_\psi(k^0)\theta(t_y-t_x)\right\}, \end{aligned} \tag{1.67}$$

where $f_\psi(k^0)$ represents the fermion distribution function. Again, particles must be substituted by quasiparticles, dressed propagators are to be adopted and self-energy corrections to the propagator modify the dispersion relations by introducing a finite width $\Gamma_\psi(k)$. For a fermion with chiral mass m_ψ, one may safely choose

$$\rho_\psi(\mathbf{k},k^0) = i\,(\not{k} + m_\psi)\left[\frac{1}{(k^0+i\epsilon+i\Gamma_\psi)^2 - \omega^2_\psi(k)} - \frac{1}{(k^0-i\epsilon-i\Gamma_\psi)^2 - \omega^2_\psi(k)}\right], \tag{1.68}$$

where $\omega^2_\psi(k) = \mathbf{k}^2 + M^2_\psi(T)$ and $M_\psi(T)$ is the effective thermal mass of the fermion in the plasma (not a chiral mass). Performing the integration over k^0 and picking up the poles of the spectral function (which is valid for quasi-particles in equilibrium or very close to equilibrium), one gets

$$\begin{aligned} G^>_\psi(\mathbf{k}, t_x - t_y) &= -\frac{i}{2\omega_\psi}\Big\{(\not{k}+m_\psi)\left[1 - f_\psi(\omega_\psi - i\Gamma_\psi)\right]\, e^{-i(\omega_\psi - i\Gamma_\psi)(t_x-t_y)} \\ &+ \gamma^0(\not{k}-m_\psi)\gamma^0\overline{f}_\psi(\omega_\psi + i\Gamma_\psi)\, e^{i(\omega_\psi+i\Gamma_\psi)(t_x-t_y)}\Big\}, \\ G^<_\psi(\mathbf{k}, t_y - t_x) &= \frac{i}{2\omega_\psi}\Big\{(\not{k}+m_\psi)\, f_\psi(\omega_\psi + i\Gamma_\psi)\, e^{-i(\omega_\psi - i\Gamma_\psi)(t_x-t_y)} \\ &+ \gamma^0(\not{k}-m_\psi)\gamma^0\left[1 - \overline{f}_\psi(\omega_\psi - i\Gamma_\psi)\right]\, e^{i(\omega_\psi+i\Gamma_\psi)(t_x-t_y)}\Big\}, \end{aligned} \tag{1.69}$$

where $k^0 = \omega_\psi$ and $f_\psi, \overline{f}_\psi$ denote the distribution function of the fermion particles and antiparticles, respectively. The expressions (1.69) are valid for $t_x - t_y > 0$.

The above definitions hold for the lepton doublets (after inserting the chiral LH projector P_L), as well as for the Majorana RH neutrinos, for which one has to assume identical particle and antiparticle distribution functions and insert the inverse of the charge conjugation matrix C in the dispersion relation.

To elucidate further the impact of the CTP approach and to see under which conditions one can obtain the classical BE from the quantum ones, one may consider the dynamics of the lightest RH neutrino N_1. To find its quantum BE we start from eq. (1.65) for the Green's function $G^{<}_{N_1}$ of the RH neutrino N_1

$$
\begin{aligned}
\left(i \overrightarrow{\not\partial}_x - M_1\right) G^{<}_{N_1}(x,y) &= -\int d^4z \left[-\Sigma^{t}_{N_1}(x,z)G^{<}_{N_1}(z,y) + \Sigma^{<}_{N_1}(x,z)G^{\bar{t}}_{N_1}(z,y)\right] \\
&= \int d^3z \int_0^t dt_z \left[\Sigma^{>}_{N_1}(x,z)G^{<}_{N_1}(z,y) - \Sigma^{<}_{N_1}(x,z)G^{>}_{N_1}(z,y)\right].
\end{aligned}
\tag{1.70}
$$

Adopting the corresponding form for the RH neutrino propagator and the center-of-mass coordinates

$$
X \equiv (t, \vec{X}) \equiv \frac{1}{2}(x+y), \quad (\tau, \vec{r}) \equiv x - y, \tag{1.71}
$$

one ends up with the following equation

$$
\begin{aligned}
\frac{\partial f_{N_1}(\mathbf{k},t)}{\partial t} &= -2\int_0^t dt_z \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2\omega_\ell(\mathbf{p})}\frac{1}{2\omega_H(\mathbf{k}-\mathbf{p})}\frac{1}{\omega_{N_1}(\mathbf{k})}\left|\mathcal{M}(N_1\to \ell H)\right|^2 \\
&\quad \times [f_{N_1}(\mathbf{k},t)(1-f_\ell(\mathbf{p},t))(1+f_H(\mathbf{k}-\mathbf{p},t)) \\
&\quad - f_\ell(\mathbf{p},t)f_H(\mathbf{k}-\mathbf{p},t)(1-f_{N_1}(\mathbf{k},t))] \\
&\quad \times \cos\left[(\omega_{N_1}(\mathbf{k}) - \omega_\ell(\mathbf{p}) - \omega_H(\mathbf{k}-\mathbf{p}))\,(t-t_z)\right] \\
&\simeq -2\int_0^t dt_z \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2\omega_\ell(\mathbf{p})}\frac{1}{2\omega_H(\mathbf{k}-\mathbf{p})}\frac{1}{\omega_{N_1}(\mathbf{k})}\left|\mathcal{M}(N_1\to \ell H)\right|^2 \\
&\quad \times \left(f_{N_1}(\mathbf{k},t) - f^{eq}_\ell(\mathbf{p})f^{eq}_H(\mathbf{k}-\mathbf{p})\right) \\
&\quad \times \cos\left[(\omega_{N_1}(\mathbf{k}) - \omega_\ell(\mathbf{p}) - \omega_H(\mathbf{k}-\mathbf{p}))\,(t-t_z)\right].
\end{aligned}
\tag{1.72}
$$

This equations holds under the assumption that the relaxation timescale for the distribution functions are longer than the timescale of the non-local kernels so that they can be extracted out of the time integral. This allows to think the distributions as functions of the center-of-mass time only. We have set to zero the damping rates of the particles in eq. (1.69) and retained only those cosines giving rise to energy delta functions that can be satisfied. Under these assumptions, the distribution function may be taken out of the time integral, leading – at large times – to the so-called Markovian description. The kinetic equation (1.72) has an obvious interpretation in terms of gain minus loss processes, but the retarded time integral and the cosine function replace the familiar energy-conserving delta functions. In the second passage, we have also made the usual assumption that all distribution functions are smaller than unity and that those of the Higgs and lepton doublets are in equilibrium and much smaller than unity, $f_\ell f_H \simeq f^{eq}_\ell f^{eq}_H$. Elastic scatterings are typically fast enough to keep kinetic equilibrium. For any distribution function we may write $f = (n/n^{eq})f^{eq}$, where n denotes the total number density. Therefore, eq. (1.72) can be re-written as

$$
\begin{aligned}
\frac{\partial n_{N_1}}{\partial t} &= -\langle \Gamma_{N_1}(t)\rangle n_{N_1} + \langle \widetilde{\Gamma}_{N_1}(t)\rangle n_{N_1}^{eq}, \\
\langle \Gamma_{N_1}(t)\rangle &= \int_0^t dt_z \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{eq}}{n_{N_1}^{eq}}\, \Gamma_{N_1}(t), \\
\Gamma_{N_1}(t) &= 2\int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\mathcal{M}(N_1\to \ell H)|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} \cos\left[(\omega_{N_1}-\omega_\ell-\omega_H)(t-t_z)\right], \\
\langle \widetilde{\Gamma}_{N_1}(t)\rangle &= \int_0^t dt_z \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{eq}}{n_{N_1}^{eq}}\, \widetilde{\Gamma}_{N_1}(t), \\
\widetilde{\Gamma}_{N_1}(t) &= 2\int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{f_\ell^{eq} f_H^{eq}}{f_{N_1}^{eq}} \frac{|\mathcal{M}(N_1\to \ell H)|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} \cos\left[(\omega_{N_1}-\omega_\ell-\omega_H)(t-t_z)\right],
\end{aligned}
\tag{1.73}
$$

where $\langle \Gamma_{N_1}(t)\rangle$ is the time-dependent thermal average of the Lorentz-dilated decay width. Integrating over large times, $t\to\infty$, thereby replacing the cosines by energy-conserving delta functions

$$
\int_0^\infty dt_z\, \cos\left[(\omega_{N_1}-\omega_\ell-\omega_H)(t-t_z)\right] = \pi\delta\left(\omega_{N_1}-\omega_\ell-\omega_H\right), \tag{1.74}
$$

we find that the two averaged rates $\langle \Gamma_{N_1}\rangle$ and $\langle \widetilde{\Gamma}_{N_1}\rangle$ coincide and we recover the usual classical BE for the RH distribution function

$$
\begin{aligned}
\frac{\partial n_{N_1}}{\partial t} &= -\langle \Gamma_{N_1}\rangle \left(n_{N_1} - n_{N_1}^{eq}\right), \\
\langle \Gamma_{N_1}\rangle &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{eq}}{n_{N_1}^{eq}} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\mathcal{M}(N_1\to \ell H)|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} (2\pi)\delta\left(\omega_{N_1}-\omega_\ell-\omega_H\right).
\end{aligned}
\tag{1.75}
$$

Taking the time interval to infinity, namely implementing Fermi's golden rule, results in neglecting memory effects, which in turn results only in on-shell processes contributing to the rate equation. The main difference between the classical and the quantum BE can be traced to memory effects and to the fact that the time evolution of the distribution function is non-Markovian. The memory of the past time evolution translates into off-shell processes.

Similarly, one can show that the equation obeyed by the asymmetry reads

$$
\begin{aligned}
\frac{\partial n_{\Delta L_\alpha}(X)}{\partial t} &= -\int d^3z \int_0^t dt_z\, \mathrm{Tr}\Big[\Sigma_{\ell_\alpha}^{>}(X,z) G_{\ell_\alpha}^{<}(z,X) - G_{\ell_\alpha}^{>}(X,z)\Sigma_{\ell_\alpha}^{<}(z,X) \\
&+ G_{\ell_\alpha}^{<}(X,z)\Sigma_{\ell_\alpha}^{>}(z,X) - \Sigma_{\ell_\alpha}^{<}(X,z) G_{\ell_\alpha}^{>}(z,X)\Big].
\end{aligned}
\tag{1.76}
$$

Proceeding as for the RH neutrino equation one finds (including for the moment only the 1-loop wave contribution to the CP asymmetry ϵ_w^α)

$$
\frac{\partial n_{\Delta L_\alpha}}{\partial t} = \epsilon_w^\alpha(t)\langle \Gamma_{N_1}\rangle \left(n_{N_1} - n_{N_1}^{eq}\right),
$$

$$
\begin{aligned}
\epsilon_w^\alpha(t) &= -\frac{4}{\langle\Gamma_{N_1}\rangle}\sum_{\beta=1}^{3}\mathrm{Im}\left(Y_{1\alpha}Y_{1\beta}Y^\dagger_{\beta 2}Y^\dagger_{\alpha 2}\right)\\
&\times \int_0^t dt_z\int_0^{t_z}dt_2\int_0^{t_2}dt_1 e^{-\Gamma_{N_2}(t_z-t_2)}e^{-\left(\Gamma_{\ell_\beta}+\Gamma_H\right)(t_2-t_1)}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{f^{eq}_{N_1}}{n^{eq}_{N_1}}\\
&\times \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1-f^{eq}_{\ell_\beta}(\mathbf{p})+f^{eq}_H(\mathbf{k}-\mathbf{p})}{2\omega_{\ell_\beta}(\mathbf{p})2\omega_H(\mathbf{k}-\mathbf{p})\omega_{N_1}(\mathbf{k})}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1-f^{eq}_{\ell_\alpha}(\mathbf{q})+f^{eq}_H(\mathbf{k}-\mathbf{q})}{2\overline{\omega}_{\ell_\alpha}(\mathbf{q})2\overline{\omega}_H(\mathbf{k}-\mathbf{q})\omega_{N_2}(\mathbf{k})}\\
&\times \sin\left(\omega_{N_1}(t-t_1)+\left(\omega_{\ell_\beta}+\omega_H\right)(t_1-t_2)+\omega_{N_2}(t_2-t_z)+\left(\overline{\omega}_{\ell_\alpha}+\overline{\omega}_H\right)(t_z-t)\right)\\
&\times \mathrm{Tr}\left(M_1P_L\,\not{p}M_2\,\not{q}\right), \qquad (1.77)
\end{aligned}
$$

where, to avoid double counting, we have not inserted the decay rates in the propagators of the initial and final states and, for simplicity, we have assumed that the damping rates of the lepton doublets and the Higgs field are constant in time. This should be a good approximation as the damping rate are to be computed for momenta of order of the mass of the RH neutrinos. As expected from first principles, we find that the CP asymmetry is a function of time and its value at a given instant depends upon the previous history of the system.

Performing the time integrals and retaining only those pieces which eventually give rise to energy-conserving delta functions in the Markovian limit, we obtain

$$
\begin{aligned}
\epsilon_w^\alpha(t) &= -\frac{4}{\langle\Gamma_{N_1}\rangle}\sum_{\beta=1}^{3}\mathrm{Im}\left(Y_{1\alpha}Y_{1\beta}Y^\dagger_{\beta 2}Y^\dagger_{\alpha 2}\right)\\
&\times \int_0^t dt_z\frac{\cos\left[\left(\omega_{N_1}-\overline{\omega}_{\ell_\alpha}-\overline{\omega}_H\right)(t-t_z)\right]}{\left(\Gamma^2_{N_2}+(\omega_{N_2}-\omega_{N_1})^2\right)\left((\Gamma_{\ell_\beta}+\Gamma_H)^2+(\omega_{N_1}-\omega_{\ell_\beta}-\omega_H)^2\right)}\\
&\times \int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{f^{eq}_{N_1}}{n^{eq}_{N_1}}(\Gamma_{\ell_\beta}+\Gamma_H)\Bigg(2\left(\omega_{N_2}-\omega_{N_1}\right)\sin^2\left[\frac{(\omega_{N_2}-\omega_{N_1})t_z}{2}\right]\\
&- \Gamma_{N_2}\sin\left[(\omega_{N_2}-\omega_{N_1})t_z\right]\Bigg)\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1-f^{eq}_{\ell_\beta}(\mathbf{p})+f^{eq}_H(\mathbf{k}-\mathbf{p})}{2\omega_{\ell_\beta}(\mathbf{p})2\omega_H(\mathbf{k}-\mathbf{p})\omega_{N_1}(\mathbf{k})}\\
&\times \int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1-f^{eq}_{\ell_\alpha}(\mathbf{q})+f^{eq}_H(\mathbf{k}-\mathbf{q})}{2\overline{\omega}_{\ell_\alpha}(\mathbf{q})2\overline{\omega}_H(\mathbf{k}-\mathbf{q})\omega_{N_2}(\mathbf{k})}\,\mathrm{Tr}\left(M_1P_L\,\not{p}M_2\,\not{q}\right). \qquad (1.78)
\end{aligned}
$$

From this expression it is already manifest that the typical timescale for the building up of the coherent CP asymmetry depends crucially on the difference in energy of the two RH neutrinos.

If we now let the upper limit of the time integral to take large values, we neglect the memory effects, the CP asymmetry picks contribution only from the on-shell processes. Taking the damping rates of the lepton doublets equal for all the flavours and the RH neutrinos nearly at rest with respect to the thermal bath, the CP asymmetry reads (now summing over all flavour indices)

$$
\begin{aligned}
\epsilon_w(t) &\simeq -\frac{\mathrm{Im}\left(YY^\dagger\right)^2_{12}}{(YY^\dagger)_{11}(YY^\dagger)_{22}}\frac{M_1}{M_2}\Gamma_{N_2}\frac{1}{(\Delta M)^2+\Gamma^2_{N_2}}\\
&\times \left(2\,\Delta M\sin^2\left[\frac{\Delta M t}{2}\right]-\Gamma_{N_2}\sin\left[\Delta M t\right]\right), \qquad (1.79)
\end{aligned}
$$

where $\Delta M=(M_2-M_1)$. The CP asymmetry (1.79) is resonantly enhanced when $\Delta M\simeq\Gamma_{N_2}$ and at the resonance point it is given by

$$\epsilon_w(t) \simeq -\frac{1}{2}\frac{\mathrm{Im}\left(YY^\dagger\right)^2_{12}}{(YY^\dagger)_{11}\,(YY^\dagger)_{22}}\left(1-\sin\left[\Delta M t\right]-\cos\left[\Delta M t\right]\right), \tag{1.80}$$

The timescale for the building up of the CP asymmetry is $\sim 1/\Delta M$. The CP asymmetry grows starting from a vanishing value and, for $t \gg (\Delta M)^{-1}$, it averages to the constant standard value. This is true if the timescale for the other processes relevant for leptogenesis is larger than $\sim 1/\Delta M$. In other words, one may define an "average" CP asymmetry

$$\langle \epsilon_w \rangle = \frac{1}{\tau_\mathrm{p}}\int_{t-\tau_\mathrm{p}}^{t} dt'\,\epsilon^{\mathrm{W}}_{N_1}(t'), \tag{1.81}$$

where τ_p represents the typical timescale of the other processes relevant for leptogenesis, *e.g.* the $\Delta L = 1$ scatterings. If $\tau_\mathrm{p} \gg 1/\Delta M \sim \Gamma^{-1}_{N_2}$, the oscillating functions in (1.80) are averaged to zero and the average CP asymmetry is given by the value used in the literature. However, the expression (1.79) should be used when $\tau_\mathrm{p} \lesssim 1/\Delta M \sim \Gamma^{-1}_{N_2}$.

The fact that the CP asymmetry is a function of time is particularly relevant in the case in which the asymmetry is generated by the decays of two heavy states which are nearly degenerate in mass and oscillate into one another with a timescale given by the inverse of the mass difference and has been studied in Refs. [86, 141]. From eq. (1.79) it is manifest that the CP asymmetry itself oscillates with the very same timescale and such a dependence may or may not be neglected depending upon the rates of the other processes in the plasma. If $\Gamma_{N_1} \gtrsim \Gamma_{N_2}$, the time dependence of the CP asymmetry may not be neglected. The expression (1.79) can also be used, once it is divided by a factor 2 (because in the wave diagram also the charged states of Higgs and lepton doublets may propagate) and the limit $M_2 \gg M_1$ is taken, for the CP asymmetry contribution from the vertex diagram

$$\epsilon_v(t) \simeq -\frac{\mathrm{Im}\left(YY^\dagger\right)^2_{12}}{16\pi\,(YY^\dagger)_{11}}\frac{M_1}{M_2}\left(2\sin^2\left[\frac{M_2 t}{2}\right]-\frac{\Gamma_{N_2}}{M_2}\sin\left[M_2 t\right]\right). \tag{1.82}$$

In this case, the timescale for this CP asymmetry is $\sim M_2$ and much larger than any other timescale in the dynamics. Therefore, one can safely average over many oscillations, getting the expression present in the literature.

What discussed here provides only one example for which a quantum Boltzmann approach is needed. In general, the lesson is that quantum BE are relevant when the typical timescale in a quantum physical process, such as the timescale for the unflavour-to-flavour transition or the timescale to build up coherently the CP asymmetry (of the order of $1/\Delta M$) is larger than the timescale for the change of the abundances.

1.5 Supersymmetric Leptogenesis

1.5.1 What's new?

Supersymmetric leptogenesis constitutes a theoretically appealing generalization of leptogenesis for the following reason: while the SM equipped with the seesaw provides the simplest way to realize leptogenesis, such a framework is plagued by an unpleasant fine-tuning problem. For a non degenerate spectrum of heavy Majorana neutrinos, successful leptogenesis requires generically a scale for the singlet neutrino masses that is much larger than the EW scale [61] but at the quantum level the gap between these two scales becomes unstable. Low-energy supersymmetry (SUSY) can naturally stabilize the required hierarchy, and this provides a sound motivation for studying leptogenesis in the framework of the supersymmetrized version of the seesaw mechanism. Supersymmetric leptogenesis, however, introduces a certain conflict between the gravitino bound on the reheat temperature and the thermal production of the heavy singlets neutrinos [142, 143, 144, 145]. In this section, we will not be concerned with the gravitino problem, nor with its possible ways out but focus on the new features SUSY brings in for leptogenesis.

The supersymmetric type-I seesaw model is described by the superpotential of the Minimal Supersymmetric SM (MSSM) with the additional terms:

$$W = \frac{1}{2} M_{pq} N_p^c N_q^c + \lambda_{\alpha p} N_p^c \ell_\alpha H_u, \tag{1.83}$$

where $p, q = 1, 2, \ldots$ label the heavy singlet states in order of increasing mass, and $\alpha = e, \mu, \tau$ labels the lepton flavour. In eq. (1.83) ℓ, H_u and N^c, are respectively the chiral superfields for the lepton and the up-type Higgs $SU(2)_L$ doublets and for the heavy $SU(2)_L$ singlet neutrinos defined according to usual conventions in terms of their LH Weyl spinor components (for example the N^c supermultiplet has scalar component $\tilde{N}^*$ and fermion component N_L^c). Finally the $SU(2)_L$ index contraction is defined as $\ell_\alpha H_u = \epsilon_{\rho\sigma} \ell_\alpha^\rho H_u^\sigma$ with $\epsilon_{12} = +1$.

Originally, the issue of MSSM leptogenesis was approached in conjunction with SM leptogenesis [51, 49] as well as in dedicated studies [146, 147]. However in these first analysis, several features that are specific of SUSY in the high temperature regime relevant for leptogenesis, in which soft SUSY breaking parameters can be effectively set to zero, had been overlooked. In that case, in spite of the large amount of new reactions, the differences between SM and MSSM leptogenesis can be resumed by means of simple counting of a few numerical factors [148, 130, 109], like for example the number of relativistic degrees of freedom in the thermal bath, the number of loop diagrams contributing to the CP asymmetries, the multiplicities of the final states in the decays of the heavy neutrinos and sneutrinos and one can estimate [109]

$$\frac{Y_{\Delta B}(\infty)^{\text{MSSM}}}{Y_{\Delta B}(\infty)^{\text{SM}}} \approx \begin{cases} \sqrt{2} & \text{(strong washout);} \\ 2\sqrt{2} & \text{(weak washout).} \end{cases} \tag{1.84}$$

Recently it was pointed out in Ref. [89] that in fact MSSM leptogenesis is rich of new and non-trivial features, and genuinely different from the simpler realization within the SM. The two important effects are:
(a) If the SUSY breaking scale does not exceed by much 1 TeV, above $T \sim 5 \times 10^7$ GeV the particle and superparticle density asymmetries do not equilibrate [149], and it is mandatory to account in the BE for the differences in the number density asymmetries of the boson and fermion degrees of freedom. This can be given in terms of a non-vanishing gaugino chemical potential.
(b) When soft SUSY breaking parameters are neglected, additional anomalous global symmetries that involve both $SU(2)_L$ and $SU(3)_C$ fermion representations emerge [150]. As a consequence, the EW and QCD sphaleron equilibrium conditions are modified with respect to the SM, and this yields a different pattern of sphaleron induced lepton-flavour mixing [67, 69, 70]. In addition, a new anomaly-free exactly conserved $\mathcal{R}$-symmetry provides an additional constraint that is not present in the SM and a careful counting reveals that *four* independent quantities, rather than the *three* of the SM case, are

required to give a complete description of the various particle asymmetries in the thermal bath, with the additional quantity corresponding to the number density asymmetry of the heavy scalar neutrinos.

Although the modifications above are interesting from the theoretical point of view, a quantitative comparison with the results obtained when the new effects are ignored shows that the corrections remain below $\mathcal{O}(1)$ [89][13]. Finally, it should be pointed out that in the supersymmetric case, the temperature in which the lepton flavour effects (see Section 1.3) come into play is enhanced by a factor of $(1+\tan^2\beta)$ since the charged Yukawa couplings are given by $h_\alpha = m_\alpha/(v_u \cos\beta)$.

The purpose of the following sections is twofold. We describe the chemical equilibrium conditions and conservation laws for MSSM in conjunction to SM. In Section 1.5.2 we list the constraints that hold independently of assuming a regime in which particle and sparticle chemical potentials equilibrate (superequilibration (SE) regime) or do not equilibrate (non-superequilibration (NSE) regime). In Section 1.5.3 we list the constraints that hold only in the SE regime, and in Section 1.5.4 the ones that hold in NSE regime. The question of NSE is irrelevant in the SM since there are no superparticles. Hence the parts relevant for the SM are Section 1.5.2 and Section 1.5.3, with the chemical potential of the gaugino set to zero, the chemical potential of the down-type higgsino replaced by the minus of the up-type higgsino (see eqs. (1.95) and (1.96)), and all the quantities related to superparticles replaced by the ones for particles.

1.5.2 General constraints

We first list in items (1), (2) and (3) below the conditions that hold in the whole temperature range $M_W \ll T \lesssim 10^{14}$ GeV. Conversely, some of the Yukawa coupling conditions given in items (4) and (5) will have to be dropped as the temperature is increased and the corresponding reactions go out of equilibrium. First we will relate the number density asymmetry of a particle $\Delta n \equiv n - \bar{n}$ for which a chemical potential can be defined to its chemical potential. For both bosons (b) and fermions (f) this relation acquires a particularly simple form in the relativistic limit $m_{b,f} \ll T$, and at first order in the chemical potential $\mu_{b,f}/T \ll 1$:

$$\Delta n_b = \frac{g_b}{3} T^2 \mu_b, \qquad \Delta n_f = \frac{g_f}{6} T^2 \mu_f \,. \tag{1.85}$$

For simplicity of notations, in the following we denote the chemical potentials with the same notation that labels the corresponding field: $\phi \equiv \mu_\phi$.

(1) At scales much higher than M_W, gauge fields have vanishing chemical potential $W = B = g = 0$ [57]. This also implies that all the particles belonging to the same $SU(2)_L$ or $SU(3)_C$ multiplets have the same chemical potential. For example $\phi(I_3 = +\frac{1}{2}) = \phi(I_3 = -\frac{1}{2})$ for a field ϕ that is a doublet of weak isospin $\vec{I}$, and similarly for color.

(2) Denoting by $\tilde{W}_R$, $\tilde{B}_R$ and $\tilde{g}_R$ the RH winos, binos and gluinos chemical potentials, and by ℓ, Q ($\tilde{\ell}$, $\tilde{Q}$) the chemical potentials of the (s)lepton and (s)quarks LH doublets, the following reactions: $\tilde{Q} + \tilde{g}_R \to Q$, $\tilde{Q} + \tilde{W}_R \to Q$, $\tilde{\ell} + \tilde{W}_R \to \ell$, $\tilde{\ell} + \tilde{B}_R \to \ell$, imply that all gauginos have the same chemical potential: $-\tilde{g} = Q - \tilde{Q} = -\tilde{W} = \ell - \tilde{\ell} = -\tilde{B}$, where $\tilde{W}$, $\tilde{B}$ and $\tilde{g}$ denote the chemical potentials of LH gauginos. It follows that the chemical potentials of the SM particles

[13] This modification would be important for certain supersymmetric leptogenesis scenarios which contain new sources of CP violation e.g. soft leptogenesis (see Section 1.6.2).

are related to those of their respective superpartners as

$$\begin{aligned}\tilde{Q},\tilde{\ell} &= Q,\ell+\tilde{g} && (1.86)\\ H_{u,d} &= \tilde{H}_{u,d}+\tilde{g} && (1.87)\\ \tilde{u},\tilde{d},\tilde{e} &= u,d,e-\tilde{g}. && (1.88)\end{aligned}$$

The last relation, in which $u, d, e \equiv u_R, d_R, e_R$ denote the RH $SU(2)_L$ singlets, follows e.g. from $\tilde{u}^c_L = u^c_L + \tilde{g}$ for the corresponding LH fields, together with $u^c_L = -u_R$, and from the analogous relation for the $SU(2)_L$ singlet squarks.

(3) Before EW symmetry breaking hypercharge is an exactly conserved quantity, and we can assume a vanishing total hypercharge for the Universe:

$$\sum_i (Y_{\Delta Q_i} + 2Y_{\Delta u_i} - Y_{\Delta d_i}) - \sum_\alpha (Y_{\Delta \ell_\alpha} + Y_{\Delta e_\alpha}) + Y_{\Delta \tilde{H}_u} - Y_{\Delta \tilde{H}_d} = 0. \qquad (1.89)$$

(4) When the reactions mediated by the lepton Yukawa couplings are faster than the Universe expansion rate[14], the following chemical equilibrium conditions are enforced:

$$\ell_\alpha - e_\alpha + \tilde{H}_d + \tilde{g} = 0, \qquad (\alpha = e,\, \mu,\, \tau). \qquad (1.90)$$

If the temperature is not too low lepton flavour equilibration (see Section 1.3.4) induced by off-diagonal slepton soft masses will not occur. We assume that this is the case, and thus we take the three ℓ_α to be independent quantities.

(5) Reactions mediated by the quark Yukawa couplings enforce the following six chemical equilibrium conditions:

$$\begin{aligned}Q_i - u_i + \tilde{H}_u + \tilde{g} &= 0, \qquad (u_i = u,\, c,\, t), && (1.91)\\ Q_i - d_i + \tilde{H}_d + \tilde{g} &= 0, \qquad (d_i = d,\, s,\, b)\,. && (1.92)\end{aligned}$$

The up-quark Yukawa coupling maintains chemical equilibrium between the LH and RH up-type quarks up to $T \sim 2 \cdot 10^6$ GeV. Note that when the Yukawa reactions of at least two families of quarks are in equilibrium, the mass basis is fixed for all the quarks and squarks. Intergeneration mixing then implies that family-changing charged-current transitions are also in equilibrium: $b_L \to c_L$ and $t_L \to s_L$ imply $Q_2 = Q_3$; $s_L \to u_L$ and $c_L \to d_L$ imply $Q_1 = Q_2$. Thus, up to temperatures $T \lesssim 10^{11}$ GeV, that are of the order of the equilibration temperature for the charm Yukawa coupling, the three quark doublets have the same chemical potential:

$$Q \equiv Q_3 = Q_2 = Q_1. \qquad (1.93)$$

At higher temperatures, when only the third family is in equilibrium, we have instead $Q \equiv Q_3 = Q_2 \neq Q_1$. Above $T \sim 10^{13}$ when (for moderate values of $\tan\beta$) also b-quark (as well as the τ-lepton) $SU(2)_L$ singlets decouple from their Yukawa reactions, all intergeneration mixing becomes negligible and $Q_3 \neq Q_2 \neq Q_1$.

[14] See Section 1.3.1 for the temperature regime when lepton Yukawa interactions are in thermal equilibrium.

1.5.3 Superequilibration regime

At relatively low temperatures, additional conditions from reactions in chemical equilibrium hold. Since the constraints below apply only in the SE regime, we number them including this label.

$6_{\rm SE}$. Equilibration of the particle-sparticle chemical potentials $\mu_\phi = \mu_{\tilde{\phi}}$ [149] is ensured when reactions like $\tilde{\ell}\tilde{\ell} \to \ell\ell$ are faster than the Universe expansion rate. These reactions are induced by gaugino interactions, but since they require a gaugino chirality flip they turn out to be proportional to its mass $m_{\tilde{g}}$, and can be neglected in the limit $m_{\tilde{g}} \to 0$.

Furthermore, since the μ parameter of the $H_u H_d$ superpotential term is expected to be of the order of the soft gaugino masses, it is reasonable to consider in the same temperature range also the effect of the higgsino mixing term, which implies that the sum of the up- and down- higgsino chemical potentials vanishes. The rates of the corresponding reactions, given approximately by $\Gamma_{\tilde{g}} \sim m_{\tilde{g}}^2/T$ and $\Gamma_\mu \sim \mu^2/T$, are faster than the Universe expansion rate up to temperatures

$$T \lesssim 5 \cdot 10^7 \left(\frac{m_{\tilde{g}},\, \mu}{500\,\mathrm{GeV}} \right)^{2/3} \mathrm{GeV}. \tag{1.94}$$

The corresponding chemical equilibrium relations enforce the conditions:

$$\tilde{g} = 0, \tag{1.95}$$

$$\tilde{H}_u + \tilde{H}_d = 0. \tag{1.96}$$

$7_{\rm SE}$. Up to temperatures given by eq. (1.94) the MSSM has the same global anomalies than the SM, that are the EW $SU(2)_L$-$U(1)_{B+L}$ mixed anomaly and the QCD chiral anomaly. They generate the effective operators $O_{EW} = \Pi_\alpha(QQQ\ell_\alpha)$ [151] and $O_{QCD} = \Pi_i(QQu^c_{Li}d^c_{Li})$ [152, 153, 151]. Above the EW phase transition reactions induced by these operators are in thermal equilibrium, and the corresponding conditions read:

$$9\,Q + \sum_\alpha \ell_\alpha = 0 \tag{1.97}$$

$$6\,Q - \sum_i (u_i + d_i) = 0\,, \tag{1.98}$$

where we have used the same chemical potential for the three quark doublets (eq. (1.93)), which is always appropriate in the SE regime below the limit eq. (1.94).

Eqs. (1.90) and (1.91)–(1.93), together with the SE conditions eqs. (1.95)–(1.96), the two anomaly conditions eqs. (1.97)–(1.98) and the hypercharge neutrality condition eq. (1.89), give $11+2+2+1 = 16$ constraints for the 18 chemical potentials. Note however that there is one redundant constraint, that we take to be the QCD sphaleron condition, since by summing up eqs. (1.91) and (1.92) and taking into account eqs. (1.93), (1.95), and (1.96) we obtain precisely eq. (1.98). Therefore, like in the SM, we have three independent chemical potentials. We can define three linear combinations of the chemical potentials corresponding to the $SU(2)_L$ anomaly-free flavour charges $\Delta_\alpha \equiv B/3 - L_\alpha$ that being anomaly-free and perturbatively conserved by the low energy MSSM Lagrangian, evolve *slowly* because the corresponding symmetries are violated only by the heavy Majorana neutrino dynamics.

Their evolution needs to be computed by means of three independent BE. In terms of the abundances eq. (1.18) the density of the Δ_α charges normalized to the entropy density can be written as:

$$Y_{\Delta_\alpha} = 3\left[\frac{1}{3}\sum_i \left(2Y_{\Delta Q_i} + Y_{\Delta u_i} + Y_{\Delta d_i}\right) - \left(2Y_{\Delta \ell_\alpha} + Y_{\Delta e_\alpha}\right) - \frac{2}{3}Y_{\Delta\tilde{g}}\right]. \tag{1.99}$$

The expression above is completely general and holds in all temperature regimes, including the NSE regime (see Section 1.5.4).

The density asymmetries of the doublet leptons and higgsinos, that weight the washout terms in the BE, can now be expressed in terms of the anomaly-free charges by means of the A matrix and C vectors introduced respectively in Ref. [67] and Ref. [69] that are defined as:

$$Y_{\Delta\ell_\alpha} = A^{\ell}_{\alpha\beta}\, Y_{\Delta_\beta}, \qquad Y_{\Delta\tilde{H}_{u,d}} = C^{\tilde{H}_{u,d}}_{\alpha}\, Y_{\Delta_\alpha}. \tag{1.100}$$

Here and in the following we will give results for the A and C matrices for the fermion states. We recall that in the SE regime the density asymmetry of a scalar boson that is in chemical equilibrium with its fermionic partner is given simply by $Y_{\Delta b} = 2\,Y_{\Delta f}$ with the factor of 2 from statistics.

First generation Yukawa reactions out of equilibrium (SE regime)

As an example let us now consider the temperatures $T \gtrsim 4\cdot 10^6(1+\tan^2\beta)\,\mathrm{GeV}$, when the d-quark Yukawa coupling can be set to zero (in order to remain within the SE regime we assume $\tan\beta \sim 1$). In this case the equilibrium dynamics is symmetric under the exchange $u \leftrightarrow d$ (both chemical potentials enter only the QCD sphaleron condition eq. (1.98) with equal weights) and so must be any physical solution of the set of constraints. Thus, the first condition in eq. (1.92) can be replaced by the condition $d = u$, and again three independent quantities suffice to determine all the particle density asymmetries. The corresponding result is:

$$A^{\ell} = \frac{1}{3\times 2148}\begin{pmatrix} -906 & 120 & 120 \\ 75 & -688 & 28 \\ 75 & 28 & -688 \end{pmatrix}, \qquad C^{\tilde{H}_u} = -C^{\tilde{H}_d} = \frac{-1}{2148}\,(37,\ 52,\ 52)\,. \tag{1.101}$$

Note that since in this regime the chemical potentials for the scalar and fermion degrees of freedom of each chiral multiplet equilibrate, the analogous results for $Y_{\Delta\ell_\alpha} + Y_{\Delta\tilde{\ell}_\alpha}$ can be obtained by simply multiplying the A matrix in eq. (1.101) by a factor of 3. This gives the same A matrix obtained in the non-supersymmetric case in the same regime (see e.g. eq. (4.12) in Ref. [69]). The C matrix (multiplied by the same factor of 3) differs from the non-supersymmetric result by a factor 1/2. This is because after substituting $\tilde{H}_d = -\tilde{H}_u$ (see eq. (1.96)) all the chemical potential conditions are formally the same than in the SM with $\tilde{H}_u$ identified with the chemical potential of the scalar Higgs, but since C expresses the result for number densities, in the SM a factor of 2 from boson statistics appears for the SM Higgs. This agrees with the analysis in Ref. [58], and is a general result that holds for SUSY within the SE regime.

1.5.4 Non-superequilibration regime

At temperatures above the limit given in eq. (1.94) the Universe expansion is fast enough that reactions induced by $m_{\tilde{g}}$ and μ do not occur. Setting to zero in the high temperature effective theory these two parameters has the following consequences:

- Condition eq. (1.95) has to be dropped, and gauginos acquire a non-vanishing chemical potential $\tilde{g} \neq 0$ (corresponding to the difference between the number of LH and RH helicity states). The chemical potentials of the members of the same matter supermultiplets are no more equal (non-superequilibration) but related as in eqs. (1.86)–(1.88).
- Condition eq. (1.96) also has to be dropped, and the chemical potentials of the up- and down-type Higgs and higgsinos do not necessarily sum up to zero.
- The MSSM gains two new global symmetries: $m_{\tilde{g}} \to 0$ yields a global R-symmetry, while $\mu \to 0$ corresponds to a global symmetry of the Peccei-Quinn (PQ) type.

Anomalous and non-anomalous symmetries

Two linear combinations R_2 and R_3 of R and PQ, having respectively only $SU(2)_L$ and $SU(3)_C$ mixed anomalies have been identified in Ref. [150]

$$R_2 = R - 2\,PQ, \qquad R_3 = R - 3\,PQ\,. \tag{1.102}$$

The authors of Ref. [150] have also constructed the effective multi-fermions operators generated by the mixed anomalies:

$$\tilde{O}_{EW} = \Pi_\alpha \left(QQQ\ell_\alpha\right)\ \tilde{H}_u \tilde{H}_d\, \tilde{W}^4\,, \tag{1.103}$$
$$\tilde{O}_{QCD} = \Pi_i \left(QQu^c d^c\right)_i\ \tilde{g}^6\,. \tag{1.104}$$

Given that three global symmetries B, L and R_2 have mixed $SU(2)_L$ anomalies (but are free of $SU(3)_C$ anomalies) we can construct two anomaly-free combinations, the first one being $B-L$ which is only violated perturbatively by $N^c\,\ell H_u$ and the second anomaly-free combination which is also an exact symmetry of the MSSM+seesaw in the NSE regime [89]

$$\mathcal{R} = \frac{5}{3}B - L + R_2, \tag{1.105}$$

and is exactly conserved. In the $SU(3)_C$ sector, besides the chiral anomaly we now have also R_3 mixed anomalies. Thus also in this case anomaly-free combinations can be constructed, and in particular we can define one combination for each quark superfield. Assigning to the LH supermultiplets chiral charge $\chi = -1$ these combinations have the form [89]:

$$\chi_{q_L} + \kappa_{q_L}\, R_3, \tag{1.106}$$

where, for example, $\kappa_{u^c_L} = \kappa_{d^c_L} = 1/3$ and $\kappa_{Q_L} = 2/3$. Note that since R_3 is perturbatively conserved by the complete MSSM+seesaw Lagrangian, when the Yukawa coupling of one quark is set to zero the corresponding charge eq. (1.106) will be exactly conserved.

Constraints in the non-superequilibration regime

In the NSE regime, the conditions listed in items 6_{SE} and 7_{SE} of the previous section have to be dropped, but new conditions arise.

6_{NSE}. The conservation law for the $\mathcal{R}$ charge yields the following global neutrality condition:

$$\begin{aligned}\mathcal{R}_{\rm tot} &= \sum_f \Delta n_f \mathcal{R}_f + \sum_b \Delta n_b \mathcal{R}_b + \Delta n_{\tilde N_1}\mathcal{R}_{\tilde N_1}\\ &= \frac{T^2}{6}\left(\sum_i (2Q_i - 5u_i + 4d_i) + 2\sum_\alpha(\ell_\alpha + e_\alpha) + 5\tilde H_d - \tilde H_u + 31\,\tilde g\right) - \Delta n_{\tilde N_1} = 0. \quad (1.107)\end{aligned}$$

The last terms in both lines of eq. (1.107) correspond to the contribution to $\mathcal{R}$-neutrality from the lightest sneutrino asymmetry $\Delta n_{\tilde N_1} = n_{\tilde N_1} - n_{\tilde N_1^*}$ with charge $\mathcal{R}_{\tilde N_1} = -\mathcal{R}_{N^c} = -1$. Note that since in general $\tilde N_1$ is not in chemical equilibrium, no chemical potential can be associated to it, and hence this constraint needs to be formulated in terms of its number density asymmetry that has to be evaluated by solving a BE for $Y_{\Delta_{\tilde N}} \equiv Y_{\tilde N_1} - Y_{\tilde N_1^*}$ (see Section 1.5.5).

7_{NSE}. The operators in eqs. (1.103)–(1.104) induce transitions that in the NSE regime are in chemical equilibrium. This enforces the generalized EW and QCD sphaleron equilibrium conditions [150]:

$$3\sum_i Q_i + \sum_\alpha \ell_\alpha + \tilde H_u + \tilde H_d + 4\,\tilde g = 0, \quad (1.108)$$

$$2\sum_i Q_i - \sum_i (u_i + d_i) + 6\,\tilde g = 0, \quad (1.109)$$

that replace eqs. (1.97) and (1.98).

8_{NSE}. The chiral-R_3 charges in eq. (1.106) are anomaly-free, but clearly they are not conserved by the quark Yukawa interactions. However, when a quark supermultiplet decouples from its Yukawa interactions an exact conservation law arises[15]. The conservation laws corresponding to these symmetries read:

$$\begin{aligned}\frac{T^2}{6}\left[3q_R + 6(q_R - \tilde g)\right] + \frac{1}{3}R_{3\,\rm tot} &= 0 \quad (1.110)\\ \frac{T^2}{6}\,2\left[3Q_L + 6(Q_L + \tilde g)\right] - \frac{2}{3}R_{3\,\rm tot} &= 0 \quad (1.111)\end{aligned}$$

and hold for $q_R = u_i,\, d_i$ and $Q_L = Q_i$ in the regimes when the appropriate Yukawa reactions are negligible. Note the factor of 2 for the Q_L chiral charge in front of the first square bracket in eq. (1.111) that is due to $SU(2)_L$ gauge multiplicity. In terms of chemical potentials and $\Delta n_{\tilde N_1}$, the total R_3 charge in eqs. (1.110) and (1.111) reads:

$$\begin{aligned}R_{3\,\rm tot} &= \frac{T^2}{6}\left(82\,\tilde g - 3\sum_i(2\,Q_i + 11\,u_i - 4\,d_i) + \sum_\alpha(16\,\ell_\alpha + 13\,e_\alpha) + 16\,\tilde H_d - 14\,\tilde H_u.\right)\\ &\quad + \Delta n_{\tilde N_1} R_{3\,\tilde N_1}, \quad (1.112)\end{aligned}$$

where $R_{3\,\tilde N_1} = -1$. As regards the leptons, since they do not couple to the QCD anomaly, by setting $h_e \to 0$ a symmetry under chiral supermultiplet rotations is directly gained for the RH leptons implying $\Delta n_e + \Delta\tilde n_e = 0$ and giving the condition:

$$e - \frac{2}{3}\tilde g = 0. \quad (1.113)$$

No analogous condition arises for the lepton doublets relevant for leptogenesis, since by assumption they remain coupled via Yukawa couplings to the heavy N's.

[15] Note that $h_{u,d} \to 0$ implies u and d decoupling, but Q_1 decoupling is ensured only if also $h_{c,s} \to 0$.

In the NSE regime there are different flavour mixing matrices for the scalar and fermion components of the leptons and Higgs supermultiplets. To express more concisely all the results, it is convenient to introduce a new C vector to describe the gaugino number density asymmetry per degree of freedom in terms of the relevant charges:

$$Y_{\Delta\tilde{g}} = C_a^{\tilde{g}}\, Y_{\Delta_a}\,, \qquad \text{with} \qquad \Delta_a = (\Delta_\alpha, \Delta_{\tilde{N}})\ . \tag{1.114}$$

First generation Yukawa reactions out of equilibrium (NSE regime)

As an example, in the temperature range between 10^8 and 10^{11} GeV, and for moderate values of $\tan\beta$, all the first generation Yukawa couplings can be set to zero. Using for u, d conditions eq. (1.110) and for e condition eq. (1.113) as are implied by $h_{u,d}$, $h_e \to 0$ we obtain:

$$\begin{aligned}
A^\ell &= \frac{1}{9\times 162332}\begin{pmatrix} -198117 & 33987 & 33987 & -8253 \\ 26634 & -147571 & 14761 & -8055 \\ 26634 & 14761 & -147571 & -8055 \end{pmatrix}, \\
C^{\tilde{g}} &= \frac{-11}{162332}\,(163,\ 165,\ 165,\ -255)\,, \\
C^{\tilde{H}_u} &= \frac{-1}{162332}\,(3918,\ 4713,\ 4713,\ 95)\ , \\
C^{\tilde{H}_d} &= \frac{1}{3\times 162332}\,(5413,\ 9712,\ 9712,\ -252)\ ,
\end{aligned} \tag{1.115}$$

where the rows correspond to $(Y_{\Delta_e},\ Y_{\Delta_\mu},\ Y_{\Delta_\tau},\ Y_{\Delta_{\tilde{N}}})$. For completeness, in eq. (1.115) we have also given the results for $C^{\tilde{H}_d}$ even if only the up-type Higgs density asymmetry is relevant for the leptogenesis processes. Note that neglecting the contribution of $\Delta n_{\tilde{N}_1}$ to the global charges $\mathcal{R}_{\rm tot}$ in eq. (1.107) and $R_{3{\rm tot}}$ in eq. (1.112) corresponds precisely to setting to zero the fourth column in all the previous matrices. Then, analogously with the SE and SM cases, within this '3-column approximation' all particle density asymmetries can be expressed just in terms of the three Y_{Δ_α} charge densities.

1.5.5 Supersymmetric Boltzmann equations

In order to illustrate how the new effects described above modify the structure of the BE, here we write down a simpler expressions in which only decays and inverse decays are included[16]:

$$sHz\frac{dY_{N_1}}{dz} = -\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\gamma_{N_1}, \tag{1.116}$$

$$sHz\frac{dY_{\tilde{N}_+}}{dz} = -\left(\frac{Y_{\tilde{N}_+}}{Y_{\tilde{N}_1}^{eq}} - 2\right)\gamma_{\tilde{N}_1}, \tag{1.117}$$

$$sHz\frac{dY_{\Delta_{\tilde{N}}}}{dz} = -\frac{Y_{\Delta_{\tilde{N}}}}{Y_{\tilde{N}_1}^{eq}}\gamma_{\tilde{N}_1} - \frac{3}{2}\gamma_{\tilde{N}_1}\sum_a C_a^{\tilde{g}}\,\frac{Y_{\Delta_a}}{Y_\ell^{eq}} + \dots\,, \tag{1.118}$$

$$sHz\frac{dY_{\Delta_\alpha}}{dz} = -\epsilon_\alpha\left[\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\gamma_{N_1} + \left(\frac{Y_{\tilde{N}_+}}{Y_{\tilde{N}_1}^{eq}} - 2\right)\gamma_{\tilde{N}_1}\right]$$

[16]The complete set of BE including decays, inverse decays and scatterings with top-quark is given in the Appendix of Ref. [89].

$$+P^0_{1\alpha}\left(\gamma_{N_1}+\frac{1}{2}\gamma_{\tilde{N}_1}\right)\sum_a\left(A^\ell_{\alpha a}+C^{\tilde{H}_u}_a+C^{\tilde{g}}_a\right)\frac{Y_{\Delta_a}}{Y^{eq}_\ell}\,, \tag{1.119}$$

where $\gamma_{\tilde{N}_1}$ is the corresponding thermally averaged decay rate for RH sneutrino $\tilde{N}_1$. In eq. (1.117) we have introduced the overall sneutrino abundance $Y_{\tilde{N}_+}=Y_{\tilde{N}_1}+Y_{\tilde{N}_1^*}$, while $Y_{\Delta_{\tilde{N}}}\equiv Y_{\tilde{N}_1}-Y_{\tilde{N}_1^*}$ in eq. (1.118) is the sneutrino density asymmetry that was already introduced in Section 1.5.4. In the washout terms we have normalized the charge densities $Y_{\Delta_a}=(Y_{\Delta_\alpha},\,Y_{\Delta_{\tilde{N}}})$ to the equilibrium density of a fermion with one degree of freedom Y^{eq}_ℓ. In eqs. (1.116) and (1.119) we have also neglected for simplicity all finite temperature effects. Taking these effects into account would imply for example that the CP asymmetry for $\tilde{N}$ decays into fermions is different from the one for decays into scalars [49], while we describe both CP asymmetries with ϵ_α. A few remarks regarding eq. (1.118) are in order. In the SE regime $\tilde{g}=0$ and thus it would seem that the sneutrino density asymmetry $Y_{\Delta_{\tilde{N}}}$ vanishes. However, this only happens for decays and inverse decays, and it is no more true when additional terms related to scattering processes, that are represented in the equation by the dots, are also included (see Ref. [147] and the Appendix of Ref. [89]). Therefore, also in the SE regime $Y_{\tilde{N}_1}$ and $Y_{\tilde{N}_1^*}$ in general differ. However, in this case recasting their equations in terms of two equations for $Y_{\tilde{N}_+}$ and $Y_{\Delta_{\tilde{N}}}$ is just a convenient parametrization. On the contrary, in the NSE regime this is mandatory, because the sneutrinos carry a globally conserved $\mathcal{R}$-charge and $Y_{\Delta_{\tilde{N}}}$ is required to formulate properly the corresponding conservation law. As we have seen, this eventually results in $Y_{\Delta_{\tilde{N}}}$ contributing to the expressions of the lepton flavour density asymmetries in terms of slowly varying quantities.

In Ref. [89] a complete numerical analysis was carried out and it was shown that numerical corrections with respect to the case when NSE effects are neglected remain at the $\mathcal{O}(1)$ level. This is because only spectator processes get affected, while the overall amount of CP violation driving leptogenesis remains the same than in previous treatments.

1.6 Beyond Type-I Seesaw and Beyond the Seesaw

There exist many variants of leptogenesis models beyond the standard type-I seesaw. In this section, we try to classify them into appropriate groups. Unavoidably there would be some overlap i.e. a hybrid model of leptogenesis which can belong to more than one group, e.g. soft leptogenesis (Section 1.6.2) from resonantly enhanced CP asymmetry could rightly fall under resonant leptogenesis (Section 1.6.1). However we try our best to categorize them according to the main features of the model and, when appropriate, they will be quoted in more than one place. Clearly, the number of beyond type-I seesaw leptogenesis models is quite large. We have not attempted in any way to be exhaustive, and we apologize in advance for the unavoidable several omissions.

1.6.1 Resonant leptogenesis

A resonant enhancement of the CP asymmetry in N_1 decay occurs when the mass difference between N_1 and N_2 is of the order of the decay widths. Such a scenario has been termed 'resonant leptogenesis', and has benefited from many studies in different formalisms[17] [50, 155, 52, 110, 156, 157, 158, 159, 75, 111, 160, 161, 162, 163, 85, 86, 164, 87, 165, 166, 167, 168] (see Ref. [169] for a recent review). The

[17] See Ref. [154] for a comparison of the different calculations.

resonant effect is related to the self energy contribution to the CP asymmetry. Consider, for simplicity, the case where only N_2 is quasi-degenerate with N_1. Then, the self-energy contribution involving the intermediate N_2, to the total CP asymmetry (we neglect important flavour effects [75]) is given by

$$\epsilon_1(\mathrm{self-energy}) = -\frac{M_1}{M_2}\frac{\Gamma_{N_2}}{M_2}\frac{M_2^2(M_2^2-M_1^2)}{(M_2^2-M_1^2)^2+M_1^2\Gamma_{N_2}^2}\frac{\mathrm{Im}[(\lambda^\dagger\lambda)_{12}^2]}{(\lambda^\dagger\lambda)_{11}(\lambda^\dagger\lambda)_{22}}. \tag{1.120}$$

The resonance condition reads

$$|M_2 - M_1| = \frac{\Gamma_{N_2}}{2}. \tag{1.121}$$

In this case

$$|\epsilon_1(\mathrm{resonance})| \simeq \frac{1}{2}\frac{|\mathcal{I}m[(\lambda^\dagger\lambda)_{12}^2]|}{(\lambda^\dagger\lambda)_{11}(\lambda^\dagger\lambda)_{22}}. \tag{1.122}$$

Thus, in the resonant case, the asymmetry is suppressed by neither the smallness of the light neutrino masses, nor the smallness of their mass splitting, nor small ratios between the singlet neutrino masses. Actually, the CP asymmetry could be of order one (more accurately, $|\epsilon_1| \leq 1/2$).

With resonant leptogenesis, the BE are different. The densities of N_1 and N_2 are followed, since both contribute to the asymmetry, and the relevant timescales are different. For instance, the typical time-scale to build up coherently the CP asymmetry is unusually long, of order $1/\Delta M$. In particular, it can be larger than the time-scale for the change of the abundance of the sterile neutrinos. This situation implies that for resonant leptogenesis quantum effects in the BE can be significant [85, 86, 170, 141] (see Section 1.4.5).

The fact that the asymmetry could be large, independently of the singlet neutrino masses, opens up the possibility of low scale resonant leptogenesis. Models along these lines have been constructed in Refs. [156, 75, 161, 171]. It is a theoretical challenge to construct models where a mass splitting as small as the decay width is naturally achieved. For attempts that utilize approximate flavour symmetries see, for example, Refs. [157, 158, 160, 164, 172, 166], while studies of this issue in the framework of minimal flavour violation can be found in Refs. [162, 163, 87]. The possibility of observing resonant CP violation due to heavy RH neutrinos at the LHC was studied in Refs. [173, 174].

1.6.2 Soft leptogenesis

The modifications to standard type-I leptogenesis due to SUSY have been discussed in Section 1.5. The important parameters there are the Yukawa couplings and the singlet neutrino parameters, which appear in the superpotential eq. (1.83). SUSY must, however, be broken. In the framework of the MSSM extended to include singlet neutrinos (MSSM+N), there are, in addition to the soft SUSY breaking terms of the MSSM, terms that involve the singlet sneutrinos $\widetilde{N}_i$, in particular bilinear (B) and trilinear (A) scalar couplings. These terms provide additional sources of lepton number violation and of CP violation. Scenarios where these terms play a dominant role in leptogenesis have been termed 'soft leptogenesis' [175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 102, 195, 196, 107] (see also Ref. [54] for a recent review).

Soft leptogenesis can take place even with a single RH neutrino because the presence of the B term implies that $\widetilde{N}$ and $\widetilde{N}^\dagger$ states mix to form two mass eigenstates with mass splitting proportional to B itself. Furthermore when $B \sim \Gamma_{\widetilde{N}}$, the CP asymmetry is resonantly enhanced realizing the resonant leptogenesis scenario (see Section 1.6.1). In the following we will consider a single generation MSSM+N. The relevant soft SUSY terms involving $\widetilde{N}$, the $SU(2)_L$ gauginos $\widetilde{\lambda}_2^{\pm,0}$, the $U(1)_Y$ gauginos

$\widetilde{\lambda}_1$ and the three sleptons $\widetilde{\ell}_\alpha$ in the basis in which charged lepton Yukawa couplings are diagonal are given by

$$\begin{aligned} -\mathcal{L}_{\rm soft} &= \widetilde{M}^2\widetilde{N}^*\widetilde{N} + \left(AY_\alpha\epsilon_{ab}\widetilde{N}\widetilde{\ell}^a_\alpha H^b_u + \frac{1}{2}BM\widetilde{N}\widetilde{N} + {\rm h.c.}\right) \\ &+\frac{1}{2}\left(m_2\overline{\widetilde{\lambda}_2^{\pm,0}}P_L\widetilde{\lambda}_2^{\pm,0} + m_1\overline{\widetilde{\lambda}_1}P_L\widetilde{\lambda}_1 + {\rm h.c.}\right), \end{aligned} \tag{1.123}$$

where for simplicity, proportionality of the bilinear and trilinear soft breaking terms to the corresponding SUSY invariant couplings has been assumed: $B_M = BM$ and $A_\alpha = AY_\alpha$. The Lagrangian derived from eqs. (1.83) and (1.123) is characterized by only three independent physical phases: $\phi_A \equiv \arg(AB^*)$, $\phi_{g_2} \equiv \frac{1}{2}\arg(Bm_2^*)$ and $\phi_{g_Y} \equiv \frac{1}{2}\arg(Bm_1^*)$ which can be assigned to A, and to the gaugino coupling operators g_2, g_Y respectively. As mentioned earlier, a crucial role in soft leptogenesis is played by the $\widetilde{N}-\widetilde{N}^\dagger$ mixing to form the mass eigenstates

$$\widetilde{N}_+ = \frac{1}{\sqrt{2}}\left(e^{i\Phi/2}\widetilde{N} + e^{-i\Phi/2}\widetilde{N}^*\right), \tag{1.124}$$

$$\widetilde{N}_- = -\frac{i}{\sqrt{2}}(e^{i\Phi/2}\widetilde{N} - e^{-i\Phi/2}\widetilde{N}^*), \tag{1.125}$$

where $\Phi \equiv \arg(BM)$ and the corresponding mass eigenvalues are $M_\pm^2 = M^2 + \widetilde{M}^2 \pm |BM|$. Without loss of generality, we can set $\Phi = 0$, which is equivalent to assigning the phases only to A and Y_α.

It has been pointed out that the CP asymmetries for the decays of $\widetilde{N}_\pm$ into scalars and fermions have opposite sign and cancel each other at the leading order [176, 177, 192], resulting in a strongly suppressed total CP asymmetry $\sim \mathcal{O}(m_{soft}^3/M^3)$ where m_{soft} is the scale of soft SUSY breaking terms. There are two possibilities that can rescue leptogenesis: Firstly, thermal effects which break SUSY can spoil this cancellation [176, 177, 192]. Secondly, non-superequilibration effects (see Section 1.5.4) which imply that lepton and slepton asymmetries differ, can also spoil this cancellation [107]. The CP asymmetries for the decays of $\widetilde{N}_\pm$ into scalars and fermions are respectively given by

$$\epsilon^s_\alpha(T) = \bar{\epsilon}_\alpha\Delta^s(T), \tag{1.126}$$

$$\epsilon^f_\alpha(T) = -\bar{\epsilon}_\alpha\Delta^f(T), \tag{1.127}$$

where $\bar{\epsilon}_\alpha$ is the temperature independent term of $\sim \mathcal{O}(m_{soft}/M)$ which contains contributions from the self-energy correction, vertex correction and the interference between the two. In the limit $T \to 0$, we have $\Delta^s(T), \Delta^f(T) \to 1/2$, and thus the inclusion of thermal effects and/or non-superequilibration is mandatory to avoid the cancellation between the asymmetries into scalars and fermions.

We can make a rough estimate of the scale relevant for soft leptogenesis by requiring $|\bar{\epsilon}| \sim m_{soft}/M \gtrsim 10^{-6}$ which gives $M \lesssim 10^9$ GeV for $m_{soft} \sim 1$ TeV. Hence soft leptogenesis always happens in the temperature regime where lepton flavour effects are relevant [190]. In general, the CP asymmetry from self-energy contribution requires $B \ll m_{soft}$ to be resonantly enhanced. However it was shown in Ref. [195] that flavour effects can greatly enhance the efficiency and eventually $B \sim m_{soft}$ is allowed. The nice feature of soft leptogenesis is that the tension with the gravitino problem gets generically relaxed and, in the lower temperature window, is completely avoided.

1.6.3 Dirac leptogenesis

The extension of the SM with singlet neutrinos allows for two different ways for generating tiny neutrino masses. The first one is the seesaw mechanism which has at least three attractive features:

- No extra symmetries (and, in particular, no global symmetries) have to be imposed.
- The extreme lightness of neutrino masses is linked to the existence of a high scale of new physics, which is well motivated for various other reasons (*e.g.* gauge unification).
- Lepton number is violated, which opens the way to leptogenesis.

The second way is to impose lepton number and give to the neutrinos Dirac masses. A priori, one might think that all three attractive features of the seesaw mechanism are lost. Indeed, one must usually impose additional symmetries. But one can still construct natural models where the tiny Yukawa couplings that are necessary for small Dirac masses are related to a small breaking of a symmetry. What is perhaps most surprising is the fact that leptogenesis could proceed successfully even if neutrinos are Dirac particles, and lepton number is not (perturbatively) broken [197, 198]. Such scenarios have been termed 'Dirac leptogenesis' [198, 199, 200, 201, 202, 203, 204, 205, 206, 207].

An implementation of the idea is the following. A CP-violating decay of a heavy particle can result in a non-zero lepton number for LH particles, and an equal and opposite non-zero lepton number for RH particles, so that the total lepton number is zero. For the charged fermions of the SM, the Yukawa interactions are fast enough that they quickly equilibrate the LH and the RH particles, and the lepton number stored in each chirality goes to zero. This is not true, however, for Dirac neutrinos. The size of their Yukawa couplings is $\lambda \lesssim 10^{-11}$, which means that equilibrium between the lepton numbers stored in LH and RH neutrinos will not be reached until the temperature falls well below the electroweak breaking scale. To see this, note that the rate of the Yukawa interactions, given by $\Gamma_\lambda \sim \lambda^2 T$, becomes significant when it equals the expansion rate of the Universe, $H \sim T^2/m_{\rm pl}$. Thus, the temperature of equilibration between LH and RH neutrinos is $T \sim \lambda^2 m_{\rm pl} \sim (\lambda/10^{-11})^2\,\text{MeV}$, that is well below the temperature when sphalerons, after having converted part of the LH lepton asymmetry into a net baryon asymmetry, are switched off.

A specific example of a supersymmetric model where Dirac neutrinos arise naturally is presented in Ref. [199]. The Majorana masses of the N-superfields are forbidden by $U(1)_L$. The neutrino Yukawa couplings are forbidden by a $U(1)_N$ symmetry where, among all the MSSM+N fields, only the N superfields are charged. The symmetry is spontaneously broken by the vacuum expectation value of a scalar field χ that can naturally be at the weak scale, $\langle\chi\rangle \sim v_u$. This breaking is communicated to the MSSM+N via extra, vector-like lepton doublet fields, $\phi + \bar\phi$, that have masses M_ϕ much larger than v_u. Consequently, the neutrino Yukawa couplings are suppressed by the small ratio $\langle\chi\rangle/M_\phi$. The CP violation arises in the decays of the vector-like leptons, whereby $\Gamma(\phi \to N H_u^c) \neq \Gamma(\bar\phi \to N^c H_u)$ and $\Gamma(\phi \to L\chi) \neq \Gamma(\bar\phi \to L^c \chi^c)$. The resulting asymmetries in N and in L are equal in magnitude and opposite in sign. Finally note that the main phenomenological implication of Dirac leptogenesis is the absence of any signal in neutrinoless double beta decays.

1.6.4 Triplet scalar (type-II) leptogenesis

One can generate seesaw masses for the light neutrinos by tree-level exchange of $SU(2)_L$-triplet scalars T [44, 208, 209, 210, 211]. The relevant new terms in the Lagrangian are

$$\mathcal{L}_T = -M_T^2|T|^2 + \frac{1}{2}\left([\lambda_L]_{\alpha\beta}\,\ell_\alpha\ell_\beta T + M_T\lambda_\phi\,\phi\phi T^* + \text{h.c.}\right). \tag{1.128}$$

Here, M_T is a real mass parameter, λ_L is a symmetric 3×3 matrix of dimensionless, complex Yukawa couplings, and λ_ϕ is a dimensionless complex coupling. Since this mechanism necessarily involves lepton number violation and allows for new CP-violating phases, it is interesting to examine it in the light of leptogenesis [212, 213, 214, 215, 216, 180, 217, 218, 219, 185, 220, 221, 222, 223, 224, 81, 225, 226, 227, 228, 229, 230, 231, 232, 233]. One obvious problem in this scenario is that, unlike singlet fermions, the triplet scalars have gauge interactions that keep them close to thermal equilibrium at temperatures $T \lesssim 10^{15}$ GeV. It turns out, however, that successful leptogenesis is possible even at a much lower temperature. This subsection is based in large part on Ref. [220] where further details and, in particular, an explicit presentation of the relevant BE can be found.

The CP asymmetry that is induced by the triplet scalar decays is defined as follows:

$$\epsilon_T \equiv 2\frac{\Gamma(\bar{T} \to \ell\ell) - \Gamma(T \to \bar{\ell}\bar{\ell})}{\Gamma_T + \Gamma_{\bar{T}}}, \tag{1.129}$$

where the overall factor of 2 comes because the triplet scalar decay produces two (anti)leptons.

To calculate ϵ_T, one should use the Lagrangian in eq. (1.128). While a single triplet is enough to produce three light massive neutrinos, there is a problem in leptogenesis if indeed this is the only source of neutrinos masses: The asymmetry is generated only at higher loops and in unacceptably small. It is still possible to produce the required lepton asymmetry from a single triplet scalar decays if there are additional sources for neutrino masses, such as type I, type III, or type II contributions from additional triplet scalars. Define $m_{\rm II}$ ($m_{\rm I}$) as the part of the light neutrino mass matrix that comes (does not come) from the contributions of the triplet scalar responsible for ϵ_T:

$$m = m_{\rm II} + m_{\rm I}. \tag{1.130}$$

Then, assuming that the particles exchanged to produce $m_{\rm I}$ are all heavier than T, we obtain the the CP asymmetry

$$\epsilon_T = \frac{1}{4\pi}\frac{M_T}{v_u^2}\sqrt{B_L B_H}\,\frac{{\rm Im}[{\rm Tr}(m_{\rm II}^\dagger m_{\rm I})]}{{\rm Tr}(m_{\rm II}^\dagger m_{\rm II})}, \tag{1.131}$$

where B_L (B_H) is the tree-level branching ratio to leptons (Higgs doublets). If these are the only decay modes, *i.e.* $B_L + B_H = 1$, then $B_L/B_H = {\rm Tr}(\lambda_L\lambda_L^\dagger)/(\lambda_H\lambda_H^\dagger)$, and there is an upper bound on the asymmetry:

$$|\epsilon_T| \leq \frac{1}{4\pi}\frac{M_T}{v_u^2}\sqrt{B_L B_H \sum_i m_{\nu_i}^2}. \tag{1.132}$$

Note that, unlike the singlet fermion case, $|\epsilon_T|$ increases with larger m_{ν_i}.

As concerns the efficiency factor, it can be close to maximal, $\eta \sim 1$, in spite of the fact that the gauge interactions tend to maintain the triplet abundance very close to thermal equilibrium. There are two necessary conditions that have to be fulfilled by the decay rates $T \to \bar{\ell}\bar{\ell}$ and $T \to \phi\phi$ in order that this will happen [220]:

1. One of the two decay rates is faster than the $T\bar{T}$ annihilation rate.
2. The other decay mode is slower than the expansion rate of the Universe.

The first condition guarantees that gauge scatterings are ineffective: the triplets decay before annihilating. The second condition guarantees that the fast decays do not washout strongly the lepton asymmetry: lepton number is violated only by the simultaneous presence of $T \to \bar{\ell}\bar{\ell}$ and $T \to \phi\phi$.

Combining a calculation of η with the upper bound on the CP asymmetry (1.132), successful leptogenesis implies a lower bound on the triplet mass M_T varying between 10^9 GeV and 10^{12} GeV, depending on the relative weight of $m_{\rm II}$ and $m_{\rm I}$ in the light neutrino mass.

Interestingly, in the supersymmetric framework, "soft leptogenesis" (see Section 1.6.2) can be successful even with the minimal set of extra fields – a single $T + \bar{T}$ – that generates both neutrino masses and the lepton asymmetry [180, 185].

1.6.5 Triplet fermion (type-III) leptogenesis

One can generate neutrino masses by the tree level exchange of $SU(2)_L$-triplet fermions T_i^a [234, 235, 236] (i denotes a heavy mass eigenstate while a is an $SU(2)_L$ index) with the Lagrangian

$$\mathcal{L}_{T^a} = [\lambda_T]_{\alpha k} \tau^a_{\rho\sigma} \ell^\rho_\alpha \phi^\sigma T^a_k - \frac{1}{2} M_i T^a_i T^a_i + \text{h.c.}. \tag{1.133}$$

Here τ^a are the Pauli matrices, M_i are real mass parameters and λ_T is a 3×3 matrix of complex Yukawa couplings.

This mechanism necessarily involves lepton number violation, and allows for new CP-violating phases so we should examine it as a possible source of leptogenesis [237, 238, 228, 239, 240, 241, 242, 243]. This subsection is based in large part on Ref. [238] where further details and the relevant BE can be found.

As concerns neutrino masses, all the qualitative features are very similar to the singlet fermion case. As concerns leptogenesis there are, however, qualitative and quantitative differences. With regard to the CP asymmetry from the lightest triplet fermion decay, the relative sign between the vertex and self-energy loop contributions is opposite to that of the singlet fermion case. Consequently, in the limit of strong hierarchy in the heavy fermion masses, the asymmetry in triplet decays is three times smaller than in the decays of the singlets. On the other hand, since the triplet has three components, the ratio between the final baryon asymmetry and $\epsilon\eta$ is three times bigger. The decay rate of the heavy fermion is the same in both cases. This, however, means that the thermally averaged decay rate is three times bigger for the triplet, as is the on-shell part of the $\Delta L = 2$ scattering rate.

A significant qualitative difference arises from the fact that the triplet has gauge interactions. The effect on the washout factor η is particularly significant for $\tilde{m} \ll 10^{-3}\,$eV, the so-called "weak washout regime" (note that this name is inappropriate for triplet fermions). The gauge interactions still drive the triplet abundance close to thermal equilibrium. A relic fraction of the triplet fermions survives. The decays of these relic triplets produce a baryon asymmetry, with

$$\eta \approx M_1/10^{13}\ \text{GeV} \quad (\text{for } \tilde{m} \ll 10^{-3}\ \text{eV}). \tag{1.134}$$

The strong dependence on M_1 results from the fact that the expansion rate of the Universe is slower at lower temperatures. On the other hand, for $\tilde{m} \gg 10^{-3}$ eV, the Yukawa interactions keep the heavy fermion abundance close to thermal equilibrium, so the difference in η between the singlet and triplet case is only $\mathcal{O}(1)$. Ignoring flavour effects, and assuming strong hierarchy between the heavy fermions, Ref. [216] obtained the lower bound

$$M_1 \gtrsim 1.5 \times 10^{10}\ \text{GeV}\,. \tag{1.135}$$

When the triplet fermion scenario is incorporated in a supersymmetric framework, and the soft breaking terms do not play a significant role, the modifications to the above analysis is by factors of $\mathcal{O}(1)$.

1.7 Conclusions

During the last few decades, a large set of experiments involving solar, atmospheric, reactor and accelerator neutrinos have converged to establish that the neutrinos are massive. The seesaw mechanism extends the Standard Model in a way that allows neutrino masses, and it provides a nice explanation of the suppression of the neutrino masses with respect to the electroweak breaking scale. Furthermore, without any addition or modification, it can also account for the observed baryon asymmetry of the Universe. The possibility of giving an explanation of two apparently unrelated experimental facts – neutrino masses and the baryon asymmetry – within a single framework that is a natural extension of the Standard Model, together with the remarkable 'coincidence' that the same neutrino mass scale suggested by neutrino oscillation data is also optimal for leptogenesis, make the idea that baryogenesis occurs through leptogenesis a very attractive one.

Leptogenesis can be quantitatively successful without any fine-tuning of the seesaw parameters. Yet, in the non-supersymmetric seesaw framework, a fine-tuning problem arises due to the large corrections to the mass-squared parameter of the Higgs potential that are proportional to the heavy Majorana neutrino masses. Supersymmetry can cure this problem, avoiding the necessity of fine tuning; however, it brings in the gravitino problem [144] that requires a low reheat temperature after inflation, in conflict with generic leptogenesis models. Thus, constructing a fully satisfactory theoretical framework that implements leptogenesis within the seesaw framework is not a straightforward task.

From the experimental side, the obvious question to ask is if it is possible to test whether the baryon asymmetry has been really produced through leptogenesis. Unfortunately it seems impossible that any direct test can be performed. To establish leptogenesis experimentally, we need to produce the heavy Majorana neutrinos and measure the CP asymmetry in their decays. However, in the most natural seesaw scenarios, these states are simply too heavy to be produced, while if they are light, then their Yukawa couplings must be very tiny, again preventing any chance of direct measurements.

Lacking the possibility of a direct proof, experiments can still provide circumstantial evidence in support of leptogenesis by establishing that (some of) the Sakharov conditions for leptogenesis are realized in nature. Planned neutrinoless double beta decay ($0\nu\beta\beta$) experiments (GERDA [244], MAJORANA [245], CUORE [246]) aim at a sensitivity to the effective $0\nu\beta\beta$ neutrino mass in the few $\times$ 10 meV range. If they succeed in establishing the Majorana nature of the light neutrinos, this will strengthen our confidence that the seesaw mechanism is at the origin of the neutrino masses and, most importantly, will establish that the first Sakharov condition for the dynamical generation of a lepton asymmetry (L violation) is realized. Proposed SuperBeam facilities [247, 248] and second generation off-axis SuperBeam experiments (T2HK [249], NOνA [250]) can discover CP violation in the leptonic sector. These experiments can only probe the Dirac phase of the neutrino mixing matrix. They cannot probe the Majorana low energy or the high energy phases, but the important point is that they can establish that the second Sakharov condition for the dynamical generation of a lepton asymmetry is satisfied. As regards the third condition, that is that the heavy neutrino decays occurred out of thermal equilibrium, it might seem the most difficult one to test experimentally. In reality the opposite is true, and in fact we already know that an absolute neutrino mass scale of the order of the solar or atmospheric mass differences is perfectly compatible with sufficiently out of equilibrium heavy neutrinos decays.

Given that we do not know how to prove that leptogenesis is the correct theory, we might ask if there is any chance to falsify it. Indeed, future neutrino experiments could weaken the case for leptogenesis, or even falsify it, mainly by establishing that the seesaw mechanism is not responsible for the observed neutrino masses. By itself, failure in revealing signals of $0\nu\beta\beta$ decays will not disprove leptogenesis. Indeed, with normal neutrino mass hierarchy one expects that the rates of lepton-number-violating processes are below experimental sensitivity. However, if neutrinos masses are quasi-degenerate or inversely hierarchical, and future measurements of the oscillation parameters will not fluctuate too much away from the present best fit values, the most sensitive $0\nu\beta\beta$ decay experiments scheduled for the near future should be able to detect a signal [251]. If instead the limit on $|m_{\beta\beta}|$ is pushed below $\sim 10\,\mathrm{meV}$ (a quite challenging task), this would suggest that either the mass hierarchy is normal, or neutrinos are not Majorana particles. The latter possibility would disprove the seesaw model and standard leptogenesis. Thus, determining the order of the neutrino mass spectrum is extremely important to shed light on the connection between $0\nu\beta\beta$ decay experiments and leptogenesis. In summary, if it is established that the neutrino mass hierarchy is inverted and at the same time no signal of $0\nu\beta\beta$ decays is detected at a level $|m_{ee}| \lesssim 10\,\mathrm{meV}$, one could conclude that the seesaw is not at the origin of the neutrino masses, and that (standard) leptogenesis is not the correct explanation of the baryon asymmetry. As concerns CP violation, a failure in detecting leptonic CP violation will not weaken the case for leptogenesis in a significant way. Instead, it would mean that the Dirac CP phase is small enough to render L-conserving CP-violating effects unobservable.

Finally, the CERN LHC has the capability of providing information that is relevant to leptogenesis, since it can play a fundamental role in establishing that the origin of the neutrino masses is not due to the seesaw mechanism, thus leaving no strong motivation for leptogenesis. This may happen in several different ways. For example (assuming that the related new physics is discovered), the LHC will be able to test if the detailed phenomenology of any of the following models is compatible with an explanation of the observed pattern of neutrino masses and mixing angles: supersymmetric R-parity violating couplings and/or L-violating bilinear terms [252, 253]; leptoquarks [254, 255]; triplet Higgses [256, 257]; new scalar particles of the type predicted in the Zee-Babu [258, 259] types of models [260, 261, 262]. It is conceivable that such discoveries can eventually exclude the seesaw mechanism and rule out leptogenesis.

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