Seven lessons for inflationary model-builders (in string theory)

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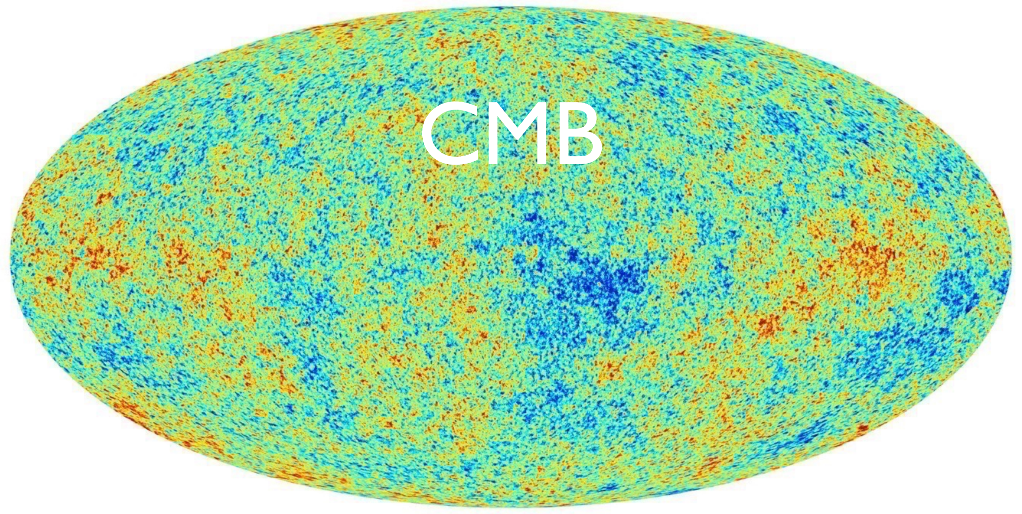
CNRS - Institut d'Astrophysique de Paris

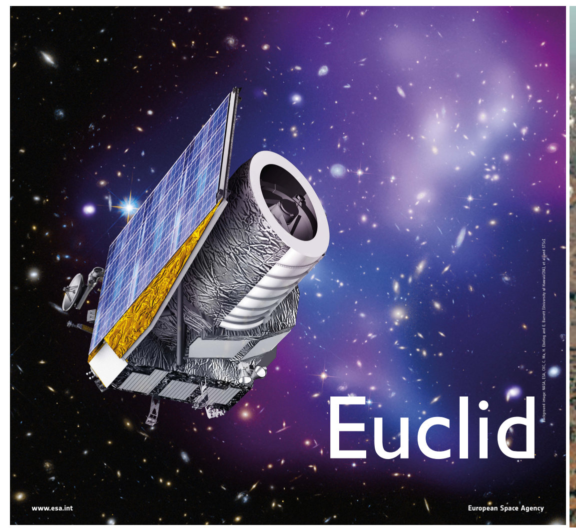
APC, String-Cosmo day, November 28th 2023

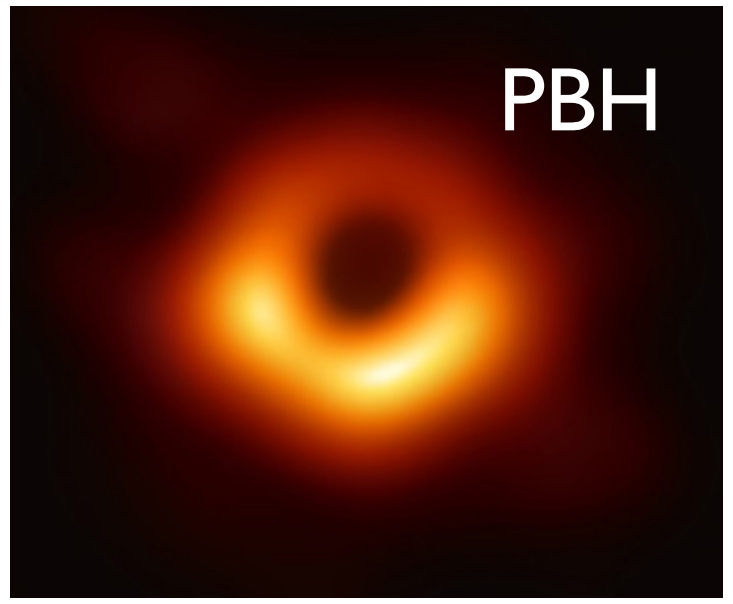
European Research Council
Established by the European Commission

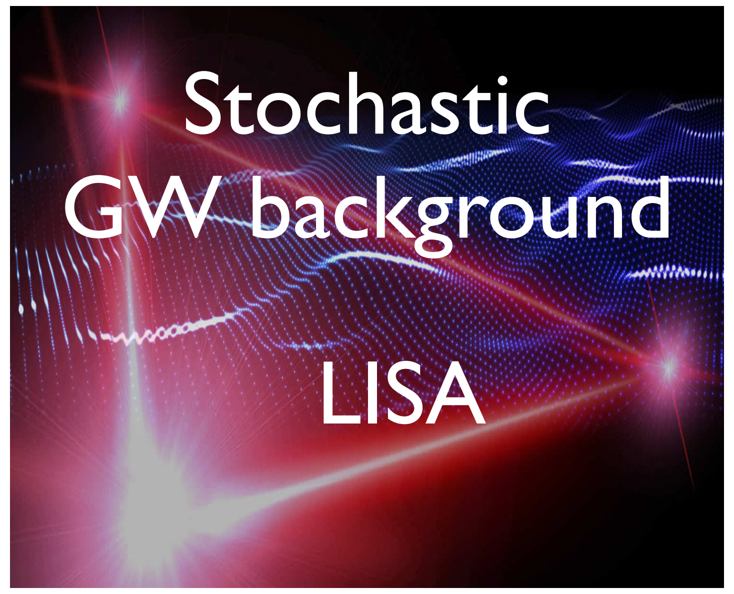
Inflation: quantum + gravitational physics, tested observationally!

+









obvious appeal for string theory

At least 3 reasons for string theorists to care about inflation

Cosmological correlation functions: probe QFT in curved backgrounds

Inflation: peculiar phenomenon sensitive to Planck-scale physics

Inflation as a cosmological collider   Energy scale $\lesssim 10^{14}\,\mathrm{GeV}$

Motivations for my talk

Hard string-theory work:

$$S_{10}[\mathcal{C}] \to S_4[\Phi(t)]$$

compactification, stabilization, hierarchies

$$M_{\rm SUSY} \ < \ H \ < \ M_{\rm KK} \ < \ M_{\rm s} \ < \ M_{\rm pl}$$

Baumann, McAllister,
Inflation and String Theory, 2014

Now,
simple inflationary predictions
from potential, isn't it?

Bad news: no!

Good news: many interesting physics!

Motivations for my talk

(some) Big questions:

$$\Lambda > 0?$$

swampland vs low-energy EFT from quantum gravity

More modest goal today:

avoid writing a wrong paper

being aware of inflationary subtleties
and help identify signatures

Disclaimer: not specific to inflation in string theory
(current inflationary constructions in string theory are low-energy QFT)

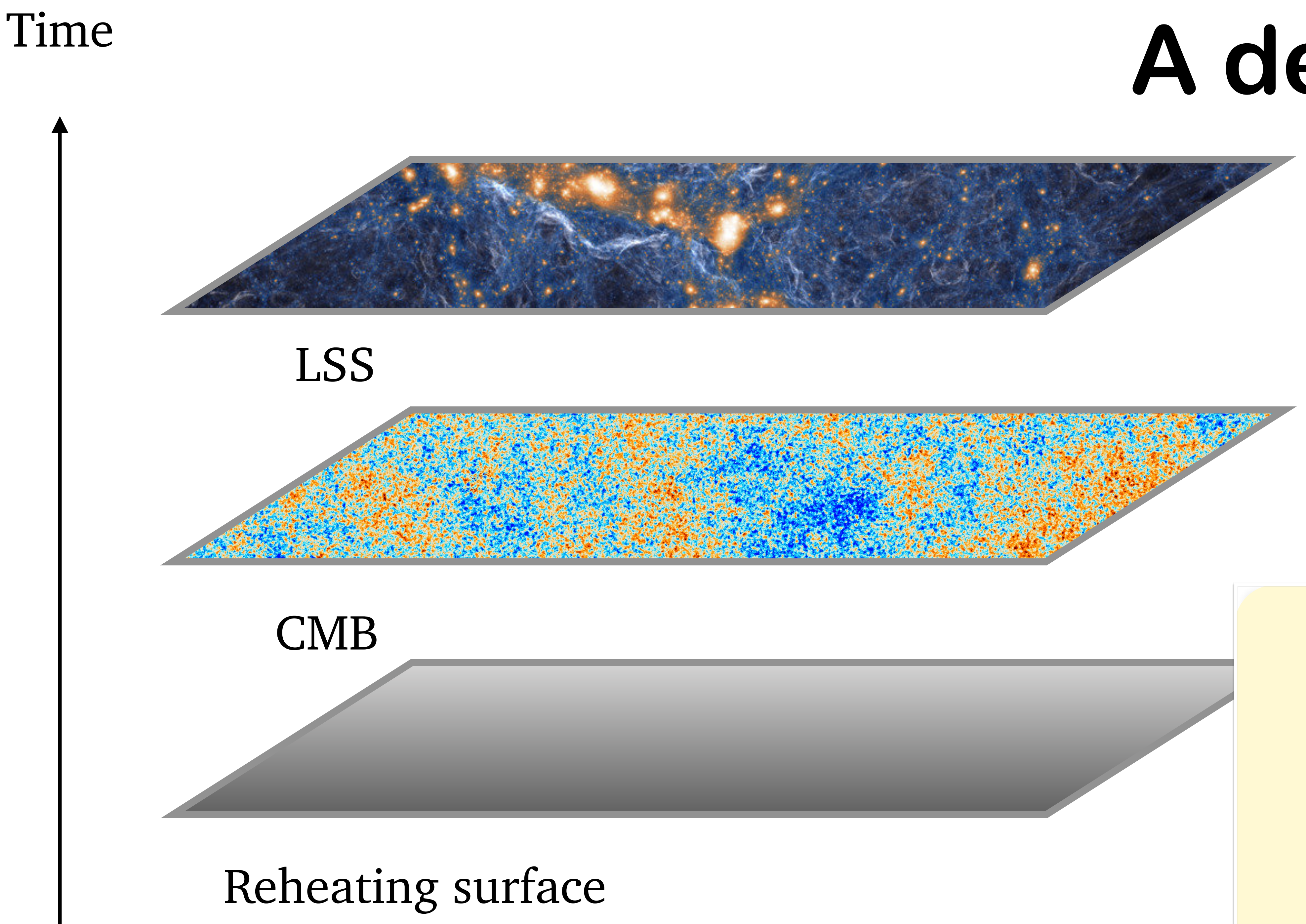
Outline

I. Light review of inflation

II. 7 lessons

I. Light review of inflation

A detective's work



Observations

observational data

Statistical properties

$$\mathbb{P}\left(\frac{\delta\rho}{\rho}, h_{ij}\right)$$

Physics of inflation?

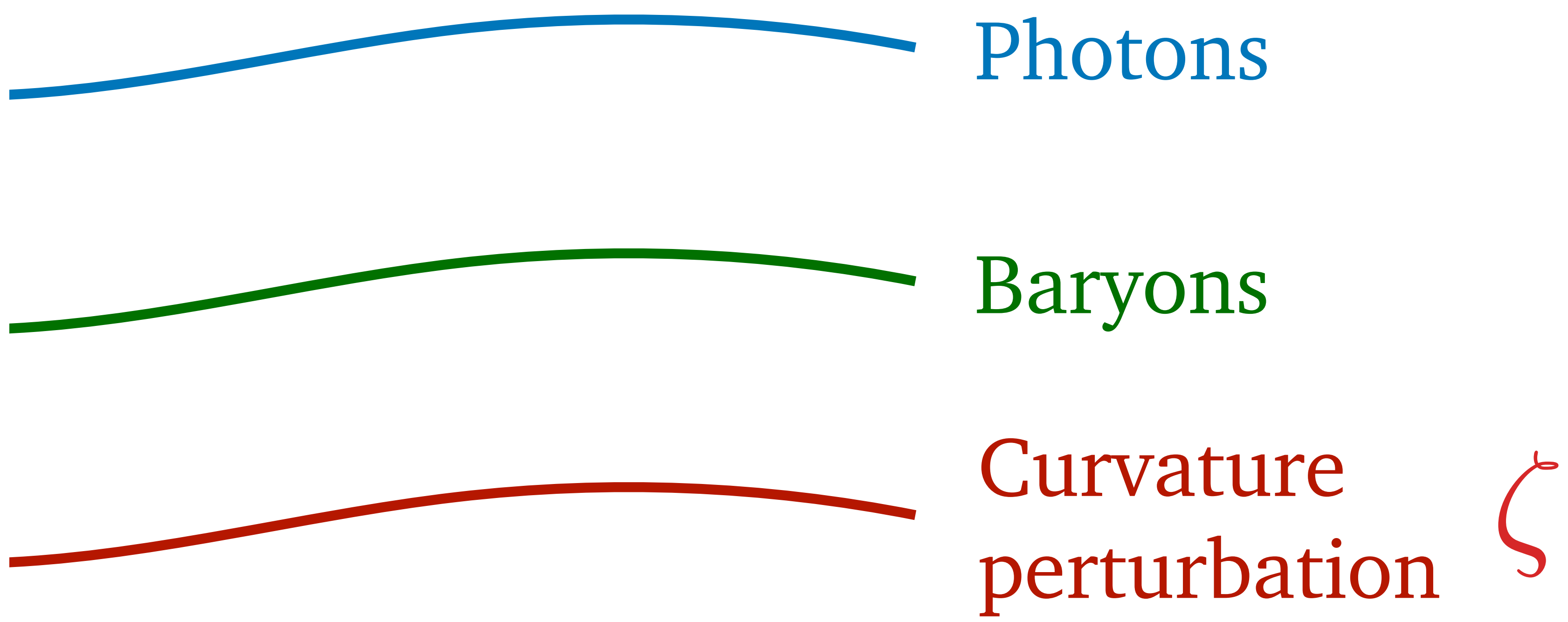
theoretical data

Clues so far

Planck [2018]

Adiabatic

$$\delta_X(\boldsymbol{x}) \propto \delta_Y(\boldsymbol{x})$$

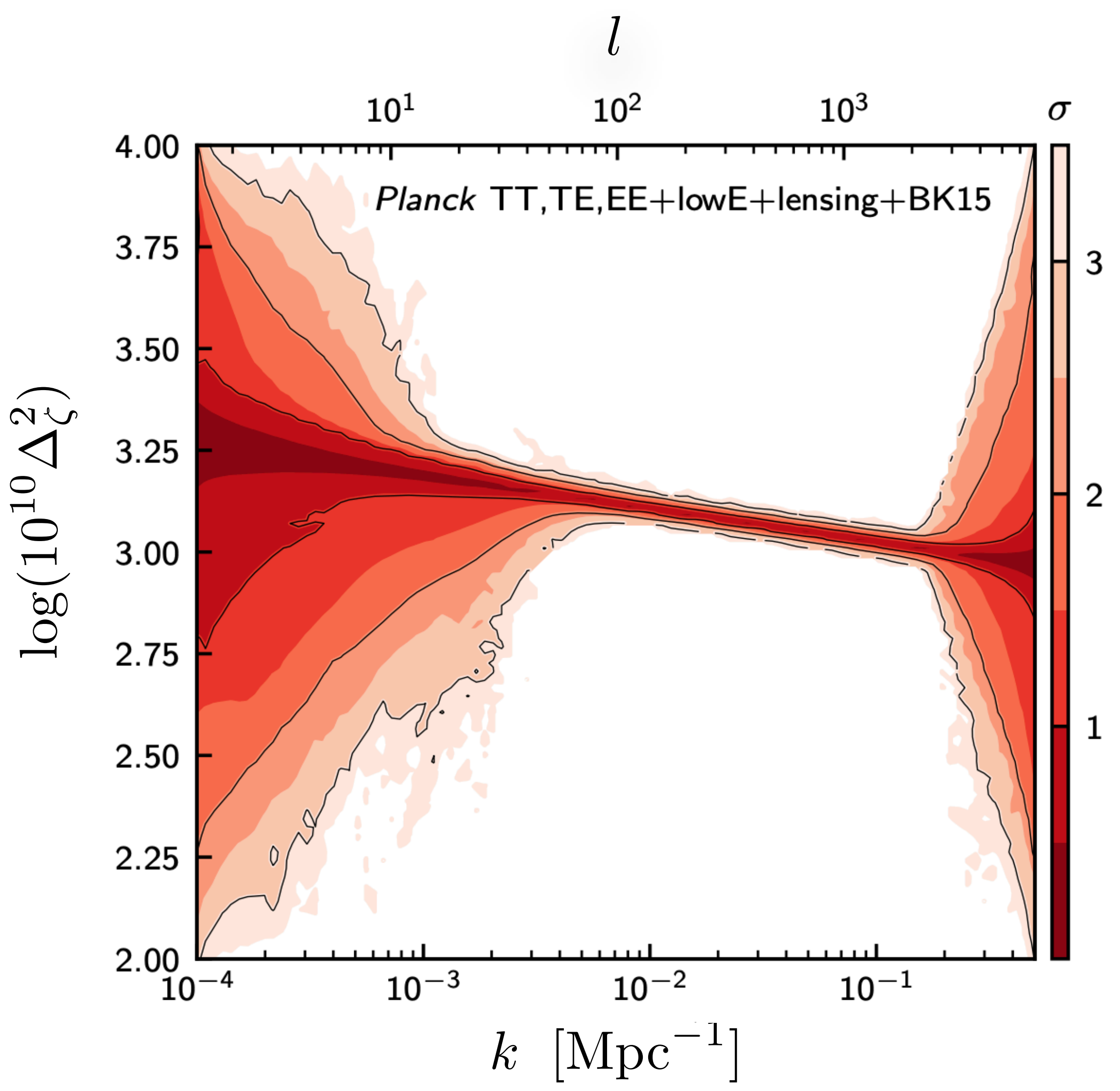


$$g_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

Single fluctuating scalar degree of freedom left over

Almost scale-invariant

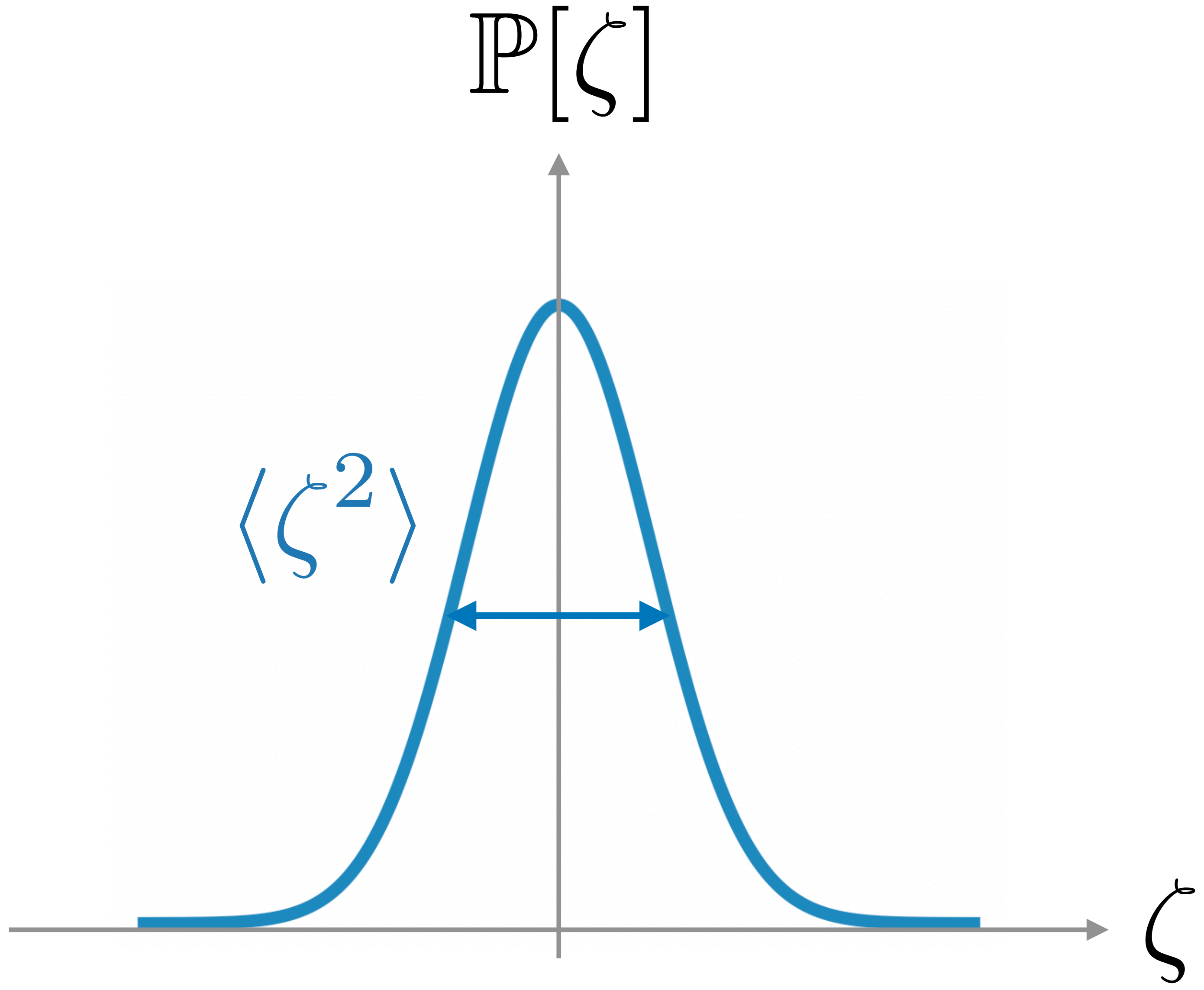
$$\Delta_\zeta^2 = \frac{k^3}{2\pi^2} \langle \zeta_{\boldsymbol{k}} \zeta_{-\boldsymbol{k}} \rangle' = A_s \left(\frac{k}{k_\star} \right)^{n_s - 1}$$



$$n_s = 0.9652 \pm 0.0042$$

Approximate time-translation invariance

Very gaussian



$$\frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} < 10^{-3}$$

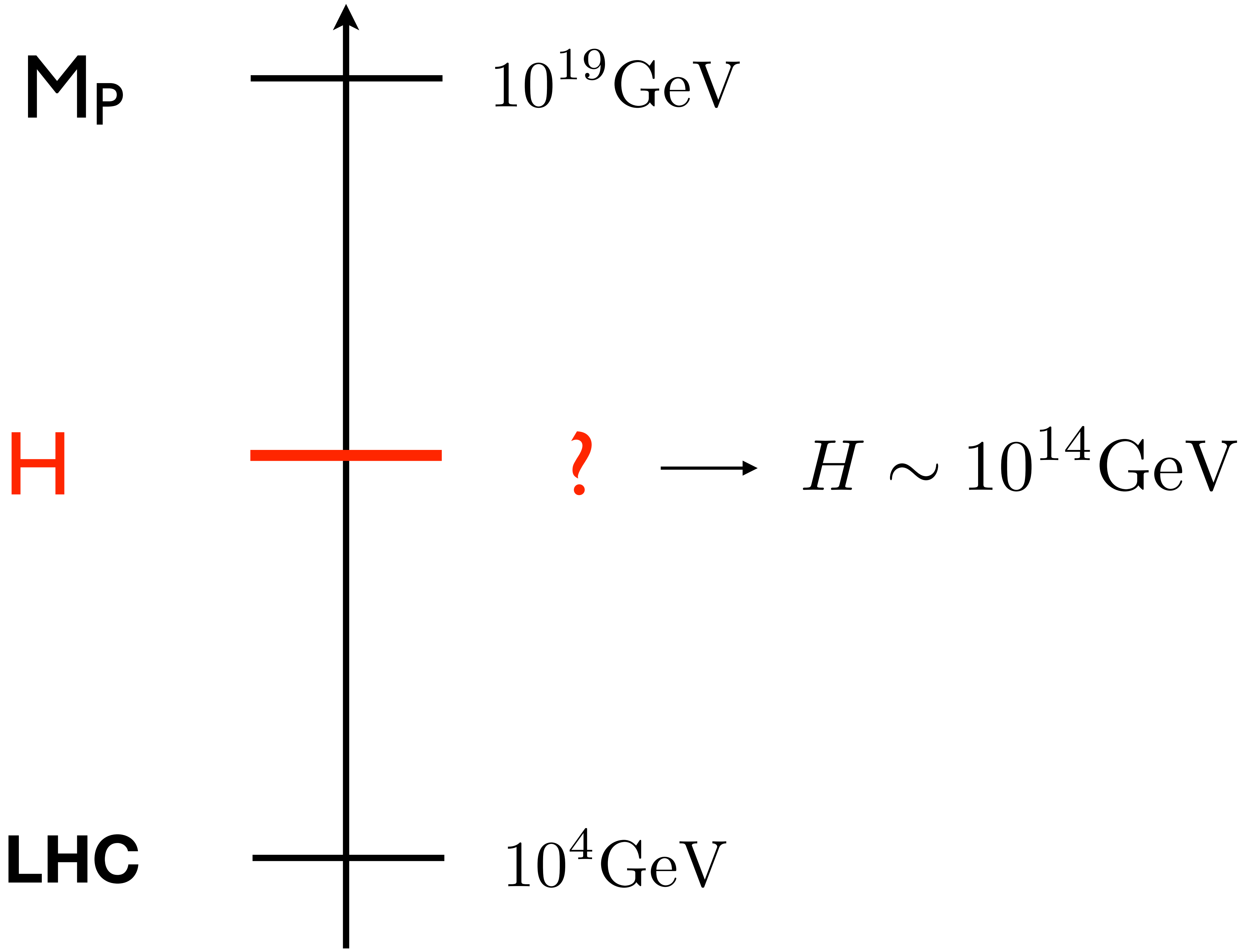
Weakly coupled theory

Clues so far

Planck [2018]

Primordial gravitational waves
from B-modes polarization of CMB

$$\frac{\langle h_{ij} h^{ij} \rangle}{\langle \zeta\zeta \rangle} \lesssim 10^{-2}$$



Detection would be spectacular
(hint about gravity at Planck scale)

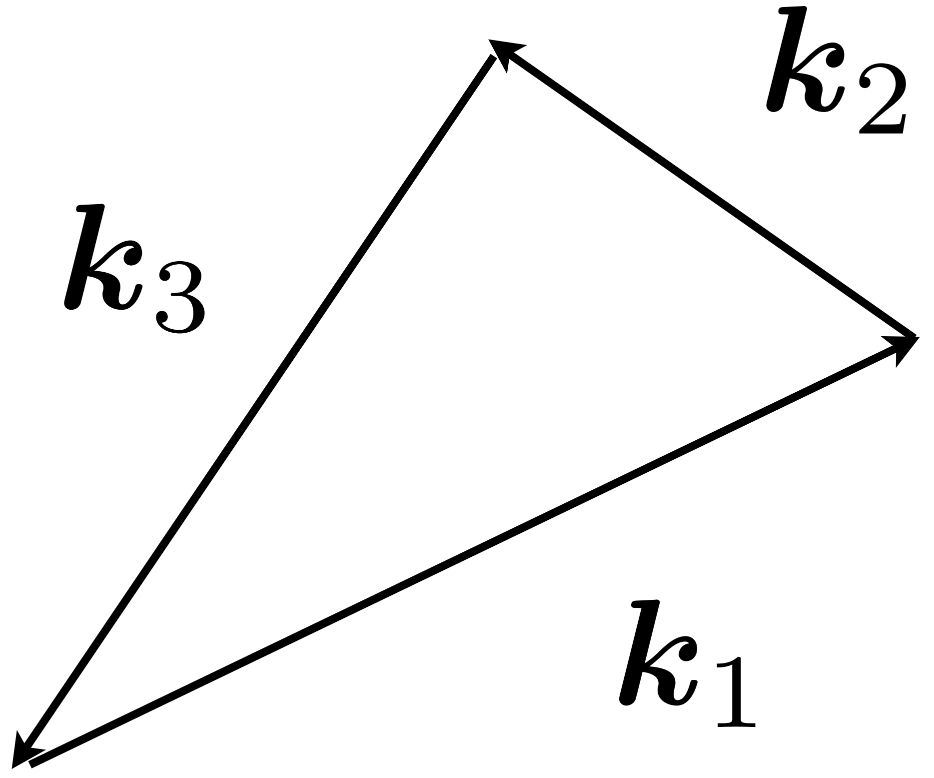
No useful theoretical lower bound:
B-modes may be forever out of reach

Bispectrum

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B_\zeta(k_1, k_2, k_3)$$

Homogeneity

Isotropy



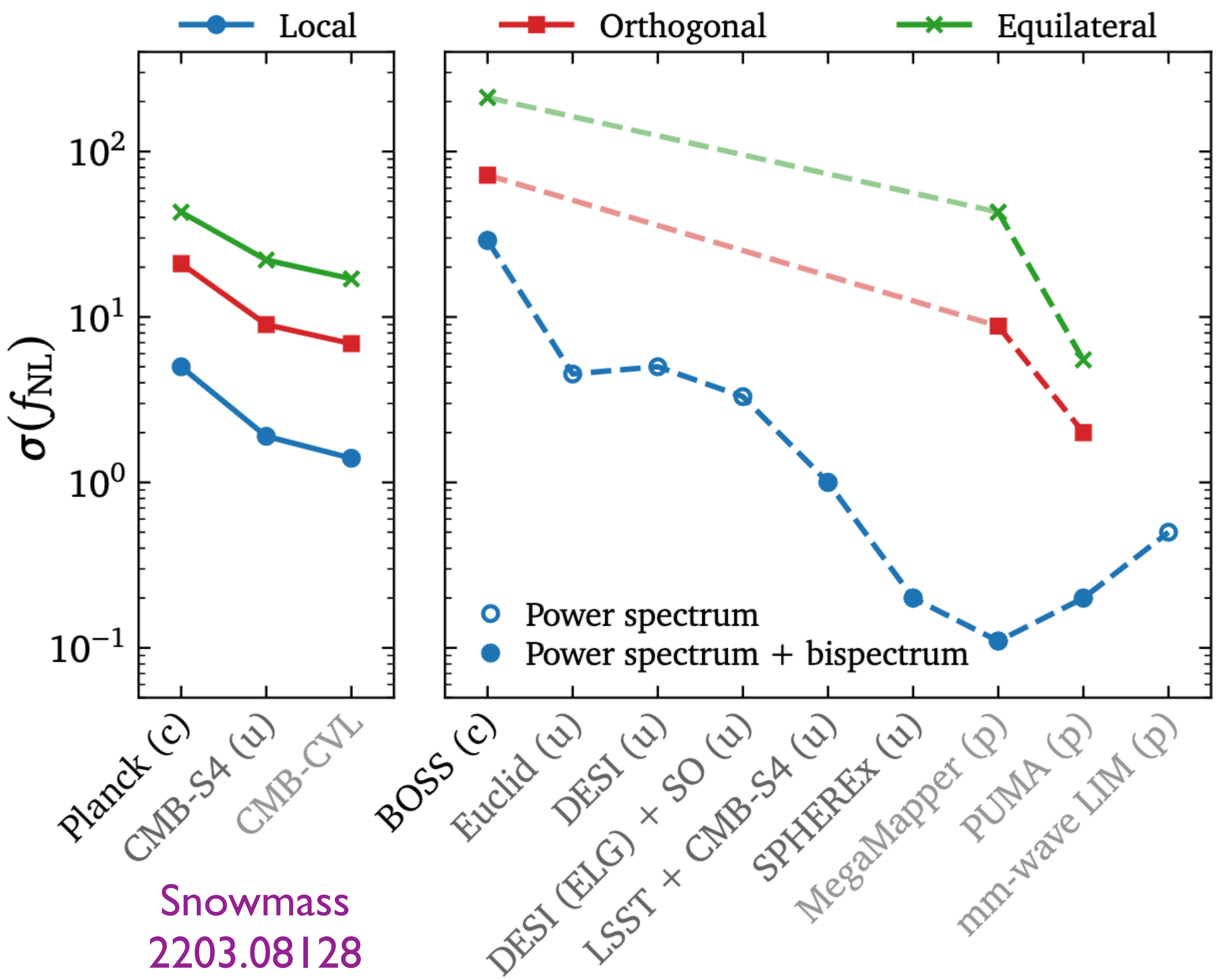
$$B_\zeta \equiv (2\pi)^4 \frac{S(k_1, k_2, k_3)}{(k_1 k_2 k_3)^2} A_s^2$$

Amplitude $S \sim f_{\rm NL}$

Scale-dependence (overall size)

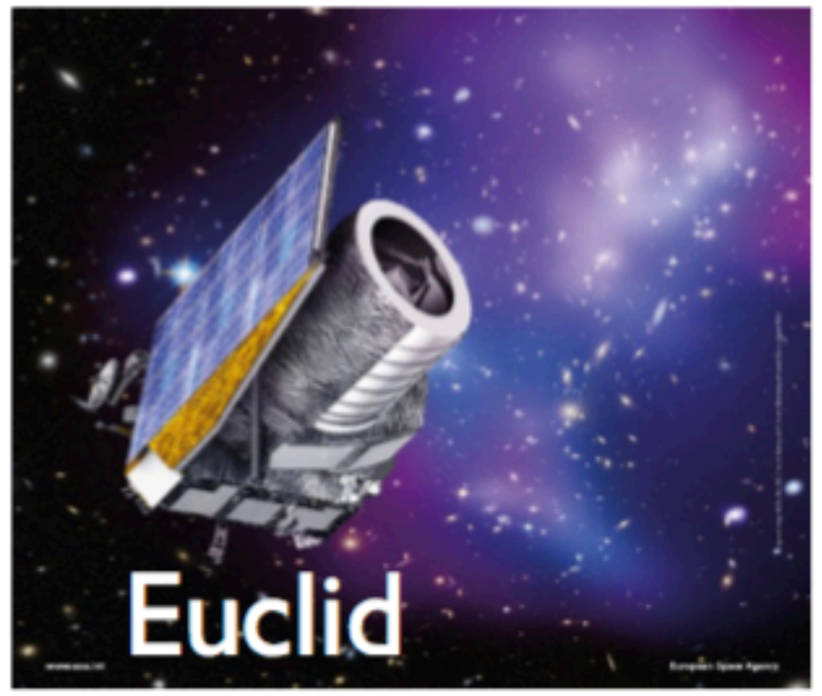
Shape dependence (configuration of triangles)

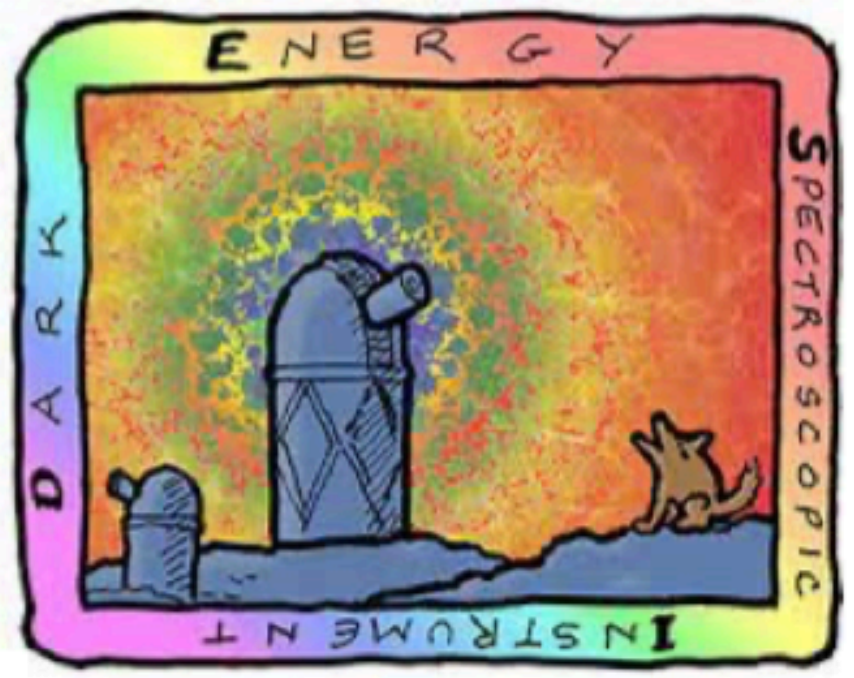
Prospects

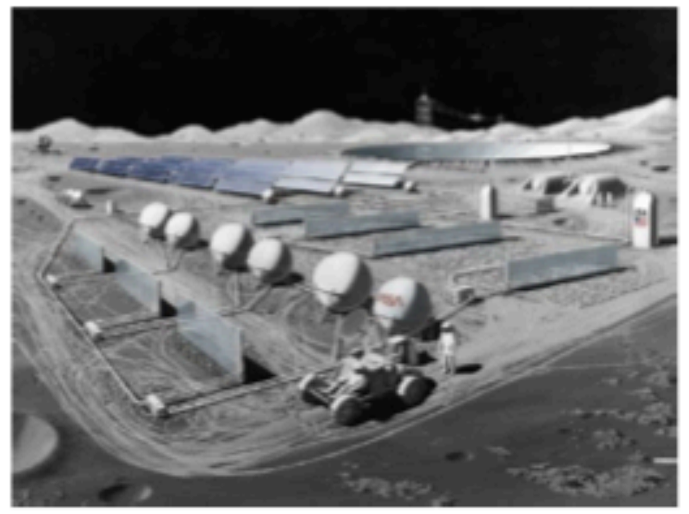


Snowmass
2203.08128

Huge efforts with CMB-S4 &
large-scale structure surveys
(scale-dependent bias, EFT of LSS,
position space maps, simulation based inference etc)

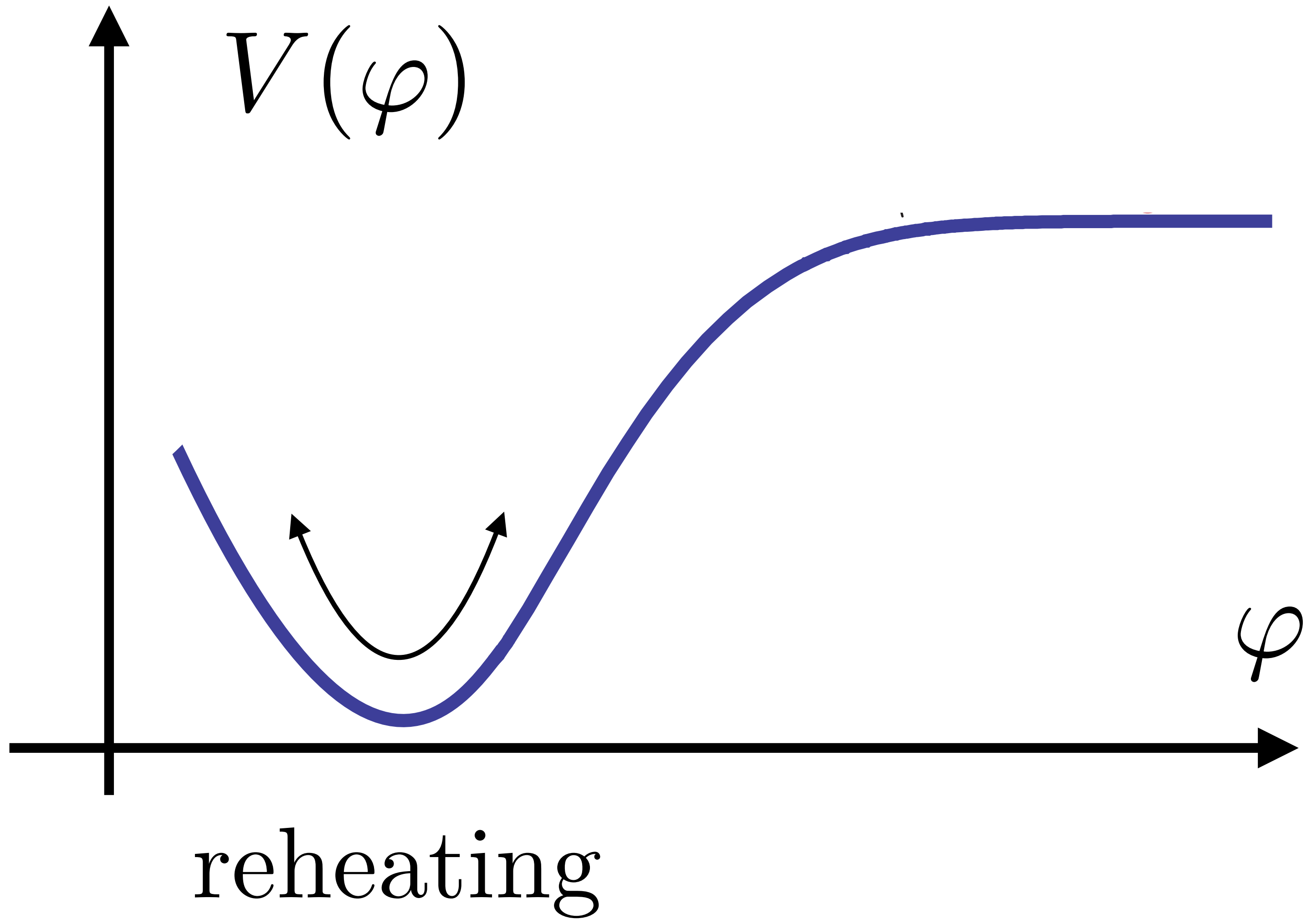






Long-term: 21cm radio-astronomy
from the far side of the moon!
(dark ages)

Simple fit: single-field slow-roll inflation, but it is not natural

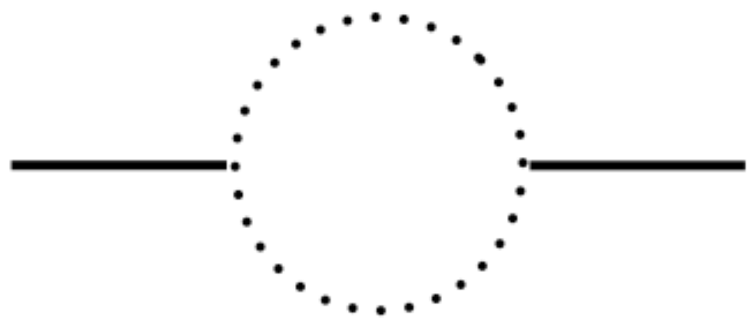


$$\eta \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$

Prolonged phase of inflation

Why is the inflaton so light? $\eta \approx \frac{m_\phi^2}{H^2} \ll 1$

like the Higgs
hierarchy problem



$$m_\phi^2 \sim \Lambda_{\text{cut-off}}^2 \gg H^2$$

Simple fit: single-field slow-roll inflation, but it is not natural

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V_0(\phi) + \sum_\delta \frac{\mathcal{O}_\delta(\phi)}{M^{\delta-4}}$$

Slow-roll action

Corrections to the low-energy effective potential

$$\frac{\Delta m_\phi^2}{H^2} \sim \left(\frac{M_{\rm Pl}}{M}\right)^2$$

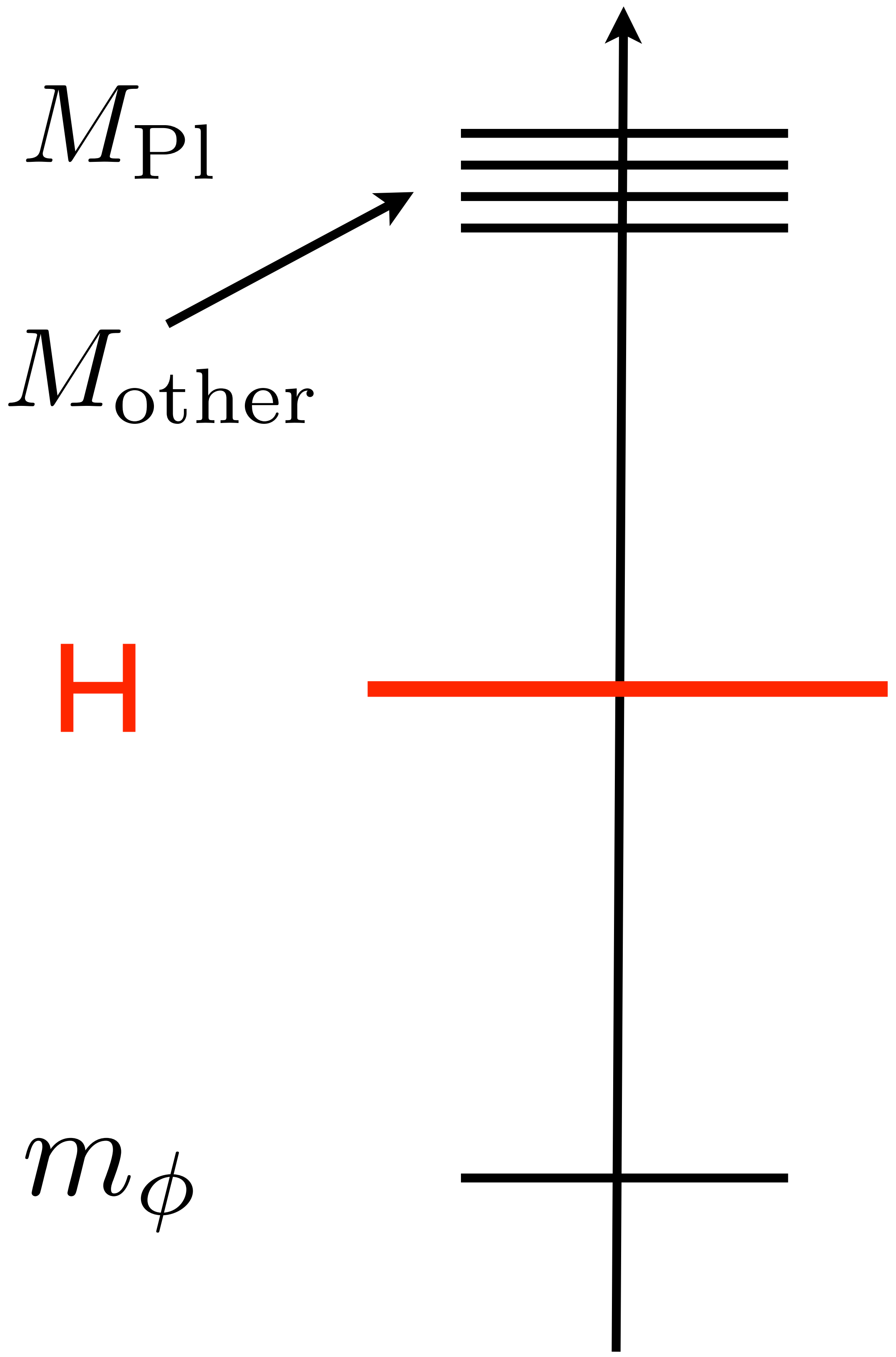
$$\Delta\eta \gtrsim 1$$

Planck-scale physics does not decouple

Symmetries do not help

Guidance from string theory?

Hope:



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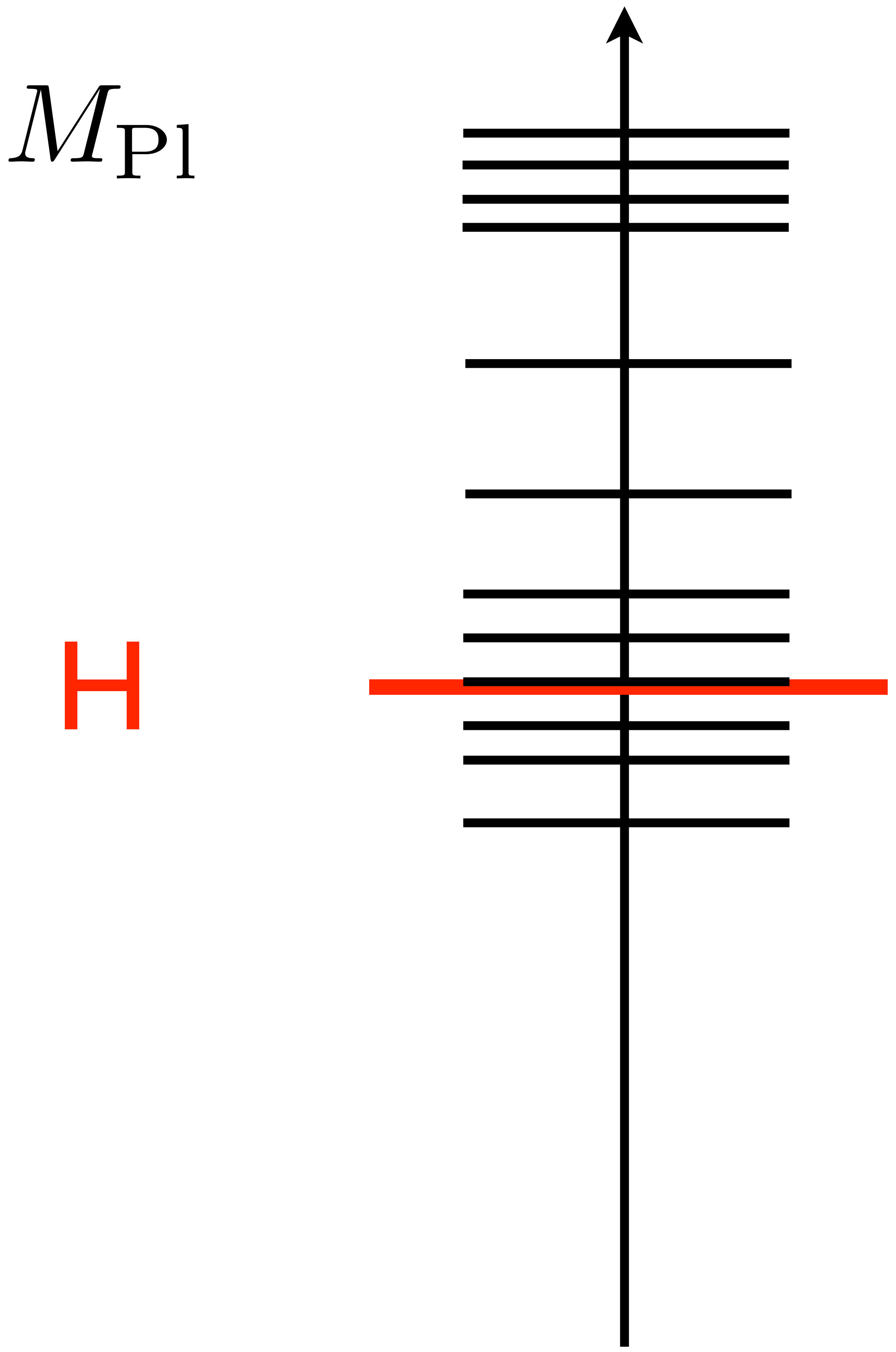


Guidance from string theory?

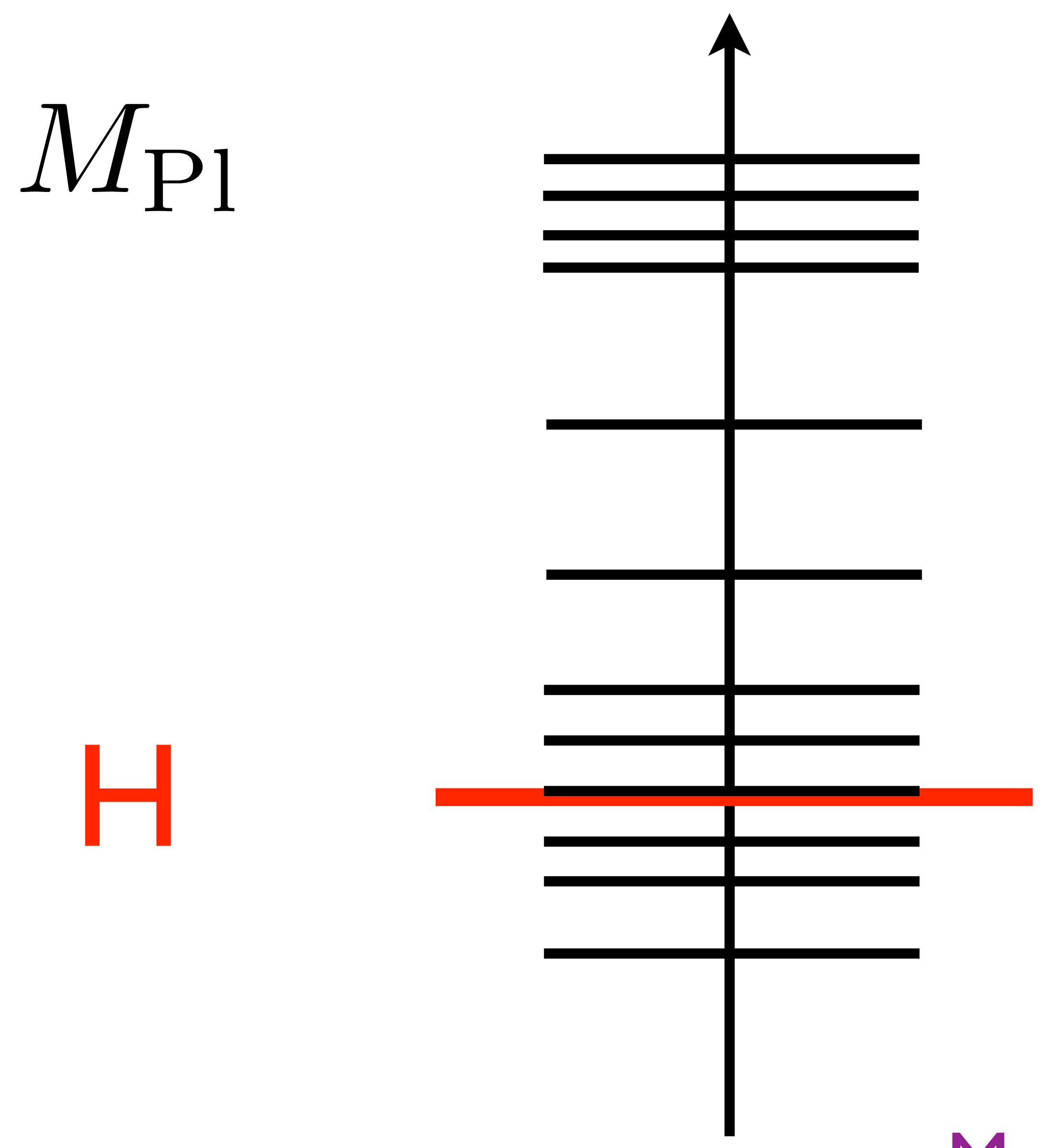
Top-down constructions:
difficult and rarely done consistently

But general picture: Baumann McAllister, 2014

- Multiple degrees of freedom
- Steep potentials
- Large couplings

→ How can data be so simple?

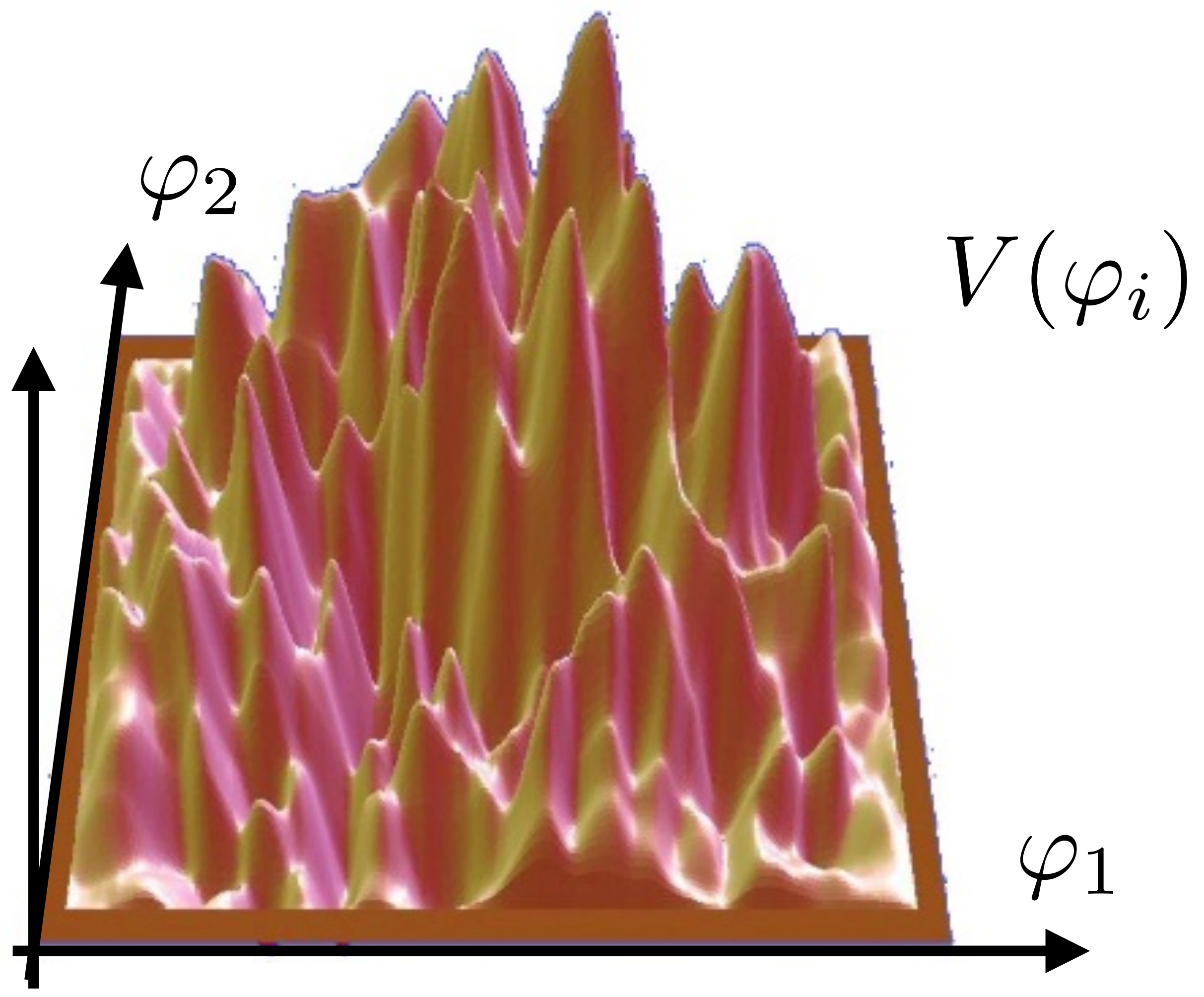
Find:



McAllister, Renaux-Petel, Xu, 2012

Single-field slow-roll: at best emergent approximate description

Some annoying facts



→ Often only 2-3 e-folds of inflation

I can do nothing for you

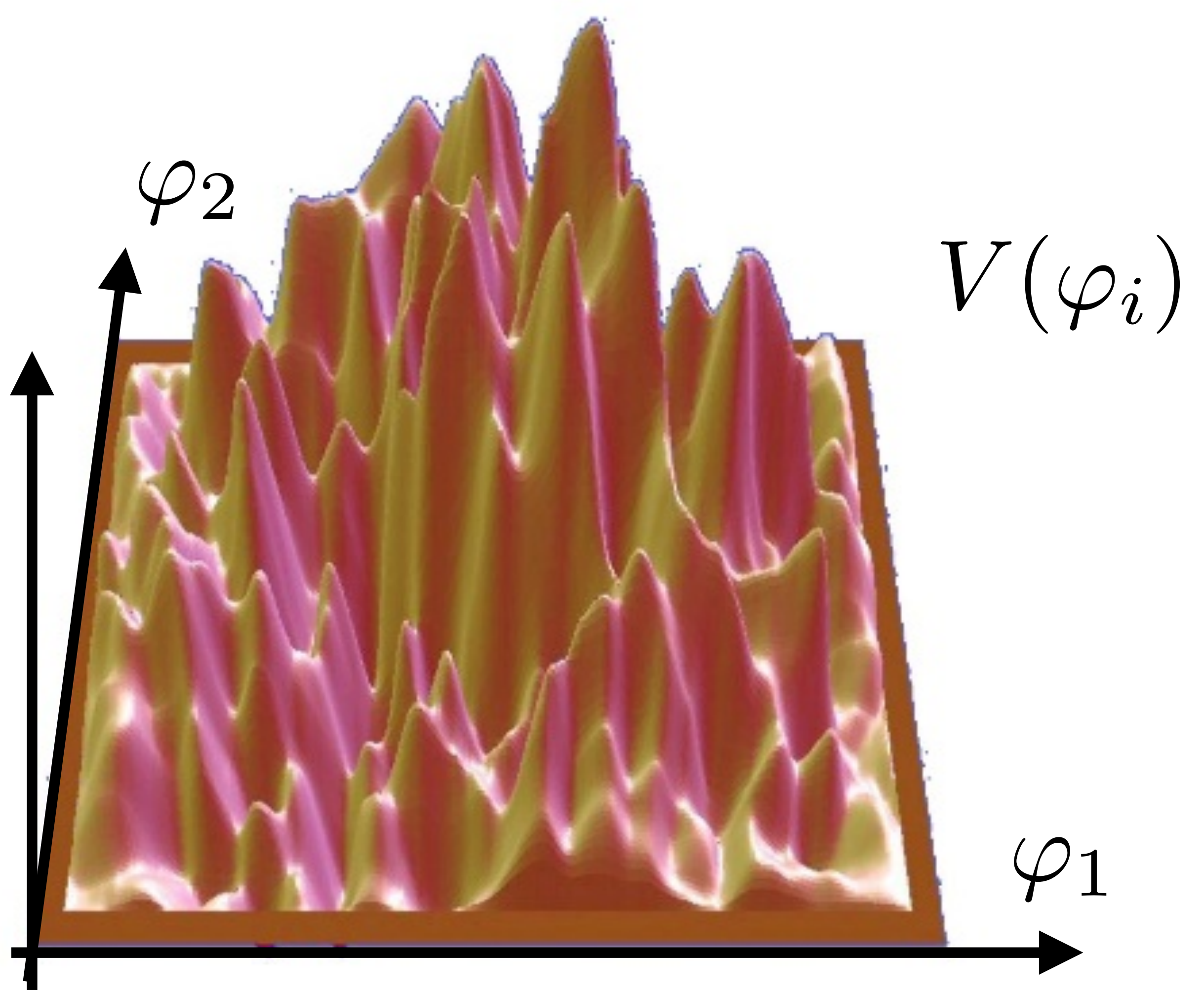
→ If you can make 7 e-folds, already non-trivial, enough for CMB and LSS

→ No unique attractor trajectory in general: depends on initial conditions

I care about our observable universe:
I consider the inflationary history followed in our part of the universe

II. 7 lessons

Lesson 1: inflation by itself is not predictive in general!



Existence of additional light degrees of freedom (beyond the effective inflaton)

$m \ll H \longrightarrow$ $\dot{\zeta} \neq 0$ on super-Hubble scales

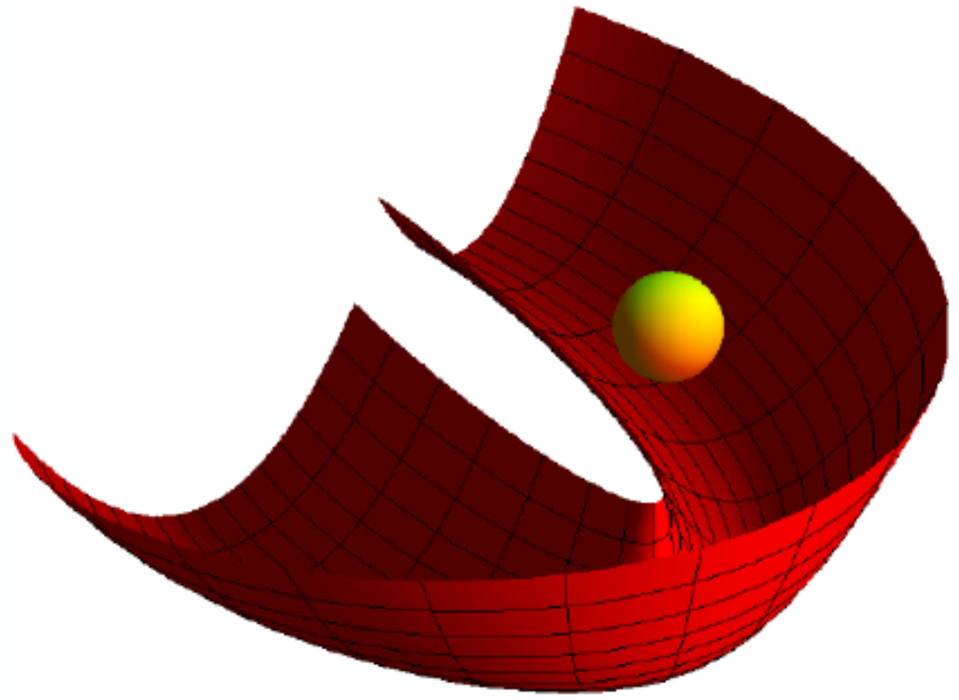
You need to take into account reheating (and you don't want that)

In the following, I consider that an adiabatic limit is reached by end of inflation

Lesson 2: integrating out is not truncating

Do not throw away
heavy fields!

$$m \gg H$$



$$\mathcal{L}(\phi, \chi) \xrightarrow[\text{Integrating out}]{\chi \text{ heavy}} \tilde{\mathcal{L}}(\phi) \neq \mathcal{L}_{\mathrm{slow-roll}}$$

$$\langle \chi \rangle = f(\phi, X = -\frac{1}{2}(\partial\phi)^2)$$

$$\mathcal{L} \supset \frac{\epsilon}{c_s^2}\left(\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right)$$

Reduced 'speed of sound' of fluctuations ...

$$+\left(\frac{1-c_s^2}{H}\right)\dot{\zeta}\frac{(\partial\zeta)^2}{a^2}$$

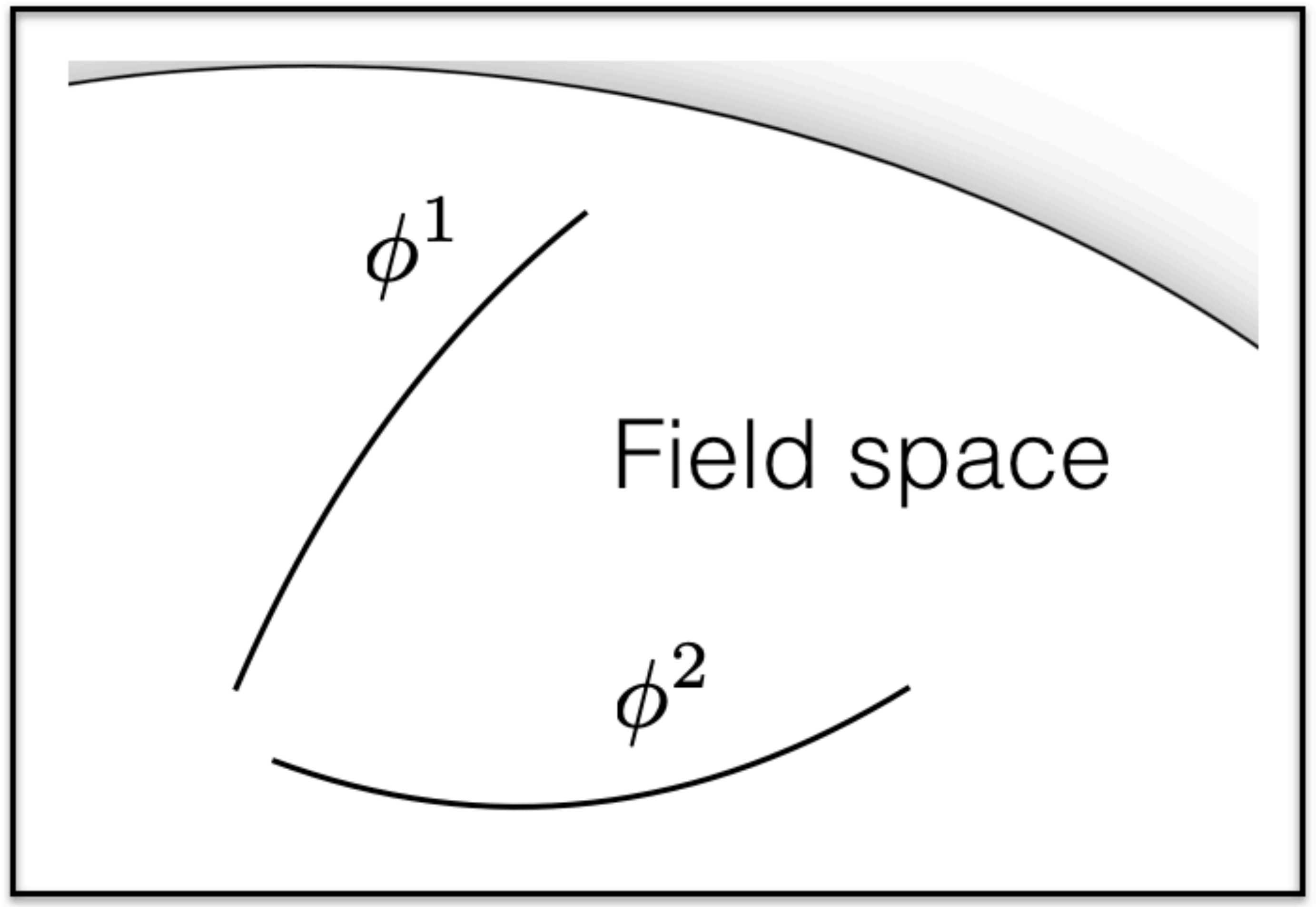
... comes with derivative interactions and non-Gaussianities

Lesson 3: curved field space matters (a lot)

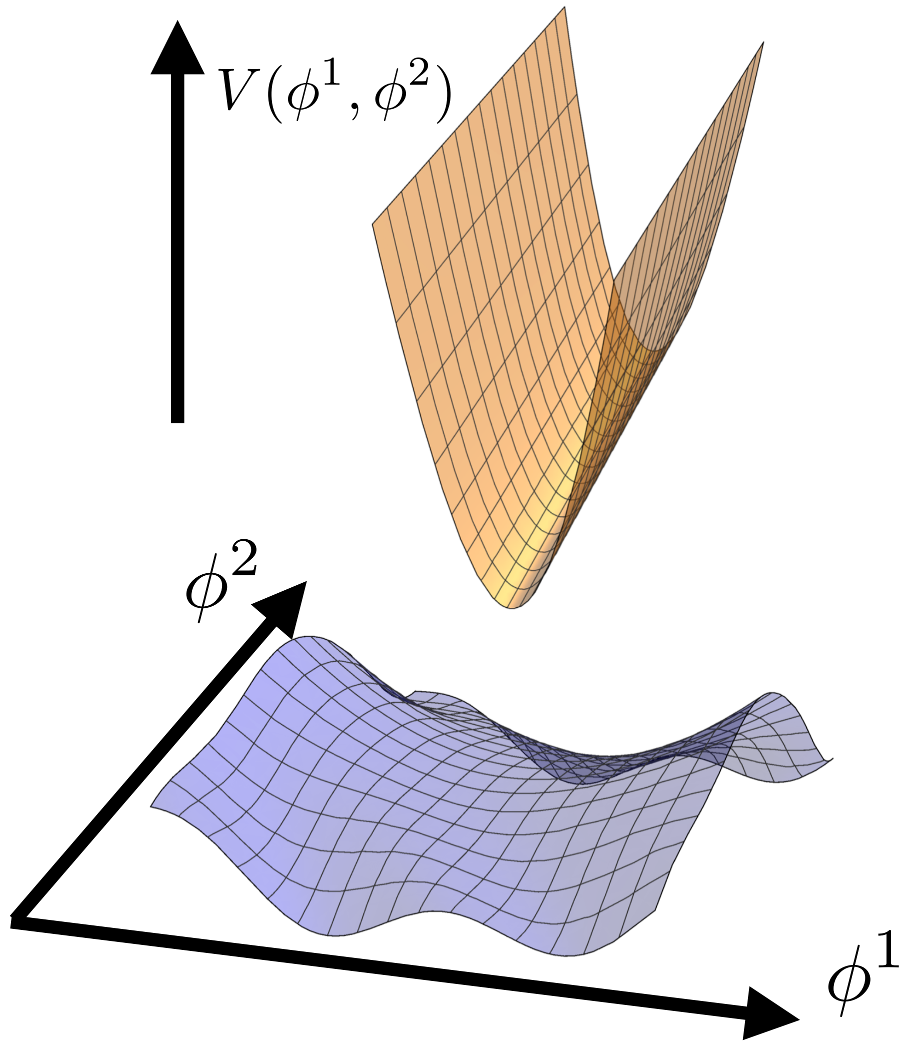
$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}G_{IJ}(\phi^K)\partial_\mu\phi^I\partial^\mu\phi^J - V(\phi^I)\right)$$

Top-down (e.g. sugra) or bottom-up (EFT)

Curved field space is generic



Invariance under field redefinitions: fields are coordinates on a field space, with metric G_{IJ}



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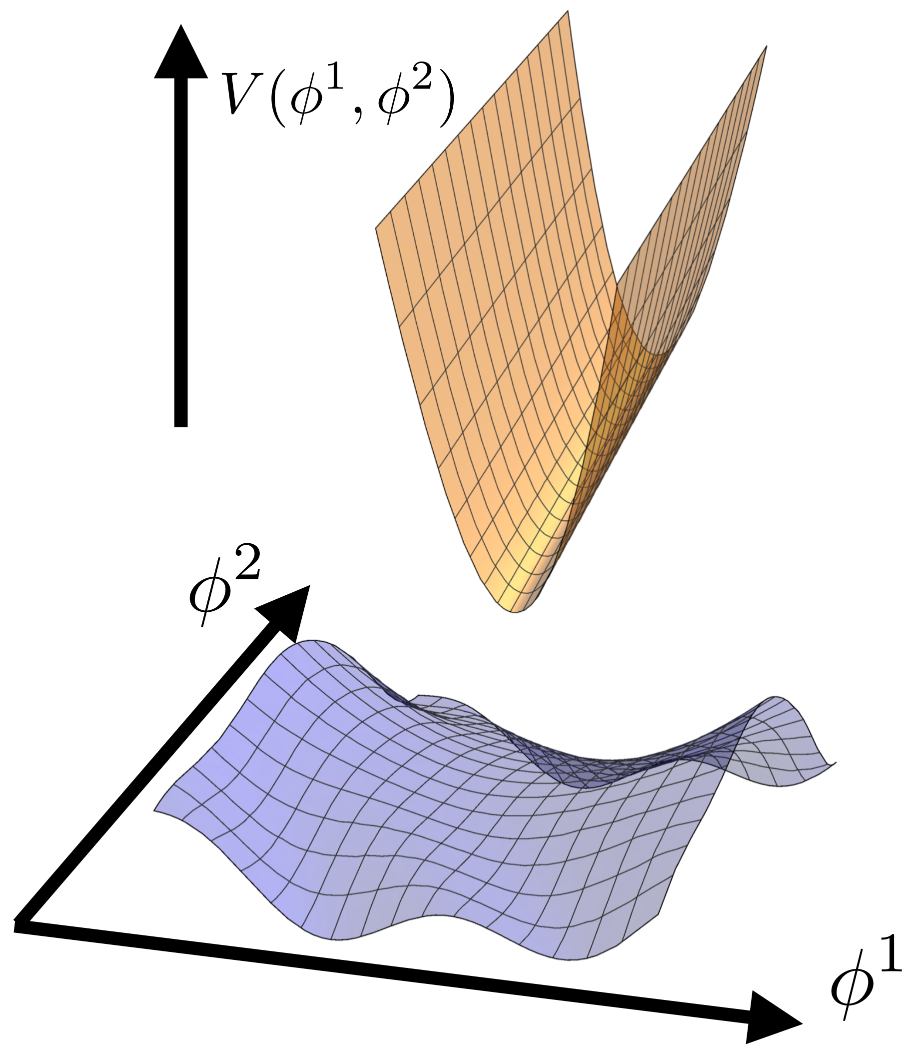
$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}G_{IJ}(\phi^K)\partial_\mu\phi^I\partial^\mu\phi^J - V(\phi^I)\right)$$

Top-down (e.g. sugra) or bottom-up (EFT)

Curved field space is generic

Studied for a long time, but impact and consequences not fully appreciated before: (e.g.)



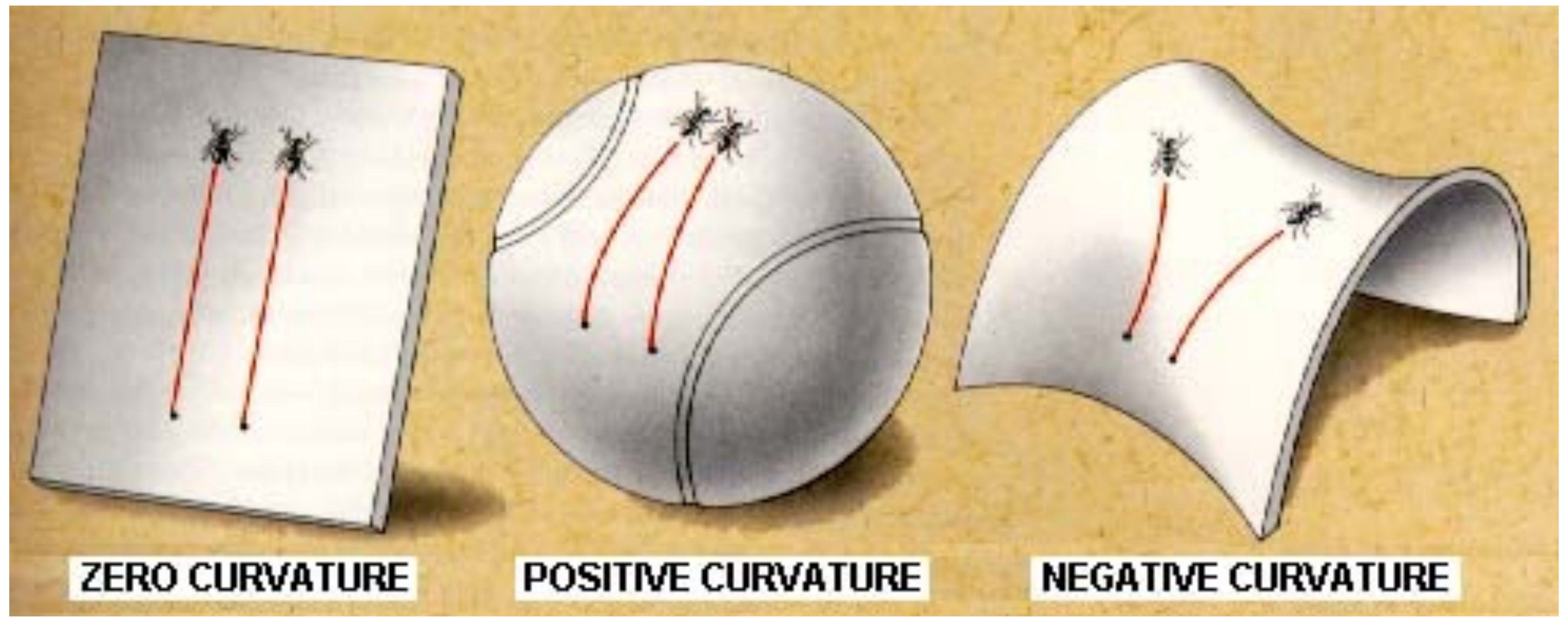
- Landscape studies with random potential

- conditions for successful inflation usually (wrongly) formulated only in terms of Hessian of potential

$$\nabla^I\nabla_J V/V$$

Geometrical destabilization of inflation

Initially neighboring geodesics tend to fall away from each other in the presence of negative curvature.

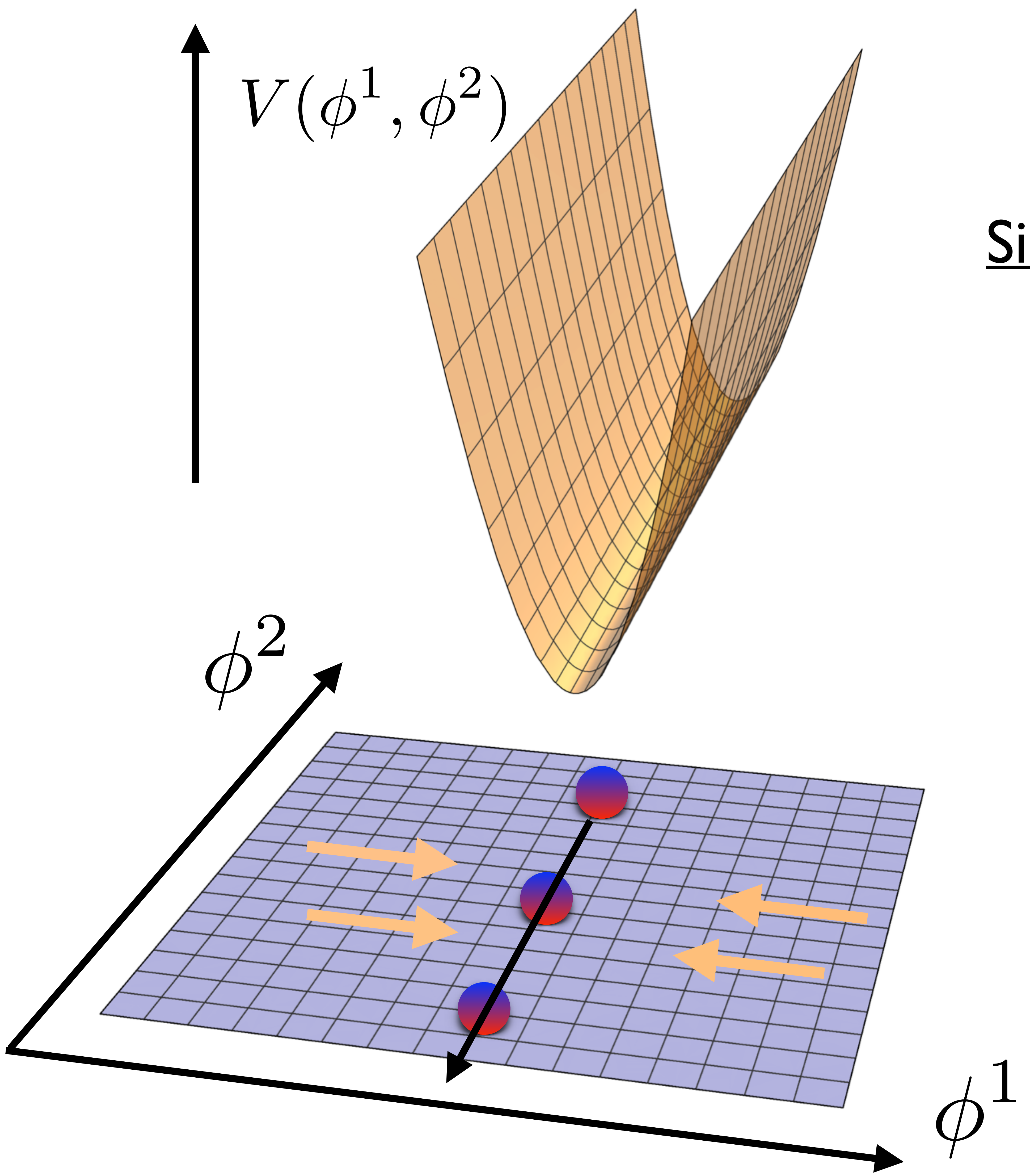


This effect applies during inflation,
it easily overcomes the effect of the potential,
and can destabilize inflationary trajectories.

Renaux-Petel, Turzynski, 16
PRL Editors' Highlight

Basic mechanism

Renaux-Petel, Turzynski, 16
PRL Editors' Highlight



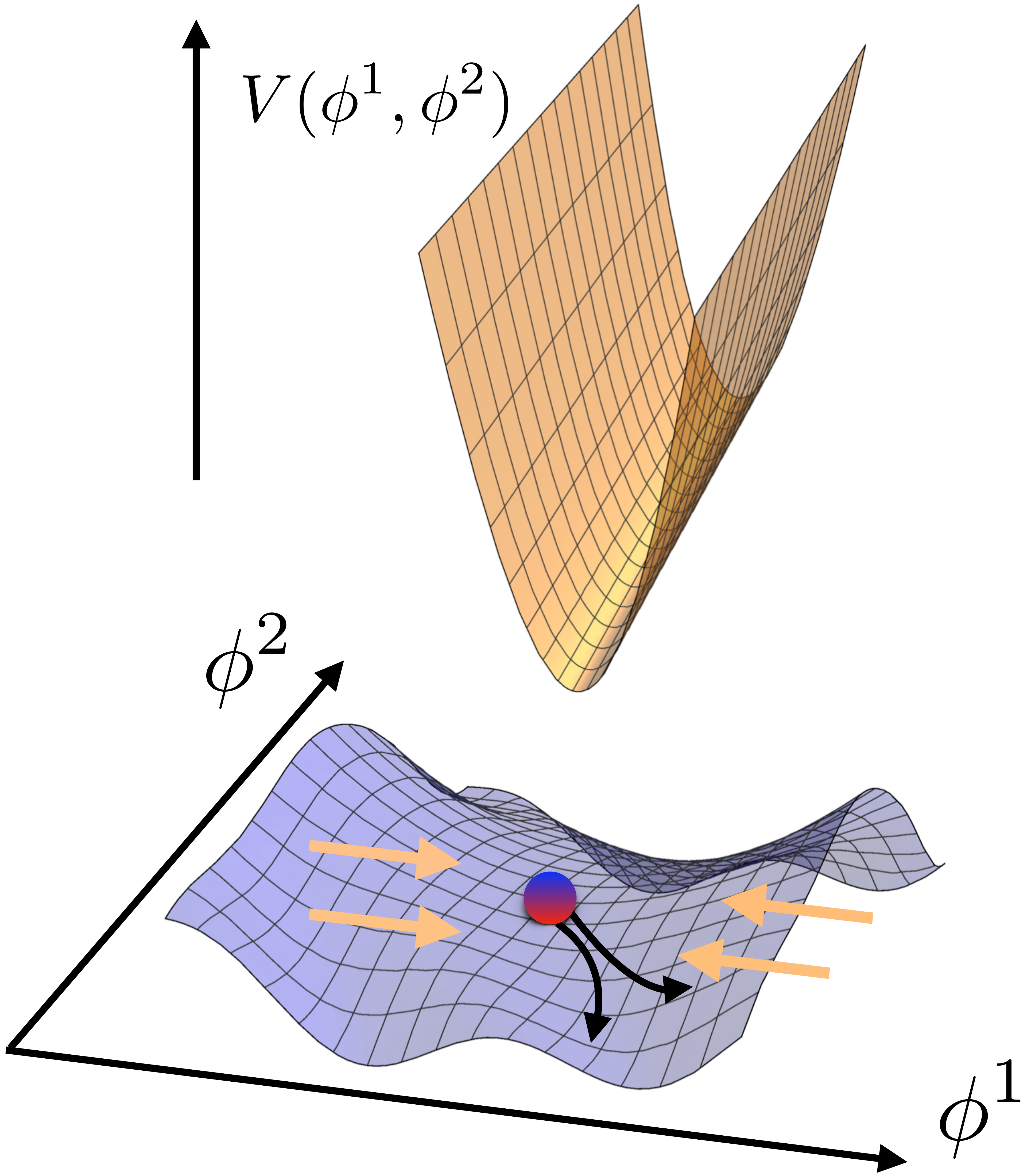
Simplest 'realistic' models (hope):

Light inflaton
+
Extra heavy fields

Effective
single-field dynamics

(valley with steep walls)

Basic mechanism



Renaux-Petel, Turzynski, 16
PRL Editors' Highlight

More realistic:

Light inflaton
+
Extra +/- heavy fields
+
Curved field space

→ Competing effects of potential and geometry

Geometrical instability

Be careful to hierarchies

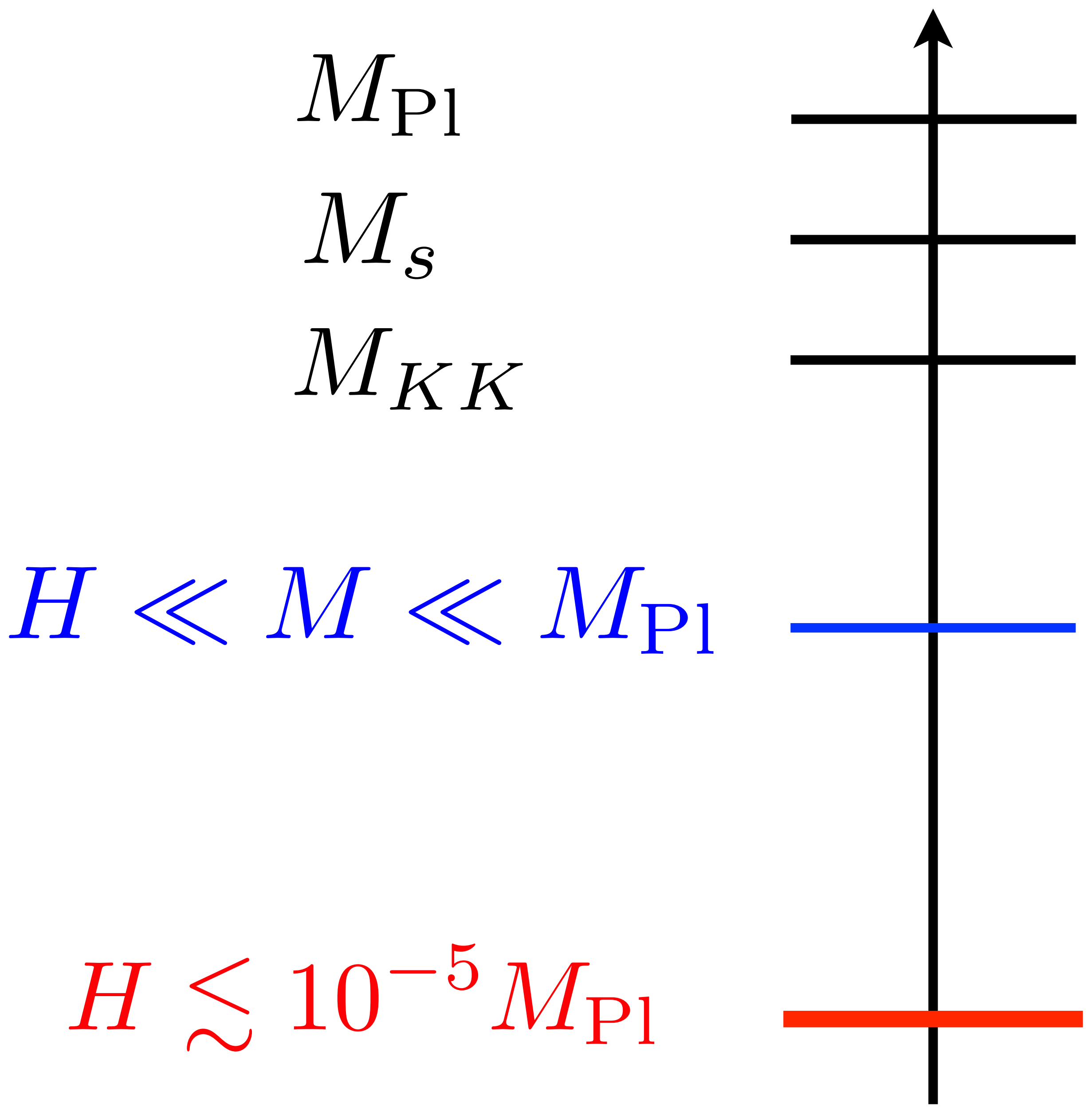
$$\ddot{Q}_s + 3H\dot{Q}_s + m^2_{s(\text{eff})} Q_s = 0$$

Fluctuations away from the background trajectory

$$\frac{m^2_{s(\text{eff})}}{H^2} \equiv \frac{V_{;ss}}{H^2} + 3\eta_\perp^2 + \epsilon\, R^{\text{field space}} M^2_{\text{Pl}}$$

Super-Hubble entropic mass squared

Rolling of the inflation in a negatively curved field space tends to induces an instability



A **large hierarchy** is generic in string theory constructions

$$R^{\text{field space}} M^2_{\text{Pl}} \sim (M_{\text{Pl}}/M)^2 \sim 10^5$$

Can easily compensate ϵ suppression

Destabilize would-be stable trajectories

Similarity with the eta-problem

$$\mathcal{L}_{\rm eff}[\phi^I] = \mathcal{L}_l[\phi^I] + \sum_i c_i \frac{\mathcal{O}_i[\phi^I, \partial\phi^I, \ldots]}{M^{\delta_i - 4}}$$

Slow-roll action

Corrections to the low-energy effective action

Correction to kinetic terms

$$\Delta\mathcal{L} = c(\partial\phi)^2 \frac{\chi^2}{M^2}$$

$$\longrightarrow \qquad \frac{\Delta m_\chi^2}{H^2} \sim c \frac{(\partial\phi)^2}{H^2 M^2} \sim c\,\epsilon \left(\frac{M_P}{M}\right)^2$$

Similarity with the eta-problem

$$\mathcal{L}_{\text{eff}}[\phi^I] = \mathcal{L}_l[\phi^I] + \sum_i c_i \frac{\mathcal{O}_i[\phi^I, \partial\phi^I, \ldots]}{M^{\delta_i - 4}}$$

Slow-roll action

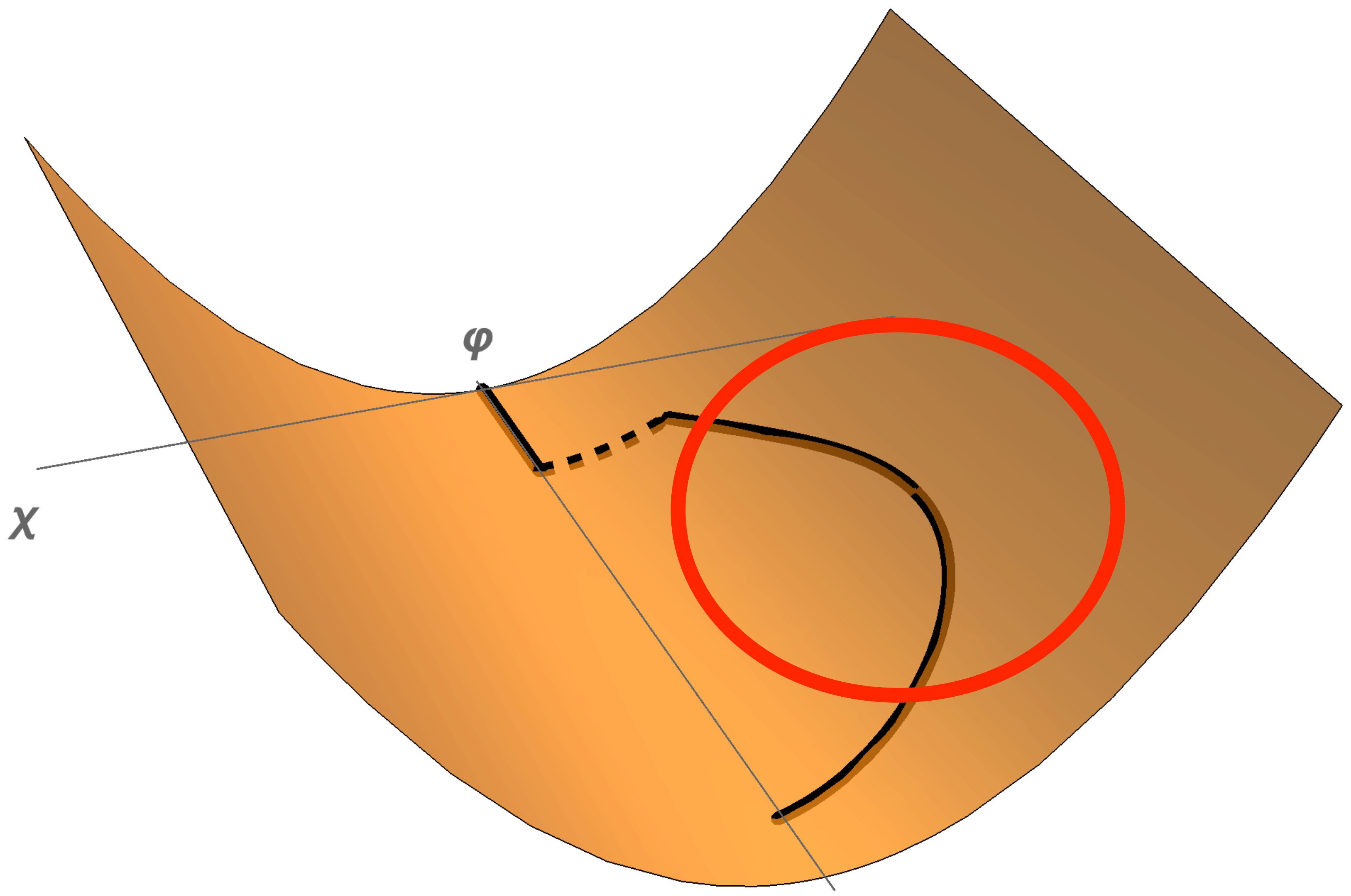
Corrections to the low-energy effective action

Geometry as important as potential, characterization of the whole action

Often ignored in explicit constructions

'Trivial field space metric for simplicity' is not possible

Lesson 4: other ways to inflate than slow-roll



Transition to a
non-standard attractor

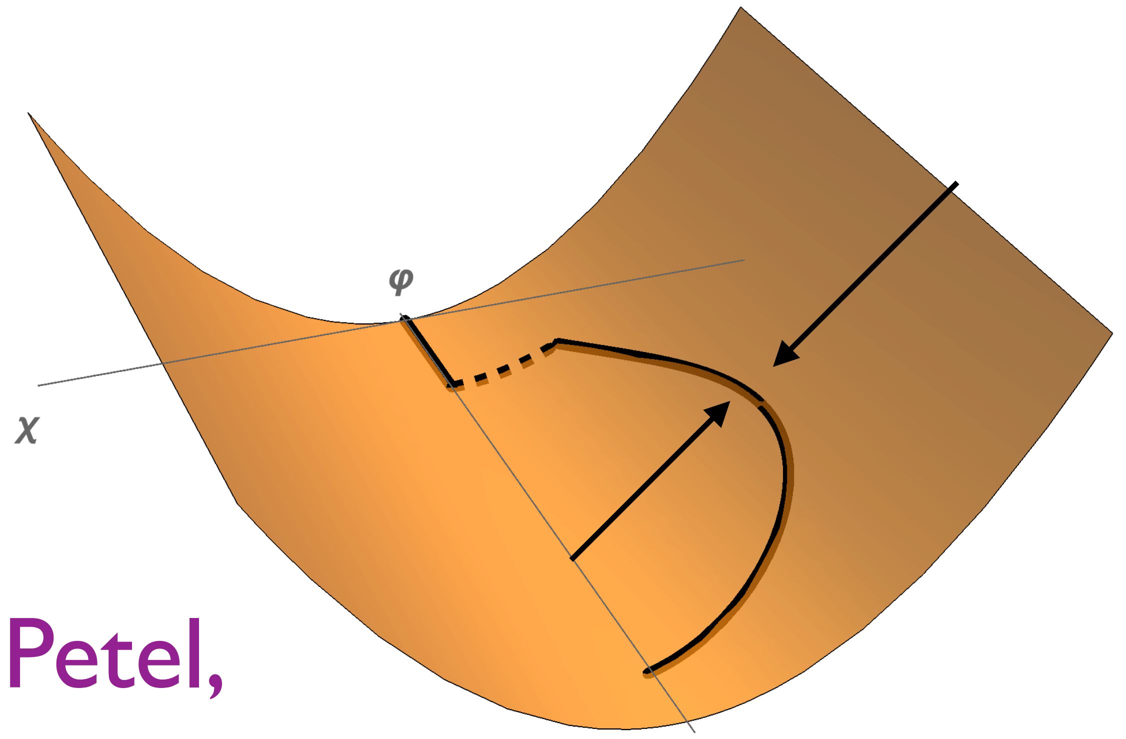
Second phase of inflation:
"sidetracked"

Garcia-Saenz, Renaux-Petel,
Ronayne,2018

$$(f_{\rm NL}^{\rm eq}, f_{\rm NL}^{\rm orth}) = \mathcal{O}(10)$$

Interesting signatures/constraints

Sidetracked inflation



Garcia-Saenz, Renaux-Petel, Ronayne, 2018

Competition potential vs geometry:

$$\eta_\perp^2 = \mathcal{O}\left(\frac{m^2}{H^2}\right) \gg 1$$

Strongly non-geodesic motion

Requirement for sidetracked inflation:
flat potentials wrt curvature scale

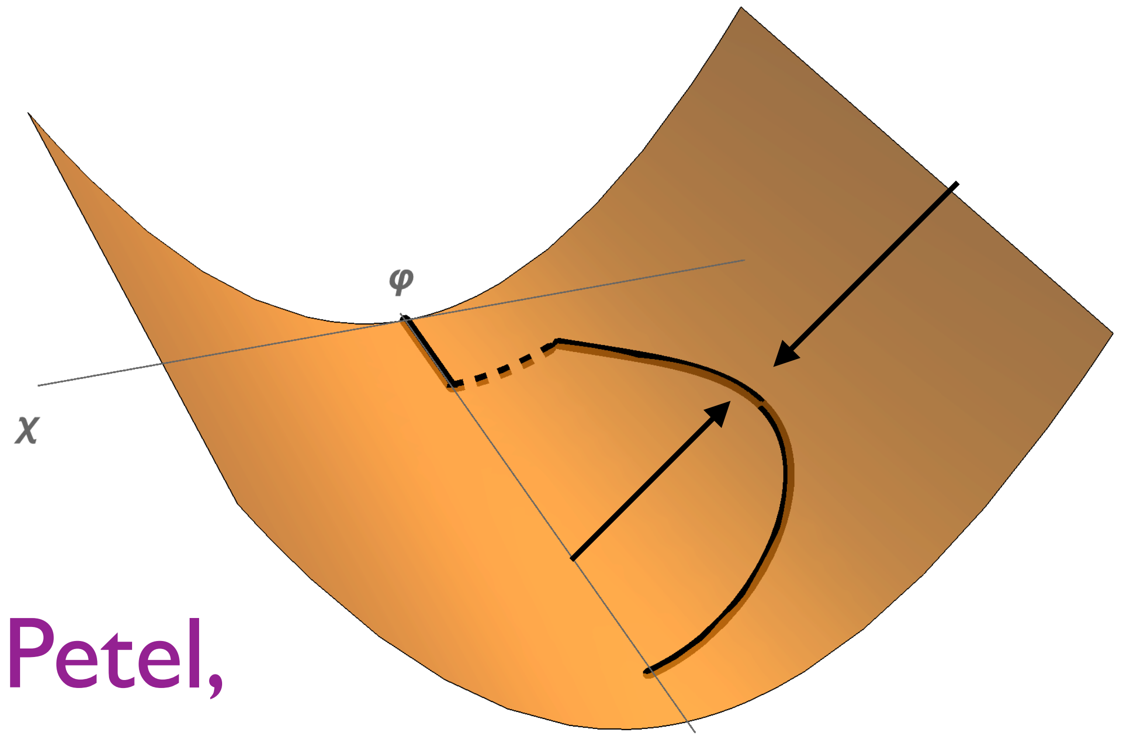
$$M\frac{V_{,\varphi}}{V} \ll 1\,, \quad M\frac{V_{,\varphi\varphi}}{V_{,\varphi}} \ll 1$$

Sustained inflation with
steep potential in Planck units
iff strongly non-geodesic motion

$$\epsilon \simeq \frac{\frac{M_{\rm Pl}^2(\nabla V)^2}{2V^2}}{1+\eta_\perp^2/9}$$

Hetz and Palma 2016
Achucarro and Palma 2018

Sidetracked inflation



Garcia-Saenz, Renaux-Petel, Ronayne, 2018

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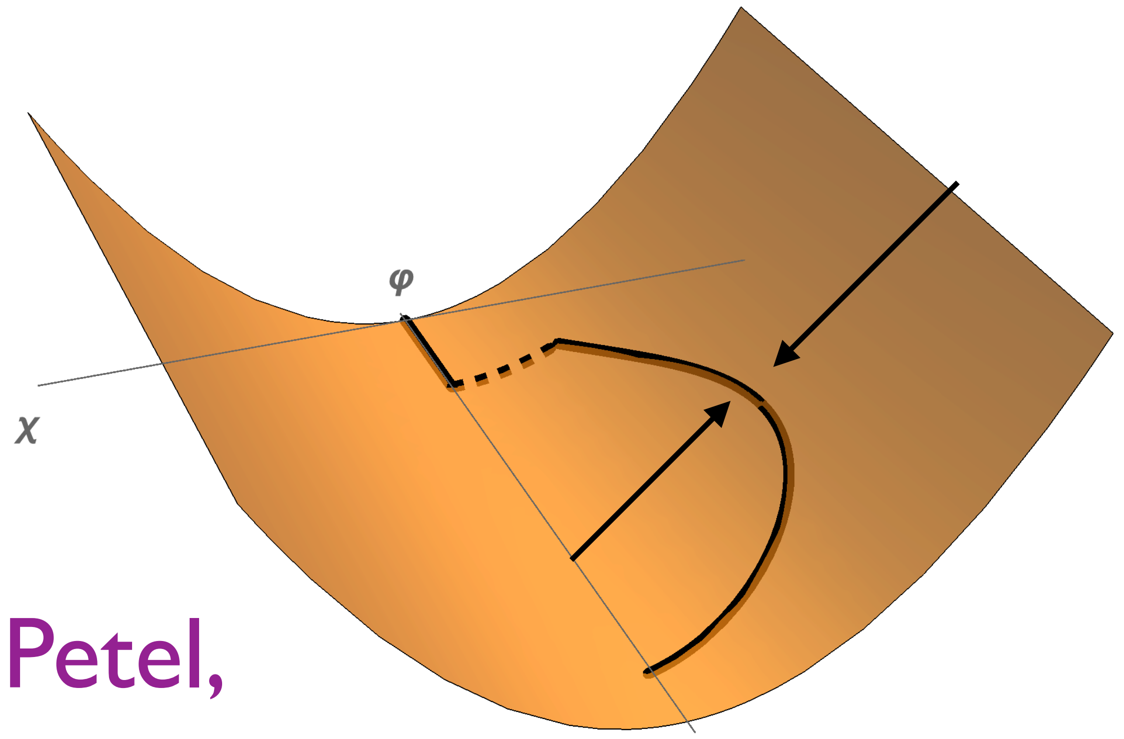
$$M\frac{V_{,\varphi}}{V} \ll 1\,, \qquad M\frac{V_{,\varphi\varphi}}{V_{,\varphi}} \ll 1$$

Strongly non-geodesic motion
in negatively curved field space
under scrutiny in recent years

sidetracked inflation
hyperinflation
angular inflation
rapid-turn
fat inflaton
...

different names
but overall
similar
mechanism

Sidetracked inflation



Garcia-Saenz, Renaux-Petel, Ronayne, 2018

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Requirement for sidetracked inflation:
flat potentials wrt curvature scale

$$M\frac{V_{,\varphi}}{V} \ll 1\,, \quad M\frac{V_{,\varphi\varphi}}{V_{,\varphi}} \ll 1$$

But with cutoff $M \ll M_{\rm Pl}$

Natural expectation to have structures over distance M

As tuned as slow-roll

Comparison with DBI

$$\mathcal{L} = -M^4 \left(\sqrt{1 - \frac{\dot\phi^2}{M^4}} - 1 \right) - V$$ with $$\frac{M}{M_{\rm Pl}} \ll 1$$ (strongly warped throat)

Can support inflation on Planck-steep potential, but at expense of hierarchy, and DBI also needs tuning

Baumann, Chen, Maldacena, McAllister …

$$\frac{M^2}{H M_{\rm Pl}} \lesssim \frac{M_{\rm Pl} V'}{V} \ll \frac{H M_{\rm Pl}}{M^2}$$

approach speed limit | DBI inflation

$$\frac{mM}{H M_{\rm Pl}} \lesssim \frac{M_{\rm Pl} V'}{V} \ll \frac{H M_{\rm Pl}}{mM}$$

geometrical destabilization | sidetracked inflation

Renaux-Petel, 2111.00989

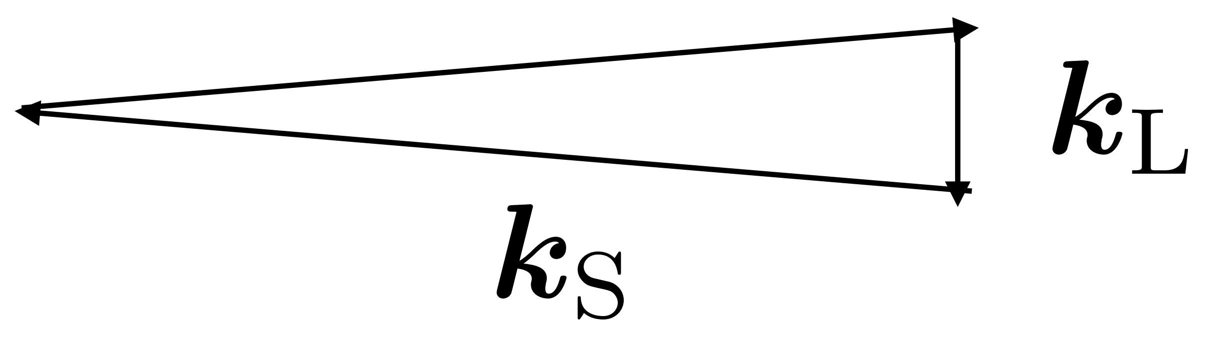
Despite ever-present tuning,
interesting signatures of inflation with strongly non-geodesic notion

Lesson 5: inflation as a cosmological collider

$$S[\zeta] \overset{?}{+} S[\text{mixing}] + S[\text{other}]$$

Ubiquitous in string theory, supergravity …

Probed in squeezed limit



$$\kappa = k_{\rm L}/k_{\rm S} \ll 1$$

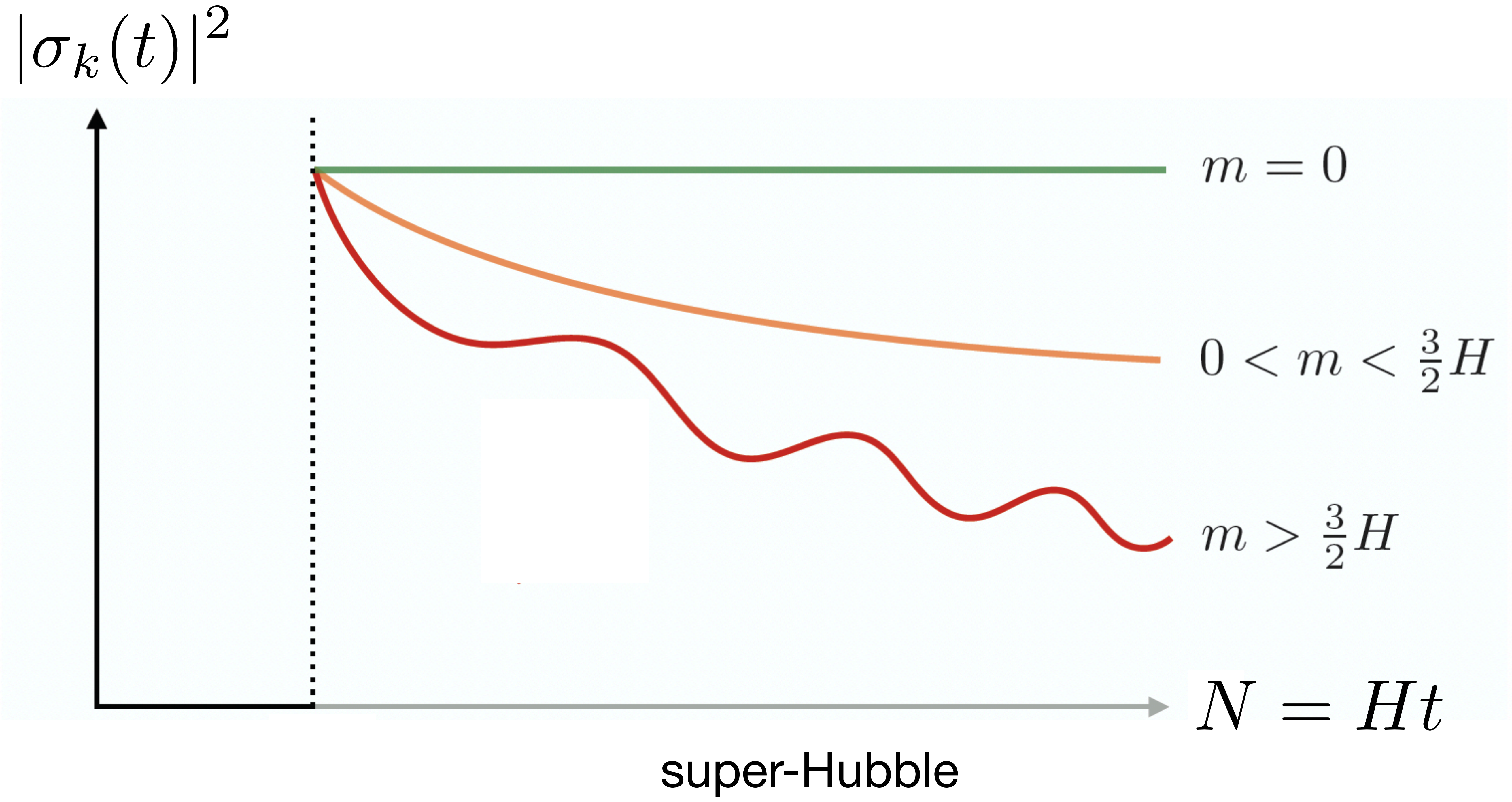
Observable squeezed limit in single-clock inflation

$$S \underset{\kappa \ll 1}{\propto} \kappa$$

Maldacena, Creminelli, Zaldarriaga, Tanaka, Urakawa, Pajer, Schmidt …

Imprints of additional degrees of freedom

$$\ddot{\sigma}_k + 3H\dot{\sigma}_k + m^2\sigma_k \simeq 0$$



$$\sigma \propto \frac{1}{a^{\frac{3}{2}-\nu}} \qquad \nu = \sqrt{9/4 - m^2/H^2}$$

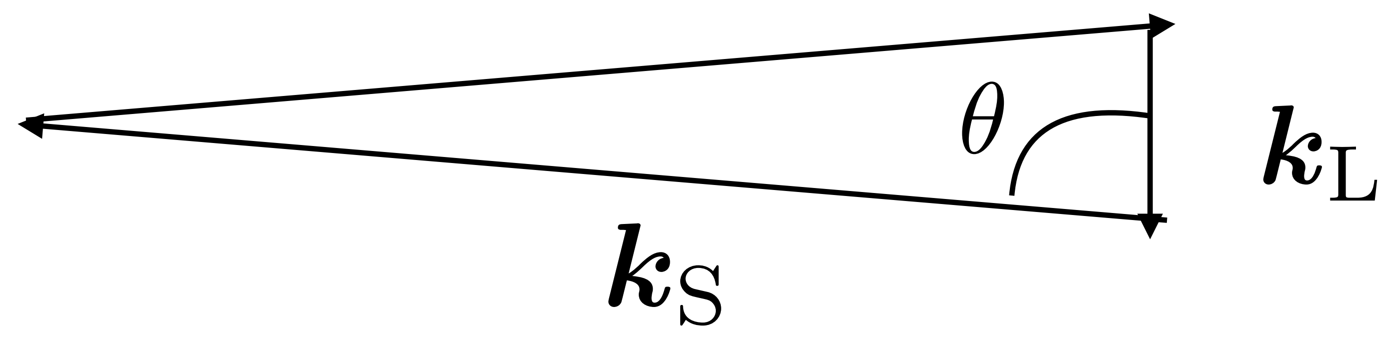
$$S \propto \kappa^{1/2-\nu}$$

$$\sigma \propto \frac{\cos(\mu N + \phi)}{a^{\frac{3}{2}}} \qquad \mu = \sqrt{m^2/H^2 - 9/4}$$

$$S \propto \kappa^{1/2} \cos(\mu \log(\kappa) + \varphi)$$

Cosmological collider physics

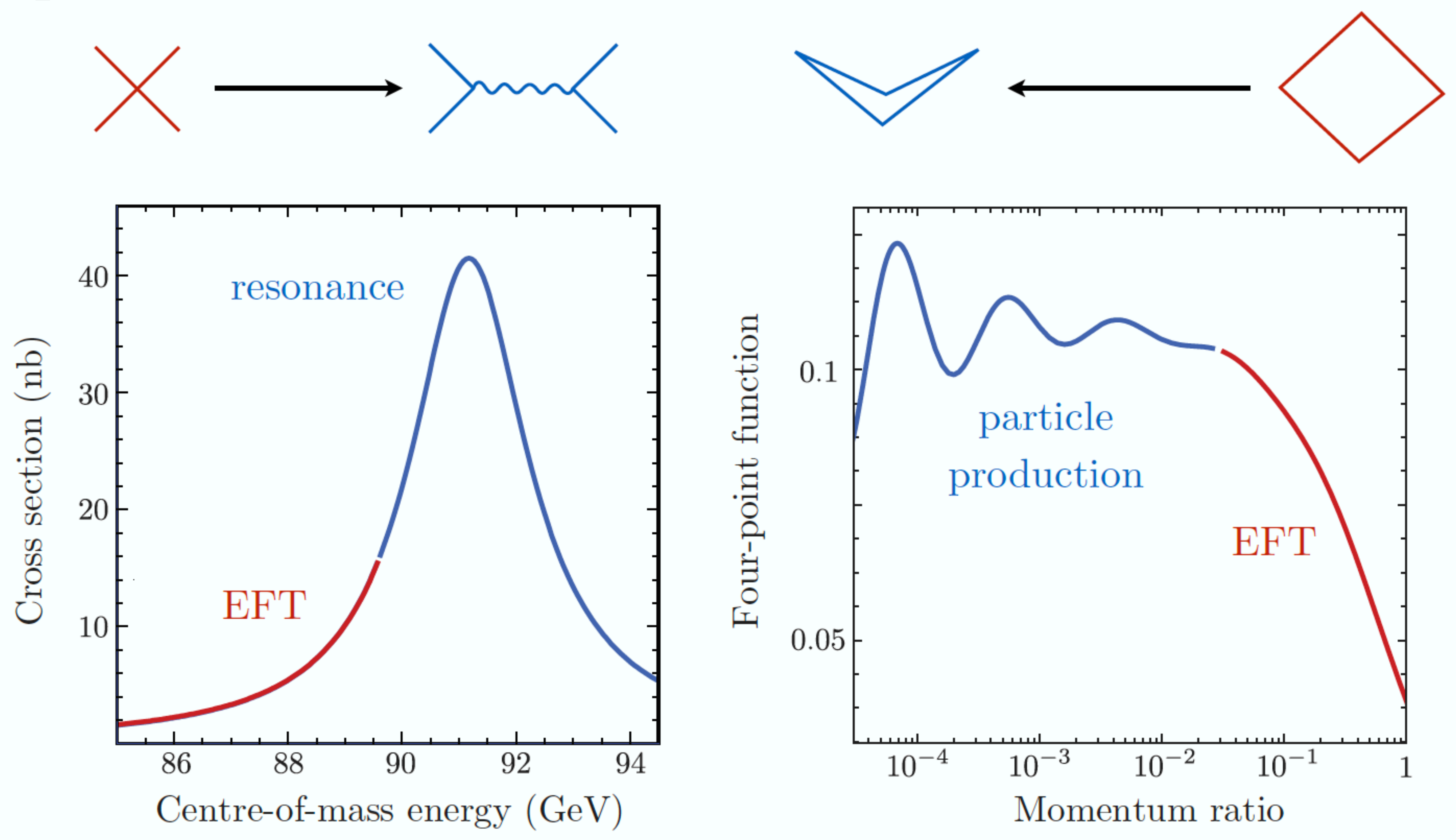
3pt



$$S \propto \kappa^{1/2} e^{-\pi\mu} \cos(\mu \log(\kappa) + \varphi) P_S(\cos\theta)$$

Mass & Spin of heavy particle

4pt



From 1811.00024

Chen, Wang 2009
Noumi, Yamaguchi, Yokohama 2012
Arkani-Hamed, Maldacena 2015
Lee, Bauman, Pimentel 2016
+ many works

Cosmological collider: « a robust probe of field content of inflation »?

$$S \propto \kappa^{1/2} e^{-\pi\mu} \underline{\cos(\mu \log(\kappa)} + \varphi) \underline{P_S(\cos\theta)}$$

As robust probe as assumptions are restrictive:
unique additional dof, weakly mixed, scale-invariant

Observed ζ ← mixing → Hidden sector

Observed ζ

Dispersion relation

mixing

Type

Strength

Time-dependence

Hidden sector

Several dofs

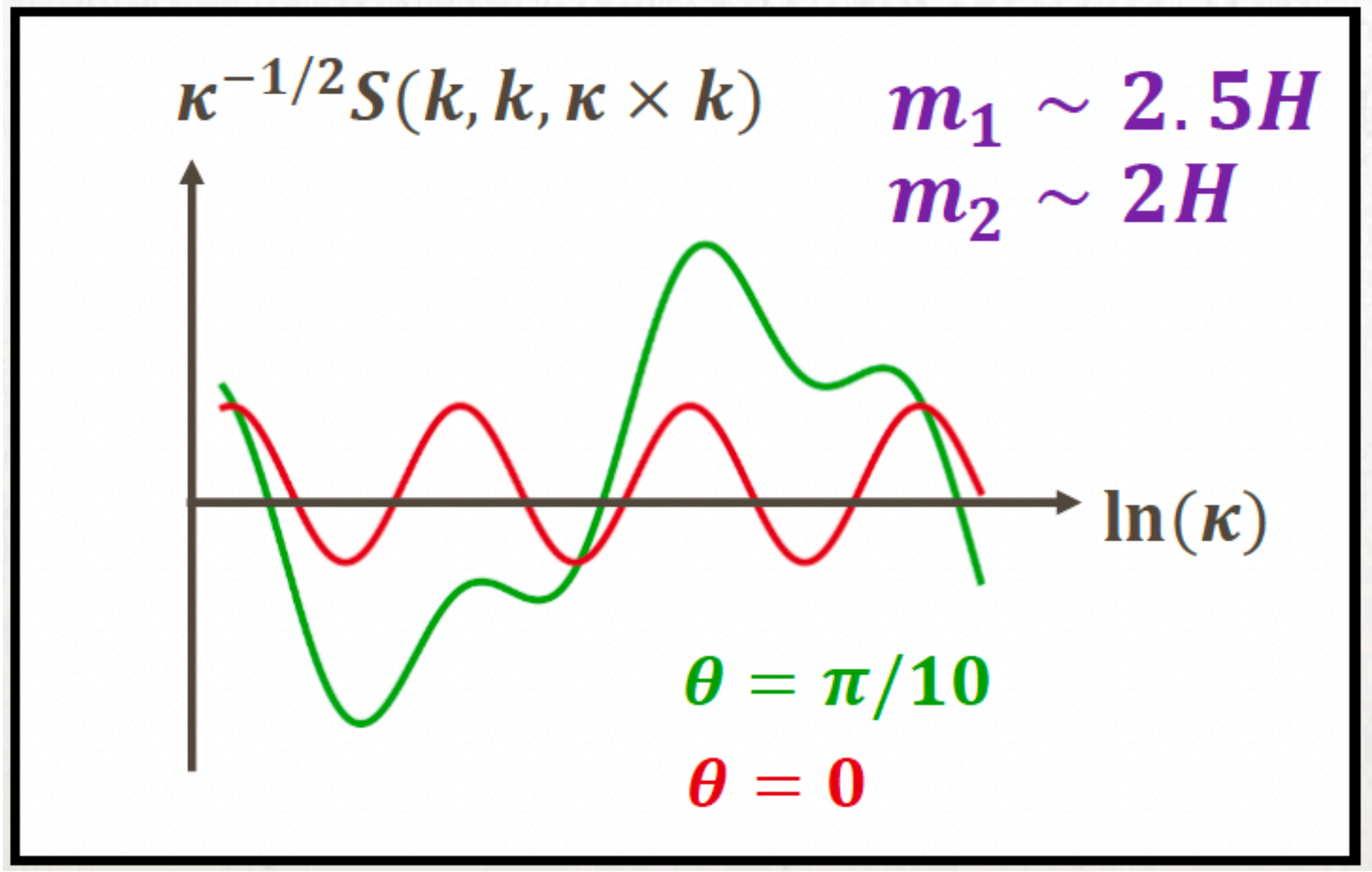
Mass and spin

Interactions

Time-dependence…

Cosmological collider signatures beyond restrictive assumptions

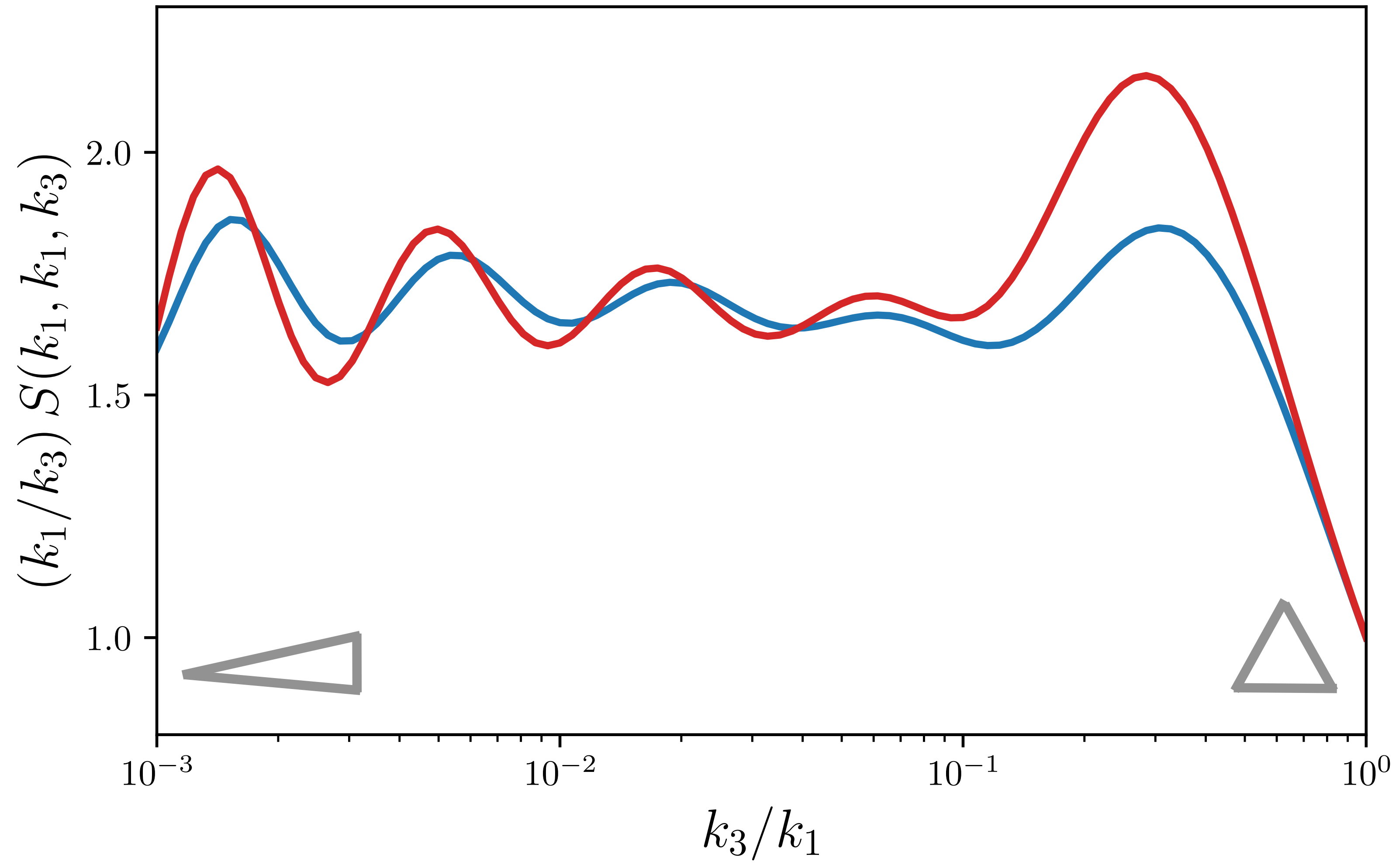
Mass mixing



Inflationary flavor oscillations

Pinol, Aoki, Renaux-Petel, Yamaguchi [2021]

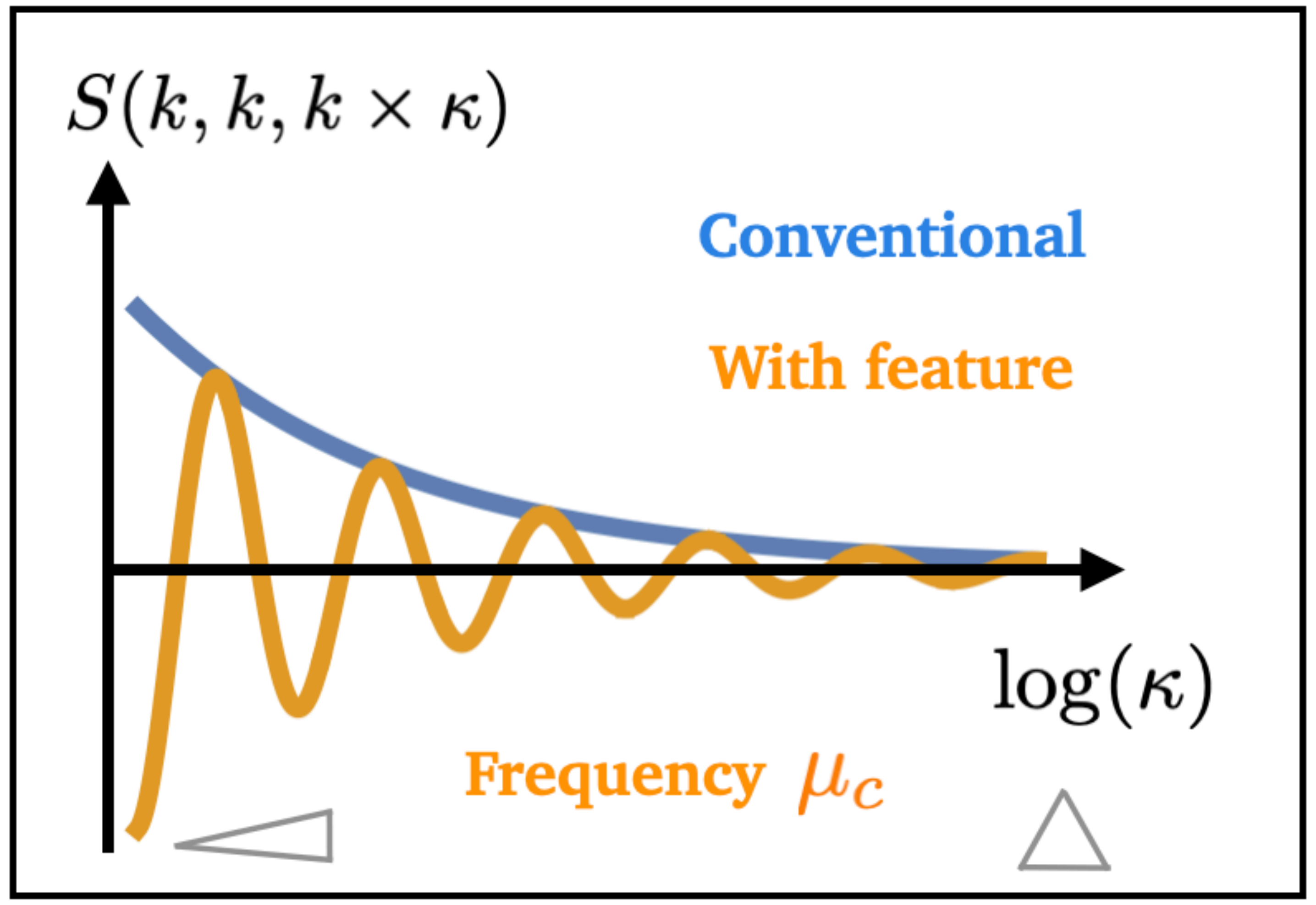
Strong mixing



Breaking degeneracies weak/strong mixing

Werth, Pinol, Renaux-Petel [2023 a,b]

Time-dependent mixing



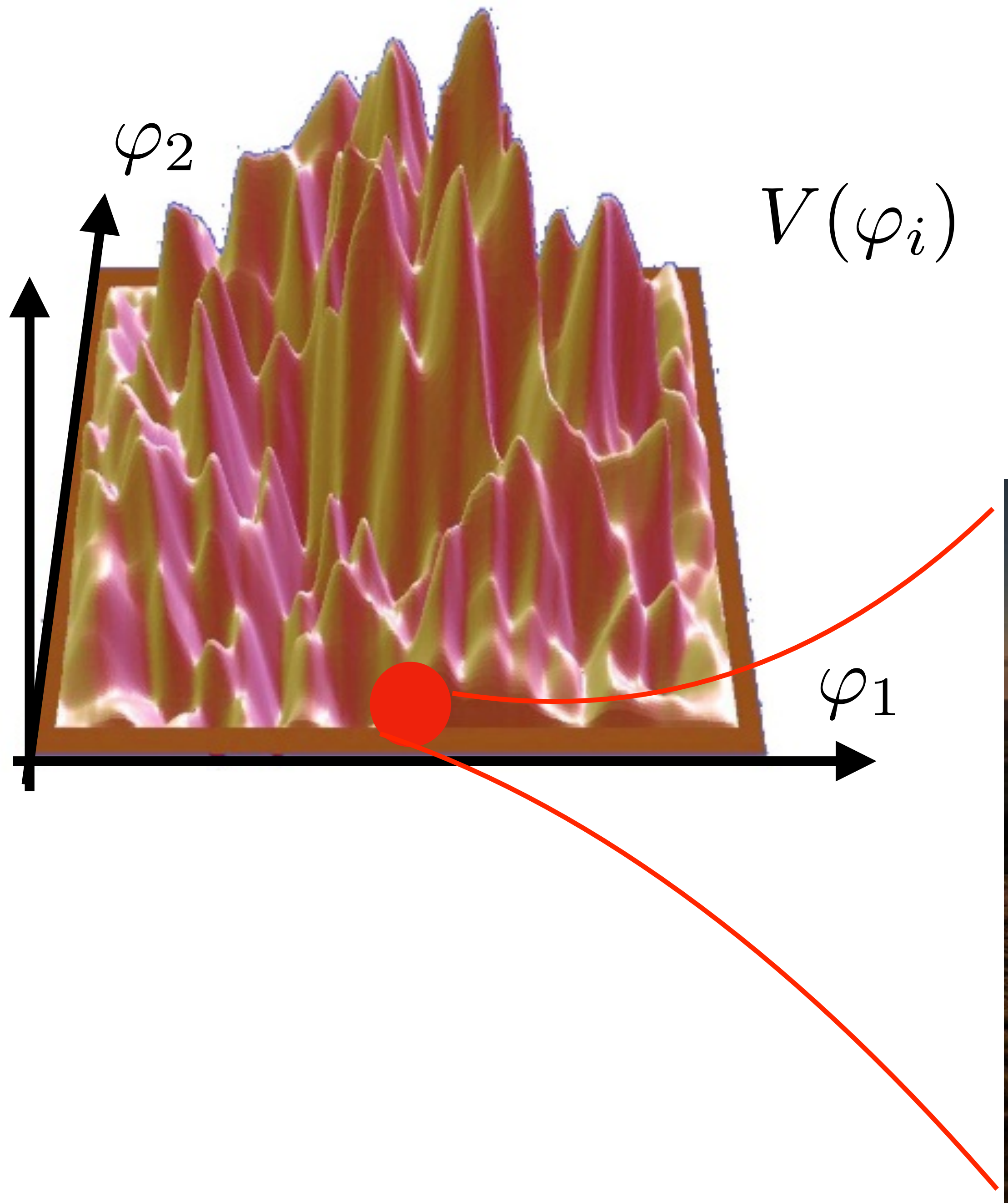
Soft limits complementary to equilateral to diagnose features

Werth, Pinol, Renaux-Petel [2023 a,b]

Lesson 6: features are well motivated

Simplicity of data

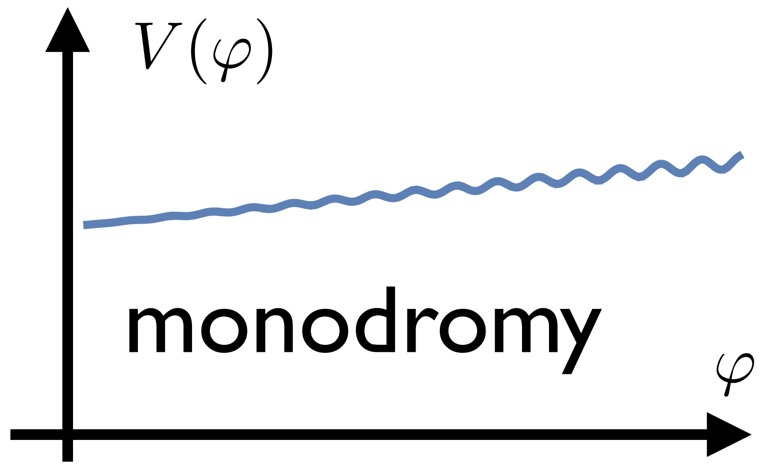
accidental



symmetry

$$\phi \to \phi + \mathrm{ct}$$

Broken by non-perturbative effects to discrete shifts



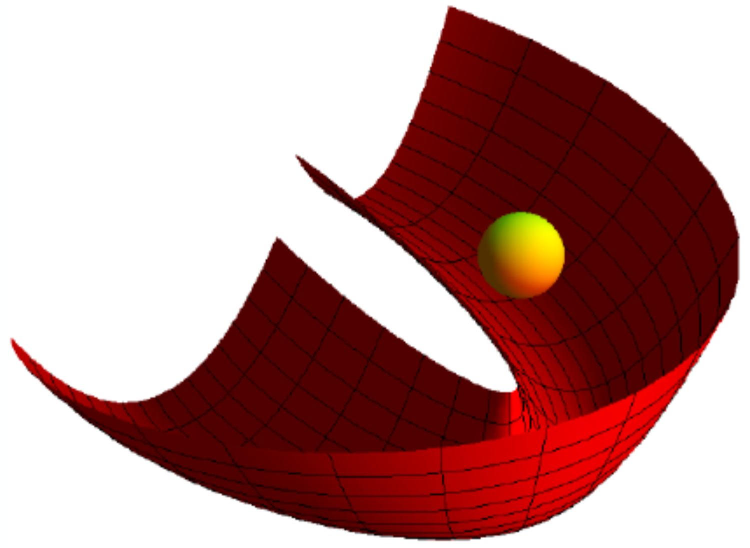
Observational consequences as features

Sharp features

$$\frac{\mathcal{P}_\zeta(k)}{\mathcal{P}_{\rm env}(k)} = \left[1 + A_{\rm lin} \cos\left(\omega_{\rm lin} k + \varphi_{\rm lin}\right)\right]$$

Localized event

step in potential
turn in field space …



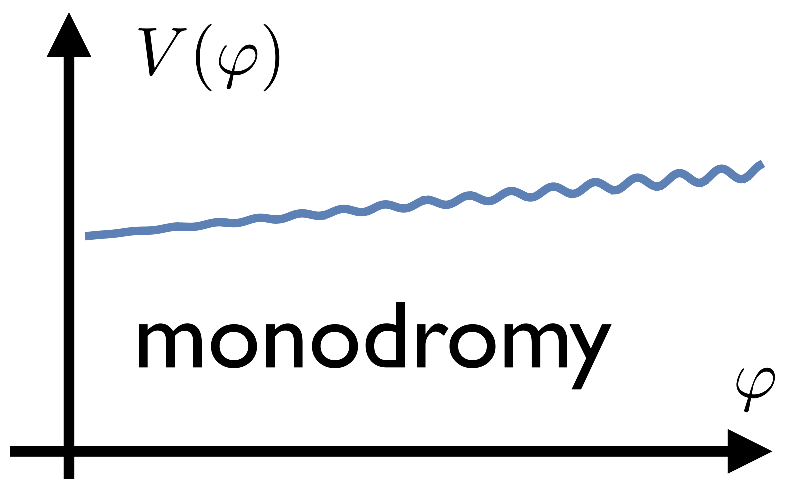
$$e^{-ik\tau_{\rm f}} = e^{ik/k_{\rm f}}$$

selection of
preferred time/scale

Resonant features

$$\left[1 + A_{\log} \cos\left(\omega_{\log} \log(k/k_\star) + \varphi_{\log}\right)\right]$$

Resonance btw
background
oscillations and
quantum modes
oscillations

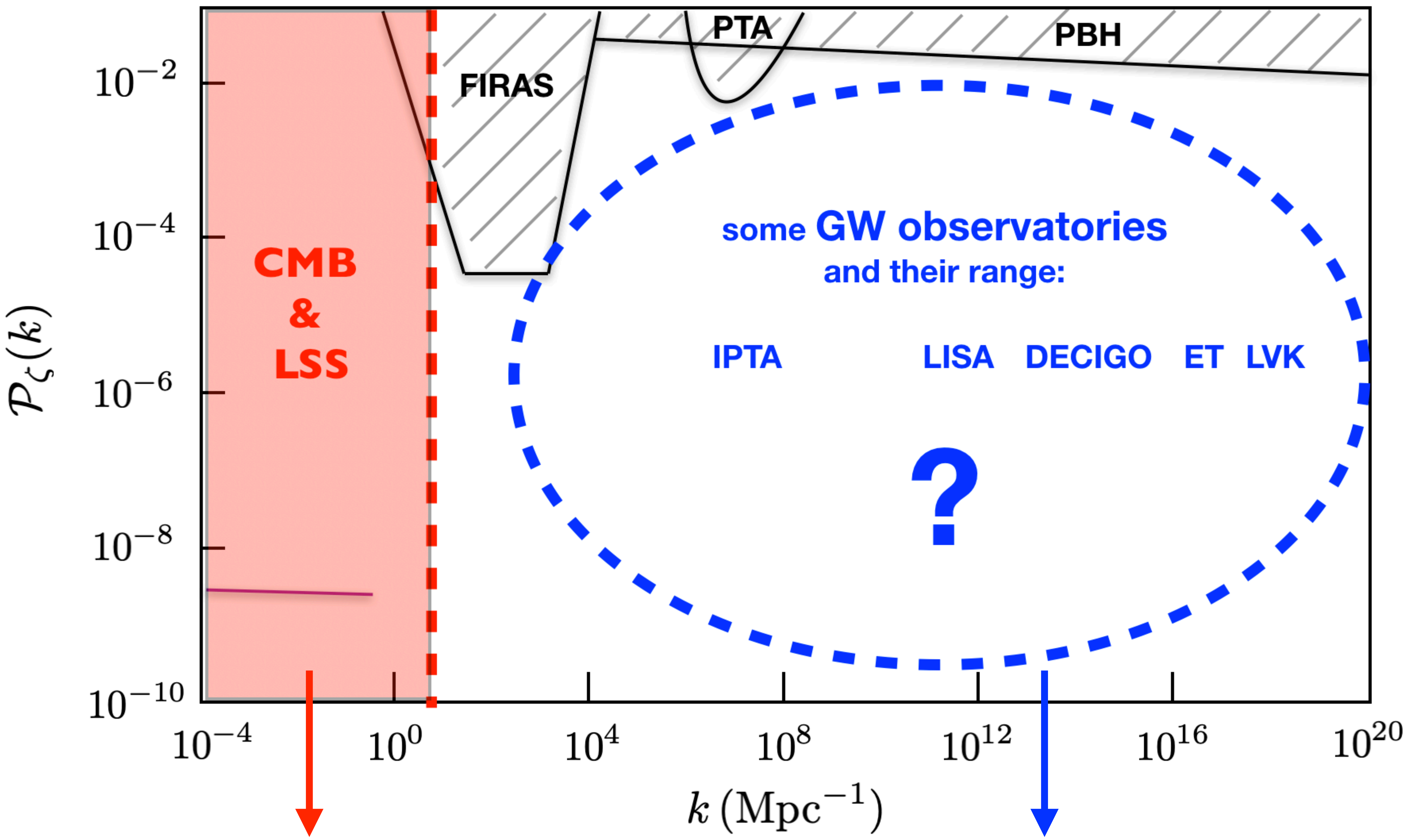


$$e^{iMt} = e^{i\frac{M}{H}N} \longrightarrow e^{i\frac{M}{H}\log(k/k_\star)}$$

$$k \propto e^{N_k}$$

similar consequences for non-Gaussianity

Lesson 7: a lot to learn from dark era of inflation



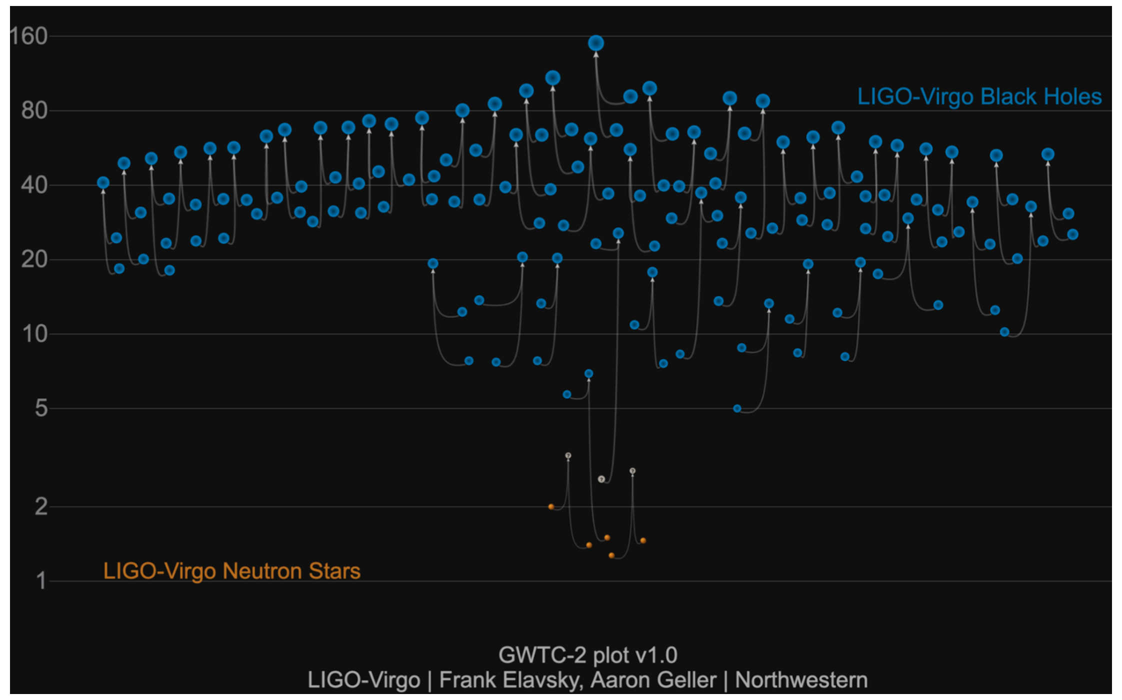
Precision physics

Non-Gaussianity, features

Exploratory physics

Probing dark inflationary era
with PBH and gravitational waves

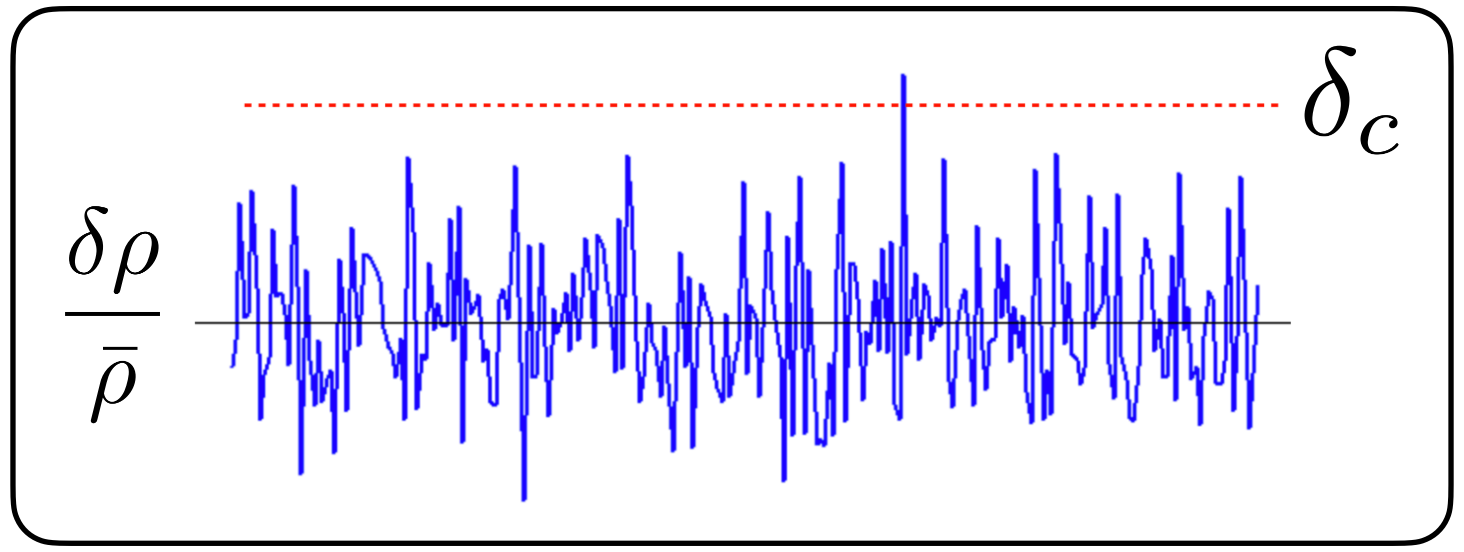
Context: GW astronomy



Have **Primordial Black Holes** been already detected? Data will tell

recent PBH review in LISA CosWG
2310.19857

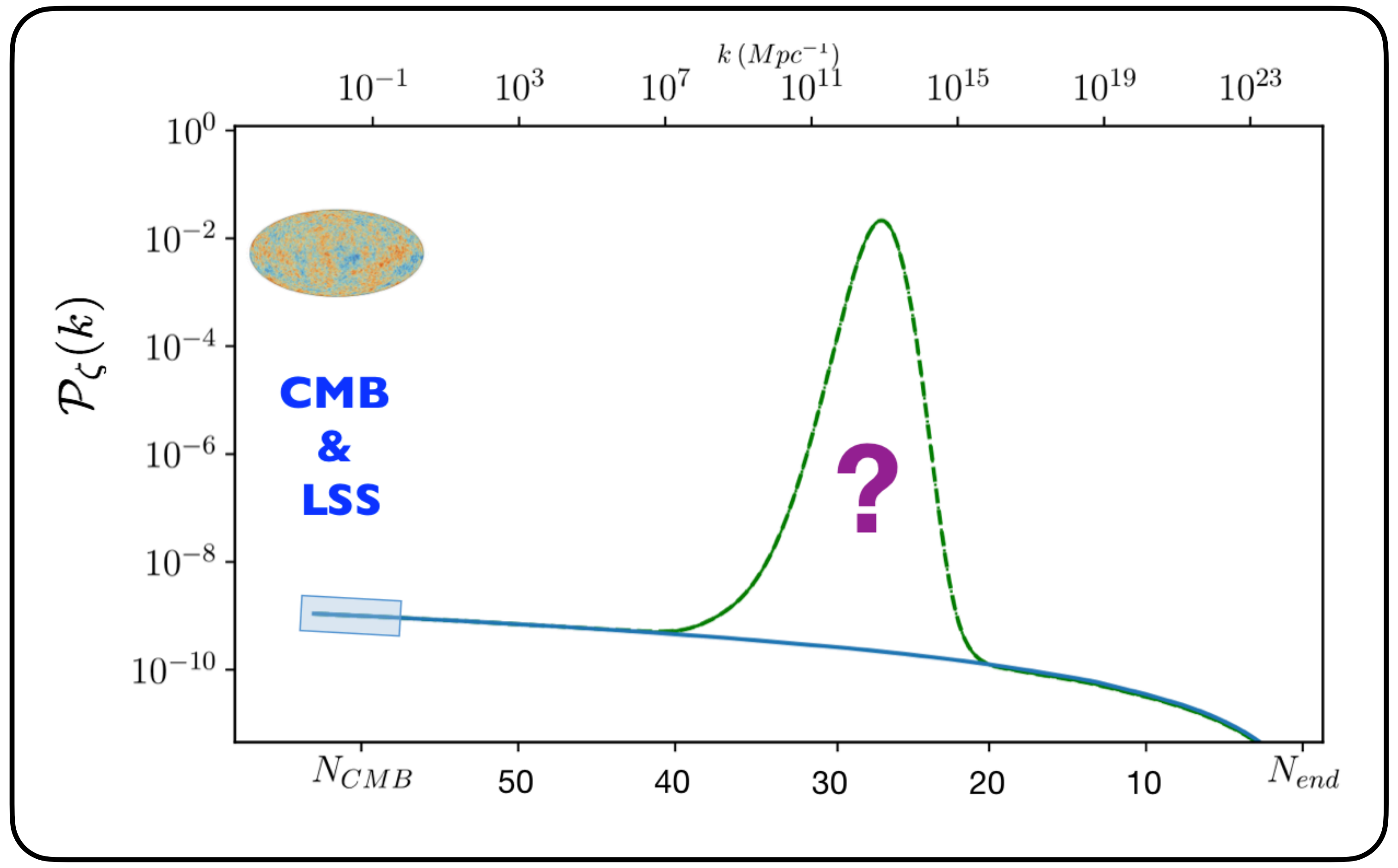
Large primordial overdensities



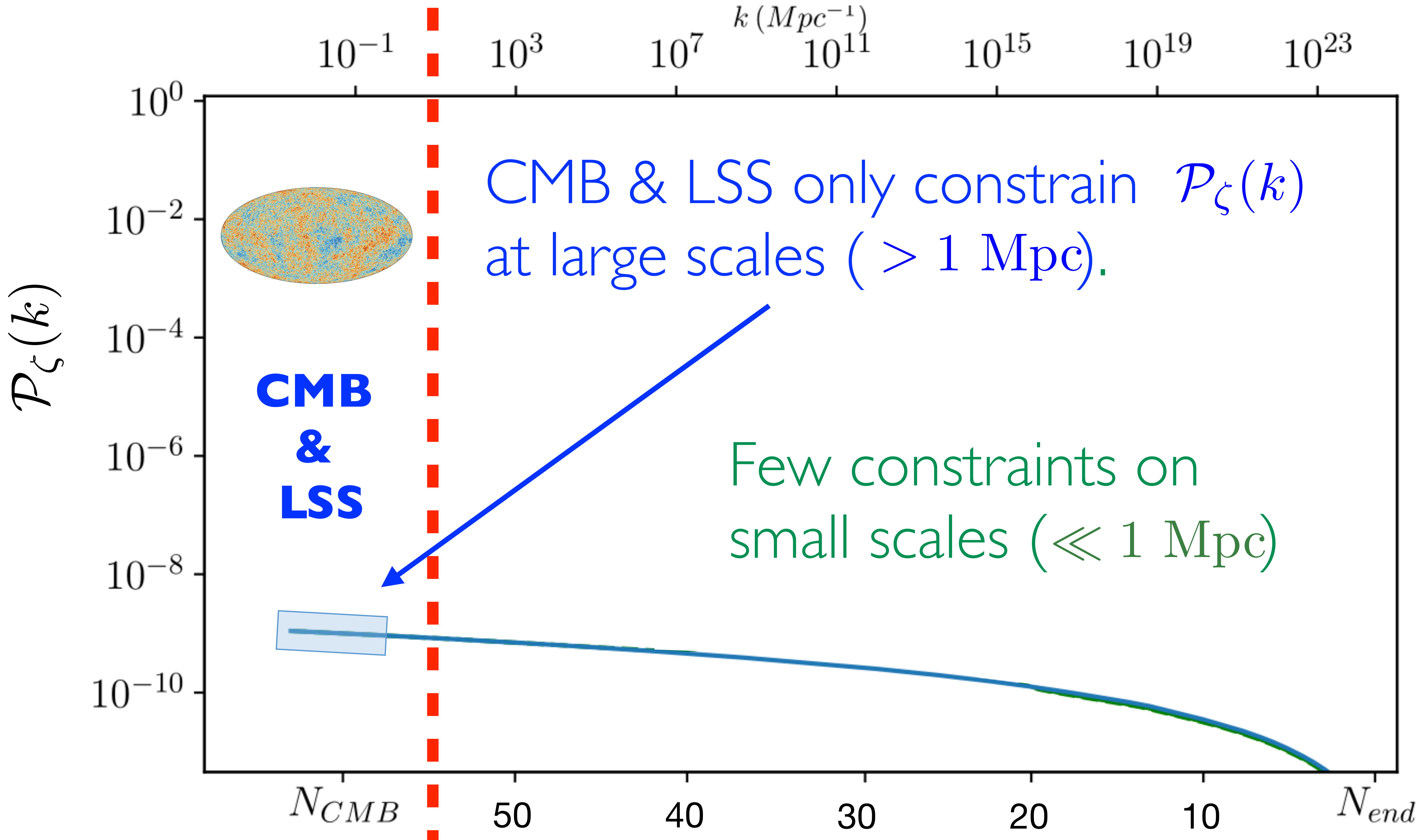
Large power spectrum on small scales and **nontrivial dynamics of inflation**

Scalar-induced **SGWB**

$$M \simeq 10^{-12} M_\odot \left(\frac{f_{\rm LISA}}{f}\right)^2$$



Inflation on small scales?

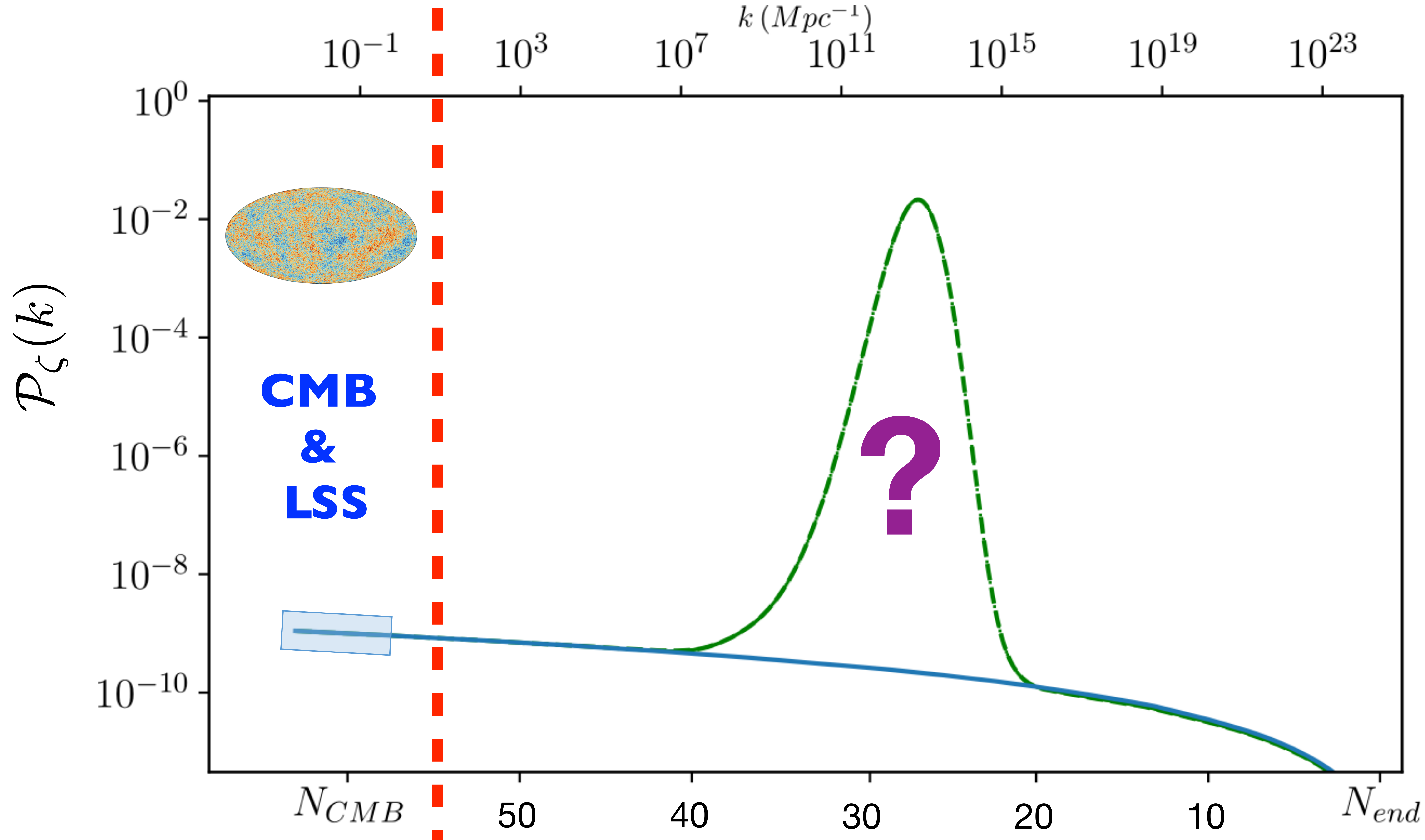


Emergent simplicity

+

Corrections

Inflation on small scales?

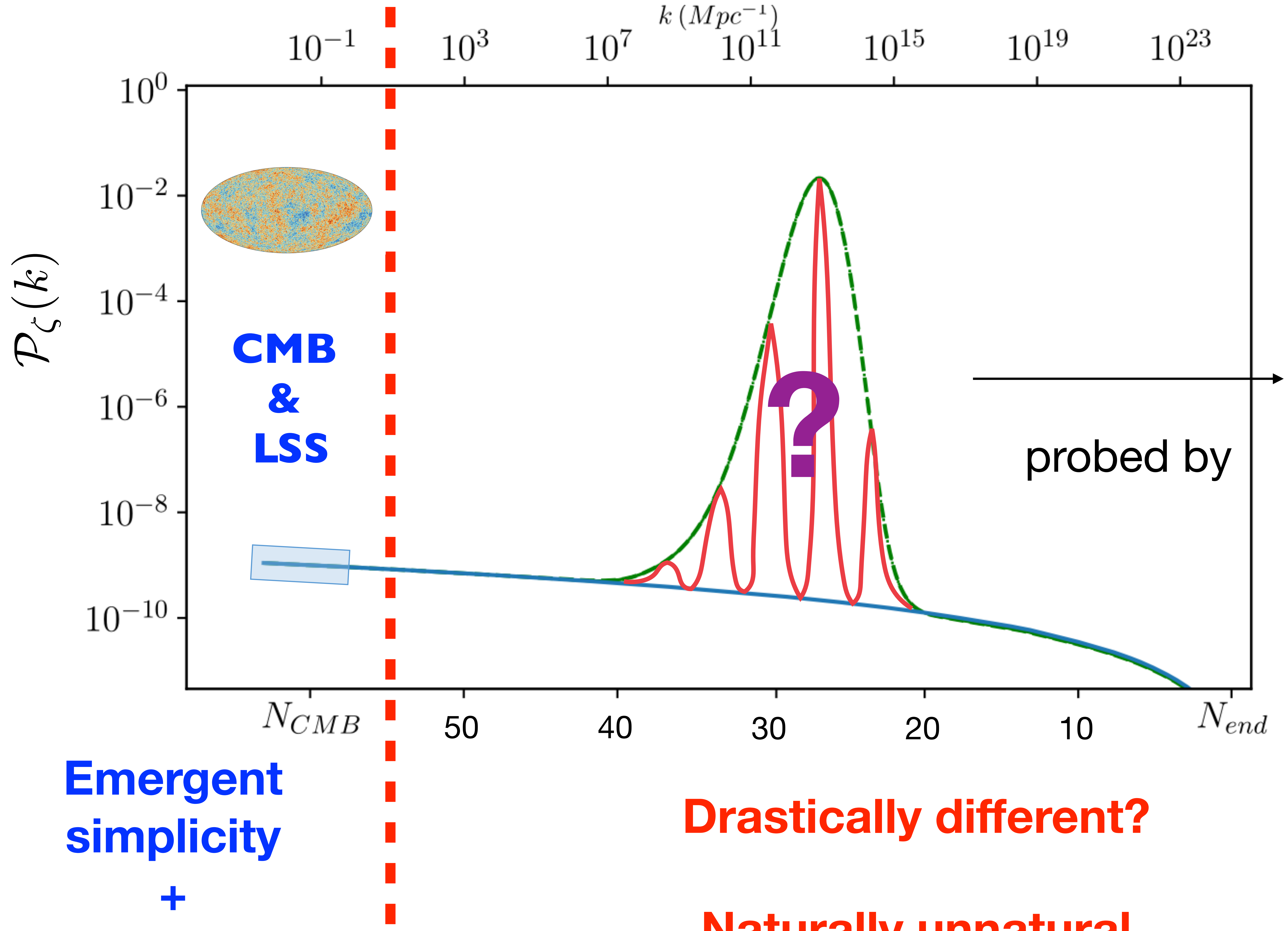


Emergent simplicity
+
Corrections

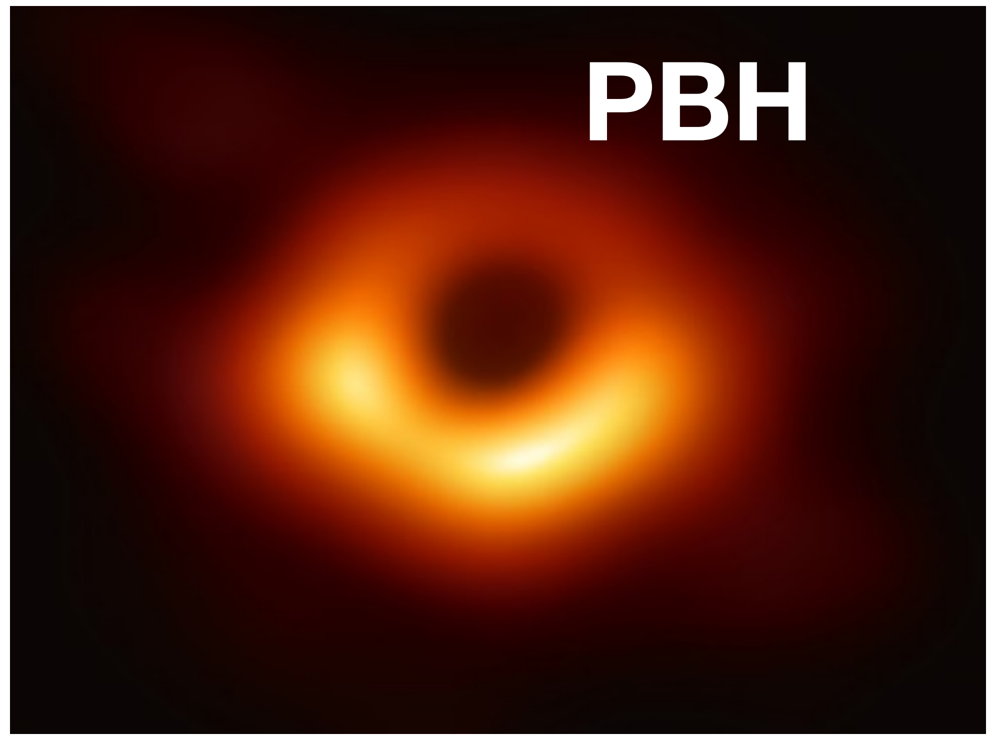
Drastically different?

Naturally unnatural

Inflation on small scales?



probed by



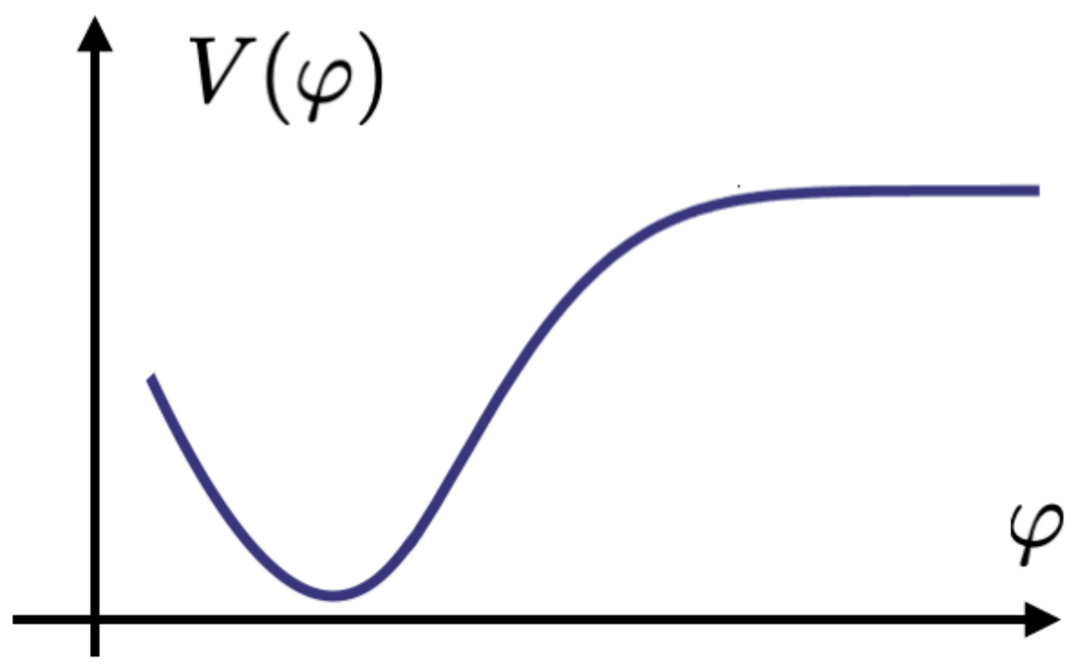
Emergent simplicity
+
Corrections

Drastically different?

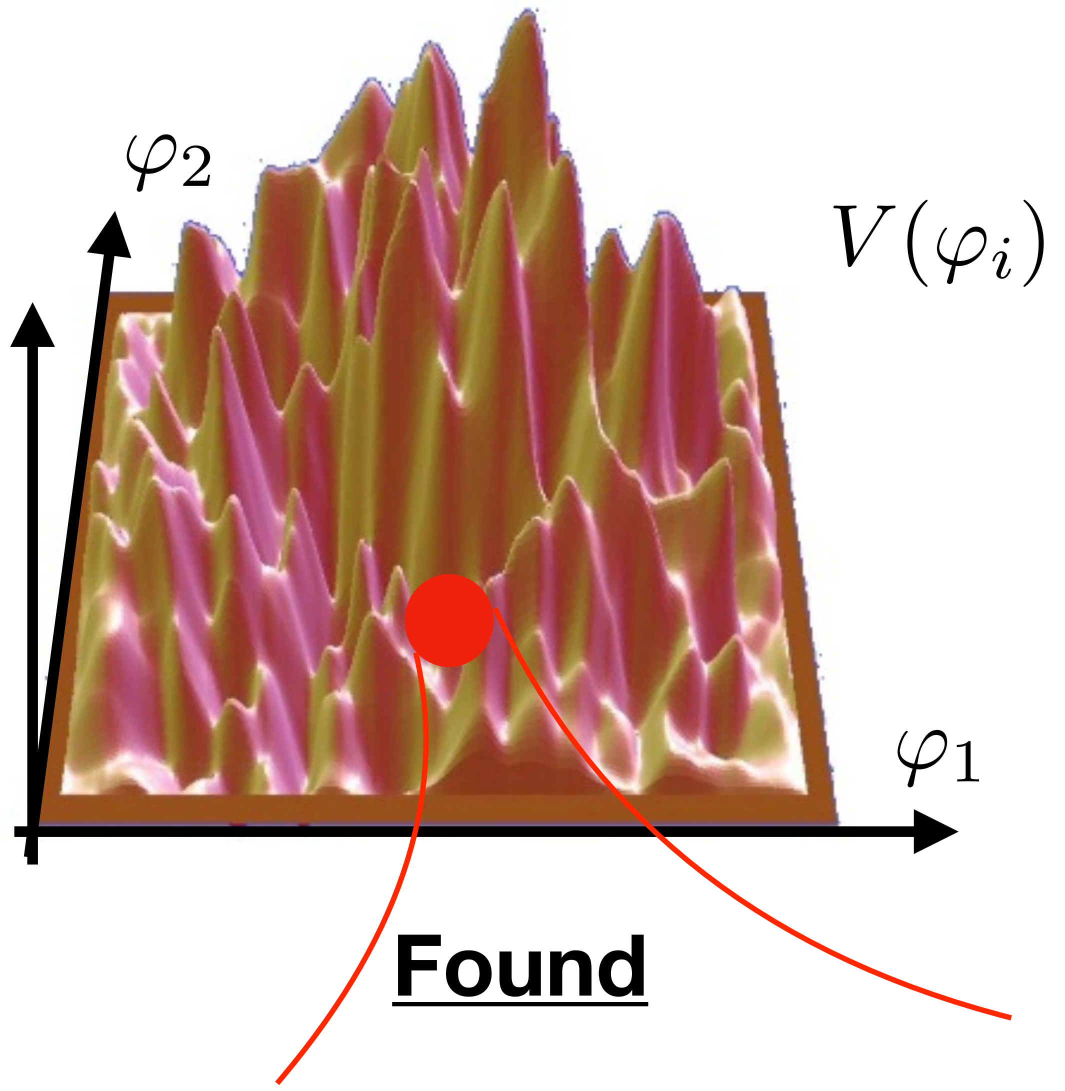
Naturally unnatural

Taking theory seriously

A prolonged phase of smooth 60 e-folds of inflation is not natural (eta-problem)



Hoped



Found

More natural for inflation to have occurred in successive phases with different properties

Ultra slow-roll, hybrid-type transitions, sharp turns, etc

GW from active sources

GW generated by nonlinear processes $\Box h \sim (\partial\zeta)^2$

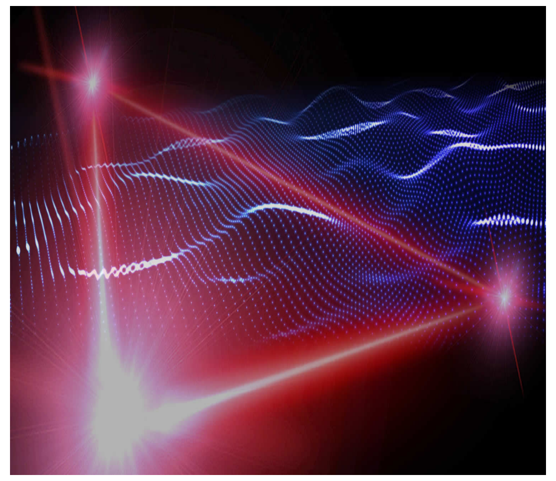
Probe of field content and dynamics of inflation: extra degrees of freedom, gauge fields, beyond slow-roll…

Frequency profile $\Omega_{\rm GW}(f)$ $\frac{f}{\rm Hz} = 1.5 \times 10^{-15} \frac{k}{\rm Mpc^{-1}}$

Chirality $h_+ \neq h_\times$

Anisotropies $\Omega_{\rm GW}(f, \hat{n})$

$$\log\left(\frac{f}{10^{-3}{\rm Hz}}\right) \simeq N_{\rm after\ CMB} - 30$$



GW observatories probe inflation on small scales

Vast activity in past years Cosmology with LISA, 2204.05434

Gravitational-Wave Primordial Cosmology

17–19 May 2021

Europe/Paris timezone

IAP

Messengers of the very early universe: Gravitational Waves and Primordial Black Holes

12–14 déc. 2022

Centro Universitario Padovano

Fuseau horaire Europe/Rome

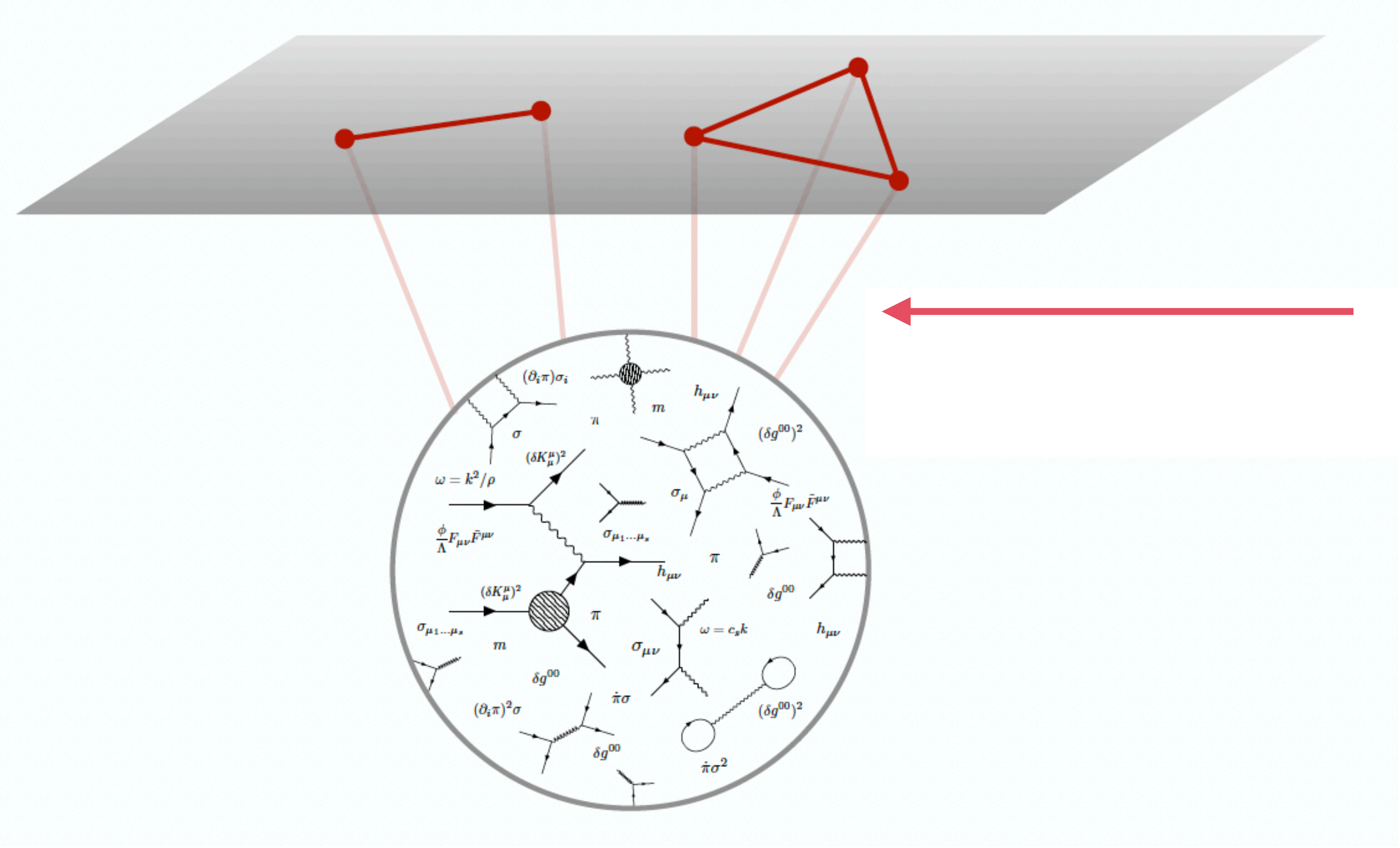
Padova

Lesson 8: we have tools doing the work for you!

Following the time evolution of cosmological correlators from their origin as quantum vacuum fluctuations to the end of inflation

Time

PyTransport
CppTransport
CosmoFLow



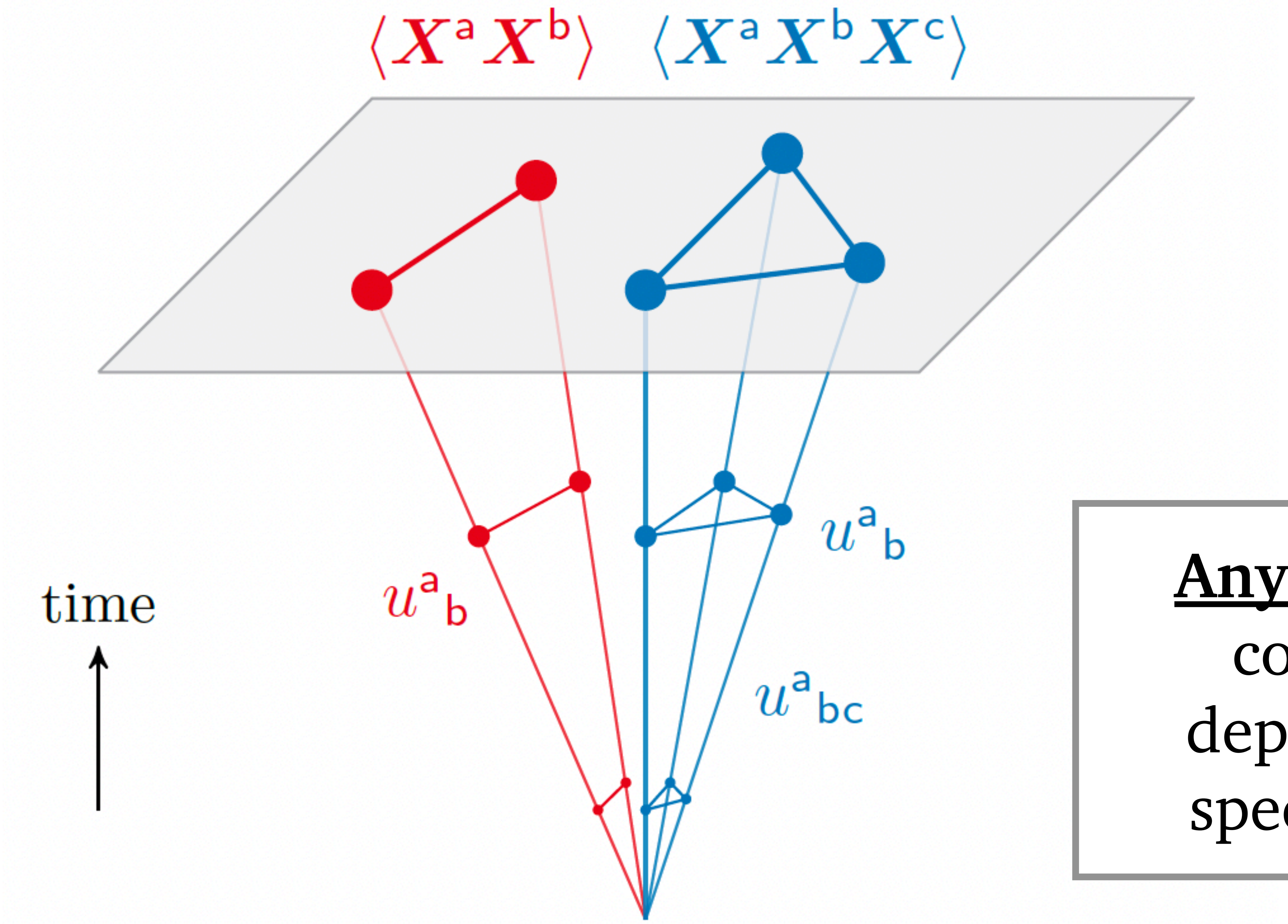
Physics encoded in time evolution

Theories → Cosmological Flow → Correlators

$$\mathcal{L}(\varphi^\alpha) \qquad \frac{\mathrm{d}}{\mathrm{d}t}\langle\varphi^\alpha\varphi^\beta\varphi^\gamma\rangle \qquad \langle\varphi^\alpha\varphi^\beta\varphi^\gamma\rangle$$

CosmoFlow

Werth, Pinol, RP, 2302.00655



Theories formulated at level of **EFT of inflationary fluctuations**

Any theory: # dofs, couplings, time-dependence, sound speeds, masses, etc → **Cosmological Flow** → **Primordial correlators**

Shift from technical considerations to unbiased exploration

Public tool for the community

Generate theoretical data

Assist theoretical understanding

Conclusions

- (Some) string theorists (tend to) oversimplify inflation

- Let us talk! Inflationary cosmology is much much richer than single- or multi-field slow-roll (theoretically and observationally)

- Some other interests in primordial cosmology I haven't mentioned: cosmological bootstrap, non-perturbative formalisms, stochastic effects, beyond local unitary EFT, reheating, lattice simulations, and much more