# Accelerated Expansion in an Open Universe

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String-Cosmo, 28 novembre 2023

## Based on:

* *Andriot, DT & Wrase*, Phys. Rev. D, arXiv 2309.03938
* *Paul Marconnet & DT*, JHEP 01 (2023) 033

# Outline

- Introduction
- Lessons from universal compactifications
- An effective point of view
- Conclusions

# Introduction

- There has been a lot of recent effort in obtaining realistic 4d cosmologies from the 10d/11d supergravities that capture the low-energy limit of string/M-theory.
- In the early 21st century accelerating 4d cosmologies from compactification were thought to be as difficult as 4d Sitter.
- The famous no-go excludes acceleration, provided:
  - absence of sources, no (or mild) singularities
  - compactness
  - two-derivative actions
  - the *Strong Energy Condition* is obeyed by the 10d/11d theory

* *Gibbons*, 1984
* *Maldacena & Nuñez*, 2000

# Introduction

- Time-dependent compactifications, however, can evade the no-go.

* *Townsend & Wohlfarth*, 2003

- Transient acceleration is in fact generic in flux compactifications (although until recently all known examples from 10d/11d compactifications were thought to give $\mathcal{O}(1)$ e-foldings).
- de Sitter space is still ruled out by the SEC (if the 4d Newton's constant is time-independent in the conventional sense).
- Late-time acceleration is not ruled out by the SEC (although no known examples from 10d/11d compactifications, if we require non-vanishing acceleration asymptotically).

* *Russo & Townsend*, 2018; 2019

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# Lessons from universal compactifications

- Recently we re-examined these statements within the framework of ***universal*** 10d compactifications.
  - Type II supergravity 10d solutions with a 4d FLRW factor.
  - Compactification on 6d Einstein, Einstein-Kähler, or CY.
  - Solutions ***independent of the*** compactification ***details***.
  - All 10d solutions are obtainable from a 1d action (consistent truncation) of 3 time-dependent scalars (the dilaton and 2 warp factors). All fluxes appear as constant coefficients in the potential.
  - In many cases there is a 4d consistent truncation to gravity+2 scalars (the dilaton and 1 warp factor). All of these admit further consistent sub-truncations to 1 scalar.

# Lessons from universal compactifications

- *Examples of transient acceleration in a near-de Sitter state with parametric control of e-foldings; rollercoaster; (semi-)eternal acceleration. They all require an open 4d universe and asymptotically vanishing acceleration, hence no event horizon.*
- *The common lore that transient acceleration always gives* $\mathcal{O}(1)$ *e-foldings is false.*

# The 4d consistent truncation (cosmological)

- A subset of the 10d solutions are derivable from

$$S_{4\mathrm{d}} = \int \mathrm{d}^4x\sqrt{g}\Big(R - 24g^{\mu\nu}\partial_\mu A\partial_\nu A - \tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(A,\phi)\Big)$$

where

$$V = \begin{cases} 72b_0^2e^{-\phi-12A} + \frac{3}{2}c_0^2e^{\phi/2-14A} & \text{CY with internal 3- and 4-form fluxes} \\ \frac{1}{2}c_\varphi^2e^{-\phi/2-18A} + \frac{1}{2}m^2e^{5\phi/2-6A} - 6\lambda e^{-8A} & \text{E with external 4-form flux} \\ \frac{3}{2}c_0^2e^{\phi/2-14A} + \frac{1}{2}m^2e^{5\phi/2-6A} - 6\lambda e^{-8A} & \text{EK with internal 4-form flux} \\ \frac{1}{2}c_\varphi^2e^{-\phi/2-18A} + \frac{3}{2}c_f^2e^{3\phi/2-10A} - 6\lambda e^{-8A} & \text{EK with internal 2-form, external 4-form} \end{cases}$$

- In the CY case: a sub-truncation, to the metric and two scalars, of the consistent truncation to the universal sector.

* *Robin Terrisse & DT*, 2019 ; *DT*, 2020

# The consistent truncation

- The 10d origin of the constants

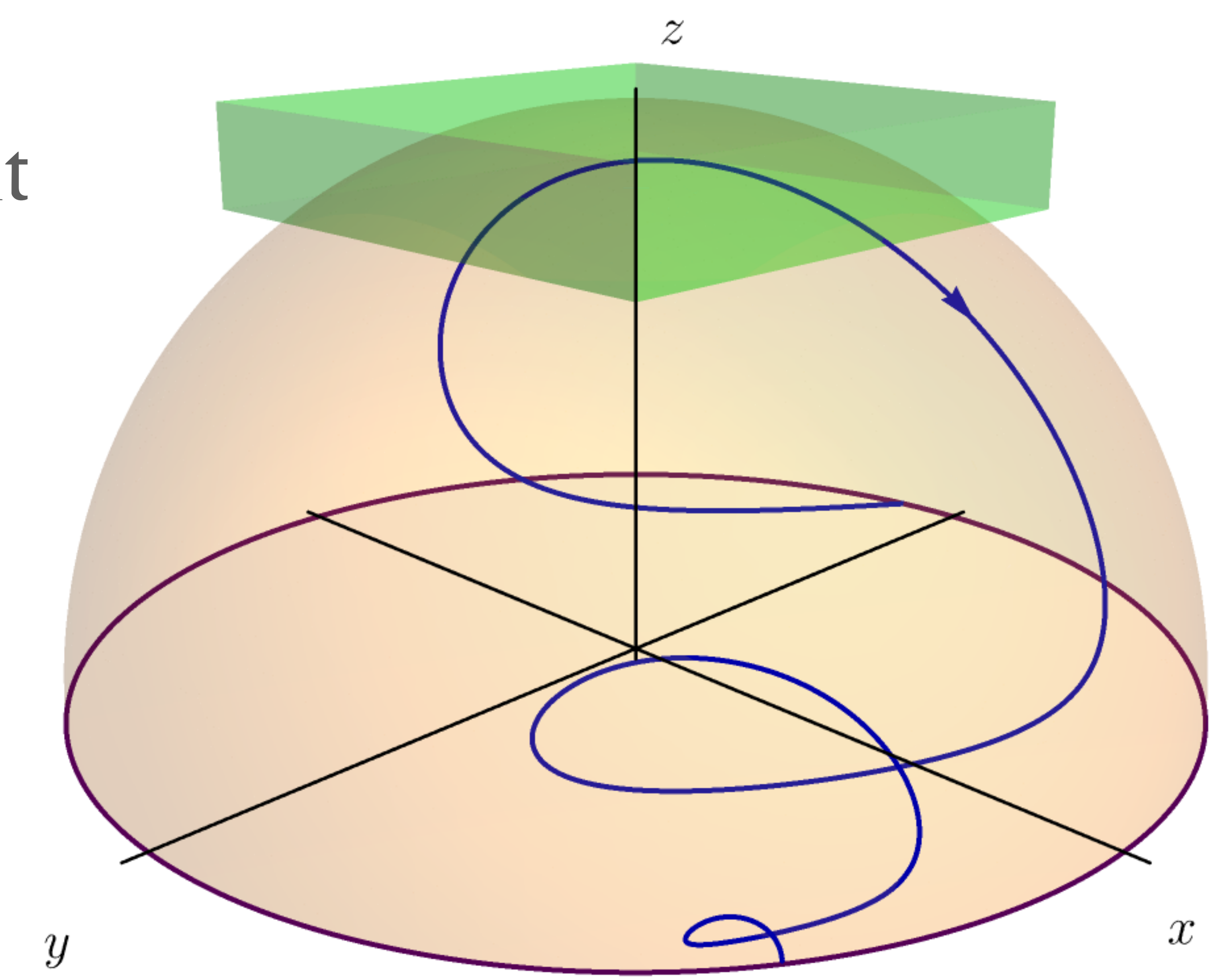
| | |
|---|---|
| $m$ | zero-form (Romans mass) |
| $c_f$ | internal two-form |
| $c_h$ | external three-form |
| $b_0$ | internal three-form |
| $c_\chi$ | mixed three-form |
| $c_\varphi$ | external four-form |
| $c_0$ | internal four-form |
| $c_{\xi\xi'}$ | mixed four-form |
| $k$ | external curvature |
| $\lambda$ | internal curvature |

# Dynamical system analysis

- Many analytic solutions (some with up to four species of flux).
  - Always possible if a single excited species of flux.
- Autonomous dynamical system if 2 excited species of flux.
  - 3 first-order equations and a constraint.
  - *Solutions correspond to trajectories in a 3d phase-space*.
  - Compactification of phase-space to (the interior of) a 3d ball.
  - The equatorial disc and the 2d sphere boundary are invariant surfaces of the dynamical flow.
  - Fixed points and trajectories on the sphere boundary or on the disc correspond to analytic solutions. *Fixed points correspond to scaling solutions*: $a(t) \propto t^p$
  - There is always an additional invariant plane (sub-truncation).

# Dynamical system analysis

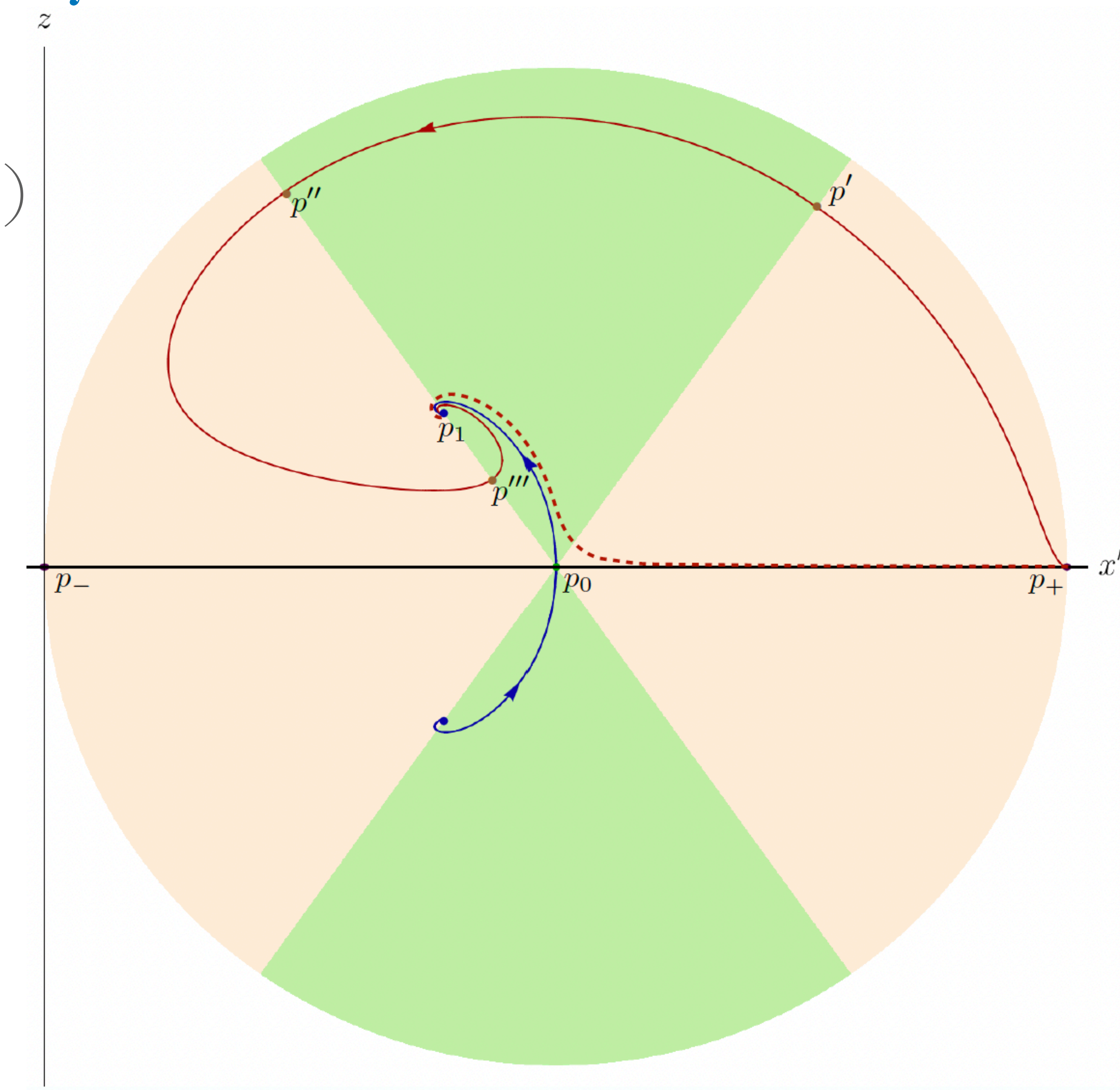
- Rephrasing the question of accelerated expansion.
  - Expanding cosmologies correspond to trajectories in the northern hemisphere (interpolating between two fixed points).
  - Acceleration is possible whenever there is a non-empty ***acceleration region*** (determined by the type of excited fluxes).
  - This explains why transient accelerated expansion is generic: it corresponds to trajectories in the northern hemisphere, passing through the acceleration region.



# Some unexpected results

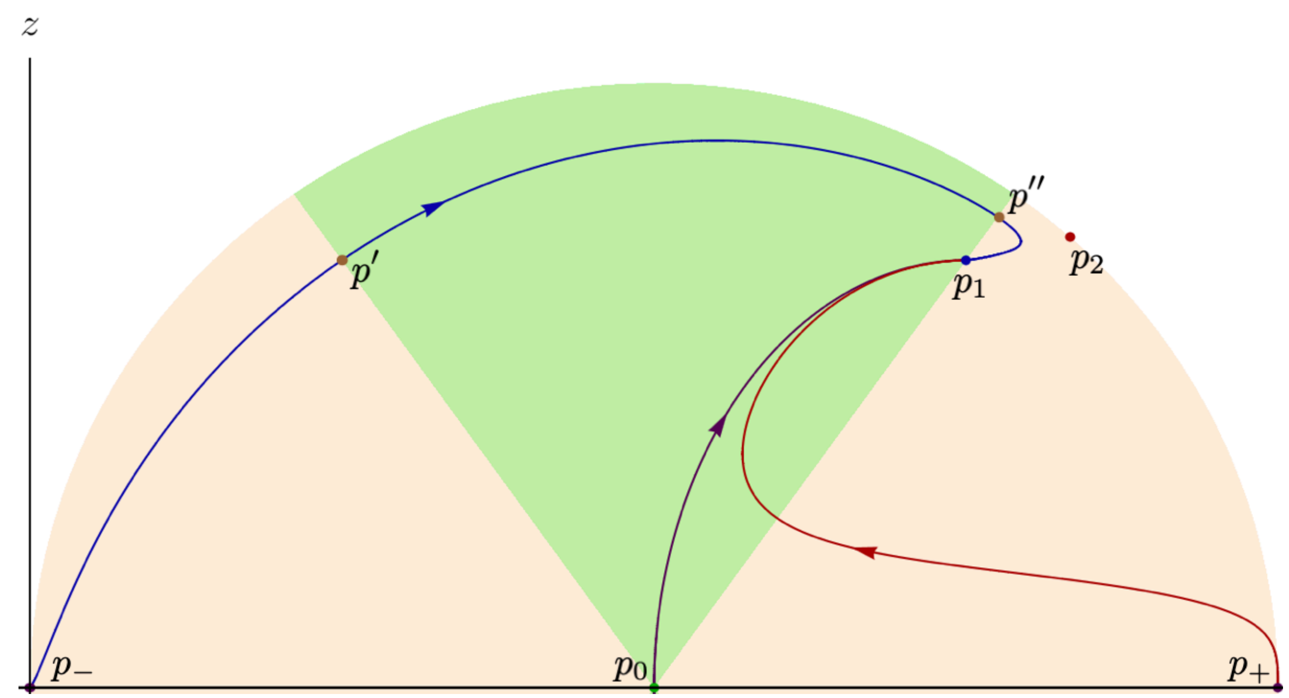
- Examples of rollercoaster cosmologies and transient acceleration with parametric control of the number of e-foldings
- Example without initial singularity
  - Accelerated contraction (expansion) for $t < 0$ ( $t > 0$ )
  - de Sitter in the neighborhood of $t = 0$

$$\mathrm{d}s^2_{\mathrm{dS}} = -\mathrm{d}t^2 + l^2 \sinh^2\left(\frac{t}{l}\right)\,\mathrm{d}\Omega^2_k$$



# Some unexpected results

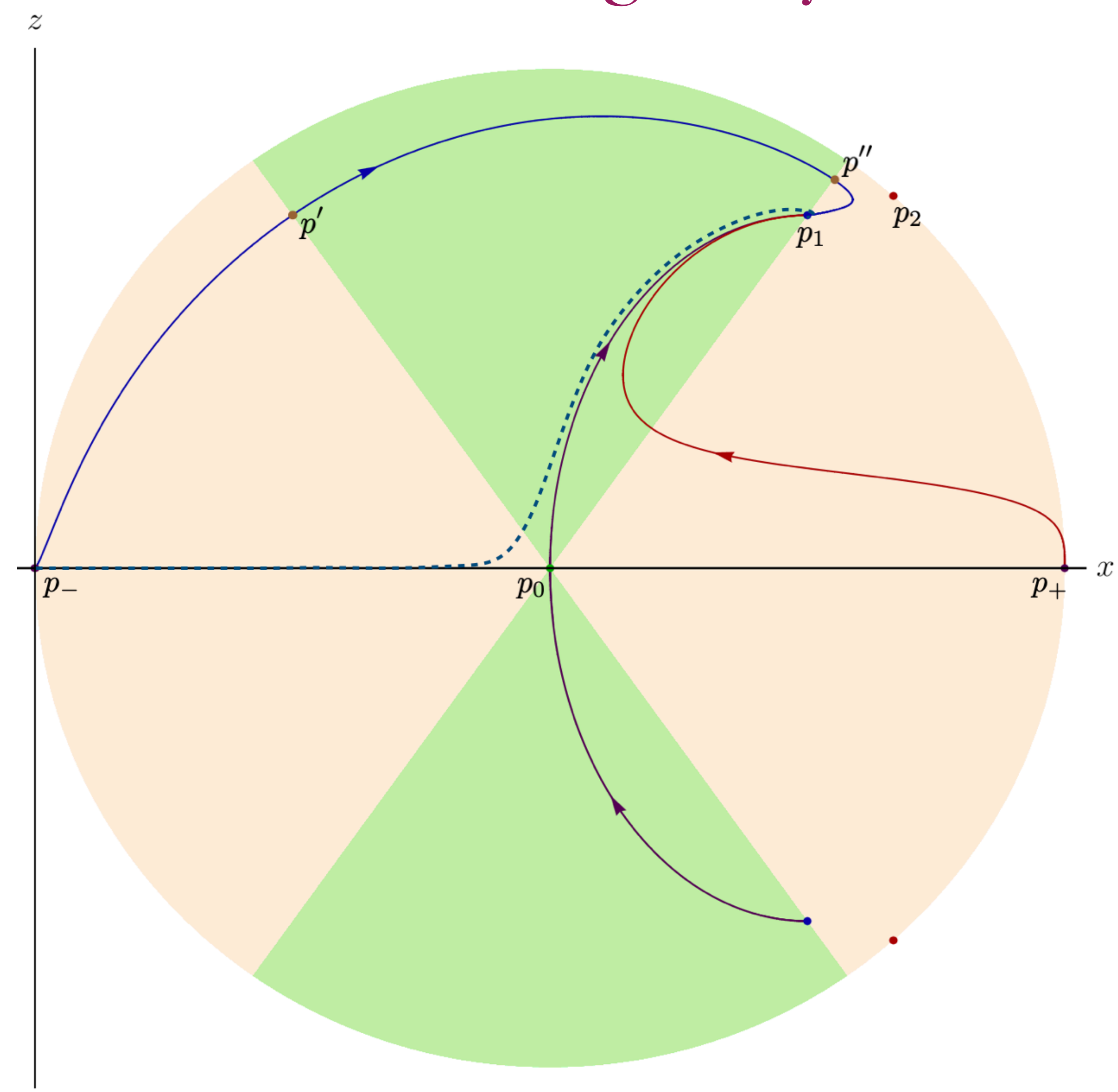
- Examples of semi-eternal and eternal acceleration
- *Andersson & Heinzle*, 2006



# Some unexpected results

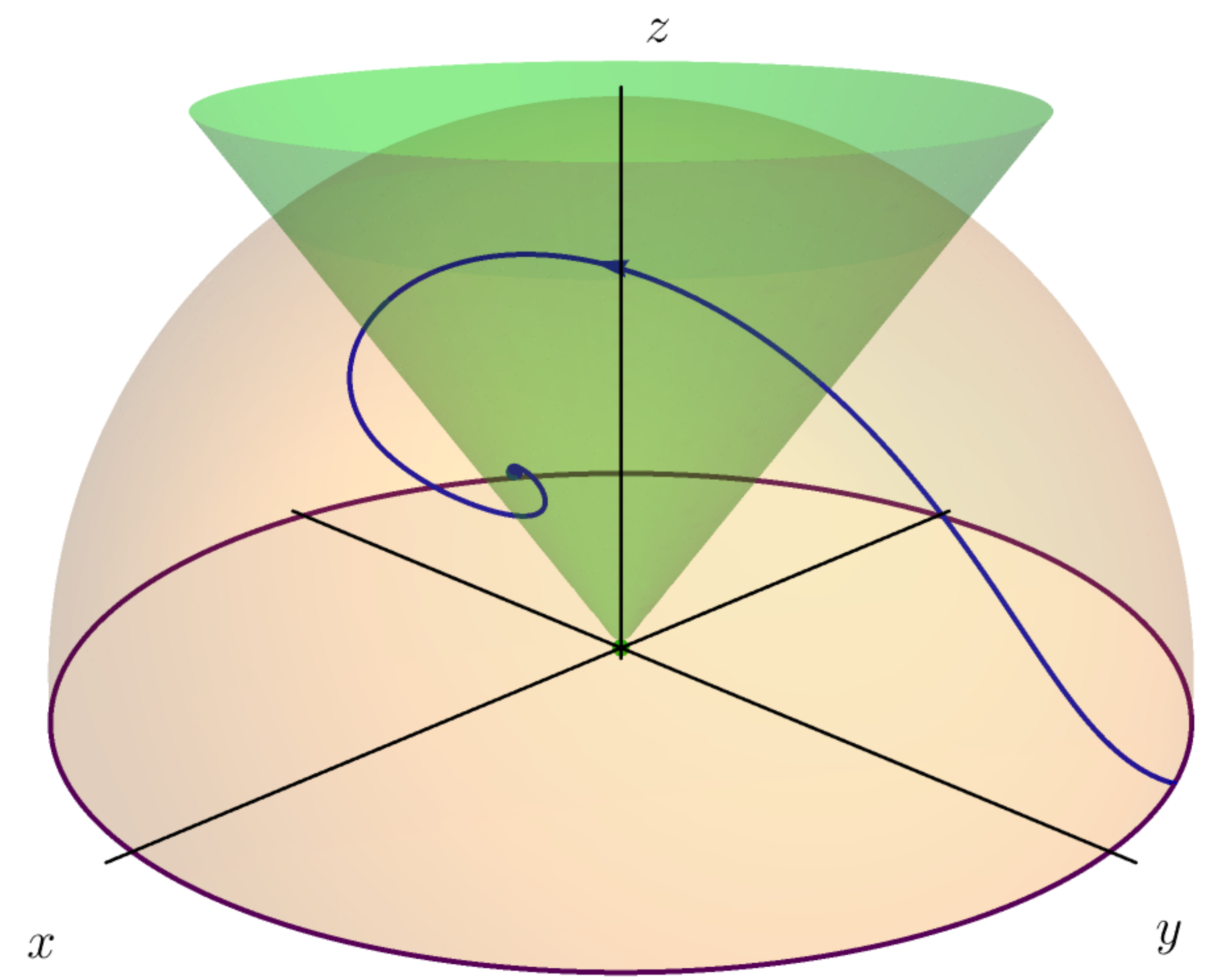
- Examples of semi-eternal and transient accelerated expansion with parametric control of the number of e-foldings
- An example of eternal acceleration without initial singularity
  - Accelerated contraction (expansion) for $t < 0 \quad (t > 0)$
  - de Sitter in the neighborhood of $t = 0$

$$ds^2_{\mathrm{dS}} = -\mathrm{d}t^2 + l^2 \sinh^2\left(\frac{t}{l}\right) \mathrm{d}\Omega_k^2$$



# Some unexpected results

- Many examples of (semi-)eternal, rollercoaster and transient accelerated expansion in a near-de Sitter space with parametric control of e-foldings.
  - They have $k = -1$
  - They have a fixed point *on the boundary of the acceleration region hence no event horizon*.



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# A $d$-dimensional model

- Action

$$S = \int \mathrm{d}^d x \sqrt{|g_d|}\left(\frac{1}{2}\mathcal{R}_d - \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - V_0\, e^{-\gamma\,\varphi}\right)$$

- Equations of motion

$$\frac{(d-1)(d-2)}{2}\left(H^2 + \frac{k}{a^2}\right) = \rho$$

$$(d-2)\frac{\ddot{a}}{a} + \frac{d-3}{d-1}\rho + p = 0 \Leftrightarrow \dot{H} - \frac{k}{a^2} + \frac{\rho+p}{d-2} = 0$$

$$\ddot{\varphi} + (d-1)H\dot{\varphi} + V' = 0$$

where

$$H = \frac{\dot{a}}{a}\ , \quad \rho = \frac{1}{2}\dot{\varphi}^2 + V\ , \quad p = \frac{1}{2}\dot{\varphi}^2 - V$$

and

$$\ddot{a} \geq 0 \ \Leftrightarrow\ w := \frac{p}{\rho} \leq -\frac{d-3}{d-1}$$

# Dynamical system

- Phase space variables

$$N = \ln a \ , \qquad x = \frac{\dot{\varphi}}{H\sqrt{(d-1)(d-2)}} \ , \qquad y = \frac{\sqrt{2V}}{H\sqrt{(d-1)(d-2)}}$$

- Equations of motion

$$\frac{\mathrm{d}x}{\mathrm{d}N} = -\frac{\sqrt{(d-1)(d-2)}}{2}\,\frac{V'}{V}\,y^2 - x\left(d-2-x^2(d-2)+y^2\right)$$

$$\frac{\mathrm{d}y}{\mathrm{d}N} = y\left(\frac{\sqrt{(d-1)(d-2)}}{2}\,\frac{V'}{V}\,x + 1 + x^2(d-2) - y^2\right)$$

- *Constraint*

$$x^2 + y^2 = 1 + \frac{k}{\dot{a}^2}$$

# Dynamical system

- Fixed points

| Fixed point $(x, y)$ | Allowed $k$ | Existence constraint | Acceleration |
|---|---|---|---|
| $P_0 :\ (0,\, 0)$ | $k = -1$ | $\dot{a}_0^2 = 1$ | no $(\ddot{a} = 0)$ |
| $P_\pm :\ (\pm 1,\, 0)$ | $k = 0$ | - | no $(\ddot{a} < 0)$ |
| $P_1 :\ \left(\frac{2}{\gamma\sqrt{(d-1)(d-2)}}, \pm\frac{2}{\gamma\sqrt{d-1}}\right)$ | $k = 0, \pm 1$ | $\gamma^2 = \frac{4}{d-2}\left(1 + \frac{k}{\dot{a}_0^2}\right)^{-1}$ | no $(\ddot{a} = 0)$ |
| $P_2 :\ \left(\frac{\gamma}{2}\sqrt{\frac{d-2}{d-1}}, \pm\sqrt{1 - \frac{\gamma^2}{4}\frac{d-2}{d-1}}\right)$ | $k = 0$ | $0 \leq \gamma^2 < 4\,\frac{d-1}{d-2}$ | iff $\gamma^2 < \frac{4}{d-2}$ |

# Dynamical system

- For $k=0,1$ existence of $P_1$ requires

$$\gamma^2 \leq \frac{4}{d-2}$$

- For $k=-1$ existence of $P_1$ requires

$$\gamma^2 > \frac{4}{d-2}$$

  - For $d \geq 10$ stable node
  - For $d < 10$ stable node if

$$\gamma^2 \leq \gamma_s^2 \equiv \frac{32}{(d-2)(10-d)}$$

  - For $d < 10$ stable spiral if

$$\gamma^2 > \gamma_s^2$$

# Dynamical system

- Acceleration

$$|y| > |x|\sqrt{d-2}$$

- Expansion

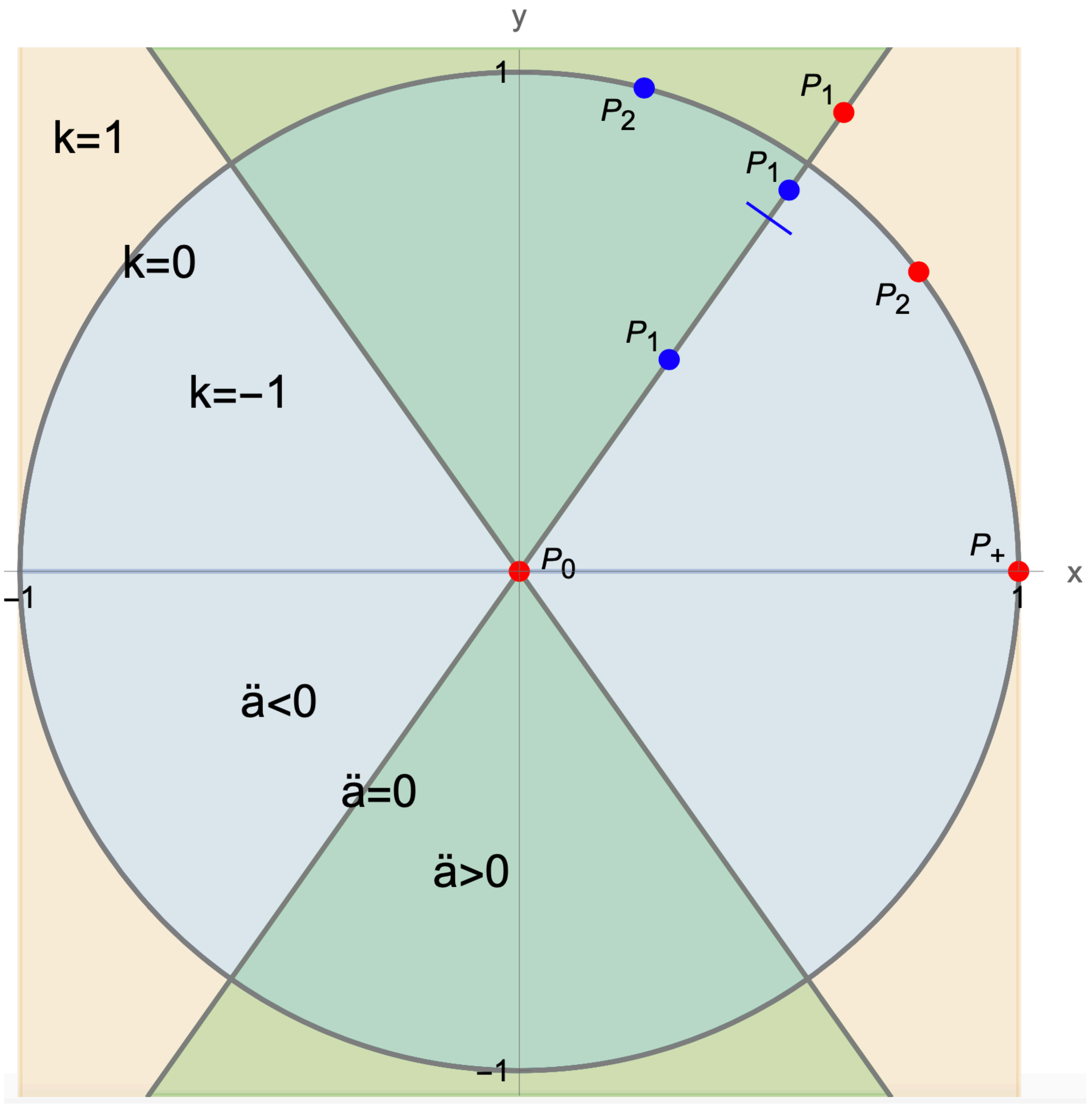
$$y > 0$$

- Open universe (*k*=-1)

$$x^2 + y^2 < 1$$

- Flat universe (*k*=0)

$$x^2 + y^2 = 1$$



# Dynamical system

- $P_I$ is a Milne universe with angular defect

$$a(t) = a_0\,(t - t_0)\ , \qquad \varphi(t) = \varphi_0 + \varphi_l\, \log(t - t_0)\ ,$$

$$a_0 = \frac{\gamma}{\sqrt{\gamma^2 - \frac{4}{d-2}}}\ , \ \varphi_0 = \frac{1}{\gamma}\log\left(\frac{\gamma^2 V_0}{2(d-2)}\right)\ , \ \varphi_l = \frac{2}{\gamma}$$

- All solutions known analytically in the vicinity of critical points, e.g.

$$a(t) = a_0\, t\left(1 + \frac{a_1}{t^p} + \ldots\right)\ ; \quad \varphi(t) = \varphi_0 + \varphi_l \log(t) + \frac{\varphi_1}{t^p} + \ldots$$

$$p^{\pm} = \frac{d-2}{2} \pm \frac{2\sqrt{2}}{\gamma}\sqrt{1 + \gamma^2\frac{(d-10)(d-2)}{32}}\ , \quad \varphi_1^{\pm} = \frac{d-1}{4}\, a_1\gamma\, p^{\pm}$$

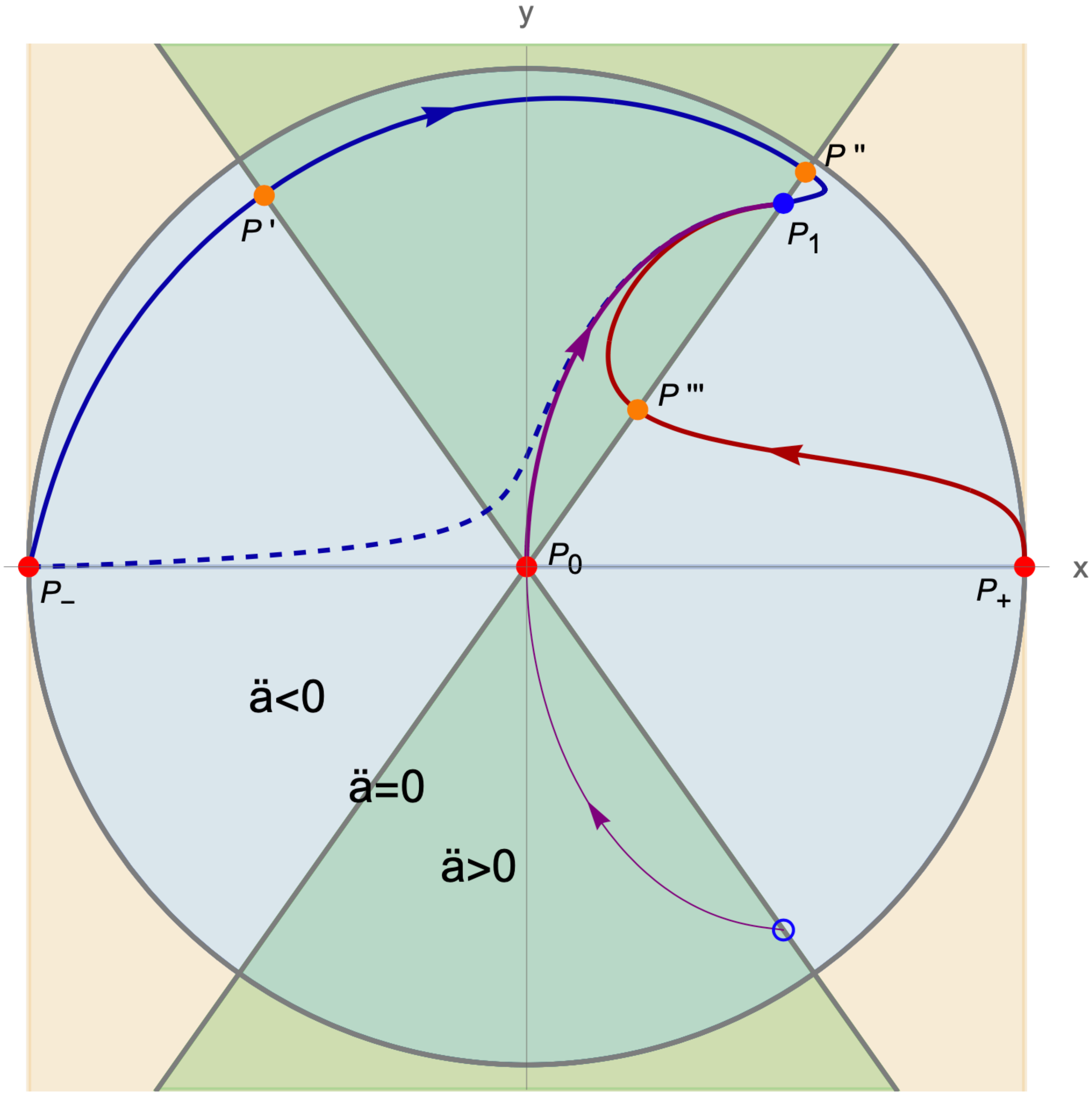
- Acceleration

$$\ddot{a}(t) = a_0 a_1 (p-1)p\,\frac{1}{t^{1+p}} + \mathcal{O}\left(t^{-(1+2p)}\right)$$

- All solutions asymptoting $P_I$ are free of (cosmic) event horizons.

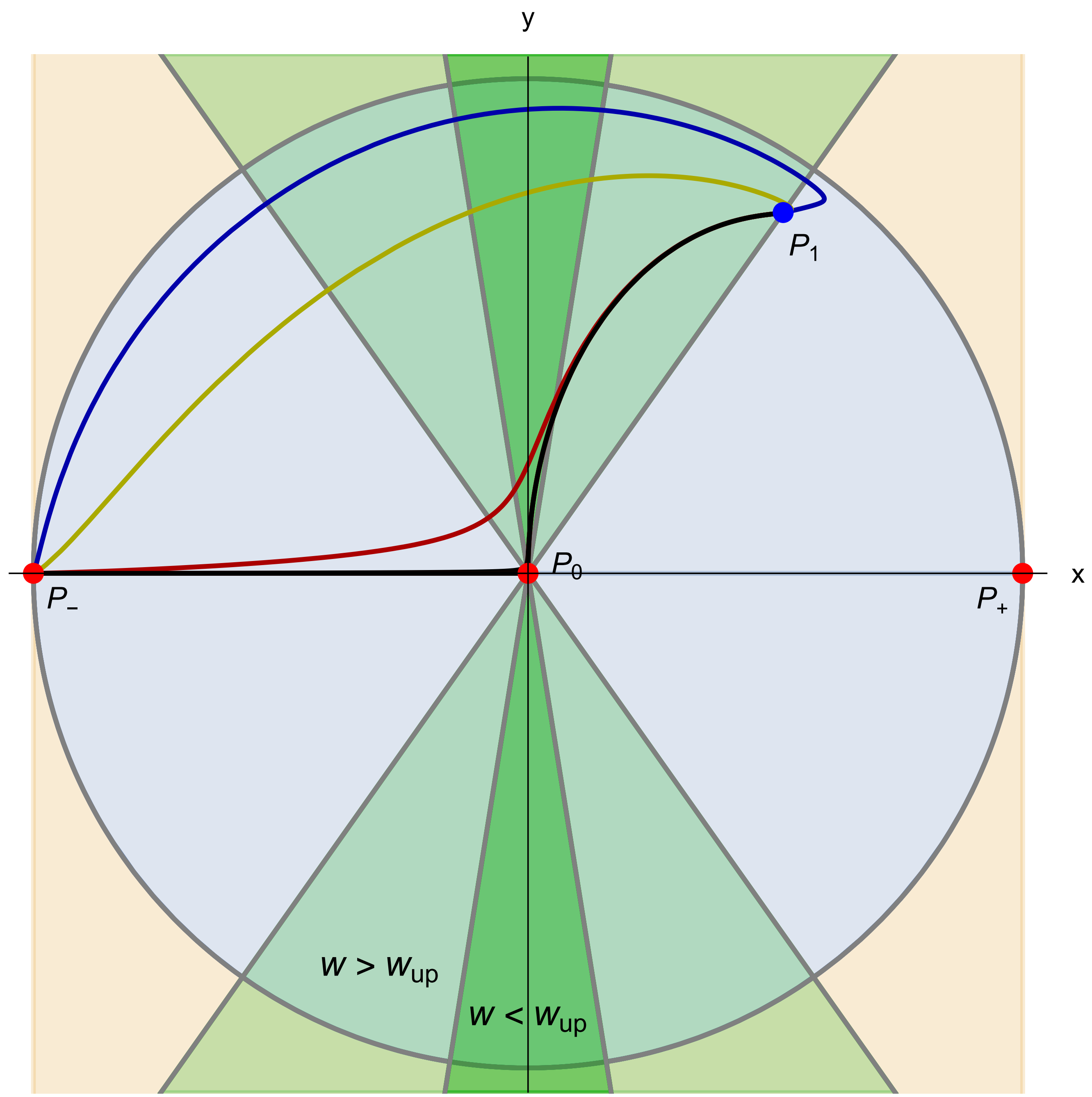
# Dynamical system

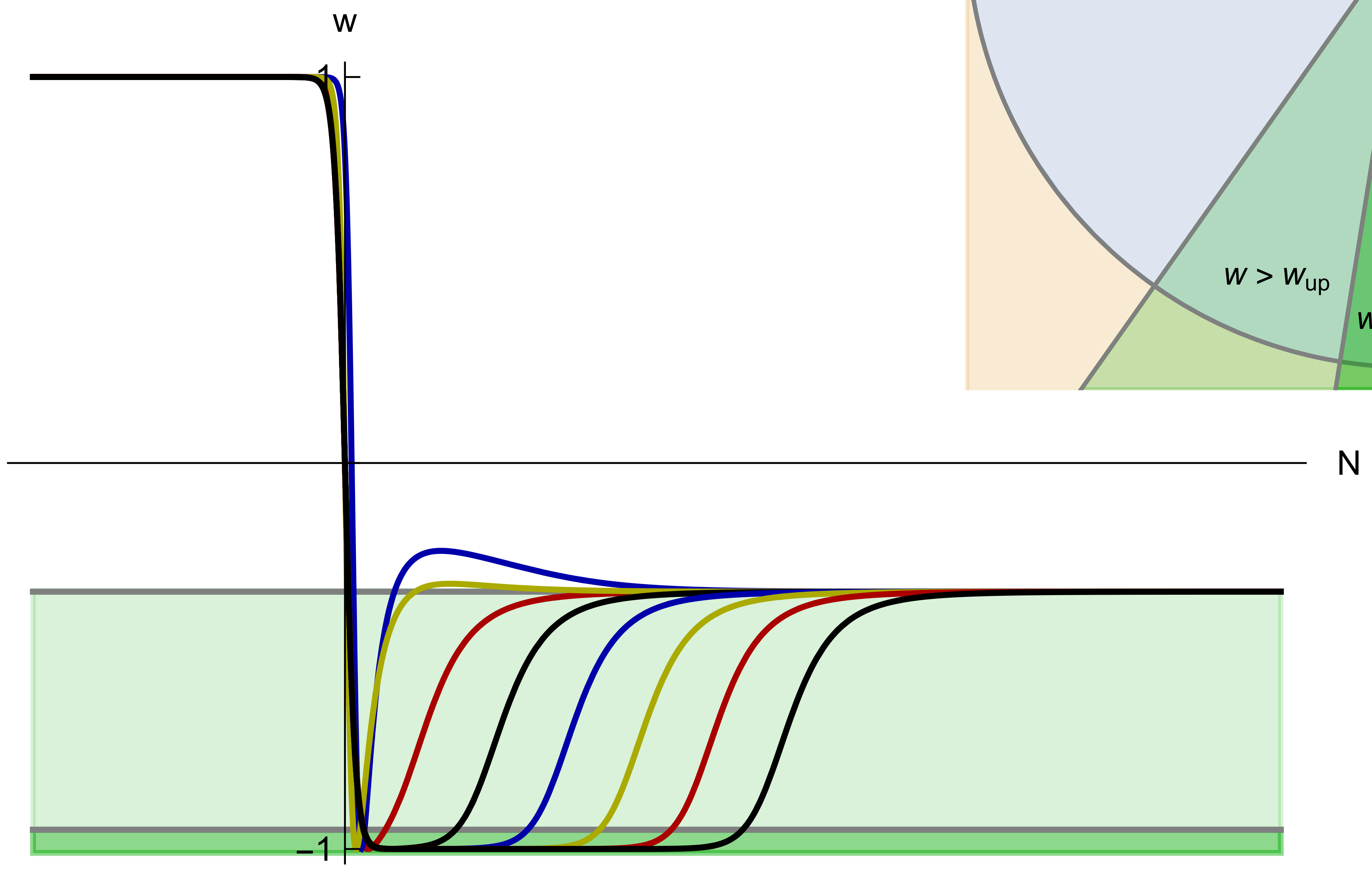
- Phase portraits of solutions asymptoting ***P₁*** stable node.



# Dynamical system

- Parametric control of e-foldings



# Dynamical system

- Uplift to a *10d* solution

$$2\sqrt{6}\,A \leftrightarrow \varphi \ ; \quad 3e^{-8A} \leftrightarrow V(\varphi) = 3e^{-\frac{4}{\sqrt{6}}\varphi} \ ; \quad \gamma = -\frac{V'(\varphi)}{V} = \frac{4}{\sqrt{6}}$$

where

$$\mathrm{d}s^2_{10} = e^{-6A}\mathrm{d}s^2_{4E} + e^{2A} g_{mn}\mathrm{d}y^m \mathrm{d}y^n$$

$$\mathrm{d}s^2_{4E} = -\mathrm{d}t^2 + a^2\mathrm{d}\Omega^2_k \ ; \quad R_{mn} = -6g_{mn}$$

$$\phi = \mathrm{cnst}\ .$$

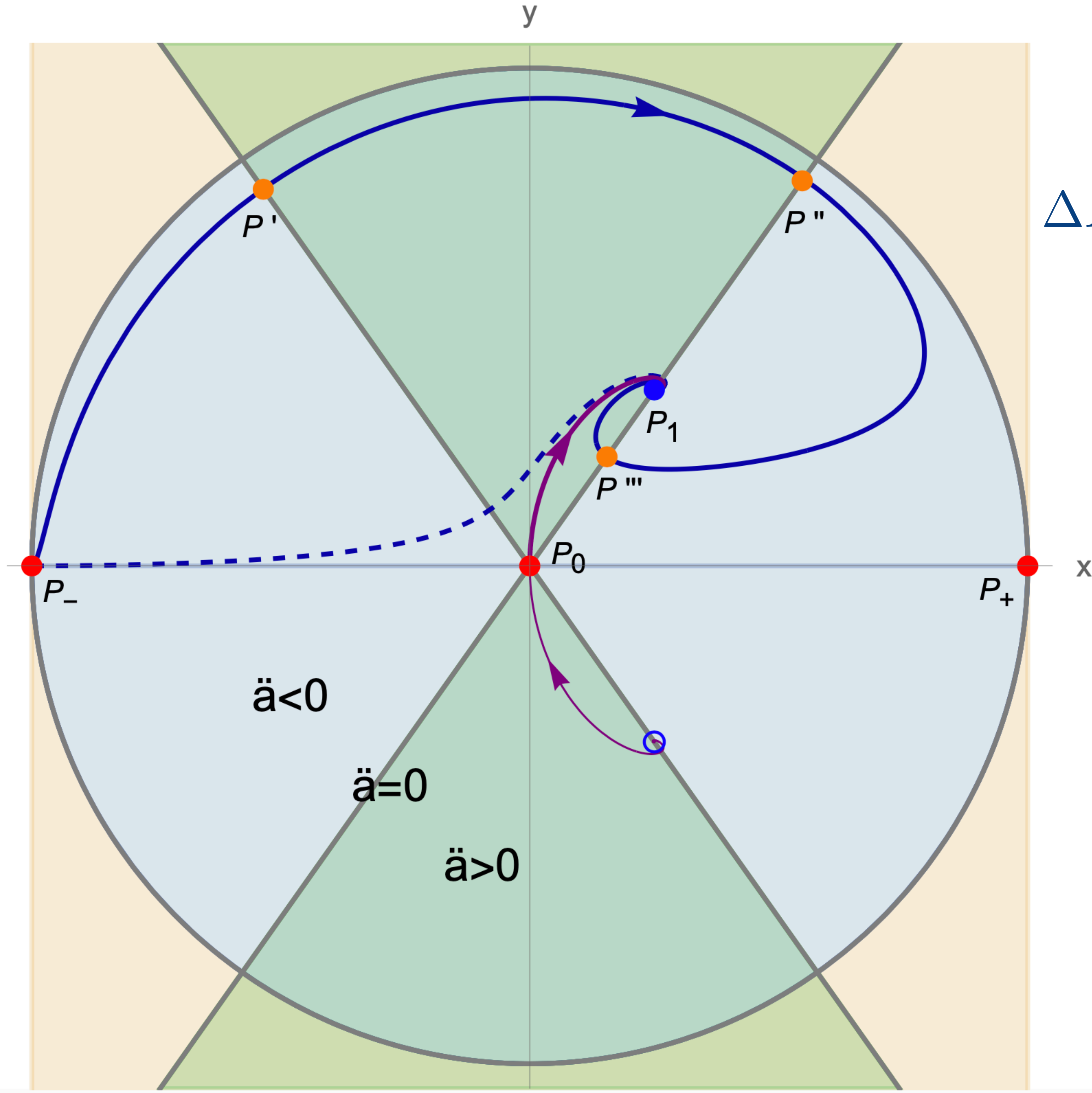
- Late time behavior

$$A \to \infty \ ; \quad g_s = \mathrm{cnst} \ ; \quad L_6 H \to 0\ .$$

$$\mathrm{d}s^2_{10} \sim \mathrm{d}T^2 + T^2(1+\dots)\mathrm{d}\Omega^2_k + T^2(1+\dots)\mathrm{d}s^2_6 \ ; \quad T \propto t^{\frac{1}{4}}$$

# Dynamical system

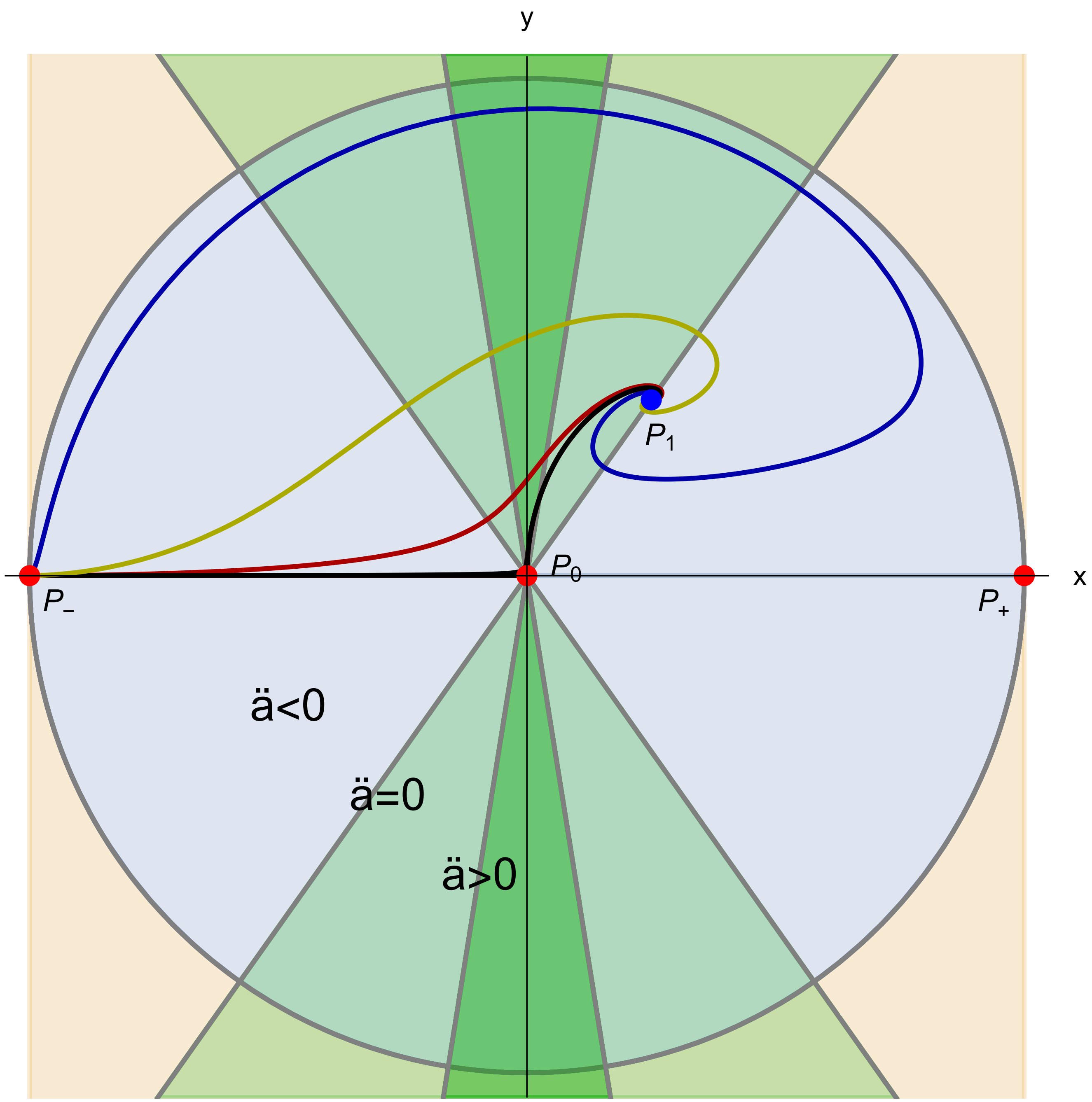
- Solutions asymptoting ***P₁*** stable spiral.

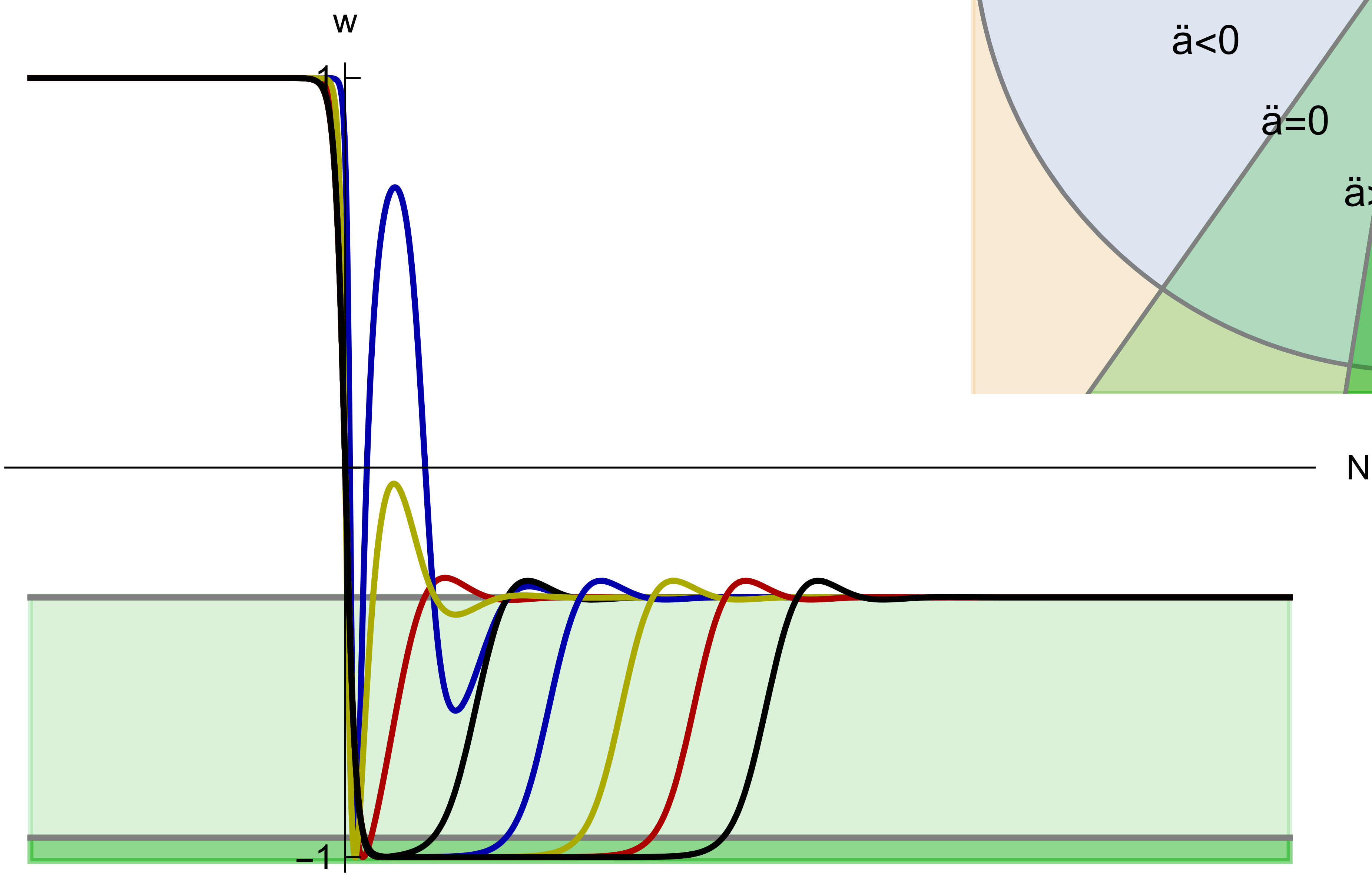



$$\Delta N = \frac{\pi}{\sqrt{2}} \frac{\gamma\,\gamma_s}{\sqrt{\gamma^2-\gamma_s^2}}$$

# Dynamical system

- Parametric control of e-foldings



# Dynamical system

- Uplift to a *10d* solution

$$\frac{4}{5}\sqrt{78}A \leftrightarrow \varphi \ ; \quad \frac{3}{4}c_f^2\, e^{-\frac{104}{5}A} \leftrightarrow V(\varphi) = \frac{3}{4}c_f^2\, e^{-\sqrt{\frac{26}{3}}\varphi}$$

$$\phi = -\frac{36}{5}A \ ; \quad \gamma = -\frac{V'(\varphi)}{V} = \sqrt{\frac{26}{3}}$$

where

$$\mathrm{d}s_{10}^2 = e^{-6A}\mathrm{d}s_{4E}^2 + e^{2A} g_{mn}\mathrm{d}y^m\mathrm{d}y^n$$

$$\mathrm{d}s_{4E}^2 = -\mathrm{d}t^2 + a^2\mathrm{d}\Omega_k^2 \ ; \quad R_{mn} = 0$$

$$F = c_f J$$

- Late time behavior

$$A \to \infty \ ; \quad g_s \to 0 \ ; \quad L_6 H \to 0 \ .$$

$$\mathrm{d}s_{10}^2 \sim \mathrm{d}T^2 + T^2(1+\dots)\mathrm{d}\Omega_k^2 + T^{\frac{10}{37}}(1+\dots)\mathrm{d}s_6^2 \ ; \quad T \propto t^{\frac{37}{52}}$$

# Conclusions

- *You can't always get what you want, but if you try sometimes, you might find you get what you need.*

* *Jagger & Richards*, Let it Bleed, 1969

- Examples of *(semi-)eternal acceleration*; *rollercoaster*; *transient acceleration in a near-de Sitter state with parametric control of e-foldings*.
  - They all have $k = -1$ *and asymptotically vanishing acceleration*.
- Nature abhors a (cosmic) event horizon ?
  - Could that « explain » the absence of de Sitter and/or eternally accelerating *scaling* solutions ?

# Conclusions

- Solutions in the classical string regime (asymptotically/for some period of time).
- No branes/orientifolds.
- Truncated modes/stability ? (Note: universal truncations capture sub-sectors of the effective theory).
- Moduli stabilization ? (Note: rigid examples are possible).
- Higher-order corrections ? (Note: classical string regime is possible).
- Realistic cosmologies ? Inflation ?

# Appendix

# Type IIA supergravity

- Action

$$S = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{g}\Big( - R + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2\cdot 2!}e^{3\phi/2}F^2 + \frac{1}{2\cdot 3!}e^{-\phi}H^2 + \frac{1}{2\cdot 4!}e^{\phi/2}G^2 + \frac{1}{2}m^2 e^{5\phi/2}\Big) + S_{\mathrm{CS}}$$

- Bianchi identities

$$\mathrm{d}F = mH \ ; \quad \mathrm{d}H = 0 \ ; \quad \mathrm{d}G = H\wedge F$$

# Metrics & times

- The 10d Einstein-frame metric

$$ds_{10}^2 = e^{2A}\left[e^{2B}(-d\eta^2 + d\Omega_k^2) + g_{mn}dy^m dy^n\right]$$

where

$$d\Omega_k^2 = \gamma_{ij}(x)dx^i dx^j \ ; \quad R_{ij}^{(3)} = 2k\gamma_{ij}$$

- The 4d Einstein-frame metric

$$ds_{4E}^2 = -a^6 d\tau^2 + a^2 d\Omega_k^2$$

where

$$a = e^{4A+B} \ ; \quad \frac{d\eta}{d\tau} = a^2$$

- The cosmological time

$$\frac{dt}{d\tau} = a^3 \ ; \quad ds_{4E}^2 = -dt^2 + a^2 d\Omega_k^2$$

# Flux Ansätze: some examples

- Calabi-Yau

$m = 0 \; ; \quad F = 0 \; ; \quad H = \frac{1}{2} b_0 \mathrm{Re}\Omega \; ; \quad G = \frac{1}{2} c_0 J \wedge J \; ; \quad R_{mn} = 0$

solution of form equations and Bianchi identities

- Einstein-Kähler with internal 2-form

$m = 0 \; ; \quad F = c_f J \; ; \quad H = 0 \; ; \quad G = 0 \; ; \quad R_{mn} = \lambda \, g_{mn}$

solution of form equations and Bianchi identities

# The 1d consistent truncation

- The remaining equations of motion (Einstein & dilaton)

$$\mathrm{d}_\tau^2 A = -\tfrac{1}{48}\left(\partial_A U - 4\partial_B U\right)$$

$$\mathrm{d}_\tau^2 B = \tfrac{1}{12}\left(\partial_A U - 3\partial_B U\right)$$

$$\mathrm{d}_\tau^2 \phi = -\partial_\phi U$$

- Constraint

$$72(\mathrm{d}_\tau A)^2 + 6(\mathrm{d}_\tau B)^2 + 48\mathrm{d}_\tau A\mathrm{d}_\tau B - \tfrac{1}{2}(\mathrm{d}_\tau \phi)^2 = U$$

# The 1d consistent truncation

- They are derivable from

$$S_{1\mathrm{d}} = \int \mathrm{d}\tau \Big\{ \frac{1}{\mathcal{N}} \Big( -72(\mathrm{d}_\tau A)^2 - 6(\mathrm{d}_\tau B)^2 - 48\mathrm{d}_\tau A \mathrm{d}_\tau B + \tfrac{1}{2}(\mathrm{d}_\tau \phi)^2 \Big) - \mathcal{N} U(A,B,\phi) \Big\}$$

where

$$U = \begin{cases} \frac{1}{2}c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2}c_h^2 e^{-\phi+12A} + \frac{3}{2}c_\chi^2 e^{\phi+4A} + c_\xi^2 e^{-\phi/2+6A} - 6ke^{16A+4B} & \text{CY} \\ 72b_0^2 e^{-\phi+12A+6B} + \frac{3}{2}c_0^2 e^{\phi/2+10A+6B} & \text{CY} \\ \frac{1}{2}c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2}m^2 e^{5\phi/2+18A+6B} - 6ke^{16A+4B} - 6\lambda e^{16A+6B} & \text{E} \\ \frac{1}{2}c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2}c_h^2 e^{-\phi+12A} + \frac{3}{2}c_\chi^2 e^{\phi+4A} - 6ke^{16A+4B} - 6\lambda e^{16A+6B} & \text{EK} \\ \frac{3}{2}c_0^2 e^{\phi/2+10A+6B} + \frac{1}{2}m^2 e^{5\phi/2+18A+6B} - 6ke^{16A+4B} - 6\lambda e^{16A+6B} & \text{EK} \\ \frac{1}{2}c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{3}{2}c_f^2 e^{3\phi/2+14A+6B} - 6ke^{16A+4B} - 6\lambda e^{16A+6B} & \text{EK} \end{cases}$$

# Dynamical system

- Vector field with ***P1*** a stable node and ***P2*** unstable.