Late-time Attractors and Cosmic Acceleration

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Based on work with:

Flavio Tonioni
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- G. Shiu, F. Tonioni, H.V. Tran, ``Accelerating universe at the end of time,'' [arXiv:2303.03418]
- G. Shiu, F. Tonioni, H.V. Tran, ``Late-time attractors and cosmic acceleration," [arXiv:2306.07327]
- G. Shiu, F. Tonioni, H.V. Tran, several ongoing works to appear.

A plea to the theorists

Nobel Prize 2011

Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory

Saul Perlmutter

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

Photo: Homewood Photography

Adam G. Riess

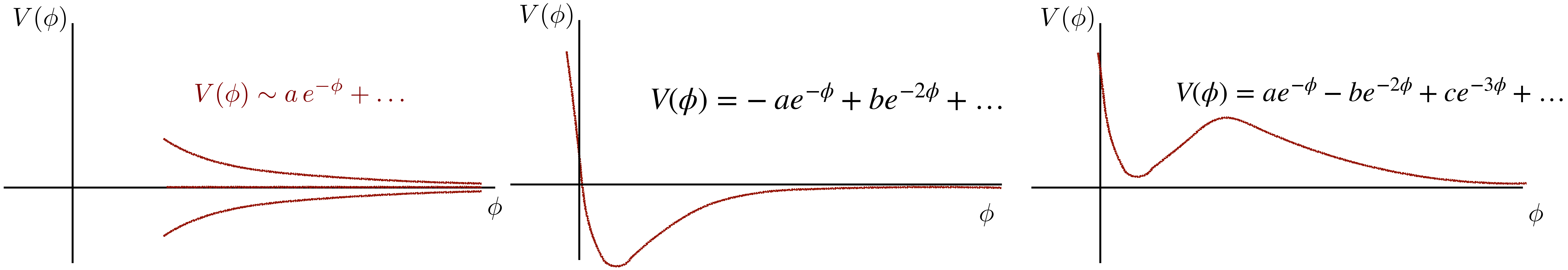
But Riess suspects that the mystery can't be solved by observations alone. "We won't really resolve it until some brilliant person, the next Einstein-like person, is able to get the idea of what's going on," he said.

So he issued **a plea to the theorists**: "Keep working," he said. "We need your help. ... It's a very juicy problem, it's hard, and **you'll win a Nobel Prize if you figure it out. In fact, I'll give you mine.**"

Oct 4, 2011 https://www.nbcnews.com/science/cosmic-log/physics-prize-highlights-puzzles-flna6c10402772

de Sitter vacua in String Theory

- Simplest possibility is $\Lambda > 0$. Sophisticated string theory scenarios for realizing dS vacua have been developed (KKLT, LVS, …), but a fully explicit construction remains elusive.
- Root of the challenge: source of cosmic acceleration should be ***derived*** (not just postulated) in a UV complete theory of gravity.
- It is a formidable task to demonstrate that the microphysics which stabilizes all moduli would lead to a theoretically controlled metastable de Sitter vacuum.
- The Dine-Seiberg problem: difficulty in finding parametrically weakly-coupled vacua.



Asymptotic runaway potentials

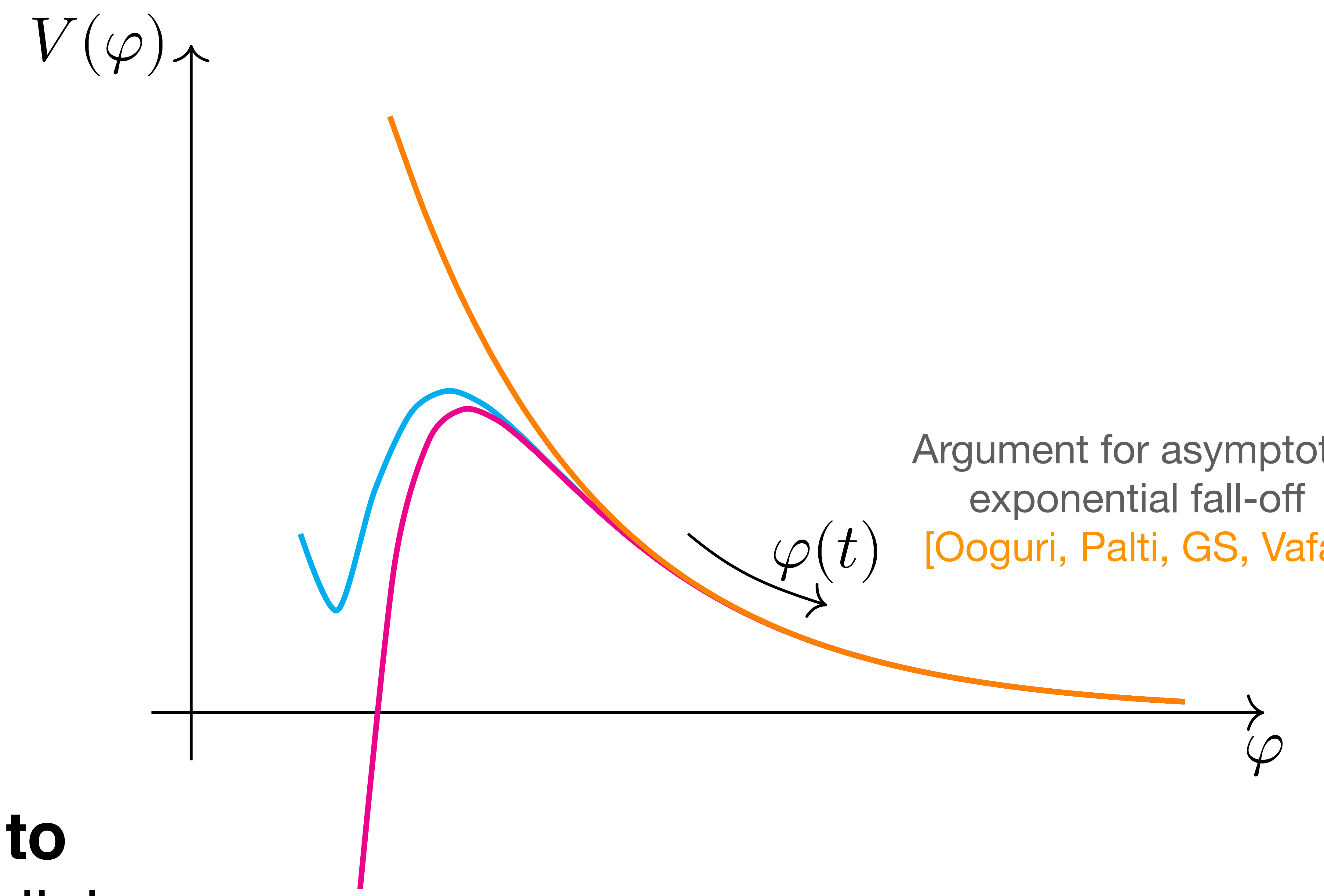
This makes runaway to the boundary of field space an interesting possibility.
[Obied, Ooguri, Spodyneiko, Vafa];[Ooguri, Palti, GS, Vafa]

Cosmic acceleration can be realized with:

- a de Sitter critical point, or
- a runaway potential with $\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$

Related to the "deceleration parameter" q:

$$q \equiv -\ddot{a}/\dot{a}^2 \qquad \epsilon = 1 + q.$$



Argument for asymptotic exponential fall-off [Ooguri, Palti, GS, Vafa]

Criterion for acceleration is in general unrelated to potential gradient. An aim or our work is to find the link (& the conditions for the link to exist) [GS, Tonioni, Tran]

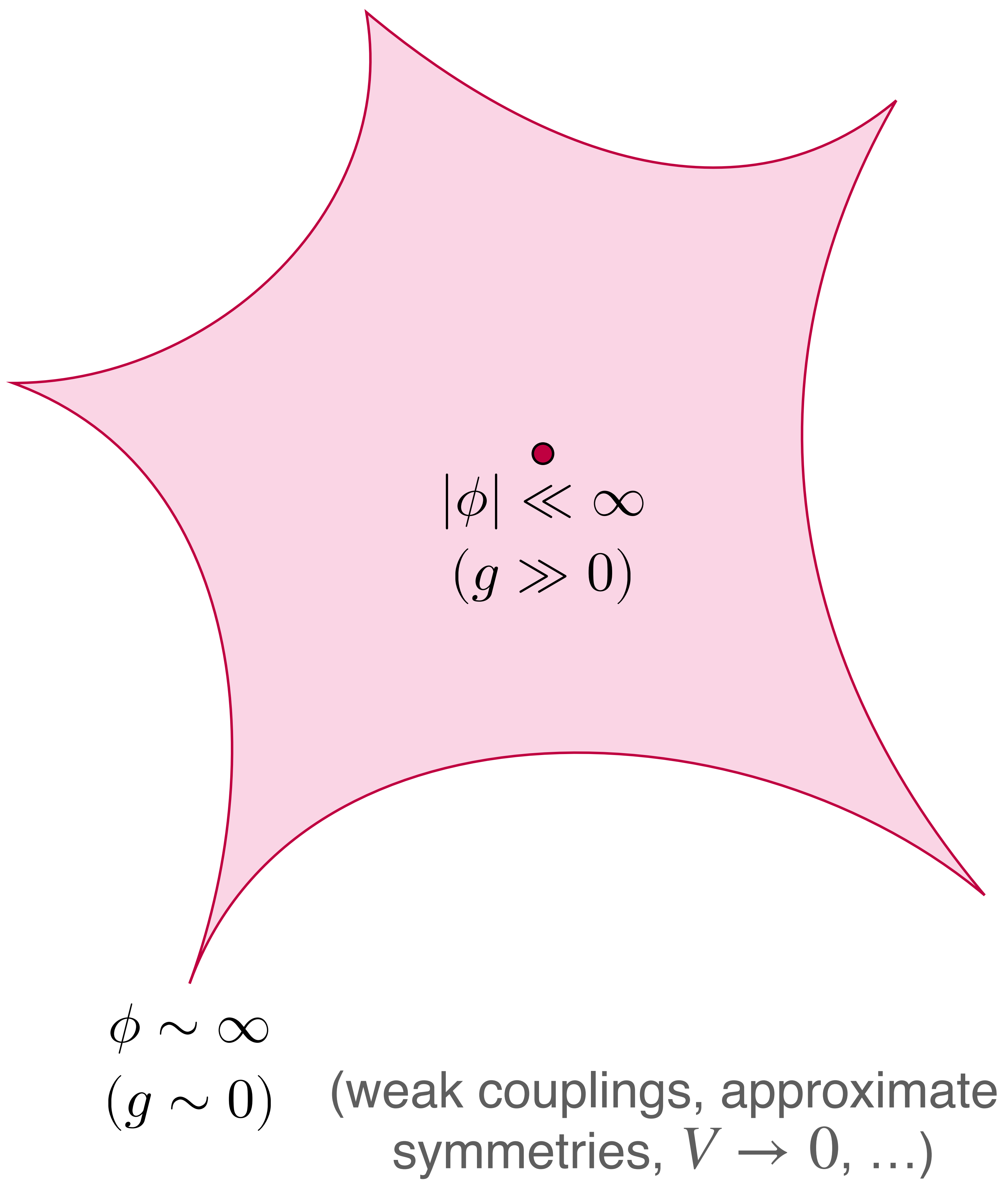
Asymptotic Dark Energy

- This possibility has recently been explored in various forms [Montero, Vafa, Valenzuela];[Rudelius];[Calderon-Infante, Ruiz, Valenzuela];[Marconnett, Tsimpis]; [GS, Tonioni, Tran x2]; [Cremonini, Gonzalo, Rajaguru, Tang, Wrase]; [Hebecker, Schreyer, Venken];[Van Riet];[Andriot, Tsimpis, Wrase];[Revello]; …

- As in many dynamical systems, the late-time regime exhibits some universal behaviors. This allows us to prove bounds on acceleration [GS, Tonioni, Tran, '23]

- Like large N expansion for QCD, studying the asymptotically late-time behavior may teach us about our current (old) universe [a la Dirac].

- Finding asymptotic dark energy in string theory is a tall order: **all non-rolling fields must be stabilized** and the runaway potential cannot be too steep.



Criterion for Cosmic Acceleration

- **How do we know if a model leads to cosmic acceleration w/o finding the on-shell solutions?**
  - (Slow-roll) inflation intuition: $\epsilon_V \ll 1, |\eta_V| \ll 1$
  - Swampland criteria are often stated in terms of gradient and/or curvature of the potential.
- **Dynamics is in general much more complicated!**
  - Time evolution does not follow gradient flow of the potential. Kinetic energy not negligible.
  - Under additional assumptions, late-time solutions approach an attractor known as scaling solutions [GS, Tonioni, Tran, 2306.07327]. Non-negligible kinetic energy yet $\epsilon = \epsilon_V$. Before an attractor is reached (which can take infinite time), $\epsilon \neq \epsilon_V$.
  - In models with 4d curvature [Andriot, Tsimpis, Wrase], ϵ_V also fails to provide the right diagnostic.
- **Our bound in** [GS, Tonioni, Tran, 2303.03418] **holds w/o knowledge of the actual solution to eoms.**

Summary of Results

[GS, Tonioni, Tran, ’23 x 3]

- We bound the rate of time variation of the Hubble parameter at late time [GS, Tonioni, Tran, ’23, STT1] The bound provides a useful diagnostic for dark energy models.

- Our bound when applied to string theoretic constructions identifies a generic obstacle to acceleration if the d-dim. dilation is one of the rolling fields. We also suggest several ways out.

- We prove conditions under which scaling solutions are **late-time attractors**. Moreover, we prove that scaling solutions **saturate** our bound on ϵ [GS, Tonioni, Tran, ’23, STT2].

- For scaling solutions, we showed $\gamma \equiv |\nabla V|/V = 2\sqrt{\epsilon/(d-2)}$ w/o assuming that a single potential term dominates or whether the kinetic or potential term dominates; in general, γ is unrelated to acceleration.

- Our results go beyond previous no-goes as we allow for quantum effects and we encompass vacua and rolling solutions (irrespective of whether the kinetic term is negligible or not).

- As a spinoff, we derived analogous bounds on ekpyrosis [GS, Tonioni, Tran, ’23, STT3, to appear].

Multi-field Cosmology

Multi-exponential potentials

- Our bound applies for any potential that takes the form (also argument by [Ooguri, Palti, GS, Vafa]):

$$V = \sum_{i=1}^{m} \Lambda_i \, \mathrm{e}^{-\kappa_d \gamma_{ia} \phi^a} .$$

after canonically normalizing the scalar fields to ϕ^a, $a = 1,\ldots,n$.

- Λ_i, γ_{ia} depend on the microscopic origin of V_i, κ_d = d-dim. gravitational coupling. The sources of potential include e.g. internal curvature, fluxes, branes/O-planes, Casimir-energy, etc.

- $\{\phi^a, a = 1,\ldots,n\}$ includes minimally the d-dim. dilaton $\tilde{\delta}$ and the string-frame volume $\tilde{\sigma}$ unless they are stabilized; related to string dilaton & Einstein-frame volume by a field rotation.

- We consider scalars rolling towards the boundary of the moduli space, the axions which have a compact field space are assumed to be stabilized. The saxions can then be canonically normalized.

- In general, the field space is curved (e.g., the axio-dilaton and the Kahler modulus). Unstabilized axions recently considered in [Revello, '23] where field space curvature cannot be ignored.

Cosmological Equations

- Non-compact d-dim. spacetime is characterized by the FLRW metric:

$$d\tilde{s}_d^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}l^2_{\mathbb{R}^{d-1}}\,,$$

- Hubble parameter: $H \equiv \frac{\dot{a}}{a}$. The proper diagnostic for cosmic **acceleration** is $\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$ to be **distinguished from the slow-roll parameter** $\epsilon_V = \frac{d-2}{4}\kappa_d^2\left(\frac{\nabla V}{V}\right)^2$.

- Scalar field equations and Friedmann equations:

$$\ddot{\phi}^a + (d-1)H\dot{\phi}^a + \frac{\partial V}{\partial \phi_a} = 0,$$

$$\frac{(d-1)(d-2)}{2}\,H^2 - \kappa_d^2\left[\frac{1}{2}\,\dot{\phi}_a\dot{\phi}^a + V\right] = 0,$$

$$\dot{H} = -\frac{\kappa_d^2}{d-2}\left[\frac{1}{2}\,\dot{\phi}_a\dot{\phi}^a - V\right] - \frac{d-1}{2}H^2,$$

Cosmological Autonomous System

- It is convenient to work with the rescaled variables:

$$x^a = \frac{\kappa_d}{\sqrt{d-1}\sqrt{d-2}} \frac{\dot{\phi}^a}{H}, \; y_i = \frac{\kappa_d \sqrt{2}}{\sqrt{d-1}\sqrt{d-2}} \frac{\sqrt{V_i}}{H}$$

- The cosmological equations can be formulated in terms of an autonomous system of ODEs given schematically as follows:

$$\frac{d\vec{z}}{dt} = g(\vec{z}) \; , \qquad \text{where } \vec{z} \equiv (x^1, \ldots, x^n, y^1, \ldots, y^m, H)$$

- Among the above ODEs is $\epsilon = -\dot{H}/H^2 = (d-1)x^2$; strategy is to bound the kinetic energy.

- Friedmann equation also takes a simple form:

$$(x)^2 + (y)^2 = 1$$

Bound on Late-time Cosmic Acceleration

- An accelerating universe can only be achieved if the total scalar potential is positive; we therefore focus on scenarios in which $V > 0$ at least asymptotically.
- Individual potential terms can be positive or negative: our proof covers general cases but for clarity, let us first show how we bound the case when $\Lambda_i > 0$ [General case in STT1].
- Rank order the exponents:

$$\gamma_\infty^a = \begin{cases} \gamma^a, & \gamma^a = \min_i {\gamma_i}^a > 0 \\ 0, & \gamma^a \leq 0 \end{cases}$$

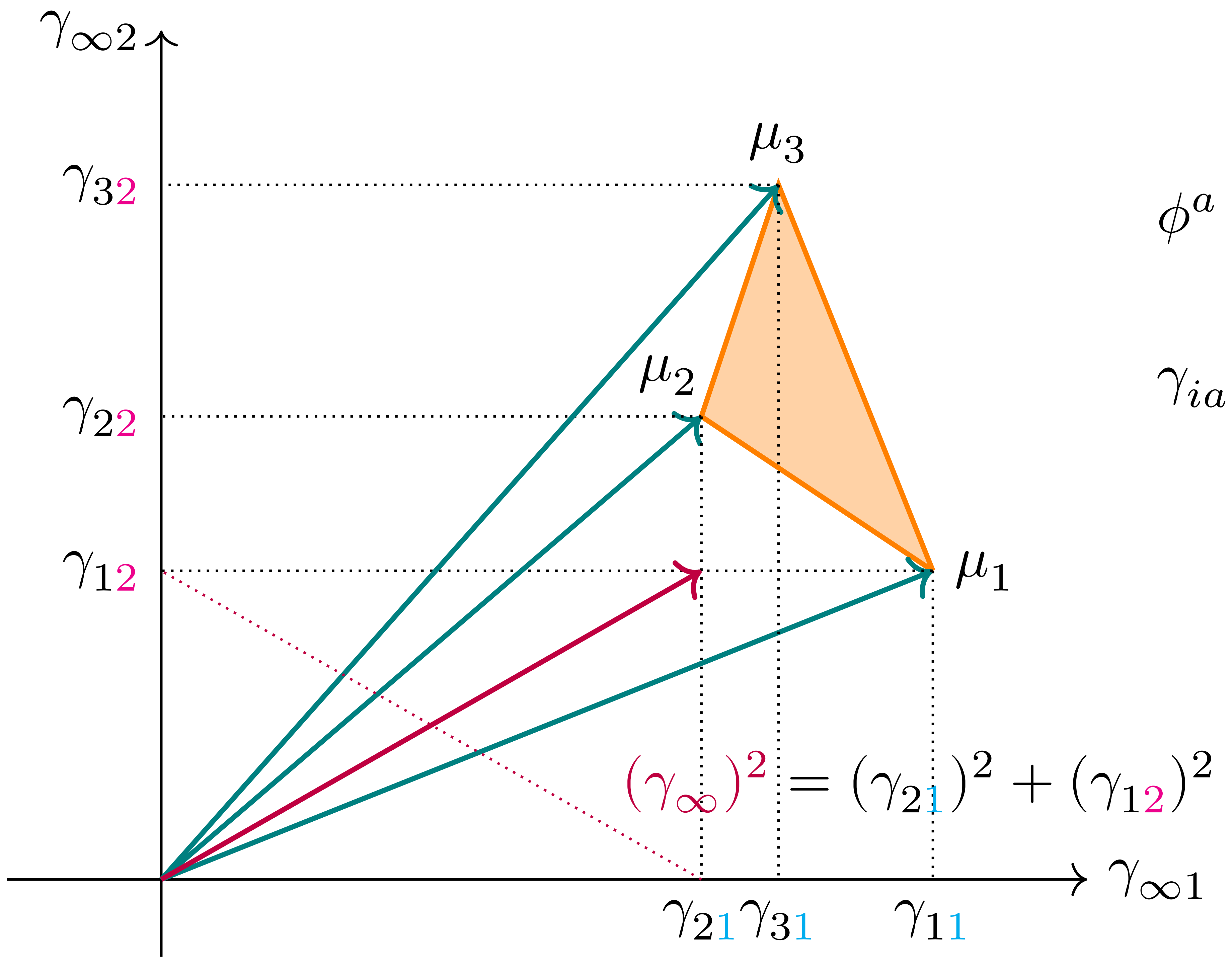
- Then we derived analytically a late-time acceleration bound:

$$d - 1 \geq \epsilon \geq \frac{d-2}{4} (\gamma_\infty)^2$$

The bound holds w/o assuming knowledge of the actual time-dependent solution.

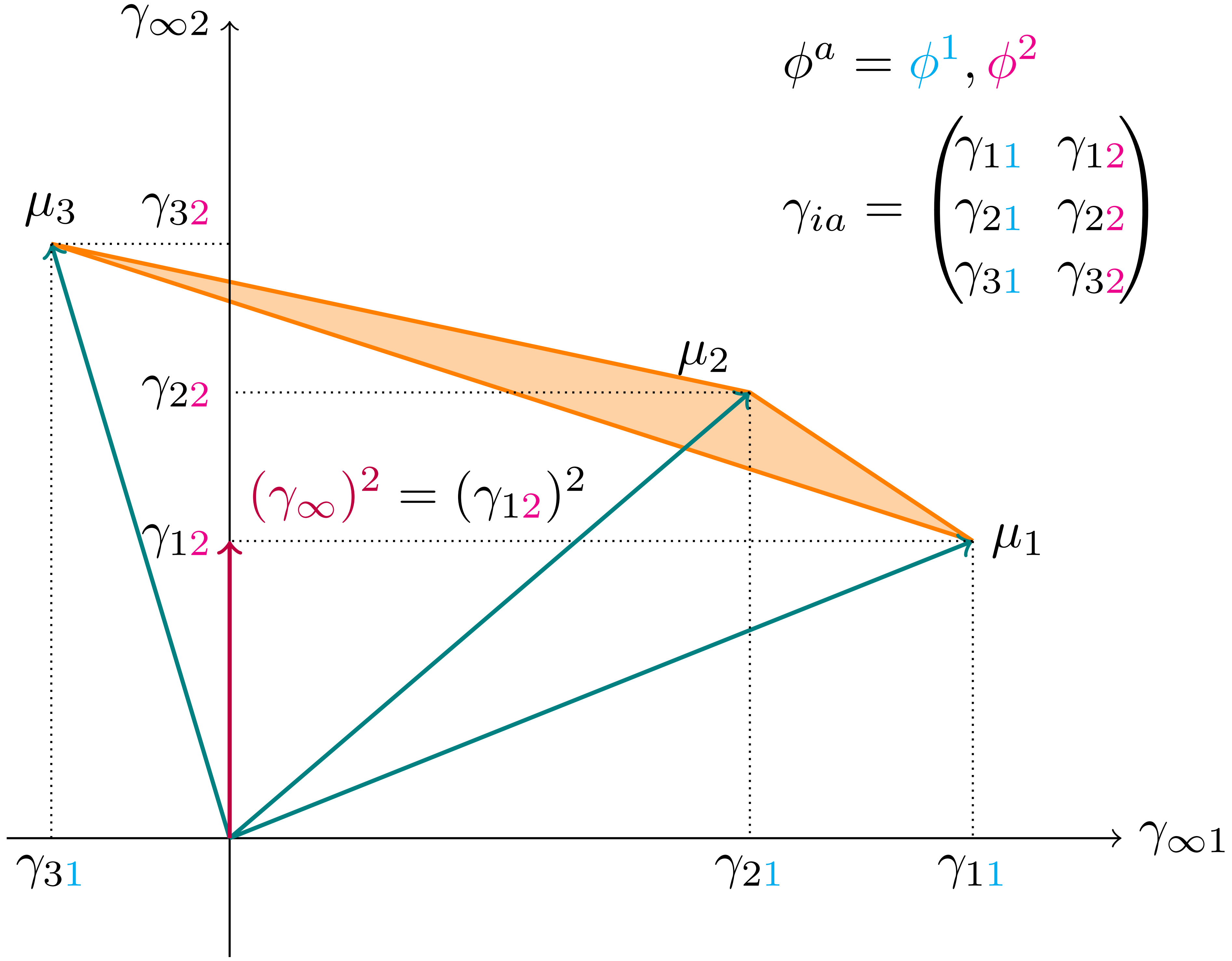
Visualizing the Acceleration Bound

- Define vectors m vectors μ_i, one for each potential term with components $(\mu_i)_a = \gamma_{ia}$



$$\phi^a = \phi^1, \phi^2$$

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \end{pmatrix}$$

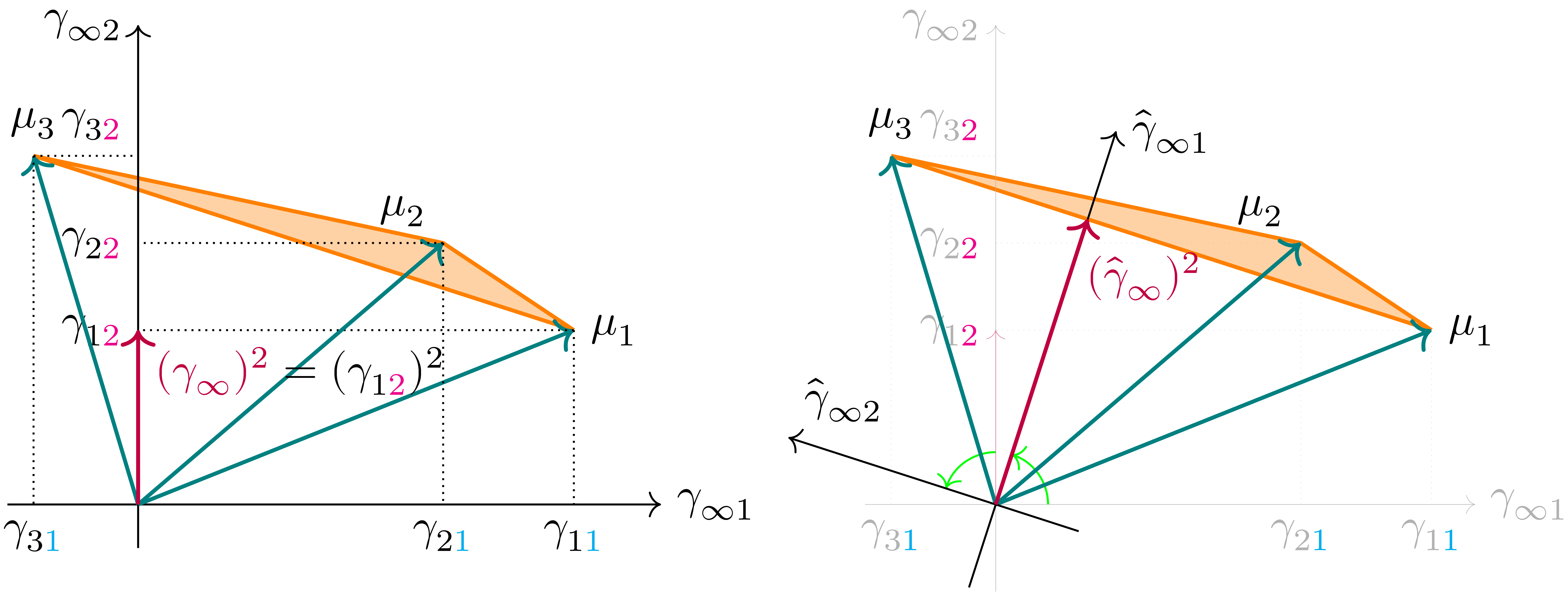


Optimizing the Acceleration Bound

- It is clear that we can find an **optimal bound** by an $O(n)$ rotation [GS, Tonioni, Tran, '23]:

$$\epsilon \geq \frac{d-2}{4} \max_{\mathrm{R}\in\mathrm{O}(n)} [\gamma_\infty(\mathrm{R})]^2$$

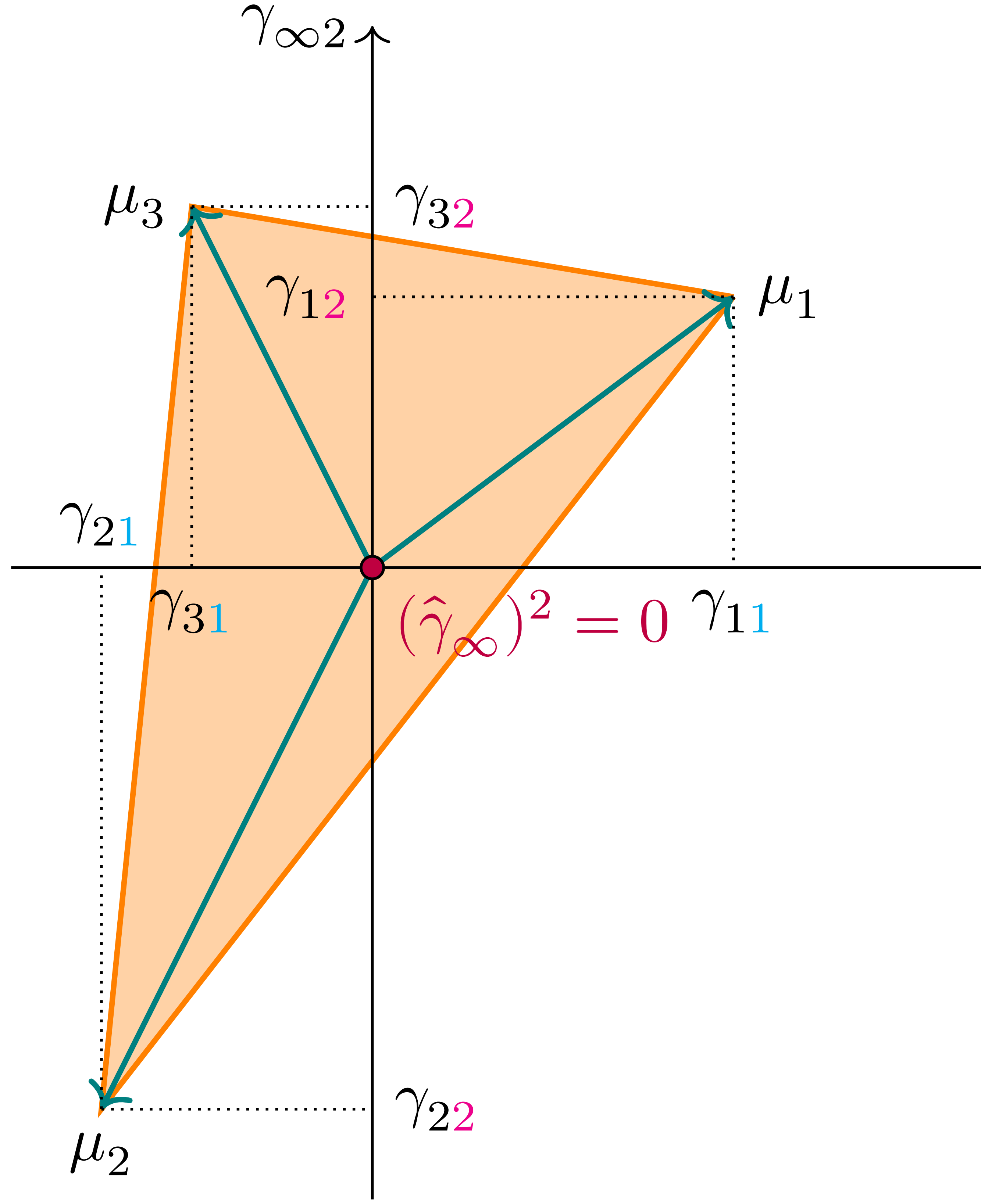
The bound is basis-independent.



How to use our bound?

[GS, Tonioni, Tran, ’23]

- Given a model, we can check if the minimum distance to the coupling convex hull is smaller than 1. Our bound does not assume any on-shell solutions nor kinetic terms are negligible.
- Easy to find bottom-up models of accelerating universe, e.g., the bound can even be trivial.
- The challenge is to find:
  - couplings like these in string theory, **and**
  - to stabilize the remaining non-rolling moduli
- Are there universal moduli that set already a strong bound?
- Each of the canonical scalars contributes positively to the bound.
- Additional rolling fields strengthen further the bound.



Obstruction by the Dilaton

- String-theoretical potentials take the form:

$$S = -\int_{\mathrm{X}_{1,9}} [A_r \wedge \star_{1,9} A_r]\, \Lambda_{10,r}\, \mathrm{e}^{-k\sigma - \chi_{\mathrm{E}}\Phi} = -\int_{\mathrm{X}_{1,d-1}} \tilde{*}_{1,d-1} \Lambda\, \mathrm{e}^{\kappa_d[\gamma_{\tilde{\delta}}(\chi_{\mathrm{E}})\tilde{\delta} - \gamma_{\tilde{\sigma}}(\chi_{\mathrm{E}}, r, k)\tilde{\sigma}]}$$

RR fields are not weighed by $e^{-\chi_E \Phi}$ (effectively set $\chi_E = 0$) but would not affect our argument.

- The d-dim. dilaton $\tilde{\delta}$ is a linear combination of the 10d dilaton Φ and Einstein frame volume.

- While the field basis choice is not unique, d-dimensional dilaton $\tilde{\delta}$ has **universal properties**:

$$\gamma_{\tilde{\delta}} = \frac{d}{\sqrt{d-2}} - \frac{1}{2}\chi_{\mathrm{E}}\sqrt{d-2} \geq \frac{2}{\sqrt{d-2}} \quad \Rightarrow \quad \epsilon \geq \frac{d-2}{4}(\gamma_\infty)^2 \geq \frac{d-2}{4}\gamma_{\tilde{\delta}}^2 \geq 1$$

- Ways out: 1) $\tilde{\delta}$ is stabilized; 2) $\tilde{\delta}$ is rolling but not in the asymptotic regions; 3) V contains at least three terms, not all of the same sign (e.g., from loop corrections).

- Non-universal couplings for other moduli: can use our bound to **constrain compactifications**.

Scaling Solutions

Scaling Solutions

- The cosmological autonomous system admits **scaling solutions** ($\epsilon =$ constant > 0):
  - scale factor takes a power law form: $a(t) \sim t^p$
  - critical points of the autonomous system: $\dot{x}^a = 0$
- **Analytic solution:** for $\mathrm{rank}\, \gamma_{ia} = m$
  - field space trajectory: $$\phi_*^a(t) = \phi_0^a + \frac{2}{\kappa_d}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}\gamma_i{}^a (M^{-1})^{ij}\right] \ln\frac{t}{t_0}, \qquad M_{ij} = \gamma_{ia}\gamma_j{}^a$$
  - scale factor: $$p = \frac{4}{d-2}\sum_{i=1}^{m}\sum_{j=1}^{m}(M^{-1})^{ij}.$$

[Copeland, Liddle, Wands, '97]
[Collinucci, Nielsen, Van Riet, '04]

- The kinetic term & every potential term have the same parametric dependence in time:

No slow-roll: $$T(t) = T(t_0)\left(\frac{t_0}{t}\right)^2, \quad V_i(t) = V_i(t_0)\left(\frac{t_0}{t}\right)^2$$

Scaling Solutions: Relevance

- Late time scale factor is bounded by power-law behavior [GS, Tonioni, Tran, '23, STT1]:

$$d - 1 \geq \epsilon \geq [(d-2)/4]\,(\hat{\gamma}_\infty)^2$$

- Scaling solutions are **perturbative late-time attractors** (linear stability) See e.g. [Hartong, Ploegh, Van Riet, Westra]

- **New result** [GS, Tonioni, Tran, '23, STT2]: we can analytically prove that if

  1. all potential terms are positive definite, i.e., $\Lambda_i > 0$, and

  2. ~~$\lambda^i = \sum_{j=1}^{m} (M^{-1})^{ij} \geq 0$, subject to $\sum_{i=1}^{m} \lambda^i > 0$. [no apparent subleading terms]~~

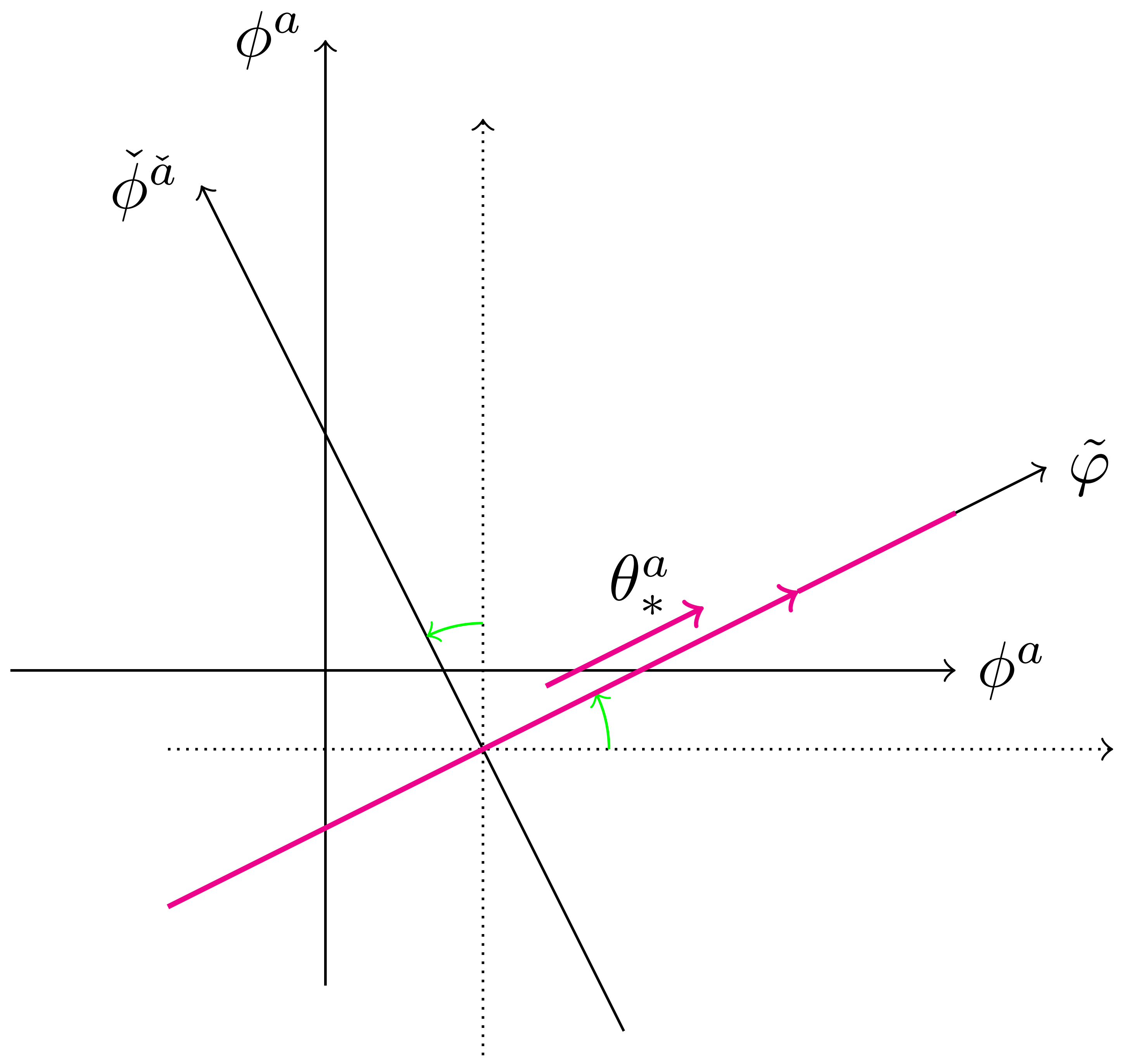
  then scaling solutions are late-time attractors, irrespective of initial conditions, and furthermore saturate the lower bound!

- We can actually drop condition 2 in the proof; see forthcoming paper [GS, Tonioni, Tran, to appear].

Scaling Solutions: Trajectory

- Straight line in field space:

$$\phi^a_*(t) = \phi^a_\infty + \frac{1}{\kappa_d}\,\alpha^a \ln\frac{t}{t_\infty}$$
$$\theta^a_* = \frac{\alpha^a}{\sqrt{\alpha_b\alpha^b}}$$



- Field space rotation such that $\check{\phi}^{\check{a}}_*(t) = \check{\phi}^{\check{a}}_\infty$ $\quad \tilde{\varphi}_*(t) = \tilde{\varphi}_\infty + \frac{1}{\kappa_d}\frac{2}{\gamma_*}\ln\frac{t}{t_\infty}$

- Potential gradient: $\gamma = \frac{\sqrt{\delta^{ab}\,\partial_a V \partial_b V}}{\kappa_d V} = -\left[\frac{1}{V(\phi_*)}\,\theta^a_*\,\frac{\partial V}{\kappa_d\,\partial\phi^a_*}(\phi_*)\right] = \frac{2}{\sqrt{d-2}}\sqrt{\epsilon}.$

In general not true unless
$\eta = -\dot{\epsilon}/(\epsilon H) = 0$
and
Ω = non-geodesity =0

Acceleration bound: Physical Interpretation

- In the optimal basis, the asymptotic potential:

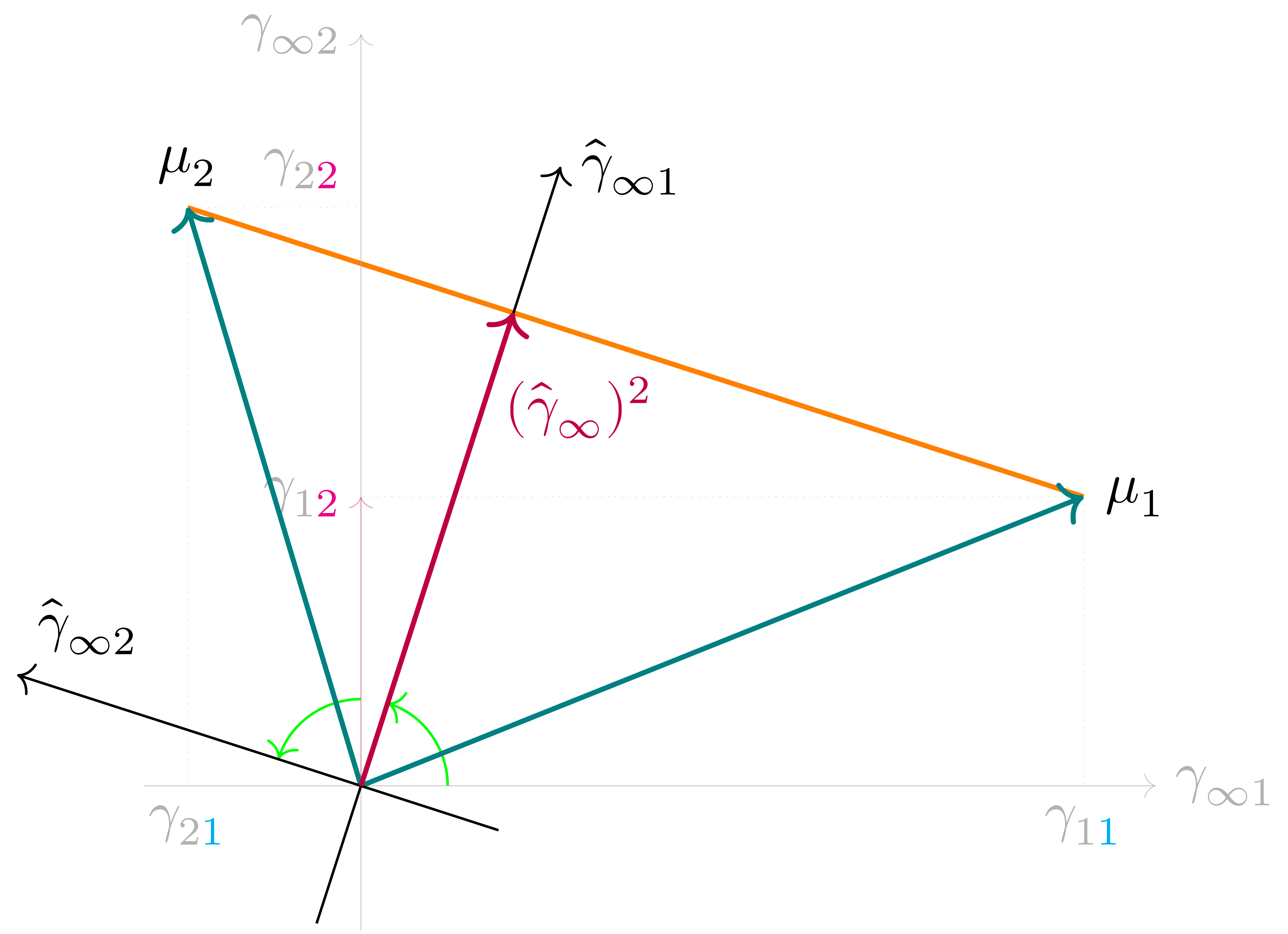
$$V = \left[\sum_{\sigma=1}^{m} \Lambda_\sigma \, \mathrm{e}^{-\kappa_d \hat{\gamma}_{\sigma \breve{a}} \hat{\phi}^{\breve{a}}} \right] \mathrm{e}^{-\kappa_d \hat{\gamma}_\infty \hat{\varphi}}$$

- Fields other than $\hat{\varphi}$ appear in the exponents with both signs and get stabilized.

- Asymptotically, we have effectively a single field, single potential on-shell:

$$\hat{V}_\infty = \hat{\Lambda}_\infty \, \mathrm{e}^{-\kappa_d \hat{\gamma}_\infty \hat{\varphi}},$$

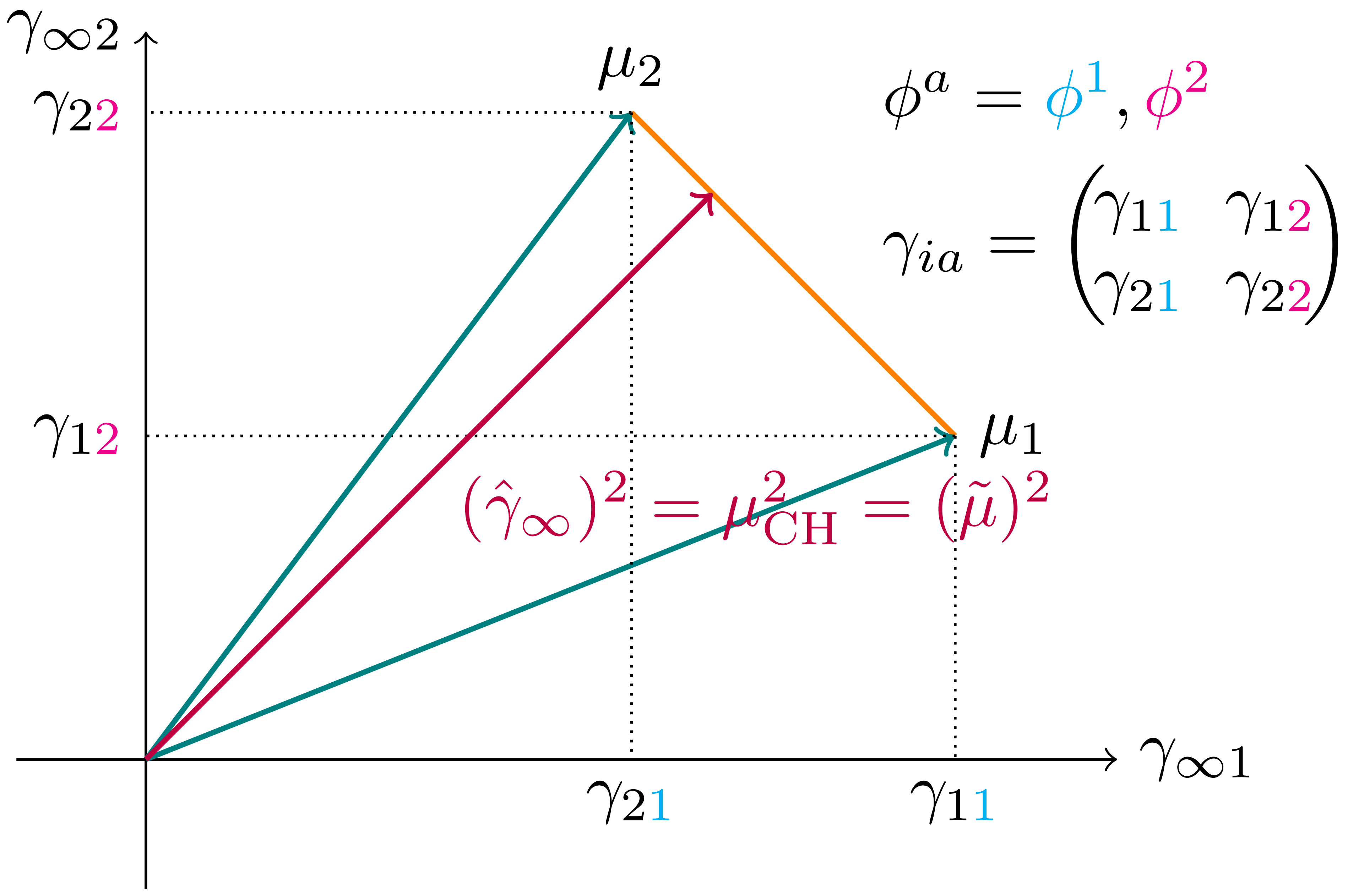
which gives $\epsilon = \dfrac{d-2}{4} \, \hat{\gamma}_\infty^2$

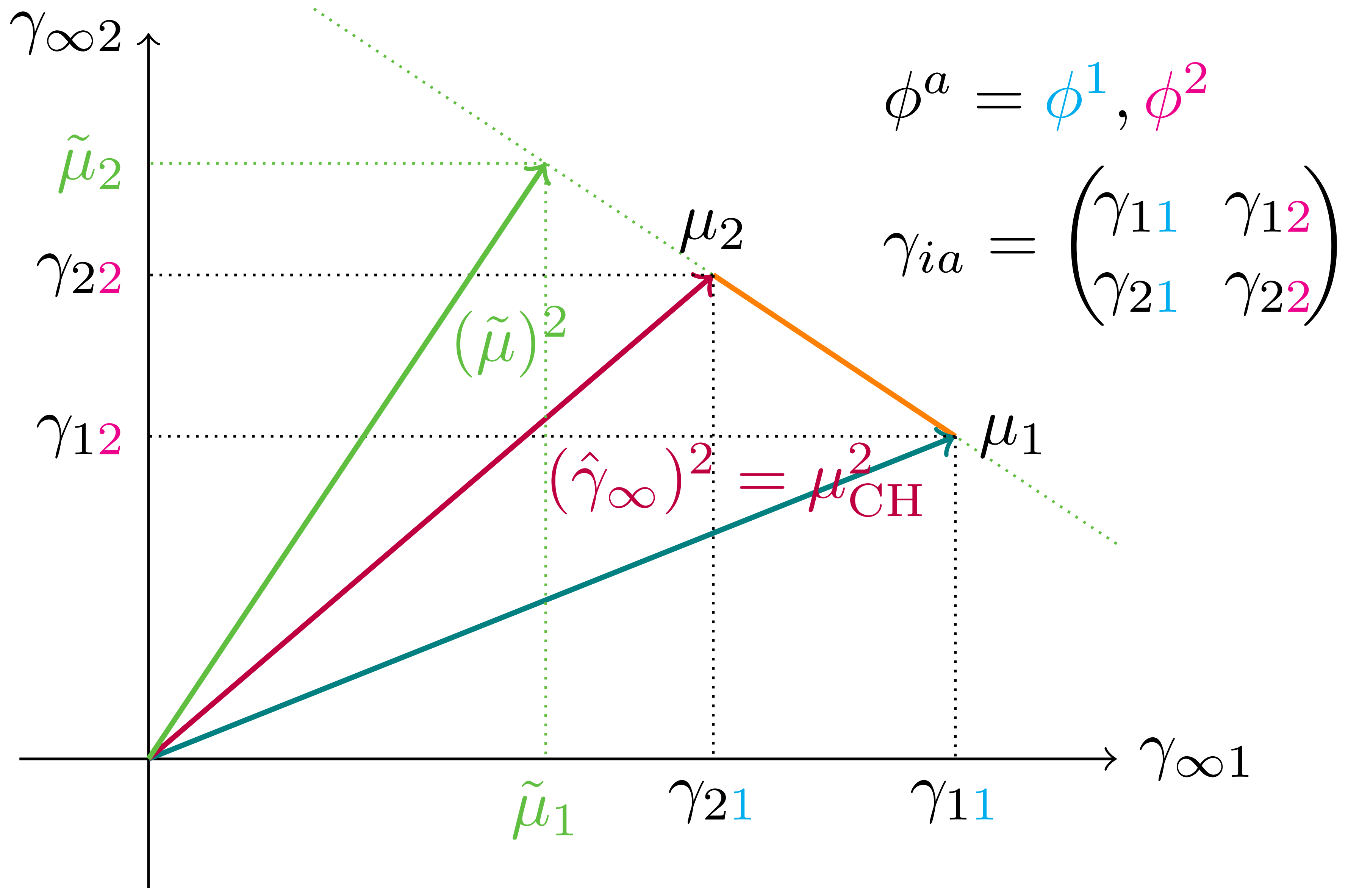


The straight line trajectory of scaling solution solves this effective single field problem and saturates the bound.

Coupling Convex Hull

[GS, Tonioni, Tran, '23]





- If the distance vector from the origin to the hyperplane containing the convex hull intersects the convex hull, we find analytically the late-time ϵ-parameter:

$$\epsilon = \frac{d-2}{4}(\hat{\gamma}_\infty)^2 = \frac{d-2}{4}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}(M^{-1})^{ij}\right]^{-1}$$

else the potential is truncated, and ϵ is given by the truncated convex hull (proof in forthcoming paper [GS, Tonioni, Tran, to appear] which dropped condition 2).

Living Dangerously

- If a Swampland bound is robust, we ought to find examples that saturate it. For exponential potentials, the Trans-Planckian Censorship Conjecture (TCC) [Bedroya, Vafa] bounds $\epsilon \geq 1$.

- Tree-level potential of the d-dim. dilaton $\hat{\delta}$ saturates this bound. If all other moduli are stabilized,

$$V = \Lambda e^{\frac{2}{\sqrt{d-2}}\kappa_d \hat{\delta}} \Rightarrow \epsilon = 1$$

- The late-time attractor has $\epsilon = 1$ (non-accelerating) but it takes infinite time to reach this attractor from an initial $\epsilon < 1$ phase (accelerating). [Note $\epsilon_V = 1$ at all time, gives wrong diagnostic].

- This mechanism was recently exploited for $k = -1$ models [Andriot, Tsimpis, Wrase] though this type accelerating solutions with no cosmological horizon can be found with $k = 0$.

- The challenge is to stabilize the remaining moduli or else any additional rolling scalars would stop acceleration [Hebecker, Schreyer, Venken] as should be clear from our bound [GS, Tonioni, Tran, '23].

- A string theory model of quintessence is yet to be constructed. Outstanding question for the future.

Summary

Summary of Results

[GS, Tonioni, Tran, '23 x 3]

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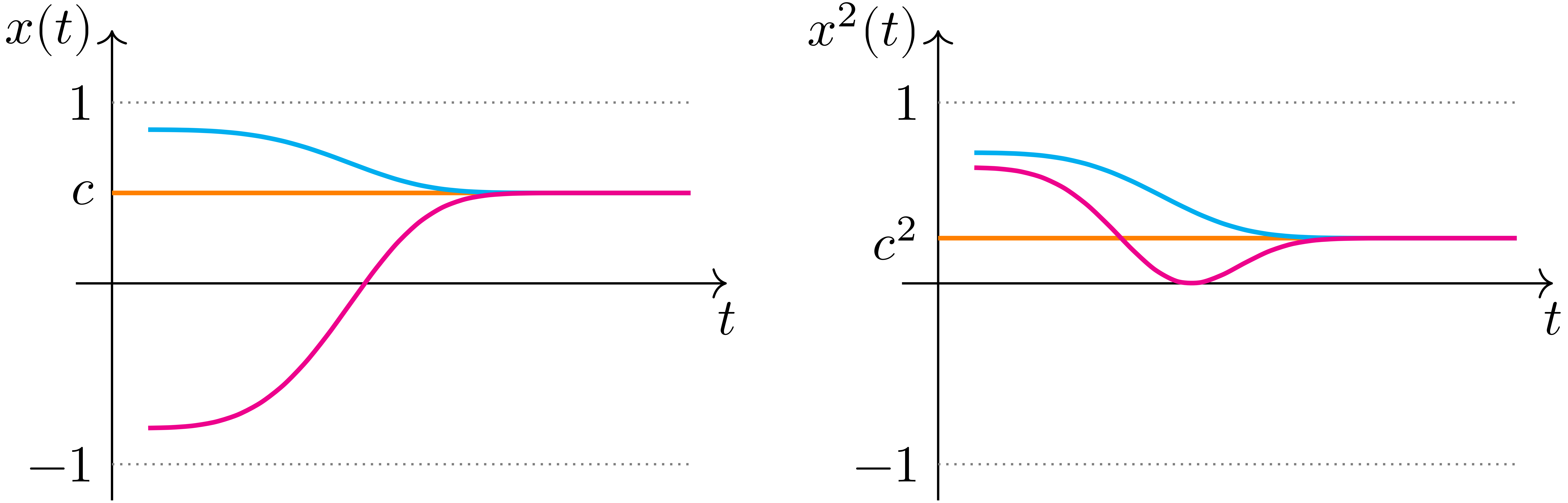
- As a spinoff, we derived analogous bounds on ekpyrosis [GS, Tonioni, Tran, '23, STT3, to appear].

Backup



x(t)
1
c
t
−1
x^2(t)
1
c^2
t
−1