# Local Non-Gaussianity, Exponential tails, and Trapped vacuum bubbles

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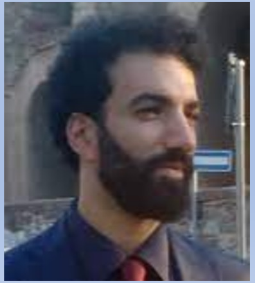


A. Escrivà, V. Atal and J. Garriga 2306.09990

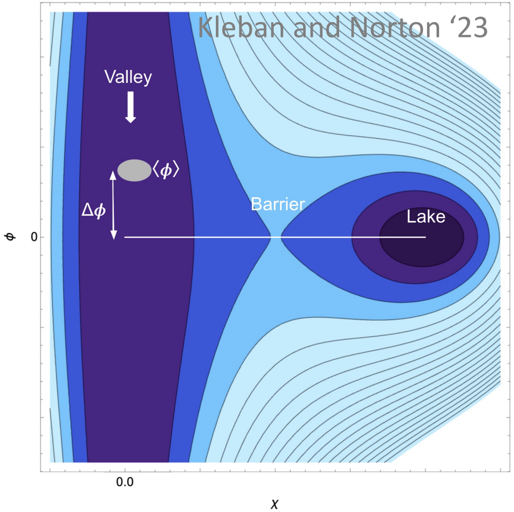
# PBH may form from…

- ❑ Collapse of **adiabatic fluctuations**
- ❑ Other mechanisms (isocurvature, cosmic strings, domain walls, Q-balls , phase transitions, etc)
- ❑ Implosion of **inflationary relics** (such as domain walls and vacuum bubbles)

J. Garriga, A. Vilenkin, J. Zhang (2015)
H. Deng, A. Vilenkin (2017)
A.Kusenko et al. *Phys.Rev.Lett.* 125 (2020) 181304
M. Kleband, C. Norton (2023)

# PBH from relic vacuum bubbles

J. Garriga, A. Vilenkin, J. Zhang. 2015

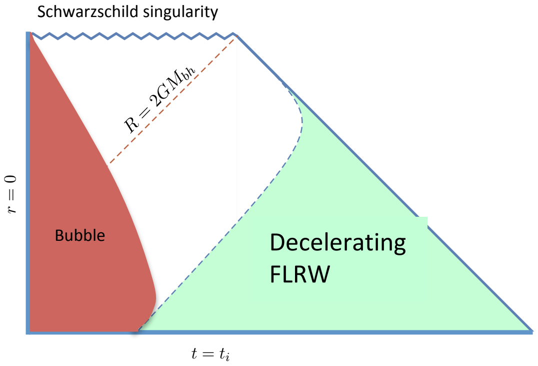



Localized **vacuum bubbles** may be produced by **tunneling** from the inflationary "valley" into a neighboring "lake"

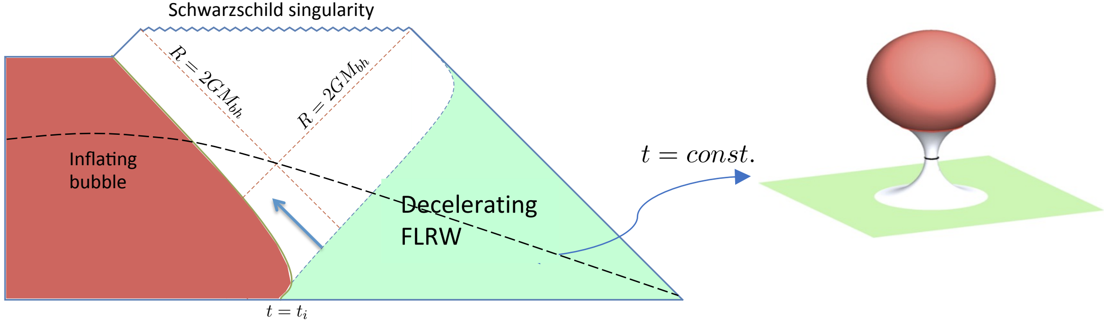
The vacuum bubble then grows, and stretches conformally during inflation.

**After inflation, it "implodes"** once its co-moving size falls within the ambient Hubble radius

**Subcritical size** $H_{lake}R_{end} \ll 1$
(**may or may not** form a PBH)




**Supercritical size** $H_{lake}R_{end} \gg 1$
**will form a PBH,** with a ¨**baby universe**¨ inside

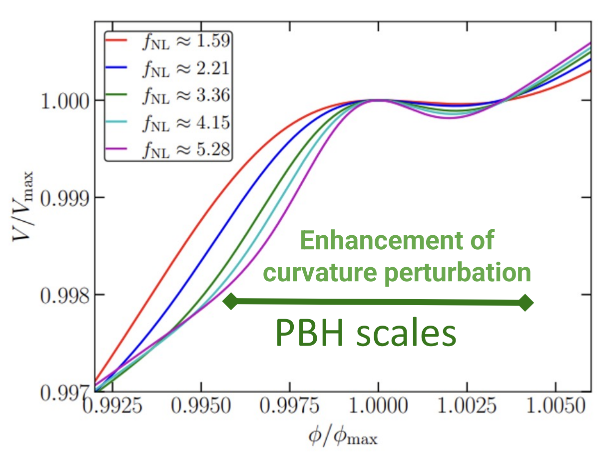



Inside the bubble: an observer sees an inflating Universe

PBH mass function can be broad (power law) or narrow (spike)

# Single field model:

Starobinsky type, with a barrier




$$f_{NL} = \frac{5}{12}\left[-3 + \sqrt{9 - 12\frac{V''(\phi_{\rm max})}{V(\phi_{max})}}\right]$$

$$\zeta \approx \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2 + \dots$$

$$\mu_\star = \frac{5}{6\, f_{NL}}$$

- **Local type non-Gaussianity** $\zeta = \zeta[\zeta_G]$ beyond perturbative expansion

$$\zeta = -\mu_* \log\left(1 - \frac{\zeta_G}{\mu_*}\right)$$

From $\delta N$ formalism

V. Atal, J. Garriga and A. M. Caballero. 2019
V. Atal, J. Cid, A. Escriva, J. Garriga. 2019
S. Pi, M. Sasaki 2023

- **Exponential tails in the PDF for** $\zeta$ are a consequence of the singularity at $\zeta_G \to \mu^* \;\Rightarrow\; \zeta \to \infty$

- **Trapped vacuum bubbles**

# "Exponential tail" is just the Jacobian near the singular point

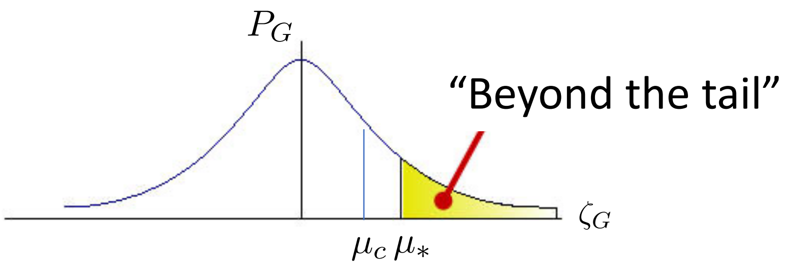
$$\zeta = -\mu_\star \ln\left(1 - \frac{\zeta_G}{\mu_\star}\right)$$

$$P[\zeta] \approx P_G[\zeta_G]\frac{d\zeta_G}{d\zeta}$$

$$\boxed{P[\zeta] \approx P_G[\mu_*]e^{-\zeta/\mu_*} \quad (\zeta \to \infty)}$$

$$\zeta_G = \mu(1 - e^{-\zeta/\mu_\star})$$

$$\frac{d\,\zeta_G}{d\,\zeta} = e^{-\zeta/\mu_\star}$$




$\zeta_G = \mu_\star$ Singular point

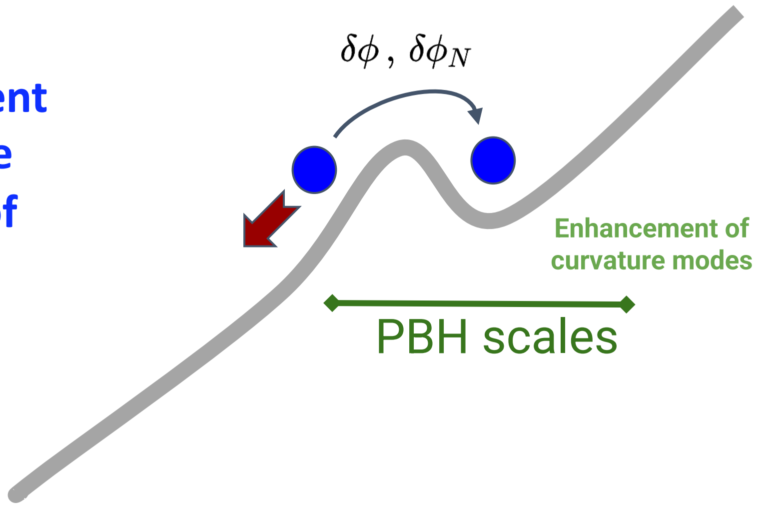
$\zeta_G \to \mu^* \;\Rightarrow\; \zeta \to \infty$

$P_G[\zeta_G]$ **normalized** $P[\zeta]$ **Not normalized!**

$$\int P[\zeta]\,D\zeta = \int_{\zeta_G<\mu_\star} P_G[\zeta_G]\,D\zeta_G < 1 \longrightarrow$$

The singularity indicates the presence of an alternative channel for PBH production, which restores unitarity.

Large backward fluctuations prevent the inflaton from overshooting the barrier, leading to the formation of localized false vacuum bubbles!



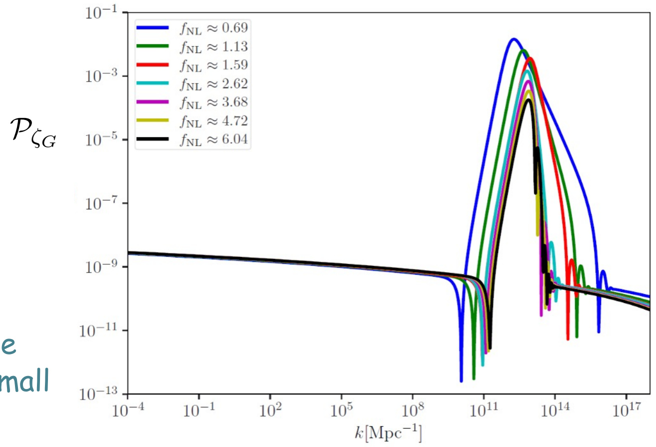

Two coexisting channels for PBH production

→ Large adiabatic fluctuations (the standard one)

→ From false vacuum bubbles → We can explore this numerically

# Adiabatic channel

Solving the MS equation numerically




BBKS: considering very large peaks -> spherically symmetric,
For a Gaussian random field the typical shape is given by

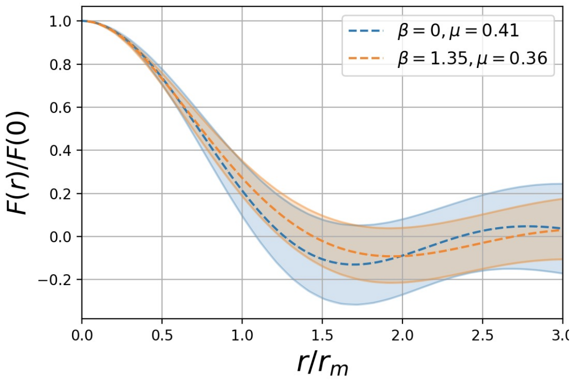
$$\zeta_G(r) = \mu_a \Psi_{\zeta_G}(r) \pm \Delta\zeta_G = \mu_a \frac{1}{\sigma_a^2} \int_{k_i}^{k_f} \mathcal{P}_{\zeta_G} \frac{\sin(kr)}{kr} d\ln k \pm \Delta\zeta_G$$

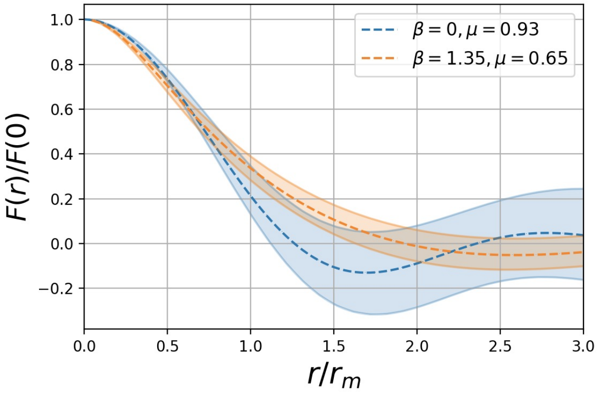
$$\nu_a = \mu_a/\sigma_a \sim \mathcal{O}(10)$$

$$< (\Delta\zeta_G)^2 >= \sigma_0^2[1 - \Psi_{\zeta_G}^2(r)]$$

The dispersion in the shape from the mean profile is small

## Shape dispersion of the **non-Gaussian** high peak



# We need the threshold and mass to estimate the PBH abundance…

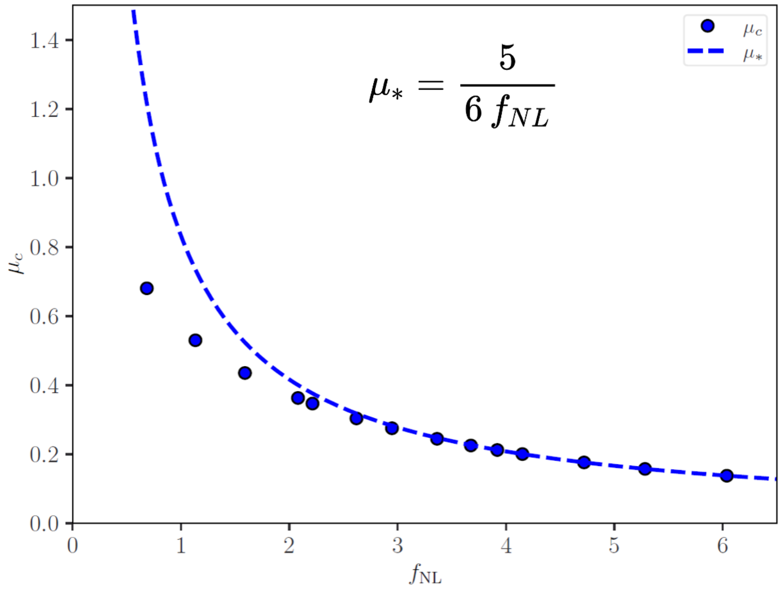
Find the value of the amplitude such that

A.Escrivà, C.Germani, R. K.Sheth (2019)

$$\bar{\mathcal{C}}_c = 2/5$$

$$\mathcal{C}(r,t) = 2\frac{M - M_b}{R} \qquad \mathcal{C} = \frac{2}{3}\left[1 - \left(1 + r\,\zeta'\right)^2\right]$$

$$\bar{\mathcal{C}}_c = \frac{3}{r_m^3(\mu_{a,c})e^{3\zeta(r_m(\mu_{a,c}))}} \int_0^{r_m(\mu_{a,c})} \mathcal{C}_c(r)(1 + r\zeta')e^{3\zeta(r)}r^2 dr.$$




PBH mass near the critical regime

$$M_{PBH}(\mu_a) \propto (\mu_a - \mu_c)^{\gamma_a}$$

# Let's move to the bubble channel->Numerical formation of bubbles

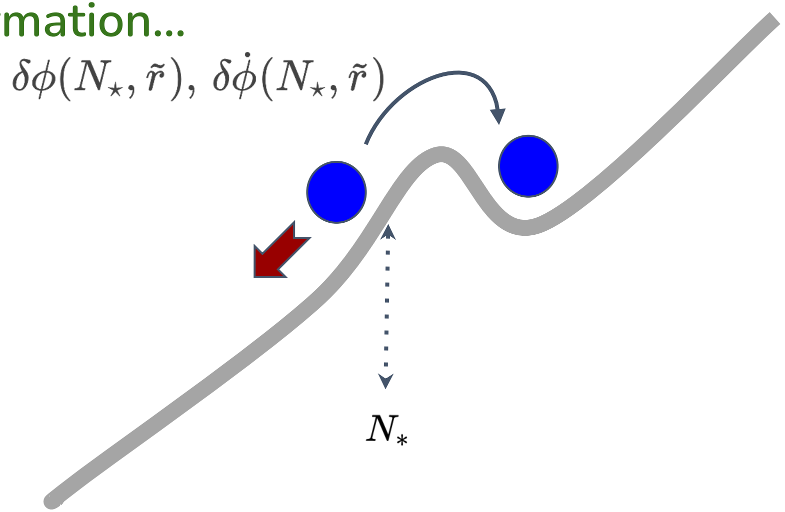
We need to solve the KG field equation taking into account a radial dependence

$$\ddot{\phi}+\dot{\phi}\left(3-\frac{1}{2}\dot{\phi}_b^2\right)-\left(\frac{a_I H_I}{a(N)H(N)}\right)^2\Delta\phi+\frac{1}{H^2}\frac{V_\phi(\phi)}{V(\phi)}=0$$

We need initial conditions for bubble formation...

In general, we can consider:

$$\phi(N_\star,\tilde{r})=\phi_b(N_\star)+\delta\phi(N_\star,\tilde{r})$$
$$\dot{\phi}(N_\star,\tilde{r})=\dot{\phi}_b(N_\star)+\delta\dot{\phi}(N_\star,\tilde{r})$$

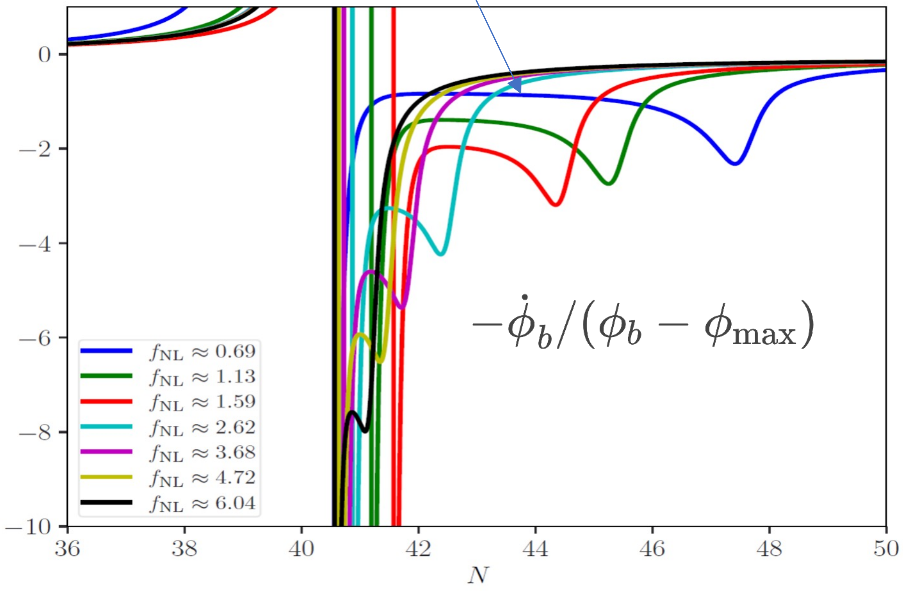



$N_*$ → When the numerics is closer to the attractor regime

$$\dot{\phi}_b\approx-\lambda_-(\phi_b-\phi_{\max}) \Longrightarrow \delta\dot{\phi}\approx-\lambda_-\delta\phi$$

## Atractor regime

$$\lambda_- = -\frac{6f_{NL}}{5}$$



# Initial conditions for bubble formation

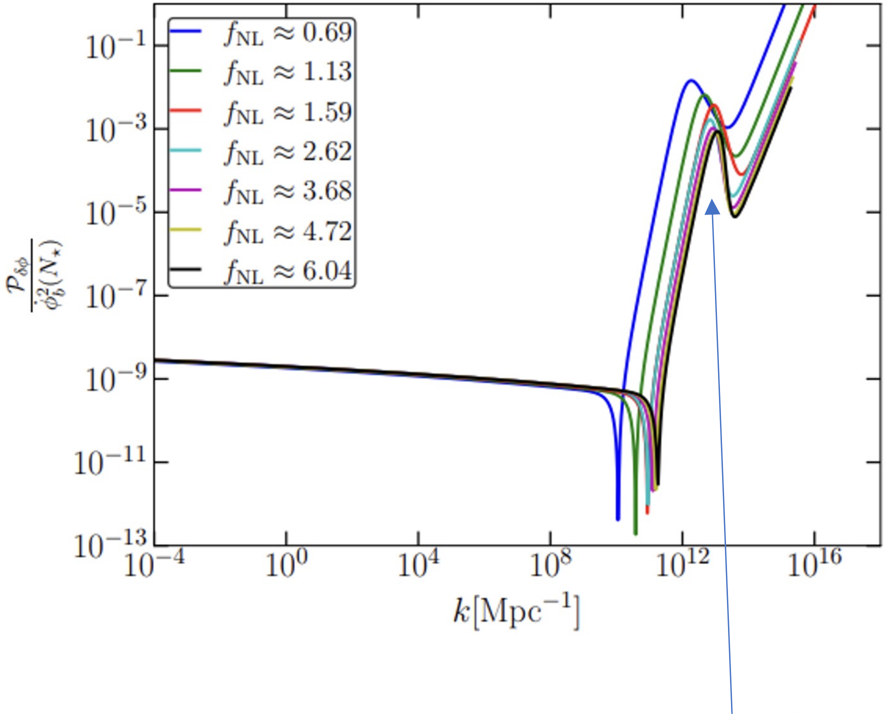
## Power spectrum for the field perturbation

$$\mathcal{P}_{\delta\phi}(N_\star, k) = \frac{k^3}{2\pi^2}\dot{\phi}_b^2(N_\star) \mid \zeta_G(N_\star, k) \mid^2$$

$$\Psi_b(N_\star, \tilde{r}) = \frac{1}{\sigma_b^2}\int_{k_i}^{k_f} \mathcal{P}_{\delta\phi}(N_\star, k)\,\mathrm{sinc}(k\tilde{r})d\ln k$$

$$\delta\phi(N_\star, \tilde{r}) = \mu_b \Psi_b(N_\star, \tilde{r})$$

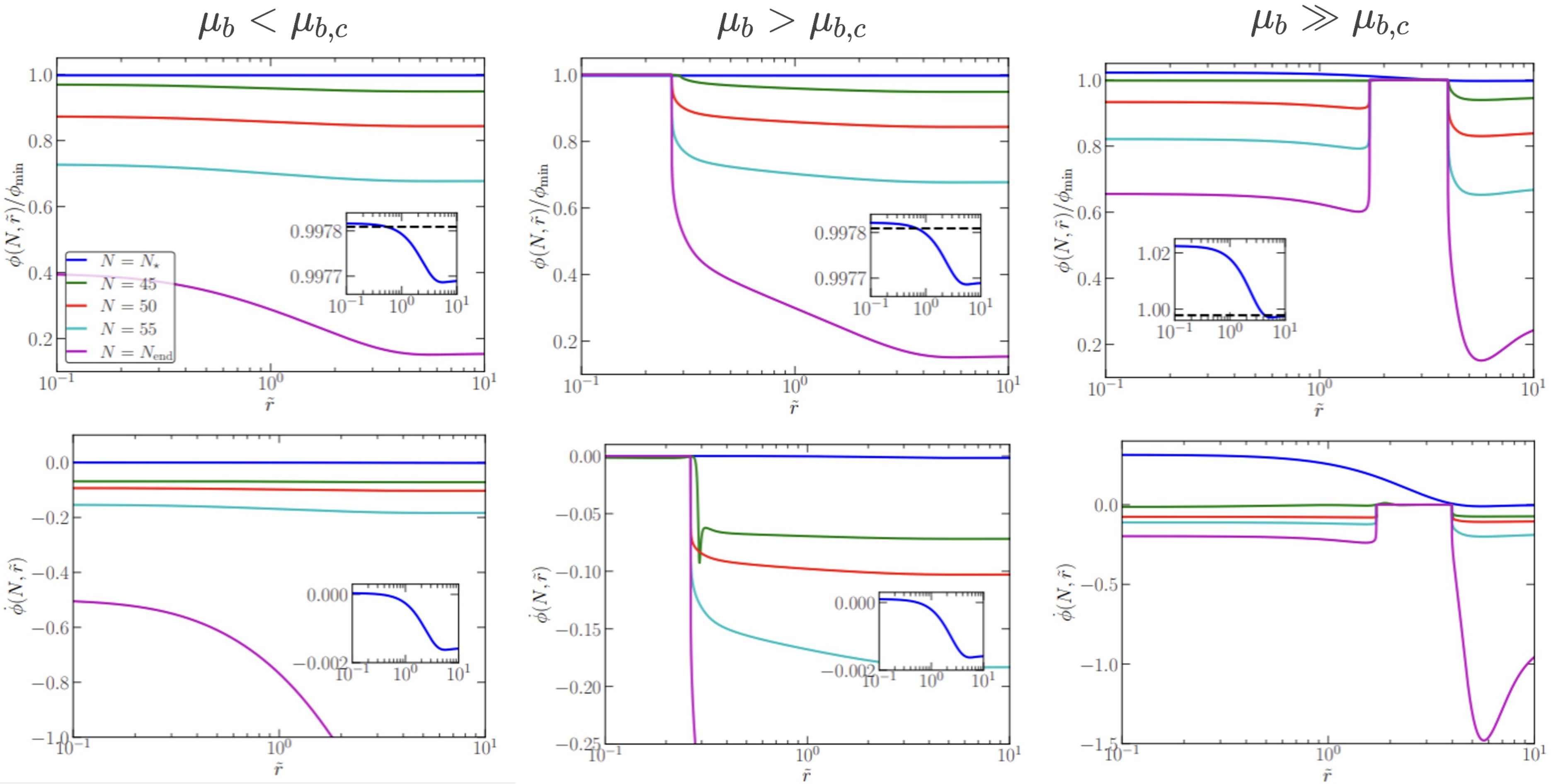
(like in the adiabatic channel)




$$\mathcal{P}_{\delta\phi}(N_\star, k) \gg \frac{k^2}{a^2(2\pi)^2}$$

# Dynamics of bubble formation

$\mu_{b,c} \approx 5 \cdot 10^{-4}$

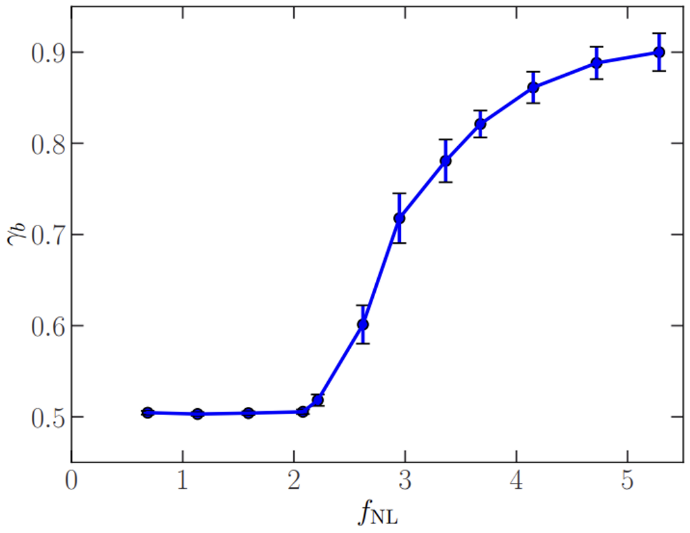


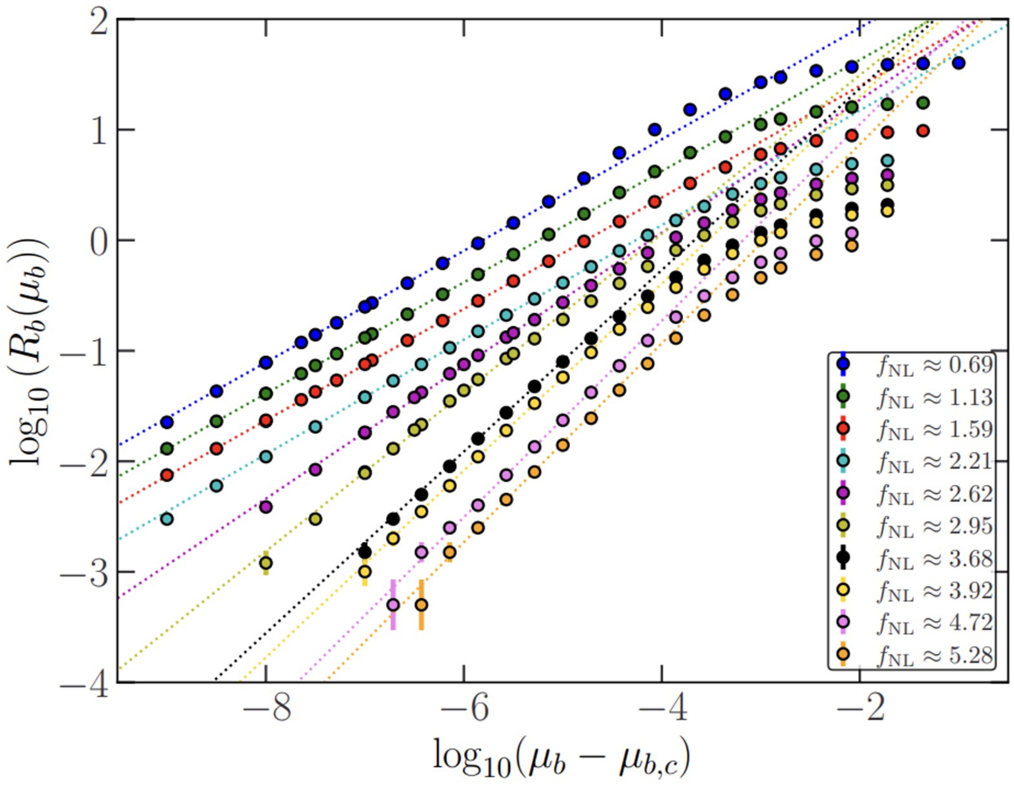
# Comoving size of the bubbles

We find a critical regime for the bubble size!

Reminiscent to the critical collapse in gravitational collapse phenomena, but gravity does not play a role

$$R_b(\mu_b) = \mathcal{K}_b(\mu_{b,c})(\mu_b - \mu_{b,c})^{\gamma_b(f_{\rm NL})}$$



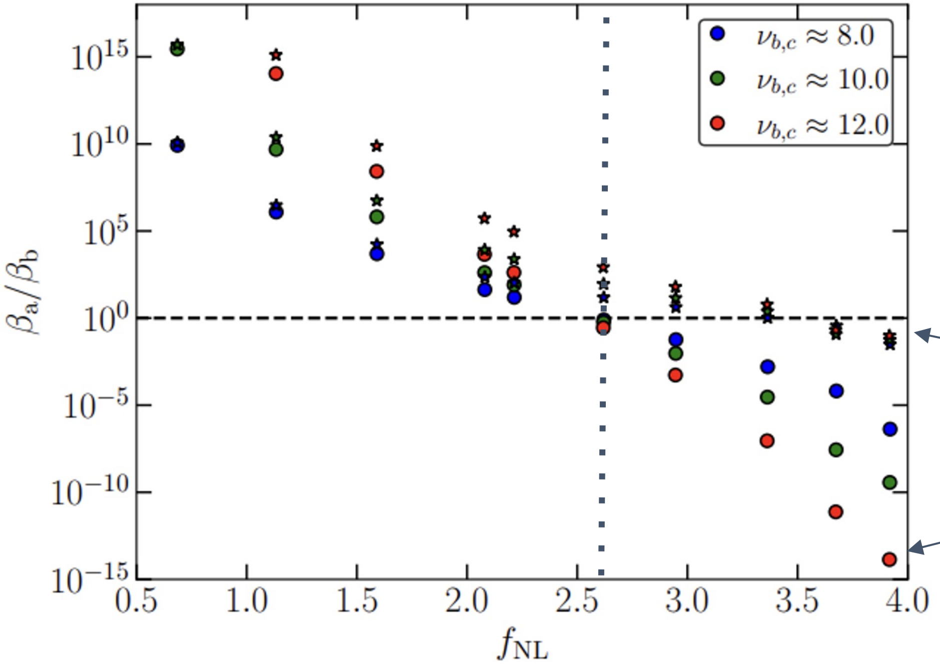



# Ratio of PBH abundance between the two channels

$$\mathcal{N}_{peak} = \left(\frac{\sigma_1}{\sqrt{3}\sigma_0}\right)^3 (\nu^3 - 3\nu) e^{-\nu^2/2}$$

$$\beta_a = \int_{\nu_{a,c}}^{\nu_*} \mathcal{N}_a(\nu) d\nu$$

$$\beta_b = \int_{\nu_{b,c}}^{\infty} \mathcal{N}_b(\nu) d\nu$$

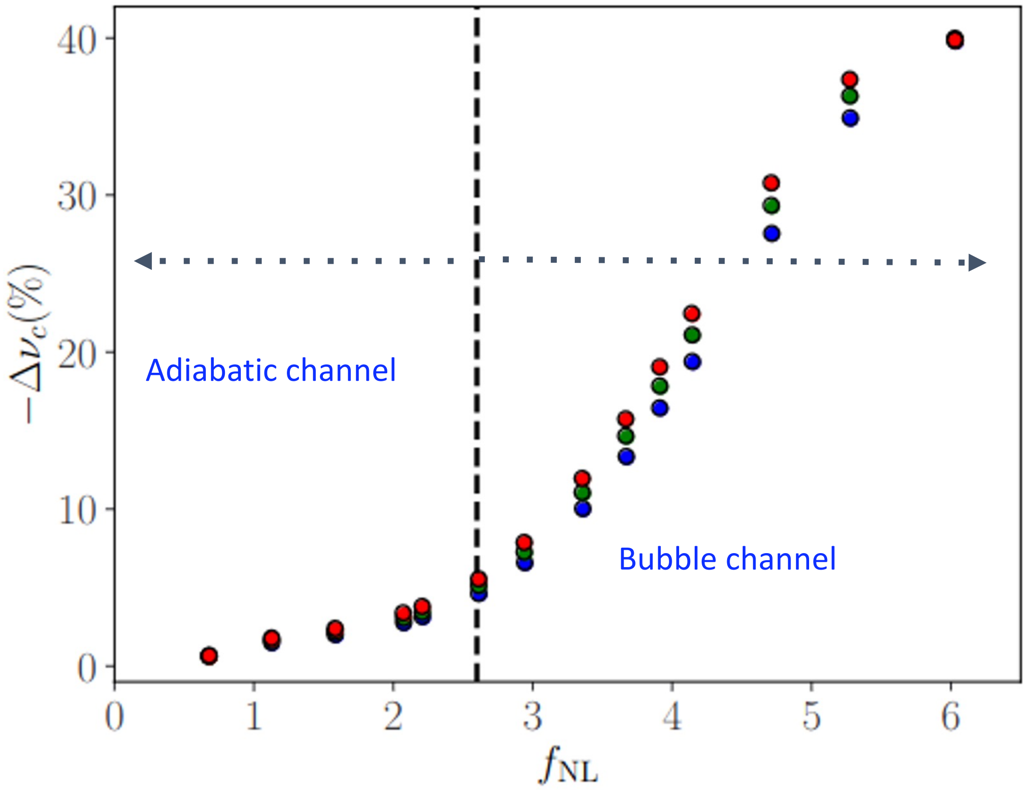



$f_{NL} \approx 2.6$ → Similar abundance in both channels

Using the naïve analytical estimate $\mu_{b,c} = \mu_\star$

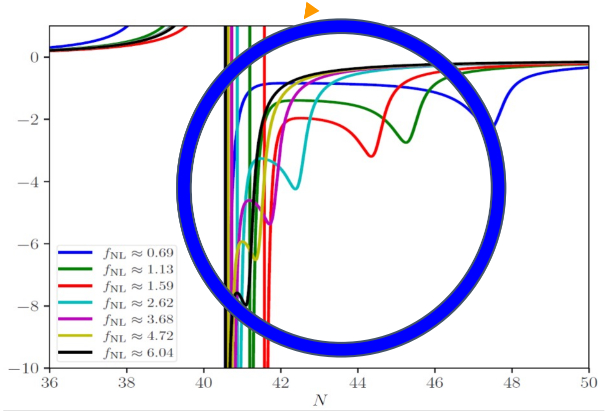
Using the numerical results (Bubbles more abundant)

# Comparison between analytical and numerical



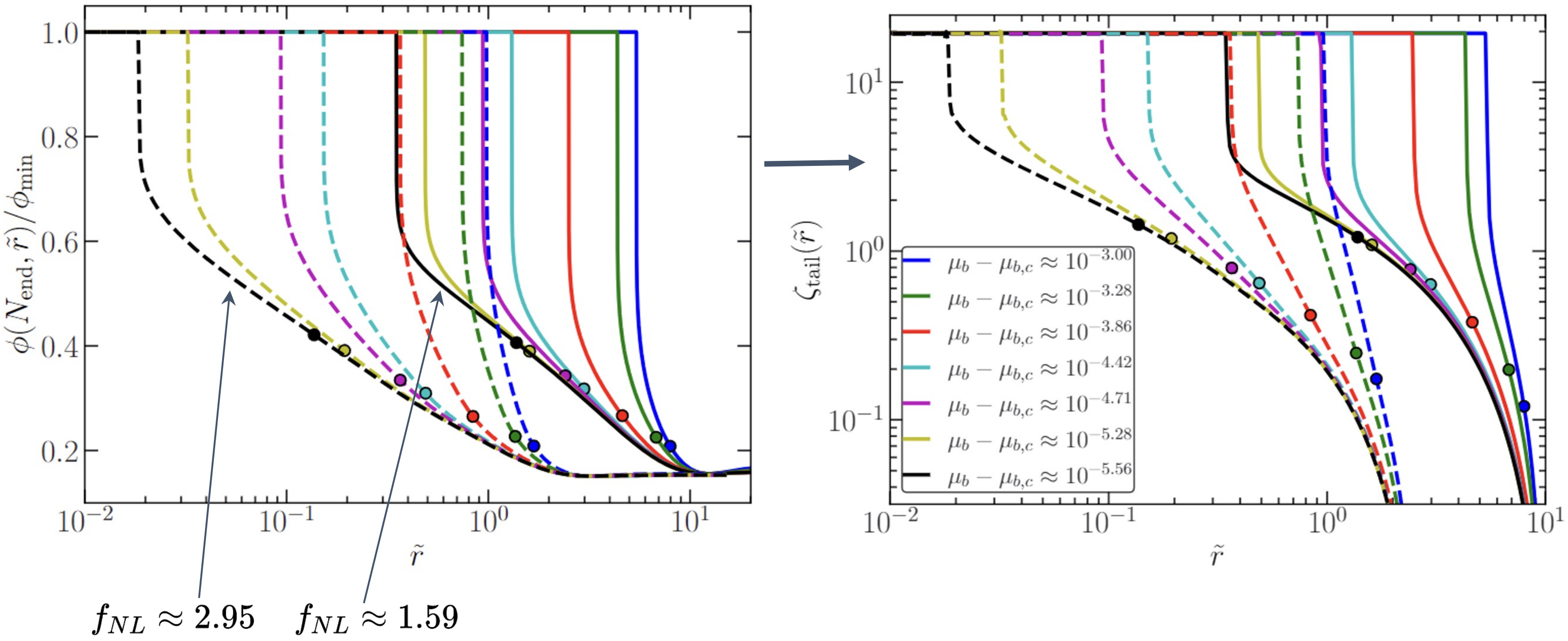

$$\Delta\nu_c(\%) = 100\frac{\nu_{b,c} - \nu_\star}{\nu_{b,c}}$$

The analytical estimate deviates for large non-gaussianity (expected, since we don't have attractor regime)

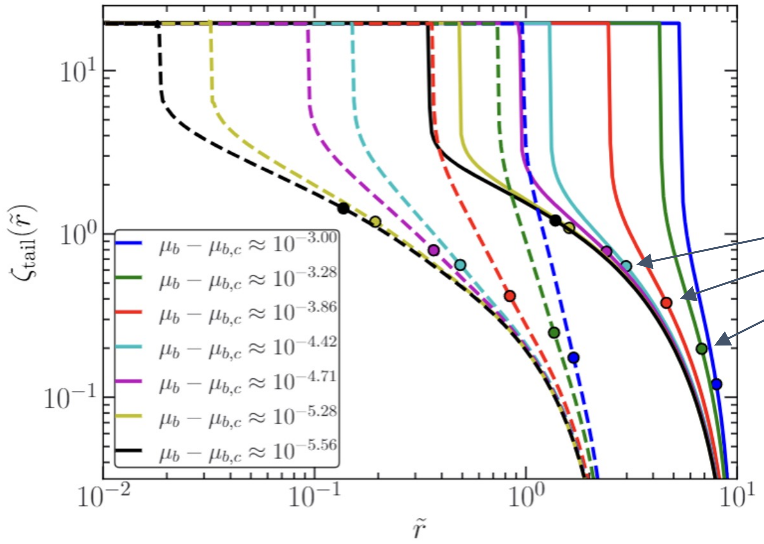


# Inflaton field at the “end of inflation”

Corresponding curvature fluctuation with delta N formalism



# Surrounding fluctuation of type II



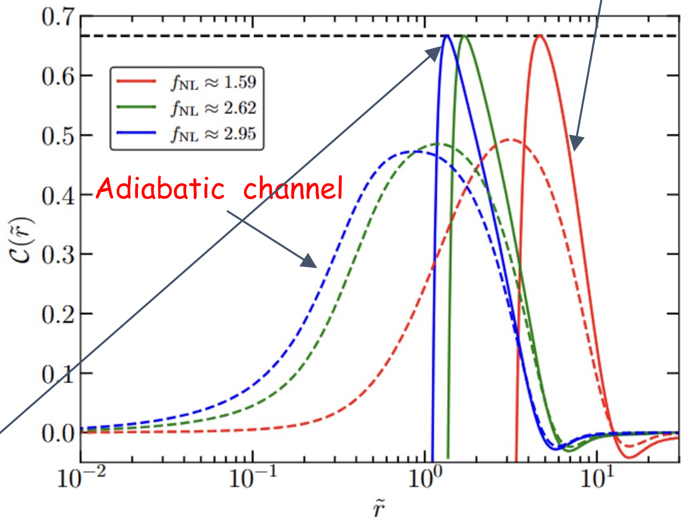

Fluctuation of type II->Areal radius non-monotonic increasing function

$$R = a\, r\, e^{\zeta_{tail}} \Rightarrow R' = a e^{\zeta_{tail}}\left(1 + r\, \zeta'_{tail}\right)$$

$$R' < 0 \Rightarrow 1 + r_{tail}\, \zeta'_{tail}(r_{tail}) = 0$$

$$\mathcal{C}(r_{tail}) = 2/3$$

Bubble channel



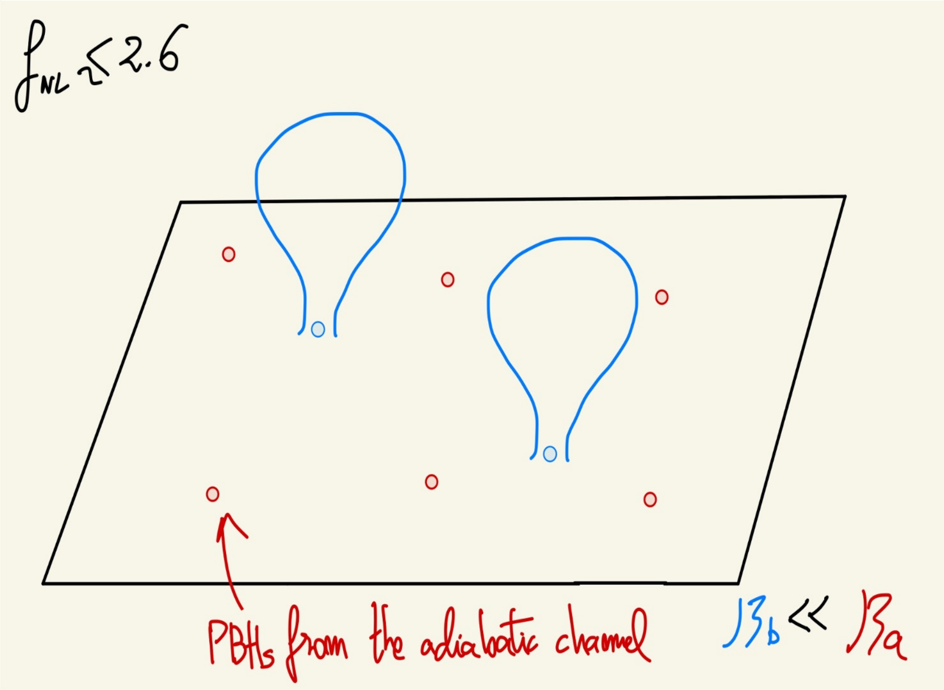

Fluctuations of type II are exceedingly suppressed in the adiabatic channel

But in the bubble channel, they actually give the dominant contribution

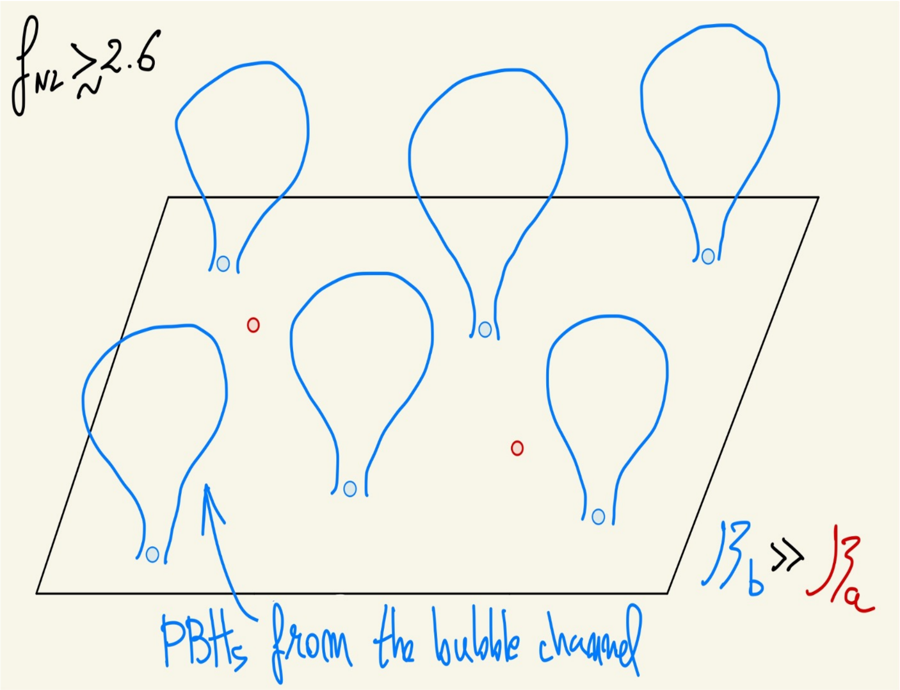
$$M_{bubble\ channel} \sim M_k(k_{tail}) \qquad k_{\rm tail}(\nu_b) = \frac{1}{r_{\rm tail}(\nu_b)\, e^{\zeta_{\rm tail}(r_{\rm tail}(\nu_b))}}$$

# Qualitative picture

Fluctuations type I-> dominant contribution

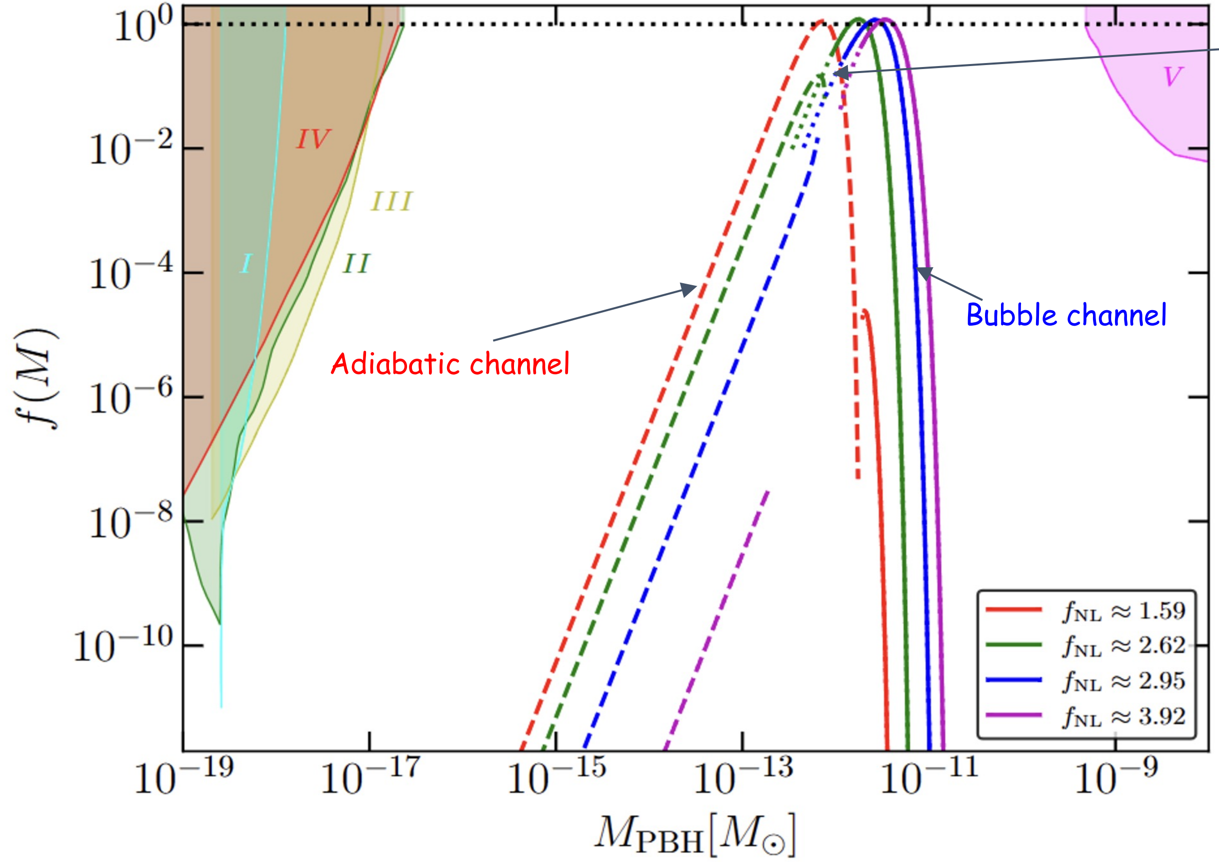



Fluctuations type II-> dominant contribution



# Mass function from both channels

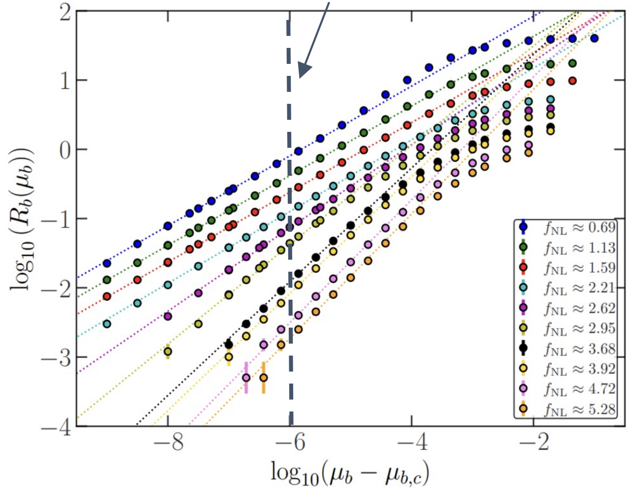
$$f_{\rm a}(M_a) = \frac{M_{\rm a}(\nu_a)\mathcal{N}_{\rm pk}(\nu_a(M_a))}{\rho_{\rm critical}\Omega_{\rm DM}}\left|\frac{d\ln M_a(\nu_a)}{d\nu_a}\right|^{-1}$$




$$\Delta\mu \sim \frac{H}{2\pi}(\Delta N)^{1/2} \gtrsim (\mu_b - \mu_{b,c})$$

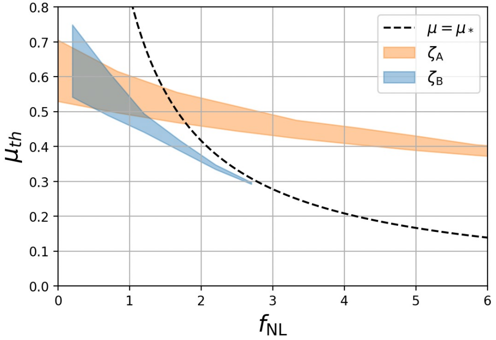
$$\mu_{b,({\rm cut-off})} - \mu_{b,c} \approx \sqrt{-\ln(R_b(\mu_b))}\frac{H(N_\star)}{2\pi}$$

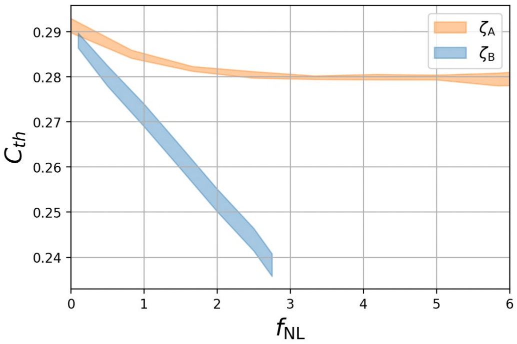
Quantum drift cut-off for very small bubbles



# Summary:

- The presence of an alternative channels for PBH production in models with local-type non-Gaussianity can easily be inferred from unitarity considerations.
- The dynamics of vacuum bubble formation has been clarified. We find a critical regime for the size of the bubbles.
- The log-relation for NG curvature fluctuation is quite accurate in predicting the bubble channel of PBH production for fnl < 2.6.
- Bubbles are more abundant than expected from the analytic log-relation. The bubble channel is dominant for fnl > 2.6.
- An adiabatic fluctuation of type II surrounds (and dominates the mass) of PBHs generated from the bubble channel.
- The mass function for the bubble channel is highly monochromatic.




$$\epsilon_1 = \dot{\phi}_b^2/2$$

$$\epsilon_2 = \dot{\phi}_b \ddot{\phi}_b/\epsilon_1$$