# Large $|\eta|$ approach to single-field inflation

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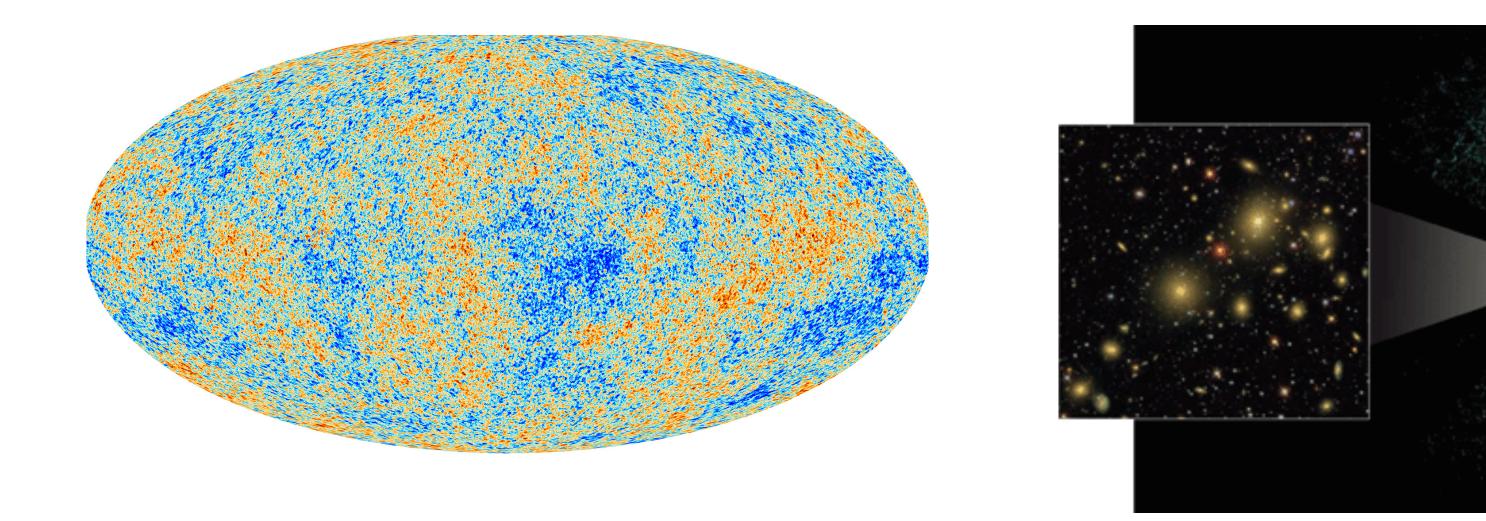




Based on 2305.11568

# Introduction

- within the first second of our universe life.
- seeds for the CMB and the LSS



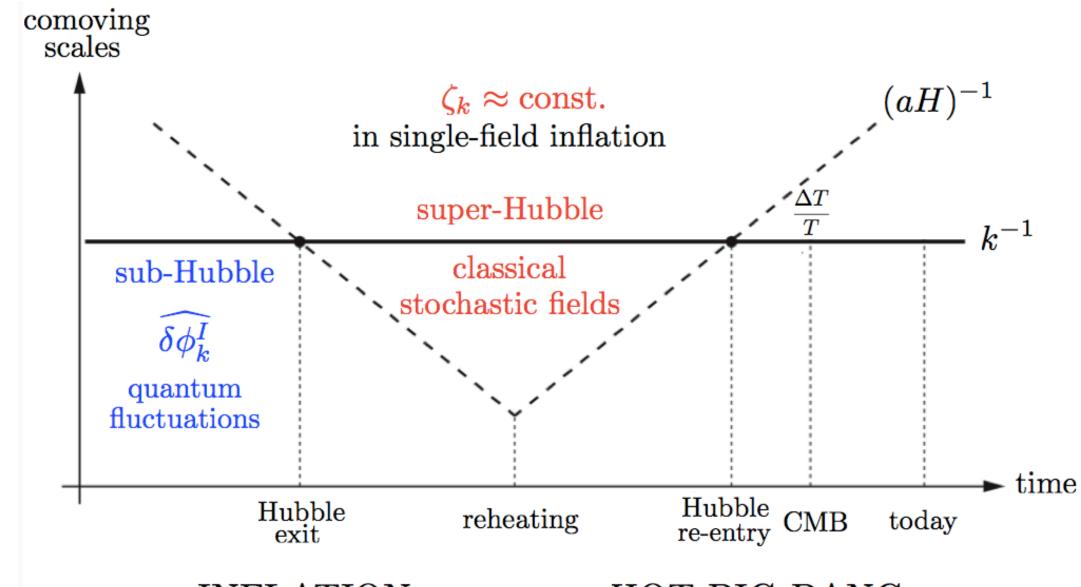
▶ Inflation is a short period of superluminal, accelerated expansion, occurred

► It solves problems of big bang cosmology: horizon, flatness, entropy problems

► Moreover, inflation provides an **elegant mechanism** for generating the **primordial** 

# Introduction

### ► Moreover, inflation provides an **elegant mechanism** for generating the **primordial** seeds for the CMB and the LSS



### INFLATION

- Then re-enter the horizon after inflation ends

HOT BIG-BANG

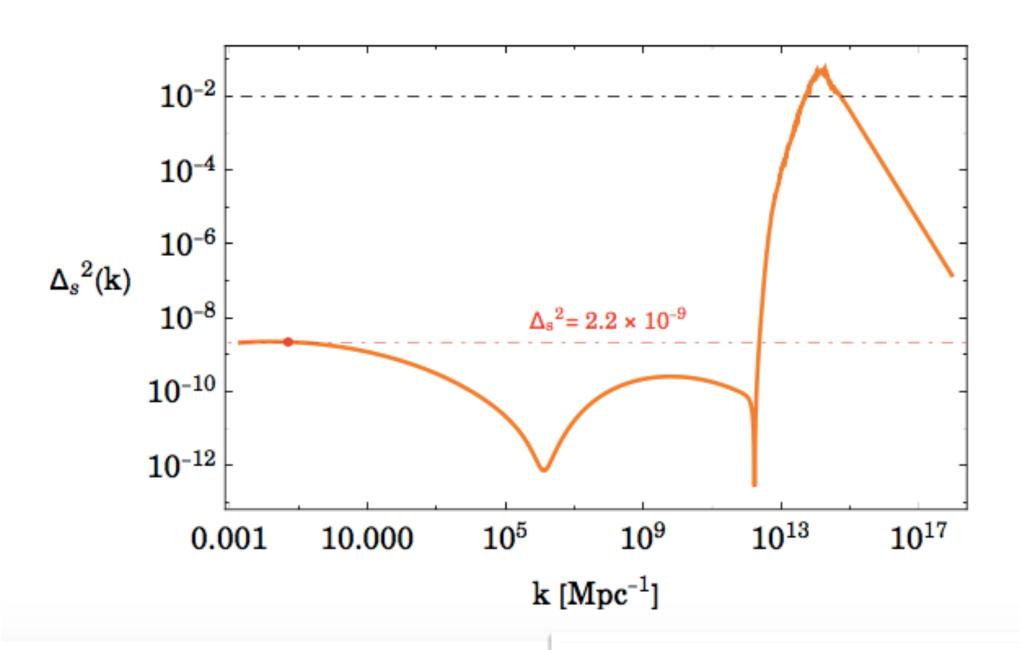
- Cosmological fluctuations are produced by quantum effects at short distances, - Their wavelength stretched beyond the horizon by the superluminal expansion.

# Dark matter and inflation

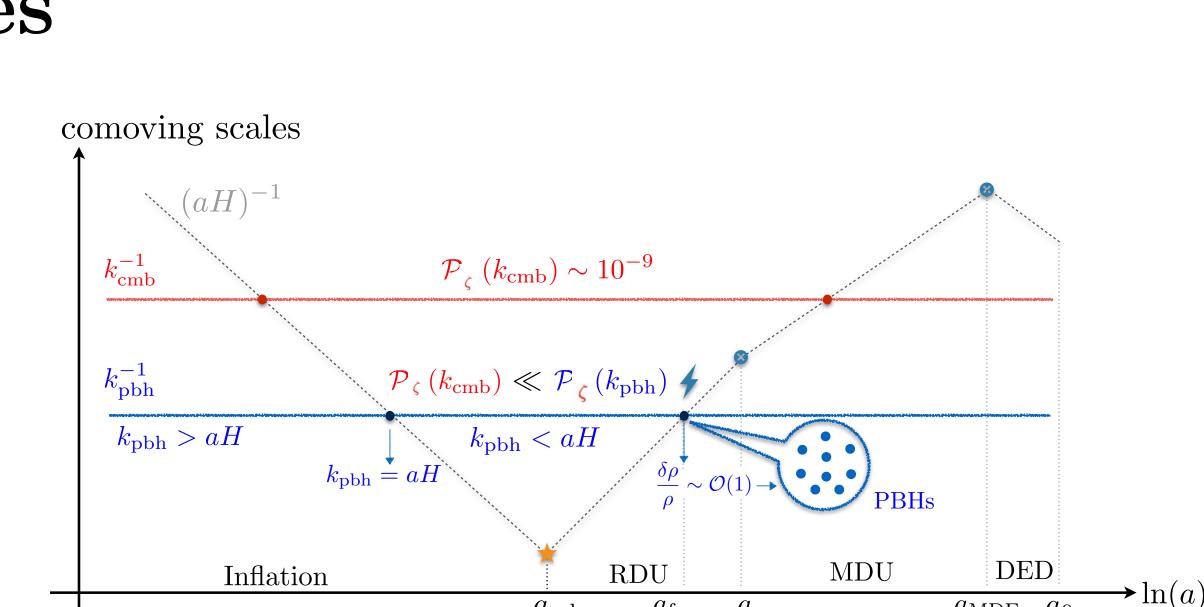
### What about dark matter? Can inflationary fluctuations source it?

### **Yes** if they increase in size at small scales

# $\triangleright$ **Primordial black holes**



The spectrum of curvature fluctuation  $\zeta$  increases towards small scales thanks to nonstandard inflationary dynamics. When re-entering the horizon during RD, curvature fluctuations source overdensities producing PBH



 $a_{\rm reh}$ 

 $a_{\mathrm{form}}$ 

 $a_{\rm eq}$ 

 $a_{\mathrm{MDE}}$   $a_{0}$ 



### The predictions of single-field inflation are very successful at CMB scales:

Fluctuations of  $\phi$  and metric  $\Rightarrow$  Curvature perturbation  $\zeta$ 

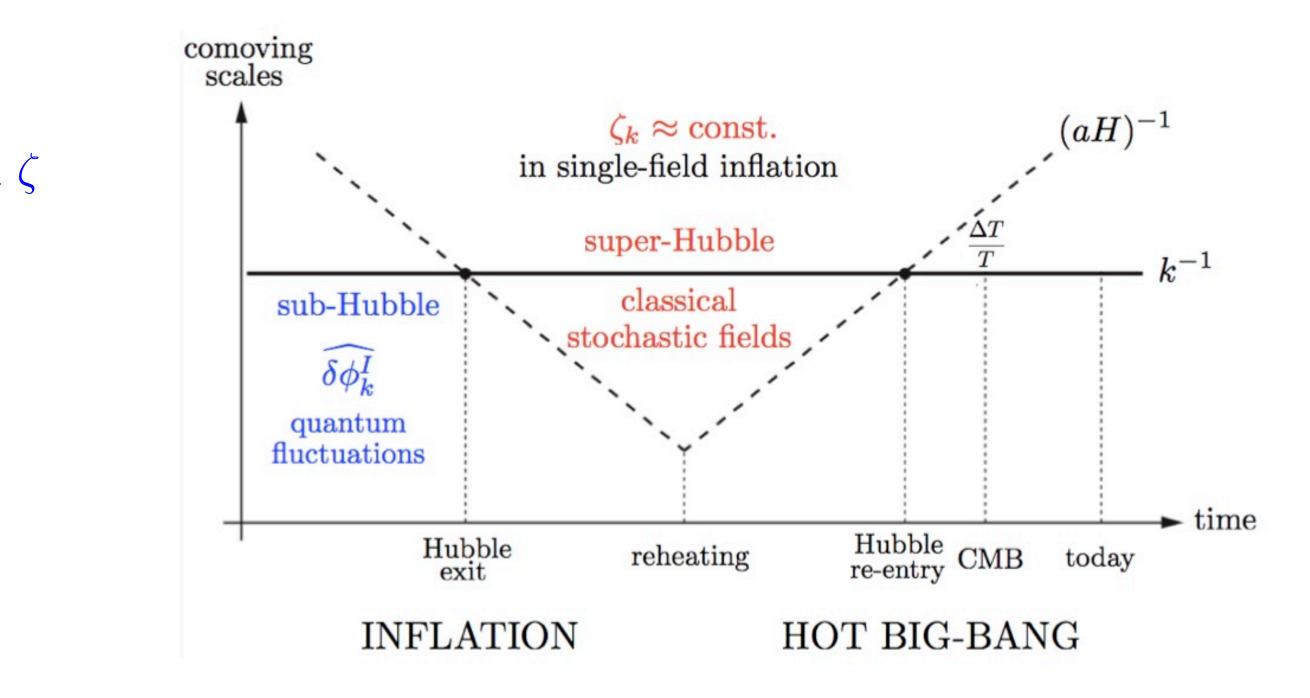
$$\Delta_{\zeta} = \frac{H^2}{8\pi^2\epsilon}$$

$$n_{\zeta} - 1 = -2\epsilon - \eta$$

**Slow-roll parameters** 

$$\begin{aligned} \epsilon &= -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \ll 1\\ \eta &= \frac{\dot{\epsilon}}{\epsilon H} = 2\epsilon + \frac{2\ddot{\phi}}{\dot{\phi} H} \ll 1 \end{aligned}$$

# **Slow-roll inflation**





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# **Slow-roll inflation**

- $\triangleright$  This is **bad** because the many existing models are degenerate.
- This is **very good** being a manifestation of EFT of inflation:  $\triangleright$ the slow-roll parameters control the spontaneous breaking of time-reparametrization invariance.

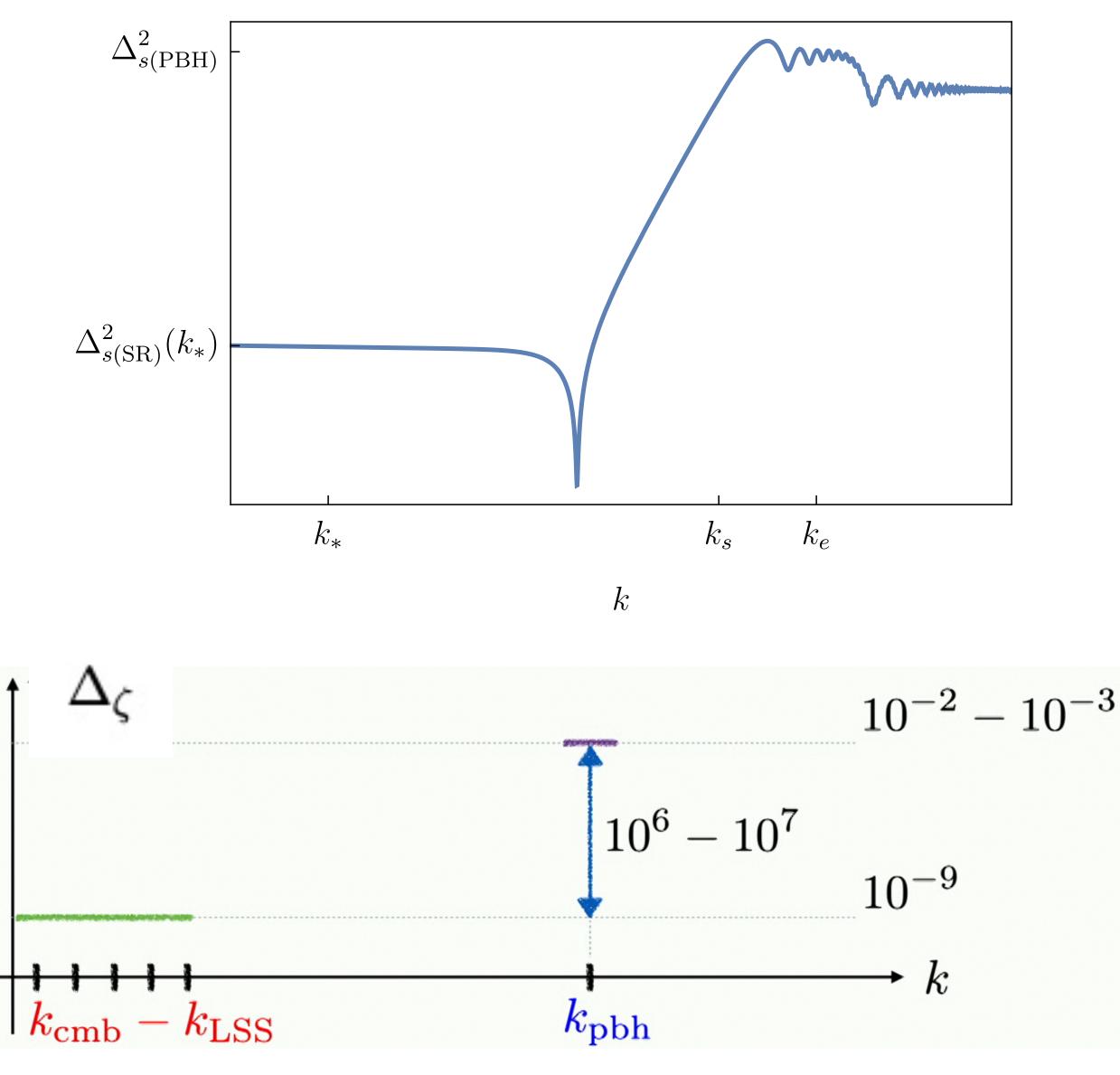
$$t \mapsto t - \pi(\boldsymbol{x}, t)$$

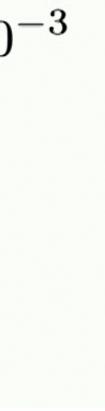
Observables depend on few parameters, controlling pattern of symmetry breaking

### We need to abandon slow-roll regime

### The parameter $\epsilon$ changes by several orders of magnitude in few e-folds

$$\Delta_{\zeta} = \frac{H^2}{8\pi^2\epsilon}$$





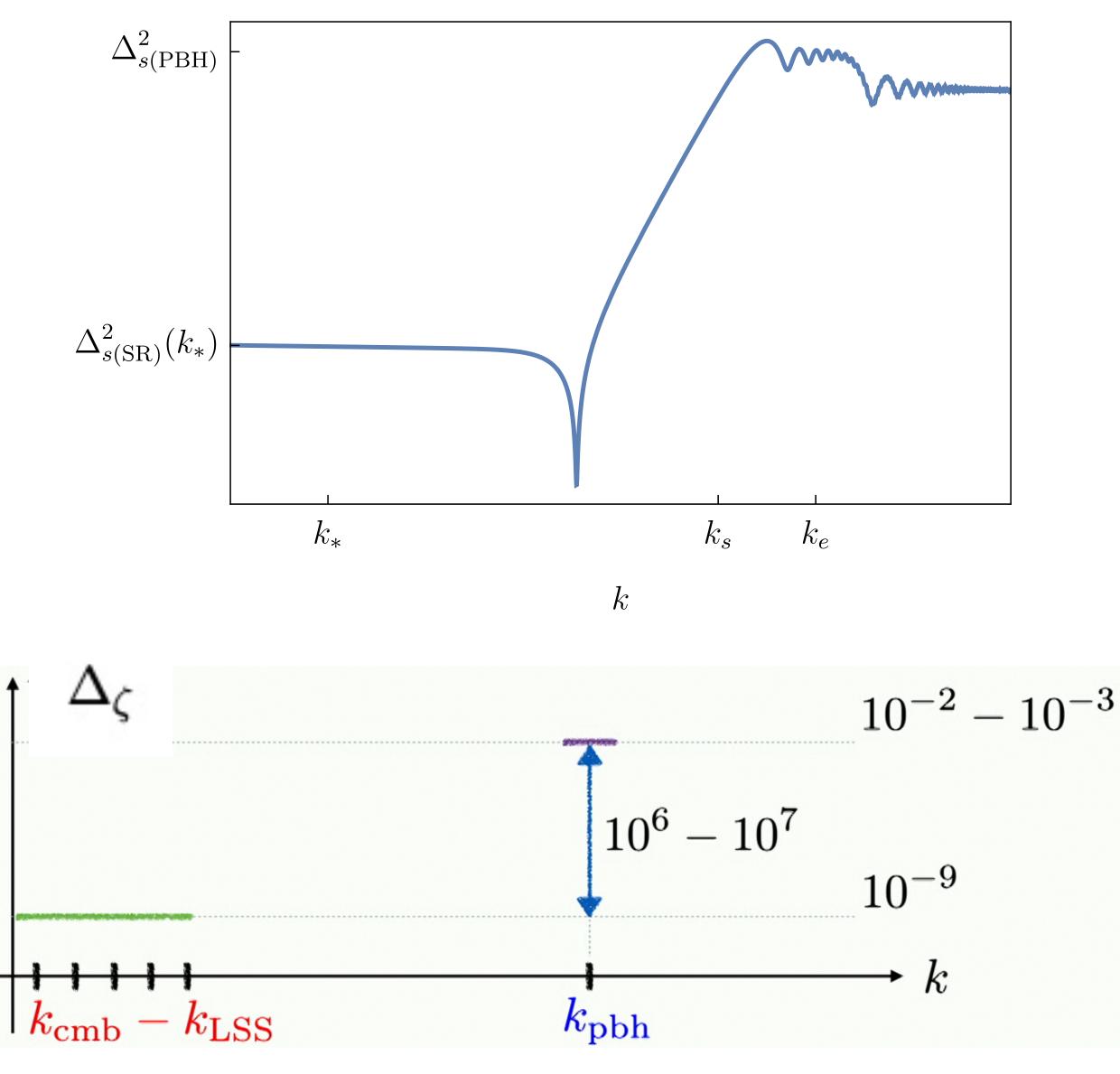
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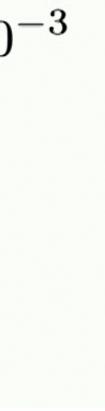
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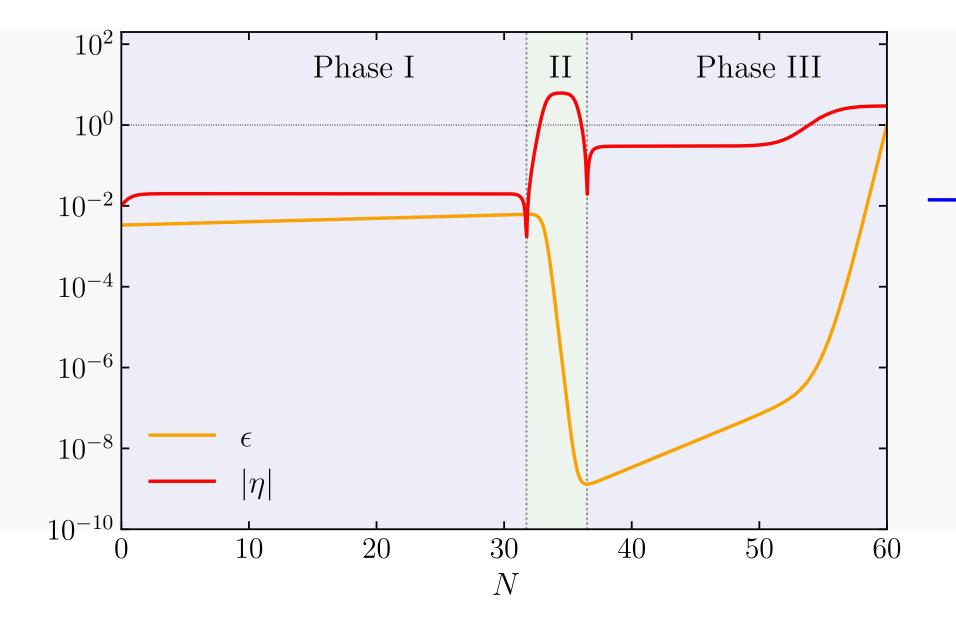
 $\eta$  must become large and negative

$$\eta = \frac{\dot{\epsilon}}{\epsilon H} = 2\epsilon + \frac{2\phi}{\dot{\phi}H}$$





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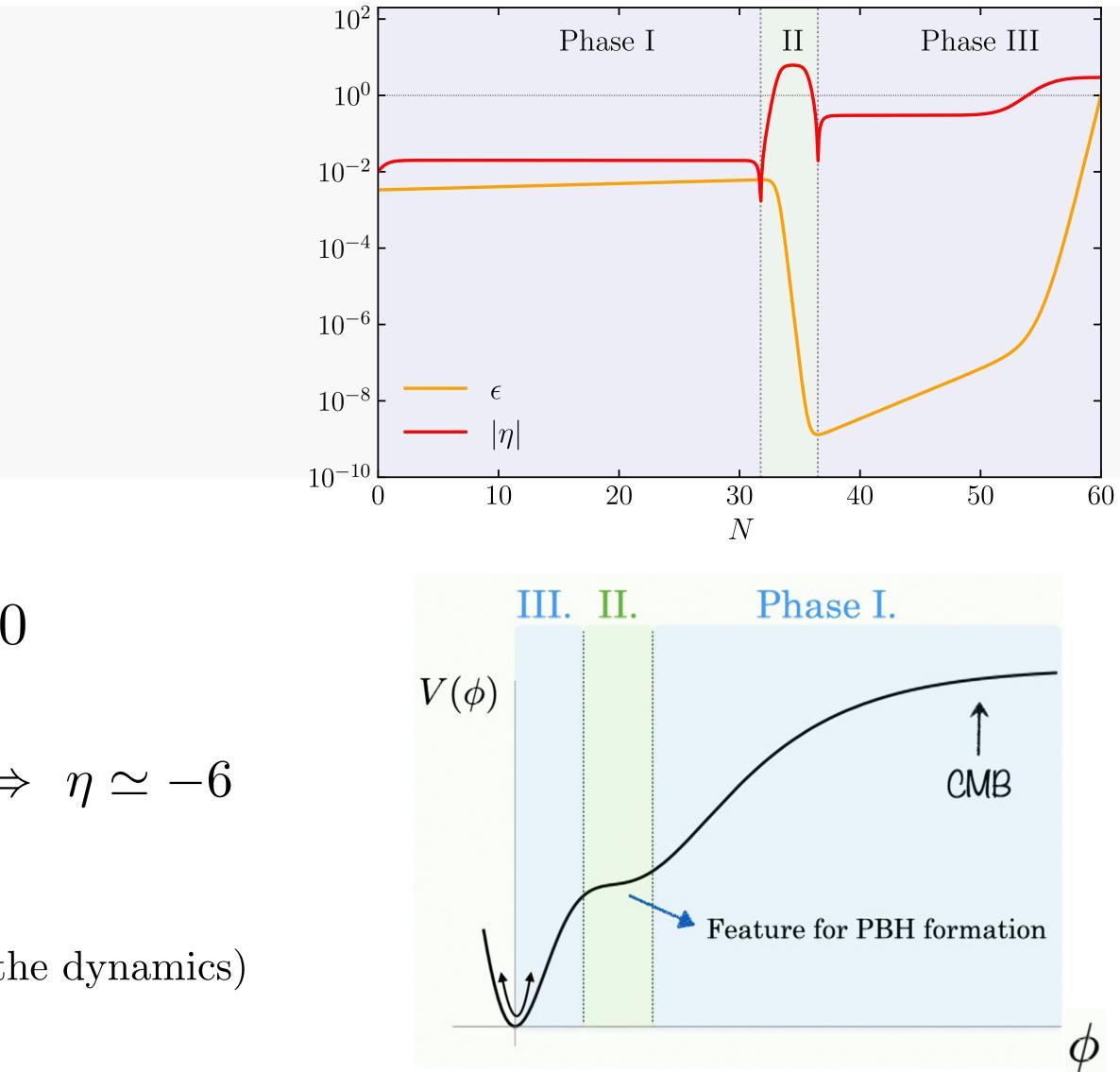


$$\eta = \frac{\dot{\epsilon}}{\epsilon H} = 2\epsilon + \frac{2\phi}{\dot{\phi}H}$$

 $\triangleright$  Ultra slow-roll inflation: V' = 0

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \implies \ddot{\phi} = -3H\dot{\phi} = 0$$

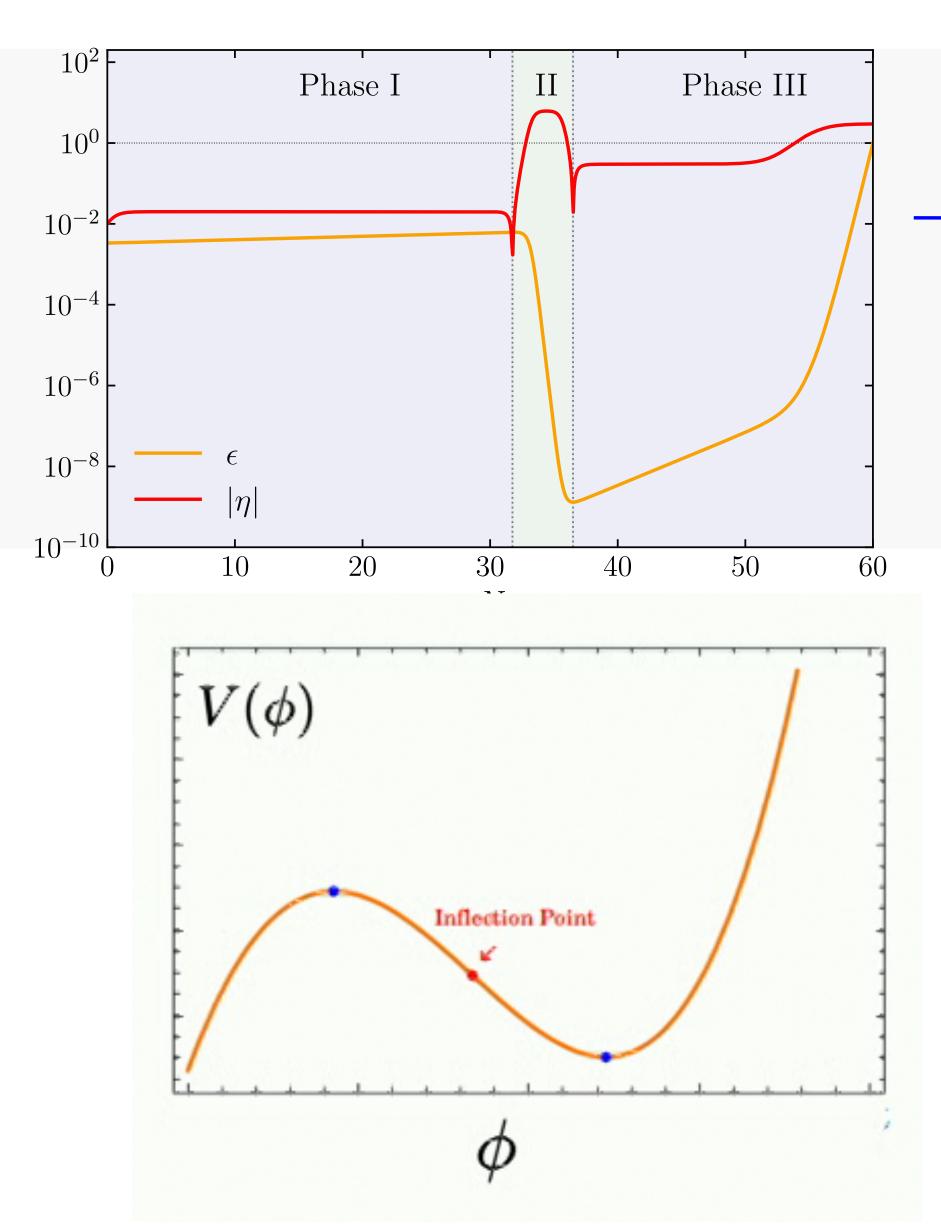
(this implies  $\phi \sim a^{-3} \Rightarrow$  decaying mode controls the dynamics) [Kinney,...,Germani-Prokopec, Dimopoulos, ...]

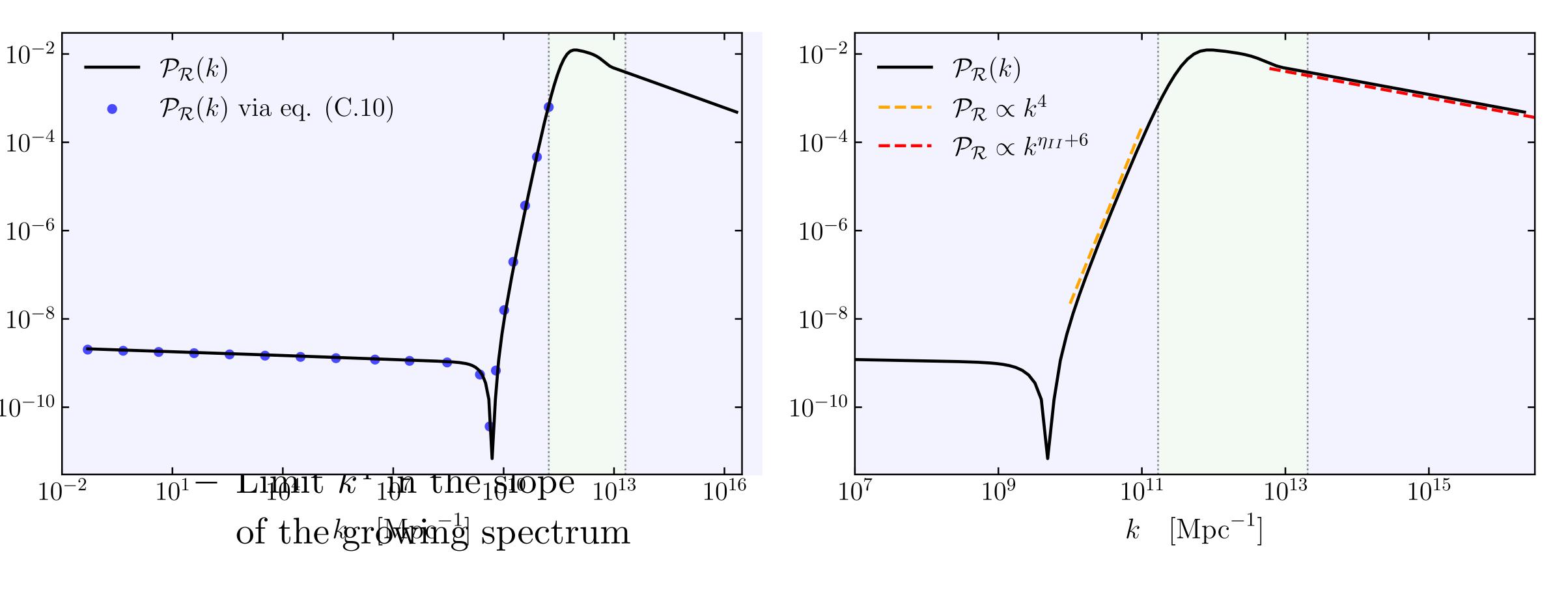


$$\eta = \frac{\dot{\epsilon}}{\epsilon H} = 2\epsilon + \frac{2\phi}{\dot{\phi}H}$$

 $\triangleright$  Constant roll inflation: V' < 0Scalar climbs a hill overshooting local minimum

$$\eta = 2\epsilon - 6 + \frac{2V'}{|\dot{\phi}|H} < -6$$



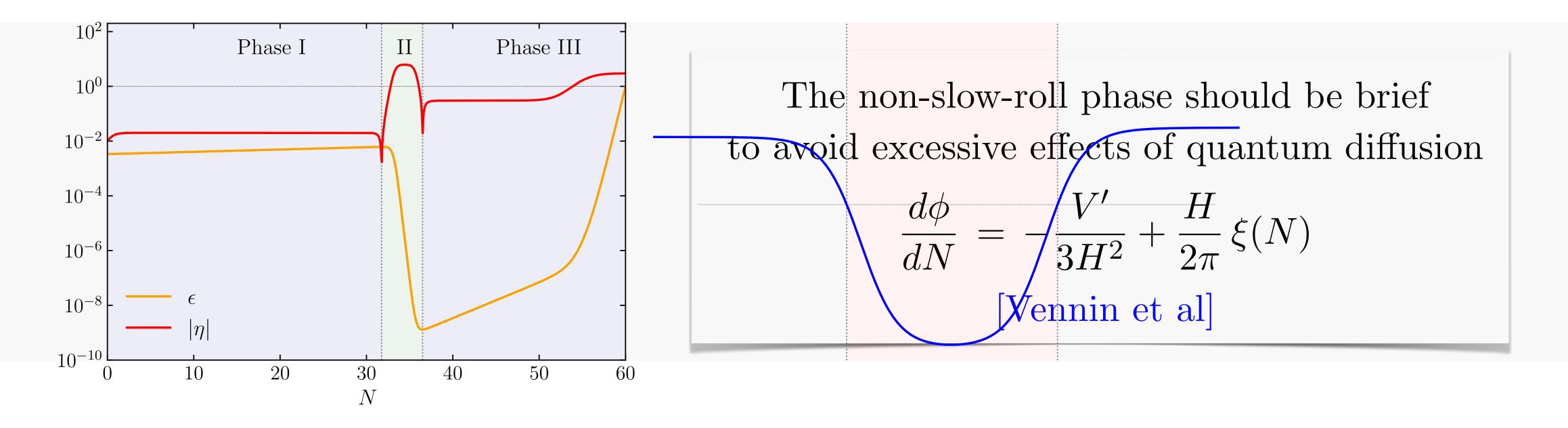


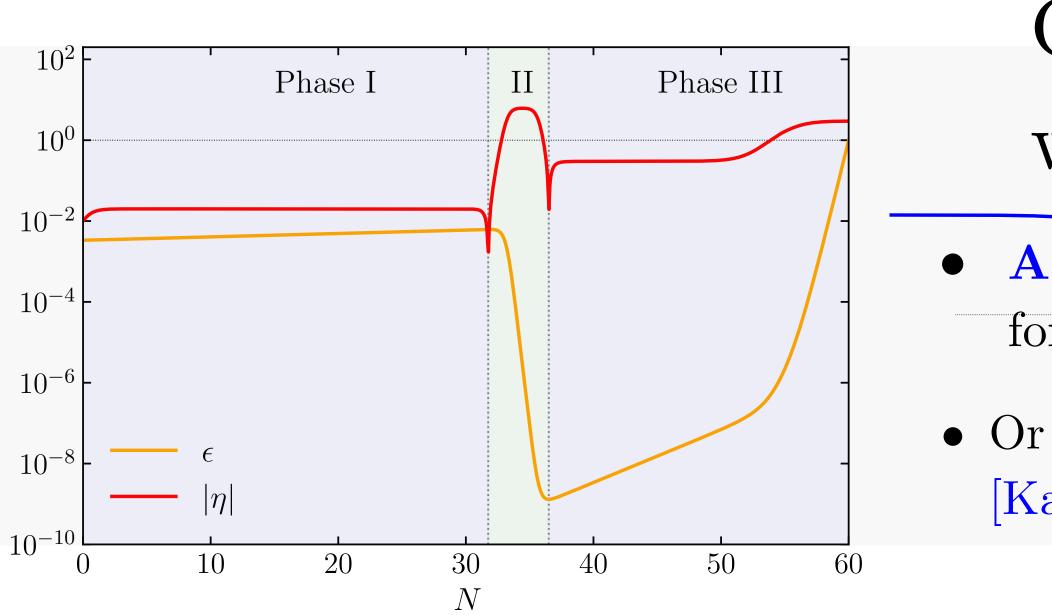
[Byrnes et al]

# Inflation and PBH

 $\Rightarrow$  We get a rapid enhancement of the spectrum

(for recent review see e.g. [Özsoy, GT])





# **Inflation and PBH**

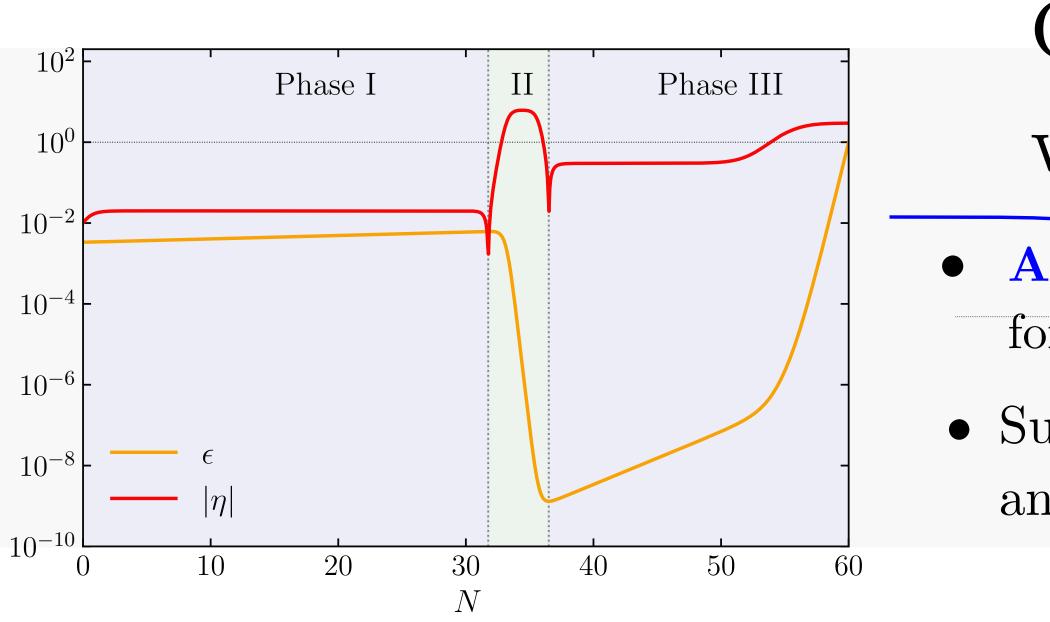
# Calculations can be carried on with the help of numerics

• Analytic control is possible

for  $\eta = -6$  and for a model of Starobinsky

• Or by designing piecewise models with constant slopes for  $\epsilon$  and  $\eta$ [Karam et al, Franciolini et al, Domenech et al, ...]

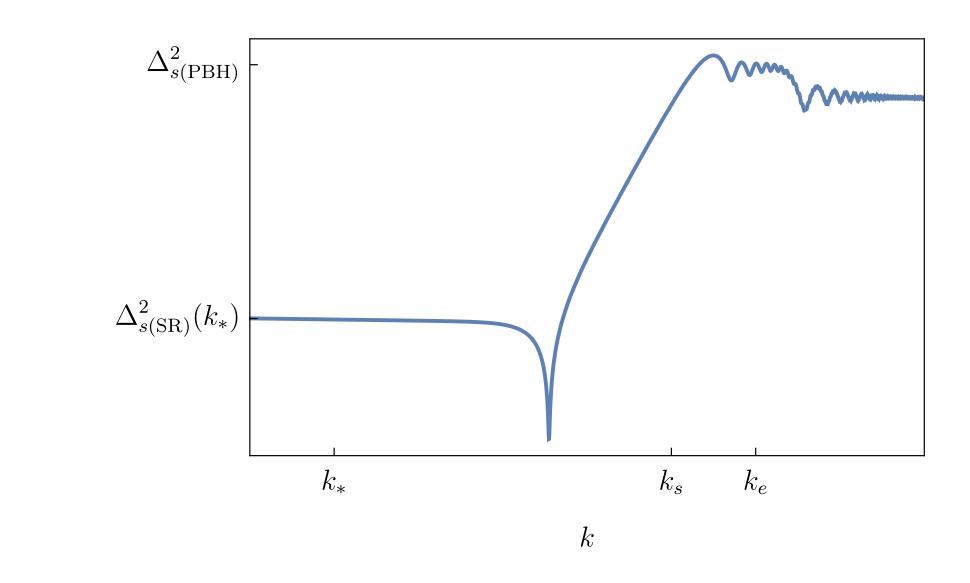




- **Good thing** Observables sensitive on details of the model.  $\triangleright$
- **Bad things** Degeneracies likely to occurr, and we lack  $\triangleright$ an analytical understanding of what is going on

- Calculations can be carried on with the help of numerics
- Analytic control is possible
  - for  $\eta = -6$  and for a model of Starobinsky
- Subtleties associated with decaying mode,
  - and connections between slow-roll and non-slow-roll phases.

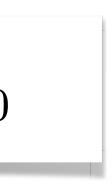






## This might lead to a **reliable** analytical framework!

## $\triangleright$ At the same time, take $\Delta N_{\rm nsr} \ll 1$ , and the product $|\eta| \Delta N_{\rm nsr} = \text{fixed} \equiv 2 \Pi_0$



## This might lead to a **reliable** analytical framework!

- $\triangleright$ 
  - in  $1/|\eta|$  and  $\epsilon$ . E.g. for the power spectrum (take  $\epsilon \ll 1$ ):

$$\frac{\Delta_{\zeta}(\kappa)}{\Delta_{\zeta}(0)} = 1 - 4\kappa \Pi_0$$

with  $\kappa = k/k_{\star}$  and  $j_1(\kappa) = \frac{\sin \kappa}{\kappa^2} - \frac{\cos \kappa}{\kappa}$ 

## At the same time, take $\Delta N_{\rm nsr} \ll 1$ , and the product $|\eta| \Delta N_{\rm nsr} = \text{fixed} \equiv 2 \Pi_0$

▷ Straightforward to solve for mode functions, and compute correlators in an expansion

 $\cos \kappa \, j_1(\kappa) + 4\kappa^2 \, \Pi_0^2 \, j_1^2(\kappa) \qquad + \mathcal{O}(1/|\eta|)$ 





## This might lead to a **reliable** analytical framework!

### $\triangleright$ At the same time, take $\Delta N_{\rm nsr} \ll 1$ , and the product $|\eta| \Delta N_{\rm nsr} = \text{fixed} \equiv 2 \Pi_0$

▷ Practically, what do we do? Whenever the end, take limit  $|\eta| \to \infty$ 

 $\triangleright$  Practically, what do we do? Whenever meeting  $\Delta N_{\rm nsr}$ , substitute with  $2\Pi_0/|\eta|$ . At



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  - in  $1/|\eta|$  and  $\epsilon$ . E.g. for the power spectrum (take  $\epsilon \ll 1$ ):

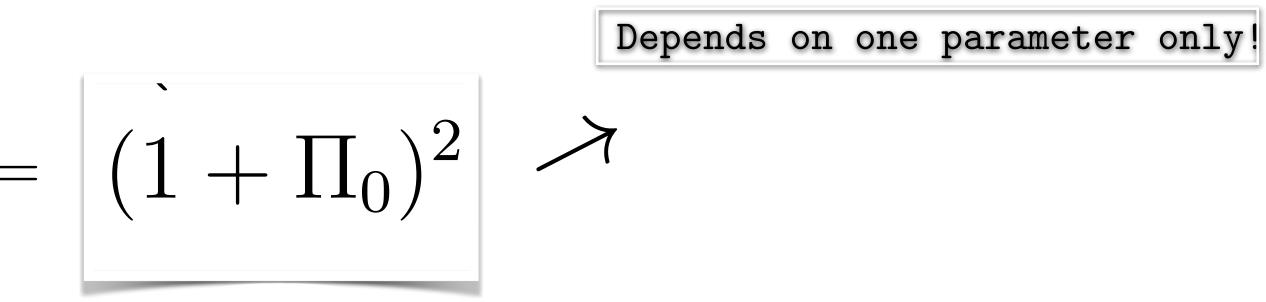
with  $\kappa = k/k_{\star}$  and K'К

 $\triangleright$ 

$$\lim_{\kappa \to \infty} \frac{\Delta_{\zeta}(\kappa)}{\Delta_{\zeta}(0)} =$$

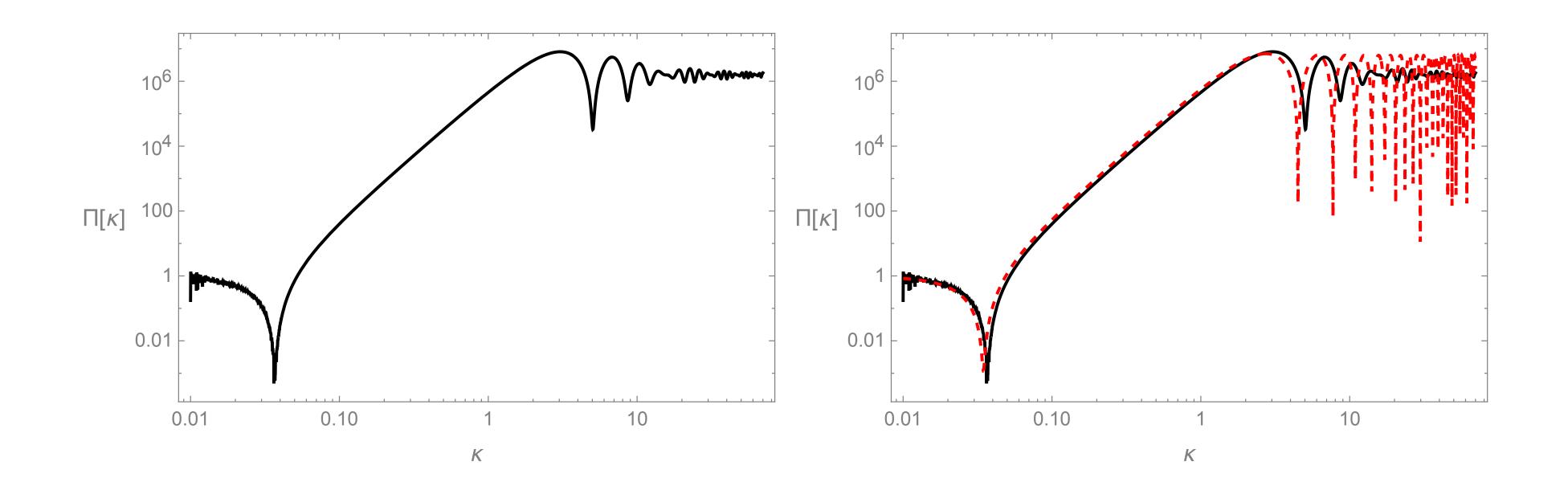
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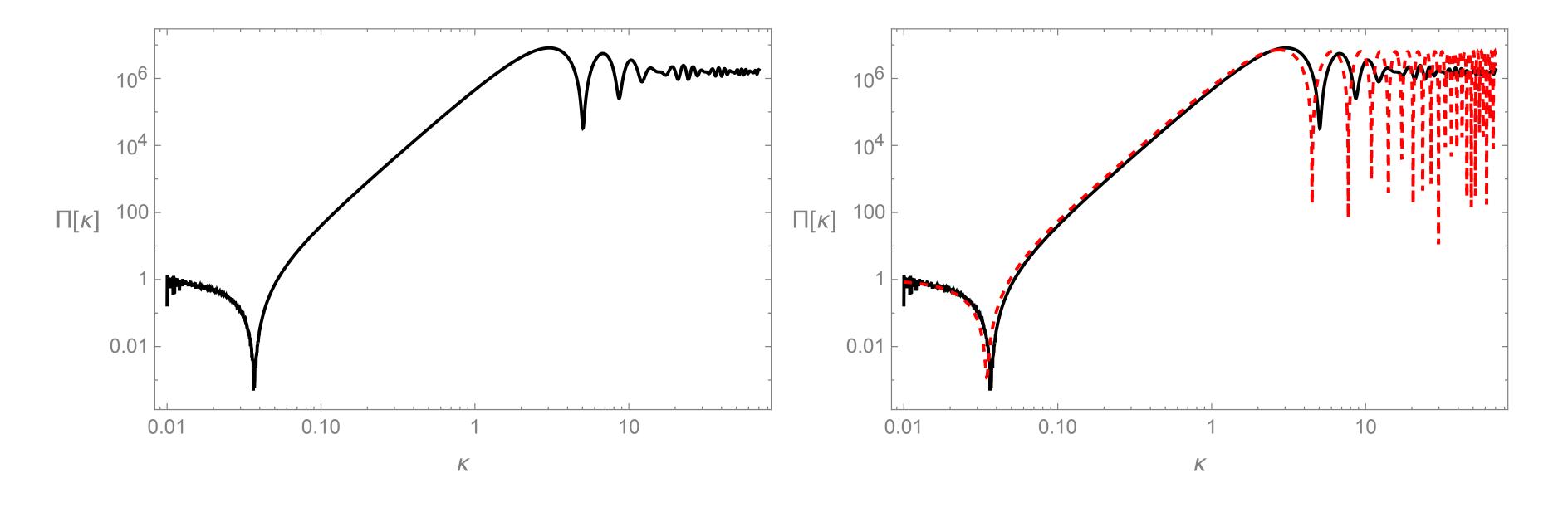


# This might lead to a **reliable** analytical framework!



### it catches pretty well the large-scale behaviour, up to the peak

 $(\mathcal{O}(1/|\eta|) \text{ corrections can be included, and improve the small-scale behaviour})$ 

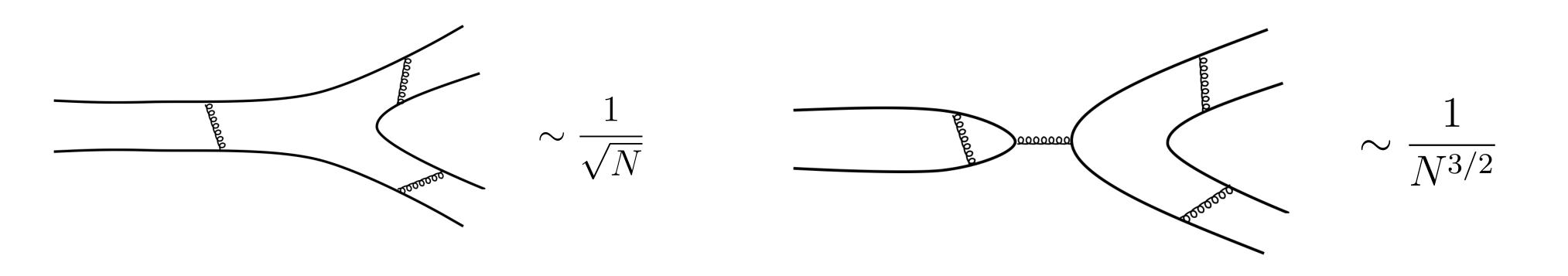


Also spectral index can be computed analytically, at leading order in  $1/|\eta|$ :

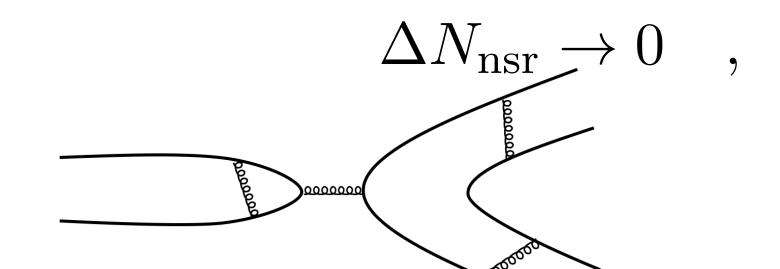
$$n_{\zeta} - 1 = \frac{2\kappa \Pi_0 \left[ (1 - 2\kappa^2) \sin (2\kappa) - 2\kappa \cos (2\kappa) \right]}{\kappa^2 + 4\kappa \Pi_0 \cos \kappa (\kappa \cos \kappa - \sin \kappa) + 4\Pi_0^2 (\kappa \cos \kappa - \sin \kappa)^2} - \frac{\Pi_0^2 \left[ 4 - (4 - 8\kappa^2) \cos (2\kappa) + 4\kappa (\kappa^2 - 2) \sin (2\kappa) \right]}{\kappa^2 + 4\kappa \Pi_0 \cos \kappa (\kappa \cos \kappa - \sin \kappa) + 4\Pi_0^2 (\kappa \cos \kappa - \sin \kappa)^2}$$

 $\triangleright$  Model studied by 't Hooft: computations simplify taking number N of colors large, and expand in 1/N. Call g the QCD coupling constant, consider limits

$$g \to 0$$
 ,  $N \to \infty$  ,  $g^2 N \equiv g_0^2 = \text{fixed}$ 



 $\triangleright$  Analogy with PBH inflationary models





 $\Delta N_{\rm nsr} \to 0$  ,  $|\eta| \to \infty$  ,  $|\eta| \Delta N_{\rm nsr} = \text{fixed}$ 



# Large $|\eta|$ limit of inflation

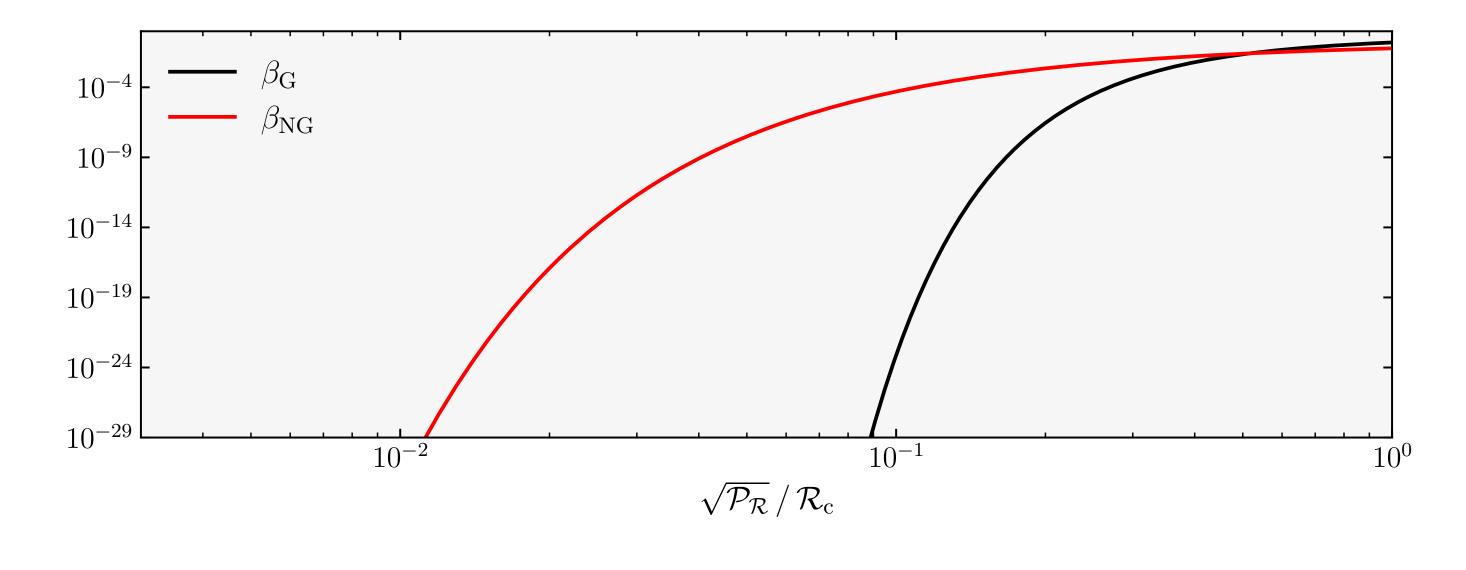
- New framework based on a novel consistent perturbative expansion. Hopefully useful for carrying on computations in the context of PBH physics
- ▷ Works in a particular limit of parameter space. The hope is to learn something useful for real-world physics. (In analogy with large-N QCD.)

# **Higher-order correlation functions**

▷ Non-Gaussian effects around the peak of the spectrum plays an important role for PBH formation. Analytic control of non-Gaussianity would be welcome!

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{s\mathbf{k_3}} \rangle = (2\pi)^3 \delta(\vec{k_1} + \vec{k_2} + \vec{k_3}) B_{\zeta}(k_1, k_2, k_3)$$

[Byrnes et al, Atal-Germani, Passaglia et al, ..., Taoso-Urbano]



We can reduce the required amplitude of  $P_{\zeta}$  for producing PBH at small scales:

# **Higher-order correlation functions** and the large- $\eta$ approach

A single dominant term in the third order Hamiltonian of single-field inflation  $\triangleright$ [Maldacena, Kristiano-Yokoyama]

$$\mathcal{H}_{\text{int}} = -\frac{1}{2} \int d^3x \, a^2(\tau) \epsilon(\tau) \, \eta'(\tau) \, \zeta^2(\tau, \vec{x}) \, \zeta'(\tau, \vec{x})$$

$$\eta'(\tau) = \Delta \eta \left[ -\delta(\tau - \tau_1) + \delta(\tau - \tau_2) \right]$$

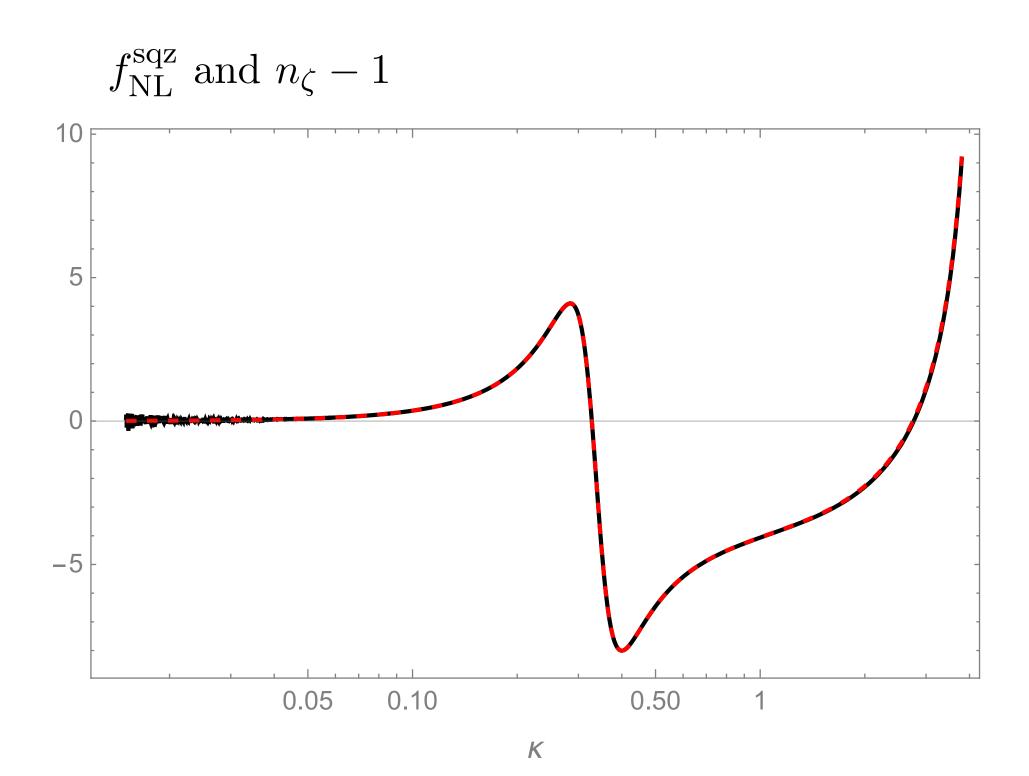
Plug mode functions and compute large- $\eta$  limit of bispectrum. At leading order in  $\triangleright$  $1/|\eta|$  one gets an analytic expression

$$B_{\zeta}(k_1, k_2, k_3) = \text{too lo$$

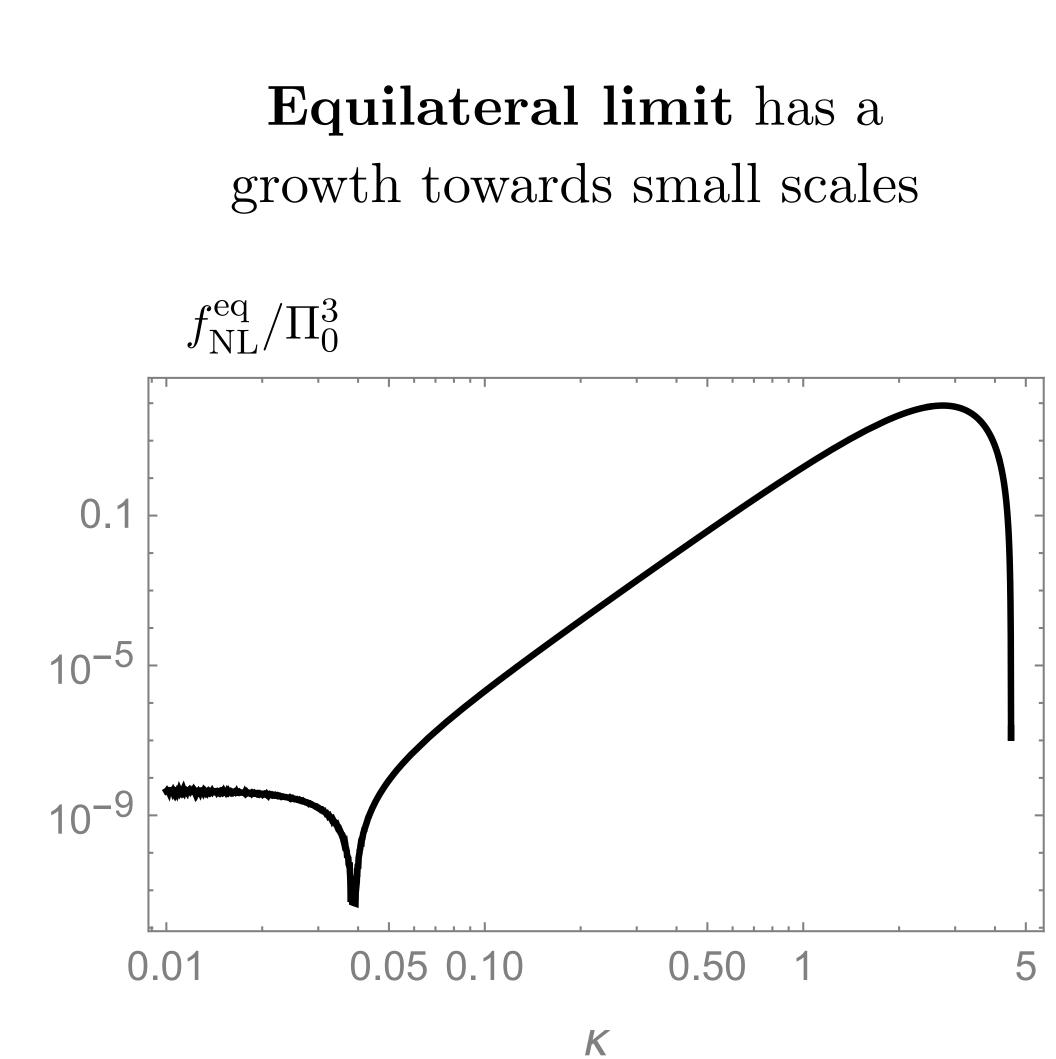
ong to fit in the slide  $\rightarrow$ Depends on one parameter only!



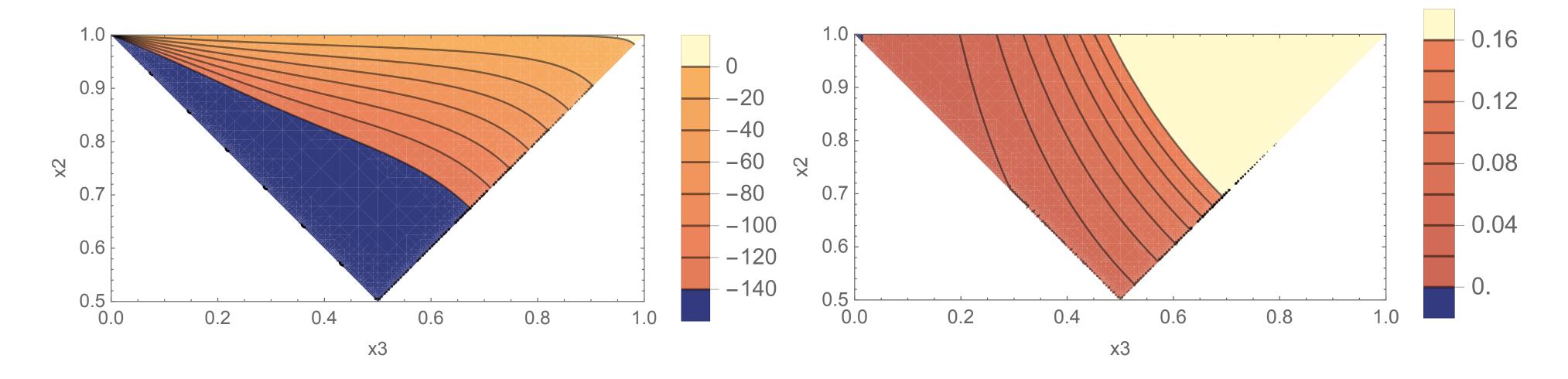
### Squeezed limit satisfies Maldacena consistency relation



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Higher-order correlation functions
and the large- |\eta| approach
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The bispectrum is strongly dependent on the scale, and at each scale it has rich shape dependence



At the dip



**Higher-order correlation functions** and the large- $|\eta|$  approach

Towards small scales

## Subtle issues: Loop corrections and PBH





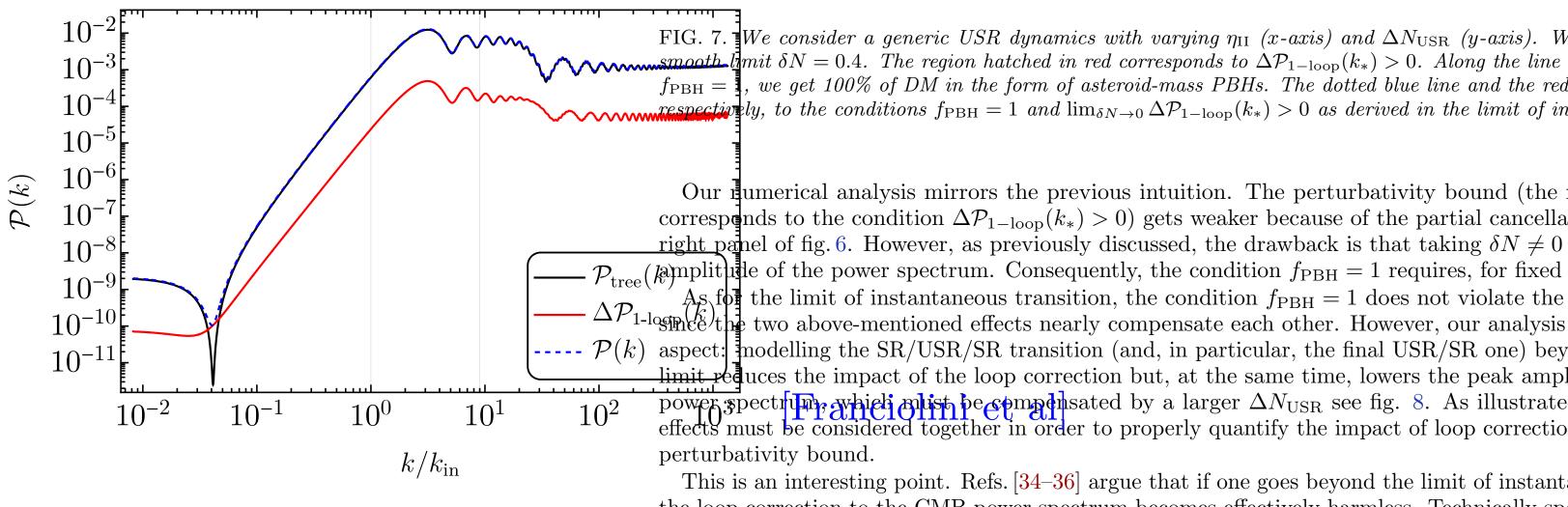
 $J - \infty$   $(I) C (I) J - \infty$ long CMB scales Schematically, one-loop correction decreases an exercited and the Boothing Boothing and the second of the Boothing Boo Subtle issues: Loop correction and manipulations are tricted to  $\xi_s$ Schematically, one-food correction to the schematic of the correction induced on the two-point correlator of to On the right side, we plot the prototypical tree-level power spectrum 3f curvature perturbation The redefinition terms are negligible about the reare ever ust en a total (another the Wing Dollation. Observable CMB modes (horizontal green band) of the figure) and, at the tree level, their correlation remains frozen from this time on. A source method and direct in-in formalismented and the latter strandbes is the source method and direct in-in formalismented for source source in the source of the side of the source in the source of the side of the source in the source of the side of the source of the side of the source is the source of the side of the source of the source of the side of the source of the sourc  $[\ldots, Weinberg, \ldots]$ which is propped by section the cather that the circle singly openation is high the sector of the se dugen pertorning integration perturbation theory (38) hor , time by parts leads to , thersource method utilizest bist an intense debate about ruling out or wether mechanism by I hefs 153-35 heffiller heffiller for that of the filler 3 setrans including the contribution of loop momenta between gia and g Trithepppycen Appars our geven de con 2 (ve had societ de la p [Kristiano-Yokoyama, Riotto, Firouzjahi, Franciolini et al, Fumagalli,...] percent level in the region of parameter space where  $f_{\rm PBH} \approx 1$ . Therefore at fir of PBHs in USR single-field inflation is not in conflict dutth perturbativ Recall the second-order and third-order actions are size by the second order and third-order actions are size to and the second order a interaction reads ment be in portant. As a concrete example, we have shown that loop correction of the Recall the second-of der and the second of t interaction  $[G]eads^{(2)}$   $[V]ection interaction interaction interaction interaction inflation. Since of the USR we are left with <math>\epsilon \ll 1$  but we need  $\epsilon = O(1)$  to end inflation. Since approximation. Romsequently, after USR we do not expect a scale-invarian applies. *ii)* Understanding the role of quartie interactions and the solution of the standard of the section o as schematically shown in  $eq(20)_{\pi}$  guartic interactions and non-

 $\langle in | \bar{T}e^{-i \int \mathcal{H}_{int}(\tau') d\tau'} \mathcal{O}(\tau) Te^{i \int \mathcal{H}_{int}(\tau') d\tau'}_{The redefinition}$  is simply in  $\mathcal{H}_{int}(\tau') d\tau'$  in  $\mathcal{H}_{int}(\tau')$ In single-field slow-roll inflation, loop corrections are small the one-loop corrections are small ▷ In PBH forming scenarios, the same mechanism that enhances which is proposed by the spectrum can also amplify loop corrections at large scales. perturbation theory (

cross the

 $\left\langle \ln \left| \bar{T}e^{-i\int \mathcal{H}_{\rm int}(\tau')d\tau'} \mathcal{O}(\tau) Te^{i\int \mathcal{H}_{\rm int}(\tau')d\tau'} \int \mathcal{U}(\tau) Te^{i\int \mathcal{H}_{\rm int}(\tau')d\tau'} \left| \ln \right\rangle \right|^{1/2} d\tau' = 0.4 \text{ while we keep } \eta_{\rm II} \text{ and } \Delta N_{\rm USR} \text{ generic as in fig. 4.} \right\rangle$ 

- [Riotto, Firouzjahi, Franciolini et al,...]; model dependent issue

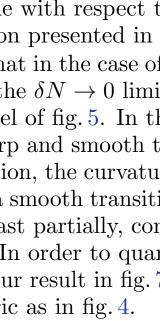


models the size of the loop correction gets reduced by one order of magnitude with respect t the limit of instantaneous SR/USR/SR transition. This confirms the intuition presented in

It should be noted, however, as evident from our discussion in section IIB, that in the case o transition the amplitude of the power spectrum gets reduced with respect to the  $\delta N \rightarrow 0$  limit Subtle issues: Loop correction fig. 3). The origin of this effect becomes evident if we consider the right panel of fig. 5. In the solution of the curvature mode  $\overline{\zeta_q}$  with  $\overline{q} = 2$  in the two cases of a sharp and smooth the curvature mode  $\overline{\zeta_q}$  with  $\overline{q} = 2$  in the two cases of a sharp and smooth the curvature mode  $\overline{\zeta_q}$  with  $\overline{q} = 2$  in the two cases of a sharp transition, the curvature mode  $\overline{\zeta_q}$  with  $\overline{q} = 2$  in the two cases of a sharp transition, the curvature mode  $\overline{\zeta_q}$  with  $\overline{q} = 2$  in the two cases of a sharp transition, the curvature mode  $\overline{\zeta_q}$  with  $\overline{q} = 2$  in the two cases of a sharp transition, the curvature mode  $\overline{\zeta_q}$  with  $\overline{q} = 2$  in the two cases of a sharp transition, the curvature mode  $\overline{\zeta_q}$  with  $\overline{q} = 2$  in the two cases of a sharp transition. longer USR phase, and its final amplitude is larger with respect to the case of a smooth transit therefore, we expect that the smaller size of the loop correction will be, at least partially, con that finite  $\delta N$  also reduces the amplitude of the tree-level power spectrum. In order to quart we repeat the analysis done in section IV A 2 but now for finite  $\delta N$ . We plot our result in fig.

•  $\eta = -6$  and sudden transition between SR and USR: loops are dangerously large, and UV quadratic divergences should be renormalized [Kristiano-Yokoyama]

• Smooth transition between SR and USR: loops can be placed under control



# Subtle issues: Loop corrections and PBH

$$\left\langle \operatorname{in} \left| \bar{T} e^{-i \int \mathcal{H}_{\operatorname{int}}(\tau') d\tau'} \mathcal{O}(\tau) T e^{i \int \mathcal{H}_{\operatorname{int}}(\tau') d\tau'} \right| \operatorname{in} \right\rangle$$

# **large-** $|\eta|$ **approach** simplifies considerably formulas in the case of a sudden transition

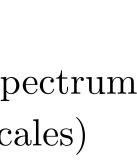
$$\Delta^{\text{loop}}(\kappa) = \Delta^{\text{tree}}(\kappa) \left[1 + L_{\text{UV}}(\kappa) + L_{\text{IR}}(\kappa)\right]$$

$$L_{\rm UV}(\kappa) = -\Delta_0 \frac{\Pi_0 \Lambda_{\rm UV}^2}{1 + \Pi_0} \left(\frac{5}{6} + \frac{3j_1(\kappa)}{6}\right)$$
$$L_{\rm IR}(\kappa) = -\frac{\Delta_0 \Pi_0}{6} \kappa^2 \ln(\mu/\Lambda_{\rm IR})$$

 $\left(\frac{\kappa}{3\kappa} - \kappa\right) \Rightarrow at \ large \ scales \ it \ can \ be \ renormalized$ 

 $\Rightarrow$  due to secular effects of superhorizon modes

 $(\Delta_0 \text{ is the spectrum})$ at large scales)



 $\left\langle \ln \left| \bar{T} e^{-i \int \mathcal{H}_{int}(\tau') d\tau' \right| \right\rangle$ 

large- $|\eta|$ 

 $\Delta^{(\text{loop})}(\kappa) = \Delta_0 - \frac{4\Delta_0 \Pi_0}{3} \left[ \right]$ 

ve



$$\mathcal{O}(\tau) T e^{i \int \mathcal{H}_{int}(\tau') d\tau'} \left| in \right\rangle$$

# **Subtle issues: Loop corrections and PBH**

 $\left\langle \ln \left| \bar{T} e^{-i \int \mathcal{H}_{int}(\tau') d\tau'} \mathcal{O}(\tau) T e^{i \int \mathcal{H}_{int}(\tau') d\tau'} \right| \ln \right\rangle$ 

- ... but recently [Fumagalli] found that we were all missing boundary terms in the interaction Hamiltonian, that once included further reduce the size of loops to  $\kappa^3$ -suppressed corrections.
- ...also [Tada et al] reach a similar conclusions exploiting the effects of boundary terms
- ....on the other hand, [Firouzjahi] repeated the computations independently and found that the inclusion of boundary terms do not help after all to suppress loop corrections...



# Conclusions

- Single-field models of inflation able to strongly enhance fluctuations at small scales can lead to interesting dark matter candidates (PBH, vector DM)
  - To properly understand their consequences, an analytical understanding of their features would be helpful.
- Since the slow-roll parameter  $|\eta|$  is larger than one for a fraction of the inflationary phase, I considered the case  $|\eta|$  large, and promoted  $1/|\eta|$  to an expansion parameter.
- Formulas simplify, and obtain analytical expressions for the two and three point functions in agreement with previous studies and with expectactions.
- It will be interesting to further apply these methods and analytical formulas to study PBH formation, including the effects of non-Gaussianities, and to the analysis of loop corrections in these scenarios.