The Formation of Primordial Black Holes with a Spectator Field

Ioanna Stamou

Université libre de Bruxelles

Based on arXiv: 2310.04174 & 2311.XXXX

Authors: Sebastien Clesse & I.S.

Structure of the presentation

The layout of this presentation:

- **Introduction.**
- **Spectator field for PBHs.**
- **Applications.**
- **Conclusions.**

Introduction

- The detection of Gravitational Waves (GWs) by the merger of binary black holes by LIGO/VIRGO has reignited interest in the study of Primordial Black Holes (PBHs).
- PBHs can explain a fraction of Dark Matter (DM) in the Universe.
- The generation of PBHs can be explained in the framework of inflation. Specifically, a substantial amplification in the scalar power spectrum can provide an explanation to PBHs.
- A drawback of models from inflation is the fine-tuning in order to obtain such amplification in power spectrum.
- Mechanism of a light quantum stochastic spectator scalar field during inflation.

Structure of the presentation

The layout of this presentation:

- **Introduction.**
- **Spectator field for PBHs.**
- **Applications.**
- **Conclusions.**

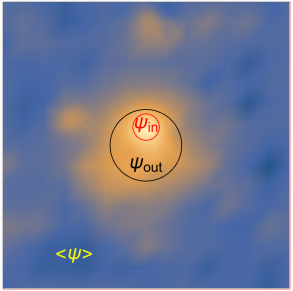
IDEA OF SPECTATOR FIELD:

- A light stochastic spectator scalar field during inflation acquires different mean values in different current Hubble patches.
- There are huge number of these patches, so these necessarily exist some in which these fluctuations leave the horizon at values required to form PBHs.
[B. Carr, S. Clesse, J.Garcia-Bellido (Mon.Not.Roy.Astron.Soc. 501)].

Schematic representation of the spectator field fluctuations: [S. Clesse, I.S.]

- Rare fluctuation leading at ψ_{out} to exit the horizon.
- Smaller fluctuation at ψ_{in} becomes super-Hubble at time $N_{\rm inf}$.

$$\delta\psi_{\rm in} \equiv \psi_{\rm in}(x, N_{\rm inf}) - \psi_{\rm out}(x, N_{\rm inf} - 1)$$
$$\delta\psi_{\rm out} \equiv \psi_{\rm out}(x, N_{\rm inf} - 1) - \langle\psi\rangle$$



During inflation

- The inflaton field drives the inflation.
- The spectator field remains frozen.

The slow-roll parameters:

$$\epsilon_1 \equiv -\frac{\mathrm{d}\ln \mathrm{H}}{\mathrm{d}N}, \quad \epsilon_2 \equiv \frac{\mathrm{d}\ln \epsilon_1}{\mathrm{d}N}, \quad \epsilon_3 \equiv \frac{\mathrm{d}\ln |\epsilon_2|}{\mathrm{d}N}. \tag{1}$$

The amplitude, $\mathrm{A_s}$, in slow-roll approximation is given:

$$A_\mathrm{s} = 2.1 \times 10^{-9} \simeq \frac{H_*^2}{8\pi^2 \epsilon_{1*} M_\mathrm{P}^2}. \tag{2}$$

The spectral index, $\mathrm{n_s}$, and the tensor-to-scalar ratio, r, are given:

$$\begin{aligned} n_\mathrm{s} =& 0.9649 \pm 0.0042 \simeq 1 - 2\epsilon_{1*} - \epsilon_{2*} \\ r =& 16\epsilon_{1*} < 0.07 \end{aligned} \tag{3}$$

In our study we consider:

- Inflationary models for the pivot scale $k_* = 0.05\,\mathrm{Mpc}^{-1}$:

Model	ϵ_{1*}	ϵ_{2*}	$\mathrm{H}_*(\mathrm{M_P})$	r
1	0.00507	0.0207	2.9×10^{-5}	0.08115
2	0.00020	0.0351	5.8×10^{-6}	0.00325

- The quantum fluctuations of ψ produced during one e-fold in a Hubble-sized region

$$\langle \delta\psi^2_{\rm in}(N_{\rm inf})\rangle \simeq \frac{H_*^2}{4\pi^2} \exp\left\{-2\frac{\epsilon_{1*}}{\epsilon_{2*}}\left[e^{\epsilon_{2*}(N_{\rm inf}-N_*)}-1\right]\right\},$$

and

$$\langle \delta\psi^2_{\rm out}(N_{\rm inf})\rangle \simeq \frac{H_*^2}{8\pi^2\epsilon_{1*}}\left[1-\exp(-2\epsilon_{1*}(N_{\rm inf}-N_*))\right] \simeq \frac{H_*^2(N_{\rm inf}-N_*)}{4\pi^2}\ .$$

➤ **After inflation**: The spectator field starts to dominate the Universe.

The equations of the spectator field ψ (cosmic time):

$$\begin{aligned} \ddot{\psi} + 3H\dot{\psi} + \frac{\partial V}{\partial \psi} = 0 \ , \\ \dot{N} = H = \sqrt{\frac{\rho}{3M_P^2}}, \end{aligned} \tag{4}$$

or equivalently (e-fold time):

$$\psi'' + \psi' \frac{1}{2\rho} \left(-\kappa \rho_{\mathrm{m,r}} e^{-\kappa N} + \psi' \frac{\mathrm{d}V}{\mathrm{d}\psi} \right) + 3\psi' + \frac{3M_{\mathrm{P}}^2}{\rho_{\mathrm{m,r}}} \frac{\mathrm{d}V}{\mathrm{d}\psi} = 0 \ , \tag{5}$$

where $\rho = \rho_{\mathrm{m,r}} e^{-\kappa N} + V(\psi)$.

• In some regions of the Universe, if the spectator field rests in a particular flat part of its potential, an extra expansion occurs. This triggers a curvature fluctuation, leading to the formation of PBHs. These are formed later when these fluctuations re-enter the Hubble radius.

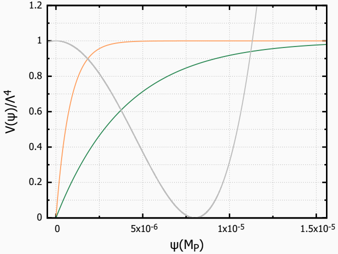
Two cases of potential are studied:

1.

$$V(\psi) = \Lambda^4 \left(1 - \exp\left[-\frac{\psi}{M}\right]\right)$$

- An extra expansion can occur.
- PBHs formation is possible.
- Model of spectator field:

Model	M($M_{\rm P}$)
1	4×10^{-6}
2	8×10^{-7}

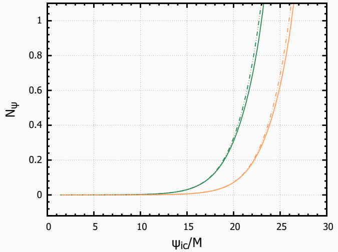


2.

$$V(\psi) = \Lambda^4 \left(1 - \frac{\psi^2}{M^2}\right)^2$$

- An extra expansion cannot occur.
- No PBHs formation.

$M = 8 \times 10^{-6} M_{\rm P}$



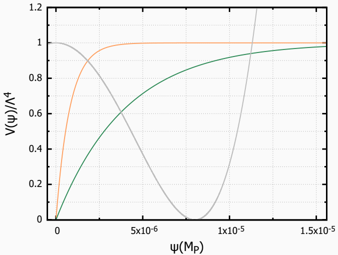
Two cases of potential are studied:

1.

$$V(\psi) = \Lambda^4 \left(1 - \exp\left[-\frac{\psi}{M}\right]\right)$$

- An extra expansion can occur.
- PBHs formation is possible.
- Model of spectator field:

Model	$\mathrm{M}(M_{\mathrm{P}})$
1	4×10^{-6}
2	8×10^{-7}

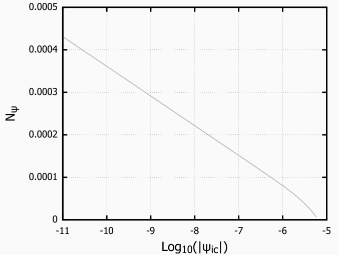


2.

$$V(\psi) = \Lambda^4 \left(1 - \frac{\psi^2}{M^2}\right)^2$$

- An extra expansion cannot occur.
- No PBHs formation.

$M = 8 \times 10^{-6} M_{\mathrm{P}}$



Spectator field for PBHs

➤ Probability distribution

- $\delta\mathrm{N}$ formalism: $\zeta(x,t) = \delta N_{\mathrm{i}}^{\mathrm{f}} \equiv N(x,t) - \bar{N}(t)$
- The probability distribution as a function of ζ:

$$P(\zeta_{\mathrm{in}} - \zeta_{\mathrm{out}}) = \int \mathrm{d}\delta\psi_{\mathrm{out}} P(\delta\psi_{\mathrm{in}}) P(\delta\psi_{\mathrm{out}}) \left. \frac{\mathrm{d}\psi}{\mathrm{d}N} \right|_{\psi_{\mathrm{out}}}, \tag{6}$$

where:

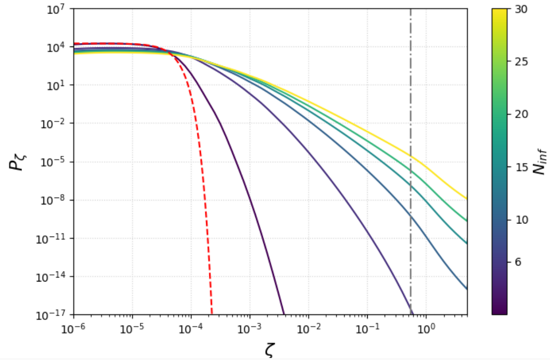
$$P(\delta\psi_{\mathrm{in}}) = \frac{1}{\sqrt{2\pi\langle\delta\psi_{\mathrm{in}}^2(N_{\mathrm{inf}})\rangle}} \exp\left[\frac{-\delta\psi_{\mathrm{in}}^2(N_{\mathrm{inf}})}{2\langle\delta\psi_{\mathrm{in}}^2(N_{\mathrm{inf}})\rangle}\right] \tag{7}$$

$$P(\delta\psi_{\mathrm{out}}) = \frac{1}{\sqrt{2\pi\langle\delta\psi_{\mathrm{out}}^2(N_{\mathrm{inf}}-1)\rangle}} \exp\left[\frac{-\delta\psi_{\mathrm{out}}^2(N_{\mathrm{inf}}-1)}{2\langle\delta\psi_{\mathrm{out}}^2(N_{\mathrm{inf}}-1)\rangle}\right].$$

Model	Inflaton $[\mathrm{H}_*(\mathrm{M_P}), \mathrm{r}]$	Spectator $[\mathrm{M}(M_{\mathrm{P}})]$
1	2.9×10^{-5}, 0.08115	4×10^{-6}
2	5.8×10^{-6}, 0.00325	8×10^{-7}

Spectator field for PBHs

➤ The probability distribution as a function of ζ:



We remark:

- The probability distribution P as a function of ζ as N_{inf} expands.
- The probability distribution should not spoil the power spectrum at CMB scales (red line corresponds to the case of a Gaussian distribution on CMB scales)

Spectator field for PBHs

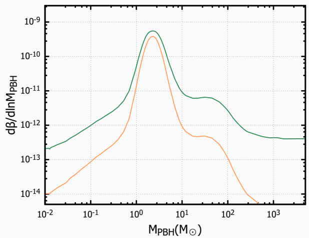
PBH formation

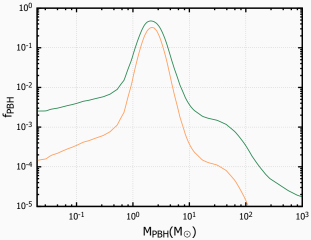
The mass fraction of PBH, $\beta(M_{\rm PBH})$, is connected to the probability distribution P:

$$\beta(M_{\rm PBH}) \equiv \frac{\mathrm{d}\rho}{\mathrm{d}\ln M_{\rm PBH}} = \int_{\zeta_{\rm cr}}^{\infty} P(\zeta)\mathrm{d}\zeta\,. \tag{8}$$

The fractional abundance of PBHs, $f_{\rm PBH}(M_{\rm PBH})$, is given a:

$$f_{\rm PBH}(M_{\rm PBH}) \approx 2.4\beta(M_{\rm PBH})\left(\frac{2.8\times10^{17}M_{\odot}}{M_{\rm PBH}}\right)^{1/2}\,. \tag{9}$$





Spectator field for PBHs

Advantages of this mechanism:

- PBHs formation without fine-tuning: PBHs form with a spectator field avoiding the need of fine-tuning

$$\epsilon_{\mathcal{O}} \equiv \frac{\mathrm{d}\log \mathcal{O}}{\mathrm{d}\log p} \sim \mathcal{O}(1)\ . \tag{10}$$

- Consistency with CMB data: Our predictions for the power spectrum evaluated from the fluctuation of spectator field respect the observable constraints at CMB scales

$$\mathcal{P}_\zeta^\psi(k) = \frac{H_*^2(k)}{4\pi^2}\left(\left.\frac{\mathrm{d}N}{\mathrm{d}\psi}\right|_{\langle N\rangle}\right)^2 \ll A_\mathrm{s}\ . \tag{11}$$

- General framework: This mechanism can be applied to a broad range of models, offering a flexible tool for cosmological exploration.

Structure of the presentation

The layout of this presentation:

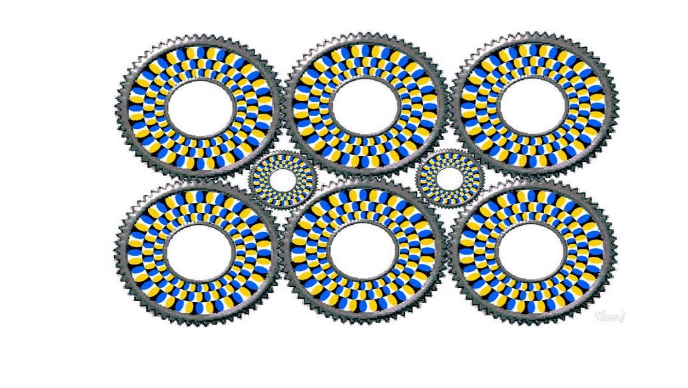
- **Introduction.**
- **Spectator field for PBHs.**
- **Applications.**
- **Conclusions.**

Applications

How can this mechanism be applied to models?

We studied two applications:

- Higgs Standard Model
- Supergravity Models



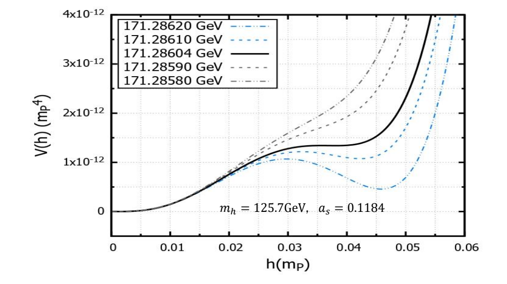
➤ Higgs Standard Model

The effective potential of the SM Higgs is given by

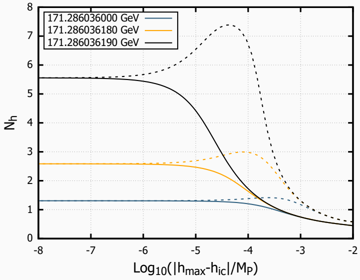
$$V(h) = \frac{\lambda(h)}{4} h^4.$$

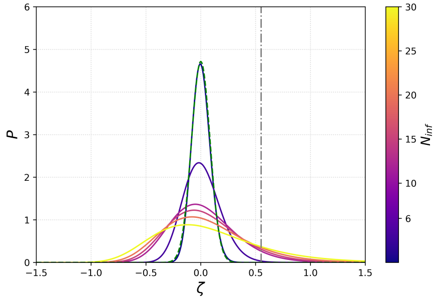
and the self-coupling $\lambda(h)$ is determined by the β function,

$$\beta_\lambda = \frac{d\lambda}{d \ln h}.$$



• If the BEH field lies exactly at the transition between metastability and stability, the potential exhibits an inflection point.





☑ The SM Higgs as spectator field leads to significant PBHs abundances without the need of extra parameters [S. Clesse, I.S.].

☐ The prediction for the power spectrum at CMB scales, which is evaluated from the mean value of the spectator field can be preserved?

The Palatini Formulation

The action in Einstein frame is given:

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left[\tilde{R} + K(h) g^{\mu\nu} \partial_\mu h \partial_\nu h + 2\Omega^{-4}(h) V(h) \right] \tag{12}$$

where

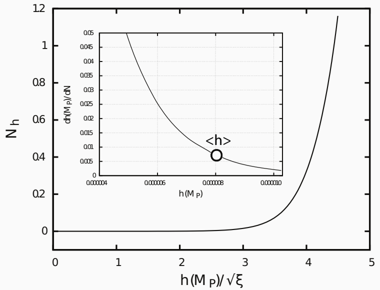
$$K(h) = \frac{1}{\Omega(h)^2}, \quad \Omega^2 = 1 + \xi h^2 \ , \tag{13}$$

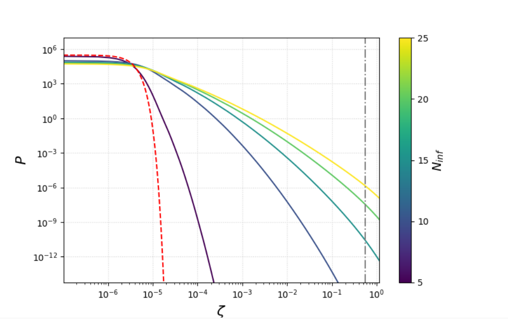
and ξ is the non-minimal coupling.

The potential with fixed the non-canonical kinetic term:

$$U(h) = \frac{1}{4\xi^2} \lambda \tanh^4 \left(\sqrt{\xi} h \right) . \tag{14}$$

The Palatini Formulation





In the framework of Palatini:

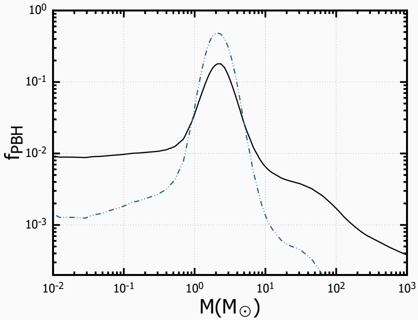
☑ The Higgs as spectator field leads to an extra expansion of the Universe.

☑ The probability distribution does not spoil the CMB power spectrum.

PBH formation

The Higgs as spectator field:

- leads to significant PBHs abundances without the need of extra parameters.
- fulfill both the condition on CMB scales and get PBH formation on small scales within the Palatini formulation.



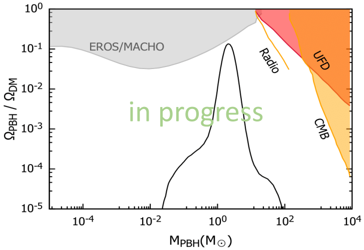
➤ No-scale supergravity

$$K = -3\ln\left(T + \bar{T} - \frac{\phi\bar{\phi}}{3}\right)$$

$$W(\phi, T)$$

• Starobinsky scalar potential is derived assuming that one field plays the role of inflaton and the other is consider a modulo field.

☐ Can this model lead to a significant fraction of PBHs to DM?
☐ Is the prediction for the power spectrum at CMB scales preserved?



Structure of the presentation

The layout of this presentation:

- **Introduction.**
- **Spectator field for PBHs.**
- **Applications.**
- **Conclusions.**

Conclusions

➤ In this presentation:

- We provide a novel mechanism of PBHs formation with the following advantages:
  1) avoiding the need of fine-tuning,
  2) consistency with constraints of CMB and explain the PBHs,
  3) applicable to other models.

- We present two applications of this mechanism:
  1) Higgs Standard Model
  2) Supergravity Model
  and we show that this mechanism can explain the production of PBHs.

- We can apply this mechanism in the other high energy frameworks.

Thank you!