ABSENCE OF ONE-LOOP EFFECTS
from small scales to large scales in non-slow-roll dynamics

JACOPO FUMAGALLI (ICCUB)

PARIS WORKSHOP on PBH & GWs, 27th November 2023

arXiv: 2305.19263, ...

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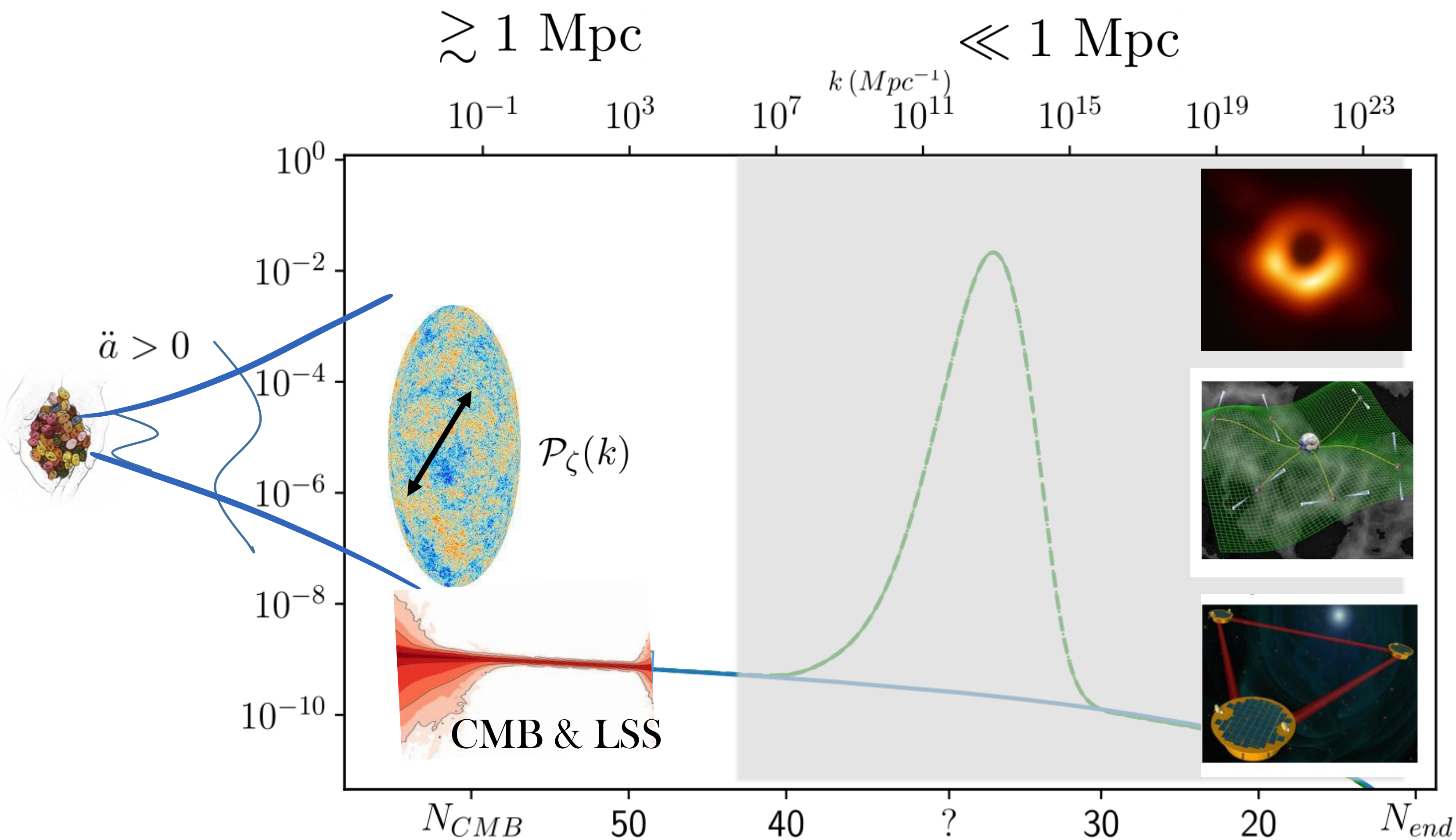
EXCELENCIA MARÍA DE MAEZTU 2020-2023

INFLATION AT SMALL SCALES



Constrained at large scales

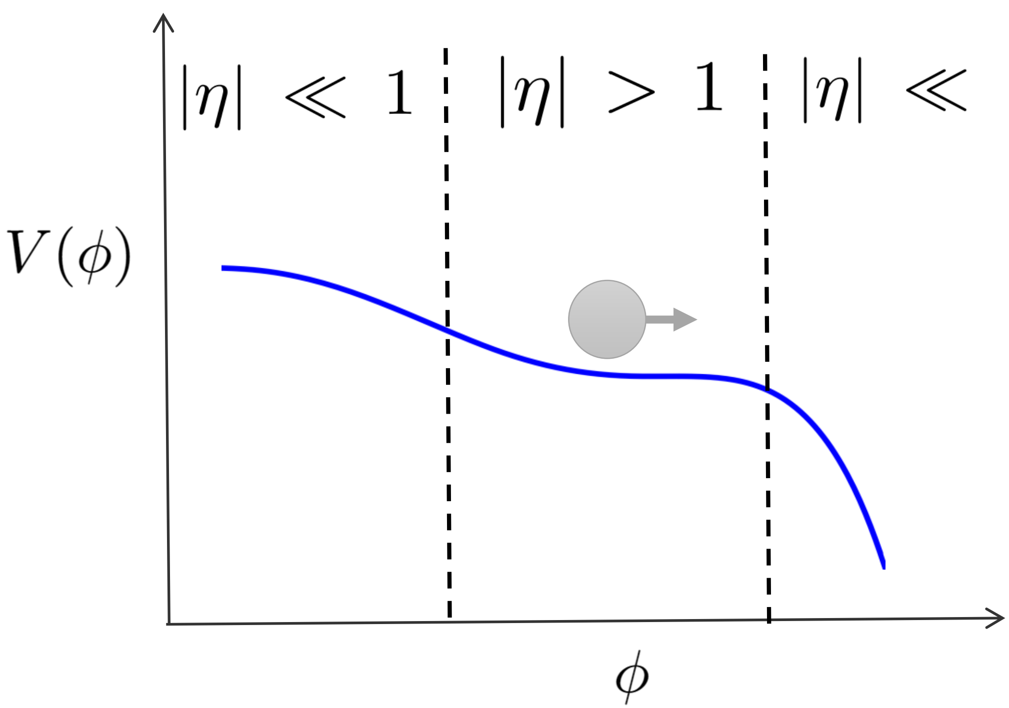
Rich phenomenology at small scales

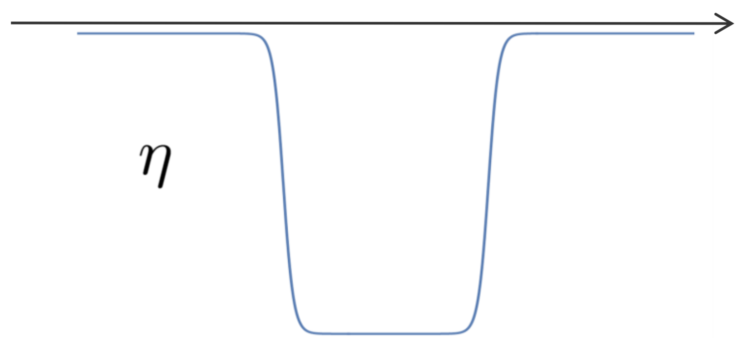
NON-SLOW ROLL PHASE

Standard way to enhance the power spectrum $|\eta| > 1$

$$\epsilon \equiv -\dot{H}/H^2 , \quad \eta \equiv \frac{d \ln \epsilon}{dN},$$

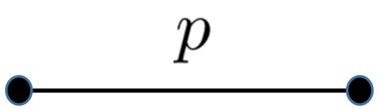
e.g. Ultra-slow-roll $\eta \simeq -6$

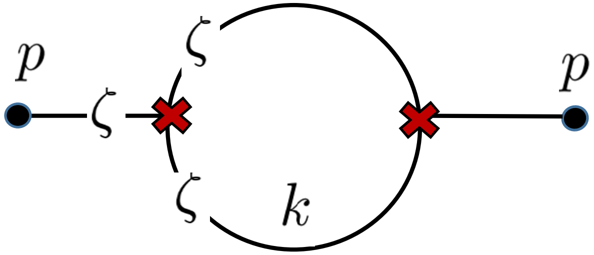


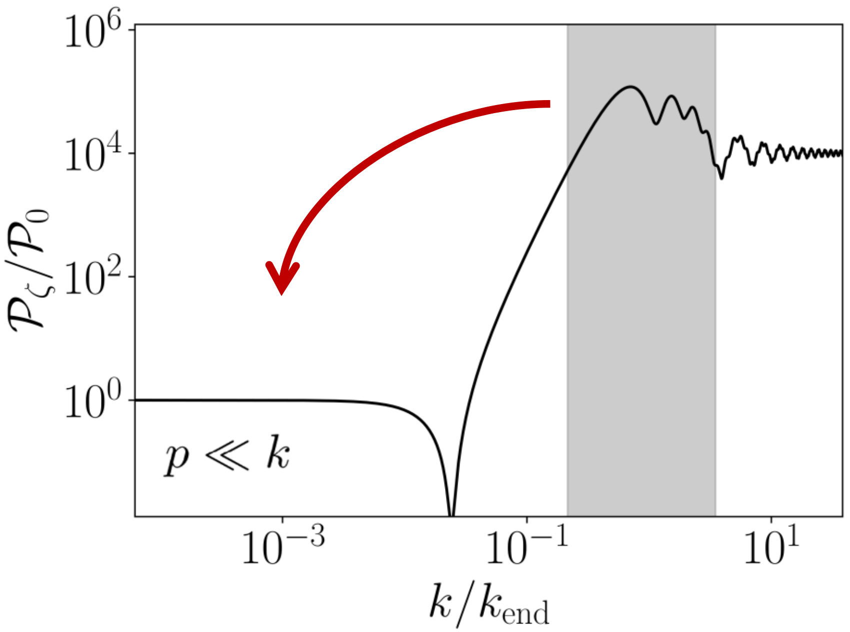


ONE-LOOP-CORRECTIONS $|\eta| > 1$

$$\mathcal{P}_\zeta = \mathcal{P}_\zeta^{\text{tree}} + \mathcal{P}_\zeta^{1-\text{loop}} + \dots,$$







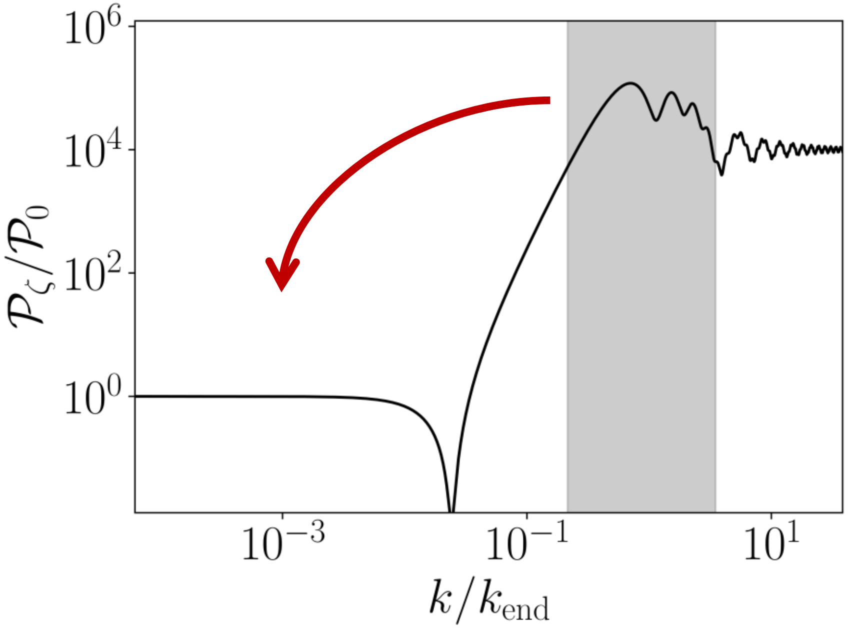
Could small scales perturbations lead to an effect on large scales?

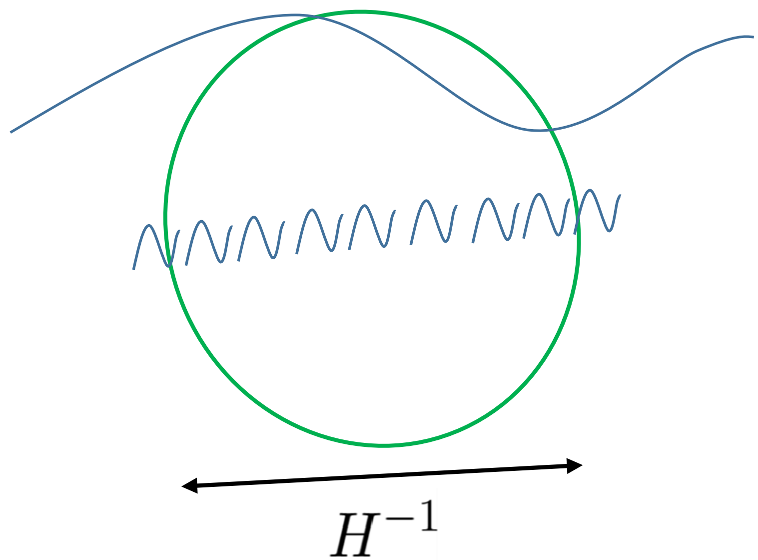
J. Kristiano and J. Yokoyama '22

ONE-LOOP CORRECTIONS

$$\mathcal{P}_\zeta^{1-\mathrm{loop}}(p) = c\,\mathcal{P}_\zeta^{\mathrm{tree}}(p)\int d\ln k\;\mathcal{P}_\zeta^{\mathrm{tree}}(k) + O\left(\frac{p^3}{k^3}\right), \qquad p \ll k$$

J. Kristiano and J. Yokoama '22, A. Riotto '23, H. Firouzjahi '23, A. Riotto and H. Firouzjahi '23, G. Franciolini et al. '23, G. Tasinato '23, S. Choudhury et al. '23 ......





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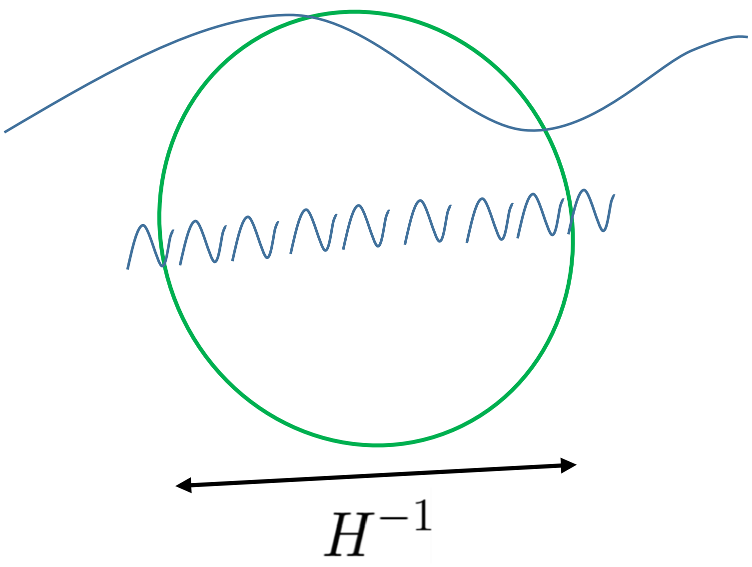
IMPLICATIONS of $c \neq 0$

- Small scales / Large scales effect which is scales independent ?



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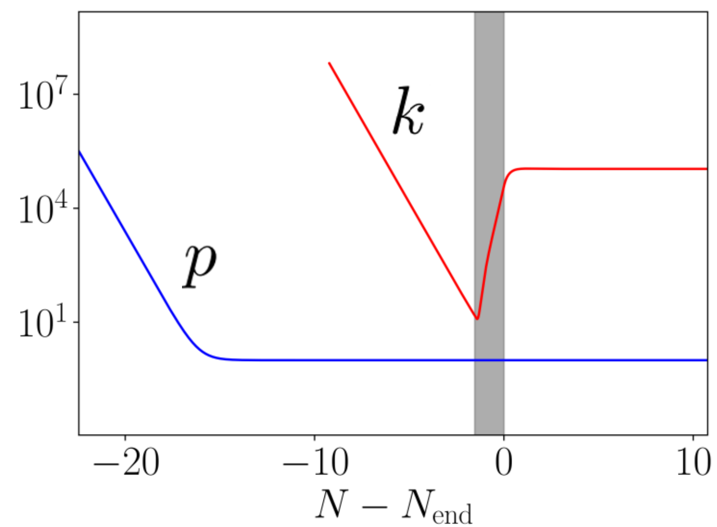

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J. Fumagalli arXiv 2305.19263

$c = 0$

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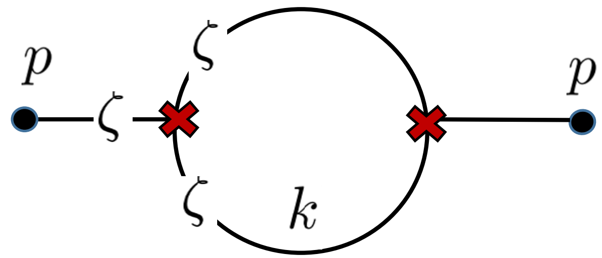
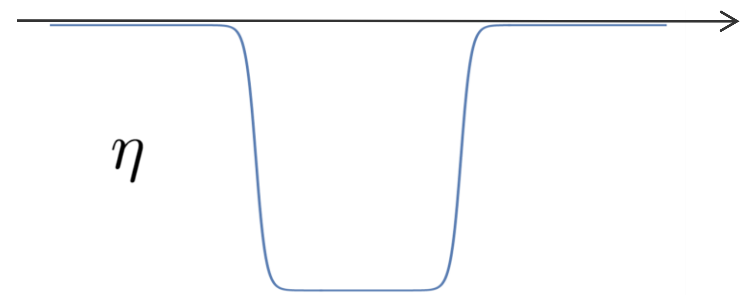
CUBIC ACTION

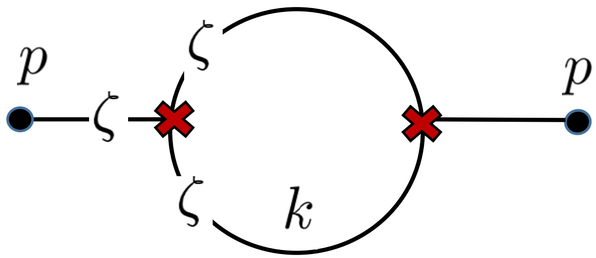


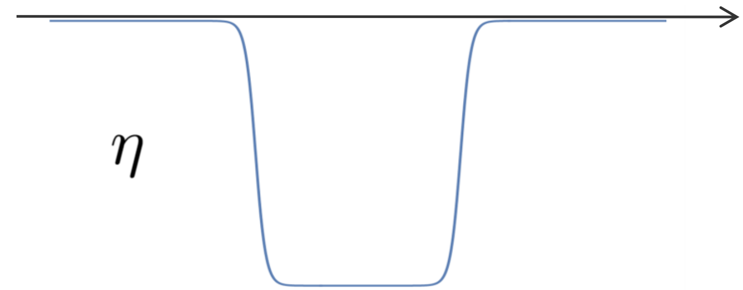


Maldacena´02 (selected terms from jump in eta) + boundary terms

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2 \frac{a^3 \epsilon}{2} \dot{\eta} \underline{\zeta^2} \dot{\zeta} + \dots$$

CUBIC ACTION & BOUNDARY TERMS





Maldacena´02 (selected terms from jump in eta) + boundary terms

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2 \frac{a^3\epsilon}{2}\dot{\eta}\zeta^2\dot{\zeta} + M_{\rm Pl}^2 \frac{d}{dt}\left[-\frac{a^3\epsilon\eta}{2}\zeta^2\dot{\zeta} + \ldots\right] + f(\zeta)\frac{\delta\mathcal{L}^{(2)}}{\delta\zeta} + \ldots$$

- Field redefinition $\zeta \to \zeta_n + f(\zeta_n)$, and then link correlators of the two variables

  Maldacena '02,...

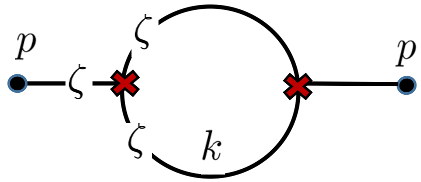
- Or work in terms of the original variable but crucially including boundary terms

F. Arroja and T. Tanaka '11,
C. Burrage, R.H. Ribeiro and D. Seery '11,...
S. Garcia-Saenz, L. Pinol and S. Renaux-Petel '20

ORIGIN OF SCALE INDEPENDENT CORRECTIONS

J. Fumagalli arXiv 2305.19263

$$\langle\hat{\zeta}_{\boldsymbol{p}}(\tau)\hat{\zeta}_{\boldsymbol{p}'}(\tau)\rangle_{1-\text{loop}} = -\int_{\tau_{\text{in}}}^{\tau} d\tau_1 \int_{\tau_{\text{in}}}^{\tau_1} d\tau_2 \langle[\mathcal{H}^{(3)}(\tau_2), [\mathcal{H}^{(3)}(\tau_1), \hat{\zeta}_{\boldsymbol{p}}(\tau)\hat{\zeta}_{\boldsymbol{p}'}(\tau)]]\rangle,$$



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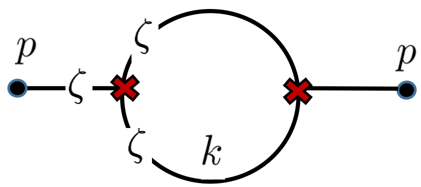

$$\mathcal{H}^{(3)} = M_{\text{Pl}}^2 \int d^3x \left(-\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' \right)$$

$$\left[\hat{\zeta}_{\boldsymbol{k}_{1,1}}\hat{\zeta}_{\boldsymbol{k}_{1,2}}\hat{\zeta}'_{\boldsymbol{k}_{1,3}}|_{\tau_2}, \left[\hat{\zeta}_{\boldsymbol{k}_{2,1}}\hat{\zeta}_{\boldsymbol{k}_{2,2}}\hat{\zeta}'_{\boldsymbol{k}_{2,3}}|_{\tau_1}, \hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}|_{\tau}\right]\right] \equiv \left[\hat{\zeta}_2^2\hat{\zeta}_2', \left[\hat{\zeta}_1^2\hat{\zeta}_1', \hat{\zeta}_{\boldsymbol{p}}\zeta_{\boldsymbol{p}'}\right]\right]$$

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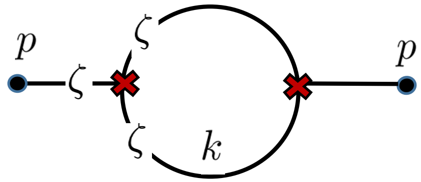

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$$\mathcal{P}_{\zeta}^{1-\text{loop}}(p) \propto p^3\langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{p}'}\rangle \propto \underline{p^3} \int d\boldsymbol{K} \underline{\left[\hat{\zeta}_1', \hat{\zeta}_{\boldsymbol{p}}\right]} \cdot \left(\left[\hat{\zeta}_2', \hat{\zeta}_1\right] \underline{\hat{\zeta}_2\hat{\zeta}_{\boldsymbol{p}}}\hat{\zeta}_2\hat{\zeta}_1 + \ldots\right)$$

— (blue) $[\hat{\zeta}_{\boldsymbol{k}}(\tau_i), \hat{\zeta}'_{\boldsymbol{p}}(\tau)] \simeq (2\pi)^3\delta(\boldsymbol{p}+\boldsymbol{k})\dfrac{i}{2a^2M_{\text{Pl}}^2\epsilon}$ independent on the long scale

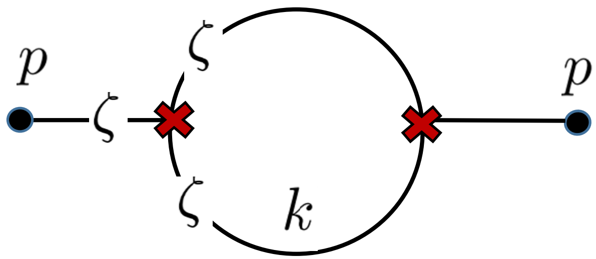
— (red) $\mathcal{P}_{\zeta}^{\text{tree}}(p)$

$\Rightarrow \propto \mathcal{P}^{\text{tree}}(p)\int d\ln k\, C(k)$

EXACT CANCELLATION 1

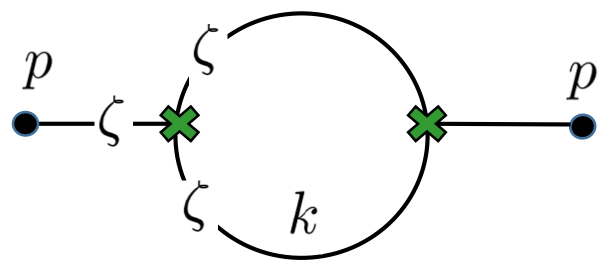
J. Fumagalli arXiv 2305.19263

$$\mathcal{H}^{(3)} = M_{\rm Pl}^2 \int d^3x \left(-\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right]\right)$$



$$\cong \frac{|\Delta\eta|^2}{4}\mathcal{P}_\zeta^{\rm tree}(p)\int \frac{d\boldsymbol{k}}{(2\pi)^3}|\zeta_k(\tau_{\rm e})|^2$$

$+$



$$\cong -\frac{|\Delta\eta|^2}{2}\frac{p^3}{2\pi^2}\int\frac{d\boldsymbol{k}}{(2\pi)^3}\int d\tau_1\left(|\zeta_p|^2(|\zeta_k|^2)' - |\zeta_p|^2(|\zeta_k|^2)' - |\zeta_k|^2(|\zeta_{|\boldsymbol{p}-\boldsymbol{k}|}|^2)'\right)$$

$$\cong 0.$$

ORIGIN OF SCALE INDEPENDENT CORRECTIONS #2

$$\mathcal{H}^{(3)} = M_{\rm Pl}^2 \int d^3x \left(-\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right]\right)$$

- H. Firouzjahi 2311.04080 : Using equivalent form of the Hamiltonian above still finds corrections to large scales due to terms I missed.

$$\mathcal{P}_\zeta^{1-\rm loop}(p) \propto \mathcal{P}^{\rm tree}(p) \int d\boldsymbol{K} \left[\hat{\zeta}'_{\boldsymbol{k}}, \hat{\zeta}_{\boldsymbol{p}}\right] \cdot (\dots)$$
$$+ \mathcal{P}^{\rm tree}(p) \int d\boldsymbol{K} \underline{\left[\hat{\zeta}_{\boldsymbol{k}}, \hat{\zeta}_{\boldsymbol{p}}\right]} \cdot (\dots)$$

Terms previously neglected and appearing once including boundary terms

— $$[\hat{\zeta}_{\boldsymbol{k}}(\tau_i), \hat{\zeta}_{\boldsymbol{p}}(\tau_0)] \simeq (2\pi)^3\delta(\boldsymbol{p}+\boldsymbol{k})\frac{i\ \tau_i}{6a^2(\tau_i)\epsilon(\tau_i)M_{\rm Pl}^2}(1+\dots)$$

also independent on the long scale (NB: statement valid also in Slow roll)

→ $$\propto \mathcal{P}^{\rm tree}(p)\int d\ln k\, C(k)$$

EXACT CANCELLATION #2

Extra terms cancel as the result of Maldacena consistency relation

Y. Tada, T. Terada and J. Tokuda 2308.04732

$$\mathcal{H}^{(3)} = M_{\rm Pl}^2 \int d^3x \left(-\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right] + \frac{d}{d\tau}\left[\frac{a\epsilon}{H}\zeta(\zeta')^2\right]\right)$$

Bispectrum

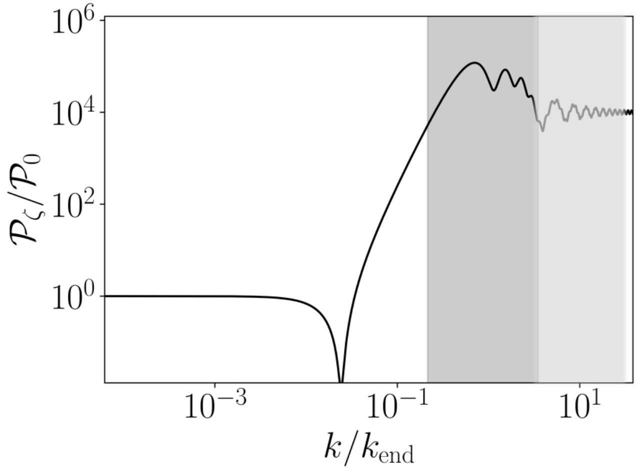
$$\mathcal{P}_\zeta^{1-\rm loop}(p) \propto \int d\tau' \int d\boldsymbol{K} \left[\hat{\zeta}_{\boldsymbol{k}}, \hat{\zeta}_{\boldsymbol{p}}\right] \cdot \mathcal{B}(\boldsymbol{p}, \boldsymbol{k}, -\boldsymbol{k}, \tau')$$

Maldacena consistency relation

$$\propto \mathcal{P}^{\rm tree}(p) \int d\tau' \int d\ln k \frac{d\mathcal{P}_\zeta}{d\ln k} \simeq \mathcal{P}^{\rm tree}(p)\mathcal{P}_\zeta(k_{max})$$

for $k_{max} \to \infty$

$$\longrightarrow 0$$

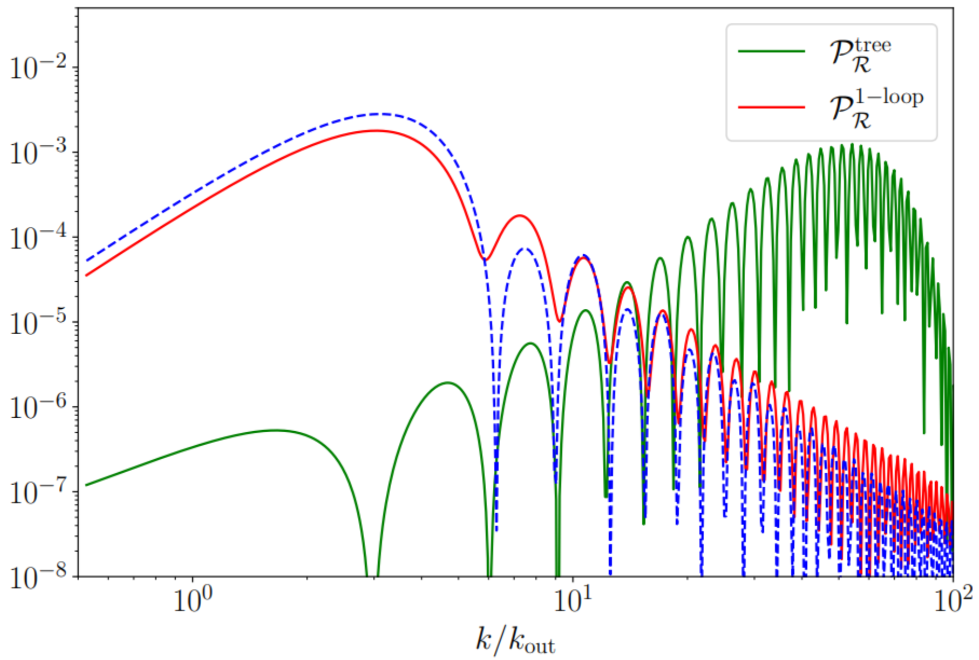


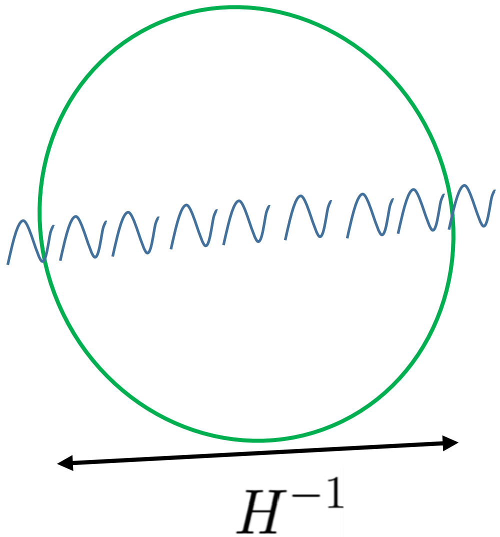
Once (as standard) sub-horizon modes are projected into the vacuum of the free theory

INFRARED RESCATTERING

See arXiv 2307.08358

JF, S. Bhattacharya, M. Peloso, S. Renaux-Petel, L. T. Witkowski





…. (near) infrared enhancement is possible but physical link associated to the ratio of the UV/IR scales

See also Sebastien's talk

CONCLUSIONS

In non-slow-roll dynamics:

$$\mathcal{P}_\zeta^{1-\text{loop}}(p) = c\, \mathcal{P}_\zeta^{\text{tree}}(p) \int d\ln k\, \mathcal{P}_\zeta^{\text{tree}}(k) + O\left(\frac{p^3}{k^3}\right), \qquad p \ll k$$

More generally $\propto \mathcal{P}^{\text{tree}}(p) \int d\ln k\, C(k)$

Future directions:

- Extending the exact cancellation including the quartic interactions
- One-loop corrections at all scales

BACK UP

$$S^{(3)}_{\text{bulk}} = \int \mathrm{d}^4x \left[a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}(\partial\zeta)(\partial\chi) + \frac{a^3\epsilon}{2}\dot{\eta}\zeta^2\dot{\zeta} + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2\right],$$

$$S^{(3)}_{\text{EoM}} = \int \mathrm{d}^4x\, 2f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1,$$

$$S^{(3)}_{\text{B}} = \int \mathrm{d}^4x \frac{\mathrm{d}}{\mathrm{d}t}\left[-9a^3H\zeta^3 + \frac{a}{H}\zeta(\partial\zeta)^2 - \frac{1}{4aH^3}(\partial\zeta)^2\partial^2\zeta - \frac{a\epsilon}{H}\zeta(\partial\zeta)^2 - \frac{a^3\epsilon}{H}\zeta\dot{\zeta}^2 + \frac{1}{2aH^2}\zeta\left(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi\right) - \frac{a\eta}{2}\zeta^2\partial^2\chi - \frac{1}{2aH}\zeta\left(\partial_i\partial_j\chi\partial_i\partial_j\chi - \partial^2\chi\partial^2\chi\right)\right],$$

with

$$\chi = a^2\epsilon\partial^{-2}\dot{\zeta}, \qquad \left.\frac{\delta L}{\delta\zeta}\right|_1 = a\left(\partial^2\dot{\chi} + H\partial^2\chi - \epsilon\partial^2\zeta\right),$$

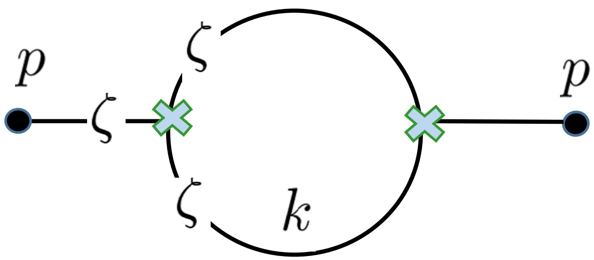
$$f(\zeta) = \frac{\eta}{4}\zeta^2 + \frac{1}{H}\zeta\dot{\zeta} + \frac{1}{4a^2H^2}\left[-(\partial\zeta)^2 + \partial^{-2}(\partial_i\partial_j(\partial_i\zeta\partial_j\zeta))\right] + \frac{1}{2a^2H}\left[(\partial\zeta)(\partial\chi) - \partial^{-2}(\partial_i\partial_j(\partial_i\zeta\partial_j\chi))\right].$$

$$\mathcal{L}^{(2)}(\zeta) = \mathcal{L}^{(2)}(\zeta_n) + f(\zeta_n)\frac{\delta\mathcal{L}^{(2)}}{\delta\zeta} + \frac{d}{dt}\left(2M_{\text{Pl}}^2\epsilon a^3\dot{\zeta}_n f(\zeta_n)\right)$$

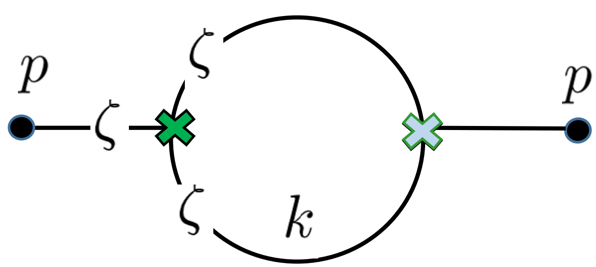
EXACT CANCELLATION 2° Method

J. Fumagalli arXiv 2305.19263

$$\mathcal{L}^{(3)} = -M_{\rm Pl}^2 \frac{1}{2} a\epsilon\eta\,\zeta^2\partial^2\zeta - M_{\rm Pl}^2 a^3\epsilon\eta\,\dot{\zeta}^2\zeta \qquad \text{(Equivalent Lagrangian)}$$



$$\cong -4M_{\rm Pl}^2|\Delta\eta|^2\mathcal{P}_\zeta^{\rm tree}(p)\int_{\tau_{\rm s}}^{\tau_{\rm e}} d\tau_1 \int_{\tau_s}^{\tau_1} d\tau_2 a^2(\tau_2)\epsilon(\tau_2)\int\frac{d\boldsymbol{k}}{(2\pi)^3}\left(k^2+|\boldsymbol{p}-\boldsymbol{k}|^2\right)$$
$$\times\ \text{Im}\left(\zeta_k^*(\tau_1)\zeta'^*_{|\boldsymbol{p}-\boldsymbol{k}|}(\tau_1)\,\zeta_k(\tau_2)\zeta_{|\boldsymbol{p}-\boldsymbol{k}|}(\tau_2)\right).$$

$+$  $\cong 0.$

INFRARED RESCATTERING

BUNCH DAVIES → EXCITED STATE

$|0\rangle_{\rm in}$ → $|0\rangle_{\rm out}$

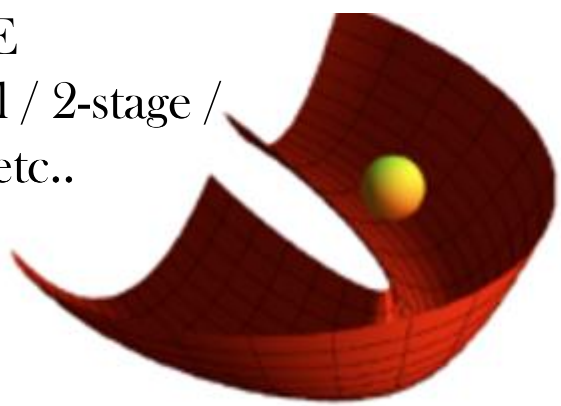
IN Region — Standard (quasi) dS evolution

OUT Region — Standard (quasi) dS evolution

τ_f → τ

SHARP FEATURE

Step in the potential / 2-stage / turn in field-space etc..



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