Einstein vs. Hawking: gravitational waves from evaporating Primordial Black Holes binaries

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Based on arXiv:2306.09069
and Eur.Phys.J.C 83 (2023) 11, 1025

Theoretical setup

- Binaries of Schwarzschild Black Holes of mass $m(t)$, orbital separation $R(t)$
- The two BHs are submitted to Hawking evaporation leading to a mass loss rate

$$\dot{m} = -\frac{\alpha_{\rm H}}{m^2}, \quad \alpha_{\rm H} \approx \mathrm{cste}, \quad \dot{m} < 0$$

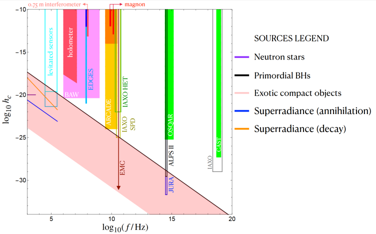
Competitive effects ?

- GWs emission leads to inspiral dynamics
- Mass loss leads to outspiral dynamics

$$\dot{R} = -3\frac{\dot{m}}{m}R$$

Hypothesis The system can be treated as Keplerian. Even at the level of a simple Newtonian analysis, there are subtle competitive effects !

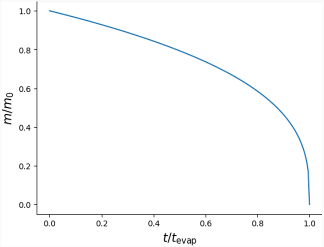
Phenomenological interest



From Aggarwal et al. (2021)

- For BHs binaries $f_{\text{ISCO}} \simeq 2200\,\text{Hz}\,\frac{M_\odot}{m_0}$
- Light PBHs are cosmological candidates for sources of high-frequency gravitational waves

- Weak mass $\implies$ Hawking evaporation comes into play $m(t) = m_0\left(1 - \frac{t}{t_{\text{ev}}}\right)^{\frac{1}{3}}$ with $t_{\text{ev}} \equiv \frac{m_0^3}{3\alpha_{\text{H}}}$
- Explosive process ? Is it true that all the physics occurs for $t \sim t_{\text{ev}}$?



Competitive effects between gravitational radiation and mass loss

Coupling mass loss to GWs emission

Back-reaction of mass loss and GWs emission on orbital energy

$$-\frac{\mathrm{d}E_{\mathrm{orbit}}}{\mathrm{d}t} = P_{\mathrm{ml}} + P_{\mathrm{GW}}$$

with

$$P_{\mathrm{ml}} = \frac{5}{2}\frac{G\dot{m}m}{R} \quad \text{and} \quad P_{\mathrm{GW}}(t) = \frac{64}{5}\frac{G^4}{c^5}\frac{m^5(t)}{R^5}$$

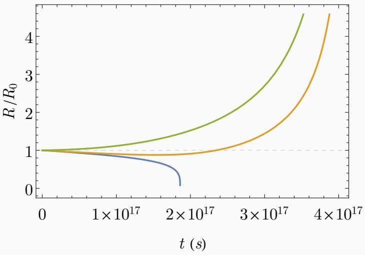
Bernoulli differential equation for the orbital separation

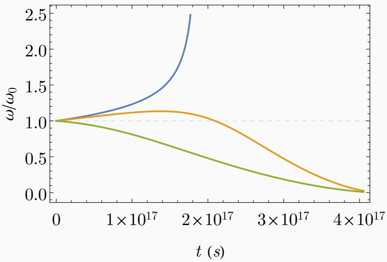
$$\dot{R} = \underbrace{-\frac{128}{5}\frac{G^3}{c^5}\frac{m^3}{R^3}}_{\mathrm{GW}} \underbrace{-3\frac{\dot{m}}{m}R}_{\mathrm{Evaporation}}$$

Two typical times t_{ev} and $t_{\mathrm{cc}} \equiv \frac{5}{512}\frac{c^5 R_0^4}{G^3 m_0^3}$ so that at initial time

$$\dot{R}/R|_{t_0=0} = -1/(4t_{\mathrm{cc}}) + 1/t_{\mathrm{ev}}.$$

Evolution of the binary system





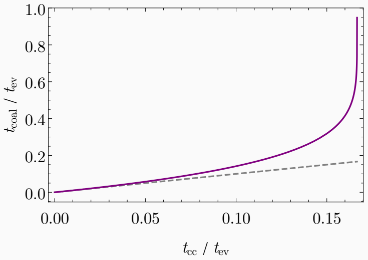
Analytic solution of Bernoulli differential equation

$$R(t) = R_0 \left(\frac{t_{\rm ev}}{t_{\rm ev}-t}\right) \left(1 + \frac{1}{6}\frac{t_{\rm ev}}{t_{\rm cc}} \left[\left(1 - \frac{t}{t_{\rm ev}}\right)^6 - 1\right]\right)^{\frac{1}{4}}$$

Three regimes showing up

- inspiralling for $t_{\rm ev} > 6t_{\rm cc}$
- outspiralling for $t_{\rm ev} < 4t_{\rm cc}$
- non-monotonic behaviour for $4t_{\rm cc} < t_{\rm ev} < 6t_{\rm cc}$

ISCO analysis



What is the imprint of Hawking evaporation in the emitted GWs?

- Inspiralling regime
  $R_0 < R_1 = 2.8\,\mathrm{Mpc}\left(\frac{m_0}{M_\odot}\right)^{\frac{3}{2}}$

- Longer time of coalescence $t_{\rm coal} = t_{\rm ev}\left(1 - \left[1 - 6\frac{t_{\rm cc}}{t_{\rm ev}}\right]^{\frac{1}{6}}\right)$
- Change in the ISCO frequency and maximum strain

$$f^{\rm H}_{\rm ISCO} \simeq f^{\rm cc}_{\rm ISCO}\left(1 - \frac{t_{\rm coal}}{t_{\rm ev}}\right)^{-\frac{1}{3}}, \quad h^{\rm H}_{\rm max} \simeq \left(1 - \frac{t_{\rm coal}}{t_{\rm ev}}\right)^{1/3} h^{\rm cc}_{\rm max}$$

Maximum effect for $t_{\rm ev} = 6t_{\rm cc}$ with $\omega(t_{\rm ISCO}) = \mathcal{O}(\omega_{\rm Planck})$ but highly fine-tuned case. Otherwise, the imprint is unobservable.

Comment on the form of $P_{\rm GW}(t)$

Full computation of GW power for circular and elliptic orbits

$$P_{\rm gw}(t) = \frac{G}{5c^5}\langle \dddot{M}_{ij}\dddot{M}_{ij} - \frac{1}{3}(\dddot{M}_{kk})^2\rangle, \quad M^{ij} = \mu x^i(t)x^j(t)$$

providing additional corrective terms. Can be neglected if

$$|m^{(n)}(t)| \ll m\omega^n, \quad m^{(n)} \equiv \frac{{\rm d}^n m}{{\rm d}t^n}$$

From Kepler's third law

$$|\dot{R}| \leq \frac{2}{3}(\omega R)\frac{|\dot{\omega}|}{\omega^2} + \frac{1}{3}\frac{|\dot{m}|}{m}R$$

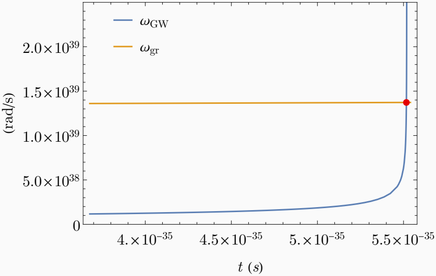
- Imposing the quasi-circularity of the orbit requires $|\dot{\omega}| \ll \omega^2$ *and* $|\dot{m}| \ll m\omega$ (= slowly varying mass condition)
- Same condition for *stable* elliptic orbits when comparing to the fundamental frequency Ω_0

Competitive effects between gravitational radiation and gravitons emission

Frequency analysis

Among particles emitted through Hawking process, gravitons create space-time excitations analogous to GWs. How do they compare ?

- Gravitons frequency $\omega_{\mathrm{gr}} = \frac{\zeta c^3}{8\pi G m_0 (1 - \frac{t}{t_{\mathrm{ev}}})^{\frac{1}{3}}}$
- ω_{GW} increases much faster than ω_{gr}

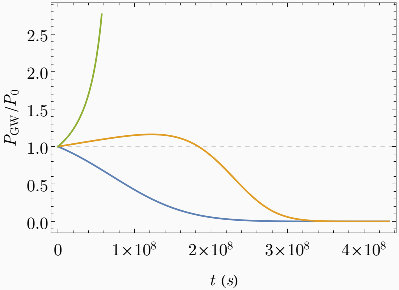


- at ISCO, convergence of both frequencies

$$\frac{\omega_{\mathrm{gr}}}{\omega_{\mathrm{GW}}}(t_{\mathrm{ISCO}}) = \left(\frac{27\zeta^2}{8\pi^2}\right)^{\frac{1}{2}} = \mathcal{O}(1).$$

with wavelength $\lambda_{\mathrm{gr}} \sim R_S$ (Schwarzschild radius)

Power comparison



- $P_{\rm gr} = \frac{\xi \alpha_{\rm H} c^2}{m_0^2 (1 - \frac{t}{t_{\rm ev}})^{\frac{2}{3}}}$
- Ratio $P_{\rm GW}(t=0)/P_{\rm gr}(t=0)$ defines a critical radius $R_{\rm G} = 8 \times 10^{-4}\,\text{Mpc}\left(\frac{m_0}{M_\odot}\right)^{\frac{7}{5}}$

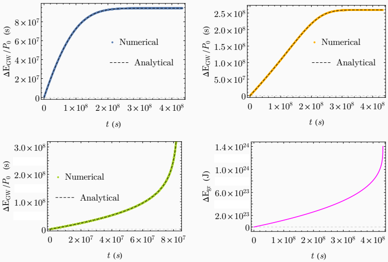
But $R_{\rm G} \neq R_1$!

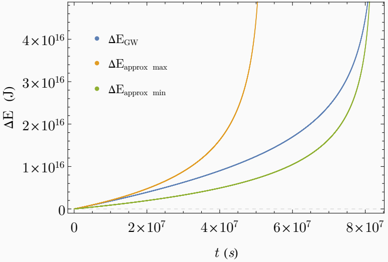
- The initial powers hierarchy does not determine the final one
- Broad spectrum of behaviours between $P_{\rm gr}/P_{\rm GW}$ and inspiralling/outspiralling dynamics
- The powers do not dictate the dynamics : but for phenomenology, inspiral $\implies P_{\rm GW} > P_{\rm gr}$

Integrated process

Instantaneous picture vs. radiated energy $\Delta E = \int P(t)\,\mathrm{d}t$

- For GW process, time-integration neither favours initial or final stages of the merging but lies in-between





- Full-integration beyond merging : $\Delta E_{GW}^{full} \lesssim \Delta E_{gr}^{full}$

A binary system of PBHs "excites" spacetime as much through gravitational waves than through gravitons.

Conclusion

Take-away messages

- Binaries of light PBHS submitted to Hawking evaporation display non-trivial and rich landscapes of behaviours
- Although "explosive", Hawking process plays a significant role from the start, as much as through the mass loss it induces than through the gravitons it (may) emit

Complements

- The Bernoulli ED method can be generalized to all power-laws evolution of $m(t)$ (*e.g.* accretion of phantom dark energy, etc) see arXiv:2306.09069
- Competitive effects between Hawking and GR at the level of cosmological expansion for a gas of PBHs, see *Eur.Phys.J.C* 83 (2023) 11, 1025

Thank you for your attention !