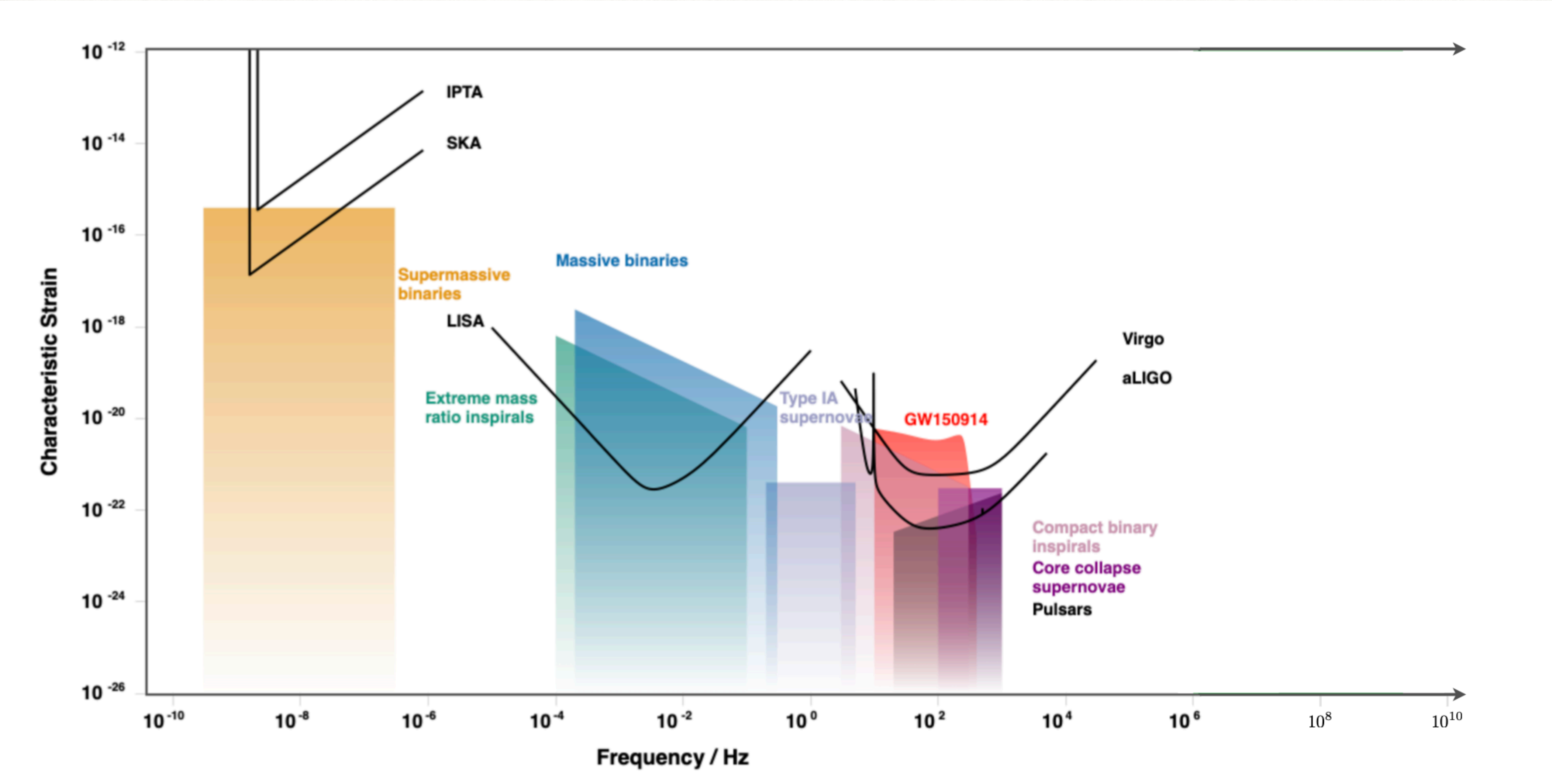
New elements on the search for high frequency gravitational waves with haloscopes/resonant cavities

Based on arXiv:2303.06006

**With Aurélien Barrau (LPSC), Juan Garcia Bellido (IFT)
and Thierry Grenet (Néel institute).**

High frequency gravitational waves



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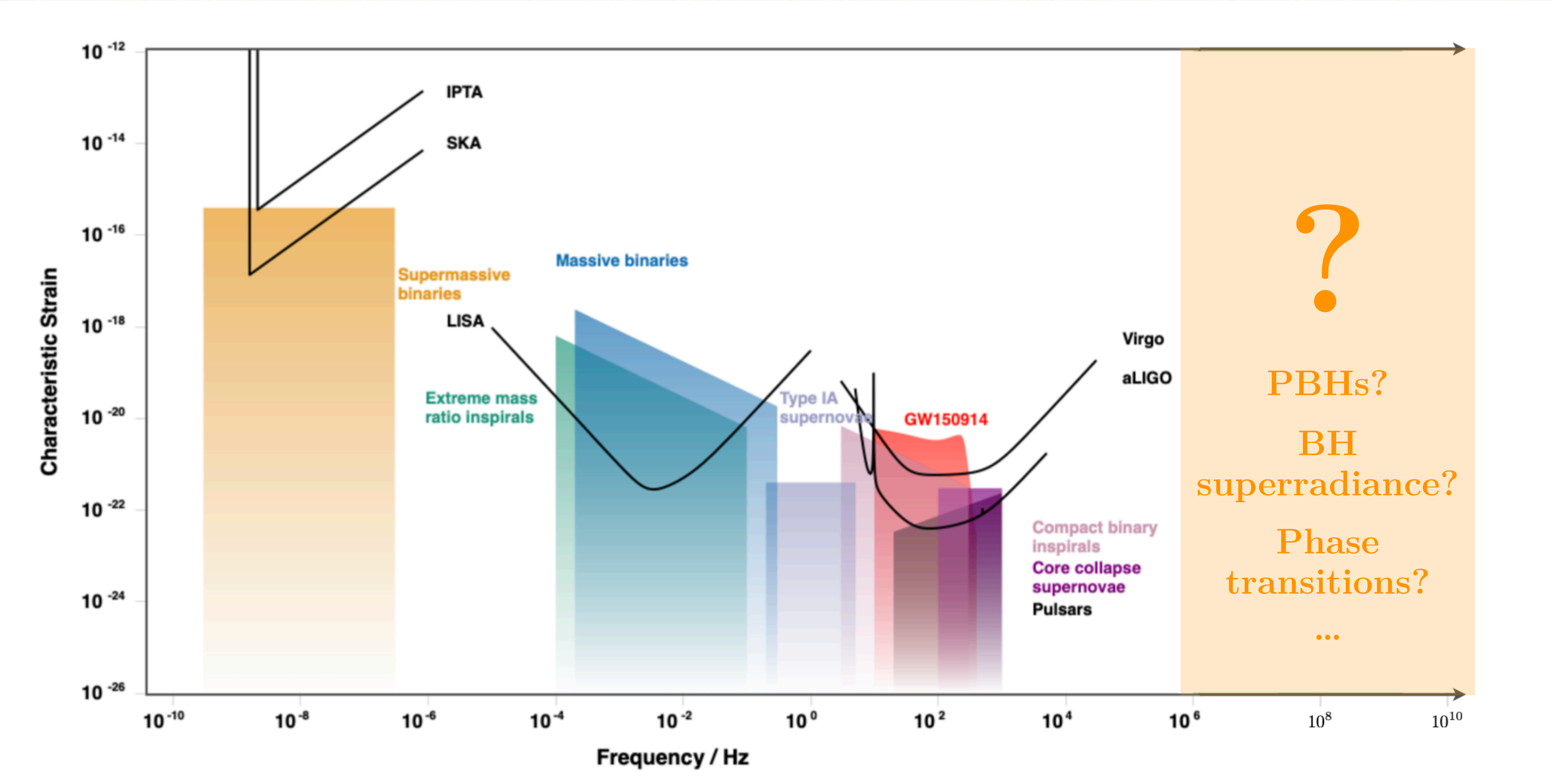

PBHs?

BH superradiance?

Phase transitions?

...

Review paper: N. Aggarwal et. al., arXiv:2011.12414

What is a haloscope?

Haloscope: experiment searching for axion dark matter in our galactic halo

Axion DM behaves like a classical oscillating field

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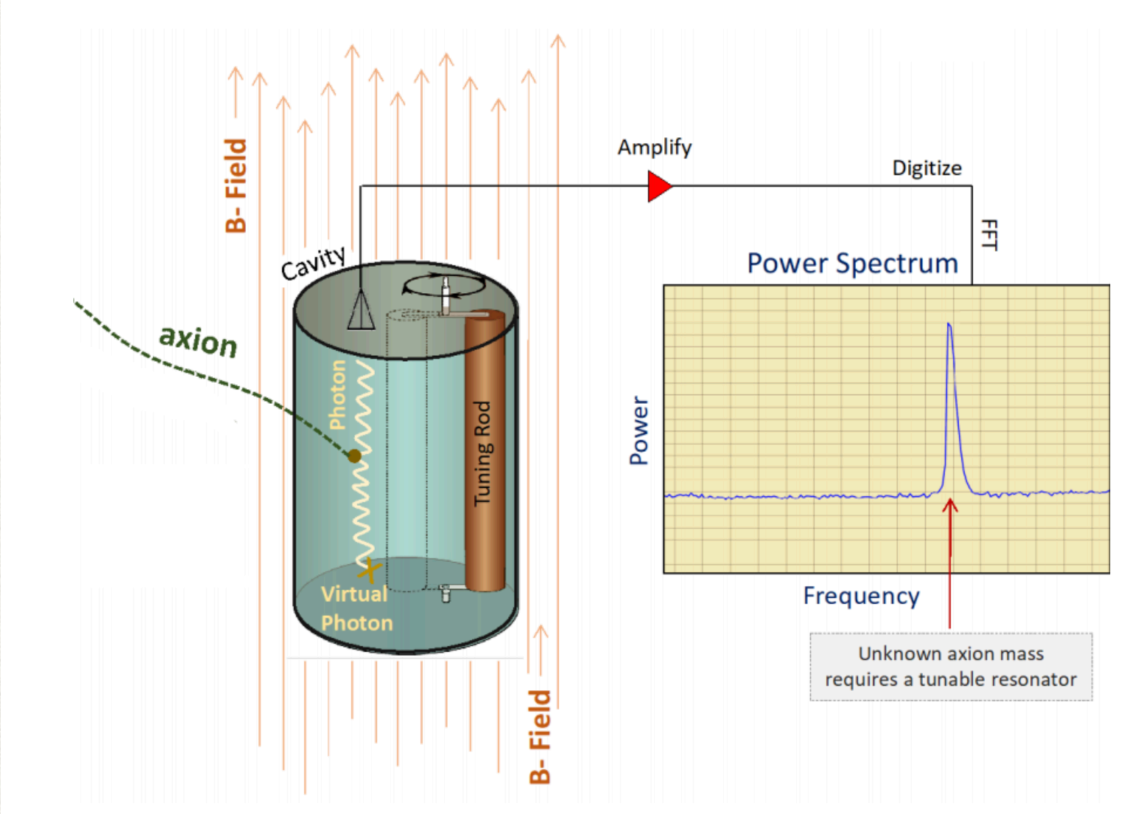

Credits: Raphael Cervantes, University of Washington

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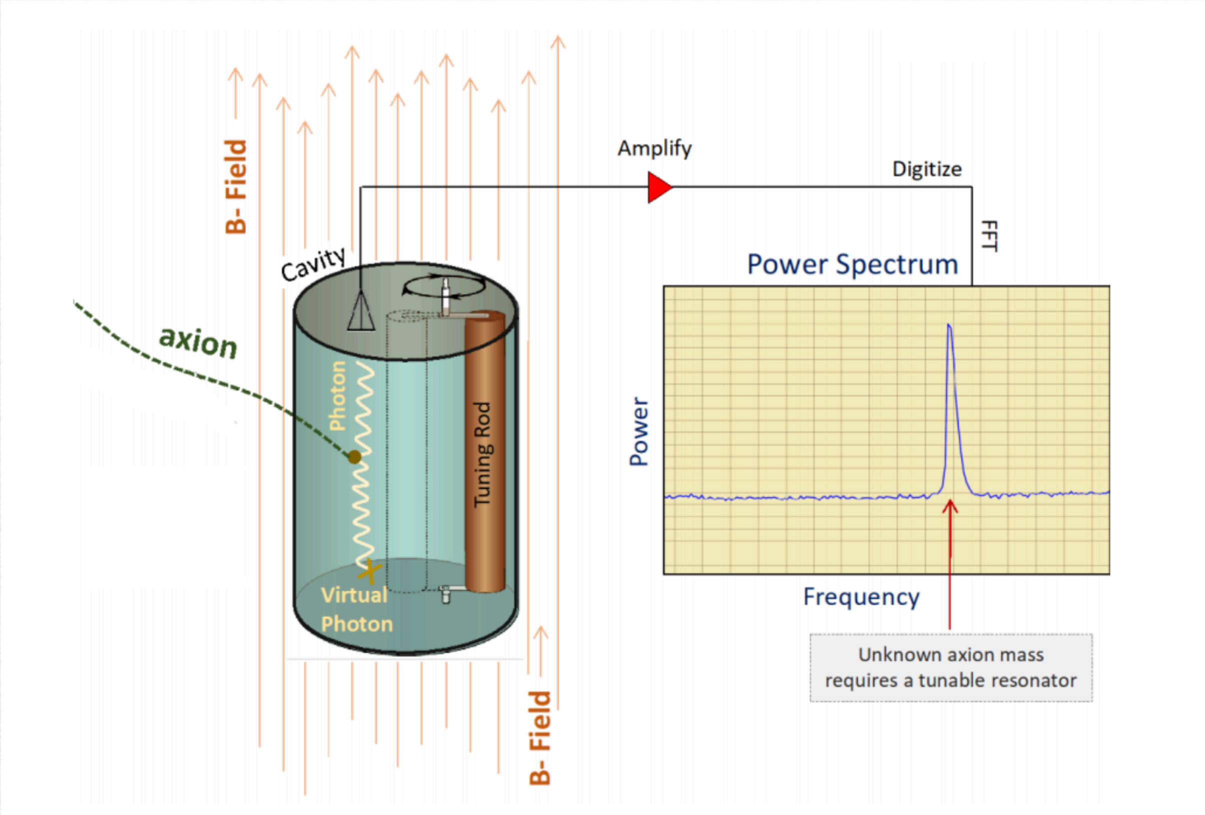

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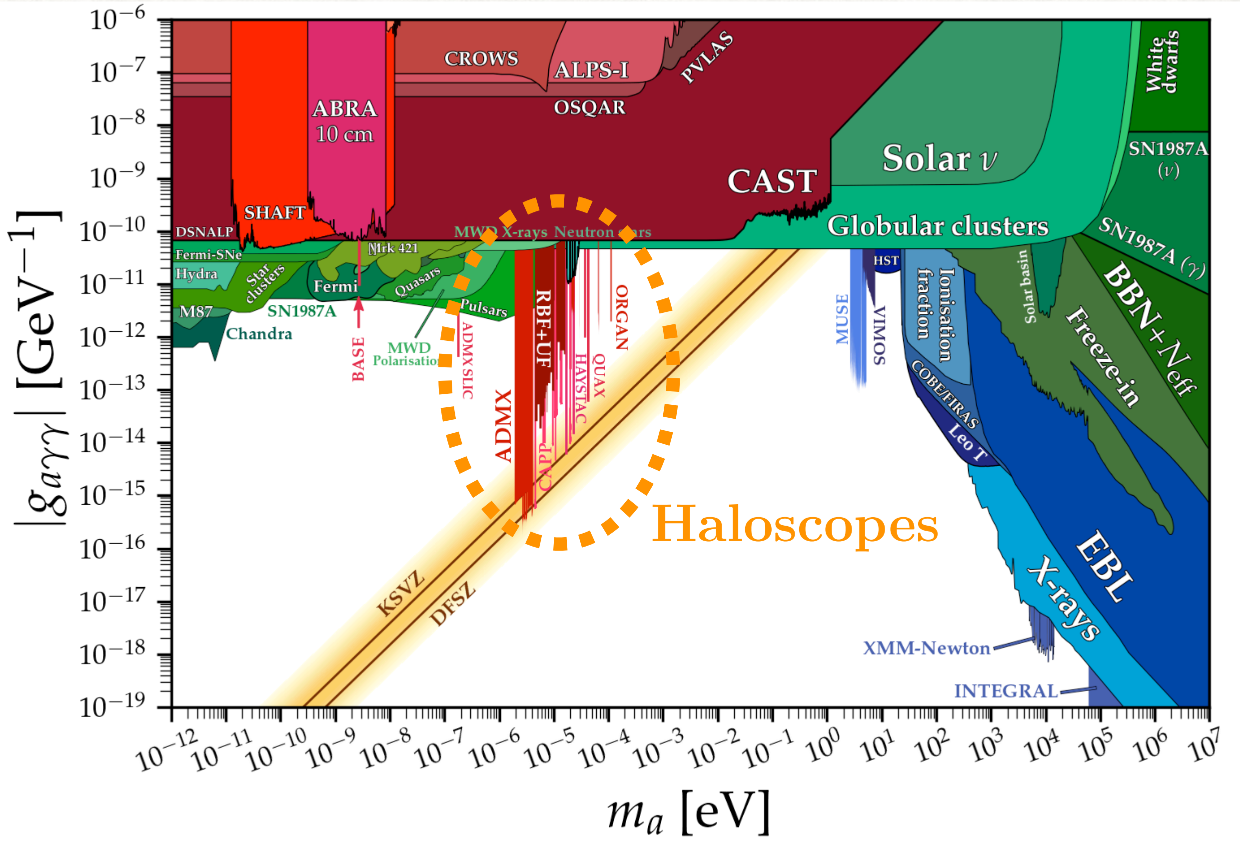

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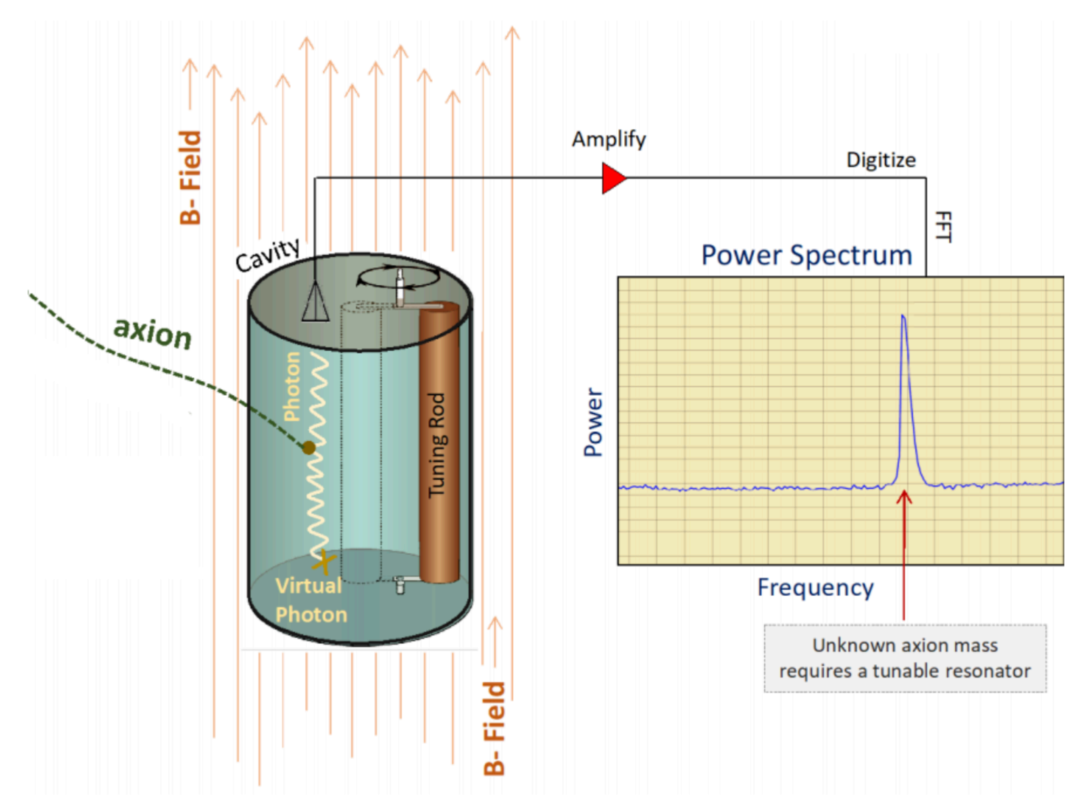

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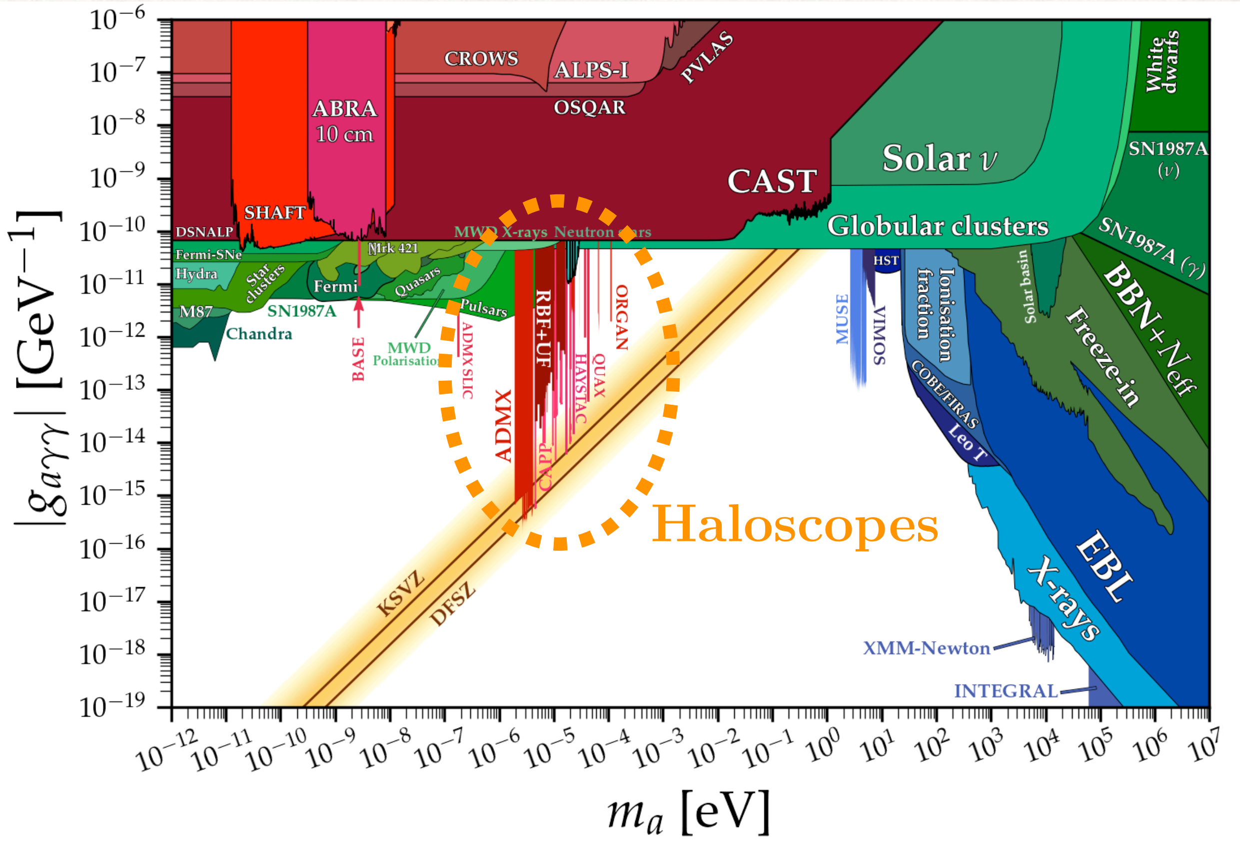

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For this study:
GrAHal platform taken as a benchmark

GrAHal
Grenoble Axion Haloscopes

T. Grenet et. al., arXiv:2110.14406

Different possible configurations

Field	Warm dia.	RF-cavity dia.	Frequency
43 T	34 mm	20 mm	11.5 GHz
40 T	50 mm	34 mm	6.76 GHz
27 T	170 mm	86 mm	2.67 GHz
17.5 T	375 mm	291 mm	0.79 GHz
9.5 T	800 mm	675 mm	0.34 GHz

Ideal for a haloscope

Axion electrodynamics

- $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - A_\mu J^\mu + \frac{1}{2}(\partial_\mu a \partial^\mu a - m^2 a^2) + \boxed{\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu}}$ Axion-photons coupling

- $\partial_\mu F^{\mu\nu} = J^\nu + g_{a\gamma\gamma}\left(\partial_\mu a\right)\tilde{F}^{\mu\nu} \longrightarrow \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{j} - g_{a\gamma\gamma}\left(\vec{B}\frac{\partial a}{\partial t} + \vec{\nabla} a \times \vec{E}\right)$ *(Maxwell-Ampère)*

Generated current: $\vec{j}_a = -g_{a\gamma\gamma}\left(\vec{B}\frac{\partial a}{\partial t} + \cancel{\vec{\nabla} a \times \vec{E}}\right)$

Current aligned along $\vec{B}$

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- Power extracted from the **cavity**: $\boxed{P_{signal} \propto g_{a\gamma\gamma}^2 B^2 Q V \frac{\rho_a}{m_a}}$ ~ **10^{-22} W** → **To be amplified!**

- Noise: $P_{noise} \propto T_{sys}$

4 key ingredients
for a good haloscope:

High magnetic fields
Good cavity (high QV)
Good amplifiers
Low temperatures

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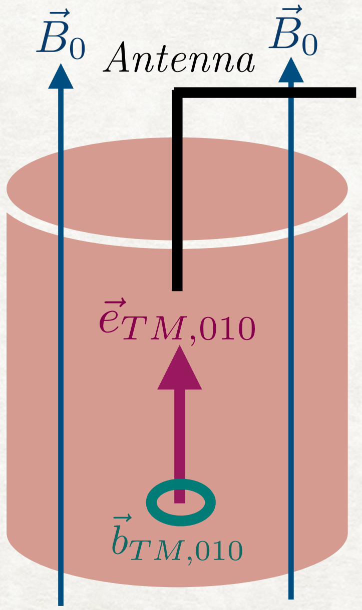
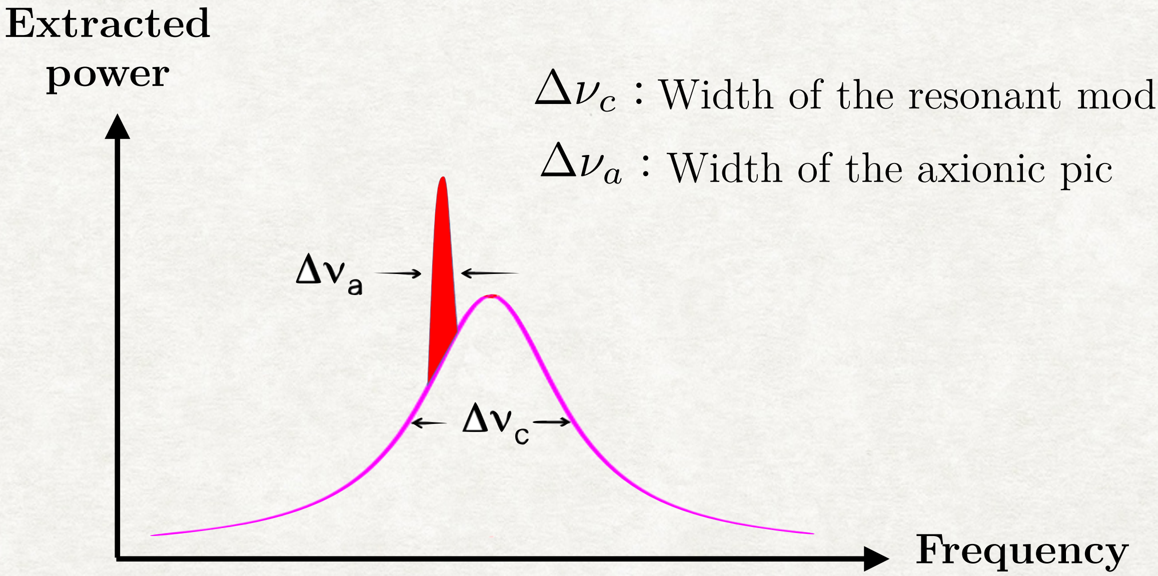




$\Delta\nu_c$: Width of the resonant mode
$\Delta\nu_a$: Width of the axionic pic

GW electrodynamics

Einstein-Maxwell action:

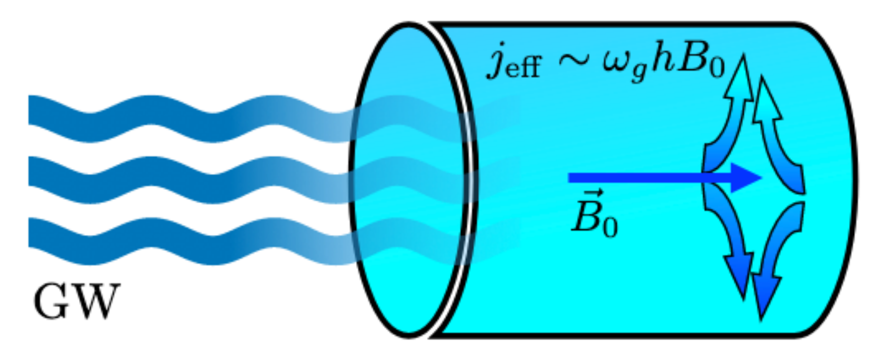
$$\boxed{S_{EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\right)}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \ , \ |h_{\mu\nu}| \ll 1$$

$$S_{EM} = -\frac{1}{4}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\int d^4x\partial_\nu\left[\frac{h}{2}F^{\mu\nu} + h^\nu_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\nu}\right]A_\mu + \mathcal{O}\left(h^2\right)$$

Effective current: $\boxed{j^\mu_{\rm eff} = \partial_\nu\left(\frac{h}{2}F^{\mu\nu} + h^\nu_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\nu}\right)}$

Result from Berlin, Blas et. al. , arXiv:2112.11465



The direction of the current depends on the GW properties $\neq$ **Axionic current!**

The search for hfGWs with resonant cavities

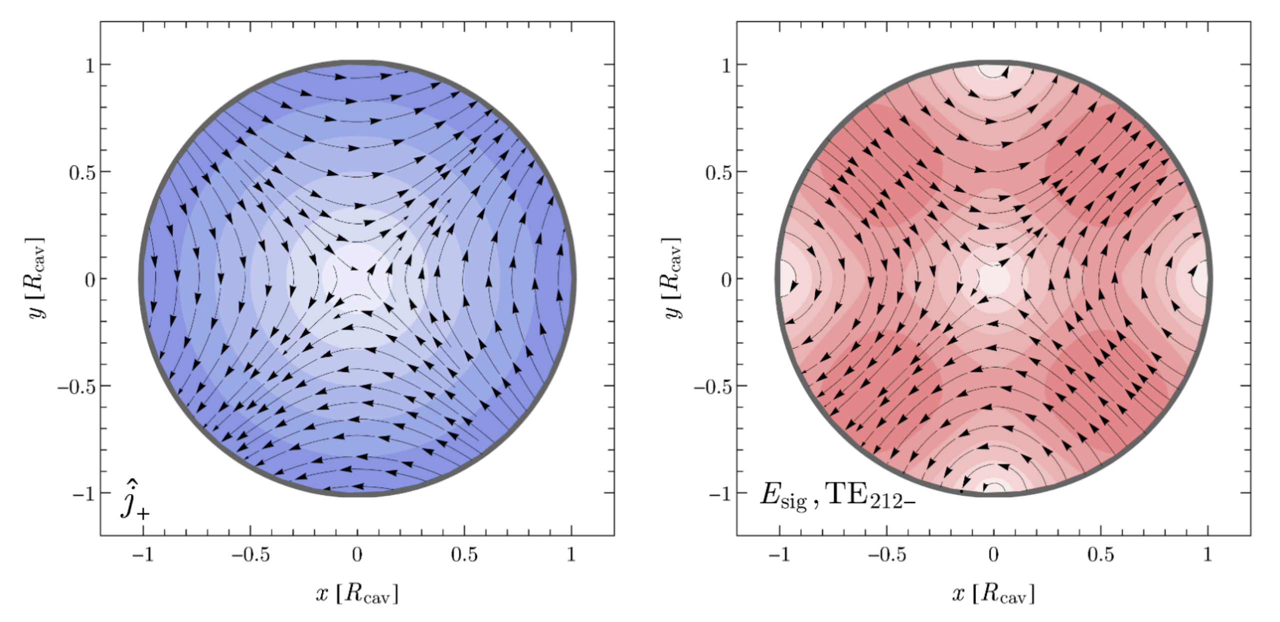
- **GW signal extracted from the cavity**

Result from Berlin et. al. , arXiv:2112.11465

$$P_{\text{sign,GW}} = \frac{1}{2}\omega_{GW}^{3} Q V_{\text{cav}}^{5/3} \left(\eta_n h B\right)^2$$

Coupling coefficient between the effective current and the cavity modes

$$\eta_n \equiv \frac{\left|\int_{V_{\text{cav}}} d^3\vec{x}\vec{E}_n^{\star}.\hat{j}_{+,\times}\right|}{V_{\text{cav}}^{1/2}\left(\int_{V_{\text{cav}}} d^3\vec{x}|\vec{E}_n|^2\right)^{1/2}}$$



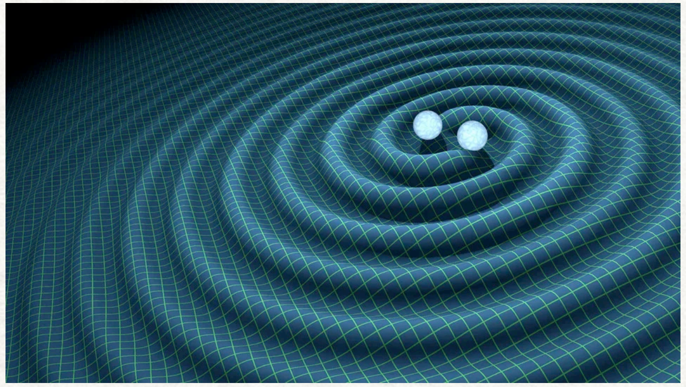
- **Signal to Noise ratio estimated by the radiometer equation:**

$$\text{SNR} \simeq \frac{P_{\text{sig}}}{k_B T_{\text{sys}}}\sqrt{\frac{t_{\text{eff}}}{\Delta\nu}}$$

The search for hfGWs with resonant cavities

- Focus on binary systems of (light) black holes

A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

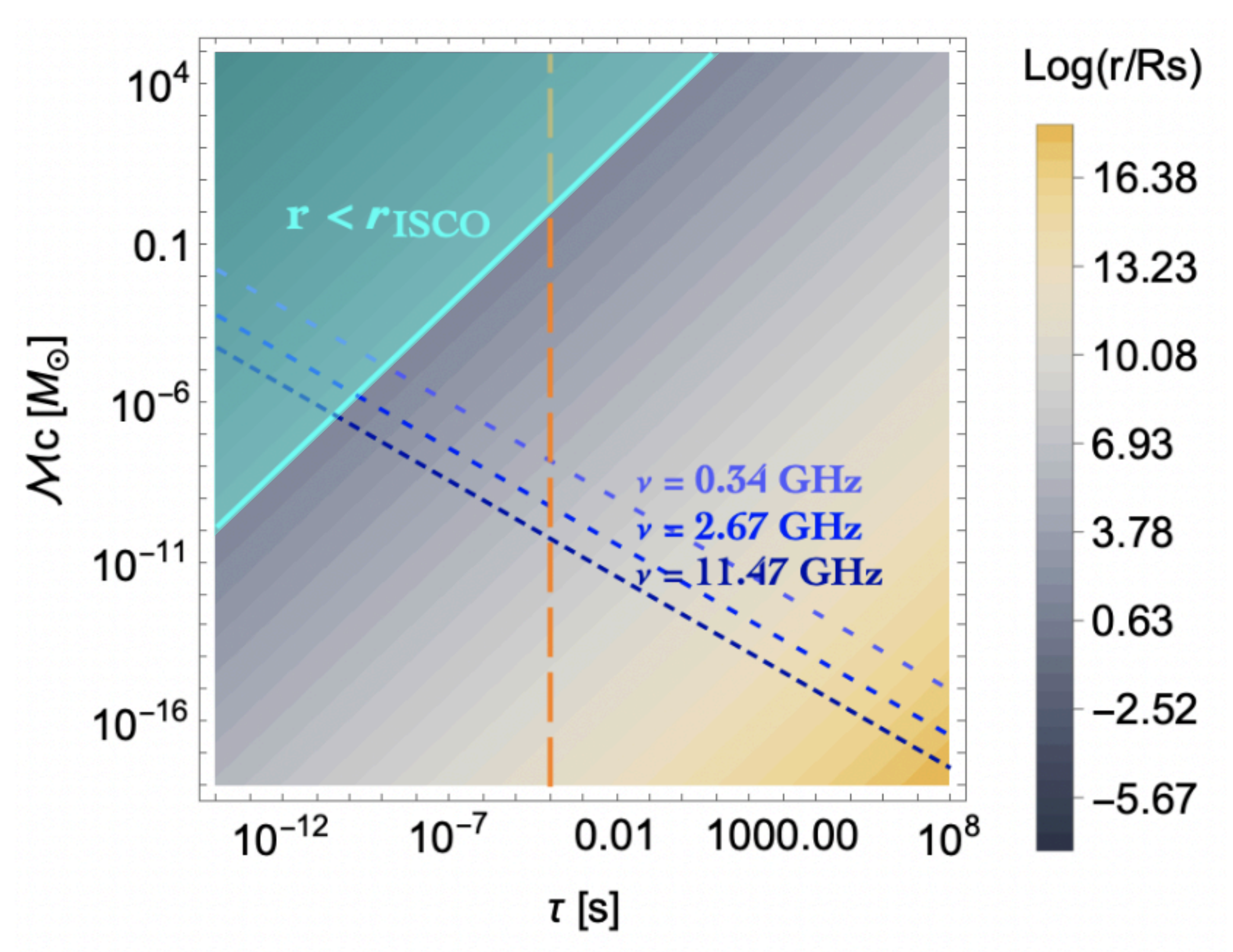


Working at fixed frequency (~ GHz)
does not fix the masses!

ν : resonant frequency
of the detector

τ : time to merger

$$\nu = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{\tau}\right)^{\frac{3}{8}}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{-\frac{5}{8}}$$



The search for hfGWs with resonant cavities

$$\boxed{\mathrm{SNR} \simeq \frac{P_{\mathrm{sig}}}{k_B T_{\mathrm{sys}}}\sqrt{\frac{t_{\mathrm{eff}}}{\Delta\nu}}} \quad + \quad \boxed{P_{\mathrm{sign,GW}} = \frac{1}{2}\omega_{GW}^3 Q V_{\mathrm{cav}}^{5/3}\left(\eta_n h B\right)^2}$$

GrAHal

Grenoble Axion Haloscopes

$\mathrm{SNR} > 1 \Rightarrow$ **Sensitivity estimates:**

$$\begin{aligned}
h &> \boxed{4.7\times10^{-22}}\times\left(\frac{0.34\ \mathrm{GHz}}{\nu}\right)^{\frac{5}{4}}\left(\frac{0.1}{\eta}\right)\left(\frac{9\ \mathrm{T}}{B_0}\right)\left(\frac{5.01\times10^{-1}\ \mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}}\left(\frac{10^5}{Q}\right)^{\frac{3}{4}}\left(\frac{T_{\mathrm{sys}}}{0.3\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{1\ \mathrm{s}}{t_{\mathrm{eff}}}\right)^{\frac{1}{4}}\\
\Leftrightarrow h &> \boxed{1.5\times10^{-21}}\times\left(\frac{2.67\ \mathrm{GHz}}{\nu}\right)^{\frac{5}{4}}\left(\frac{0.1}{\eta}\right)\left(\frac{27\ \mathrm{T}}{B_0}\right)\left(\frac{1.83\times10^{-3}\ \mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}}\left(\frac{10^5}{Q}\right)^{\frac{3}{4}}\left(\frac{T_{\mathrm{sys}}}{0.4\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{1\ \mathrm{s}}{t_{\mathrm{eff}}}\right)^{\frac{1}{4}}\\
\Leftrightarrow h &> \boxed{4.8\times10^{-21}}\times\left(\frac{11.47\ \mathrm{GHz}}{\nu}\right)^{\frac{5}{4}}\left(\frac{0.1}{\eta}\right)\left(\frac{43\ \mathrm{T}}{B_0}\right)\left(\frac{4.93\times10^{-5}\ \mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}}\left(\frac{10^5}{Q}\right)^{\frac{3}{4}}\left(\frac{T_{\mathrm{sys}}}{1.0\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{1\ \mathrm{s}}{t_{\mathrm{eff}}}\right)^{\frac{1}{4}}
\end{aligned}$$

Extremely encouraging

The search for hfGWs with resonant cavities

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Extremely encouraging

But …

What about this value?

The search for hfGWs with resonant cavities

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Hypothesis made:

The signal must remain coherent and located in the experimental frequency bandwidth during at least 1s

Is it really possible?

The search for hfGWs with resonant cavities

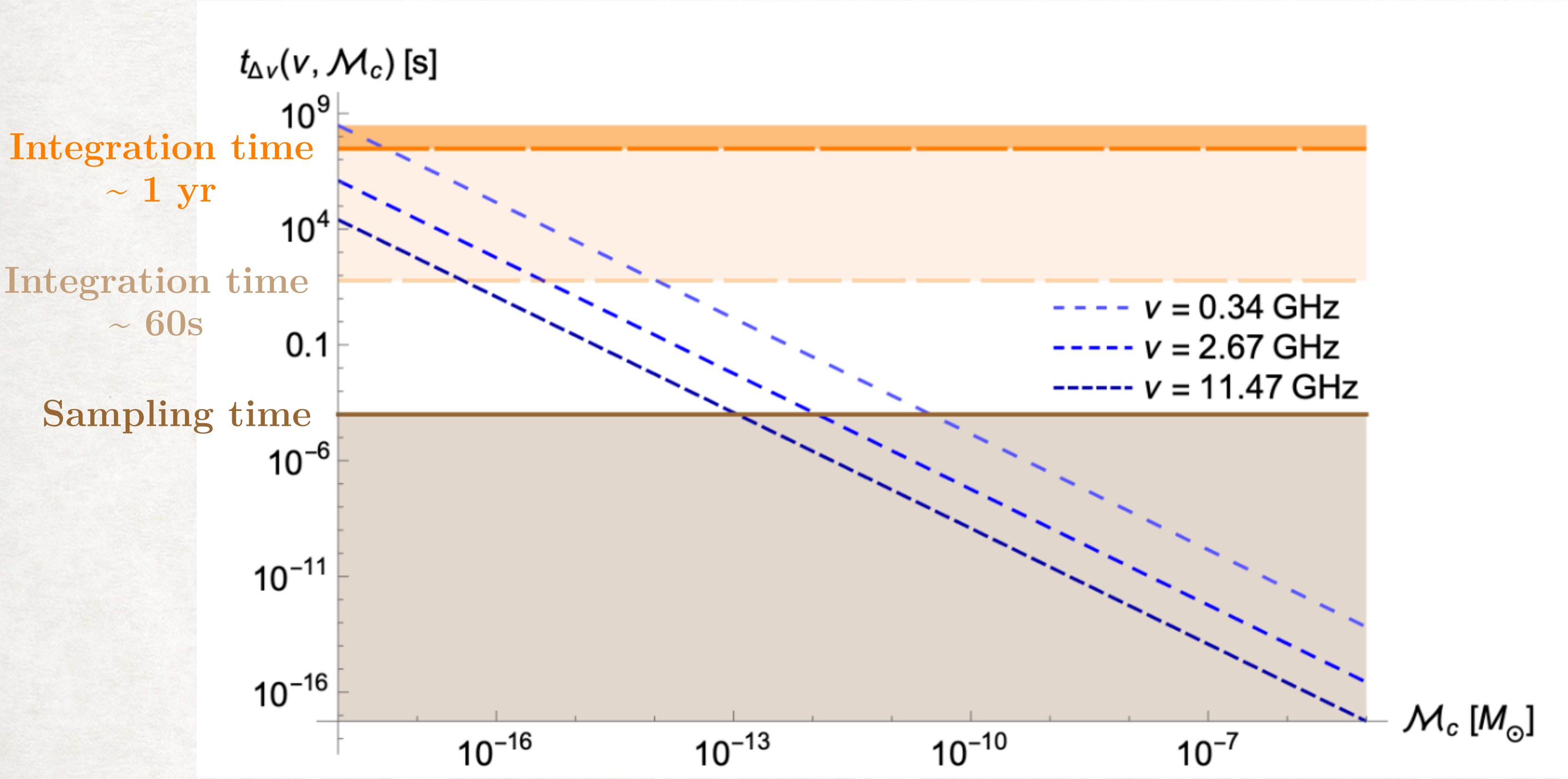
- **The frequency of GWs coming from binary systems drifts with time**

$$\dot{f}(\nu) = \frac{96}{5}\pi^{\frac{8}{3}}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{\frac{5}{3}}\nu^{\frac{11}{3}}$$

- **Time during which the signal drifts in the frequency sensitivity bandwidth:**

$$t_{\Delta\nu} \sim \frac{\Delta\nu}{\dot{f}(\nu)} = \frac{\nu}{Q\dot{f}(\nu)}$$

$$t_{\Delta\nu} \sim \frac{5}{96}\pi^{-\frac{8}{3}}\nu^{-\frac{8}{3}}Q^{-1}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{-\frac{5}{3}}$$



Fast decrease of the signal duration with the mass

The heavier the BHs, the closer they are to their merging

The search for hfGWs with resonant cavities

$$\mathrm{SNR} \simeq \frac{P_{\mathrm{sig}}}{k_B T_{\mathrm{sys}}}\sqrt{\frac{t_{\mathrm{eff}}}{\Delta\nu}} > 1$$

3 different regimes:

1) Effective time given by the signal frequency drift through the frequency bandwidth of the cavity

$$t_{\mathrm{eff}} = t_{\Delta\nu}$$

$$h > 2.0 \times 10^{-21} \times \left(\frac{2.67\ \mathrm{GHz}}{\nu}\right)^{\frac{7}{12}} \left(\frac{0.1}{\eta}\right) \left(\frac{27\ \mathrm{T}}{B_0}\right) \left(\frac{1.83 \times 10^{-3}\mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}} \left(\frac{10^5}{Q}\right)^{\frac{1}{2}} \left(\frac{T_{\mathrm{sys}}}{0.4\ \mathrm{K}}\right)^{\frac{1}{2}} \left(\frac{\mathcal{M}c}{10^{-14} M_\odot}\right)^{\frac{5}{12}}$$

2) Effective time limited by the duration of the experiment *Very small chirp masses*

The signal would spend "more time than available" within the cavity bandwidth

$$t_{\Delta\nu} > t_{\mathrm{max}} \Rightarrow t_{\mathrm{eff}} = t_{\mathrm{max}}$$

$$h > 5.3 \times 10^{-22} \times \left(\frac{2.67\ \mathrm{GHz}}{\nu}\right)^{\frac{5}{4}} \left(\frac{0.1}{\eta}\right) \left(\frac{27\ \mathrm{T}}{B_0}\right) \left(\frac{1.83 \times 10^{-3}\ \mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}} \left(\frac{10^5}{Q}\right)^{\frac{3}{4}} \left(\frac{T_{\mathrm{sys}}}{0.4\ \mathrm{K}}\right)^{\frac{1}{2}} \left(\frac{60\ \mathrm{s}}{t_{\mathrm{max}}}\right)^{\frac{1}{4}}$$

3) Effective time limited by the sampling rate *Highest chirp masses accessible*

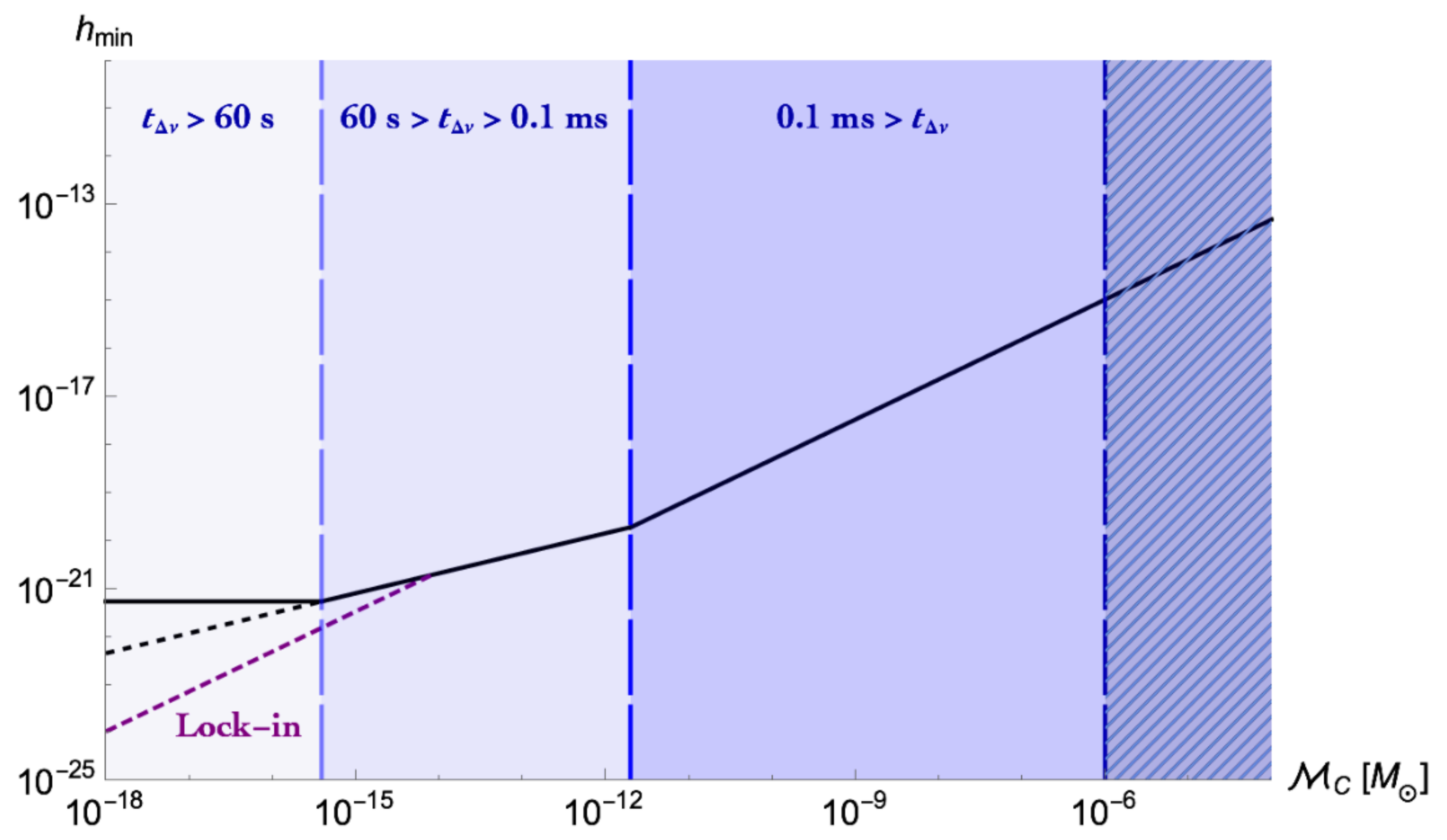
The time spent by the signal within the experimental bandwidth is smaller than the inverse sampling frequency

$$t_{\mathrm{eff}} = t_{\Delta\nu}^2 / t_{\mathrm{min}}$$

$$h > 3.3 \times 10^{-18} \times \left(\frac{2.67\ \mathrm{GHz}}{\nu}\right)^{\frac{1}{6}} \left(\frac{0.1}{\eta}\right) \left(\frac{27\ \mathrm{T}}{B_0}\right) \left(\frac{1.83 \times 10^{-3}\ \mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}} \left(\frac{T_{\mathrm{sys}}}{0.4\ \mathrm{K}}\right)^{\frac{1}{2}} \left(\frac{\mathcal{M}c}{10^{-9} M_\odot}\right)^{\frac{5}{6}}$$

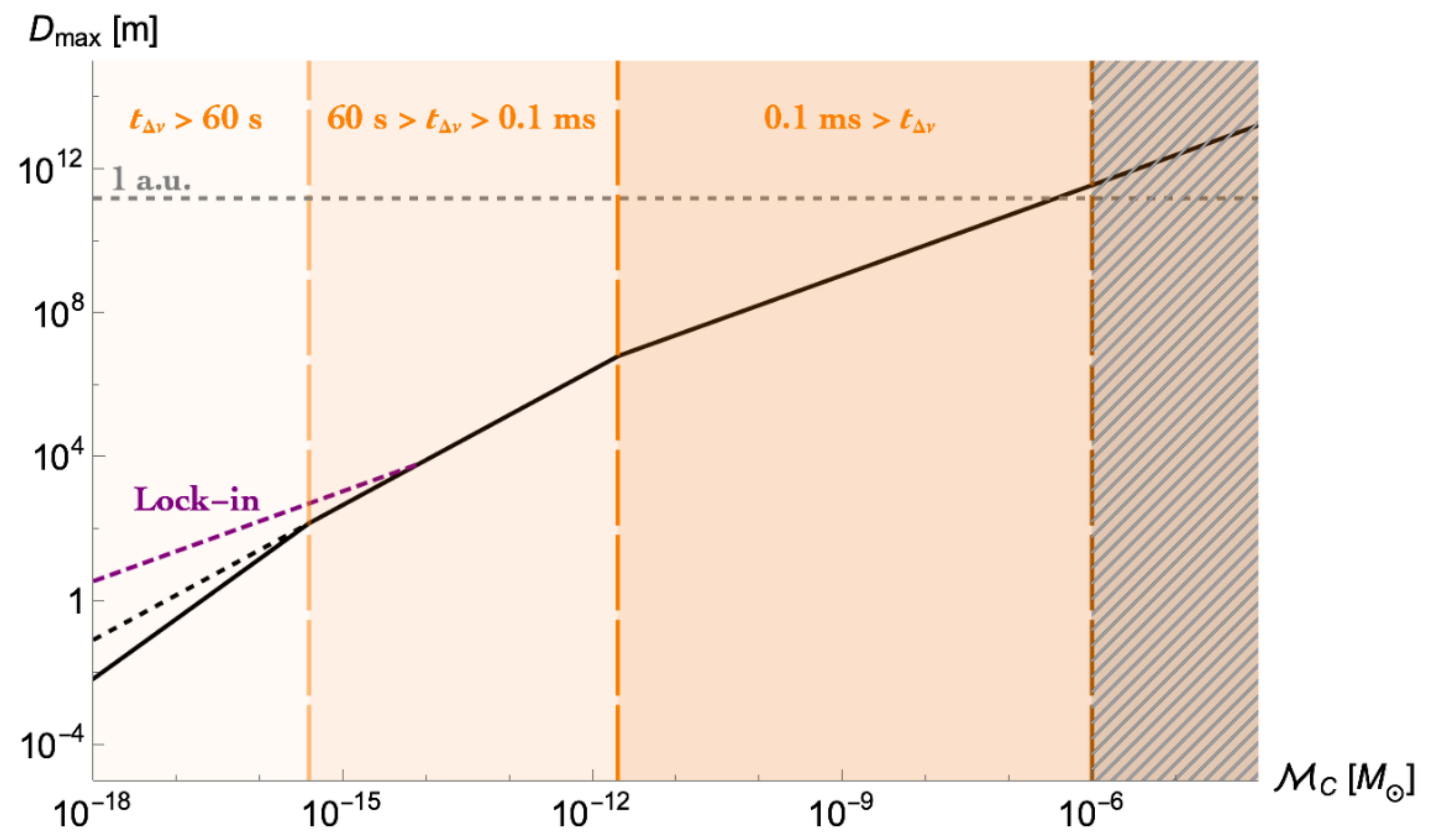
The search for hfGWs with resonant cavities

Strain sensitivity



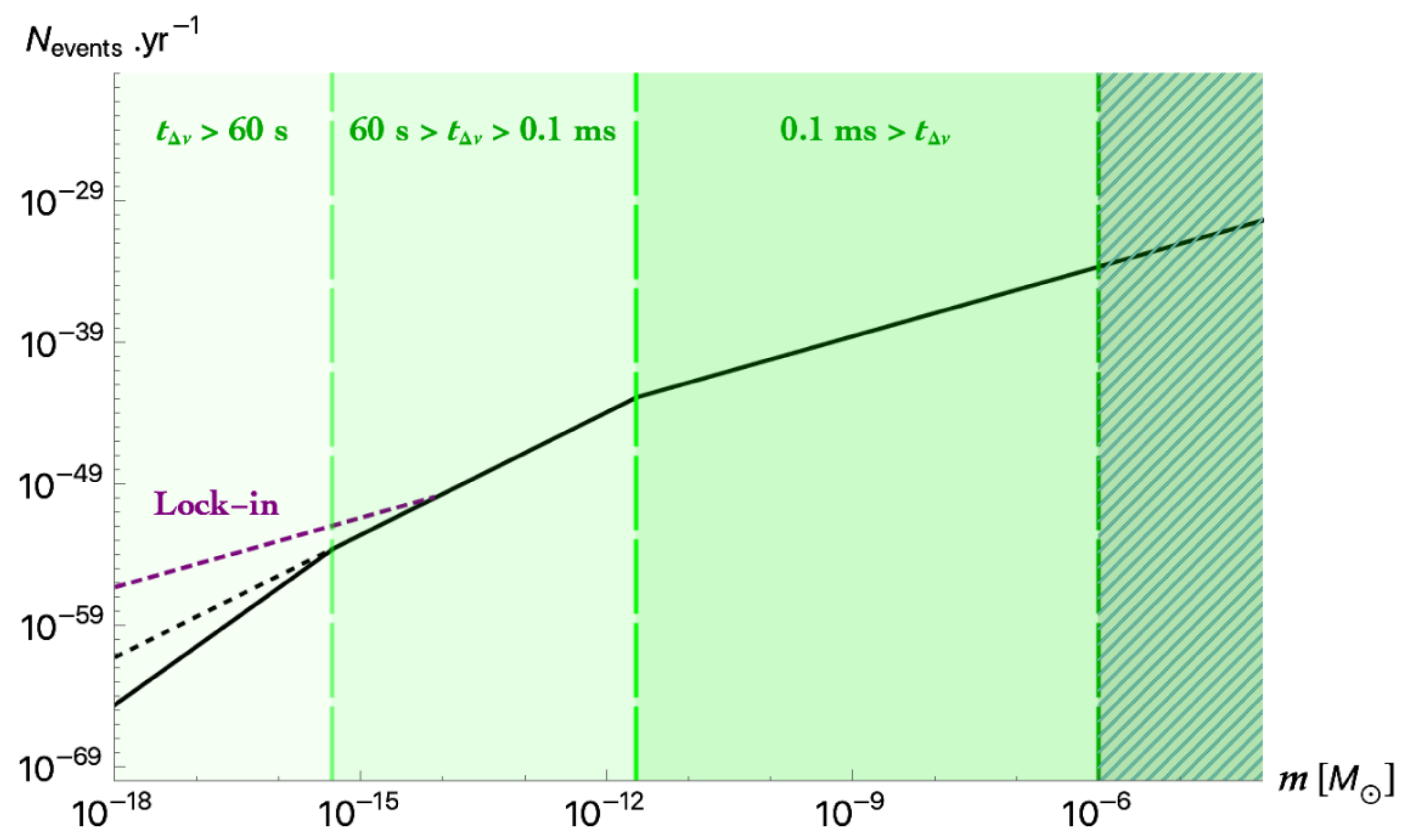
The search for hfGWs with resonant cavities

Accessible distance



The search for hfGWs with resonant cavities

Number of expected events



The search for hfGWs with resonant cavities

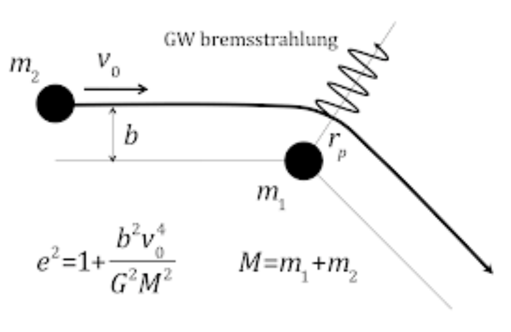
Take away message:

Time analyses are mandatory to derive realistic estimates

Not a small correction but a huge effect

Drastically reduces the sensitivity, thus the accessible distance

- **Possibilities to increase the signal:**
  - **Time domain analyses**
  - **Coupling to different modes in the cavity**
  - **Eccentric orbits** → Boost the emitted power by a factor $F(e) = (1-e^2)^{-7/2}(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4)$
  - **Hyperbolic encounters**



J.G. Bellido et al, arXiv:1711.09702

« See Martin Teuscher's talk »

Thank you!