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Gravitational Wave signatures of No-scale Supergravity in NANOgrav and beyond

Based on S. Basilakos , D.V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, **C. Tzerefos** (2307.08601)
and S. Basilakos , D.V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, **C. Tzerefos** (2309.15820)

Haris Tzerefos

National and Kapodistrian University of Athens
chtzeref@phys.uoa.gr

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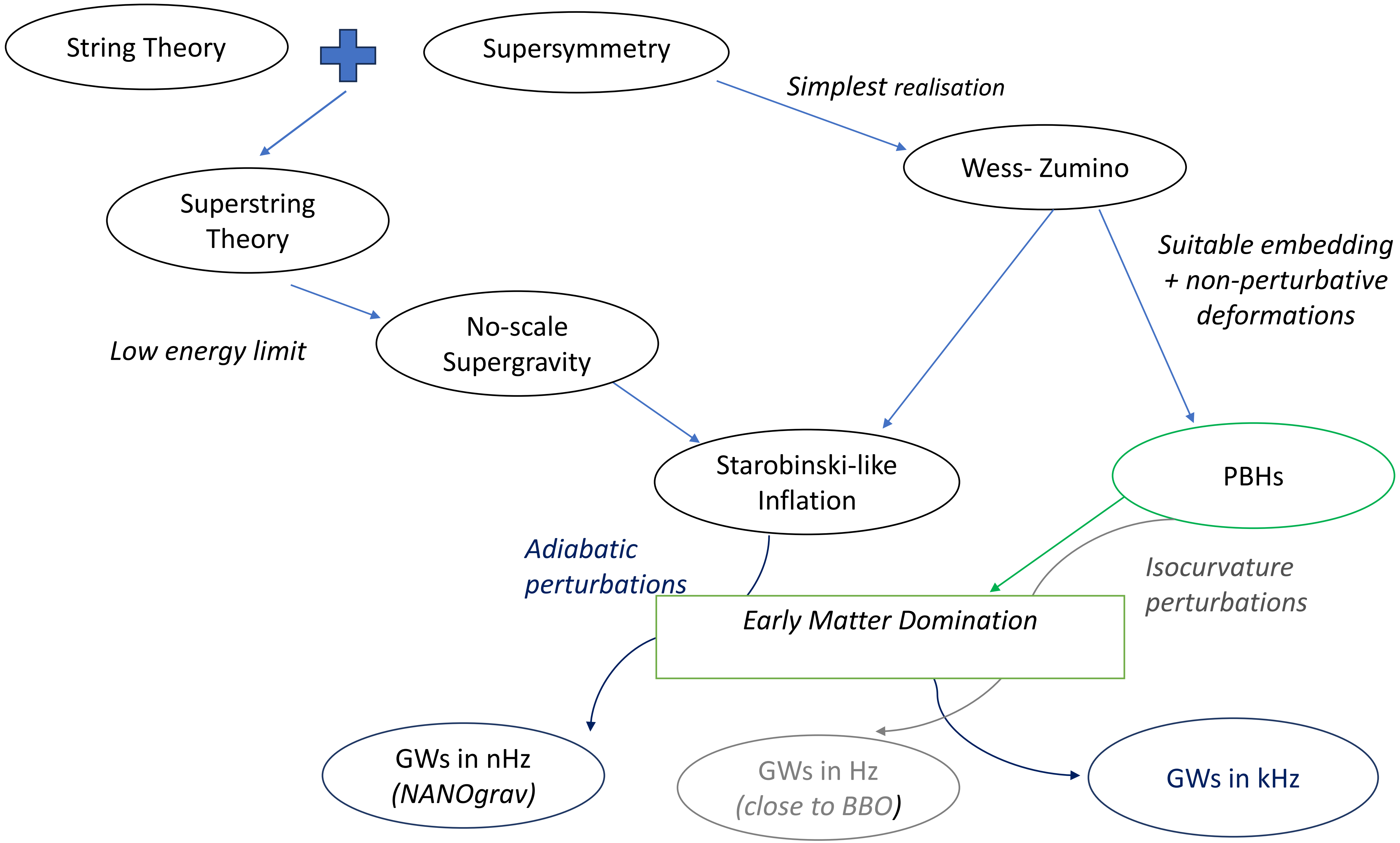

String Theory
Supersymmetry
Simplest realisation
Superstring Theory
Wess- Zumino
Suitable embedding + non-perturbative deformations
Low energy limit
No-scale Supergravity
Starobinski-like Inflation
PBHs
Adiabatic perturbations
Isocurvature perturbations
Early Matter Domination
GWs in nHz (NANOgrav)
GWs in Hz (close to BBO)
GWs in kHz

Theoretical introduction

- **Supergravity (SUGRA)** is a quantum field theory in which global supersymmetry has been promoted to a *local* symmetry. Therefore, its *gauging* describes *gravitation*.

- **No-scale supergravity** is a particular class of SUGRA which is characterized by the *absence of any external scales*, hence its name every relevant energy scale is a function of M_{pl} only. Its significant perks include:

✓ It has been explicitly demonstrated that it naturally arises as the *low energy limit of superstring theory* [Antoniadis, Ellis, Floratos, Nanopoulos, Tomaras, PLB, 1987]

✓ It cures the cosmological constant problem in the sense that it naturally providing *vanishing cosmological constant at the tree level* [Cremmer, Ferrara, Kounnas, Nanopoulos, PLB, 1983]

✓ Through its framework it can produce *Starobinsky-like inflation*, compatible with the Planck data [Ellis, Nanopoulos, Olive, JCAP, 2013]

✓ It can provide an efficient mechanism for *reheating*, the generation of *neutrino masses* and *leptogenesis* [Antoniadis, Nanopoulos, Rizos, JCAP ,2021]

Theoretical introduction

- The most general (N=1) SUGRA is characterized by two functions: The Kahler potential K, which is a Hermitian function of the matter scalar field and quantifies its geometry, and a holomorphic function of the fields called superpotential W. V is the scalar potential :

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(K_{i\bar{j}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{j}} - V\right)$$ with $$V = e^K\left(\mathcal{D}_{\bar{i}}\bar{W}K^{\bar{i}j}\mathcal{D}_jW - 3|W|^2\right) + \frac{\tilde{g}^2}{2}(K^iT^a\Phi_i)^2$$

and $K_{i\bar{j}}(\Phi,\bar{\Phi}) = \dfrac{\partial^2 K}{\partial\Phi^i\partial\bar{\Phi}^{\bar{j}}}$, $\mathcal{D}_iW \equiv \partial_iW + K_iW$ and $i = \{\phi, T\}$ which are chiral superfields.

- The simplest globally supersymmetric model is the Wess-Zumino one, which is characterized by one single chiral superfield φ and the following superpotential: $W = \frac{\hat{\mu}}{2}\varphi^2 - \frac{\lambda}{3}\varphi^3$, with a mass term $\hat{\mu}$ and a trilinear coupling term λ

No-scale Wess-Zumino (NSWZ) SUGRA

- In order to facilitate early universe inflationary scenarios, we shall embed this model in the context of $SU(2,1)/SU(2)\times U(1)$ no-scale supergravity by matching the T field to the modulus field and the φ field to the inflaton. The corresponding Kahler potential for this construction is

$$K = -3\ln\left(T+\bar{T}-\frac{\varphi\bar{\varphi}}{3}\right)$$

- Remarkably, by setting $T=\bar{T}=\frac{c}{2}$, $\mathrm{Im}\varphi=0$ and making a transformation of φ in order to obtain a canonical kinetic term, one obtains Starobinski inflation for $\lambda/\mu=1/3$ and $\hat{\mu}=\mu\sqrt{c/3}$

[Ellis, Nanopoulos, Olive, PRL, 2013],

[Nanopoulos, Spanos, Stamou, PRD, 2020]

$$V(\chi)=\frac{\mu^2}{4}\left(1-e^{-\sqrt{\frac{2}{3}}\chi}\right)^2$$

$$\varphi=\sqrt{3c}\,\tanh\left(\frac{\chi}{\sqrt{3}}\right)$$

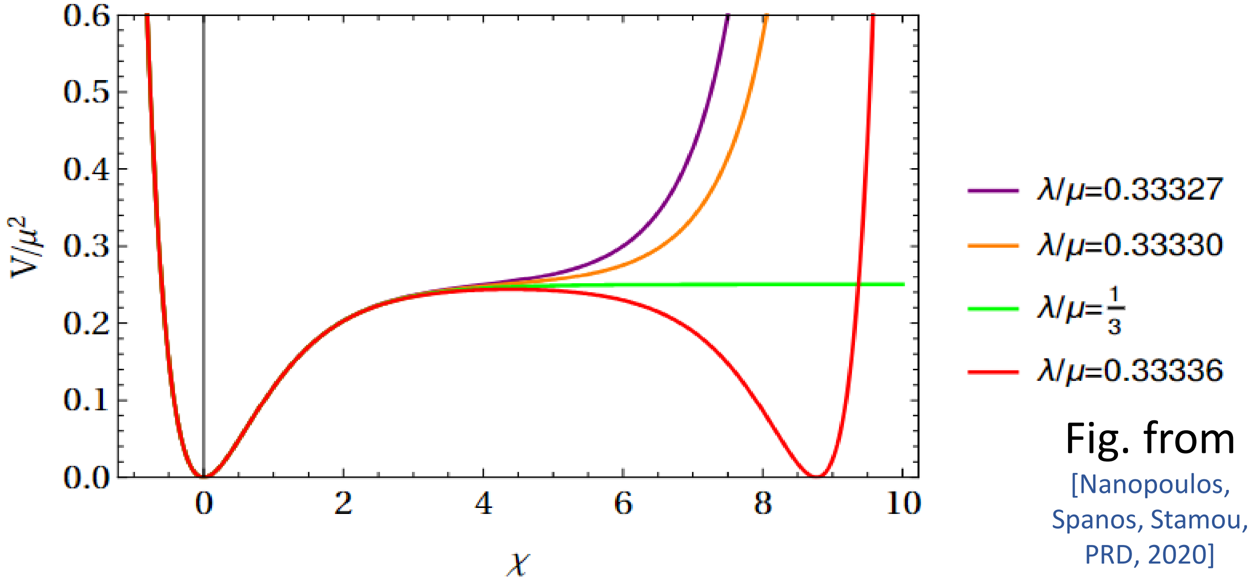


Fig. from [Nanopoulos, Spanos, Stamou, PRD, 2020]

NSWZ SUGRA inflection point inflation

- A common mechanism to produce PBHs is via the use of inflationary potentials with inflection points aka points where $V''(\chi_{\text{inflection}}) = V'(\chi_{\text{inflection}}) \simeq 0$ which induce the so-called **ultra slow roll inflation** (USR)

- To realize such set-ups, one can introduce the following **non-perturbative deformations** to the Kahler potential first introduced in [Nanopoulos, Spanos, Stamou, PRD, 2020]:

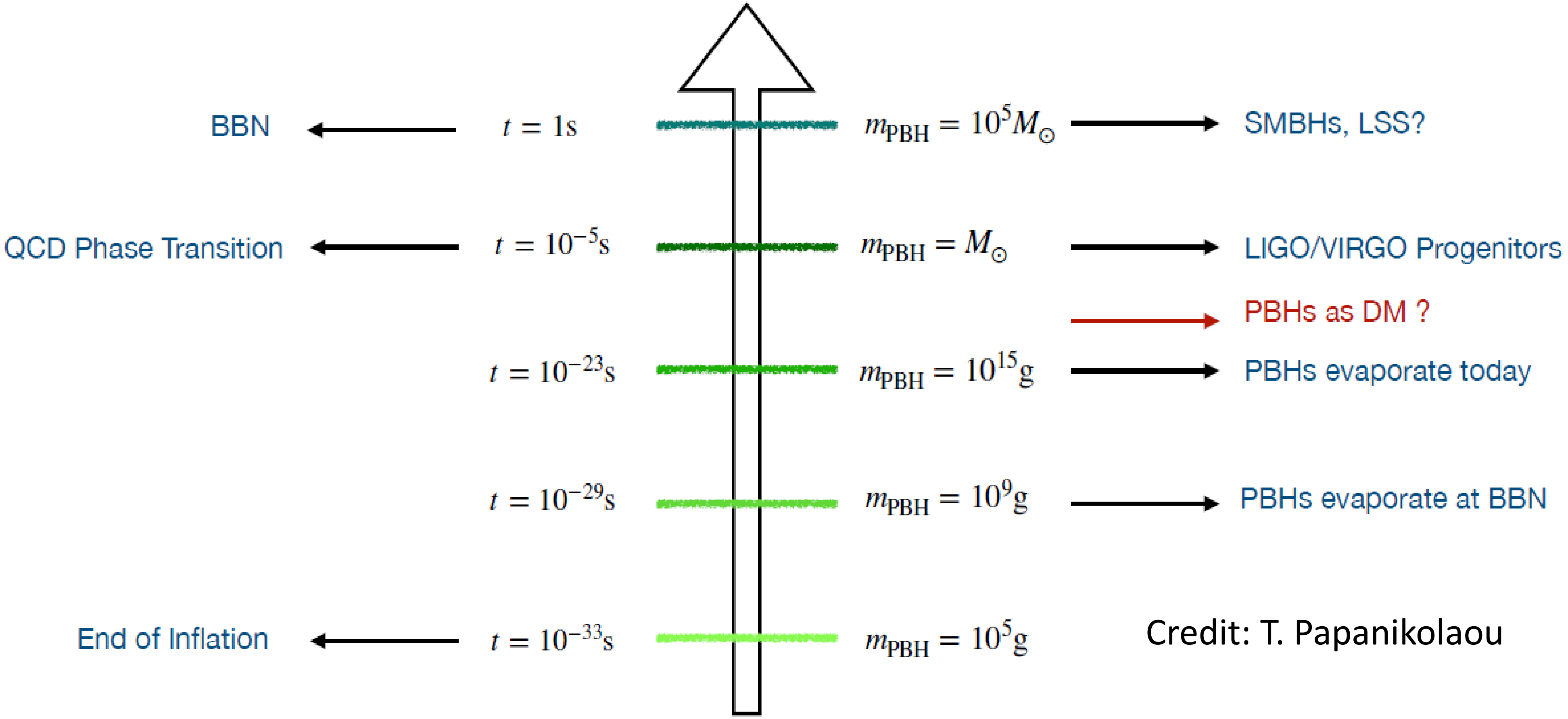
$$K = -3\ln\left[T + \bar{T} - \frac{\varphi\bar{\varphi}}{3} + a\, e^{-b(\varphi+\bar{\varphi})^2}(\varphi+\bar{\varphi})^4\right]$$ with a and b real constants

- At the end, one obtains the following potential

$$V(\phi) = \frac{3e^{12b\phi^2}\phi^2(c\mu^2 - 2\sqrt{3c}\,\lambda\,\mu\,\phi + 3\lambda^2\,\phi^2)}{\left[-48a\phi^4 + e^{4b\phi^2}(-3c+\phi^2)\right]^2\,\left[e^{4b\phi^2} - 24\,a\,\phi^2(6+4b\,\phi^2(-9+8b\,\phi^2))\right]}$$

PBHs

Primordial Black Holes (PBHs) form in the early universe out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region, $m_{\rm PBH} = \gamma M_{\rm H} \propto H^{-1}$ where $\gamma \sim O(1)$ (a nice review [Carr et al, 2020])



Credit: T. Papanikolaou

Perks of ultra-light PBHs

We will consider **ultra-light PBHs** for which $m_{\rm PBH} < 10^9 {\rm g}$ Some of their perks include:

✓They can induce an early matter dominated era (eMD) since $\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a$ and **evaporate before BBN**. Their evaporation **drives the reheating process**

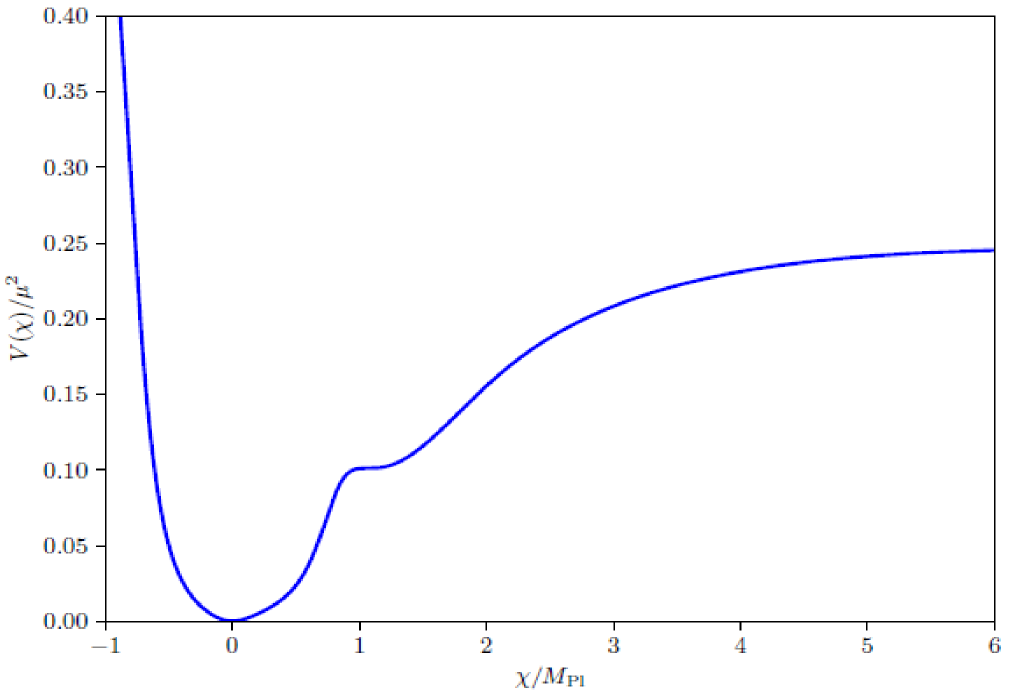
(e.g. [Martin, Papanikolaou, Vennin, JCAP, 2020])

✓This eMD era enhances the magnitude of the curvature perturbation and consequently gives rise to **scalar induced gravitational waves (SIGWs)** with very interesting phenomenology. For instance, one can constrain the underlying gravity theory

(e.g. [Papanikolaou, Tzerefos, Basilakos, Saridakis, JCAP, 2021 and EPJC, 2022])

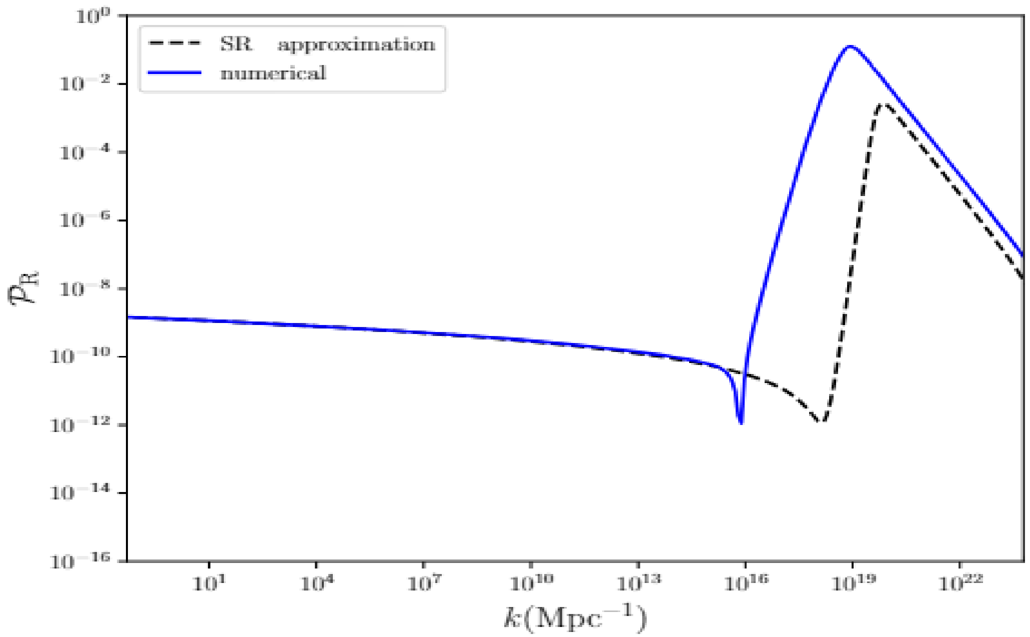
PBHs in no-scale SUGRA

Our modified potential gives rise to the following power spectrum given our choice of fiducial parameters:



$$a = -1,\ b = 22.35,\ c = 0.065,\ \mu = 2 \times 10^{-5}$$

$$\lambda/\mu = 0.3333449 \text{ and } \phi_0 = 0.4295$$

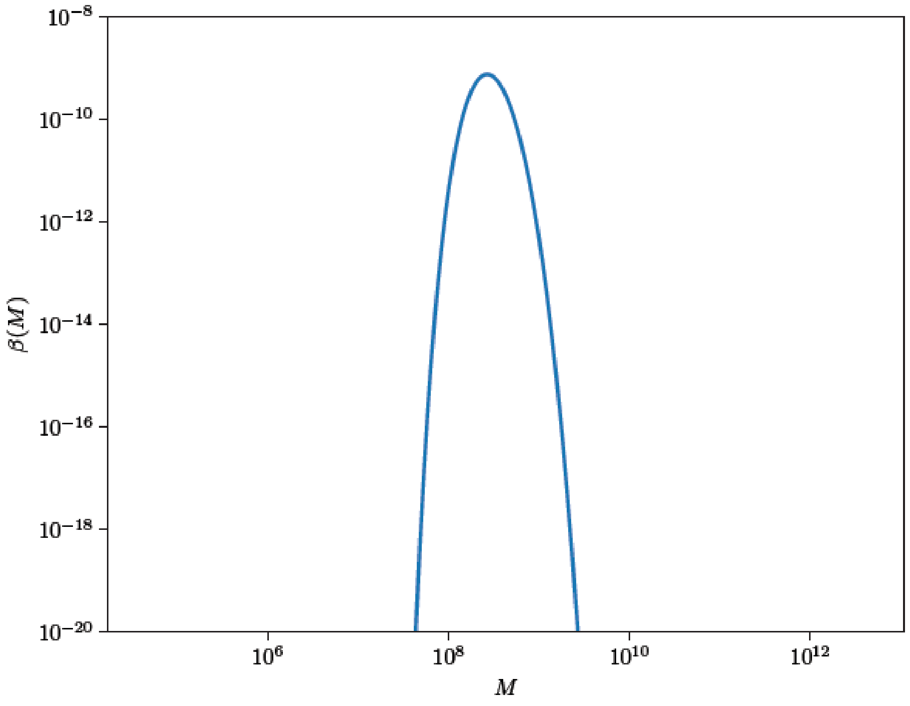


$$M_{\rm PBH} = 17 M_\odot \left(\frac{k}{10^6 \mathrm{Mpc}^{-1}}\right)^{-2} \sim 10^8 \mathrm{g}$$

eMD driven by PBHs

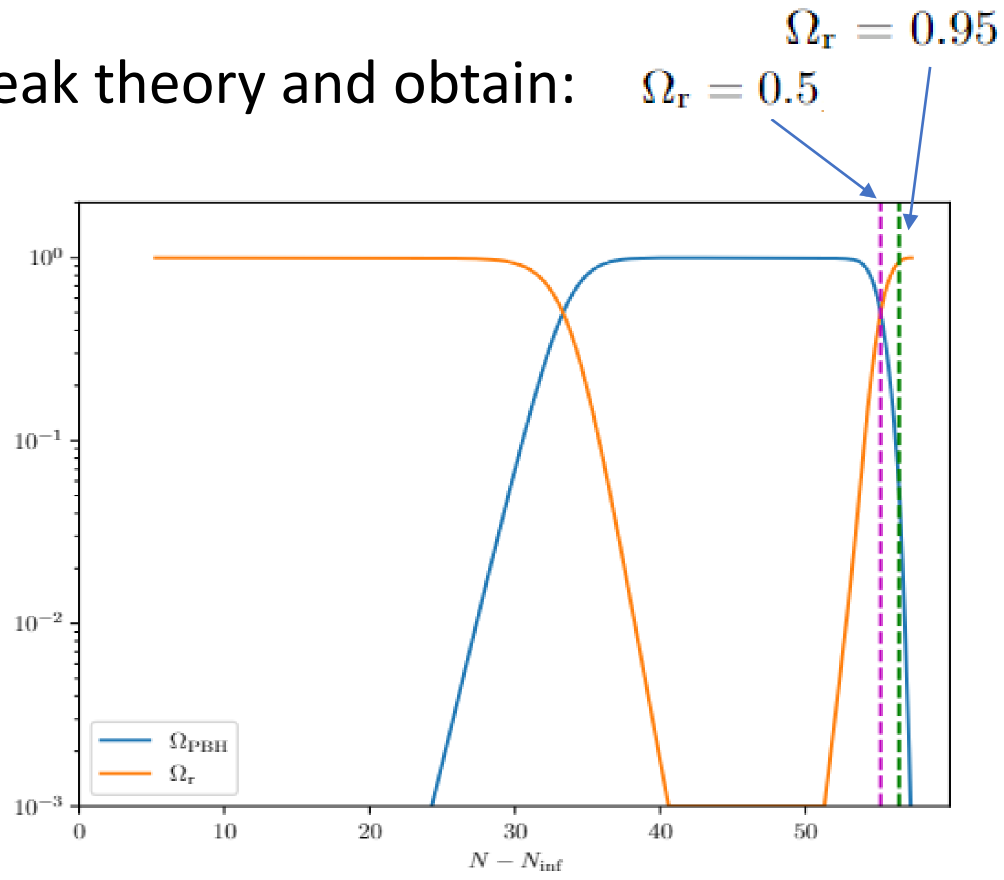
- Since $\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a$ an eMD era driven by them arises

- To find their mass function $\beta(M) \equiv \frac{1}{\rho_{\rm tot}} \frac{{\rm d}\rho_{\rm PBH}}{{\rm d}\ln M}$ we use peak theory and obtain:



Taking into account their Hawking evaporation

$$M(t) = M_{\rm f}\left\{1 - \frac{t - t_{\rm f}}{\Delta t_{\rm evap}(M_{\rm f})}\right\}^{1/3}$$



Note: We treat mass function as monochromatic $\longrightarrow$ eMD to lRD sudden

Essentials of the Scalar Induced GWs (SIGWs)

- Working in the Newtonian gauge, the 2nd order tensor perturbations are described as follows

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\Phi)\mathrm{d}\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

- Their equation of motion in fourier space is $h_{\boldsymbol{k}}^{s,\prime\prime} + 2\mathcal{H}h_{\boldsymbol{k}}^{s,\prime} + k^2 h_{\boldsymbol{k}}^s = 4S_{\boldsymbol{k}}^s$

- The source term is $S_{\boldsymbol{k}}^s = \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^{3/2}} e_{ij}^s(\boldsymbol{k})q_iq_j\left[2\Phi_{\boldsymbol{q}}\Phi_{\boldsymbol{k}-\boldsymbol{q}} + \frac{4}{3(1+w_{\mathrm{tot}})}(\mathcal{H}^{-1}\Phi'_{\boldsymbol{q}} + \Phi_{\boldsymbol{q}})(\mathcal{H}^{-1}\Phi'_{\boldsymbol{k}-\boldsymbol{q}} + \Phi_{\boldsymbol{k}-\boldsymbol{q}})\right]$

- At the end, the spectral abundance of GWs can be given by

$$\Omega_{\mathrm{GW}}(\eta,k) \equiv \frac{1}{\bar{\rho}_{\mathrm{tot}}}\frac{\mathrm{d}\rho_{\mathrm{GW}}(\eta,k)}{\mathrm{d}\ln k} = \frac{1}{24}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2\overline{\mathcal{P}_h^{(s)}(\eta,k)}$$ and $$\mathcal{P}_h^{(s)}(\eta,k) \equiv \frac{k^3|h_k|^2}{2\pi^2} \propto \int \mathrm{d}v \int \mathrm{d}u I^2(u,v,x)\mathcal{P}_\Phi(kv)\mathcal{P}_\Phi(ku)$$

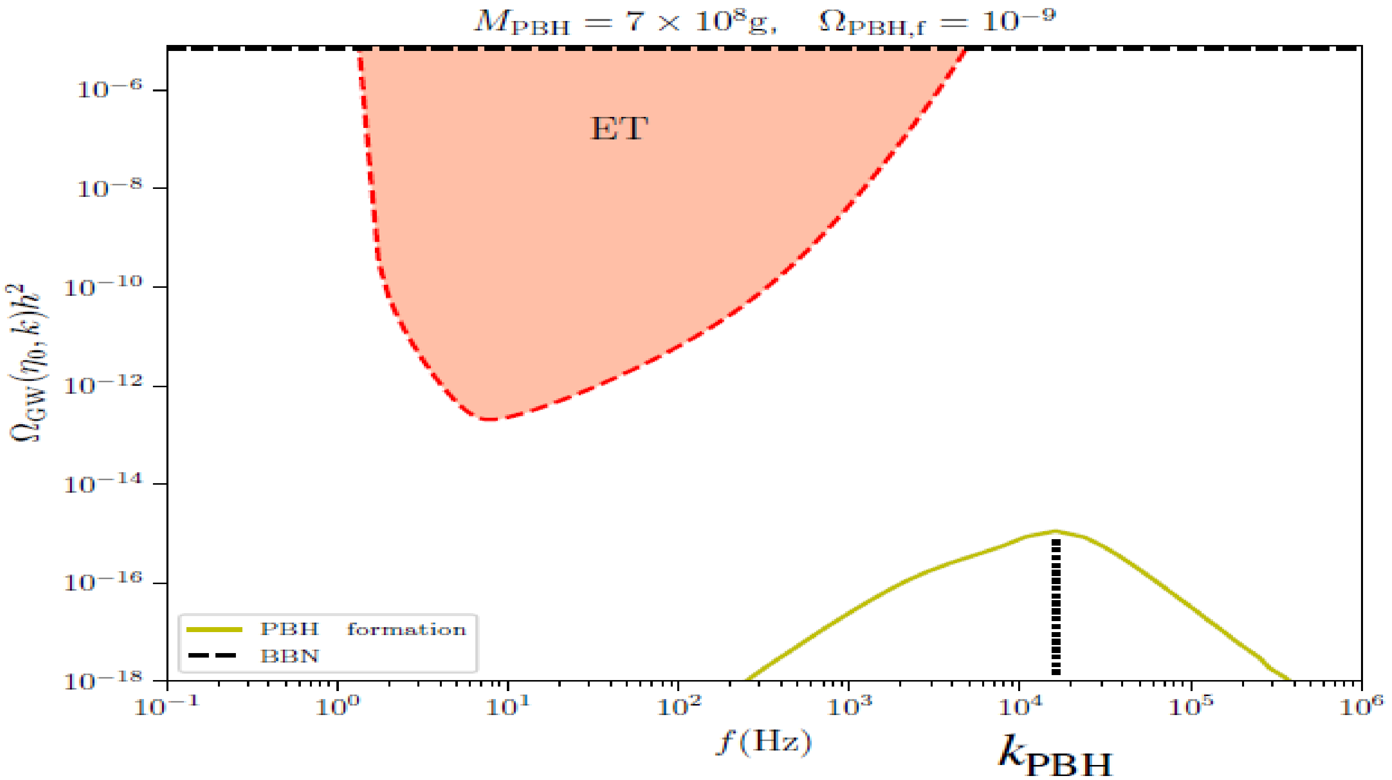
The kernel $I(u,v,x)$ is complicated function containing the info for eMD $\longrightarrow$ lRD eras (see [Inomata et al, PRD, 2019])

The relevant GW sources and their spectrum

A) Inflationary adiabatic perturbations $\longrightarrow$ GWs with two peaks

i) GWs are produced by the enhancement of $\mathcal{P}_{\mathcal{R}}(k)$ (peaked at $10^{19}\mathrm{Mpc}^{-1}$) at PBHs scales peaked at the **kHz range** [Kohri, Terada, PRD, 2018]

$$\Omega_{\mathrm{GW}}^{\mathrm{form}}(\eta_0, k) = \frac{a_{\mathrm{d}}}{a_{\mathrm{evap}}} c_g \Omega_{\mathrm{r}}^{(0)} \Omega_{\mathrm{GW}}(\eta_{\mathrm{f}}, k), \text{ with } c_g = \frac{g_{*\rho,*}}{g_{*\rho,0}} \left(\frac{g_{*\mathrm{S},0}}{g_{*\mathrm{S},*}}\right)^{4/3} \sim 0.4 \text{ and } \Omega_{\mathrm{r}}^{(0)} \simeq 10^{-4}$$



Inflationary adiabatic perturbations

ii) It is related to the ***resonant amplification*** of curvature perturbations of scales entering the horizon during the eMD era. Specifically, since Φ' goes **quickly** from 0 (since Φ=*constant* in any MD) to $\Phi' \neq 0$ during lRD s, there is a resonant enhancement of GWs mainly sourced by the term $\mathcal{H}^{-2}\Phi'^2$

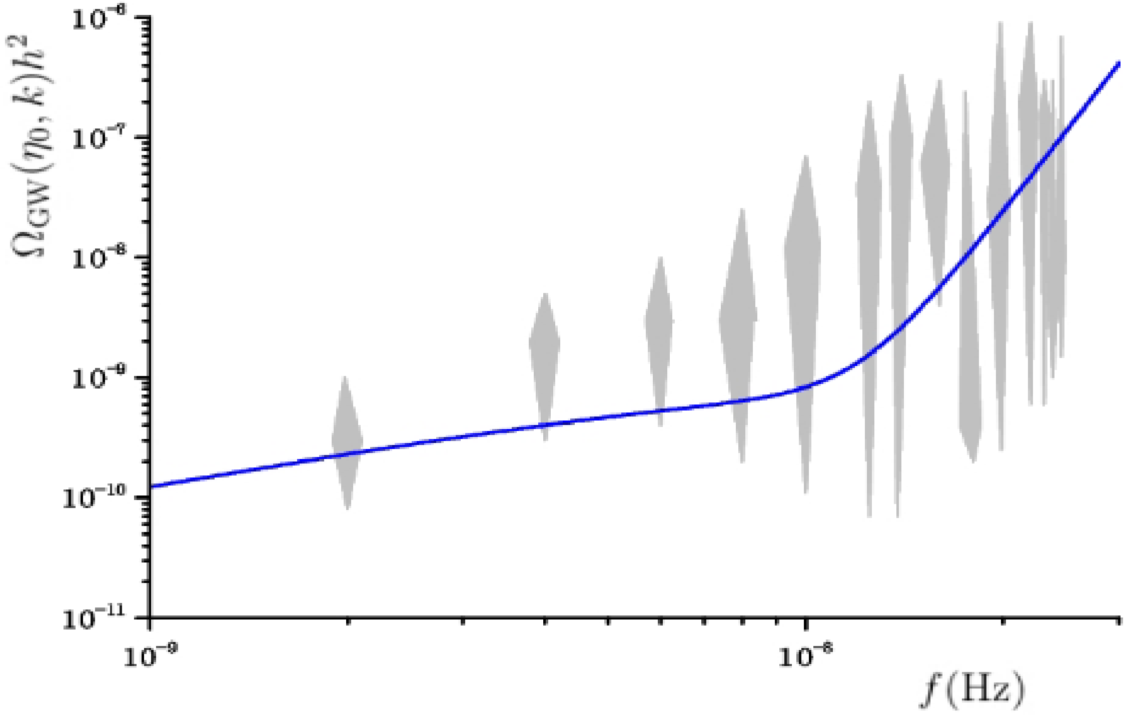
Also, since during a MD $\delta \sim \alpha$, we need to ensure that we are working in the perturbative regime $\longrightarrow$ we introduce a non linear cut- off scale : $\delta_{k_{NL}}(\eta_r) = 1$

At the end, $f_{GW\,peak} = c\,\frac{k_{NL}}{2\pi a_0} \sim nHz$

Inflationary adiabatic perturbations

This GW signal peaks at the **nHz frequency range** and is in agreement with **NANOGrav/PTA GW data**.

$$\Omega_{\rm GW}^{\rm res}(\eta_0, k) = c_g \Omega_{\rm r}^{(0)} \Omega_{\rm GW}(\eta_{\rm lRD}, k)$$



SIGWs from Poisson fluctuations of a gas of PBHs

- Random distribution of PBHs + same mass $\longrightarrow$ they follow Poisson statistics :

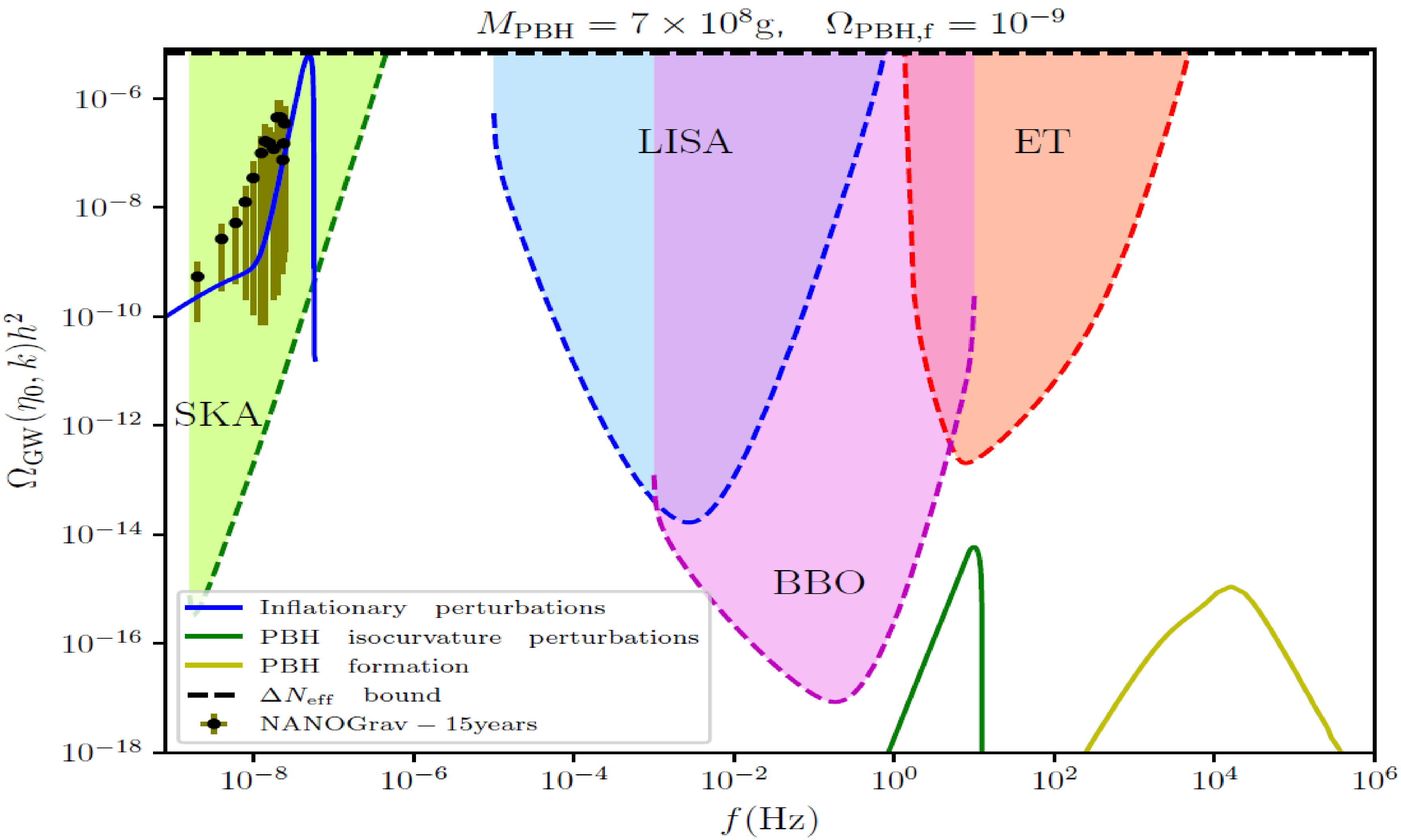
$$\mathcal{P}_\delta(k) = \frac{k^3}{2\pi^2} P_\delta(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3 \Theta(k_{\rm UV} - k)$$

- Since ρ_{PBH} is inhomogeneous and ρ_{tot} is homogenous $\longrightarrow$ $\delta_{\rm PBH}$ is an **isocurvature perturbation**

- $\delta_{\rm PBH}$ generated in the eRD era will be converted in an eMD era to a curvature perturbation ζ_{PBH} associated with the scalar potential [Papanikolaou, Vennin, Langlois, JCAP, 2021]

$$\mathcal{P}_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3 \left(5 + \frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}$$

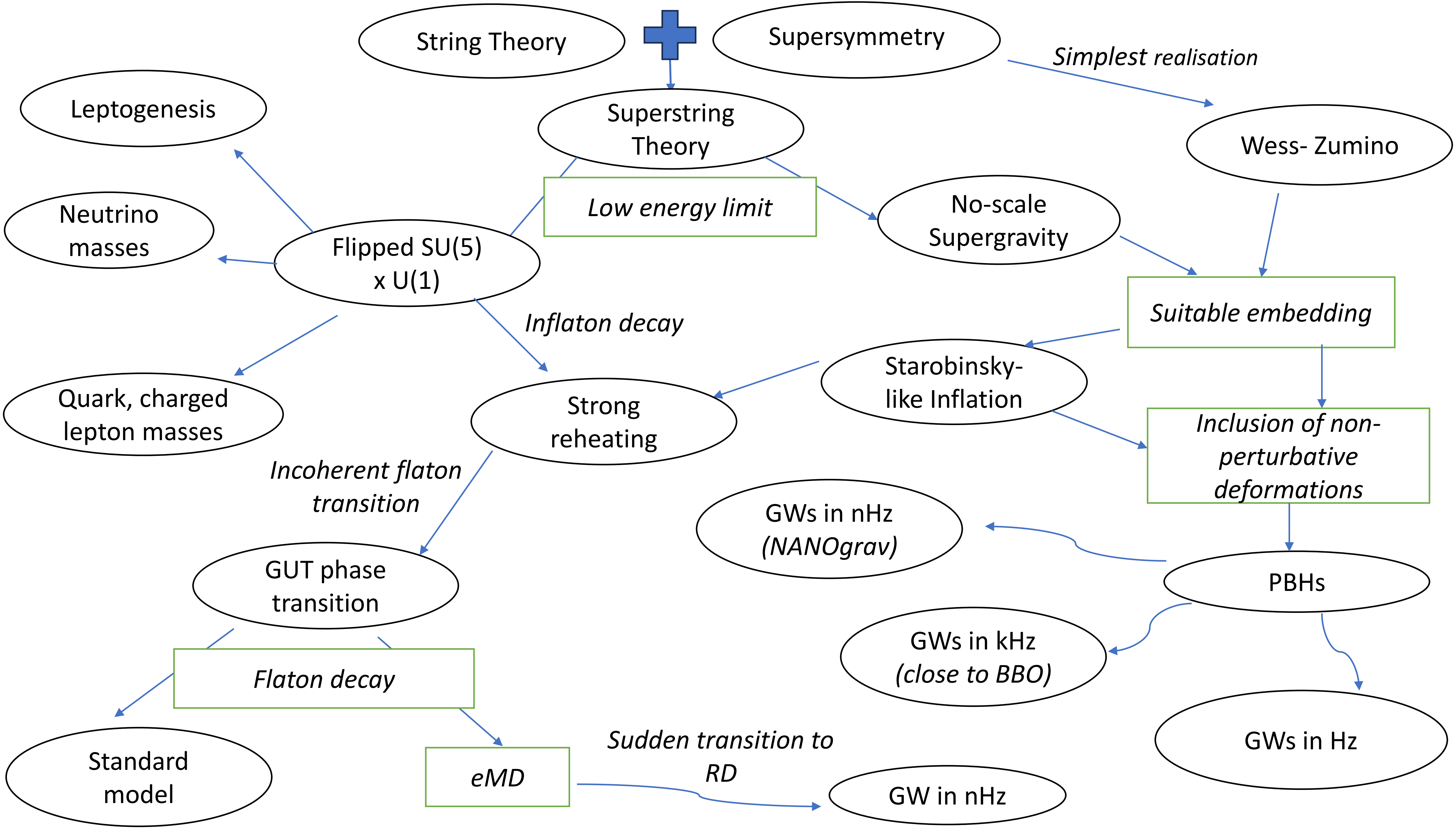
The complete three-peaked signal



A simultaneous detection of all three peaks could an indication in favor of no-scale SUGRA, or the presence of eMDs by PBHs in general

Further application

We can use a similar set-up to obtain GW signals in other theories as well


String Theory
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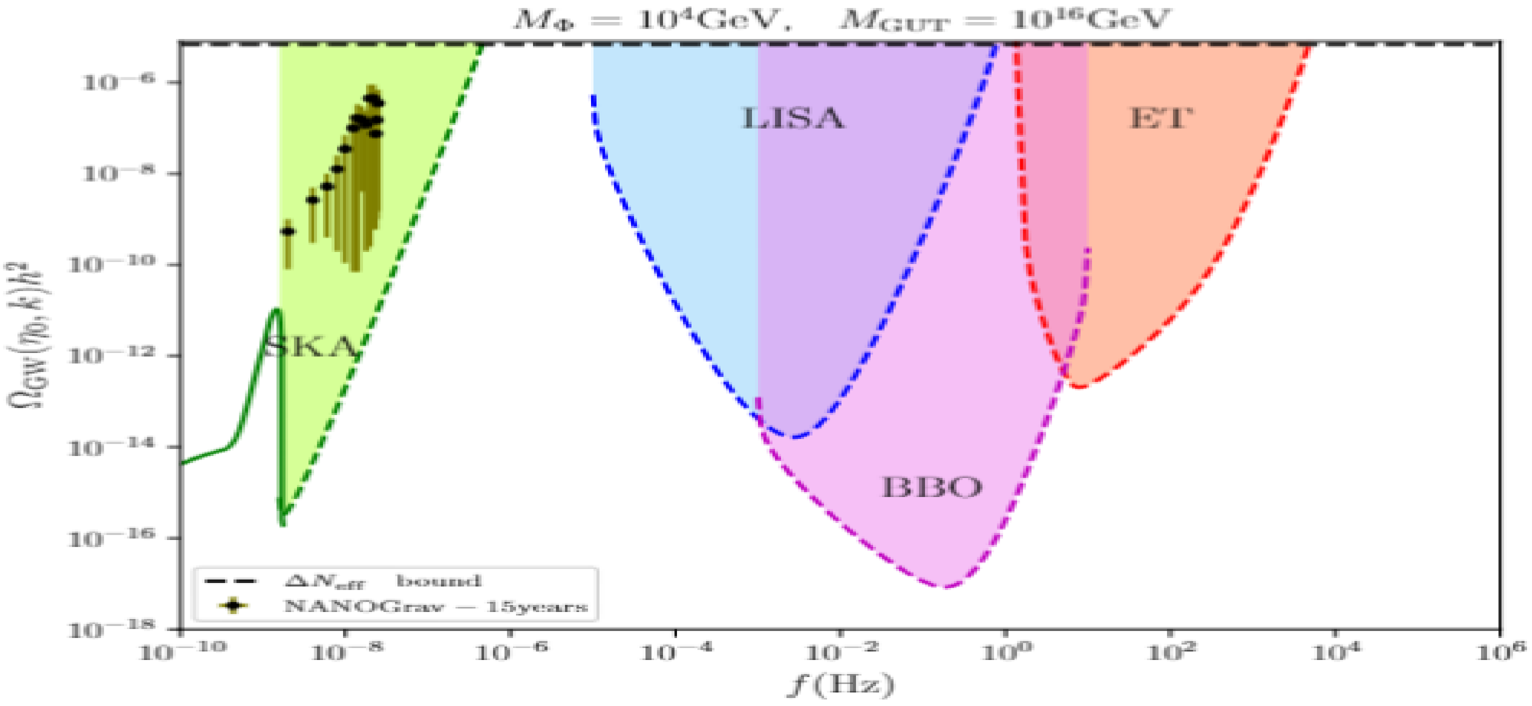
Low energy limit
No-scale Supergravity
Neutrino masses
Flipped SU(5) x U(1)
Suitable embedding
Inflaton decay
Starobinsky-like Inflation
Quark, charged lepton masses
Strong reheating
Inclusion of non-perturbative deformations
Incoherent flaton transition
GWs in nHz (NANOgrav)
GUT phase transition
PBHs
GWs in kHz (close to BBO)
Flaton decay
GWs in Hz
Sudden transition to RD
Standard model
eMD
GW in nHz

GW signal from an eMD era in flipped SU(5)

This time, instead of the PBH, the eMD is induced by the flaton field, a field responsible for the GUT phase transition (eg [Ellis, Garcia, Nagata, Nanopoulos, Olive, JCAP, 2019])

Again, the transition from eMD to lRD is *sudden* $\longrightarrow$ there is the same resonant amplification, with frequency:

$$f_{\rm GW,peak} = \frac{k_{\rm NL}}{2\pi a_0} = \frac{k}{2\pi a_{\rm d\Phi}}\frac{a_{\rm d\Phi}}{a_{\rm eq}}\frac{a_{\rm eq}}{a_0} = \frac{k}{2\pi a_{\rm d\Phi}}\left(\frac{\rho_{\rm eq}}{\rho_{\rm d\Phi}}\right)^{1/4}\left(\frac{\rho_0}{\rho_{\rm eq}}\right)^{1/3}$$
$$= 1.5\times 10^{-9}\left(\frac{\lambda_{1,3,5,7}}{0.5}\right)^2\left(\frac{m_\Phi}{10^4{\rm GeV}}\right)^{3/2}\left(\frac{10^{16}{\rm GeV}}{M_{\rm GUT}}\right).$$



Conclusions

- We worked within **NSWZ**, a framework which gives rise to **Starobinski inflation** compatible with the Planck data, namely $n_s = 0.96$ and $r < 0.004$.

- Through the deformed Kahler potential and our choice of fiducial parameters, we obtain **ultra-light PBHs** which give rise to an **eMD** and evaporate before BBN.

- We derived the GWs power spectrum produced by i) **adiabatic inflationary curvature perturbations** and ii) **isocurvature perturbations** due to fluctuations of the number density of PBHs . Both processes are **amplified by the eMD** driven by the PBHs.

- The produced GW signal has a characteristic **three-peak form**: At **nHz**, **Hz** and **kHz**, in agreement with the **PTA data**. The simultaneous detection of **all three peaks** can constitute a clear indication of the plausibility of this model.
- The idea of this set-up can be extended to other models which can accommodate **eMDs** with **sudden** transition to lRD.
- There is further unexplored phenomenology: during the eMD there could be another phase of PBH formation etc
- In general, this framework and its extensions can provide a new portal to exploring and potentially discriminating between otherwise viable models.

Appendix: SU(5) flipped outline