Gravitational waves and Planck constraints from PBH dark matter seeded by multifield inflation

Sarah Geller †

Paris Workshop on Primordial Black Holes and Gravitational Waves

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NSF MPS postdoc fellow, SCIPP, UCSC, (MIT) †

PREVIOUSLY ON P·B·H·S A·N·D G·W·S

- fundamental work on MFI models: Renaux-Patel with Langlois, Steer, Tanaka, Tasinato, McAllister, Xu, Turzynski:2008-2015)
- single-field attractor behavior in MFI models: (Kaiser and Sfakianakis:2013, f.b. Linde and Kallosh: 2013)
- Other more specific multifield models studied e.g.
  - ➡ Higgs (Bezrukov,Shaposhnikov:2008, Greenwood et.al.2013, others)
  - ➡ hybrid (Garcia-Bellido with Wands and Linde:1996, with Lyth:2011 and Cleese:2015)
  - ➡ $\alpha-$attractor models (Kallosh, Linde, and others: 2013-)
- Single field plateau (Garcia-Bellido and Ruiz-Morales: 2017)

Variously studied USR, PBH production, isocurvature models, a and CMB constraints in these specific models or more general toy models

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Do PBHs that can account for all of DM occur as a result of collapse of density perturbations ***from MFI with non-minimal couplings***?

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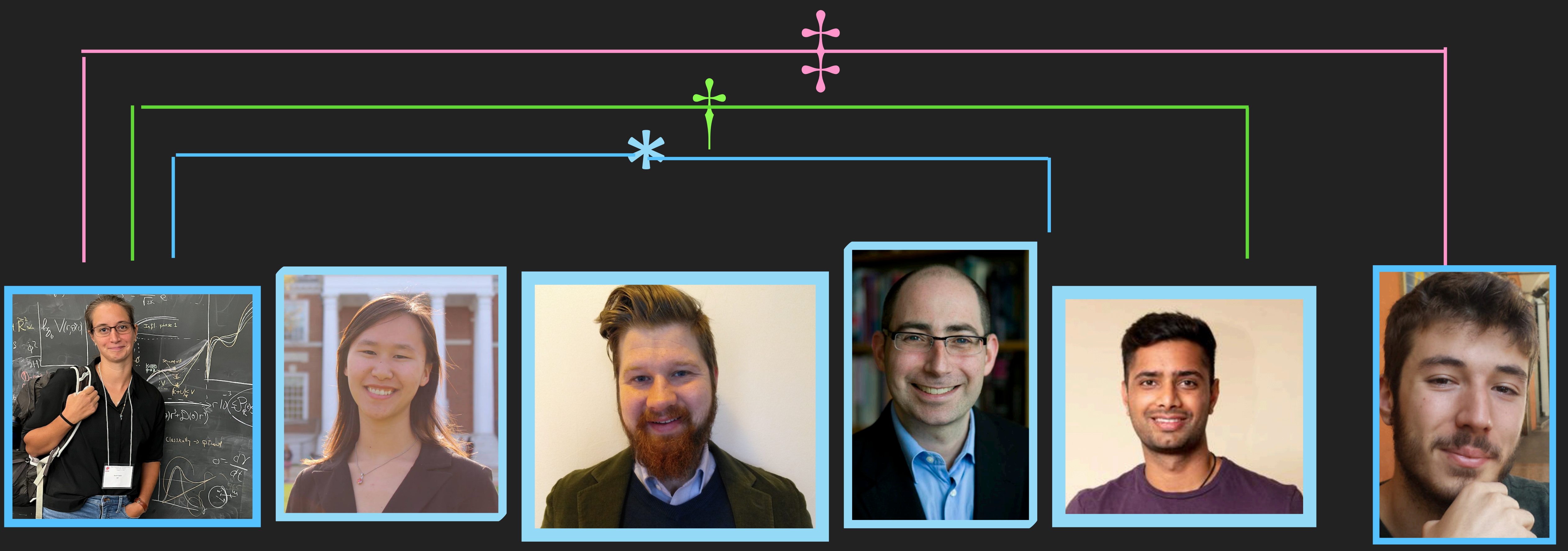
What is the ***predicted SGWB signature and SNRs for new/old observations***? * † ‡



(* 2205.04471 *Geller*, Qin, McDonough, Kaiser)

(† 2303.02168 Qin, *Geller*, Balaji, McDonough, Kaiser)

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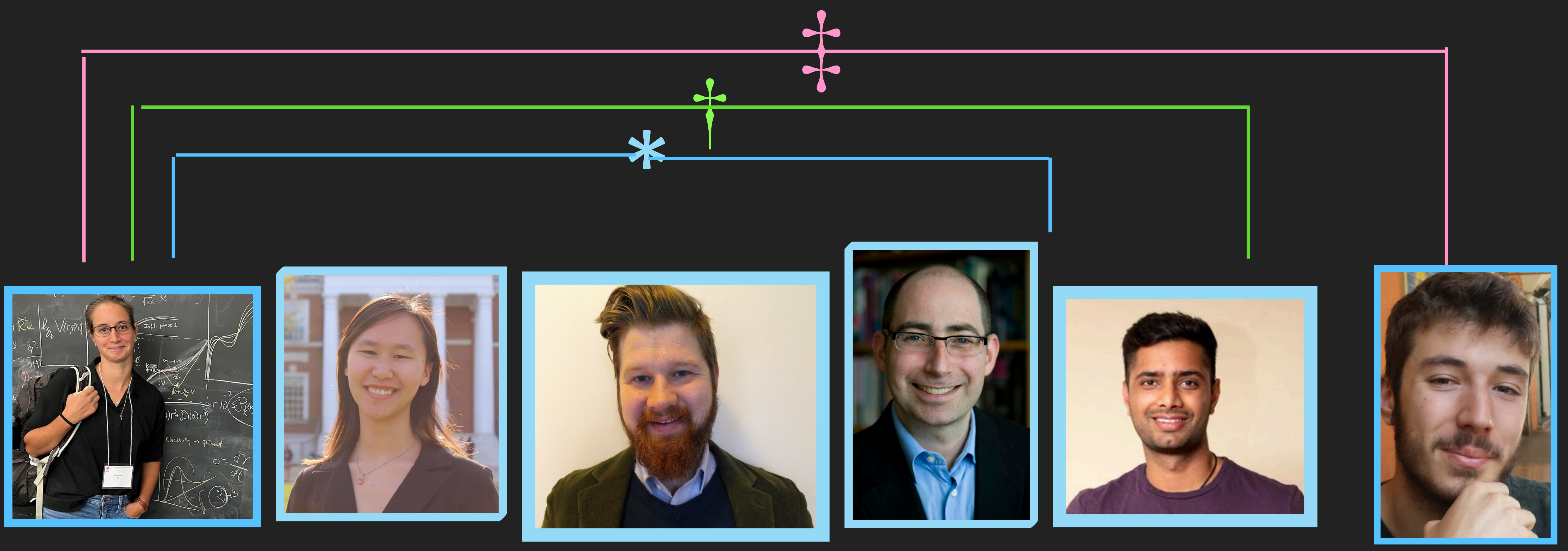
✔ To understand how ***multifield*** inflation with ***non-minimal couplings*** can generate PBHs



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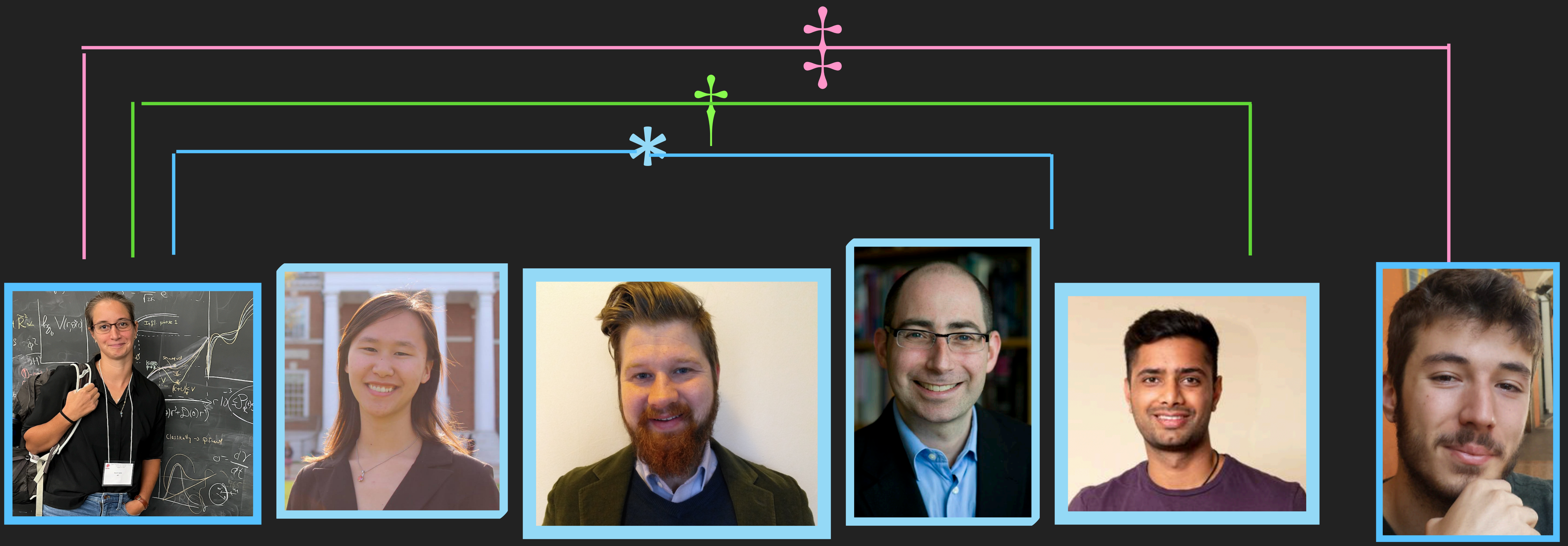

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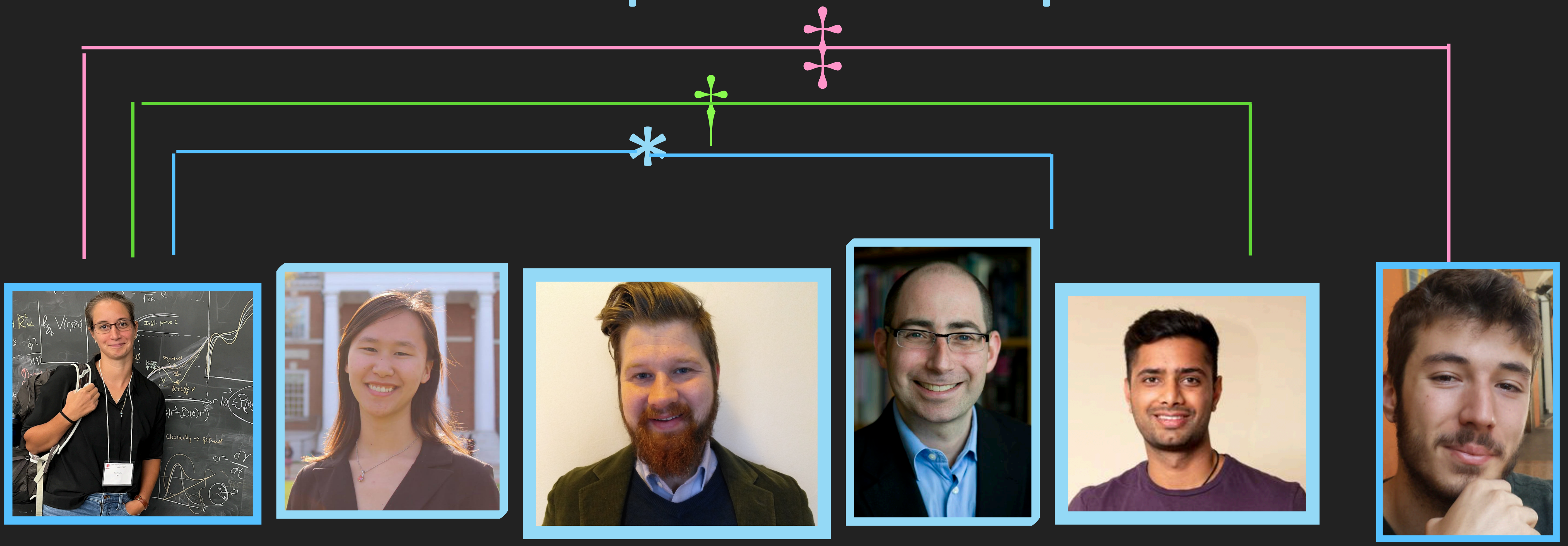
WE NEED:

- ✓ (To understand how ***multifield*** inflation with ***non-minimal coupli*** CAVEAT generate PBHs)
- ✓ To understand the origin of gravitational waves from PBH fo CAVEAT tion
- Explore interplay of CMB and PBH constraints at early/late times and how they impact the available parameter space



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Realistic and **generic ingredients** from high energy theory

Multifield Models $\sim \phi^I(x^\mu)$

- Field theories (FTs) at high energies generically have > 1 scalar d.o.f.
- BSM theories have even more! e.g. MSSM
- in some types of inflation, avoids topological instabilities

Non-minimal couplings $f(\phi^I) \supset \xi_I \left(\phi^I\right)^2$

- Self interacting ϕ^I in curved spacetime induce non-minimal couplings (loop corrections)
- RG flow of couplings ↑ with no UV fixed point.
- EFT thinking: all well-behaved dim-4 operators consistent with symmetries should be included in the action.

Jordan Frame Action:

$$\tilde{S} = \int d^4x\sqrt{-\tilde{g}}\left[f\left(\phi^I\right)\tilde{R} - \frac{1}{2}\delta_{IJ}\tilde{g}^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \tilde{V}\left(\phi^I\right)\right]$$ **with** $$\ddagger\, f(\phi^I) = \frac{1}{2}\left[M_{\rm pl}^2 + \sum_{I=1}^{N}\xi_I(\phi^I(x^\mu))^2\right]$$

non-minimal couplings

Conformal Transformation

$$\tilde{g}^{\mu\nu} \to g^{\mu\nu} = \Omega^{-2}(x)\tilde{g}^{\mu\nu}$$

Einstein Frame Action:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\mathcal{G}_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$

Einstein Frame

trade-off: non-canonical kinetic terms but usual Einstein-Hilbert (gravitational coupling) term.

Induces curvature on field space, $\mathcal{G}_{IJ}(\Phi^K)$

Stretches potential by factor of $\dfrac{M_{\rm pl}^4}{4f^2(\phi^I)}$

$\Longrightarrow$EOM: $\mathcal{D}_t\dot{\varphi}^I + 3H\dot{\varphi}^I + \mathcal{G}^{IK}V_{,K} = 0$

Jordan Frame (Effective) Action:

$$\tilde{\mathbf{S}} = \int \mathbf{d^4x}\sqrt{-\tilde{\mathbf{g}}}\left[\mathbf{f}\left(\phi^{\mathbf{I}}\right)\tilde{\mathbf{R}} - \frac{\mathbf{1}}{\mathbf{2}}\delta_{\mathbf{IJ}}\tilde{\mathbf{g}}^{\mu\nu}\partial_\mu\phi^{\mathbf{I}}\partial_\nu\phi^{\mathbf{J}} - \tilde{\mathbf{V}}\left(\phi^{\mathbf{I}}\right)\right]$$ with $\ddagger\, f(\phi^I) = \frac{1}{2}\left[M_{\text{pl}}^2 + \sum_{I=1}^{N}\xi_I(\phi^I(x^\mu))^2\right]$

The SUGRA "UV" embedding:

$\mathcal{N} = 1$, Four-dimensional supergravity with 2 chiral superfields $\mathbf{\Phi}(x,\theta)^I = \varpi^I + \sqrt{2}\theta\eta^I + \theta\theta F^I$

$\varpi^I \to \frac{1}{\sqrt{2}}(\phi^I + i\psi^I)$

Model specified by: Superpotential (C-W) $\tilde{W} = \mu b_{IJ}\Phi^I\Phi^J + c_{IJK}\Phi_I\Phi_J\Phi_K$

Khäler potential: $\tilde{K}(\Phi,\bar{\Phi}) = -\frac{1}{2}\sum_{I=1}^{2}(\Phi^I - \bar{\Phi}^{\bar{I}})^2$

Jordan frame effective potential:

$$\tilde{V} = \exp\left(\frac{\tilde{K}}{M_{\text{pl}}^2}\right)\left[|D\tilde{W}|^2 - 3M_{\text{pl}}^{-2}|\tilde{W}|^2\right]\Bigg|_{\Phi^I\to\varpi^I,\bar{\Phi}^{\bar{I}}\to\bar{\varpi}^{\bar{I}}}$$

$\to$ scales of interest $\tilde{V} < M_{\text{pl}}^4$

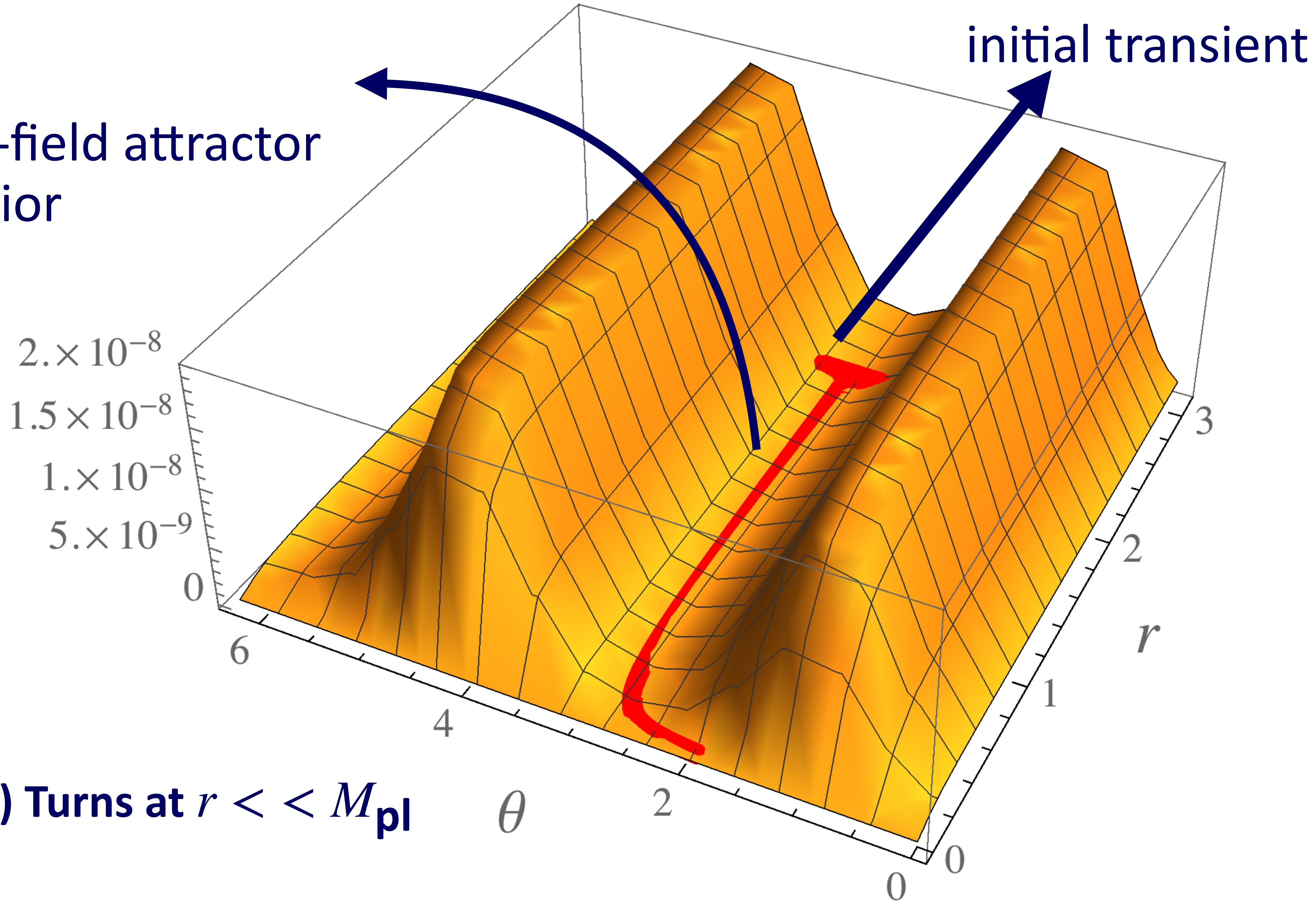
The 2-field inflaton potential and (exact) field space trajectories when $b_1 = b_2 = b$ and $c_3 = c_2$

$$V(r,\theta) = \frac{1}{4f^2(r,\theta)}\left(\mathscr{B}(\theta)r^2 + \mathscr{C}(\theta)r^3 + \mathscr{D}(\theta)r^4\right)$$

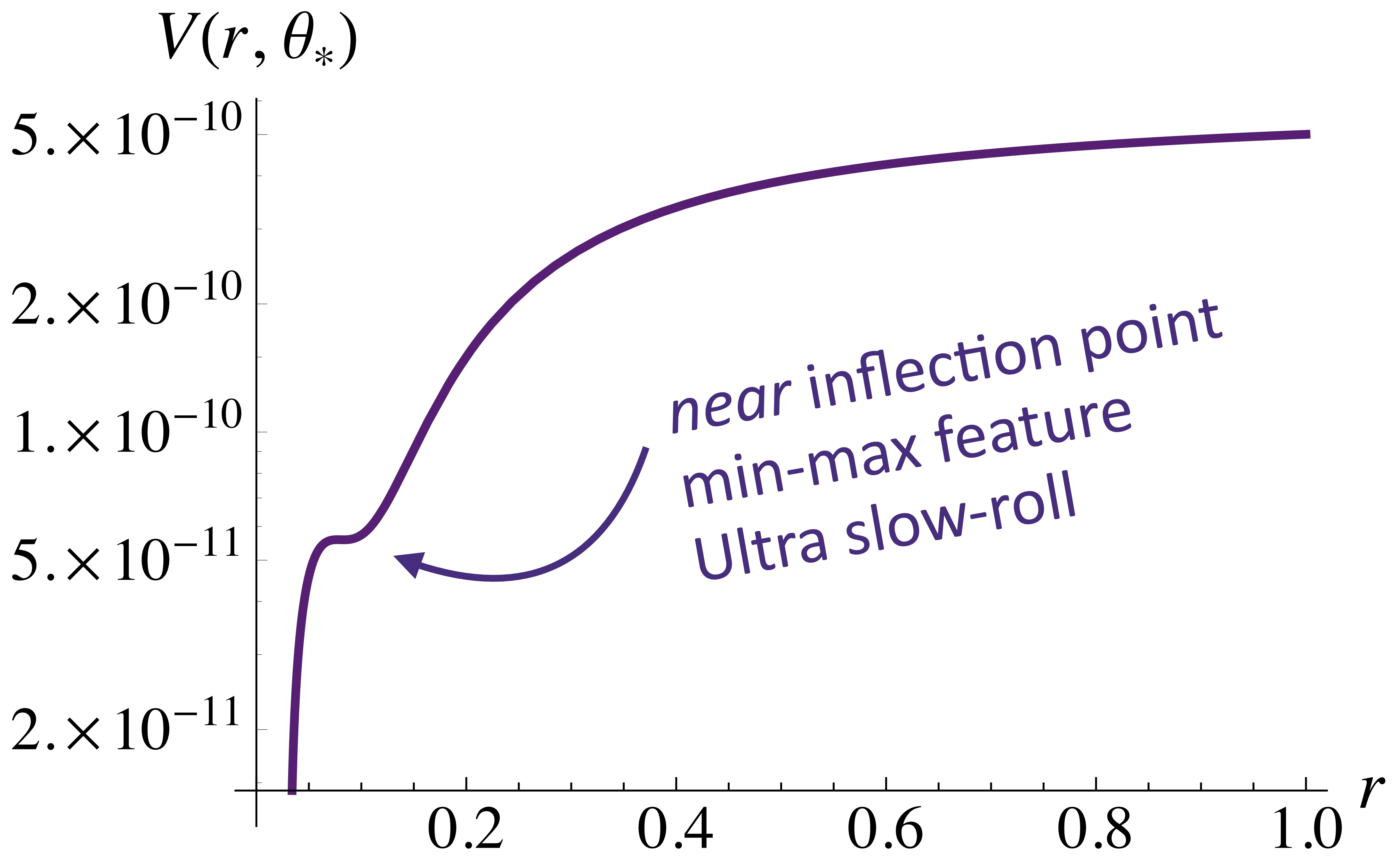
where $\mathscr{B}, \mathscr{C},$ and $\mathscr{D}$ depend on (b, c_1, c_2, c_4)

Exact field-space trajectories $\theta_*^{\pm}(r)$ are analytic solutions of $\partial_\theta V(r,\theta_*) = 0$

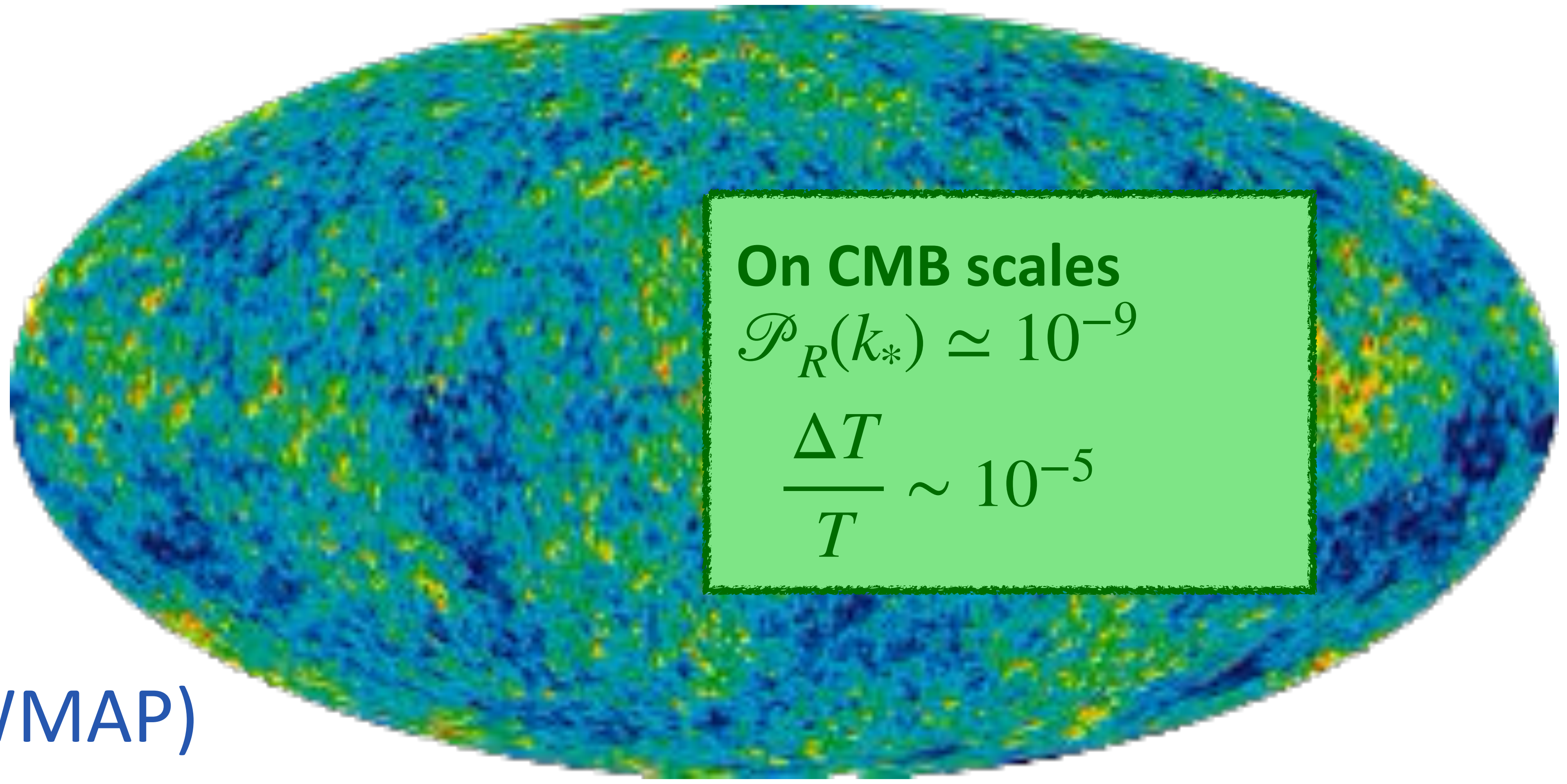
single-field attractor behavior



(slight) Turns at $r << M_{\text{pl}}$



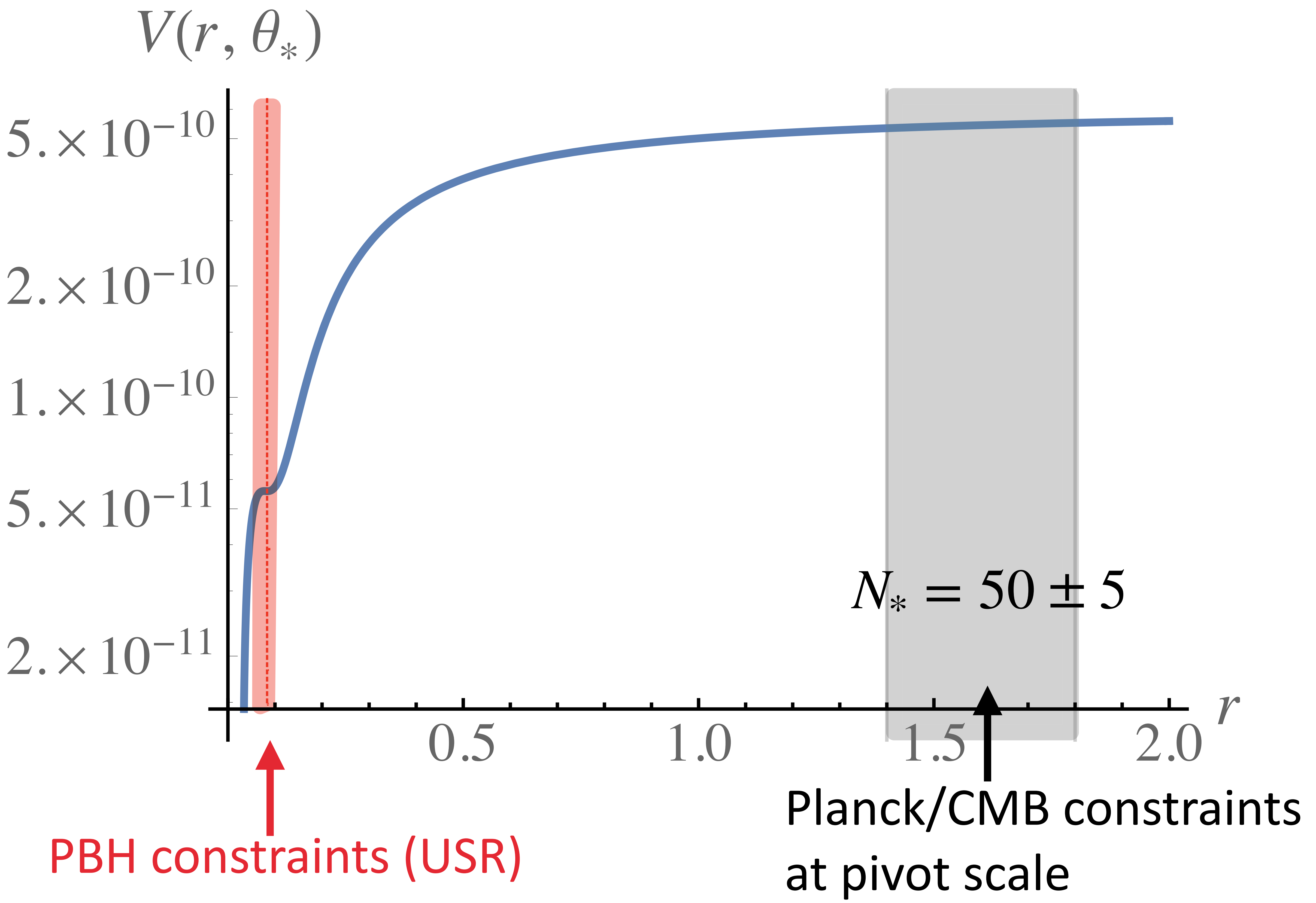
Cosmic Microwave Background vs PBH formation constraints



(WMAP)

Planck 2018: gives constraints at "pivot scale"
$k_* = .05\text{Mpc}^{-1} \simeq N_* = 55 \pm 5$ e-folds

For a scalar 2-field potential, $V(r, \theta)$
(single-field attractor model)



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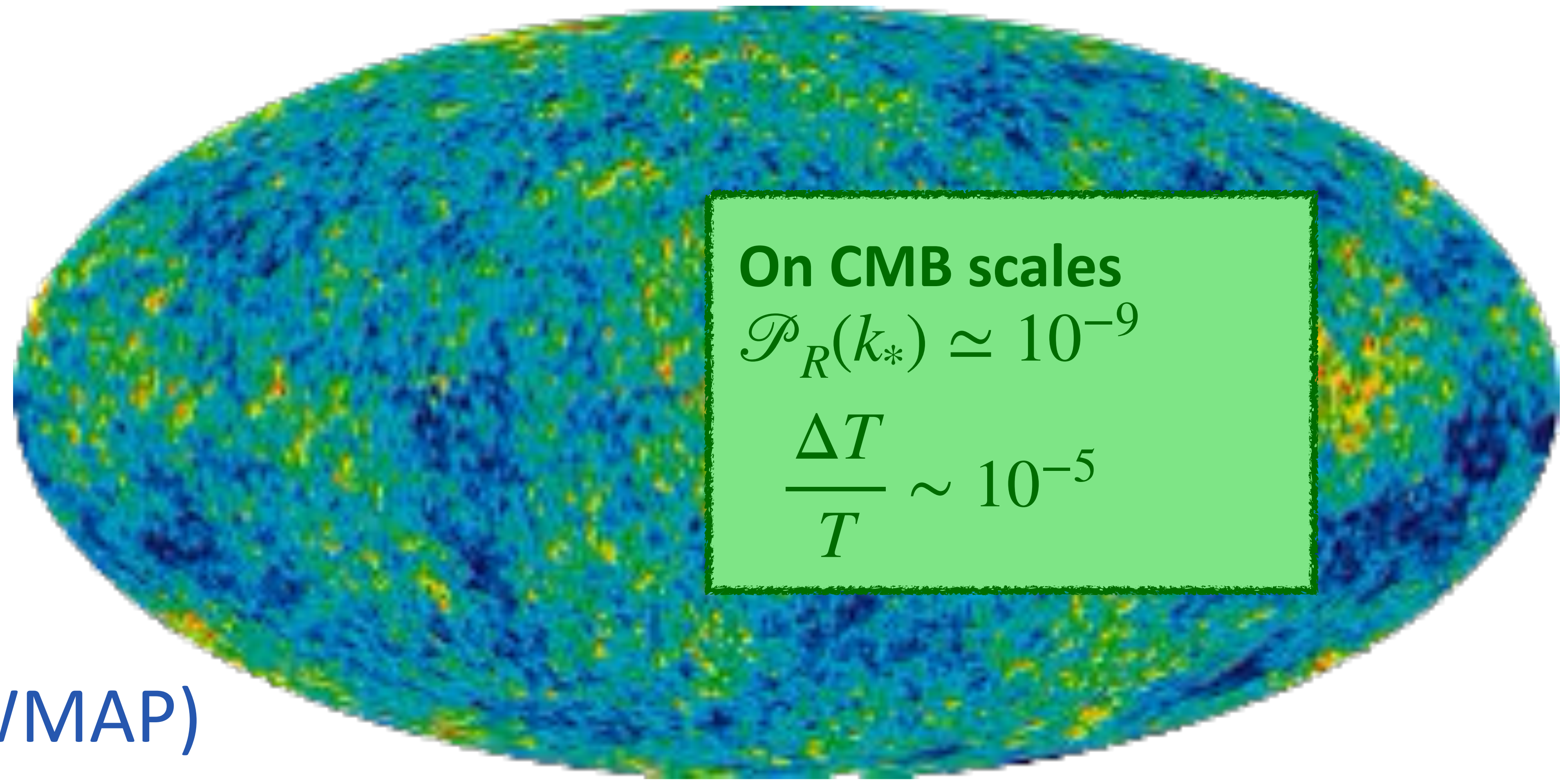
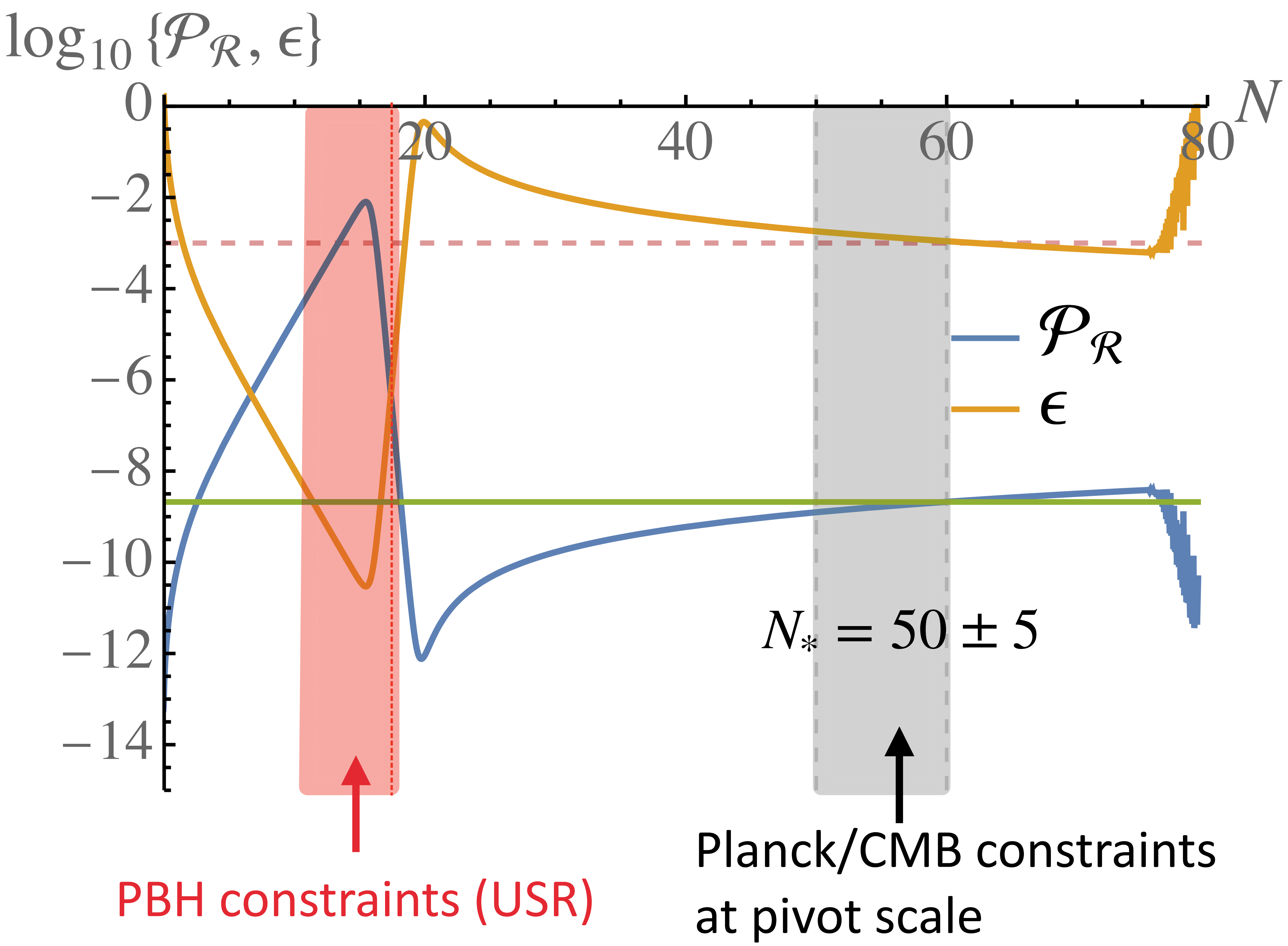


(WMAP)

Visualized on the power spectrum

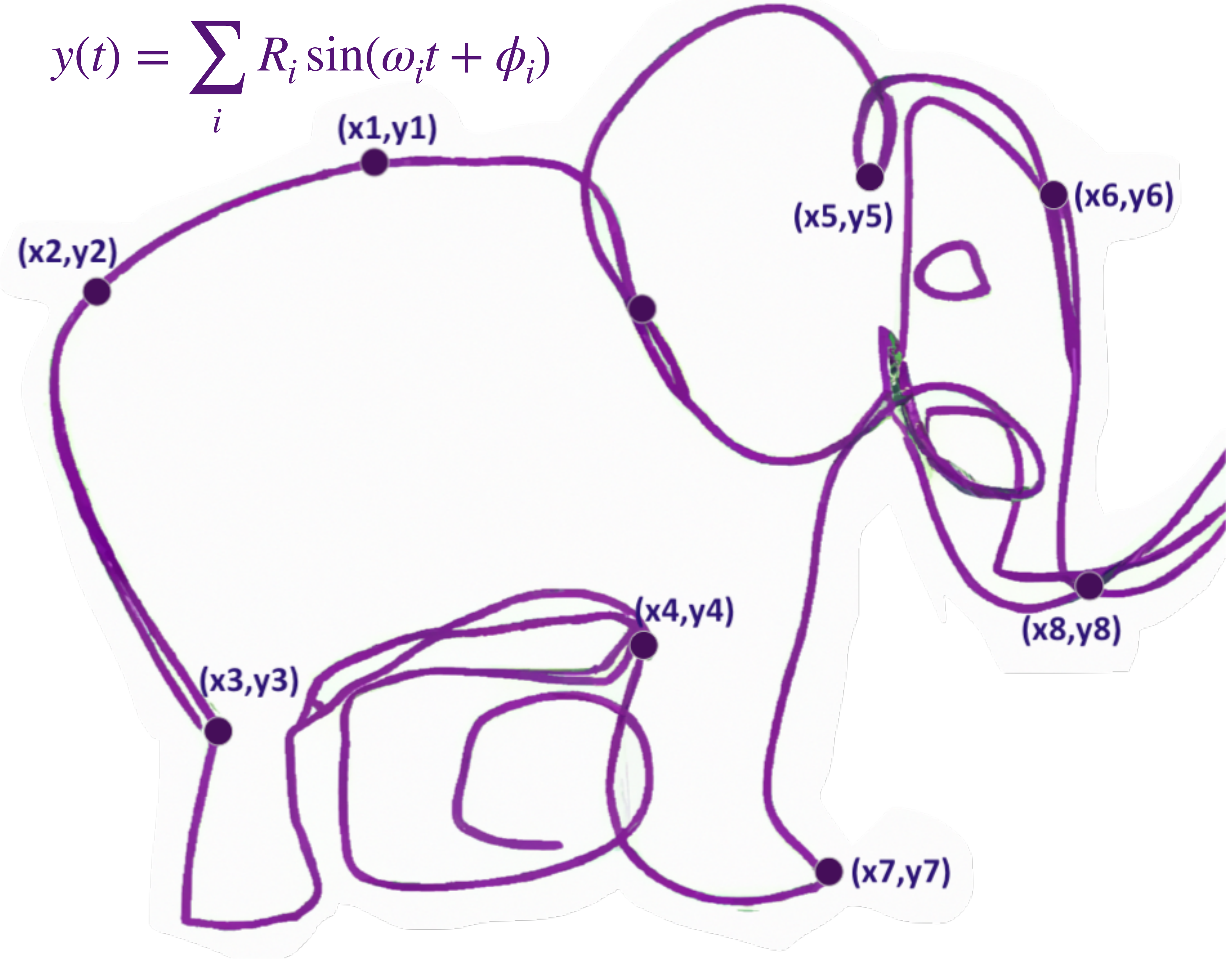


Planck 2018: gives constraints at "pivot scale"
$k_* = .05\text{Mpc}^{-1} \simeq N_* = 55 \pm 5$ e-folds

"With four parameters I can fit an elephant and with five I can make him wiggle his trunk"
Enrico Fermi to John Von Neumman
(https://www.nature.com/articles/427297a)

$$x(t) = \sum_i R_i \cos(\omega_i t + \phi_i)$$

$$y(t) = \sum_i R_i \sin(\omega_i t + \phi_i)$$



Observables & Constraints

CMB

- $n_s(k_*)$ spectral index
- $\alpha(k_*)$ running of spectral index
- $r(k_*)$ tensor-to-scalar ratio
- A_s normalization at k_*

N_* Number of e-folds prior to end of inflation, at k_*

- $\beta_{\text{iso}}(k_*)$ primordial isocurvature perturbations
- f_{NL} primordial non-Gaussianities (bispectra)

PBH

- $\mathscr{P}_R(k_{\text{pbh}})$ Peak amplitude of power spectrum
- ΔN e-folds remaining after $\log(\mathscr{P}_R) \geq -3$

How we match observables & constraints | **Observables & Constraints**

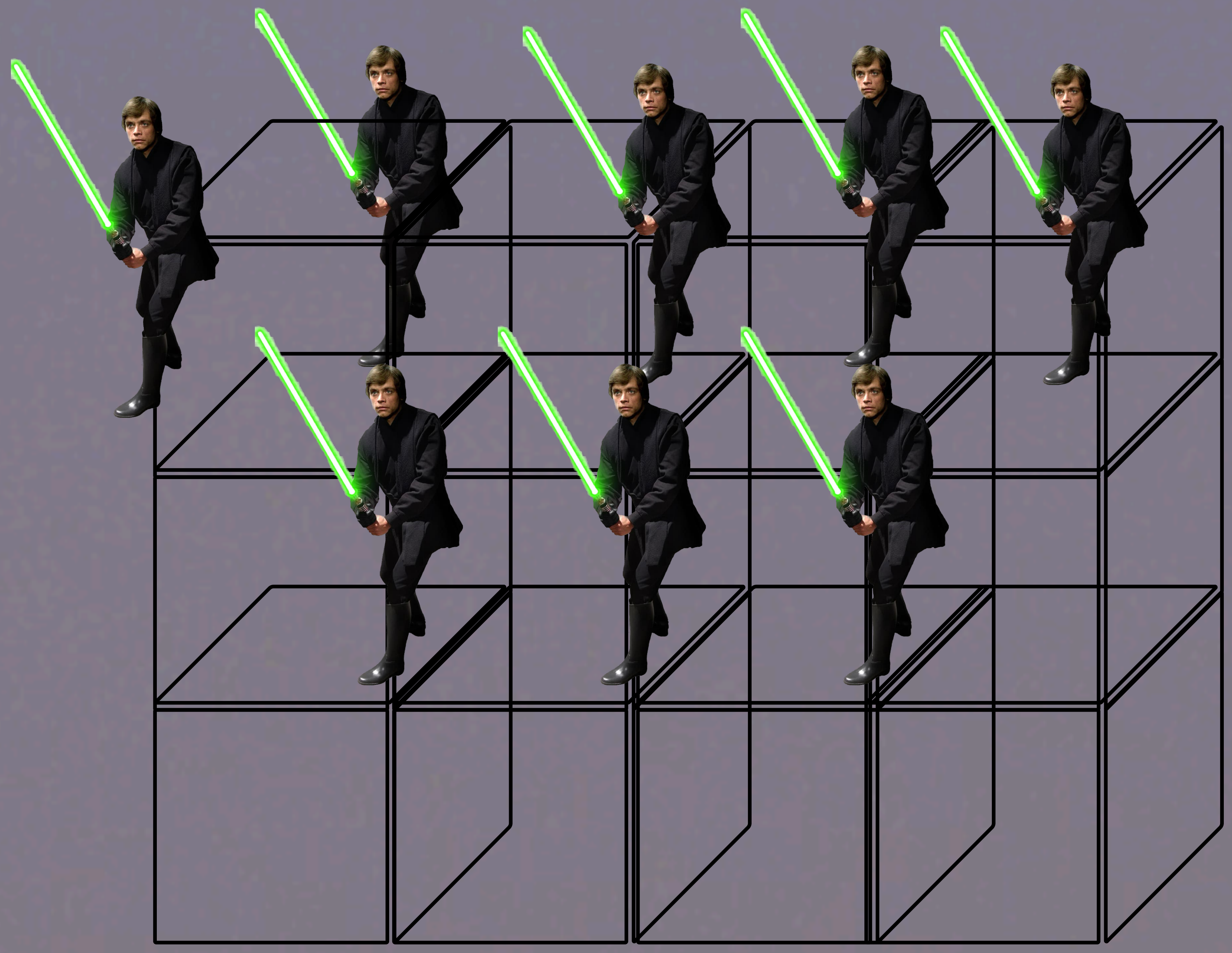
1. Use Gaussian priors ↔ *Planck* 2018, Bicep/Keck constraints on ΛCDM →
   - $n_s(k_*)$ spectral index
   - $\alpha(k_*)$ running of spectral index
   - $r(k_*)$ tensor-to-scalar ratio
   - A_s normalization at k_*

2. We *choose* value of $N_* \in [55 \pm 5]$ to *optimize* best fit to CMB observables →
   - N_* Number of e-folds prior to end of inflation, at k_*

3. Already exponentially suppressed →
   - $\beta_{\text{iso}}(k_*)$ primordial isocurvature perturbations
   - f_{NL} primordial non-Gaussianities (bispectra)

4. Enforce minimal requirement that model produces PBHs with $\frac{\Omega_{\text{pbh}}}{\Omega_{\text{DM}}} \sim \mathcal{O}(1)$ →
   - $\mathcal{P}_R(k_{\text{pbh}})$ Peak amplitude of power spectrum
   - ΔN e-folds remaining after $\log(\mathcal{P}_R) \geq -3$

MCMC, 200 walkers each taking 10,000 steps through a 4-dim parameter space



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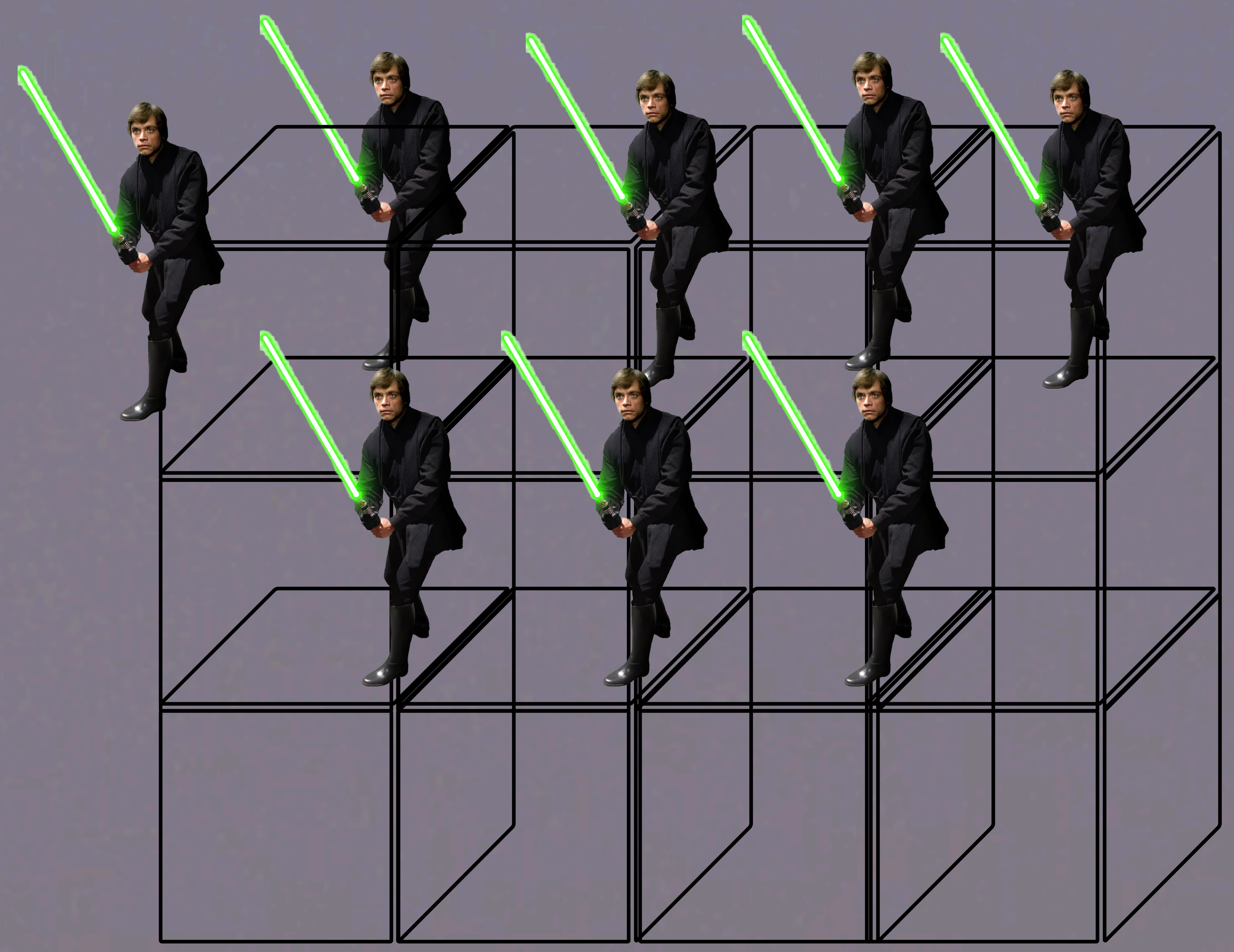

Posterior distributions on

$n_s(k_*), A_s(k_*), N_*, \alpha(k_*), r(k_*)$

***optimizing* over possible reheating scenarios**

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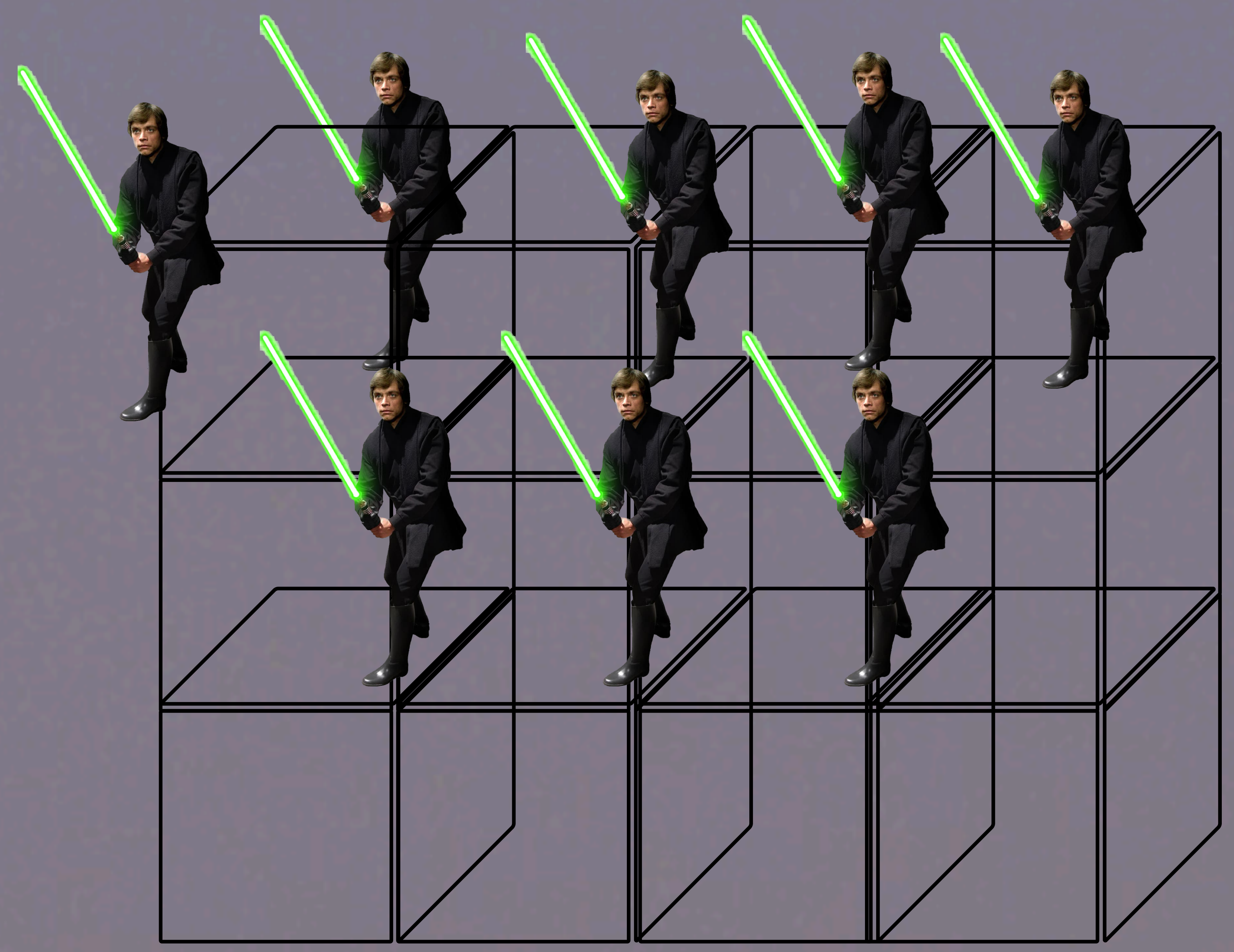

Posterior distributions on $n_s(k_*), A_s(k_*), N_*, \alpha(k_*), r(k_*)$ ***optimizing*** **over possible reheating scenarios**

Physics is driven primarily by fits to $n_s(k_*)$ **and** N_*

At higher values of N_***, prefer higher** n_s **as** N_* **decreases,** n_s **decreases**

Also correlation in ***range of*** N_* **with** n_s

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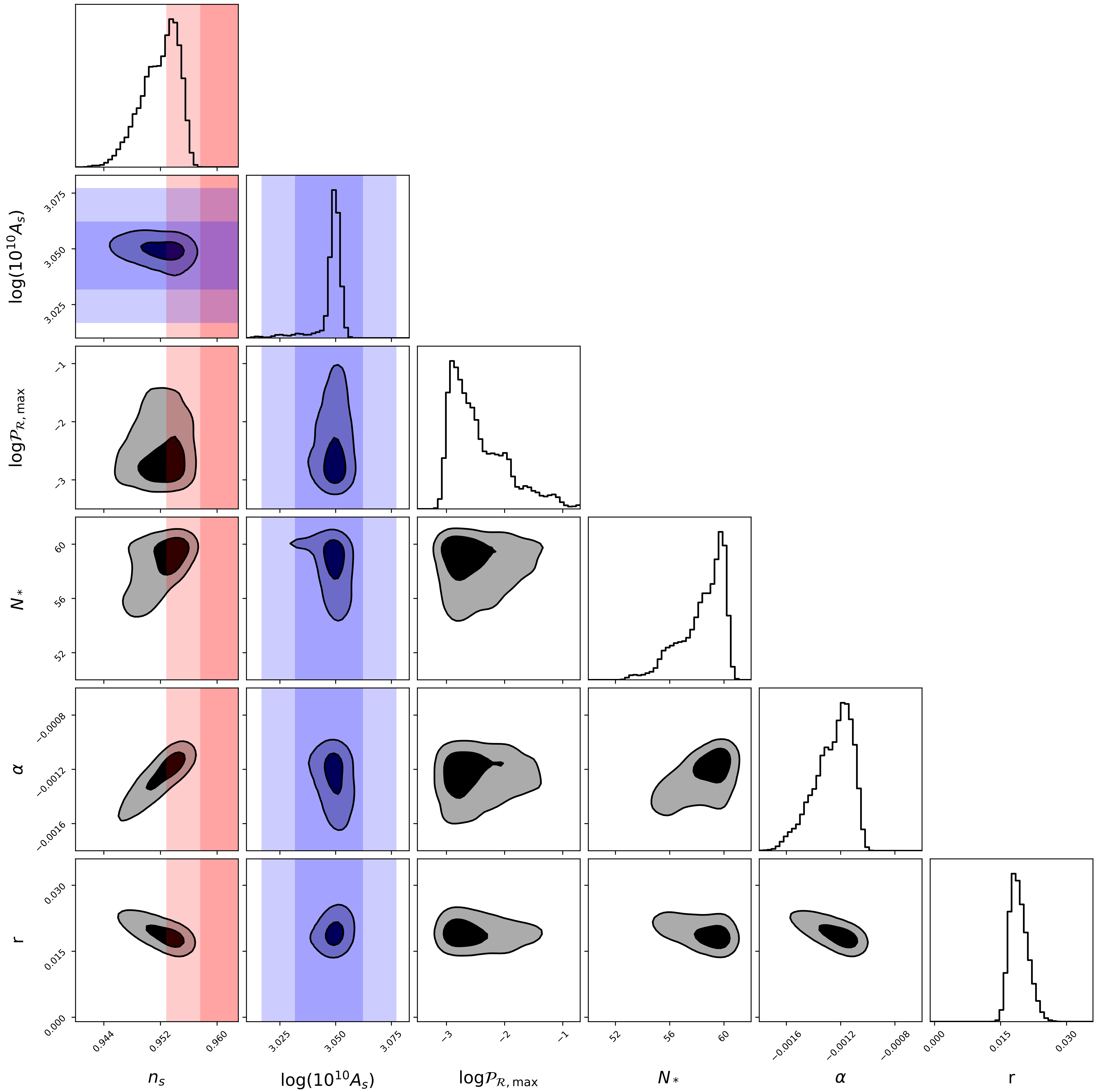
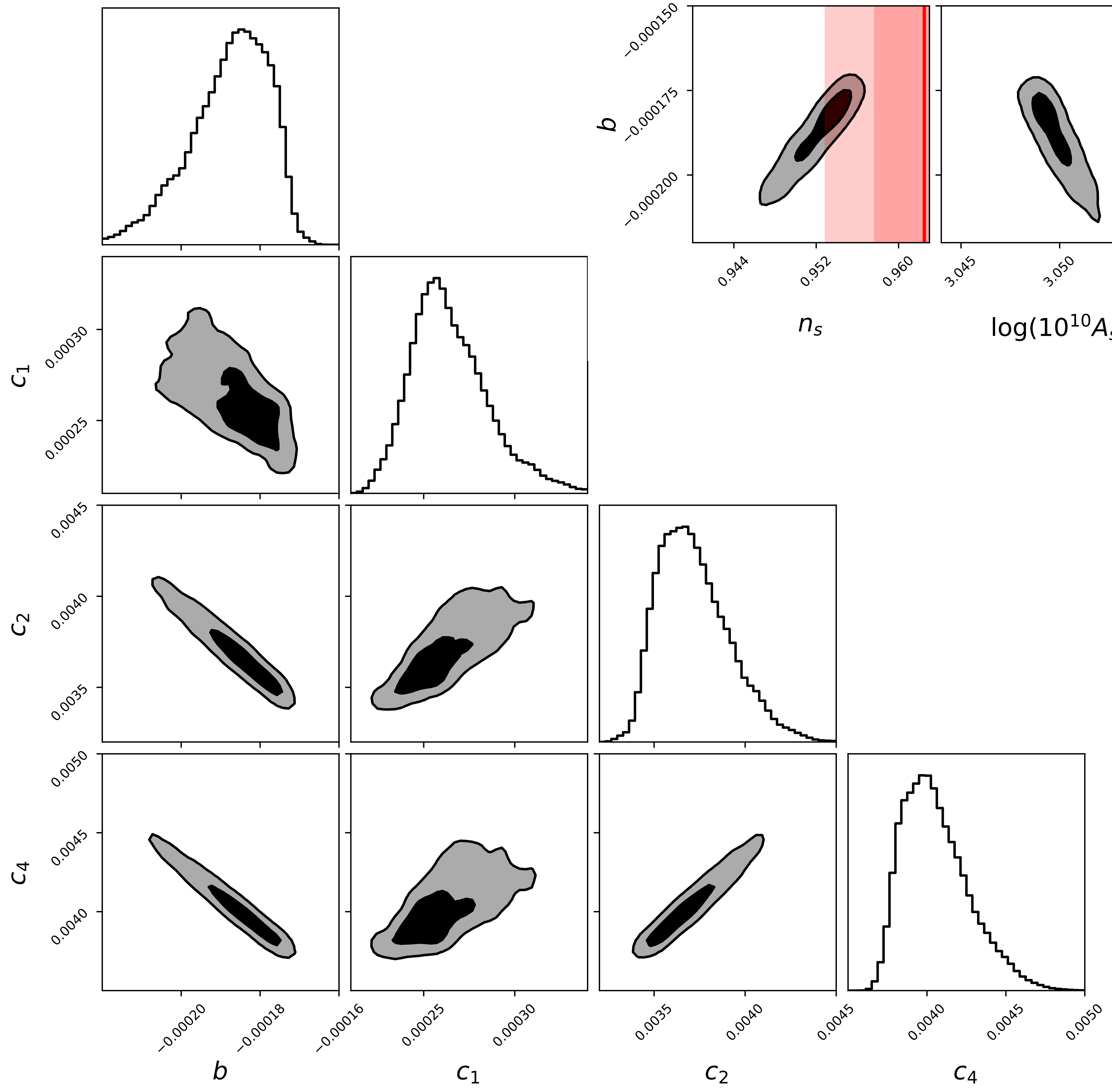
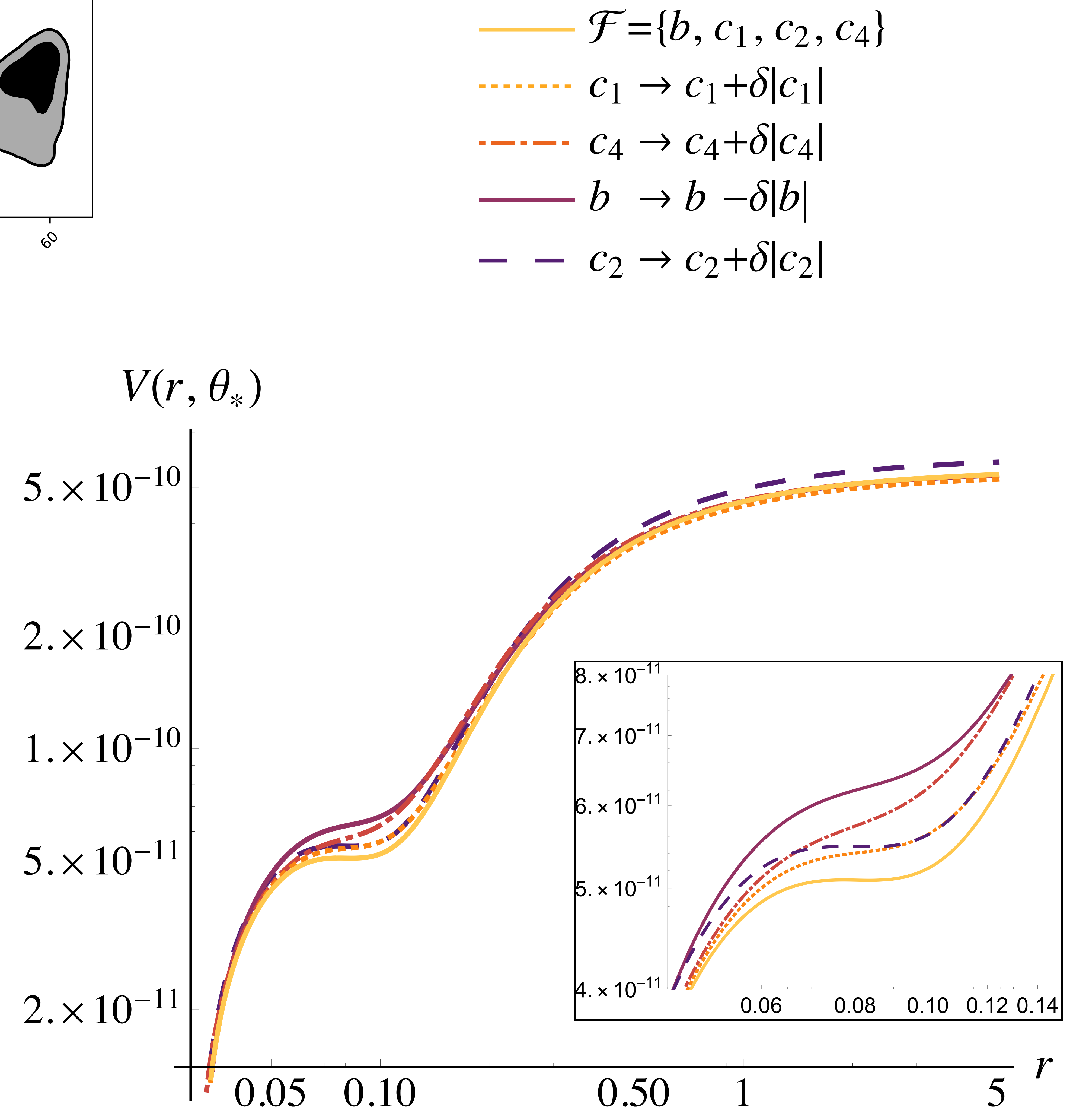
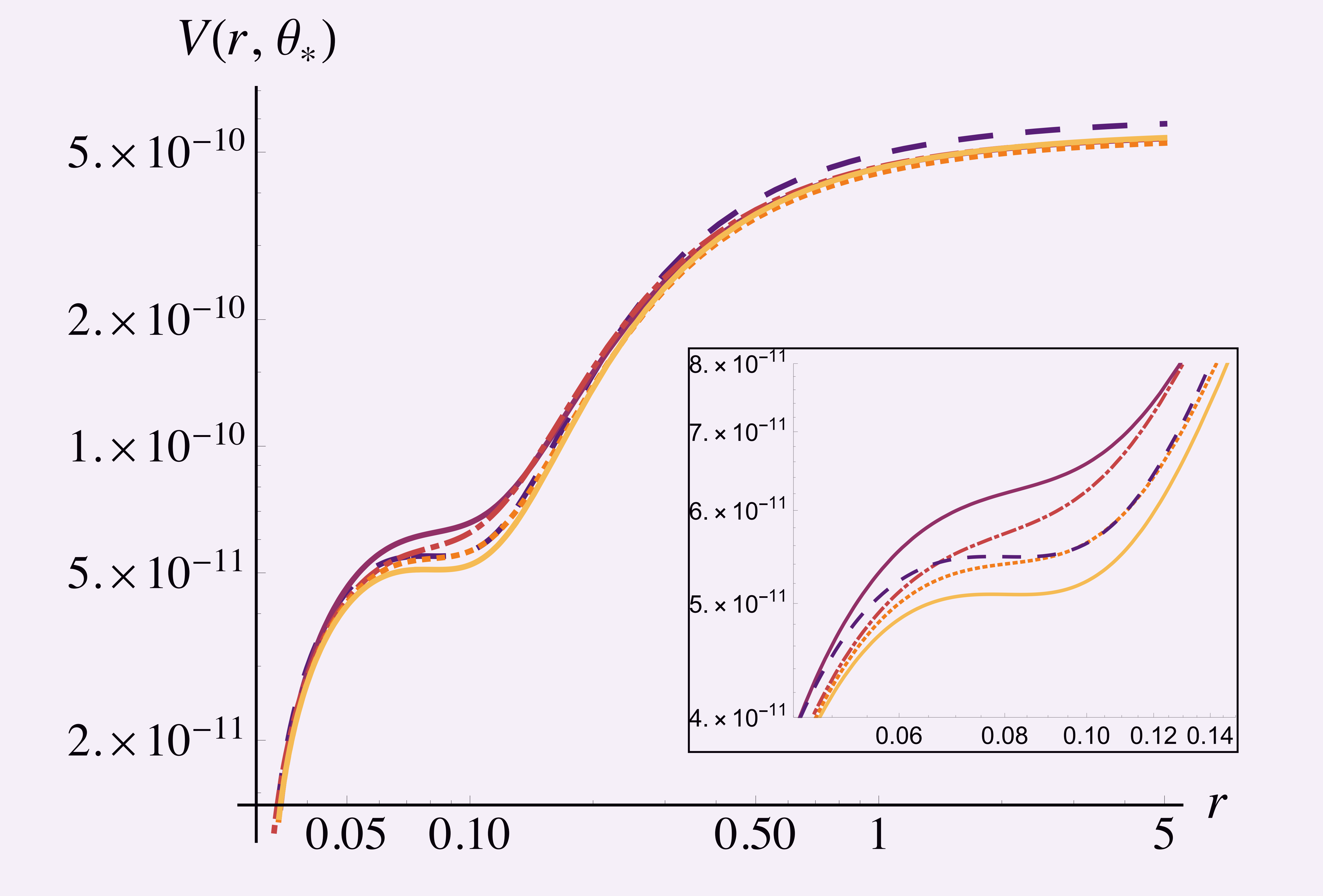


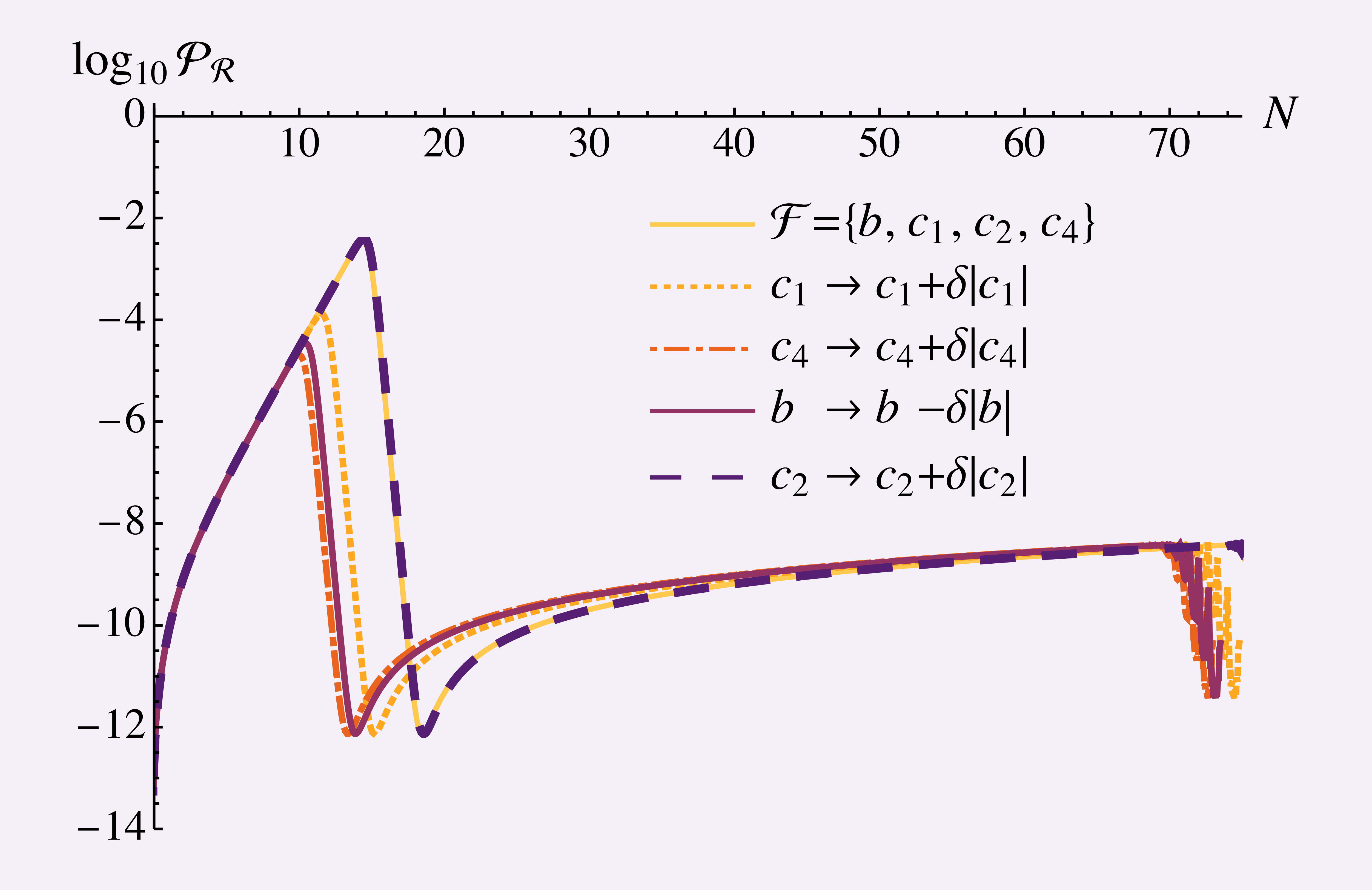

b
n_s
$\log(10^{10}A_s)$
N_*
c_1
c_2
c_4


$\mathcal{F} = \{b, c_1, c_2, c_4\}$
$c_1 \to c_1 + \delta|c_1|$
$c_4 \to c_4 + \delta|c_4|$
$b \to b - \delta|b|$
$c_2 \to c_2 + \delta|c_2|$
$V(r, \theta_*)$
r

- Visualize degeneracy in 4-dimensional parameter space (b, c_1, c_2, c_4) by varying one parameter at a time to obtain self-similar potential and power spectrum.

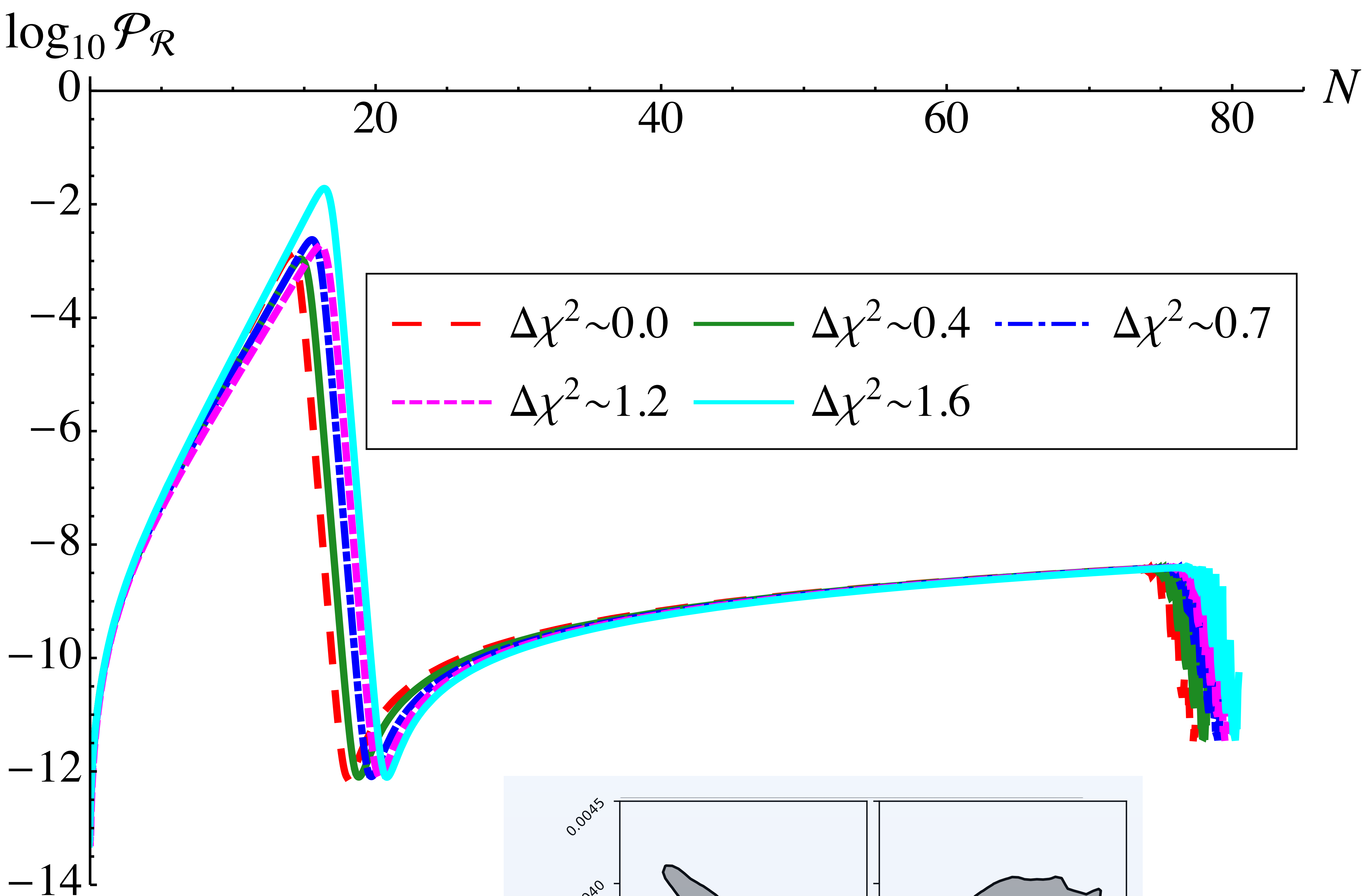
- Degeneracy $\equiv \Delta\chi^2_{\text{tot}} \leq .01$

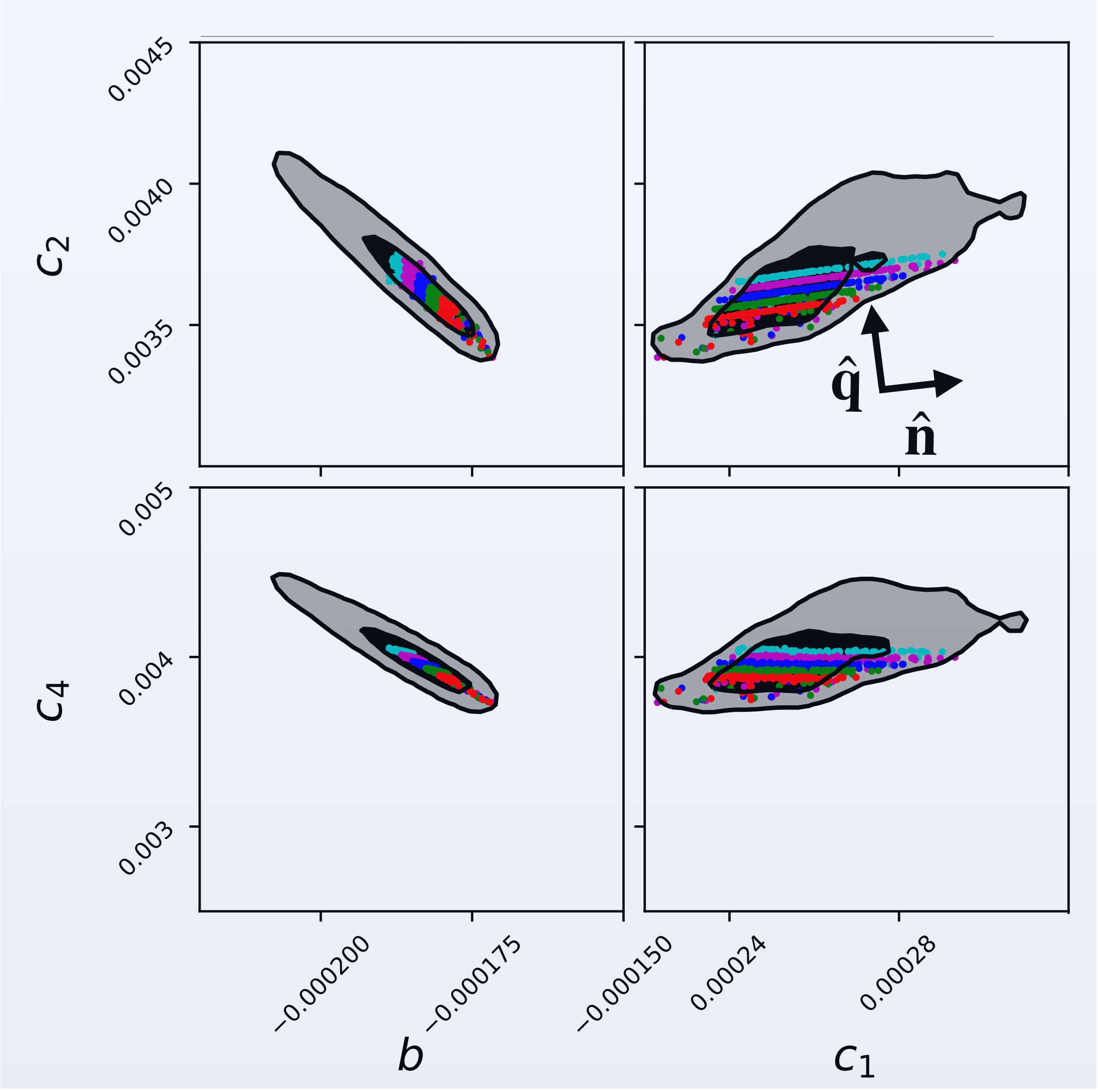




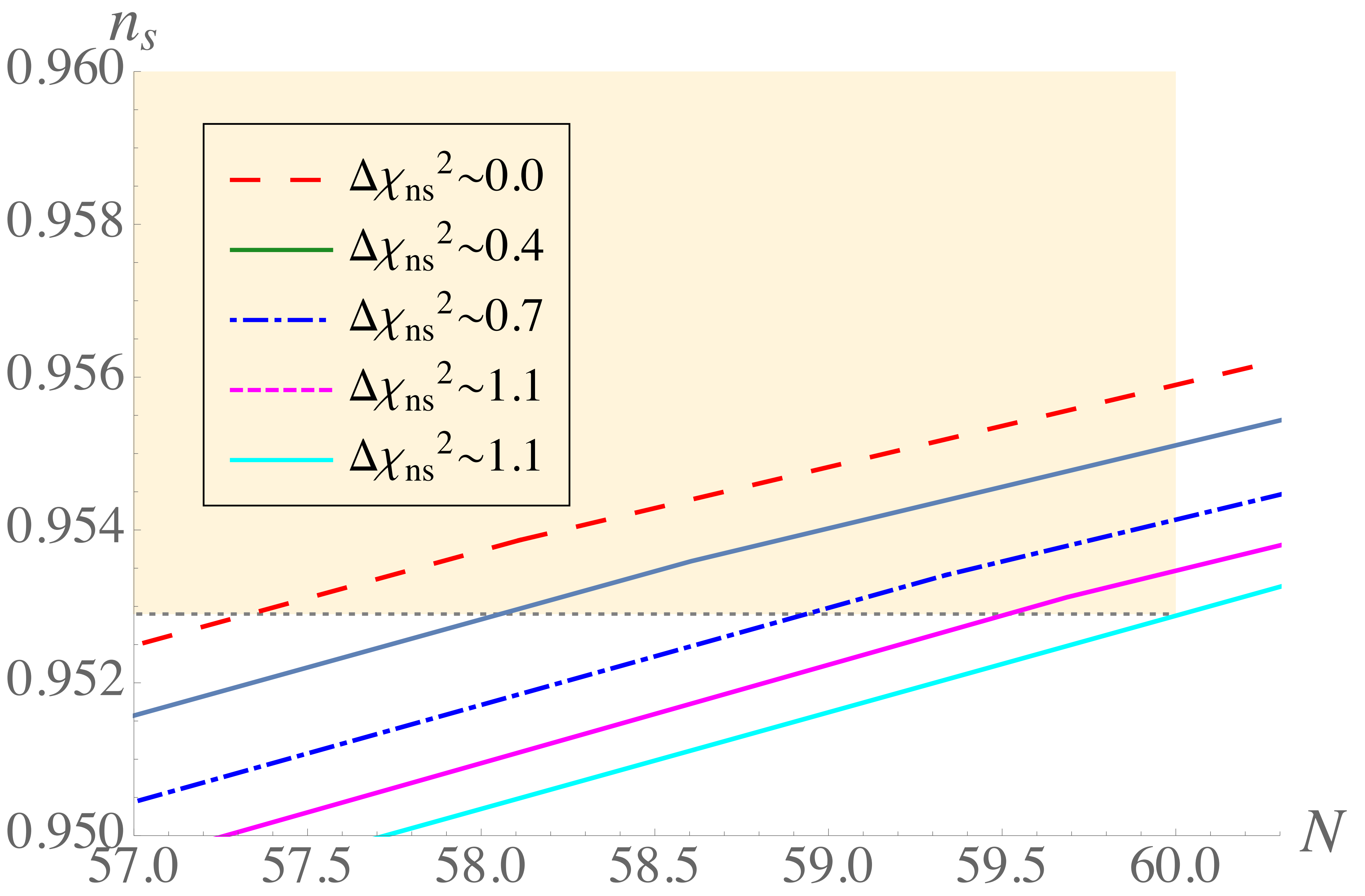
Potential variations $\delta \simeq 10^{-2}$

Power spectrum variations $\delta \simeq 10^{-6}$





- Identify five example super-sets of degenerate points
- $\hat{n}$ degeneracy direction ($\Delta\chi^2_{\text{tot}} \sim$ constant)
- $\hat{q}$ orthogonal direction



$$\Omega_{\text{GW},0}h^2 \approx 1.62 \times 10^{-5} \left(\frac{1}{24} \left(\frac{k}{aH} \right)^2 \overline{\mathcal{P}_h(k,\tau)} \right)$$

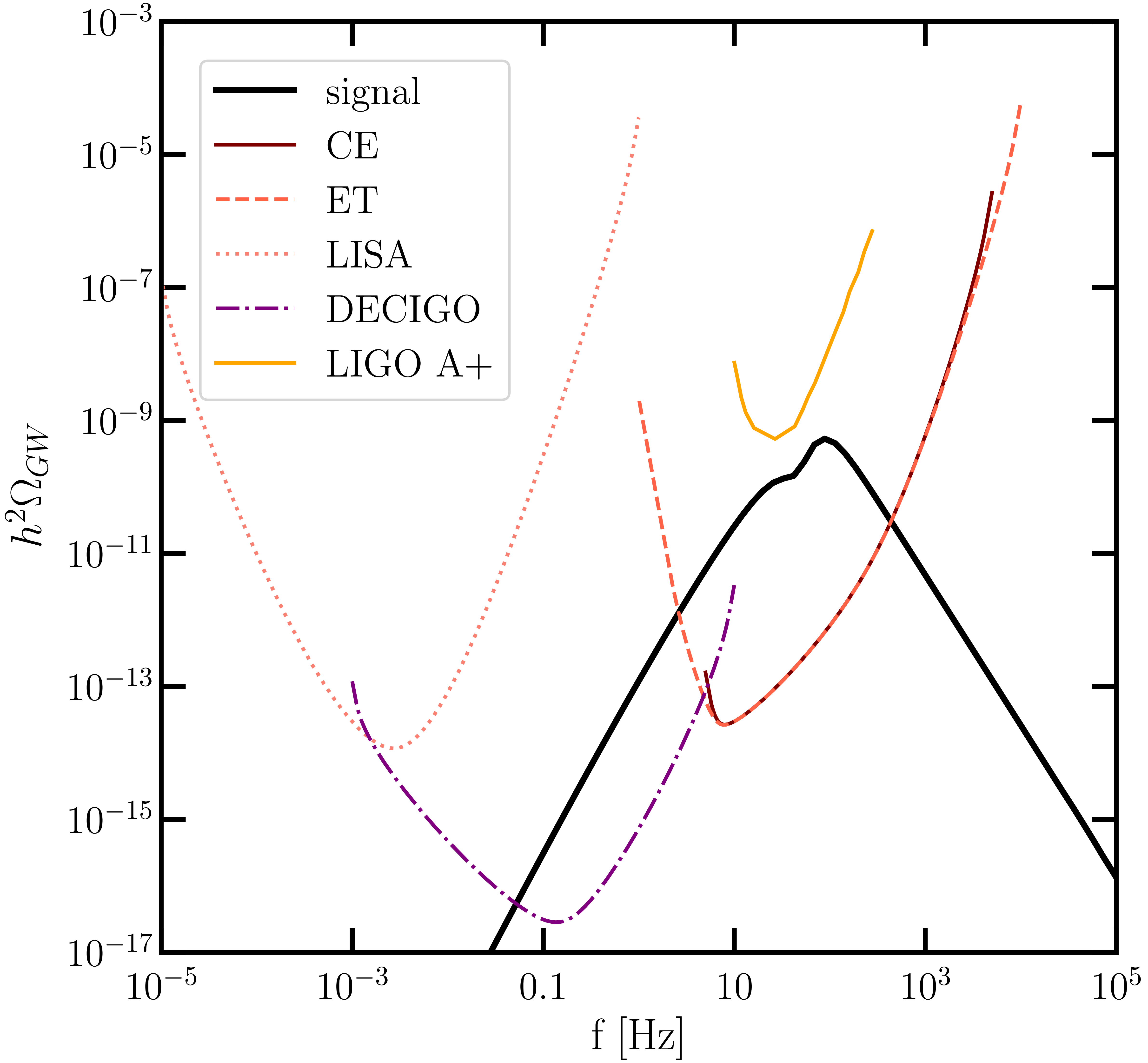
SGWB signal is detectible if SNR $\rho \geq 1$

Signal to noise of various GW observatories:

$$\rho = \sqrt{2t_{\text{obs}} \int_{f_{min}}^{f_{max}} df \left(\frac{\Omega_{\text{GW},0}(f)}{\Omega_{\text{noise}}(f)} \right)^2}$$

t_{obs} = run time of experiment

$\xi = 100, b = -1.8 \times 10^{-4}, c_1 = 2.5 \times 10^{-4}, c_2 = 3.570913 \times 10^{-3}, c_4 = 3.9 \times 10^{-3}$



Spectral density vs. integrated power-law sensitivity curves

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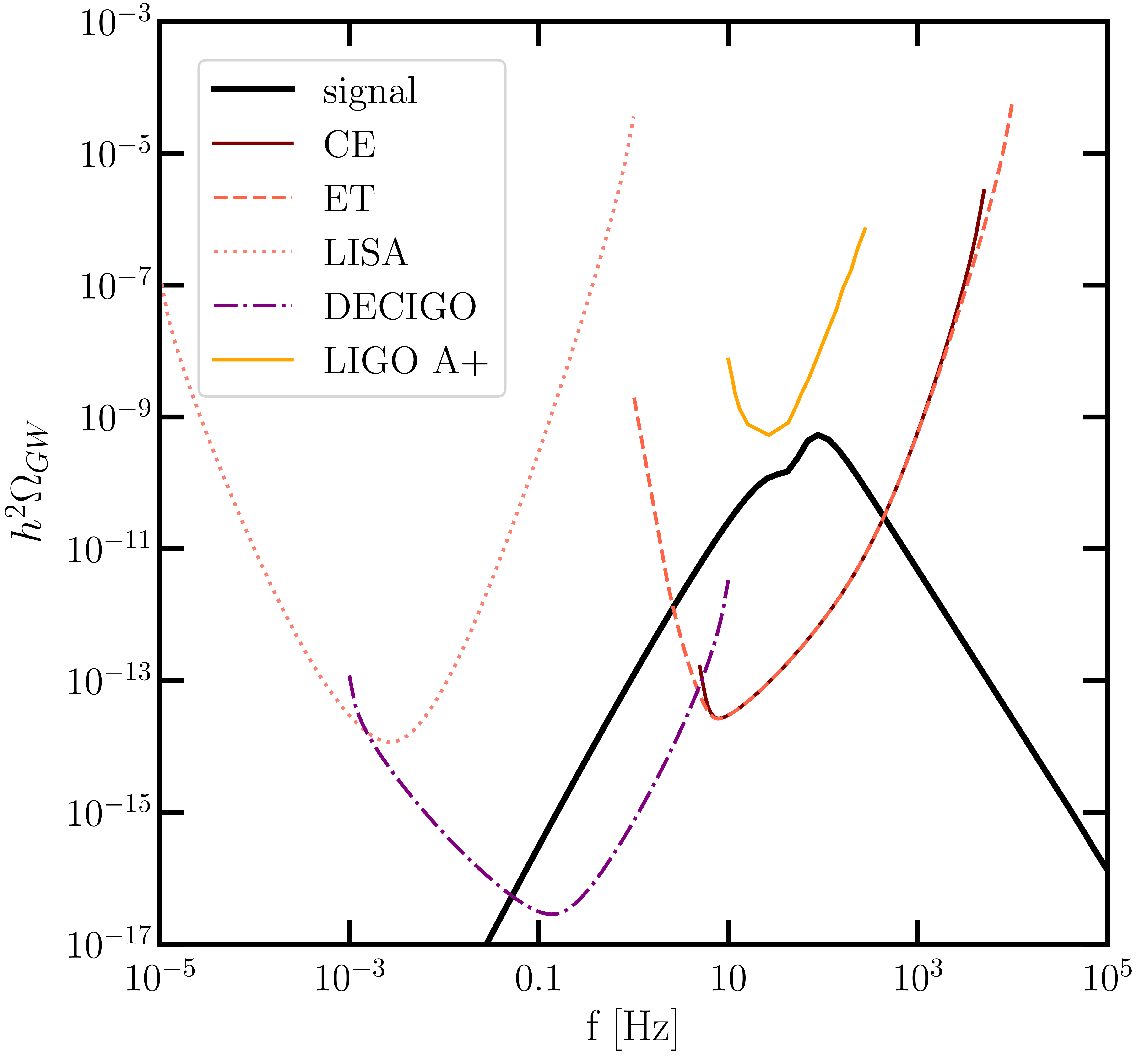
Signal to noise of various GW observatories:

Experiment	$\log_{10}\rho$	68% CL	95% CL
LIGO A+	−2.25	$^{+1.18}_{-0.74}$	$^{+3.02}_{-1.34}$
LISA	−6.52	$^{+3.28}_{-3.33}$	$^{+6.99}_{-3.47}$
ET	2.04	$^{+1.35}_{-0.95}$	$^{+3.23}_{-1.58}$
DECIGO	1.91	$^{+1.91}_{-1.49}$	$^{+4.49}_{-2.22}$
CE	2.32	$^{+1.17}_{-0.99}$	$^{+3.15}_{-1.55}$

$\xi = 100, b = -1.8 \times 10^{-4}, c_1 = 2.5 \times 10^{-4}, c_2 = 3.570913 \times 10^{-3}, c_4 = 3.9 \times 10^{-3}$



Spectral density vs. integrated power-law sensitivity curves

Extra and Q/A Slides

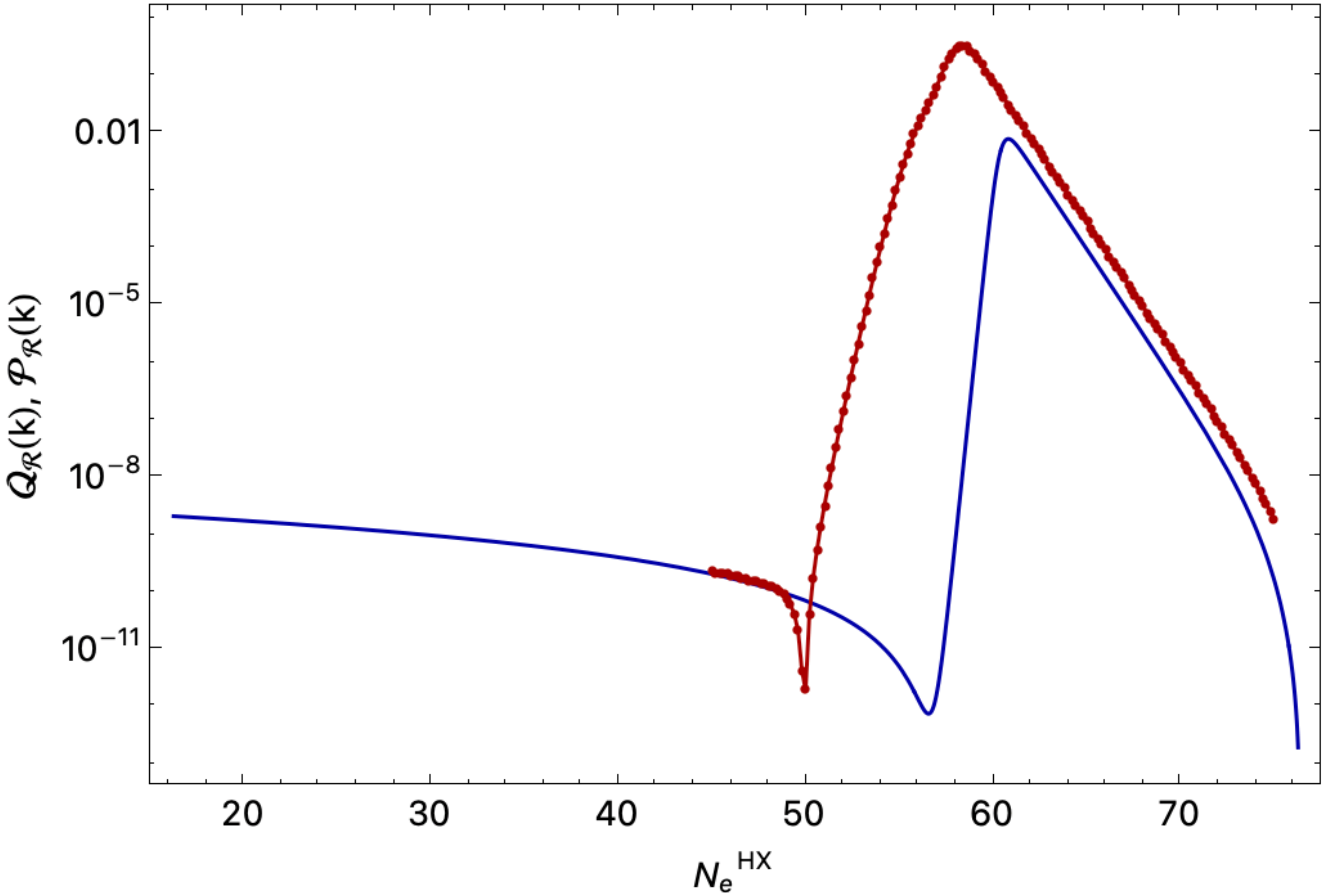
Treating the Ultra-slow roll dynamics carefully by solving the equations of motion for perturbations numerically *without* using SR approximation:

- Taking into account the growth of so-called “decaying modes”

-These are strongly suppressed for many ($\mathcal{O}(50)$) e-folds before growing for about 2.5 e-folds of USR.

Red= no SR approx.

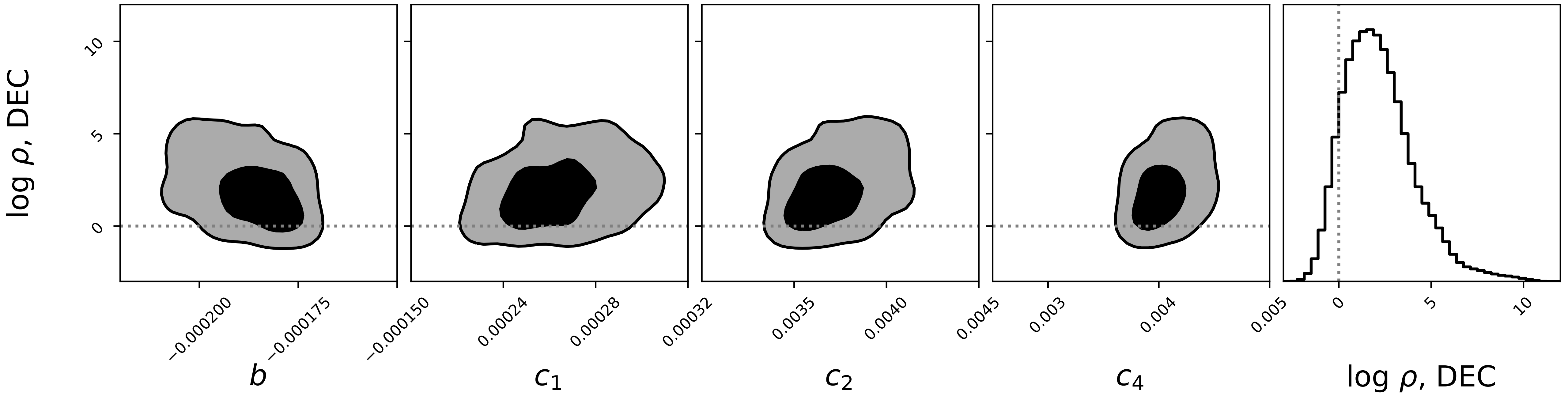
Blue= SR approx. sol for analytic trajectories



Comments

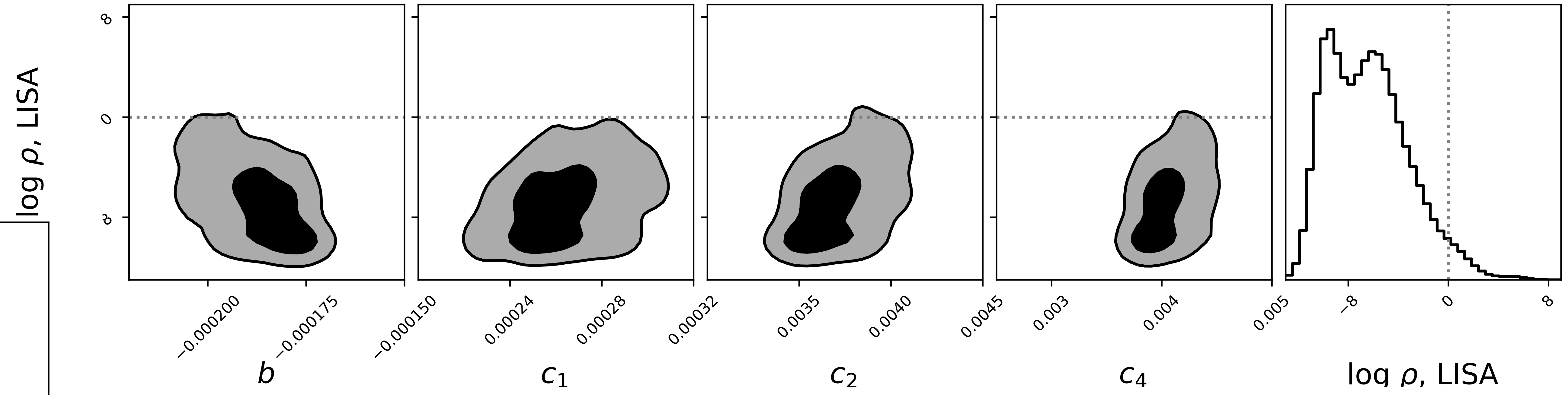
We know the growth of “decaying modes” during USR doesn’t lead to excessive $\mathscr{P}_{\mathscr{R}}$ amplification, only “helps” (earlier and slightly higher peak $\Longrightarrow$ “deeper” into the DM mass range

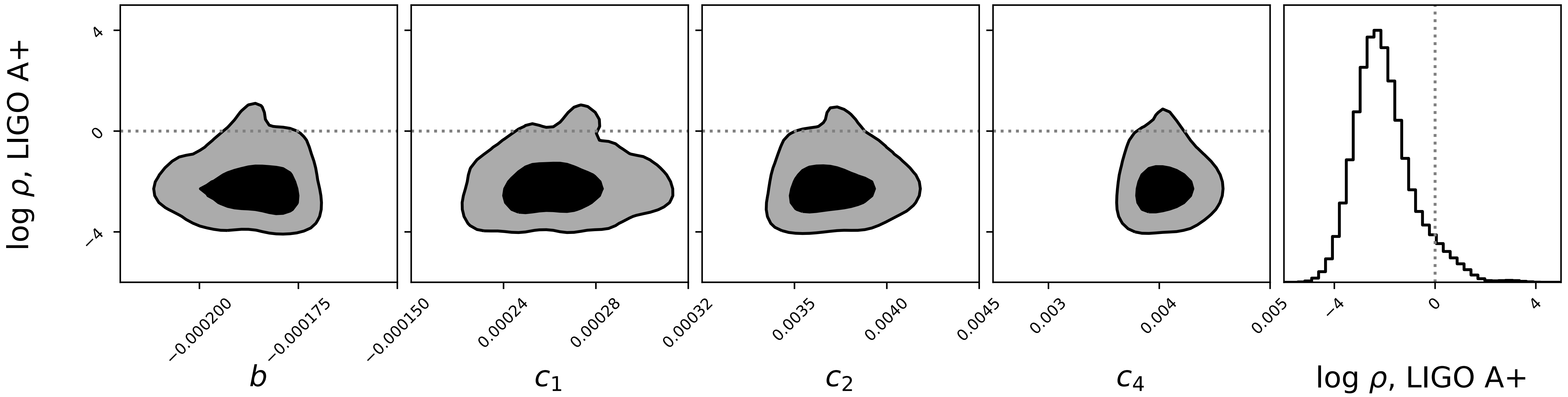
We have not fully reconciled our analytic estimate for $k_{\rm pbh}$ with numerical result, which differ by some order 1-10 factor



DECIGO

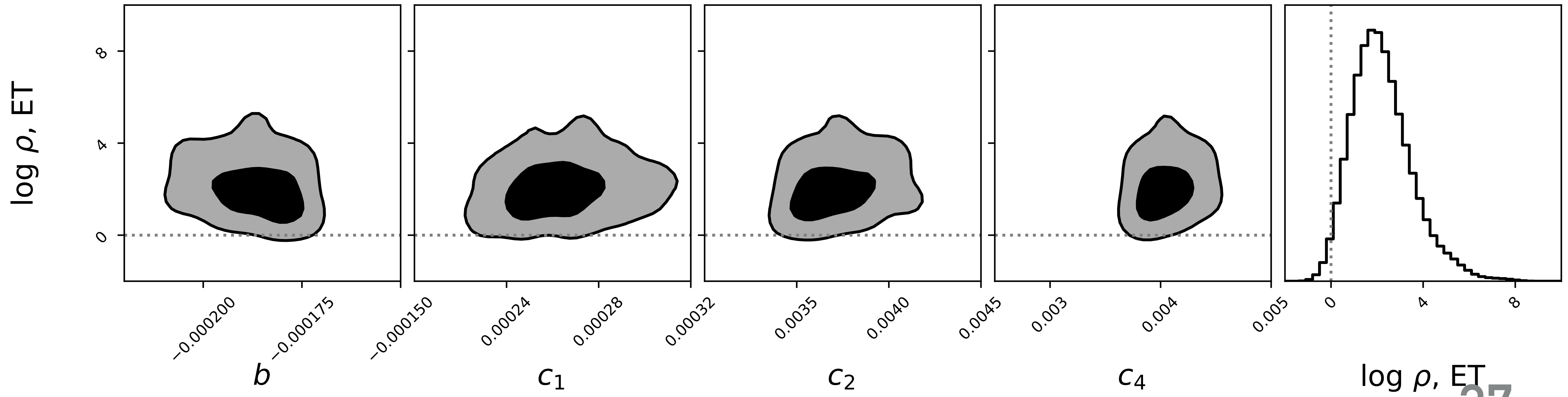
LISA





LIGO A+

Einstein Telescope



b/c_1	$-5.05^{+0.03}_{-0.05} \times 10^{-2}$
c_1/c_2	$6.84^{+0.32}_{-0.26} \times 10^{-2}$
c_2/c_4	$1.096^{+0.009}_{-0.008}$

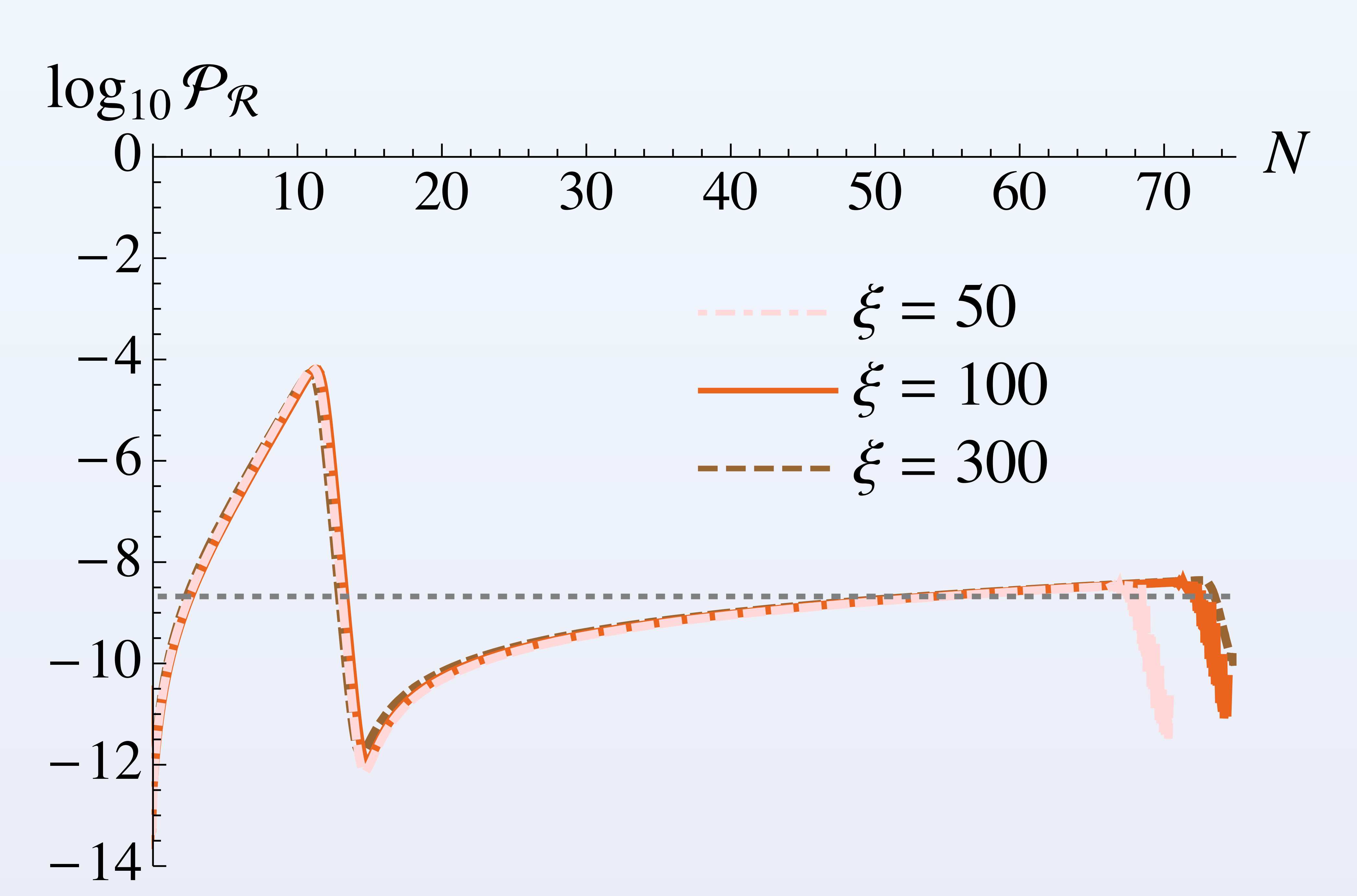
$b = y\hat{b}, c_i = y\hat{c}_i, y > 0$

Scaling relations:

Fixing $\hat{b}\sqrt{\xi}$ = constant, $\frac{\xi}{y}$ = constant

$V(r, \theta_*)$ and $\mathscr{P}_R$

show self-similarity at various values of ξ



Horizontal axis: Number of e-folds before end of inflation. Inflation ends at N=0.

Power Spectrum Peaks in (Our) 2-field Model

Adiabatic and Isocurvature modes **decouple** for $\omega = 0$

Large turns $\Longrightarrow$ **transfer of power** from isocurvature modes to adiabatic modes

$$\mathscr{R}_k = \frac{H}{\dot{\sigma}} Q_\sigma = \frac{Q_\sigma}{M_{\rm pl}\sqrt{2\epsilon}}$$

$$\mathscr{P}_{\mathbf{R}}(\mathbf{k}) \equiv \frac{\mathbf{k}^3}{2\pi^2} |\mathscr{R}_{\mathbf{k}}|^2$$

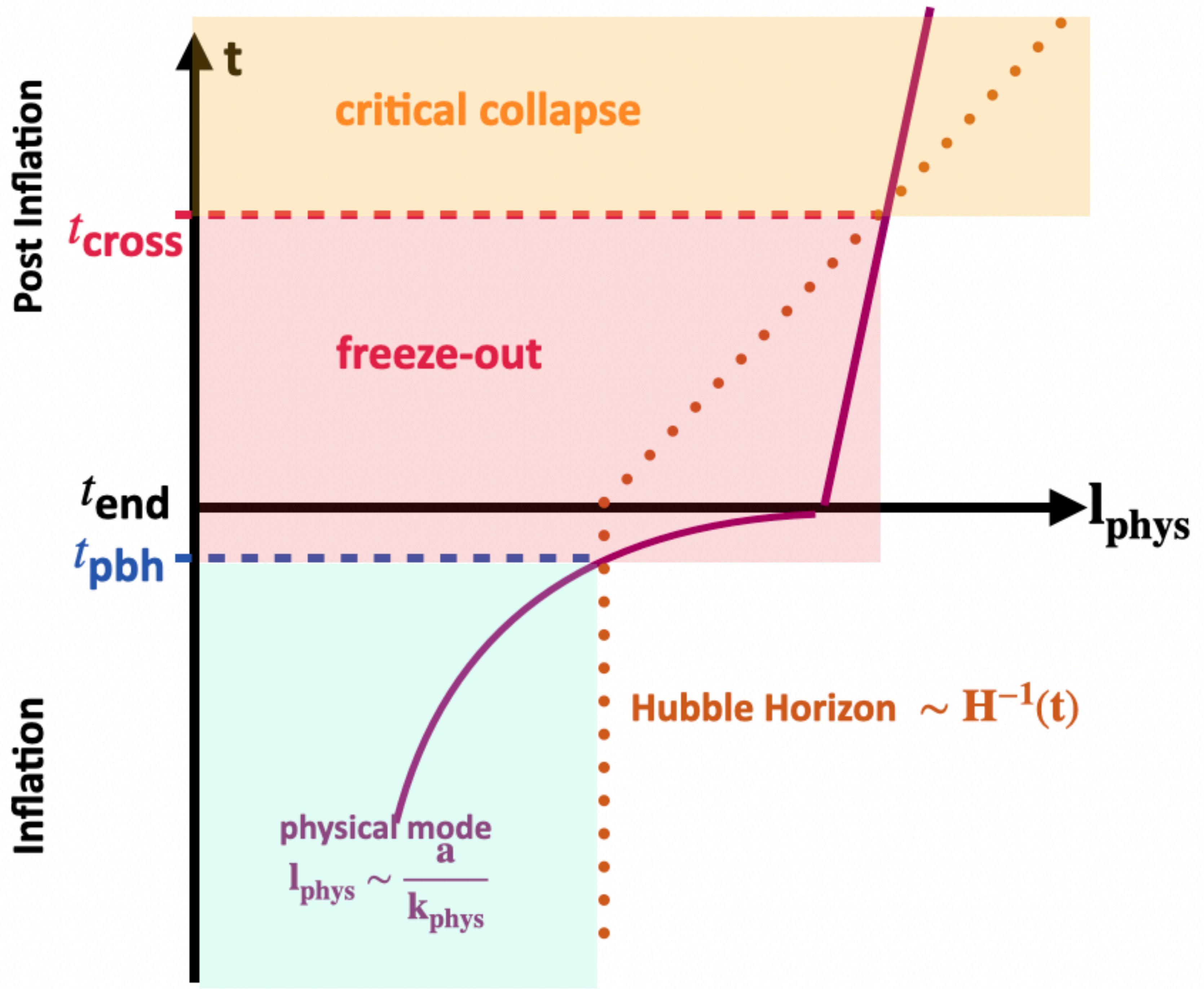
Multifield effects **heavily constrained** by experiment but just around pivot scale!

Main idea: multi-field model with **slight** turns while keeping isocurvature modes small -$\mathscr{P}_{\mathbf{R}}$ **amplified for modes** $k_{\rm pbh}(t_{USR})$

How do you make a black hole?

Numerator gets larger:
(1) tachyonic modes (hybrid inflation)
(2) turns in field space (multifield seeds)

Denominator gets smaller:
Brief phase of Ultra slow-roll



Power Spectrum Peaks in Our 2-field Model

(Multifield) Gauge Invariant Mukhanov-Sasaki variables

$$Q^I = \delta\phi^I + \frac{\dot{\varphi}^I}{H}\psi$$

Split into two modes: Adiabatic and Isocurvature

$$Q^I = \underbrace{\hat{\sigma}^I Q_\sigma}_{\text{Adiabatic}} + \underbrace{\sqrt{|\mathscr{G}_{IJ}|}\,\epsilon^{IJ}\hat{\sigma}_J Q_s}_{\text{Isocurvature}}$$

In multifield inflation: trajectory can turn and perturbations can couple

Covariant turn rate vector:

$$\omega^{\mathbf{I}} \equiv \mathscr{D}_t\hat{\sigma}^I = \dot{\phi}^J \mathscr{D}_J \hat{\sigma}^I \quad \textbf{where} \quad \hat{\sigma}^I \equiv \frac{\dot{\varphi}}{\sqrt{\mathscr{G}_{IJ}\dot{\varphi}^I\dot{\varphi}^J}}$$

Adiabatic:
fields have equal fraction over/under-densities

Isocurvature:
overall density uniform not in chemical equilibrium

under-density

over-density

Vegemite over-densities

Margarine over-density

inspiration: Katelin Schutz

$\omega^2 << \mathbf{H}^2 \rightarrow$ (only slight turning)

$\frac{\mu_s^2}{\mathbf{H}^2} >> 1 \rightarrow$ Isocurvature modes heavy

More on critical collapse criteria and PBH masses…

$k_{\rm PBH} = a(t_c)H(t_c)$ Corresponds to threshold for $\mathscr{P}_R(k_{\rm PBH}) \geq 10^{-3}$

Relate the mode that leads to collapse to the resultant PBH mass via:

$$\frac{k_{\rm pbh}}{3.2 \times 10^{-5}{\rm Mpc}^{-1}} \sim \left(\frac{30 M_\odot}{M_{\rm pbh}}\right)^{1/2} \left(\frac{g_*(T_c)}{106.85}\right)^{-1/12}$$

Original calculation due to Carr using estimate from Jean's instability: in radiation dominated epoch, collapse requires fractional over-density $\frac{\delta\rho}{\bar{\rho}} \geq \delta_c \gtrsim c_s^2$, where $c_s^2 = w = 1/3$ relates to radiation fluid EOS. Found $\delta_c \sim .4$. In reality, gets GR corrections and depends on initial curvature perturbation profile.

Better approach: Use the compaction function which gives $\delta_c \sim .4 - .66$

$$\mathscr{C} = \frac{2(M - M_{\rm bg})}{R(t, r)},$$

i.e.2x mass excess/circumferential radius

The Field Space in Multifield Inflation

Jordan Frame:

$$\tilde{S} = \int d^4x \sqrt{-\tilde{g}} \left[f\left(\phi^I\right) \tilde{R} - \frac{1}{2} \delta_{IJ} \tilde{g}^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - \tilde{V}\left(\phi^I\right) \right]$$

Conformal (stretching) Transformation

$$\tilde{g}^{\mu\nu} \rightarrow g^{\mu\nu} = \Omega^{-2}(x) \tilde{g}^{\mu\nu}$$

Einstein Frame:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} R - \frac{1}{2} \mathcal{G}_{IJ} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - V(\phi^I) \right]$$

Induces non-canonical kinetic terms $\Longrightarrow$ curved field space

Kaiser 1003.1159v2

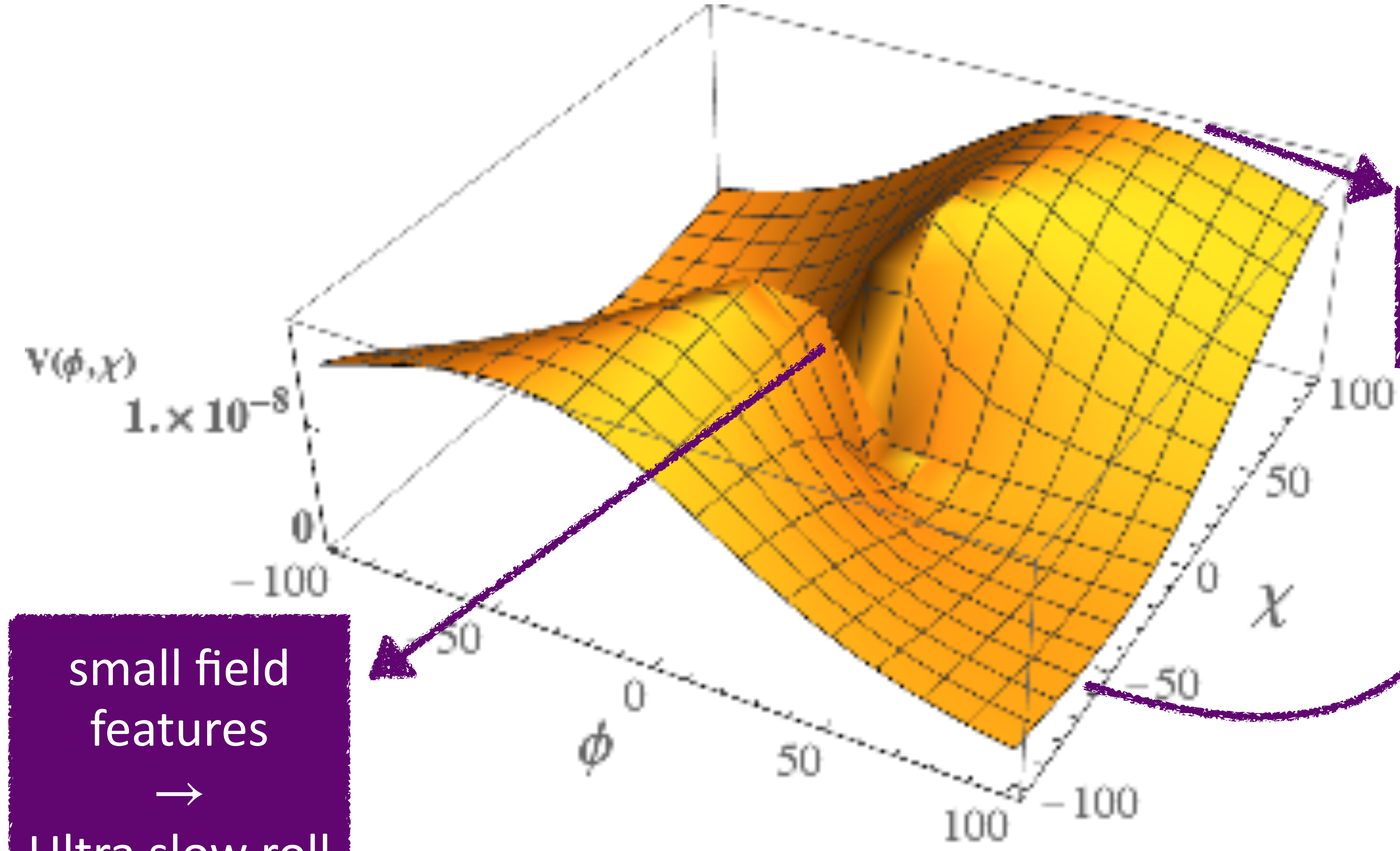
Field space metric:

$$\mathcal{G}_{IJ}(\phi^K) = \frac{M_{\rm pl}^2}{2} \left[\delta_{IJ} + \frac{3}{f(\phi^K)} f_{,I} f_{,J} \right]$$

$$\tilde{V}(\phi^I) \rightarrow V(\phi^I) = \frac{M_{\rm pl}^4}{4 f^2(\phi^I)} \tilde{V}(\phi^I)$$

Potential gets **stretched**

Consistent with CMB anisotropies



~plateau at large field values

small field features → Ultra slow roll

Ingredients from High Energy Theory

Multiple fields and Non-minimal Couplings

(to make f dimensionless)

$$\ddagger\ \mathbf{f}(\phi) = \frac{f(\hat{\phi} M_{\rm pl})}{M_{\rm pl}^2}$$

Multifield Models $\sim \phi^I(x^\mu)$

- Field theories (FTs) at high energies generically have > 1 scalar d.o.f., even the SM
- BSM theories have even more, e.g. Minimally Supersymmetric Standard Model ∋ 7 Chiral Superfields

Non-minimal Couplings

- Self interacting scalar fields in curved spacetime generically induce non-minimal couplings
- EFT point of view: well-behaved dim 4 operators that should be included in S
- RG: The couplings increase with energy scale with no UV fixed point

$$\tilde{\mathbf{S}} = \int d^4\mathbf{x}\sqrt{-\tilde{\mathbf{g}}}\left[\mathbf{f}\left(\phi^{\mathbf{I}}\right)\tilde{\mathbf{R}} - \frac{1}{2}\delta_{\mathbf{IJ}}\tilde{\mathbf{g}}^{\mu\nu}\partial_\mu\phi^{\mathbf{I}}\partial_\nu\phi^{\mathbf{J}} - \tilde{\mathbf{V}}\left(\phi^{\mathbf{I}}\right)\right]$$

$$\ddagger\ f(\phi^I) = \frac{1}{2}\left[M_{\rm pl}^2 + \sum_{I=1}^{N}\xi_I(\phi^I(x^\mu))^2\right]$$

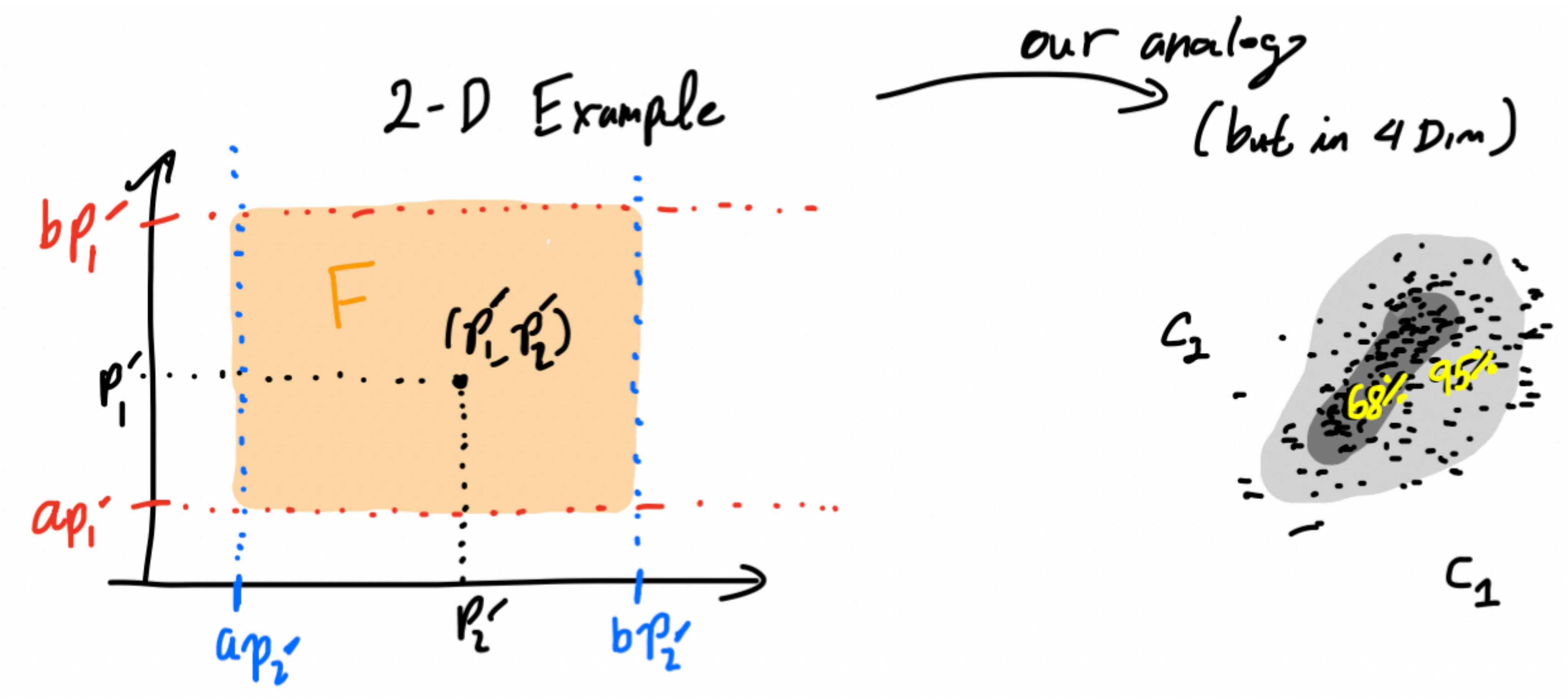
non-minimal couplings

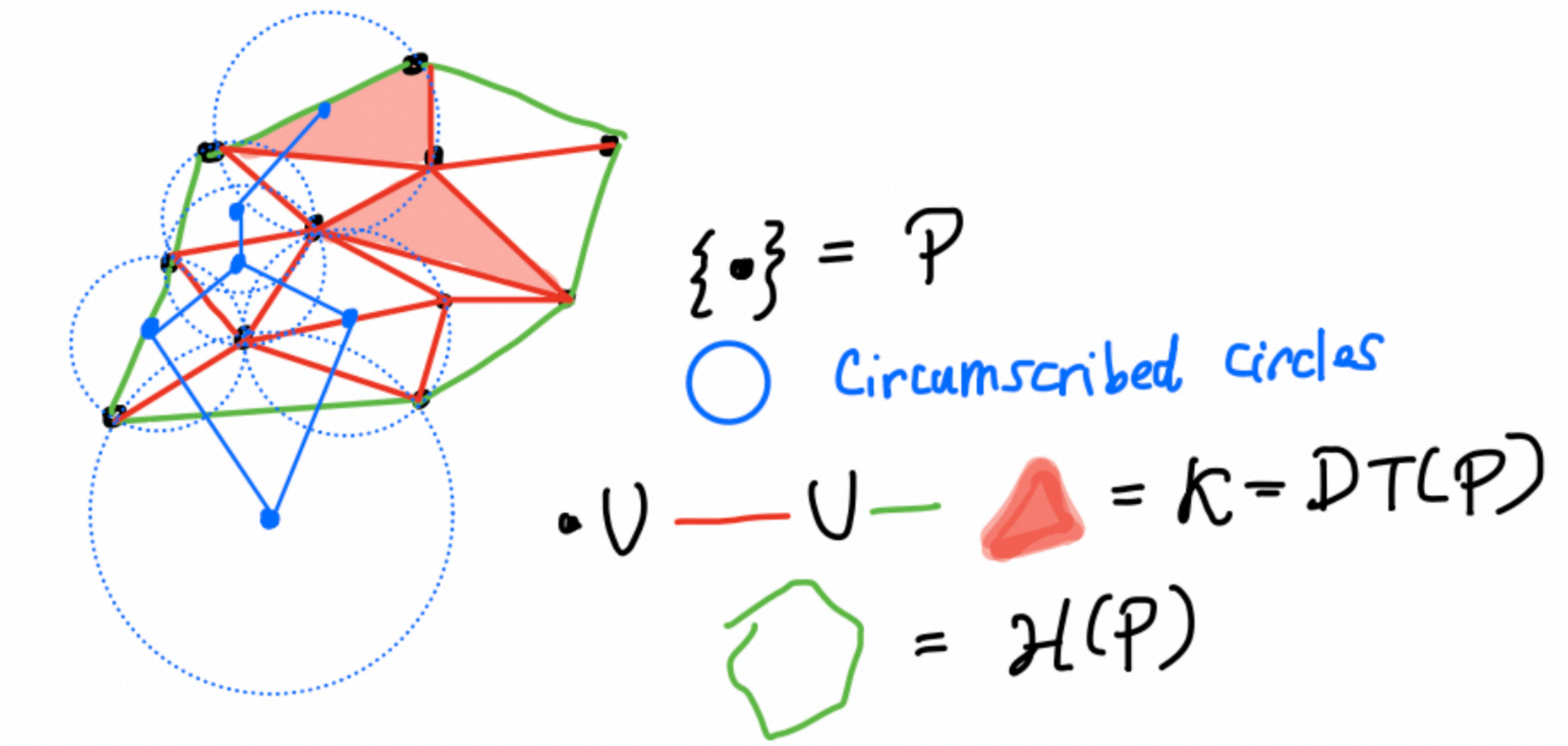
IMPLEMENTING A QUANTITATIVE MEASURE OF FINE TUNING OF MODEL PARAMETERS

To do Bayesian comparison between different models (with same number of d.o.f), we really need to compute the weighted volume of the degeneracy region in parameter space.

Problem: our degeneracy region is full of holes (not simply connected), because of the constraint that we must avoid USR that leads to dominant quantum diffusion effects.

Idea: Use a combination of convex-hull wrapping + recursive Voronoi tessellation to converge on the true volume of the degeneracy region and implement a measure of fine tuning, such as that proposed in (0705.2241) (Athron and Miller, 2007).





This is ongoing work… I have made some more progress and would be glad to chat about it more!

- We can get a better idea of what degree of fine tuning is required for model parameters by looking at ratios of the couplings, b, c_i

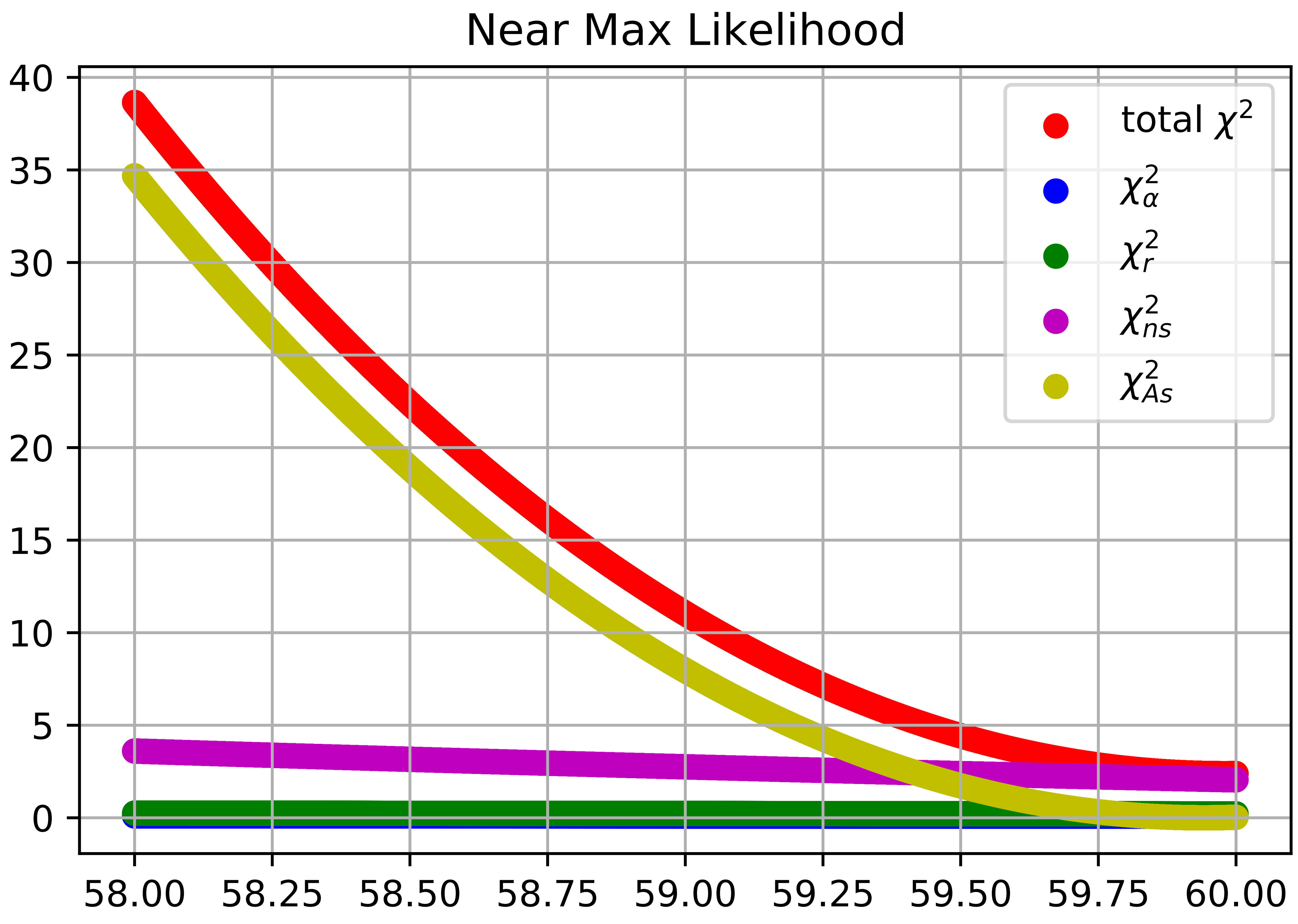
- We perform an MCMC sampling as a feasible/less expensive alternative to computing the full **Bayesian evidence** which is the job of computing the integral of likelihood, weighted by the prior over parameter space, normalized by prior-weighted volume of parameter space.

Fewer degeneracies amongst ***ratios of model parameters.***

b/c_1	$-5.05^{+0.03}_{-0.05} \times 10^{-2}$
c_1/c_2	$6.84^{+0.32}_{-0.26} \times 10^{-2}$
c_2/c_4	$1.096^{+0.009}_{-0.008}$

- Degeneracy region in cosmological parameter spaces like ours are generally localized (rather than perfect lines for instance)… the degeneracy regions change the Bayesian integral.

What is driving the range of values for optimal reheating histories (N_*)?



One might think we'd always favor larger N_* but it turns out to drive A_s away from *Planck* central value just enough to not be the case... instead, more central n_s restricts range of optimal N_*