

Theoretical consistency with enhanced density fluctuations

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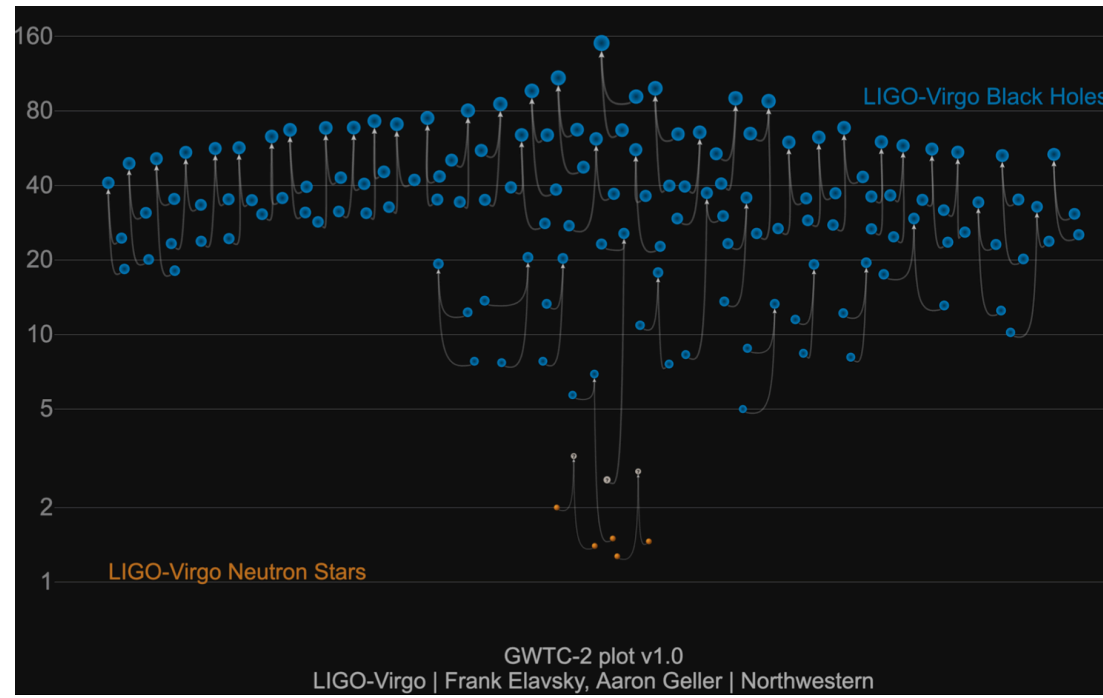


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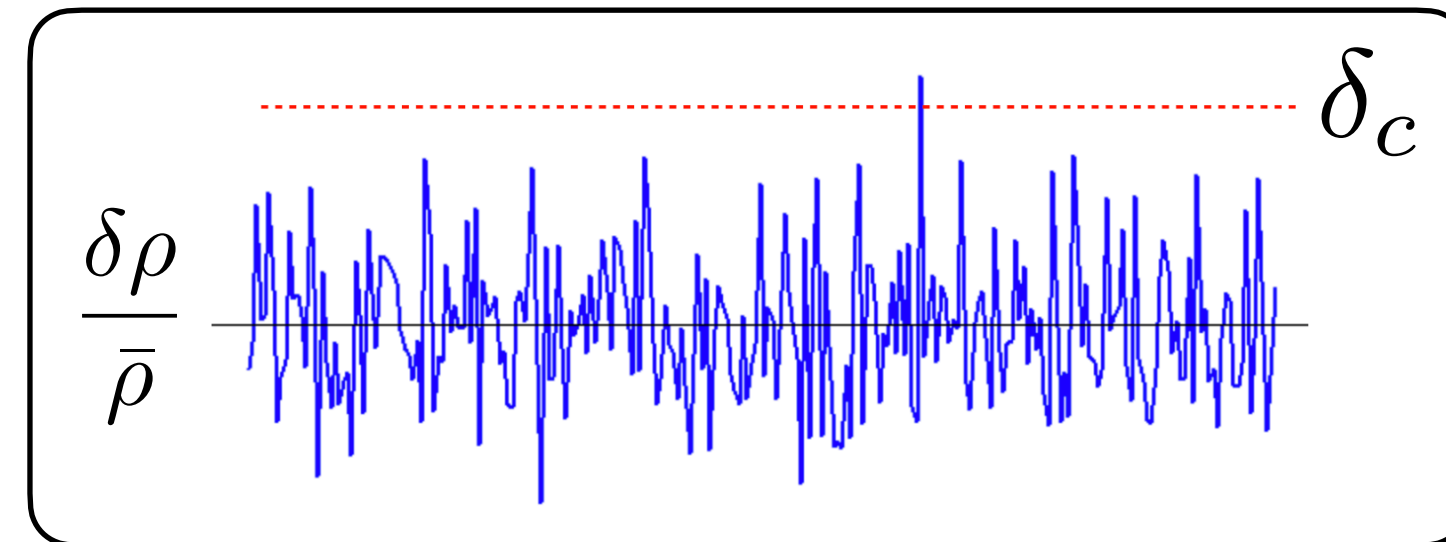
Context: GW astronomy



Have **Primordial Black Holes** been already detected? Data will tell

recent PBH review in LISA CosWG
[2310.19857](#)

Large primordial overdensities

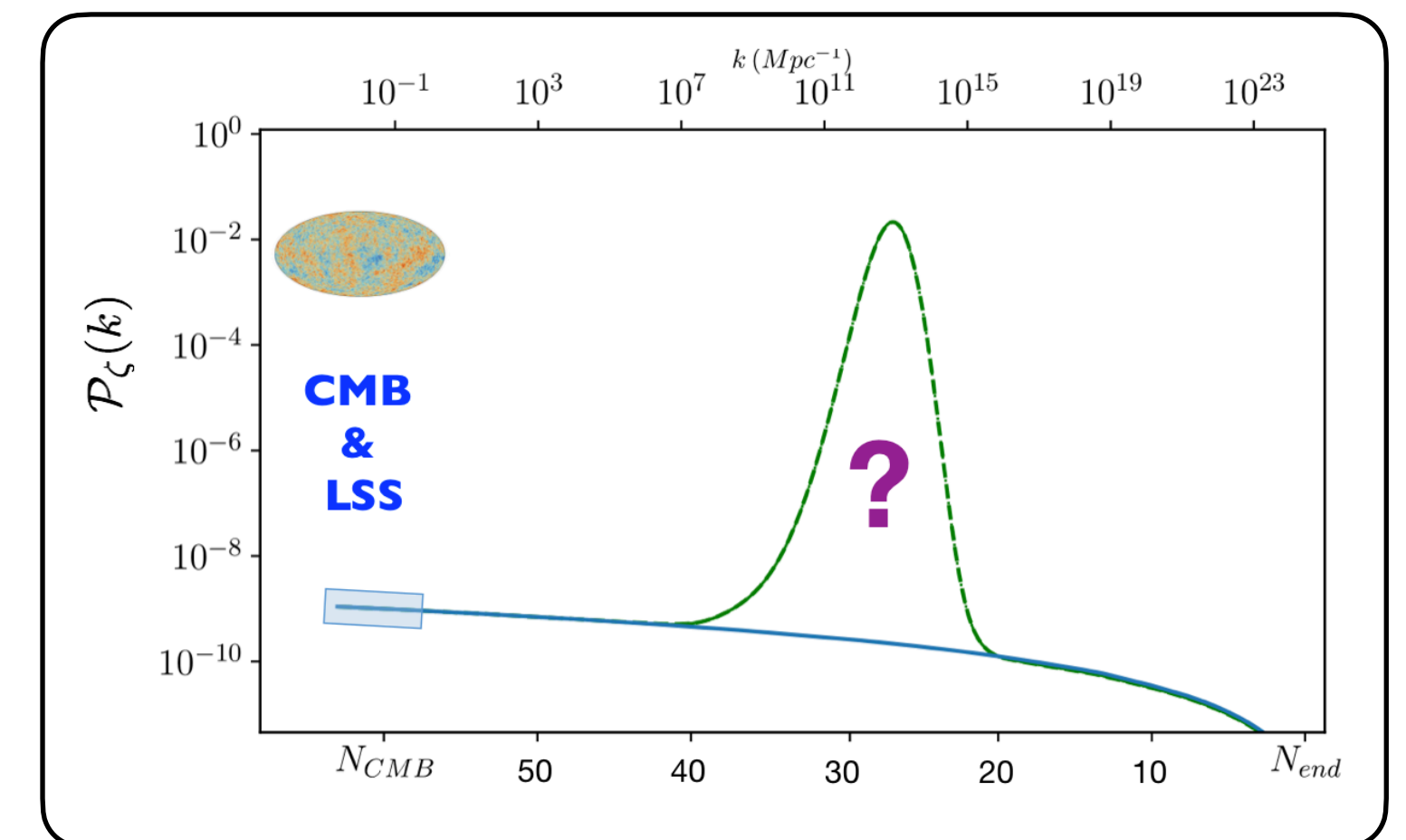


Large power spectrum on small scales and **nontrivial dynamics of inflation**

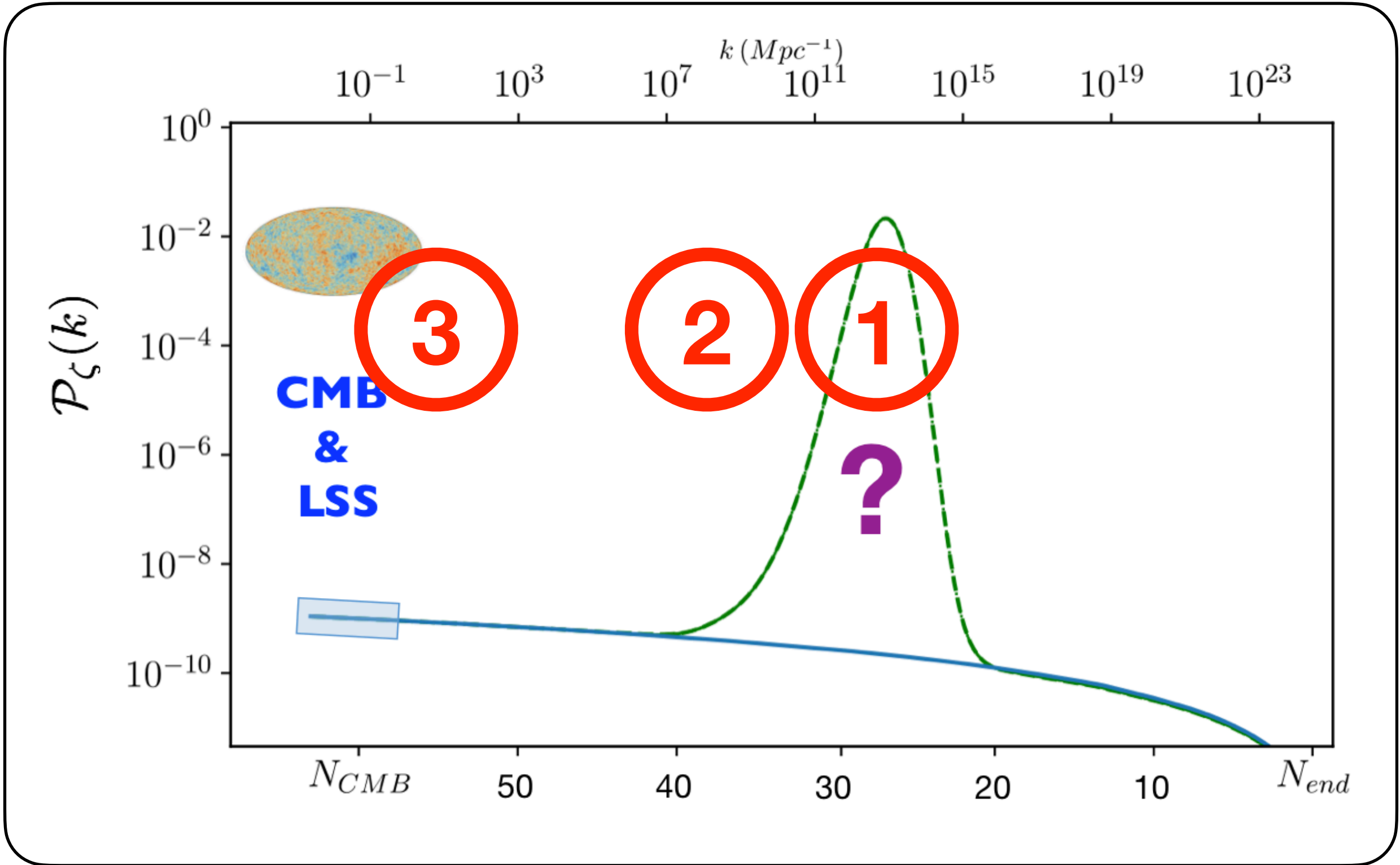
Scalar-induced **SGWB**

$$M \simeq 10^{-12} M_{\odot} \left(\frac{f_{\text{LISA}}}{f} \right)^2$$

Bartolo et al 2019



A tale of three scales



1

slow-roll violation
particle production
enhanced fluctuations ...



theoretical consistency?

2

Infrared rescattering

3

Effects on CMB scales

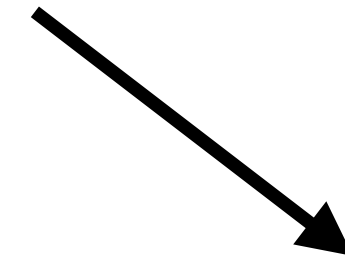
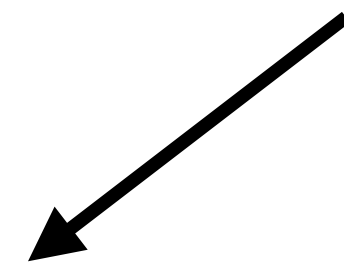
Questions

slow-roll violation, particle production, enhanced fluctuations...



Validity of standard perturbation theory?

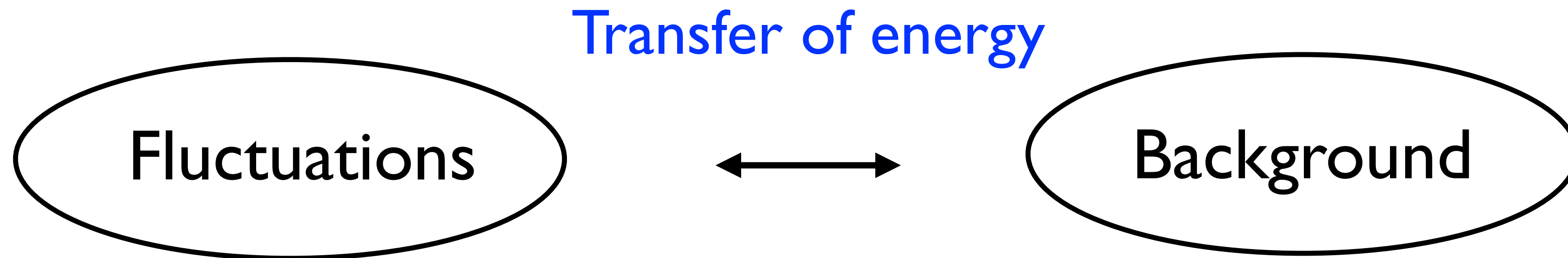
(distinct questions)



Backreaction?

**Loss of
perturbative control?**

Backreaction



Energy in fluctuations comes from background!

Typically slows down the inflaton

Retro-action on dynamics of fluctuations

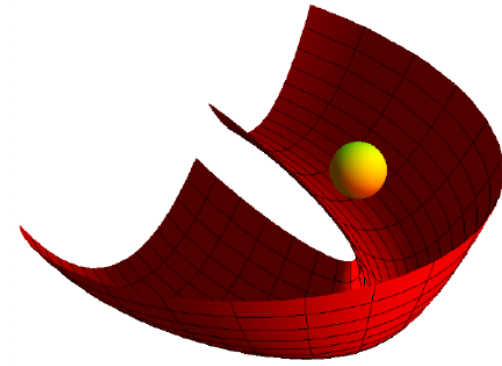
N.B: comparison point:
background kinetic energy

Non-trivial but within scope of analytical methods

Particle production

Sharp feature

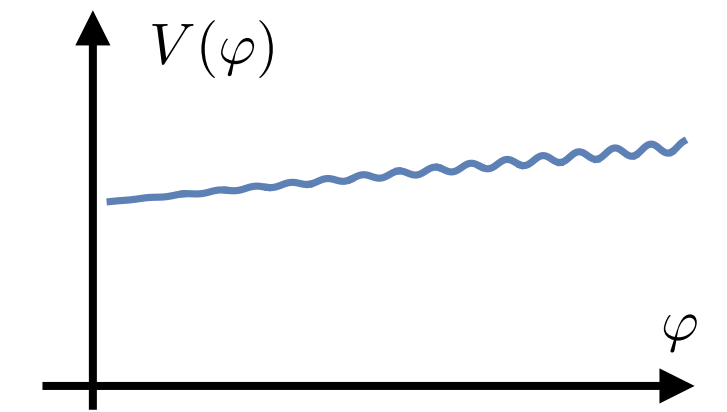
step in potential,
turn in field space ...



$$\Delta t \ll H^{-1} \longrightarrow \omega \sim 1/\Delta t \gg H$$

Resonant feature

Resonance btw
background oscillations
and quantum modes oscillations



$$\omega \gg H$$

Excitation of
sub-Hubble modes



Langage and intuition
of particles useful

Particle production

The produced **particles carry energy** (and pressure)

Backreaction

$$\zeta \sim \alpha \zeta^{\text{BD}} + \beta \zeta^{*\text{BD}}$$

$$\rho = \frac{1}{a^4} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} k |\beta_k|^2$$

energy density of
relativistic particles

Homan, Tolley, 2007
Fumagalli, RP, Witkowski, 2012.02761

Gravitational waves

$$\square h_{ij} = T_{ij}^{\text{TT}}$$

amount of GW bounded
by backreaction / energy conservation

Inomata, 2109.06192

Fumagalli, Palma, RP, Sypsas,
Witkowski, Zenteno, 2111.14664,

Loss of perturbative control (some generic spirit)

Time-dependent coupling in EFT of inflation $M(t)$ characteristic time scale of variation Δt

Governs
structure of interactions

$$M(t + \pi) = M(t) + \pi \dot{M}(t) + \dots$$

↑

Goldstone boson of broken time-diff invariance

expansion ok if $\pi < \Delta t$ at energy $\omega \sim 1/\Delta t$ excited by time-dependence

Loss of perturbative control (some generic spirit)

e.g., canonical single-field inflation

$$\mathcal{L}/a^3 = \frac{f_\pi^4}{2} \dot{\pi}^2 + \dots \quad \text{with} \quad f_\pi^4 = 2M_{\text{Pl}}^2 |\dot{H}|$$



$$\Delta t > 1/f_\pi$$

ω

f_π

H

$\mathcal{P}_\zeta \sim \left(\frac{H}{f_\pi}\right)^4$

the higher \mathcal{P}_ζ the smaller f_π/H
the easier perturbation theory breaks down

see, e.g., Bartolo, Cannone, Matarrese 13,14
Adshead, Hu, 14

Breakdown of perturbation theory

$$\mathcal{L} \sim (\delta\varphi)^2 + (\delta\varphi)^3 + (\delta\varphi)^4 + \dots$$

2 diagnostic tools

non-Gaussianity

$$\frac{\mathcal{L}_3}{\mathcal{L}_2} \sim f_{\text{NL}} \mathcal{P}_\zeta^{1/2} \sim 1$$

useful tools:

PyTransport
CppTransport

nonlinear
sigma models

see [Iacconi's talk](#) and [2304.14260](#)

CosmoFlow

EFT based approach

[Werth, Pinol, RP, 2302.00655](#) and to appear

loops

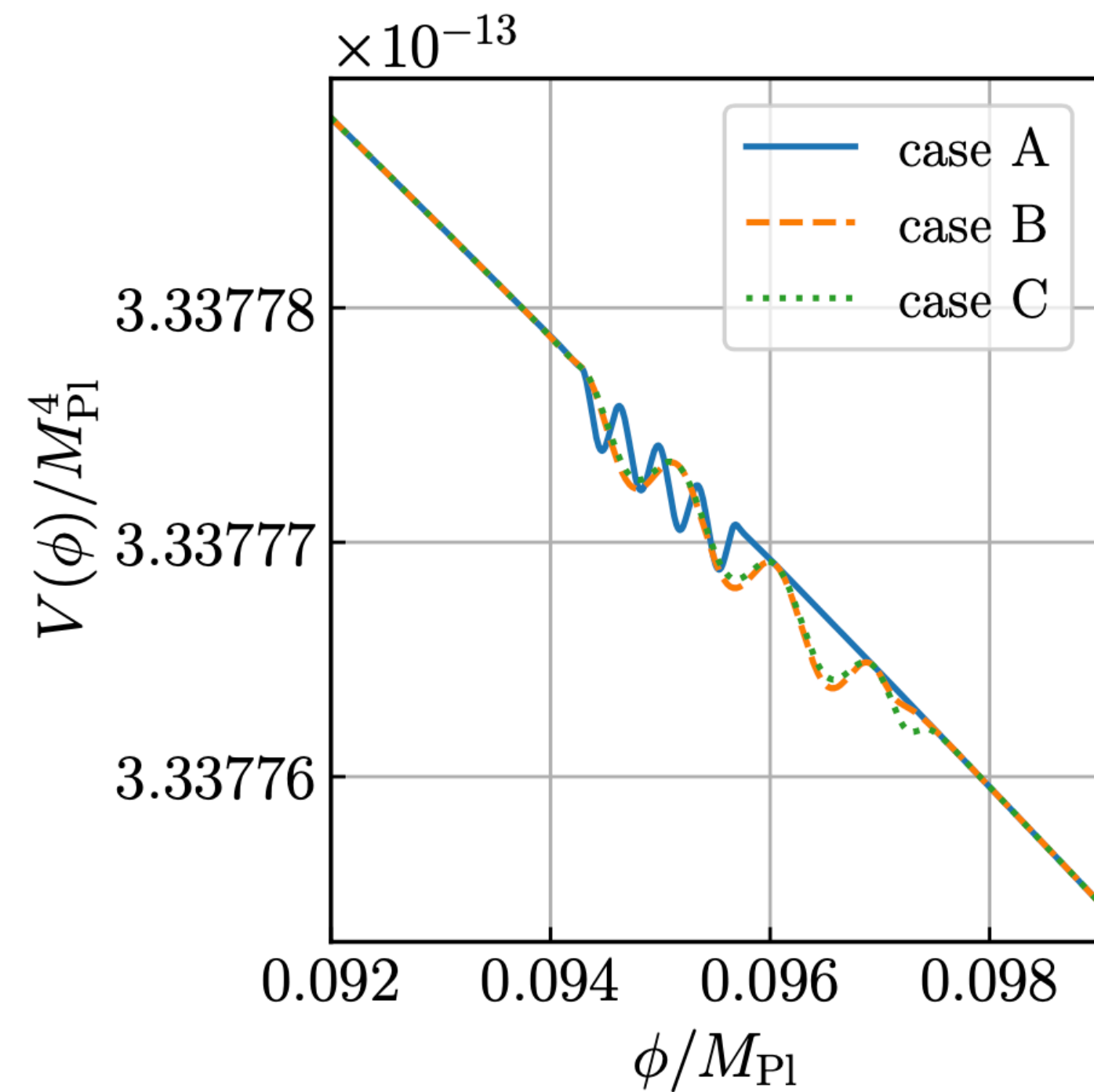
$$\langle \delta\phi^2 \rangle = \text{---} + \text{---} \circ \text{---} + \text{---} \circ \text{---} + \dots$$

$\mathcal{P}_{\text{tree}}$

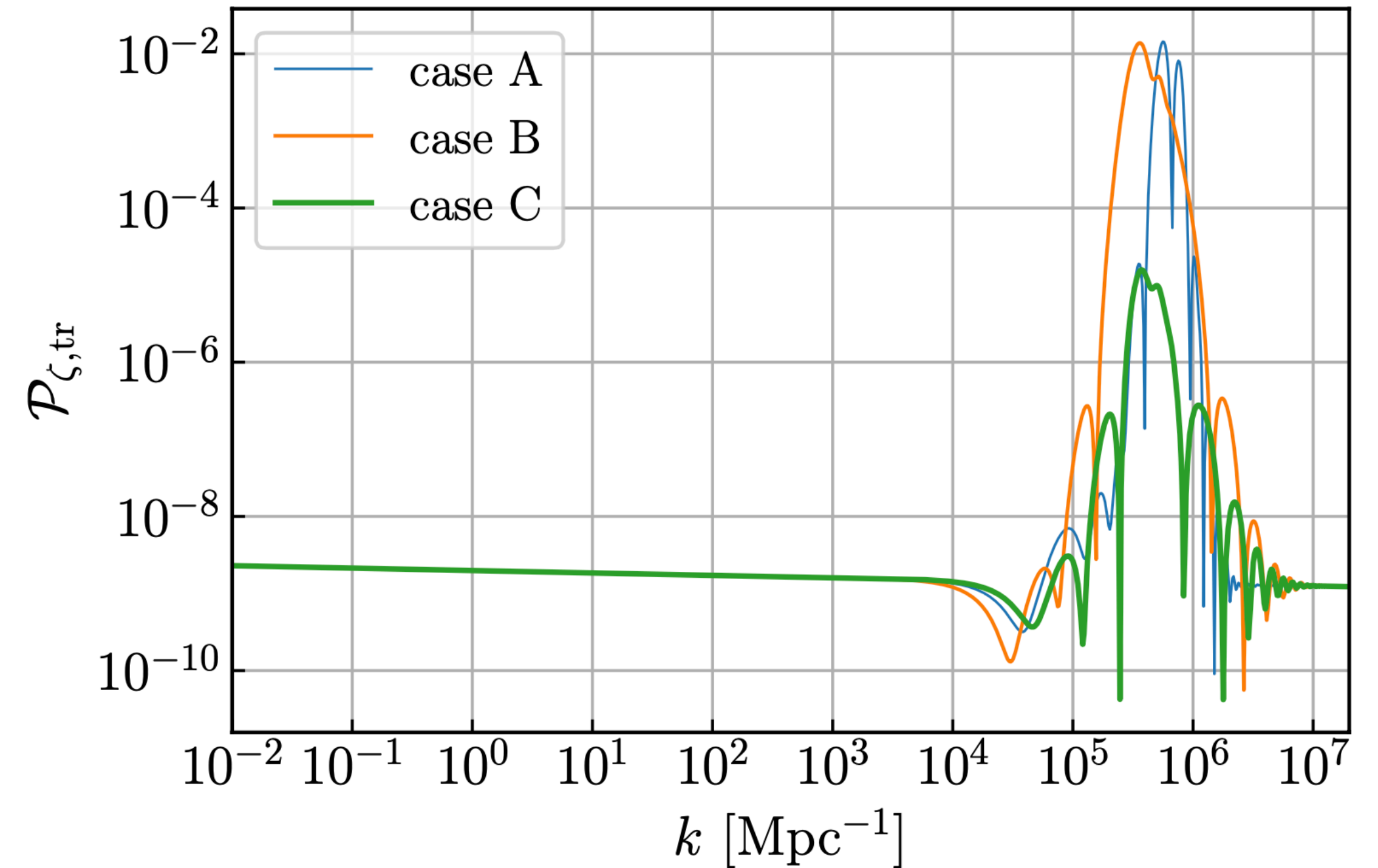
$\mathcal{P}_{1\text{-loop}}$

[Inomata, Braglia, Chen, RP, 2211.02586](#)

Example of Resonant Amplification



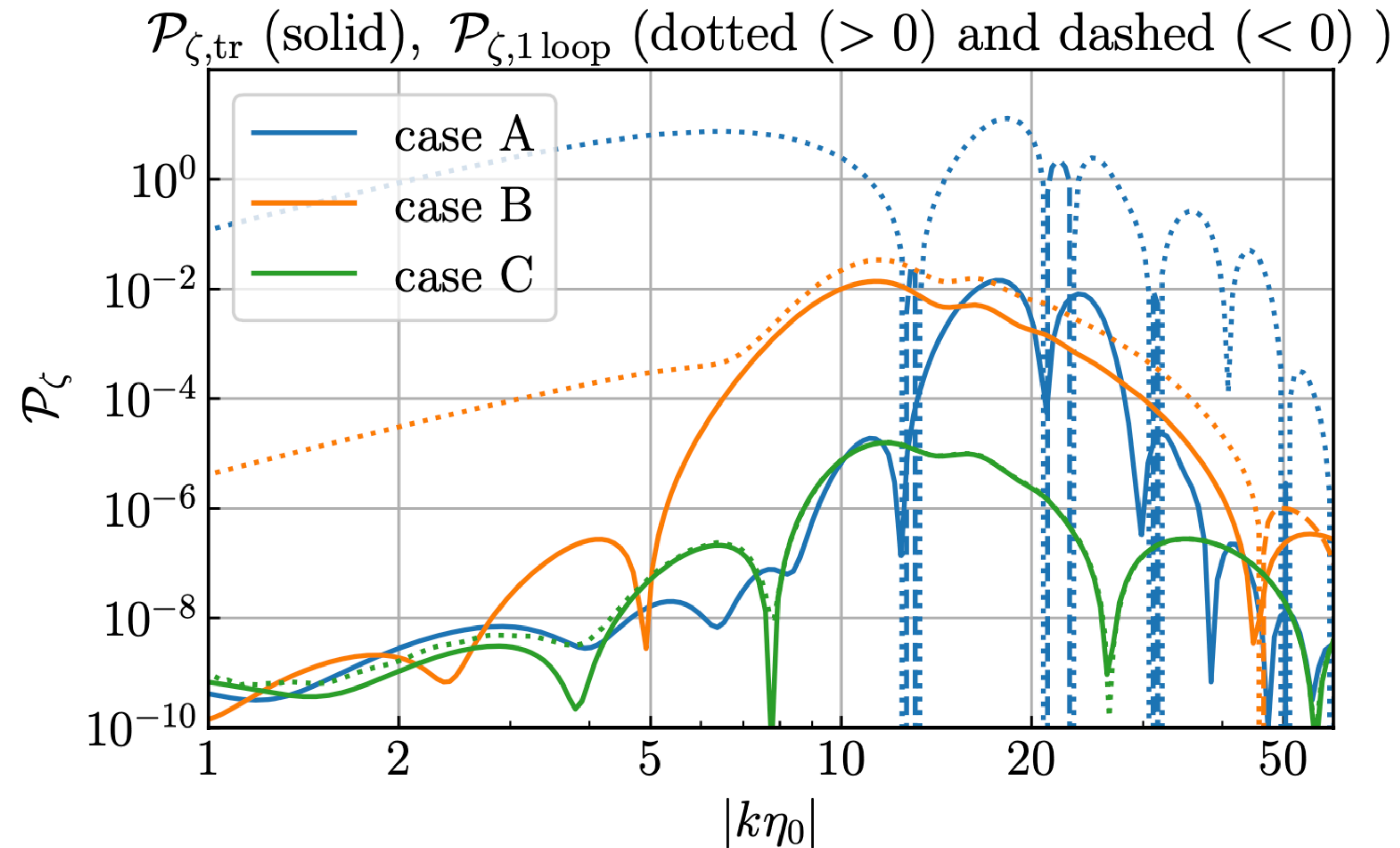
Floquet-type
resonance



slow-roll + transient **periodic modulations**
(resonant NG setup)

Tree-level power spectrum

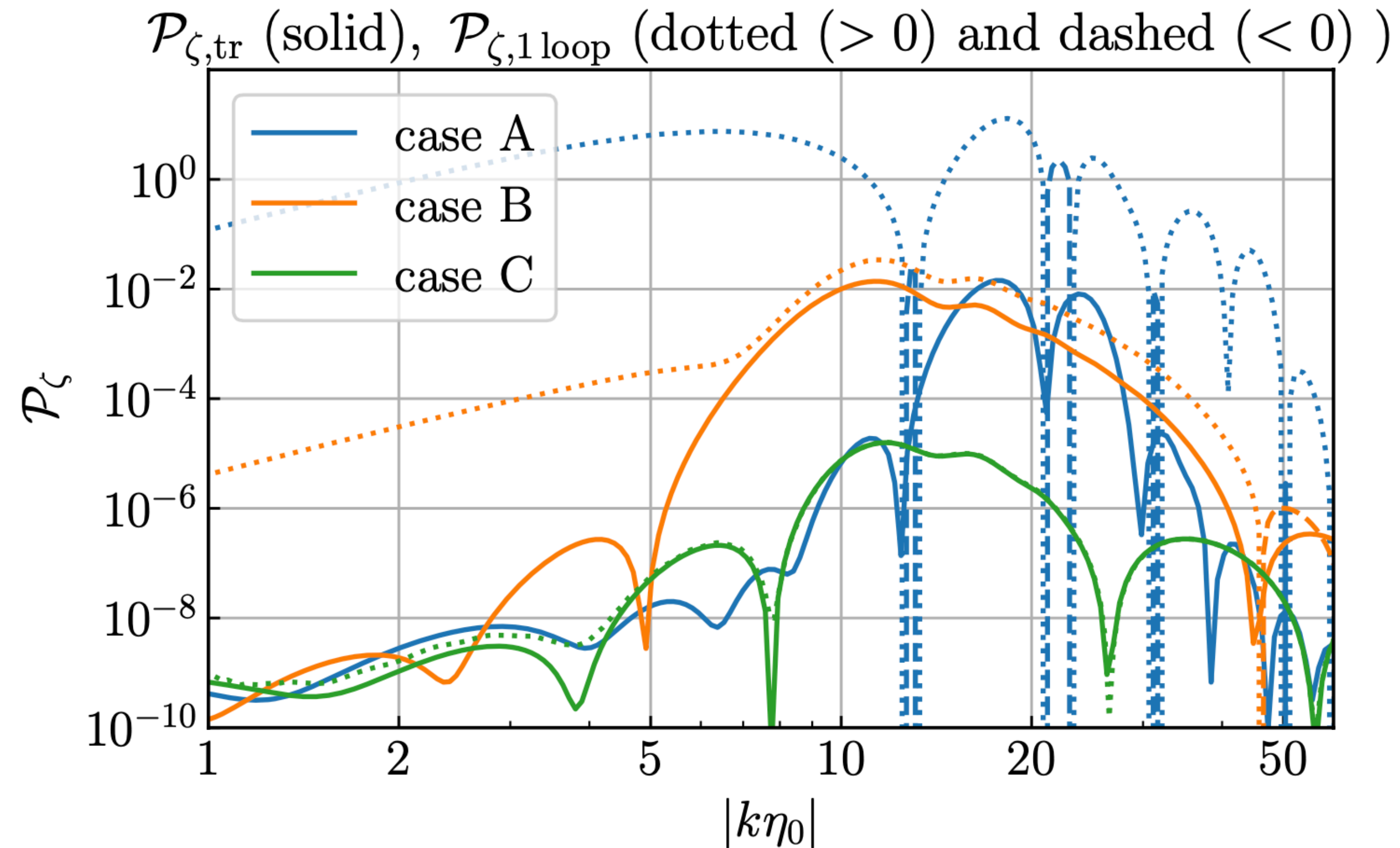
(first) First-principles numerical 1-loop computation



Standard background + numerical mode functions +

numerical computations
of loop integrals
(natural cutoffs)

(first) First-principles numerical 1-loop computation



Standard Perturbation Theory

C: under control

B: marginal

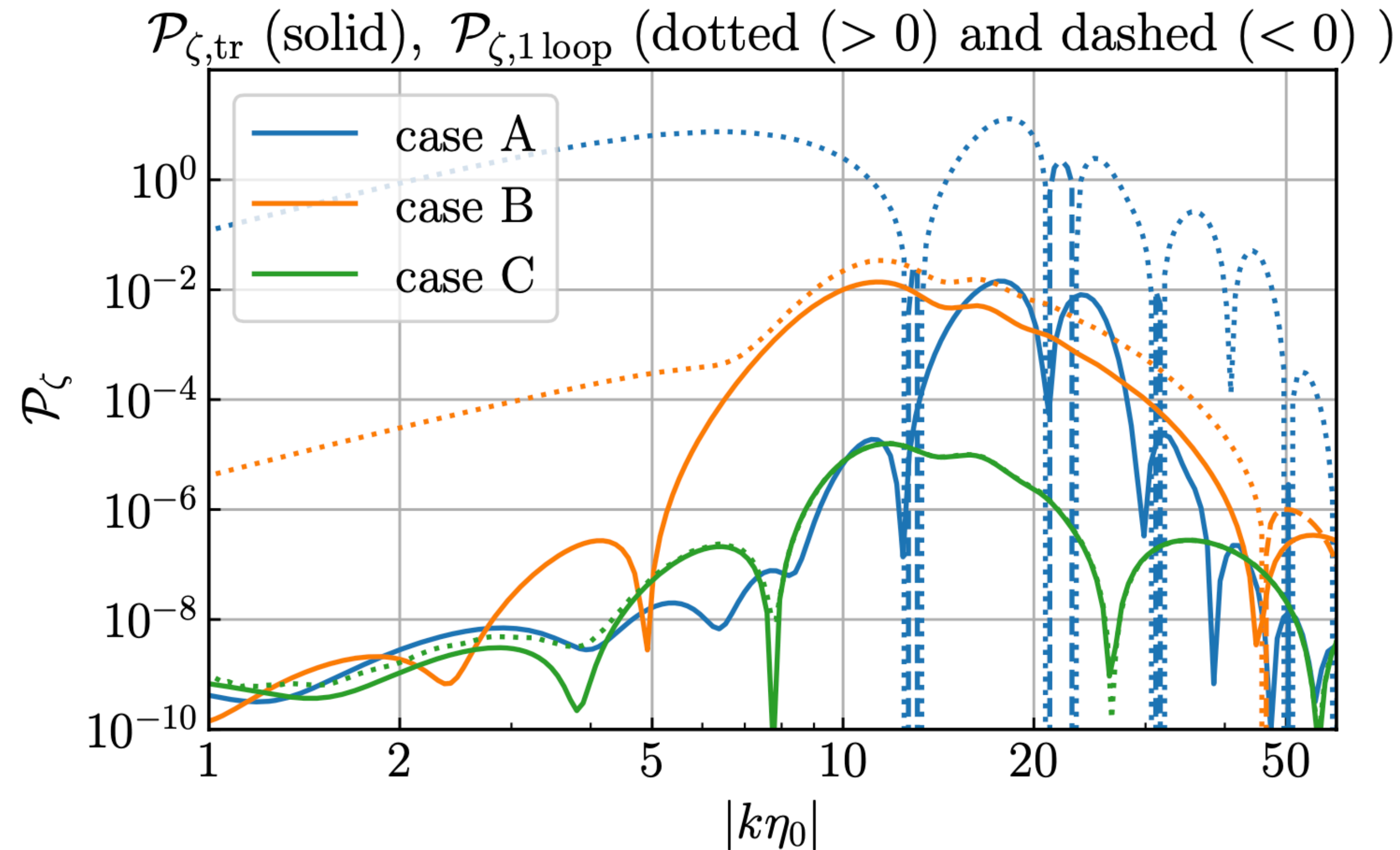
A: out of control

Models with PBH always
out of control (in our study!)

Standard background + numerical mode functions +

numerical computations
of loop integrals
(natural cutoffs)

(first) First-principles numerical 1-loop computation



Qualitative **analytical**
understanding as well

Cases A and B:
backreaction also an issue

numerical computations
of **loop integrals**
(natural cutoffs)

Standard background + numerical mode functions +

Lattice simulations to the rescue

Not ultimate answers to all questions (classical vs quantum) but

Extremely useful approach

Fully nonlinear eom
for scalar fields

in (almost) FLRW background
sourced by average
full energy density and pressure

Lattice simulations to the rescue

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Backreaction

see Angelo Caravano's review talk, and **preliminary results for resonant amplification setup**

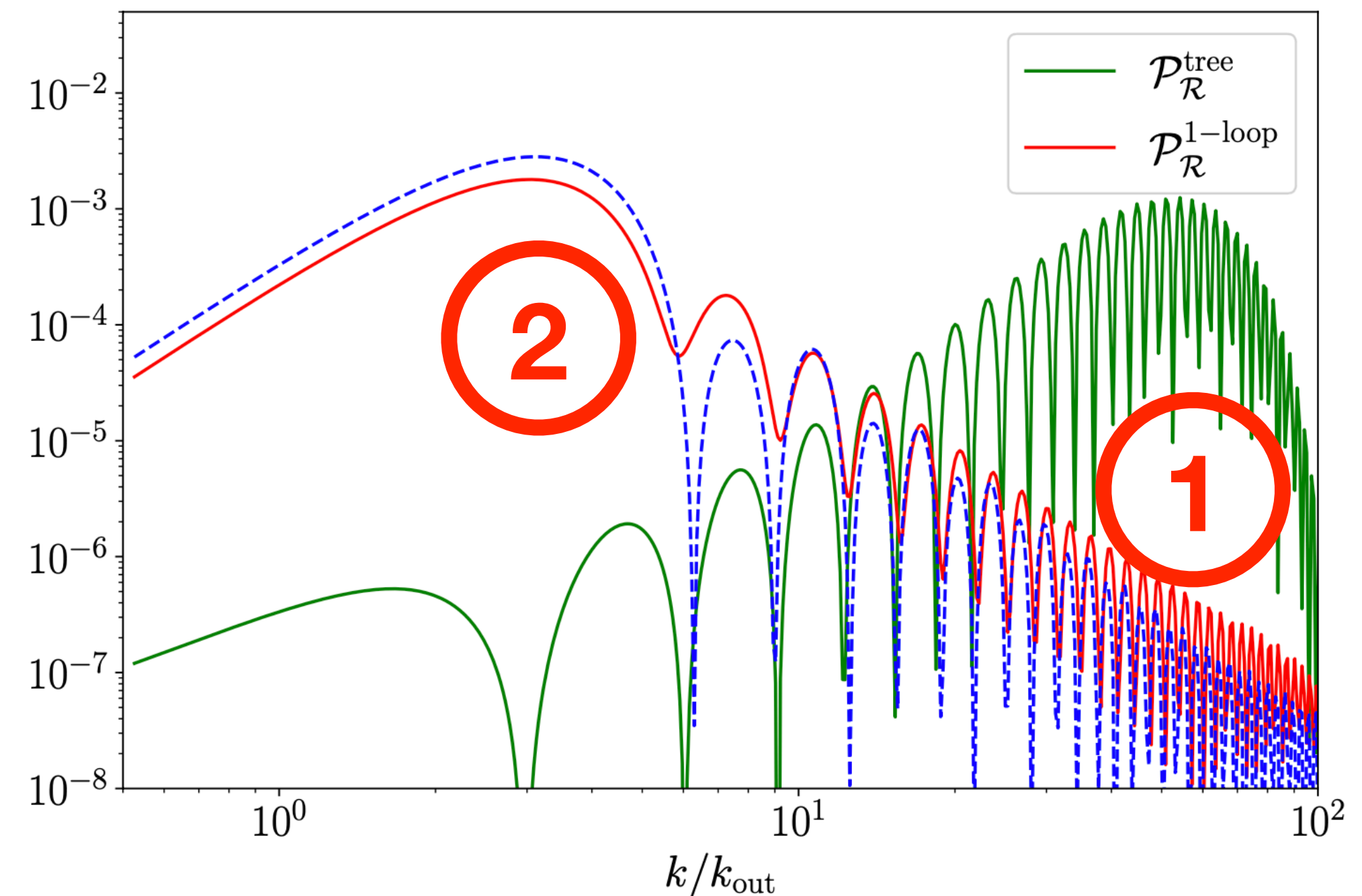
Infrared rescattering

Is $\mathcal{P}_{1\text{-loop}} > \mathcal{P}_{\text{tree}}$ always a problem for SPT?

No! Depends on which scales we discuss

Short reason: some phenomena start at loop level

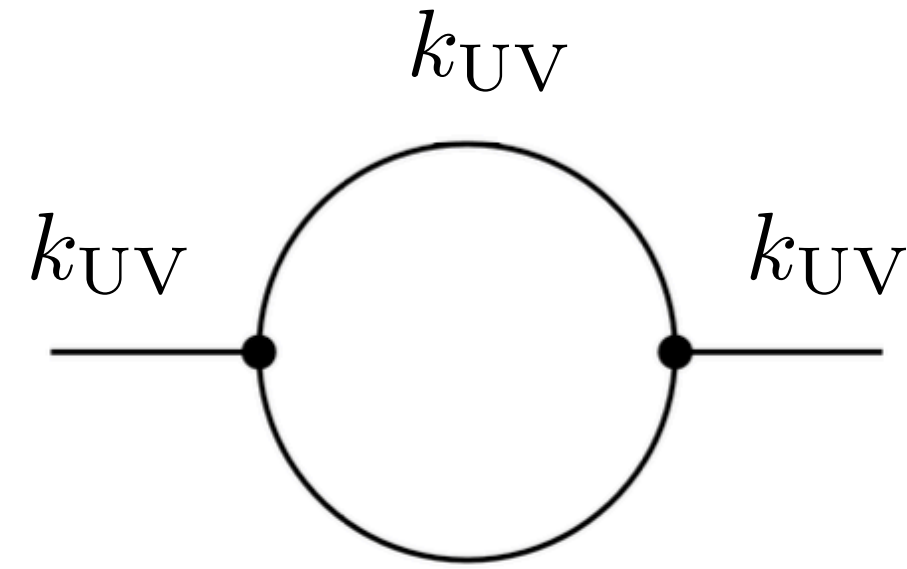
$\mathcal{P}_{2\text{-loop}} < \mathcal{P}_{1\text{-loop}}$
enough then



Infrared rescattering

see Jacopo Fumagalli's talk

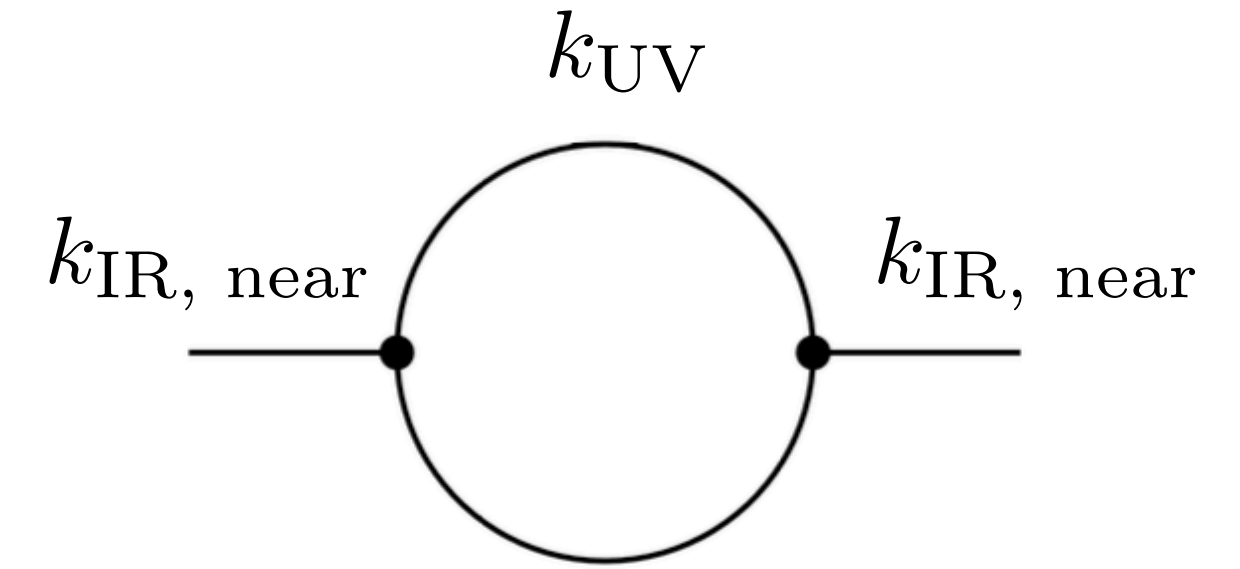
1



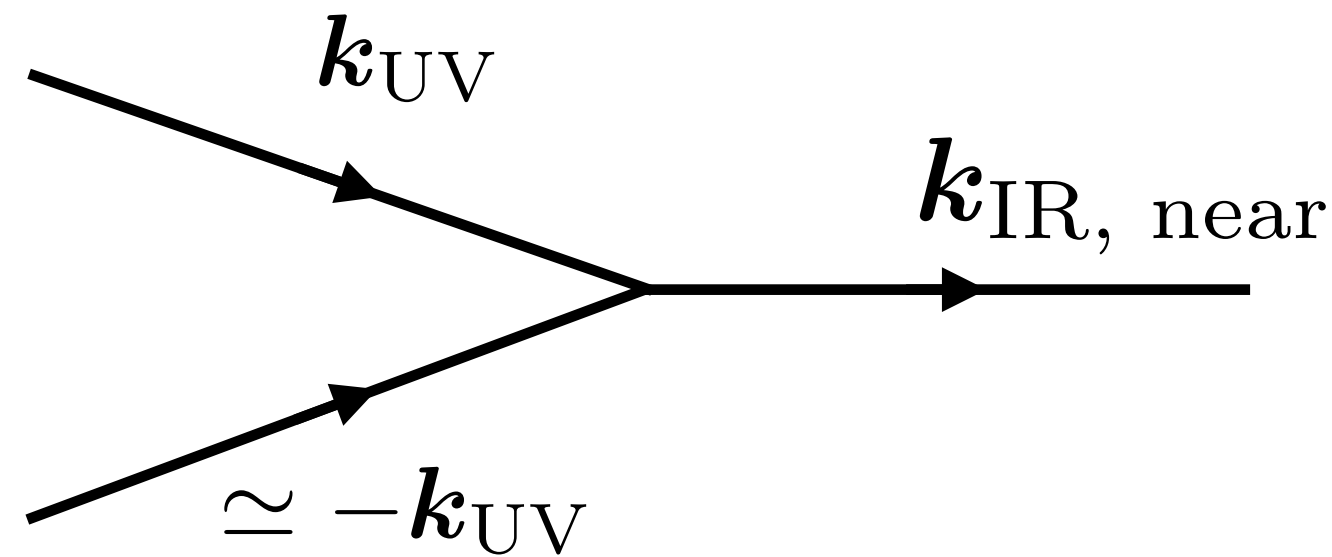
better be under control

vs

2



Particles created inside the horizon



Interact and quickly decay into modes with $\lambda \sim H^{-1}$

Generic analytical results (single-field, spectator, multi-field)

Universal: relevant even with gravitational interactions

Similar effect for GW production

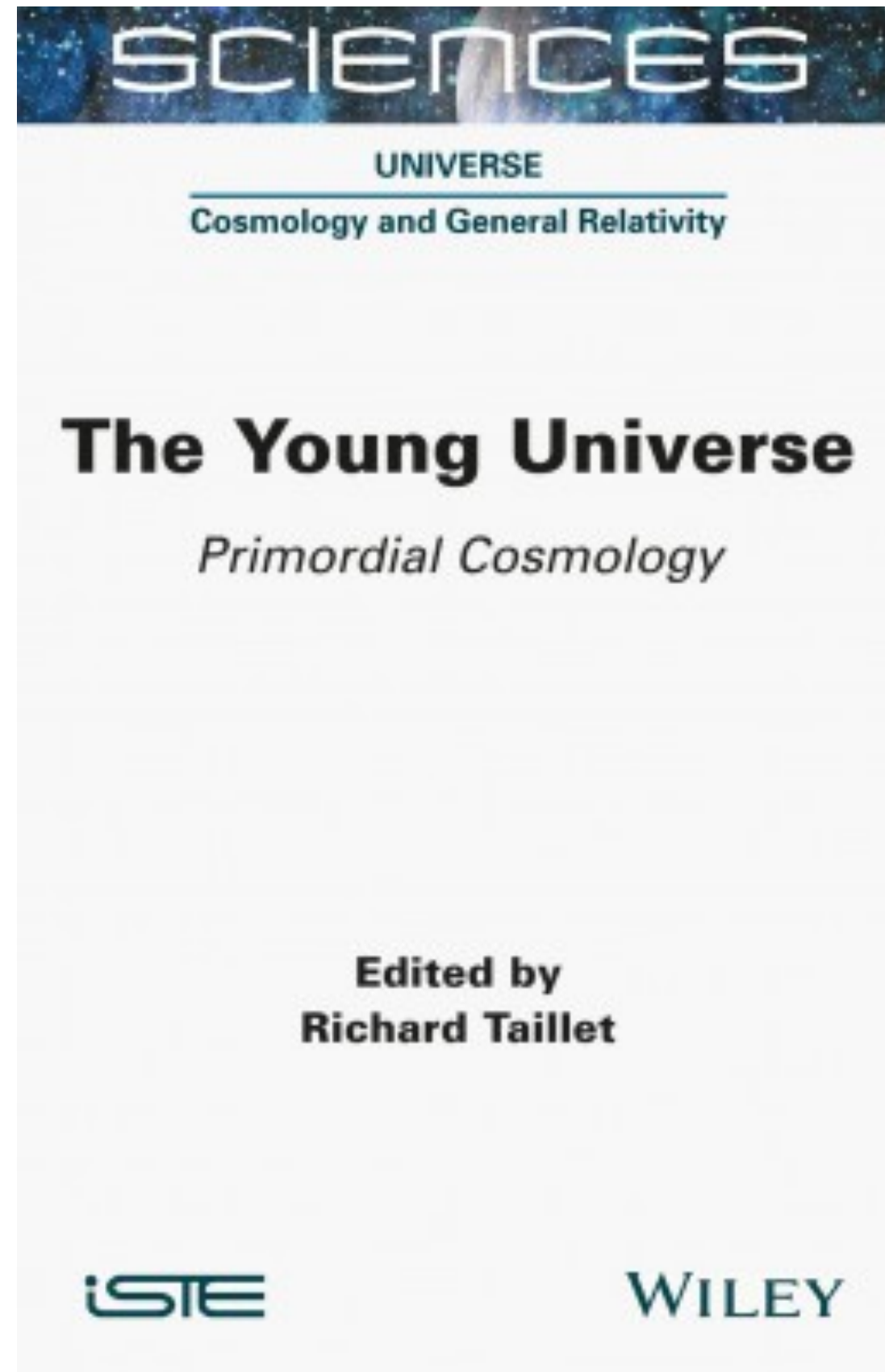
2307.08358, Fumagalli, Bhattacharya, Peloso, RP, Witkowski

2111.14664, Fumagalli, Palma, RP Sypsas, Witkowski, Zenteno

Conclusions

- **Backreaction** (ok, compute) **and perturbative control** (more tricky): ever-present threats in models with enhanced fluctuations
- **First-principle numerical computations of loop effects** with enhanced fluctuations (perturbative control always problematic for setups with PBH)
- **First lattice simulation** of it: way beyond standard perturbation theory!
- **Infrared rescattering**: IR cascade of power. Generic effect, of relevance for PBH

Thank you!



Oct 22, 348 pages

undergraduate & graduate textbook, 4 authors :

1. A Thermal History of the Universe and Primordial Nucleosynthesis, Pierre Salati.
2. Cosmological Microwave Background, Julien Lesgourgues.
3. Cosmological Inflation, Sébastien Renaux-Petel.
4. Dark Matter, Richard Taillet.